Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.7/109-1.2.1.7-b

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**207**]. This is test number [109].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.52 (206)	0.48 (1)
Mathematica	98.55 (204)	1.45 (3)
Maple	87.44 (181)	12.56 (26)
Fricas	87.44 (181)	12.56 (26)
Reduce	85.51 (177)	14.49 (30)
Giac	84.54 (175)	15.46 (32)
Maxima	61.84 (128)	38.16 (79)
Sympy	39.61 (82)	60.39 (125)
Mupad	36.23 (75)	63.77 (132)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

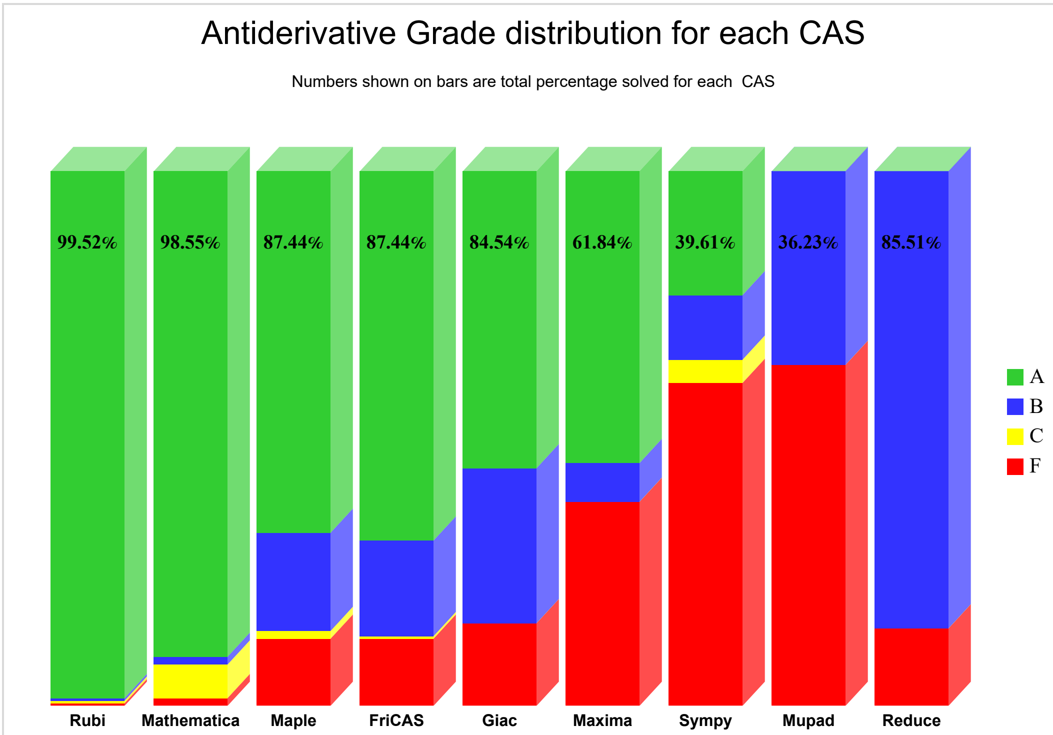
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

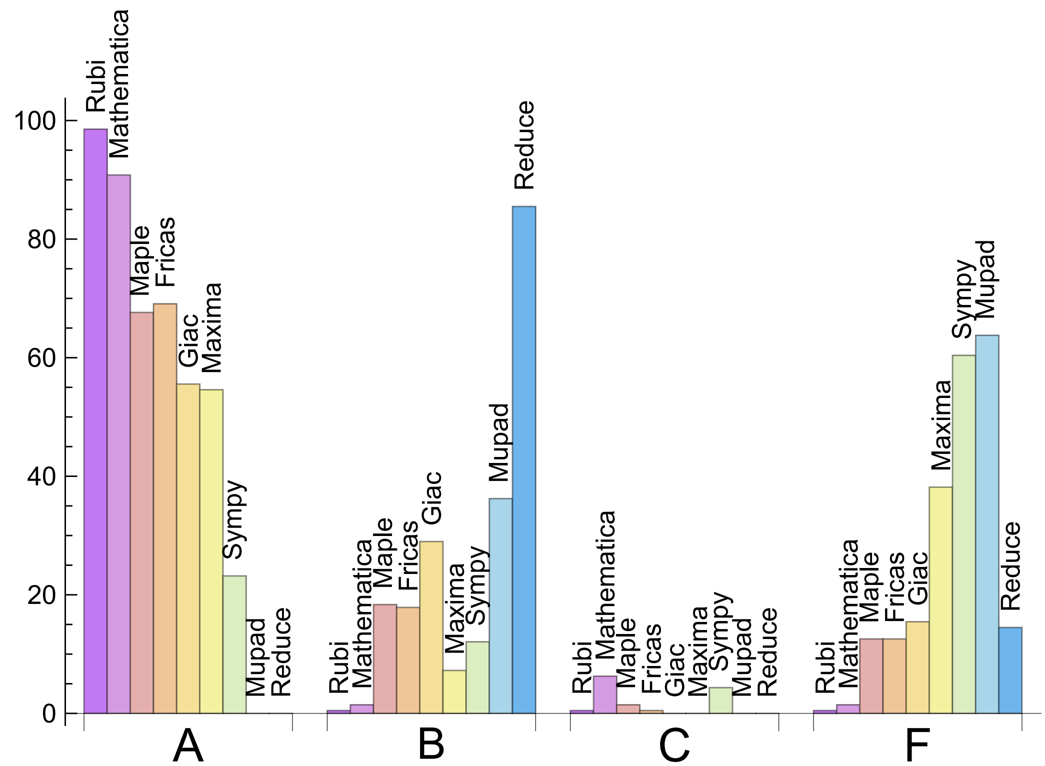
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.551	0.483	0.483	0.483
Mathematica	90.821	1.449	6.280	1.449
Fricas	69.082	17.874	0.483	12.560
Maple	67.633	18.357	1.449	12.560
Giac	55.556	28.986	0.000	15.459
Maxima	54.589	7.246	0.000	38.164
Sympy	23.188	12.077	4.348	60.386
Mupad	0.000	36.232	0.000	63.768
Reduce	0.000	85.507	0.000	14.493

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Fricas	26	23.08	76.92	0.00
Maple	26	23.08	76.92	0.00
Reduce	30	100.00	0.00	0.00
Giac	32	65.62	18.75	15.62
Maxima	79	25.32	0.00	74.68
Sympy	125	83.20	16.00	0.80
Mupad	132	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.10
Giac	0.46
Maple	0.57
Rubi	1.07
Fricas	2.11
Reduce	2.61
Sympy	3.67
Mathematica	4.12
Mupad	13.34

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	238.91	3.22	144.00	0.99
Rubi	285.84	2.37	188.50	1.05
Fricas	692.11	37.30	172.00	1.27
Mathematica	753.50	2.85	135.50	0.91
Maple	934.12	7.12	179.00	0.91
Mupad	1351.76	56.73	297.00	1.43
Giac	1475.51	4.96	225.00	1.19
Reduce	2259.72	44.20	323.00	2.43
Sympy	3722.12	12.07	232.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

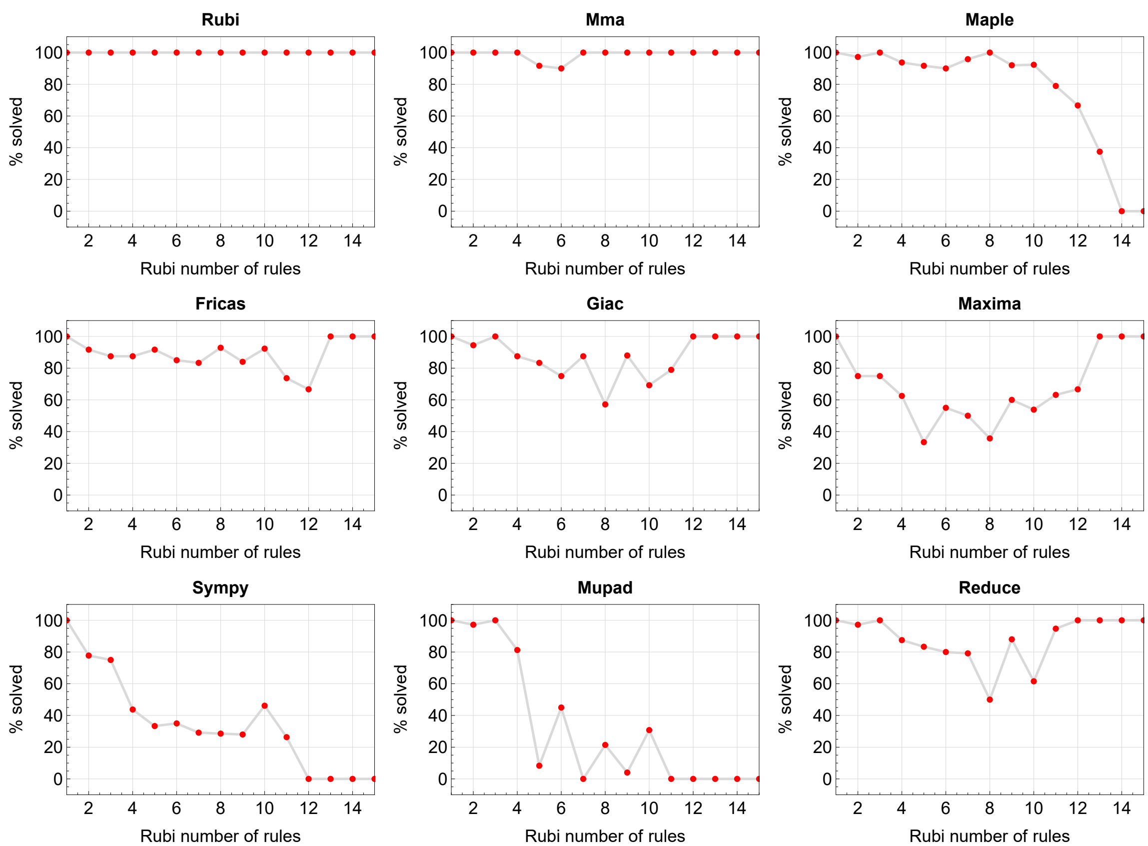


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

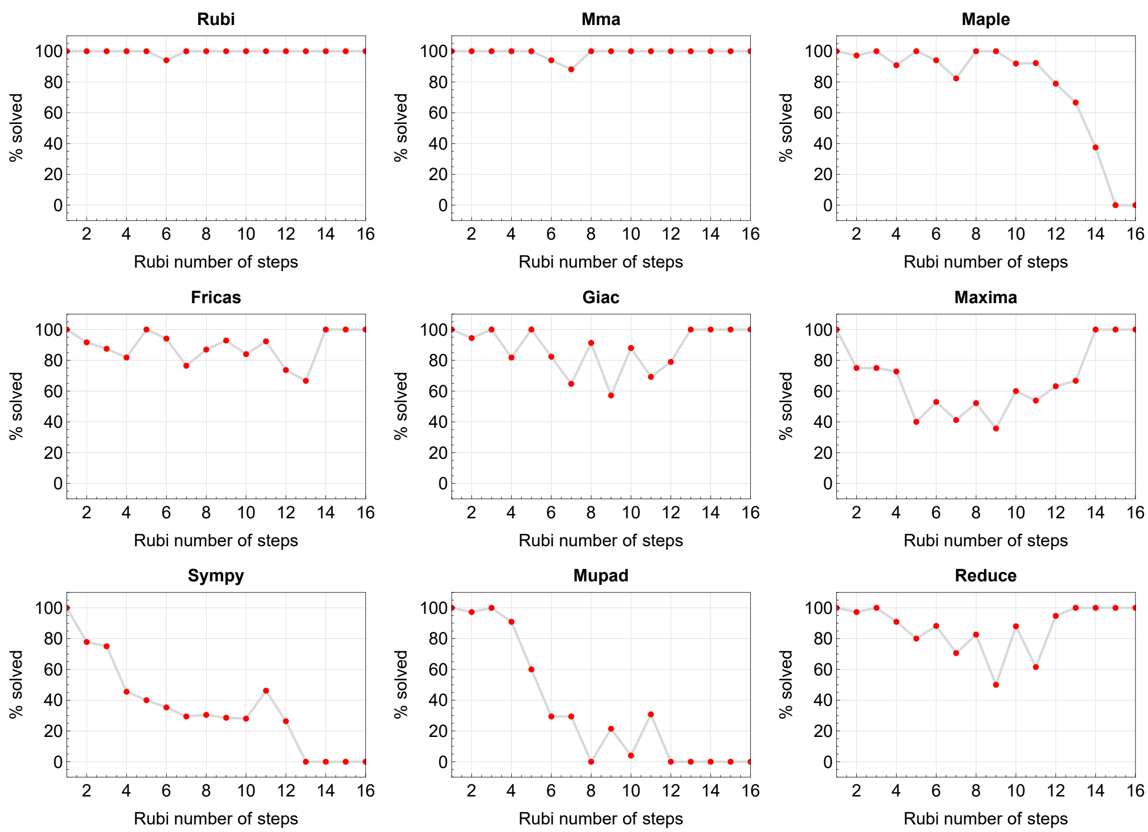


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

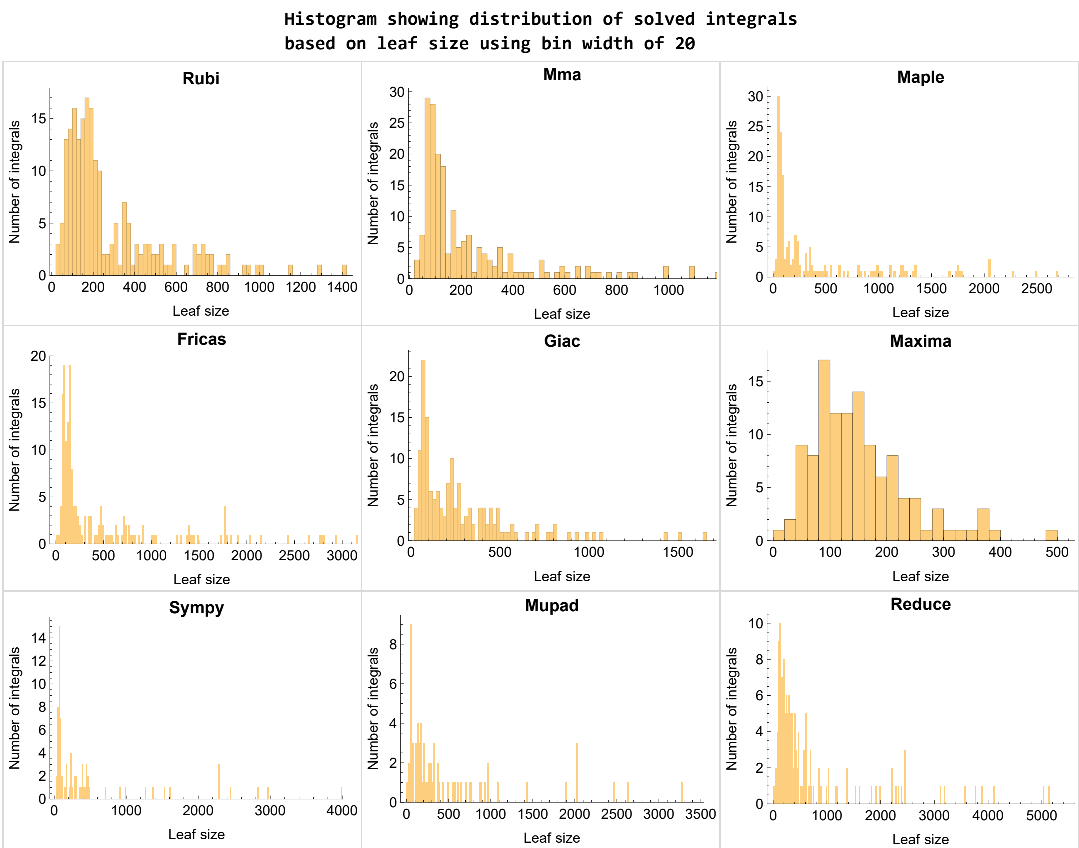


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

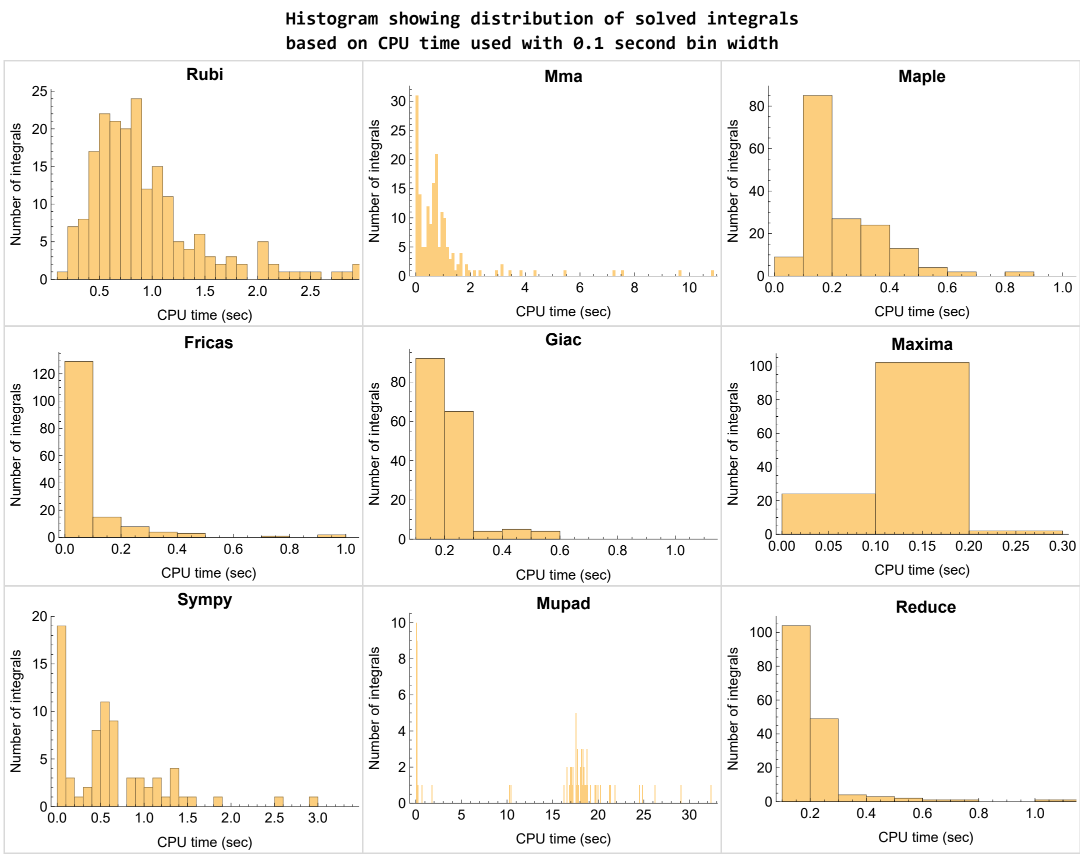


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

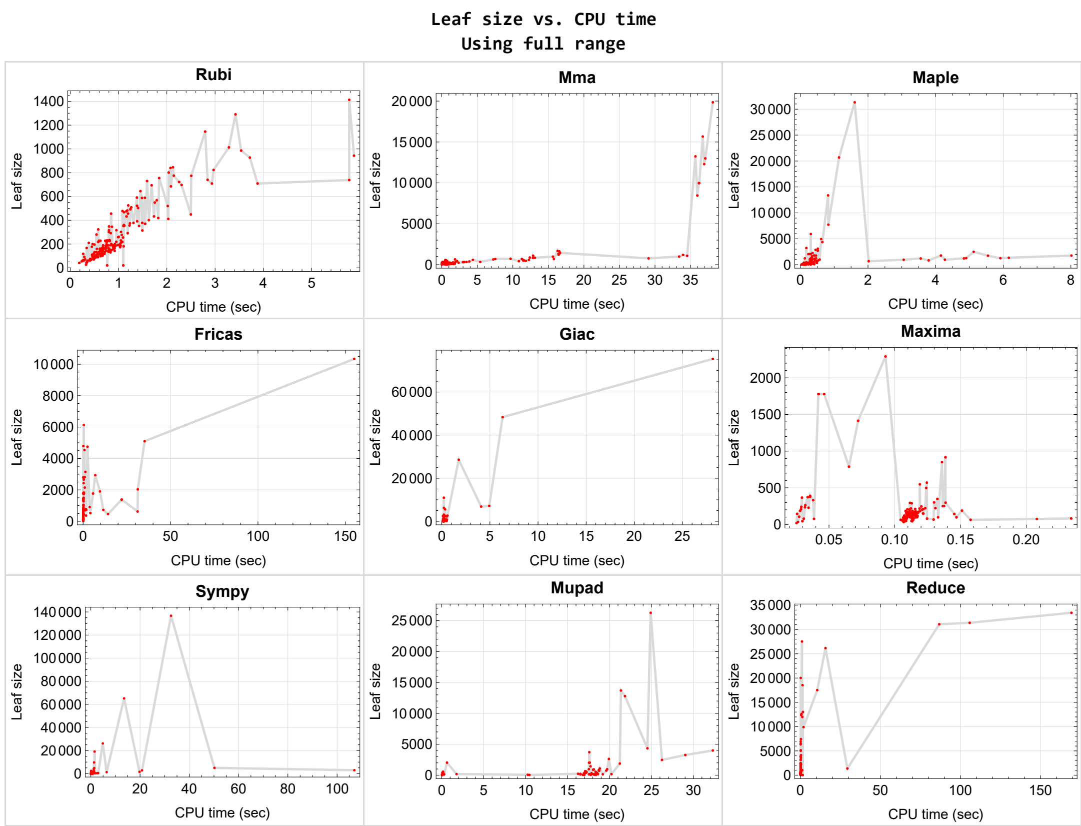


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{3, 96, 99, 100, 101\}$

Mathematica $\{3, 100\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

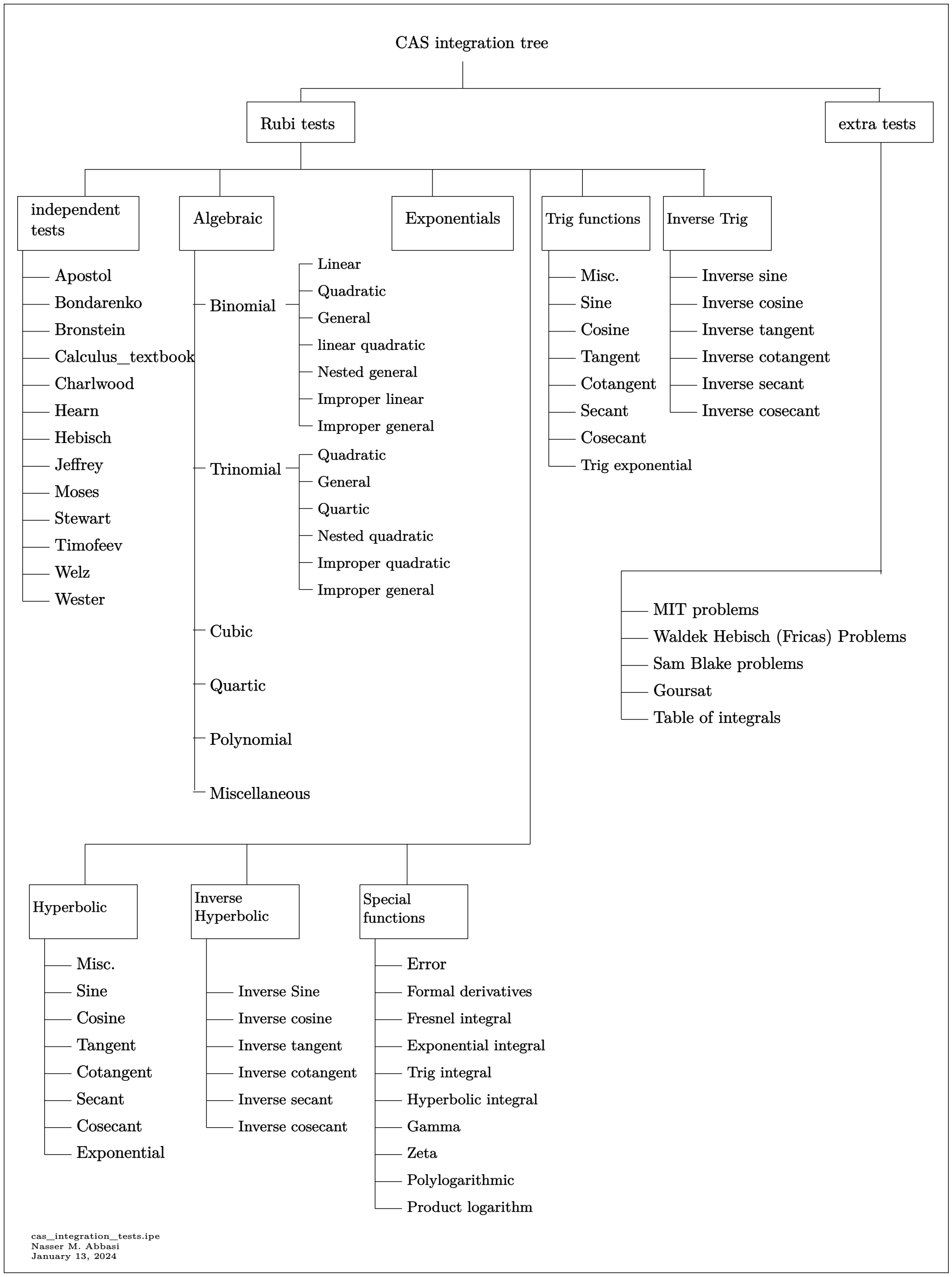
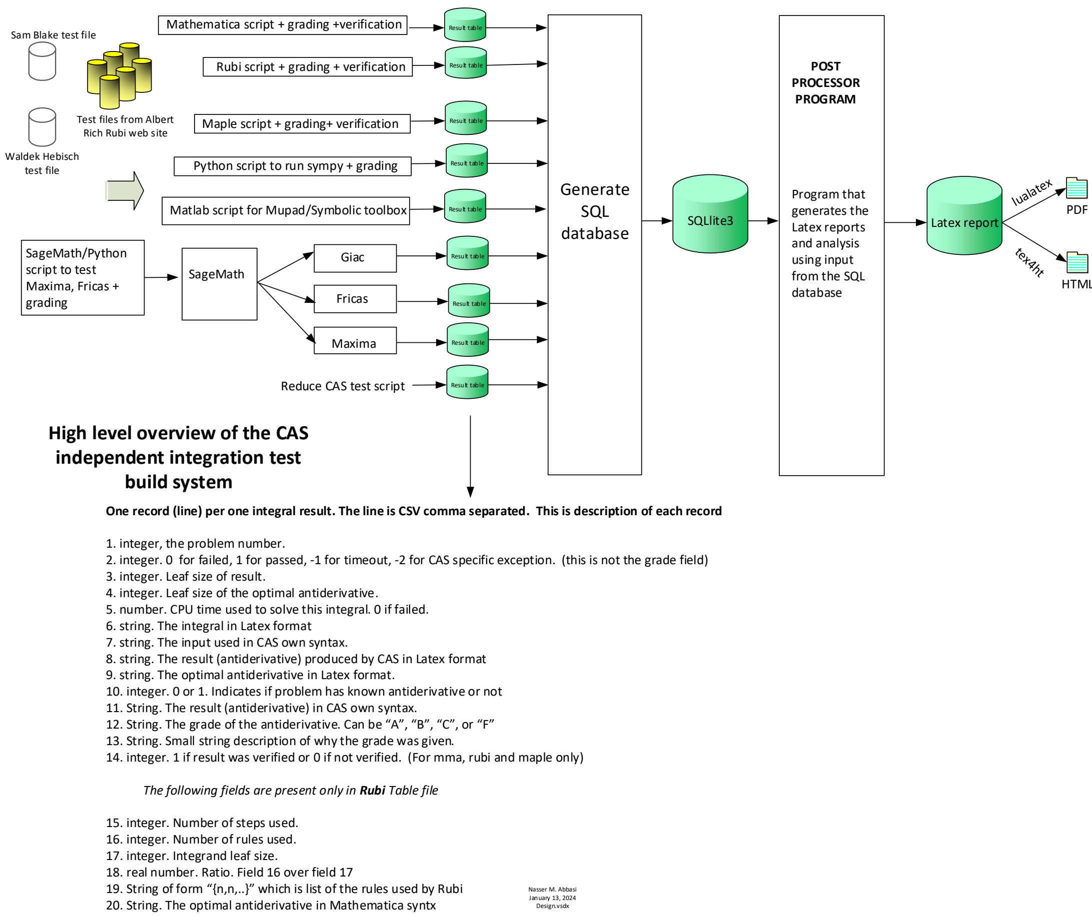


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

B grade { 113 }

C grade { 1 }

F normal fail { 110 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

B grade { 111, 112, 113 }

C grade { 1, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106 }

F normal fail { 107, 108, 109 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 36, 37, 38, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 71, 72, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 97, 99, 100, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 167, 168, 169, 170, 179, 180, 181, 186, 187, 188, 189, 192, 193, 194, 195, 196, 198, 199, 200 }

B grade { 14, 15, 29, 30, 31, 32, 33, 34, 35, 39, 40, 41, 42, 43, 44, 68, 69, 73, 74, 75, 76, 95, 96, 98, 101, 102, 103, 104, 105, 106, 111, 112, 113, 201, 202, 203, 204, 205 }

C grade { 1, 92, 197 }

F normal fail { 3, 107, 108, 109, 206, 207 }

F(-1) timedout fail { 161, 162, 163, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 190, 191 }

F(-2) exception fail { }

Fricas

A grade { 4, 5, 6, 7, 8, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 36, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 101, 102, 103, 104, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 197, 198, 199, 200 }

B grade { 2, 11, 12, 13, 33, 34, 35, 69, 70, 71, 72, 73, 74, 75, 76, 90, 98, 99, 100, 105, 106, 111, 112, 113, 145, 150, 151, 152, 153, 157, 158, 196, 201, 202, 203, 204, 205 }

C grade { 1 }

F normal fail { 3, 107, 108, 109, 206, 207 }

F(-1) timedout fail { 9, 10, 14, 15, 27, 28, 29, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 67, 68 }

F(-2) exception fail { }

Maxima

A grade { 16, 17, 18, 19, 20, 21, 22, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 93, 94, 114, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 197, 198, 199, 200 }

B grade { 89, 90, 110, 111, 112, 113, 152, 158, 178, 194, 195, 196, 203, 204, 205 }

C grade { }

F normal fail { 1, 3, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 202, 206, 207 }

F(-1) timedout fail { }

F(-2) exception fail { 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 115, 116, 117, 118, 119, 120, 121, 122, 123, 201 }

Giac

A grade { 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 36, 45, 46, 47, 48, 51, 52, 53, 54, 57, 58, 59, 60, 63, 64, 65, 66, 70, 71, 72, 73, 77, 78, 79, 80, 83, 84, 85, 86, 89, 90, 91, 92, 114, 115, 116, 117, 119, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 169, 170, 171, 173, 179, 180, 181, 182, 186, 187, 188, 189, 190, 194, 195, 196, 197, 201, 202 }

B grade { 10, 14, 15, 29, 30, 32, 33, 34, 35, 39, 40, 43, 44, 49, 50, 55, 56, 61, 62, 69, 74, 76, 81, 82, 87, 88, 93, 94, 111, 112, 113, 120, 121, 122, 123, 162, 163, 164, 165, 166, 167, 168, 172, 174, 175, 176, 177, 178, 183, 184, 185, 191, 192, 193, 198, 199, 200, 203, 204, 205 }

C grade { }

F normal fail { 1, 2, 3, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 206, 207 }

F(-1) timedout fail { 28, 31, 38, 41, 42, 75 }

F(-2) exception fail { 27, 37, 67, 68, 118 }

Mupad

A grade { }

B grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45, 46, 47, 73, 91, 111, 112, 113, 114, 119, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 203, 204, 205 }

C grade { }

F normal fail { }

F(-1) timedout fail { 3, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 115, 116, 117, 118, 120, 121, 122, 123, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 206, 207 }

F(-2) exception fail { }

Sympy

A grade { 16, 17, 18, 19, 20, 21, 22, 45, 46, 47, 51, 52, 53, 57, 58, 59, 65, 77, 78, 79, 114, 115, 116, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 149, 156, 159, 160, 169, 170, 179, 180, 181 }

B grade { 4, 5, 6, 7, 11, 12, 13, 23, 24, 25, 26, 33, 34, 35, 36, 63, 64, 66, 111, 112, 113, 117, 203, 204, 205 }

C grade { 139, 140, 141, 146, 147, 148, 153, 154, 155 }

F normal fail { 1, 2, 27, 28, 29, 30, 31, 32, 37, 38, 39, 40, 41, 42, 43, 44, 48, 49, 50, 54, 55, 56, 60, 61, 62, 67, 68, 69, 70, 71, 72, 73, 74, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 110, 118, 119, 120, 121, 122, 123, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 176, 177, 178, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

F(-1) timedout fail { 8, 9, 10, 14, 15, 75, 76, 107, 109, 143, 144, 145, 150, 151, 152, 157, 158, 202, 206, 207 }

F(-2) exception fail { 3 }

Reduce

A grade { }

B grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 39, 40, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205 }

C grade { }

F normal fail { 3, 23, 24, 33, 34, 35, 43, 44, 63, 64, 70, 71, 74, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 206, 207 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	C	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1	228	259	669	0	6132	0	0	6499	3714
N.S.	1	228.00	259.00	669.00	0.00	6132.00	0.00	0.00	6499.00	3714.00
time (sec)	N/A	0.626	1.090	2.020	0.000	0.362	0.000	0.000	0.194	17.587

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	217	219	324	0	465	0	0	1034	1089
N.S.	1	1.01	1.02	1.51	0.00	2.17	0.00	0.00	4.83	5.09
time (sec)	N/A	0.580	0.473	0.489	0.000	14.244	0.000	0.000	0.213	18.840

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	228	165	0	0	0	0	0	0	0
N.S.	1	1.22	0.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.682	0.000	0.000	0.000	0.000	0.000	1.836	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	585	869	0	2150	4972	805	2328	967
N.S.	1	1.00	0.99	1.47	0.00	3.64	8.41	1.36	3.94	1.64
time (sec)	N/A	1.381	0.509	0.447	0.000	0.934	50.206	0.203	0.175	19.718

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	345	453	0	1273	2839	433	1379	557
N.S.	1	1.00	0.99	1.30	0.00	3.66	8.16	1.24	3.96	1.60
time (sec)	N/A	0.804	0.299	0.310	0.000	0.293	20.798	0.184	0.177	18.332

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	173	192	0	654	1265	189	673	273
N.S.	1	1.00	0.98	1.08	0.00	3.69	7.15	1.07	3.80	1.54
time (sec)	N/A	0.446	0.150	0.268	0.000	0.153	6.438	0.155	0.189	18.462

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	95	93	0	302	488	89	253	132
N.S.	1	1.00	1.03	1.01	0.00	3.28	5.30	0.97	2.75	1.43
time (sec)	N/A	0.291	0.062	0.237	0.000	0.081	1.095	0.197	0.169	0.288

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	193	179	0	625	0	199	593	2467
N.S.	1	1.00	0.98	0.91	0.00	3.19	0.00	1.02	3.03	12.59
time (sec)	N/A	0.500	0.174	0.332	0.000	31.154	0.000	0.176	0.172	26.244

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	281	346	0	0	0	459	2385	3991
N.S.	1	1.00	0.89	1.09	0.00	0.00	0.00	1.45	7.55	12.63
time (sec)	N/A	0.802	0.484	0.251	0.000	0.000	0.000	0.200	0.171	32.304

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	504	629	0	0	0	1065	6343	12784
N.S.	1	1.00	0.99	1.24	0.00	0.00	0.00	2.09	12.46	25.12
time (sec)	N/A	1.264	0.618	0.487	0.000	0.000	0.000	0.145	0.189	21.835

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	301	398	628	0	2771	2966	547	3190	742
N.S.	1	0.70	0.93	1.47	0.00	6.49	6.95	1.28	7.47	1.74
time (sec)	N/A	0.800	0.771	0.309	0.000	0.286	107.002	0.246	0.191	19.611

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	200	225	308	0	1413	1535	271	1526	376
N.S.	1	0.86	0.97	1.33	0.00	6.09	6.62	1.17	6.58	1.62
time (sec)	N/A	0.463	0.372	0.264	0.000	0.136	19.774	0.232	0.181	18.508

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	118	114	133	0	632	459	125	571	203
N.S.	1	1.03	0.99	1.16	0.00	5.50	3.99	1.09	4.97	1.77
time (sec)	N/A	0.269	0.095	0.200	0.000	0.087	1.129	0.163	0.168	0.163

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	456	405	809	0	0	0	886	5037	13698
N.S.	1	1.13	1.00	2.01	0.00	0.00	0.00	2.20	12.50	33.99
time (sec)	N/A	1.196	0.780	0.431	0.000	0.000	0.000	0.171	0.179	21.355

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	665	709	650	1345	0	0	0	1506	20026	26278
N.S.	1	1.07	0.98	2.02	0.00	0.00	0.00	2.26	30.11	39.52
time (sec)	N/A	3.879	1.869	0.325	0.000	0.000	0.000	0.189	0.215	24.896

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	60	53	51	75	60	51	116	55
N.S.	1	1.05	0.97	0.85	0.82	1.21	0.97	0.82	1.87	0.89
time (sec)	N/A	0.281	0.038	0.299	0.113	0.076	0.067	0.205	0.170	16.988

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	60	55	46	46	70	54	46	111	48
N.S.	1	1.09	1.00	0.84	0.84	1.27	0.98	0.84	2.02	0.87
time (sec)	N/A	0.264	0.028	0.103	0.113	0.071	0.073	0.191	0.173	0.045

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	55	52	45	43	60	53	43	106	59
N.S.	1	1.06	1.00	0.87	0.83	1.15	1.02	0.83	2.04	1.13
time (sec)	N/A	0.238	0.022	0.105	0.108	0.072	0.090	0.180	0.180	16.820

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	39	34	32	41	41	32	70	35
N.S.	1	1.00	0.95	0.83	0.78	1.00	1.00	0.78	1.71	0.85
time (sec)	N/A	0.187	0.022	0.105	0.107	0.067	0.072	0.155	0.160	10.450

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	56	48	47	72	54	48	122	58
N.S.	1	1.09	1.00	0.86	0.84	1.29	0.96	0.86	2.18	1.04
time (sec)	N/A	0.254	0.029	0.109	0.109	0.071	0.086	0.175	0.178	0.060

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	64	61	55	54	85	65	55	136	68
N.S.	1	1.05	1.00	0.90	0.89	1.39	1.07	0.90	2.23	1.11
time (sec)	N/A	0.292	0.026	0.107	0.107	0.072	0.085	0.188	0.173	10.243

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	66	60	63	98	71	63	147	75
N.S.	1	1.04	0.97	0.88	0.93	1.44	1.04	0.93	2.16	1.10
time (sec)	N/A	0.309	0.034	0.109	0.104	0.074	0.095	0.139	0.169	0.102

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	914	755	1093	1757	0	2817	4820	1657	32	3262
N.S.	1	0.83	1.20	1.92	0.00	3.08	5.27	1.81	0.04	3.57
time (sec)	N/A	1.842	12.853	0.462	0.000	0.901	1.404	0.226	200.027	29.028

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	478	705	1027	0	1791	2440	983	32	1881
N.S.	1	0.83	1.23	1.79	0.00	3.11	4.24	1.71	0.06	3.27
time (sec)	N/A	1.082	11.265	0.356	0.000	0.472	1.220	0.184	200.019	21.224

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	280	345	488	0	1009	993	478	1363	877
N.S.	1	0.87	1.07	1.52	0.00	3.13	3.08	1.48	4.23	2.72
time (sec)	N/A	0.544	5.414	0.278	0.000	0.257	1.041	0.217	29.382	18.743

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	167	174	197	0	465	384	204	557	320
N.S.	1	0.95	0.99	1.13	0.00	2.66	2.19	1.17	3.18	1.83
time (sec)	N/A	0.340	0.120	0.237	0.000	0.098	0.514	0.176	0.300	17.843

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	345	324	515	0	0	0	0	13015	0
N.S.	1	1.07	1.00	1.59	0.00	0.00	0.00	0.00	40.29	0.00
time (sec)	N/A	0.864	3.173	0.329	0.000	0.000	0.000	0.000	1.621	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	482	430	704	0	0	0	0	3565	0
N.S.	1	1.06	0.95	1.55	0.00	0.00	0.00	0.00	7.87	0.00
time (sec)	N/A	1.181	10.852	0.326	0.000	0.000	0.000	0.000	0.363	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	446	468	500	1230	0	0	0	2364	7427	0
N.S.	1	1.05	1.12	2.76	0.00	0.00	0.00	5.30	16.65	0.00
time (sec)	N/A	1.112	11.468	0.464	0.000	0.000	0.000	0.455	0.344	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	645	574	3090	0	0	0	6846	11991	0
N.S.	1	1.33	1.18	6.37	0.00	0.00	0.00	14.12	24.72	0.00
time (sec)	N/A	1.453	12.269	0.517	0.000	0.000	0.000	4.091	1.204	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	456	693	4940	0	0	0	0	17516	0
N.S.	1	0.91	1.39	9.90	0.00	0.00	0.00	0.00	35.10	0.00
time (sec)	N/A	0.846	15.780	0.616	0.000	0.000	0.000	0.000	10.536	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	826	730	1334	7714	0	0	0	28577	31077	0
N.S.	1	0.88	1.62	9.34	0.00	0.00	0.00	34.60	37.62	0.00
time (sec)	N/A	1.591	16.412	0.828	0.000	0.000	0.000	1.763	86.772	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1153	801	1683	2870	0	4751	19122	2902	32	0
N.S.	1	0.69	1.46	2.49	0.00	4.12	16.58	2.52	0.03	0.00
time (sec)	N/A	2.038	16.291	0.450	0.000	2.492	1.521	0.260	200.032	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	743	525	877	1676	0	3145	9687	1802	32	0
N.S.	1	0.71	1.18	2.26	0.00	4.23	13.04	2.43	0.04	0.00
time (sec)	N/A	1.202	12.385	0.345	0.000	1.287	1.336	0.232	200.027	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	323	608	936	0	1833	3990	925	30	0
N.S.	1	0.79	1.48	2.28	0.00	4.47	9.73	2.26	0.07	0.00
time (sec)	N/A	0.580	11.405	0.279	0.000	0.419	1.170	0.288	200.027	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	210	293	398	0	839	1360	403	1037	0
N.S.	1	0.89	1.24	1.69	0.00	3.56	5.76	1.71	4.39	0.00
time (sec)	N/A	0.385	0.266	0.228	0.000	0.116	0.571	0.282	1.136	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	694	728	990	0	0	0	0	31379	0
N.S.	1	1.05	1.10	1.49	0.00	0.00	0.00	0.00	47.26	0.00
time (sec)	N/A	1.685	9.682	0.333	0.000	0.000	0.000	0.000	105.792	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	750	775	641	1324	0	0	0	0	4115	0
N.S.	1	1.03	0.85	1.77	0.00	0.00	0.00	0.00	5.49	0.00
time (sec)	N/A	2.145	7.248	0.345	0.000	0.000	0.000	0.000	0.400	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	819	840	817	1781	0	0	0	2628	9892	0
N.S.	1	1.03	1.00	2.17	0.00	0.00	0.00	3.21	12.08	0.00
time (sec)	N/A	2.076	12.837	0.408	0.000	0.000	0.000	0.599	1.976	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	829	846	983	2697	0	0	0	7155	18539	0
N.S.	1	1.02	1.19	3.25	0.00	0.00	0.00	8.63	22.36	0.00
time (sec)	N/A	2.127	15.642	0.517	0.000	0.000	0.000	4.936	1.356	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	808	1146	1447	4351	0	0	0	0	26169	0
N.S.	1	1.42	1.79	5.38	0.00	0.00	0.00	0.00	32.39	0.00
time (sec)	N/A	2.794	16.638	0.651	0.000	0.000	0.000	0.000	15.697	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	871	1291	1363	13372	0	0	0	0	33446	0
N.S.	1	1.48	1.56	15.35	0.00	0.00	0.00	0.00	38.40	0.00
time (sec)	N/A	3.422	16.565	0.820	0.000	0.000	0.000	0.000	169.395	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	549	1222	20684	0	0	0	48343	32	0
N.S.	1	0.83	1.85	31.34	0.00	0.00	0.00	73.25	0.05	0.00
time (sec)	N/A	1.750	16.435	1.144	0.000	0.000	0.000	6.321	200.030	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1062	823	1636	31330	0	0	0	75375	32	0
N.S.	1	0.77	1.54	29.50	0.00	0.00	0.00	70.97	0.03	0.00
time (sec)	N/A	2.969	16.534	1.606	0.000	0.000	0.000	28.154	200.029	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	163	80	60	126	83	76	78	138	170
N.S.	1	1.14	0.56	0.42	0.88	0.58	0.53	0.55	0.97	1.19
time (sec)	N/A	0.648	0.747	0.315	0.111	0.081	0.485	0.162	0.199	18.626

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	133	75	55	109	78	70	73	122	153
N.S.	1	1.13	0.64	0.47	0.92	0.66	0.59	0.62	1.03	1.30
time (sec)	N/A	0.546	0.602	0.128	0.110	0.074	0.501	0.222	0.194	18.044

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	103	70	50	92	73	63	68	106	136
N.S.	1	1.11	0.75	0.54	0.99	0.78	0.68	0.73	1.14	1.46
time (sec)	N/A	0.453	0.464	0.119	0.108	0.087	0.500	0.223	0.189	17.784

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	104	70	96	115	0	126	188	0
N.S.	1	1.06	1.03	0.69	0.95	1.14	0.00	1.25	1.86	0.00
time (sec)	N/A	0.572	0.414	0.181	0.113	0.085	0.000	0.271	0.231	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	114	110	87	103	133	0	380	175	0
N.S.	1	1.06	1.02	0.81	0.95	1.23	0.00	3.52	1.62	0.00
time (sec)	N/A	0.559	0.674	0.169	0.112	0.090	0.000	0.395	0.210	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	111	87	114	149	0	251	249	0
N.S.	1	1.05	0.97	0.76	0.99	1.30	0.00	2.18	2.17	0.00
time (sec)	N/A	0.577	0.741	0.169	0.113	0.096	0.000	0.244	0.213	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	183	90	70	155	93	90	88	170	0
N.S.	1	1.16	0.57	0.44	0.98	0.59	0.57	0.56	1.08	0.00
time (sec)	N/A	0.734	1.039	0.132	0.109	0.084	0.556	0.209	0.236	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	161	85	65	138	88	82	83	154	0
N.S.	1	1.14	0.60	0.46	0.98	0.62	0.58	0.59	1.09	0.00
time (sec)	N/A	0.614	0.939	0.131	0.111	0.079	0.529	0.205	0.226	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	131	80	60	121	83	76	78	138	0
N.S.	1	1.13	0.69	0.52	1.04	0.72	0.66	0.67	1.19	0.00
time (sec)	N/A	0.504	0.692	0.127	0.107	0.082	0.540	0.206	0.249	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	135	114	80	125	125	0	136	220	0
N.S.	1	1.09	0.92	0.65	1.01	1.01	0.00	1.10	1.77	0.00
time (sec)	N/A	0.657	0.652	0.163	0.110	0.082	0.000	0.237	0.264	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	142	121	97	132	143	0	570	207	0
N.S.	1	1.08	0.92	0.74	1.01	1.09	0.00	4.35	1.58	0.00
time (sec)	N/A	0.676	0.723	0.173	0.111	0.081	0.000	0.454	0.274	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	153	121	97	143	159	0	261	281	0
N.S.	1	1.11	0.88	0.70	1.04	1.15	0.00	1.89	2.04	0.00
time (sec)	N/A	0.670	0.926	0.171	0.114	0.082	0.000	0.271	0.252	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	219	100	80	184	103	104	98	202	0
N.S.	1	1.16	0.53	0.42	0.97	0.54	0.55	0.52	1.07	0.00
time (sec)	N/A	0.761	1.315	0.151	0.111	0.073	0.671	0.224	0.418	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	189	95	75	167	98	97	93	186	0
N.S.	1	1.15	0.58	0.46	1.02	0.60	0.59	0.57	1.13	0.00
time (sec)	N/A	0.680	1.190	0.145	0.110	0.075	0.632	0.217	0.277	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	159	90	70	150	93	88	88	170	0
N.S.	1	1.14	0.65	0.50	1.08	0.67	0.63	0.63	1.22	0.00
time (sec)	N/A	0.552	1.066	0.121	0.111	0.077	0.639	0.213	0.249	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	163	124	90	154	135	0	146	252	0
N.S.	1	1.11	0.84	0.61	1.05	0.92	0.00	0.99	1.71	0.00
time (sec)	N/A	0.708	0.934	0.166	0.114	0.087	0.000	0.232	0.237	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	170	131	107	161	153	0	760	239	0
N.S.	1	1.10	0.85	0.69	1.05	0.99	0.00	4.94	1.55	0.00
time (sec)	N/A	0.752	1.021	0.174	0.118	0.083	0.000	0.543	0.220	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	177	131	107	172	169	0	271	313	0
N.S.	1	1.10	0.81	0.66	1.07	1.05	0.00	1.68	1.94	0.00
time (sec)	N/A	0.745	1.195	0.191	0.115	0.096	0.000	0.257	0.219	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	674	709	588	810	0	1435	1613	798	32	0
N.S.	1	1.05	0.87	1.20	0.00	2.13	2.39	1.18	0.05	0.00
time (sec)	N/A	2.934	4.380	0.526	0.000	0.301	1.355	0.249	200.031	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	432	341	437	0	861	910	442	32	0
N.S.	1	1.05	0.83	1.06	0.00	2.09	2.21	1.07	0.08	0.00
time (sec)	N/A	1.735	1.953	0.378	0.000	0.176	1.116	0.160	200.020	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	232	178	192	0	461	437	201	608	0
N.S.	1	1.07	0.82	0.88	0.00	2.12	2.01	0.93	2.80	0.00
time (sec)	N/A	0.819	0.894	0.285	0.000	0.204	0.975	0.268	0.198	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	98	86	0	227	226	93	231	0
N.S.	1	1.04	0.84	0.74	0.00	1.96	1.95	0.80	1.99	0.00
time (sec)	N/A	0.491	0.056	0.242	0.000	0.084	0.357	0.226	0.201	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	188	187	251	0	0	0	0	12390	0
N.S.	1	1.05	1.04	1.40	0.00	0.00	0.00	0.00	69.22	0.00
time (sec)	N/A	0.792	0.977	0.305	0.000	0.000	0.000	0.000	0.513	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	277	236	485	0	0	0	0	2284	0
N.S.	1	1.16	0.99	2.03	0.00	0.00	0.00	0.00	9.56	0.00
time (sec)	N/A	1.069	1.606	0.289	0.000	0.000	0.000	0.000	0.419	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	356	471	1013	0	2034	0	2279	5126	0
N.S.	1	1.06	1.40	3.01	0.00	6.05	0.00	6.78	15.26	0.00
time (sec)	N/A	1.228	11.831	0.353	0.000	31.268	0.000	0.532	0.267	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	520	694	991	0	2937	0	1028	32	0
N.S.	1	0.78	1.04	1.49	0.00	4.41	0.00	1.54	0.05	0.00
time (sec)	N/A	2.019	7.528	0.570	0.000	6.946	0.000	0.367	200.021	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	302	390	542	0	1769	0	565	32	0
N.S.	1	0.75	0.97	1.34	0.00	4.39	0.00	1.40	0.08	0.00
time (sec)	N/A	1.051	3.882	0.411	0.000	5.674	0.000	0.484	200.026	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	199	194	307	0	905	0	263	1610	0
N.S.	1	0.96	0.93	1.48	0.00	4.35	0.00	1.26	7.74	0.00
time (sec)	N/A	0.752	1.318	0.319	0.000	3.737	0.000	0.497	0.251	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	111	106	201	0	429	0	118	574	143
N.S.	1	1.03	0.98	1.86	0.00	3.97	0.00	1.09	5.31	1.32
time (sec)	N/A	0.454	0.233	0.248	0.000	0.414	0.000	0.511	0.228	19.191

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	225	235	547	0	1905	0	708	32	0
N.S.	1	1.02	1.07	2.49	0.00	8.66	0.00	3.22	0.15	0.00
time (sec)	N/A	0.820	1.646	0.295	0.000	9.637	0.000	0.443	200.039	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	419	518	1115	0	5098	0	0	12617	0
N.S.	1	1.02	1.26	2.71	0.00	12.40	0.00	0.00	30.70	0.00
time (sec)	N/A	1.823	11.598	0.287	0.000	35.172	0.000	0.000	0.712	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	699	738	847	2268	0	10340	0	5562	27527	0
N.S.	1	1.06	1.21	3.24	0.00	14.79	0.00	7.96	39.38	0.00
time (sec)	N/A	5.777	13.107	0.361	0.000	155.057	0.000	0.349	1.006	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	135	70	50	97	73	63	68	106	0
N.S.	1	1.12	0.58	0.42	0.81	0.61	0.52	0.57	0.88	0.00
time (sec)	N/A	0.600	0.580	0.138	0.133	0.083	0.491	0.202	0.212	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	105	65	45	80	68	56	63	90	0
N.S.	1	1.11	0.68	0.47	0.84	0.72	0.59	0.66	0.95	0.00
time (sec)	N/A	0.528	0.484	0.129	0.125	0.074	0.483	0.204	0.214	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	60	40	63	63	49	58	74	0
N.S.	1	1.07	0.86	0.57	0.90	0.90	0.70	0.83	1.06	0.00
time (sec)	N/A	0.412	0.313	0.122	0.158	0.073	0.586	0.142	0.172	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	83	93	60	67	105	0	116	158	0
N.S.	1	1.06	1.19	0.77	0.86	1.35	0.00	1.49	2.03	0.00
time (sec)	N/A	0.507	0.379	0.158	0.130	0.078	0.000	0.249	0.196	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	99	67	74	123	0	191	145	0
N.S.	1	1.04	1.19	0.81	0.89	1.48	0.00	2.30	1.75	0.00
time (sec)	N/A	0.505	0.475	0.160	0.208	0.081	0.000	0.302	0.226	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	94	77	68	82	96	0	204	154	0
N.S.	1	1.06	0.87	0.76	0.92	1.08	0.00	2.29	1.73	0.00
time (sec)	N/A	0.466	0.514	0.138	0.234	0.079	0.000	0.230	0.215	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	70	50	97	97	0	67	199	0
N.S.	1	1.04	0.68	0.49	0.94	0.94	0.00	0.65	1.93	0.00
time (sec)	N/A	0.628	0.837	0.144	0.147	0.085	0.000	0.207	0.249	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	65	45	80	92	0	62	184	0
N.S.	1	1.02	0.79	0.55	0.98	1.12	0.00	0.76	2.24	0.00
time (sec)	N/A	0.516	0.719	0.141	0.113	0.079	0.000	0.171	0.217	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	60	40	63	87	0	57	168	0
N.S.	1	1.02	0.95	0.63	1.00	1.38	0.00	0.90	2.67	0.00
time (sec)	N/A	0.404	0.574	0.125	0.114	0.076	0.000	0.159	0.216	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	51	64	96	0	91	411	0
N.S.	1	1.00	1.13	0.82	1.03	1.55	0.00	1.47	6.63	0.00
time (sec)	N/A	0.408	0.538	0.125	0.106	0.070	0.000	0.223	0.218	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	90	63	96	106	0	168	217	0
N.S.	1	1.03	1.03	0.72	1.10	1.22	0.00	1.93	2.49	0.00
time (sec)	N/A	0.481	0.551	0.129	0.116	0.076	0.000	0.232	0.248	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	87	68	145	126	0	223	278	0
N.S.	1	1.04	0.78	0.61	1.29	1.12	0.00	1.99	2.48	0.00
time (sec)	N/A	0.635	0.649	0.139	0.114	0.075	0.000	0.170	0.203	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	90	70	50	202	117	0	67	290	0
N.S.	1	1.05	0.81	0.58	2.35	1.36	0.00	0.78	3.37	0.00
time (sec)	N/A	0.554	0.926	0.144	0.111	0.073	0.000	0.219	0.220	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	65	45	185	112	0	62	274	0
N.S.	1	1.04	0.96	0.66	2.72	1.65	0.00	0.91	4.03	0.00
time (sec)	N/A	0.490	0.766	0.135	0.113	0.079	0.000	0.219	0.214	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	76	51	0	28	114	49
N.S.	1	1.00	0.70	0.64	1.62	1.09	0.00	0.60	2.43	1.04
time (sec)	N/A	0.347	0.584	0.125	0.039	0.071	0.000	0.184	0.229	18.387

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	80	80	93	126	0	101	714	0
N.S.	1	1.06	0.94	0.94	1.09	1.48	0.00	1.19	8.40	0.00
time (sec)	N/A	0.465	0.675	0.122	0.107	0.082	0.000	0.193	0.241	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	118	92	73	125	141	0	233	341	0
N.S.	1	1.07	0.84	0.66	1.14	1.28	0.00	2.12	3.10	0.00
time (sec)	N/A	0.636	0.788	0.132	0.114	0.072	0.000	0.261	0.227	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	146	105	78	174	156	0	233	403	0
N.S.	1	1.08	0.78	0.58	1.29	1.16	0.00	1.73	2.99	0.00
time (sec)	N/A	0.806	0.890	0.142	0.113	0.084	0.000	0.241	0.220	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	880	927	15669	1736	0	1023	0	0	32	0
N.S.	1	1.05	17.81	1.97	0.00	1.16	0.00	0.00	0.04	0.00
time (sec)	N/A	3.720	36.718	5.549	0.000	0.107	0.000	0.000	200.043	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	654	685	9965	1202	0	764	0	0	32	0
N.S.	1	1.05	15.24	1.84	0.00	1.17	0.00	0.00	0.05	0.00
time (sec)	N/A	2.085	36.164	3.559	0.000	0.098	0.000	0.000	200.024	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	726	774	13240	1215	0	814	0	0	32	0
N.S.	1	1.07	18.24	1.67	0.00	1.12	0.00	0.00	0.04	0.00
time (sec)	N/A	2.506	35.689	4.837	0.000	0.099	0.000	0.000	200.042	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	691	722	8456	1340	0	1385	0	0	0	0
N.S.	1	1.04	12.24	1.94	0.00	2.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.255	35.958	6.165	0.000	0.126	0.000	0.000	50.014	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	976	1013	12997	1766	0	2430	0	0	32	0
N.S.	1	1.04	13.32	1.81	0.00	2.49	0.00	0.00	0.03	0.00
time (sec)	N/A	3.286	37.077	4.157	0.000	0.297	0.000	0.000	200.036	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1342	1413	19853	2484	0	4543	0	0	32	0
N.S.	1	1.05	14.79	1.85	0.00	3.39	0.00	0.00	0.02	0.00
time (sec)	N/A	5.778	38.135	5.125	0.000	0.776	0.000	0.000	200.029	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	740	9972	1261	0	761	0	0	32	0
N.S.	1	1.07	14.39	1.82	0.00	1.10	0.00	0.00	0.05	0.00
time (sec)	N/A	2.844	36.225	4.904	0.000	0.098	0.000	0.000	200.030	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	568	992	955	0	571	0	0	32	0
N.S.	1	1.07	1.88	1.81	0.00	1.08	0.00	0.00	0.06	0.00
time (sec)	N/A	1.792	33.397	3.056	0.000	0.090	0.000	0.000	200.035	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	476	1080	823	0	451	0	0	32	0
N.S.	1	1.06	2.40	1.83	0.00	1.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.161	34.526	3.794	0.000	0.090	0.000	0.000	200.027	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	522	772	965	0	708	0	0	32	0
N.S.	1	1.07	1.59	1.99	0.00	1.46	0.00	0.00	0.07	0.00
time (sec)	N/A	1.383	29.091	4.274	0.000	0.100	0.000	0.000	200.038	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	660	697	1194	1248	0	1305	0	0	0	0
N.S.	1	1.06	1.81	1.89	0.00	1.98	0.00	0.00	0.00	0.00
time (sec)	N/A	2.306	33.954	5.914	0.000	0.133	0.000	0.000	20.025	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	922	986	12295	1757	0	2656	0	0	0	0
N.S.	1	1.07	13.34	1.91	0.00	2.88	0.00	0.00	0.00	0.00
time (sec)	N/A	3.539	36.920	8.017	0.000	0.371	0.000	0.000	94.898	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	508	503	0	0	0	0	0	0	0	0
N.S.	1	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.405	0.000	0.000	0.000	0.000	0.000	0.000	180.390	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	494	492	0	0	0	0	0	0	32	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	1.245	0.000	0.000	0.000	0.000	0.000	0.000	200.026	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	588	589	0	0	0	0	0	0	254	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.546	0.000	0.000	0.000	0.000	0.000	0.000	0.165	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	B	A	F	F	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	137	125	331	356	0	0	242	0
N.S.	1	0.00	0.91	0.83	2.21	2.37	0.00	0.00	1.61	0.00
time (sec)	N/A	0.000	0.676	0.189	0.038	0.084	0.000	0.000	0.183	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	167	2052	1779	1779	2281	2467	2459	2026
N.S.	1	1.00	8.35	102.60	88.95	88.95	114.05	123.35	122.95	101.30
time (sec)	N/A	0.764	0.463	0.270	0.046	0.078	0.147	0.199	0.167	17.552

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	167	2052	1779	1779	2281	2467	2459	2026
N.S.	1	1.00	8.35	102.60	88.95	88.95	114.05	123.35	122.95	101.30
time (sec)	N/A	1.098	0.071	0.236	0.042	0.082	0.155	0.202	0.179	17.637

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	943	167	2052	1779	1779	2281	2467	2459	2026
N.S.	1	47.15	8.35	102.60	88.95	88.95	114.05	123.35	122.95	101.30
time (sec)	N/A	5.876	0.039	0.306	0.042	0.083	0.149	0.127	0.194	0.628

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	18	18	20	20	18	18
N.S.	1	1.00	1.00	0.73	0.69	0.69	0.77	0.77	0.69	0.69
time (sec)	N/A	0.328	0.006	0.091	0.025	0.060	0.048	0.214	1.497	0.052

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	370	282	314	0	701	700	321	887	0
N.S.	1	1.07	0.82	0.91	0.00	2.03	2.02	0.93	2.56	0.00
time (sec)	N/A	1.552	1.587	0.357	0.000	0.176	0.850	0.195	0.675	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	261	199	210	0	499	479	221	619	0
N.S.	1	1.07	0.81	0.86	0.00	2.04	1.96	0.90	2.53	0.00
time (sec)	N/A	1.003	0.997	0.290	0.000	0.121	0.834	0.167	0.348	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	190	139	134	0	341	347	144	401	0
N.S.	1	1.07	0.79	0.76	0.00	1.93	1.96	0.81	2.27	0.00
time (sec)	N/A	0.757	0.670	0.280	0.000	0.099	0.661	0.196	0.154	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	168	135	223	0	733	0	0	1196	0
N.S.	1	1.08	0.87	1.44	0.00	4.73	0.00	0.00	7.72	0.00
time (sec)	N/A	0.896	0.849	0.271	0.000	1.827	0.000	0.000	0.261	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	152	146	156	0	703	0	170	208	166
N.S.	1	1.09	1.05	1.12	0.00	5.06	0.00	1.22	1.50	1.19
time (sec)	N/A	0.832	1.240	0.310	0.000	1.148	0.000	0.182	0.194	20.220

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	175	170	134	0	783	0	346	300	0
N.S.	1	1.10	1.07	0.84	0.00	4.92	0.00	2.18	1.89	0.00
time (sec)	N/A	0.894	1.658	0.286	0.000	1.534	0.000	0.241	0.186	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	201	178	150	0	365	0	680	440	0
N.S.	1	1.08	0.96	0.81	0.00	1.96	0.00	3.66	2.37	0.00
time (sec)	N/A	0.924	1.336	0.342	0.000	1.676	0.000	0.188	0.194	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	291	237	236	0	525	0	1433	686	0
N.S.	1	1.08	0.88	0.87	0.00	1.94	0.00	5.31	2.54	0.00
time (sec)	N/A	1.190	2.370	0.362	0.000	4.156	0.000	0.192	0.295	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	401	328	350	0	727	0	2155	982	0
N.S.	1	1.08	0.88	0.94	0.00	1.96	0.00	5.81	2.65	0.00
time (sec)	N/A	1.628	3.487	0.484	0.000	11.541	0.000	0.208	0.587	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	258	212	200	206	206	230	237	235	196
N.S.	1	0.97	0.80	0.75	0.77	0.77	0.86	0.89	0.88	0.74
time (sec)	N/A	0.993	0.042	0.091	0.029	0.062	0.037	0.170	0.168	0.136

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	157	136	139	145	145	158	160	158	137
N.S.	1	1.04	0.90	0.92	0.96	0.96	1.05	1.06	1.05	0.91
time (sec)	N/A	0.730	0.036	0.092	0.026	0.058	0.032	0.182	0.171	18.245

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	78	79	79	87	83	81	77
N.S.	1	1.00	1.00	0.84	0.85	0.85	0.94	0.89	0.87	0.83
time (sec)	N/A	0.559	0.016	0.052	0.031	0.058	0.023	0.148	0.159	18.166

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	34	34	34	37	34	33	34
N.S.	1	1.00	1.00	0.81	0.81	0.81	0.88	0.81	0.79	0.81
time (sec)	N/A	0.336	0.003	0.033	0.026	0.064	0.024	0.139	0.166	0.028

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	179	220	228	230	235	249	283	260
N.S.	1	1.00	0.79	0.96	1.00	1.01	1.03	1.09	1.24	1.14
time (sec)	N/A	0.724	0.064	0.100	0.034	0.065	0.253	0.184	0.165	0.070

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	223	232	234	319	238	325	392	363
N.S.	1	1.00	0.98	1.02	1.03	1.40	1.04	1.43	1.72	1.59
time (sec)	N/A	0.788	0.087	0.092	0.032	0.065	0.447	0.161	0.174	18.364

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	204	225	240	360	248	237	452	297
N.S.	1	1.00	0.88	0.97	1.04	1.56	1.07	1.03	1.96	1.29
time (sec)	N/A	0.762	0.066	0.089	0.029	0.064	0.853	0.164	0.179	0.104

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	391	277	254	263	263	298	305	303	251
N.S.	1	1.48	1.05	0.96	0.99	0.99	1.12	1.15	1.14	0.95
time (sec)	N/A	1.385	0.045	0.110	0.032	0.064	0.038	0.148	0.174	18.481

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	201	201	177	185	185	206	206	204	175
N.S.	1	1.03	1.03	0.91	0.95	0.95	1.06	1.06	1.05	0.90
time (sec)	N/A	0.850	0.029	0.102	0.028	0.059	0.031	0.118	0.179	0.138

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	100	105	105	112	107	105	101
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.93	0.88	0.87	0.83
time (sec)	N/A	0.670	0.019	0.095	0.027	0.056	0.025	0.143	0.176	18.167

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	44	44	44	56	44	43	44
N.S.	1	1.00	1.00	0.73	0.73	0.73	0.93	0.73	0.72	0.73
time (sec)	N/A	0.362	0.002	0.089	0.030	0.055	0.031	0.178	0.168	0.039

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	262	354	366	368	372	416	462	434
N.S.	1	1.00	0.74	1.01	1.04	1.05	1.06	1.18	1.31	1.23
time (sec)	N/A	1.103	0.130	0.101	0.029	0.066	0.510	0.160	0.159	0.084

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	342	366	372	490	393	491	607	939
N.S.	1	1.00	0.97	1.04	1.05	1.39	1.11	1.39	1.72	2.66
time (sec)	N/A	1.104	0.145	0.102	0.034	0.067	0.685	0.159	0.165	18.134

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	311	359	378	545	394	392	699	771
N.S.	1	1.00	0.88	1.01	1.07	1.54	1.11	1.11	1.97	2.18
time (sec)	N/A	1.121	0.103	0.103	0.036	0.074	1.376	0.154	0.201	18.712

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	344	361	390	587	401	383	750	560
N.S.	1	1.00	0.96	1.00	1.08	1.63	1.11	1.06	2.08	1.56
time (sec)	N/A	1.112	0.133	0.106	0.036	0.068	2.540	0.210	0.223	0.156

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	178	222	206	206	450	221	286	397
N.S.	1	1.00	0.81	1.00	0.93	0.93	2.04	1.00	1.29	1.80
time (sec)	N/A	0.743	0.126	0.276	0.110	0.072	0.696	0.145	0.197	17.214

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	130	147	141	141	303	146	187	223
N.S.	1	1.00	0.83	0.94	0.90	0.90	1.94	0.94	1.20	1.43
time (sec)	N/A	0.616	0.085	0.145	0.109	0.074	0.501	0.188	0.230	16.244

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	86	83	84	84	163	82	99	107
N.S.	1	1.00	0.87	0.84	0.85	0.85	1.65	0.83	1.00	1.08
time (sec)	N/A	0.493	0.056	0.132	0.114	0.074	0.321	0.157	0.166	0.074

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	50	44	43	43	61	43	42	45
N.S.	1	1.00	0.89	0.79	0.77	0.77	1.09	0.77	0.75	0.80
time (sec)	N/A	0.355	0.021	0.109	0.113	0.065	0.057	0.177	0.186	0.042

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	146	142	160	171	0	159	210	713
N.S.	1	1.00	0.87	0.85	0.95	1.02	0.00	0.95	1.25	4.24
time (sec)	N/A	0.685	0.120	0.256	0.109	0.101	0.000	0.140	0.175	18.762

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	233	213	294	416	0	357	599	312
N.S.	1	1.00	1.00	0.91	1.26	1.79	0.00	1.53	2.57	1.34
time (sec)	N/A	0.808	0.168	0.276	0.112	0.144	0.000	0.164	0.170	17.053

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	278	298	498	698	0	438	1163	493
N.S.	1	1.00	0.88	0.94	1.57	2.20	0.00	1.38	3.67	1.56
time (sec)	N/A	0.956	0.455	0.300	0.124	0.204	0.000	0.155	0.195	17.262

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	189	209	214	212	350	444	215	607	333
N.S.	1	0.84	0.93	0.95	0.94	1.56	1.97	0.96	2.70	1.48
time (sec)	N/A	0.904	0.162	0.159	0.116	0.077	1.371	0.174	0.199	0.161

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	143	150	145	147	245	298	146	421	211
N.S.	1	0.89	0.94	0.91	0.92	1.53	1.86	0.91	2.63	1.32
time (sec)	N/A	0.740	0.124	0.148	0.122	0.070	0.954	0.179	0.207	16.530

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	98	96	87	90	147	165	88	246	115
N.S.	1	0.95	0.93	0.84	0.87	1.43	1.60	0.85	2.39	1.12
time (sec)	N/A	0.630	0.071	0.125	0.109	0.070	0.595	0.120	0.188	16.523

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	66	59	50	52	78	65	52	117	52
N.S.	1	1.05	0.94	0.79	0.83	1.24	1.03	0.83	1.86	0.83
time (sec)	N/A	0.428	0.038	0.111	0.106	0.066	0.073	0.139	0.192	0.053

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	230	186	214	289	479	0	296	851	330
N.S.	1	1.03	0.83	0.96	1.29	2.14	0.00	1.32	3.80	1.47
time (sec)	N/A	1.050	0.163	0.259	0.113	0.132	0.000	0.155	0.167	16.957

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	314	270	303	548	910	0	584	1837	601
N.S.	1	1.00	0.86	0.97	1.75	2.91	0.00	1.87	5.87	1.92
time (sec)	N/A	1.493	0.261	0.290	0.119	0.173	0.000	0.198	0.187	17.167

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	411	363	400	851	1499	0	648	3116	887
N.S.	1	1.00	0.88	0.97	2.07	3.64	0.00	1.57	7.56	2.15
time (sec)	N/A	2.031	0.417	0.177	0.136	0.277	0.000	0.136	0.198	18.032

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	181	209	209	222	441	469	210	855	299
N.S.	1	0.79	0.91	0.91	0.97	1.92	2.04	0.91	3.72	1.30
time (sec)	N/A	1.107	0.299	0.158	0.131	0.078	2.995	0.175	0.169	0.164

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	142	146	146	155	302	304	145	579	203
N.S.	1	0.86	0.88	0.88	0.94	1.83	1.84	0.88	3.51	1.23
time (sec)	N/A	0.919	0.188	0.151	0.113	0.072	1.842	0.163	0.196	0.129

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	116	107	91	101	172	163	91	322	125
N.S.	1	1.05	0.97	0.83	0.92	1.56	1.48	0.83	2.93	1.14
time (sec)	N/A	0.671	0.087	0.128	0.113	0.073	0.905	0.160	0.185	17.538

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	53	47	56	75	61	46	123	55
N.S.	1	1.12	0.83	0.73	0.88	1.17	0.95	0.72	1.92	0.86
time (sec)	N/A	0.443	0.042	0.107	0.108	0.066	0.077	0.144	0.196	17.757

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	352	282	359	571	1052	0	495	1994	641
N.S.	1	1.07	0.86	1.09	1.74	3.20	0.00	1.50	6.06	1.95
time (sec)	N/A	1.425	0.313	0.303	0.124	0.225	0.000	0.175	0.196	17.089

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	449	389	473	916	1734	0	806	3775	965
N.S.	1	1.01	0.88	1.07	2.07	3.91	0.00	1.82	8.52	2.18
time (sec)	N/A	2.497	0.461	0.175	0.139	0.356	0.000	0.203	0.206	17.533

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	163	80	60	126	83	76	78	138	170
N.S.	1	1.14	0.56	0.42	0.88	0.58	0.53	0.55	0.97	1.19
time (sec)	N/A	0.796	1.683	0.170	0.109	0.077	0.610	0.165	0.215	1.767

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	144	75	55	109	78	70	73	122	153
N.S.	1	1.16	0.60	0.44	0.88	0.63	0.56	0.59	0.98	1.23
time (sec)	N/A	0.622	0.615	0.125	0.108	0.073	0.451	0.128	0.176	17.585

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	165	103	0	128	125	0	129	166	0
N.S.	1	1.11	0.69	0.00	0.86	0.84	0.00	0.87	1.11	0.00
time (sec)	N/A	0.908	0.705	180.000	0.113	0.084	0.000	0.186	0.206	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	163	110	0	132	143	0	531	207	0
N.S.	1	1.09	0.74	0.00	0.89	0.96	0.00	3.56	1.39	0.00
time (sec)	N/A	0.906	0.789	180.000	0.116	0.085	0.000	0.235	0.192	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	158	110	0	143	159	0	258	281	0
N.S.	1	1.05	0.73	0.00	0.95	1.05	0.00	1.71	1.86	0.00
time (sec)	N/A	0.894	0.912	180.000	0.145	0.080	0.000	0.214	0.193	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	172	110	0	160	173	0	304	355	0
N.S.	1	1.09	0.70	0.00	1.01	1.09	0.00	1.92	2.25	0.00
time (sec)	N/A	0.896	1.041	180.000	0.116	0.083	0.000	0.181	0.195	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	183	110	0	181	189	0	327	429	0
N.S.	1	1.11	0.67	0.00	1.10	1.15	0.00	1.98	2.60	0.00
time (sec)	N/A	0.933	0.966	180.000	0.119	0.090	0.000	0.222	0.198	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	183	110	0	222	203	0	387	503	0
N.S.	1	1.11	0.67	0.00	1.35	1.23	0.00	2.35	3.05	0.00
time (sec)	N/A	0.923	1.084	180.000	0.123	0.085	0.000	0.195	0.208	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	189	86	88	250	156	0	405	402	0
N.S.	1	1.12	0.51	0.52	1.48	0.92	0.00	2.40	2.38	0.00
time (sec)	N/A	0.872	1.130	0.410	0.137	0.081	0.000	0.202	0.181	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	219	91	93	301	171	0	456	464	0
N.S.	1	1.13	0.47	0.48	1.55	0.88	0.00	2.35	2.39	0.00
time (sec)	N/A	0.953	1.569	0.171	0.130	0.092	0.000	0.170	0.190	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	191	90	70	155	93	90	88	170	0
N.S.	1	1.15	0.54	0.42	0.93	0.56	0.54	0.53	1.02	0.00
time (sec)	N/A	0.838	1.065	0.138	0.112	0.074	0.675	0.212	0.237	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	172	85	65	138	88	83	83	154	0
N.S.	1	1.17	0.58	0.44	0.94	0.60	0.56	0.56	1.05	0.00
time (sec)	N/A	0.673	0.932	0.122	0.110	0.076	0.461	0.165	0.234	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	191	113	0	157	135	0	139	198	0
N.S.	1	1.11	0.66	0.00	0.91	0.78	0.00	0.81	1.15	0.00
time (sec)	N/A	0.978	1.025	180.000	0.117	0.084	0.000	0.227	0.188	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	172	184	120	0	161	153	0	707	239	0
N.S.	1	1.07	0.70	0.00	0.94	0.89	0.00	4.11	1.39	0.00
time (sec)	N/A	1.028	1.123	180.000	0.115	0.085	0.000	0.273	0.174	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	186	120	0	172	169	0	268	313	0
N.S.	1	1.07	0.69	0.00	0.99	0.97	0.00	1.54	1.80	0.00
time (sec)	N/A	1.042	1.284	180.000	0.118	0.094	0.000	0.149	0.166	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	196	120	0	189	183	0	314	387	0
N.S.	1	1.08	0.66	0.00	1.04	1.01	0.00	1.73	2.14	0.00
time (sec)	N/A	1.019	1.237	180.000	0.151	0.087	0.000	0.206	0.170	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	207	120	0	210	199	0	503	461	0
N.S.	1	1.10	0.64	0.00	1.12	1.06	0.00	2.68	2.45	0.00
time (sec)	N/A	1.045	1.159	180.000	0.122	0.094	0.000	0.247	0.189	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	218	120	0	251	213	0	406	535	0
N.S.	1	1.16	0.64	0.00	1.34	1.13	0.00	2.16	2.85	0.00
time (sec)	N/A	1.067	1.321	180.000	0.137	0.096	0.000	0.169	0.172	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	218	120	0	297	229	0	452	609	0
N.S.	1	1.12	0.62	0.00	1.52	1.17	0.00	2.32	3.12	0.00
time (sec)	N/A	1.078	1.440	180.000	0.139	0.095	0.000	0.200	0.163	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	218	120	0	348	243	0	489	683	0
N.S.	1	1.12	0.62	0.00	1.78	1.25	0.00	2.51	3.50	0.00
time (sec)	N/A	1.076	1.843	180.000	0.132	0.100	0.000	0.268	0.174	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	165	75	55	114	78	70	73	122	0
N.S.	1	1.14	0.52	0.38	0.79	0.54	0.48	0.50	0.84	0.00
time (sec)	N/A	0.918	0.749	0.135	0.111	0.071	0.576	0.126	0.180	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	133	70	50	96	73	60	68	105	0
N.S.	1	1.11	0.58	0.42	0.80	0.61	0.50	0.57	0.88	0.00
time (sec)	N/A	0.716	0.548	0.122	0.107	0.076	0.612	0.177	0.193	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	65	45	80	68	56	63	90	0
N.S.	1	1.15	0.64	0.45	0.79	0.67	0.55	0.62	0.89	0.00
time (sec)	N/A	0.570	0.495	0.115	0.108	0.070	0.412	0.151	0.169	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	137	93	0	99	115	0	119	134	0
N.S.	1	1.16	0.79	0.00	0.84	0.97	0.00	1.01	1.14	0.00
time (sec)	N/A	0.825	0.520	180.000	0.112	0.082	0.000	0.189	0.168	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	130	100	0	103	133	0	339	175	0
N.S.	1	1.07	0.82	0.00	0.84	1.09	0.00	2.78	1.43	0.00
time (sec)	N/A	0.833	0.761	180.000	0.116	0.080	0.000	0.176	0.186	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	134	100	0	114	149	0	248	249	0
N.S.	1	1.05	0.78	0.00	0.89	1.16	0.00	1.94	1.95	0.00
time (sec)	N/A	0.803	0.724	180.000	0.117	0.089	0.000	0.216	0.167	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	100	0	131	163	0	285	323	0
N.S.	1	1.10	0.74	0.00	0.97	1.21	0.00	2.11	2.39	0.00
time (sec)	N/A	0.855	0.739	180.000	0.116	0.086	0.000	0.174	0.171	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	154	76	78	149	125	0	164	278	0
N.S.	1	1.11	0.55	0.56	1.07	0.90	0.00	1.18	2.00	0.00
time (sec)	N/A	0.800	0.783	0.141	0.121	0.077	0.000	0.180	0.166	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	139	75	55	114	102	0	72	215	0
N.S.	1	1.12	0.60	0.44	0.92	0.82	0.00	0.58	1.73	0.00
time (sec)	N/A	0.781	0.764	0.157	0.110	0.077	0.000	0.170	0.172	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	118	70	50	97	97	0	67	199	0
N.S.	1	1.15	0.68	0.49	0.94	0.94	0.00	0.65	1.93	0.00
time (sec)	N/A	0.595	0.665	0.123	0.109	0.074	0.000	0.200	0.179	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	90	65	45	80	92	0	62	183	0
N.S.	1	1.10	0.79	0.55	0.98	1.12	0.00	0.76	2.23	0.00
time (sec)	N/A	0.464	0.694	0.122	0.109	0.074	0.000	0.125	0.167	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	102	93	0	99	149	0	118	355	0
N.S.	1	1.01	0.92	0.00	0.98	1.48	0.00	1.17	3.51	0.00
time (sec)	N/A	0.669	0.756	180.000	0.117	0.087	0.000	0.210	0.176	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	114	110	0	116	157	0	225	323	0
N.S.	1	1.06	1.02	0.00	1.07	1.45	0.00	2.08	2.99	0.00
time (sec)	N/A	0.683	0.713	180.000	0.116	0.081	0.000	0.214	0.182	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	118	76	68	149	126	0	220	278	0
N.S.	1	1.05	0.68	0.61	1.33	1.12	0.00	1.96	2.48	0.00
time (sec)	N/A	0.630	0.655	0.135	0.113	0.076	0.000	0.187	0.162	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	143	81	73	217	141	0	271	340	0
N.S.	1	1.04	0.59	0.53	1.58	1.03	0.00	1.98	2.48	0.00
time (sec)	N/A	0.809	0.745	0.133	0.119	0.080	0.000	0.217	0.177	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	118	75	55	219	122	0	71	305	0
N.S.	1	1.12	0.71	0.52	2.09	1.16	0.00	0.68	2.90	0.00
time (sec)	N/A	0.711	0.747	0.144	0.112	0.080	0.000	0.158	0.179	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	94	70	50	202	117	0	66	289	0
N.S.	1	1.09	0.81	0.58	2.35	1.36	0.00	0.77	3.36	0.00
time (sec)	N/A	0.515	0.907	0.125	0.121	0.075	0.000	0.116	0.176	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	65	45	185	112	0	62	273	0
N.S.	1	1.04	0.96	0.66	2.72	1.65	0.00	0.91	4.01	0.00
time (sec)	N/A	0.428	0.647	0.120	0.108	0.077	0.000	0.198	0.174	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	90	69	81	110	126	0	92	433	0
N.S.	1	1.06	0.81	0.95	1.29	1.48	0.00	1.08	5.09	0.00
time (sec)	N/A	0.574	0.703	0.135	0.112	0.083	0.000	0.136	0.185	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	120	81	73	127	141	0	206	340	0
N.S.	1	1.09	0.74	0.66	1.15	1.28	0.00	1.87	3.09	0.00
time (sec)	N/A	0.648	0.923	0.140	0.114	0.077	0.000	0.263	0.183	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	146	94	78	178	155	0	228	402	0
N.S.	1	1.08	0.70	0.58	1.32	1.15	0.00	1.69	2.98	0.00
time (sec)	N/A	0.818	0.859	0.138	0.116	0.077	0.000	0.217	0.167	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	173	91	83	246	170	0	279	464	0
N.S.	1	1.08	0.57	0.52	1.54	1.06	0.00	1.74	2.90	0.00
time (sec)	N/A	1.032	1.005	0.142	0.120	0.080	0.000	0.202	0.189	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	377	315	1114	0	1373	0	463	2208	0
N.S.	1	1.06	0.89	3.15	0.00	3.88	0.00	1.31	6.24	0.00
time (sec)	N/A	1.310	3.108	0.430	0.000	22.002	0.000	0.170	0.325	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	374	316	1147	0	1385	0	488	2215	0
N.S.	1	1.06	0.90	3.25	0.00	3.92	0.00	1.38	6.27	0.00
time (sec)	N/A	1.236	2.972	0.415	0.000	22.070	0.000	0.192	0.186	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	537	5924	2292	4795	136733	10965	6933	4341
N.S.	1	1.00	0.91	10.07	3.90	8.15	232.54	18.65	11.79	7.38
time (sec)	N/A	1.485	0.626	0.309	0.093	0.122	32.575	0.222	0.170	24.510

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	391	3222	1414	2796	65193	6226	3894	2625
N.S.	1	1.00	0.91	7.46	3.27	6.47	150.91	14.41	9.01	6.08
time (sec)	N/A	1.166	0.426	0.178	0.072	0.101	13.520	0.221	0.161	19.918

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	261	1220	788	1448	26165	3099	1907	1425
N.S.	1	1.00	0.89	4.18	2.70	4.96	89.61	10.61	6.53	4.88
time (sec)	N/A	0.839	0.313	0.132	0.065	0.087	4.872	0.216	0.178	17.730

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	254	255	221	0	0	0	0	0	1244	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	4.90	0.00
time (sec)	N/A	1.093	1.098	0.000	0.000	0.000	0.000	0.000	0.172	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	376	378	441	0	0	0	0	0	0	0
N.S.	1	1.01	1.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.491	2.149	0.000	0.000	0.000	0.000	0.000	0.395	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [60] had the largest ratio of [.4062500000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	2	2	228.00	32	0.062
2	A	4	4	1.01	47	0.085
3	A	7	7	1.22	69	0.101
4	A	2	2	1.00	30	0.067
5	A	2	2	1.00	30	0.067
6	A	2	2	1.00	28	0.071
7	A	2	2	1.00	23	0.087
8	A	2	2	1.00	30	0.067
9	A	2	2	1.00	30	0.067
10	A	2	2	1.00	30	0.067
11	A	3	3	0.70	30	0.100
12	A	6	5	0.86	28	0.179
13	A	5	4	1.03	23	0.174
14	A	4	4	1.13	30	0.133
15	A	3	3	1.07	30	0.100
16	A	4	4	1.05	20	0.200
17	A	3	3	1.09	20	0.150
18	A	7	6	1.06	18	0.333
19	A	5	4	1.00	17	0.235
20	A	3	3	1.09	20	0.150
21	A	3	3	1.05	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.04	20	0.150
23	A	11	10	0.83	32	0.312
24	A	9	8	0.83	32	0.250
25	A	7	6	0.87	30	0.200
26	A	7	6	0.95	25	0.240
27	A	10	9	1.07	32	0.281
28	A	10	9	1.06	32	0.281
29	A	11	10	1.05	32	0.312
30	A	10	9	1.33	32	0.281
31	A	7	6	0.91	32	0.188
32	A	9	8	0.88	32	0.250
33	A	12	11	0.69	32	0.344
34	A	10	9	0.71	32	0.281
35	A	8	7	0.79	30	0.233
36	A	8	7	0.89	25	0.280
37	A	12	11	1.05	32	0.344
38	A	12	11	1.03	32	0.344
39	A	13	12	1.03	32	0.375
40	A	12	11	1.02	32	0.344
41	A	12	11	1.42	32	0.344
42	A	12	11	1.48	32	0.344
43	A	8	7	0.83	32	0.219
44	A	10	9	0.77	32	0.281
45	A	11	10	1.14	32	0.312
46	A	9	8	1.13	32	0.250
47	A	7	6	1.11	30	0.200
48	A	10	9	1.06	32	0.281
49	A	10	9	1.06	32	0.281
50	A	10	9	1.05	32	0.281
51	A	12	11	1.16	32	0.344
52	A	10	9	1.14	32	0.281
53	A	8	7	1.13	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	12	11	1.09	32	0.344
55	A	12	11	1.08	32	0.344
56	A	12	11	1.11	32	0.344
57	A	12	11	1.16	32	0.344
58	A	11	10	1.15	32	0.312
59	A	9	8	1.14	30	0.267
60	A	14	13	1.11	32	0.406
61	A	14	13	1.10	32	0.406
62	A	14	13	1.10	32	0.406
63	A	10	9	1.05	32	0.281
64	A	8	7	1.05	32	0.219
65	A	6	5	1.07	30	0.167
66	A	6	5	1.04	25	0.200
67	A	8	7	1.05	32	0.219
68	A	8	7	1.16	32	0.219
69	A	6	5	1.06	32	0.156
70	A	8	7	0.78	32	0.219
71	A	6	5	0.75	32	0.156
72	A	6	5	0.96	30	0.167
73	A	5	4	1.03	25	0.160
74	A	5	4	1.02	32	0.125
75	A	6	5	1.02	32	0.156
76	A	8	7	1.06	32	0.219
77	A	9	8	1.12	32	0.250
78	A	8	7	1.11	32	0.219
79	A	6	5	1.07	30	0.167
80	A	8	7	1.06	32	0.219
81	A	8	7	1.04	32	0.219
82	A	6	5	1.06	32	0.156
83	A	10	9	1.04	32	0.281
84	A	8	7	1.02	32	0.219
85	A	6	5	1.02	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	5	4	1.00	32	0.125
87	A	6	5	1.03	32	0.156
88	A	8	7	1.04	32	0.219
89	A	8	7	1.05	32	0.219
90	A	7	6	1.04	32	0.188
91	A	3	3	1.00	30	0.100
92	A	7	6	1.06	32	0.188
93	A	8	7	1.07	32	0.219
94	A	10	9	1.08	32	0.281
95	A	11	10	1.05	34	0.294
96	A	9	8	1.05	34	0.235
97	A	9	8	1.07	34	0.235
98	A	9	8	1.04	34	0.235
99	A	9	8	1.04	34	0.235
100	A	11	10	1.05	34	0.294
101	A	11	10	1.07	34	0.294
102	A	9	8	1.07	34	0.235
103	A	7	6	1.06	34	0.176
104	A	7	6	1.07	34	0.176
105	A	9	8	1.06	34	0.235
106	A	11	10	1.07	34	0.294
107	A	7	6	0.99	30	0.200
108	A	6	5	1.00	32	0.156
109	A	7	6	1.00	34	0.176
110	F	0	0	N/A	0.000	N/A
111	A	1	1	1.00	47	0.021
112	A	2	2	1.00	75	0.027
113	B	2	2	47.15	97	0.021
114	A	3	3	1.00	16	0.188
115	A	10	9	1.07	33	0.273
116	A	8	7	1.07	31	0.226
117	A	8	7	1.07	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	10	9	1.08	33	0.273
119	A	9	8	1.09	33	0.242
120	A	10	9	1.10	33	0.273
121	A	8	7	1.08	33	0.212
122	A	10	9	1.08	33	0.273
123	A	12	11	1.08	33	0.333
124	A	2	2	0.97	36	0.056
125	A	2	2	1.04	36	0.056
126	A	2	2	1.00	34	0.059
127	A	2	2	1.00	29	0.069
128	A	2	2	1.00	36	0.056
129	A	2	2	1.00	36	0.056
130	A	2	2	1.00	36	0.056
131	A	2	2	1.48	38	0.053
132	A	2	2	1.03	38	0.053
133	A	2	2	1.00	36	0.056
134	A	2	2	1.00	31	0.065
135	A	2	2	1.00	38	0.053
136	A	2	2	1.00	38	0.053
137	A	2	2	1.00	38	0.053
138	A	2	2	1.00	38	0.053
139	A	2	2	1.00	38	0.053
140	A	2	2	1.00	38	0.053
141	A	2	2	1.00	36	0.056
142	A	2	2	1.00	31	0.065
143	A	2	2	1.00	38	0.053
144	A	2	2	1.00	38	0.053
145	A	2	2	1.00	38	0.053
146	A	4	4	0.84	38	0.105
147	A	4	4	0.89	38	0.105
148	A	4	4	0.95	36	0.111
149	A	4	4	1.05	31	0.129

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	1.03	38	0.105
151	A	4	4	1.00	38	0.105
152	A	4	4	1.00	38	0.105
153	A	6	6	0.79	38	0.158
154	A	6	6	0.86	38	0.158
155	A	10	9	1.05	36	0.250
156	A	7	6	1.12	31	0.194
157	A	6	6	1.07	38	0.158
158	A	6	6	1.01	38	0.158
159	A	11	10	1.14	38	0.263
160	A	11	10	1.16	33	0.303
161	A	14	13	1.11	40	0.325
162	A	14	13	1.09	40	0.325
163	A	14	13	1.05	40	0.325
164	A	12	11	1.09	40	0.275
165	A	14	13	1.11	40	0.325
166	A	13	12	1.11	40	0.300
167	A	10	9	1.12	40	0.225
168	A	13	12	1.13	40	0.300
169	A	12	11	1.15	38	0.289
170	A	12	11	1.17	33	0.333
171	A	15	14	1.11	40	0.350
172	A	16	15	1.07	40	0.375
173	A	15	14	1.07	40	0.350
174	A	14	13	1.08	40	0.325
175	A	15	14	1.10	40	0.350
176	A	15	14	1.16	40	0.350
177	A	15	14	1.12	40	0.350
178	A	16	15	1.12	40	0.375
179	A	11	10	1.14	40	0.250
180	A	10	9	1.11	38	0.237
181	A	10	9	1.15	33	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	12	11	1.16	40	0.275
183	A	12	11	1.07	40	0.275
184	A	12	11	1.05	40	0.275
185	A	11	10	1.10	40	0.250
186	A	9	8	1.11	40	0.200
187	A	11	10	1.12	40	0.250
188	A	9	8	1.15	38	0.211
189	A	8	7	1.10	33	0.212
190	A	10	9	1.01	40	0.225
191	A	10	9	1.06	40	0.225
192	A	8	7	1.05	40	0.175
193	A	10	9	1.04	40	0.225
194	A	10	9	1.12	40	0.225
195	A	8	7	1.09	38	0.184
196	A	7	6	1.04	33	0.182
197	A	7	6	1.06	40	0.150
198	A	8	7	1.09	40	0.175
199	A	10	9	1.08	40	0.225
200	A	12	11	1.08	40	0.275
201	A	7	6	1.06	35	0.171
202	A	7	6	1.06	36	0.167
203	A	2	2	1.00	38	0.053
204	A	2	2	1.00	38	0.053
205	A	2	2	1.00	36	0.056
206	A	2	2	1.00	38	0.053
207	A	4	4	1.01	38	0.105

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(2+3x+5x^2)} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 1

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)} \, dx = 0$$

output
```
0
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 1.09 (sec) , antiderivative size = 259, normalized size of antiderivative = 259.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)} \, dx = \frac{1}{775}\left(\frac{310C\sqrt{d+ex}}{e} + \frac{\left(50i\sqrt{31}A+\left(-155-15i\sqrt{31}\right)B+\left(93-11i\sqrt{31}\right)C\right)\arctan\left(\frac{\sqrt{10}\sqrt{-10d+3e-i\sqrt{31}e}\sqrt{d+ex}}{10d-3e+i\sqrt{31}e}\right)}{\sqrt{-d+\frac{1}{10}\left(3-i\sqrt{31}\right)e}} + \frac{\left(-50i\sqrt{31}A+5i\left(31i+3\sqrt{31}\right)B+\left(93+11i\sqrt{31}\right)C\right)\arctan\left(\frac{\sqrt{10}\sqrt{-10d+3e+i\sqrt{31}e}\sqrt{d+ex}}{10d-3e-i\sqrt{31}e}\right)}{\sqrt{-d+\frac{1}{10}\left(3+i\sqrt{31}\right)e}}\right)$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(2 + 3*x + 5*x^2)),x]
```

output
```
((310*C*Sqrt[d + e*x])/e + (((50*I)*Sqrt[31]*A + (-155 - (15*I)*Sqrt[31])*
B + (93 - (11*I)*Sqrt[31])*C)*ArcTan[(Sqrt[10]*Sqrt[-10*d + 3*e - I*Sqrt[3
1]*e]*Sqrt[d + e*x])/(10*d - 3*e + I*Sqrt[31]*e)])/Sqrt[-d + ((3 - I*Sqrt[
31])*e)/10] + (((-50*I)*Sqrt[31]*A + (5*I)*(31*I + 3*Sqrt[31])*B + (93 + (
11*I)*Sqrt[31])*C)*ArcTan[(Sqrt[10]*Sqrt[-10*d + 3*e + I*Sqrt[31]*e]*Sqrt[
d + e*x])/(10*d - 3*e - I*Sqrt[31]*e)])/Sqrt[-d + ((3 + I*Sqrt[31])*e)/10]
)/775
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 228.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(5x^2+3x+2)\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-\frac{i(50A-15B-11C)}{5\sqrt{31}}+B-\frac{3C}{5}}{\left(10x-i\sqrt{31}+3\right)\sqrt{d+ex}}+\frac{\frac{i(50A-15B-11C)}{5\sqrt{31}}+B-\frac{3C}{5}}{\left(10x+i\sqrt{31}+3\right)\sqrt{d+ex}}+\frac{C}{5\sqrt{d+ex}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{2}{155}}\left(50iA-5\left(-\sqrt{31}+3i\right)B-\left(3\sqrt{31}+11i\right)C\right)\text{arctanh}\left(\frac{\sqrt{10}\sqrt{d+ex}}{\sqrt{10d+i\left(-\sqrt{31}+3i\right)e}}\right)}{5\sqrt{10d-\left(3+i\sqrt{31}\right)e}}+\frac{\sqrt{\frac{2}{155}}\left(50iA-5\left(\sqrt{31}+3i\right)B-\left(-3\sqrt{31}+11i\right)C\right)\text{arctanh}\left(\frac{\sqrt{10}\sqrt{d+ex}}{\sqrt{10d+i\left(\sqrt{31}+3i\right)e}}\right)}{5\sqrt{10d+i\left(\sqrt{31}+3i\right)e}}+\frac{2C\sqrt{d+ex}}{5e}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(2 + 3*x + 5*x^2)),x]
```

output
```
(2*C*Sqrt[d + e*x])/(5*e) - (Sqrt[2/155]*((50*I)*A - 5*(3*I - Sqrt[31])*B
- (11*I + 3*Sqrt[31])*C)*ArcTanh[(Sqrt[10]*Sqrt[d + e*x])/Sqrt[10*d + I*(3
*I - Sqrt[31])*e]])/(5*Sqrt[10*d - (3 + I*Sqrt[31])*e]) + (Sqrt[2/155]*((5
0*I)*A - 5*(3*I + Sqrt[31])*B - (11*I - 3*Sqrt[31])*C)*ArcTanh[(Sqrt[10]*S
qrt[d + e*x])/Sqrt[10*d + I*(3*I + Sqrt[31])*e]])/(5*Sqrt[10*d + I*(3*I +
Sqrt[31])*e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 2.02 (sec) , antiderivative size = 669, normalized size of antiderivative = 669.00

method	result
pseudoelliptic	$-\frac{-31e\left(\frac{\left(B-\frac{3C}{5}\right)\sqrt{5}\,\sqrt{5d^2-3de+2e^2}}{5}+\left(A-\frac{2C}{5}\right)e-\left(B-\frac{3C}{5}\right)d\right)\arctan\left(\frac{2\sqrt{5}\,\sqrt{ex+d}-\sqrt{2\sqrt{5}\,\sqrt{5d^2-3de+2e^2}+10d-3e}}{\sqrt{2\sqrt{5}\,\sqrt{5d^2-3de+2e^2}-10d+3e}}\right)-}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(5*x^2+3*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/31*(-31*e*(1/5*(B-3/5*C)*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)+(A-2/5*C)*e-
(B-3/5*C)*d)*arctan((2*5^(1/2)*(e*x+d)^(1/2)-(2*5^(1/2)*(5*d^2-3*d*e+2*e^2
)^(1/2)+10*d-3*e)^(1/2))/(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)-10*d+3*e)^(1
/2))-31*e*(1/5*(B-3/5*C)*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)+(A-2/5*C)*e-(B-
3/5*C)*d)*arctan((2*5^(1/2)*(e*x+d)^(1/2)+(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(
1/2)+10*d-3*e)^(1/2))/(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)-10*d+3*e)^(1/2)
)+(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)-10*d+3*e)^(1/2)*((5^(1/2)*(A-3/10*B
-11/50*C)*(5*d^2-3*d*e+2*e^2)^(1/2)+(-2*B+3/5*C+3/2*A)*e-5*(A-3/10*B-11/50
*C)*d)*(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)+10*d-3*e)^(1/2)*ln(5^(1/2)*(e*
x+d)-(e*x+d)^(1/2)*(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)+10*d-3*e)^(1/2)+(5
*d^2-3*d*e+2*e^2)^(1/2))-(5^(1/2)*(A-3/10*B-11/50*C)*(5*d^2-3*d*e+2*e^2)^(
1/2)+(-2*B+3/5*C+3/2*A)*e-5*(A-3/10*B-11/50*C)*d)*(2*5^(1/2)*(5*d^2-3*d*e+
2*e^2)^(1/2)+10*d-3*e)^(1/2)*ln(5^(1/2)*(e*x+d)+(e*x+d)^(1/2)*(2*5^(1/2)*(
5*d^2-3*d*e+2*e^2)^(1/2)+10*d-3*e)^(1/2)+(5*d^2-3*d*e+2*e^2)^(1/2))-62/5*C
*(e*x+d)^(1/2)*(5*d^2-3*d*e+2*e^2)^(1/2)))/(2*5^(1/2)*(5*d^2-3*d*e+2*e^2)^
(1/2)-10*d+3*e)^(1/2)/(5*d^2-3*d*e+2*e^2)^(1/2)/e
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 0.36 (sec) , antiderivative size = 6132, normalized size of antiderivative = 6132.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(5x^2+3x+2)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(5*x**2+3*x+2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*(5*x**2 + 3*x + 2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{ex+d}(5\,x^2+3\,x+2)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(e*x + d)*(5*x^2 + 3*x + 2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{ex+d}(5\,x^2+3\,x+2)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 17.59 (sec) , antiderivative size = 3714, normalized size of antiderivative = 3714.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(3*x + 5*x^2 + 2)),x)
```

output
```
log(((d + e*x)^(1/2)*(440*B^2*e^2 - 2000*A^2*e^2 + (632*C^2*e^2)/5 + 1200*
A*B*e^2 + 880*A*C*e^2 - 1008*B*C*e^2) - ((A^2*e*375i - A^2*d*1250i + B^2*d
*275i + B^2*e*150i + C^2*d*79i - C^2*e*126i + A*B*d*750i - A*B*e*1000i + A
*C*d*550i + A*C*e*300i - B*C*d*630i + B*C*e*220i - 125*31^(1/2)*A^2*e - 75
*31^(1/2)*B^2*d + 50*31^(1/2)*B^2*e + 33*31^(1/2)*C^2*d - 2*31^(1/2)*C^2*e
 + 250*31^(1/2)*A*B*d - 150*31^(1/2)*A*C*d + 100*31^(1/2)*A*C*e - 10*31^(1
/2)*B*C*d - 60*31^(1/2)*B*C*e)/(7750*(d^2*5i - d*e*3i + e^2*2i)))^(1/2)*(6
200*A*e^3 - 2480*C*e^3 + (62000*d*e^2 - 18600*e^3)*(d + e*x)^(1/2)*((A^2*e
*375i - A^2*d*1250i + B^2*d*275i + B^2*e*150i + C^2*d*79i - C^2*e*126i + A
*B*d*750i - A*B*e*1000i + A*C*d*550i + A*C*e*300i - B*C*d*630i + B*C*e*220
i - 125*31^(1/2)*A^2*e - 75*31^(1/2)*B^2*d + 50*31^(1/2)*B^2*e + 33*31^(1/
2)*C^2*d - 2*31^(1/2)*C^2*e + 250*31^(1/2)*A*B*d - 150*31^(1/2)*A*C*d + 10
0*31^(1/2)*A*C*e - 10*31^(1/2)*B*C*d - 60*31^(1/2)*B*C*e)/(7750*(d^2*5i -
d*e*3i + e^2*2i)))^(1/2) - 6200*B*d*e^2 + 3720*C*d*e^2))*((A^2*e*375i - A^
2*d*1250i + B^2*d*275i + B^2*e*150i + C^2*d*79i - C^2*e*126i + A*B*d*750i
- A*B*e*1000i + A*C*d*550i + A*C*e*300i - B*C*d*630i + B*C*e*220i - 125*31
^(1/2)*A^2*e - 75*31^(1/2)*B^2*d + 50*31^(1/2)*B^2*e + 33*31^(1/2)*C^2*d -
 2*31^(1/2)*C^2*e + 250*31^(1/2)*A*B*d - 150*31^(1/2)*A*C*d + 100*31^(1/2)
*A*C*e - 10*31^(1/2)*B*C*d - 60*31^(1/2)*B*C*e)/(7750*(d^2*5i - d*e*3i + e
^2*2i)))^(1/2) - 80*B^3*e^2 + (96*C^3*e^2)/5 + 120*A*B^2*e^2 - 200*A^2*...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6499, normalized size of antiderivative = 6499.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(2+3x+5x^2)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(5*x^2+3*x+2),x)
```

output
```
( - 500*sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e)*sqrt(5*
d**2 - 3*d*e + 2*e**2)*atan((sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5)
+ 10*d - 3*e) - 2*sqrt(d + e*x)*sqrt(5))/sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e*
*2)*sqrt(5) - 10*d + 3*e))*a*d + 150*sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*
sqrt(5) - 10*d + 3*e)*sqrt(5*d**2 - 3*d*e + 2*e**2)*atan((sqrt(2*sqrt(5*d*
*2 - 3*d*e + 2*e**2)*sqrt(5) + 10*d - 3*e) - 2*sqrt(d + e*x)*sqrt(5))/sqrt
(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e))*a*e + 150*sqrt(2*s
qrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e)*sqrt(5*d**2 - 3*d*e + 2
*e**2)*atan((sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) + 10*d - 3*e) -
2*sqrt(d + e*x)*sqrt(5))/sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10
*d + 3*e))*b*d - 200*sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d +
 3*e)*sqrt(5*d**2 - 3*d*e + 2*e**2)*atan((sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e
**2)*sqrt(5) + 10*d - 3*e) - 2*sqrt(d + e*x)*sqrt(5))/sqrt(2*sqrt(5*d**2 -
 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e))*b*e + 110*sqrt(2*sqrt(5*d**2 - 3*d
*e + 2*e**2)*sqrt(5) - 10*d + 3*e)*sqrt(5*d**2 - 3*d*e + 2*e**2)*atan((sqr
t(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) + 10*d - 3*e) - 2*sqrt(d + e*x)*
sqrt(5))/sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e))*c*d +
 60*sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) - 10*d + 3*e)*sqrt(5*d**2
 - 3*d*e + 2*e**2)*atan((sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**2)*sqrt(5) + 10
*d - 3*e) - 2*sqrt(d + e*x)*sqrt(5))/sqrt(2*sqrt(5*d**2 - 3*d*e + 2*e**...
```

3.2 $\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 214

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2(fg^2-egh+dh^2)}{3h^3(2cg-bh)(g+hx)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}} - \frac{2(12c^2fg^3+3b^2h^2(3fg-eh)-2bch(10fg^2-h(eg+2dh))-h(6bceh^2-3b^2fh^2+4c^2(fg^2-h(eg+2$$
$$\overline{3h^3(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

output
```
2/3*(d*h^2-e*g*h+f*g^2)/h^3/(-b*h+2*c*g)/(h*x+g)/(-g*(-b*h+c*g)+b*h^2*x+c*
h^2*x^2)^(1/2)-2/3*(12*c^2*f*g^3+3*b^2*h^2*(-e*h+3*f*g)-2*b*c*h*(10*f*g^2-
h*(2*d*h+e*g))-h*(6*b*c*e*h^2-3*b^2*f*h^2+4*c^2*(f*g^2-h*(2*d*h+e*g)))*x)/
h^3/(-b*h+2*c*g)^3/(-g*(-b*h+c*g)+b*h^2*x+c*h^2*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2b^2h^2(-h(2eg+dh+3ehx)+f(8g^2+12ghx+3h^2x^2)}{}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2
*x^2)^(3/2)),x]
```

output
```
(2*b^2*h^2*(-(h*(2*e*g + d*h + 3*e*h*x)) + f*(8*g^2 + 12*g*h*x + 3*h^2*x^2
)) + 8*c^2*(f*g^2*(2*g^2 + 2*g*h*x - h^2*x^2) + h*(e*g*(g^2 + g*h*x + h^2*
x^2) + d*h*(-g^2 + 2*g*h*x + 2*h^2*x^2))) - 4*b*c*h*(2*f*g^2*(4*g + 5*h*x)
 + h*(-2*d*h*(2*g + h*x) + e*(g^2 + 2*g*h*x + 3*h^2*x^2))))/(3*h^3*(-2*c*g
 + b*h)^3*(g + h*x)*Sqrt[(g + h*x)*(-(c*g) + b*h + c*h*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.085$, Rules used = {2169, 27, 1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(bgh+bh^2x-cg^2+ch^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2169}$$

$$-\frac{\int \frac{h^3(bfg-2cdh+(2cfg-2ceh+bfh)x)}{2(g+hx)\left(cx^2h^2+bxh^2-g(cg-bh)\right)^{3/2}}dx}{ch^4}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{bfg-2cdh+(2cfg-2ceh+bfh)x}{(g+hx)\left(cx^2h^2+bxh^2-g(cg-bh)\right)^{3/2}}dx}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$$\Big\downarrow \text{1220}$$

$$-\frac{\frac{\left(-3b^2fh^2+6bceh^2+c^2\left(4fg^2-4h(2dh+eg)\right)\right)\int \frac{1}{\left(cx^2h^2+bxh^2-g(cg-bh)\right)^{3/2}}dx}{3h(2cg-bh)}-\frac{4c\left(dh^2-egh+fg^2\right)}{3h^2(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{-\frac{2(b+2cx)\left(-3b^2fh^2+6bceh^2+c^2\left(4fg^2-4h(2dh+eg)\right)\right)}{3h(2cg-bh)^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}-\frac{4c\left(dh^2-egh+fg^2\right)}{3h^2(g+hx)(2cg-bh)\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}}{2ch}-\frac{f}{ch^3\sqrt{-g(cg-bh)+bh^2x+ch^2x^2}}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*(-(c*g^2) + b*g*h + b*h^2*x + c*h^2*x^2)^
(3/2)),x]
```

output
```
-(f/(c*h^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2])) - ((-2*(6*b*c*e*
h^2 - 3*b^2*f*h^2 + c^2*(4*f*g^2 - 4*h*(e*g + 2*d*h)))*(b + 2*c*x))/(3*h*(
2*c*g - b*h)^3*Sqrt[-(g*(c*g - b*h)) + b*h^2*x + c*h^2*x^2]) - (4*c*(f*g^2
 - e*g*h + d*h^2))/(3*h^2*(2*c*g - b*h)*(g + h*x)*Sqrt[-(g*(c*g - b*h)) +
b*h^2*x + c*h^2*x^2]))/(2*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 2169
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d -
b*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.51

method	result
gosper	$-\frac{2(chx+bh-cg)(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4x+4bcegh^3x+2}{3(b^3h^3-6b^2cgh^2+12bc^2g}$
orering	$-\frac{2(chx+bh-cg)(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4x+4bcegh^3x+2}{3(b^3h^3-6b^2cgh^2+12bc^2g}$
trager	$-\frac{2(-3b^2fh^4x^2+6bceh^4x^2-8c^2dh^4x^2-4c^2egh^3x^2+4c^2fg^2h^2x^2+3b^2eh^4x-12b^2fgh^3x-4bcdh^4x+4bcegh^3x+20bcfg^2h^2x-8}{3h^3(b^2h^2-4bc}$
default	$\frac{\frac{2eh(2ch^2x+bh^2)}{(4ch^2(bgh-cg^2)-b^2h^4)\sqrt{ch^2x^2+bh^2x+bgh-cg^2}}+fh\left(-\frac{1}{ch^2\sqrt{ch^2x^2+bh^2x+bgh-cg^2}}-\frac{b(2ch^2x+bh^2)}{c(4ch^2(bgh-cg^2)-b^2h^4)\sqrt{ch^2x^2+bh^2x+bgh}}\right.}{h^2}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x,method=_
RETURNVERBOSE)
```

output
```
-2/3*(c*h*x+b*h-c*g)*(-3*b^2*f*h^4*x^2+6*b*c*e*h^4*x^2-8*c^2*d*h^4*x^2-4*c
^2*e*g*h^3*x^2+4*c^2*f*g^2*h^2*x^2+3*b^2*e*h^4*x-12*b^2*f*g*h^3*x-4*b*c*d*
h^4*x+4*b*c*e*g*h^3*x+20*b*c*f*g^2*h^2*x-8*c^2*d*g*h^3*x-4*c^2*e*g^2*h^2*x
-8*c^2*f*g^3*h*x+b^2*d*h^4+2*b^2*e*g*h^3-8*b^2*f*g^2*h^2-8*b*c*d*g*h^3+2*b
*c*e*g^2*h^2+16*b*c*f*g^3*h+4*c^2*d*g^2*h^2-4*c^2*e*g^3*h-8*c^2*f*g^4)/(b^
3*h^3-6*b^2*c*g*h^2+12*b*c^2*g^2*h-8*c^3*g^3)/h^3/(c*h^2*x^2+b*h^2*x+b*g*h
-c*g^2)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(206) = 412$.

Time = 14.24 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.17

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{2\left(8\,c^2fg^4-b^2dh^4+4\left(c^2e-4\,bcf\right)g^3h-2\left(2\,c^2d+bce\right.\right.}{3\left(8\,c^4g^6h^3-20\,bc^3g^5h^4+18\,b^2c^2g^4h^5-7\right.}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="fricas")
```

output
```
2/3*(8*c^2*f*g^4 - b^2*d*h^4 + 4*(c^2*e - 4*b*c*f)*g^3*h - 2*(2*c^2*d + b*
c*e - 4*b^2*f)*g^2*h^2 + 2*(4*b*c*d - b^2*e)*g*h^3 - (4*c^2*f*g^2*h^2 - 4*
c^2*e*g*h^3 - (8*c^2*d - 6*b*c*e + 3*b^2*f)*h^4)*x^2 + (8*c^2*f*g^3*h + 4*
(c^2*e - 5*b*c*f)*g^2*h^2 + 4*(2*c^2*d - b*c*e + 3*b^2*f)*g*h^3 + (4*b*c*d
 - 3*b^2*e)*h^4)*x)*sqrt(c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)/(8*c^4*g^6*h
^3 - 20*b*c^3*g^5*h^4 + 18*b^2*c^2*g^4*h^5 - 7*b^3*c*g^3*h^6 + b^4*g^2*h^7
 - (8*c^4*g^3*h^6 - 12*b*c^3*g^2*h^7 + 6*b^2*c^2*g*h^8 - b^3*c*h^9)*x^3 -
(8*c^4*g^4*h^5 - 4*b*c^3*g^3*h^6 - 6*b^2*c^2*g^2*h^7 + 5*b^3*c*g*h^8 - b^4
*h^9)*x^2 + (8*c^4*g^5*h^4 - 28*b*c^3*g^4*h^5 + 30*b^2*c^2*g^3*h^6 - 13*b^
3*c*g^2*h^7 + 2*b^4*g*h^8)*x)
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{\left((g+hx)\left(bh-cg+chx\right)\right)^{\frac{3}{2}}(g+hx)}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*h**2*x**2+b*h**2*x+b*g*h-c*g**2)**(3/2
),x)
```

output
```
Integral((d + e*x + f*x**2)/(((g + h*x)*(b*h - c*g + c*h*x))**(3/2)*(g + h
*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \int \frac{fx^2+ex+d}{\left(ch^2x^2+bh^2x-cg^2+bgh\right)^{\frac{3}{2}}(hx+g)}\,dx$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x, a
lgorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)/((c*h^2*x^2 + b*h^2*x - c*g^2 + b*g*h)^(3/2)*(
h*x + g)), x)
```

Mupad [B] (verification not implemented)

Time = 18.84 (sec) , antiderivative size = 1089, normalized size of antiderivative = 5.09

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \frac{16\,c^2\,f\,g^4\,\sqrt{-c\,g^2+b\,g\,h+c\,h^2\,x^2+b\,h^2\,x}-2\,b^2\,d\,h^4\,\sqrt{}}{}$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(3/
2)),x)
```

output
```
(16*c^2*f*g^4*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 2*b^2*d*h^4*(b
*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*c^2*d*g^2*h^2*(b*h^2*x - c*g
^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*b^2*f*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*
x^2 + b*g*h)^(1/2) + 16*c^2*d*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h
)^(1/2) + 6*b^2*f*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 4*
b^2*e*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^3*h*(b
*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 6*b^2*e*h^4*x*(b*h^2*x - c*g^2
 + c*h^2*x^2 + b*g*h)^(1/2) + 8*b*c*d*h^4*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
 b*g*h)^(1/2) - 8*c^2*f*g^2*h^2*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^
(1/2) - 4*b*c*e*g^2*h^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 12*b
*c*e*h^4*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*d*g*h^3*
x*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 24*b^2*f*g*h^3*x*(b*h^2*x
- c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) + 16*c^2*f*g^3*h*x*(b*h^2*x - c*g^2 + c
*h^2*x^2 + b*g*h)^(1/2) + 8*c^2*e*g^2*h^2*x*(b*h^2*x - c*g^2 + c*h^2*x^2 +
 b*g*h)^(1/2) + 8*c^2*e*g*h^3*x^2*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1
/2) + 16*b*c*d*g*h^3*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 32*b*c*
f*g^3*h*(b*h^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 8*b*c*e*g*h^3*x*(b*h
^2*x - c*g^2 + c*h^2*x^2 + b*g*h)^(1/2) - 40*b*c*f*g^2*h^2*x*(b*h^2*x - c*
g^2 + c*h^2*x^2 + b*g*h)^(1/2))/(3*b^4*g^2*h^7 + 24*c^4*g^6*h^3 + 3*b^4*h^
9*x^2 - 60*b*c^3*g^5*h^4 - 21*b^3*c*g^3*h^6 + 3*b^3*c*h^9*x^3 + 24*c^4*...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1034, normalized size of antiderivative = 4.83

$$\int \frac{d+ex+fx^2}{(g+hx)\left(-cg^2+bgh+bh^2x+ch^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*h^2*x^2+b*h^2*x+b*g*h-c*g^2)^(3/2),x)
```

output

```
(2*( - 6*sqrt(c)*sqrt(b*h - c*g + c*h*x)*b**2*f*g**2*h**2 - 12*sqrt(c)*sqr
t(b*h - c*g + c*h*x)*b**2*f*g*h**3*x - 6*sqrt(c)*sqrt(b*h - c*g + c*h*x)*b
**2*f*h**4*x**2 + 6*sqrt(c)*sqrt(b*h - c*g + c*h*x)*b*c*e*g**2*h**2 + 12*s
qrt(c)*sqrt(b*h - c*g + c*h*x)*b*c*e*g*h**3*x + 6*sqrt(c)*sqrt(b*h - c*g +
 c*h*x)*b*c*e*h**4*x**2 + 12*sqrt(c)*sqrt(b*h - c*g + c*h*x)*b*c*f*g**3*h
+ 24*sqrt(c)*sqrt(b*h - c*g + c*h*x)*b*c*f*g**2*h**2*x + 12*sqrt(c)*sqrt(b
*h - c*g + c*h*x)*b*c*f*g*h**3*x**2 - 8*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c*
*2*d*g**2*h**2 - 16*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c**2*d*g*h**3*x - 8*sq
rt(c)*sqrt(b*h - c*g + c*h*x)*c**2*d*h**4*x**2 - 4*sqrt(c)*sqrt(b*h - c*g
+ c*h*x)*c**2*e*g**3*h - 8*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c**2*e*g**2*h**
2*x - 4*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c**2*e*g*h**3*x**2 - 8*sqrt(c)*sqr
t(b*h - c*g + c*h*x)*c**2*f*g**4 - 16*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c**2
*f*g**3*h*x - 8*sqrt(c)*sqrt(b*h - c*g + c*h*x)*c**2*f*g**2*h**2*x**2 - sq
rt(g + h*x)*b**2*c*d*h**4 - 2*sqrt(g + h*x)*b**2*c*e*g*h**3 - 3*sqrt(g + h
*x)*b**2*c*e*h**4*x + 8*sqrt(g + h*x)*b**2*c*f*g**2*h**2 + 12*sqrt(g + h*x
)*b**2*c*f*g*h**3*x + 3*sqrt(g + h*x)*b**2*c*f*h**4*x**2 + 8*sqrt(g + h*x)
*b*c**2*d*g*h**3 + 4*sqrt(g + h*x)*b*c**2*d*h**4*x - 2*sqrt(g + h*x)*b*c**
2*e*g**2*h**2 - 4*sqrt(g + h*x)*b*c**2*e*g*h**3*x - 6*sqrt(g + h*x)*b*c**2
*e*h**4*x**2 - 16*sqrt(g + h*x)*b*c**2*f*g**3*h - 20*sqrt(g + h*x)*b*c**2*
f*g**2*h**2*x - 4*sqrt(g + h*x)*c**3*d*g**2*h**2 + 8*sqrt(g + h*x)*c**3...
```

3.3 $\int (d+ex)^m (-cd^2+bde+be^2x+ce^2x^2)^p ((-cd+be)f$

Optimal result

Integrand size = 69, antiderivative size = 187

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) dx$$
$$= \frac{g(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p}}{ce^2(3+m+2p)}$$
$$+ \frac{(beg(1+m+p)+c(dg(1-m)-ef(3+m+2p)))(d+ex)^{-1+m}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{2+p} \text{H}}{ce^2(2cd-be)(1+m+p)(3+m+2p)}$$

output
```
g*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b*e^2*x+c*e^2*x^2)^(2+p)/c/e^2/(3+m+2*p)+(
b*e*g*(1+m+p)+c*(d*g*(1-m)-e*f*(3+m+2*p)))*(e*x+d)^(-1+m)*(-d*(-b*e+c*d)+b
*e^2*x+c*e^2*x^2)^(2+p)*hypergeom([1, 3+m+2*p],[2+m+p],c*(e*x+d)/(-b*e+2*c
*d))/c/e^2/(-b*e+2*c*d)/(1+m+p)/(3+m+2*p)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.88

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) dx$$
$$= \frac{(d+ex)^m(-cd+be+cex)^2(-((d+ex)(-be+c(d-ex))))^p \left(ceg(d+ex)+\frac{e(cdg(-1+m)-beg(1+m+p)+cef}{}\right.}{c^2e^3(3+m+2p)}$$

input
```
Integrate[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d)
+ b*e)*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

output
```
((d + e*x)^m*(-(c*d) + b*e + c*e*x)^2*(-((d + e*x)*(-(b*e) + c*(d - e*x)))
)^p*(c*e*g*(d + e*x) + (e*(c*d*g*(-1 + m) - b*e*g*(1 + m + p) + c*e*f*(3 +
 m + 2*p))*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*Hypergeometric2F1[-m - p
, 2 + p, 3 + p, (-(c*d) + b*e + c*e*x)/(-2*c*d + b*e)])/(2 + p)))/(c^2*e^3
*(3 + m + 2*p))
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.101$, Rules used = {2163, 27, 1221, 1139, 1138, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m \left(bde+be^2x-cd^2+ce^2x^2\right)^p \left(x(beg-cdg+cef)+f(be-cd)+cegx^2\right) dx$$

$$\Big\downarrow \text{2163}$$

$$de\int \frac{(d+ex)^{m-1}(f+gx)\left(cx^2e^2+bxe^2-d(cd-be)\right)^{p+1}}{de}dx$$

$$\Big\downarrow \text{27}$$

$$\int (f+gx)(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+1}dx$$

$\Big\downarrow$ 1221

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(beg(m+p+1)+cdg(1-m)-cef(m+2p+3))\int(d+ex)^{m-1}\left(cx^2e^2+bxe^2-d(cd-be)\right)^{p+1}dx}{ce(m+2p+3)}$$

$\Big\downarrow$ 1139

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m}(beg(m+p+1)+cdg(1-m)-cef(m+2p+3))\int\left(\frac{ex}{d}+1\right)^{m-1}\left(cx^2e^2+bxe^2-d(cd-\right.}{cde(m+2p+3)}$$

$\Big\downarrow$ 1138

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m\left(\frac{ex}{d}+1\right)^{-m-p}(cdex-d(cd-be))^{-p}\left(-d(cd-be)+be^2x+ce^2x^2\right)^p(beg(m+p+1)+cdg(1-m)-}{cde(m+2p+3)}$$

$\Big\downarrow$ 80

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(cdex-d(cd-be))^{-p}\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+cdg(1-m)-}{cde(m+2p+3)}$$

$\Big\downarrow$ 79

$$\frac{g(d+ex)^{m-1}\left(-d(cd-be)+be^2x+ce^2x^2\right)^{p+2}}{ce^2(m+2p+3)}-\frac{(d+ex)^m(cdex-d(cd-be))^2\left(-d(cd-be)+be^2x+ce^2x^2\right)^p\left(\frac{c(d+ex)}{2cd-be}\right)^{-m-p}(beg(m+p+1)+cdg(1-m)-}{c^2d^2e^2(p+2)(m+2p+3)}$$

input
```
Int[(d + e*x)^m*(-(c*d^2) + b*d*e + b*e^2*x + c*e^2*x^2)^p*((-(c*d) + b*e)
*f + (c*e*f - c*d*g + b*e*g)*x + c*e*g*x^2),x]
```

output
```
(g*(d + e*x)^(-1 + m)*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^(2 + p))/(c
*e^2*(3 + m + 2*p)) - ((c*d*g*(1 - m) + b*e*g*(1 + m + p) - c*e*f*(3 + m +
 2*p))*(d + e*x)^m*((c*(d + e*x))/(2*c*d - b*e))^(-m - p)*(-(d*(c*d - b*e)
) + c*d*e*x)^2*(-(d*(c*d - b*e)) + b*e^2*x + c*e^2*x^2)^p*Hypergeometric2F
1[-m - p, 2 + p, 3 + p, (c*d - b*e - c*e*x)/(2*c*d - b*e)])/(c^2*d^2*e^2*(
2 + p)*(3 + m + 2*p))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 1138
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^m*((a + b*x + c*x^2)^FracPart[p]/((1 + e*(x/d))^FracPart[p]
*(a/d + (c*x)/e)^FracPart[p]))   Int[(1 + e*(x/d))^(m + p)*(a/d + (c/e)*x)^
p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
&& (IntegerQ[m] || GtQ[d, 0]) &&  !(IGtQ[m, 0] && (IntegerQ[3*p] || Integer
Q[4*p]))
```

rule 1139
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[d^IntPart[m]*((d + e*x)^FracPart[m]/(1 + e*(x/d))^FracPart[m]
)   Int[(1 + e*(x/d))^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IntegerQ[m] || GtQ[d, 0])
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

rule 2163
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[d*e   Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*
e + c*d*x, x]*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2
, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Maple [F]

$$\int (ex+d)^m \left(c\,e^2x^2+b\,e^2x+bde-c\,d^2\right)^p \left((be-cd)\,f+(beg-cdg+cef)\,x+ceg\,x^2\right)dx$$

input
```
int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-c*d*g+
c*e*f)*x+c*e*g*x^2),x)
```

output
```
int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-c*d*g+
c*e*f)*x+c*e*g*x^2),x)
```

Fricas [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\, dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-
c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="fricas")
```

output
```
integral((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - (c*d - b*e)*g)*x)*(c*e^2*x^
2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Sympy [F(-2)]

Exception generated.

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f + (cef-cdg+beg)x+cegx^2\right) dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((e*x+d)**m*(c*e**2*x**2+b*e**2*x+b*d*e-c*d**2)**p*((b*e-c*d)*f+(
b*e*g-c*d*g+c*e*f)*x+c*e*g*x**2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right) dx$$
$$= \int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\, dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-
c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="maxima")
```

output
```
integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x
^2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m\left(-cd^2+bde+be^2x+ce^2x^2\right)^p\left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx$$
$$=\int \left(cegx^2-(cd-be)f+(cef-cdg+beg)x\right)\left(ce^2x^2+be^2x-cd^2+bde\right)^p(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-
c*d*g+c*e*f)*x+c*e*g*x^2),x, algorithm="giac")
```

output
```
integrate((c*e*g*x^2 - (c*d - b*e)*f + (c*e*f - c*d*g + b*e*g)*x)*(c*e^2*x
^2 + b*e^2*x - c*d^2 + b*d*e)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\left(-cd^2+bde+be^2x+ce^2x^2\right)^p\left((-cd+be)f+(cef-cdg+beg)x+cegx^2\right)\,dx=\int (d+e\,x)^m\left(c\,e\,g\,x^2+\left(b\,e\,g-c\,d\,g+c\,e\,f\right)\,x+f\,\left(b\,e-c\,d\right)\right)\left(-c\,d^2+b\,d\,e+c\,e^2\,x^2+b\,e^2\,x\right)^p\,dx$$

input
```
int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c
*e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p,x)
```

output
```
int((d + e*x)^m*(f*(b*e - c*d) + x*(b*e*g - c*d*g + c*e*f) + c*e*g*x^2)*(c
*e^2*x^2 - c*d^2 + b*d*e + b*e^2*x)^p, x)
```

Reduce [F]

$$\int (d+ex)^m \left(-cd^2+bde+be^2x+ce^2x^2\right)^p \left((-cd+be)f + (cef-cdg+beg)x+cegx^2\right) dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*e^2*x^2+b*e^2*x+b*d*e-c*d^2)^p*((b*e-c*d)*f+(b*e*g-c*d*g+
c*e*f)*x+c*e*g*x^2),x)
```

output
```
( - (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**3*d*e**3*
g*p**2 - (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**3*d*
e**3*g*p + (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**3*
e**4*g*m*p**2*x + (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**
p*b**3*e**4*g*m*p*x + (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**
2)**p*b**3*e**4*g*p**3*x + (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**
2*x**2)**p*b**3*e**4*g*p**2*x - (d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 +
c*e**2*x**2)**p*b**2*c*d**2*e**2*g*m**2 - 5*(d + e*x)**m*(b*d*e + b*e**2*x
 - c*d**2 + c*e**2*x**2)**p*b**2*c*d**2*e**2*g*m*p - 3*(d + e*x)**m*(b*d*e
 + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d**2*e**2*g*m - (d + e*x)**m
*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d**2*e**2*g*p**2 + (d
 + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d*e**3*f*m*
*3 + 6*(d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d*
e**3*f*m**2*p + 5*(d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**
p*b**2*c*d*e**3*f*m**2 + 12*(d + e*x)**m*(b*d*e + b*e**2*x - c*d**2 + c*e*
*2*x**2)**p*b**2*c*d*e**3*f*m*p**2 + 19*(d + e*x)**m*(b*d*e + b*e**2*x - c
*d**2 + c*e**2*x**2)**p*b**2*c*d*e**3*f*m*p + 6*(d + e*x)**m*(b*d*e + b*e*
*2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d*e**3*f*m + 8*(d + e*x)**m*(b*d*e
+ b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d*e**3*f*p**3 + 18*(d + e*x)*
*m*(b*d*e + b*e**2*x - c*d**2 + c*e**2*x**2)**p*b**2*c*d*e**3*f*p**2 + ...
```

3.4 $\int \frac{(d+ex)^3(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 591

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx =$$

$$-\frac{(b^3e^3h - c^3d(3e^2f + 3deg + d^2h) - bce^2(beg + 3bdh + 2aeh) + c^2e(ae(eg + 3dh) + b(e^2f + 3deg + 3c}{c^4}$$

$$+\frac{e(b^2e^2h + c^2(e^2f + 3deg + 3d^2h) - ce(beg + 3bdh + aeh))\,x^2}{2c^3}$$

$$+\frac{e^2(ceg + 3cdh - beh)x^3}{3c^2} + \frac{e^3hx^4}{4c}$$

$$-\frac{(2c^5d^3f - b^5e^3h + b^3ce^2(beg + 3bdh + 5aeh) - c^4d(bd(3ef + dg) + 2a(3e^2f + 3deg + d^2h)) - bc^2e(5a}{}$$

$$+\frac{(c^4d^2(3ef + dg) + b^4e^3h - b^2ce^2(beg + 3bdh + 3aeh) + c^2e(a^2e^2h + 2abe(eg + 3dh) + b^2(e^2f + 3deg}{2c^5}$$

output
```
-(b^3*e^3*h-c^3*d*(d^2*h+3*d*e*g+3*e^2*f)-b*c*e^2*(2*a*e*h+3*b*d*h+b*e*g)+
c^2*e*(a*e*(3*d*h+e*g)+b*(3*d^2*h+3*d*e*g+e^2*f)))*x/c^4+1/2*e*(b^2*e^2*h+
c^2*(3*d^2*h+3*d*e*g+e^2*f)-c*e*(a*e*h+3*b*d*h+b*e*g))*x^2/c^3+1/3*e^2*(-b
*e*h+3*c*d*h+c*e*g)*x^3/c^2+1/4*e^3*h*x^4/c-(2*c^5*d^3*f-b^5*e^3*h+b^3*c*e
^2*(5*a*e*h+3*b*d*h+b*e*g)-c^4*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h+3*d*e*g+3*e^2
*f))-b*c^2*e*(5*a^2*e^2*h+4*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))
+c^3*(2*a^2*e^2*(3*d*h+e*g)+b^2*d*(d^2*h+3*d*e*g+3*e^2*f)+3*a*b*e*(3*d^2*h
+3*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(
1/2)+1/2*(c^4*d^2*(d*g+3*e*f)+b^4*e^3*h-b^2*c*e^2*(3*a*e*h+3*b*d*h+b*e*g)+
c^2*e*(a^2*e^2*h+2*a*b*e*(3*d*h+e*g)+b^2*(3*d^2*h+3*d*e*g+e^2*f))-c^3*(b*d
*(d^2*h+3*d*e*g+3*e^2*f)+a*e*(3*d^2*h+3*d*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^5
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 585, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2}\,dx$$

$$= \frac{12c(-b^3e^3h + c^3d(3e^2f + 3deg + d^2h) + bce^2(beg + 3bdh + 2aeh) - c^2e(ae(eg + 3dh) + b(e^2f + 3deg +}{}$$

input
```
Integrate[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(12*c*(-(b^3*e^3*h) + c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) + b*c*e^2*(b*e*g +
 3*b*d*h + 2*a*e*h) - c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^
2*h)))*x + 6*c^2*e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e
*g + 3*b*d*h + a*e*h))*x^2 + 4*c^3*e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3 + 3*c
^4*e^3*h*x^4 + (12*(2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*b*d*h +
 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2*h)) -
b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^
2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d^2*h) +
 3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] + 6*(c^4*d^2*(3*e*f + d*g) + b^4*e^3*h - b^2*c*e^2
*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g + 3*d*h) +
b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g + d^2*h) +
a*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + x*(b + c*x)])/(12*c^5)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3\left(f+gx+hx^2\right)}{a+bx+cx^2}dx$$

$$\big\downarrow \text{2159}$$

$$\int \left(\frac{x\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\left(3d^2h+3deg\right.\right.\right.}{}\right.$$

$$\big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^3\left(2a^2e^2(3dh+eg)+3abe\left(3d^2h+3deg+e^2f\right)+b^2d\left(d^2h+3deg+3e^2f\right)\right)-bc^2e\left(5a^2e^2\right.\right.}{}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(c^2e\left(a^2e^2h+2abe(3dh+eg)+b^2\left(3d^2h+3deg+e^2f\right)\right)-b^2ce^2(3aeh+3bdh+beg)-c^3\left(ae\right.\right.}{2c^5}$$
$$\frac{x\left(c^2e\left(ae(3dh+eg)+b\left(3d^2h+3deg+e^2f\right)\right)-bce^2(2aeh+3bdh+beg)+b^3e^3h+c^3(-d)\left(d^2h+3deg+3e^2f\right.\right.}{c^4}$$
$$\frac{ex^2\left(-ce(aeh+3bdh+beg)+b^2e^2h+c^2\left(3d^2h+3deg+e^2f\right)\right)}{2c^3}+\frac{e^2x^3(-beh+3cdh+ceg)}{3c^2}+\frac{e^3hx^4}{4c}$$

input
```
Int[((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
-(((b^3*e^3*h - c^3*d*(3*e^2*f + 3*d*e*g + d^2*h) - b*c*e^2*(b*e*g + 3*b*d
*h + 2*a*e*h) + c^2*e*(a*e*(e*g + 3*d*h) + b*(e^2*f + 3*d*e*g + 3*d^2*h)))
*x)/c^4) + (e*(b^2*e^2*h + c^2*(e^2*f + 3*d*e*g + 3*d^2*h) - c*e*(b*e*g +
3*b*d*h + a*e*h))*x^2)/(2*c^3) + (e^2*(c*e*g + 3*c*d*h - b*e*h)*x^3)/(3*c^
2) + (e^3*h*x^4)/(4*c) - ((2*c^5*d^3*f - b^5*e^3*h + b^3*c*e^2*(b*e*g + 3*
b*d*h + 5*a*e*h) - c^4*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f + 3*d*e*g + d^2
*h)) - b*c^2*e*(5*a^2*e^2*h + 4*a*b*e*(e*g + 3*d*h) + b^2*(e^2*f + 3*d*e*g
 + 3*d^2*h)) + c^3*(2*a^2*e^2*(e*g + 3*d*h) + b^2*d*(3*e^2*f + 3*d*e*g + d
^2*h) + 3*a*b*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2
 - 4*a*c]])/(c^5*Sqrt[b^2 - 4*a*c]) + ((c^4*d^2*(3*e*f + d*g) + b^4*e^3*h
- b^2*c*e^2*(b*e*g + 3*b*d*h + 3*a*e*h) + c^2*e*(a^2*e^2*h + 2*a*b*e*(e*g
+ 3*d*h) + b^2*(e^2*f + 3*d*e*g + 3*d^2*h)) - c^3*(b*d*(3*e^2*f + 3*d*e*g
+ d^2*h) + a*e*(e^2*f + 3*d*e*g + 3*d^2*h)))*Log[a + b*x + c*x^2])/(2*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 869, normalized size of antiderivative = 1.47

method	result
default	$\frac{\frac{1}{4} h e^3 x^4 c^3 - \frac{1}{3} b c^2 e^3 h x^3 + c^3 d e^2 h x^3 + \frac{1}{3} c^3 e^3 g x^3 - \frac{1}{2} a c^2 e^3 h x^2 + \frac{1}{2} b^2 c e^3 h x^2 - \frac{3}{2} b c^2 d e^2 h x^2 - \frac{1}{2} b c^2 e^3 g x^2 + \frac{3}{2} c^3 d^2 e h x^2 + \frac{3}{2} c^3 d e^2 g x}{}$
risch	Expression too large to display

input
```
int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/4*h*e^3*x^4*c^3-1/3*b*c^2*e^3*h*x^3+c^3*d*e^2*h*x^3+1/3*c^3*e^3*g
*x^3-1/2*a*c^2*e^3*h*x^2+1/2*b^2*c*e^3*h*x^2-3/2*b*c^2*d*e^2*h*x^2-1/2*b*c
^2*e^3*g*x^2+3/2*c^3*d^2*e*h*x^2+3/2*c^3*d*e^2*g*x^2+1/2*c^3*e^3*f*x^2+2*x
*a*b*c*e^3*h-3*x*a*c^2*d*e^2*h-x*e^3*c^2*a*g-x*b^3*e^3*h+3*x*b^2*c*d*e^2*h
+x*b^2*c*e^3*g-3*x*b*c^2*d^2*e*h-3*x*b*c^2*d*e^2*g-x*b*c^2*e^3*f+x*c^3*d^3
*h+3*x*e*c^3*g*d^2+3*x*e^2*c^3*d*f)+1/c^4*(1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^
3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^
3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*
d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*
g+3*c^4*d^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-2*a^2*b*c*e^3*h+3*a^2*c^2*d*e^2*h+a
^2*c^2*e^3*g+a*b^3*e^3*h-3*a*b^2*c*d*e^2*h-a*b^2*c*e^3*g+3*a*b*c^2*d^2*e*h
+3*a*b*c^2*d*e^2*g+a*b*c^2*e^3*f-a*c^3*d^3*h-3*a*c^3*d^2*e*g-3*a*c^3*d*e^2
*f+c^4*d^3*f-1/2*(a^2*c^2*e^3*h-3*a*b^2*c*e^3*h+6*a*b*c^2*d*e^2*h+2*a*b*c^
2*e^3*g-3*a*c^3*d^2*e*h-3*a*c^3*d*e^2*g-a*c^3*e^3*f+b^4*e^3*h-3*b^3*c*d*e^
2*h-b^3*c*e^3*g+3*b^2*c^2*d^2*e*h+3*b^2*c^2*d*e^2*g+b^2*c^2*e^3*f-b*c^3*d^
3*h-3*b*c^3*d^2*e*g-3*b*c^3*d*e^2*f+c^4*d^3*g+3*c^4*d^2*e*f)*b/c)/(4*a*c-b
^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 2150, normalized size of antiderivative = 3.64

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*c^4 - 4*a*c^5)*e^3*h*x^4 + 4*((b^2*c^4 - 4*a*c^5)*e^3*g + (3
*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*h)*x^3 + 6*((b^2*c
^4 - 4*a*c^5)*e^3*f + (3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)
*e^3)*g + (3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (
b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*h)*x^2 - 6*sqrt(b^2 - 4*a*c)*((2*c
^5*d^3 - 3*b*c^4*d^2*e + 3*(b^2*c^3 - 2*a*c^4)*d*e^2 - (b^3*c^2 - 3*a*b*c^
3)*e^3)*f - (b*c^4*d^3 - 3*(b^2*c^3 - 2*a*c^4)*d^2*e + 3*(b^3*c^2 - 3*a*b*
c^3)*d*e^2 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^3)*g + ((b^2*c^3 - 2*a*c^
4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^
3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3)*h)*log((2*c^2*x^2 + 2*b*c*
x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 12*(
(3*(b^2*c^4 - 4*a*c^5)*d*e^2 - (b^3*c^3 - 4*a*b*c^4)*e^3)*f + (3*(b^2*c^4
- 4*a*c^5)*d^2*e - 3*(b^3*c^3 - 4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3
+ 4*a^2*c^4)*e^3)*g + ((b^2*c^4 - 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d
^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2
+ 8*a^2*b*c^3)*e^3)*h)*x + 6*((3*(b^2*c^4 - 4*a*c^5)*d^2*e - 3*(b^3*c^3 -
4*a*b*c^4)*d*e^2 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e^3)*f + ((b^2*c^4
- 4*a*c^5)*d^3 - 3*(b^3*c^3 - 4*a*b*c^4)*d^2*e + 3*(b^4*c^2 - 5*a*b^2*c^3
+ 4*a^2*c^4)*d*e^2 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^3)*g - ((b^3*c^
3 - 4*a*b*c^4)*d^3 - 3*(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d^2*e + 3*(b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4972 vs. $2(619) = 1238$.

Time = 50.21 (sec) , antiderivative size = 4972, normalized size of antiderivative = 8.41

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x**3*(-b*e**3*h/(3*c**2) + d*e**2*h/c + e**3*g/(3*c)) + x**2*(-a*e**3*h/(2
*c**2) + b**2*e**3*h/(2*c**3) - 3*b*d*e**2*h/(2*c**2) - b*e**3*g/(2*c**2)
+ 3*d**2*e*h/(2*c) + 3*d*e**2*g/(2*c) + e**3*f/(2*c)) + x*(2*a*b*e**3*h/c*
*3 - 3*a*d*e**2*h/c**2 - a*e**3*g/c**2 - b**3*e**3*h/c**4 + 3*b**2*d*e**2*
h/c**3 + b**2*e**3*g/c**3 - 3*b*d**2*e*h/c**2 - 3*b*d*e**2*g/c**2 - b*e**3
*f/c**2 + d**3*h/c + 3*d**2*e*g/c + 3*d*e**2*f/c) + (-sqrt(-4*a*c + b**2)*
(5*a**2*b*c**2*e**3*h - 6*a**2*c**3*d*e**2*h - 2*a**2*c**3*e**3*g - 5*a*b*
*3*c*e**3*h + 12*a*b**2*c**2*d*e**2*h + 4*a*b**2*c**2*e**3*g - 9*a*b*c**3*
d**2*e*h - 9*a*b*c**3*d*e**2*g - 3*a*b*c**3*e**3*f + 2*a*c**4*d**3*h + 6*a
*c**4*d**2*e*g + 6*a*c**4*d*e**2*f + b**5*e**3*h - 3*b**4*c*d*e**2*h - b**
4*c*e**3*g + 3*b**3*c**2*d**2*e*h + 3*b**3*c**2*d*e**2*g + b**3*c**2*e**3*
f - b**2*c**3*d**3*h - 3*b**2*c**3*d**2*e*g - 3*b**2*c**3*d*e**2*f + b*c**
4*d**3*g + 3*b*c**4*d**2*e*f - 2*c**5*d**3*f)/(2*c**5*(4*a*c - b**2)) + (a
**2*c**2*e**3*h - 3*a*b**2*c*e**3*h + 6*a*b*c**2*d*e**2*h + 2*a*b*c**2*e**
3*g - 3*a*c**3*d**2*e*h - 3*a*c**3*d*e**2*g - a*c**3*e**3*f + b**4*e**3*h
- 3*b**3*c*d*e**2*h - b**3*c*e**3*g + 3*b**2*c**2*d**2*e*h + 3*b**2*c**2*d
*e**2*g + b**2*c**2*e**3*f - b*c**3*d**3*h - 3*b*c**3*d**2*e*g - 3*b*c**3*
d*e**2*f + c**4*d**3*g + 3*c**4*d**2*e*f)/(2*c**5))*log(x + (2*a**3*c**2*e
**3*h - 4*a**2*b**2*c*e**3*h + 9*a**2*b*c**2*d*e**2*h + 3*a**2*b*c**2*e**3
*g - 6*a**2*c**3*d**2*e*h - 6*a**2*c**3*d*e**2*g - 2*a**2*c**3*e**3*f +...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^3\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$
$$= \frac{3 c^3e^3hx^4 + 4 c^3e^3gx^3 + 12 c^3de^2hx^3 - 4 bc^2e^3hx^3 + 6 c^3e^3fx^2 + 18 c^3de^2gx^2 - 6 bc^2e^3gx^2 + 18 c^3d^2ehx^2}{}$$
$$+ \frac{(3 c^4d^2ef - 3 bc^3de^2f + b^2c^2e^3f - ac^3e^3f + c^4d^3g - 3 bc^3d^2eg + 3 b^2c^2de^2g - 3 ac^3de^2g - b^3ce^3g + 2 a}{}$$
$$+ \frac{(2 c^5d^3f - 3 bc^4d^2ef + 3 b^2c^3de^2f - 6 ac^4de^2f - b^3c^2e^3f + 3 abc^3e^3f - bc^4d^3g + 3 b^2c^3d^2eg - 6 ac^4d^2e}{}$$

input
```
integrate((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/12*(3*c^3*e^3*h*x^4 + 4*c^3*e^3*g*x^3 + 12*c^3*d*e^2*h*x^3 - 4*b*c^2*e^3
*h*x^3 + 6*c^3*e^3*f*x^2 + 18*c^3*d*e^2*g*x^2 - 6*b*c^2*e^3*g*x^2 + 18*c^3
*d^2*e*h*x^2 - 18*b*c^2*d*e^2*h*x^2 + 6*b^2*c*e^3*h*x^2 - 6*a*c^2*e^3*h*x^
2 + 36*c^3*d*e^2*f*x - 12*b*c^2*e^3*f*x + 36*c^3*d^2*e*g*x - 36*b*c^2*d*e^
2*g*x + 12*b^2*c*e^3*g*x - 12*a*c^2*e^3*g*x + 12*c^3*d^3*h*x - 36*b*c^2*d^
2*e*h*x + 36*b^2*c*d*e^2*h*x - 36*a*c^2*d*e^2*h*x - 12*b^3*e^3*h*x + 24*a*
b*c*e^3*h*x)/c^4 + 1/2*(3*c^4*d^2*e*f - 3*b*c^3*d*e^2*f + b^2*c^2*e^3*f -
a*c^3*e^3*f + c^4*d^3*g - 3*b*c^3*d^2*e*g + 3*b^2*c^2*d*e^2*g - 3*a*c^3*d*
e^2*g - b^3*c*e^3*g + 2*a*b*c^2*e^3*g - b*c^3*d^3*h + 3*b^2*c^2*d^2*e*h -
3*a*c^3*d^2*e*h - 3*b^3*c*d*e^2*h + 6*a*b*c^2*d*e^2*h + b^4*e^3*h - 3*a*b^
2*c*e^3*h + a^2*c^2*e^3*h)*log(c*x^2 + b*x + a)/c^5 + (2*c^5*d^3*f - 3*b*c
^4*d^2*e*f + 3*b^2*c^3*d*e^2*f - 6*a*c^4*d*e^2*f - b^3*c^2*e^3*f + 3*a*b*c
^3*e^3*f - b*c^4*d^3*g + 3*b^2*c^3*d^2*e*g - 6*a*c^4*d^2*e*g - 3*b^3*c^2*d
*e^2*g + 9*a*b*c^3*d*e^2*g + b^4*c*e^3*g - 4*a*b^2*c^2*e^3*g + 2*a^2*c^3*e
^3*g + b^2*c^3*d^3*h - 2*a*c^4*d^3*h - 3*b^3*c^2*d^2*e*h + 9*a*b*c^3*d^2*e
*h + 3*b^4*c*d*e^2*h - 12*a*b^2*c^2*d*e^2*h + 6*a^2*c^3*d*e^2*h - b^5*e^3*
h + 5*a*b^3*c*e^3*h - 5*a^2*b*c^2*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

Mupad [B] (verification not implemented)

Time = 19.72 (sec) , antiderivative size = 967, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx$$

$$= x^3 \left(\frac{g\,e^3+3\,d\,h\,e^2}{3\,c}-\frac{b\,e^3\,h}{3\,c^2}\right)+x\left(\frac{h\,d^3+3\,g\,d^2\,e+3\,f\,d\,e^2}{c}+\frac{b\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{c}+\frac{a\,e^3\,h}{c^2}\right)}{c}-\frac{a\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{c}\right)$$
$$-x^2\left(\frac{b\left(\frac{g\,e^3+3\,d\,h\,e^2}{c}-\frac{b\,e^3\,h}{c^2}\right)}{2\,c}-\frac{3\,h\,d^2\,e+3\,g\,d\,e^2+f\,e^3}{2\,c}+\frac{a\,e^3\,h}{2\,c^2}\right)$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-4\,h\,a^3\,c^3\,e^3+13\,h\,a^2\,b^2\,c^2\,e^3-24\,h\,a^2\,b\,c^3\,d\,e^2-8\,g\,a^2\,b\,c^3\,e^3+12\,h\,a^2\,c^4\,d^2\,e+12\right.}{}$$
$$+\frac{e^3\,h\,x^4}{4\,c}$$
$$+\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(-5\,h\,a^2\,b\,c^2\,e^3+6\,h\,a^2\,c^3\,d\,e^2+2\,g\,a^2\,c^3\,e^3+5\,h\,a\,b^3\,c\,e^3-12\,h\,a\,b^2\,c^2\,d\,e^2\right.}{}$$

input

```
int(((d + e*x)^3*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x^3*((e^3*g + 3*d*e^2*h)/(3*c) - (b*e^3*h)/(3*c^2)) + x*((d^3*h + 3*d*e^2*
f + 3*d^2*e*g)/c + (b*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2))/c - (e^
3*f + 3*d*e^2*g + 3*d^2*e*h)/c + (a*e^3*h)/c^2))/c - (a*((e^3*g + 3*d*e^2*
h)/c - (b*e^3*h)/c^2))/c) - x^2*((b*((e^3*g + 3*d*e^2*h)/c - (b*e^3*h)/c^2
))/(2*c) - (e^3*f + 3*d*e^2*g + 3*d^2*e*h)/(2*c) + (a*e^3*h)/(2*c^2)) - (l
og(a + b*x + c*x^2)*(b^6*e^3*h + 4*a^2*c^4*e^3*f + b^2*c^4*d^3*g + b^4*c^2
*e^3*f - 4*a^3*c^3*e^3*h - b^3*c^3*d^3*h - 4*a*c^5*d^3*g - b^5*c*e^3*g + 4
*a*b*c^4*d^3*h - 7*a*b^4*c*e^3*h - 12*a*c^5*d^2*e*f - 3*b^5*c*d*e^2*h - 5*
a*b^2*c^3*e^3*f + 6*a*b^3*c^2*e^3*g - 8*a^2*b*c^3*e^3*g + 12*a^2*c^4*d*e^2
*g + 3*b^2*c^4*d^2*e*f - 3*b^3*c^3*d*e^2*f + 12*a^2*c^4*d^2*e*h - 3*b^3*c^
3*d^2*e*g + 3*b^4*c^2*d*e^2*g + 3*b^4*c^2*d^2*e*h + 13*a^2*b^2*c^2*e^3*h +
 12*a*b*c^4*d*e^2*f + 12*a*b*c^4*d^2*e*g - 15*a*b^2*c^3*d*e^2*g - 15*a*b^2
*c^3*d^2*e*h + 18*a*b^3*c^2*d*e^2*h - 24*a^2*b*c^3*d*e^2*h))/(2*(4*a*c^6 -
 b^2*c^5)) + (e^3*h*x^4)/(4*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*
a*c - b^2)^(1/2))*(2*c^5*d^3*f - b^5*e^3*h + 2*a^2*c^3*e^3*g - b^3*c^2*e^3
*f + b^2*c^3*d^3*h - 2*a*c^4*d^3*h - b*c^4*d^3*g + b^4*c*e^3*g + 3*a*b*c^3
*e^3*f + 5*a*b^3*c*e^3*h - 6*a*c^4*d*e^2*f - 6*a*c^4*d^2*e*g - 3*b*c^4*d^2
*e*f + 3*b^4*c*d*e^2*h - 4*a*b^2*c^2*e^3*g - 5*a^2*b*c^2*e^3*h + 3*b^2*c^3
*d*e^2*f + 6*a^2*c^3*d*e^2*h + 3*b^2*c^3*d^2*e*g - 3*b^3*c^2*d*e^2*g - 3*b
^3*c^2*d^2*e*h + 9*a*b*c^3*d*e^2*g + 9*a*b*c^3*d^2*e*h - 12*a*b^2*c^2*d...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2328, normalized size of antiderivative = 3.94

$$\int \frac{(d+ex)^3 (f+gx+hx^2)}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

output
```
( - 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2
*e**3*h + 72*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*
c**3*d*e**2*h + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*c**3*e**3*g + 60*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a*b**3*c*e**3*h - 144*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a*b**2*c**2*d*e**2*h - 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a*b**2*c**2*e**3*g + 108*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*a*b*c**3*d**2*e*h + 108*sqrt(4*a*c - b**2)*atan((b
 + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*d*e**2*g + 36*sqrt(4*a*c - b**2)*at
an((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*e**3*f - 24*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**4*d**3*h - 72*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**4*d**2*e*g - 72*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**4*d*e**2*f - 12*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**5*e**3*h + 36*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*d*e**2*h + 12*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*c*e**3*g - 36*sqrt(4*a
*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*d**2*e*h - 36*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*d*e**2*g -
 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c**2*e**3
*f + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*...
```

3.5 $\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 348

$$\begin{aligned}
&\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx \\
&= \frac{(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x}{c^3} \\
&\quad+\frac{e(ceg+2cdh-beh)x^2}{2c^2}+\frac{e^2hx^3}{3c} \\
&\quad-\frac{(2c^4d^2f+b^4e^2h-b^2ce(beg+2bdh+4aeh)-c^3(bd(2ef+dg)+2a(e^2f+2deg+d^2h))+c^2(2a^2e^2h}{c^4\sqrt{b^2-4ac}} \\
&\quad+\frac{(c^3d(2ef+dg)-b^3e^2h+bce(beg+2bdh+2aeh)-c^2(ae(eg+2dh)+b(e^2f+2deg+d^2h)))\log(a}{2c^4}
\end{aligned}$$

output
```
(b^2*e^2*h+c^2*(d^2*h+2*d*e*g+e^2*f)-c*e*(a*e*h+2*b*d*h+b*e*g))*x/c^3+1/2*
e*(-b*e*h+2*c*d*h+c*e*g)*x^2/c^2+1/3*e^2*h*x^3/c-(2*c^4*d^2*f+b^4*e^2*h-b^
2*c*e*(4*a*e*h+2*b*d*h+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a*(d^2*h+2*d*e*g+e^2*
f))+c^2*(2*a^2*e^2*h+3*a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*d*e*g+e^2*f)))*arcta
nh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)+1/2*(c^3*d*(d*g+2*
e*f)-b^3*e^2*h+b*c*e*(2*a*e*h+2*b*d*h+b*e*g)-c^2*(a*e*(2*d*h+e*g)+b*(d^2*h
+2*d*e*g+e^2*f)))*ln(c*x^2+b*x+a)/c^4
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$$

$$= \frac{6c(b^2e^2h+c^2(e^2f+2deg+d^2h)-ce(beg+2bdh+aeh))\,x+3c^2e(ceg+2cdh-beh)x^2+2c^3e^2hx^3+\cdots}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(6*c*(b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a
*e*h))*x + 3*c^2*e*(c*e*g + 2*c*d*h - b*e*h)*x^2 + 2*c^3*e^2*h*x^3 + (6*(2
*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b*d*(2
*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a*b*e*
(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 3*(c^3*d*(2*e*f + d*g) - b^3*e^2*h + b*
c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2*f + 2*d*
e*g + d^2*h)))*Log[a + x*(b + c*x)])/(6*c^4)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-ce(aeh+2bdh+beg)+b^2e^2h+c^2(d^2h+2deg+e^2f)}{c^3}+\frac{x\left(-c^2\left(ae(2dh+eg)+b\left(d^2h+2deg+e^2f\right)\right)+\cdots\right.}{}\right.$$

$\downarrow$ 2009

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(c^2\left(2a^2e^2h+3abe(2dh+eg)+b^2\left(d^2h+2deg+e^2f\right)\right)-b^2ce(4aeh+2bdh+beg)-c^3(2a}{c^4\sqrt{b^2-4ac}}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(-c^2\left(ae(2dh+eg)+b\left(d^2h+2deg+e^2f\right)\right)+bce(2aeh+2bdh+beg)+b^3\left(-e^2\right)h+c^3d(dg}{2c^4}$$
$$\frac{x\left(-ce(aeh+2bdh+beg)+b^2e^2h+c^2\left(d^2h+2deg+e^2f\right)\right)}{c^3}+\frac{ex^2(-beh+2cdh+ceg)}{2c^2}+\frac{e^2hx^3}{3c}$$

input
```
Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
((b^2*e^2*h + c^2*(e^2*f + 2*d*e*g + d^2*h) - c*e*(b*e*g + 2*b*d*h + a*e*h
))*x)/c^3 + (e*(c*e*g + 2*c*d*h - b*e*h)*x^2)/(2*c^2) + (e^2*h*x^3)/(3*c)
 - ((2*c^4*d^2*f + b^4*e^2*h - b^2*c*e*(b*e*g + 2*b*d*h + 4*a*e*h) - c^3*(b
*d*(2*e*f + d*g) + 2*a*(e^2*f + 2*d*e*g + d^2*h)) + c^2*(2*a^2*e^2*h + 3*a
*b*e*(e*g + 2*d*h) + b^2*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/S
qrt[b^2 - 4*a*c]])/(c^4*Sqrt[b^2 - 4*a*c]) + ((c^3*d*(2*e*f + d*g) - b^3*e
^2*h + b*c*e*(b*e*g + 2*b*d*h + 2*a*e*h) - c^2*(a*e*(e*g + 2*d*h) + b*(e^2
*f + 2*d*e*g + d^2*h)))*Log[a + b*x + c*x^2])/(2*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.30

method	result
default	$-\frac{-\frac{1}{3}h\,e^2x^3c^2+\frac{1}{2}bc\,e^2h\,x^2-c^2deh\,x^2-\frac{1}{2}c^2e^2g\,x^2+xac\,e^2h-x\,b^2e^2h+2xbcdeh+xbc\,e^2g-x\,c^2d^2h-2x\,c^2egd-x\,e^2c^2f}{c^3}+\frac{\left(2abc\,e^2h\right.}{}$
risch	Expression too large to display

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/3*h*e^2*x^3*c^2+1/2*b*c*e^2*h*x^2-c^2*d*e*h*x^2-1/2*c^2*e^2*g*x
^2+x*a*c*e^2*h-x*b^2*e^2*h+2*x*b*c*d*e*h+x*b*c*e^2*g-x*c^2*d^2*h-2*x*c^2*e
*g*d-x*e^2*c^2*f)+1/c^3*(1/2*(2*a*b*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*
e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*
d^2*g+2*c^3*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*c*e^2*h-a*b^2*e^2*h+2*a*b*c*d*
e*h+a*b*c*e^2*g-a*c^2*d^2*h-2*a*c^2*d*e*g-a*c^2*e^2*f+c^3*d^2*f-1/2*(2*a*b
*c*e^2*h-2*a*c^2*d*e*h-a*c^2*e^2*g-b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g-b*c
^2*d^2*h-2*b*c^2*d*e*g-b*c^2*e^2*f+c^3*d^2*g+2*c^3*d*e*f)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1273, normalized size of antiderivative = 3.66

$$\int \frac{(d+ex)^2\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a*c^4)*e^2*g + (2*
(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x^2 + 3*sqrt(b^2 -
 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*e^2)*f - (b*c^3*d^
2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)*g + ((b^2*c^2 - 2
*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^
2)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x +
b))/(c*x^2 + b*x + a)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*
c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*g + ((b^2*c^3 - 4*a*c^4)*d^2 - 2*(b^
3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^2)*h)*x + 3*(
(2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*f + ((b^2*c^3 - 4*
a*c^4)*d^2 - 2*(b^3*c^2 - 4*a*b*c^3)*d*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^
3)*e^2)*g - ((b^3*c^2 - 4*a*b*c^3)*d^2 - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^
3)*d*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*h)*log(c*x^2 + b*x + a))/(b^
2*c^4 - 4*a*c^5), 1/6*(2*(b^2*c^3 - 4*a*c^4)*e^2*h*x^3 + 3*((b^2*c^3 - 4*a
*c^4)*e^2*g + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*c^2 - 4*a*b*c^3)*e^2)*h)*x
^2 - 6*sqrt(-b^2 + 4*a*c)*((2*c^4*d^2 - 2*b*c^3*d*e + (b^2*c^2 - 2*a*c^3)*
e^2)*f - (b*c^3*d^2 - 2*(b^2*c^2 - 2*a*c^3)*d*e + (b^3*c - 3*a*b*c^2)*e^2)
*g + ((b^2*c^2 - 2*a*c^3)*d^2 - 2*(b^3*c - 3*a*b*c^2)*d*e + (b^4 - 4*a*b^2
*c + 2*a^2*c^2)*e^2)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*
c)) + 6*((b^2*c^3 - 4*a*c^4)*e^2*f + (2*(b^2*c^3 - 4*a*c^4)*d*e - (b^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2839 vs. $2(359) = 718$.

Time = 20.80 (sec) , antiderivative size = 2839, normalized size of antiderivative = 8.16

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x**2*(-b*e**2*h/(2*c**2) + d*e*h/c + e**2*g/(2*c)) + x*(-a*e**2*h/c**2 + b
**2*e**2*h/c**3 - 2*b*d*e*h/c**2 - b*e**2*g/c**2 + d**2*h/c + 2*d*e*g/c +
e**2*f/c) + (-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h
+ 6*a*b*c**2*d*e*h + 3*a*b*c**2*e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g
- 2*a*c**3*e**2*f + b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c*
*2*d**2*h + 2*b**2*c**2*d*e*g + b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**
3*d*e*f + 2*c**4*d**2*f)/(2*c**4*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c
**2*d*e*h - a*c**2*e**2*g - b**3*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g -
 b*c**2*d**2*h - 2*b*c**2*d*e*g - b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e
*f)/(2*c**4))*log(x + (-3*a**2*b*c*e**2*h + 4*a**2*c**2*d*e*h + 2*a**2*c**
2*e**2*g + a*b**3*e**2*h - 2*a*b**2*c*d*e*h - a*b**2*c*e**2*g + a*b*c**2*d
**2*h + 2*a*b*c**2*d*e*g + a*b*c**2*e**2*f + 4*a*c**4*(-sqrt(-4*a*c + b**2
)*(2*a**2*c**2*e**2*h - 4*a*b**2*c*e**2*h + 6*a*b*c**2*d*e*h + 3*a*b*c**2*
e**2*g - 2*a*c**3*d**2*h - 4*a*c**3*d*e*g - 2*a*c**3*e**2*f + b**4*e**2*h
- 2*b**3*c*d*e*h - b**3*c*e**2*g + b**2*c**2*d**2*h + 2*b**2*c**2*d*e*g +
b**2*c**2*e**2*f - b*c**3*d**2*g - 2*b*c**3*d*e*f + 2*c**4*d**2*f)/(2*c**4
*(4*a*c - b**2)) + (2*a*b*c*e**2*h - 2*a*c**2*d*e*h - a*c**2*e**2*g - b**3
*e**2*h + 2*b**2*c*d*e*h + b**2*c*e**2*g - b*c**2*d**2*h - 2*b*c**2*d*e*g
- b*c**2*e**2*f + c**3*d**2*g + 2*c**3*d*e*f)/(2*c**4)) - 2*a*c**3*d**2*g
- 4*a*c**3*d*e*f - b**2*c**3*(-sqrt(-4*a*c + b**2)*(2*a**2*c**2*e**2*h ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$= \frac{2\,c^2e^2hx^3 + 3\,c^2e^2gx^2 + 6\,c^2dehx^2 - 3\,bce^2hx^2 + 6\,c^2e^2fx + 12\,c^2degx - 6\,bce^2gx + 6\,c^2d^2hx - 12\,bcdeh}{6\,c^3}$$
$$+ \frac{(2\,c^3def - bc^2e^2f + c^3d^2g - 2\,bc^2deg + b^2ce^2g - ac^2e^2g - bc^2d^2h + 2\,b^2cdeh - 2\,ac^2deh - b^3e^2h + 2\,a}{2\,c^4}$$
$$+ \frac{(2\,c^4d^2f - 2\,bc^3def + b^2c^2e^2f - 2\,ac^3e^2f - bc^3d^2g + 2\,b^2c^2deg - 4\,ac^3deg - b^3ce^2g + 3\,abc^2e^2g + b^2c^2}{\sqrt{-b^2 + 4\,ac}}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/6*(2*c^2*e^2*h*x^3 + 3*c^2*e^2*g*x^2 + 6*c^2*d*e*h*x^2 - 3*b*c*e^2*h*x^2
 + 6*c^2*e^2*f*x + 12*c^2*d*e*g*x - 6*b*c*e^2*g*x + 6*c^2*d^2*h*x - 12*b*c
*d*e*h*x + 6*b^2*e^2*h*x - 6*a*c*e^2*h*x)/c^3 + 1/2*(2*c^3*d*e*f - b*c^2*e
^2*f + c^3*d^2*g - 2*b*c^2*d*e*g + b^2*c*e^2*g - a*c^2*e^2*g - b*c^2*d^2*h
 + 2*b^2*c*d*e*h - 2*a*c^2*d*e*h - b^3*e^2*h + 2*a*b*c*e^2*h)*log(c*x^2 +
b*x + a)/c^4 + (2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*
f - b*c^3*d^2*g + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^
2*e^2*g + b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h
+ b^4*e^2*h - 4*a*b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*arctan((2*c*x + b)/sqrt(-
b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = x^2\left(\frac{g\,e^2+2\,d\,h\,e}{2\,c}-\frac{b\,e^2\,h}{2\,c^2}\right)$$
$$-x\left(\frac{b\left(\frac{g\,e^2+2\,d\,h\,e}{c}-\frac{b\,e^2\,h}{c^2}\right)}{c}-\frac{h\,d^2+2\,g\,d\,e+f\,e^2}{c}+\frac{a\,e^2\,h}{c^2}\right)$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\left(-8\,h\,a^2\,b\,c^2\,e^2+8\,h\,a^2\,c^3\,d\,e+4\,g\,a^2\,c^3\,e^2+6\,h\,a\,b^3\,c\,e^2-10\,h\,a\,b^2\,c^2\,d\,e-5\,g\,a\,b^2\,c\right.}{}$$
$$+\frac{e^2\,h\,x^3}{3\,c}$$
$$+\frac{\operatorname{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(2\,h\,a^2\,c^2\,e^2-4\,h\,a\,b^2\,c\,e^2+6\,h\,a\,b\,c^2\,d\,e+3\,g\,a\,b\,c^2\,e^2-2\,h\,a\,c^3\,d^2-4\,g\,a\,c\right.}{}$$

input
```
int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x^2*((e^2*g + 2*d*e*h)/(2*c) - (b*e^2*h)/(2*c^2)) - x*((b*((e^2*g + 2*d*e*
h)/c - (b*e^2*h)/c^2))/c - (e^2*f + d^2*h + 2*d*e*g)/c + (a*e^2*h)/c^2) -
(log(a + b*x + c*x^2)*(4*a^2*c^3*e^2*g - b^5*e^2*h + b^2*c^3*d^2*g - b^3*c
^2*e^2*f - b^3*c^2*d^2*h - 4*a*c^4*d^2*g + b^4*c*e^2*g + 4*a*b*c^3*e^2*f +
 4*a*b*c^3*d^2*h + 6*a*b^3*c*e^2*h + 2*b^2*c^3*d*e*f + 8*a^2*c^3*d*e*h - 2
*b^3*c^2*d*e*g - 5*a*b^2*c^2*e^2*g - 8*a^2*b*c^2*e^2*h - 8*a*c^4*d*e*f + 2
*b^4*c*d*e*h + 8*a*b*c^3*d*e*g - 10*a*b^2*c^2*d*e*h))/(2*(4*a*c^5 - b^2*c^
4)) + (e^2*h*x^3)/(3*c) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b
^2)^(1/2))*(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^
2*c^2*d^2*h - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g +
3*a*b*c^2*e^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*
c^3*d*e*f - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c^4*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1379, normalized size of antiderivative = 3.96

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{a+bx+cx^2} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

output
```
(12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*e**2
*h - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*e
**2*h + 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**
2*d*e*h + 18*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c
**2*e**2*g - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*
c**3*d**2*h - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*c**3*d*e*g - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*c**3*e**2*f + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b
**4*e**2*h - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*
*3*c*d*e*h - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**
3*c*e**2*g + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**
2*c**2*d**2*h + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b**2*c**2*d*e*g + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**2*c**2*e**2*f - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b
**2))*b*c**3*d**2*g - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*b*c**3*d*e*f + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*c**4*d**2*f + 24*log(a + b*x + c*x**2)*a**2*b*c**2*e**2*h - 24*log(
a + b*x + c*x**2)*a**2*c**3*d*e*h - 12*log(a + b*x + c*x**2)*a**2*c**3*e**
2*g - 18*log(a + b*x + c*x**2)*a*b**3*c*e**2*h + 30*log(a + b*x + c*x**2)*
a*b**2*c**2*d*e*h + 15*log(a + b*x + c*x**2)*a*b**2*c**2*e**2*g - 12*lo...
```

3.6 $\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx = \frac{(ceg+cdh-beh)x}{c^2}+\frac{ehx^2}{2c} - \frac{\left(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)+bc(beg+bdh+3aeh)\right)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}} + \frac{\left(c^2(ef+dg)+b^2eh-c(beg+bdh+aeh)\right)\log\left(a+bx+cx^2\right)}{2c^3}$$

output
```
(-b*e*h+c*d*h+c*e*g)*x/c^2+1/2*e*h*x^2/c-(2*c^3*d*f-b^3*e*h-c^2*(2*a*d*h+2
*a*e*g+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*arctanh((2*c*x+b)/(-4*a*c+b
^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)+1/2*(c^2*(d*g+e*f)+b^2*e*h-c*(a*e*h+b*d*
h+b*e*g))*ln(c*x^2+b*x+a)/c^3
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$
$$= \frac{2c(ceg+cdh-beh)x+c^2ehx^2-\frac{2\left(-2c^3df+b^3eh+c^2(bef+bdg+2aeg+2adh)-bc(beg+bdh+3aeh)\right)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(c^2}{2c^3}$$

input
```
Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
(2*c*(c*e*g + c*d*h - b*e*h)*x + c^2*e*h*x^2 - (2*(-2*c^3*d*f + b^3*e*h +
c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) - b*c*(b*e*g + b*d*h + 3*a*e*h))*A
rcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c^2*(e*f + d*
g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + x*(b + c*x)])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)+abeh-ac(dh+eg)+c^2df}{c^2\left(a+bx+cx^2\right)}+\frac{-beh+cdh+ceg}{c^2}+\frac{ehx}{c}\right)d$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c^2(2adh+2aeg+bdg+bef)+bc(3aeh+bdh+beg)+b^3(-e)h+2c^3df\right)}{c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)\left(-c(aeh+bdh+beg)+b^2eh+c^2(dg+ef)\right)}{2c^3}+\frac{x(-beh+cdh+ceg)}{c^2}+\frac{ehx^2}{2c}$$

input
```
Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x]
```

output
```
((c*e*g + c*d*h - b*e*h)*x)/c^2 + (e*h*x^2)/(2*c) - ((2*c^3*d*f - b^3*e*h
- c^2*(b*e*f + b*d*g + 2*a*e*g + 2*a*d*h) + b*c*(b*e*g + b*d*h + 3*a*e*h))
*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((c^2*(
e*f + d*g) + b^2*e*h - c*(b*e*g + b*d*h + a*e*h))*Log[a + b*x + c*x^2])/(2
*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.08

method	result
default	$-\frac{-\frac{1}{2}eh\,x^2c+xbeh-xcdh-cegx}{c^2}+\frac{\frac{\left(-aceh+b^2eh-bcdh-bceg+c^2dg+c^2ef\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(abeh-acdh-aceg+c^2df-\frac{\left(-aceh+b^2eh\cdots\right.}{}\right.}{\sqrt{\cdots}}}{c^2}$
risch	Expression too large to display

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*e*h*x^2*c+x*b*e*h-x*c*d*h-c*e*g*x)+1/c^2*(1/2*(-a*c*e*h+b^2*e
*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)/c*ln(c*x^2+b*x+a)+2*(a*b*e*h-a*c*d*h-a
*c*e*g+c^2*d*f-1/2*(-a*c*e*h+b^2*e*h-b*c*d*h-b*c*e*g+c^2*d*g+c^2*e*f)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.69

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{a+bx+cx^2}\,dx$$
$$= \left[\frac{(b^2c^2-4\,ac^3)ehx^2+\sqrt{b^2-4\,ac}((2\,c^3d-bc^2e)f-(bc^2d-(b^2c-2\,ac^2)e)g+((b^2c-2\,ac^2)d-(b^3-3\right.$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 + sqrt(b^2 - 4*a*c)*((2*c^3*d - b*c^2*e)
*f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 - 3*a
*b*c)*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + 2*((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*
a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*
c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^
4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)
, 1/2*((b^2*c^2 - 4*a*c^3)*e*h*x^2 - 2*sqrt(-b^2 + 4*a*c)*((2*c^3*d - b*c^
2*e)*f - (b*c^2*d - (b^2*c - 2*a*c^2)*e)*g + ((b^2*c - 2*a*c^2)*d - (b^3 -
 3*a*b*c)*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*
((b^2*c^2 - 4*a*c^3)*e*g + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)
*h)*x + ((b^2*c^2 - 4*a*c^3)*e*f + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*g - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e)*h)*
log(c*x^2 + b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. $2(182) = 364$.

Time = 6.44 (sec) , antiderivative size = 1265, normalized size of antiderivative = 7.15

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
x*(-b*e*h/c**2 + d*h/c + e*g/c) + (-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2*a
*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d*g
 - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*h
+ b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a**2*c*e*h
 - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(-sqrt(-4*a*c + b**2)*(3*
a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e
*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*
e*h - b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) - 2*a*
c**2*d*g - 2*a*c**2*e*f - b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e*h - 2
*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b*c**2*d
*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h - b**2*e*
h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3)) + b*c**2*d*f)/(3*a*
b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g
 - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)) + (sqrt(-4*a*c + b**2)*(3*a*b*c*
e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h + b**2*c*e*g - b
*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**2)) - (a*c*e*h -
b**2*e*h + b*c*d*h + b*c*e*g - c**2*d*g - c**2*e*f)/(2*c**3))*log(x + (2*a
**2*c*e*h - a*b**2*e*h + a*b*c*d*h + a*b*c*e*g + 4*a*c**3*(sqrt(-4*a*c + b
**2)*(3*a*b*c*e*h - 2*a*c**2*d*h - 2*a*c**2*e*g - b**3*e*h + b**2*c*d*h +
b**2*c*e*g - b*c**2*d*g - b*c**2*e*f + 2*c**3*d*f)/(2*c**3*(4*a*c - b**...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)\,(f+gx+hx^2)}{a+bx+cx^2}\,dx = \frac{cehx^2+2\,cegx+2\,cdhx-2\,behx}{2\,c^2} + \frac{(c^2ef+c^2dg-bceg-bcdh+b^2eh-aceh)\log\left(cx^2+bx+a\right)}{2\,c^3} + \frac{(2\,c^3df-bc^2ef-bc^2dg+b^2ceg-2\,ac^2eg+b^2cdh-2\,ac^2dh-b^3eh+3\,abceh)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^3}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
1/2*(c*e*h*x^2 + 2*c*e*g*x + 2*c*d*h*x - 2*b*e*h*x)/c^2 + 1/2*(c^2*e*f + c
^2*d*g - b*c*e*g - b*c*d*h + b^2*e*h - a*c*e*h)*log(c*x^2 + b*x + a)/c^3 +
 (2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d*h
- 2*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*
c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B] (verification not implemented)

Time = 18.46 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx = x\left(\frac{d\,h+e\,g}{c}-\frac{b\,e\,h}{c^2}\right)$$
$$-\frac{\ln\left(c\,x^2+b\,x+a\right)\left(b^4\,e\,h-4\,a\,c^3\,d\,g-4\,a\,c^3\,e\,f-b^3\,c\,d\,h-b^3\,c\,e\,g+b^2\,c^2\,d\,g+b^2\,c^2\,e\,f+4\,a^2\,c^2\,e\,h-\right.}{2\left(4\,a\,c^4-b^2\,c^3\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\right.}{c^3\,\sqrt{4\,a\,c-b^2}}$$
$$+\frac{e\,h\,x^2}{2\,c}$$

input
```
int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2),x)
```

output
```
x*((d*h + e*g)/c - (b*e*h)/c^2) - (log(a + b*x + c*x^2)*(b^4*e*h - 4*a*c^3
*d*g - 4*a*c^3*e*f - b^3*c*d*h - b^3*c*e*g + b^2*c^2*d*g + b^2*c^2*e*f + 4
*a^2*c^2*e*h + 4*a*b*c^2*d*h + 4*a*b*c^2*e*g - 5*a*b^2*c*e*h))/(2*(4*a*c^4
 - b^2*c^3)) - (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2))*
(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f -
 b^2*c*d*h - b^2*c*e*g - 3*a*b*c*e*h))/(c^3*(4*a*c - b^2)^(1/2)) + (e*h*x^
2)/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 673, normalized size of antiderivative = 3.80

$$\int \frac{(d+ex)(f+gx+hx^2)}{a+bx+cx^2}\,dx$$
$$=\frac{-\log(c\,x^2+bx+a)\,b^4eh+\log(c\,x^2+bx+a)\,b^3cdh+\log(c\,x^2+bx+a)\,b^3ceg-\log(c\,x^2+bx+a)\,b^2c^2\ldots}{}$$

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

output
```
(6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*e*h - 4*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*d*h - 4*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*e*g - 2*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*e*h + 2*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*d*h + 2*sqrt(4*a*c - b*
*2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*e*g - 2*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*d*g - 2*sqrt(4*a*c - b**2)*at
an((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*e*f + 4*sqrt(4*a*c - b**2)*atan(
(b + 2*c*x)/sqrt(4*a*c - b**2))*c**3*d*f - 4*log(a + b*x + c*x**2)*a**2*c*
*2*e*h + 5*log(a + b*x + c*x**2)*a*b**2*c*e*h - 4*log(a + b*x + c*x**2)*a*
b*c**2*d*h - 4*log(a + b*x + c*x**2)*a*b*c**2*e*g + 4*log(a + b*x + c*x**2
)*a*c**3*d*g + 4*log(a + b*x + c*x**2)*a*c**3*e*f - log(a + b*x + c*x**2)*
b**4*e*h + log(a + b*x + c*x**2)*b**3*c*d*h + log(a + b*x + c*x**2)*b**3*c
*e*g - log(a + b*x + c*x**2)*b**2*c**2*d*g - log(a + b*x + c*x**2)*b**2*c*
*2*e*f - 8*a*b*c**2*e*h*x + 8*a*c**3*d*h*x + 8*a*c**3*e*g*x + 4*a*c**3*e*h
*x**2 + 2*b**3*c*e*h*x - 2*b**2*c**2*d*h*x - 2*b**2*c**2*e*g*x - b**2*c**2
*e*h*x**2)/(2*c**3*(4*a*c - b**2))
```

3.7 $\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx=\frac{hx}{c}-\frac{(2c^2f-bcg+b^2h-2ach)\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log(a+bx+cx^2)}{2c^2}$$

output
```
h*x/c-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))
/c^2/(-4*a*c+b^2)^(1/2)+1/2*(-b*h+c*g)*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx=\frac{hx}{c}+\frac{(2c^2f-bcg+b^2h-2ach)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}+\frac{(cg-bh)\log(a+bx+cx^2)}{2c^2}$$

input
```
Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

output
```
(h*x)/c + ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^
2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])
/(2*c^2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(\frac{-ah+x(cg-bh)+cf}{c\,(a+bx+cx^2)}+\frac{h}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2ach+b^2h-bcg+2c^2f\right)}{c^2\sqrt{b^2-4ac}}+\frac{(cg-bh)\log\left(a+bx+cx^2\right)}{2c^2}+\frac{hx}{c}$$

input
```
Int[(f + g*x + h*x^2)/(a + b*x + c*x^2),x]
```

output
```
(h*x)/c - ((2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]) + ((c*g - b*h)*Log[a + b*x + c*x^2])/
(2*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{hx}{c}+\frac{\frac{(-bh+cg)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-ah+cf-\frac{(-bh+cg)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c}$	93
risch	Expression too large to display	1649

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
h*x/c+1/c*(1/2*(-b*h+c*g)/c*ln(c*x^2+b*x+a)+2*(-a*h+c*f-1/2*(-b*h+c*g)*b/c
)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 3.28

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx$$
$$= \left[\frac{2\,(b^2c-4\,ac^2)hx-(2\,c^2f-bcg+(b^2-2\,ac)h)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,cx+b)}{cx^2+bx+a}\right)+((b}{2\,(b^2c^2-4\,ac^3)}\right.$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2*c - 4*a*c^2)*h*x - (2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*
c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*c)*h)*l
og(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), 1/2*(2*(b^2*c - 4*a*c^2)*h*x - 2
*(2*c^2*f - b*c*g + (b^2 - 2*a*c)*h)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*g - (b^3 - 4*a*b*
c)*h)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(88) = 176$.

Time = 1.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.30

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)} - \frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ac}{2c^2\cdot(}\right.}{2ach-b^2h+bcg-2c^2f}\right.$$
$$+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)} - \frac{bh-cg}{2c^2}\right)\log\left(x+\frac{-abh-4ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2h+bcg-2c^2f)}{2c^2\cdot(4ac-b^2)}-\frac{bh-cg}{2c^2}\right)+2acg+b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ach-b^2}{2c^2\cdot(4ac-}\right.}{2ach-b^2h+bcg-2c^2f}\right.$$
$$+\frac{hx}{c}$$

input
```
integrate((h*x**2+g*x+f)/(c*x**2+b*x+a),x)
```

output
```
(-sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c
 - b**2)) - (b*h - c*g)/(2*c**2))*log(x + (-a*b*h - 4*a*c**2*(-sqrt(-4*a*c
 + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (
b*h - c*g)/(2*c**2)) + 2*a*c*g + b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*h - b
**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2))
- b*c*f)/(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)) + (sqrt(-4*a*c + b**2)*(2*
a*c*h - b**2*h + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(
2*c**2))*log(x + (-a*b*h - 4*a*c**2*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h
 + b*c*g - 2*c**2*f)/(2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) + 2*a
*c*g + b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*h - b**2*h + b*c*g - 2*c**2*f)/(
2*c**2*(4*a*c - b**2)) - (b*h - c*g)/(2*c**2)) - b*c*f)/(2*a*c*h - b**2*h
+ b*c*g - 2*c**2*f)) + h*x/c
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{hx}{c} + \frac{(cg-bh)\log\left(cx^2+bx+a\right)}{2\,c^2} + \frac{(2\,c^2f-bcg+b^2h-2\,ach)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}c^2}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
h*x/c + 1/2*(c*g - b*h)*log(c*x^2 + b*x + a)/c^2 + (2*c^2*f - b*c*g + b^2*
h - 2*a*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^
2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.43

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{h\,x}{c} + \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(h\,b^3-g\,b^2\,c-4\,a\,h\,b\,c+4\,a\,g\,c^2\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)} + \frac{\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c\right)}{c^2\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((f + g*x + h*x^2)/(a + b*x + c*x^2),x)
```

output
```
(h*x)/c + (log(a + b*x + c*x^2)*(b^3*h + 4*a*c^2*g - b^2*c*g - 4*a*b*c*h))
/(2*(4*a*c^3 - b^2*c^2)) + (atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c -
b^2)^(1/2))*(2*c^2*f + b^2*h - 2*a*c*h - b*c*g))/(c^2*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.75

$$\int \frac{f+gx+hx^2}{a+bx+cx^2}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ach+2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)b^2h-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bcg+4}{}$$

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*h + 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*h - 2*sqrt(4*a*
c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*g + 4*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*f - 4*log(a + b*x + c*x**2)*a*
b*c*h + 4*log(a + b*x + c*x**2)*a*c**2*g + log(a + b*x + c*x**2)*b**3*h -
log(a + b*x + c*x**2)*b**2*c*g + 8*a*c**2*h*x - 2*b**2*c*h*x)/(2*c**2*(4*a
*c - b**2))
```

3.8 $\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx = -\frac{(2c^2df+b(bd-ae)h-c(bef+bdg-2aeg+2adh))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{(e^2f-deg+d^2h)\log(d+ex)}{e\,(cd^2-bde+ae^2)} - \frac{(cef-cdg+bdh-aeh)\log\left(a+bx+cx^2\right)}{2c\,(cd^2-bde+ae^2)}$$

output
```
-(2*c^2*d*f+b*(-a*e+b*d)*h-c*(2*a*d*h-2*a*e*g+b*d*g+b*e*f))*arctanh((2*c*x
+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)/(a*e^2-b*d*e+c*d^2)+(d^2*h-d*
e*g+e^2*f)*ln(e*x+d)/e/(a*e^2-b*d*e+c*d^2)-1/2*(-a*e*h+b*d*h-c*d*g+c*e*f)*
ln(c*x^2+b*x+a)/c/(a*e^2-b*d*e+c*d^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.98

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx = \frac{-2e(-2c^2df+b(-bd+ae)h+c(bef+bdg-2aeg+2adh))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+2c\sqrt{-b^2+4ac}(e^2f-}{2c\sqrt{-b^2+4ac}\,e\,(cd^2+e(-b}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

output
```
(-2*e*(-2*c^2*d*f + b*(-(b*d) + a*e)*h + c*(b*e*f + b*d*g - 2*a*e*g + 2*a*
d*h))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + 2*c*Sqrt[-b^2 + 4*a*c]*(e^2
*f - d*e*g + d^2*h)*Log[d + e*x] - Sqrt[-b^2 + 4*a*c]*e*(c*e*f - c*d*g + b
*d*h - a*e*h)*Log[a + x*(b + c*x)])/(2*c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(
-(b*d) + a*e)))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-x(-aeh+bdh-cdg+cef)-adh+aeg-bef+cdf}{(a+bx+cx^2)(ae^2-bde+cd^2)}+\frac{d^2h-deg+e^2f}{(d+ex)(ae^2-bde+cd^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df\right)}{c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{\log\left(a+bx+cx^2\right)(-aeh+bdh-cdg+cef)}{2c\,(ae^2-bde+cd^2)}+\frac{\log(d+ex)\left(d^2h-deg+e^2f\right)}{e\,(ae^2-bde+cd^2)}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x]
```

output
```
-(((2*c^2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*A
rcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e
 + a*e^2))) + ((e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(e*(c*d^2 - b*d*e + a
*e^2)) - ((c*e*f - c*d*g + b*d*h - a*e*h)*Log[a + b*x + c*x^2])/(2*c*(c*d^
2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{(aeh-bdh+cdg-cef)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-adh+aeg-bef+cdf-\frac{(aeh-bdh+cdg-cef)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,e^2-bde+c\,d^2}+\frac{(d^2h-deg+e^2f)\ln(ex+d)}{e(a\,e^2-bde+c\,d^2)}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)*(1/2*(a*e*h-b*d*h+c*d*g-c*e*f)/c*ln(c*x^2+b*x+a)+2*(
-a*d*h+a*e*g-b*e*f+c*d*f-1/2*(a*e*h-b*d*h+c*d*g-c*e*f)*b/c)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))+(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/e/(
a*e^2-b*d*e+c*d^2)
```

Fricas [A] (verification not implemented)

Time = 31.15 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.19

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$$

$$= \Bigg[-\frac{\sqrt{b^2-4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2}}{cx^2+bx+a}\right.}{}$$

$$-\frac{2\,\sqrt{-b^2+4\,ac}((2\,c^2de-bce^2)f-(bcde-2\,ace^2)g-(abe^2-(b^2-2\,ac)de)h)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,cx}{b^2-4\,ac}\right.}{}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(b^2 - 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*
g - (a*b*e^2 - (b^2 - 2*a*c)*d*e)*h)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*
c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + ((b^2*c - 4*a*c^2)
*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*a*b*c)*d*e - (a*b^2 - 4*a^2*c
)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a
*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*x + d))/((b^2*c^2 - 4*a*c^3)*
d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), -1/2*(2*sq
rt(-b^2 + 4*a*c)*((2*c^2*d*e - b*c*e^2)*f - (b*c*d*e - 2*a*c*e^2)*g - (a*b
*e^2 - (b^2 - 2*a*c)*d*e)*h)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 -
 4*a*c)) + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + ((b^3 - 4*
a*b*c)*d*e - (a*b^2 - 4*a^2*c)*e^2)*h)*log(c*x^2 + b*x + a) - 2*((b^2*c -
4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g + (b^2*c - 4*a*c^2)*d^2*h)*log(e*
x + d))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c
- 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)}\,dx &= -\frac{(cef-cdg+bdh-aeh)\log\left(cx^2+bx+a\right)}{2\,(c^2d^2-bcde+ace^2)}+\frac{(e^2f-deg+d^2h)\log\left(|ex+d|\right)}{cd^2e-bde^2+ae^3}\\&\quad+\frac{(2\,c^2df-bcef-bcdg+2\,aceg+b^2dh-2\,acdh-abeh)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(c^2d^2-bcde+ace^2)\sqrt{-b^2+4\,ac}}\end{aligned}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(c*e*f - c*d*g + b*d*h - a*e*h)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d
*e + a*c*e^2) + (e^2*f - d*e*g + d^2*h)*log(abs(e*x + d))/(c*d^2*e - b*d*e
^2 + a*e^3) + (2*c^2*d*f - b*c*e*f - b*c*d*g + 2*a*c*e*g + b^2*d*h - 2*a*c
*d*h - a*b*e*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e
 + a*c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 26.24 (sec) , antiderivative size = 2467, normalized size of antiderivative = 12.59

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)}\,dx=\text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)),x)
```

output
```
(log(a^2*b*e^4*g - 2*a*b^2*e^4*f - 2*a^3*e^4*h + 6*a^2*c*e^4*f - 4*a*c^2*d
^4*h + b^2*c*d^4*h + b^3*d^3*e*h - 2*b^3*e^4*f*x + a^2*e^4*g*(b^2 - 4*a*c)
^(1/2) + a*b^2*d*e^3*g + 6*a*c^2*d^3*e*g + b*c^2*d^3*e*f + 3*a^2*b*d*e^3*h
 - 10*a^2*c*d*e^3*g - 2*b^2*c*d^3*e*g + a*b^2*e^4*g*x - a^2*b*e^4*h*x - 2*
a^2*c*e^4*g*x + b^3*d*e^3*g*x + 2*c^3*d^3*e*f*x - 3*a^2*d*e^3*h*(b^2 - 4*a
*c)^(1/2) - c^2*d^3*e*f*(b^2 - 4*a*c)^(1/2) - b^2*d^3*e*h*(b^2 - 4*a*c)^(1
/2) - 2*b^2*e^4*f*x*(b^2 - 4*a*c)^(1/2) - a^2*e^4*h*x*(b^2 - 4*a*c)^(1/2)
- 2*c^2*d^4*h*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2*f - 4*a*b^2*d^2*e^2
*h + b^2*c*d^2*e^2*f + 10*a^2*c*d^2*e^2*h - b^3*d^2*e^2*h*x - 2*a*b*e^4*f*
(b^2 - 4*a*c)^(1/2) - b*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d*e^3*f - 3*
a*b*c*d^3*e*h + 7*a*b*c*e^4*f*x - 5*c^2*d^2*e^2*f*x*(b^2 - 4*a*c)^(1/2) -
b^2*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + a*b*d*e^3*g*(b^2 - 4*a*c)^(1/2) + 7*
a*c*d*e^3*f*(b^2 - 4*a*c)^(1/2) + 5*a*c*d^3*e*h*(b^2 - 4*a*c)^(1/2) + 2*b*
c*d^3*e*g*(b^2 - 4*a*c)^(1/2) + a*b*e^4*g*x*(b^2 - 4*a*c)^(1/2) + 3*a*c*e^
4*f*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^2*e^2*g - 14*a*c^2*d*e^3*f*x + 5*b^2
*c*d*e^3*f*x - 10*a*c^2*d^3*e*h*x - b*c^2*d^3*e*g*x + 6*a^2*c*d*e^3*h*x +
3*b^2*c*d^3*e*h*x + 2*a*b*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - 7*a*c*d^2*e^2*g*
(b^2 - 4*a*c)^(1/2) - b*c*d^2*e^2*f*(b^2 - 4*a*c)^(1/2) + b^2*d*e^3*g*x*(b
^2 - 4*a*c)^(1/2) + 3*c^2*d^3*e*g*x*(b^2 - 4*a*c)^(1/2) + 14*a*c^2*d^2*e^2
*g*x - 3*b*c^2*d^2*e^2*f*x - 2*b^2*c*d^2*e^2*g*x + 5*a*c*d^2*e^2*h*x*(b...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 593, normalized size of antiderivative = 3.03

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ab\,e^2h-4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)acdeh+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)ac}{}$$

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a),x)
```

output

```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*e**2*h -
 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d*e*h + 4*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*e**2*g + 2*sqrt
(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*d*e*h - 2*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*d*e*g - 2*sqrt(4*a*c
- b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*c*e**2*f + 4*sqrt(4*a*c - b
**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*c**2*d*e*f + 4*log(a + b*x + c*x
**2)*a**2*c*e**2*h - log(a + b*x + c*x**2)*a*b**2*e**2*h - 4*log(a + b*x +
 c*x**2)*a*b*c*d*e*h + 4*log(a + b*x + c*x**2)*a*c**2*d*e*g - 4*log(a + b*
x + c*x**2)*a*c**2*e**2*f + log(a + b*x + c*x**2)*b**3*d*e*h - log(a + b*x
 + c*x**2)*b**2*c*d*e*g + log(a + b*x + c*x**2)*b**2*c*e**2*f + 8*log(d +
e*x)*a*c**2*d**2*h - 8*log(d + e*x)*a*c**2*d*e*g + 8*log(d + e*x)*a*c**2*e
**2*f - 2*log(d + e*x)*b**2*c*d**2*h + 2*log(d + e*x)*b**2*c*d*e*g - 2*log
(d + e*x)*b**2*c*e**2*f)/(2*c*e*(4*a**2*c*e**2 - a*b**2*e**2 - 4*a*b*c*d*e
 + 4*a*c**2*d**2 + b**3*d*e - b**2*c*d**2))
```

3.9 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 316

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = -\frac{e^2f-deg+d^2h}{e(cd^2-bde+ae^2)(d+ex)} - \frac{(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))\,\text{arcta}}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^2} + \frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^2} - \frac{(cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h))\log(a+bx+cx^2)}{2(cd^2-bde+ae^2)^2}$$

output
```
-(d^2*h-d*e*g+e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)-(2*c^2*d^2*f+2*a^2*e^2*
h-a*b*e*(2*d*h+e*g)+b^2*(d^2*h+e^2*f)-c*(b*d*(d*g+2*e*f)+2*a*(d^2*h-2*d*e*
g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a*e^2
-b*d*e+c*d^2)^2+(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))*ln(e*
x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*(c*d*(-d*g+2*e*f)+a*e*(-2*d*h+e*g)-b*(-d^2*
h+e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx$$
$$= \frac{-\frac{2(cd^2+e(-bd+ae))(e^2f-deg+d^2h)}{e(d+ex)}+\frac{2(2c^2d^2f+2a^2e^2h-abe(eg+2dh)+b^2(e^2f+d^2h)-c(bd(2ef+dg)+2a(e^2f-2deg+d^2h)))\arctan\left(\cdots\right.}{\sqrt{-b^2+4ac}}}{\cdots}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

output
```
((-2*(c*d^2 + e*(-(b*d) + a*e))*(e^2*f - d*e*g + d^2*h))/(e*(d + e*x)) + (
2*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) -
 c*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)
/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(c*d*(2*e*f - d*g) + a*e*(e*g
 - 2*d*h) + b*(-(e^2*f) + d^2*h))*Log[d + e*x] + (c*d*(-2*e*f + d*g) + a*e
*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e
*(-(b*d) + a*e))^2)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \ 2159$$
$$\int \left(\frac{e^2(a^2h-abg+b^2f)-cx(ae(eg-2dh)-b(e^2f-d^2h)+cd(2ef-dg))-c(a(d^2h-2deg+e^2f)+2bdef\cdots}{(a+bx+cx^2)\,(ae^2-bde+cd^2)^2}\right.$$
$$\Big\downarrow \ 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^2}$$

$$\frac{\log\left(a+bx+cx^2\right)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^2}-$$

$$\frac{d^2h-deg+e^2f}{e(d+ex)\left(ae^2-bde+cd^2\right)}+\frac{\log(d+ex)\left(ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^2}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x]
```

output
```
-((e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x))) - ((2*c^2
*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h) - c*(b*d*
(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b
^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2) + ((c*d*(2*e*f
 - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))*Log[d + e*x])/(c*d^2 - b*
d*e + a*e^2)^2 - ((c*d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*
h))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{\left(2acdeh-ac\,e^2g-bc\,d^2h+bc\,e^2f+c^2d^2g-2c^2def\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(a^2e^2h-ab\,e^2g-ac\,d^2h+2acdeg-ac\,e^2f+b^2e^2f-2bcdef+c^2d^2f-\frac{(2acde}{\sqrt{4ac-b^2}}\right.}{}}{\left(a\,e^2-bde+c\,d^2\right)^2}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)^2*(1/2*(2*a*c*d*e*h-a*c*e^2*g-b*c*d^2*h+b*c*e^2*f+c^
2*d^2*g-2*c^2*d*e*f)/c*ln(c*x^2+b*x+a)+2*(a^2*e^2*h-a*b*e^2*g-a*c*d^2*h+2*
a*c*d*e*g-a*c*e^2*f+b^2*e^2*f-2*b*c*d*e*f+c^2*d^2*f-1/2*(2*a*c*d*e*h-a*c*e
^2*g-b*c*d^2*h+b*c*e^2*f+c^2*d^2*g-2*c^2*d*e*f)*b/c)/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-(d^2*h-d*e*g+e^2*f)/e/(a*e^2-b*d*e+c*d^2
)/(e*x+d)-(2*a*d*e*h-a*e^2*g-b*d^2*h+b*e^2*f+c*d^2*g-2*c*d*e*f)/(a*e^2-b*d
*e+c*d^2)^2*ln(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.45

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)}\,dx =$$

$$-\frac{(2\,cdef-be^2f-cd^2g+ae^2g+bd^2h-2\,adeh)\log\left(c-\frac{2\,cd}{ex+d}+\frac{cd^2}{(ex+d)^2}+\frac{be}{ex+d}-\frac{bde}{(ex+d)^2}+\frac{ae^2}{(ex+d)^2}\right)}{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)}$$

$$-\frac{\frac{e^3f}{ex+d}-\frac{de^2g}{ex+d}+\frac{d^2eh}{ex+d}}{cd^2e^2-bde^3+ae^4}$$

$$+\frac{(2\,c^2d^2e^2f-2\,bcde^3f+b^2e^4f-2\,ace^4f-bcd^2e^2g+4\,acde^3g-abe^4g+b^2d^2e^2h-2\,acd^2e^2h-2\,abd\ldots}{(c^2d^4-2\,bcd^3e+b^2d^2e^2+2\,acd^2e^2-2\,abde^3+a^2e^4)\sqrt{-b^2+\ldots}}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(2*c*d*e*f - b*e^2*f - c*d^2*g + a*e^2*g + b*d^2*h - 2*a*d*e*h)*log(c
 - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2
 + a*e^2/(e*x + d)^2)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*a*b*d*e^3 + a^2*e^4) - (e^3*f/(e*x + d) - d*e^2*g/(e*x + d) + d^2*e*h
/(e*x + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + (2*c^2*d^2*e^2*f - 2*b*c*d*e^3
*f + b^2*e^4*f - 2*a*c*e^4*f - b*c*d^2*e^2*g + 4*a*c*d*e^3*g - a*b*e^4*g +
 b^2*d^2*e^2*h - 2*a*c*d^2*e^2*h - 2*a*b*d*e^3*h + 2*a^2*e^4*h)*arctan((2*
c*d - 2*c*d^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sq
rt(-b^2 + 4*a*c)*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
 - 2*a*b*d*e^3 + a^2*e^4)*sqrt(-b^2 + 4*a*c)*e^2)
```

Mupad [B] (verification not implemented)

Time = 32.30 (sec) , antiderivative size = 3991, normalized size of antiderivative = 12.63

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)),x)
```

output

```
(log(d + e*x)*(e^2*(a*g - b*f) + d^2*(b*h - c*g) - d*e*(2*a*h - 2*c*f)))/(
a^2*e^4 + c^2*d^4 + b^2*d^2*e^2 - 2*a*b*d*e^3 - 2*b*c*d^3*e + 2*a*c*d^2*e^
2) + (log(2*a*b^3*e^4*f - 2*b^2*c^2*d^4*g - 2*a^2*b^2*e^4*g + 6*a*c^3*d^4*
g + b*c^3*d^4*f + a^3*b*e^4*h + 6*a^3*c*e^4*g + 2*b^3*c*d^4*h + 2*b^4*e^4*
f*x + 2*c^4*d^4*f*x - c^3*d^4*f*(b^2 - 4*a*c)^(1/2) + a^3*e^4*h*(b^2 - 4*a
*c)^(1/2) - 7*a^2*b*c*e^4*f - 7*a*b*c^2*d^4*h - 16*a*c^3*d^3*e*f - 16*a^3*
c*d*e^3*h - 2*a*b^3*e^4*g*x - 2*a*c^3*d^4*h*x - b*c^3*d^4*g*x - 2*a^3*c*e^
4*h*x + 2*a*b^2*e^4*f*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*e^4*g*(b^2 - 4*a*c)^(1
/2) - a^2*c*e^4*f*(b^2 - 4*a*c)^(1/2) + a*c^2*d^4*h*(b^2 - 4*a*c)^(1/2) +
2*b*c^2*d^4*g*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d^4*h*(b^2 - 4*a*c)^(1/2) + 2*
b^3*e^4*f*x*(b^2 - 4*a*c)^(1/2) + 3*c^3*d^4*g*x*(b^2 - 4*a*c)^(1/2) + 16*a
^2*c^2*d*e^3*f - a*b^3*d^2*e^2*h + 2*a^2*b^2*d*e^3*h + 2*b^2*c^2*d^3*e*f -
 b^3*c*d^2*e^2*f + 16*a^2*c^2*d^3*e*h + 2*a^2*c^2*e^4*f*x + a^2*b^2*e^4*h*
x + b^2*c^2*d^4*h*x - b^4*d^2*e^2*h*x - 20*a^2*c^2*d^2*e^2*g + 14*a*c^2*d^
2*e^2*f*(b^2 - 4*a*c)^(1/2) - a*b^2*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) + b^2*c*
d^2*e^2*f*(b^2 - 4*a*c)^(1/2) - 14*a^2*c*d^2*e^2*h*(b^2 - 4*a*c)^(1/2) - b
^3*d^2*e^2*h*x*(b^2 - 4*a*c)^(1/2) + 10*b^2*c^2*d^2*e^2*f*x + 28*a^2*c^2*d
^2*e^2*h*x - 6*a*b^2*c*d*e^3*f + 4*a*b*c^2*d^3*e*g + 4*a^2*b*c*d*e^3*g - 6
*a*b^2*c*d^3*e*h - 8*a*b^2*c*e^4*f*x + 7*a^2*b*c*e^4*g*x + 2*a*b^3*d*e^3*h
*x + 16*a*c^3*d^3*e*g*x - 4*b*c^3*d^3*e*f*x - 8*b^3*c*d*e^3*f*x + 2*b^3...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2385, normalized size of antiderivative = 7.55

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)} \, dx = \text{Too large to display}$$

input

```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a),x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*d**2*e**2*
h + 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*d*e**3*
h*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*d**3*e
*h - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*d**2*e*
*2*g - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*d**2*
e**2*h*x - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*d
*e**3*g*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*
d**4*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**
3*e*g - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**3
*e*h*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d**
2*e**2*f + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*d
**2*e**2*g*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
*c*d*e**3*f*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
b**2*d**4*h + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b*
*2*d**3*e*h*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
b**2*d**2*e**2*f + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2
))*b**2*d*e**3*f*x - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*b*c*d**4*g - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*b*c*d**3*e*f - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*b*c*d**3*e*g*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*...
```

3.10 $\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 509

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx$$

$$= -\frac{e^2f-deg+d^2h}{2e\,(cd^2-bde+ae^2)\,(d+ex)^2} - \frac{cd(2ef-dg)+ae(eg-2dh)-b(e^2f-d^2h)}{(cd^2-bde+ae^2)^2\,(d+ex)}$$

$$-\frac{(2c^3d^3f-be^3(b^2f-abg+a^2h)-c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(2a^2e^2(eg-3dh) -}{\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)^3}$$

$$+\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(d+ex)}{(cd^2-bde+ae^2)^3}$$

$$-\frac{(c^2d^2(3ef-dg)+e^3(b^2f-abg+a^2h)-ace(e^2f-3deg+3d^2h)-bc(3de^2f-d^3h))\log(a+bx+c}{2\,(cd^2-bde+ae^2)^3}$$

output
```
-1/2*(d^2*h-d*e*g+e^2*f)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^2-(c*d*(-d*g+2*e*f)
+a*e*(-2*d*h+e*g)-b*(-d^2*h+e^2*f))/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)-(2*c^3*d
^3*f-b*e^3*(a^2*h-a*b*g+b^2*f)-c^2*d*(b*d*(d*g+3*e*f)+2*a*(d^2*h-3*d*e*g+3
*e^2*f))-c*(2*a^2*e^2*(-3*d*h+e*g)-3*a*b*e*(-d^2*h-d*e*g+e^2*f)-b^2*(d^3*h
+3*d*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/(a*
e^2-b*d*e+c*d^2)^3+(c^2*d^2*(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-a*c*e*(3*
d^2*h-3*d*e*g+e^2*f)-b*c*(-d^3*h+3*d*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)
^3-1/2*(c^2*d^2*(-d*g+3*e*f)+e^3*(a^2*h-a*b*g+b^2*f)-a*c*e*(3*d^2*h-3*d*e*
g+e^2*f)-b*c*(-d^3*h+3*d*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d^2)^3
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.99

$$\int \frac{f+gx+hx^2}{(d+ex)^3(a+bx+cx^2)}\,dx$$
$$= -\frac{e^2f-deg+d^2h}{2e(cd^2+e(-bd+ae))(d+ex)^2}+\frac{cd(-2ef+dg)+ae(-eg+2dh)+b(e^2f-d^2h)}{(cd^2+e(-bd+ae))^2(d+ex)}$$
$$+\frac{(-2c^3d^3f+be^3(b^2f-abg+a^2h)+c^2d(bd(3ef+dg)+2a(3e^2f-3deg+d^2h))-c(-2a^2e^2(eg-3d}{\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))^3}$$
$$-\frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(d+ex)}{(cd^2+e(-bd+ae))^3}$$
$$+\frac{(c^2d^2(-3ef+dg)-e^3(b^2f-abg+a^2h)+ace(e^2f-3deg+3d^2h)+bc(3de^2f-d^3h))\log(a+x(b+}{2(cd^2+e(-bd+ae))^3}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 + e*(-(b*d) + a*e))*(d + e*x)^2) +
(c*d*(-2*e*f + d*g) + a*e*(-(e*g) + 2*d*h) + b*(e^2*f - d^2*h))/((c*d^2 +
e*(-(b*d) + a*e))^2*(d + e*x)) + ((-2*c^3*d^3*f + b*e^3*(b^2*f - a*b*g + a
^2*h) + c^2*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(-
2*a^2*e^2*(e*g - 3*d*h) + 3*a*b*e*(e^2*f - d*e*g - d^2*h) + b^2*(3*d*e^2*f
 + d^3*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e))^3) - ((c^2*d^2*(-3*e*f + d*g) - e^3*(b^2*f - a*b*
g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e^2*f - d^3*h))*
Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 + ((c^2*d^2*(-3*e*f + d*g) - e^
3*(b^2*f - a*b*g + a^2*h) + a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) + b*c*(3*d*e
^2*f - d^3*h))*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{e\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{(d+ex)\left(ae^2-bde+cd^2\right)^3}+\frac{-cx\left(e^3\left(a^2h-\right.\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-c\left(2a^2e^2(eg-3dh)-3abe\left(d^2(-h)-deg+e^2f\right)-\left(b^2\left(d^3h+3de^2f\right)\right)\right)-be^3\left(a^2h-abg\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}$$
$$\frac{\log\left(a+bx+cx^2\right)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{2\left(ae^2-bde+cd^2\right)^3}+$$
$$\frac{\log(d+ex)\left(e^3\left(a^2h-abg+b^2f\right)-ace\left(3d^2h-3deg+e^2f\right)-bc\left(3de^2f-d^3h\right)+c^2d^2(3ef-dg)\right)}{\left(ae^2-bde+cd^2\right)^3}-$$
$$\frac{d^2h-deg+e^2f}{2e(d+ex)^2\left(ae^2-bde+cd^2\right)}-\frac{ae(eg-2dh)-b\left(e^2f-d^2h\right)+cd(2ef-dg)}{(d+ex)\left(ae^2-bde+cd^2\right)^2}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x]
```

output
```
-1/2*(e^2*f - d*e*g + d^2*h)/(e*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (c*
d*(2*e*f - d*g) + a*e*(e*g - 2*d*h) - b*(e^2*f - d^2*h))/((c*d^2 - b*d*e +
 a*e^2)^2*(d + e*x)) - ((2*c^3*d^3*f - b*e^3*(b^2*f - a*b*g + a^2*h) - c^2
*d*(b*d*(3*e*f + d*g) + 2*a*(3*e^2*f - 3*d*e*g + d^2*h)) - c*(2*a^2*e^2*(e
*g - 3*d*h) - 3*a*b*e*(e^2*f - d*e*g - d^2*h) - b^2*(3*d*e^2*f + d^3*h)))*
ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e
+ a*e^2)^3) + ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^2*h) - a*c*
e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[d + e*x])/(c*
d^2 - b*d*e + a*e^2)^3 - ((c^2*d^2*(3*e*f - d*g) + e^3*(b^2*f - a*b*g + a^
2*h) - a*c*e*(e^2*f - 3*d*e*g + 3*d^2*h) - b*c*(3*d*e^2*f - d^3*h))*Log[a
+ b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.24

method	result
default	$\frac{\frac{\left(-a^{2} c e^{3} h+a b c e^{3} g+3 a c^{2} d^{2} e h-3 a c^{2} d e^{2} g+a c^{2} e^{3} f-b^{2} c e^{3} f-b c^{2} d^{3} h+3 b c^{2} d e^{2} f+c^{3} d^{3} g-3 c^{3} d^{2} e f\right) \ln\left(c x^{2}+b x+a\right)}{2 c}+\frac{2\left(-a^{2} b e^{3} h+3 a^{2} c d e^{2}\right.}{}}{}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)^3*(1/2*(-a^2*c*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a
*c^2*d*e^2*g+a*c^2*e^3*f-b^2*c*e^3*f-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g
-3*c^3*d^2*e*f)/c*ln(c*x^2+b*x+a)+2*(-a^2*b*e^3*h+3*a^2*c*d*e^2*h-a^2*c*e^
3*g+a*b^2*e^3*g-3*a*b*c*d*e^2*g+2*a*b*c*e^3*f-a*c^2*d^3*h+3*a*c^2*d^2*e*g-
3*a*c^2*d*e^2*f-b^3*e^3*f+3*b^2*c*d*e^2*f-3*b*c^2*d^2*e*f+c^3*d^3*f-1/2*(-
a^2*c*e^3*h+a*b*c*e^3*g+3*a*c^2*d^2*e*h-3*a*c^2*d*e^2*g+a*c^2*e^3*f-b^2*c*
e^3*f-b*c^2*d^3*h+3*b*c^2*d*e^2*f+c^3*d^3*g-3*c^3*d^2*e*f)*b/c)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2*(d^2*h-d*e*g+e^2*f)/e/(a*
e^2-b*d*e+c*d^2)/(e*x+d)^2+(a^2*e^3*h-a*b*e^3*g-3*a*c*d^2*e*h+3*a*c*d*e^2*
g-a*c*e^3*f+b^2*e^3*f+b*c*d^3*h-3*b*c*d*e^2*f-c^2*d^3*g+3*c^2*d^2*e*f)/(a*
e^2-b*d*e+c*d^2)^3*ln(e*x+d)+(2*a*d*e*h-a*e^2*g-b*d^2*h+b*e^2*f+c*d^2*g-2*
c*d*e*f)/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx=\text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**3/(c*x**2+b*x+a),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. $2(501) = 1002$.

Time = 0.14 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.09

$$\int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} dx = \text{Too large to display}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*(3*c^2*d^2*e*f - 3*b*c*d*e^2*f + b^2*e^3*f - a*c*e^3*f - c^2*d^3*g +
3*a*c*d*e^2*g - a*b*e^3*g + b*c*d^3*h - 3*a*c*d^2*e*h + a^2*e^3*h)*log(c*x
^2 + b*x + a)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2
 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a
^2*b*d*e^5 + a^3*e^6) + (3*c^2*d^2*e^2*f - 3*b*c*d*e^3*f + b^2*e^4*f - a*c
*e^4*f - c^2*d^3*e*g + 3*a*c*d*e^3*g - a*b*e^4*g + b*c*d^3*e*h - 3*a*c*d^2
*e^2*h + a^2*e^4*h)*log(abs(e*x + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2
*c*d^4*e^3 + 3*a*c^2*d^4*e^3 - b^3*d^3*e^4 - 6*a*b*c*d^3*e^4 + 3*a*b^2*d^2
*e^5 + 3*a^2*c*d^2*e^5 - 3*a^2*b*d*e^6 + a^3*e^7) + (2*c^3*d^3*f - 3*b*c^2
*d^2*e*f + 3*b^2*c*d*e^2*f - 6*a*c^2*d*e^2*f - b^3*e^3*f + 3*a*b*c*e^3*f -
 b*c^2*d^3*g + 6*a*c^2*d^2*e*g - 3*a*b*c*d*e^2*g + a*b^2*e^3*g - 2*a^2*c*e
^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - 3*a*b*c*d^2*e*h + 6*a^2*c*d*e^2*h - a
^2*b*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^6 - 3*b*c^2*d^5
*e + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3
*a*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6)*sqrt(-b^2 + 4*
a*c)) - 1/2*(5*c^2*d^4*e^2*f - 8*b*c*d^3*e^3*f + 3*b^2*d^2*e^4*f + 6*a*c*d
^2*e^4*f - 4*a*b*d*e^5*f + a^2*e^6*f - 3*c^2*d^5*e*g + 4*b*c*d^4*e^2*g - b
^2*d^3*e^3*g - 2*a*c*d^3*e^3*g + a^2*d*e^5*g + c^2*d^6*h - b^2*d^4*e^2*h -
 2*a*c*d^4*e^2*h + 4*a*b*d^3*e^3*h - 3*a^2*d^2*e^4*h + 2*(2*c^2*d^3*e^3*f
- 3*b*c*d^2*e^4*f + b^2*d*e^5*f + 2*a*c*d*e^5*f - a*b*e^6*f - c^2*d^4*e...
```

Mupad [B] (verification not implemented)

Time = 21.84 (sec) , antiderivative size = 12784, normalized size of antiderivative = 25.12

$$\int \frac{f+gx+hx^2}{(d+ex)^3\,(a+bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^3*(a + b*x + c*x^2)),x)
```

output
```
symsum(log(root(24*a^6*b*c*d*e^11*z^3 + 24*a*b*c^6*d^11*e*z^3 + 240*a^4*b*
c^3*d^5*e^7*z^3 + 240*a^3*b*c^4*d^7*e^5*z^3 + 120*a^5*b*c^2*d^3*e^9*z^3 +
120*a^2*b*c^5*d^9*e^3*z^3 - 54*a^5*b^2*c*d^2*e^10*z^3 - 54*a*b^2*c^5*d^10*
e^2*z^3 + 50*a^4*b^3*c*d^3*e^9*z^3 + 50*a*b^3*c^4*d^9*e^3*z^3 - 36*a^2*b^5
*c*d^5*e^7*z^3 - 36*a*b^5*c^2*d^7*e^5*z^3 + 26*a*b^6*c*d^6*e^6*z^3 - 340*a
^3*b^2*c^3*d^6*e^6*z^3 - 225*a^4*b^2*c^2*d^4*e^8*z^3 - 225*a^2*b^2*c^4*d^8
*e^4*z^3 + 180*a^3*b^3*c^2*d^5*e^7*z^3 + 180*a^2*b^3*c^3*d^7*e^5*z^3 - 30*
a^2*b^4*c^2*d^6*e^6*z^3 - 6*b^7*c*d^7*e^5*z^3 - 6*b^3*c^5*d^11*e*z^3 - 6*a
^5*b^3*d*e^11*z^3 - 6*a*b^7*d^5*e^7*z^3 - 20*b^5*c^3*d^9*e^3*z^3 + 15*b^6*
c^2*d^8*e^4*z^3 + 15*b^4*c^4*d^10*e^2*z^3 - 80*a^4*c^4*d^6*e^6*z^3 - 60*a^
5*c^3*d^4*e^8*z^3 - 60*a^3*c^5*d^8*e^4*z^3 - 24*a^6*c^2*d^2*e^10*z^3 - 24*
a^2*c^6*d^10*e^2*z^3 - 20*a^3*b^5*d^3*e^9*z^3 + 15*a^4*b^4*d^2*e^10*z^3 +
15*a^2*b^6*d^4*e^8*z^3 - 4*a^7*c*e^12*z^3 - 4*a*c^7*d^12*z^3 + b^8*d^6*e^6
*z^3 + b^2*c^6*d^12*z^3 + a^6*b^2*e^12*z^3 - 9*a^3*b^2*c*d*e^5*g*h*z - 9*a
*b^2*c^3*d^5*e*g*h*z - 30*a^3*b*c^2*d*e^5*f*h*z + 9*a^2*b^3*c*d*e^5*f*h*z
+ 3*a*b^4*c*d^2*e^4*f*h*z + 27*a*b*c^4*d^4*e^2*f*g*z + 6*a^2*b^2*c^2*d^3*e
^3*g*h*z - 33*a^2*b^2*c^2*d^2*e^4*f*h*z + 18*a*b*c^4*d^5*e*f*h*z - 12*a*b^
4*c*d*e^5*f*g*z + 27*a^3*b*c^2*d^2*e^4*g*h*z + 27*a^2*b*c^3*d^4*e^2*g*h*z
- 3*a^2*b^3*c*d^2*e^4*g*h*z - 3*a*b^3*c^2*d^4*e^2*g*h*z + 52*a^2*b*c^3*d^3
*e^3*f*h*z - 4*a*b^3*c^2*d^3*e^3*f*h*z - 3*a*b^2*c^3*d^4*e^2*f*h*z - 93...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6343, normalized size of antiderivative = 12.46

$$\int \frac{f + gx + hx^2}{(d + ex)^3 (a + bx + cx^2)} \, dx = \text{Too large to display}$$

input
```
int((h*x^2+g*x+f)/(e*x+d)^3/(c*x^2+b*x+a),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*d**3*
e**4*h - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
d**2*e**5*h*x - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a**2*b*d*e**6*h*x**2 + 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**2*c*d**4*e**3*h - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**2*c*d**3*e**4*g + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a**2*c*d**3*e**4*h*x - 8*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**2*c*d**2*e**5*g*x + 12*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c*d**2*e**5*h*x**2 - 4*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c*d*e**6*g*x**2 + 2*sqrt(
4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*d**3*e**4*g + 4*
sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*d**2*e**5*g
*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*d*e*
*6*g*x**2 - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*
c*d**5*e**2*h - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a*b*c*d**4*e**3*g - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a*b*c*d**4*e**3*h*x + 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a*b*c*d**3*e**4*f - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*a*b*c*d**3*e**4*g*x - 6*sqrt(4*a*c - b**2)*atan((b + 2*c*x
)/sqrt(4*a*c - b**2))*a*b*c*d**3*e**4*h*x**2 + 12*sqrt(4*a*c - b**2)*at...
```

3.11 $\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 427

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{e^2hx}{c^2}$$
$$-\frac{ab^3e^2h-ab^2ce(eg+2dh)-2ac^2(cd(2ef+dg)-ae(eg+2dh))+bc(c^2d^2f-3a^2e^2h+ac(e^2f+2de}{}$$
$$+\frac{(4c^4d^2f-2b^4e^2h-6ac^2e(beg+2bdh+2aeh)+b^2ce(beg+2bdh+12aeh)-c^3(2bd(2ef+dg)-4a(}{c^3\,(b^2-4ac)^{3/2}}$$
$$+\frac{e(ceg+2cdh-2beh)\log(a+bx+cx^2)}{2c^3}$$

output
```
e^2*h*x/c^2-(a*b^3*e^2*h-a*b^2*c*e*(2*d*h+e*g)-2*a*c^2*(c*d*(d*g+2*e*f)-a*
e*(2*d*h+e*g))+b*c*(c^2*d^2*f-3*a^2*e^2*h+a*c*(d^2*h+2*d*e*g+e^2*f))+(2*c^
4*d^2*f+b^4*e^2*h-b^2*c*e*(4*a*e*h+2*b*d*h+b*e*g)-c^3*(b*d*(d*g+2*e*f)+2*a
*(d^2*h+2*d*e*g+e^2*f))+c^2*(2*a^2*e^2*h+3*a*b*e*(2*d*h+e*g)+b^2*(d^2*h+2*
d*e*g+e^2*f)))*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)+(4*c^4*d^2*f-2*b^4*e^2*h-
6*a*c^2*e*(2*a*e*h+2*b*d*h+b*e*g)+b^2*c*e*(12*a*e*h+2*b*d*h+b*e*g)-c^3*(2*
b*d*(d*g+2*e*f)-4*a*(d^2*h+2*d*e*g+e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)
^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/2*e*(-2*b*e*h+2*c*d*h+c*e*g)*ln(c*x^2+b*x
+a)/c^3
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{2ce^2hx - \frac{2\left(b^4e^2hx+b^3e(aeh-c(eg+2dh)x)+b^2c\left(c\left(e^2f+2deg+d^2h\right)x-ae(eg+2dh+4ehx)\right)+2c^2\left(c^2d^2fx-ac\left(e^2fx+2de(f+gx)+d^2(g+hx\right.\right.\right.}{(b^2-4ac)(a+x(b+cx))}}{}$$

input
```
Integrate[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
(2*c*e^2*h*x - (2*(b^4*e^2*h*x + b^3*e*(a*e*h - c*(e*g + 2*d*h)*x) + b^2*c
*(c*(e^2*f + 2*d*e*g + d^2*h)*x - a*e*(e*g + 2*d*h + 4*e*h*x)) + 2*c^2*(c^
2*d^2*f*x - a*c*(e^2*f*x + 2*d*e*(f + g*x) + d^2*(g + h*x)) + a^2*e*(2*d*h
 + e*(g + h*x))) + b*c*(-3*a^2*e^2*h + c^2*d*(-2*e*f*x + d*(f - g*x)) + a*
c*(d^2*h + e^2*(f + 3*g*x) + 2*d*e*(g + 3*h*x)))))/((b^2 - 4*a*c)*(a + x*(
b + c*x))) + (2*(4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e*g + 2*b*d*h +
2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) + c^3*(-2*b*d*(2*e*f + d*g
) + 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(3/2) + e*(c*e*g + 2*c*d*h - 2*b*e*h)*Log[a + x*(b + c*x)
])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.70, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2175, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{(d+ex)\left(2cdf-2bef-bdg+4aeg+2adh-\frac{2abeh}{c}-e\left(\frac{2hb^2}{c}-gb+2cf-6ah\right)x\right)}{cx^2+bx+a}dx}{b^2-4ac}$$

$\Big\downarrow$ 1200

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \left(\frac{2d^2fc^3-\left(bd(2ef+dg)-2a\left(hd^2+2egd+e^2f\right)\right)c^2-ae(beg+2bdh+6aeh)c+2ab^2e^2h-\left(b^2-4ac\right)e(ceg+2cdh-2beh)x}{c^2\left(cx^2+bx+a\right)}-\frac{e^2\left(2hb^2-cgb+2c^2f-6\right.}{c^2}\right.}{b^2-4ac}$$

$\Big\downarrow$ 2009

$$\frac{(d+ex)^2\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(b^2ce(12aeh+2bdh+beg)-c^3\left(2bd(dg+2ef)-4a\left(d^2h+2deg+e^2f\right)\right)-6ac^2e(2aeh+2bdh+beg)-2b^4e^2h+4c^4d^2f\right)}{c^3\sqrt{b^2-4ac}}-\frac{e(b}{}}{b^2-4ac}$$

input
```
Int[((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)^2*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a
*c*h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (-((e^2*(2*c^2*f - b*c*g +
 2*b^2*h - 6*a*c*h)*x)/c^2) - ((4*c^4*d^2*f - 2*b^4*e^2*h - 6*a*c^2*e*(b*e
*g + 2*b*d*h + 2*a*e*h) + b^2*c*e*(b*e*g + 2*b*d*h + 12*a*e*h) - c^3*(2*b*
d*(2*e*f + d*g) - 4*a*(e^2*f + 2*d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sqrt
[b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*e*(c*e*g + 2*c*d*
h - 2*b*e*h)*Log[a + b*x + c*x^2])/(2*c^3))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.47

method	result
default	$\frac{e^2 h x}{c^2} - \frac{-\frac{\left(2a^2c^2e^2h-4a\,b^2c\,e^2h+6ab\,c^2deh+3ab\,c^2e^2g-2a\,c^3d^2h-4a\,c^3deg-2a\,c^3e^2f+b^4e^2h-2b^3cdeh-b^3c\,e^2g+b^2c^2d^2h+2b^2c^2deg+b^2c^2}{c\left(4ac-b^2\right)}}{}$
risch	Expression too large to display

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
e^2*h*x/c^2-1/c^2*((-(2*a^2*c^2*e^2*h-4*a*b^2*c*e^2*h+6*a*b*c^2*d*e*h+3*a*
b*c^2*e^2*g-2*a*c^3*d^2*h-4*a*c^3*d*e*g-2*a*c^3*e^2*f+b^4*e^2*h-2*b^3*c*d*
e*h-b^3*c*e^2*g+b^2*c^2*d^2*h+2*b^2*c^2*d*e*g+b^2*c^2*e^2*f-b*c^3*d^2*g-2*
b*c^3*d*e*f+2*c^4*d^2*f)/c/(4*a*c-b^2)*x+(3*a^2*b*c*e^2*h-4*a^2*c^2*d*e*h-
2*a^2*c^2*e^2*g-a*b^3*e^2*h+2*a*b^2*c*d*e*h+a*b^2*c*e^2*g-a*b*c^2*d^2*h-2*
a*b*c^2*d*e*g-a*b*c^2*e^2*f+2*a*c^3*d^2*g+4*a*c^3*d*e*f-b*c^3*d^2*f)/c/(4*
a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a*b*c*e^2*h-8*a*c^2*d*e*h-4*
a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)/c*ln(c*x^2+b*x+a)+2*(6*
a^2*c*e^2*h-2*a*b^2*e^2*h+2*a*b*c*d*e*h+a*b*c*e^2*g-2*a*c^2*d^2*h-4*a*c^2*
d*e*g-2*a*c^2*e^2*f+b*c^2*d^2*g+2*b*c^2*d*e*f-2*c^3*d^2*f-1/2*(8*a*b*c*e^2
*h-8*a*c^2*d*e*h-4*a*c^2*e^2*g-2*b^3*e^2*h+2*b^2*c*d*e*h+b^2*c*e^2*g)*b/c)
/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(420) = 840$.

Time = 0.29 (sec) , antiderivative size = 2771, normalized size of antiderivative = 6.49

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e^2*h*x^3 + 2*(b^5*c - 8*a*b^
3*c^2 + 16*a^2*b*c^3)*e^2*h*x^2 + ((4*(c^5*d^2 - b*c^4*d*e + a*c^4*e^2)*f
- (2*b*c^4*d^2 - 8*a*c^4*d*e - (b^3*c^2 - 6*a*b*c^3)*e^2)*g + 2*(2*a*c^4*d
^2 + (b^3*c^2 - 6*a*b*c^3)*d*e - (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*e^2)*h)
*x^2 + 4*(a*c^4*d^2 - a*b*c^3*d*e + a^2*c^3*e^2)*f - (2*a*b*c^3*d^2 - 8*a^
2*c^3*d*e - (a*b^3*c - 6*a^2*b*c^2)*e^2)*g + 2*(2*a^2*c^3*d^2 + (a*b^3*c -
 6*a^2*b*c^2)*d*e - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*e^2)*h + (4*(b*c^4*d
^2 - b^2*c^3*d*e + a*b*c^3*e^2)*f - (2*b^2*c^3*d^2 - 8*a*b*c^3*d*e - (b^4*
c - 6*a*b^2*c^2)*e^2)*g + 2*(2*a*b*c^3*d^2 + (b^4*c - 6*a*b^2*c^2)*d*e - (
b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*e^2)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2
 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a
)) - 2*((b^3*c^3 - 4*a*b*c^4)*d^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*d*e + (a*b^3
*c^2 - 4*a^2*b*c^3)*e^2)*f + 2*(2*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c
^2 - 4*a^2*b*c^3)*d*e + (a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e^2)*g - 2*(
(a*b^3*c^2 - 4*a^2*b*c^3)*d^2 - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*d*
e + (a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*e^2)*h - 2*((2*(b^2*c^4 - 4*a*c^5
)*d^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d*e + (b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*
e^2)*f - ((b^3*c^3 - 4*a*b*c^4)*d^2 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4
)*d*e + (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e^2)*g + ((b^4*c^2 - 6*a*b^2*
c^3 + 8*a^2*c^4)*d^2 - 2*(b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*d*e + (b^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2966 vs. $2(445) = 890$.

Time = 107.00 (sec) , antiderivative size = 2966, normalized size of antiderivative = 6.95

$$\int \frac{(d+ex)^2 (f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a*
*2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*
g - 4*a*c**3*d**2*h - 8*a*c**3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2
*b**3*c*d*e*h - b**3*c*e**2*g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*
d**2*f)/(2*c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*
log(x + (-10*a**2*b*c*e**2*h - 16*a**2*c**4*(-e*(2*b*e*h - 2*c*d*h - c*e*g
)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c*e
**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c**3
*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*g
+ 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**3
- 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 16*a**2*c**2*d*e*h + 8*a**2*
c**2*e**2*g + 2*a*b**3*e**2*h + 8*a*b**2*c**3*(-e*(2*b*e*h - 2*c*d*h - c*e
*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*(12*a**2*c**2*e**2*h - 12*a*b**2*c
*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c**2*e**2*g - 4*a*c**3*d**2*h - 8*a*c*
*3*d*e*g - 4*a*c**3*e**2*f + 2*b**4*e**2*h - 2*b**3*c*d*e*h - b**3*c*e**2*
g + 2*b*c**3*d**2*g + 4*b*c**3*d*e*f - 4*c**4*d**2*f)/(2*c**3*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - 2*a*b**2*c*d*e*h - a*b**2*
c*e**2*g - 2*a*b*c**2*d**2*h - 4*a*b*c**2*d*e*g - 2*a*b*c**2*e**2*f - b**4
*c**2*(-e*(2*b*e*h - 2*c*d*h - c*e*g)/(2*c**3) - sqrt(-(4*a*c - b**2)**3)*
(12*a**2*c**2*e**2*h - 12*a*b**2*c*e**2*h + 12*a*b*c**2*d*e*h + 6*a*b*c...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)^2(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{e^2hx}{c^2} - \frac{(4c^4d^2f - 4bc^3def + 4ac^3e^2f - 2bc^3d^2g + 8ac^3deg + b^3ce^2g - 6abc^2e^2g + 4ac^3d^2h + 2b^3cdeh - 12}{(b^2c^3 - 4ac^4)\sqrt{-b^2+4ac}} + \frac{(ce^2g + 2cdeh - 2be^2h)\log(cx^2+bx+a)}{2c^3} - \frac{\frac{(2c^4d^2f - 2bc^3def + b^2c^2e^2f - 2ac^3e^2f - bc^3d^2g + 2b^2c^2deg - 4ac^3deg - b^3ce^2g + 3abc^2e^2g + b^2c^2d^2h - 2ac^3d^2h - 2b^3cdeh + 6abc^2deh + b^4e^2h}{c}}{(cx^2 + b}$$

input
```
integrate((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
e^2*h*x/c^2 - (4*c^4*d^2*f - 4*b*c^3*d*e*f + 4*a*c^3*e^2*f - 2*b*c^3*d^2*g
 + 8*a*c^3*d*e*g + b^3*c*e^2*g - 6*a*b*c^2*e^2*g + 4*a*c^3*d^2*h + 2*b^3*c
*d*e*h - 12*a*b*c^2*d*e*h - 2*b^4*e^2*h + 12*a*b^2*c*e^2*h - 12*a^2*c^2*e^
2*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2
 + 4*a*c)) + 1/2*(c*e^2*g + 2*c*d*e*h - 2*b*e^2*h)*log(c*x^2 + b*x + a)/c^
3 - ((2*c^4*d^2*f - 2*b*c^3*d*e*f + b^2*c^2*e^2*f - 2*a*c^3*e^2*f - b*c^3*
d^2*g + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - b^3*c*e^2*g + 3*a*b*c^2*e^2*g +
b^2*c^2*d^2*h - 2*a*c^3*d^2*h - 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h + b^4*e^2*
h - 4*a*b^2*c*e^2*h + 2*a^2*c^2*e^2*h)*x/c + (b*c^3*d^2*f - 4*a*c^3*d*e*f
+ a*b*c^2*e^2*f - 2*a*c^3*d^2*g + 2*a*b*c^2*d*e*g - a*b^2*c*e^2*g + 2*a^2*
c^2*e^2*g + a*b*c^2*d^2*h - 2*a*b^2*c*d*e*h + 4*a^2*c^2*d*e*h + a*b^3*e^2*
h - 3*a^2*b*c*e^2*h)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 19.61 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$$

$$= \frac{\frac{-3\,h\,a^2\,b\,c\,e^2+4\,h\,a^2\,c^2\,d\,e+2\,g\,a^2\,c^2\,e^2+h\,a\,b^3\,e^2-2\,h\,a\,b^2\,c\,d\,e-g\,a\,b^2\,c\,e^2+h\,a\,b\,c^2\,d^2+2\,g\,a\,b\,c^2\,d\,e+f\,a\,b\,c^2\,e^2-2\,g\,a\,c^3\,d^2-4\,f\,a\,c^3\,d\,e+f\,b\,c^3\,d}{c\,(4\,a\,c-b^2)}}{}$$

$$+ \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,h\,a^3\,b\,c^3\,e^2+64\,g\,a^3\,c^4\,e^2+128\,d\,h\,a^3\,c^4\,e+96\,h\,a^2\,b^3\,c^2\,e^2-48\,g\,a^2\,b^2\,c^3\,e^2-\right.}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5\right.}$$

$$+ \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(-12\,h\,a^2\,c^2\,e^2+12\,h\,a\,b^2\,c\,e^2-12\,h\,a\,b\,c^2\,d\,e-6\,g\,a\,b\,c^2\,e^2+4\,h\,a\,c^3\,d\right.}{c^3\,\left(4\,a\,c-\right.}$$

$$+ \frac{e^2\,h\,x}{c^2}$$

input
```
int(((d + e*x)^2*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

output
```
((2*a^2*c^2*e^2*g - 2*a*c^3*d^2*g + b*c^3*d^2*f + a*b^3*e^2*h + a*b*c^2*e^
2*f + a*b*c^2*d^2*h - a*b^2*c*e^2*g - 3*a^2*b*c*e^2*h + 4*a^2*c^2*d*e*h -
4*a*c^3*d*e*f + 2*a*b*c^2*d*e*g - 2*a*b^2*c*d*e*h)/(c*(4*a*c - b^2)) + (x*
(2*c^4*d^2*f + b^4*e^2*h + b^2*c^2*e^2*f + 2*a^2*c^2*e^2*h + b^2*c^2*d^2*h
 - 2*a*c^3*e^2*f - 2*a*c^3*d^2*h - b*c^3*d^2*g - b^3*c*e^2*g + 3*a*b*c^2*e
^2*g - 4*a*b^2*c*e^2*h + 2*b^2*c^2*d*e*g - 4*a*c^3*d*e*g - 2*b*c^3*d*e*f -
 2*b^3*c*d*e*h + 6*a*b*c^2*d*e*h))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b
*c^2*x) + (log(a + b*x + c*x^2)*(2*b^7*e^2*h + 64*a^3*c^4*e^2*g - b^6*c*e^
2*g - 24*a*b^5*c*e^2*h + 128*a^3*c^4*d*e*h + 12*a*b^4*c^2*e^2*g - 128*a^3*
b*c^3*e^2*h - 2*b^6*c*d*e*h - 48*a^2*b^2*c^3*e^2*g + 96*a^2*b^3*c^2*e^2*h
+ 24*a*b^4*c^2*d*e*h - 96*a^2*b^2*c^3*d*e*h))/(2*(64*a^3*c^6 - b^6*c^3 + 1
2*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*
c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(4*c^4*d^2*f - 2*b^4*e^2*h - 1
2*a^2*c^2*e^2*h + 4*a*c^3*e^2*f + 4*a*c^3*d^2*h - 2*b*c^3*d^2*g + b^3*c*e^
2*g - 6*a*b*c^2*e^2*g + 12*a*b^2*c*e^2*h + 8*a*c^3*d*e*g - 4*b*c^3*d*e*f +
 2*b^3*c*d*e*h - 12*a*b*c^2*d*e*h))/(c^3*(4*a*c - b^2)^(3/2)) + (e^2*h*x)/
c^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3190, normalized size of antiderivative = 7.47

$$\int \frac{(d+ex)^2\,(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

output
```
( - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2
*e**2*h + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*
b**3*c*e**2*h - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b**2*c**2*d*e*h - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**2*b**2*c**2*e**2*g - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*a**2*b**2*c**2*e**2*h*x + 8*sqrt(4*a*c - b**2)*atan((b +
 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*d**2*h + 16*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*d*e*g + 8*sqrt(4*a*c - b**2)
*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*e**2*f - 24*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*e**2*h*x**2 - 4*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**5*e**2*h + 4*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*d*e*h + 2*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*e**2*g + 2
4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**4*c*e**2*h*
x - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c**2
*d*e*h*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*
*3*c**2*e**2*g*x + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**3*c**2*e**2*h*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*a*b**2*c**3*d**2*g + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/s
qrt(4*a*c - b**2))*a*b**2*c**3*d**2*h*x - 8*sqrt(4*a*c - b**2)*atan((b ...
```

3.12 $\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 232

$$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \frac{ab^2eh-bc(cdf+aeg+adh)+2ac(c(ef+dg)-aeh)-(2c^3df-b^3eh-c^2(bef+bdg+2aeg+2adh)}{c^2(b^2-4ac)(a+bx+cx^2)} + \frac{(4c^3df+b^3eh-6abceh-2c^2(b(ef+dg)-2a(eg+dh)))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}} + \frac{eh\log(a+bx+cx^2)}{2c^2}$$

output

```
(a*b^2*e*h-b*c*(a*d*h+a*e*g+c*d*f)+2*a*c*(c*(d*g+e*f)-a*e*h)-(2*c^3*d*f-b^
3*e*h-c^2*(2*a*d*h+2*a*e*g+b*d*g+b*e*f)+b*c*(3*a*e*h+b*d*h+b*e*g))*x)/c^2/
(-4*a*c+b^2)/(c*x^2+b*x+a)+(4*c^3*d*f+b^3*e*h-6*a*b*c*e*h-2*c^2*(b*(d*g+e*
f)-2*a*(d*h+e*g)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^
(3/2)+1/2*e*h*ln(c*x^2+b*x+a)/c^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{-\frac{2\left(-b^3ehx+b^2(-aeh+c(eg+dh)x)+bc(adh-cefx+cd(f-gx)+ae(g+3hx))+2c\left(a^2eh+c^2dfx-ac(e(f+gx)+d(g+hx))\right)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2\left(4c^3df+b^3eh-\right.}{}}{2c^2}$$

input

```
Integrate[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output

```
((-2*(-(b^3*e*h*x) + b^2*(-(a*e*h) + c*(e*g + d*h)*x) + b*c*(a*d*h - c*e*f
*x + c*d*(f - g*x) + a*e*(g + 3*h*x)) + 2*c*(a^2*e*h + c^2*d*f*x - a*c*(e*
(f + g*x) + d*(g + h*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*(4*c^3*
d*f + b^3*e*h - 6*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + e*h*Log[a + x*
(b + c*x)])/(2*c^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2175, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx$$

$$\Big\downarrow \ 2175$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{2cdf-b(ef+dg)-\frac{abeh}{c}+2a(eg+dh)+\left(4a-\frac{b^2}{c}\right)ehx}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{1142}$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{\frac{\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)\int\frac{1}{cx^2+bx+a}dx}{2c^2}-\frac{eh\,(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}}{b^2-4ac}$$

$$\Big\downarrow \text{1083}$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{-\frac{eh\,(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}-\frac{\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)\int\frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{c^2}}{b^2-4ac}$$

$$\Big\downarrow \text{219}$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{-\frac{eh\,(b^2-4ac)\int\frac{b+2cx}{cx^2+bx+a}dx}{2c^2}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\sqrt{b^2-4ac}}}{b^2-4ac}$$

$$\Big\downarrow \text{1103}$$

$$\frac{(d+ex)\left(c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)\right)}{c\,(b^2-4ac)\,(a+bx+cx^2)}-\frac{-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-2c^2(b(dg+ef)-2a(dh+eg))-6abceh+b^3eh+4c^3df\right)}{c^2\sqrt{b^2-4ac}}-\frac{eh\,(b^2-4ac)\log\left(a+bx+cx^2\right)}{2c^2}}{b^2-4ac}$$

input
```
Int[((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x]
```

output
```
((d + e*x)*(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c
*h)*x))/(c*(b^2 - 4*a*c)*(a + b*x + c*x^2)) - (-(((4*c^3*d*f + b^3*e*h - 6
*a*b*c*e*h - 2*c^2*(b*(e*f + d*g) - 2*a*(e*g + d*h)))*ArcTanh[(b + 2*c*x)/
Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c])) - ((b^2 - 4*a*c)*e*h*Log[a +
b*x + c*x^2])/(2*c^2))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.33

method	result
default	$\frac{\frac{\left(3abceh-2a\,c^2dh-2a\,c^2eg-b^3eh+b^2cdh+b^2ceg-b\,c^2dg-b\,c^2ef+2c^3df\right)x}{c^2\left(4ac-b^2\right)}+\frac{2a^2ceh-a\,b^2eh+abcdh+abceg-2a\,c^2dg-2a\,c^2ef+b\,c^2df}{\left(4ac-b^2\right)c^2}}{c\,x^2+bx+a}+\frac{\left(4ace\right.}{}$
risch	Expression too large to display

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
((3*a*b*c*e*h-2*a*c^2*d*h-2*a*c^2*e*g-b^3*e*h+b^2*c*d*h+b^2*c*e*g-b*c^2*d*
g-b*c^2*e*f+2*c^3*d*f)/c^2/(4*a*c-b^2)*x+(2*a^2*c*e*h-a*b^2*e*h+a*b*c*d*h+
a*b*c*e*g-2*a*c^2*d*g-2*a*c^2*e*f+b*c^2*d*f)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+a
)+1/c/(4*a*c-b^2)*(1/2*(4*a*c*e*h-b^2*e*h)/c*ln(c*x^2+b*x+a)+2*(-a*b*e*h+2
*a*c*d*h+2*a*c*e*g-b*c*d*g-b*c*e*f+2*c^2*d*f-1/2*(4*a*c*e*h-b^2*e*h)*b/c)/
(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. 2(227) = 454.

Time = 0.14 (sec) , antiderivative size = 1413, normalized size of antiderivative = 6.09

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[1/2*(((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d +
 (b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^2*
d - 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c^3
*d - b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4 -
6*a*b^2*c)*e)*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*((b^3*c^2 - 4*a
*b*c^3)*d - 2*(a*b^2*c^2 - 4*a^2*c^3)*e)*f + 2*(2*(a*b^2*c^2 - 4*a^2*c^3)*
d - (a*b^3*c - 4*a^2*b*c^2)*e)*g - 2*((a*b^3*c - 4*a^2*b*c^2)*d - (a*b^4 -
 6*a^2*b^2*c + 8*a^3*c^2)*e)*h - 2*((2*(b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 -
4*a*b*c^3)*e)*f - ((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*
c^3)*e)*g + ((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d - (b^5 - 7*a*b^3*c + 12*a
^2*b*c^2)*e)*h)*x + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*e*h*x^2 + (b^5 - 8
*a*b^3*c + 16*a^2*b*c^2)*e*h*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*e*h)*l
og(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
 1/2*(2*((2*(2*c^4*d - b*c^3*e)*f - 2*(b*c^3*d - 2*a*c^3*e)*g + (4*a*c^3*d
 + (b^3*c - 6*a*b*c^2)*e)*h)*x^2 + 2*(2*a*c^3*d - a*b*c^2*e)*f - 2*(a*b*c^
2*d - 2*a^2*c^2*e)*g + (4*a^2*c^2*d + (a*b^3 - 6*a^2*b*c)*e)*h + (2*(2*b*c
^3*d - b^2*c^2*e)*f - 2*(b^2*c^2*d - 2*a*b*c^2*e)*g + (4*a*b*c^2*d + (b^4
- 6*a*b^2*c)*e)*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(236) = 472$.

Time = 19.77 (sec) , antiderivative size = 1535, normalized size of antiderivative = 6.62

$$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)*(h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output

```
(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a
*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(
64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-16*a**2
*c**3*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h
 - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*
c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 8*a**2*c*
e*h + 8*a*b**2*c**2*(e*h/(2*c**2) - sqrt(-(4*a*c - b**2)**3)*(6*a*b*c*e*h
- 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c**2*e*f - 4
*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
)) - a*b**2*e*h - 2*a*b*c*d*h - 2*a*b*c*e*g - b**4*c*(e*h/(2*c**2) - sqrt(
-(4*a*c - b**2)**3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h
+ 2*b*c**2*d*g + 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6))) + b**2*c*d*g + b**2*c*e*f - 2*b*c**2*d
*f)/(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g +
 2*b*c**2*e*f - 4*c**3*d*f)) + (e*h/(2*c**2) + sqrt(-(4*a*c - b**2)**3)*(6
*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g + 2*b*c
**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**
4*c - b**6)))*log(x + (-16*a**2*c**3*(e*h/(2*c**2) + sqrt(-(4*a*c - b**2)*
*3)*(6*a*b*c*e*h - 4*a*c**2*d*h - 4*a*c**2*e*g - b**3*e*h + 2*b*c**2*d*g +
 2*b*c**2*e*f - 4*c**3*d*f)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input

```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{eh\log\left(cx^2+bx+a\right)}{2\,c^2} - \frac{\left(4\,c^3df - 2\,bc^2ef - 2\,bc^2dg + 4\,ac^2eg + 4\,ac^2dh + b^3eh - 6\,abceh\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2c^2 - 4\,ac^3\right)\sqrt{-b^2+4\,ac}} - \frac{bc^2df - 2\,ac^2ef - 2\,ac^2dg + abceg + abcdh - ab^2eh + 2\,a^2ceh + \left(2\,c^3df - bc^2ef - bc^2dg + b^2ceg - 2\,a\right.}{\left(cx^2+bx+a\right)\left(b^2-4\,ac\right)c^2}$$

input
```
integrate((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*e*h*log(c*x^2 + b*x + a)/c^2 - (4*c^3*d*f - 2*b*c^2*e*f - 2*b*c^2*d*g
+ 4*a*c^2*e*g + 4*a*c^2*d*h + b^3*e*h - 6*a*b*c*e*h)*arctan((2*c*x + b)/sq
rt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d*f -
2*a*c^2*e*f - 2*a*c^2*d*g + a*b*c*e*g + a*b*c*d*h - a*b^2*e*h + 2*a^2*c*e*
h + (2*c^3*d*f - b*c^2*e*f - b*c^2*d*g + b^2*c*e*g - 2*a*c^2*e*g + b^2*c*d
*h - 2*a*c^2*d*h - b^3*e*h + 3*a*b*c*e*h)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a
*c)*c^2)
```

Mupad [B] (verification not implemented)

Time = 18.51 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex)\left(f+gx+hx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx = \frac{\frac{b\,c^2\,d\,f-2\,a\,c^2\,e\,f-2\,a\,c^2\,d\,g-a\,b^2\,e\,h+2\,a^2\,c\,e\,h+a\,b\,c\,d\,h+a\,b\,c\,e\,g}{c^2\,\left(4\,a\,c-b^2\right)} - \frac{x\,\left(b^3\,e\,h-2\,c^3\,d\,f+2\,a\,c^2\,d\,h+2\,a\,c^2\,e\,g+b\,c^2\,d\,g+b\,c^2\,e\,f-b^2\,c\,d\,h-b^2\,c\,e\,g\right.}{c^2\,\left(4\,a\,c-b^2\right)}}{c\,x^2+b\,x+a} - \frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,e\,h\,a^3\,c^3+48\,e\,h\,a^2\,b^2\,c^2-12\,e\,h\,a\,b^4\,c+e\,h\,b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)} + \frac{\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c-4\,a\,b\,c^2}{c\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,c^3\,d\,f+b^3\,e\,h+4\,a\,c^2\,d\,h+4\,a\,c^2\,e\,g-2\,b\,c^2\,d\,g-2\,b\,c^2\,e\,f-6\,a\,b\,c\,e\right.}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

input
```
int(((d + e*x)*(f + g*x + h*x^2))/(a + b*x + c*x^2)^2,x)
```

output
```
((b*c^2*d*f - 2*a*c^2*e*f - 2*a*c^2*d*g - a*b^2*e*h + 2*a^2*c*e*h + a*b*c*
d*h + a*b*c*e*g)/(c^2*(4*a*c - b^2)) - (x*(b^3*e*h - 2*c^3*d*f + 2*a*c^2*d
*h + 2*a*c^2*e*g + b*c^2*d*g + b*c^2*e*f - b^2*c*d*h - b^2*c*e*g - 3*a*b*c
*e*h))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6
*e*h - 64*a^3*c^3*e*h + 48*a^2*b^2*c^2*e*h - 12*a*b^4*c*e*h))/(2*(64*a^3*c
^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (atan((2*c*x)/(4*a*c - b^
2)^(1/2) - (b^3*c - 4*a*b*c^2)/(c*(4*a*c - b^2)^(3/2)))*(4*c^3*d*f + b^3*e
*h + 4*a*c^2*d*h + 4*a*c^2*e*g - 2*b*c^2*d*g - 2*b*c^2*e*f - 6*a*b*c*e*h))
/(c^2*(4*a*c - b^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1526, normalized size of antiderivative = 6.58

$$\int \frac{(d+ex)(f+gx+hx^2)}{(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)*(h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

output

```
( - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c
*e*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c*
*2*d*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*
c**2*e*g + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**
4*e*h - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*
c*e*h*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2
*c**2*d*g + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*
*2*c**2*d*h*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*
a*b**2*c**2*e*f + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a*b**2*c**2*e*g*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a*b**2*c**2*e*h*x**2 + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*a*b*c**3*d*f + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a*b*c**3*d*h*x**2 + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqr
t(4*a*c - b**2))*a*b*c**3*e*g*x**2 + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*b**5*e*h*x + 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sq
rt(4*a*c - b**2))*b**4*c*e*h*x**2 - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/
sqrt(4*a*c - b**2))*b**3*c**2*d*g*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x
)/sqrt(4*a*c - b**2))*b**3*c**2*e*f*x + 8*sqrt(4*a*c - b**2)*atan((b + 2*c
*x)/sqrt(4*a*c - b**2))*b**2*c**3*d*f*x - 4*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*b**2*c**3*d*g*x**2 - 4*sqrt(4*a*c - b**2)*ata...
```

3.13 $\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{2acg-b(cf+ah)-(2c^2f-bcg+b^2h-2ach)\,x}{c\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{2(2cf-bg+2ah)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

output
```
(2*a*c*g-b*(a*h+c*f)-(-2*a*c*h+b^2*h-b*c*g+2*c^2*f)*x)/c/(-4*a*c+b^2)/(c*x
^2+b*x+a)+2*(2*a*h-b*g+2*c*f)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{abh+2c^2fx+b^2hx+bc(f-gx)-2ac(g+hx)}{c\,(-b^2+4ac)\,(a+x(b+cx))} - \frac{2(-2cf+bg-2ah)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

input
```
Integrate[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(a*b*h + 2*c^2*f*x + b^2*h*x + b*c*(f - g*x) - 2*a*c*(g + h*x))/(c*(-b^2 +
 4*a*c)*(a + x*(b + c*x))) - (2*(-2*c*f + b*g - 2*a*h)*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{\int \frac{2cf-bg+2ah}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2ah-bg+2cf)\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{2(2ah-bg+2cf)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}+\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2ah-bg+2cf)}{\left(b^2-4ac\right)^{3/2}}+\frac{c\left(2ag-b\left(\frac{ah}{c}+f\right)\right)-x\left(-2ach+b^2h-bcg+2c^2f\right)}{c\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

input
```
Int[(f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x]
```

output
```
(c*(2*a*g - b*(f + (a*h)/c)) - (2*c^2*f - b*c*g + b^2*h - 2*a*c*h)*x)/(c*(
b^2 - 4*a*c)*(a + b*x + c*x^2)) + (2*(2*c*f - b*g + 2*a*h)*ArcTanh[(b + 2*
c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.16

method	result
default	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2(2ah-bg+2cf)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ach-b^2h+bcg-2c^2f\right)x}{c\left(4ac-b^2\right)}+\frac{abh-2acg+bcf}{c\left(4ac-b^2\right)}}{c\,x^2+bx+a}+\frac{2\ln\left((-8a\,c^2+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)ah}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left((-8a\,c^2+2b^2c)x+(-4a\right.}{(-4ac+b}$

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(-(2*a*c*h-b^2*h+b*c*g-2*c^2*f)/c/(4*a*c-b^2)*x+1/c*(a*b*h-2*a*c*g+b*c*f)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2*(2*a*h-b*g+2*c*f)/(4*a*c-b^2)^(3/2)*arctan((2
*c*x+b)/(4*a*c-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(111) = 222.

Time = 0.09 (sec) , antiderivative size = 632, normalized size of antiderivative = 5.50

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx$$
$$=\left[-\frac{(2\,ac^2f-abcg+2\,a^2ch+(2\,c^3f-bc^2g+2\,ac^2h)x^2+(2\,bc^2f-b^2cg+2\,abch)x)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2}{}\right.}{ab^4c-8\,a^2b^2c^2+}\right.$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
[-((2*a*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2
+ (2*b*c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 +
2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
+ (b^3*c - 4*a*b*c^2)*f - 2*(a*b^2*c - 4*a^2*c^2)*g + (a*b^3 - 4*a^2*b*c)*
h + (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b*c^2)*g + (b^4 - 6*a*b^2*c +
8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^
2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x), (2*(2*a
*c^2*f - a*b*c*g + 2*a^2*c*h + (2*c^3*f - b*c^2*g + 2*a*c^2*h)*x^2 + (2*b*
c^2*f - b^2*c*g + 2*a*b*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^3*c - 4*a*b*c^2)*f + 2*(a*b^2*c - 4*a^
2*c^2)*g - (a*b^3 - 4*a^2*b*c)*h - (2*(b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a
*b*c^2)*g + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*h)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(107) = 214$.

Time = 1.13 (sec) , antiderivative size = 459, normalized size of antiderivative = 3.99

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = -\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg$$

$$+2cf)\log\left(x+\frac{-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2ab}{4ach-2bcg+4c^2f}\right.$$

$$+\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg$$

$$+2cf)\log\left(x+\frac{16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\cdot(2ah-bg+2cf)+2abh}{4ach-2bcg+4c^2f}\right.$$

$$+\frac{abh-2acg+bcf+x(-2ach+b^2h-bcg+2c^2f)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x\,(4abc^2-b^3c)}$$

input
```
integrate((h*x**2+g*x+f)/(c*x**2+b*x+a)**2,x)
```

output
```
-sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (-16*a**2*c**2*s
qrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 8*a*b**2*c*sqrt(-1/(4*a*
c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h - b**4*sqrt(-1/(4*a*c - b**2
)**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g + 4*c**
2*f)) + sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f)*log(x + (16*a**2*
c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) - 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(2*a*h - b*g + 2*c*f) + 2*a*b*h + b**4*sqrt(-1/(4*a*c
- b**2)**3)*(2*a*h - b*g + 2*c*f) - b**2*g + 2*b*c*f)/(4*a*c*h - 2*b*c*g +
 4*c**2*f)) + (a*b*h - 2*a*c*g + b*c*f + x*(-2*a*c*h + b**2*h - b*c*g + 2*
c**2*f))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*(4*a*b*
c**2 - b**3*c))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = -\frac{2\,(2\,cf-bg+2\,ah)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}} - \frac{2\,c^2fx-bcgx+b^2hx-2\,achx+bcf-2\,acg+abh}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

input
```
integrate((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(2*c*f - b*g + 2*a*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*
a*c)*sqrt(-b^2 + 4*a*c)) - (2*c^2*f*x - b*c*g*x + b^2*h*x - 2*a*c*h*x + b*
c*f - 2*a*c*g + a*b*h)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.77

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{\frac{a\,b\,h-2\,a\,c\,g+b\,c\,f}{c\,(4\,a\,c-b^2)}+\frac{x\,(h\,b^2-g\,b\,c+2\,f\,c^2-2\,a\,h\,c)}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a} - \frac{2\,\mathrm{atan}\left(\frac{\left(\frac{(b^3-4\,a\,b\,c)\,(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{5/2}}-\frac{2\,c\,x\,(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{3/2}}\right)\,(4\,a\,c-b^2)}{2\,a\,h-b\,g+2\,c\,f}\right)\,(2\,a\,h-b\,g+2\,c\,f)}{(4\,a\,c-b^2)^{3/2}}$$

input
```
int((f + g*x + h*x^2)/(a + b*x + c*x^2)^2,x)
```

output
```
((a*b*h - 2*a*c*g + b*c*f)/(c*(4*a*c - b^2)) + (x*(2*c^2*f + b^2*h - 2*a*c
*h - b*c*g))/(c*(4*a*c - b^2)))/(a + b*x + c*x^2) - (2*atan(((((b^3 - 4*a*
b*c)*(2*a*h - b*g + 2*c*f))/(4*a*c - b^2)^(5/2) - (2*c*x*(2*a*h - b*g + 2*
c*f))/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a*h - b*g + 2*c*f))*(2*a*h -
b*g + 2*c*f))/(4*a*c - b^2)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.97

$$\int \frac{f+gx+hx^2}{(a+bx+cx^2)^2}\,dx = \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a^2bh-2\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2g+4\sqrt{4ac-b^2}\,atan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)a\,b^2hx}{}$$

input
```
int((h*x^2+g*x+f)/(c*x^2+b*x+a)^2,x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*h - 2*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*g + 4*sqrt(4*
a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*h*x + 4*sqrt(4*a*c
 - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*f + 4*sqrt(4*a*c - b**
2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*h*x**2 - 2*sqrt(4*a*c - b**2
)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*g*x + 4*sqrt(4*a*c - b**2)*ata
n((b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*f*x - 2*sqrt(4*a*c - b**2)*atan((
b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*g*x**2 + 4*sqrt(4*a*c - b**2)*atan((
b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*f*x**2 + 8*a**3*c*h - 2*a**2*b**2*h
- 4*a**2*b*c*g - 8*a**2*c**2*f + 8*a**2*c**2*h*x**2 + a*b**3*g + 6*a*b**2*
c*f - 6*a*b**2*c*h*x**2 + 4*a*b*c**2*g*x**2 - 8*a*c**3*f*x**2 - b**4*f + b
**4*h*x**2 - b**3*c*g*x**2 + 2*b**2*c**2*f*x**2)/(b*(16*a**3*c**2 - 8*a**2
*b**2*c + 16*a**2*b*c**2*x + 16*a**2*c**3*x**2 + a*b**4 - 8*a*b**3*c*x - 8
*a*b**2*c**2*x**2 + b**5*x + b**4*c*x**2))
```

3.14 $\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 403

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2} \, dx = \frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-((2cd-be)(cf-ah)-(bd-2ae)(cg-bh))x}{(b^2-4ac)\left(cd^2-bde+ae^2\right)\left(a+bx+cx^2\right)} + \frac{(4c^3d^3f+be(4abdeh-2a^2e^2h+b^2(e^2f-deg-d^2h))-2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))}{(b^2-4ac)^{3/2}\left(cd^2-bde+ae\right.} + \frac{e(e^2f-deg+d^2h)\log(d+ex)}{\left(cd^2-bde+ae^2\right)^2} - \frac{e(e^2f-deg+d^2h)\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^2}$$

output

```
(b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f)-((-b*e+2*c*d)*(-a*
h+c*f)-(-2*a*e+b*d)*(-b*h+c*g))*x)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^2
+b*x+a)+(4*c^3*d^3*f+b*e*(4*a*b*d*e*h-2*a^2*e^2*h+b^2*(-d^2*h-d*e*g+e^2*f)
)-2*c^2*d*(b*d*(d*g+3*e*f)-2*a*(d^2*h-d*e*g+3*e^2*f))+2*c*e*(2*b^2*d^2*g+2
*a^2*e*(-d*h+e*g)-a*b*(d^2*h+d*e*g+3*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e^2-b*d*e+c*d^2)^2+e*(d^2*h-d*e*g+e^2*f)*l
n(e*x+d)/(a*e^2-b*d*e+c*d^2)^2-1/2*e*(d^2*h-d*e*g+e^2*f)*ln(c*x^2+b*x+a)/(
a*e^2-b*d*e+c*d^2)^2
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{f+gx+hx^2}{(d+ex)\,(a+bx+cx^2)^2}\,dx$$
$$= \frac{-2a^2eh+2c^2dfx+b^2(-ef+dhx)+bc(-efx+d(f-gx))+ab(dh+e(g-hx))+2ac(e(f+gx)-c}{(b^2-4ac)\,(-cd^2+e(bd-ae))\,(a+x(b+cx))}$$
$$-\frac{(-4c^3d^3f+2c^2d(bd(3ef+dg)-2a(3e^2f-deg+d^2h))+be(-4abdeh+2a^2e^2h+b^2(-e^2f+deg+}{(-b^2+4ac)^{3/2}\,(cd^2+e(-}$$
$$+\frac{e(e^2f-deg+d^2h)\log(d+ex)}{(cd^2+e(-bd+ae))^2}-\frac{e(e^2f-deg+d^2h)\log(a+x(b+cx))}{2\,(cd^2+e(-bd+ae))^2}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

output
```
(-2*a^2*e*h + 2*c^2*d*f*x + b^2*(-(e*f) + d*h*x) + b*c*(-(e*f*x) + d*(f -
g*x)) + a*b*(d*h + e*(g - h*x)) + 2*a*c*(e*(f + g*x) - d*(g + h*x)))/((b^2
 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*(a + x*(b + c*x))) - ((-4*c^3*d^3*f +
 2*c^2*d*(b*d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + b*e*(-4*a*b
*d*e*h + 2*a^2*e^2*h + b^2*(-(e^2*f) + d*e*g + d^2*h)) + 2*c*e*(-2*b^2*d^2
*g + 2*a^2*e*(-(e*g) + d*h) + a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTan[(b +
2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(c*d^2 + e*(-(b*d) + a*e
))^2) + (e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e)
)^2 - (e*(e^2*f - d*e*g + d^2*h)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*
d) + a*e))^2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx$$

$$\big\downarrow \; 2177$$

$$\frac{-x(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2e}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{\int \frac{2c^2fd^2-bc(ef+dg)d-be(bef-bdg+adh)+2ac\left(hd^2-egd+2e^2f\right)+e\left(2dfc^2-(bef+bdg-2aeg+2adh)c+b(bd-ae)h\right)x}{\left(cd^2-bed+ae^2\right)(d+ex)\left(cx^2+bx+a\right)}dx}{b^2-4ac}$$

$$\big\downarrow \; 27$$

$$\frac{-x(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2e}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{\int \frac{2c^2fd^2-bc(ef+dg)d-be(bef-bdg+adh)+2ac\left(hd^2-egd+2e^2f\right)+e\left(2dfc^2-(bef+bdg-2aeg+2adh)c+b(bd-ae)h\right)x}{(d+ex)\left(cx^2+bx+a\right)}dx}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\big\downarrow \; 1200$$

$$\frac{-x(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2e}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{\int \Big(\frac{2c^3fd^3-c^2\left(bd(3ef+dg)-2a\left(hd^2-egd+3e^2f\right)\right)d+be^2\left(-eha^2+2bdha+b^2(ef-dg)\right)+ce\left(2e(eg-dh)a^2-b\left(3hd^2-egd+5e^2f\right)a+2b^2d^2g\right)+c(b^2}{\left(cd^2-bed+ae^2\right)\left(cx^2+bx+a\right)}}{\left(b^2-4ac\right)\left(ae^2-bde+cd^2\right)}$$

$$\big\downarrow \; 2009$$

$$\frac{-x(-c(2adh-2aeg+bdg+bef)+bh(bd-ae)+2c^2df)-b(adh+aeg+cdf)-2a(-aeh-cdg+cef)+b^2e}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(ae^2-bde+cd^2\right)}$$

$$\frac{-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(2ce\left(2a^2e(eg-dh)-ab\left(d^2h+deg+3e^2f\right)+2b^2d^2g\right)+be\left(-2a^2e^2h+4abdeh+b^2\left(d^2(-h)-deg+e^2f\right)\right)-2c^2d\left(bd(dg+3ef)\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)}}{\left(b^2-4ac\right)\left(ae^2-bde\right.}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x]
```

output
```
(b^2*e*f - b*(c*d*f + a*e*g + a*d*h) - 2*a*(c*e*f - c*d*g - a*e*h) - (2*c^
2*d*f + b*(b*d - a*e)*h - c*(b*e*f + b*d*g - 2*a*e*g + 2*a*d*h))*x)/((b^2
- 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)) - (-(((4*c^3*d^3*f + b
*e*(4*a*b*d*e*h - 2*a^2*e^2*h + b^2*(e^2*f - d*e*g - d^2*h)) - 2*c^2*d*(b*
d*(3*e*f + d*g) - 2*a*(3*e^2*f - d*e*g + d^2*h)) + 2*c*e*(2*b^2*d^2*g + 2*
a^2*e*(e*g - d*h) - a*b*(3*e^2*f + d*e*g + d^2*h)))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2))) - ((b^2 - 4
*a*c)*e*(e^2*f - d*e*g + d^2*h)*Log[d + e*x])/(c*d^2 - b*d*e + a*e^2) + ((
b^2 - 4*a*c)*e*(e^2*f - d*e*g + d^2*h)*Log[a + b*x + c*x^2])/(2*(c*d^2 - b
*d*e + a*e^2)))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(397) = 794$.

Time = 0.43 (sec) , antiderivative size = 809, normalized size of antiderivative = 2.01

method	result
default	$-\frac{\frac{\left(a^2 b e^3 h+2a^2 c d e^2 h-2a^2 c e^3 g-2a b^2 d e^2 h-abc d^2 e h+3abcd e^2 g+abc e^3 f+2a c^2 d^3 h-2a c^2 d^2 e g-2a c^2 d e^2 f+b^3 d^2 e h-b^2 c d^3 h-b^2 c d^2 e g-b^2}{4ac-b^2}}{}$
risch	Expression too large to display

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(a*e^2-b*d*e+c*d^2)^2*(((a^2*b*e^3*h+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*
b^2*d*e^2*h-a*b*c*d^2*e*h+3*a*b*c*d*e^2*g+a*b*c*e^3*f+2*a*c^2*d^3*h-2*a*c^
2*d^2*e*g-2*a*c^2*d*e^2*f+b^3*d^2*e*h-b^2*c*d^3*h-b^2*c*d^2*e*g-b^2*c*d*e^
2*f+b*c^2*d^3*g+3*b*c^2*d^2*e*f-2*c^3*d^3*f)/(4*a*c-b^2)*x+(2*a^3*e^3*h-3*
a^2*b*d*e^2*h-a^2*b*e^3*g+2*a^2*c*d^2*e*h+2*a^2*c*d*e^2*g-2*a^2*c*e^3*f+a*
b^2*d^2*e*h+a*b^2*d*e^2*g+a*b^2*e^3*f-a*b*c*d^3*h-3*a*b*c*d^2*e*g+a*b*c*d*
e^2*f+2*a*c^2*d^3*g-2*a*c^2*d^2*e*f-b^3*d*e^2*f+2*b^2*c*d^2*e*f-b*c^2*d^3*
f)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(4*a*c^2*d^2*e*h-4*a*c^2*
d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^2*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)/c*ln(c*x^2+
b*x+a)+2*(a^2*b*e^3*h+2*a^2*c*d*e^2*h-2*a^2*c*e^3*g-2*a*b^2*d*e^2*h+3*a*b*
c*d^2*e*h-a*b*c*d*e^2*g+5*a*b*c*e^3*f-2*a*c^2*d^3*h+2*a*c^2*d^2*e*g-6*a*c^
2*d*e^2*f+b^3*d*e^2*g-b^3*e^3*f-2*b^2*c*d^2*e*g+b*c^2*d^3*g+3*b*c^2*d^2*e*
f-2*c^3*d^3*f-1/2*(4*a*c^2*d^2*e*h-4*a*c^2*d*e^2*g+4*a*c^2*e^3*f-b^2*c*d^2
*e*h+b^2*c*d*e^2*g-b^2*c*e^3*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))))+e*(d^2*h-d*e*g+e^2*f)*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2} dx = \text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2} dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f + gx + hx^2}{(d + ex)(a + bx + cx^2)^2} dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(397) = 794$.

Time = 0.17 (sec) , antiderivative size = 886, normalized size of antiderivative = 2.20

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(e^3*f - d*e^2*g + d^2*e*h)*log(c*x^2 + b*x + a)/(c^2*d^4 - 2*b*c*d^3
*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*a*b*d*e^3 + a^2*e^4) + (e^4*f - d*e^3
*g + d^2*e^2*h)*log(abs(e*x + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3
 + 2*a*c*d^2*e^3 - 2*a*b*d*e^4 + a^2*e^5) - (4*c^3*d^3*f - 6*b*c^2*d^2*e*f
 + 12*a*c^2*d*e^2*f + b^3*e^3*f - 6*a*b*c*e^3*f - 2*b*c^2*d^3*g + 4*b^2*c*
d^2*e*g - 4*a*c^2*d^2*e*g - b^3*d*e^2*g - 2*a*b*c*d*e^2*g + 4*a^2*c*e^3*g
+ 4*a*c^2*d^3*h - b^3*d^2*e*h - 2*a*b*c*d^2*e*h + 4*a*b^2*d*e^2*h - 4*a^2*
c*d*e^2*h - 2*a^2*b*e^3*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^
2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*
b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*
b^2*e^4 - 4*a^3*c*e^4)*sqrt(-b^2 + 4*a*c)) - (b*c^2*d^3*f - 2*b^2*c*d^2*e*
f + 2*a*c^2*d^2*e*f + b^3*d*e^2*f - a*b*c*d*e^2*f - a*b^2*e^3*f + 2*a^2*c*
e^3*f - 2*a*c^2*d^3*g + 3*a*b*c*d^2*e*g - a*b^2*d*e^2*g - 2*a^2*c*d*e^2*g
+ a^2*b*e^3*g + a*b*c*d^3*h - a*b^2*d^2*e*h - 2*a^2*c*d^2*e*h + 3*a^2*b*d*
e^2*h - 2*a^3*e^3*h + (2*c^3*d^3*f - 3*b*c^2*d^2*e*f + b^2*c*d*e^2*f + 2*a
*c^2*d*e^2*f - a*b*c*e^3*f - b*c^2*d^3*g + b^2*c*d^2*e*g + 2*a*c^2*d^2*e*g
 - 3*a*b*c*d*e^2*g + 2*a^2*c*e^3*g + b^2*c*d^3*h - 2*a*c^2*d^3*h - b^3*d^2
*e*h + a*b*c*d^2*e*h + 2*a*b^2*d*e^2*h - 2*a^2*c*d*e^2*h - a^2*b*e^3*h)*x)
/((c*d^2 - b*d*e + a*e^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 21.36 (sec) , antiderivative size = 13698, normalized size of antiderivative = 33.99

$$\int \frac{f+gx+hx^2}{(d+ex)\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)*(a + b*x + c*x^2)^2),x)
```

output
```
symsum(log(root(768*a^5*b*c^4*d^3*e^5*z^3 + 768*a^4*b*c^5*d^5*e^3*z^3 - 19
2*a^5*b^3*c^2*d*e^7*z^3 - 192*a^2*b^3*c^5*d^7*e*z^3 - 68*a^3*b^6*c*d^2*e^6
*z^3 - 68*a*b^6*c^3*d^6*e^2*z^3 + 36*a^2*b^7*c*d^3*e^5*z^3 + 36*a*b^7*c^2*
d^5*e^3*z^3 + 256*a^6*b*c^3*d*e^7*z^3 + 256*a^3*b*c^6*d^7*e*z^3 + 48*a^4*b
^5*c*d*e^7*z^3 + 48*a*b^5*c^4*d^7*e*z^3 - 480*a^4*b^2*c^4*d^4*e^4*z^3 + 44
0*a^3*b^4*c^3*d^4*e^4*z^3 - 320*a^4*b^3*c^3*d^3*e^5*z^3 - 320*a^3*b^3*c^4*
d^5*e^3*z^3 + 240*a^4*b^4*c^2*d^2*e^6*z^3 + 240*a^2*b^4*c^4*d^6*e^2*z^3 -
192*a^5*b^2*c^3*d^2*e^6*z^3 - 192*a^3*b^2*c^5*d^6*e^2*z^3 - 90*a^2*b^6*c^2
*d^4*e^4*z^3 - 48*a^3*b^5*c^2*d^3*e^5*z^3 - 48*a^2*b^5*c^3*d^5*e^3*z^3 - 4
*b^9*c*d^5*e^3*z^3 - 4*b^7*c^3*d^7*e*z^3 - 4*a^3*b^7*d*e^7*z^3 - 4*a*b^9*d
^3*e^5*z^3 - 12*a^5*b^4*c*e^8*z^3 - 12*a*b^4*c^5*d^8*z^3 + 6*b^8*c^2*d^6*e
^2*z^3 - 384*a^5*c^5*d^4*e^4*z^3 - 256*a^6*c^4*d^2*e^6*z^3 - 256*a^4*c^6*d
^6*e^2*z^3 + 6*a^2*b^8*d^2*e^6*z^3 + 48*a^6*b^2*c^2*e^8*z^3 + 48*a^2*b^2*c
^6*d^8*z^3 - 64*a^7*c^3*e^8*z^3 - 64*a^3*c^7*d^8*z^3 + b^10*d^4*e^4*z^3 +
b^6*c^4*d^8*z^3 + a^4*b^6*e^8*z^3 - 28*a*b^4*c*d^3*e^3*g*h*z - 10*a^3*b^2*
c*d*e^5*g*h*z - 10*a*b^2*c^3*d^5*e*g*h*z + 16*a*b^4*c*d^2*e^4*f*h*z + 14*a
^2*b^3*c*d*e^5*f*h*z + 4*a*b*c^4*d^4*e^2*f*g*z + 84*a^2*b^2*c^2*d^3*e^3*g*
h*z - 108*a^2*b^2*c^2*d^2*e^4*f*h*z + 16*a*b*c^4*d^5*e*f*h*z - 20*a*b^4*c*
d*e^5*f*g*z + 8*a^2*b^3*c*d^2*e^4*g*h*z + 8*a*b^3*c^2*d^4*e^2*g*h*z - 4*a^
3*b*c^2*d^2*e^4*g*h*z - 4*a^2*b*c^3*d^4*e^2*g*h*z + 16*a^2*b*c^3*d^3*e^...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 5037, normalized size of antiderivative = 12.50

$$\int \frac{f+gx+hx^2}{(d+ex)(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((h*x^2+g*x+f)/(e*x+d)/(c*x^2+b*x+a)^2,x)
```

output

```
( - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*e*
*3*h - 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c*
d*e**2*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*
b*c*e**3*g + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**
2*b**3*d*e**2*h - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2)
)*a**2*b**3*e**3*h*x - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c -
b**2))*a**2*b**2*c*d**2*e*h - 4*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4
*a*c - b**2))*a**2*b**2*c*d*e**2*g - 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*a**2*b**2*c*d*e**2*h*x - 12*sqrt(4*a*c - b**2)*atan((
b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*e**3*f + 8*sqrt(4*a*c - b**2)*a
tan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*e**3*g*x - 4*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b**2*c*e**3*h*x**2 + 8*sqr
t(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*d**3*h -
8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**2*d**2
*e*g + 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c
**2*d*e**2*f - 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a
**2*b*c**2*d*e**2*h*x**2 + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*
c - b**2))*a**2*b*c**2*e**3*g*x**2 - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*x)
/sqrt(4*a*c - b**2))*a*b**4*d**2*e*h - 2*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a*b**4*d*e**2*g + 8*sqrt(4*a*c - b**2)*atan((b +...
```

3.15 $\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 665

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)^2}\,dx =$$

$$-\frac{e(2c^2d^2f-bcd(2ef+dg)+2a^2e^2h-abe(eg+2dh)+b^2(2e^2f-deg+2d^2h)-2ac(3e^2f-4deg+3}{(b^2-4ac)\,(cd^2-bde+ae^2)^2\,(d+ex)}$$

$$+\frac{b^2ef-b(cdf+aeg+adh)-2a(cef-cdg-aeh)-((2cd-be)(cf-ah)-(bd-2ae)(cg-bh))x}{(b^2-4ac)\,(cd^2-bde+ae^2)\,(d+ex)\,(a+bx+cx^2)}$$

$$+\frac{(4c^4d^4f-b^3e^3(2bef-bdg-aeg+2adh)-2c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))-6c^2e(4a}{}$$

$$-\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log(d+ex)}{(cd^2-bde+ae^2)^3}$$

$$+\frac{e(e^2(2bef-bdg-aeg+2adh)-cd(4e^2f-3deg+2d^2h))\log\left(a+bx+cx^2\right)}{2\left(cd^2-bde+ae^2\right)^3}$$

output
```
-e*(2*c^2*d^2*f-b*c*d*(d*g+2*e*f)+2*a^2*e^2*h-a*b*e*(2*d*h+e*g)+b^2*(2*d^2
*h-d*e*g+2*e^2*f)-2*a*c*(3*d^2*h-4*d*e*g+3*e^2*f))/(-4*a*c+b^2)/(a*e^2-b*d
*e+c*d^2)^2/(e*x+d)+(b^2*e*f-b*(a*d*h+a*e*g+c*d*f)-2*a*(-a*e*h-c*d*g+c*e*f
)-((-b*e+2*c*d)*(-a*h+c*f)-(-2*a*e+b*d)*(-b*h+c*g))*x)/(-4*a*c+b^2)/(a*e^2
-b*d*e+c*d^2)/(e*x+d)/(c*x^2+b*x+a)+(4*c^4*d^4*f-b^3*e^3*(2*a*d*h-a*e*g-b*
d*g+2*b*e*f)-2*c^3*d^2*(b*d*(d*g+4*e*f)-2*a*(d^2*h-2*d*e*g+6*e^2*f))-6*c^2
*e*(4*a*b*d*e^2*f-b^2*d^3*g+2*a^2*e*(2*d^2*h-2*d*e*g+e^2*f))-c*e*(6*a^2*b*
e^3*g-4*a^3*e^3*h-b^3*d*(-2*d^2*h-3*d*e*g+4*e^2*f)-6*a*b^2*e*(2*d^2*h-d*e*
g+2*e^2*f)))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(a*e
^2-b*d*e+c*d^2)^3-e*(e^2*(2*a*d*h-a*e*g-b*d*g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*
g+4*e^2*f))*ln(e*x+d)/(a*e^2-b*d*e+c*d^2)^3+1/2*e*(e^2*(2*a*d*h-a*e*g-b*d*
g+2*b*e*f)-c*d*(2*d^2*h-3*d*e*g+4*e^2*f))*ln(c*x^2+b*x+a)/(a*e^2-b*d*e+c*d
^2)^3
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 650, normalized size of antiderivative = 0.98

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)^2}\,dx = -\frac{e(e^2f-deg+d^2h)}{(cd^2+e(-bd+ae))^2\,(d+ex)}$$
$$+\frac{-b^3e^2f+b^2(ae^2g-c(-2def+e^2fx+d^2hx))+b(-a^2e^2h+c^2d(-df+2efx+dgx)+ac(-d^2h+e}{(b^2-4ac)\,(cd^2+e(}$$
$$-\frac{(4c^4d^4f+b^3e^3(-2bef+bdg+aeg-2adh)-2c^3d^2(bd(4ef+dg)-2a(6e^2f-2deg+d^2h))-6c^2e(}{}$$
$$+\frac{(e^3(-2bef+bdg+aeg-2adh)+cde(4e^2f-3deg+2d^2h))\log(d+ex)}{(cd^2+e(-bd+ae))^3}$$
$$-\frac{(e^3(-2bef+bdg+aeg-2adh)+cde(4e^2f-3deg+2d^2h))\log(a+x(b+cx))}{2\,(cd^2+e(-bd+ae))^3}$$

input
```
Integrate[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

output

```
-((e*(e^2*f - d*e*g + d^2*h))/((c*d^2 + e*(-(b*d) + a*e))^2*(d + e*x))) +
(-(b^3*e^2*f) + b^2*(a*e^2*g - c*(-2*d*e*f + e^2*f*x + d^2*h*x)) + b*(-(a^
2*e^2*h) + c^2*d*(-(d*f) + 2*e*f*x + d*g*x) + a*c*(-(d^2*h) + e^2*(3*f + g
*x) - 2*d*e*(g - h*x))) + 2*c*(-(c^2*d^2*f*x) + a*c*(e^2*f*x - 2*d*e*(f +
g*x) + d^2*(g + h*x)) - a^2*e*(-2*d*h + e*(g + h*x))))/((b^2 - 4*a*c)*(c*d
^2 + e*(-(b*d) + a*e))^2*(a + x*(b + c*x))) - ((4*c^4*d^4*f + b^3*e^3*(-2*
b*e*f + b*d*g + a*e*g - 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*g) - 2*a*(6*e
^2*f - 2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g + 2*a^2*e*(e
^2*f - 2*d*e*g + 2*d^2*h)) + c*e*(-6*a^2*b*e^3*g + 4*a^3*e^3*h + b^3*d*(4*
e^2*f - 3*d*e*g - 2*d^2*h) + 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^2*h)))*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/((-b^2 + 4*a*c)^(3/2)*(-(c*d^2) + e*(b*
d - a*e))^3) + ((e^3*(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f
 - 3*d*e*g + 2*d^2*h))*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^3 - ((e^3*
(-2*b*e*f + b*d*g + a*e*g - 2*a*d*h) + c*d*e*(4*e^2*f - 3*d*e*g + 2*d^2*h)
)*Log[a + x*(b + c*x)])/(2*(c*d^2 + e*(-(b*d) + a*e))^3)
```

Rubi [A] (verified)

Time = 3.88 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2177, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}dx$$

$$\Big\downarrow \text{ 2177}$$

$$-\frac{\int \frac{\frac{ce^2\left(\left(hd^2+e^2f\right)b^2-ae(eg+2dh)b+2c^2d^2f+2a^2e^2h-c\left(bd(2ef+dg)+2a\left(hd^2-2egd+e^2f\right)\right)\right)x^2}{\left(cd^2-bed+ae^2\right)^2}+\frac{e\left(4c^3fd^3-2c^2\left(bd(2ef+dg)-2a\left(-hd^2+egd+e^2f\right)\right)d-c\left(\right.}{}}{}}{} \qquad \frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-\right.}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left(\right.}$$

$$\Big\downarrow \text{ 2159}$$

$$-\frac{\int \left(\frac{(b^2-4ac)\left(e^2(2bef-bdg-aeg+2adh)-cd(2hd^2-3egd+4e^2f)\right)e^2}{(cd^2-bed+ae^2)^3(d+ex)}-\frac{(b^2-4ac)\left(hd^2-egd+e^2f\right)e^2}{(cd^2-bed+ae^2)^2(d+ex)^2}+\frac{2c^4fd^4-c^3\left(bd(4ef+dg)-2a(hd^2-\right.}{}\right.}{}$$

$$\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h-\right.}{(b^2-4ac)\left(a+bx+cx^2\right)(\ldots}$$

$\Big\downarrow$ 2009

$$-\frac{cx\left(2a^2e^2h-c\left(2a\left(d^2h-2deg+e^2f\right)+bd(dg+2ef)\right)-abe(2dh+eg)+b^2\left(d^2h+e^2f\right)+2c^2d^2f\right)+b\left(a^2e^2h\right.}{(b^2-4ac)\left(a+bx+cx^2\right)}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-6c^2e\left(2a^2e\left(2d^2h-2deg+e^2f\right)+4abde^2f-b^2d^3g\right)-ce\left(-4a^3e^3h+6a^2be^3g-6ab^2e\left(2d^2h-deg+2e^2f\right)+b^3(-d)\left(-2d^2h-\right.\right.\right.}{\sqrt{b^2-4ac}\left(ae^2-bde+cd^2\right)^3}$$

input
```
Int[(f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x]
```

output
```
-((b^3*e^2*f - b^2*e*(2*c*d*f + a*e*g) + 2*a*c*(c*d*(2*e*f - d*g) + a*e*(e
*g - 2*d*h)) + b*(c^2*d^2*f + a^2*e^2*h - a*c*(3*e^2*f - 2*d*e*g - d^2*h))
 + c*(2*c^2*d^2*f + 2*a^2*e^2*h - a*b*e*(e*g + 2*d*h) + b^2*(e^2*f + d^2*h
) - c*(b*d*(2*e*f + d*g) + 2*a*(e^2*f - 2*d*e*g + d^2*h)))*x)/((b^2 - 4*a*
c)*(c*d^2 - b*d*e + a*e^2)^2*(a + b*x + c*x^2))) - (((b^2 - 4*a*c)*e*(e^2*
f - d*e*g + d^2*h))/((c*d^2 - b*d*e + a*e^2)^2*(d + e*x)) - ((4*c^4*d^4*f
- b^3*e^3*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - 2*c^3*d^2*(b*d*(4*e*f + d*
g) - 2*a*(6*e^2*f - 2*d*e*g + d^2*h)) - 6*c^2*e*(4*a*b*d*e^2*f - b^2*d^3*g
 + 2*a^2*e*(e^2*f - 2*d*e*g + 2*d^2*h)) - c*e*(6*a^2*b*e^3*g - 4*a^3*e^3*h
 - b^3*d*(4*e^2*f - 3*d*e*g - 2*d^2*h) - 6*a*b^2*e*(2*e^2*f - d*e*g + 2*d^
2*h)))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c*d^2 -
 b*d*e + a*e^2)^3) + ((b^2 - 4*a*c)*e*(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*
d*h) - c*d*(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[d + e*x])/(c*d^2 - b*d*e + a
*e^2)^3 - ((b^2 - 4*a*c)*e*(e^2*(2*b*e*f - b*d*g - a*e*g + 2*a*d*h) - c*d*
(4*e^2*f - 3*d*e*g + 2*d^2*h))*Log[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a
*e^2)^3))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(659) = 1318$.

Time = 0.32 (sec) , antiderivative size = 1345, normalized size of antiderivative = 2.02

method	result	size
default	Expression too large to display	1345
risch	Expression too large to display	8771

input
```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a*e^2-b*d*e+c*d^2)^3*((c*(2*a^3*e^4*h-4*a^2*b*d*e^3*h-a^2*b*e^4*g+4*a^2
*c*d*e^3*g-2*a^2*c*e^4*f+3*a*b^2*d^2*e^2*h+a*b^2*d*e^3*g+a*b^2*e^4*f-6*a*b
*c*d^2*e^2*g-2*a*c^2*d^4*h+4*a*c^2*d^3*e*g-b^3*d^3*e*h-b^3*d*e^3*f+b^2*c*d
^4*h+b^2*c*d^3*e*g+3*b^2*c*d^2*e^2*f-b*c^2*d^4*g-4*b*c^2*d^3*e*f+2*c^3*d^4
*f)/(4*a*c-b^2)*x+(a^3*b*e^4*h-4*a^3*c*d*e^3*h+2*a^3*c*e^4*g-a^2*b^2*d*e^3
*h-a^2*b^2*e^4*g+6*a^2*b*c*d^2*e^2*h-3*a^2*b*c*e^4*f-4*a^2*c^2*d^3*e*h+4*a
^2*c^2*d*e^3*f+a*b^3*d*e^3*g+a*b^3*e^4*f-a*b^2*c*d^3*e*h-3*a*b^2*c*d^2*e^2
*g+a*b^2*c*d*e^3*f+a*b*c^2*d^4*h+4*a*b*c^2*d^3*e*g-6*a*b*c^2*d^2*e^2*f-2*a
*c^3*d^4*g+4*a*c^3*d^3*e*f-b^4*d*e^3*f+3*b^3*c*d^2*e^2*f-3*b^2*c^2*d^3*e*f
+b*c^3*d^4*f)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^2*d*e
^3*h-4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8*a
*b*c^2*e^4*f-8*a*c^3*d^3*e*h+12*a*c^3*d^2*e^2*g-16*a*c^3*d*e^3*f+b^3*c*d*e
^3*g-2*b^3*c*e^4*f+2*b^2*c^2*d^3*e*h-3*b^2*c^2*d^2*e^2*g+4*b^2*c^2*d*e^3*f
)/c*ln(c*x^2+b*x+a)+2*(-20*a*b*c^2*d*e^3*f+4*a^2*b*c*d*e^3*h+6*a*b^2*c*d^2
*e^2*h-5*a*b^2*c*d*e^3*g-4*a*b*c^2*d^3*e*h+6*a*b*c^2*d^2*e^2*g+2*a^3*c*e^4
*h-6*a^2*c^2*e^4*f+a*b^3*e^4*g+2*a*c^3*d^4*h+b^4*d*e^3*g-b*c^3*d^4*g+2*c^4
*d^4*f-5*a^2*b*c*e^4*g-12*a^2*c^2*d^2*e^2*h+12*a^2*c^2*d*e^3*g-2*a*b^3*d*e
^3*h+10*a*b^2*c*e^4*f-4*a*c^3*d^3*e*g+12*a*c^3*d^2*e^2*f-3*b^3*c*d^2*e^2*g
+4*b^3*c*d*e^3*f+3*b^2*c^2*d^3*e*g-4*b*c^3*d^3*e*f-1/2*(8*a^2*c^2*d*e^3*h-
4*a^2*c^2*e^4*g-2*a*b^2*c*d*e^3*h+a*b^2*c*e^4*g-4*a*b*c^2*d*e^3*g+8*a*b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2\,(a+bx+cx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((h*x**2+g*x+f)/(e*x+d)**2/(c*x**2+b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(658) = 1316$.

Time $= 0.19$ (sec) , antiderivative size $= 1506$, normalized size of antiderivative $= 2.26$

$$\int \frac{f+gx+hx^2}{(d+ex)^2(a+bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(4*c*d*e^3*f - 2*b*e^4*f - 3*c*d^2*e^2*g + b*d*e^3*g + a*e^4*g + 2*c*
d^3*e*h - 2*a*d*e^3*h)*log(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(
e*x + d) - b*d*e/(e*x + d)^2 + a*e^2/(e*x + d)^2)/(c^3*d^6 - 3*b*c^2*d^5*e
 + 3*b^2*c*d^4*e^2 + 3*a*c^2*d^4*e^2 - b^3*d^3*e^3 - 6*a*b*c*d^3*e^3 + 3*a
*b^2*d^2*e^4 + 3*a^2*c*d^2*e^4 - 3*a^2*b*d*e^5 + a^3*e^6) - (e^7*f/(e*x +
d) - d*e^6*g/(e*x + d) + d^2*e^5*h/(e*x + d))/(c^2*d^4*e^4 - 2*b*c*d^3*e^5
 + b^2*d^2*e^6 + 2*a*c*d^2*e^6 - 2*a*b*d*e^7 + a^2*e^8) - (4*c^4*d^4*e^2*f
 - 8*b*c^3*d^3*e^3*f + 24*a*c^3*d^2*e^4*f + 4*b^3*c*d*e^5*f - 24*a*b*c^2*d
*e^5*f - 2*b^4*e^6*f + 12*a*b^2*c*e^6*f - 12*a^2*c^2*e^6*f - 2*b*c^3*d^4*e
^2*g + 6*b^2*c^2*d^3*e^3*g - 8*a*c^3*d^3*e^3*g - 3*b^3*c*d^2*e^4*g + b^4*d
*e^5*g - 6*a*b^2*c*d*e^5*g + 24*a^2*c^2*d*e^5*g + a*b^3*e^6*g - 6*a^2*b*c*
e^6*g + 4*a*c^3*d^4*e^2*h - 2*b^3*c*d^3*e^3*h + 12*a*b^2*c*d^2*e^4*h - 24*
a^2*c^2*d^2*e^4*h - 2*a*b^3*d*e^5*h + 4*a^3*c*e^6*h)*arctan((2*c*d - 2*c*d
^2/(e*x + d) - b*e + 2*b*d*e/(e*x + d) - 2*a*e^2/(e*x + d))/(sqrt(-b^2 + 4
*a*c)*e))/((b^2*c^3*d^6 - 4*a*c^4*d^6 - 3*b^3*c^2*d^5*e + 12*a*b*c^3*d^5*e
 + 3*b^4*c*d^4*e^2 - 9*a*b^2*c^2*d^4*e^2 - 12*a^2*c^3*d^4*e^2 - b^5*d^3*e^
3 - 2*a*b^3*c*d^3*e^3 + 24*a^2*b*c^2*d^3*e^3 + 3*a*b^4*d^2*e^4 - 9*a^2*b^2
*c*d^2*e^4 - 12*a^3*c^2*d^2*e^4 - 3*a^2*b^3*d*e^5 + 12*a^3*b*c*d*e^5 + a^3
*b^2*e^6 - 4*a^4*c*e^6)*sqrt(-b^2 + 4*a*c)*e^2) - ((2*c^4*d^3*e*f - 3*b*c^
3*d^2*e^2*f + 3*b^2*c^2*d*e^3*f - 6*a*c^3*d*e^3*f - b^3*c*e^4*f + 3*a*b...
```

Mupad [B] (verification not implemented)

Time = 24.90 (sec) , antiderivative size = 26278, normalized size of antiderivative = 39.52

$$\int \frac{f+gx+hx^2}{(d+ex)^2\left(a+bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
int((f + g*x + h*x^2)/((d + e*x)^2*(a + b*x + c*x^2)^2),x)
```

output
```
((a*b^2*e^3*f - 2*a*c^2*d^3*g + b*c^2*d^3*f - 4*a^2*c*e^3*f + b^3*d*e^2*f
- 2*a*b^2*d*e^2*g + 4*a*c^2*d^2*e*f + a*b^2*d^2*e*h + a^2*b*d*e^2*h + 6*a^
2*c*d*e^2*g - 2*b^2*c*d^2*e*f - 8*a^2*c*d^2*e*h + a*b*c*d^3*h - 3*a*b*c*d*
e^2*f + 2*a*b*c*d^2*e*g)/(4*a*c^3*d^4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^
2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e -
8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^2) + (x*(2*b^3*e^3*f +
 2*c^3*d^3*f - a*b^2*e^3*g - 2*a*c^2*d^3*h - b*c^2*d^3*g + a^2*b*e^3*h + 2
*a^2*c*e^3*g + b^2*c*d^3*h - b^3*d*e^2*g + b^3*d^2*e*h + 2*a*c^2*d*e^2*f +
 2*a*c^2*d^2*e*g - b*c^2*d^2*e*f - b^2*c*d*e^2*f - 2*a^2*c*d*e^2*h - 7*a*b
*c*e^3*f + 5*a*b*c*d*e^2*g - 5*a*b*c*d^2*e*h))/(4*a*c^3*d^4 + 4*a^3*c*e^4
- a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d*
e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 + 2*a*b^2*c*d^2*e^
2) - (x^2*(6*a*c^2*e^3*f - 2*b^2*c*e^3*f - 2*a^2*c*e^3*h - 2*c^3*d^2*e*f -
 8*a*c^2*d*e^2*g + 2*b*c^2*d*e^2*f + 6*a*c^2*d^2*e*h + b*c^2*d^2*e*g + b^2
*c*d*e^2*g - 2*b^2*c*d^2*e*h + a*b*c*e^3*g + 2*a*b*c*d*e^2*h))/(4*a*c^3*d^
4 + 4*a^3*c*e^4 - a^2*b^2*e^4 - b^2*c^2*d^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*
e^2 + 2*a*b^3*d*e^3 + 2*b^3*c*d^3*e - 8*a*b*c^2*d^3*e - 8*a^2*b*c*d*e^3 +
2*a*b^2*c*d^2*e^2))/(a*d + x*(a*e + b*d) + x^2*(b*e + c*d) + c*e*x^3) + sy
msum(log((x*(36*a^2*c^5*e^7*f^2 + 4*b^4*c^3*e^7*f^2 + 4*a^4*c^3*e^7*h^2 +
4*c^7*d^4*e^3*f^2 + a^2*b^2*c^3*e^7*g^2 + 64*a^2*c^5*d^2*e^5*g^2 + 12*b...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20026, normalized size of antiderivative = 30.11

$$\int \frac{f + gx + hx^2}{(d + ex)^2 (a + bx + cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int((h*x^2+g*x+f)/(e*x+d)^2/(c*x^2+b*x+a)^2,x)
```

output

```
(8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c*d*e**5
*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*b*c*e*
*6*h*x + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**4*c*
*2*d**2*e**4*h + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))
*a**4*c**2*d*e**5*h*x - 12*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c
- b**2))*a**3*b**2*c*d*e**5*g + 8*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt
(4*a*c - b**2))*a**3*b**2*c*d*e**5*h*x - 12*sqrt(4*a*c - b**2)*atan((b + 2
*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*e**6*g*x + 8*sqrt(4*a*c - b**2)*atan
((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b**2*c*e**6*h*x**2 - 48*sqrt(4*a*c -
 b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*d**3*e**3*h + 36*s
qrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*d**2*e*
*4*g - 40*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c
**2*d**2*e**4*h*x - 24*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**3*b*c**2*d*e**5*f + 36*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*
a*c - b**2))*a**3*b*c**2*d*e**5*g*x + 16*sqrt(4*a*c - b**2)*atan((b + 2*c*
x)/sqrt(4*a*c - b**2))*a**3*b*c**2*d*e**5*h*x**2 - 24*sqrt(4*a*c - b**2)*a
tan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*e**6*f*x + 8*sqrt(4*a*c -
b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*b*c**2*e**6*h*x**3 - 48*sq
rt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c**3*d**4*e**2*
h + 48*sqrt(4*a*c - b**2)*atan((b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c**...
```

3.16 $\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx = 3x+\frac{x^2}{2}+\frac{2(2-x)}{3(1-x+x^2)}+\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

output
```
3*x+1/2*x^2+2*(2-x)/(3*x^2-3*x+3)+10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
+2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2} dx = 3x+\frac{x^2}{2}-\frac{2(-2+x)}{3(1-x+x^2)}-\frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+2\log\left(1-x+x^2\right)$$

input
```
Integrate[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
3*x + x^2/2 - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/Sqrt[
3]])/(3*Sqrt[3]) + 2*Log[1 - x + x^2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2191, 25, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (x^2+x+1)}{(x^2-x+1)^2} \, dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3} \int -\frac{-3x^3-6x^2-6x+2}{x^2-x+1}dx + \frac{2(2-x)}{3(x^2-x+1)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{3} \int \frac{-3x^3-6x^2-6x+2}{x^2-x+1}dx$$

$$\Big\downarrow \text{ 2188}$$

$$\frac{2(2-x)}{3(x^2-x+1)} - \frac{1}{3} \int \left(\frac{11-12x}{x^2-x+1} - 3x - 9\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}\left(\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3x^2}{2} + 6\log\left(x^2-x+1\right) + 9x\right) + \frac{2(2-x)}{3(x^2-x+1)}$$

input
```
Int[(x^3*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(2*(2 - x))/(3*(1 - x + x^2)) + (9*x + (3*x^2)/2 + (10*ArcTan[(1 - 2*x)/Sq
rt[3]])/Sqrt[3] + 6*Log[1 - x + x^2])/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^2}{2}+3x+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(x^2-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	53
risch	$\frac{x^2}{2}+3x+\frac{-\frac{2x}{3}+\frac{4}{3}}{x^2-x+1}+2\ln\left(4x^2-4x+4\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	55

input
```
int(x^3*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+3*x+(-2/3*x+4/3)/(x^2-x+1)+2*ln(x^2-x+1)-10/9*3^(1/2)*arctan(1/3*(
2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx$$
$$= \frac{9\,x^4+45\,x^3-20\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-45\,x^2+36\,(x^2-x+1)\log\left(x^2-x+1\right)+42 ...}{18\,(x^2-x+1)}$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(9*x^4 + 45*x^3 - 20*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x -
1)) - 45*x^2 + 36*(x^2 - x + 1)*log(x^2 - x + 1) + 42*x + 24)/(x^2 - x + 1
)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{x^2}{2}+3x+\frac{4-2x}{3x^2-3x+3}+2\log\left(x^2-x+1\right) - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**3*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
x**2/2 + 3*x + (4 - 2*x)/(3*x**2 - 3*x + 3) + 2*log(x**2 - x + 1) - 10*sqr
t(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,(x^2-x+1)} + 2\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(
x^2 - x + 1) + 2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{1}{2}\,x^2 - \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + 3\,x - \frac{2\,(x-2)}{3\,(x^2-x+1)} + 2\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^3*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
1/2*x^2 - 10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 2/3*(x - 2)/(
x^2 - x + 1) + 2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = 3\,x+2\,\ln\left(x^2-x+1\right)-\frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1} - \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}+\frac{x^2}{2}$$

input
```
int((x^3*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
3*x + 2*log(x^2 - x + 1) - ((2*x)/3 - 4/3)/(x^2 - x + 1) - (10*3^(1/2)*ata
n((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9 + x^2/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.87

$$\int \frac{x^3(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{-20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2+20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x-20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)+36\log(x^2-x+1)\,x^2-36\log(x^2-}{18x^2-18x+18}$$

input
```
int(x^3*(x^2+x+1)/(x^2-x+1)^2,x)
```

output
```
( - 20*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 + 20*sqrt(3)*atan((2*x - 1)/sq
rt(3))*x - 20*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 36*log(x**2 - x + 1)*x**2
- 36*log(x**2 - x + 1)*x + 36*log(x**2 - x + 1) + 9*x**4 + 45*x**3 - 3*x**
2 + 66)/(18*(x**2 - x + 1))
```

3.17 $\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} \, dx = x + \frac{2(1-2x)}{3(1-x+x^2)} - \frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
x+2*(1-2*x)/(3*x^2-3*x+3)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+3/2*ln(x
^2-x+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2} \, dx = x - \frac{2(-1+2x)}{3(1-x+x^2)} + \frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
x - (2*(-1 + 2*x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*S
qrt[3]) + (3*Log[1 - x + x^2])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2191, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(x^2+x+1\right)}{\left(x^2-x+1\right)^2} \, dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{3} \int \frac{3 x^2+6 x+2}{x^2-x+1}dx+\frac{2(1-2 x)}{3 \left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{3} \int \left(3-\frac{1-9 x}{x^2-x+1}\right)dx+\frac{2(1-2 x)}{3 \left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{7 \arctan \left(\frac{1-2 x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{9}{2} \log \left(x^2-x+1\right)+3 x\right)+\frac{2(1-2 x)}{3 \left(x^2-x+1\right)}$$

input
```
Int[(x^2*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(2*(1 - 2*x))/(3*(1 - x + x^2)) + (3*x - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqr
t[3] + (9*Log[1 - x + x^2])/2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
default	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	46
risch	$x+\frac{-\frac{4x}{3}+\frac{2}{3}}{x^2-x+1}+\frac{3\ln\left(4x^2-4x+4\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	48

input
```
int(x^2*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+(-4/3*x+2/3)/(x^2-x+1)+3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3*(2*x-1)*3^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{18\,x^3+14\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-18\,x^2+27\,(x^2-x+1)\log\left(x^2-x+1\right)-6\,x+12}{18\,(x^2-x+1)}$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(18*x^3 + 14*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 18
*x^2 + 27*(x^2 - x + 1)*log(x^2 - x + 1) - 6*x + 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{2-4x}{3x^2-3x+3}+\frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x**2*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
x + (2 - 4*x)/(3*x**2 - 3*x + 3) + 3*log(x**2 - x + 1)/2 + 7*sqrt(3)*atan(
2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)+x-\frac{2\,(2\,x-1)}{3\,(x^2-x+1)}+\frac{3}{2}\,\log\left(x^2-x+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1
) + 3/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x - \frac{2(2x-1)}{3(x^2-x+1)}+\frac{3}{2}\log\left(x^2-x+1\right)$$

input
```
integrate(x^2*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x - 2/3*(2*x - 1)/(x^2 - x + 1
) + 3/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx = x+\frac{3\ln\left(x^2-x+1\right)}{2}-\frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x^2*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
x + (3*log(x^2 - x + 1))/2 - ((4*x)/3 - 2/3)/(x^2 - x + 1) + (7*3^(1/2)*at
an((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{x^2(1+x+x^2)}{(1-x+x^2)^2}\,dx$$
$$= \frac{14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 - 14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x + 14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right) + 27\log(x^2-x+1)\,x^2 - 27\log(x^2-x}{18x^2-18x+18}$$

input
```
int(x^2*(x^2+x+1)/(x^2-x+1)^2,x)
```

output
```
(14*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 - 14*sqrt(3)*atan((2*x - 1)/sqrt(
3))*x + 14*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 27*log(x**2 - x + 1)*x**2 - 2
7*log(x**2 - x + 1)*x + 27*log(x**2 - x + 1) + 18*x**3 - 24*x**2 + 6)/(18*
(x**2 - x + 1))
```

3.18 $\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx = -\frac{2(1+x)}{3(1-x+x^2)} - \frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log(1-x+x^2)$$

output
```
(-2-2*x)/(3*x^2-3*x+3)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+1/2*ln(x^2
-x+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2} \, dx = -\frac{2(1+x)}{3(1-x+x^2)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\log(1-x+x^2)$$

input
```
Integrate[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(-2*(1 + x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
]) + Log[1 - x + x^2]/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2191, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(x^2+x+1)}{(x^2-x+1)^2}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{3}\int \frac{3x+4}{x^2-x+1}dx-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{11}{2}\int \frac{1}{x^2-x+1}dx+\frac{3}{2}\int -\frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{11}{2}\int \frac{1}{x^2-x+1}dx-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx-11\int \frac{1}{-(2x-1)^2-3}d(2x-1)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(\frac{11\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\int \frac{1-2x}{x^2-x+1}dx\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(\frac{11\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{3}{2}\log\left(x^2-x+1\right)\right)-\frac{2(x+1)}{3\left(x^2-x+1\right)}$$

input
```
Int[(x*(1 + x + x^2))/(1 - x + x^2)^2,x]
```

output
```
(-2*(1 + x))/(3*(1 - x + x^2)) + ((11*ArcTan[(-1 + 2*x)/Sqrt[3]])/Sqrt[3]
+ (3*Log[1 - x + x^2])/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(x^2-x+1)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	45
risch	$\frac{-\frac{2x}{3}-\frac{2}{3}}{x^2-x+1}+\frac{\ln(4x^2-4x+4)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	47

input
```
int(x*(x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(-2/3*x-2/3)/(x^2-x+1)+1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{22\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+9(x^2-x+1)\log(x^2-x+1)-12x-12}{18(x^2-x+1)}$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 9*(x^2 - x
+ 1)*log(x^2 - x + 1) - 12*x - 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{-2x-2}{3x^2-3x+3} + \frac{\log(x^2-x+1)}{2} + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate(x*(x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
(-2*x - 2)/(3*x**2 - 3*x + 3) + log(x**2 - x + 1)/2 + 11*sqrt(3)*atan(2*sq
rt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2}\log(x^2-x+1)$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1
/2*log(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{2(x+1)}{3(x^2-x+1)} + \frac{1}{2}\log(x^2-x+1)$$

input
```
integrate(x*(x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 2/3*(x + 1)/(x^2 - x + 1) + 1
/2*log(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 16.82 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{\ln(x^2-x+1)}{2} - \frac{2x}{3(x^2-x+1)} - \frac{2}{3(x^2-x+1)} + \frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x*(x + x^2 + 1))/(x^2 - x + 1)^2,x)
```

output
```
log(x^2 - x + 1)/2 - (2*x)/(3*(x^2 - x + 1)) - 2/(3*(x^2 - x + 1)) + (11*3
^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.04

$$\int \frac{x(1+x+x^2)}{(1-x+x^2)^2}\,dx = \frac{22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 - 22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x + 22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right) + 9\log(x^2-x+1)\,x^2 - 9\log(x^2-x+}{18x^2-18x+18}$$

input
```
int(x*(x^2+x+1)/(x^2-x+1)^2,x)
```

output
```
(22*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 - 22*sqrt(3)*atan((2*x - 1)/sqrt(
3))*x + 22*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 9*log(x**2 - x + 1)*x**2 - 9*
log(x**2 - x + 1)*x + 9*log(x**2 - x + 1) - 12*x**2 - 24)/(18*(x**2 - x +
1))
```

3.19 $\int \frac{1+x+x^2}{(1-x+x^2)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{1+x+x^2}{(1-x+x^2)^2} dx = -\frac{2(2-x)}{3(1-x+x^2)} - \frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

output
```
(-4+2*x)/(3*x^2-3*x+3)-10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1+x+x^2}{(1-x+x^2)^2} dx = \frac{2(-2+x)}{3(1-x+x^2)} + \frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

input
```
Integrate[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

output
```
(2*(-2 + x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{(x^2-x+1)^2}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{1}{3}\int \frac{5}{x^2-x+1}dx-\frac{2(2-x)}{3\,(x^2-x+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{3}\int \frac{1}{x^2-x+1}dx-\frac{2(2-x)}{3\,(x^2-x+1)}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{10}{3}\int \frac{1}{-(2x-1)^2-3}d(2x-1)-\frac{2(2-x)}{3\,(x^2-x+1)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{10\arctan\left(\frac{2x-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2(2-x)}{3\,(x^2-x+1)}$$

input
```
Int[(1 + x + x^2)/(1 - x + x^2)^2,x]
```

output
```
(-2*(2 - x))/(3*(1 - x + x^2)) + (10*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt[3
])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	34
risch	$\frac{\frac{2x}{3}-\frac{4}{3}}{x^2-x+1}+\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	34

input
```
int((x^2+x+1)/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
(2/3*x-4/3)/(x^2-x+1)+10/9*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{2\left(5\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+3x-6\right)}{9\left(x^2-x+1\right)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
2/9*(5*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3*x - 6)/(x^2
 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{2x-4}{3x^2-3x+3}+\frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/(x**2-x+1)**2,x)
```

output
```
(2*x - 4)/(3*x**2 - 3*x + 3) + 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/
9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{\left(1-x+x^2\right)^2}\,dx=\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{2\left(x-2\right)}{3\left(x^2-x+1\right)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{10}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right) + \frac{2\,(x-2)}{3\,(x^2-x+1)}$$

input
```
integrate((x^2+x+1)/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(x - 2)/(x^2 - x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{\frac{2\,x}{3}-\frac{4}{3}}{x^2-x+1} + \frac{10\,\sqrt{3}\,\mathrm{atan}\left(\frac{2\,\sqrt{3}\,x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
int((x + x^2 + 1)/(x^2 - x + 1)^2,x)
```

output
```
((2*x)/3 - 4/3)/(x^2 - x + 1) + (10*3^(1/2)*atan((2*3^(1/2)*x)/3 - 3^(1/2)
/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int \frac{1+x+x^2}{(1-x+x^2)^2}\,dx = \frac{10\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 - 10\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x + 10\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right) + 6x^2 - 6}{9x^2-9x+9}$$

input
```
int((x^2+x+1)/(x^2-x+1)^2,x)
```

output
```
(2*(5*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 - 5*sqrt(3)*atan((2*x - 1)/sqrt
(3))*x + 5*sqrt(3)*atan((2*x - 1)/sqrt(3)) + 3*x**2 - 3))/(9*(x**2 - x + 1
))
```

3.20 $\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2} dx = -\frac{2(1-2x)}{3\left(1-x+x^2\right)} - \frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)$$

output
```
(-2+4*x)/(3*x^2-3*x+3)-11/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+ln(x)-1/2*
ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2} dx = \frac{2(-1+2x)}{3\left(1-x+x^2\right)} + \frac{11\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

output
```
(2*(-1 + 2*x))/(3*(1 - x + x^2)) + (11*ArcTan[(-1 + 2*x)/Sqrt[3]])/(3*Sqrt
[3]) + Log[x] - Log[1 - x + x^2]/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2177, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x(x^2-x+1)^2}dx$$

$$\Big\downarrow \; 2177$$

$$\frac{1}{3}\int \frac{4x+3}{x(x^2-x+1)}dx - \frac{2(1-2x)}{3(x^2-x+1)}$$

$$\Big\downarrow \; 1200$$

$$\frac{1}{3}\int \left(\frac{7-3x}{x^2-x+1}+\frac{3}{x}\right)dx - \frac{2(1-2x)}{3(x^2-x+1)}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}\left(-\frac{11\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\log\left(x^2-x+1\right)+3\log(x)\right)-\frac{2(1-2x)}{3(x^2-x+1)}$$

input
```
Int[(1 + x + x^2)/(x*(1 - x + x^2)^2),x]
```

output
```
(-2*(1 - 2*x))/(3*(1 - x + x^2)) + ((-11*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt[3
] + 3*Log[x] - (3*Log[1 - x + x^2])/2)/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result	size
default	$\ln\left(x\right)-\frac{-\frac{4x}{3}+\frac{2}{3}}{x^{2}-x+1}-\frac{\ln\left(x^{2}-x+1\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	48
risch	$\frac{\frac{4x}{3}-\frac{2}{3}}{x^{2}-x+1}-\frac{\ln\left(4x^{2}-4x+4\right)}{2}+\frac{11\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}+\ln\left(x\right)$	49

input
```
int((x^2+x+1)/x/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(x)-(-4/3*x+2/3)/(x^2-x+1)-1/2*ln(x^2-x+1)+11/9*3^(1/2)*arctan(1/3*(2*x-
1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.29

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{22\,\sqrt{3}(x^2-x+1)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-9\,(x^2-x+1)\log\left(x^2-x+1\right)+18\,(x^2-x+1)\log\left(x\right)+}{18\,(x^2-x+1)}$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(22*sqrt(3)*(x^2 - x + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) - 9*(x^2 - x
+ 1)*log(x^2 - x + 1) + 18*(x^2 - x + 1)*log(x) + 24*x - 12)/(x^2 - x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{4x-2}{3x^2-3x+3}+\log\left(x\right)-\frac{\log\left(x^2-x+1\right)}{2}+\frac{11\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/x/(x**2-x+1)**2,x)
```

output
```
(4*x - 2)/(3*x**2 - 3*x + 3) + log(x) - log(x**2 - x + 1)/2 + 11*sqrt(3)*a
tan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{1+x+x^2}{x(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(2x-1)}{3(x^2-x+1)} - \frac{1}{2}\log\left(x^2-x+1\right) + \log(x)$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
 1/2*log(x^2 - x + 1) + log(x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1+x+x^2}{x(1-x+x^2)^2}\,dx = \frac{11}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) + \frac{2(2x-1)}{3(x^2-x+1)} - \frac{1}{2}\log\left(x^2-x+1\right) + \log(|x|)$$

input
```
integrate((x^2+x+1)/x/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
11/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 2/3*(2*x - 1)/(x^2 - x + 1) -
 1/2*log(x^2 - x + 1) + log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \ln(x) + \frac{\frac{4x}{3}-\frac{2}{3}}{x^2-x+1} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,11\mathrm{i}}{18}\right)$$

input
```
int((x + x^2 + 1)/(x*(x^2 - x + 1)^2),x)
```

output
```
log(x) + ((4*x)/3 - 2/3)/(x^2 - x + 1) - log(x - (3^(1/2)*1i)/2 - 1/2)*((3
^(1/2)*11i)/18 + 1/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*11i)/18 -
1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.18

$$\int \frac{1+x+x^2}{x\left(1-x+x^2\right)^2}\,dx = \frac{22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 - 22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x + 22\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right) - 9\log(x^2-x+1)\,x^2 + 9\log(x^2-x+}{18x^2-18x+18}$$

input
```
int((x^2+x+1)/x/(x^2-x+1)^2,x)
```

output
```
(22*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 - 22*sqrt(3)*atan((2*x - 1)/sqrt(
3))*x + 22*sqrt(3)*atan((2*x - 1)/sqrt(3)) - 9*log(x**2 - x + 1)*x**2 + 9*
log(x**2 - x + 1)*x - 9*log(x**2 - x + 1) + 18*log(x)*x**2 - 18*log(x)*x +
 18*log(x) + 24*x**2 + 12)/(18*(x**2 - x + 1))
```

3.21 $\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}-\frac{7\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

output
```
-1/x+2*(1+x)/(3*x^2-3*x+3)-7/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2)+3*ln(x)
-3/2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = -\frac{1}{x}+\frac{2(1+x)}{3\left(1-x+x^2\right)}+\frac{7\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}+3\log(x)-\frac{3}{2}\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

output
```
-x^(-1) + (2*(1 + x))/(3*(1 - x + x^2)) + (7*ArcTan[(-1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) + 3*Log[x] - (3*Log[1 - x + x^2])/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = $\{2177, 2159, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^2 (x^2-x+1)^2} \, dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3} \int \frac{2x^2+6x+3}{x^2 (x^2-x+1)} dx + \frac{2(x+1)}{3 (x^2-x+1)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3} \int \left(\frac{8-9x}{x^2-x+1} + \frac{9}{x} + \frac{3}{x^2}\right) dx + \frac{2(x+1)}{3 (x^2-x+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(-\frac{7 \arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{9}{2} \log\left(x^2-x+1\right) - \frac{3}{x} + 9 \log(x)\right) + \frac{2(x+1)}{3 (x^2-x+1)}$$

input
```
Int[(1 + x + x^2)/(x^2*(1 - x + x^2)^2),x]
```

output
```
(2*(1 + x))/(3*(1 - x + x^2)) + (-3/x - (7*ArcTan[(1 - 2*x)/Sqrt[3]])/Sqrt
[3] + 9*Log[x] - (9*Log[1 - x + x^2])/2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{x}+3\ln\left(x\right)-\frac{-\frac{2x}{3}-\frac{2}{3}}{x^{2}-x+1}-\frac{3\ln\left(x^{2}-x+1\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	55
risch	$\frac{-\frac{1}{3}x^{2}+\frac{5}{3}x-1}{x\left(x^{2}-x+1\right)}+3\ln\left(x\right)-\frac{3\ln\left(49x^{2}-49x+49\right)}{2}+\frac{7\sqrt{3}\,\arctan\left(\frac{2\left(7x-\frac{7}{2}\right)\sqrt{3}}{21}\right)}{9}$	59

input
```
int((x^2+x+1)/x^2/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x+3*ln(x)-(-2/3*x-2/3)/(x^2-x+1)-3/2*ln(x^2-x+1)+7/9*3^(1/2)*arctan(1/3
*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{14\sqrt{3}(x^3-x^2+x)\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right)-6\,x^2-27\,(x^3-x^2+x)\log\left(x^2-x+1\right)+54\,(x^3-x^2+x}{18\,(x^3-x^2+x)}$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
1/18*(14*sqrt(3)*(x^3 - x^2 + x)*arctan(1/3*sqrt(3)*(2*x - 1)) - 6*x^2 - 2
7*(x^3 - x^2 + x)*log(x^2 - x + 1) + 54*(x^3 - x^2 + x)*log(x) + 30*x - 18
)/(x^3 - x^2 + x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{-x^2+5x-3}{3x^3-3x^2+3x}+3\log\left(x\right) - \frac{3\log\left(x^2-x+1\right)}{2}+\frac{7\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

input
```
integrate((x**2+x+1)/x**2/(x**2-x+1)**2,x)
```

output
```
(-x**2 + 5*x - 3)/(3*x***3 - 3*x***2 + 3*x) + 3*log(x) - 3*log(x**2 - x + 1)
/2 + 7*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x^2-5x+3}{3\left(x^3-x^2+x\right)} - \frac{3}{2}\log\left(x^2-x+1\right) + 3\log\left(x\right)$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
 + x) - 3/2*log(x^2 - x + 1) + 3*log(x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{7}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{x^2-5x+3}{3\left(x^3-x^2+x\right)} - \frac{3}{2}\log\left(x^2-x+1\right) + 3\log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^2/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
7/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/3*(x^2 - 5*x + 3)/(x^3 - x^2
 + x) - 3/2*log(x^2 - x + 1) + 3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.11

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = 3\,\ln(x) - \frac{\frac{x^2}{3}-\frac{5\,x}{3}+1}{x^3-x^2+x} - \ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right) + \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{3}{2}+\frac{\sqrt{3}\,7\mathrm{i}}{18}\right)$$

input
```
int((x + x^2 + 1)/(x^2*(x^2 - x + 1)^2),x)
```

output
```
3*log(x) - (x^2/3 - (5*x)/3 + 1)/(x - x^2 + x^3) - log(x - (3^(1/2)*1i)/2
- 1/2)*((3^(1/2)*7i)/18 + 3/2) + log(x + (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*7
i)/18 - 3/2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.23

$$\int \frac{1+x+x^2}{x^2\left(1-x+x^2\right)^2}\,dx = \frac{14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^3 - 14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2 + 14\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x - 27\log(x^2-x+1)\,x^3 + 27\log(x^2}{18x\,(x^2-x}$$

input
```
int((x^2+x+1)/x^2/(x^2-x+1)^2,x)
```

output
```
(14*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**3 - 14*sqrt(3)*atan((2*x - 1)/sqrt(
3))*x**2 + 14*sqrt(3)*atan((2*x - 1)/sqrt(3))*x - 27*log(x**2 - x + 1)*x**
3 + 27*log(x**2 - x + 1)*x**2 - 27*log(x**2 - x + 1)*x + 54*log(x)*x**3 -
54*log(x)*x**2 + 54*log(x)*x - 6*x**3 + 24*x - 18)/(18*x*(x**2 - x + 1))
```

3.22 $\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = -\frac{1}{2x^2}-\frac{3}{x}+\frac{2(2-x)}{3(1-x+x^2)}+\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x)-2\log\left(1-x+x^2\right)$$

output
```
-1/2/x^2-3/x+2*(2-x)/(3*x^2-3*x+3)+10/9*arctan(1/3*(1-2*x)*3^(1/2))*3^(1/2
)+4*ln(x)-2*ln(x^2-x+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{1+x+x^2}{x^3(1-x+x^2)^2}\,dx = -\frac{1}{2x^2}-\frac{3}{x}-\frac{2(-2+x)}{3(1-x+x^2)}-\frac{10\arctan\left(\frac{-1+2x}{\sqrt{3}}\right)}{3\sqrt{3}} + 4\log(x)-2\log\left(1-x+x^2\right)$$

input
```
Integrate[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

output
```
-1/2*1/x^2 - 3/x - (2*(-2 + x))/(3*(1 - x + x^2)) - (10*ArcTan[(-1 + 2*x)/
Sqrt[3]])/(3*Sqrt[3]) + 4*Log[x] - 2*Log[1 - x + x^2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2177, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x+1}{x^3\left(x^2-x+1\right)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{3}\int \frac{-2x^3+6x^2+6x+3}{x^3\left(x^2-x+1\right)}dx+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{3}\int \left(\frac{1-12x}{x^2-x+1}+\frac{12}{x}+\frac{9}{x^2}+\frac{3}{x^3}\right)dx+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{10\arctan\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2x^2}-6\log\left(x^2-x+1\right)-\frac{9}{x}+12\log(x)\right)+\frac{2(2-x)}{3\left(x^2-x+1\right)}$$

input
```
Int[(1 + x + x^2)/(x^3*(1 - x + x^2)^2),x]
```

output
```
(2*(2 - x))/(3*(1 - x + x^2)) + (-3/(2*x^2) - 9/x + (10*ArcTan[(1 - 2*x)/S
qrt[3]])/Sqrt[3] + 12*Log[x] - 6*Log[1 - x + x^2])/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2x^{2}}-\frac{3}{x}+4\ln\left(x\right)-\frac{\frac{2x}{3}-\frac{4}{3}}{x^{2}-x+1}-2\ln\left(x^{2}-x+1\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)}{9}$	60
risch	$\frac{-\frac{11}{3}x^{3}+\frac{23}{6}x^{2}-\frac{5}{2}x-\frac{1}{2}}{x^{2}\left(x^{2}-x+1\right)}+4\ln\left(x\right)-2\ln\left(100x^{2}-100x+100\right)-\frac{10\sqrt{3}\,\arctan\left(\frac{(10x-5)\sqrt{3}}{15}\right)}{9}$	64

input
```
int((x^2+x+1)/x^3/(x^2-x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-3/x+4*ln(x)-(2/3*x-4/3)/(x^2-x+1)-2*ln(x^2-x+1)-10/9*3^(1/2)*arct
an(1/3*(2*x-1)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.44

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{66\,x^3+20\,\sqrt{3}(x^4-x^3+x^2)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x-1)\right)-69\,x^2+36\,(x^4-x^3+x^2)\log\left(x^2-x+1\right)-72}{18\,(x^4-x^3+x^2)}$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="fricas")
```

output
```
-1/18*(66*x^3 + 20*sqrt(3)*(x^4 - x^3 + x^2)*arctan(1/3*sqrt(3)*(2*x - 1))
 - 69*x^2 + 36*(x^4 - x^3 + x^2)*log(x^2 - x + 1) - 72*(x^4 - x^3 + x^2)*l
og(x) + 45*x + 9)/(x^4 - x^3 + x^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = 4\log\left(x\right)-2\log\left(x^2-x+1\right) - \frac{10\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}+\frac{-22x^3+23x^2-15x-3}{6x^4-6x^3+6x^2}$$

input
```
integrate((x**2+x+1)/x**3/(x**2-x+1)**2,x)
```

output
```
4*log(x) - 2*log(x**2 - x + 1) - 10*sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3
)/9 + (-22*x**3 + 23*x**2 - 15*x - 3)/(6*x**4 - 6*x**3 + 6*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3-23x^2+15x+3}{6\left(x^4-x^3+x^2\right)} - 2\log\left(x^2-x+1\right) + 4\log\left(x\right)$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="maxima")
```

output
```
-10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x
+ 3)/(x^4 - x^3 + x^2) - 2*log(x^2 - x + 1) + 4*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = -\frac{10}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{22x^3-23x^2+15x+3}{6\left(x^2-x+1\right)x^2} - 2\log\left(x^2-x+1\right) + 4\log\left(|x|\right)$$

input
```
integrate((x^2+x+1)/x^3/(x^2-x+1)^2,x, algorithm="giac")
```

output
```
-10/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) - 1/6*(22*x^3 - 23*x^2 + 15*x
+ 3)/((x^2 - x + 1)*x^2) - 2*log(x^2 - x + 1) + 4*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = 4\,\ln\left(x\right)+\ln\left(x-\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(-2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right) - \ln\left(x-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(2+\frac{\sqrt{3}\,5\mathrm{i}}{9}\right)-\frac{\frac{11\,x^3}{3}-\frac{23\,x^2}{6}+\frac{5\,x}{2}+\frac{1}{2}}{x^4-x^3+x^2}$$

input
```
int((x + x^2 + 1)/(x^3*(x^2 - x + 1)^2),x)
```

output
```
4*log(x) + log(x - (3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 - 2) - log(x + (3
^(1/2)*1i)/2 - 1/2)*((3^(1/2)*5i)/9 + 2) - ((5*x)/2 - (23*x^2)/6 + (11*x^3
)/3 + 1/2)/(x^2 - x^3 + x^4)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.16

$$\int \frac{1+x+x^2}{x^3\left(1-x+x^2\right)^2}\,dx = \frac{-20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^4+20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^3-20\sqrt{3}\,atan\left(\frac{2x-1}{\sqrt{3}}\right)x^2-36\log(x^2-x+1)\,x^4+36\log(x}{18x^2\,(x}$$

input
```
int((x^2+x+1)/x^3/(x^2-x+1)^2,x)
```

output
```
( - 20*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**4 + 20*sqrt(3)*atan((2*x - 1)/sq
rt(3))*x**3 - 20*sqrt(3)*atan((2*x - 1)/sqrt(3))*x**2 - 36*log(x**2 - x +
1)*x**4 + 36*log(x**2 - x + 1)*x**3 - 36*log(x**2 - x + 1)*x**2 + 72*log(x
)*x**4 - 72*log(x)*x**3 + 72*log(x)*x**2 - 66*x**4 + 3*x**2 - 45*x - 9)/(1
8*x**2*(x**2 - x + 1))
```

3.23 $\int (g+hx)^3\sqrt{a+bx+cx^2}(d+ex+fx^2)\ dx$

Optimal result

Integrand size = 32, antiderivative size = 914

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\ dx$$

$$= \frac{(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg+dh) + 2bg(eg+3dh)) + 6b^3ch^2(20afh + 7b(3fg+eh))}{}$$

$$+ \frac{\left(\frac{33b^2fh}{c} + c\left(28eg - \frac{12fg^2}{h} + 56dh\right) - 2(8bfg + 21beh + 16afh)\right)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{280c^2}$$

$$+ \frac{\left(14ce - 11bf - \frac{6cfg}{h}\right)(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{84c^2} + \frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}$$

$$+ \frac{\left(1155b^4fh^3 - \frac{128c^4(3fg^4 - 7g^2h(eg+12dh))}{h} - 42b^2ch^2(78afh + 35b(3fg+eh)) + 8c^2h(128a^2fh^2 + 343abh}{}$$

$$- \frac{(b^2-4ac)\left(256c^5dg^3 - 33b^5fh^3 - 64c^4g(afg^2 + 3ah(eg+dh) + 2bg(eg+3dh)) + 6b^3ch^2(20afh + 7b\right.}{}$$

output
```
1/1024*(256*c^5*d*g^3-33*b^5*f*h^3-64*c^4*g*(a*f*g^2+3*a*h*(d*h+e*g)+2*b*g
*(3*d*h+e*g))+6*b^3*c*h^2*(20*a*f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h
^2+14*a*b*h*(e*h+3*f*g)+7*b^2*(d*h^2+3*e*g*h+3*f*g^2))+16*c^3*(2*a^2*h^2*(
e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))
*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^6+1/280*(33*b^2*f*h/c+c*(28*e*g-12*f*g^2/
h+56*d*h)-32*a*f*h-42*b*e*h-16*b*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2+1/
84*(14*c*e-11*b*f-6*c*f*g/h)*(h*x+g)^3*(c*x^2+b*x+a)^(3/2)/c^2+1/7*f*(h*x+
g)^4*(c*x^2+b*x+a)^(3/2)/c/h+1/13440*(1155*b^4*f*h^3-128*c^4*(3*f*g^4-7*g^
2*h*(12*d*h+e*g))/h-42*b^2*c*h^2*(78*a*f*h+35*b*(e*h+3*f*g))+8*c^2*h*(128*
a^2*f*h^2+343*a*b*h*(e*h+3*f*g)+b^2*(537*f*g^2+245*h*(d*h+3*e*g)))-16*c^3*
(16*a*h*(15*f*g^2+7*h*(d*h+3*e*g))+b*g*(17*f*g^2+21*h*(25*d*h+19*e*g)))-6*
c*(231*b^3*f*h^3-6*b*c*h^2*(74*a*f*h+49*b*e*h+59*b*f*g)+16*c^3*(3*f*g^3-7*
g*h*(7*d*h+e*g))+8*c^2*h*(5*b*f*g^2+7*b*h*(7*d*h+9*e*g)+a*h*(35*e*h+41*f*g
)))*x)*(c*x^2+b*x+a)^(3/2)/c^5-1/2048*(-4*a*c+b^2)*(256*c^5*d*g^3-33*b^5*f
*h^3-64*c^4*g*(a*f*g^2+3*a*h*(d*h+e*g)+2*b*g*(3*d*h+e*g))+6*b^3*c*h^2*(20*
a*f*h+7*b*(e*h+3*f*g))-8*b*c^2*h*(10*a^2*f*h^2+14*a*b*h*(e*h+3*f*g)+7*b^2*
(d*h^2+3*e*g*h+3*f*g^2))+16*c^3*(2*a^2*h^2*(e*h+3*f*g)+5*b^2*g*(f*g^2+3*h*
(d*h+e*g))+6*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 12.85 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.20

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
Integrate[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^6*f*h^3 + 210*b^5*c*h^2*(63*f*g
+ 21*e*h + 11*f*h*x) - 84*b^4*c*h*(-260*a*f*h^2 + 35*c*h*(6*e*g + 2*d*h +
e*h*x) + c*f*(210*g^2 + 105*g*h*x + 22*h^2*x^2)) - 16*b^2*c^2*(2163*a^2*f*
h^3 - 2*a*c*h*(7*h*(345*e*g + 115*d*h + 56*e*h*x) + 3*f*(805*g^2 + 392*g*h
*x + 81*h^2*x^2)) + 2*c^2*(7*d*h*(180*g^2 + 75*g*h*x + 14*h^2*x^2) + 21*e*
(20*g^3 + 25*g^2*h*x + 14*g*h^2*x^2 + 3*h^3*x^3) + f*x*(175*g^3 + 294*g^2*
h*x + 189*g*h^2*x^2 + 44*h^3*x^3))) + 16*b^3*c^2*(-42*a*h^2*(35*e*h + 3*f*
(35*g + 6*h*x)) + c*(f*(525*g^3 + 735*g^2*h*x + 441*g*h^2*x^2 + 99*h^3*x^3
) + 7*h*(5*d*h*(45*g + 7*h*x) + 3*e*(75*g^2 + 35*g*h*x + 7*h^2*x^2)))) + 3
2*b*c^3*(a^2*h^2*(2373*f*g + 791*e*h + 397*f*h*x) - 2*a*c*(f*(455*g^3 + 60
9*g^2*h*x + 357*g*h^2*x^2 + 79*h^3*x^3) + 7*h*(d*h*(195*g + 29*h*x) + e*(1
95*g^2 + 87*g*h*x + 17*h^2*x^2))) + 4*c^2*(21*d*(10*g^3 + 10*g^2*h*x + 5*g
*h^2*x^2 + h^3*x^3) + x*(7*e*(10*g^3 + 15*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^
3) + f*x*(35*g^3 + 63*g^2*h*x + 42*g*h^2*x^2 + 10*h^3*x^3)))) + 64*c^3*(12
8*a^3*f*h^3 - a^2*c*h*(7*h*(96*e*g + 32*d*h + 15*e*h*x) + f*(672*g^2 + 315
*g*h*x + 64*h^2*x^2)) + 2*a*c^2*(7*d*h*(120*g^2 + 45*g*h*x + 8*h^2*x^2) +
7*e*(40*g^3 + 45*g^2*h*x + 24*g*h^2*x^2 + 5*h^3*x^3) + 3*f*x*(35*g^3 + 56*
g^2*h*x + 35*g*h^2*x^2 + 8*h^3*x^3)) + 4*c^3*x*(21*d*(10*g^3 + 20*g^2*h*x
+ 15*g*h^2*x^2 + 4*h^3*x^3) + x*(7*e*(20*g^3 + 45*g^2*h*x + 36*g*h^2*x^2 +
 10*h^3*x^3) + 3*f*x*(35*g^3 + 84*g^2*h*x + 70*g*h^2*x^2 + 20*h^3*x^3))...
```

Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^3(3bfg-14cdh+8afh+(6cfg-14ceh+11bfh)x)\sqrt{cx^2+bx+a}dx}{7ch^2}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-\frac{\int (g+hx)^3(3bfg-14cdh+8afh+(6cfg-14ceh+11bfh)x)\sqrt{cx^2+bx+a}dx}{14ch}$$

$\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\int -\frac{3}{2}(g+hx)^2(11fghb^2+22afh^2b-2cg(3fg+7eh)b+4ch(14cdg-5afg-7aeh)+(-4(3fg^2-7h(eg+2dh))c^2-2h(8bfg+21beh+16afh)c+33b^2fh$$

$$6c$$

$$14ch$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\int(g+hx)^2(11fghb^2+22afh^2b-2cg(3fg+7eh)b+4ch(14cdg-5afg-7aeh)+(-4(3fg^2-7h(e$$

$$4c$$

$$14ch$$

$\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\int-\frac{1}{2}(g+hx)(99fgh^2b^3+2(66afh^3-cgh(79fg+63eh))b^2+4c(6cfg^3+14ch(4eg+3dh)g-ah^2(95fg+42e$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\int(g+hx$$

$\downarrow$ 1225

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}--\frac{35h(1$$

$\downarrow$ 1087

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\big(1}{}$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\big(1}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{3/2}}{7ch}-$$

$$\frac{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}(11bfh-14ceh+6cfg)}{6c}-\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}\left(-2ch(16afh+21beh+8bfg)+33b^2fh^2-4c^2\left(3fg^2-7h(2dh+eg)\right)\right)}{5c}-\frac{35h\Big(}{}$$

input
```
Int[(g + h*x)^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^4*(a + b*x + c*x^2)^(3/2))/(7*c*h) - (((6*c*f*g - 14*c*e*h +
11*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (((33*b^2*f*h^2 - 2
*c*h*(8*b*f*g + 21*b*e*h + 16*a*f*h) - 4*c^2*(3*f*g^2 - 7*h*(e*g + 2*d*h))
)*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (-1/24*((1155*b^4*f*h^4 - 1
28*c^4*(3*f*g^4 - 7*g^2*h*(e*g + 12*d*h)) - 42*b^2*c*h^3*(78*a*f*h + 35*b*
(3*f*g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 343*a*b*h*(3*f*g + e*h) + b^2*
(537*f*g^2 + 245*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(15*f*g^2 + 7*h*(3*e
*g + d*h)) + b*g*(17*f*g^2 + 21*h*(19*e*g + 25*d*h))) - 6*c*h*(231*b^3*f*h
^3 - 6*b*c*h^2*(59*b*f*g + 49*b*e*h + 74*a*f*h) + 16*c^3*(3*f*g^3 - 7*g*h*
(e*g + 7*d*h)) + 8*c^2*h*(5*b*f*g^2 + 7*b*h*(9*e*g + 7*d*h) + a*h*(41*f*g
+ 35*e*h)))*x)*(a + b*x + c*x^2)^(3/2))/c^2 - (35*h*(256*c^5*d*g^3 - 33*b^
5*f*h^3 - 64*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 2*b*g*(e*g + 3*d*h)) + 6
*b^3*c*h^2*(20*a*f*h + 7*b*(3*f*g + e*h)) - 8*b*c^2*h*(10*a^2*f*h^2 + 14*a
*b*h*(3*f*g + e*h) + 7*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 16*c^3*(2*a^2*h^
2*(3*f*g + e*h) + 5*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 6*a*b*h*(3*f*g^2 + h
*(3*e*g + d*h))))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a
*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/
(16*c^2))/(10*c))/(4*c))/(14*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.92

method	result	size
risch	Expression too large to display	1757
default	Expression too large to display	2090

input
```
int((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/107520*(15360*c^6*f*h^3*x^6+1280*b*c^5*f*h^3*x^5+17920*c^6*e*h^3*x^5+537
60*c^6*f*g*h^2*x^5+3072*a*c^5*f*h^3*x^4-1408*b^2*c^4*f*h^3*x^4+1792*b*c^5*
e*h^3*x^4+5376*b*c^5*f*g*h^2*x^4+21504*c^6*d*h^3*x^4+64512*c^6*e*g*h^2*x^4
+64512*c^6*f*g^2*h*x^4-5056*a*b*c^4*f*h^3*x^3+4480*a*c^5*e*h^3*x^3+13440*a
*c^5*f*g*h^2*x^3+1584*b^3*c^3*f*h^3*x^3-2016*b^2*c^4*e*h^3*x^3-6048*b^2*c^
4*f*g*h^2*x^3+2688*b*c^5*d*h^3*x^3+8064*b*c^5*e*g*h^2*x^3+8064*b*c^5*f*g^2
*h*x^3+80640*c^6*d*g*h^2*x^3+80640*c^6*e*g^2*h*x^3+26880*c^6*f*g^3*x^3-409
6*a^2*c^4*f*h^3*x^2+7776*a*b^2*c^3*f*h^3*x^2-7616*a*b*c^4*e*h^3*x^2-22848*
a*b*c^4*f*g*h^2*x^2+7168*a*c^5*d*h^3*x^2+21504*a*c^5*e*g*h^2*x^2+21504*a*c
^5*f*g^2*h*x^2-1848*b^4*c^2*f*h^3*x^2+2352*b^3*c^3*e*h^3*x^2+7056*b^3*c^3*
f*g*h^2*x^2-3136*b^2*c^4*d*h^3*x^2-9408*b^2*c^4*e*g*h^2*x^2-9408*b^2*c^4*f
*g^2*h*x^2+13440*b*c^5*d*g*h^2*x^2+13440*b*c^5*e*g^2*h*x^2+4480*b*c^5*f*g^
3*x^2+107520*c^6*d*g^2*h*x^2+35840*c^6*e*g^3*x^2+12704*a^2*b*c^3*f*h^3*x-6
720*a^2*c^4*e*h^3*x-20160*a^2*c^4*f*g*h^2*x-12096*a*b^3*c^2*f*h^3*x+12544*
a*b^2*c^3*e*h^3*x+37632*a*b^2*c^3*f*g*h^2*x-12992*a*b*c^4*d*h^3*x-38976*a*
b*c^4*e*g*h^2*x-38976*a*b*c^4*f*g^2*h*x+40320*a*c^5*d*g*h^2*x+40320*a*c^5*
e*g^2*h*x+13440*a*c^5*f*g^3*x+2310*b^5*c*f*h^3*x-2940*b^4*c^2*e*h^3*x-8820
*b^4*c^2*f*g*h^2*x+3920*b^3*c^3*d*h^3*x+11760*b^3*c^3*e*g*h^2*x+11760*b^3*
c^3*f*g^2*h*x-16800*b^2*c^4*d*g*h^2*x-16800*b^2*c^4*e*g^2*h*x-5600*b^2*c^4
*f*g^3*x+26880*b*c^5*d*g^2*h*x+8960*b*c^5*e*g^3*x+53760*c^6*d*g^3*x+819...
```

Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 2817, normalized size of antiderivative = 3.08

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
[1/430080*(105*(16*(16*(b^2*c^5 - 4*a*c^6)*d - 8*(b^3*c^4 - 4*a*b*c^5)*e +
 (5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*f)*g^3 - 24*(16*(b^3*c^4 - 4*a*b*
c^5)*d - 2*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e + (7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*f)*g^2*h + 6*(8*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*
c^5)*d - 4*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e + (21*b^6*c - 140*a
*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*f)*g*h^2 - (8*(7*b^5*c^2 - 40*a*b
^3*c^3 + 48*a^2*b*c^4)*d - 2*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 -
 64*a^3*c^4)*e + (33*b^7 - 252*a*b^5*c + 560*a^2*b^3*c^2 - 320*a^3*b*c^3)*
f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) + 4*(15360*c^7*f*h^3*x^6 + 1280*(42*c^7*f*g*h^
2 + (14*c^7*e + b*c^6*f)*h^3)*x^5 + 128*(504*c^7*f*g^2*h + 42*(12*c^7*e +
b*c^6*f)*g*h^2 + (168*c^7*d + 14*b*c^6*e - (11*b^2*c^5 - 24*a*c^6)*f)*h^3)
*x^4 + 560*(48*b*c^6*d - 8*(3*b^2*c^5 - 8*a*c^6)*e + (15*b^3*c^4 - 52*a*b*
c^5)*f)*g^3 - 168*(80*(3*b^2*c^5 - 8*a*c^6)*d - 10*(15*b^3*c^4 - 52*a*b*c^
5)*e + (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*f)*g^2*h + 42*(40*(15*b
^3*c^4 - 52*a*b*c^5)*d - 4*(105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*e +
 (315*b^5*c^2 - 1680*a*b^3*c^3 + 1808*a^2*b*c^4)*f)*g*h^2 - (56*(105*b^4*c
^3 - 460*a*b^2*c^4 + 256*a^2*c^5)*d - 14*(315*b^5*c^2 - 1680*a*b^3*c^3 + 1
808*a^2*b*c^4)*e + (3465*b^6*c - 21840*a*b^4*c^2 + 34608*a^2*b^2*c^3 - 819
2*a^3*c^4)*f)*h^3 + 16*(1680*c^7*f*g^3 + 504*(10*c^7*e + b*c^6*f)*g^2*h...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4820 vs. $2(959) = 1918$.

Time = 1.40 (sec) , antiderivative size = 4820, normalized size of antiderivative = 5.27

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)**3*(c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**6/7 + x**5*(b*f*h**3/14 + c*e
*h**3 + 3*c*f*g*h**2)/(6*c) + x**4*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 -
 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*
h**2 + 3*c*f*g**2*h)/(5*c) + x**3*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3
/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**
2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*
h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*
c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*g**3)/(4*c) + x**2*(a*d*h**3 + 3*a*
e*g*h**2 + 3*a*f*g**2*h - 4*a*(a*f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b
*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2
+ 3*c*f*g**2*h)/(5*c) + 3*b*d*g*h**2 + 3*b*e*g**2*h + b*f*g**3 - 7*b*(a*e*
h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(6*c) +
b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*f*h**3/7 + b*e*h**3 + 3*b*
f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*g*h**2)/(12*c) + c*d*h**3
+ 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d*g*h**2 + 3*c*e*g**2*h + c*f*
g**3)/(8*c) + 3*c*d*g**2*h + c*e*g**3)/(3*c) + x*(3*a*d*g*h**2 + 3*a*e*g**
2*h + a*f*g**3 - 3*a*(a*e*h**3 + 3*a*f*g*h**2 - 5*a*(b*f*h**3/14 + c*e*h**
3 + 3*c*f*g*h**2)/(6*c) + b*d*h**3 + 3*b*e*g*h**2 + 3*b*f*g**2*h - 9*b*(a*
f*h**3/7 + b*e*h**3 + 3*b*f*g*h**2 - 11*b*(b*f*h**3/14 + c*e*h**3 + 3*c*f*
g*h**2)/(12*c) + c*d*h**3 + 3*c*e*g*h**2 + 3*c*f*g**2*h)/(10*c) + 3*c*d...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1657, normalized size of antiderivative = 1.81

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*f*h^3*x + (42*c^6*f*g*h
^2 + 14*c^6*e*h^3 + b*c^5*f*h^3)/c^6)*x + (504*c^6*f*g^2*h + 504*c^6*e*g*h
^2 + 42*b*c^5*f*g*h^2 + 168*c^6*d*h^3 + 14*b*c^5*e*h^3 - 11*b^2*c^4*f*h^3
+ 24*a*c^5*f*h^3)/c^6)*x + (1680*c^6*f*g^3 + 5040*c^6*e*g^2*h + 504*b*c^5*
f*g^2*h + 5040*c^6*d*g*h^2 + 504*b*c^5*e*g*h^2 - 378*b^2*c^4*f*g*h^2 + 840
*a*c^5*f*g*h^2 + 168*b*c^5*d*h^3 - 126*b^2*c^4*e*h^3 + 280*a*c^5*e*h^3 + 9
9*b^3*c^3*f*h^3 - 316*a*b*c^4*f*h^3)/c^6)*x + (4480*c^6*e*g^3 + 560*b*c^5*
f*g^3 + 13440*c^6*d*g^2*h + 1680*b*c^5*e*g^2*h - 1176*b^2*c^4*f*g^2*h + 26
88*a*c^5*f*g^2*h + 1680*b*c^5*d*g*h^2 - 1176*b^2*c^4*e*g*h^2 + 2688*a*c^5*
e*g*h^2 + 882*b^3*c^3*f*g*h^2 - 2856*a*b*c^4*f*g*h^2 - 392*b^2*c^4*d*h^3 +
 896*a*c^5*d*h^3 + 294*b^3*c^3*e*h^3 - 952*a*b*c^4*e*h^3 - 231*b^4*c^2*f*h
^3 + 972*a*b^2*c^3*f*h^3 - 512*a^2*c^4*f*h^3)/c^6)*x + (26880*c^6*d*g^3 +
4480*b*c^5*e*g^3 - 2800*b^2*c^4*f*g^3 + 6720*a*c^5*f*g^3 + 13440*b*c^5*d*g
^2*h - 8400*b^2*c^4*e*g^2*h + 20160*a*c^5*e*g^2*h + 5880*b^3*c^3*f*g^2*h -
 19488*a*b*c^4*f*g^2*h - 8400*b^2*c^4*d*g*h^2 + 20160*a*c^5*d*g*h^2 + 5880
*b^3*c^3*e*g*h^2 - 19488*a*b*c^4*e*g*h^2 - 4410*b^4*c^2*f*g*h^2 + 18816*a*
b^2*c^3*f*g*h^2 - 10080*a^2*c^4*f*g*h^2 + 1960*b^3*c^3*d*h^3 - 6496*a*b*c^
4*d*h^3 - 1470*b^4*c^2*e*h^3 + 6272*a*b^2*c^3*e*h^3 - 3360*a^2*c^4*e*h^3 +
 1155*b^5*c*f*h^3 - 6048*a*b^3*c^2*f*h^3 + 6352*a^2*b*c^3*f*h^3)/c^6)*x +
(26880*b*c^5*d*g^3 - 13440*b^2*c^4*e*g^3 + 35840*a*c^5*e*g^3 + 8400*b^3...
```

Mupad [B] (verification not implemented)

Time = 29.03 (sec) , antiderivative size = 3262, normalized size of antiderivative = 3.57

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((g + h*x)^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g^3*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (8*a^3*f*h^3*(a + b*x + c*
x^2)^(1/2))/(105*c^3) - (33*b^6*f*h^3*(a + b*x + c*x^2)^(1/2))/(1024*c^6)
+ (d*h^3*x^2*(a + b*x + c*x^2)^(3/2))/(5*c) + (e*h^3*x^3*(a + b*x + c*x^2)
^(3/2))/(6*c) + (f*h^3*x^4*(a + b*x + c*x^2)^(3/2))/(7*c) - (a*f*g^3*((x/2
 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x
+ c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^3*log((b/2 + c*x
)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^3*l
og((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c
^(5/2)) - (2*a*d*h^3*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2)
)*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a
+ b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*f*g^3*((log((b + 2*c*x)/c^(1
/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a
+ c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (e
*g^3*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)
 + (33*b^7*f*h^3*log(b + 2*c^(1/2)*(a + b*x + c*x^2)^(1/2) + 2*c*x))/(2048
*c^(13/2)) + (f*g^3*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*e*h^3*((5*b*((lo
g((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^
(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24
*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x/2 + b/(4*c))*(a
 + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1...
```

Reduce [F]

$$\int (g+hx)^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int (hx+g)^3\sqrt{c\,x^2+bx+a}\,\left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^3*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

3.24 $\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 575

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg+2dh))+8c\ldots}{512c^5}$$
$$+\frac{\left(4ce-3bf-\frac{2cfg}{h}\right)(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{20c^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$
$$-\frac{\left(105b^3fh^2+\frac{64c^3(fg^3-2gh(eg+5dh))}{h}-28bch(7afh+5b(2fg+eh))+8c^2(7bfg^2+25bh(2eg+dh)+16\ldots\right.}{\ldots}$$
$$-\frac{(b^2-4ac)\left(128c^4dg^2+21b^4fh^2-28b^2ch(2bfg+beh+2afh)-32c^3(afg^2+ah(2eg+dh)+2bg(eg\ldots\right.}{1024c^{11/2}\ldots}$$

output
```
1/512*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g)-32*c^
3*(a*f*g^2+a*h*(d*h+2*e*g)+2*b*g*(2*d*h+e*g))+8*c^2*(2*a^2*f*h^2+6*a*b*h*(
e*h+2*f*g)+5*b^2*(f*g^2+h*(d*h+2*e*g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5
+1/20*(4*c*e-3*b*f-2*c*f*g/h)*(h*x+g)^2*(c*x^2+b*x+a)^(3/2)/c^2+1/6*f*(h*x
+g)^3*(c*x^2+b*x+a)^(3/2)/c/h-1/960*(105*b^3*f*h^2+64*c^3*(f*g^3-2*g*h*(5*
d*h+e*g))/h-28*b*c*h*(7*a*f*h+5*b*(e*h+2*f*g))+8*c^2*(7*b*f*g^2+25*b*h*(d*
h+2*e*g)+16*a*h*(e*h+2*f*g))-6*c*(21*b^2*f*h^2-4*c*h*(5*a*f*h+7*b*e*h+2*b*
f*g)-8*c^2*(f*g^2-h*(5*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(3/2)/c^4-1/1024*(-4*
a*c+b^2)*(128*c^4*d*g^2+21*b^4*f*h^2-28*b^2*c*h*(2*a*f*h+b*e*h+2*b*f*g)-32
*c^3*(a*f*g^2+a*h*(d*h+2*e*g)+2*b*g*(2*d*h+e*g))+8*c^2*(2*a^2*f*h^2+6*a*b*
h*(e*h+2*f*g)+5*b^2*(f*g^2+h*(d*h+2*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 11.26 (sec) , antiderivative size = 705, normalized size of antiderivative = 1.23

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \frac{3840c^{9/2}dg^2(b+2cx)\sqrt{a+x(b+cx)}+5120c^{9/2}g(eg+2dh)(a+x(b+cx))^{3/2}+3840c^{9/2}(fg^2+h(2eg}{}$$

input
```
Integrate[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(3840*c^(9/2)*d*g^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + 5120*c^(9/2)*g*(e*
g + 2*d*h)*(a + x*(b + c*x))^(3/2) + 3840*c^(9/2)*(f*g^2 + h*(2*e*g + d*h)
)*x*(a + x*(b + c*x))^(3/2) + 3072*c^(9/2)*h*(2*f*g + e*h)*x^2*(a + x*(b +
 c*x))^(3/2) + 2560*c^(9/2)*f*h^2*x^3*(a + x*(b + c*x))^(3/2) - 1920*c^4*(
b^2 - 4*a*c)*d*g^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
- 960*b*c^3*g*(e*g + 2*d*h)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] -
 (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]) + 4
*c*h*(2*f*g + e*h)*(-16*c^(3/2)*(-35*b^2 + 32*a*c + 42*b*c*x)*(a + x*(b +
c*x))^(3/2) - 15*b*(7*b^2 - 12*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b +
 c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])) - 40*c^2*(f*g^2 + h*(2*e*g + d*h))*(80*b*c^(3/2)*(a + x*(b + c*x))^(
3/2) - 3*(5*b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - f*h
^2*(2304*b*c^(7/2)*x^2*(a + x*(b + c*x))^(3/2) + 16*c^(3/2)*(105*b^3 - 196
*a*b*c - 126*b^2*c*x + 120*a*c^2*x)*(a + x*(b + c*x))^(3/2) - 15*(21*b^4 -
 56*a*b^2*c + 16*a^2*c^2)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (
b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(15
360*c^(11/2))
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{3}{2}h(g+hx)^2(bfg-4cdh+2afh+(2cfg-4ceh+3bfh)x)\sqrt{cx^2+bx+a}dx}{6ch^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\int (g+hx)^2(bfg-4cdh+2afh+(2cfg-4ceh+3bfh)x)\sqrt{cx^2+bx+a}dx}{4ch}$$

$\Big\downarrow$ 1236

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{\int -\frac{1}{2}(g+hx)\left(9fghb^2+12afh^2b-4cg(fg+3eh)b+4ch(10cdg-3afg-4aeh)+\left(-8\left(fg^2-h(2eg+5dh)\right)c^2-4h(2bfg+7beh+5afh)c+21b^2fh^2\right)x\right)\sqrt{\cdots}}{5c}}{4ch}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{\int (g+hx)\left(9fghb^2+12afh^2b-4cg(fg+3eh)b+4ch(10cdg-3afg-4aeh)+\left(-8\left(fg^2-h(2eg+5dh)\cdots\right)\right)\right)}{10c}}{4ch}$$

$\Big\downarrow$ 1225

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+\cdots\right)\right)}{16c^2}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+\cdots\right)\right)}{\cdots}}{16c^2}}{\cdots}$$

$\Big\downarrow$ 1092

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{\frac{5h\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b^2\left(h(dh+2eg)+fg^2\right)\right)-28b^2ch(2afh+beh+2bfg)-32c^3\left(ah(dh+2eg)+\cdots\right)\right)}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 219

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}{6ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}(3bfh-4ceh+2cfg)}{5c}-\frac{5h\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)\left(8c^2\left(2a^2fh^2+6abh(eh+2fg)+5b\right.\right.}{}}{}$$

input
```
Int[(g + h*x)^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + b*x + c*x^2)^(3/2))/(6*c*h) - ((((2*c*f*g - 4*c*e*h + 3
*b*f*h)*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c) - (-1/24*((105*b^3*f*h^
3 + 64*c^3*(f*g^3 - 2*g*h*(e*g + 5*d*h)) - 28*b*c*h^2*(7*a*f*h + 5*b*(2*f*
g + e*h)) + 8*c^2*h*(7*b*f*g^2 + 25*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*
h)) - 6*c*h*(21*b^2*f*h^2 - 4*c*h*(2*b*f*g + 7*b*e*h + 5*a*f*h) - 8*c^2*(f
*g^2 - h*(2*e*g + 5*d*h)))*x)*(a + b*x + c*x^2)^(3/2))/c^2 + (5*h*(128*c^4
*d*g^2 + 21*b^4*f*h^2 - 28*b^2*c*h*(2*b*f*g + b*e*h + 2*a*f*h) - 32*c^3*(a
*f*g^2 + a*h*(2*e*g + d*h) + 2*b*g*(e*g + 2*d*h)) + 8*c^2*(2*a^2*f*h^2 + 6
*a*b*h*(2*f*g + e*h) + 5*b^2*(f*g^2 + h*(2*e*g + d*h))))*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c^2))/(10*c))/(4*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 1027, normalized size of antiderivative = 1.79

method	result	size
risch	Expression too large to display	1027
default	Expression too large to display	1208

input

```
int((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output

```
1/7680*(1280*c^5*f*h^2*x^5+128*b*c^4*f*h^2*x^4+1536*c^5*e*h^2*x^4+3072*c^5
*f*g*h*x^4+320*a*c^4*f*h^2*x^3-144*b^2*c^3*f*h^2*x^3+192*b*c^4*e*h^2*x^3+3
84*b*c^4*f*g*h*x^3+1920*c^5*d*h^2*x^3+3840*c^5*e*g*h*x^3+1920*c^5*f*g^2*x^
3-544*a*b*c^3*f*h^2*x^2+512*a*c^4*e*h^2*x^2+1024*a*c^4*f*g*h*x^2+168*b^3*c
^2*f*h^2*x^2-224*b^2*c^3*e*h^2*x^2-448*b^2*c^3*f*g*h*x^2+320*b*c^4*d*h^2*x
^2+640*b*c^4*e*g*h*x^2+320*b*c^4*f*g^2*x^2+5120*c^5*d*g*h*x^2+2560*c^5*e*g
^2*x^2-480*a^2*c^3*f*h^2*x+896*a*b^2*c^2*f*h^2*x-928*a*b*c^3*e*h^2*x-1856*
a*b*c^3*f*g*h*x+960*a*c^4*d*h^2*x+1920*a*c^4*e*g*h*x+960*a*c^4*f*g^2*x-210
*b^4*c*f*h^2*x+280*b^3*c^2*e*h^2*x+560*b^3*c^2*f*g*h*x-400*b^2*c^3*d*h^2*x
-800*b^2*c^3*e*g*h*x-400*b^2*c^3*f*g^2*x+1280*b*c^4*d*g*h*x+640*b*c^4*e*g^
2*x+3840*c^5*d*g^2*x+1808*a^2*b*c^2*f*h^2-1024*a^2*c^3*e*h^2-2048*a^2*c^3*
f*g*h-1680*a*b^3*c*f*h^2+1840*a*b^2*c^2*e*h^2+3680*a*b^2*c^2*f*g*h-2080*a*
b*c^3*d*h^2-4160*a*b*c^3*e*g*h-2080*a*b*c^3*f*g^2+5120*a*c^4*d*g*h+2560*a*
c^4*e*g^2+315*b^5*f*h^2-420*b^4*c*e*h^2-840*b^4*c*f*g*h+600*b^3*c^2*d*h^2+
1200*b^3*c^2*e*g*h+600*b^3*c^2*f*g^2-1920*b^2*c^3*d*g*h-960*b^2*c^3*e*g^2+
1920*b*c^4*d*g^2)*(c*x^2+b*x+a)^(1/2)/c^5+1/1024*(64*a^3*c^3*f*h^2-240*a^2
*b^2*c^2*f*h^2+192*a^2*b*c^3*e*h^2+384*a^2*b*c^3*f*g*h-128*a^2*c^4*d*h^2-2
56*a^2*c^4*e*g*h-128*a^2*c^4*f*g^2+140*a*b^4*c*f*h^2-160*a*b^3*c^2*e*h^2-3
20*a*b^3*c^2*f*g*h+192*a*b^2*c^3*d*h^2+384*a*b^2*c^3*e*g*h+192*a*b^2*c^3*f
*g^2-512*a*b*c^4*d*g*h-256*a*b*c^4*e*g^2+512*a*c^5*d*g^2-21*b^6*f*h^2+2...
```

Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1791, normalized size of antiderivative = 3.11

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
[-1/30720*(15*(8*(16*(b^2*c^4 - 4*a*c^5)*d - 8*(b^3*c^3 - 4*a*b*c^4)*e + (
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g^2 - 8*(16*(b^3*c^3 - 4*a*b*c^4
)*d - 2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + (7*b^5*c - 40*a*b^3*c^
2 + 48*a^2*b*c^3)*f)*g*h + (8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*d -
4*(7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e + (21*b^6 - 140*a*b^4*c + 240*
a^2*b^2*c^2 - 64*a^3*c^3)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 -
 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*h^2*
x^5 + 128*(24*c^6*f*g*h + (12*c^6*e + b*c^5*f)*h^2)*x^4 + 16*(120*c^6*f*g^
2 + 24*(10*c^6*e + b*c^5*f)*g*h + (120*c^6*d + 12*b*c^5*e - (9*b^2*c^4 - 2
0*a*c^5)*f)*h^2)*x^3 + 40*(48*b*c^5*d - 8*(3*b^2*c^4 - 8*a*c^5)*e + (15*b^
3*c^3 - 52*a*b*c^4)*f)*g^2 - 8*(80*(3*b^2*c^4 - 8*a*c^5)*d - 10*(15*b^3*c^
3 - 52*a*b*c^4)*e + (105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*c^4)*f)*g*h + (
40*(15*b^3*c^3 - 52*a*b*c^4)*d - 4*(105*b^4*c^2 - 460*a*b^2*c^3 + 256*a^2*
c^4)*e + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f)*h^2 + 8*(40*(8*c
^6*e + b*c^5*f)*g^2 + 8*(80*c^6*d + 10*b*c^5*e - (7*b^2*c^4 - 16*a*c^5)*f)
*g*h + (40*b*c^5*d - 4*(7*b^2*c^4 - 16*a*c^5)*e + (21*b^3*c^3 - 68*a*b*c^4
)*f)*h^2)*x^2 + 2*(40*(48*c^6*d + 8*b*c^5*e - (5*b^2*c^4 - 12*a*c^5)*f)*g^
2 + 8*(80*b*c^5*d - 10*(5*b^2*c^4 - 12*a*c^5)*e + (35*b^3*c^3 - 116*a*b*c^
4)*f)*g*h - (40*(5*b^2*c^4 - 12*a*c^5)*d - 4*(35*b^3*c^3 - 116*a*b*c^4)*e
+ (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f)*h^2)*x)*sqrt(c*x^2 + b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2440 vs. $2(597) = 1194$.

Time = 1.22 (sec) , antiderivative size = 2440, normalized size of antiderivative = 4.24

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)**2*(c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**5/6 + x**4*(b*f*h**2/12 + c*e
*h**2 + 2*c*f*g*h)/(5*c) + x**3*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(
b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g*
*2)/(4*c) + x**2*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c
*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h*
*2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**
2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**2)/(3*c) + x*(a*d*h**
2 + 2*a*e*g*h + a*f*g**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b
*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**
2)/(4*c) + 2*b*d*g*h + b*e*g**2 - 5*b*(a*e*h**2 + 2*a*f*g*h - 4*a*(b*f*h**
2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g*h + b*f*g**2 - 7*b
*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2 + 2*c*f*
g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) + 2*c*d*g*h + c*e*g**
2)/(6*c) + c*d*g**2)/(2*c) + (2*a*d*g*h + a*e*g**2 - 2*a*(a*e*h**2 + 2*a*f
*g*h - 4*a*(b*f*h**2/12 + c*e*h**2 + 2*c*f*g*h)/(5*c) + b*d*h**2 + 2*b*e*g
*h + b*f*g**2 - 7*b*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12
+ c*e*h**2 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(8*c) +
2*c*d*g*h + c*e*g**2)/(3*c) + b*d*g**2 - 3*b*(a*d*h**2 + 2*a*e*g*h + a*f*g
**2 - 3*a*(a*f*h**2/6 + b*e*h**2 + 2*b*f*g*h - 9*b*(b*f*h**2/12 + c*e*h**2
 + 2*c*f*g*h)/(10*c) + c*d*h**2 + 2*c*e*g*h + c*f*g**2)/(4*c) + 2*b*d*g...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.71

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f*h^2*x + (24*c^5*f*g*h + 12*
c^5*e*h^2 + b*c^4*f*h^2)/c^5)*x + (120*c^5*f*g^2 + 240*c^5*e*g*h + 24*b*c^
4*f*g*h + 120*c^5*d*h^2 + 12*b*c^4*e*h^2 - 9*b^2*c^3*f*h^2 + 20*a*c^4*f*h^
2)/c^5)*x + (320*c^5*e*g^2 + 40*b*c^4*f*g^2 + 640*c^5*d*g*h + 80*b*c^4*e*g
*h - 56*b^2*c^3*f*g*h + 128*a*c^4*f*g*h + 40*b*c^4*d*h^2 - 28*b^2*c^3*e*h^
2 + 64*a*c^4*e*h^2 + 21*b^3*c^2*f*h^2 - 68*a*b*c^3*f*h^2)/c^5)*x + (1920*c
^5*d*g^2 + 320*b*c^4*e*g^2 - 200*b^2*c^3*f*g^2 + 480*a*c^4*f*g^2 + 640*b*c
^4*d*g*h - 400*b^2*c^3*e*g*h + 960*a*c^4*e*g*h + 280*b^3*c^2*f*g*h - 928*a
*b*c^3*f*g*h - 200*b^2*c^3*d*h^2 + 480*a*c^4*d*h^2 + 140*b^3*c^2*e*h^2 - 4
64*a*b*c^3*e*h^2 - 105*b^4*c*f*h^2 + 448*a*b^2*c^2*f*h^2 - 240*a^2*c^3*f*h
^2)/c^5)*x + (1920*b*c^4*d*g^2 - 960*b^2*c^3*e*g^2 + 2560*a*c^4*e*g^2 + 60
0*b^3*c^2*f*g^2 - 2080*a*b*c^3*f*g^2 - 1920*b^2*c^3*d*g*h + 5120*a*c^4*d*g
*h + 1200*b^3*c^2*e*g*h - 4160*a*b*c^3*e*g*h - 840*b^4*c*f*g*h + 3680*a*b^
2*c^2*f*g*h - 2048*a^2*c^3*f*g*h + 600*b^3*c^2*d*h^2 - 2080*a*b*c^3*d*h^2
- 420*b^4*c*e*h^2 + 1840*a*b^2*c^2*e*h^2 - 1024*a^2*c^3*e*h^2 + 315*b^5*f*
h^2 - 1680*a*b^3*c*f*h^2 + 1808*a^2*b*c^2*f*h^2)/c^5) + 1/1024*(128*b^2*c^
4*d*g^2 - 512*a*c^5*d*g^2 - 64*b^3*c^3*e*g^2 + 256*a*b*c^4*e*g^2 + 40*b^4*
c^2*f*g^2 - 192*a*b^2*c^3*f*g^2 + 128*a^2*c^4*f*g^2 - 128*b^3*c^3*d*g*h +
512*a*b*c^4*d*g*h + 80*b^4*c^2*e*g*h - 384*a*b^2*c^3*e*g*h + 256*a^2*c^4*e
*g*h - 56*b^5*c*f*g*h + 320*a*b^3*c^2*f*g*h - 384*a^2*b*c^3*f*g*h + 40*...
```

Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 1881, normalized size of antiderivative = 3.27

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
int((g + h*x)^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g^2*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (e*h^2*x^2*(a + b*x + c*x^
2)^(3/2))/(5*c) + (f*h^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c) - (a*d*h^2*((x
/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*
x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) - (a*f*g^2*((x/2 + b/
(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x
^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c) + (d*g^2*log((b/2 + c*x)/c^(
1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*g^2*log((b
 + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2
)) - (2*a*e*h^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^
3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x
 + c*x^2)^(1/2))/(24*c^2)))/(5*c) - (5*b*d*h^2*((log((b + 2*c*x)/c^(1/2) +
 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x
^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*
g^2*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c)
)/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1
/2))/(24*c^2)))/(8*c) + (e*g^2*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*
x + c*x^2)^(1/2))/(24*c^2) + (d*h^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (f*
g^2*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*f*h^2*((5*b*((log((b + 2*c*x)/c^
(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(
a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c)...
```

Reduce [F]

$$\int (g+hx)^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \int \left(hx+g\right)^2\sqrt{c\,x^2+bx+a}\left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^2*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

3.25 $\int (g+hx)\sqrt{a+bx+cx^2}(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 322

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh)))\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^4}$$
$$+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}$$
$$+\frac{(35b^2fh^2-c^2(48fg^2-80h(eg+dh))-2ch(16afh+25b(fg+eh))-6ch(6cfg-10ceh+7bfh)x)}{240c^3h}$$
$$-\frac{(b^2-4ac)\,(32c^3dg-7b^3fh-8c^2(2beg+afg+2bdh+aeh)+2bc(6afh+5b(fg+eh)))\,\text{arctanh}\Big(\frac{}{2\sqrt{}}}{256c^{9/2}}$$

output
```
1/128*(32*c^3*d*g-7*b^3*f*h-8*c^2*(a*e*h+a*f*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a
*f*h+5*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^4+1/5*f*(h*x+g)^2*(c*
x^2+b*x+a)^(3/2)/c/h+1/240*(35*b^2*f*h^2-c^2*(48*f*g^2-80*h*(d*h+e*g))-2*c
*h*(16*a*f*h+25*b*(e*h+f*g))-6*c*h*(7*b*f*h-10*c*e*h+6*c*f*g)*x)*(c*x^2+b*
x+a)^(3/2)/c^3/h-1/256*(-4*a*c+b^2)*(32*c^3*d*g-7*b^3*f*h-8*c^2*(a*e*h+a*f
*g+2*b*d*h+2*b*e*g)+2*b*c*(6*a*f*h+5*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 5.41 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^4fh+10b^3c(15fg+15eh+7fhx)-4b^2c(-115afh+c(60eg+60dh+25fg}{}$$

input
```
Integrate[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^4*f*h + 10*b^3*c*(15*f*g + 15*e*h +
 7*f*h*x) - 4*b^2*c*(-115*a*f*h + c*(60*e*g + 60*d*h + 25*f*g*x + 25*e*h*x
 + 14*f*h*x^2)) + 8*b*c^2*(20*c*d*(3*g + h*x) - a*(65*f*g + 65*e*h + 29*f*
h*x) + 2*c*x*(5*e*(2*g + h*x) + f*x*(5*g + 3*h*x))) + 16*c^2*(-16*a^2*f*h
+ a*c*(40*d*h + 5*e*(8*g + 3*h*x) + f*x*(15*g + 8*h*x)) + 2*c^2*x*(10*d*(3
*g + 2*h*x) + x*(5*e*(4*g + 3*h*x) + 3*f*x*(5*g + 4*h*x))))) + 15*(b^2 - 4
*a*c)*(-32*c^3*d*g + 7*b^3*f*h + 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h)
 - 2*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt
[a + x*(b + c*x)])])/(1920*c^(9/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2184, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 2184$$

$$\frac{\int -\frac{1}{2}h(g+hx)(3bfg-10cdh+4afh+(6cfg-10ceh+7bfh)x)\sqrt{cx^2+bx+a}dx}{5ch^2}+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{\int (g+hx)(3bfg-10cdh+4afh+(6cfg-10ceh+7bfh)x)\sqrt{cx^2+bx+a}dx}{10ch}$$

$$\Big\downarrow \text{1225}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\int \sqrt{cx^2+bx+a}dx}{16c^2}-\frac{\left(a+bx+cx^2\right)^{3/2}(-2ch(16afh+25b(eh+fg))}{}}{10ch}$$

$$\Big\downarrow \text{1087}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c^2}-\frac{(a+bx+cx}{}}{10ch}$$

$$\Big\downarrow \text{1092}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c^2}-}{10ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}{5ch}-\frac{-\frac{5h\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)(-8c^2(aeh+afg+2bdh+2beg)+2bc(6afh+5b(eh+fg))-7b^3fh+32c^3dg)}{16c^2}-}{10ch}$$

input
```
Int[(g + h*x)*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^2*(a + b*x + c*x^2)^(3/2))/(5*c*h) - (-1/24*((35*b^2*f*h^2 -
16*c^2*(3*f*g^2 - 5*h*(e*g + d*h)) - 2*c*h*(16*a*f*h + 25*b*(f*g + e*h)) -
 6*c*h*(6*c*f*g - 10*c*e*h + 7*b*f*h)*x)*(a + b*x + c*x^2)^(3/2))/c^2 - (5
*h*(32*c^3*d*g - 7*b^3*f*h - 8*c^2*(2*b*e*g + a*f*g + 2*b*d*h + a*e*h) + 2
*b*c*(6*a*f*h + 5*b*(f*g + e*h)))*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*
c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(3/2))))/(16*c^2))/(10*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{\left(-384hf\,c^4x^4-48b\,c^3fh\,x^3-480c^4eh\,x^3-480c^4fg\,x^3-128a\,c^3fh\,x^2+56b^2c^2fh\,x^2-80b\,c^3eh\,x^2-80b\,c^3fg\,x^2-640c^4dh\,x^2-640c\cdots\right.}{\cdots}$
default	$dg\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{(4ac-b^2)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+(dh+eg)\left(\frac{(c\,x^2+bx+a)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+\cdots}}{4c}\cdots\right.}{\cdots}\cdots\right.$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(-384*c^4*f*h*x^4-48*b*c^3*f*h*x^3-480*c^4*e*h*x^3-480*c^4*f*g*x^3
-128*a*c^3*f*h*x^2+56*b^2*c^2*f*h*x^2-80*b*c^3*e*h*x^2-80*b*c^3*f*g*x^2-64
0*c^4*d*h*x^2-640*c^4*e*g*x^2+232*a*b*c^2*f*h*x-240*a*c^3*e*h*x-240*a*c^3*
f*g*x-70*b^3*c*f*h*x+100*b^2*c^2*e*h*x+100*b^2*c^2*f*g*x-160*b*c^3*d*h*x-1
60*b*c^3*e*g*x-960*c^4*d*g*x+256*a^2*c^2*f*h-460*a*b^2*c*f*h+520*a*b*c^2*e
*h+520*a*b*c^2*f*g-640*a*c^3*d*h-640*a*c^3*e*g+105*b^4*f*h-150*b^3*c*e*h-1
50*b^3*c*f*g+240*b^2*c^2*d*h+240*b^2*c^2*e*g-480*b*c^3*d*g)*(c*x^2+b*x+a)^
(1/2)/c^4+1/256*(48*a^2*b*c^2*f*h-32*a^2*c^3*e*h-32*a^2*c^3*f*g-40*a*b^3*c
*f*h+48*a*b^2*c^2*e*h+48*a*b^2*c^2*f*g-64*a*b*c^3*d*h-64*a*b*c^3*e*g+128*a
*c^4*d*g+7*b^5*f*h-10*b^4*c*e*h-10*b^4*c*f*g+16*b^3*c^2*d*h+16*b^3*c^2*e*g
-32*b^2*c^3*d*g)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1009, normalized size of antiderivative = 3.13

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8*(b^3*c^2 - 4*a*b*c^3)*e + (5
*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (16*(b^3*c^2 - 4*a*b*c^3)*d - 2
*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7*b^5 - 40*a*b^3*c + 48*a^2*b*
c^2)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h*x^4 + 48*(10*c^5*f*g + (10
*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f)*g + (80*c^5*d + 10*b*
c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b*c^4*d - 8*(3*b^2*c^3 -
 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*(3*b^2*c^3 - 8*a*c^4)*d
 - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c - 460*a*b^2*c^2 + 256*a^2*c
^3)*f)*h + 2*(10*(48*c^5*d + 8*b*c^4*e - (5*b^2*c^3 - 12*a*c^4)*f)*g + (80
*b*c^4*d - 10*(5*b^2*c^3 - 12*a*c^4)*e + (35*b^3*c^2 - 116*a*b*c^3)*f)*h)*
x)*sqrt(c*x^2 + b*x + a))/c^5, 1/3840*(15*(2*(16*(b^2*c^3 - 4*a*c^4)*d - 8
*(b^3*c^2 - 4*a*b*c^3)*e + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*g - (1
6*(b^3*c^2 - 4*a*b*c^3)*d - 2*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*e + (7
*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*f)*h)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(384*c^5*f*h*x^4
 + 48*(10*c^5*f*g + (10*c^5*e + b*c^4*f)*h)*x^3 + 8*(10*(8*c^5*e + b*c^4*f
)*g + (80*c^5*d + 10*b*c^4*e - (7*b^2*c^3 - 16*a*c^4)*f)*h)*x^2 + 10*(48*b
*c^4*d - 8*(3*b^2*c^3 - 8*a*c^4)*e + (15*b^3*c^2 - 52*a*b*c^3)*f)*g - (80*
(3*b^2*c^3 - 8*a*c^4)*d - 10*(15*b^3*c^2 - 52*a*b*c^3)*e + (105*b^4*c -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(332) = 664$.

Time = 1.04 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.08

$$\int (g + hx)\sqrt{a + bx + cx^2}(d + ex + fx^2)\ dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**4/5 + x**3*(b*f*h/10 + c*e*h + c
*f*g)/(4*c) + x**2*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*
g)/(8*c) + c*d*h + c*e*g)/(3*c) + x*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h
 + c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*
h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(2*c) + (a*d*h
 + a*e*g - 2*a*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(
8*c) + c*d*h + c*e*g)/(3*c) + b*d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 +
 c*e*h + c*f*g)/(4*c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b
*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/c
) + (a*d*g - a*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*c) + b*d
*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)
/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(2*c) - b*(a*d*h + a*e*g - 2*a*(a*f
*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h + c*f*g)/(8*c) + c*d*h + c*e*
g)/(3*c) + b*d*g - 3*b*(a*e*h + a*f*g - 3*a*(b*f*h/10 + c*e*h + c*f*g)/(4*
c) + b*d*h + b*e*g - 5*b*(a*f*h/5 + b*e*h + b*f*g - 7*b*(b*f*h/10 + c*e*h
+ c*f*g)/(8*c) + c*d*h + c*e*g)/(6*c) + c*d*g)/(4*c))/(2*c))*Piecewise((lo
g(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c)
, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), N
e(c, 0)), (2*(f*h*(a + b*x)**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-3*a*f*h +
 b*e*h + b*f*g)/(7*b**3) + (a + b*x)**(5/2)*(3*a**2*f*h - 2*a*b*e*h - 2...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\sqrt{a+bx+cx^2}\,(d+ex+fx^2)\ dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.48

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx+a}\Bigg(2\Bigg(4\Bigg(6\left(8\,fhx+\frac{10\,c^4fg+10\,c^4eh+bc^3fh}{c^4}\right)x+\frac{80\,c^4eg+10\,bc^3fg+80\,c^4dh+$$
$$+\frac{(32\,b^2c^3dg-128\,ac^4dg-16\,b^3c^2eg+64\,abc^3eg+10\,b^4cfg-48\,ab^2c^2fg+32\,a^2c^3fg-16\,b^3c^2dh+64$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h*x + (10*c^4*f*g + 10*c^4*e*h
+ b*c^3*f*h)/c^4)*x + (80*c^4*e*g + 10*b*c^3*f*g + 80*c^4*d*h + 10*b*c^3*e
*h - 7*b^2*c^2*f*h + 16*a*c^3*f*h)/c^4)*x + (480*c^4*d*g + 80*b*c^3*e*g -
50*b^2*c^2*f*g + 120*a*c^3*f*g + 80*b*c^3*d*h - 50*b^2*c^2*e*h + 120*a*c^3
*e*h + 35*b^3*c*f*h - 116*a*b*c^2*f*h)/c^4)*x + (480*b*c^3*d*g - 240*b^2*c
^2*e*g + 640*a*c^3*e*g + 150*b^3*c*f*g - 520*a*b*c^2*f*g - 240*b^2*c^2*d*h
 + 640*a*c^3*d*h + 150*b^3*c*e*h - 520*a*b*c^2*e*h - 105*b^4*f*h + 460*a*b
^2*c*f*h - 256*a^2*c^2*f*h)/c^4) + 1/256*(32*b^2*c^3*d*g - 128*a*c^4*d*g -
 16*b^3*c^2*e*g + 64*a*b*c^3*e*g + 10*b^4*c*f*g - 48*a*b^2*c^2*f*g + 32*a^
2*c^3*f*g - 16*b^3*c^2*d*h + 64*a*b*c^3*d*h + 10*b^4*c*e*h - 48*a*b^2*c^2*
e*h + 32*a^2*c^3*e*h - 7*b^5*f*h + 40*a*b^3*c*f*h - 48*a^2*b*c^2*f*h)*log(
abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 18.74 (sec) , antiderivative size = 877, normalized size of antiderivative = 2.72

$$\int (g+hx)\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
int((g + h*x)*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*g*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (2*a*f*h*((log((b + 2*c*x)/c
^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*
(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(5*c) -
 (5*b*e*h*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*
a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x
^2)^(1/2))/(24*c^2)))/(8*c) - (5*b*f*g*((log((b + 2*c*x)/c^(1/2) + 2*(a +
b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*
b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))/(8*c) + (d*h*(8*c*(a +
c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*g*(8*c*(a
 + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2) + (e*h*x*(a
 + b*x + c*x^2)^(3/2))/(4*c) + (f*g*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (7*
b*f*h*((5*b*((log((b + 2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 -
4*a*b*c))/(16*c^(5/2)) + ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c
*x^2)^(1/2))/(24*c^2)))/(8*c) - (x*(a + b*x + c*x^2)^(3/2))/(4*c) + (a*((x
/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*
x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2))))/(4*c)))/(10*c) + (f*h*x^2*(
a + b*x + c*x^2)^(3/2))/(5*c) - (a*e*h*((x/2 + b/(4*c))*(a + b*x + c*x^2)^
(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))
/(2*c^(3/2))))/(4*c) - (a*f*g*((x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) + (
log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c - b^2/4))/(2*c^...
```

Reduce [B] (verification not implemented)

Time = 29.38 (sec) , antiderivative size = 1363, normalized size of antiderivative = 4.23

$$\int (g + hx)\sqrt{a + bx + cx^2}\left(d + ex + fx^2\right) dx = \text{Too large to display}$$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
( - 512*sqrt(a + b*x + c*x**2)*a**2*c**3*f*h + 920*sqrt(a + b*x + c*x**2)*
a*b**2*c**2*f*h - 1040*sqrt(a + b*x + c*x**2)*a*b*c**3*e*h - 1040*sqrt(a +
 b*x + c*x**2)*a*b*c**3*f*g - 464*sqrt(a + b*x + c*x**2)*a*b*c**3*f*h*x +
1280*sqrt(a + b*x + c*x**2)*a*c**4*d*h + 1280*sqrt(a + b*x + c*x**2)*a*c**
4*e*g + 480*sqrt(a + b*x + c*x**2)*a*c**4*e*h*x + 480*sqrt(a + b*x + c*x**
2)*a*c**4*f*g*x + 256*sqrt(a + b*x + c*x**2)*a*c**4*f*h*x**2 - 210*sqrt(a
+ b*x + c*x**2)*b**4*c*f*h + 300*sqrt(a + b*x + c*x**2)*b**3*c**2*e*h + 30
0*sqrt(a + b*x + c*x**2)*b**3*c**2*f*g + 140*sqrt(a + b*x + c*x**2)*b**3*c
**2*f*h*x - 480*sqrt(a + b*x + c*x**2)*b**2*c**3*d*h - 480*sqrt(a + b*x +
c*x**2)*b**2*c**3*e*g - 200*sqrt(a + b*x + c*x**2)*b**2*c**3*e*h*x - 200*s
qrt(a + b*x + c*x**2)*b**2*c**3*f*g*x - 112*sqrt(a + b*x + c*x**2)*b**2*c*
*3*f*h*x**2 + 960*sqrt(a + b*x + c*x**2)*b*c**4*d*g + 320*sqrt(a + b*x + c
*x**2)*b*c**4*d*h*x + 320*sqrt(a + b*x + c*x**2)*b*c**4*e*g*x + 160*sqrt(a
 + b*x + c*x**2)*b*c**4*e*h*x**2 + 160*sqrt(a + b*x + c*x**2)*b*c**4*f*g*x
**2 + 96*sqrt(a + b*x + c*x**2)*b*c**4*f*h*x**3 + 1920*sqrt(a + b*x + c*x*
*2)*c**5*d*g*x + 1280*sqrt(a + b*x + c*x**2)*c**5*d*h*x**2 + 1280*sqrt(a +
 b*x + c*x**2)*c**5*e*g*x**2 + 960*sqrt(a + b*x + c*x**2)*c**5*e*h*x**3 +
960*sqrt(a + b*x + c*x**2)*c**5*f*g*x**3 + 768*sqrt(a + b*x + c*x**2)*c**5
*f*h*x**4 + 720*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
/sqrt(4*a*c - b**2))*a**2*b*c**2*f*h - 480*sqrt(c)*log((2*sqrt(c)*sqrt(...
```

3.26 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 175

$$\begin{aligned}\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx &= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} \\ &\quad+\frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2}+\frac{fx(a+bx+cx^2)^{3/2}}{4c} \\ &\quad-\frac{\left(b^2-4ac\right)\left(16c^2d+5b^2f-4c(2be+af)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}}\end{aligned}$$

output
```
1/64*(-4*a*c*f+5*b^2*f-8*b*c*e+16*c^2*d)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3
+1/24*(-5*b*f+8*c*e)*(c*x^2+b*x+a)^(3/2)/c^2+1/4*f*x*(c*x^2+b*x+a)^(3/2)/c
-1/128*(-4*a*c+b^2)*(16*c^2*d+5*b^2*f-4*c*(a*f+2*b*e))*arctanh(1/2*(2*c*x+
b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= \frac{\sqrt{c}\sqrt{a+x(b+cx)}(15b^3f-2b^2c(12e+5fx)+4bc(-13af+2c(6d+2ex+fx^2))+8c^2(a(8e+3fx)+}{192c^{7/2}}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c*
(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e
+ a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(192*c^(7
/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2192, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{1}{2}(8cd-2af+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{4c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (2(4cd-af)+(8ce-5bf)x)\sqrt{cx^2+bx+a}dx}{8c}+\frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{(-4acf+5b^2f-8bce+16c^2d)\int \sqrt{cx^2+bx+a}dx}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

$\downarrow$ 1087

$$\frac{\frac{(-4acf+5b^2f-8bce+16c^2d)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

$\downarrow$ 1092

$$\frac{\frac{(-4acf+5b^2f-8bce+16c^2d)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

$\downarrow$ 219

$$\frac{\frac{\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)(-4acf+5b^2f-8bce+16c^2d)}{2c}+\frac{(a+bx+cx^2)^{3/2}(8ce-5bf)}{3c}}{8c}+\frac{fx(a+bx+cx^2)^{3/2}}{4c}$$

input
```
Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(3/2))/(4*c) + (((8*c*e - 5*b*f)*(a + b*x + c*x^2)^
(3/2))/(3*c) + ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(((b + 2*c*x)*Sqr
t[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2]])]/(8*c^(3/2))))/(2*c))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\left(-48 f\,c^{3} x^{3}-8 b\,c^{2} f\,x^{2}-64 c^{3} e\,x^{2}-24 a\,c^{2} f x+10 b^{2} c f x-16 b\,c^{2} e x-96 c^{3} d x+52 a b c f-64 a\,c^{2} e-15 b^{3} f+24 b^{2} c e-48 b\,c^{2} d\right) \sqrt{c\,x^{2}+b x}}{192 c^{3}}$
default	$d\left(\frac{\left(2 c x+b\right) \sqrt{c\,x^{2}+b x+a}}{4 c}+\frac{\left(4 a c-b^{2}\right) \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{8 c^{\frac{3}{2}}}\right)+e\left(\frac{\left(c\,x^{2}+b x+a\right)^{\frac{3}{2}}}{3 c}-\frac{b\left(\frac{\left(2 c x+b\right) \sqrt{c\,x^{2}+b x+a}}{4 c}+\frac{\left(4 a c-b\right.}{}\right.}{2 c}\right.$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*c^3*f*x^3-8*b*c^2*f*x^2-64*c^3*e*x^2-24*a*c^2*f*x+10*b^2*c*f*x
-16*b*c^2*e*x-96*c^3*d*x+52*a*b*c*f-64*a*c^2*e-15*b^3*f+24*b^2*c*e-48*b*c^
2*d)*(c*x^2+b*x+a)^(1/2)/c^3-1/128*(16*a^2*c^2*f-24*a*b^2*c*f+32*a*b*c^2*e
-64*a*c^3*d+5*b^4*f-8*b^3*c*e+16*b^2*c^2*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.66

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\left[\frac{3\left(16\left(b^2c^2-4\,ac^3\right)d-8\left(b^3c-4\,abc^2\right)e+\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 2
4*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^
3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c -
52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqr
t(c*x^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4
*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c
^4*f*x^3 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3
)*e + (15*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 1
2*a*c^3)*f)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(168) = 336$.

Time = 0.51 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.19

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fx^3}{4}+\frac{x^2\left(\frac{bf}{8}+ce\right)}{3c}+\frac{x\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{2c}+\frac{ae-\frac{2a\left(\frac{bf}{8}+ce\right)}{3c}+bd-\frac{3b\left(\frac{af}{4}+be-\frac{5b\left(\frac{bf}{8}+ce\right)}{6c}+cd\right)}{4c}}{c}\right)+ \\ \frac{2\left(\frac{f(a+bx)^{\frac{7}{2}}}{7b^2}+\frac{(a+bx)^{\frac{5}{2}}(-2af+be)}{5b^2}+\frac{(a+bx)^{\frac{3}{2}}\left(a^2f-abe+b^2d\right)}{3b^2}\right)}{b} \\ \sqrt{a}\left(dx+\frac{ex^2}{2}+\frac{fx^3}{3}\right) \end{cases}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*x**3/4 + x**2*(b*f/8 + c*e)/(3*c) + x
*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) + (a*e - 2*a*(b*f/8 +
 c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(4*c
))/c) + (a*d - a*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)/(6*c) + c*d)/(2*c) - b*(
a*e - 2*a*(b*f/8 + c*e)/(3*c) + b*d - 3*b*(a*f/4 + b*e - 5*b*(b*f/8 + c*e)
/(6*c) + c*d)/(4*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c)
 + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(f*(a + b*x)**(7/2)/
(7*b**2) + (a + b*x)**(5/2)*(-2*a*f + b*e)/(5*b**2) + (a + b*x)**(3/2)*(a*
*2*f - a*b*e + b**2*d)/(3*b**2))/b, Ne(b, 0)), (sqrt(a)*(d*x + e*x**2/2 +
f*x**3/3), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.17

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,fx+\frac{8\,c^3e+bc^2f}{c^3}\right)x+\frac{48\,c^3d+8\,bc^2e-5\,b^2cf+12\,ac^2f}{c^3}\right)x+\frac{48\,bc^2d-\right.$$
$$+\frac{\left(16\,b^2c^2d-64\,ac^3d-8\,b^3ce+32\,abc^2e+5\,b^4f-24\,ab^2cf+16\,a^2c^2f\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\right.\right.}{128\,c^{\frac{7}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (8*c^3*e + b*c^2*f)/c^3)*x + (4
8*c^3*d + 8*b*c^2*e - 5*b^2*c*f + 12*a*c^2*f)/c^3)*x + (48*b*c^2*d - 24*b^
2*c*e + 64*a*c^2*e + 15*b^3*f - 52*a*b*c*f)/c^3) + 1/128*(16*b^2*c^2*d - 6
4*a*c^3*d - 8*b^3*c*e + 32*a*b*c^2*e + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2
*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 17.84 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.83

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$= d\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a} - \frac{a\,f\,\left(\left(\frac{x}{2}+\frac{b}{4\,c}\right)\,\sqrt{c\,x^2+b\,x+a}+\frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}}\right)}{4\,c} + \frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)\,\left(a\,c-\frac{b^2}{4}\right)}{2\,c^{3/2}} + \frac{e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}} - \frac{5\,b\,f\,\left(\frac{\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x+a}\right)\,\left(b^3-4\,a\,b\,c\right)}{16\,c^{5/2}}+\frac{\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2}\right)}{8\,c} + \frac{e\,\left(-3\,b^2+2\,c\,x\,b+8\,c\,\left(c\,x^2+a\right)\right)\,\sqrt{c\,x^2+b\,x+a}}{24\,c^2} + \frac{f\,x\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}{4\,c}$$

input
```
int((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
d*(x/2 + b/(4*c))*(a + b*x + c*x^2)^(1/2) - (a*f*((x/2 + b/(4*c))*(a + b*x
 + c*x^2)^(1/2) + (log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2))*(a*c
 - b^2/4))/(2*c^(3/2))))/(4*c) + (d*log((b/2 + c*x)/c^(1/2) + (a + b*x + c
*x^2)^(1/2))*(a*c - b^2/4))/(2*c^(3/2)) + (e*log((b + 2*c*x)/c^(1/2) + 2*(
a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2)) - (5*b*f*((log((b +
2*c*x)/c^(1/2) + 2*(a + b*x + c*x^2)^(1/2))*(b^3 - 4*a*b*c))/(16*c^(5/2))
+ ((8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(24*c^2)))
/(8*c) + (e*(8*c*(a + c*x^2) - 3*b^2 + 2*b*c*x)*(a + b*x + c*x^2)^(1/2))/(
24*c^2) + (f*x*(a + b*x + c*x^2)^(3/2))/(4*c)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.18

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\frac{-104\sqrt{c\,x^2+bx+a}\,ab\,c^2f+128\sqrt{c\,x^2+bx+a}\,a\,c^3e+48\sqrt{c\,x^2+bx+a}\,a\,c^3fx+30\sqrt{c\,x^2+bx+a}\,b^3}{}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
( - 104*sqrt(a + b*x + c*x**2)*a*b*c**2*f + 128*sqrt(a + b*x + c*x**2)*a*c
**3*e + 48*sqrt(a + b*x + c*x**2)*a*c**3*f*x + 30*sqrt(a + b*x + c*x**2)*b
**3*c*f - 48*sqrt(a + b*x + c*x**2)*b**2*c**2*e - 20*sqrt(a + b*x + c*x**2
)*b**2*c**2*f*x + 96*sqrt(a + b*x + c*x**2)*b*c**3*d + 32*sqrt(a + b*x + c
*x**2)*b*c**3*e*x + 16*sqrt(a + b*x + c*x**2)*b*c**3*f*x**2 + 192*sqrt(a +
 b*x + c*x**2)*c**4*d*x + 128*sqrt(a + b*x + c*x**2)*c**4*e*x**2 + 96*sqrt
(a + b*x + c*x**2)*c**4*f*x**3 - 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x +
c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*f + 72*sqrt(c)*log((2*s
qrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*f
- 96*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c
 - b**2))*a*b*c**2*e + 192*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**3*d - 15*sqrt(c)*log((2*sqrt(c)*sqrt(
a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**4*f + 24*sqrt(c)*log
((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c
*e - 48*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*
a*c - b**2))*b**2*c**2*d)/(384*c**4)
```

3.27 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 323

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx =& -\frac{\left(b^2fh+c^2\left(8eg-\frac{8fg^2}{h}-8dh\right)+2bc(fg-eh)+2c(2cfg-2ceh+bfh)x\right)\sqrt{a+bx+cx^2}}{8c^2h^2}\\ &+\frac{f(a+bx+cx^2)^{3/2}}{3ch}\\ &+\frac{\left(4c(2cg-bh)(bfg-2cdh)-\frac{(2cfg-2ceh+bfh)(8c^2g^2-b^2h^2-4ch(bg-ah))}{h}\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}h^3}\\ &+\frac{\sqrt{cg^2-bgh+ah^2}(fg^2-h(eg-dh))\,\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^4}\end{aligned}$$

output
```
-1/8*(b^2*f*h+c^2*(8*e*g-8*f*g^2/h-8*d*h)+2*b*c*(-e*h+f*g)+2*c*(b*f*h-2*c*
e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(1/2)/c^2/h^2+1/3*f*(c*x^2+b*x+a)^(3/2)/c/h+
1/16*(4*c*(-b*h+2*c*g)*(b*f*g-2*c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(8*c^2*g^2-
b^2*h^2-4*c*h*(-a*h+b*g))/h)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(5/2)/h^3+(a*h^2-b*g*h+c*g^2)^(1/2)*(f*g^2-h*(-d*h+e*g))*arctanh(1
/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2
))/h^4
```

Mathematica [A] (verified)

Time = 3.17 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

$$= \frac{\frac{h\sqrt{a+x(b+cx)}\left(-3b^2fh^2+2ch(4afh+b(-3fg+3eh+fhx))+4c^2\left(3h(-2eg+2dh+ehx)+f\left(6g^2-3ghx+2h^2x^2\right)\right)\right)}{c^2}+48\sqrt{-cg^2+h(bg-}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
((h*Sqrt[a + x*(b + c*x)]*(-3*b^2*f*h^2 + 2*c*h*(4*a*f*h + b*(-3*f*g + 3*e
*h + f*h*x)) + 4*c^2*(3*h*(-2*e*g + 2*d*h + e*h*x) + f*(6*g^2 - 3*g*h*x +
2*h^2*x^2))))/c^2 + 48*Sqrt[-(c*g^2) + h*(b*g - a*h)]*(f*g^2 + h*(-(e*g) +
 d*h))*ArcTan[(Sqrt[-(c*g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sq
rt[a + x*(b + c*x)])] - (3*(-(b^3*f*h^3) + 2*b*c*h^2*(-(b*f*g) + b*e*h + 2
*a*f*h) + 16*c^3*(f*g^3 + g*h*(-(e*g) + d*h)) - 8*c^2*h*(b*f*g^2 + b*h*(-(
e*g) + d*h) + a*h*(-(f*g) + e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a
+ x*(b + c*x)])])/c^(5/2))/(24*h^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{g+hx}\,dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{\int -\frac{3h(bfg-2cdh+(2cfg-2ceh+bfh)x)\sqrt{cx^2+bx+a}}{2(g+hx)}dx}{3ch^2}+\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\int\frac{(bfg-2cdh+(2cfg-2ceh+bfh)x)\sqrt{cx^2+bx+a}}{g+hx}dx}{2ch}$$

$\downarrow$ 1231

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\int\frac{4ch(bg-2ah)(bfg-2cdh)-g\left(-hb^2+4cgb-4ach\right)(2cfg-2c}{}}{}}{2ch}$$

$\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\int\frac{4ch(bg-2ah)(bfg-2cdh)-g\left(-hb^2+4cgb-4ach\right)(2cfg-2c}{}}{}}{2ch}$$

$\downarrow$ 1269

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(b}{h}}{}}{2ch}$$

$\downarrow$ 1092

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{2\left(4ch(2cg-bh)(bfg-2cdh)-\left(-4ch(bg-ah)-b^2h^2+8c^2g^2\right)(}{h}}{}}{2ch}$$

$\downarrow$ 219

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{16c^2\left(ah^2-bgh+cg^2\right)\left(fg^2-h(eg-dh)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+bx-}}}{h}}{2ch}$$

$\downarrow$ 1154

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\right.}{\sqrt{c}h}}{}}{}$$

$\Big\downarrow$ 219

$$\frac{f\left(a+bx+cx^2\right)^{3/2}}{3ch}-$$

$$\frac{\frac{\sqrt{a+bx+cx^2}(4ch(bfg-2cdh)+2chx(bfh-2ceh+2cfg)-(4cg-bh)(bfh-2ceh+2cfg))}{4ch^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(2cg-bh)(bfg-2cdh)-\right.}{\sqrt{c}h}}{}}{2ch}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + b*x + c*x^2)^(3/2))/(3*c*h) - (((4*c*h*(b*f*g - 2*c*d*h) - (4*c*g
 - b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 2*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x)*
Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h
) - (2*c*f*g - 2*c*e*h + b*f*h)*(8*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - a*h)))
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + (16
*c^2*Sqrt[c*g^2 - b*g*h + a*h^2]*(f*g^2 - h*(e*g - d*h))*ArcTanh[(b*g - 2*
a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2
])])/h)/(8*c*h^2))/(2*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.59

method	result
risch	$\frac{(8f\,h^2c^2x^2+2bcf\,h^2x+12c^2e\,h^2x-12c^2fghx+8acf\,h^2-3b^2f\,h^2+6bce\,h^2-6bcfgh+24c^2d\,h^2-24c^2egh+24c^2f\,g^2)\sqrt{c\,x^2+bx+a}}{24c^2h^3} -$
default	$\frac{eh\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)+fh\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\right.}{8c^{\frac{3}{2}}}\right.}{2c}\right.}{h^2}$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*c^2*f*h^2*x^2+2*b*c*f*h^2*x+12*c^2*e*h^2*x-12*c^2*f*g*h*x+8*a*c*f*
h^2-3*b^2*f*h^2+6*b*c*e*h^2-6*b*c*f*g*h+24*c^2*d*h^2-24*c^2*e*g*h+24*c^2*f
*g^2)*(c*x^2+b*x+a)^(1/2)/c^2/h^3-1/16/c^2/h^3*((4*a*b*c*f*h^3-8*a*c^2*e*h
^3+8*a*c^2*f*g*h^2-b^3*f*h^3+2*b^2*c*e*h^3-2*b^2*c*f*g*h^2-8*b*c^2*d*h^3+8
*b*c^2*e*g*h^2-8*b*c^2*f*g^2*h+16*c^3*d*g*h^2-16*c^3*e*g^2*h+16*c^3*f*g^3)
/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+16*(a*d*h^4-a*e*g*h
^3+a*f*g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*h+c*f*g
^4)*c^2/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+
(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-
2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

Reduce [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 13015, normalized size of antiderivative = 40.29

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{g+hx}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

output
```
( - 48*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a
*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*
g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)
*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*s
qrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*
g*h - 8*c**2*g**2))*b*c**3*d*h**3 + 48*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h
+ c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 -
 b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((
2*sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*
h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g -
 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2))*b*c**3*e*g*h**2 - 48*s
qrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 -
b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sq
rt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h
 + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*s
qrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*
c**2*g**2))*b*c**3*f*g**2*h + 96*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g*
*2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*
h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt
(c)*sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**...
```

3.28 $\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 453

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx =$$

$$-\frac{\left(bfh(bg-ah)-4c^2g\left(2eg-\frac{3fg^2}{h}-dh\right)+4ach(2fg-eh)-bc(13fg^2-8egh+4dh^2)+2ch\left(2ceg\right.\right.}{4ch^2\left(cg^2-bgh+ah^2\right)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{h\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(b^2fh^2+4ch(2bfg-beh-afh)-8c^2(3fg^2-h(2eg-dh))\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}h^4}$$

$$-\frac{\left(2c(3fg^3-gh(2eg-dh))-h(5bfg^2-bh(3eg-dh)-2ah(2fg-eh))\right)\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx}}\right.}{2h^4\sqrt{cg^2-bgh+ah^2}}$$

output
```
-1/4*(b*f*h*(-a*h+b*g)-4*c^2*g*(2*e*g-3*f*g^2/h-d*h)+4*a*c*h*(-e*h+2*f*g)-
b*c*(4*d*h^2-8*e*g*h+13*f*g^2)+2*c*h*(2*c*e*g+b*f*g-3*c*f*g^2/h-2*c*d*h-a*
f*h)*x)*(c*x^2+b*x+a)^(1/2)/c/h^2/(a*h^2-b*g*h+c*g^2)-(f*g^2-h*(-d*h+e*g))
*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/8*(b^2*f*h^2+4*c*h*(-
a*f*h-b*e*h+2*b*f*g)-8*c^2*(3*f*g^2-h*(-d*h+2*e*g)))*arctanh(1/2*(2*c*x+b)
/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^4-1/2*(2*c*(3*f*g^3-g*h*(-d*h+2*e*
g))-h*(5*b*f*g^2-b*h*(-d*h+3*e*g)-2*a*h*(-e*h+2*f*g)))*arctanh(1/2*(b*g-2*
a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/(a*
h^2-b*g*h+c*g^2)^(1/2)
```

Mathematica [A] (verified)

Time = 10.85 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

$$= \frac{-8(2fg-eh)\sqrt{a+x(b+cx)} + \frac{2fh(b+2cx)\sqrt{a+x(b+cx)}}{c} - \frac{8(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{g+hx} + \frac{(-b^2+4ac)fh\operatorname{arctanh}\left(\cdots\right.}{c^{3/2}}\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
(-8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)] + (2*f*h*(b + 2*c*x)*Sqrt[a + x*(b
 + c*x)])/c - (8*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/(g + h*
x) + ((-b^2 + 4*a*c)*f*h*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*
x)])])/c^(3/2) + (4*(f*g^2 + h*(-(e*g) + d*h))*(2*Sqrt[c]*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h +
2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c
*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)]))/h + (4*(2*f*g - e*h)*((2*c*g - b*
h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*Sqrt
[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqr
t[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(Sqrt[c]*h))/(8*h^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2181, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^2}dx$$

$$\Big\downarrow \ 2181$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+2cdg-3beg-2afg+bdh+2aeh-2\left(-\frac{3cfg^2}{h}+2ceg+bfg-2cdh-afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)}dx}{ah^2-bgh+cg^2}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{3fg^2}{h}+3eg-dh\right)-2\left(-\frac{3cfg^2}{h}+2ceg+bfg-2cdh-afh\right)x\right)\sqrt{cx^2+bx+a}}{g+hx}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1231$$

$$\frac{-\frac{\int \frac{\left(cg^2-bhg+ah^2\right)\left(fghb^2-4c\left(3fg^2-h(2eg-dh)\right)b+4ach(3fg-2eh)+\left(-8\left(3fg^2-h(2eg-dh)\right)c^2+4h(2bfg-beh-afh)c+b^2fh^2\right)x\right)}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{4ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2\right.}{}}{2\left(ah^2-bgh+cg\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{\left(ah^2-bgh+cg^2\right)\int \frac{fghb^2-4c\left(3fg^2-h(2eg-dh)\right)b+4ach(3fg-2eh)+\left(-8\left(3fg^2-h(2eg-dh)\right)c^2+4h(2bfg-beh-afh)c+b^2fh^2\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{4ch^3}-\frac{\sqrt{a+bx+cx^2}\left(2c\right.}{}}{2\left(ah^2-bgh+cg\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1269

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}+\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-eh)-bh(3eg-\right.\right.}{h}\right.}{4ch^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1092

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{2\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}+\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg\right.\right.}{}\right.}{4ch^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{4c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-eh)-bh(3eg-dh)+5bfg^2\right)\right)\int\frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(\right.}{}\right.}{4ch^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1154

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{\sqrt{c}h}-\frac{8c\left(2c\left(3fg^3-gh(2eg-dh)\right)-h\left(-2ah(2fg-\right.\right.}{}\right.}{4ch^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4ch(-afh-beh+2bfg)+b^2fh^2-8c^2\left(3fg^2-h(2eg-dh)\right)\right)}{\sqrt{c}h}+\frac{4c\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+c}}\right.}{}\right.}{4ch^3}$$

$$\frac{(a+bx+cx^2)^{3/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
-(((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x))) + (-1/2*((b*f*h*(b*g - a*h) - 4*c^2*g*(2*e*g - (3*f*g^2)/
h - d*h) + 4*a*c*h*(2*f*g - e*h) - b*c*(13*f*g^2 - 4*h*(2*e*g - d*h)) + 2*
c*h*(2*c*e*g + b*f*g - (3*c*f*g^2)/h - 2*c*d*h - a*f*h)*x)*Sqrt[a + b*x +
c*x^2])/(c*h^2) - ((c*g^2 - b*g*h + a*h^2)*(((b^2*f*h^2 + 4*c*h*(2*b*f*g -
 b*e*h - a*f*h) - 8*c^2*(3*f*g^2 - h*(2*e*g - d*h)))*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + (4*c*(2*c*(3*f*g^3 - g*h*
(2*e*g - d*h)) - h*(5*b*f*g^2 - b*h*(3*e*g - d*h) - 2*a*h*(2*f*g - e*h)))*
ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqr
t[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2])))/(4*c*h^3))/(2*(c*g
^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.55

method	result
risch	$\frac{(2xfhc+bfh+4che-8cfg)\sqrt{c x^2+bx+a}}{4c h^3}+\frac{\frac{\left(4acf h^2-b^2 f h^2+4bce h^2-8bcfgh+8c^2 d h^2-16c^2 egh+24c^2 f g^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{h\sqrt{c}}-$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*h*x+b*f*h+4*c*e*h-8*c*f*g)*(c*x^2+b*x+a)^(1/2)/c/h^3+1/8/c/h^3*
((4*a*c*f*h^2-b^2*f*h^2+4*b*c*e*h^2-8*b*c*f*g*h+8*c^2*d*h^2-16*c^2*e*g*h+2
4*c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-8*c/h^2
*(a*e*h^3-2*a*f*g*h^2+b*d*h^3-2*b*e*g*h^2+3*b*f*g^2*h-2*c*d*g*h^2+3*c*e*g^
2*h-4*c*f*g^3)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h
^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b
*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+8*c*(a*d*h^4-
a*e*g*h^3+a*f*g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*
h+c*f*g^4)/h^3*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+
c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-
2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^2}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 3565, normalized size of antiderivative = 7.87

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

output
```
(8*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
- b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**2*e*g*h**3 + 8*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**2*e*h**4*x - 16*sqrt(a*h
**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c
*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**2*f*g**2*h**2 - 16*sqrt(a*h**
2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g
**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**2*f*g*h**3*x + 4*sqrt(a*h**2 -
b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)
 - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**2*d*g*h**3 + 4*sqrt(a*h**2 - b*g*h
+ c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a
*h + b*g - b*h*x + 2*c*g*x)*b*c**2*d*h**4*x - 12*sqrt(a*h**2 - b*g*h + c*g
**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h +
b*g - b*h*x + 2*c*g*x)*b*c**2*e*g**2*h**2 - 12*sqrt(a*h**2 - b*g*h + c*g**
2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*
g - b*h*x + 2*c*g*x)*b*c**2*e*g*h**3*x + 20*sqrt(a*h**2 - b*g*h + c*g**2)*
log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g -
 b*h*x + 2*c*g*x)*b*c**2*f*g**3*h + 20*sqrt(a*h**2 - b*g*h + c*g**2)*log(2
*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*
x + 2*c*g*x)*b*c**2*f*g**2*h**2*x - 8*sqrt(a*h**2 - b*g*h + c*g**2)*log...
```

3.29 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 446

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx =$$

$$-\frac{\left(11bfg^2-bh(3eg+dh)-\frac{4cg^2(3fg-eh)}{h}-4ah(3fg-eh)+2h\left(ceg+2bfg-\frac{3cfg^2}{h}-cdh-2afh\right)x\right)}{4h^2\left(cg^2-bgh+ah^2\right)(g+hx)}$$

$$-\frac{\left(fg^2-h(eg-dh)\right)\left(a+bx+cx^2\right)^{3/2}}{2h\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(6cfg-2ceh-bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}h^4}$$

$$+\frac{(8c^2g^3(3fg-eh)-4ch(bg^2(10fg-3eh)-ah(9fg^2-3egh+dh^2))+h^2(8a^2fh^2-4abh(6fg-eh)}{8h^4\left(cg^2-bgh+ah^2\right)^{3/2}}$$

output
```
-1/4*(11*b*f*g^2-b*h*(d*h+3*e*g)-4*c*g^2*(-e*h+3*f*g)/h-4*a*h*(-e*h+3*f*g)
+2*h*(c*e*g+2*b*f*g-3*c*f*g^2/h-c*d*h-2*a*f*h)*x)*(c*x^2+b*x+a)^(1/2)/h^2/
(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h
/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/2*(-b*f*h-2*c*e*h+6*c*f*g)*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/h^4+1/8*(8*c^2*g^3*(-e*h+3*f*
g)-4*c*h*(b*g^2*(-3*e*h+10*f*g)-a*h*(d*h^2-3*e*g*h+9*f*g^2))+h^2*(8*a^2*f*
h^2-4*a*b*h*(-e*h+6*f*g)+b^2*(15*f*g^2-h*(d*h+3*e*g))))*arctanh(1/2*(b*g-2
*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/(a
*h^2-b*g*h+c*g^2)^(3/2)
```

Mathematica [A] (verified)

Time = 11.47 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx$$

$$= \frac{8f\sqrt{a+x(b+cx)} + \frac{8(2fg-eh)\sqrt{a+x(b+cx)}}{g+hx} + \frac{2h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2} + \frac{(-b^2+4ac)h(fg^2+\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
(8*f*Sqrt[a + x*(b + c*x)] + (8*(2*f*g - e*h)*Sqrt[a + x*(b + c*x)])/(g +
h*x) + (2*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c
*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 + 4
*a*c)*h*(f*g^2 + h*(-(e*g) + d*h))*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x)
)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-
(b*g) + a*h))^(3/2) - (4*(2*f*g - e*h)*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x
+ b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/
Sqrt[c*g^2 + h*(-(b*g) + a*h)]))/h + (4*f*(((-2*c*g + b*h)*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*
g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/h)/(8*h^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2181, 27, 1230, 25, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^3}\,dx$$

$$\Big\downarrow \ 2181$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+4cdg-3beg-4afg-bdh+4aeh-2\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^2}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(4cdg-4afg+4aeh-b\left(-\frac{3fg^2}{h}+3eg+dh\right)-2\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^2}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1230$$

$$\frac{-\frac{\int -\frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^2}-\frac{\sqrt{a+bx\cdots}}{\cdots}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{h(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^2}-\frac{\sqrt{a+bx+cx^2}\cdots}{\cdots}}{4\left(ah^2-bgh+cg^2\right)}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{h\left(2(2bg-2ah)\left(-\frac{3cfg^2}{h}+ceg+2bfg-cdh-2afh\right)+b\left(3bfg^2-bh(3eg+dh)+4h(cdg-afg+aeh)\right)\right)-4(6cfg-2ceh-bfh)\left(cg^2-bhg+ah^2\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2h^3}-\frac{\sqrt{a+bx+cx^2}\cdots}{\cdots}}{4\left(ah^2-bgh+cg^2\right)}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{4\left(ah^2-bg\cdots\right.}{\cdots}}{2h^3}\cdots}{4\left(ah^{\cdots}\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{8\left(ah^2-bg\cdots\right.}{\cdots}}{2h^3}\cdots}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}-\frac{4\arctan\cdots}{\cdots}}{2h^3}\cdots}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$-\frac{\frac{2\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3fg-eh)\right)\int\frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2}{cx^2+b}}}{h}}{2h^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2-4abh(6fg-eh)+b^2\left(15fg^2-h(dh+3eg)\right)\right)-4ch\left(bg^2(10fg-3eh)-ah\left(dh^2-3egh+9fg^2\right)\right)+8c^2g^3(3f}{h\sqrt{ah^2-bgh+cg^2}}}{2h^3}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + (-(((11*b*f*g^2 - b*h*(3*e*g + d*h) - (4*c*g^2*(3*f
*g - e*h))/h - 4*a*h*(3*f*g - e*h) + 2*h*(c*e*g + 2*b*f*g - (3*c*f*g^2)/h
- c*d*h - 2*a*f*h)*x)*Sqrt[a + b*x + c*x^2])/(h^2*(g + h*x))) + ((-4*(6*c*
f*g - 2*c*e*h - b*f*h)*(c*g^2 - b*g*h + a*h^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) + ((8*c^2*g^3*(3*f*g - e*h) - 4*c
*h*(b*g^2*(10*f*g - 3*e*h) - a*h*(9*f*g^2 - 3*e*g*h + d*h^2)) + h^2*(8*a^2
*f*h^2 - 4*a*b*h*(6*f*g - e*h) + b^2*(15*f*g^2 - h*(3*e*g + d*h))))*ArcTan
h[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*h^3))/(4*(c*g^2 - b*g*
h + a*h^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1229 vs. $2(418) = 836$.

Time = 0.46 (sec) , antiderivative size = 1230, normalized size of antiderivative = 2.76

method	result	size
risch	Expression too large to display	1230
default	Expression too large to display	2166

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
f/h^3*(c*x^2+b*x+a)^(1/2)+1/2/h^3*((b*f*h+2*c*e*h-6*c*f*g)/h*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-2/h^2*(a*f*h^2+b*e*h^2-3*b*f*g*h+c*
d*h^2-3*c*e*g*h+6*c*f*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*
g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x
+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+2
/h^3*(a*e*h^3-2*a*f*g*h^2+b*d*h^3-2*b*e*g*h^2+3*b*f*g^2*h-2*c*d*g*h^2+3*c*
e*g^2*h-4*c*f*g^3)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2
*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*
g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(
b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2
*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+2*(a*d*h^4-a*e*g
*h^3+a*f*g^2*h^2-b*d*g*h^3+b*e*g^2*h^2-b*f*g^3*h+c*d*g^2*h^2-c*e*g^3*h+c*f
*g^4)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)
/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c
*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/
h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*
(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/
h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(418) = 836$.

Time = 0.46 (sec) , antiderivative size = 2364, normalized size of antiderivative = 5.30

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/4*(24*c^2*f*g^4 - 8*c^2*e*g^3*h - 40*b*c*f*g^3*h + 12*b*c*e*g^2*h^2 + 15
*b^2*f*g^2*h^2 + 36*a*c*f*g^2*h^2 - 3*b^2*e*g*h^3 - 12*a*c*e*g*h^3 - 24*a*
b*f*g*h^3 - b^2*d*h^4 + 4*a*c*d*h^4 + 4*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-(
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a
*h^2))/((c*g^2*h^4 - b*g*h^5 + a*h^6)*sqrt(-c*g^2 + b*g*h - a*h^2)) + sqrt
(c*x^2 + b*x + a)*f/h^3 + 1/4*(24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^
2*f*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^2 - 32*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 8*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3
*b*c*e*g^2*h^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 2
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 8*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
3*b^2*e*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 + (sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h
^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 40*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*c^(5/2)*f*g^5 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^2*c^(5/2)*e*g^4*h - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2
)*f*g^4*h + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 + 20
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 + 3*(sqrt(c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^3}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 7427, normalized size of antiderivative = 16.65

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^3} \, dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

output
```
(8*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
- b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*f*g**2*h**4 + 16
*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*f*g*h**5*x + 8*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*f*h**6*x**2 + 4*sqrt(a
*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h +
 c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c*e*g**2*h**4 + 8*sqrt(a*h**
2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g
**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c*e*g*h**5*x + 4*sqrt(a*h**2 - b
*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)
- 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c*e*h**6*x**2 - 24*sqrt(a*h**2 - b*g*
h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2
*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c*f*g**3*h**3 - 48*sqrt(a*h**2 - b*g*h +
 c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*
h + b*g - b*h*x + 2*c*g*x)*a*b*c*f*g**2*h**4*x - 24*sqrt(a*h**2 - b*g*h +
c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h
 + b*g - b*h*x + 2*c*g*x)*a*b*c*f*g*h**5*x**2 + 4*sqrt(a*h**2 - b*g*h + c*
g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h +
 b*g - b*h*x + 2*c*g*x)*a*c**2*d*g**2*h**4 + 8*sqrt(a*h**2 - b*g*h + c*...
```

3.30 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 485

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = -\frac{f\sqrt{a+bx+cx^2}}{h^3(g+hx)}$$
$$-\frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\,(bg-2ah+(2cg-bh)x)\sqrt{a+bx+ca}}{8h^2\,(cg^2-bgh+ah^2)^2\,(g+hx)^2}$$
$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{3h\,(cg^2-bgh+ah^2)\,(g+hx)^3}+\frac{\sqrt{c}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^4}$$
$$-\frac{(16c^3fg^5-8c^2gh(5bfg^3-5afg^2h+adh^3)-bh^3(8a^2fh^2-2abh(6fg+eh)+b^2(5fg^2+egh+dh^2))}{16h^4\,(cg}$$

output
```
-f*(c*x^2+b*x+a)^(1/2)/h^3/(h*x+g)-1/8*(2*c*(-d*g*h^2+f*g^3)-h*(3*b*f*g^2-
b*h*(d*h+e*g)-2*a*h*(-e*h+2*f*g)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a
)^(1/2)/h^2/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^
2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3+c^(1/2)*f*arctanh(1/2*(2*c*
x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4-1/16*(16*c^3*f*g^5-8*c^2*g*h*(a*d*h^
3-5*a*f*g^2*h+5*b*f*g^3)-b*h^3*(8*a^2*f*h^2-2*a*b*h*(e*h+6*f*g)+b^2*(d*h^2
+e*g*h+5*f*g^2))+2*c*h^2*(4*a^2*h^2*(-e*h+4*f*g)-2*a*b*h*(-d*h^2-e*g*h+15*
f*g^2)+b^2*(d*g*h^2+15*f*g^3)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*
h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^4/(a*h^2-b*g*h+c*g^2)^(5/2)
```

Mathematica [A] (verified)

Time = 12.27 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

$$= \frac{-\frac{48f\sqrt{a+x(b+cx)}}{g+hx} - \frac{16h^2(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(cg^2+h(-bg+ah))(g+hx)^3} + \frac{12h(-2fg+eh)\sqrt{a+x(b+cx)}(-2ah+2cgx+b(g-hx))}{(cg^2+h(-bg+ah))(g+hx)^2} - \frac{6(b^2-4ac)h(-}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
((-48*f*Sqrt[a + x*(b + c*x)])/(g + h*x) - (16*h^2*(f*g^2 + h*(-(e*g) + d*
h))*(a + x*(b + c*x))^(3/2))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^3) + (1
2*h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)))
/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) - (6*(b^2 - 4*a*c)*h*(-2*f*g + e
*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a
*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) + (3*h*(2*c
*g - b*h)*(f*g^2 + h*(-(e*g) + d*h))*((2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2
*c*g*x + b*(g - h*x)))/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)^2) + ((-b^2 +
 4*a*c)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g)
 + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(c*g^
2 + h*(-(b*g) + a*h)) + (24*f*(2*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])] - ((2*c*g - b*h)*ArcTanh[(-2*a*h + 2*c*g*x + b*(g -
h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*g^
2 + h*(-(b*g) + a*h)]))/h)/(48*h^3)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.33, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2181, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^4}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{3\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^3}dx}{3\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^3}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1229$$

$$-\frac{\int -\frac{16cfx\left(cg^2-bhg+ah^2\right)^2+h\left(\frac{8bc^2fg^4}{h}+8a^2bfh^3+4abch\left(7fg^2-h(eg+dh)\right)+b^3h\left(5fg^2+h(eg+dh)\right)-8ac\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2b^2\left(a(6fg+eh)\cdots\right.\right.}{2h(g+hx)\sqrt{cx^2+bx+a}}}{4h^2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{16cfx\left(cg^2-bhg+ah^2\right)^2+h\left(\frac{8bc^2fg^4}{h}+8a^2bfh^3+4abch\left(7fg^2-h(eg+dh)\right)+b^3h\left(5fg^2+h(eg+dh)\right)-8ac\left(a(2fg-eh)h^2+c\left(fg^3-dgh^2\right)\right)-2b^2\left(a(6fg+eh)h^2+\cdots\right.\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{16cf\left(ah^2-bgh+cg^2\right)^2\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)+b^2\left(dh^2+\cdots\right.\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{32cf\left(ah^2-bgh+cg^2\right)^2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{16\sqrt{c}f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)^2}{h}-\frac{\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6f\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b^2\left(dgh^2+15fg^3\right)\right)-bh^3\left(8a^2fh^2-2abh(eh+6fg)+b^2\left(dh^2+egh+5fg^2\right)\right)-8c^2gh\left(adh^3-5afg^2h+5bfg^3\right)\right.}{h}}{8h^3(ah^2-bgh+c}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{16\sqrt{c}f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)^2}{h}-\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(2ch^2\left(4a^2h^2(4fg-eh)-2abh\left(-dh^2-egh+15fg^2\right)+b\right.\right.}{h}}{8h^3(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
-1/3*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^3) + (-1/4*(((8*c^2*f*g^5)/h + 4*a^2*e*h^4 + 4*a*c*g*h*(
3*f*g^2 + d*h^2) + b^2*g*h*(5*f*g^2 + h*(e*g + d*h)) - 2*b*(a*h^2*(3*f*g^2
 + 2*e*g*h + d*h^2) + c*(7*f*g^4 + d*g^2*h^2)) + (8*f*(c*g^2 - h*(b*g - a*
h))^2 + (2*c*g - b*h)*(2*c*(f*g^3 - d*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d
*h) - 2*a*h*(2*f*g - e*h))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h
 + a*h^2)*(g + h*x)^2) + ((16*Sqrt[c]*f*(c*g^2 - b*g*h + a*h^2)^2*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h - ((16*c^3*f*g^5 - 8*c^2
*g*h*(5*b*f*g^3 - 5*a*f*g^2*h + a*d*h^3) - b*h^3*(8*a^2*f*h^2 - 2*a*b*h*(6
*f*g + e*h) + b^2*(5*f*g^2 + e*g*h + d*h^2)) + 2*c*h^2*(4*a^2*h^2*(4*f*g -
 e*h) - 2*a*b*h*(15*f*g^2 - e*g*h - d*h^2) + b^2*(15*f*g^3 + d*g*h^2)))*Ar
cTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[
a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(8*h^3*(c*g^2 - b*g*h
 + a*h^2)))/(2*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3089 vs. $2(457) = 914$.

Time = 0.52 (sec) , antiderivative size = 3090, normalized size of antiderivative = 6.37

method	result	size
default	Expression too large to display	3090

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
f/h^4*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/
h)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*((
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-
2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)
/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2
/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)
/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x
+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+2*c/(a*h^2-b*g*h+c*g^2)*h^
2*(1/4*(2*c*(x+g/h)+(b*h-2*c*g)/h)/c*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^
2/h^2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-
2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+(e*h-2*f*g)/h^5*(-1/2/(
a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2
-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-
b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c
*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+g/h)^2*c+(b*h-
2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h*ln((1/2*
(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6846 vs. $2(457) = 914$.

Time = 4.09 (sec) , antiderivative size = 6846, normalized size of antiderivative = 14.12

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

output
```
-1/8*(16*c^3*f*g^5 - 40*b*c^2*f*g^4*h + 30*b^2*c*f*g^3*h^2 + 40*a*c^2*f*g^
3*h^2 - 5*b^3*f*g^2*h^3 - 60*a*b*c*f*g^2*h^3 + 2*b^2*c*d*g*h^4 - 8*a*c^2*d
*g*h^4 - b^3*e*g*h^4 + 4*a*b*c*e*g*h^4 + 12*a*b^2*f*g*h^4 + 32*a^2*c*f*g*h
^4 - b^3*d*h^5 + 4*a*b*c*d*h^5 + 2*a*b^2*e*h^5 - 8*a^2*c*e*h^5 - 8*a^2*b*f
*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*
g^2 + b*g*h - a*h^2))/((c^2*g^4*h^4 - 2*b*c*g^3*h^5 + b^2*g^2*h^6 + 2*a*c*
g^2*h^6 - 2*a*b*g*h^7 + a^2*h^8)*sqrt(-c*g^2 + b*g*h - a*h^2)) - sqrt(c)*f
*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/h^4 - 1/24*(
144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*f*g^5*h^2 - 48*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*c^3*e*g^4*h^3 - 312*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b*c^2*f*g^4*h^3 + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*
e*g^3*h^4 + 198*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*f*g^3*h^4 + 26
4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 48*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*b^2*c*e*g^2*h^5 - 96*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^5*a*c^2*e*g^2*h^5 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*f
*g^2*h^5 - 300*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 - 6*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 24*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*c^2*d*g*h^6 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
5*b^3*e*g*h^6 + 84*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*e*g*h^6 + 6
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^2*f*g*h^6 + 96*(sqrt(c)*x -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^4}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 11991, normalized size of antiderivative = 24.72

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

output
```
(24*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*f*g**3*h**5 + 7
2*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
 b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*f*g**2*h**6*x + 7
2*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
 b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*f*g*h**7*x**2 + 2
4*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
 b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*f*h**8*x**3 + 24*
sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b
*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*e*g**3*h**5 + 72*sq
rt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g
*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*e*g**2*h**6*x + 72*sq
rt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g
*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*e*g*h**7*x**2 + 24*sq
rt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g
*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*e*h**8*x**3 - 96*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*f*g**4*h**4 - 288*sqrt(
a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
+ c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*f*g**3*h**5*x - 288*s...
```

3.31 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 499

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$
$$= \frac{(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh))+b^2(5fg^2+h(3eg}{64\,(cg^2-bgh+ah^2)^3\,(g+hx)^2}$$
$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{3/2}}{4h\,(cg^2-bgh+ah^2)\,(g+hx)^4}$$
$$+\frac{(6cfg^3+2cgh(eg-5dh)+8ah^2(2fg-eh)-bh(11fg^2-h(3eg+5dh)))\,(a+bx+cx^2)^{3/2}}{24h\,(cg^2-bgh+ah^2)^2\,(g+hx)^3}$$
$$-\frac{(b^2-4ac)\,(16c^2dg^2+16a^2fh^2-8abh(2fg+eh)-4c(afg^2-ah(5eg-dh)+2bg(eg+2dh))+b^2(5}{128\,(cg^2-bgh+ah^2)^{7/2}}$$

output
```
1/64*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)-4*c*(a*f*g^2-a*h*(-d*h
+5*e*g)+2*b*g*(2*d*h+e*g))+b^2*(5*f*g^2+h*(5*d*h+3*e*g)))*(b*g-2*a*h+(-b*h
+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2-1/4*(f*g^2-
h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4+1/24*(6*
c*f*g^3+2*c*g*h*(-5*d*h+e*g)+8*a*h^2*(-e*h+2*f*g)-b*h*(11*f*g^2-h*(5*d*h+3
*e*g)))*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^3-1/128*(-4*a*
c+b^2)*(16*c^2*d*g^2+16*a^2*f*h^2-8*a*b*h*(e*h+2*f*g)-4*c*(a*f*g^2-a*h*(-d
*h+5*e*g)+2*b*g*(2*d*h+e*g))+b^2*(5*f*g^2+h*(5*d*h+3*e*g)))*arctanh(1/2*(b
*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a
*h^2-b*g*h+c*g^2)^(7/2)
```

Mathematica [A] (verified)

Time = 15.78 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

$$= \frac{-48h(cg^2+h(-bg+ah))^{5/2}\left(fg^2+h(-eg+dh)\right)(a+x(b+cx))^{3/2}+64h(2fg-eh)\left(cg^2+h(-bg+\right.}{}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
(-48*h*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(f*g^2 + h*(-(e*g) + d*h))*(a + x*
(b + c*x))^(3/2) + 64*h*(2*f*g - e*h)*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g
+ h*x)*(a + x*(b + c*x))^(3/2) + 48*f*(c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g
+ h*x)^2*Sqrt[a + x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) - (f*g^2 +
 h*(-(e*g) + d*h))*(g + h*x)*(40*h*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h)
)^(3/2)*(a + x*(b + c*x))^(3/2) + 3*(8*c^2*g^2 + (5*b^2*h^2)/2 - 2*c*h*(4*
b*g + a*h))*(g + h*x)*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c
*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) - (b^2 - 4*a*c)*(g + h*x)^2*ArcTanh[
(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[
a + x*(b + c*x)])])) - 24*(b^2 - 4*a*c)*f*(c*g^2 + h*(-(b*g) + a*h))^2*(g
+ h*x)^4*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g
) + a*h)]*Sqrt[a + x*(b + c*x)])] + 12*(2*c*g - b*h)*(2*f*g - e*h)*(c*g^2
+ h*(-(b*g) + a*h))*(g + h*x)^2*(-2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a
+ x*(b + c*x)]*(-2*a*h + 2*c*g*x + b*(g - h*x)) + (b^2 - 4*a*c)*(g + h*x)^
2*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h
)]*Sqrt[a + x*(b + c*x)])]))/(192*h^2*(c*g^2 + h*(-(b*g) + a*h))^(7/2)*(g
+ h*x)^4)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2181, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^5}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+8cdg-3beg-8afg-5bdh+8aeh+2\left(\frac{3cfg^2}{h}+ceg-4bfg-cdh+4afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^4}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(8cdg-8afg+8aeh-b\left(-\frac{3fg^2}{h}+3eg+5dh\right)-2\left(4bfg-4afh-c\left(\frac{3fg^2}{h}+eg-dh\right)\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^4}dx}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1228

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\int \frac{\sqrt{cx^2+bx+a}}{(g+hx)^3}dx}{2(ah^2-bgh+cg^2)}+\frac{(a+bx+cx^2)^{3/2}(8}{}}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1152

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{(b}{}\right.}{2(ah^2-bgh+cg^2)}}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-8abh(eh+2fg)+b^2\left(h(5dh+3eg)+5fg^2\right)+16c^2dg^2\right)\left(\frac{\left(b^2-4ac\right)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah}{cx}}}{4\left(ah^2-\right.}\right.}{2(ah^2-bgh+cg^2)}}{8(}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)\left(16a^2fh^2-4c\left(-ah(5eg-dh)+afg^2+2bg(2dh+eg)\right)-\right.}{2(ah^2-bgh+cg^2)}}{8\left(ah^2-bg\right.}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^4) + (((6*c*f*g^3 + 2*c*g*h*(e*g - 5*d*h) + 8*a*h^2*(2*f
*g - e*h) - b*h*(11*f*g^2 - h*(3*e*g + 5*d*h)))*(a + b*x + c*x^2)^(3/2))/(
3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) + ((16*c^2*d*g^2 + 16*a^2*f*h^2 -
 8*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(5*e*g - d*h) + 2*b*g*(e*g + 2
*d*h)) + b^2*(5*f*g^2 + h*(3*e*g + 5*d*h)))*(((b*g - 2*a*h + (2*c*g - b*h)
*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2
 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(2*(c*
g^2 - b*g*h + a*h^2)))/(8*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4939 vs. $2(477) = 954$.

Time = 0.62 (sec) , antiderivative size = 4940, normalized size of antiderivative = 9.90

method	result	size
default	Expression too large to display	4940

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)-1/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2
)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(
a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(((x+g
/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*
g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(
x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+2*c/(a*h^2-b*g*h+c*g^2)*h^2*(1
/4*(2*c*(x+g/h)+(b*h-2*c*g)/h)/c*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^
2)/c^(3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*
g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))))+1/2*c/(a*h^2-b*g*h+c*g^2)*h
^2*(((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*
(b*h-2*c*g)/h*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2
*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2
)/h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2
*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)
/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))))+(e*h-2*f*g)/h^6*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^5}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 10.54 (sec) , antiderivative size = 17516, normalized size of antiderivative = 35.10

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

output

```
(192*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**4*h**2 +
768*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**3*h**3*x +
 1152*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h*
*2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**2*h**4*x
**2 + 768*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(
a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g*h**5*
x**3 + 192*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt
(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*h**6*x
**4 - 48*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a
*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f*g**4*
h**2 - 192*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt
(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f*g**
3*h**3*x - 288*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*
sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f
*g**2*h**4*x**2 - 192*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c
*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2
*b**2*f*g*h**5*x**3 - 48*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x
+ c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x...
```

3.32 $\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 826

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$= \frac{(32c^3dg^3 - 8c^2g(afg^2 - 3ah(2eg-dh) + 2bg(eg+3dh)) + 2c(4a^2h^2(6fg-eh) - 6abh(3fg^2 + h(3eg -}{}$$

$$- \frac{(fg^2 - h(eg-dh))\,(a+bx+cx^2)^{3/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5}$$

$$+ \frac{(6cfg^3 + 2cgh(2eg-7dh) + 10ah^2(2fg-eh) - bh(13fg^2 - h(3eg+7dh)))\,(a+bx+cx^2)^{3/2}}{40h\,(cg^2-bgh+ah^2)^2\,(g+hx)^4}$$

$$+ \frac{(4c^2(3fg^4 + g^2h(2eg-27dh)) - 5h^2(16a^2fh^2 - 2abh(6fg+5eh) + b^2(3fg^2+3egh+7dh^2)) - 2ch(}{240h\,(cg^2-bgh+ah^2)^3\,(g+}$$

$$- \frac{(b^2-4ac)\,(32c^3dg^3 - 8c^2g(afg^2 - 3ah(2eg-dh) + 2bg(eg+3dh)) + 2c(4a^2h^2(6fg-eh) - 6abh(3}{}$$

output
```
1/128*(32*c^3*d*g^3-8*c^2*g*(a*f*g^2-3*a*h*(-d*h+2*e*g)+2*b*g*(3*d*h+e*g))
+2*c*(4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(3*f*g^2+h*(-d*h+3*e*g))+b^2*(5*f*g^3
+3*g*h*(5*d*h+2*e*g)))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(3*f*g^
2+h*(7*d*h+3*e*g))))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2
-b*g*h+c*g^2)^4/(h*x+g)^2-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(3/2)/h/(
a*h^2-b*g*h+c*g^2)/(h*x+g)^5+1/40*(6*c*f*g^3+2*c*g*h*(-7*d*h+2*e*g)+10*a*h
^2*(-e*h+2*f*g)-b*h*(13*f*g^2-h*(7*d*h+3*e*g)))*(c*x^2+b*x+a)^(3/2)/h/(a*h
^2-b*g*h+c*g^2)^2/(h*x+g)^4+1/240*(4*c^2*(3*f*g^4+g^2*h*(-27*d*h+2*e*g))-5
*h^2*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(7*d*h^2+3*e*g*h+3*f*g^2))-2*
c*h*(b*g*(-54*d*h^2-21*e*g*h+16*f*g^2)-2*a*h*(8*d*h^2-33*e*g*h+18*f*g^2)))
*(c*x^2+b*x+a)^(3/2)/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^3-1/256*(-4*a*c+b^2)*
(32*c^3*d*g^3-8*c^2*g*(a*f*g^2-3*a*h*(-d*h+2*e*g)+2*b*g*(3*d*h+e*g))+2*c*(
4*a^2*h^2*(-e*h+6*f*g)-6*a*b*h*(3*f*g^2+h*(-d*h+3*e*g))+b^2*(5*f*g^3+3*g*h
*(5*d*h+2*e*g)))-b*h*(16*a^2*f*h^2-2*a*b*h*(5*e*h+6*f*g)+b^2*(3*f*g^2+h*(7
*d*h+3*e*g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(9/2)
```

Mathematica [A] (verified)

Time = 16.41 (sec) , antiderivative size = 1334, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6} \, dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(h*
(c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*f*g - e*h)*(a + b*x + c*x^2)*Sq
rt[a + x*(b + c*x)])/(4*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^4) - (f*(a + b
*x + c*x^2)*Sqrt[a + x*(b + c*x)])/(3*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
3) - ((-2*f*g + e*h)*Sqrt[a + x*(b + c*x)]*(((c*g*h - (h*(-8*c*g + 5*b*h))
/2)*(a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^3) - ((-
2*(a*c*h^2 + (c*g*(-8*c*g + 5*b*h))/2) + b*(c*g*h + (h*(-8*c*g + 5*b*h))/2
))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*
g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*
g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqr
t[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(2*(c*g^2 - b*g*
h + a*h^2))))/(4*h^2*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) - ((f*
g^2 - e*g*h + d*h^2)*Sqrt[a + x*(b + c*x)]*(-1/4*((-2*c*g*h + (h*(-10*c*g
+ 7*b*h))/2)*(a + b*x + c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^4
) - ((((-7*c*g*h*(2*c*g - b*h))/2 - (h*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(4
7*b*g + 16*a*h)))/4)*(a + b*x + c*x^2)^(3/2))/(3*(c*g^2 - b*g*h + a*h^2)*(
g + h*x)^3) - ((-2*((-7*a*c*h^2*(2*c*g - b*h))/2 + (c*g*(80*c^2*g^2 + 35*b
^2*h^2 - 2*c*h*(47*b*g + 16*a*h)))/4) + b*((-7*c*g*h*(2*c*g - b*h))/2 + (h
*(80*c^2*g^2 + 35*b^2*h^2 - 2*c*h*(47*b*g + 16*a*h)))/4))*(((b*g - 2*a*h +
 (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g ...
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 730, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2181, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}{(g+hx)^6}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{3bfg^2}{h}+10cdg-3beg-10afg-7bdh+10aeh+2\left(\frac{3cfg^2}{h}+2ceg-5bfg-2cdh+5afh\right)x\right)\sqrt{cx^2+bx+a}}{2(g+hx)^5}dx}{5\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(10cdg-b\left(-\frac{3fg^2}{h}+3eg+7dh\right)-10a(fg-eh)-2\left(5bfg-5afh-c\left(\frac{3fg^2}{h}+2eg-2dh\right)\right)x\right)\sqrt{cx^2+bx+a}}{(g+hx)^5}dx}{10\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1237$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}\left(10ah^2(2fg-eh)-bh\left(13fg^2-h(7dh+3eg)\right)+2cgh(2eg-7dh)+6cfg^3\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\int -\frac{\left(h\left(5\left(3fg^2+h(3eg+7dh)\right)b^2-2cg\left(-\frac{3fg^2}{h}+18eg+47d\right.\right.}{}}{10\left(ah^2-\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\left(h\left(5\left(3fg^2+h(3eg+7dh)\right)b^2-2cg\left(-\frac{3fg^2}{h}+18eg+47dh\right)b-10ah(6fg+5eh)b+80c^2dg^2+80a^2fh^2-16ac\left(2fg^2-h(7eg-2dh)\right)\right)+2c\left(6cfg^3+2ch(2eg-7dh)g+1\right.}{(g+hx)^4}}{8h\left(ah^2-bgh+cg^2\right)}}{10\left(ah^2-\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1228$$

$$\frac{5h\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh+2eg)+5fg^3\right)\right)-bh\left(16a^2fh^2-2abh(5eh+6fg)+b^2\left(h(7dh+3eg)+3fg^2\right)\right)-8c^2g\left(-3ah(2eg-dh)\right.\right.}{2\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \ 1152$$

$$\frac{5h\left(2c\left(4a^2h^2(6fg-eh)-6abh\left(h(3eg-dh)+3fg^2\right)+b^2\left(3gh(5dh+2eg)+5fg^3\right)\right)-bh\left(16a^2fh^2-2abh(5eh+6fg)+b^2\left(h(7dh+3eg)+3fg^2\right)\right)-8c^2g\left(-3ah(2eg-dh)\right.\right.}{2\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 1154$$

$$5h\Big(2c\Big(4a^2h^2(6fg-eh)-6abh\Big(h(3eg-dh)+3fg^2\Big)+b^2\Big(3gh(5dh+2eg)+5fg^3\Big)\Big)-bh\Big(16a^2fh^2-2abh(5eh+6fg)+b^2\Big(h(7dh+3eg)+3fg^2\Big)\Big)-8c^2g\Big(-3ah(2eg-dh)\ldots$$

$$2\Big(ah^2-b\ldots$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 219$$

$$5h\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)\Big(2c\Big(4a^2h^2(6fg-eh)-6abh\Big(h(3eg-dh)+3fg^2\Big)+b^2\Big(3gh(5\ldots$$

$$2\left(ah^2-bgh+cg^2\right)$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(3/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^5) + (((6*c*f*g^3 + 2*c*g*h*(2*e*g - 7*d*h) + 10*a*h^2*(
2*f*g - e*h) - b*h*(13*f*g^2 - h*(3*e*g + 7*d*h)))*(a + b*x + c*x^2)^(3/2)
)/(4*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + (((4*c^2*(3*f*g^4 + g^2*h*(2
*e*g - 27*d*h)) - 5*h^2*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f
*g^2 + 3*e*g*h + 7*d*h^2)) - 2*c*h*(b*g*(16*f*g^2 - 21*e*g*h - 54*d*h^2) -
 2*a*h*(18*f*g^2 - 33*e*g*h + 8*d*h^2)))*(a + b*x + c*x^2)^(3/2))/(3*(c*g^
2 - b*g*h + a*h^2)*(g + h*x)^3) + (5*h*(32*c^3*d*g^3 - 8*c^2*g*(a*f*g^2 -
3*a*h*(2*e*g - d*h) + 2*b*g*(e*g + 3*d*h)) + 2*c*(4*a^2*h^2*(6*f*g - e*h)
- 6*a*b*h*(3*f*g^2 + h*(3*e*g - d*h)) + b^2*(5*f*g^3 + 3*g*h*(2*e*g + 5*d*
h))) - b*h*(16*a^2*f*h^2 - 2*a*b*h*(6*f*g + 5*e*h) + b^2*(3*f*g^2 + h*(3*e
*g + 7*d*h))))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4
*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*
h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])
])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(2*(c*g^2 - b*g*h + a*h^2)))/(8*h*(
c*g^2 - b*g*h + a*h^2)))/(10*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 7713 vs. 2(800) = 1600.

Time = 0.83 (sec) , antiderivative size = 7714, normalized size of antiderivative = 9.34

method	result	size
default	Expression too large to display	7714

input
```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28577 vs. 2(800) = 1600.

Time = 1.76 (sec) , antiderivative size = 28577, normalized size of antiderivative = 34.60

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

output
```
-1/128*(32*b^2*c^3*d*g^3 - 128*a*c^4*d*g^3 - 16*b^3*c^2*e*g^3 + 64*a*b*c^3
*e*g^3 + 10*b^4*c*f*g^3 - 48*a*b^2*c^2*f*g^3 + 32*a^2*c^3*f*g^3 - 48*b^3*c
^2*d*g^2*h + 192*a*b*c^3*d*g^2*h + 12*b^4*c*e*g^2*h - 192*a^2*c^3*e*g^2*h
- 3*b^5*f*g^2*h - 24*a*b^3*c*f*g^2*h + 144*a^2*b*c^2*f*g^2*h + 30*b^4*c*d*
g*h^2 - 144*a*b^2*c^2*d*g*h^2 + 96*a^2*c^3*d*g*h^2 - 3*b^5*e*g*h^2 - 24*a*
b^3*c*e*g*h^2 + 144*a^2*b*c^2*e*g*h^2 + 12*a*b^4*f*g*h^2 - 192*a^3*c^2*f*g
*h^2 - 7*b^5*d*h^3 + 40*a*b^3*c*d*h^3 - 48*a^2*b*c^2*d*h^3 + 10*a*b^4*e*h^
3 - 48*a^2*b^2*c*e*h^3 + 32*a^3*c^2*e*h^3 - 16*a^2*b^3*f*h^3 + 64*a^3*b*c*
f*h^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c
*g^2 + b*g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a
*c^3*g^6*h^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b
^2*c*g^4*h^4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 +
6*a^2*b^2*g^2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2
 + b*g*h - a*h^2)) + 1/1920*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^2
*c^3*d*g^3*h^8 - 1920*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*c^4*d*g^3*h^
8 - 240*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*e*g^3*h^8 + 960*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b*c^3*e*g^3*h^8 + 150*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^9*b^4*c*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^9*a*b^2*c^2*f*g^3*h^8 + 480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^
2*c^3*f*g^3*h^8 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^3*c^2*d*g...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}{{\left(g+h\,x\right)}^6}\,dx$$

input
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 86.77 (sec) , antiderivative size = 31077, normalized size of antiderivative = 37.62

$$\int \frac{\sqrt{a+bx+cx^2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input

```
int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

output

```
(960*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*
h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g**5*h*
*3 + 4800*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sq
rt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g*
*4*h**4*x + 9600*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x
**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b
*c*f*g**3*h**5*x**2 + 9600*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a +
 b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g
*x)*a**3*b*c*f*g**2*h**6*x**3 + 4800*sqrt(a*h**2 - b*g*h + c*g**2)*log( -
2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h
*x + 2*c*g*x)*a**3*b*c*f*g*h**7*x**4 + 960*sqrt(a*h**2 - b*g*h + c*g**2)*l
og( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g
 - b*h*x + 2*c*g*x)*a**3*b*c*f*h**8*x**5 + 480*sqrt(a*h**2 - b*g*h + c*g**
2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h +
 b*g - b*h*x + 2*c*g*x)*a**3*c**2*e*g**5*h**3 + 2400*sqrt(a*h**2 - b*g*h +
 c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2
*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c**2*e*g**4*h**4*x + 4800*sqrt(a*h**2 -
 b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g
**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c**2*e*g**3*h**5*x**2 + 4800*sq
rt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2...
```

3.33 $\int (g+hx)^3 (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 1153

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

output
```
-1/32768*(-4*a*c+b^2)*(1536*c^5*d*g^3-143*b^5*f*h^3-256*c^4*g*(a*f*g^2+3*a
*h*(d*h+e*g)+3*b*g*(3*d*h+e*g))+22*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g))-48
*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2))+3
2*c^3*(3*a^2*h^2*(e*h+3*f*g)+14*b^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*
g^2+h*(d*h+3*e*g))))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^7+1/12288*(1536*c^5*d
*g^3-143*b^5*f*h^3-256*c^4*g*(a*f*g^2+3*a*h*(d*h+e*g)+3*b*g*(3*d*h+e*g))+2
2*b^3*c*h^2*(20*a*f*h+9*b*(e*h+3*f*g))-48*b*c^2*h*(5*a^2*f*h^2+9*a*b*h*(e*
h+3*f*g)+6*b^2*(d*h^2+3*e*g*h+3*f*g^2))+32*c^3*(3*a^2*h^2*(e*h+3*f*g)+14*b
^2*g*(f*g^2+3*h*(d*h+e*g))+12*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*(2*c*x+b)*(c
*x^2+b*x+a)^(3/2)/c^6+1/2016*(143*b^2*f*h/c+12*c*(9*e*g-5*f*g^2/h+24*d*h)-
128*a*f*h-198*b*e*h-48*b*f*g)*(h*x+g)^2*(c*x^2+b*x+a)^(5/2)/c^2+1/144*(18*
c*e-13*b*f-10*c*f*g/h)*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c^2+1/9*f*(h*x+g)^4*(
c*x^2+b*x+a)^(5/2)/c/h+1/80640*(3003*b^4*f*h^3-192*c^4*(5*f*g^4-3*g^2*h*(6
4*d*h+3*e*g))/h-198*b^2*c*h^2*(38*a*f*h+21*b*(e*h+3*f*g))+8*c^2*h*(256*a^2
*f*h^2+837*a*b*h*(e*h+3*f*g)+b^2*(1553*f*g^2+756*h*(d*h+3*e*g)))-16*c^3*(3
2*a*h*(17*f*g^2+9*h*(d*h+3*e*g))+b*g*(13*f*g^2+9*h*(196*d*h+141*e*g)))-10*
c*(429*b^3*f*h^3-22*b*c*h^2*(34*a*f*h+27*b*e*h+29*b*f*g)+16*c^3*(5*f*g^3-9
*g*h*(12*d*h+e*g))+8*c^2*h*(a*h*(63*e*h+61*f*g)+3*b*(f*g^2+6*h*(6*d*h+7*e*
g))))*x)*(c*x^2+b*x+a)^(5/2)/c^5+1/65536*(-4*a*c+b^2)^2*(1536*c^5*d*g^3-14
3*b^5*f*h^3-256*c^4*g*(a*f*g^2+3*a*h*(d*h+e*g)+3*b*g*(3*d*h+e*g))+22*b^...
```

Mathematica [A] (verified)

Time = 16.29 (sec) , antiderivative size = 1683, normalized size of antiderivative = 1.46

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
Integrate[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(72*h*(f*g^2 + h*(-(e*g) + d*h))*(g + h*x)^2*(a + x*(b + c*x))^(5/2) + 63*
h*(-2*f*g + e*h)*(g + h*x)^3*(a + x*(b + c*x))^(5/2) + 56*f*h*(g + h*x)^4*
(a + x*(b + c*x))^(5/2) - (f*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-45045*b^8*
h^5 + 2310*b^7*c*h^4*(135*g + 13*h*x) - 84*b^6*c*h^3*(-5225*a*h^2 + c*(108
00*g^2 + 2475*g*h*x + 286*h^2*x^2)) + 72*b^5*c^2*h^2*(-7*a*h^2*(5475*g + 5
17*h*x) + 2*c*(9800*g^3 + 4200*g^2*h*x + 1155*g*h^2*x^2 + 143*h^3*x^3)) -
16*b^4*c^2*h*(86499*a^2*h^4 - 9*a*c*h^2*(50400*g^2 + 11235*g*h*x + 1276*h^
2*x^2) + 2*c^2*(37800*g^4 + 29400*g^3*h*x + 15120*g^2*h^2*x^2 + 4455*g*h^3
*x^3 + 572*h^4*x^4)) - 128*b*c^4*(a^3*h^4*(41355*g + 3701*h*x) - 6*a^2*c*h
^2*(22680*g^3 + 8760*g^2*h*x + 2265*g*h^2*x^2 + 269*h^3*x^3) + 40*a*c^2*(6
30*g^5 + 434*g^4*h*x - 1036*g^3*h^2*x^2 - 2292*g^2*h^3*x^3 - 1675*g*h^4*x^
4 - 433*h^5*x^5) + 80*c^3*x^2*(378*g^5 + 1162*g^4*h*x + 1288*g^3*h^2*x^2 +
 456*g^2*h^3*x^3 - 131*g*h^4*x^4 - 91*h^5*x^5)) - 256*c^4*(1024*a^4*h^5 -
a^3*c*h^3*(23040*g^2 + 4725*g*h*x + 512*h^2*x^2) + 80*c^4*g*x^3*(126*g^4 +
 448*g^3*h*x + 616*g^2*h^2*x^2 + 384*g*h^3*x^3 + 91*h^4*x^4) - 2*a^2*c^2*h
*(-17920*g^4 - 3640*g^3*h*x + 7680*g^2*h^2*x^2 + 7385*g*h^3*x^3 + 2048*h^4
*x^4) + 40*a*c^3*x*(630*g^5 + 1792*g^4*h*x + 2044*g^3*h^2*x^2 + 960*g^2*h^
3*x^3 + 49*g*h^4*x^4 - 64*h^5*x^5)) + 32*b^3*c^3*(9*a^2*h^4*(25515*g + 235
3*h*x) - 4*a*c*h^2*(79800*g^3 + 32760*g^2*h*x + 8775*g*h^2*x^2 + 1067*h^3*
x^3) + 40*c^2*(378*g^5 + 630*g^4*h*x + 588*g^3*h^2*x^2 + 324*g^2*h^3*x^...
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 801, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^3(5bfg-18cdh+8afh+(10cfg-18ceh+13bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{9ch^2}+\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\int (g+hx)^3(5bfg-18cdh+8afh+(10cfg-18ceh+13bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{18ch}$$

$\big\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{\int -\frac{1}{2}(g+hx)^2\left(65fghb^2+78afh^2b-30cg(fg+3eh)b+4ch(72cdg-17afg-27aeh)+\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+1\cdots\right.\right.}{8c}}{18ch}$$

$\big\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\int (g+hx)^2\left(65fghb^2+78afh^2b-30cg(fg+3eh)b+4ch(72cdg-17afg-27aeh)+\left(-12\left(5fg^2-\cdots\right.\right.\right.}{16c}}{18ch}$$

$\big\downarrow$ 1236

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\int -\frac{1}{2}(g+hx)\left(715fgh^2b^3+2\left(286afh^3-5cgh(115fg+99eh)\right)b^2-4c\left(ah^2(481fg+198eh)-30cg\left(fg^2+3h\cdots\right.\right.\right.}{\cdots}}{\cdots}$$

$\big\downarrow$ 27

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{\int(\cdots}{\cdots}}{\cdots}}{\cdots}$$

$\big\downarrow$ 1225

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{-\cdots}{\cdots}}{\cdots}}{\cdots}$$

$\big\downarrow$ 1087

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-$$

$$\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{-}{}}{}}{}$$

$\downarrow$ 1087

$$\frac{f(g+hx)^4\left(a+bx+cx^2\right)^{5/2}}{9ch}-$$

$$\frac{\frac{(g+hx)^3\left(a+bx+cx^2\right)^{5/2}(13bfh-18ceh+10cfg)}{8c}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}\left(-2ch(64afh+99beh+24bfg)+143b^2fh^2-12c^2\left(5fg^2-3h(8dh+3eg)\right)\right)}{7c}-\frac{-}{}}{}}{}$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\left(cx^2+bx+a\right)^{5/2}}{9ch}-$$

$$\frac{\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(cx^2+bx+a\right)^{5/2}}{8c}-\frac{\frac{\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+143b^2fh^2\right)(g+hx)^2\left(cx^2+bx+a\right)^{5/2}}{7c}-\frac{-}{}}{}}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\left(cx^2+bx+a\right)^{5/2}}{9ch}-$$

$$\frac{\frac{(10cfg-18ceh+13bfh)(g+hx)^3\left(cx^2+bx+a\right)^{5/2}}{8c}-\frac{\frac{\left(-12\left(5fg^2-3h(3eg+8dh)\right)c^2-2h(24bfg+99beh+64afh)c+143b^2fh^2\right)(g+hx)^2\left(cx^2+bx+a\right)^{5/2}}{7c}-\frac{-}{}}{}}{}$$

input
```
Int[(g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^4*(a + b*x + c*x^2)^(5/2))/(9*c*h) - (((10*c*f*g - 18*c*e*h +
 13*b*f*h)*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(8*c) - (((143*b^2*f*h^2 -
 2*c*h*(24*b*f*g + 99*b*e*h + 64*a*f*h) - 12*c^2*(5*f*g^2 - 3*h*(3*e*g + 8
*d*h)))*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) - (-1/20*((3003*b^4*f*h
^4 - 192*c^4*(5*f*g^4 - 3*g^2*h*(3*e*g + 64*d*h)) - 198*b^2*c*h^3*(38*a*f*
h + 21*b*(3*f*g + e*h)) + 8*c^2*h^2*(256*a^2*f*h^2 + 837*a*b*h*(3*f*g + e*
h) + b^2*(1553*f*g^2 + 756*h*(3*e*g + d*h))) - 16*c^3*h*(32*a*h*(17*f*g^2
+ 9*h*(3*e*g + d*h)) + b*g*(13*f*g^2 + 9*h*(141*e*g + 196*d*h))) - 10*c*h*
(429*b^3*f*h^3 - 22*b*c*h^2*(29*b*f*g + 27*b*e*h + 34*a*f*h) + 16*c^3*(5*f
*g^3 - 9*g*h*(e*g + 12*d*h)) + 8*c^2*h*(a*h*(61*f*g + 63*e*h) + 3*b*(f*g^2
 + 6*h*(7*e*g + 6*d*h))))*x)*(a + b*x + c*x^2)^(5/2))/c^2 - (21*h*(1536*c^
5*d*g^3 - 143*b^5*f*h^3 - 256*c^4*g*(a*f*g^2 + 3*a*h*(e*g + d*h) + 3*b*g*(
e*g + 3*d*h)) + 22*b^3*c*h^2*(20*a*f*h + 9*b*(3*f*g + e*h)) - 48*b*c^2*h*(
5*a^2*f*h^2 + 9*a*b*h*(3*f*g + e*h) + 6*b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) +
 32*c^3*(3*a^2*h^2*(3*f*g + e*h) + 14*b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 12
*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2)
)/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((
b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^
(3/2))))/(16*c)))/(8*c^2))/(14*c))/(16*c))/(18*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2869 vs. 2(1116) = 2232.

Time = 0.45 (sec) , antiderivative size = 2870, normalized size of antiderivative = 2.49

method	result	size
default	Expression too large to display	2870
risch	Expression too large to display	3146

input
```
int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^3*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+
b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))+g^2*(3*d*h+e*g)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1
/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*
x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2)))))+h^2*(e*h+3*f*g)*(1/8*x^3*(c*x^2+b*x+a)^(5/2)/c-11/16*b/c*(1/
7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c
*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3
/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)
/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(
1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))
)-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a
)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c
-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))-3/8*a/c*(1/6
*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*
(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2
+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)
^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c
*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2374 vs. 2(1116) = 2232.

Time = 2.49 (sec) , antiderivative size = 4751, normalized size of antiderivative = 4.12

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19122 vs. 2(1210) = 2420.

Time = 1.52 (sec) , antiderivative size = 19122, normalized size of antiderivative = 16.58

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output

```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**3*x**8/9 + x**7*(19*b*c*f*h**3/1
8 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + x**6*(10*a*c*f*h**3/9 + b**2*f*
h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**
3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**
2*h)/(7*c) + x**5*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6*a*c*f*g*h**2 - 7*a*(19*
b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*c) + b**2*e*h**3 + 3*b**
2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c*f*g**2*h - 13*b*(10*a*c
*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h
**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*
h**2 + 3*c**2*f*g**2*h)/(14*c) + 3*c**2*d*g*h**2 + 3*c**2*e*g**2*h + c**2*
f*g**3)/(6*c) + x**4*(a**2*f*h**3 + 2*a*b*e*h**3 + 6*a*b*f*g*h**2 + 2*a*c*
d*h**3 + 6*a*c*e*g*h**2 + 6*a*c*f*g**2*h - 6*a*(10*a*c*f*h**3/9 + b**2*f*h
**3 + 2*b*c*e*h**3 + 6*b*c*f*g*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3
 + 3*c**2*f*g*h**2)/(16*c) + c**2*d*h**3 + 3*c**2*e*g*h**2 + 3*c**2*f*g**2
*h)/(7*c) + b**2*d*h**3 + 3*b**2*e*g*h**2 + 3*b**2*f*g**2*h + 6*b*c*d*g*h*
*2 + 6*b*c*e*g**2*h + 2*b*c*f*g**3 - 11*b*(2*a*b*f*h**3 + 2*a*c*e*h**3 + 6
*a*c*f*g*h**2 - 7*a*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(8*
c) + b**2*e*h**3 + 3*b**2*f*g*h**2 + 2*b*c*d*h**3 + 6*b*c*e*g*h**2 + 6*b*c
*f*g**2*h - 13*b*(10*a*c*f*h**3/9 + b**2*f*h**3 + 2*b*c*e*h**3 + 6*b*c*f*g
*h**2 - 15*b*(19*b*c*f*h**3/18 + c**2*e*h**3 + 3*c**2*f*g*h**2)/(16*c) ...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \text{Exception raised: ValueError}$$

input

```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output

```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2902 vs. $2(1116) = 2232$.

Time = 0.26 (sec) , antiderivative size = 2902, normalized size of antiderivative = 2.52

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/10321920*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*(16*c*f*h^3*x + (5
4*c^9*f*g*h^2 + 18*c^9*e*h^3 + 19*b*c^8*f*h^3)/c^8)*x + (864*c^9*f*g^2*h +
 864*c^9*e*g*h^2 + 918*b*c^8*f*g*h^2 + 288*c^9*d*h^3 + 306*b*c^8*e*h^3 + 3
*b^2*c^7*f*h^3 + 320*a*c^8*f*h^3)/c^8)*x + (1344*c^9*f*g^3 + 4032*c^9*e*g^
2*h + 4320*b*c^8*f*g^2*h + 4032*c^9*d*g*h^2 + 4320*b*c^8*e*g*h^2 + 54*b^2*
c^7*f*g*h^2 + 4536*a*c^8*f*g*h^2 + 1440*b*c^8*d*h^3 + 18*b^2*c^7*e*h^3 + 1
512*a*c^8*e*h^3 - 13*b^3*c^6*f*h^3 + 60*a*b*c^7*f*h^3)/c^8)*x + (16128*c^9
*e*g^3 + 17472*b*c^8*f*g^3 + 48384*c^9*d*g^2*h + 52416*b*c^8*e*g^2*h + 864
*b^2*c^7*f*g^2*h + 55296*a*c^8*f*g^2*h + 52416*b*c^8*d*g*h^2 + 864*b^2*c^7
*e*g*h^2 + 55296*a*c^8*e*g*h^2 - 594*b^3*c^6*f*g*h^2 + 2808*a*b*c^7*f*g*h^
2 + 288*b^2*c^7*d*h^3 + 18432*a*c^8*d*h^3 - 198*b^3*c^6*e*h^3 + 936*a*b*c^
7*e*h^3 + 143*b^4*c^5*f*h^3 - 804*a*b^2*c^6*f*h^3 + 768*a^2*c^7*f*h^3)/c^8
)*x + (161280*c^9*d*g^3 + 177408*b*c^8*e*g^3 + 4032*b^2*c^7*f*g^3 + 188160
*a*c^8*f*g^3 + 532224*b*c^8*d*g^2*h + 12096*b^2*c^7*e*g^2*h + 564480*a*c^8
*e*g^2*h - 7776*b^3*c^6*f*g^2*h + 38016*a*b*c^7*f*g^2*h + 12096*b^2*c^7*d*
g*h^2 + 564480*a*c^8*d*g*h^2 - 7776*b^3*c^6*e*g*h^2 + 38016*a*b*c^7*e*g*h^
2 + 5346*b^4*c^5*f*g*h^2 - 30672*a*b^2*c^6*f*g*h^2 + 30240*a^2*c^7*f*g*h^2
 - 2592*b^3*c^6*d*h^3 + 12672*a*b*c^7*d*h^3 + 1782*b^4*c^5*e*h^3 - 10224*a
*b^2*c^6*e*h^3 + 10080*a^2*c^7*e*h^3 - 1287*b^5*c^4*f*h^3 + 8536*a*b^3*c^5
*f*h^3 - 12912*a^2*b*c^6*f*h^3)/c^8)*x + (483840*b*c^8*d*g^3 + 16128*b^...
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \int (g+h\,x)^3\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^3*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int (g+hx)^3 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \int (hx+g)^3 \left(c\,x^2+bx+a\right)^{\frac{3}{2}} \left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^3*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

3.34 $\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 743

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx =$$

$$-\frac{(b^2-4ac)\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg\right.}{16384c^6}$$

$$+\frac{\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg+2dh))+\right.}{6144c^5}$$

$$+\frac{\left(16ce-11bf-\frac{10cfg}{h}\right)(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{112c^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$-\frac{\left(693b^3fh^2+\frac{96c^3(5fg^3-8gh(eg+7dh))}{h}-36bch(31afh+28b(2fg+eh))+8c^2(31bfg^2+196bh(2eg+dh)+\right.}{}$$

$$+\frac{(b^2-4ac)^2\left(768c^4dg^2+99b^4fh^2-72b^2ch(4bfg+2beh+3afh)-128c^3(afg^2+ah(2eg+dh)+3bg(eg\right.}{32768c^{13/}}$$

output
```
-1/16384*(-4*a*c+b^2)*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*
e*h+4*b*f*g)-128*c^3*(a*f*g^2+a*h*(d*h+2*e*g)+3*b*g*(2*d*h+e*g))+16*c^2*(3
*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(f*g^2+h*(d*h+2*e*g))))*(2*c*x+b)*(
c*x^2+b*x+a)^(1/2)/c^6+1/6144*(768*c^4*d*g^2+99*b^4*f*h^2-72*b^2*c*h*(3*a*
f*h+2*b*e*h+4*b*f*g)-128*c^3*(a*f*g^2+a*h*(d*h+2*e*g)+3*b*g*(2*d*h+e*g))+1
6*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b^2*(f*g^2+h*(d*h+2*e*g))))*(2*
c*x+b)*(c*x^2+b*x+a)^(3/2)/c^5+1/112*(16*c*e-11*b*f-10*c*f*g/h)*(h*x+g)^2*
(c*x^2+b*x+a)^(5/2)/c^2+1/8*f*(h*x+g)^3*(c*x^2+b*x+a)^(5/2)/c/h-1/13440*(6
93*b^3*f*h^2+96*c^3*(5*f*g^3-8*g*h*(7*d*h+e*g))/h-36*b*c*h*(31*a*f*h+28*b*
(e*h+2*f*g))+8*c^2*(31*b*f*g^2+196*b*h*(d*h+2*e*g)+96*a*h*(e*h+2*f*g))-10*
c*(99*b^2*f*h^2-8*c^2*(5*f*g^2-4*h*(7*d*h+2*e*g))-12*c*h*(7*a*f*h+2*b*(6*e
*h+f*g)))*x)*(c*x^2+b*x+a)^(5/2)/c^4+1/32768*(-4*a*c+b^2)^2*(768*c^4*d*g^2
+99*b^4*f*h^2-72*b^2*c*h*(3*a*f*h+2*b*e*h+4*b*f*g)-128*c^3*(a*f*g^2+a*h*(d
*h+2*e*g)+3*b*g*(2*d*h+e*g))+16*c^2*(3*a^2*f*h^2+12*a*b*h*(e*h+2*f*g)+14*b
^2*(f*g^2+h*(d*h+2*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/
2))/c^(13/2)
```

Mathematica [A] (verified)

Time = 12.38 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.18

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{430080dg^2(b+2cx)(a+x(b+cx))^{3/2}+688128g(eg+2dh)(a+x(b+cx))^{5/2}+573440(fg^2 \cdots}{\cdots}$$

input
```
Integrate[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(430080*d*g^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 688128*g*(e*g + 2*d*h)
*(a + x*(b + c*x))^(5/2) + 573440*(f*g^2 + h*(2*e*g + d*h))*x*(a + x*(b +
c*x))^(5/2) + 491520*h*(2*f*g + e*h)*x^2*(a + x*(b + c*x))^(5/2) + 430080*
f*h^2*x^3*(a + x*(b + c*x))^(5/2) + (80640*(b^2 - 4*a*c)*d*g^2*(-2*Sqrt[c]
*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (13440*b*g*(e*g + 2*d*h)*(16*c
^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b
 + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (48*h*(2*f*g + e*h)*(-256*c^(5/2
)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*
a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*
Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(f*g^2 + h*(2*e*
g + d*h))*(1792*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*
c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(
b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(7/2) - (3*f*h^2*(112640*b*c^(9/2)*x^2
*(a + x*(b + c*x))^(5/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x
+ 280*a*c^2*x)*(a + x*(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*
c^2)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*...
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.71, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^2(5bfg-16cdh+6afh+(10cfg-16ceh+11bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{8ch^2}+\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\int (g+hx)^2(5bfg-16cdh+6afh+(10cfg-16ceh+11bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{16ch}$$

$\Big\downarrow$ 1236

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\frac{\int -\frac{1}{2}(g+hx)\left(55fghb^2+44afh^2b-20cg(fg+4eh)b+4ch(56cdg-11afg-16aeh)+\left(-8\left(5fg^2-4h(2eg+7dh)\right)c^2-12h(7afh+2b(fg+6eh))c+99b^2\cdots\right.\right.}{7c}\cdots}{16ch}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{\int (g+hx)\left(55fghb^2+44afh^2b-20cg(fg+4eh)b+4ch(56cdg-11afg-16aeh)+\left(-8\left(5fg^2-4h\cdots\right.\right.\right.}{14c}\cdots}{16ch}$$

$\Big\downarrow$ 1225

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{\frac{7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(\cdots\right.\right.}{24c^2}\cdots}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{\frac{7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1087

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{\frac{7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 1092

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{7h\left(16c^2\left(3a^2fh^2+12abh(eh+2fg)+14b^2\left(h(dh+2eg)+fg^2\right)\right)-72b^2ch(3afh+2beh+4bfg)-128c^3\left(ah(\right.\right.}{}}{}$$

$\downarrow$ 219

$$\frac{f(g+hx)^3\left(a+bx+cx^2\right)^{5/2}}{8ch}-$$

$$\frac{\frac{(g+hx)^2\left(a+bx+cx^2\right)^{5/2}(11bfh-16ceh+10cfg)}{7c}-\frac{7h\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{8c^{3/2}}\right.}{16c}\right.}{}}{}$$

input
```
Int[(g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^3*(a + b*x + c*x^2)^(5/2))/(8*c*h) - ((((10*c*f*g - 16*c*e*h +
 11*b*f*h)*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c) - (-1/60*((693*b^3*f
*h^3 + 96*c^3*(5*f*g^3 - 8*g*h*(e*g + 7*d*h)) - 36*b*c*h^2*(31*a*f*h + 28*
b*(2*f*g + e*h)) + 8*c^2*h*(31*b*f*g^2 + 196*b*h*(2*e*g + d*h) + 96*a*h*(2
*f*g + e*h)) - 10*c*h*(99*b^2*f*h^2 - 8*c^2*(5*f*g^2 - 4*h*(2*e*g + 7*d*h)
) - 12*c*h*(7*a*f*h + 2*b*(f*g + 6*e*h)))*x)*(a + b*x + c*x^2)^(5/2))/c^2
+ (7*h*(768*c^4*d*g^2 + 99*b^4*f*h^2 - 72*b^2*c*h*(4*b*f*g + 2*b*e*h + 3*a
*f*h) - 128*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + 3*b*g*(e*g + 2*d*h)) + 16*c
^2*(3*a^2*f*h^2 + 12*a*b*h*(2*f*g + e*h) + 14*b^2*(f*g^2 + h*(2*e*g + d*h)
)))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b +
 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(24*c^2))/(14*c
))/(16*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. 2(713) = 1426.

Time = 0.34 (sec) , antiderivative size = 1676, normalized size of antiderivative = 2.26

method	result	size
default	Expression too large to display	1676
risch	Expression too large to display	1933

input
```
int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g^2*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+
b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))))+g*(2*d*h+e*g)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8
*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^
2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2)))))+h*(e*h+2*f*g)*(1/7*x^2*(c*x^2+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(
c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c
*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/
4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*
(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(
c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))))))+(d*h^2+2*e*g*h+f*g^2)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b
/c*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)
+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+
b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)
^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1571 vs. $2(713) = 1426$.

Time = 1.29 (sec) , antiderivative size = 3145, normalized size of antiderivative = 4.23

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9687 vs. 2(779) = 1558.

Time = 1.34 (sec) , antiderivative size = 9687, normalized size of antiderivative = 13.04

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h**2*x**7/8 + x**6*(17*b*c*f*h**2/1
6 + c**2*e*h**2 + 2*c**2*f*g*h)/(7*c) + x**5*(9*a*c*f*h**2/8 + b**2*f*h**2
 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c
**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + x**4
*(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2
*e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2
+ 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*
e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h
)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h
 + c**2*e*g**2)/(5*c) + x**3*(a**2*f*h**2 + 2*a*b*e*h**2 + 4*a*b*f*g*h + 2
*a*c*d*h**2 + 4*a*c*e*g*h + 2*a*c*f*g**2 - 5*a*(9*a*c*f*h**2/8 + b**2*f*h*
*2 + 2*b*c*e*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2
*c**2*f*g*h)/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(6*c) + b*
*2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2 + 4*b*c*d*g*h + 2*b*c*e*g**2 - 9*b*
(2*a*b*f*h**2 + 2*a*c*e*h**2 + 4*a*c*f*g*h - 6*a*(17*b*c*f*h**2/16 + c**2*
e*h**2 + 2*c**2*f*g*h)/(7*c) + b**2*e*h**2 + 2*b**2*f*g*h + 2*b*c*d*h**2 +
 4*b*c*e*g*h + 2*b*c*f*g**2 - 11*b*(9*a*c*f*h**2/8 + b**2*f*h**2 + 2*b*c*e
*h**2 + 4*b*c*f*g*h - 13*b*(17*b*c*f*h**2/16 + c**2*e*h**2 + 2*c**2*f*g*h)
/(14*c) + c**2*d*h**2 + 2*c**2*e*g*h + c**2*f*g**2)/(12*c) + 2*c**2*d*g*h
+ c**2*e*g**2)/(10*c) + c**2*d*g**2)/(4*c) + x**2*(a**2*e*h**2 + 2*a**2...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1802 vs. 2(713) = 1426.

Time = 0.23 (sec) , antiderivative size = 1802, normalized size of antiderivative = 2.43

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f*h^2*x + (32*c^
8*f*g*h + 16*c^8*e*h^2 + 17*b*c^7*f*h^2)/c^7)*x + (224*c^8*f*g^2 + 448*c^8
*e*g*h + 480*b*c^7*f*g*h + 224*c^8*d*h^2 + 240*b*c^7*e*h^2 + 3*b^2*c^6*f*h
^2 + 252*a*c^7*f*h^2)/c^7)*x + (2688*c^8*e*g^2 + 2912*b*c^7*f*g^2 + 5376*c
^8*d*g*h + 5824*b*c^7*e*g*h + 96*b^2*c^6*f*g*h + 6144*a*c^7*f*g*h + 2912*b
*c^7*d*h^2 + 48*b^2*c^6*e*h^2 + 3072*a*c^7*e*h^2 - 33*b^3*c^5*f*h^2 + 156*
a*b*c^6*f*h^2)/c^7)*x + (26880*c^8*d*g^2 + 29568*b*c^7*e*g^2 + 672*b^2*c^6
*f*g^2 + 31360*a*c^7*f*g^2 + 59136*b*c^7*d*g*h + 1344*b^2*c^6*e*g*h + 6272
0*a*c^7*e*g*h - 864*b^3*c^5*f*g*h + 4224*a*b*c^6*f*g*h + 672*b^2*c^6*d*h^2
 + 31360*a*c^7*d*h^2 - 432*b^3*c^5*e*h^2 + 2112*a*b*c^6*e*h^2 + 297*b^4*c^
4*f*h^2 - 1704*a*b^2*c^5*f*h^2 + 1680*a^2*c^6*f*h^2)/c^7)*x + (80640*b*c^7
*d*g^2 + 2688*b^2*c^6*e*g^2 + 86016*a*c^7*e*g^2 - 1568*b^3*c^5*f*g^2 + 806
4*a*b*c^6*f*g^2 + 5376*b^2*c^6*d*g*h + 172032*a*c^7*d*g*h - 3136*b^3*c^5*e
*g*h + 16128*a*b*c^6*e*g*h + 2016*b^4*c^4*f*g*h - 11904*a*b^2*c^5*f*g*h +
12288*a^2*c^6*f*g*h - 1568*b^3*c^5*d*h^2 + 8064*a*b*c^6*d*h^2 + 1008*b^4*c
^4*e*h^2 - 5952*a*b^2*c^5*e*h^2 + 6144*a^2*c^6*e*h^2 - 693*b^5*c^3*f*h^2 +
 4680*a*b^3*c^4*f*h^2 - 7248*a^2*b*c^5*f*h^2)/c^7)*x + (26880*b^2*c^6*d*g^
2 + 537600*a*c^7*d*g^2 - 13440*b^3*c^5*e*g^2 + 75264*a*b*c^6*e*g^2 + 7840*
b^4*c^4*f*g^2 - 48384*a*b^2*c^5*f*g^2 + 53760*a^2*c^6*f*g^2 - 26880*b^3*c^
5*d*g*h + 150528*a*b*c^6*d*g*h + 15680*b^4*c^4*e*g*h - 96768*a*b^2*c^5*...
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx=\int (g+h\,x)^2\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^2*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int (g+hx)^2 \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)\,dx = \int (hx+g)^2 \left(c\,x^2+bx+a\right)^{\frac{3}{2}} \left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^2*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

3.35 $\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 410

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{(b^2-4ac)\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\sqrt{a-}}{1024c^5}$$
$$+\frac{\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)(b+2cx)\left(a+bx+cx^2\right)^3}{384c^4}$$
$$+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$
$$+\frac{\left(63b^2fh-48acfh+24c^2\left(7eg-\frac{5fg^2}{h}+7dh\right)-98bc(fg+eh)-10c(10cfg-14ceh+9bfh)x\right)(a+ba}{840c^3}$$
$$+\frac{\left(b^2-4ac\right)^2\left(48c^3dg-9b^3fh-8c^2(3beg+afg+3bdh+aeh)+2bc(6afh+7b(fg+eh))\right)\text{arctanh}\left(\frac{}{2\sqrt{c}\sqrt{}}\right.}{2048c^{11/2}}$$

output
```
-1/1024*(-4*a*c+b^2)*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*
e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^5+1/38
4*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d*h+3*b*e*g)+2*b*c*(6*a*f*h
+7*b*(e*h+f*g)))*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^4+1/7*f*(h*x+g)^2*(c*x^2+
b*x+a)^(5/2)/c/h+1/840*(63*b^2*f*h-48*a*c*f*h+24*c^2*(7*e*g-5*f*g^2/h+7*d*
h)-98*b*c*(e*h+f*g)-10*c*(9*b*f*h-14*c*e*h+10*c*f*g)*x)*(c*x^2+b*x+a)^(5/2
)/c^3+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g-9*b^3*f*h-8*c^2*(a*e*h+a*f*g+3*b*d
*h+3*b*e*g)+2*b*c*(6*a*f*h+7*b*(e*h+f*g)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 11.40 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.48

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx = \frac{26880dg(b+2cx)(a+x(b+cx))^{3/2}+43008(eg+dh)(a+x(b+cx))^{5/2}+35840(fg+eh)x(a$$

input
```
Integrate[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(26880*d*g*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 43008*(e*g + d*h)*(a + x*
(b + c*x))^(5/2) + 35840*(f*g + e*h)*x*(a + x*(b + c*x))^(5/2) + 30720*f*h
*x^2*(a + x*(b + c*x))^(5/2) + (5040*(b^2 - 4*a*c)*d*g*(-2*Sqrt[c]*(b + 2*
c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (840*b*(e*g + d*h)*(16*c^(3/2)*(b + 2*
c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
x*(b + c*x)])])))/c^(5/2) + (3*f*h*(-256*c^(5/2)*(-21*b^2 + 16*a*c + 30*b*
c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x
)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
+ x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]))))/c^(9/2) - (14*(f*g + e*h)*(1792*b*c^(5/2)*(a + x*(b + c*x)
)^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2
) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 -
4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(7/2))
/(215040*c)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2184, 27, 1225, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \; 2184$$

$$\frac{\int -\frac{1}{2}h(g+hx)(5bfg-14cdh+4afh+(10cfg-14ceh+9bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{7ch^2}+\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}$$

$$\Big\downarrow \; 27$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{\int (g+hx)(5bfg-14cdh+4afh+(10cfg-14ceh+9bfh)x)\left(cx^2+bx+a\right)^{3/2}dx}{14ch}$$

$$\Big\downarrow \; 1225$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{24c^2}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(-2ch(24afh+49b(eh+f\right.}{}}{14ch}$$

$$\Big\downarrow \; 1087$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-\frac{-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{24c^2}-\frac{(a+bx}{}}{14ch}$$

$$\Big\downarrow \; 1087$$

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-$$

$$-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4\right.}{}\right.}{16c}\right.}{24c^2}$$

$$14c$$

$\downarrow$ 1092

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-$$

$$-\frac{7h\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(6afh+7b(eh+fg))-9b^3fh+48c^3dg\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4\right.}{}\right.}{16c}\right.}{24c^2}$$

$\downarrow$ 219

$$\frac{f(g+hx)^2\left(a+bx+cx^2\right)^{5/2}}{7ch}-$$

$$-\frac{7h\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-8c^2(aeh+afg+3bdh+3beg)+2bc(\right.}{24c^2}$$

input
```
Int[(g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^2*(a + b*x + c*x^2)^(5/2))/(7*c*h) - (-1/60*((63*b^2*f*h^2 -
24*c^2*(5*f*g^2 - 7*h*(e*g + d*h)) - 2*c*h*(24*a*f*h + 49*b*(f*g + e*h)) -
 10*c*h*(10*c*f*g - 14*c*e*h + 9*b*f*h)*x)*(a + b*x + c*x^2)^(5/2))/c^2 -
(7*h*(48*c^3*d*g - 9*b^3*f*h - 8*c^2*(3*b*e*g + a*f*g + 3*b*d*h + a*e*h) +
 2*b*c*(6*a*f*h + 7*b*(f*g + e*h)))*(((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))
/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*c) - ((b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(
3/2))))/(16*c)))/(24*c^2))/(14*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(384) = 768$.

Time = 0.28 (sec) , antiderivative size = 936, normalized size of antiderivative = 2.28

method	result
default	Expression too large to display
risch	$-\frac{(-15360c^6fh\,x^6-19200b\,c^5fh\,x^5-17920c^6eh\,x^5-17920c^6fg\,x^5-24576a\,c^5fh\,x^4-384b^2c^4fh\,x^4-23296b\,c^5eh\,x^4-23296b\,c^5fg}{}$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)
/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))))+(d*h+e*g)*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*
x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+
a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
)))))+(e*h+f*g)*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2+b*x+a)^(
5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/
4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2
)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/
c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))+h*f*(1/7*x^2*(c*x^2
+b*x+a)^(5/2)/c-9/14*b/c*(1/6*x*(c*x^2+b*x+a)^(5/2)/c-7/12*b/c*(1/5*(c*x^2
+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b
^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-1/6*a/c*(1/8*(2*c*x+b)/c*(c*x^2+b*
x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*
a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))))-2/7*a/c*(1
/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16
*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(384) = 768$.

Time = 0.42 (sec) , antiderivative size = 1833, normalized size of antiderivative = 4.47

$$\int (g + hx) \left(a + bx + cx^2\right)^{3/2} \left(d + ex + fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/430080*(105*(2*(24*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 12*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c
^3 - 64*a^3*c^4)*f)*g - (24*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(
7*b^6*c - 60*a*b^4*c^2 + 144*a^2*b^2*c^3 - 64*a^3*c^4)*e + 3*(3*b^7 - 28*a
*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*f)*h)*sqrt(c)*log(-8*c^2*x^2 - 8*b
*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(153
60*c^7*f*h*x^6 + 1280*(14*c^7*f*g + (14*c^7*e + 15*b*c^6*f)*h)*x^5 + 128*(
14*(12*c^7*e + 13*b*c^6*f)*g + (168*c^7*d + 182*b*c^6*e + 3*(b^2*c^5 + 64*
a*c^6)*f)*h)*x^4 + 16*(14*(120*c^7*d + 132*b*c^6*e + (3*b^2*c^5 + 140*a*c^
6)*f)*g + (1848*b*c^6*d + 14*(3*b^2*c^5 + 140*a*c^6)*e - 3*(9*b^3*c^4 - 44
*a*b*c^5)*f)*h)*x^3 + 8*(14*(360*b*c^6*d + 12*(b^2*c^5 + 32*a*c^6)*e - (7*
b^3*c^4 - 36*a*b*c^5)*f)*g + (168*(b^2*c^5 + 32*a*c^6)*d - 14*(7*b^3*c^4 -
 36*a*b*c^5)*e + 3*(21*b^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*f)*h)*x^2 -
14*(120*(3*b^3*c^4 - 20*a*b*c^5)*d - 12*(15*b^4*c^3 - 100*a*b^2*c^4 + 128*
a^2*c^5)*e + (105*b^5*c^2 - 760*a*b^3*c^3 + 1296*a^2*b*c^4)*f)*g + (168*(1
5*b^4*c^3 - 100*a*b^2*c^4 + 128*a^2*c^5)*d - 14*(105*b^5*c^2 - 760*a*b^3*c
^3 + 1296*a^2*b*c^4)*e + 3*(315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3
- 2048*a^3*c^4)*f)*h + 2*(14*(120*(b^2*c^5 + 20*a*c^6)*d - 12*(5*b^3*c^4 -
 28*a*b*c^5)*e + (35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5)*f)*g - (168*(5
*b^3*c^4 - 28*a*b*c^5)*d - 14*(35*b^4*c^3 - 216*a*b^2*c^4 + 240*a^2*c^5...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3990 vs. $2(434) = 868$.

Time = 1.17 (sec) , antiderivative size = 3990, normalized size of antiderivative = 9.73

$$\int (g + hx) \left(a + bx + cx^2\right)^{3/2} \left(d + ex + fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*h*x**6/7 + x**5*(15*b*c*f*h/14 + c*
*2*e*h + c**2*f*g)/(6*c) + x**4*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*
c*f*g - 11*b*(15*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**
2*e*g)/(5*c) + x**3*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/1
4 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e
*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*
h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*
g)/(4*c) + x**2*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2*a*c*e*g
- 4*a*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/1
4 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(5*c) + b**2*d*h +
b**2*e*g + 2*b*c*d*g - 7*b*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*
c*f*h/14 + c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h +
2*b*c*e*g - 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15
*b*c*f*h/14 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) +
c**2*d*g)/(8*c))/(3*c) + x*(a**2*e*h + a**2*f*g + 2*a*b*d*h + 2*a*b*e*g +
2*a*c*d*g - 3*a*(2*a*b*f*h + 2*a*c*e*h + 2*a*c*f*g - 5*a*(15*b*c*f*h/14 +
c**2*e*h + c**2*f*g)/(6*c) + b**2*e*h + b**2*f*g + 2*b*c*d*h + 2*b*c*e*g -
 9*b*(8*a*c*f*h/7 + b**2*f*h + 2*b*c*e*h + 2*b*c*f*g - 11*b*(15*b*c*f*h/14
 + c**2*e*h + c**2*f*g)/(12*c) + c**2*d*h + c**2*e*g)/(10*c) + c**2*d*g)/(
4*c) + b**2*d*g - 5*b*(a**2*f*h + 2*a*b*e*h + 2*a*b*f*g + 2*a*c*d*h + 2...
```

Maxima [F(-2)]

Exception generated.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(384) = 768$.

Time = 0.29 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.26

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*c*f*h*x + (14*c^7*f*g +
 14*c^7*e*h + 15*b*c^6*f*h)/c^6)*x + (168*c^7*e*g + 182*b*c^6*f*g + 168*c^
7*d*h + 182*b*c^6*e*h + 3*b^2*c^5*f*h + 192*a*c^6*f*h)/c^6)*x + (1680*c^7*
d*g + 1848*b*c^6*e*g + 42*b^2*c^5*f*g + 1960*a*c^6*f*g + 1848*b*c^6*d*h +
42*b^2*c^5*e*h + 1960*a*c^6*e*h - 27*b^3*c^4*f*h + 132*a*b*c^5*f*h)/c^6)*x
 + (5040*b*c^6*d*g + 168*b^2*c^5*e*g + 5376*a*c^6*e*g - 98*b^3*c^4*f*g + 5
04*a*b*c^5*f*g + 168*b^2*c^5*d*h + 5376*a*c^6*d*h - 98*b^3*c^4*e*h + 504*a
*b*c^5*e*h + 63*b^4*c^3*f*h - 372*a*b^2*c^4*f*h + 384*a^2*c^5*f*h)/c^6)*x
+ (1680*b^2*c^5*d*g + 33600*a*c^6*d*g - 840*b^3*c^4*e*g + 4704*a*b*c^5*e*g
 + 490*b^4*c^3*f*g - 3024*a*b^2*c^4*f*g + 3360*a^2*c^5*f*g - 840*b^3*c^4*d
*h + 4704*a*b*c^5*d*h + 490*b^4*c^3*e*h - 3024*a*b^2*c^4*e*h + 3360*a^2*c^
5*e*h - 315*b^5*c^2*f*h + 2184*a*b^3*c^3*f*h - 3504*a^2*b*c^4*f*h)/c^6)*x
- (5040*b^3*c^4*d*g - 33600*a*b*c^5*d*g - 2520*b^4*c^3*e*g + 16800*a*b^2*c
^4*e*g - 21504*a^2*c^5*e*g + 1470*b^5*c^2*f*g - 10640*a*b^3*c^3*f*g + 1814
4*a^2*b*c^4*f*g - 2520*b^4*c^3*d*h + 16800*a*b^2*c^4*d*h - 21504*a^2*c^5*d
*h + 1470*b^5*c^2*e*h - 10640*a*b^3*c^3*e*h + 18144*a^2*b*c^4*e*h - 945*b^
6*c*f*h + 7560*a*b^4*c^2*f*h - 16464*a^2*b^2*c^3*f*h + 6144*a^3*c^4*f*h)/c
^6) - 1/2048*(48*b^4*c^3*d*g - 384*a*b^2*c^4*d*g + 768*a^2*c^5*d*g - 24*b^
5*c^2*e*g + 192*a*b^3*c^3*e*g - 384*a^2*b*c^4*e*g + 14*b^6*c*f*g - 120*a*b
^4*c^2*f*g + 288*a^2*b^2*c^3*f*g - 128*a^3*c^4*f*g - 24*b^5*c^2*d*h + 1...
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int (g+hx)\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)\,dx=\int (hx+g)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}\left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)*(c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

3.36 $\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx$

Optimal result

Integrand size = 25, antiderivative size = 236

$$\begin{aligned}\int (a+bx+cx^2)^{3/2} (d+ex+fx^2)\, dx = & -\frac{(b^2-4ac)\,(24c^2d+7b^2f-4c(3be+af))\,(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} \\ & +\frac{(24c^2d-12bce+7b^2f-4acf)\,(b+2cx)\,(a+bx+cx^2)^{3/2}}{192c^3} \\ & +\frac{(12ce-7bf)\,(a+bx+cx^2)^{5/2}}{60c^2}+\frac{fx(a+bx+cx^2)^{5/2}}{6c} \\ & +\frac{(b^2-4ac)^2\,(24c^2d+7b^2f-4c(3be+af))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}}\end{aligned}$$

output
```
-1/512*(-4*a*c+b^2)*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e))*(2*c*x+b)*(c*x^2+b*
x+a)^(1/2)/c^4+1/192*(-4*a*c*f+7*b^2*f-12*b*c*e+24*c^2*d)*(2*c*x+b)*(c*x^2
+b*x+a)^(3/2)/c^3+1/60*(-7*b*f+12*c*e)*(c*x^2+b*x+a)^(5/2)/c^2+1/6*f*x*(c*
x^2+b*x+a)^(5/2)/c+1/1024*(-4*a*c+b^2)^2*(24*c^2*d+7*b^2*f-4*c*(a*f+3*b*e)
)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.24

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \frac{\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5f+10b^4c(18e+7fx)-8b^3c(45cd-95af+cx(15e+7fx))+48}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5*f + 10*b^4*c*(18*e + 7*f*x) - 8*b
^3*c*(45*c*d - 95*a*f + c*x*(15*e + 7*f*x)) + 48*b^2*c^2*(-(a*(25*e + 9*f*
x)) + c*x*(5*d + x*(2*e + f*x))) + 16*b*c^2*(-81*a^2*f + 6*a*c*(25*d + x*(
7*e + 3*f*x)) + 4*c^2*x^2*(45*d + x*(33*e + 26*f*x))) + 32*c^3*(3*a^2*(16*
e + 5*f*x) + 4*c^2*x^3*(15*d + 2*x*(6*e + 5*f*x)) + 2*a*c*x*(75*d + x*(48*
e + 35*f*x)))) + 15*(b^2 - 4*a*c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f
))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(7680*c^(9/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2192, 27, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \; 2192$$

$$\frac{\int \frac{1}{2}(12cd-2af+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{6c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (2(6cd-af)+(12ce-7bf)x)\left(cx^2+bx+a\right)^{3/2}dx}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\int \left(cx^2+bx+a\right)^{3/2}dx}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\int \sqrt{cx^2+bx+a}dx}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12ce-7bf)}{5c}}{12c}+\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{\left(a+bx+cx^2\right)^{5/2}(12}{5c}}{12c}$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\frac{\left(-4acf+7b^2f-12bce+24c^2d\right)\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c}\right)}{16c}\right)}{2c}+(a+bx}{12c}$$
$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8c}-\frac{3\left(b^2-4ac\right)\left(\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4c}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}}\right)}{16c}\right)\left(-4acf+7b^2f-12bce+24c^2d\right)}{2c}+\frac{(a+bx\cdots}{\cdots}}{12c}$$

$$\frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

output
```
(f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + (((12*c*e - 7*b*f)*(a + b*x + c*x^2)
^(5/2))/(5*c) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(((b + 2*c*x)*(
a + b*x + c*x^2)^(3/2))/(8*c) - (3*(b^2 - 4*a*c)*(((b + 2*c*x)*Sqrt[a + b*
x + c*x^2])/(4*c) - ((b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(8*c^(3/2))))/(16*c)))/(2*c))/(12*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{\left(-1280c^5 f\,x^5-1664b\,c^4 f\,x^4-1536c^5 e\,x^4-2240a\,c^4 f\,x^3-48b^2c^3 f\,x^3-2112b\,c^4 e\,x^3-1920c^5 d\,x^3-288ab\,c^3 f\,x^2-3072a\,c^4 e\,x^2+5\right.}{}$
default	$d\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+e\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{5c}\right.$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^4*(-1280*c^5*f*x^5-1664*b*c^4*f*x^4-1536*c^5*e*x^4-2240*a*c^4*f*
x^3-48*b^2*c^3*f*x^3-2112*b*c^4*e*x^3-1920*c^5*d*x^3-288*a*b*c^3*f*x^2-307
2*a*c^4*e*x^2+56*b^3*c^2*f*x^2-96*b^2*c^3*e*x^2-2880*b*c^4*d*x^2-480*a^2*c
^3*f*x+432*a*b^2*c^2*f*x-672*a*b*c^3*e*x-4800*a*c^4*d*x-70*b^4*c*f*x+120*b
^3*c^2*e*x-240*b^2*c^3*d*x+1296*a^2*b*c^2*f-1536*a^2*c^3*e-760*a*b^3*c*f+1
200*a*b^2*c^2*e-2400*a*b*c^3*d+105*b^5*f-180*b^4*c*e+360*b^3*c^2*d)*(c*x^2
+b*x+a)^(1/2)-1/1024*(64*a^3*c^3*f-144*a^2*b^2*c^2*f+192*a^2*b*c^3*e-384*a
^2*c^4*d+60*a*b^4*c*f-96*a*b^3*c^2*e+192*a*b^2*c^3*d-7*b^6*f+12*b^5*c*e-24
*b^4*c^2*d)/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 839, normalized size of antiderivative = 3.56

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^
3*c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)
*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c
^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 +
 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*
x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 12
8*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^
2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*
b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*
b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x +
a*c)) - 2*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*
d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^
2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 2
0*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*
c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12
*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(230) = 460$.

Time = 0.57 (sec) , antiderivative size = 1360, normalized size of antiderivative = 5.76

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(c*f*x**5/6 + x**4*(13*b*c*f/12 + c**2*e
)/(5*c) + x**3*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/
(10*c) + c**2*d)/(4*c) + x**2*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2
*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13
*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(3*c) + x*(a**2*f + 2*a*b*e +
2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(
10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12
 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e -
9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(2*c) + (a**2*e
+ 2*a*b*d - 2*a*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b*
*2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c*
*2*e)/(10*c) + c**2*d)/(8*c))/(3*c) - 3*b*(a**2*f + 2*a*b*e + 2*a*c*d - 3*
a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2
*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e - 4*a*(13*b*c*f/12 + c**2*e)/(
5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c))/(4*c))/c) + (a**2*d - a*(a**
2*f + 2*a*b*e + 2*a*c*d - 3*a*(7*a*c*f/6 + b**2*f + 2*b*c*e - 9*b*(13*b*c*
f/12 + c**2*e)/(10*c) + c**2*d)/(4*c) + b**2*d - 5*b*(2*a*b*f + 2*a*c*e -
4*a*(13*b*c*f/12 + c**2*e)/(5*c) + b**2*e + 2*b*c*d - 7*b*(7*a*c*f/6 + b**
2*f + 2*b*c*e - 9*b*(13*b*c*f/12 + c**2*e)/(10*c) + c**2*d)/(8*c))/(6*c...
```

Maxima [F(-2)]

Exception generated.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.71

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cfx+\frac{12\,c^6e+13\,bc^5f}{c^5}\right)x+\frac{120\,c^6d+132\,bc^5e+3\,b^2c^4 \cdots}{c^5}\right.\right.\right.\right. - \frac{(24\,b^4c^2d-192\,ab^2c^3d+384\,a^2c^4d-12\,b^5ce+96\,ab^3c^2e-192\,a^2bc^3e+7\,b^6f-60\,ab^4cf+144\,a^2b^2c^2f \cdots}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (12*c^6*e + 13*b*c^5*
f)/c^5)*x + (120*c^6*d + 132*b*c^5*e + 3*b^2*c^4*f + 140*a*c^5*f)/c^5)*x +
 (360*b*c^5*d + 12*b^2*c^4*e + 384*a*c^5*e - 7*b^3*c^3*f + 36*a*b*c^4*f)/c
^5)*x + (120*b^2*c^4*d + 2400*a*c^5*d - 60*b^3*c^3*e + 336*a*b*c^4*e + 35*
b^4*c^2*f - 216*a*b^2*c^3*f + 240*a^2*c^4*f)/c^5)*x - (360*b^3*c^3*d - 240
0*a*b*c^4*d - 180*b^4*c^2*e + 1200*a*b^2*c^3*e - 1536*a^2*c^4*e + 105*b^5*
c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f)/c^5) - 1/1024*(24*b^4*c^2*d - 19
2*a*b^2*c^3*d + 384*a^2*c^4*d - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^
3*e + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^3*c^3*f)*log(abs(2
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx = \int \left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right) \,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 1037, normalized size of antiderivative = 4.39

$$\int (a+bx+cx^2)^{3/2} (d+ex+fx^2) \, dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

output
```
( - 2592*sqrt(a + b*x + c*x**2)*a**2*b*c**3*f + 3072*sqrt(a + b*x + c*x**2
)*a**2*c**4*e + 960*sqrt(a + b*x + c*x**2)*a**2*c**4*f*x + 1520*sqrt(a + b
*x + c*x**2)*a*b**3*c**2*f - 2400*sqrt(a + b*x + c*x**2)*a*b**2*c**3*e - 8
64*sqrt(a + b*x + c*x**2)*a*b**2*c**3*f*x + 4800*sqrt(a + b*x + c*x**2)*a*
b*c**4*d + 1344*sqrt(a + b*x + c*x**2)*a*b*c**4*e*x + 576*sqrt(a + b*x + c
*x**2)*a*b*c**4*f*x**2 + 9600*sqrt(a + b*x + c*x**2)*a*c**5*d*x + 6144*sqr
t(a + b*x + c*x**2)*a*c**5*e*x**2 + 4480*sqrt(a + b*x + c*x**2)*a*c**5*f*x
**3 - 210*sqrt(a + b*x + c*x**2)*b**5*c*f + 360*sqrt(a + b*x + c*x**2)*b**
4*c**2*e + 140*sqrt(a + b*x + c*x**2)*b**4*c**2*f*x - 720*sqrt(a + b*x + c
*x**2)*b**3*c**3*d - 240*sqrt(a + b*x + c*x**2)*b**3*c**3*e*x - 112*sqrt(a
 + b*x + c*x**2)*b**3*c**3*f*x**2 + 480*sqrt(a + b*x + c*x**2)*b**2*c**4*d
*x + 192*sqrt(a + b*x + c*x**2)*b**2*c**4*e*x**2 + 96*sqrt(a + b*x + c*x**
2)*b**2*c**4*f*x**3 + 5760*sqrt(a + b*x + c*x**2)*b*c**5*d*x**2 + 4224*sqr
t(a + b*x + c*x**2)*b*c**5*e*x**3 + 3328*sqrt(a + b*x + c*x**2)*b*c**5*f*x
**4 + 3840*sqrt(a + b*x + c*x**2)*c**6*d*x**3 + 3072*sqrt(a + b*x + c*x**2
)*c**6*e*x**4 + 2560*sqrt(a + b*x + c*x**2)*c**6*f*x**5 - 960*sqrt(c)*log(
(2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**3*c*
*3*f + 2160*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqr
t(4*a*c - b**2))*a**2*b**2*c**2*f - 2880*sqrt(c)*log((2*sqrt(c)*sqrt(a + b
*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*b*c**3*e + 5760*sqrt...
```

3.37 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 664

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{\left(3b^4fh^4+6b^2ch^3(bfg-beh-2afh)-32c^3h(5bg-4ah)(fg^2\right.}{}$$

$$-\frac{\left(3b^2fh+16c^2\left(eg-\frac{fg^2}{h}-dh\right)+6bc(fg-eh)+6c(2cfg-2ceh+bfh)x\right)(a+bx+cx^2)^{3/2}}{48c^2h^2}$$

$$+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$-\frac{\left(4c(2cg-bh)(8ch(bg-2ah)(bfg-2cdh)-g(8bcg-3b^2h-4ach)(2cfg-2ceh+bfh))+\frac{2\left(\frac{1}{2}(b^2-4ac)\right.}{}\right.}{256c^{7/2}h}$$

$$+\frac{(cg^2-bgh+ah^2)^{3/2}(fg^2-h(eg-dh))\,\mathrm{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

output
```
1/128*(3*b^4*f*h^4+6*b^2*c*h^3*(-2*a*f*h-b*e*h+b*f*g)-32*c^3*h*(-4*a*h+5*b
*g)*(f*g^2-h*(-d*h+e*g))+128*c^4*(f*g^4-g^2*h*(-d*h+e*g))-8*b*c^2*h^2*(3*a
*h*(-e*h+f*g)-2*b*(d*h^2-e*g*h+f*g^2))+2*c*h^2*(8*c*(-b*h+2*c*g)*(b*f*g-2*
c*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g))
/h)*x)*(c*x^2+b*x+a)^(1/2)/c^3/h^5-1/48*(3*b^2*f*h+16*c^2*(e*g-f*g^2/h-d*h
)+6*b*c*(-e*h+f*g)+6*c*(b*f*h-2*c*e*h+2*c*f*g)*x)*(c*x^2+b*x+a)^(3/2)/c^2/
h^2+1/5*f*(c*x^2+b*x+a)^(5/2)/c/h-1/256*(4*c*(-b*h+2*c*g)*(8*c*h*(-2*a*h+b
*g)*(b*f*g-2*c*d*h)-g*(-4*a*c*h-3*b^2*h+8*b*c*g)*(b*f*h-2*c*e*h+2*c*f*g))+
2*(1/2*(-4*a*c+b^2)*h^2-2*c*g*(-b*h+2*c*g))*(8*c*h*(-b*h+2*c*g)*(b*f*g-2*c
*d*h)-(b*f*h-2*c*e*h+2*c*f*g)*(16*c^2*g^2-3*b^2*h^2-4*c*h*(-3*a*h+2*b*g)))
/h)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/h^5+(a*h^2-
b*g*h+c*g^2)^(3/2)*(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g
)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6
```

Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(45b^4fh^4-30b^2ch^3(10afh+b(-3fg+3eh+fhx))+12c^2h^2\left(32a^2fh^2+2ab\right.\right.}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
((h*Sqrt[a + x*(b + c*x)]*(45*b^4*f*h^4 - 30*b^2*c*h^3*(10*a*f*h + b*(-3*f
*g + 3*e*h + f*h*x)) + 12*c^2*h^2*(32*a^2*f*h^2 + 2*a*b*h*(-25*f*g + 25*e*
h + 7*f*h*x) + b^2*(5*h*(-4*e*g + 4*d*h + e*h*x) + f*(20*g^2 - 5*g*h*x + 2
*h^2*x^2))) + 32*c^4*(f*(60*g^4 - 30*g^3*h*x + 20*g^2*h^2*x^2 - 15*g*h^3*x
^3 + 12*h^4*x^4) + 5*h*(2*d*h*(6*g^2 - 3*g*h*x + 2*h^2*x^2) + e*(-12*g^3 +
 6*g^2*h*x - 4*g*h^2*x^2 + 3*h^3*x^3))) + 16*c^3*h*(a*h*(5*h*(-32*e*g + 32
*d*h + 15*e*h*x) + f*(160*g^2 - 75*g*h*x + 48*h^2*x^2)) + b*(f*(-150*g^3 +
 70*g^2*h*x - 45*g*h^2*x^2 + 33*h^3*x^3) + 5*h*(2*d*h*(-15*g + 7*h*x) + e*
(30*g^2 - 14*g*h*x + 9*h^2*x^2))))))/c^3 + 3840*Sqrt[-(c*g^2) + h*(b*g - a
*h)]*(c*g^2 + h*(-(b*g) + a*h))*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[-(
c*g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])]
- (15*(3*b^5*f*h^5 - 6*b^3*c*h^4*(-(b*f*g) + b*e*h + 4*a*f*h) - 384*c^4*g*
h*(b*g - a*h)*(f*g^2 + h*(-(e*g) + d*h)) + 256*c^5*(f*g^5 + g^3*h*(-(e*g)
+ d*h)) + 16*b*c^2*h^3*(3*a^2*f*h^2 + 3*a*b*h*(-(f*g) + e*h) + b^2*(f*g^2
- e*g*h + d*h^2)) + 96*c^3*h^2*(a^2*h^2*(f*g - e*h) + b^2*g*(f*g^2 - e*g*h
 + d*h^2) - 2*a*b*h*(f*g^2 - e*g*h + d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a
] + Sqrt[a + x*(b + c*x)])])/c^(7/2))/(1920*h^6)
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{5h(bfg-2cdh+(2cfg-2ceh+bfh)x)(cx^2+bx+a)^{3/2}}{2(g+hx)}dx}{5ch^2}+\frac{f(a+bx+cx^2)^{5/2}}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(a+bx+cx^2)^{5/2}}{5ch}-\frac{\int \frac{(bfg-2cdh+(2cfg-2ceh+bfh)x)(cx^2+bx+a)^{3/2}}{g+hx}dx}{2ch}$$

$$\Big\downarrow \text{1231}$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\int\frac{\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-3hb^2+8cgb-4ach\right)(2}{}}{2ch}$$

$\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\int\frac{\left(8ch(bg-2ah)(bfg-2cdh)-g\left(-3hb^2+8cgb-4ach\right)(2}{}}{2ch}$$

$\downarrow$ 1231

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}$$

$\downarrow$ 27

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}$$

$\downarrow$ 1269

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}$$

$\downarrow$ 1092

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$
$$\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}$$

$$\Big\downarrow \; 219$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 1154$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 219$$

$$\frac{f\left(a+bx+cx^2\right)^{5/2}}{5ch}-$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{3/2}(8ch(bfg-2cdh)+6chx(bfh-2ceh+2cfg)-(8cg-3bh)(bfh-2ceh+2cfg))}{24ch^2}-\frac{\sqrt{a+bx+cx^2}\left(2chx\left(8ch(2cg-bh)(bfg-2cdh)-\left(-4ch(2\right.\right.\right.}{}}{}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x]
```

output
```
(f*(a + b*x + c*x^2)^(5/2))/(5*c*h) - (((8*c*h*(b*f*g - 2*c*d*h) - (8*c*g
- 3*b*h)*(2*c*f*g - 2*c*e*h + b*f*h) + 6*c*h*(2*c*f*g - 2*c*e*h + b*f*h)*x
)*(a + b*x + c*x^2)^(3/2))/(24*c*h^2) - (((3*b^4*f*h^4 + 6*b^2*c*h^3*(b*f*
g - b*e*h - 2*a*f*h) - 32*c^3*h*(5*b*g - 4*a*h)*(f*g^2 - h*(e*g - d*h)) +
128*c^4*(f*g^4 - g^2*h*(e*g - d*h)) - 8*b*c^2*h^2*(3*a*h*(f*g - e*h) - 2*b
*(f*g^2 - e*g*h + d*h^2)) + 2*c*h*(8*c*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h) -
 (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g^2 - 3*b^2*h^2 - 4*c*h*(2*b*g - 3*a*
h)))*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(2*c*g - b*h)*(8*c*h*(
b*g - 2*a*h)*(b*f*g - 2*c*d*h) - g*(8*b*c*g - 3*b^2*h - 4*a*c*h)*(2*c*f*g
- 2*c*e*h + b*f*h)) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(8*c
*h*(2*c*g - b*h)*(b*f*g - 2*c*d*h) - (2*c*f*g - 2*c*e*h + b*f*h)*(16*c^2*g
^2 - 3*b^2*h^2 - 4*c*h*(2*b*g - 3*a*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - (256*c^3*(c*g^2 - b*g*h + a*h^2)^(3/
2)*(f*g^2 - h*(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt
[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/h)/(8*c*h^2))/(16*c*h^2))
/(2*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 990, normalized size of antiderivative = 1.49

method	result
default	$eh\left(\frac{(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{8c}+\frac{3\left(4ac-b^2\right)\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+fh\left(\frac{\left(c\,x^2+bx+a\right)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx}{}\right.}{}\right.$
risch	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/h^2*(e*h*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))))+f*h*(1/5*(c*x^2+b*x+a)^(5/2)/c-1/2*b/c*(1/8*(2*c
*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2*c*x+b)/c*(c*x^2+b*x
+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))))-f*g*(1/8*(2*c*x+b)/c*(c*x^2+b*x+a)^(3/2)+3/16*(4*a*c-b^2)/c*(1/4*(2
*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2)))))+(d*h^2-e*g*h+f*g^2)/h^3*(1/3*((x+g/h)^2*c+(b*h-
2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(3/2)+1/2*(b*h-2*c*g)/h*(1/4*(2*
c*(x+g/h)+(b*h-2*c*g)/h)/c*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)+1/8*(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/c^(
3/2)*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)))+(a*h^2-b*g*h+c*g^2)/h^2*(((x+g/h)^
2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)/h
*ln((1/2*(b*h-2*c*g)/h+c*(x+g/h))/c^(1/2)+((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/
h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/c^(1/2)-(a*h^2-b*g*h+c*g^2)/h^2/((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{g+hx}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g),x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{g+h\,x}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x),x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x), x)
```

Reduce [B] (verification not implemented)

Time = 105.79 (sec) , antiderivative size = 31379, normalized size of antiderivative = 47.26

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{g+hx}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g),x)
```

output
```
( - 3840*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt
(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**
2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**
2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8
*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*
c*g*h - 8*c**2*g**2))*a*b*c**4*d*h**5 + 3840*sqrt(4*sqrt(c)*sqrt(a*h**2 -
b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*
h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*
atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*s
qrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)
*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2))*a*b*c**4*e*g*h**
4 - 3840*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt
(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**
2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**
2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8
*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*
c*g*h - 8*c**2*g**2))*a*b*c**4*f*g**2*h**3 + 7680*sqrt(4*sqrt(c)*sqrt(a*h*
*2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4
*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g
**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*s...
```

3.38 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 750

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx =$$

$$-\frac{(3b^3fh^3+4bch^2(4bfg-2beh-3afh)+64c^3(5fg^3-gh(4eg-3dh))-16c^2h(19bfg^2-bh(14eg-9dh$$

$$-\frac{\left(3bfh(bg-ah)+\frac{8c^2(5fg^3-gh(4eg-3dh))}{h}-c(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh))+6ch\left(4ceg+b\right.\right.}{24ch^2(cg^2-bgh+ah^2)}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{h(cg^2-bgh+ah^2)(g+hx)}$$

$$+\frac{(3b^4fh^4+8b^2ch^3(2bfg-beh-3afh)+128c^4(5fg^4-g^2h(4eg-3dh))+48c^2h^2(a^2fh^2-2abh(2fg-e}{}$$

$$-\frac{\sqrt{cg^2-bgh+ah^2}(2c(5fg^3-gh(4eg-3dh))-h(7bfg^2-bh(5eg-3dh)-2ah(2fg-eh)))\,\mathrm{arctanh}\left(\right.}{2h^6}$$

output
```
-1/64*(3*b^3*f*h^3+4*b*c*h^2*(-3*a*f*h-2*b*e*h+4*b*f*g)+64*c^3*(5*f*g^3-g*
h*(-3*d*h+4*e*g))-16*c^2*h*(19*b*f*g^2-b*h*(-9*d*h+14*e*g)-4*a*h*(-e*h+2*f
*g))+2*c*h*(3*b^2*f*h^2+4*c*h*(-3*a*f*h-2*b*e*h+4*b*f*g)-16*c^2*(5*f*g^2-h
*(-3*d*h+4*e*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^2/h^5-1/24*(3*b*f*h*(-a*h+b*g)+
8*c^2*(5*f*g^3-g*h*(-3*d*h+4*e*g))/h-c*(43*b*f*g^2-8*b*h*(-3*d*h+4*e*g)-8*
a*h*(-e*h+2*f*g))+6*c*h*(4*c*e*g+b*f*g-5*c*f*g^2/h-4*c*d*h-a*f*h)*x)*(c*x^
2+b*x+a)^(3/2)/c/h^2/(a*h^2-b*g*h+c*g^2)-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a
)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+1/128*(3*b^4*f*h^4+8*b^2*c*h^3*(-3*a
*f*h-b*e*h+2*b*f*g)+128*c^4*(5*f*g^4-g^2*h*(-3*d*h+4*e*g))+48*c^2*h^2*(a^2
*f*h^2-2*a*b*h*(-e*h+2*f*g)+b^2*(d*h^2-2*e*g*h+3*f*g^2))+192*c^3*h*(a*h*(d
*h^2-2*e*g*h+3*f*g^2)-b*g*(2*d*h^2-3*e*g*h+4*f*g^2)))*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/h^6-1/2*(a*h^2-b*g*h+c*g^2)^(1/2)*(
2*c*(5*f*g^3-g*h*(-3*d*h+4*e*g))-h*(7*b*f*g^2-b*h*(-3*d*h+5*e*g)-2*a*h*(-e
*h+2*f*g)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/h^6
```

Mathematica [A] (verified)

Time = 7.25 (sec) , antiderivative size = 641, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \frac{h\sqrt{a+x(b+cx)}\left(-9b^3fh^3(g+hx)+6bch^2(g+hx)(10afh+b(-8fg+4eh+fhx))-16c^3\left(f(6\right.\right.}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
((h*Sqrt[a + x*(b + c*x)]*(-9*b^3*f*h^3*(g + h*x) + 6*b*c*h^2*(g + h*x)*(1
0*a*f*h + b*(-8*f*g + 4*e*h + f*h*x)) - 16*c^3*(f*(60*g^4 + 30*g^3*h*x - 1
0*g^2*h^2*x^2 + 5*g*h^3*x^3 - 3*h^4*x^4) - 2*h*(3*d*h*(-6*g^2 - 3*g*h*x +
h^2*x^2) + 2*e*(12*g^3 + 6*g^2*h*x - 2*g*h^2*x^2 + h^3*x^3))) + 8*c^2*h*(a
*h*(8*h*(7*e*g - 3*d*h + 4*e*h*x) + f*(-88*g^2 - 49*g*h*x + 15*h^2*x^2)) +
 b*(f*(114*g^3 + 62*g^2*h*x - 19*g*h^2*x^2 + 9*h^3*x^3) + 2*h*(3*d*h*(9*g
+ 5*h*x) + e*(-42*g^2 - 23*g*h*x + 7*h^2*x^2))))))/(c^2*(g + h*x)) - 192*S
qrt[-(c*g^2) + h*(b*g - a*h)]*(2*c*(5*f*g^3 + g*h*(-4*e*g + 3*d*h)) + h*(-
7*b*f*g^2 + b*h*(5*e*g - 3*d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqrt[-(c*
g^2) + h*(b*g - a*h)]*x)/(Sqrt[a]*(g + h*x) - g*Sqrt[a + x*(b + c*x)])] +
(3*(3*b^4*f*h^4 - 8*b^2*c*h^3*(-2*b*f*g + b*e*h + 3*a*f*h) + 128*c^4*(5*f*
g^4 + g^2*h*(-4*e*g + 3*d*h)) + 48*c^2*h^2*(a^2*f*h^2 + 2*a*b*h*(-2*f*g +
e*h) + b^2*(3*f*g^2 - 2*e*g*h + d*h^2)) - 192*c^3*h*(-(a*h*(3*f*g^2 - 2*e*
g*h + d*h^2)) + b*g*(4*f*g^2 - 3*e*g*h + 2*d*h^2)))*ArcTanh[(Sqrt[c]*x)/(-
Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(5/2))/(192*h^6)
```

Rubi [A] (verified)

Time = 2.14 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+2cdg-5beg-2afg+3bdh+2aeh-2\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)}dx}{ah^2-bgh+cg^2}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{h(g+hx)(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{5fg^2}{h}+5eg-3dh\right)-2\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{g+hx}dx}{2\left(ah^2-bgh+cg^2\right)}-$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1231

$$-\frac{\int \frac{\left(cg^2-bhg+ah^2\right)\left(3fghb^2-8c\left(5fg^2-h(4eg-3dh)\right)b+4ach(5fg-4eh)+\left(-16\left(5fg^2-h(4eg-3dh)\right)c^2+4h(4bfg-2beh-3afh)c+3b^2fh^2\right)x\right)\sqrt{cx^2+bx+a}}{h(g+hx)}dx}{8ch^2}$$

$2\left(ah^2-\right.$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 27

$$-\frac{\left(ah^2-bgh+cg^2\right)\int \frac{\left(3fghb^2-8c\left(5fg^2-h(4eg-3dh)\right)b+4ach(5fg-4eh)+\left(-16\left(5fg^2-h(4eg-3dh)\right)c^2+4h(4bfg-2beh-3afh)c+3b^2fh^2\right)x\right)\sqrt{cx^2+bx+a}}{g+hx}dx}{8ch^3}$$

$2\left(ah^2\right.$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 1231

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\right.}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$\big\downarrow$ 27

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\right.}{4ch^2}\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\cdots\right.}{4ch^2}\cdots}{\cdots}$$

$$\frac{(a+bx+cx^2)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\cdots\right.}{4ch^2}\cdots}{\cdots}$$

$$\frac{(a+bx+cx^2)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\cdots\right.}{4ch^2}\cdots}{\cdots}$$

$$\frac{(a+bx+cx^2)^{5/2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$-\frac{\left(\frac{8\left(5fg^3-gh(4eg-3dh)\right)c^2}{h}-\left(43bfg^2-8bh(4eg-3dh)-8ah(2fg-eh)\right)c+6h\left(-\frac{5cfg^2}{h}+4ceg+bfg-4cdh-afh\right)xc+3bfh(bg-ah)\right)(cx^2+bx+a)\cdots}{12ch^2}\cdots$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{h\left(cg^2-bhg+ah^2\right)(g+hx)}$$

$$\Big\downarrow \; 219$$

$$-\frac{(ah^2-bgh+cg^2)\left(\frac{\sqrt{a+bx+cx^2}\left(2chx\left(4ch(-3afh-2beh+4bfg)+3b^2fh^2-16c^2\left(5fg^2-h(4eg-3dh)\right)\right)-16c^2h\left(-4ah(2fg-eh)-bh(14eg-9dh)+19bfg^2\right)+4b\cdots}{4ch^2}\right.}{\cdots}$$

$$\frac{(a+bx+cx^2)^{5/2}\,(fg^2-h(eg-dh))}{h(g+hx)\,(ah^2-bgh+cg^2)}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x]
```

output
```
-(((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h + a*
h^2)*(g + h*x))) + (-1/12*((3*b*f*h*(b*g - a*h) + (8*c^2*(5*f*g^3 - g*h*(4
*e*g - 3*d*h)))/h - c*(43*b*f*g^2 - 8*b*h*(4*e*g - 3*d*h) - 8*a*h*(2*f*g -
 e*h)) + 6*c*h*(4*c*e*g + b*f*g - (5*c*f*g^2)/h - 4*c*d*h - a*f*h)*x)*(a +
 b*x + c*x^2)^(3/2))/(c*h^2) - ((c*g^2 - b*g*h + a*h^2)*(((3*b^3*f*h^3 + 4
*b*c*h^2*(4*b*f*g - 2*b*e*h - 3*a*f*h) + 64*c^3*(5*f*g^3 - g*h*(4*e*g - 3*
d*h)) - 16*c^2*h*(19*b*f*g^2 - b*h*(14*e*g - 9*d*h) - 4*a*h*(2*f*g - e*h))
 + 2*c*h*(3*b^2*f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h) - 16*c^2*(5*f*
g^2 - h*(4*e*g - 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/(4*c*h^2) - (((4*c*h*(
2*c*g - b*h)*(3*b^2*f*g*h + 4*a*c*h*(5*f*g - 4*e*h) - 8*b*c*(5*f*g^2 - h*(
4*e*g - 3*d*h))) - 2*(4*c^2*g^2 - (b^2*h^2)/2 - 2*c*h*(b*g - a*h))*(3*b^2*
f*h^2 + 4*c*h*(4*b*f*g - 2*b*e*h - 3*a*f*h) - 16*c^2*(5*f*g^2 - h*(4*e*g -
 3*d*h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c
]*h) - (64*c^2*Sqrt[c*g^2 - b*g*h + a*h^2]*(2*c*(5*f*g^3 - g*h*(4*e*g - 3*
d*h)) - h*(7*b*f*g^2 - b*h*(5*e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh
[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b
*x + c*x^2])])/h)/(8*c*h^2)))/(8*c*h^3))/(2*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 1324, normalized size of antiderivative = 1.77

method	result	size
risch	Expression too large to display	1324
default	Expression too large to display	1853

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192/c^2*(48*c^3*f*h^3*x^3+72*b*c^2*f*h^3*x^2+64*c^3*e*h^3*x^2-128*c^3*f*
g*h^2*x^2+120*a*c^2*f*h^3*x+6*b^2*c*f*h^3*x+112*b*c^2*e*h^3*x-224*b*c^2*f*
g*h^2*x+96*c^3*d*h^3*x-192*c^3*e*g*h^2*x+288*c^3*f*g^2*h*x+60*a*b*c*f*h^3+
256*a*c^2*e*h^3-512*a*c^2*f*g*h^2-9*b^3*f*h^3+24*b^2*c*e*h^3-48*b^2*c*f*g*
h^2+240*b*c^2*d*h^3-480*b*c^2*e*g*h^2+720*b*c^2*f*g^2*h-384*c^3*d*g*h^2+57
6*c^3*e*g^2*h-768*c^3*f*g^3)*(c*x^2+b*x+a)^(1/2)/h^5+1/128/h^5/c^2*((48*a^
2*c^2*f*h^4-24*a*b^2*c*f*h^4+96*a*b*c^2*e*h^4-192*a*b*c^2*f*g*h^3+192*a*c^
3*d*h^4-384*a*c^3*e*g*h^3+576*a*c^3*f*g^2*h^2+3*b^4*f*h^4-8*b^3*c*e*h^4+16
*b^3*c*f*g*h^3+48*b^2*c^2*d*h^4-96*b^2*c^2*e*g*h^3+144*b^2*c^2*f*g^2*h^2-3
84*b*c^3*d*g*h^3+576*b*c^3*e*g^2*h^2-768*b*c^3*f*g^3*h+384*c^4*d*g^2*h^2-5
12*c^4*e*g^3*h+640*c^4*f*g^4)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)/c^(1/2)-128*c^2/h^2*(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6
*a*b*f*g^2*h^3-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4
+3*b^2*e*g^2*h^3-4*b^2*f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g^3*h^2+10*b*c*f*
g^4*h-4*c^2*d*g^3*h^2+5*c^2*e*g^4*h-6*c^2*f*g^5)/((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2))/(x+g/h))+128*c^2*(a^2*d*h^6-a^2*e*g*h^5+a^2*f*g^2*h^4-2*a*b*d*
g*h^5+2*a*b*e*g^2*h^4-2*a*b*f*g^3*h^3+2*a*c*d*g^2*h^4-2*a*c*e*g^3*h^3+2*a*
c*f*g^4*h^2+b^2*d*g^2*h^4-b^2*e*g^3*h^3+b^2*f*g^4*h^2-2*b*c*d*g^3*h^3+2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**2,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*h-2*c*g>0)', see `assume?` for
 more deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\int \frac{\left(c\,x^2+b\,x+a\right)^{3/2}\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^2}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 4115, normalized size of antiderivative = 5.49

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^2}\,dx=\text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^2,x)
```

output
```
(384*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**3*e*g*h**3 + 384
*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**3*e*h**4*x - 768*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**3*f*g**2*h**2 - 768*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**3*f*g*h**3*x + 576*sqrt(a
*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h +
 c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*d*g*h**3 + 576*sqrt(a*h**
2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g
**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*d*h**4*x - 960*sqrt(a*h**2 -
b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)
 - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*e*g**2*h**2 - 960*sqrt(a*h**2 - b
*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)
- 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*e*g*h**3*x + 1344*sqrt(a*h**2 - b*
g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) -
 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*f*g**3*h + 1344*sqrt(a*h**2 - b*g*h
 + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*
a*h + b*g - b*h*x + 2*c*g*x)*b*c**3*f*g**2*h**2*x - 1152*sqrt(a*h**2 - ...
```

3.39 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 819

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx =$$

$$-\frac{\left(b^2fh^2(bg-ah)+8c^3g^2\left(6eg-\frac{10fg^2}{h}-3dh\right)-2c^2(2ah(19fg^2-9egh+3dh^2)-3bg(22fg^2-12egh+ \right.}{}$$

$$-\frac{\left(31bfg^2+4cg\left(6eg-\frac{10fg^2}{h}-3dh\right)-3bh(5eg-dh)-4ah(7fg-3eh)+2h\left(3ceg+2bfg-\frac{5cfg^2}{h}-3c\right.\right.}{12h^2(cg^2-bgh+ah^2)(g+hx)}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{2h(cg^2-bgh+ah^2)(g+hx)^2}$$

$$-\frac{(b^3fh^3+6bch^2(3bfg-beh-2afh)+16c^3(10fg^3-3gh(2eg-dh))-24c^2h(6bfg^2-bh(3eg-dh)-}{16c^{3/2}h^6}$$

$$+\frac{(8c^2(10fg^4-3g^2h(2eg-dh))-4ch(28bfg^3-3bgh(5eg-2dh)-ah(19fg^2-9egh+3dh^2))+h^2(8a^2}{8h^6\sqrt{cg^2-bgh+}}$$

output
```
-1/8*(b^2*f*h^2*(-a*h+b*g)+8*c^3*g^2*(6*e*g-10*f*g^2/h-3*d*h)-2*c^2*(2*a*h
*(3*d*h^2-9*e*g*h+19*f*g^2)-3*b*g*(5*d*h^2-12*e*g*h+22*f*g^2))-c*h*(8*a^2*
f*h^2-18*a*b*h*(-e*h+3*f*g)+b^2*(53*f*g^2-6*h*(-d*h+4*e*g)))+2*c*(b*f*h^2*
(-a*h+b*g)+2*c^2*(10*f*g^3-3*g*h*(-d*h+2*e*g))+c*h*(2*a*h*(-3*e*h+7*f*g)-3
*b*(d*h^2-3*e*g*h+6*f*g^2)))*x)*(c*x^2+b*x+a)^(1/2)/c/h^4/(a*h^2-b*g*h+c*g
^2)-1/12*(31*b*f*g^2+4*c*g*(6*e*g-10*f*g^2/h-3*d*h)-3*b*h*(-d*h+5*e*g)-4*a
*h*(-3*e*h+7*f*g)+2*h*(3*c*e*g+2*b*f*g-5*c*f*g^2/h-3*c*d*h-2*a*f*h)*x)*(c*
x^2+b*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/2*(f*g^2-h*(-d*h+e*g))*
(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^2-1/16*(b^3*f*h^3+6*b*c*
h^2*(-2*a*f*h-b*e*h+3*b*f*g)+16*c^3*(10*f*g^3-3*g*h*(-d*h+2*e*g))-24*c^2*h
*(6*b*f*g^2-b*h*(-d*h+3*e*g)-a*h*(-e*h+3*f*g)))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^6+1/8*(8*c^2*(10*f*g^4-3*g^2*h*(-d*h+2*
e*g))-4*c*h*(28*b*f*g^3-3*b*g*h*(-2*d*h+5*e*g)-a*h*(3*d*h^2-9*e*g*h+19*f*g
^2))+h^2*(8*a^2*f*h^2-4*a*b*h*(-3*e*h+10*f*g)+b^2*(35*f*g^2-3*h*(-d*h+5*e*
g))))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*
x^2+b*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(1/2)
```

Mathematica [A] (verified)

Time = 12.84 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \frac{4f(a+x(b+cx))^{3/2} - \frac{6(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(g+hx)^2} + \frac{12(2fg-eh)(\cdots}{g+\cdots}}{\cdots}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
(4*f*(a + x*(b + c*x))^(3/2) - (6*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c
*x))^(3/2))/(g + h*x)^2 + (12*(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g +
h*x) + (9*(-2*f*g + e*h)*((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(h*(-4*c*g
 + 3*b*h + 2*c*h*x)*Sqrt[a + x*(b + c*x)] + 2*(2*c*g - b*h)*Sqrt[c*g^2 + h
*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2
+ h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])))/(2*Sqrt[c]*h^3) + (9*(f*g^2
 + h*(-(e*g) + d*h))*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) -
 (Sqrt[a + x*(b + c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a
*h + b*h*x)))/h^2 + (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^
2 + 4*c*h*(-2*b*g + a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) +
 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b
+ c*x)])])/(2*h^3)))/(-(c*g^2) + h*(b*g - a*h)) - (3*f*((2*c*g - b*h)*(8*c
^2*g^2 - b^2*h^2 + 4*c*h*(-2*b*g + 3*a*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + x*(b + c*x)])] + 2*Sqrt[c]*(h*Sqrt[a + x*(b + c*x)]*(-(b^2*h^2) +
 4*c^2*g*(-2*g + h*x) - 2*c*h*(-5*b*g + 4*a*h + b*h*x)) + 8*c*(c*g^2 + h*(
-(b*g) + a*h))^(3/2)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*
g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])))/(4*c^(3/2)*h^3))/(12*h^
3)
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 1230, 25, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+4cdg-5beg-4afg+bdh+4aeh-2\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^2}dx}{2(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{2h(g+hx)^2(ah^2-bgh+cg^2)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\left(4cdg-4afg+4aeh-b\left(-\frac{5fg^2}{h}+5eg-dh\right)-2\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^2}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 1230$$

$$-\frac{\int -\frac{\Big(h\Big(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\Big)\Big)-4\Big(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\Big(2ah(7fg-3eh)- \cdots}{h(g+hx)}}{2h^2} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\Big(h\Big(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\Big)\Big)-4\Big(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\Big(2ah(7fg-3eh)-3b\Big(6 \cdots}{h(g+hx)}}{2h^2} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{\Big(h\Big(2(4bg-2ah)\left(-\frac{5cfg^2}{h}+3ceg+2bfg-3cdh-2afh\right)+3b\left(5bfg^2-bh(5eg-dh)+4h(cdg-afg+aeh)\right)\Big)\Big)-4\Big(2\left(10fg^3-3gh(2eg-dh)\right)c^2+h\Big(2ah(7fg-3eh)-3b\Big(6 \cdots}{g+hx}}{2h^3} \cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 1231$$

$$-\frac{\frac{\int \frac{2\left(cg^2-bhg+ah^2\right)\Big(fgh^2b^3-2ch\left(26fg^2-3h(4eg-dh)\right)b^2+4c\left(20cfg^3-6ch(2eg-dh)g+ah^2(17fg-6eh)\right)b-8ach\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)+\Big(16\Big(10fg \cdots}{(g+hx)\sqrt{cx^2+bx+a}}}{4ch^2} \cdots}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow 27$$

$$-\frac{\left(ah^2-bgh+cg^2\right)\int \frac{fgh^2b^3-2ch\left(26fg^2-3h(4eg-dh)\right)b^2+4c\left(20cfg^3-6ch(2eg-dh)g+ah^2(17fg-6eh)\right)b-8ach\left(10cfg^2+2afh^2-3ch(2eg-dh)\right)+\left(16\left(10fg^3-\cdots\right.\right.}{(g+hx)\sqrt{cx^2+bx+a}}}{2ch^2}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1269

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\left(-24c^2h\left(-ah(3fg-eh)-bh(3eg-dh)+6bfg^2\right)+6bch^2(-2afh-beh+3bfg)+b^3fh^3+16c^3\left(10fg^3-3gh(2eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{2c(\cdots}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1092

$$-\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8\cdots\right.\right.}{ch^2}\cdots$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

$\downarrow$ 219

$$-\frac{\left(ah^2-bgh+cg^2\right)\left(\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24c^2h\left(-ah(3fg-eh)-bh(3eg-dh)+6bfg^2\right)+6bch^2(-2afh-beh+3bfg)+b^3fh^3+16c^3\left(10fg^3-3gh(2eg-\cdots\right.\right.}{\sqrt{c}h}\cdots\right.}{\cdots}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1154

$$-\frac{\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8a\right.}{ch^2}}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

$\downarrow$ 219

$$-\frac{\frac{\sqrt{cx^2+bx+a}\left(-8\left(10fg^4-3g^2h(2eg-dh)\right)c^3-2h\left(2ah\left(19fg^2-9ehg+3dh^2\right)-3bg\left(22fg^2-12ehg+5dh^2\right)\right)c^2-h^2\left(\left(53fg^2-6h(4eg-dh)\right)b^2-18ah(3fg-eh)b+8a\right.}{ch^2}}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{2h\left(cg^2-bhg+ah^2\right)(g+hx)^2}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x]
```

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^2) + (-1/3*((31*b*f*g^2 + 4*c*g*(6*e*g - (10*f*g^2)/h -
3*d*h) - 3*b*h*(5*e*g - d*h) - 4*a*h*(7*f*g - 3*e*h) + 2*h*(3*c*e*g + 2*b*
f*g - (5*c*f*g^2)/h - 3*c*d*h - 2*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(h^2*
(g + h*x)) + (-(((b^2*f*h^3*(b*g - a*h) - 8*c^3*(10*f*g^4 - 3*g^2*h*(2*e*g
 - d*h)) - 2*c^2*h*(2*a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2) - 3*b*g*(22*f*g^2
 - 12*e*g*h + 5*d*h^2)) - c*h^2*(8*a^2*f*h^2 - 18*a*b*h*(3*f*g - e*h) + b^
2*(53*f*g^2 - 6*h*(4*e*g - d*h))) + 2*c*h*(b*f*h^2*(b*g - a*h) + 2*c^2*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) + c*h*(2*a*h*(7*f*g - 3*e*h) - 3*b*(6*f*g^2
- 3*e*g*h + d*h^2)))*x)*Sqrt[a + b*x + c*x^2])/(c*h^2)) - ((c*g^2 - b*g*h
+ a*h^2)*(((b^3*f*h^3 + 6*b*c*h^2*(3*b*f*g - b*e*h - 2*a*f*h) + 16*c^3*(10
*f*g^3 - 3*g*h*(2*e*g - d*h)) - 24*c^2*h*(6*b*f*g^2 - b*h*(3*e*g - d*h) -
a*h*(3*f*g - e*h)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[c]*h) - (2*c*(8*c^2*(10*f*g^4 - 3*g^2*h*(2*e*g - d*h)) - 4*c*h*(28
*b*f*g^3 - 3*b*g*h*(5*e*g - 2*d*h) - a*h*(19*f*g^2 - 9*e*g*h + 3*d*h^2)) +
 h^2*(8*a^2*f*h^2 - 4*a*b*h*(10*f*g - 3*e*h) + b^2*(35*f*g^2 - 3*h*(5*e*g
- d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h +
a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2])))/(2*c*h^2
))/(2*h^3))/(4*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1780 vs. $2(787) = 1574$.

Time = 0.41 (sec) , antiderivative size = 1781, normalized size of antiderivative = 2.17

method	result	size
risch	Expression too large to display	1781
default	Expression too large to display	3600

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*c^2*f*h^2*x^2+14*b*c*f*h^2*x+12*c^2*e*h^2*x-36*c^2*f*g*h*x+32*a*
c*f*h^2+3*b^2*f*h^2+30*b*c*e*h^2-90*b*c*f*g*h+24*c^2*d*h^2-72*c^2*e*g*h+14
4*c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/h^5+1/16/h^5/c*((12*a*b*c*f*h^3+24*a*c^2*
e*h^3-72*a*c^2*f*g*h^2-b^3*f*h^3+6*b^2*c*e*h^3-18*b^2*c*f*g*h^2+24*b*c^2*d
*h^3-72*b*c^2*e*g*h^2+144*b*c^2*f*g^2*h-48*c^3*d*g*h^2+96*c^3*e*g^2*h-160*
c^3*f*g^3)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+16*c/h^3*
(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6*a*b*f*g^2*h^3-4*a*c*d
*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4+3*b^2*e*g^2*h^3-4*b^2
*f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g^3*h^2+10*b*c*f*g^4*h-4*c^2*d*g^3*h^2+
5*c^2*e*g^4*h-6*c^2*f*g^5)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*
c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(
a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^
2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*
c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+16*c*(a^
2*d*h^6-a^2*e*g*h^5+a^2*f*g^2*h^4-2*a*b*d*g*h^5+2*a*b*e*g^2*h^4-2*a*b*f*g^
3*h^3+2*a*c*d*g^2*h^4-2*a*c*e*g^3*h^3+2*a*c*f*g^4*h^2+b^2*d*g^2*h^4-b^2*e*
g^3*h^3+b^2*f*g^4*h^2-2*b*c*d*g^3*h^3+2*b*c*e*g^4*h^2-2*b*c*f*g^5*h+c^2*d*
g^4*h^2-c^2*e*g^5*h+c^2*f*g^6)/h^4*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2
*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*
h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx = \int \frac{(a + bx + cx^2)^{\frac{3}{2}} (d + ex + fx^2)}{(g + hx)^3} dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**3,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2628 vs. $2(787) = 1574$.

Time = 0.60 (sec) , antiderivative size = 2628, normalized size of antiderivative = 3.21

$$\int \frac{(a + bx + cx^2)^{3/2} (d + ex + fx^2)}{(g + hx)^3} dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*x*(4*c*f*x/h^3 - (18*c^3*f*g*h^14 - 6*c^3*e*
h^15 - 7*b*c^2*f*h^15)/(c^2*h^18)) + (144*c^3*f*g^2*h^13 - 72*c^3*e*g*h^14
 - 90*b*c^2*f*g*h^14 + 24*c^3*d*h^15 + 30*b*c^2*e*h^15 + 3*b^2*c*f*h^15 +
32*a*c^2*f*h^15)/(c^2*h^18)) + 1/4*(80*c^2*f*g^4 - 48*c^2*e*g^3*h - 112*b*
c*f*g^3*h + 24*c^2*d*g^2*h^2 + 60*b*c*e*g^2*h^2 + 35*b^2*f*g^2*h^2 + 76*a*
c*f*g^2*h^2 - 24*b*c*d*g*h^3 - 15*b^2*e*g*h^3 - 36*a*c*e*g*h^3 - 40*a*b*f*
g*h^3 + 3*b^2*d*h^4 + 12*a*c*d*h^4 + 12*a*b*e*h^4 + 8*a^2*f*h^4)*arctan(-(
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h - a
*h^2))/(sqrt(-c*g^2 + b*g*h - a*h^2)*h^6) + 1/16*(160*c^3*f*g^3 - 96*c^3*e
*g^2*h - 144*b*c^2*f*g^2*h + 48*c^3*d*g*h^2 + 72*b*c^2*e*g*h^2 + 18*b^2*c*
f*g*h^2 + 72*a*c^2*f*g*h^2 - 24*b*c^2*d*h^3 - 6*b^2*c*e*h^3 - 24*a*c^2*e*h
^3 + b^3*f*h^3 - 12*a*b*c*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*sqrt(c) + b))/(c^(3/2)*h^6) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^3*c^2*f*g^4*h - 32*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*e*g^3*h^
2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 + 24*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*c^2*d*g^2*h^3 + 36*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*b*c*e*g^2*h^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*
g^2*h^3 + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 - 24*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*d*g*h^4 - 9*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^3*b^2*e*g*h^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^3}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 9892, normalized size of antiderivative = 12.08

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^3}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^3,x)
```

output
```
(48*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*f*g**2*h**4
+ 96*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*f*g*h**5*x
+ 48*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*f*h**6*x**2
 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h*
*2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*e*g**2*h**4
 + 144*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h
**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*e*g*h**5*x
 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h*
*2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*e*h**6*x**2
 - 240*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h
**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*f*g**3*h**
3 - 480*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*
h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*f*g**2*h*
*4*x - 240*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt
(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*c**2*f*g*h*
*5*x**2 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sq
rt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c**3*d*g...
```

3.40 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 829

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx =$$

$$-\frac{\left(12a^2fh^3-8c^2g^2\left(4eg-\frac{10fg^2}{h}-dh\right)-6abh^2(7fg-eh)+4ach(23fg^2-2h(4eg-dh))-6bcg(18fg^2\right.}{}$$

$$-\frac{\left(17bfg^2+2cg\left(4eg-\frac{10fg^2}{h}-dh\right)-bh(5eg+dh)-6ah(3fg-eh)+2h\left(2ceg+3bfg-\frac{5cfg^2}{h}-2cdh\right.\right.}{12h^2\left(cg^2-bgh+ah^2\right)(g+hx)^2}$$

$$-\frac{(fg^2-h(eg-dh))\left(a+bx+cx^2\right)^{5/2}}{3h\left(cg^2-bgh+ah^2\right)(g+hx)^3}$$

$$+\frac{(3b^2fh^2-12ch(4bfg-beh-afh)+8c^2(10fg^2-h(4eg-dh)))\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}h^6}$$

$$-\frac{(16c^3(10fg^5-g^3h(4eg-dh))-bh^3(24a^2fh^2-6abh(10fg-eh)+b^2(35fg^2-5egh-dh^2))+6ch^2(4a\right.}{}$$

output
```
-1/8*(12*a^2*f*h^3-8*c^2*g^2*(4*e*g-10*f*g^2/h-d*h)-6*a*b*h^2*(-e*h+7*f*g)
+4*a*c*h*(23*f*g^2-2*h*(-d*h+4*e*g))-6*b*c*g*(18*f*g^2-h*(-d*h+6*e*g))+b^2
*h*(29*f*g^2-h*(d*h+5*e*g))+2*(3*b*f*h^2*(-a*h+b*g)+2*c^2*(10*f*g^3-g*h*(-
d*h+4*e*g))-c*h*(22*b*f*g^2-b*h*(-d*h+7*e*g)-6*a*h*(-e*h+3*f*g)))*x)*(c*x^
2+b*x+a)^(1/2)/h^4/(a*h^2-b*g*h+c*g^2)/(h*x+g)-1/12*(17*b*f*g^2+2*c*g*(4*e
*g-10*f*g^2/h-d*h)-b*h*(d*h+5*e*g)-6*a*h*(-e*h+3*f*g)+2*h*(2*c*e*g+3*b*f*g
-5*c*f*g^2/h-2*c*d*h-3*a*f*h)*x)*(c*x^2+b*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^
2)/(h*x+g)^2-1/3*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c
*g^2)/(h*x+g)^3+1/8*(3*b^2*f*h^2-12*c*h*(-a*f*h-b*e*h+4*b*f*g)+8*c^2*(10*f
*g^2-h*(-d*h+4*e*g)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
^(1/2)/h^6-1/16*(16*c^3*(10*f*g^5-g^3*h*(-d*h+4*e*g))-b*h^3*(24*a^2*f*h^2-
6*a*b*h*(-e*h+10*f*g)+b^2*(-d*h^2-5*e*g*h+35*f*g^2))+6*c*h^2*(4*a^2*h^2*(-
e*h+4*f*g)+b^2*g*(d*h^2-10*e*g*h+35*f*g^2)-2*a*b*h*(d*h^2-7*e*g*h+25*f*g^2
))-24*c^2*g*h*(b*g*(d*h^2-5*e*g*h+14*f*g^2)-a*h*(d*h^2-4*e*g*h+11*f*g^2)))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(3/2)
```

Mathematica [A] (verified)

Time = 15.64 (sec) , antiderivative size = 983, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \frac{-\frac{4(fg^2+h(-eg+dh))(a+x(b+cx))^{3/2}}{(g+hx)^3}+\frac{6(2fg-eh)(a+x(b+cx))^{3/2}}{(g+hx)^2}-\frac{12f(a+x(b-}{g+h}}{}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
((-4*(f*g^2 + h*(-(e*g) + d*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^3 + (6*
(2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 - (12*f*(a + x*(b + c*x
))^(3/2))/(g + h*x) + (9*f*(2*h*(-4*c*g + 3*b*h + 2*c*h*x)*Sqrt[a + x*(b +
 c*x)] + ((8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g + a*h))*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] + 4*(2*c*g - b*h)*Sqrt[c*g^2
+ h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g
^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])]))/(2*h^3) + (9*(-2*f*g + e*
h)*(((-2*c*g + b*h)*(a + x*(b + c*x))^(3/2))/(g + h*x) - (Sqrt[a + x*(b +
c*x)]*(b^2*h^2 + 2*c^2*g*(2*g - h*x) + c*h*(-5*b*g + 2*a*h + b*h*x)))/h^2
+ (4*Sqrt[c]*(2*c*g - b*h)*(c*g^2 + h*(-(b*g) + a*h))*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + (8*c^2*g^2 + b^2*h^2 + 4*c*h*(-2*b*g
+ a*h))*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x +
 b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(2*h^3)
))/(-(c*g^2) + h*(b*g - a*h)) + (3*(f*g^2 + h*(-(e*g) + d*h))*((4*(2*c*g -
 b*h)*(c*g^2 + h*(-(b*g) + a*h))*(a + x*(b + c*x))^(3/2))/(g + h*x)^2 + (2
*(-4*c^2*g^2 + b^2*h^2 + 4*c*h*(b*g - 2*a*h))*(a + x*(b + c*x))^(3/2))/(g
+ h*x) + (-2*c*h*Sqrt[a + x*(b + c*x)]*(b^3*h^3 + 4*c^3*g^2*(2*g - h*x) +
b*c*h^2*(5*b*g - 10*a*h + b*h*x) - 2*c^2*h*(b*g*(7*g - 2*h*x) + 2*a*h*(-3*
g + 2*h*x))) + 16*c^(5/2)*(c*g^2 + h*(-(b*g) + a*h))^2*ArcTanh[(b + 2*c*x)
/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] + c*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-...
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1230, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+6cdg-5beg-6afg-bdh+6aeh-2\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^3}dx}{3(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{3h(g+hx)^3(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(6cdg-6afg+6aeh-b\left(-\frac{5fg^2}{h}+5eg+dh\right)-2\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^3}dx}{6\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1230$$

$$-\frac{3\int -\frac{2\left(h\left((4bg-4ah)\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)+b\left(5bfg^2-bh(5eg+dh)+6h(cdg-afg+aeh)\right)\right)-2\left(2\left(10fg^3-gh(4eg-dh)\right)c^2-h\left(22bfg^2-bh(7eg-\cdots\right.\right.\right.}{h(g+hx)^2}}{8h^2}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{\left(h\left((4bg-4ah)\left(-\frac{5cfg^2}{h}+2ceg+3bfg-2cdh-3afh\right)+b\left(5bfg^2-bh(5eg+dh)+6h(cdg-afg+aeh)\right)\right)-2\left(2\left(10fg^3-gh(4eg-dh)\right)c^2-h\left(22bfg^2-bh(7eg-dh)-\cdots\right.\right.\right.}{(g+hx)^2}}{4h^3}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1230$$

$$3\left(-\frac{\int -\frac{h^2\left(29fg^2-h(5eg+dh)\right)b^3-6\left(a(9fg-eh)h^3+cg\left(18fg^2-h(6eg-dh)\right)h\right)b^2+4\left(6a^2fh^4+3ac\left(15fg^2-h(5eg-dh)\right)h^2+2c^2\left(10fg^4-g^2h(4eg-dh)\right)\right)b-8a\cdots}{(g+hx)\sqrt{cx^2+bx+a}}}{2h^2}\cdots\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 25$$

$$3\left(\frac{\int \frac{h^2\left(29fg^2-h(5eg+dh)\right)b^3-6\left(a(9fg-eh)h^3+cg\left(18fg^2-h(6eg-dh)\right)h\right)b^2+4\left(6a^2fh^4+3ac\left(15fg^2-h(5eg-dh)\right)h^2+2c^2\left(10fg^4-g^2h(4eg-dh)\right)\right)b-8ach\cdots}{(g+hx)\sqrt{cx^2+bx+a}}}{2h^2}\cdots\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1269}$$

$$3\left(\frac{\frac{2\left(ah^2-bgh+cg^2\right)\left(-12ch(-afh-beh+4bfg)+3b^2fh^2+8c^2\left(10fg^2-h(4eg-dh)\right)\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{\left(6ch^2\left(4a^2h^2(4fg-eh)-2abh\left(dh^2-7egh+25fg^2\right)+b\right.\right.}{}}{}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1092}$$

$$3\left(\frac{\frac{4\left(cg^2-bhg+ah^2\right)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(1\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

$$\Big\downarrow \text{ 219}$$

$$3\left(\frac{\frac{2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)\left(-12ch(-afh-beh+4bfg)+3b^2fh^2+8c^2\left(10fg^2-h(4eg-dh)\right)\right)}{\sqrt{c}h}-\frac{\left(6ch^2\left(4a^2h^2(4fg-eh)-2abh\left(dh^2-7eg\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{3h(g+hx)^3\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{ 1154}$$

$$3\left(\frac{\frac{2\left(cg^2-bhg+ah^2\right)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{\sqrt{c}h}+\frac{2\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(1\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

$$\Big\downarrow \text{ 219}$$

$$3\left(\frac{\frac{2(cg^2-bhg+ah^2)\left(8\left(10fg^2-h(4eg-dh)\right)c^2-12h(4bfg-beh-afh)c+3b^2fh^2\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{\sqrt{c}h}-\frac{\left(16\left(10fg^5-g^3h(4eg-dh)\right)c^3-24gh\left(bg\left(14\right.\right.\right.}{}}{}\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{3h\left(cg^2-bhg+ah^2\right)(g+hx)^3}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x]
```

output
```
-1/3*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^3) + (-1/2*((17*b*f*g^2 + 2*c*g*(4*e*g - (10*f*g^2)/h -
d*h) - b*h*(5*e*g + d*h) - 6*a*h*(3*f*g - e*h) + 2*h*(2*c*e*g + 3*b*f*g -
(5*c*f*g^2)/h - 2*c*d*h - 3*a*f*h)*x)*(a + b*x + c*x^2)^(3/2))/(h^2*(g + h
*x)^2) + (3*(-(((8*c^2*(10*f*g^4 - g^2*h*(4*e*g - d*h)) - 2*c*h*(3*b*g*(18
*f*g^2 - 6*e*g*h + d*h^2) - 2*a*h*(23*f*g^2 - 8*e*g*h + 2*d*h^2)) + h^2*(1
2*a^2*f*h^2 - 6*a*b*h*(7*f*g - e*h) + b^2*(29*f*g^2 - h*(5*e*g + d*h))) +
2*h*(3*b*f*h^2*(b*g - a*h) + 2*c^2*(10*f*g^3 - g*h*(4*e*g - d*h)) - c*h*(2
2*b*f*g^2 - b*h*(7*e*g - d*h) - 6*a*h*(3*f*g - e*h)))*x)*Sqrt[a + b*x + c*
x^2])/(h^2*(g + h*x))) + ((2*(c*g^2 - b*g*h + a*h^2)*(3*b^2*f*h^2 - 12*c*h
*(4*b*f*g - b*e*h - a*f*h) + 8*c^2*(10*f*g^2 - h*(4*e*g - d*h)))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - ((16*c^3*(10*
f*g^5 - g^3*h*(4*e*g - d*h)) - b*h^3*(24*a^2*f*h^2 - 6*a*b*h*(10*f*g - e*h
) + b^2*(35*f*g^2 - 5*e*g*h - d*h^2)) + 6*c*h^2*(4*a^2*h^2*(4*f*g - e*h) +
 b^2*g*(35*f*g^2 - 10*e*g*h + d*h^2) - 2*a*b*h*(25*f*g^2 - 7*e*g*h + d*h^2
)) - 24*c^2*g*h*(b*g*(14*f*g^2 - 5*e*g*h + d*h^2) - a*h*(11*f*g^2 - 4*e*g*
h + d*h^2)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
 + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*h^2
)))/(4*h^3))/(6*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2696 vs. $2(797) = 1594$.

Time = 0.52 (sec) , antiderivative size = 2697, normalized size of antiderivative = 3.25

method	result	size
risch	Expression too large to display	2697
default	Expression too large to display	6060

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*h*x+5*b*f*h+4*c*e*h-16*c*f*g)*(c*x^2+b*x+a)^(1/2)/h^5+1/8/h^5*(
(12*a*c*f*h^2+3*b^2*f*h^2+12*b*c*e*h^2-48*b*c*f*g*h+8*c^2*d*h^2-32*c^2*e*g
*h+80*c^2*f*g^2)/h*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-8/h
^2*(2*a*b*f*h^3+2*a*c*e*h^3-8*a*c*f*g*h^2+b^2*e*h^3-4*b^2*f*g*h^2+2*b*c*d*
h^3-8*b*c*e*g*h^2+20*b*c*f*g^2*h-4*c^2*d*g*h^2+10*c^2*e*g^2*h-20*c^2*f*g^3
)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+8/h^3*(a^2*f*h^4+2*a*b*e*h
^4-6*a*b*f*g*h^3+2*a*c*d*h^4-6*a*c*e*g*h^3+12*a*c*f*g^2*h^2+b^2*d*h^4-3*b^
2*e*g*h^3+6*b^2*f*g^2*h^2-6*b*c*d*g*h^3+12*b*c*e*g^2*h^2-20*b*c*f*g^3*h+6*
c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^4)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+
g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2
*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(
a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x
+g/h)))+8/h^4*(a^2*e*h^5-2*a^2*f*g*h^4+2*a*b*d*h^5-4*a*b*e*g*h^4+6*a*b*f*g
^2*h^3-4*a*c*d*g*h^4+6*a*c*e*g^2*h^3-8*a*c*f*g^3*h^2-2*b^2*d*g*h^4+3*b^2*e
*g^2*h^3-4*b^2*f*g^3*h^2+6*b*c*d*g^2*h^3-8*b*c*e*g^3*h^2+10*b*c*f*g^4*h-4*
c^2*d*g^3*h^2+5*c^2*e*g^4*h-6*c^2*f*g^5)*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+
g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**4,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7155 vs. $2(796) = 1592$.

Time = 4.94 (sec) , antiderivative size = 7155, normalized size of antiderivative = 8.63

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*c*f*x/h^4 - (16*c^2*f*g*h^10 - 4*c^2*e*h^11 -
 5*b*c*f*h^11)/(c*h^15)) - 1/8*(160*c^3*f*g^5 - 64*c^3*e*g^4*h - 336*b*c^2
*f*g^4*h + 16*c^3*d*g^3*h^2 + 120*b*c^2*e*g^3*h^2 + 210*b^2*c*f*g^3*h^2 +
264*a*c^2*f*g^3*h^2 - 24*b*c^2*d*g^2*h^3 - 60*b^2*c*e*g^2*h^3 - 96*a*c^2*e
*g^2*h^3 - 35*b^3*f*g^2*h^3 - 300*a*b*c*f*g^2*h^3 + 6*b^2*c*d*g*h^4 + 24*a
*c^2*d*g*h^4 + 5*b^3*e*g*h^4 + 84*a*b*c*e*g*h^4 + 60*a*b^2*f*g*h^4 + 96*a^
2*c*f*g*h^4 + b^3*d*h^5 - 12*a*b*c*d*h^5 - 6*a*b^2*e*h^5 - 24*a^2*c*e*h^5
- 24*a^2*b*f*h^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)
*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c*g^2*h^6 - b*g*h^7 + a*h^8)*sqrt(-c*g
^2 + b*g*h - a*h^2)) - 1/24*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3
*f*g^5*h^2 - 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*c^3*e*g^4*h^3 - 912
*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*f*g^4*h^3 + 144*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*c^3*d*g^3*h^4 + 504*(sqrt(c)*x - sqrt(c*x^2 + b*x
 + a))^5*b*c^2*e*g^3*h^4 + 522*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c
*f*g^3*h^4 + 552*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*c^2*f*g^3*h^4 - 2
16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b*c^2*d*g^2*h^5 - 252*(sqrt(c)*x
- sqrt(c*x^2 + b*x + a))^5*b^2*c*e*g^2*h^5 - 288*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^5*a*c^2*e*g^2*h^5 - 87*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^
3*f*g^2*h^5 - 540*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b*c*f*g^2*h^5 +
78*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^2*c*d*g*h^6 + 120*(sqrt(c)*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^4}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^4, x)
```

Reduce [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 18539, normalized size of antiderivative = 22.36

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^4,x)
```

output
```
(72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*f*g**3*h**5 +
 216*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*f*g**2*h**6*
x + 216*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*
h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*f*g*h**7*
x**2 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(
a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*f*h**8*
x**3 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(
a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*e*g**3
*h**5 + 216*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqr
t(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*e*g*
*2*h**6*x + 216*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)
*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c**2*
e*g*h**7*x**2 + 72*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x*
*2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*c*
*2*e*h**8*x**3 - 288*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*
x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*
c**2*f*g**4*h**4 - 864*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x +
c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*...
```

3.41 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 808

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \frac{(10cfg-2ceh-3bfh)\sqrt{a+bx+cx^2}}{2h^5(g+hx)}$$

$$+\frac{(16c^2g^3(5fg-eh)+h^2(48a^2fh^2-8abh(14fg-eh)+b^2(61fg^2-5egh-3dh^2))-4ch(6bg^2(6fg-eh)}{64h^4(cg^2-bgh+ah^2)^2(g+hx)}$$

$$+\frac{\left(\frac{8cg^2(5fg-eh)}{h}+8ah(5fg-eh)-b(37fg^2-5egh-3dh^2)-6h\left(ceg+4bfg-\frac{5cfg^2}{h}-cdh-4afh\right)x\right)(a}{24h^2(cg^2-bgh+ah^2)(g+hx)^3}$$

$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{4h(cg^2-bgh+ah^2)(g+hx)^4}$$

$$-\frac{\sqrt{c}(10cfg-2ceh-3bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2h^6}$$

$$+\frac{(128c^4g^5(5fg-eh)-64c^3g^3h(bg(28fg-5eh)-5ah(5fg-eh))+8ch^3(24a^3fh^3-12a^2bh^2(10fg-eh}{}$$

output
```
1/2*(-3*b*f*h-2*c*e*h+10*c*f*g)*(c*x^2+b*x+a)^(1/2)/h^5/(h*x+g)+1/64*(16*c
^2*g^3*(-e*h+5*f*g)+h^2*(48*a^2*f*h^2-8*a*b*h*(-e*h+14*f*g)+b^2*(-3*d*h^2-
5*e*g*h+61*f*g^2))-4*c*h*(6*b*g^2*(-e*h+6*f*g)-a*h*(3*d*h^2-7*e*g*h+35*f*g
^2)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/h^4/(a*h^2-b*g*h+c*g^
2)^2/(h*x+g)^2+1/24*(8*c*g^2*(-e*h+5*f*g)/h+8*a*h*(-e*h+5*f*g)-b*(-3*d*h^2
-5*e*g*h+37*f*g^2)-6*h*(c*e*g+4*b*f*g-5*c*f*g^2/h-c*d*h-4*a*f*h)*x)*(c*x^2
+b*x+a)^(3/2)/h^2/(a*h^2-b*g*h+c*g^2)/(h*x+g)^3-1/4*(f*g^2-h*(-d*h+e*g))*(
c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^4-1/2*c^(1/2)*(-3*b*f*h-2
*c*e*h+10*c*f*g)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/h^6+1/
128*(128*c^4*g^5*(-e*h+5*f*g)-64*c^3*g^3*h*(b*g*(-5*e*h+28*f*g)-5*a*h*(-e*
h+5*f*g))+8*c*h^3*(24*a^3*f*h^3-12*a^2*b*h^2*(-e*h+10*f*g)-5*b^3*g^2*(-e*h
+14*f*g)+3*a*b^2*h*(-d*h^2-5*e*g*h+55*f*g^2))-48*c^2*h^2*(10*a*b*g^2*h*(-e
*h+6*f*g)-5*b^2*g^3*(-e*h+7*f*g)-a^2*h^2*(d*h^2-5*e*g*h+25*f*g^2))+b^2*h^4
*(48*a^2*f*h^2-8*a*b*h*(e*h+10*f*g)+b^2*(3*d*h^2+5*e*g*h+35*f*g^2)))*arcta
nh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^
(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(5/2)
```

Mathematica [A] (verified)

Time = 16.64 (sec) , antiderivative size = 1447, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
((2*f*g - e*h)*(a + x*(b + c*x))^(3/2))/(3*h^3*(g + h*x)^3) - (f*(a + x*(b
 + c*x))^(3/2))/(2*h^3*(g + h*x)^2) + ((f*g^2 - h*(e*g - d*h))*(b*g - 2*a*
h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2 - h*(b*g - a*h
))*(g + h*x)^4) + (3*f*(a + x*(b + c*x))^(3/2)*(((-2*c*g + b*h)*(a + b*x +
 c*x^2)^(3/2))/((-(c*g^2) + b*g*h - a*h^2)*(g + h*x)) + (((-2*c*(2*c*g - b
*h)*(2*c*g - (b*h)/2) + c*h*(4*b*c*g - b^2*h - 4*a*c*h) + 2*c^2*h*(2*c*g -
 b*h)*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-8*c^(3/2)*(2*c*g - b*h)*(c*
g^2 - h*(b*g - a*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/h - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(8*c^2*g*(2*c*g - b*h)*(c*g^2 - h*(b
*g - a*h)) - 2*c*h*(4*b*c*g - b^2*h - 4*a*c*h)*(c*g^2 - h*(b*g - a*h)))*Ar
cTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sq
rt[a + b*x + c*x^2])])/(h*(4*c*g^2 - 4*b*g*h + 4*a*h^2)))/(4*c*h^2))/(-(c*
g^2) + b*g*h - a*h^2)))/(4*h^3*(a + b*x + c*x^2)^(3/2)) + ((-2*f*g + e*h)*
(a + x*(b + c*x))^(3/2)*(-1/2*((-2*c*g + b*h)*(a + b*x + c*x^2)^(3/2))/((c
*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g - b*h)) + (h*(2*b*c*g
 + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*g^2) + b*g*h - a*h^
2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g
 - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h^2 + 4*b*c*(2*c*g^2
 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b*g - 2*a*h))*x)*Sqr
t[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 - h*(b*g - a*h))^2...
```

Rubi [A] (verified)

Time = 2.79 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1229, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+8cdg-5beg-8afg-3bdh+8aeh-2\left(-\frac{5cfg^2}{h}+ceg+4bfg-cdh-4afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^4}dx}{4(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{4h(g+hx)^4(ah^2-bgh+cg^2)}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(8cdg-8afg+8aeh-b\left(-\frac{5fg^2}{h}+5eg+3dh\right)-2\left(-\frac{5cfg^2}{h}+ceg+4bfg-cdh-4afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^4}dx}{8\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1229

$$-\int -\frac{\left(h\left(h\left(35fg^2+h(5eg+3dh)\right)b^3-4\left(31cfg^3-ch(5eg-3dh)g+2ah^2(10fg+eh)\right)b^2+48a^2fh^3b+\frac{16c^2g^3(5fg-eh)b}{h}+4ach\left(61fg^2-h(17eg+3dh)\right)b-16ac\left(5cf\right.\right.\right.}{}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 27

$$\int \frac{\left(h^2\left(35fg^2+h(5eg+3dh)\right)b^3-4h\left(31cfg^3-ch(5eg-3dh)g+2ah^2(10fg+eh)\right)b^2+4\left(12a^2fh^4+ac\left(61fg^2-h(17eg+3dh)\right)h^2+4c^2g^3(5fg-eh)\right)b-16ach\left(5cfg^3-\right.}{8}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{4h(g+hx)^4\left(ah^2-bgh+cg^2\right)}$$

$\downarrow$ 1230

$$\frac{\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg^2+5ehg+3dh\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 25

$$\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 1269

$$\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 1092

$$\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 219

$$\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 1154

$$\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

$\downarrow$ 219

$$\frac{\sqrt{cx^2+bx+a}\left(64c^3(5fg-eh)g^4-16c^2h(bg(41fg-7eh)-8ah(5fg-eh))g^2+4ch^2\left(2b^2(46fg-5eh)g^2+16a^2h^2(5fg-eh)-abh\left(173fg^2-25ehg-3dh^2\right)\right)-bh^3\left(\left(35fg\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{4h\left(cg^2-bhg+ah^2\right)(g+hx)^4}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x]
```

output
```
-1/4*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^4) + (-1/12*(((16*c^2*g^4*(5*f*g - e*h))/h - h*(16*a^2*h
^2*(f*g - 2*e*h) - b^2*g*(35*f*g^2 + 5*e*g*h + 3*d*h^2) + 4*a*b*h*(7*f*g^2
 + 7*e*g*h + 3*d*h^2)) - 4*c*g*(b*g*(31*f*g^2 - 5*e*g*h + 3*d*h^2) - a*h*(
25*f*g^2 - 5*e*g*h + 9*d*h^2)) + 3*h*(4*a*c*h*(17*f*g^2 - h*(5*e*g - d*h))
 + (8*c^2*(5*f*g^4 - g^2*h*(e*g + d*h)))/h - 8*b*c*g*(9*f*g^2 - h*(2*e*g +
 d*h)) + h*(16*a^2*f*h^2 - 8*a*b*h*(6*f*g - e*h) + b^2*(29*f*g^2 - 5*e*g*h
 - 3*d*h^2)))*x)*(a + b*x + c*x^2)^(3/2))/(h^2*(c*g^2 - b*g*h + a*h^2)*(g
+ h*x)^3) + (((64*c^3*g^4*(5*f*g - e*h) - 16*c^2*g^2*h*(b*g*(41*f*g - 7*e*
h) - 8*a*h*(5*f*g - e*h)) + 4*c*h^2*(2*b^2*g^2*(46*f*g - 5*e*h) + 16*a^2*h
^2*(5*f*g - e*h) - a*b*h*(173*f*g^2 - 25*e*g*h - 3*d*h^2)) - b*h^3*(48*a^2
*f*h^2 - 8*a*b*h*(10*f*g + e*h) + b^2*(35*f*g^2 + 5*e*g*h + 3*d*h^2)) + 2*
c*h*(16*c^2*g^3*(5*f*g - e*h) - 4*c*h*(6*b*g^2*(6*f*g - e*h) - a*h*(35*f*g
^2 - h*(7*e*g - 3*d*h))) + h^2*(48*a^2*f*h^2 - 8*a*b*h*(14*f*g - e*h) + b^
2*(61*f*g^2 - h*(5*e*g + 3*d*h))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(g + h*x
)) + ((-64*Sqrt[c]*(10*c*f*g - 2*c*e*h - 3*b*f*h)*(c*g^2 - b*g*h + a*h^2)^
2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h + ((128*c^4*g^
5*(5*f*g - e*h) - 64*c^3*g^3*h*(b*g*(28*f*g - 5*e*h) - 5*a*h*(5*f*g - e*h)
) + 8*c*h^3*(24*a^3*f*h^3 - 12*a^2*b*h^2*(10*f*g - e*h) - 5*b^3*g^2*(14*f*
g - e*h) + 3*a*b^2*h*(55*f*g^2 - 5*e*g*h - d*h^2)) - 48*c^2*h^2*(10*a*b...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4350 vs. $2(772) = 1544$.

Time = 0.65 (sec) , antiderivative size = 4351, normalized size of antiderivative = 5.38

method	result	size
risch	Expression too large to display	4351
default	Expression too large to display	10038

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x,method=_RETURNVERBOSE)
```

output
```
f/h^5*(c*x^2+b*x+a)^(1/2)*c+1/2/h^5*(c^(1/2)*(3*b*f*h+2*c*e*h-10*c*f*g)/h*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-2/h^2*(2*a*c*f*h^2+b^2*f*h^2+2
*b*c*e*h^2-10*b*c*f*g*h+c^2*d*h^2-5*c^2*e*g*h+15*c^2*f*g^2)/((a*h^2-b*g*h+
c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+2/h^3*(2*a*b*f*h^3+2*a*c*e*h^3-8*a*c*f*g*h
^2+b^2*e*h^3-4*b^2*f*g*h^2+2*b*c*d*h^3-8*b*c*e*g*h^2+20*b*c*f*g^2*h-4*c^2*
d*g*h^2+10*c^2*e*g^2*h-20*c^2*f*g^3)*(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-
2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-
b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*(
(x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))
)+2/h^4*(a^2*f*h^4+2*a*b*e*h^4-6*a*b*f*g*h^3+2*a*c*d*h^4-6*a*c*e*g*h^3+12*
a*c*f*g^2*h^2+b^2*d*h^4-3*b^2*e*g*h^3+6*b^2*f*g^2*h^2-6*b*c*d*g*h^3+12*b*c
*e*g^2*h^2-20*b*c*f*g^3*h+6*c^2*d*g^2*h^2-10*c^2*e*g^3*h+15*c^2*f*g^4)*(-1
/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*
h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^5}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**5,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^5}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^5, x)
```

Reduce [B] (verification not implemented)

Time = 15.70 (sec) , antiderivative size = 26169, normalized size of antiderivative = 32.39

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^5}\,dx=\text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^5,x)
```

output

```
(576*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**4*h**6 +
2304*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**3*h**7*x
+ 3456*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h
**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**2*h**8*
x**2 + 2304*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqr
t(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g*h**
9*x**3 + 576*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sq
rt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*h**1
0*x**4 + 144*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sq
rt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f*g
**4*h**6 + 576*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*
sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f
*g**3*h**7*x + 864*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x*
*2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*
*2*f*g**2*h**8*x**2 + 576*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x
 + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*
a**2*b**2*f*g*h**9*x**3 + 144*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a +
 b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*...
```

3.42 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 871

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = -\frac{cf\sqrt{a+bx+cx^2}}{h^5(g+hx)}$$

$$-\frac{(32c^3fg^5-8c^2gh(10bfg^3-11afg^2h+3adh^3)+2ch^2(4a^2h^2(10fg-3eh)-6abh(11fg^2-egh-dh^2)\cdots}{128h^4\cdots}$$

$$-\frac{f(a+bx+cx^2)^{3/2}}{3h^3(g+hx)^3}$$

$$-\frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\,(bg-2ah+(2cg-bh)x)\,(a+bx+cx^2)^{3\cdots}}{16h^2\,(cg^2-bgh+ah^2)^2\,(g+hx)^4}$$

$$-\frac{(fg^2-h(eg-dh))\,(a+bx+cx^2)^{5/2}}{5h\,(cg^2-bgh+ah^2)\,(g+hx)^5}+\frac{c^{3/2}f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{h^6}$$

$$-\frac{(256c^5fg^7-896c^4fg^5h(bg-ah)+32c^3gh^2(35b^2fg^4-70abfg^3h+a^2h^2(35fg^2-3dh^2))-16c^2h^3(35b^3\cdots}{\cdots}$$

output
```
-c*f*(c*x^2+b*x+a)^(1/2)/h^5/(h*x+g)-1/128*(32*c^3*f*g^5-8*c^2*g*h*(3*a*d*
h^3-11*a*f*g^2*h+10*b*f*g^3)+2*c*h^2*(4*a^2*h^2*(-3*e*h+10*f*g)-6*a*b*h*(-
d*h^2-e*g*h+11*f*g^2)+b^2*(3*d*g*h^2+29*f*g^3))-b*h^3*(16*a^2*f*h^2-2*a*b*
h*(3*e*h+10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g))))*(b*g-2*a*h+(-b*h+2*c*g)*x)*
(c*x^2+b*x+a)^(1/2)/h^4/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^2-1/3*f*(c*x^2+b*x+a
)^(3/2)/h^3/(h*x+g)^3-1/16*(2*c*(-d*g*h^2+f*g^3)-h*(3*b*f*g^2-b*h*(d*h+e*g
)-2*a*h*(-e*h+2*f*g)))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/h^2/
(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^4-1/5*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/
2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^5+c^(3/2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/h^6-1/256*(256*c^5*f*g^7-896*c^4*f*g^5*h*(-a*h+b*g)+
32*c^3*g*h^2*(35*b^2*f*g^4-70*a*b*f*g^3*h+a^2*h^2*(-3*d*h^2+35*f*g^2))-16*
c^2*h^3*(35*b^3*f*g^4-6*a^3*h^3*(-e*h+6*f*g)+3*a^2*b*h^2*(-d*h^2-e*g*h+35*
f*g^2)-3*a*b^2*g*h*(d*h^2+35*f*g^2))+b^3*h^5*(16*a^2*f*h^2-2*a*b*h*(3*e*h+
10*f*g)+b^2*(7*f*g^2+3*h*(d*h+e*g)))-2*b*c*h^4*(96*a^3*f*h^3-24*a^2*b*h^2*
(e*h+8*f*g)-b^3*(-3*d*g*h^2+35*f*g^3)+4*a*b^2*h*(35*f*g^2+3*h*(d*h+e*g))))
*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/h^6/(a*h^2-b*g*h+c*g^2)^(7/2)
```

Mathematica [A] (verified)

Time = 16.57 (sec) , antiderivative size = 1363, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output
```
-1/3*(f*(a + x*(b + c*x))^(3/2))/(h^3*(g + h*x)^3) - ((2*f*g - e*h)*(b*g -
 2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2 - h*(b*g
- a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(
b + c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^5) + ((2*c*g - b*h
)*(f*g^2 - e*g*h + d*h^2)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*g
- b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4)
 - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2
])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g
) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))
/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x +
c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(-1/2*((-2*c*g + b*h)*(a + b*x
+ c*x^2)^(3/2))/((c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - (((-(c*g*(2*c*g -
b*h)) + (h*(2*b*c*g + b^2*h - 8*a*c*h))/2)*(a + b*x + c*x^2)^(3/2))/((-(c*
g^2) + b*g*h - a*h^2)*(g + h*x)) + (((-(c*(2*c*g - (b*h)/2)*(4*c^2*g^2 - b
^2*h^2 - 4*c*h*(b*g - 2*a*h))) + (c*h*(-10*b^2*c*g*h + 8*a*c^2*g*h - b^3*h
^2 + 4*b*c*(2*c*g^2 + 3*a*h^2)))/2 + c^2*h*(4*c^2*g^2 - b^2*h^2 - 4*c*h*(b
*g - 2*a*h))*x)*Sqrt[a + b*x + c*x^2])/(2*c*h^2) - ((-16*c^(5/2)*(c*g^2 -
h*(b*g - a*h))^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h
 - (4*Sqrt[c*g^2 - b*g*h + a*h^2]*(-(c*h*(c*g^2 - b*g*h + a*h^2)*(8*b*c...
```

Rubi [A] (verified)

Time = 3.42 (sec) , antiderivative size = 1291, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1229, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

$$\Big\downarrow \ 2181$$

$$-\frac{\int -\frac{5\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^5}dx}{5\left(ah^2-bgh+cg^2\right)}-\frac{(a+bx+cx^2)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^5}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1229$$

$$-\frac{\int -\frac{\left(h^2\left(7fg^2+3h(eg+dh)\right)b^3-2\left(a(10fg+3eh)h^3+c\left(13fhg^3+3dh^3g\right)\right)b^2+4\left(4c^2fg^4+4a^2fh^4+ach^2\left(17fg^2-3h(eg+dh)\right)\right)b-24ach\left(a(2fg-eh)h^2+c\left(fg^3-\cdots\right.\right.\cdots}{2h(g+hx)^3}\cdots}{8h^2(ah^2-bgh+cg^2)}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(h^2\left(7fg^2+3h(eg+dh)\right)b^3-2\left(a(10fg+3eh)h^3+c\left(13fhg^3+3dh^3g\right)\right)b^2+4\left(4c^2fg^4+4a^2fh^4+ach^2\left(17fg^2-3h(eg+dh)\right)\right)b-24ach\left(a(2fg-eh)h^2+c\left(fg^3-dgh\cdots\right.\right.\cdots}{(g+hx)^3}\cdots}{16h^3(ah^2-bgh+cg^2)}\cdots$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{5h(g+hx)^5\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1229$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\cdots\right.\right.\cdots}{\cdots}\cdots$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\cdots\right.\right.\cdots}{\cdots}\cdots$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 1269$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 1092$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 219$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 1154$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

$$\Big\downarrow \; 219$$

$$-\frac{\sqrt{cx^2+bx+a}\left(128c^4fg^7-32c^3fh(11bg-10ah)g^5+8c^2h^2\left(38b^2fg^4-abh\left(65fg^2+3dh^2\right)g+2a^2h^2\left(13fg^2+3dh^2\right)\right)g-2ch^3\left(\left(35fg^4-3dg^2h^2\right)b^3-2ag^2h(34fg+3\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{5h\left(cg^2-bhg+ah^2\right)(g+hx)^5}$$

input

```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x]
```

output

```
-1/5*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^5) + (-1/24*(((16*c^2*f*g^5)/h - 2*c*g*(13*b*f*g^3 - 10*
a*f*g^2*h + 3*b*d*g*h^2 - 6*a*d*h^3) - h*(4*a^2*h^2*(2*f*g - 3*e*h) - b^2*
(7*f*g^3 + 3*g*h*(e*g + d*h)) + 2*a*b*h*(f*g^2 + 3*h*(2*e*g + d*h))) + (16
*f*(c*g^2 - h*(b*g - a*h))^2 + 3*(2*c*g - b*h)*(2*c*(f*g^3 - d*g*h^2) - h*
(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h))))*x)*(a + b*x + c*x^2)
^(3/2))/(h^2*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) + (-1/4*((128*c^4*f*g^7
- 32*c^3*f*g^5*h*(11*b*g - 10*a*h) + 8*c^2*g*h^2*(38*b^2*f*g^4 + 2*a^2*h^2
*(13*f*g^2 + 3*d*h^2) - a*b*g*h*(65*f*g^2 + 3*d*h^2)) - 2*c*h^3*(8*a^3*h^3
*(2*f*g - 3*e*h) - 2*a*b^2*g^2*h*(34*f*g + 3*e*h) + 4*a^2*b*h^2*(5*f*g^2 +
 6*e*g*h + 3*d*h^2) + b^3*(35*f*g^4 - 3*d*g^2*h^2)) - b*h^4*(b*g - 2*a*h)*
(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*h*(e*g + d*h))
) + h*(128*c*f*(c*g^2 - h*(b*g - a*h))^3 + (2*c*g - b*h)*(32*c^3*f*g^5 - 8
*c^2*g*h*(10*b*f*g^3 - 11*a*f*g^2*h + 3*a*d*h^3) + 2*c*h^2*(4*a^2*h^2*(10*
f*g - 3*e*h) - 6*a*b*h*(11*f*g^2 - e*g*h - d*h^2) + b^2*(29*f*g^3 + 3*d*g*
h^2)) - b*h^3*(16*a^2*f*h^2 - 2*a*b*h*(10*f*g + 3*e*h) + b^2*(7*f*g^2 + 3*
h*(e*g + d*h)))))*x)*Sqrt[a + b*x + c*x^2])/(h^2*(c*g^2 - b*g*h + a*h^2)*(
g + h*x)^2) - ((-256*c^(3/2)*f*(c*g^2 - b*g*h + a*h^2)^3*ArcTanh[(b + 2*c*
x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/h + ((256*c^5*f*g^7 - 896*c^4*f*g^5
*h*(b*g - a*h) + 32*c^3*g*h^2*(35*b^2*f*g^4 - 70*a*b*f*g^3*h + a^2*h^2*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 13371 vs. 2(835) = 1670.

Time = 0.82 (sec) , antiderivative size = 13372, normalized size of antiderivative = 15.35

method	result	size
default	Expression too large to display	13372

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^6}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**6,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^6}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^6, x)
```

Reduce [B] (verification not implemented)

Time = 169.39 (sec) , antiderivative size = 33446, normalized size of antiderivative = 38.40

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^6}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^6,x)
```

output

```
(2880*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h*
*2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g**5*h**7
 + 14400*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a
*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g**4*h
**8*x + 28800*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*s
qrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g
**3*h**9*x**2 + 28800*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c
*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3
*b*c*f*g**2*h**10*x**3 + 14400*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a
+ b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*
g*x)*a**3*b*c*f*g*h**11*x**4 + 2880*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sq
rt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x +
 2*c*g*x)*a**3*b*c*f*h**12*x**5 + 1440*sqrt(a*h**2 - b*g*h + c*g**2)*log(2
*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*
x + 2*c*g*x)*a**3*c**2*e*g**5*h**7 + 7200*sqrt(a*h**2 - b*g*h + c*g**2)*lo
g(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b
*h*x + 2*c*g*x)*a**3*c**2*e*g**4*h**8*x + 14400*sqrt(a*h**2 - b*g*h + c*g*
*2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b
*g - b*h*x + 2*c*g*x)*a**3*c**2*e*g**3*h**9*x**2 + 14400*sqrt(a*h**2 - b*g
*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)...
```

3.43 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx$

Optimal result

Integrand size = 32, antiderivative size = 660

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7} dx =$$
$$-\frac{(b^2-4ac)(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7f}{512(cg^2-bgh+ah^2)^4(g+hx)^2}$$
$$+\frac{(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7fg^2+h(5eg}{192(cg^2-bgh+ah^2)^3(g+hx)^4}$$
$$-\frac{(fg^2-h(eg-dh))(a+bx+cx^2)^{5/2}}{6h(cg^2-bgh+ah^2)(g+hx)^6}$$
$$+\frac{(2c(5fg^3+gh(eg-7dh))-h(17bfg^2-bh(5eg+7dh)-12ah(2fg-eh)))(a+bx+cx^2)^{5/2}}{60h(cg^2-bgh+ah^2)^2(g+hx)^5}$$
$$+\frac{(b^2-4ac)^2(24c^2dg^2+24a^2fh^2-12abh(2fg+eh)-4c(afg^2-ah(7eg-dh)+3bg(eg+2dh))+b^2(7f}{1024(cg^2-bgh+ah^2)^{9/2}}$$

output
```
-1/512*(-4*a*c+b^2)*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)-4*c*(a
*f*g^2-a*h*(-d*h+7*e*g)+3*b*g*(2*d*h+e*g))+b^2*(7*f*g^2+h*(7*d*h+5*e*g)))*
(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+
g)^2+1/192*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)-4*c*(a*f*g^2-a*
h*(-d*h+7*e*g)+3*b*g*(2*d*h+e*g))+b^2*(7*f*g^2+h*(7*d*h+5*e*g)))*(b*g-2*a*
h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^4-1/6*
(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)^6+1
/60*(2*c*(5*f*g^3+g*h*(-7*d*h+e*g))-h*(17*b*f*g^2-b*h*(7*d*h+5*e*g)-12*a*h
*(-e*h+2*f*g)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^5+1/10
24*(-4*a*c+b^2)^2*(24*c^2*d*g^2+24*a^2*f*h^2-12*a*b*h*(e*h+2*f*g)-4*c*(a*f
*g^2-a*h*(-d*h+7*e*g)+3*b*g*(2*d*h+e*g))+b^2*(7*f*g^2+h*(7*d*h+5*e*g)))*ar
ctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+
a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(9/2)
```

Mathematica [A] (verified)

Time = 16.44 (sec) , antiderivative size = 1222, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output
```
(f*(b*g - 2*a*h + (2*c*g - b*h)*x)*(a + x*(b + c*x))^(3/2))/(8*h^2*(c*g^2
- h*(b*g - a*h))*(g + h*x)^4) - ((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)
*(a + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) + ((2*
f*g - e*h)*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - h*(b*g
 - a*h))*(g + h*x)^5) + ((2*c*g - b*h)*(-2*f*g + e*h)*(a + x*(b + c*x))^(3
/2)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 -
 b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g -
b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + (
(b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b
*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c
*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2
- b*g*h + a*h^2)*(a + b*x + c*x^2)^(3/2)) - ((f*g^2 - e*g*h + d*h^2)*(a +
x*(b + c*x))^(3/2)*(((c*g*h - (h*(-12*c*g + 7*b*h))/2)*(a + b*x + c*x^2)^(
5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) - ((-2*(a*c*h^2 + (c*g*(-12*
c*g + 7*b*h))/2) + b*(c*g*h + (h*(-12*c*g + 7*b*h))/2))*(((b*g - 2*a*h + (
2*c*g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h
*x)^4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x +
 c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[
(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*...
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2181, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+12cdg-5beg-12afg-7bdh+12aeh+2\left(\frac{5cfg^2}{h}+ceg-6bfg-cdh+6afh\right)x\right)(cx^2+bx+a)^{3/2}}{2(g+hx)^6}dx}{6(ah^2-bgh+cg^2)}-\frac{(a+bx+cx^2)^{5/2}(fg^2-h(eg-dh))}{6h(g+hx)^6(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(12cdg-b\left(-\frac{5fg^2}{h}+5eg+7dh\right)-12a(fg-eh)-2\left(6bfg-6afh-c\left(\frac{5fg^2}{h}+eg-dh\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^6}dx}{12\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\int \frac{\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^5}dx}{2(ah^2-bgh+cg^2)}+\frac{(a+bx+cx^2)^{5}}{}}{12\left(ah^2-bgh+cg^2\right)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1152$$

$$\frac{\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}\right.}{2(ah^2-bgh+cg^2)}}{12\left(ah^2-bgh+cg\right.}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1152$$

$$\frac{\left(24a^2fh^2-4c\left(-ah(7eg-dh)+afg^2+3bg(2dh+eg)\right)-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}\right.}{2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(24a^2fh^2-4c(-ah(7eg-dh)+afg^2+3bg(2dh+eg))-12abh(eh+2fg)+b^2\left(h(7dh+5eg)+7fg^2\right)+24c^2dg^2\right)\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}\right.}{2(ah^2-bgh+cg^2}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{\left(a+bx+cx^2\right)^{3/2}(-2ah+x(2cg-bh)+bg)}{8(g+hx)^4\left(ah^2-bgh+cg^2\right)}-\frac{3\left(b^2-4ac\right)\left(\frac{\sqrt{a+bx+cx^2}(-2ah+x(2cg-bh)+bg)}{4(g+hx)^2\left(ah^2-bgh+cg^2\right)}-\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{8\left(ah^2-bgh+cg^2\right)^{3/2}}\right)}{16\left(ah^2-bgh+cg^2\right)}\right)\Big(}{2(ah^2-bgh+cg^2)}$$

$$\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}$$

input

```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x]
```

output

```
-1/6*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^6) + (((2*c*(5*f*g^3 + g*h*(e*g - 7*d*h)) - h*(17*b*f*g^
2 - b*h*(5*e*g + 7*d*h) - 12*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2))/
(5*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + ((24*c^2*d*g^2 + 24*a^2*f*h^2
- 12*a*b*h*(2*f*g + e*h) - 4*c*(a*f*g^2 - a*h*(7*e*g - d*h) + 3*b*g*(e*g +
 2*d*h)) + b^2*(7*f*g^2 + h*(5*e*g + 7*d*h)))*(((b*g - 2*a*h + (2*c*g - b*
h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (
3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(
4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a
*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2]
)])/(8*(c*g^2 - b*g*h + a*h^2)^(3/2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*
(c*g^2 - b*g*h + a*h^2)))/(12*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 20683 vs. 2(634) = 1268.

Time = 1.14 (sec) , antiderivative size = 20684, normalized size of antiderivative = 31.34

method	result	size
default	Expression too large to display	20684

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^7}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**7,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**7, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48343 vs. 2(634) = 1268.

Time = 6.32 (sec) , antiderivative size = 48343, normalized size of antiderivative = 73.25

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x, algorithm="giac")
```

output
```
1/512*(24*b^4*c^2*d*g^2 - 192*a*b^2*c^3*d*g^2 + 384*a^2*c^4*d*g^2 - 12*b^5
*c*e*g^2 + 96*a*b^3*c^2*e*g^2 - 192*a^2*b*c^3*e*g^2 + 7*b^6*f*g^2 - 60*a*b
^4*c*f*g^2 + 144*a^2*b^2*c^2*f*g^2 - 64*a^3*c^3*f*g^2 - 24*b^5*c*d*g*h + 1
92*a*b^3*c^2*d*g*h - 384*a^2*b*c^3*d*g*h + 5*b^6*e*g*h - 12*a*b^4*c*e*g*h
- 144*a^2*b^2*c^2*e*g*h + 448*a^3*c^3*e*g*h - 24*a*b^5*f*g*h + 192*a^2*b^3
*c*f*g*h - 384*a^3*b*c^2*f*g*h + 7*b^6*d*h^2 - 60*a*b^4*c*d*h^2 + 144*a^2*
b^2*c^2*d*h^2 - 64*a^3*c^3*d*h^2 - 12*a*b^5*e*h^2 + 96*a^2*b^3*c*e*h^2 - 1
92*a^3*b*c^2*e*h^2 + 24*a^2*b^4*f*h^2 - 192*a^3*b^2*c*f*h^2 + 384*a^4*c^2*
f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c
*g^2 + b*g*h - a*h^2))/((c^4*g^8 - 4*b*c^3*g^7*h + 6*b^2*c^2*g^6*h^2 + 4*a
*c^3*g^6*h^2 - 4*b^3*c*g^5*h^3 - 12*a*b*c^2*g^5*h^3 + b^4*g^4*h^4 + 12*a*b
^2*c*g^4*h^4 + 6*a^2*c^2*g^4*h^4 - 4*a*b^3*g^3*h^5 - 12*a^2*b*c*g^3*h^5 +
6*a^2*b^2*g^2*h^6 + 4*a^3*c*g^2*h^6 - 4*a^3*b*g*h^7 + a^4*h^8)*sqrt(-c*g^2
 + b*g*h - a*h^2)) + 1/7680*(15360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*
c^6*f*g^8*h^5 - 61440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b*c^5*f*g^7*h
^6 + 92160*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*b^2*c^4*f*g^6*h^7 + 6144
0*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^11*a*c^5*f*g^6*h^7 - 61440*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^11*b^3*c^3*f*g^5*h^8 - 184320*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^11*a*b*c^4*f*g^5*h^8 + 15360*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^11*b^4*c^2*f*g^4*h^9 + 184320*(sqrt(c)*x - sqrt(c*x^2 + b*x +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}{(g+h\,x)^7}\,dx$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7,x)
```

output
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^7, x)
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^7}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}(f\,x^2+ex+d)}{(hx+g)^7}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

output
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^7,x)
```

3.44 $\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1062

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

output
```
-1/1024*(-4*a*c+b^2)*(48*c^3*d*g^3-8*c^2*g*(a*f*g^2-a*h*(-3*d*h+8*e*g)+3*b
*g*(3*d*h+e*g))-b*h*(24*a^2*f*h^2-2*a*b*h*(7*e*h+10*f*g)+b^2*(5*f*g^2+h*(9
*d*h+5*e*g)))+2*c*(4*a^2*h^2*(-e*h+8*f*g)-2*a*b*h*(13*f*g^2+h*(-3*d*h+13*e
*g))+b^2*(7*f*g^3+g*h*(21*d*h+10*e*g))))*(b*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2
+b*x+a)^(1/2)/(a*h^2-b*g*h+c*g^2)^5/(h*x+g)^2+1/384*(48*c^3*d*g^3-8*c^2*g*
(a*f*g^2-a*h*(-3*d*h+8*e*g)+3*b*g*(3*d*h+e*g))-b*h*(24*a^2*f*h^2-2*a*b*h*(
7*e*h+10*f*g)+b^2*(5*f*g^2+h*(9*d*h+5*e*g)))+2*c*(4*a^2*h^2*(-e*h+8*f*g)-2
*a*b*h*(13*f*g^2+h*(-3*d*h+13*e*g))+b^2*(7*f*g^3+g*h*(21*d*h+10*e*g))))*(b
*g-2*a*h+(-b*h+2*c*g)*x)*(c*x^2+b*x+a)^(3/2)/(a*h^2-b*g*h+c*g^2)^4/(h*x+g)
^4-1/7*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x
+g)^7+1/84*(2*c*(5*f*g^3+g*h*(-9*d*h+2*e*g))-h*(19*b*f*g^2-b*h*(9*d*h+5*e*
g)-14*a*h*(-e*h+2*f*g)))*(c*x^2+b*x+a)^(5/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+
g)^6+1/840*(4*c^2*(5*f*g^4+g^2*h*(-51*d*h+2*e*g))-7*h^2*(24*a^2*f*h^2-2*a*
b*h*(7*e*h+10*f*g)+b^2*(9*d*h^2+5*e*g*h+5*f*g^2))-2*c*h*(3*b*g*(-34*d*h^2-
15*e*g*h+8*f*g^2)-2*a*h*(12*d*h^2-61*e*g*h+26*f*g^2)))*(c*x^2+b*x+a)^(5/2)
/h/(a*h^2-b*g*h+c*g^2)^3/(h*x+g)^5+1/2048*(-4*a*c+b^2)^2*(48*c^3*d*g^3-8*c
^2*g*(a*f*g^2-a*h*(-3*d*h+8*e*g)+3*b*g*(3*d*h+e*g))-b*h*(24*a^2*f*h^2-2*a*
b*h*(7*e*h+10*f*g)+b^2*(5*f*g^2+h*(9*d*h+5*e*g)))+2*c*(4*a^2*h^2*(-e*h+8*f
*g)-2*a*b*h*(13*f*g^2+h*(-3*d*h+13*e*g))+b^2*(7*f*g^3+g*h*(21*d*h+10*e*g))
))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*...
```

Mathematica [A] (verified)

Time = 16.53 (sec) , antiderivative size = 1636, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
Integrate[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

output
```
-1/7*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(
h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^7) + ((2*f*g - e*h)*(a + b*x + c*x^2)*
(a + x*(b + c*x))^(3/2))/(6*h*(c*g^2 - h*(b*g - a*h))*(g + h*x)^6) - (f*(a
 + b*x + c*x^2)*(a + x*(b + c*x))^(3/2))/(5*h*(c*g^2 - b*g*h + a*h^2)*(g +
 h*x)^5) + (f*(2*c*g - b*h)*(a + x*(b + c*x))^(3/2)*(((b*g - 2*a*h + (2*c*
g - b*h)*x)*(a + b*x + c*x^2)^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^
4) - (3*(b^2 - 4*a*c)*(((b*g - 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x
^2])/(4*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b
*g) + 2*a*h - (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))
))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*h^2*(c*g^2 - b*g*h + a*h^2)*(a + b*x
+ c*x^2)^(3/2)) - ((-2*f*g + e*h)*(a + x*(b + c*x))^(3/2)*(((c*g*h - (h*(-
12*c*g + 7*b*h))/2)*(a + b*x + c*x^2)^(5/2))/(5*(c*g^2 - b*g*h + a*h^2)*(g
 + h*x)^5) - ((-2*(a*c*h^2 + (c*g*(-12*c*g + 7*b*h))/2) + b*(c*g*h + (h*(-
12*c*g + 7*b*h))/2))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)^(
3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g -
2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^2)
*(g + h*x)^2) + ((b^2 - 4*a*c)*ArcTanh[(-(b*g) + 2*a*h - (2*c*g - b*h)*x)/
(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*
g*h + a*h^2]*(4*c*g^2 - 4*b*g*h + 4*a*h^2))))/(16*(c*g^2 - b*g*h + a*h^...
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 823, normalized size of antiderivative = 0.77, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1237, 27, 1228, 1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}{(g+hx)^8}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\left(\frac{5bfg^2}{h}+14cdg-5beg-14afg-9bdh+14aeh+2\left(\frac{5cfg^2}{h}+2ceg-7bfg-2cdh+7afh\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{2(g+hx)^7}dx}{7\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\left(14cdg-b\left(-\frac{5fg^2}{h}+5eg+9dh\right)-14a(fg-eh)-2\left(7bfg-7afh-c\left(\frac{5fg^2}{h}+2eg-2dh\right)\right)x\right)\left(cx^2+bx+a\right)^{3/2}}{(g+hx)^7}dx}{14\left(ah^2-bgh+cg^2\right)}-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1237$$

$$\frac{\frac{\left(a+bx+cx^2\right)^{5/2}\left(2c\left(gh(2eg-9dh)+5fg^3\right)-h\left(-14ah(2fg-eh)-bh(9dh+5eg)+19bfg^2\right)\right)}{6h(g+hx)^6\left(ah^2-bgh+cg^2\right)}-\int -\frac{\left(h\left(7\left(5fg^2+h(5eg+9dh)\right)b^2-2cg\left(-\frac{5fg^2}{h}+40eg+9\right.\right.\right.}{}}{14\,(a}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{\left(h\left(7\left(5fg^2+h(5eg+9dh)\right)b^2-2cg\left(-\frac{5fg^2}{h}+40eg+93dh\right)b-14ah(10fg+7eh)b+168c^2dg^2+168a^2fh^2-24ac\left(2fg^2-h(9eg-2dh)\right)\right)+2c\left(2c\left(5fg^3+h(2eg-9dh)\right.\right.\right.}{(g+hx)^6}}{12h\left(ah^2-bgh+cg^2\right)}}{14\,(ah}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{7h\left(2c\left(4a^2h^2(8fg-eh)-2abh\left(h(13eg-3dh)+13fg^2\right)+b^2\left(gh(21dh+10eg)+7fg^3\right)\right)-bh\left(24a^2fh^2-2abh(7eh+10fg)+b^2\left(h(9dh+5eg)+5fg^2\right)\right)-8c^2g\left(-ah(8eg-\right.\right.}{2\left(ah^2-bgh+cg^2\right)}}{}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1152$$

$$\frac{\frac{7h\left(2c\left(4a^2h^2(8fg-eh)-2abh\left(h(13eg-3dh)+13fg^2\right)+b^2\left(gh(21dh+10eg)+7fg^3\right)\right)-bh\left(24a^2fh^2-2abh(7eh+10fg)+b^2\left(h(9dh+5eg)+5fg^2\right)\right)-8c^2g\left(-ah(8eg-\right.\right.}{2\left(ah^2-bgh+cg^2\right)}}{}$$

$$-\frac{\left(a+bx+cx^2\right)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1152

$$7h\left(2c\left(4a^2h^2(8fg-eh)-2abh\left(h(13eg-3dh)+13fg^2\right)+b^2\left(gh(21dh+10eg)+7fg^3\right)\right)-bh\left(24a^2fh^2-2abh(7eh+10fg)+b^2\left(h(9dh+5eg)+5fg^2\right)\right)-8c^2g\left(-ah(8eg-\right.\right.$$

$$\frac{(a+bx+cx^2)^{5/2}\left(fg^2-h(eg-dh)\right)}{7h(g+hx)^7\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1154

$$\frac{\left(2c(5fg^3+h(2eg-9dh)g)-h\left(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)\right)\right)\left(cx^2+bx+a\right)^{5/2}}{6h(cg^2-bhg+ah^2)(g+hx)^6}+\frac{\left(4\left(5fg^4+h(2eg-51dh)g^2\right)c^2-2h\left(3bg\left(8fg^2-15ehg-34dh\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{7h\left(cg^2-bhg+ah^2\right)(g+hx)^7}$$

$\Big\downarrow$ 219

$$\frac{\left(2c(5fg^3+h(2eg-9dh)g)-h\left(19bfg^2-bh(5eg+9dh)-14ah(2fg-eh)\right)\right)\left(cx^2+bx+a\right)^{5/2}}{6h(cg^2-bhg+ah^2)(g+hx)^6}+\frac{\left(4\left(5fg^4+h(2eg-51dh)g^2\right)c^2-2h\left(3bg\left(8fg^2-15ehg-34dh\right.\right.\right.}{}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\left(cx^2+bx+a\right)^{5/2}}{7h\left(cg^2-bhg+ah^2\right)(g+hx)^7}$$

input
```
Int[((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x]
```

output
```
-1/7*((f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(5/2))/(h*(c*g^2 - b*g*h +
 a*h^2)*(g + h*x)^7) + (((2*c*(5*f*g^3 + g*h*(2*e*g - 9*d*h)) - h*(19*b*f*
g^2 - b*h*(5*e*g + 9*d*h) - 14*a*h*(2*f*g - e*h)))*(a + b*x + c*x^2)^(5/2)
)/(6*h*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^6) + (((4*c^2*(5*f*g^4 + g^2*h*(2
*e*g - 51*d*h)) - 7*h^2*(24*a^2*f*h^2 - 2*a*b*h*(10*f*g + 7*e*h) + b^2*(5*
f*g^2 + 5*e*g*h + 9*d*h^2)) - 2*c*h*(3*b*g*(8*f*g^2 - 15*e*g*h - 34*d*h^2)
 - 2*a*h*(26*f*g^2 - 61*e*g*h + 12*d*h^2)))*(a + b*x + c*x^2)^(5/2))/(5*(c
*g^2 - b*g*h + a*h^2)*(g + h*x)^5) + (7*h*(48*c^3*d*g^3 - 8*c^2*g*(a*f*g^2
 - a*h*(8*e*g - 3*d*h) + 3*b*g*(e*g + 3*d*h)) - b*h*(24*a^2*f*h^2 - 2*a*b*
h*(10*f*g + 7*e*h) + b^2*(5*f*g^2 + h*(5*e*g + 9*d*h))) + 2*c*(4*a^2*h^2*(
8*f*g - e*h) - 2*a*b*h*(13*f*g^2 + h*(13*e*g - 3*d*h)) + b^2*(7*f*g^3 + g*
h*(10*e*g + 21*d*h))))*(((b*g - 2*a*h + (2*c*g - b*h)*x)*(a + b*x + c*x^2)
^(3/2))/(8*(c*g^2 - b*g*h + a*h^2)*(g + h*x)^4) - (3*(b^2 - 4*a*c)*(((b*g
- 2*a*h + (2*c*g - b*h)*x)*Sqrt[a + b*x + c*x^2])/(4*(c*g^2 - b*g*h + a*h^
2)*(g + h*x)^2) - ((b^2 - 4*a*c)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(
2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(8*(c*g^2 - b*g*h +
 a*h^2)^(3/2))))/(16*(c*g^2 - b*g*h + a*h^2))))/(2*(c*g^2 - b*g*h + a*h^2)
))/(12*h*(c*g^2 - b*g*h + a*h^2)))/(14*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31329 vs. 2(1032) = 2064.

Time = 1.61 (sec) , antiderivative size = 31330, normalized size of antiderivative = 29.50

method	result	size
default	Expression too large to display	31330

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \int \frac{(a+bx+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}{(g+hx)^8}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)/(h*x+g)**8,x)
```

output
```
Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)/(g + h*x)**8, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75375 vs. 2(1032) = 2064.

Time = 28.15 (sec) , antiderivative size = 75375, normalized size of antiderivative = 70.97

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x, algorithm="giac")
```

output
```
1/1024*(48*b^4*c^3*d*g^3 - 384*a*b^2*c^4*d*g^3 + 768*a^2*c^5*d*g^3 - 24*b^
5*c^2*e*g^3 + 192*a*b^3*c^3*e*g^3 - 384*a^2*b*c^4*e*g^3 + 14*b^6*c*f*g^3 -
 120*a*b^4*c^2*f*g^3 + 288*a^2*b^2*c^3*f*g^3 - 128*a^3*c^4*f*g^3 - 72*b^5*
c^2*d*g^2*h + 576*a*b^3*c^3*d*g^2*h - 1152*a^2*b*c^4*d*g^2*h + 20*b^6*c*e*
g^2*h - 96*a*b^4*c^2*e*g^2*h - 192*a^2*b^2*c^3*e*g^2*h + 1024*a^3*c^4*e*g^
2*h - 5*b^7*f*g^2*h - 12*a*b^5*c*f*g^2*h + 336*a^2*b^3*c^2*f*g^2*h - 832*a
^3*b*c^3*f*g^2*h + 42*b^6*c*d*g*h^2 - 360*a*b^4*c^2*d*g*h^2 + 864*a^2*b^2*
c^3*d*g*h^2 - 384*a^3*c^4*d*g*h^2 - 5*b^7*e*g*h^2 - 12*a*b^5*c*e*g*h^2 + 3
36*a^2*b^3*c^2*e*g*h^2 - 832*a^3*b*c^3*e*g*h^2 + 20*a*b^6*f*g*h^2 - 96*a^2
*b^4*c*f*g*h^2 - 192*a^3*b^2*c^2*f*g*h^2 + 1024*a^4*c^3*f*g*h^2 - 9*b^7*d*
h^3 + 84*a*b^5*c*d*h^3 - 240*a^2*b^3*c^2*d*h^3 + 192*a^3*b*c^3*d*h^3 + 14*
a*b^6*e*h^3 - 120*a^2*b^4*c*e*h^3 + 288*a^3*b^2*c^2*e*h^3 - 128*a^4*c^3*e*
h^3 - 24*a^2*b^5*f*h^3 + 192*a^3*b^3*c*f*h^3 - 384*a^4*b*c^2*f*h^3)*arctan
(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g*h
- a*h^2))/((c^5*g^10 - 5*b*c^4*g^9*h + 10*b^2*c^3*g^8*h^2 + 5*a*c^4*g^8*h^
2 - 10*b^3*c^2*g^7*h^3 - 20*a*b*c^3*g^7*h^3 + 5*b^4*c*g^6*h^4 + 30*a*b^2*c
^2*g^6*h^4 + 10*a^2*c^3*g^6*h^4 - b^5*g^5*h^5 - 20*a*b^3*c*g^5*h^5 - 30*a^
2*b*c^2*g^5*h^5 + 5*a*b^4*g^4*h^6 + 30*a^2*b^2*c*g^4*h^6 + 10*a^3*c^2*g^4*
h^6 - 10*a^2*b^3*g^3*h^7 - 20*a^3*b*c*g^3*h^7 + 10*a^3*b^2*g^2*h^8 + 5*a^4
*c*g^2*h^8 - 5*a^4*b*g*h^9 + a^5*h^10)*sqrt(-c*g^2 + b*g*h - a*h^2)) - ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \text{Hanged}$$

input
```
int(((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2))/(g + h*x)^8,x)
```

output
```
\text{Hanged}
```

Reduce [F]

$$\int \frac{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}{(g+hx)^8}\,dx = \int \frac{(c\,x^2+bx+a)^{\frac{3}{2}}(f\,x^2+ex+d)}{(hx+g)^8}dx$$

input
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

output
```
int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)/(h*x+g)^8,x)
```

3.45 $\int (1+2x)^3\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 143

$$\begin{aligned}\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx &= \frac{5393(1-6x)\sqrt{2-x+3x^2}}{15552}+\frac{17}{105}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\ &\quad+\frac{67}{378}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{2}{21}(1+2x)^4\left(2-x+3x^2\right)^{3/2}\\ &\quad-\frac{(75295+26982x)\left(2-x+3x^2\right)^{3/2}}{68040}+\frac{124039\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{31104\sqrt{3}}\end{aligned}$$

output
```
5393/15552*(1-6*x)*(3*x^2-x+2)^(1/2)+17/105*(1+2*x)^2*(3*x^2-x+2)^(3/2)+67
/378*(1+2*x)^3*(3*x^2-x+2)^(3/2)+2/21*(1+2*x)^4*(3*x^2-x+2)^(3/2)-1/68040*
(75295+26982*x)*(3*x^2-x+2)^(3/2)+124039/93312*arcsinh(1/23*(1-6*x)*23^(1/
2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{6\sqrt{2-x+3x^2}(-543069+1493894x+3280872x^2+5497776x^3+7491456x^4+6462720x^5+2488320x}{3265920}$$

input
```
Integrate[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-543069 + 1493894*x + 3280872*x^2 + 5497776*x^3 +
7491456*x^4 + 6462720*x^5 + 2488320*x^6) + 4341365*Sqrt[3]*Log[1 - 6*x + 2
*Sqrt[6 - 3*x + 9*x^2]])/3265920
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$
$$\Big\downarrow \text{ 2184}$$
$$\frac{1}{84}\int -4(8-67x)(2x+1)^3\sqrt{3x^2-x+2}dx+\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$
$$\Big\downarrow \text{ 27}$$
$$\frac{2}{21}(2x+1)^4\left(3x^2-x+2\right)^{3/2}-\frac{1}{21}\int (8-67x)(2x+1)^3\sqrt{3x^2-x+2}dx$$
$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{21}\left(\frac{67}{18}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{18}\int \frac{3}{2}(565-612x)(2x+1)^2\sqrt{3x^2-x+2}dx\right)+$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{67}{18}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{12}\int (565-612x)(2x+1)^2\sqrt{3x^2-x+2}dx\right)+$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{204}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{15}\int 3(2x+1)(2998x+4151)\sqrt{3x^2-x+2}dx\right)+\frac{67}{18}\left(3x^2-x+2\right)$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{204}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{5}\int (2x+1)(2998x+4151)\sqrt{3x^2-x+2}dx\right)+\frac{67}{18}\left(3x^2-x+2\right)^{3/2}$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\int \sqrt{3x^2-x+2}dx-\frac{1}{54}(26982x+75295)\left(3x^2-x+2\right)^{3/2}\right)+\frac{204}{5}\left(3x^2-x+2\right)^{3/2}(2x$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+75295)\left(3x^2-x+\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{21}\left(\frac{1}{12}\left(\frac{1}{5}\left(-\frac{188755}{36}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{54}(26982x+75295)\left(3x^2-x+2\right)^3\right.\right.\right.$$

$$\frac{2}{21}\left(3x^2-x+2\right)^{3/2}(2x+1)^4$$

input
```
Int[(1 + 2*x)^3*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(3/2))/21 + ((67*(1 + 2*x)^3*(2 - x + 3*x^2
)^(3/2))/18 + ((204*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2))/5 + (-1/54*((75295
+ 26982*x)*(2 - x + 3*x^2)^(3/2)) - (188755*(-1/12*((1 - 6*x)*Sqrt[2 - x +
 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/36)/5)/12)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(2488320x^6+6462720x^5+7491456x^4+5497776x^3+3280872x^2+1493894x-543069)\sqrt{3x^2-x+2}}{544320} - \frac{124039\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\right.}{23}\right.}{93312}$
trager	$\left(\frac{32}{7}x^6+\frac{748}{63}x^5+\frac{1858}{135}x^4+\frac{38179}{3780}x^3+\frac{19529}{3240}x^2+\frac{746947}{272160}x-\frac{60341}{60480}\right)\sqrt{3x^2-x+2} - \frac{124039\ \mathrm{RootOf}\left(\_Z\right.}{}$
default	$-\frac{5393(6x-1)\sqrt{3x^2-x+2}}{15552} - \frac{124039\sqrt{3}\ \mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{93312} - \frac{45739(3x^2-x+2)^{\frac{3}{2}}}{68040} + \frac{7849x(3x^2-x+2)^{\frac{3}{2}}}{3780} + \frac{1594x^2(3x}{31}$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/544320*(2488320*x^6+6462720*x^5+7491456*x^4+5497776*x^3+3280872*x^2+1493
894*x-543069)*(3*x^2-x+2)^(1/2)-124039/93312*3^(1/2)*arcsinh(6/23*23^(1/2)
*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{544320}\left(2488320\,x^6+6462720\,x^5+7491456\,x^4+5497776\,x^3+3280872\,x^2+1493894\,x-543069\right)\sqrt{3\,x}$$
$$+\frac{124039}{186624}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/544320*(2488320*x^6 + 6462720*x^5 + 7491456*x^4 + 5497776*x^3 + 3280872*
x^2 + 1493894*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/186624*sqrt(3)*log(
4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\sqrt{3x^2-x+2}\cdot\left(\frac{32x^6}{7}+\frac{748x^5}{63}+\frac{1858x^4}{135}+\frac{38179x^3}{3780}+\frac{19529x^2}{3240}+\frac{746947x}{272160}-\frac{60341}{60480}\right)$$
$$-\frac{124039\sqrt{3}\,\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{93312}$$

input
```
integrate((1+2*x)**3*(3*x**2-x+2)**(1/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**6/7 + 748*x**5/63 + 1858*x**4/135 + 38179*x**3
/3780 + 19529*x**2/3240 + 746947*x/272160 - 60341/60480) - 124039*sqrt(3)*
asinh(6*sqrt(23)*(x - 1/6)/23)/93312
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{32}{21}\left(3x^2-x+2\right)^{\frac{3}{2}}x^4+\frac{844}{189}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3+\frac{1594}{315}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2$$
$$+\frac{7849}{3780}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{45739}{68040}\left(3x^2-x+2\right)^{\frac{3}{2}}-\frac{5393}{2592}\sqrt{3x^2-x+2}\,x$$
$$-\frac{124039}{93312}\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{5393}{15552}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
32/21*(3*x^2 - x + 2)^(3/2)*x^4 + 844/189*(3*x^2 - x + 2)^(3/2)*x^3 + 1594
/315*(3*x^2 - x + 2)^(3/2)*x^2 + 7849/3780*(3*x^2 - x + 2)^(3/2)*x - 45739
/68040*(3*x^2 - x + 2)^(3/2) - 5393/2592*sqrt(3*x^2 - x + 2)*x - 124039/93
312*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 5393/15552*sqrt(3*x^2 - x +
 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{544320}\left(2\left(12\left(6\left(8\left(30\left(72\,x+187\right)x+6503\right)x+38179\right)x+136703\right)x+746947\right)x-543069\right)\sqrt{3\,x^2-x}$$
$$+\frac{124039}{93312}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/544320*(2*(12*(6*(8*(30*(72*x + 187)*x + 6503)*x + 38179)*x + 136703)*x
+ 746947)*x - 543069)*sqrt(3*x^2 - x + 2) + 124039/93312*sqrt(3)*log(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1594\,x^2\,\left(3\,x^2-x+2\right)^{3/2}}{315}+\frac{844\,x^3\,\left(3\,x^2-x+2\right)^{3/2}}{189}$$
$$+\frac{32\,x^4\,\left(3\,x^2-x+2\right)^{3/2}}{21}-\frac{137057\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{136080}$$
$$-\frac{5959\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{1890}-\frac{45739\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{1632960}$$
$$+\frac{7849\,x\,\left(3\,x^2-x+2\right)^{3/2}}{3780}-\frac{1051997\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{3265920}$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(1594*x^2*(3*x^2 - x + 2)^(3/2))/315 + (844*x^3*(3*x^2 - x + 2)^(3/2))/189
 + (32*x^4*(3*x^2 - x + 2)^(3/2))/21 - (137057*3^(1/2)*log((3*x^2 - x + 2)
^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/136080 - (5959*(x/2 - 1/12)*(3*x^2 - x
+ 2)^(1/2))/1890 - (45739*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/16329
60 + (7849*x*(3*x^2 - x + 2)^(3/2))/3780 - (1051997*3^(1/2)*log(2*(3*x^2 -
 x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/3265920
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int (1+2x)^3\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$= \frac{32\sqrt{3x^2-x+2}\,x^6}{7}+\frac{748\sqrt{3x^2-x+2}\,x^5}{63}+\frac{1858\sqrt{3x^2-x+2}\,x^4}{135}+\frac{38179\sqrt{3x^2-x+2}\,x^3}{3780}+\frac{19529\sqrt{3x^2-x+2}\,x^2}{3240}+\frac{746947\sqrt{3x^2-x+2}\,x}{272160}-\frac{60341\sqrt{3x^2-x+2}}{60480}-\frac{124039\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{93312}$$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x)
```

output
```
(14929920*sqrt(3*x**2 - x + 2)*x**6 + 38776320*sqrt(3*x**2 - x + 2)*x**5 +
 44948736*sqrt(3*x**2 - x + 2)*x**4 + 32986656*sqrt(3*x**2 - x + 2)*x**3 +
 19685232*sqrt(3*x**2 - x + 2)*x**2 + 8963364*sqrt(3*x**2 - x + 2)*x - 325
8414*sqrt(3*x**2 - x + 2) - 4341365*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sq
rt(3) + 6*x - 1)/sqrt(23)))/3265920
```

3.46 $\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 118

$$\begin{aligned}\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx &= \frac{235(1-6x)\sqrt{2-x+3x^2}}{1296}+\frac{1}{5}(1+2x)^2\left(2-x+3x^2\right)^{3/2}\\ &\quad+\frac{1}{9}(1+2x)^3\left(2-x+3x^2\right)^{3/2}+\frac{1}{810}(25+306x)\left(2-x+3x^2\right)^{3/2}+\frac{5405\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2592\sqrt{3}}\end{aligned}$$

output
```
235/1296*(1-6*x)*(3*x^2-x+2)^(1/2)+1/5*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/9*(1+
2*x)^3*(3*x^2-x+2)^(3/2)+1/810*(25+306*x)*(3*x^2-x+2)^(3/2)+5405/7776*arcs
inh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.64

$$\int (1+2x)^2\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx = \frac{6\sqrt{2-x+3x^2}(5607+14638x+22344x^2+33552x^3+35712x^4+17280x^5)+27025\sqrt{3}\log\left(1-6x+2\right.}{38880}$$

input
```
Integrate[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(5607 + 14638*x + 22344*x^2 + 33552*x^3 + 35712*x^4
 + 17280*x^5) + 27025*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/3888
0
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{72}\int -12(1-18x)(2x+1)^2\sqrt{3x^2-x+2}dx+\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}(2x+1)^3\left(3x^2-x+2\right)^{3/2}-\frac{1}{6}\int (1-18x)(2x+1)^2\sqrt{3x^2-x+2}dx$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{1}{15}\int 12(11-17x)(2x+1)\sqrt{3x^2-x+2}dx\right)+\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\int (11-17x)(2x+1)\sqrt{3x^2-x+2}dx\right)+\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\int\sqrt{3x^2-x+2}dx-\frac{1}{108}(306x+25)\left(3x^2-x+2\right)^{3/2}\right)\right)+$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{108}(306x+\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+\right.\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{6}\left(\frac{6}{5}(2x+1)^2\left(3x^2-x+2\right)^{3/2}-\frac{4}{5}\left(\frac{1175}{72}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{108}(306x+25\right.\right.$$
$$\frac{1}{9}\left(3x^2-x+2\right)^{3/2}(2x+1)^3$$

input
```
Int[(1 + 2*x)^2*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2))/9 + ((6*(1 + 2*x)^2*(2 - x + 3*x^2)^(3
/2))/5 - (4*(-1/108*((25 + 306*x)*(2 - x + 3*x^2)^(3/2)) + (1175*(-1/12*((
1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt
[3])))/72))/5)/6
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(17280x^5+35712x^4+33552x^3+22344x^2+14638x+5607)\sqrt{3x^2-x+2}}{6480}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$
trager	$\left(\frac{8}{3}x^5+\frac{248}{45}x^4+\frac{233}{45}x^3+\frac{931}{270}x^2+\frac{7319}{3240}x+\frac{623}{720}\right)\sqrt{3x^2-x+2}-\frac{5405\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2\right.\right.}{777}$
default	$-\frac{235(6x-1)\sqrt{3x^2-x+2}}{1296}-\frac{5405\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}+\frac{277(3x^2-x+2)^{\frac{3}{2}}}{810}+\frac{83x(3x^2-x+2)^{\frac{3}{2}}}{45}+\frac{32x^2(3x^2-x+2)^{\frac{3}{2}}}{15}$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/6480*(17280*x^5+35712*x^4+33552*x^3+22344*x^2+14638*x+5607)*(3*x^2-x+2)^
(1/2)-5405/7776*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{6480}\left(17280\,x^5+35712\,x^4+33552\,x^3+22344\,x^2+14638\,x+5607\right)\sqrt{3\,x^2-x+2} + \frac{5405}{15552}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/6480*(17280*x^5 + 35712*x^4 + 33552*x^3 + 22344*x^2 + 14638*x + 5607)*sq
rt(3*x^2 - x + 2) + 5405/15552*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(
6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{8x^5}{3}+\frac{248x^4}{45}+\frac{233x^3}{45}+\frac{931x^2}{270}+\frac{7319x}{3240}+\frac{623}{720}\right) - \frac{5405\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{7776}$$

input
```
integrate((1+2*x)**2*(3*x**2-x+2)**(1/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**5/3 + 248*x**4/45 + 233*x**3/45 + 931*x**2/270
+ 7319*x/3240 + 623/720) - 5405*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/777
6
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8}{9}\left(3x^2-x+2\right)^{\frac{3}{2}}x^3 + \frac{32}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2 + \frac{83}{45}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{277}{810}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{235}{216}\sqrt{3x^2-x+2}x - \frac{5405}{7776}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{235}{1296}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/9*(3*x^2 - x + 2)^(3/2)*x^3 + 32/15*(3*x^2 - x + 2)^(3/2)*x^2 + 83/45*(3
*x^2 - x + 2)^(3/2)*x + 277/810*(3*x^2 - x + 2)^(3/2) - 235/216*sqrt(3*x^2
 - x + 2)*x - 5405/7776*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 235/129
6*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.62

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{6480}\left(2\left(12\left(6\left(8\left(15x+31\right)x+233\right)x+931\right)x+7319\right)x+5607\right)\sqrt{3x^2-x+2} + \frac{5405}{7776}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/6480*(2*(12*(6*(8*(15*x + 31)*x + 233)*x + 931)*x + 7319)*x + 5607)*sqrt
(3*x^2 - x + 2) + 5405/7776*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2
 - x + 2)) + 1)
```

Mupad [B] (verification not implemented)

Time = 18.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.30

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{32\,x^2\,\left(3\,x^2-x+2\right)^{3/2}}{15}+\frac{8\,x^3\,\left(3\,x^2-x+2\right)^{3/2}}{9}-\frac{2783\,\sqrt{3}\,\ln\left(\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(3\,x-\frac{1}{2}\right)}{3}\right)}{3240}-\frac{121\,\left(\frac{x}{2}-\frac{1}{12}\right)\,\sqrt{3\,x^2-x+2}}{45}+\frac{277\,\sqrt{3\,x^2-x+2}\,\left(72\,x^2-6\,x+45\right)}{19440}+\frac{83\,x\,\left(3\,x^2-x+2\right)^{3/2}}{45}+\frac{6371\,\sqrt{3}\,\ln\left(2\,\sqrt{3\,x^2-x+2}+\frac{\sqrt{3}\,\left(6\,x-1\right)}{3}\right)}{38880}$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(32*x^2*(3*x^2 - x + 2)^(3/2))/15 + (8*x^3*(3*x^2 - x + 2)^(3/2))/9 - (278
3*3^(1/2)*log((3*x^2 - x + 2)^(1/2) + (3^(1/2)*(3*x - 1/2))/3))/3240 - (12
1*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/45 + (277*(3*x^2 - x + 2)^(1/2)*(72*
x^2 - 6*x + 45))/19440 + (83*x*(3*x^2 - x + 2)^(3/2))/45 + (6371*3^(1/2)*l
og(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)*(6*x - 1))/3))/38880
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int (1+2x)^2\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{8\sqrt{3x^2-x+2}\,x^5}{3}+\frac{248\sqrt{3x^2-x+2}\,x^4}{45}+\frac{233\sqrt{3x^2-x+2}\,x^3}{45}+\frac{931\sqrt{3x^2-x+2}\,x^2}{270}+\frac{7319\sqrt{3x^2-x+2}\,x}{3240}+\frac{623\sqrt{3x^2-x+2}}{720}-\frac{5405\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{7776}$$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x)
```

output
```
(103680*sqrt(3*x**2 - x + 2)*x**5 + 214272*sqrt(3*x**2 - x + 2)*x**4 + 201
312*sqrt(3*x**2 - x + 2)*x**3 + 134064*sqrt(3*x**2 - x + 2)*x**2 + 87828*s
qrt(3*x**2 - x + 2)*x + 33642*sqrt(3*x**2 - x + 2) - 27025*sqrt(3)*log((2*
sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/38880
```

3.47 $\int (1+2x)\sqrt{2-x+3x^2}(1+3x+4x^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 93

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{19(1-6x)\sqrt{2-x+3x^2}}{2592} + \frac{2}{15}(1+2x)^2\left(2-x+3x^2\right)^{3/2} + \frac{(745+738x)\left(2-x+3x^2\right)^{3/2}}{1620} + \frac{437\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{5184\sqrt{3}}$$

output
```
19/2592*(1-6*x)*(3*x^2-x+2)^(1/2)+2/15*(1+2*x)^2*(3*x^2-x+2)^(3/2)+1/1620*
(745+738*x)*(3*x^2-x+2)^(3/2)+437/15552*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(
1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{6\sqrt{2-x+3x^2}(15471+17374x+24072x^2+31536x^3+20736x^4)+2185\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+\right.}{77760}$$

input
```
Integrate[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(15471 + 17374*x + 24072*x^2 + 31536*x^3 + 20736*x^
4) + 2185*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/77760
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2184, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)\,dx$$
$$\Big\downarrow \text{2184}$$
$$\frac{1}{60}\int 4(2x+1)(41x+2)\sqrt{3x^2-x+2}dx+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{15}\int (2x+1)(41x+2)\sqrt{3x^2-x+2}dx+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$
$$\Big\downarrow \text{1225}$$
$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\int \sqrt{3x^2-x+2}dx\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+\cdots}\right)\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{15}\left(\frac{1}{108}(738x+745)\left(3x^2-x+2\right)^{3/2}-\frac{95}{72}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)\right)+\frac{2}{15}\left(3x^2-x+2\right)^{3/2}(2x+1)^2$$

input
```
Int[(1 + 2*x)*Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2))/15 + (((745 + 738*x)*(2 - x + 3*x^2)
^(3/2))/108 - (95*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1
 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/72)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(20736x^4+31536x^3+24072x^2+17374x+15471)\sqrt{3x^2-x+2}}{12960}-\frac{437\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{15552}$
trager	$\left(\frac{8}{5}x^4+\frac{73}{30}x^3+\frac{1003}{540}x^2+\frac{8687}{6480}x+\frac{191}{160}\right)\sqrt{3x^2-x+2}-\frac{437\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{Root}\right.}{15552}$
default	$-\frac{19(6x-1)\sqrt{3x^2-x+2}}{2592}-\frac{437\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{15552}+\frac{961(3x^2-x+2)^{\frac{3}{2}}}{1620}+\frac{89x(3x^2-x+2)^{\frac{3}{2}}}{90}+\frac{8x^2(3x^2-x+2)^{\frac{3}{2}}}{15}$

input
```
int((1+2*x)*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/12960*(20736*x^4+31536*x^3+24072*x^2+17374*x+15471)*(3*x^2-x+2)^(1/2)-43
7/15552*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{1}{12960}\left(20736\,x^4+31536\,x^3+24072\,x^2+17374\,x+15471\right)\sqrt{3\,x^2-x+2} + \frac{437}{31104}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/12960*(20736*x^4 + 31536*x^3 + 24072*x^2 + 17374*x + 15471)*sqrt(3*x^2 -
 x + 2) + 437/31104*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) -
72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{8x^4}{5}+\frac{73x^3}{30}+\frac{1003x^2}{540}+\frac{8687x}{6480}+\frac{191}{160}\right) - \frac{437\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{15552}$$

input
```
integrate((1+2*x)*(3*x**2-x+2)**(1/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**4/5 + 73*x**3/30 + 1003*x**2/540 + 8687*x/6480
+ 191/160) - 437*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/15552
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8}{15}\left(3x^2-x+2\right)^{\frac{3}{2}}x^2+\frac{89}{90}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{961}{1620}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{19}{432}\sqrt{3x^2-x+2}x - \frac{437}{15552}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)+\frac{19}{2592}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/15*(3*x^2 - x + 2)^(3/2)*x^2 + 89/90*(3*x^2 - x + 2)^(3/2)*x + 961/1620*
(3*x^2 - x + 2)^(3/2) - 19/432*sqrt(3*x^2 - x + 2)*x - 437/15552*sqrt(3)*a
rcsinh(1/23*sqrt(23)*(6*x - 1)) + 19/2592*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{1}{12960}\left(2\left(12\left(18\left(48x+73\right)x+1003\right)x+8687\right)x+15471\right)\sqrt{3x^2-x+2}+\frac{437}{15552}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/12960*(2*(12*(18*(48*x + 73)*x + 1003)*x + 8687)*x + 15471)*sqrt(3*x^2 -
 x + 2) + 437/15552*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
)) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.46

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx$$
$$=\frac{8x^2\left(3x^2-x+2\right)^{3/2}}{15}-\frac{253\sqrt{3}\,\ln\left(\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(3x-\frac{1}{2}\right)}{3}\right)}{810}-\frac{44\left(\frac{x}{2}-\frac{1}{12}\right)\sqrt{3x^2-x+2}}{45}+\frac{961\sqrt{3x^2-x+2}\left(72x^2-6x+45\right)}{38880}+\frac{89x\left(3x^2-x+2\right)^{3/2}}{90}+\frac{22103\sqrt{3}\,\ln\left(2\sqrt{3x^2-x+2}+\frac{\sqrt{3}\left(6x-1\right)}{3}\right)}{77760}$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1),x)
```

output
```
(8*x^2*(3*x^2 - x + 2)^(3/2))/15 - (253*3^(1/2)*log((3*x^2 - x + 2)^(1/2)
+ (3^(1/2)*(3*x - 1/2))/3))/810 - (44*(x/2 - 1/12)*(3*x^2 - x + 2)^(1/2))/
45 + (961*(3*x^2 - x + 2)^(1/2)*(72*x^2 - 6*x + 45))/38880 + (89*x*(3*x^2
- x + 2)^(3/2))/90 + (22103*3^(1/2)*log(2*(3*x^2 - x + 2)^(1/2) + (3^(1/2)
*(6*x - 1))/3))/77760
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int (1+2x)\sqrt{2-x+3x^2}\left(1+3x+4x^2\right)\,dx = \frac{8\sqrt{3x^2-x+2}\,x^4}{5}+\frac{73\sqrt{3x^2-x+2}\,x^3}{30}+\frac{1003\sqrt{3x^2-x+2}\,x^2}{540}+\frac{8687\sqrt{3x^2-x+2}\,x}{6480}+\frac{191\sqrt{3x^2-x+2}}{160}-\frac{437\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{15552}$$

input
```
int((1+2*x)*(3*x^2-x+2)^(1/2)*(4*x^2+3*x+1),x)
```

output
```
(124416*sqrt(3*x**2 - x + 2)*x**4 + 189216*sqrt(3*x**2 - x + 2)*x**3 + 144
432*sqrt(3*x**2 - x + 2)*x**2 + 104244*sqrt(3*x**2 - x + 2)*x + 92826*sqrt
(3*x**2 - x + 2) - 2185*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x
- 1)/sqrt(23)))/77760
```

3.48 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 101

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{72}(13+30x)\sqrt{2-x+3x^2} + \frac{2}{9}\left(2-x+3x^2\right)^{3/2} - \frac{43\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{144\sqrt{3}} - \frac{1}{8}\sqrt{13}\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/72*(13+30*x)*(3*x^2-x+2)^(1/2)+2/9*(3*x^2-x+2)^(3/2)-43/432*arcsinh(1/23
*(1-6*x)*23^(1/2))*3^(1/2)-1/8*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x
^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{432}\left(6\sqrt{2-x+3x^2}\,(45+14x+48x^2) + 108\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) - 43\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

input
```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(45 + 14*x + 48*x^2) + 108*Sqrt[13]*ArcTanh[(Sqrt[3
] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 43*Sqrt[3]*Log[1 - 6*
x + 2*Sqrt[6 - 3*x + 9*x^2]])/432
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1231, 25, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{2x+1}\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{36}\int \frac{12(5x+4)\sqrt{3x^2-x+2}}{2x+1}dx + \frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(5x+4)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 1231

$$\frac{1}{3}\left(\frac{1}{24}(30x+13)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{86x+277}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 25

$$\frac{1}{3}\left(\frac{1}{48}\int \frac{86x+277}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 1269

$$\frac{1}{3}\left(\frac{1}{48}\left(43\int \frac{1}{\sqrt{3x^2-x+2}}dx+234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+ \frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 1090

$$\frac{1}{3}\left(\frac{1}{48}\left(234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{43\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+ \frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 222

$$\frac{1}{3}\left(\frac{1}{48}\left(234\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+ \frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 1154

$$\frac{1}{3}\left(\frac{1}{48}\left(\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-468\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+ \frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

$\big\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{48}\left(\frac{43\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-18\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{24}\sqrt{3x^2-x+2}(30x+13)\right)+\frac{2}{9}\left(3x^2-x+2\right)^{3/2}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(2*(2 - x + 3*x^2)^(3/2))/9 + (((13 + 30*x)*Sqrt[2 - x + 3*x^2])/24 + ((43
*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - 18*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*
Sqrt[13]*Sqrt[2 - x + 3*x^2])])/48)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

method	result
risch	$\frac{(48x^2+14x+45)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{432}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2(\frac{9}{2}-4x)\sqrt{13}}{13\sqrt{12(\frac{1}{2}+x)^2+5-16x}}\right)}{8}$
default	$\frac{5(6x-1)\sqrt{3x^2-x+2}}{72}+\frac{43\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,(x-\frac{1}{6})}{23}\right)}{432}+\frac{2(3x^2-x+2)^{\frac{3}{2}}}{9}+\frac{\sqrt{12(\frac{1}{2}+x)^2+5-16x}}{8}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2(\frac{9}{2}-\ldots}{13\sqrt{12(\frac{1}{2}\ldots}}\right)}{8}$
trager	$\left(\frac{2}{3}x^2+\frac{7}{36}x+\frac{5}{8}\right)\sqrt{3x^2-x+2}+\frac{43\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2\ldots}\right.}{432}$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/72*(48*x^2+14*x+45)*(3*x^2-x+2)^(1/2)+43/432*3^(1/2)*arcsinh(6/23*23^(1/
2)*(x-1/6))-1/8*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-1
6*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{1}{72}\left(48\,x^2+14\,x+45\right)\sqrt{3\,x^2-x+2} \\ &\quad+\frac{43}{864}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right) \\ &\quad+\frac{1}{16}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

output
```
1/72*(48*x^2 + 14*x + 45)*sqrt(3*x^2 - x + 2) + 43/864*sqrt(3)*log(-4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 1/16*sqrt(13)*lo
g(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x
^2 + 4*x + 1))
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((3*x**2-x+2)**(1/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{2}{9}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{5}{12}\sqrt{3x^2-x+2}\,x \\ &\quad + \frac{43}{432}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &\quad + \frac{1}{8}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|} - \frac{9\sqrt{23}}{23\,|2x+1|}\right) \\ &\quad + \frac{13}{72}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

output
```
2/9*(3*x^2 - x + 2)^(3/2) + 5/12*sqrt(3*x^2 - x + 2)*x + 43/432*sqrt(3)*ar
csinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/8*sqrt(13)*arcsinh(8/23*sqrt(23
)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 13/72*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{72}\,(2\,(24\,x+7)x+45)\sqrt{3\,x^2-x+2} - \frac{43}{432}\,\sqrt{3}\log\left(-6\,\sqrt{3}x+\sqrt{3}+6\,\sqrt{3\,x^2-x+2}\right) + \frac{1}{8}\,\sqrt{13}\log\left(-\frac{\left|-4\,\sqrt{3}x-2\,\sqrt{13}-2\,\sqrt{3}+4\,\sqrt{3\,x^2-x+2}\right|}{2\,(2\,\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\,\sqrt{3\,x^2-x+2})}\right)$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

output
```
1/72*(2*(24*x + 7)*x + 45)*sqrt(3*x^2 - x + 2) - 43/432*sqrt(3)*log(-6*sqr
t(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/8*sqrt(13)*log(-1/2*abs(-4*s
qrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x -
sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,(4\,x^2+3\,x+1)}{2\,x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{1+2x}\,dx$$
$$= \frac{\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6ix-i}{\sqrt{39}-4}\right)i}{8} + \frac{2\sqrt{3x^2-x+2}\,x^2}{3}$$
$$+ \frac{7\sqrt{3x^2-x+2}\,x}{36} + \frac{5\sqrt{3x^2-x+2}}{8}$$
$$+ \frac{\sqrt{13}\log\left(24\sqrt{3x^2-x+2}\,\sqrt{3}\,x-4\sqrt{3x^2-x+2}\,\sqrt{3}+8\sqrt{39}+72x^2-24x-30\right)}{16}$$
$$- \frac{\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{3}+\sqrt{39}+6x+3\right)}{8} + \frac{43\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{432}$$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x),x)
```

output
```
(54*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39)
 - 4))*i + 288*sqrt(3*x**2 - x + 2)*x**2 + 84*sqrt(3*x**2 - x + 2)*x + 270
*sqrt(3*x**2 - x + 2) + 27*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*x
- 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30) - 54*
sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) + sqrt(39) + 6*x + 3) + 43*sqr
t(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/432
```

3.49 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 108

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{1}{156}(67-96x)\sqrt{2-x+3x^2}-\frac{(2-x+3x^2)^{3/2}}{13(1+2x)} - \frac{11\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{17\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{8\sqrt{13}}$$

output
```
-1/156*(67-96*x)*(3*x^2-x+2)^(1/2)-(3*x^2-x+2)^(3/2)/(13+26*x)-11/18*arcsi
nh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+17/104*13^(1/2)*arctanh(1/26*(9-8*x)*13^
(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\sqrt{2-x+3x^2}(-7-2x+12x^2)}{12+24x} - \frac{17\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4\sqrt{13}} - \frac{11\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
(Sqrt[2 - x + 3*x^2]*(-7 - 2*x + 12*x^2))/(12 + 24*x) - (17*ArcTanh[(Sqrt[
3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(4*Sqrt[13]) - (11*Lo
g[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}\left(4x^2+3x+1\right)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{(64x+15)\sqrt{3x^2-x+2}}{2(2x+1)}dx-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\int \frac{(64x+15)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{26}\left(-\frac{1}{48}\int \frac{52(7-88x)}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\left(-\frac{13}{12}\int \frac{7-88x}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-44\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$\Big\downarrow$ 1090

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{44\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$\Big\downarrow$ 222

$$\frac{1}{26}\left(-\frac{13}{12}\left(51\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$\Big\downarrow$ 1154

$$\frac{1}{26}\left(-\frac{13}{12}\left(-102\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

$\Big\downarrow$ 219

$$\frac{1}{26}\left(-\frac{13}{12}\left(-\frac{44\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{51\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)-\frac{1}{6}\sqrt{3x^2-x+2}(67-96x)\right)-\frac{\left(3x^2-x+2\right)^{3/2}}{13(2x+1)}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(3/2)/(1 + 2*x) + (-1/6*((67 - 96*x)*Sqrt[2 - x + 3*
x^2]) - (13*((-44*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (51*ArcTanh[(9 -
 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13]))/12)/26
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result
risch	$\frac{36x^4-18x^3+5x^2+3x-14}{12(1+2x)\sqrt{3x^2-x+2}}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{17\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{104}$
default	$\frac{(6x-1)\sqrt{3x^2-x+2}}{12}+\frac{11\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}-\frac{\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}{26\left(\frac{1}{2}+x\right)}-\frac{17\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}{104}+\frac{17\sqrt{13}\,\operatorname{arctanh}\left(\right.}{}$
trager	$\frac{(12x^2-2x-7)\sqrt{3x^2-x+2}}{12+24x}+\frac{11\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}\right)}{18}-\frac{17\,\mathrm{RootO}}{}$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(36*x^4-18*x^3+5*x^2+3*x-14)/(1+2*x)/(3*x^2-x+2)^(1/2)+11/18*3^(1/2)*
arcsinh(6/23*23^(1/2)*(x-1/6))+17/104*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(
1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$
$$=\frac{572\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+153\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{}}{}\right.}{1872\,(2x+1)}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/1872*(572*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 153*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 156*(12*
x^2 - 2*x - 7)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+2)**(1/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\begin{aligned}\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{2}\sqrt{3x^2-x+2}x \\ &\quad + \frac{11}{18}\sqrt{3}\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &\quad - \frac{17}{104}\sqrt{13}\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) \\ &\quad - \frac{1}{3}\sqrt{3x^2-x+2} - \frac{\sqrt{3x^2-x+2}}{4(2x+1)}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(3*x^2 - x + 2)*x + 11/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*s
qrt(23)) - 17/104*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqr
t(23)/abs(2*x + 1)) - 1/3*sqrt(3*x^2 - x + 2) - 1/4*sqrt(3*x^2 - x + 2)/(2
*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(85) = 170$.

Time = 0.39 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.52

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx$$
$$= \frac{17}{104}\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$
$$-\frac{11}{18}\sqrt{3}\log\left(\frac{\left|-2\,\sqrt{3}+2\,\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{2\,\sqrt{13}}{2\,x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)}\right)\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$
$$-\frac{1}{8}\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)$$
$$+\frac{67\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)^3\operatorname{sgn}\left(\frac{1}{2\,x+1}\right)-57\,\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)^2\operatorname{sgn}\left(\frac{1}{2\,x+1}\right.}{12\left(\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right.\right.}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

output
```
17/104*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 11/18*sqrt(3)*log(1/2*abs(-2*sqr
t(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(
sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))*s
gn(1/(2*x + 1)) - 1/8*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(2*x +
 1)) + 1/12*(67*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x +
 1))^3*sgn(1/(2*x + 1)) - 57*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 129*(sqrt(-8/(2*x + 1) + 1
3/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 27*sqrt(13)*sg
n(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
+ 1))^2 - 3)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{\sqrt{3\,x^2-x+2}\,(4\,x^2+3\,x+1)}{(2\,x+1)^2}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{936\sqrt{3x^2-x+2}\,x^2-156\sqrt{3x^2-x+2}\,x-546\sqrt{3x^2-x+2}+306\sqrt{13}\log\left(-2\sqrt{3x^2-x+2}\,\sqrt{13}+8\right.}{}$$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^2,x)
```

output
```
(936*sqrt(3*x**2 - x + 2)*x**2 - 156*sqrt(3*x**2 - x + 2)*x - 546*sqrt(3*x
**2 - x + 2) + 306*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x -
 9)*x + 153*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 3
06*sqrt(13)*log(2*x + 1)*x - 153*sqrt(13)*log(2*x + 1) + 1144*sqrt(3)*log(
 - 2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 572*sqrt(3)*log( - 2*sqrt
(3*x**2 - x + 2)*sqrt(3) - 6*x + 1))/(936*(2*x + 1))
```

3.50 $\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} dx$

Optimal result

Integrand size = 32, antiderivative size = 115

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} dx = \frac{11(7+10x)\sqrt{2-x+3x^2}}{104(1+2x)} - \frac{(2-x+3x^2)^{3/2}}{26(1+2x)^2} + \frac{11\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8\sqrt{3}} - \frac{803\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{208\sqrt{13}}$$

output
```
11*(7+10*x)*(3*x^2-x+2)^(1/2)/(104+208*x)-1/26*(3*x^2-x+2)^(3/2)/(1+2*x)^2
+11/24*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-803/2704*13^(1/2)*arctanh(1/
26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3} dx = \frac{\frac{39\sqrt{2-x+3x^2}(69+268x+208x^2)}{(1+2x)^2} + 2409\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right) + 1859\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+\right.}{4056}$$

input
```
Integrate[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((39*Sqrt[2 - x + 3*x^2]*(69 + 268*x + 208*x^2))/(1 + 2*x)^2 + 2409*Sqrt[1
3]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 185
9*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/4056
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2181, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3x^2-x+2}(4x^2+3x+1)}{(2x+1)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{11(10x+3)\sqrt{3x^2-x+2}}{2(2x+1)^2}dx-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{52}\int \frac{(10x+3)\sqrt{3x^2-x+2}}{(2x+1)^2}dx-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{11}{52}\left(\frac{(10x+7)\sqrt{3x^2-x+2}}{2(2x+1)}-\frac{1}{8}\int -\frac{2(47-52x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{52}\left(\frac{1}{4}\int \frac{47-52x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-26\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1090

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{26\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 222

$$\frac{11}{52}\left(\frac{1}{4}\left(73\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1154

$$\frac{11}{52}\left(\frac{1}{4}\left(-146\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 219

$$\frac{11}{52}\left(\frac{1}{4}\left(-\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{73\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)+\frac{\sqrt{3x^2-x+2}(10x+7)}{2(2x+1)}\right)-\frac{(3x^2-x+2)^{3/2}}{26(2x+1)^2}$$

input
```
Int[(Sqrt[2 - x + 3*x^2]*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(3/2)/(1 + 2*x)^2 + (11*(((7 + 10*x)*Sqrt[2 - x + 3*
x^2])/(2*(1 + 2*x)) + ((-26*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (73*Ar
cTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/4))/52
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

method	result
risch	$\frac{624x^4+596x^3+355x^2+467x+138}{104(1+2x)^2\sqrt{3x^2-x+2}}-\frac{11\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{2704}$
trager	$\frac{(208x^2+268x+69)\sqrt{3x^2-x+2}}{104(1+2x)^2}-\frac{11\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}\right)}{24}+\frac{803\,\mathrm{R}}{}$
default	$\frac{803\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}{2704}-\frac{11\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{24}-\frac{803\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{2704}-\frac{\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right.}{104\left(\frac{1}{2}+x\right)^2}$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/104*(624*x^4+596*x^3+355*x^2+467*x+138)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-11/2
4*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-803/2704*13^(1/2)*arctanh(2/13*(9
/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$$
$$= \frac{3718\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+2409\sqrt{13}(4x^2+4x+}{16224\,(4x^2+4x+}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/16224*(3718*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25) + 2409*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*
sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4
*x + 1)) + 156*(208*x^2 + 268*x + 69)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x +
1)
```

Sympy [F]

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx=\int \frac{\sqrt{3x^2-x+2}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((3*x**2-x+2)**(1/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

output
```
Integral(sqrt(3*x**2 - x + 2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx=-\frac{11}{24}\sqrt{3}\,\operatorname{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{803}{2704}\sqrt{13}\,\operatorname{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{55}{104}\sqrt{3\,x^2-x+2}-\frac{(3\,x^2-x+2)^{\frac{3}{2}}}{26\,(4\,x^2+4\,x+1)}+\frac{11\sqrt{3\,x^2-x+2}}{52\,(2\,x+1)}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
-11/24*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 803/2704*sqrt(13
)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 55/
104*sqrt(3*x^2 - x + 2) - 1/26*(3*x^2 - x + 2)^(3/2)/(4*x^2 + 4*x + 1) + 1
1/52*sqrt(3*x^2 - x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(92) = 184$.

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{11}{24}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{803}{2704}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right) + \frac{1}{2}\sqrt{3x^2-x+2} + \frac{318\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-69\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-1241\sqrt{3}x+649\sqrt{3}+1241\sqrt{3x^2-}}{104\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

output
```
11/24*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 803/
2704*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(
3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))
) + 1/2*sqrt(3*x^2 - x + 2) + 1/104*(318*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
^3 - 69*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1241*sqrt(3)*x + 649
*sqrt(3) + 1241*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^
2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\sqrt{3x^2-x+2}\,(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(1/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.17

$$\int \frac{\sqrt{2-x+3x^2}(1+3x+4x^2)}{(1+2x)^3}\,dx$$
$$= \frac{16224\sqrt{3x^2-x+2}\,x^2+20904\sqrt{3x^2-x+2}\,x+5382\sqrt{3x^2-x+2}+9636\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{1\right.}}{}$$

input
```
int((3*x^2-x+2)^(1/2)*(4*x^2+3*x+1)/(1+2*x)^3,x)
```

output
```
(16224*sqrt(3*x**2 - x + 2)*x**2 + 20904*sqrt(3*x**2 - x + 2)*x + 5382*sqr
t(3*x**2 - x + 2) + 9636*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*
x - 9)*x**2 + 9636*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)
*x + 2409*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 9636*s
qrt(13)*log(2*x + 1)*x**2 - 9636*sqrt(13)*log(2*x + 1)*x - 2409*sqrt(13)*l
og(2*x + 1) + 14872*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*
x**2 + 14872*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 371
8*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1))/(8112*(4*x**2 + 4
*x + 1))
```

3.51 $\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 158

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1255639(1-6x)\sqrt{2-x+3x^2}}{4478976} + \frac{54593(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{11(283-5850x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{913}{486}x^2\left(2-x+3x^2\right)^{5/2} + \frac{77}{81}x^3\left(2-x+3x^2\right)^{5/2} + \frac{2}{27}(1+2x)^4\left(2-x+3x^2\right)^{5/2} + \frac{28879697\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{8957952\sqrt{3}}$$

output
```
1255639/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)+54593/559872*(1-6*x)*(3*x^2-x+2)
^(3/2)-11/58320*(283-5850*x)*(3*x^2-x+2)^(5/2)+913/486*x^2*(3*x^2-x+2)^(5/
2)+77/81*x^3*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^4*(3*x^2-x+2)^(5/2)+28879697/2
6873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} (1+3x +4x^2)\ dx = \frac{6\sqrt{2-x+3x^2}(12499587+84014278x+201289704x^2+421626672x^3+649452672x^4+7112}{}$$

input
```
Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(12499587 + 84014278*x + 201289704*x^2 + 421626672*
x^3 + 649452672*x^4 + 711210240*x^5 + 635765760*x^6 + 510105600*x^7 + 2388
78720*x^8) + 144398485*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/134
369280
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2184, 27, 1267, 27, 2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{3/2} \left(4x^2+3x+1\right) dx$$

$$\Big\downarrow \ 2184$$

$$\frac{1}{108}\int 308x(2x+1)^3 \left(3x^2-x+2\right)^{3/2}dx+\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \ 27$$

$$\frac{77}{27}\int x(2x+1)^3 \left(3x^2-x+2\right)^{3/2}dx+\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \ 1267$$

$$\frac{77}{27}\left(\frac{1}{24}\int 4x\left(3x^2-x+2\right)^{3/2}\left(83x^2+24x+6\right)dx+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{77}{27}\left(\frac{1}{6}\int x\left(3x^2-x+2\right)^{3/2}\left(83x^2+24x+6\right)dx+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{2184}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{21}\int -\frac{1}{2}(412-1755x)x\left(3x^2-x+2\right)^{3/2}dx+\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}-\frac{1}{42}\int(412-1755x)x\left(3x^2-x+2\right)^{3/2}dx\right)+\frac{1}{3}\left(3x^2-x+2\right)^{5/2}x^3\right)+$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{1225}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\int\left(3x^2-x+2\right)^{3/2}dx-\frac{1}{60}(283-5850x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{83}{21}x^2\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{3}\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\int\sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{60}(283-5850x)\left(3x^2-x+2\right)^{5}\right.\right.\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)\right.\right.\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$$\Big\downarrow \text{1090}$$

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\right.\right.\right.\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

$\downarrow$ 222

$$\frac{77}{27}\left(\frac{1}{6}\left(\frac{1}{42}\left(-\frac{4963}{24}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\right.\right.\right.$$
$$\frac{2}{27}\left(3x^2-x+2\right)^{5/2}(2x+1)^4$$

input
```
Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(5/2))/27 + (77*((x^3*(2 - x + 3*x^2)^(5/2)
)/3 + ((83*x^2*(2 - x + 3*x^2)^(5/2))/21 + (-1/60*((283 - 5850*x)*(2 - x +
 3*x^2)^(5/2)) - (4963*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/
12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24
*Sqrt[3])))/16))/24)/42)/6))/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1267
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_
) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g^n*(d + e*x)^(m + n - 1)*((a + b
*x + c*x^2)^(p + 1)/(c*e^(n - 1)*(m + n + 2*p + 1))), x] + Simp[1/(c*e^n*(m
 + n + 2*p + 1))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p*ExpandToSum[c*e^n*(m
 + n + 2*p + 1)*(f + g*x)^n - c*g^n*(m + n + 2*p + 1)*(d + e*x)^n - g^n*(d
+ e*x)^(n - 2)*(b*d*e*(p + 1) + a*e^2*(m + n - 1) - c*d^2*(m + n + 2*p + 1)
 - e*(2*c*d - b*e)*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && IGtQ[n, 1] && IntegerQ[m] && NeQ[m + n + 2*p + 1, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(238878720x^8+510105600x^7+635765760x^6+711210240x^5+649452672x^4+421626672x^3+201289704x^2+84014278x+12499587)\sqrt{}}{22394880}$
trager	$\left(\frac{32}{3}x^8+\frac{205}{9}x^7+\frac{511}{18}x^6+\frac{20579}{648}x^5+\frac{563761}{19440}x^4+\frac{2927963}{155520}x^3+\frac{8387071}{933120}x^2+\frac{42007139}{11197440}x+\frac{1388843}{2488320}\right)\sqrt{3x^2}$
default	$-\frac{54593(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{559872}-\frac{1255639(6x-1)\sqrt{3x^2-x+2}}{4478976}-\frac{28879697\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+\frac{1207(3x^2-x+2)^{\frac{5}{2}}}{58320}+$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/22394880*(238878720*x^8+510105600*x^7+635765760*x^6+711210240*x^5+649452
672*x^4+421626672*x^3+201289704*x^2+84014278*x+12499587)*(3*x^2-x+2)^(1/2)
-28879697/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx=\frac{1}{22394880}\left(238878720\,x^8+510105600\,x^7+635765760\,x^6+711210240\,x^5+649452672\,x^4+4\right.$$
$$+\frac{28879697}{53747712}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/22394880*(238878720*x^8 + 510105600*x^7 + 635765760*x^6 + 711210240*x^5
+ 649452672*x^4 + 421626672*x^3 + 201289704*x^2 + 84014278*x + 12499587)*s
qrt(3*x^2 - x + 2) + 28879697/53747712*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 -
x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{32x^8}{3}+\frac{205x^7}{9}+\frac{511x^6}{18}+\frac{20579x^5}{648}+\frac{563761x^4}{19440}+\frac{2927963x^3}{155520}+\frac{8387071x^2}{933120}+\frac{42007139x}{11197440}+\frac{1388843}{2488320}\right)-\frac{28879697\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

input
```
integrate((1+2*x)**3*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**8/3 + 205*x**7/9 + 511*x**6/18 + 20579*x**5/64
8 + 563761*x**4/19440 + 2927963*x**3/155520 + 8387071*x**2/933120 + 420071
39*x/11197440 + 1388843/2488320) - 28879697*sqrt(3)*asinh(6*sqrt(23)*(x -
1/6)/23)/26873856
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.98

$$\int (1+2x)^3\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{32}{27}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^4+\frac{269}{81}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3+\frac{1777}{486}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{1099}{648}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x+\frac{1207}{58320}\left(3\,x^2-x+2\right)^{\frac{5}{2}}-\frac{54593}{93312}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x+\frac{54593}{559872}\left(3\,x^2-x+2\right)^{\frac{3}{2}}-\frac{1255639}{746496}\sqrt{3\,x^2-x+2}x-\frac{28879697}{26873856}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right)+\frac{1255639}{4478976}\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
32/27*(3*x^2 - x + 2)^(5/2)*x^4 + 269/81*(3*x^2 - x + 2)^(5/2)*x^3 + 1777/
486*(3*x^2 - x + 2)^(5/2)*x^2 + 1099/648*(3*x^2 - x + 2)^(5/2)*x + 1207/58
320*(3*x^2 - x + 2)^(5/2) - 54593/93312*(3*x^2 - x + 2)^(3/2)*x + 54593/55
9872*(3*x^2 - x + 2)^(3/2) - 1255639/746496*sqrt(3*x^2 - x + 2)*x - 288796
97/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 1255639/4478976*sqr
t(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \frac{1}{22394880} (2 (12 (6 (8 (30 (36 (2 (96 x+205)x+511)x+20579)x+563761)x+2927963)x+8 + \frac{28879697}{26873856} \sqrt{3} \log \left(-2 \sqrt{3}\left(\sqrt{3}x-\sqrt{3 x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/22394880*(2*(12*(6*(8*(30*(36*(2*(96*x + 205)*x + 511)*x + 20579)*x + 56
3761)*x + 2927963)*x + 8387071)*x + 42007139)*x + 12499587)*sqrt(3*x^2 - x
 + 2) + 28879697/26873856*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 -
 x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) dx = \int (2 x+1)^3 \left(3 x^2-x+2\right)^{3/2} \left(4 x^2+3 x+1\right) dx$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{32\sqrt{3x^2-x+2}\,x^8}{3} + \frac{205\sqrt{3x^2-x+2}\,x^7}{9} + \frac{511\sqrt{3x^2-x+2}\,x^6}{18} + \frac{20579\sqrt{3x^2-x+2}\,x^5}{648} + \frac{563761\sqrt{3x^2-x+2}\,x^4}{19440} + \frac{2927963\sqrt{3x^2-x+2}\,x^3}{155520} + \frac{8387071\sqrt{3x^2-x+2}\,x^2}{933120} + \frac{42007139\sqrt{3x^2-x+2}\,x}{11197440} + \frac{1388843\sqrt{3x^2-x+2}}{2488320} - \frac{28879697\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{26873856}$$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

output
```
(1433272320*sqrt(3*x**2 - x + 2)*x**8 + 3060633600*sqrt(3*x**2 - x + 2)*x*
*7 + 3814594560*sqrt(3*x**2 - x + 2)*x**6 + 4267261440*sqrt(3*x**2 - x + 2
)*x**5 + 3896716032*sqrt(3*x**2 - x + 2)*x**4 + 2529760032*sqrt(3*x**2 - x
 + 2)*x**3 + 1207738224*sqrt(3*x**2 - x + 2)*x**2 + 504085668*sqrt(3*x**2
- x + 2)*x + 74997522*sqrt(3*x**2 - x + 2) - 144398485*sqrt(3)*log((2*sqrt
(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/134369280
```

3.52 $\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 141

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{2093(1-6x)\sqrt{2-x+3x^2}}{27648} + \frac{91(1-6x)\left(2-x+3x^2\right)^{3/2}}{3456} + \frac{8}{63}(1+2x)^2\left(2-x+3x^2\right)^{5/2} + \frac{1}{12}(1+2x)^3\left(2-x+3x^2\right)^{5/2} + \frac{13(29+50x)\left(2-x+3x^2\right)^{5/2}}{2520} + \frac{48139\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{55296\sqrt{3}}$$

output

```
2093/27648*(1-6*x)*(3*x^2-x+2)^(1/2)+91/3456*(1-6*x)*(3*x^2-x+2)^(3/2)+8/6
3*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/12*(1+2*x)^3*(3*x^2-x+2)^(5/2)+13/2520*(29
+50*x)*(3*x^2-x+2)^(5/2)+48139/165888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(1517367+2735918x+5694024x^2+10119792x^3+12173952x^4+10656000x^5}{5806080}$$

input
```
Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(1517367 + 2735918*x + 5694024*x^2 + 10119792*x^3 +
 12173952*x^4 + 10656000*x^5 + 9262080*x^6 + 5806080*x^7) + 1684865*Sqrt[3
]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5806080
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{3/2} \left(4x^2+3x+1\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\int 4(2x+1)^2(64x+5)\left(3x^2-x+2\right)^{3/2}dx+\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{24}\int (2x+1)^2(64x+5)\left(3x^2-x+2\right)^{3/2}dx+\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{24}\left(\frac{1}{21}\int -13(19-90x)(2x+1)\left(3x^2-x+2\right)^{3/2}dx+\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\int (19-90x)(2x+1)\left(3x^2-x+2\right)^{3/2}dx\right)+\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{5}(50x+29)\left(3x^2-x+2\right)^{5/2}\right)\right)+\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{5}(50x\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2\right.\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

$$\Big\downarrow \; 222$$

$$\frac{1}{24}\left(\frac{64}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}-\frac{13}{21}\left(\frac{49}{2}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6\right.\right.\right.$$
$$\frac{1}{12}\left(3x^2-x+2\right)^{5/2}(2x+1)^3$$

input
```
Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2))/12 + ((64*(1 + 2*x)^2*(2 - x + 3*x^2)^
(5/2))/21 - (13*(-1/5*((29 + 50*x)*(2 - x + 3*x^2)^(5/2)) + (49*(-1/24*((1
 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]
) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/16))/2))/21)/24
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.46

method	result
risch	$\frac{(5806080x^7+9262080x^6+10656000x^5+12173952x^4+10119792x^3+5694024x^2+2735918x+1517367)\sqrt{3x^2-x+2}}{967680} - \frac{48139\sqrt{3}\,\text{arc}$
trager	$\left(6x^7+\frac{67}{7}x^6+\frac{925}{84}x^5+\frac{4529}{360}x^4+\frac{210829}{20160}x^3+\frac{33893}{5760}x^2+\frac{1367959}{483840}x+\frac{505789}{322560}\right)\sqrt{3x^2-x+2}+\frac{48139\,\text{Ro}$
default	$-\frac{91(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{3456}-\frac{2093(6x-1)\sqrt{3x^2-x+2}}{27648}-\frac{48139\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}+\frac{907(3x^2-x+2)^{\frac{5}{2}}}{2520}+\frac{319x(3x^2-}{252}$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/967680*(5806080*x^7+9262080*x^6+10656000*x^5+12173952*x^4+10119792*x^3+5
694024*x^2+2735918*x+1517367)*(3*x^2-x+2)^(1/2)-48139/165888*3^(1/2)*arcsi
nh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{1}{967680} \left(5806080\, x^7 + 9262080\, x^6 + 10656000\, x^5 + 12173952\, x^4 + 10119792\, x^3 + 5694024\, x^2 \right. + \frac{48139}{331776} \sqrt{3} \log \left(4\, \sqrt{3}\sqrt{3\, x^2 - x + 2}(6\, x - 1) - 72\, x^2 + 24\, x - 25\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/967680*(5806080*x^7 + 9262080*x^6 + 10656000*x^5 + 12173952*x^4 + 101197
92*x^3 + 5694024*x^2 + 2735918*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/33
1776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x -
 25)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.58

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \sqrt{3x^2-x+2} \cdot \left(6x^7 + \frac{67x^6}{7} + \frac{925x^5}{84} + \frac{4529x^4}{360} + \frac{210829x^3}{20160} + \frac{33893x^2}{5760} + \frac{1367959x}{483840} + \frac{505789}{322560}\right) - \frac{48139\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{165888}$$

input
```
integrate((1+2*x)**2*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(6*x**7 + 67*x**6/7 + 925*x**5/84 + 4529*x**4/360 + 2
10829*x**3/20160 + 33893*x**2/5760 + 1367959*x/483840 + 505789/322560) - 4
8139*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/165888
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \frac{2}{3}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^3 + \frac{95}{63}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x^2 + \frac{319}{252}\left(3\,x^2-x+2\right)^{\frac{5}{2}}x + \frac{907}{2520}\left(3\,x^2-x+2\right)^{\frac{5}{2}} - \frac{91}{576}\left(3\,x^2-x+2\right)^{\frac{3}{2}}x + \frac{91}{3456}\left(3\,x^2-x+2\right)^{\frac{3}{2}} - \frac{2093}{4608}\sqrt{3\,x^2-x+2}\,x - \frac{48139}{165888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) + \frac{2093}{27648}\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
2/3*(3*x^2 - x + 2)^(5/2)*x^3 + 95/63*(3*x^2 - x + 2)^(5/2)*x^2 + 319/252*
(3*x^2 - x + 2)^(5/2)*x + 907/2520*(3*x^2 - x + 2)^(5/2) - 91/576*(3*x^2 -
 x + 2)^(3/2)*x + 91/3456*(3*x^2 - x + 2)^(3/2) - 2093/4608*sqrt(3*x^2 - x
 + 2)*x - 48139/165888*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2093/276
48*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \frac{1}{967680}\left(2\left(12\left(2\left(8\left(30\left(12\left(42\,x+67\right)x+925\right)x+31703\right)x+210829\right)x+237251\right)x+1367959 \right. + \frac{48139}{165888}\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/967680*(2*(12*(2*(8*(30*(12*(42*x + 67)*x + 925)*x + 31703)*x + 210829)*
x + 237251)*x + 1367959)*x + 1517367)*sqrt(3*x^2 - x + 2) + 48139/165888*s
qrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = \int (2\,x+1)^2\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.09

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right)\,dx = 6\sqrt{3x^2-x+2}\,x^7 + \frac{67\sqrt{3x^2-x+2}\,x^6}{7} + \frac{925\sqrt{3x^2-x+2}\,x^5}{84} + \frac{4529\sqrt{3x^2-x+2}\,x^4}{360} + \frac{210829\sqrt{3x^2-x+2}\,x^3}{20160} + \frac{33893\sqrt{3x^2-x+2}\,x^2}{5760} + \frac{1367959\sqrt{3x^2-x+2}\,x}{483840} + \frac{505789\sqrt{3x^2-x+2}}{322560} - \frac{48139\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{165888}$$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

output
```
(34836480*sqrt(3*x**2 - x + 2)*x**7 + 55572480*sqrt(3*x**2 - x + 2)*x**6 +
 63936000*sqrt(3*x**2 - x + 2)*x**5 + 73043712*sqrt(3*x**2 - x + 2)*x**4 +
 60718752*sqrt(3*x**2 - x + 2)*x**3 + 34164144*sqrt(3*x**2 - x + 2)*x**2 +
 16415508*sqrt(3*x**2 - x + 2)*x + 9104202*sqrt(3*x**2 - x + 2) - 1684865*
sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/5806080
```

3.53 $\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 30, antiderivative size = 116

$$\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = -\frac{1633(1-6x)\sqrt{2-x+3x^2}}{20736} - \frac{71(1-6x)\left(2-x+3x^2\right)^{3/2}}{2592} + \frac{2}{21}(1+2x)^2 \left(2-x+3x^2\right)^{5/2} + \frac{1}{378}(109+102x)\left(2-x+3x^2\right)^{5/2} - \frac{37559 \text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{41472\sqrt{3}}$$

output
```
-1633/20736*(1-6*x)*(3*x^2-x+2)^(1/2)-71/2592*(1-6*x)*(3*x^2-x+2)^(3/2)+2/
21*(1+2*x)^2*(3*x^2-x+2)^(5/2)+1/378*(109+102*x)*(3*x^2-x+2)^(5/2)-37559/1
24416*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int (1+2x) \left(2-x+3x^2\right)^{3/2} \left(1+3x+4x^2\right) \, dx = \frac{6\sqrt{2-x+3x^2}(203337+275410x+531384x^2+744336x^3+653184x^4+518400x^5+497664x}{870912}$$

input
```
Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(203337 + 275410*x + 531384*x^2 + 744336*x^3 + 6531
84*x^4 + 518400*x^5 + 497664*x^6) - 262913*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6
- 3*x + 9*x^2]])/870912
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(3x^2-x+2\right)^{3/2}\left(4x^2+3x+1\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{84}\int 4(2x+1)(51x+10)\left(3x^2-x+2\right)^{3/2}dx+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{21}\int (2x+1)(51x+10)\left(3x^2-x+2\right)^{3/2}dx+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{21}\left(\frac{497}{36}\int \left(3x^2-x+2\right)^{3/2}dx+\frac{1}{18}(102x+109)\left(3x^2-x+2\right)^{5/2}\right)+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x+109)\left(3x^2-x+2\right)^{5/2}\right)+\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{23}{24}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x\right.$$
$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+\right.\right.\right.$$
$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{21}\left(\frac{497}{36}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{18}(102x+\right.$$
$$\frac{2}{21}(2x+1)^2\left(3x^2-x+2\right)^{5/2}$$

input
```
Int[(1 + 2*x)*(2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2))/21 + (((109 + 102*x)*(2 - x + 3*x^2)
^(5/2))/18 + (497*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((
1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt
[3])))/16))/36)/21
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.52

method	result
risch	$\frac{(497664x^6+518400x^5+653184x^4+744336x^3+531384x^2+275410x+203337)\sqrt{3x^2-x+2}}{145152}+\frac{37559\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}$
trager	$\left(\frac{24}{7}x^6+\frac{25}{7}x^5+\frac{9}{2}x^4+\frac{1723}{336}x^3+\frac{3163}{864}x^2+\frac{137705}{72576}x+\frac{7531}{5376}\right)\sqrt{3x^2-x+2}+\frac{37559\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln(\ldots}{}$
default	$\frac{71(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{2592}+\frac{1633(6x-1)\sqrt{3x^2-x+2}}{20736}+\frac{37559\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}+\frac{145(3x^2-x+2)^{\frac{5}{2}}}{378}+\frac{41x(3x^2-x+\ldots}{63}$

input
```
int((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/145152*(497664*x^6+518400*x^5+653184*x^4+744336*x^3+531384*x^2+275410*x+
203337)*(3*x^2-x+2)^(1/2)+37559/124416*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/
6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)dx=\frac{1}{145152}\left(497664\,x^6+518400\,x^5+653184\,x^4+744336\,x^3+531384\,x^2+275410\,x+203337\right)\sqrt{\ldots}+\frac{37559}{248832}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/145152*(497664*x^6 + 518400*x^5 + 653184*x^4 + 744336*x^3 + 531384*x^2 +
 275410*x + 203337)*sqrt(3*x^2 - x + 2) + 37559/248832*sqrt(3)*log(-4*sqrt
(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(\frac{24x^6}{7}+\frac{25x^5}{7}+\frac{9x^4}{2}+\frac{1723x^3}{336}+\frac{3163x^2}{864}+\frac{137705x}{72576}+\frac{7531}{5376}\right)+\frac{37559\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{124416}$$

input
```
integrate((1+2*x)*(3*x**2-x+2)**(3/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(24*x**6/7 + 25*x**5/7 + 9*x**4/2 + 1723*x**3/336 + 3
163*x**2/864 + 137705*x/72576 + 7531/5376) + 37559*sqrt(3)*asinh(6*sqrt(23
)*(x - 1/6)/23)/124416
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{8}{21}\left(3x^2-x+2\right)^{\frac{5}{2}}x^2+\frac{41}{63}\left(3x^2-x+2\right)^{\frac{5}{2}}x+\frac{145}{378}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{71}{432}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{71}{2592}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{1633}{3456}\sqrt{3x^2-x+2}x+\frac{37559}{124416}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{1633}{20736}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/21*(3*x^2 - x + 2)^(5/2)*x^2 + 41/63*(3*x^2 - x + 2)^(5/2)*x + 145/378*(
3*x^2 - x + 2)^(5/2) + 71/432*(3*x^2 - x + 2)^(3/2)*x - 71/2592*(3*x^2 - x
 + 2)^(3/2) + 1633/3456*sqrt(3*x^2 - x + 2)*x + 37559/124416*sqrt(3)*arcsi
nh(1/23*sqrt(23)*(6*x - 1)) - 1633/20736*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{145152}\,(2\,(12\,(18\,(24\,(2\,(24\,x+25)x+63)x+1723)x+22141)x+137705)x+203337)\sqrt{3\,x^2-x+2}-\frac{37559}{124416}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/145152*(2*(12*(18*(24*(2*(24*x + 25)*x + 63)*x + 1723)*x + 22141)*x + 13
7705)*x + 203337)*sqrt(3*x^2 - x + 2) - 37559/124416*sqrt(3)*log(-2*sqrt(3
)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx=\int (2\,x+1)\,\left(3\,x^2-x+2\right)^{3/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)*(3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.19

$$\int (1+2x)\left(2-x+3x^2\right)^{3/2}\left(1+3x+4x^2\right)\,dx = \frac{24\sqrt{3x^2-x+2}\,x^6}{7} + \frac{25\sqrt{3x^2-x+2}\,x^5}{7} + \frac{9\sqrt{3x^2-x+2}\,x^4}{2} + \frac{1723\sqrt{3x^2-x+2}\,x^3}{336} + \frac{3163\sqrt{3x^2-x+2}\,x^2}{864} + \frac{137705\sqrt{3x^2-x+2}\,x}{72576} + \frac{7531\sqrt{3x^2-x+2}}{5376} + \frac{37559\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{124416}$$

input
```
int((1+2*x)*(3*x^2-x+2)^(3/2)*(4*x^2+3*x+1),x)
```

output
```
(2985984*sqrt(3*x**2 - x + 2)*x**6 + 3110400*sqrt(3*x**2 - x + 2)*x**5 + 3
919104*sqrt(3*x**2 - x + 2)*x**4 + 4466016*sqrt(3*x**2 - x + 2)*x**3 + 318
8304*sqrt(3*x**2 - x + 2)*x**2 + 1652460*sqrt(3*x**2 - x + 2)*x + 1220022*
sqrt(3*x**2 - x + 2) + 262913*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3)
+ 6*x - 1)/sqrt(23)))/870912
```

3.54 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 124

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{(869+402x)\sqrt{2-x+3x^2}}{1152} + \frac{1}{144}(7+30x)\left(2-x+3x^2\right)^{3/2} + \frac{2}{15}\left(2-x+3x^2\right)^{5/2} + \frac{2203\,\mathrm{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{2304\sqrt{3}} - \frac{13}{32}\sqrt{13}\,\mathrm{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/1152*(869+402*x)*(3*x^2-x+2)^(1/2)+1/144*(7+30*x)*(3*x^2-x+2)^(3/2)+2/15
*(3*x^2-x+2)^(5/2)+2203/6912*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-13/32*
13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(7977+1058x+9624x^2-1008x^3+6912x^4)+28$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(7977 + 1058*x + 9624*x^2 - 1008*x^3 + 6912*x^4) +
28080*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqr
t[13]] + 11015*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/34560
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{2x+1}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{60}\int \frac{20(5x+4)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{(5x+4)(3x^2-x+2)^{3/2}}{2x+1}dx+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{1}{48}(30x+7)(3x^2-x+2)^{3/2}-\frac{1}{96}\int -\frac{3(134x+223)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{32}\int \frac{(134x+223)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{48}(30x+7)(3x^2-x+2)^{3/2}\right)+\frac{2}{15}(3x^2-x+2)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{12}(402x+869)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(9965-4406x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{48}(30x+7)\left(3x^2-x+2\right)^{3/2}\right)+\frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\int \frac{9965-4406x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)+\frac{1}{48}(30x+7)\left(3x^2-x+2\right)^{3/2}\right)+\frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 1269

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-2203\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)\right. \quad \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 1090

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2203\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+\right.\right. \quad \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 222

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(12168\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right)+\right. \quad \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 1154

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(-24336\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right.\right. \quad \frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{32}\left(\frac{1}{24}\left(-\frac{2203\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-936\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(402x+869)\right.\right.$$

$$\frac{2}{15}\left(3x^2-x+2\right)^{5/2}$$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(2*(2 - x + 3*x^2)^(5/2))/15 + (((7 + 30*x)*(2 - x + 3*x^2)^(3/2))/48 + ((
(869 + 402*x)*Sqrt[2 - x + 3*x^2])/12 + ((-2203*ArcSinh[(-1 + 6*x)/Sqrt[23
]])/Sqrt[3] - 936*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^
2])])/24)/32)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.65

method	result
risch	$\frac{(6912x^4-1008x^3+9624x^2+1058x+7977)\sqrt{3x^2-x+2}}{5760}-\frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912}-\frac{13\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-}}\right.}{32}$
trager	$\left(\frac{6}{5}x^4-\frac{7}{40}x^3+\frac{401}{240}x^2+\frac{529}{2880}x+\frac{2659}{1920}\right)\sqrt{3x^2-x+2}+\frac{2203\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\right.}{6912}$
default	$\frac{5(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{144}+\frac{115(6x-1)\sqrt{3x^2-x+2}}{1152}-\frac{2203\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{6912}+\frac{2(3x^2-x+2)^{\frac{5}{2}}}{15}+\frac{\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right.}{12}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/5760*(6912*x^4-1008*x^3+9624*x^2+1058*x+7977)*(3*x^2-x+2)^(1/2)-2203/691
2*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-13/32*13^(1/2)*arctanh(2/13*(9/2-
4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}\left(6912\,x^4-1008\,x^3+9624\,x^2+1058\,x+7977\right)\sqrt{3\,x^2-x+}$$
$$+\frac{2203}{13824}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$
$$+\frac{13}{64}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

output
```
1/5760*(6912*x^4 - 1008*x^3 + 9624*x^2 + 1058*x + 7977)*sqrt(3*x^2 - x + 2
) + 2203/13824*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^
2 + 24*x - 25) + 13/64*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x
- 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx=\int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx=\frac{2}{15}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{5}{24}\left(3x^2-x+2\right)^{\frac{3}{2}}x+\frac{7}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{67}{192}\sqrt{3x^2-x+2}x-\frac{2203}{6912}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)+\frac{13}{32}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|}-\frac{9\sqrt{23}}{23\,|2x+1|}\right)+\frac{869}{1152}\sqrt{3x^2-x+2}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

output
```
2/15*(3*x^2 - x + 2)^(5/2) + 5/24*(3*x^2 - x + 2)^(3/2)*x + 7/144*(3*x^2 -
 x + 2)^(3/2) + 67/192*sqrt(3*x^2 - x + 2)*x - 2203/6912*sqrt(3)*arcsinh(6
/23*sqrt(23)*x - 1/23*sqrt(23)) + 13/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/a
bs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 869/1152*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{5760}\left(2\left(12\left(6\left(48\,x-7\right)x+401\right)x+529\right)x+7977\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{2203}{6912}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right)$$
$$+\frac{13}{32}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

output
```
1/5760*(2*(12*(6*(48*x - 7)*x + 401)*x + 529)*x + 7977)*sqrt(3*x^2 - x + 2
) + 2203/6912*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2))
+ 13/32*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sq
rt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3\,x^2-x+2)^{3/2}\,(4\,x^2+3\,x+1)}{2\,x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.77

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{13\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6ix-i}{\sqrt{39}-4}\right)i}{32} + \frac{6\sqrt{3x^2-x+2}\,x^4}{5} - \frac{7\sqrt{3x^2-x+2}\,x^3}{40} + \frac{401\sqrt{3x^2-x+2}\,x^2}{240} + \frac{529\sqrt{3x^2-x+2}\,x}{2880} + \frac{2659\sqrt{3x^2-x+2}}{1920} + \frac{13\sqrt{13}\log\left(24\sqrt{3x^2-x+2}\,\sqrt{3}\,x-4\sqrt{3x^2-x+2}\,\sqrt{3}+8\sqrt{39}+72x^2-24x-30\right)}{64} - \frac{13\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{3}+\sqrt{39}+6x+3\right)}{32} - \frac{2203\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{6912}$$

input

```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x),x)
```

output

```
(14040*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(
39) - 4))*i + 41472*sqrt(3*x**2 - x + 2)*x**4 - 6048*sqrt(3*x**2 - x + 2)*
x**3 + 57744*sqrt(3*x**2 - x + 2)*x**2 + 6348*sqrt(3*x**2 - x + 2)*x + 478
62*sqrt(3*x**2 - x + 2) + 7020*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3
)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30) -
 14040*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) + sqrt(39) + 6*x + 3) -
 11015*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/3
4560
```

3.55 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 131

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2} dx = -\frac{1}{192}(349-294x)\sqrt{2-x+3x^2}-\frac{1}{104}(23-38x)\left(2-x+3x^2\right)^{3/2} -\frac{(2-x+3x^2)^{5/2}}{13(1+2x)}-\frac{2327\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{384\sqrt{3}}+\frac{25}{32}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
-1/192*(349-294*x)*(3*x^2-x+2)^(1/2)-1/104*(23-38*x)*(3*x^2-x+2)^(3/2)-(3*
x^2-x+2)^(5/2)/(13+26*x)-2327/1152*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+
25/32*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}(-493-332x+564x^2-96x^3+288x^4)}{1+2x} - 1800\sqrt{13}\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x\right.}{1152}$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(-493 - 332*x + 564*x^2 - 96*x^3 + 288*x^4))/(1 +
2*x) - 1800*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2
])/Sqrt[13]] - 2327*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1152
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{(76x+13)(3x^2-x+2)^{3/2}}{2(2x+1)}dx - \frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\int \frac{(76x+13)(3x^2-x+2)^{3/2}}{2x+1}dx - \frac{(3x^2-x+2)^{5/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{26}\left(-\frac{1}{96}\int \frac{156(1-98x)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{1}{4}(23-38x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 27

$$\frac{1}{26}\left(-\frac{13}{8}\int \frac{(1-98x)\sqrt{3x^2-x+2}}{2x+1}dx-\frac{1}{4}(23-38x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 1231

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{1}{12}(349-294x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{26(121-358x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{1}{4}(23-38x)\left(3x^2-x+2\right)^{3/2}\right)$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 27

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\int \frac{121-358x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}(23-38x)\left(3x^2-x+2\right)^{3/2}\right)-$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 1269

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-179\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 1090

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{179\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-29\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 222

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(300\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 1154

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(-600\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

$\big\downarrow$ 219

$$\frac{1}{26}\left(-\frac{13}{8}\left(\frac{13}{24}\left(-\frac{179\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-\frac{300\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(349-294x)\right)-\frac{1}{4}(2\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{13(2x+1)}$$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(5/2)/(1 + 2*x) + (-1/4*((23 - 38*x)*(2 - x + 3*x^2)
^(3/2)) - (13*(((349 - 294*x)*Sqrt[2 - x + 3*x^2])/12 + (13*((-179*ArcSinh
[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - (300*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2
 - x + 3*x^2])])/Sqrt[13]))/24))/8)/26
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.74

method	result
risch	$\frac{864x^6-576x^5+2364x^4-1752x^3-19x^2-171x-986}{192(1+2x)\sqrt{3x^2-x+2}}+\frac{2327\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1152}+\frac{25\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16}}\right.}{32}$
trager	$\frac{\left(288x^4-96x^3+564x^2-332x-493\right)\sqrt{3x^2-x+2}}{192+384x}-\frac{2327\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_\right.\right.}{1152}$
default	$\frac{(6x-1)\left(3x^2-x+2\right)^{\frac{3}{2}}}{24}+\frac{23(6x-1)\sqrt{3x^2-x+2}}{192}+\frac{2327\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1152}-\frac{25\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}{156}+\frac{13(6x-1)\sqrt{}}{}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/192*(864*x^6-576*x^5+2364*x^4-1752*x^3-19*x^2-171*x-986)/(1+2*x)/(3*x^2-
x+2)^(1/2)+2327/1152*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+25/32*13^(1/2)
*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{2327\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24\right.}{}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/2304*(2327*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1
) - 72*x^2 + 24*x - 25) + 900*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^
2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 12*(288
*x^4 - 96*x^3 + 564*x^2 - 332*x - 493)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{4}(3x^2-x+2)^{\frac{3}{2}}x - \frac{1}{8}(3x^2-x+2)^{\frac{3}{2}} \\ &+ \frac{49}{32}\sqrt{3x^2-x+2}x + \frac{2327}{1152}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{25}{32}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) \\ &- \frac{349}{192}\sqrt{3x^2-x+2} - \frac{(3x^2-x+2)^{\frac{3}{2}}}{4(2x+1)}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/4*(3*x^2 - x + 2)^(3/2)*x - 1/8*(3*x^2 - x + 2)^(3/2) + 49/32*sqrt(3*x^2
 - x + 2)*x + 2327/1152*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) -
 25/32*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) - 349/192*sqrt(3*x^2 - x + 2) - 1/4*(3*x^2 - x + 2)^(3/2)/(2*x +
1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(104) = 208$.

Time = 0.45 (sec) , antiderivative size = 570, normalized size of antiderivative = 4.35

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

output
```
25/32*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 2327/1152*sqrt(3)*log(1/2*abs(-2*
sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1)
)/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))
)*sgn(1/(2*x + 1)) - 13/32*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*sgn(1/(
2*x + 1)) + 1/192*(5929*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13
)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 7272*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2
*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^6*sgn(1/(2*x + 1)) + 25101*(sqrt(-8/(
2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) -
48*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))
^4*sgn(1/(2*x + 1)) + 112359*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sq
rt(13)/(2*x + 1))^3*sgn(1/(2*x + 1)) - 69336*sqrt(13)*(sqrt(-8/(2*x + 1) +
 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 71955*(sqr
t(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)
) + 24624*sqrt(13)*sgn(1/(2*x + 1)))/((sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^2 - 3)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3\,x^2-x+2)^{3/2}\,(4\,x^2+3\,x+1)}{(2\,x+1)^2}\,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.58

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{1728\sqrt{3x^2-x+2}\,x^4-576\sqrt{3x^2-x+2}\,x^3+3384\sqrt{3x^2-x+2}$$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^2,x)
```

output
```
(1728*sqrt(3*x**2 - x + 2)*x**4 - 576*sqrt(3*x**2 - x + 2)*x**3 + 3384*sqr
t(3*x**2 - x + 2)*x**2 - 1992*sqrt(3*x**2 - x + 2)*x - 2958*sqrt(3*x**2 -
x + 2) + 1800*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x
 + 900*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 1800*s
qrt(13)*log(2*x + 1)*x - 900*sqrt(13)*log(2*x + 1) + 4654*sqrt(3)*log( - 2
*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 2327*sqrt(3)*log( - 2*sqrt(3*
x**2 - x + 2)*sqrt(3) - 6*x + 1))/(1152*(2*x + 1))
```

3.56 $\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 138

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{624}(1858-771x)\sqrt{2-x+3x^2} + \frac{(151+122x)(2-x+3x^2)^{3/2}}{312(1+2x)} - \frac{(2-x+3x^2)^{5/2}}{26(1+2x)^2} + \frac{1519\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{192\sqrt{3}} - \frac{1153\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{64\sqrt{13}}$$

output
```
1/624*(1858-771*x)*(3*x^2-x+2)^(1/2)+(151+122*x)*(3*x^2-x+2)^(3/2)/(312+62
4*x)-1/26*(3*x^2-x+2)^(5/2)/(1+2*x)^2+1519/576*arcsinh(1/23*(1-6*x)*23^(1/
2))*3^(1/2)-1153/832*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1
/2))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{\frac{156\sqrt{2-x+3x^2}(182+627x+390x^2-68x^3+96x^4)}{(1+2x)^2}+20754\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}\right.}{7488}$$

input
```
Integrate[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((156*Sqrt[2 - x + 3*x^2]*(182 + 627*x + 390*x^2 - 68*x^3 + 96*x^4))/(1 +
2*x)^2 + 20754*Sqrt[13]*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*
x^2])/Sqrt[13]] + 19747*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/74
88
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2181, 27, 1230, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{3/2}(4x^2+3x+1)}{(2x+1)^3}dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{1}{26}\int -\frac{(122x+31)(3x^2-x+2)^{3/2}}{2(2x+1)^2}dx-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{52}\int \frac{(122x+31)(3x^2-x+2)^{3/2}}{(2x+1)^2}dx-\frac{(3x^2-x+2)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \; 1230$$

$$\frac{1}{52}\left(\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}-\frac{1}{8}\int -\frac{2(639-1028x)\sqrt{3x^2-x+2}}{2x+1}dx\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{52}\left(\frac{1}{4}\int \frac{(639-1028x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{1}{3}(1858-771x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{104(970-1519x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\int \frac{970-1519x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(122x+151)\left(3x^2-x+2\right)^{3/2}}{6(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519}{2}\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)-\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{2\sqrt{69}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-77\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 222

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(\frac{3459}{2}\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(1\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 1154

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(-3459\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

$\Big\downarrow$ 219

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{6}\left(-\frac{1519\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-\frac{3459\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}\right)+\frac{1}{3}\sqrt{3x^2-x+2}(1858-771x)\right)+\frac{(12\right.$$

$$\frac{\left(3x^2-x+2\right)^{5/2}}{26(2x+1)^2}$$

input
```
Int[((2 - x + 3*x^2)^(3/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(5/2)/(1 + 2*x)^2 + (((151 + 122*x)*(2 - x + 3*x^2)^
(3/2))/(6*(1 + 2*x)) + (((1858 - 771*x)*Sqrt[2 - x + 3*x^2])/3 + (13*((-15
19*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(2*Sqrt[3]) - (3459*ArcTanh[(9 - 8*x)/(2*
Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13])))/6)/4)/52
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.70

method	result
risch	$\frac{288x^6-300x^5+1430x^4+1355x^3+699x^2+1072x+364}{48(1+2x)^2\sqrt{3x^2-x+2}} - \frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576} - \frac{1153\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+}}\right.}{832}$
trager	$\frac{(96x^4-68x^3+390x^2+627x+182)\sqrt{3x^2-x+2}}{48(1+2x)^2} + \frac{1153\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\mathrm{RootOf}\left(\_\right.}{1+2x}\right.}{832}$
default	$\frac{1153\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}{4056} - \frac{257(6x-1)\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}{1248} - \frac{1519\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{576} + \frac{1153\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}{832}$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/48*(288*x^6-300*x^5+1430*x^4+1355*x^3+699*x^2+1072*x+364)/(1+2*x)^2/(3*x
^2-x+2)^(1/2)-1519/576*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1153/832*13^
(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{19747\sqrt{3}(4x^2+4x+1)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2\right.}{}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/14976*(19747*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)
*(6*x - 1) - 72*x^2 + 24*x - 25) + 10377*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(
4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 +
 4*x + 1)) + 312*(96*x^4 - 68*x^3 + 390*x^2 + 627*x + 182)*sqrt(3*x^2 - x
+ 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{\frac{3}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((3*x**2-x+2)**(3/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

output
```
Integral((3*x**2 - x + 2)**(3/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx &= \frac{61}{312}\,(3\,x^2-x+2)^{\frac{3}{2}} - \frac{(3\,x^2-x+2)^{\frac{5}{2}}}{26\,(4\,x^2+4\,x+1)} \\ &\quad - \frac{257}{208}\,\sqrt{3\,x^2-x+2}x - \frac{1519}{576}\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\,\sqrt{23}x - \frac{1}{23}\,\sqrt{23}\right) \\ &\quad + \frac{1153}{832}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right) \\ &\quad + \frac{929}{312}\,\sqrt{3\,x^2-x+2} + \frac{15\,(3\,x^2-x+2)^{\frac{3}{2}}}{52\,(2\,x+1)}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
61/312*(3*x^2 - x + 2)^(3/2) - 1/26*(3*x^2 - x + 2)^(5/2)/(4*x^2 + 4*x + 1
) - 257/208*sqrt(3*x^2 - x + 2)*x - 1519/576*sqrt(3)*arcsinh(6/23*sqrt(23)
*x - 1/23*sqrt(23)) + 1153/832*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x +
1) - 9/23*sqrt(23)/abs(2*x + 1)) + 929/312*sqrt(3*x^2 - x + 2) + 15/52*(3*
x^2 - x + 2)^(3/2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(111) = 222$.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.89

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{96}\left(2\left(24\,x-41\right)x+265\right)\sqrt{3\,x^2-x+2}$$
$$+\frac{1519}{576}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$
$$+\frac{1153}{832}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right)$$
$$+\frac{446\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3-85\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1993\sqrt{3}x+1009\sqrt{3}+1993\sqrt{3\,x^2-x}}{32\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

output
```
1/96*(2*(24*x - 41)*x + 265)*sqrt(3*x^2 - x + 2) + 1519/576*sqrt(3)*log(-2
*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 1153/832*sqrt(13)*log(-1
/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*s
qrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 1/32*(446*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^3 - 85*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
))^2 - 1993*sqrt(3)*x + 1009*sqrt(3) + 1993*sqrt(3*x^2 - x + 2))/(2*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)
) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{\left(3\,x^2-x+2\right)^{3/2}\left(4\,x^2+3\,x+1\right)}{\left(2\,x+1\right)^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(3/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.04

$$\int \frac{(2-x+3x^2)^{3/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{14976\sqrt{3x^2-x+2}\,x^4 - 10608\sqrt{3x^2-x+2}\,x^3 + 60840\sqrt{3x^2-}}{}$$

input
```
int((3*x^2-x+2)^(3/2)*(4*x^2+3*x+1)/(1+2*x)^3,x)
```

output
```
(14976*sqrt(3*x**2 - x + 2)*x**4 - 10608*sqrt(3*x**2 - x + 2)*x**3 + 60840
*sqrt(3*x**2 - x + 2)*x**2 + 97812*sqrt(3*x**2 - x + 2)*x + 28392*sqrt(3*x
**2 - x + 2) + 41508*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x -
9)*x**2 + 41508*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x
+ 10377*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 41508*sq
rt(13)*log(2*x + 1)*x**2 - 41508*sqrt(13)*log(2*x + 1)*x - 10377*sqrt(13)*
log(2*x + 1) + 78988*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)
*x**2 + 78988*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 19
747*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1))/(7488*(4*x**2 +
 4*x + 1))
```

3.57 $\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx$

Optimal result

Integrand size = 32, antiderivative size = 189

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{2692081(1-6x)\sqrt{2-x+3x^2}}{11943936} + \frac{117047(1-6x)\left(2-x+3x^2\right)^{3/2}}{1492992} + \frac{5089(1-6x)\left(2-x+3x^2\right)^{5/2}}{155520} - \frac{(26353-21350x)\left(2-x+3x^2\right)^{7/2}}{498960} + \frac{133(1+2x)^2\left(2-x+3x^2\right)^{7/2}}{1485} + \frac{29}{330}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{2}{33}(1+2x)^4\left(2-x+3x^2\right)^{7/2} + \frac{61917863\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{23887872\sqrt{3}}$$

output
```
2692081/11943936*(1-6*x)*(3*x^2-x+2)^(1/2)+117047/1492992*(1-6*x)*(3*x^2-x
+2)^(3/2)+5089/155520*(1-6*x)*(3*x^2-x+2)^(5/2)-1/498960*(26353-21350*x)*(
3*x^2-x+2)^(7/2)+133/1485*(1+2*x)^2*(3*x^2-x+2)^(7/2)+29/330*(1+2*x)^3*(3*
x^2-x+2)^(7/2)+2/33*(1+2*x)^4*(3*x^2-x+2)^(7/2)+61917863/71663616*arcsinh(
1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.53

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \frac{6\sqrt{2-x+3x^2}(9173509857+26646633218x+72088585464x^2+161269204752x^3+2636361}{}$$

input
```
Integrate[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(9173509857 + 26646633218*x + 72088585464*x^2 + 161
269204752*x^3 + 263636134272*x^4 + 347247744768*x^5 + 415908006912*x^6 + 4
19978151936*x^7 + 308846297088*x^8 + 207681159168*x^9 + 120394874880*x^10)
 + 23838377255*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/27590492160
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2184, 27, 1236, 27, 1236, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right)\,dx$$

$$\Big\downarrow \ 2184$$

$$\frac{1}{132}\int 4(2x+1)^3(87x+8)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 27$$

$$\frac{1}{33}\int (2x+1)^3(87x+8)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1236$$

$$\frac{1}{33}\left(\frac{1}{30}\int -\frac{9}{2}(111-532x)(2x+1)^2\left(3x^2-x+2\right)^{5/2}dx+\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}\right)+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 27$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\int (111-532x)(2x+1)^2\left(3x^2-x+2\right)^{5/2}dx\right)+\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1236$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\int (5391-3050x)(2x+1)\left(3x^2-x+2\right)^{5/2}dx-\frac{532}{27}(2x+1)^2\left(3x^2\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1225$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\int \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{84}(26353-21350x)\left(3x^2-x+2\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3}\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1087$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x}\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

$$\Big\downarrow 222$$

$$\frac{1}{33}\left(\frac{29}{10}(2x+1)^3\left(3x^2-x+2\right)^{7/2}-\frac{3}{20}\left(\frac{1}{27}\left(\frac{55979}{8}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right.\right.\right.\right.\right.\right.$$
$$\frac{2}{33}\left(3x^2-x+2\right)^{7/2}(2x+1)^4$$

input
```
Int[(1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^4*(2 - x + 3*x^2)^(7/2))/33 + ((29*(1 + 2*x)^3*(2 - x + 3*x^2
)^(7/2))/10 - (3*((-532*(1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + (((26353 -
 21350*x)*(2 - x + 3*x^2)^(7/2))/84 + (55979*(-1/36*((1 - 6*x)*(2 - x + 3*
x^2)^(5/2)) + (115*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*(
(1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqr
t[3])))/16))/72))/8)/27))/20)/33
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(120394874880x^{10}+207681159168x^9+308846297088x^8+419978151936x^7+415908006912x^6+347247744768x^5+263636134272x^4 \ldots}{4598415360}$
trager	$\left(\frac{288}{11}x^{10}+\frac{2484}{55}x^9+\frac{3694}{55}x^8+\frac{120557}{1320}x^7+\frac{557147}{6160}x^6+\frac{50238389}{665280}x^5+\frac{32692973}{570240}x^4+\frac{1119925033}{31933440}x^3+\frac{429098\ldots}{273715\ldots}\right.$
default	$-\frac{5089(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{155520}-\frac{117047(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{1492992}-\frac{2692081(6x-1)\sqrt{3x^2-x+2}}{11943936}-\frac{61917863\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\,(x-\ldots}{23}\right.}{71663616}$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/4598415360*(120394874880*x^10+207681159168*x^9+308846297088*x^8+41997815
1936*x^7+415908006912*x^6+347247744768*x^5+263636134272*x^4+161269204752*x
^3+72088585464*x^2+26646633218*x+9173509857)*(3*x^2-x+2)^(1/2)-61917863/71
663616*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.54

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{1}{4598415360}\left(120394874880\,x^{10}+207681159168\,x^9+308846297088\,x^8+419978151936\,x^7+\ldots\right. + \frac{61917863}{143327232}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/4598415360*(120394874880*x^10 + 207681159168*x^9 + 308846297088*x^8 + 41
9978151936*x^7 + 415908006912*x^6 + 347247744768*x^5 + 263636134272*x^4 +
161269204752*x^3 + 72088585464*x^2 + 26646633218*x + 9173509857)*sqrt(3*x^
2 - x + 2) + 61917863/143327232*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.55

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2} \cdot \left(\frac{288x^{10}}{11}+\frac{2484x^9}{55}+\frac{3694x^8}{55}+\frac{120557x^7}{1320}+\frac{557147x^6}{6160}+\frac{50238389x^5}{665280}+\frac{32692973x^4}{570240}+\frac{1119925033x^3}{31933440}+\frac{429098723x^2}{27371520}+\frac{13323316609x}{2299207680}+\frac{1019278873}{510935040}\right) - \frac{61917863\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{71663616}$$

input
```
integrate((1+2*x)**3*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(288*x**10/11 + 2484*x**9/55 + 3694*x**8/55 + 120557*
x**7/1320 + 557147*x**6/6160 + 50238389*x**5/665280 + 32692973*x**4/570240
 + 1119925033*x**3/31933440 + 429098723*x**2/27371520 + 13323316609*x/2299
207680 + 1019278873/510935040) - 61917863*sqrt(3)*asinh(6*sqrt(23)*(x - 1/
6)/23)/71663616
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.97

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \frac{32}{33}\left(3x^2-x+2\right)^{\frac{7}{2}}x^4 + \frac{436}{165}\left(3x^2-x+2\right)^{\frac{7}{2}}x^3 + \frac{4258}{1485}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2 + \frac{10073}{7128}\left(3x^2-x+2\right)^{\frac{7}{2}}x + \frac{92423}{498960}\left(3x^2-x+2\right)^{\frac{7}{2}} - \frac{5089}{25920}\left(3x^2-x+2\right)^{\frac{5}{2}}x + \frac{5089}{155520}\left(3x^2-x+2\right)^{\frac{5}{2}} - \frac{117047}{248832}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{117047}{1492992}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{2692081}{1990656}\sqrt{3x^2-x+2}\,x - \frac{61917863}{71663616}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{2692081}{11943936}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
32/33*(3*x^2 - x + 2)^(7/2)*x^4 + 436/165*(3*x^2 - x + 2)^(7/2)*x^3 + 4258
/1485*(3*x^2 - x + 2)^(7/2)*x^2 + 10073/7128*(3*x^2 - x + 2)^(7/2)*x + 924
23/498960*(3*x^2 - x + 2)^(7/2) - 5089/25920*(3*x^2 - x + 2)^(5/2)*x + 508
9/155520*(3*x^2 - x + 2)^(5/2) - 117047/248832*(3*x^2 - x + 2)^(3/2)*x + 1
17047/1492992*(3*x^2 - x + 2)^(3/2) - 2692081/1990656*sqrt(3*x^2 - x + 2)*
x - 61917863/71663616*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 2692081/1
1943936*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int (1+2x)^3 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{1}{4598415360} \left(2 \left(12 \left(6 \left(8 \left(6 \left(36 \left(14 \left(48 \left(18 \left(40 x+69\right) x+1847\right) x+120557\right) x+1671441\right) x+50 \right.\right.\right.\right.\right.\right. + \frac{61917863}{71663616}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/4598415360*(2*(12*(6*(8*(6*(36*(14*(48*(18*(40*x + 69)*x + 1847)*x + 120
557)*x + 1671441)*x + 50238389)*x + 228850811)*x + 1119925033)*x + 3003691
061)*x + 13323316609)*x + 9173509857)*sqrt(3*x^2 - x + 2) + 61917863/71663
616*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2) \, dx = \int (2x+1)^3 (3x^2-x+2)^{5/2} (4x^2+3x+1) \, dx$$

input
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07

$$\int (1+2x)^3 (2-x+3x^2)^{5/2} (1+3x+4x^2) \, dx = \frac{288\sqrt{3x^2-x+2}\, x^{10}}{11} + \frac{2484\sqrt{3x^2-x+2}\, x^9}{55} + \frac{3694\sqrt{3x^2-x+2}\, x^8}{55} + \frac{120557\sqrt{3x^2-x+2}\, x^7}{1320} + \frac{557147\sqrt{3x^2-x+2}\, x^6}{6160} + \frac{50238389\sqrt{3x^2-x+2}\, x^5}{665280} + \frac{32692973\sqrt{3x^2-x+2}\, x^4}{570240} + \frac{1119925033\sqrt{3x^2-x+2}\, x^3}{31933440} + \frac{429098723\sqrt{3x^2-x+2}\, x^2}{27371520} + \frac{13323316609\sqrt{3x^2-x+2}\, x}{2299207680} + \frac{1019278873\sqrt{3x^2-x+2}}{510935040} - \frac{61917863\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{71663616}$$

input
```
int((1+2*x)^3*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

output

```
(722369249280*sqrt(3*x**2 - x + 2)*x**10 + 1246086955008*sqrt(3*x**2 - x +
 2)*x**9 + 1853077782528*sqrt(3*x**2 - x + 2)*x**8 + 2519868911616*sqrt(3*
x**2 - x + 2)*x**7 + 2495448041472*sqrt(3*x**2 - x + 2)*x**6 + 20834864686
08*sqrt(3*x**2 - x + 2)*x**5 + 1581816805632*sqrt(3*x**2 - x + 2)*x**4 + 9
67615228512*sqrt(3*x**2 - x + 2)*x**3 + 432531512784*sqrt(3*x**2 - x + 2)*
x**2 + 159879799308*sqrt(3*x**2 - x + 2)*x + 55041059142*sqrt(3*x**2 - x +
 2) - 23838377255*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/s
qrt(23)))/27590492160
```

3.58 $\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 164

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = -\frac{154997(1-6x)\sqrt{2-x+3x^2}}{4478976} - \frac{6739(1-6x)\left(2-x+3x^2\right)^{3/2}}{559872} - \frac{293(1-6x)\left(2-x+3x^2\right)^{5/2}}{58320} + \frac{37}{405}(1+2x)^2\left(2-x+3x^2\right)^{7/2} + \frac{1}{15}(1+2x)^3\left(2-x+3x^2\right)^{7/2} + \frac{(2731+3430x)\left(2-x+3x^2\right)^{7/2}}{17010} - \frac{3564931\text{ar}}{8957}$$

output
```
-154997/4478976*(1-6*x)*(3*x^2-x+2)^(1/2)-6739/559872*(1-6*x)*(3*x^2-x+2)^
(3/2)-293/58320*(1-6*x)*(3*x^2-x+2)^(5/2)+37/405*(1+2*x)^2*(3*x^2-x+2)^(7/
2)+1/15*(1+2*x)^3*(3*x^2-x+2)^(7/2)+1/17010*(2731+3430*x)*(3*x^2-x+2)^(7/2
)-3564931/26873856*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{6\sqrt{2-x+3x^2}(387182961+692659234x+1693765752x^2+3096104976x^3+4171579776x^4+$$

input
```
Integrate[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(387182961 + 692659234*x + 1693765752*x^2 + 3096104
976*x^3 + 4171579776*x^4 + 4996802304*x^5 + 5671627776*x^6 + 4427716608*x^
7 + 2675441664*x^8 + 2257403904*x^9) - 124772585*Sqrt[3]*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/940584960
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2184, 27, 1236, 27, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^2 \left(3x^2-x+2\right)^{5/2} \left(4x^2+3x+1\right) dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{120}\int 4(2x+1)^2(74x+13)\left(3x^2-x+2\right)^{5/2}dx+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{30}\int (2x+1)^2(74x+13)\left(3x^2-x+2\right)^{5/2}dx+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1236}$$

$$\frac{1}{30}\left(\frac{1}{27}\int 2(2x+1)(980x+9)\left(3x^2-x+2\right)^{5/2}dx+\frac{74}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\left(\frac{2}{27}\int (2x+1)(980x+9)\left(3x^2-x+2\right)^{5/2}dx+\frac{74}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1225$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\int \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{42}(3430x+2731)\left(3x^2-x+2\right)^{7/2}\right)+\frac{74}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}\right)+\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{42}(3430x+2731)\left(3x^2-x+2\right)^{7/\cdots}\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\cdots\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)\cdots\right.\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\,(\cdots\right.\right.\right.\right.$$
$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{30}\left(\frac{2}{27}\left(\frac{293}{4}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\right.\right.\right.\right.$$

$$\frac{1}{15}(2x+1)^3\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[(1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
((1 + 2*x)^3*(2 - x + 3*x^2)^(7/2))/15 + ((74*(1 + 2*x)^2*(2 - x + 3*x^2)^
(7/2))/27 + (2*(((2731 + 3430*x)*(2 - x + 3*x^2)^(7/2))/42 + (293*(-1/36*(
(1 - 6*x)*(2 - x + 3*x^2)^(5/2)) + (115*(-1/24*((1 - 6*x)*(2 - x + 3*x^2)^
(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^2]) + (23*ArcSinh[(-1 + 6*
x)/Sqrt[23]])/(24*Sqrt[3])))/16))/72))/4))/27)/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.46

method	result
risch	$\frac{(2257403904x^9+2675441664x^8+4427716608x^7+5671627776x^6+4996802304x^5+4171579776x^4+3096104976x^3+1693765752x^2+}{156764160}$
trager	$\left(\frac{72}{5}x^9+\frac{256}{15}x^8+\frac{1271}{45}x^7+\frac{22793}{630}x^6+\frac{722917}{22680}x^5+\frac{517309}{19440}x^4+\frac{21500729}{1088640}x^3+\frac{10081939}{933120}x^2+\frac{346329617}{78382080}x+\right.$
default	$\frac{293(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{58320}+\frac{6739(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{559872}+\frac{154997(6x-1)\sqrt{3x^2-x+2}}{4478976}+\frac{3564931\sqrt{3}\,\mathrm{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}+$

input
```
int((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/156764160*(2257403904*x^9+2675441664*x^8+4427716608*x^7+5671627776*x^6+4
996802304*x^5+4171579776*x^4+3096104976*x^3+1693765752*x^2+692659234*x+387
182961)*(3*x^2-x+2)^(1/2)+3564931/26873856*3^(1/2)*arcsinh(6/23*23^(1/2)*(
x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.60

$$\int (1+2x)^2\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)dx=\frac{1}{156764160}\left(2257403904\,x^9+2675441664\,x^8+4427716608\,x^7+5671627776\,x^6+4996802304\right.$$
$$+\frac{3564931}{53747712}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/156764160*(2257403904*x^9 + 2675441664*x^8 + 4427716608*x^7 + 5671627776
*x^6 + 4996802304*x^5 + 4171579776*x^4 + 3096104976*x^3 + 1693765752*x^2 +
 692659234*x + 387182961)*sqrt(3*x^2 - x + 2) + 3564931/53747712*sqrt(3)*l
og(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \sqrt{3x^2-x+2} \cdot \left(\frac{72x^9}{5}+\frac{256x^8}{15}+\frac{1271x^7}{45}+\frac{22793x^6}{630}+\frac{722917x^5}{22680}+\frac{517309x^4}{19440}+\frac{21500729x^3}{1088640}+\frac{10081939x^2}{933120}+\frac{346329617x}{78382080}+\frac{43020329}{17418240}\right)+\frac{3564931\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{26873856}$$

input
```
integrate((1+2*x)**2*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(72*x**9/5 + 256*x**8/15 + 1271*x**7/45 + 22793*x**6/
630 + 722917*x**5/22680 + 517309*x**4/19440 + 21500729*x**3/1088640 + 1008
1939*x**2/933120 + 346329617*x/78382080 + 43020329/17418240) + 3564931*sqr
t(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/26873856
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right)\,dx = \frac{8}{15}\left(3x^2-x+2\right)^{\frac{7}{2}}x^3+\frac{472}{405}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2+\frac{235}{243}\left(3x^2-x+2\right)^{\frac{7}{2}}x+\frac{5419}{17010}\left(3x^2-x+2\right)^{\frac{7}{2}}+\frac{293}{9720}\left(3x^2-x+2\right)^{\frac{5}{2}}x-\frac{293}{58320}\left(3x^2-x+2\right)^{\frac{5}{2}}+\frac{6739}{93312}\left(3x^2-x+2\right)^{\frac{3}{2}}x-\frac{6739}{559872}\left(3x^2-x+2\right)^{\frac{3}{2}}+\frac{154997}{746496}\sqrt{3x^2-x+2}x+\frac{3564931}{26873856}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right)-\frac{154997}{4478976}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/15*(3*x^2 - x + 2)^(7/2)*x^3 + 472/405*(3*x^2 - x + 2)^(7/2)*x^2 + 235/2
43*(3*x^2 - x + 2)^(7/2)*x + 5419/17010*(3*x^2 - x + 2)^(7/2) + 293/9720*(
3*x^2 - x + 2)^(5/2)*x - 293/58320*(3*x^2 - x + 2)^(5/2) + 6739/93312*(3*x
^2 - x + 2)^(3/2)*x - 6739/559872*(3*x^2 - x + 2)^(3/2) + 154997/746496*sq
rt(3*x^2 - x + 2)*x + 3564931/26873856*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x
- 1)) - 154997/4478976*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \frac{1}{156764160} (2 (12 (6 (8 (6 (36 (14 (24 (27 x + 32)x + 1271)x + 22793)x + 722917)x + 3621163) - \frac{3564931}{26873856} \sqrt{3} \log \left(-2 \sqrt{3} \left(\sqrt{3}x - \sqrt{3 x^2 - x + 2}\right) + 1\right)$$

input
```
integrate((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/156764160*(2*(12*(6*(8*(6*(36*(14*(24*(27*x + 32)*x + 1271)*x + 22793)*x
 + 722917)*x + 3621163)*x + 21500729)*x + 70573573)*x + 346329617)*x + 387
182961)*sqrt(3*x^2 - x + 2) - 3564931/26873856*sqrt(3)*log(-2*sqrt(3)*(sqr
t(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) dx = \int (2 x + 1)^2 \left(3 x^2 - x + 2\right)^{5/2} \left(4 x^2 + 3 x + 1\right) dx$$

input
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13

$$\int (1+2x)^2 \left(2-x+3x^2\right)^{5/2} \left(1+3x+4x^2\right) \, dx = \frac{72\sqrt{3x^2-x+2}\,x^9}{5} + \frac{256\sqrt{3x^2-x+2}\,x^8}{15} + \frac{1271\sqrt{3x^2-x+2}\,x^7}{45} + \frac{22793\sqrt{3x^2-x+2}\,x^6}{630} + \frac{722917\sqrt{3x^2-x+2}\,x^5}{22680} + \frac{517309\sqrt{3x^2-x+2}\,x^4}{19440} + \frac{21500729\sqrt{3x^2-x+2}\,x^3}{1088640} + \frac{10081939\sqrt{3x^2-x+2}\,x^2}{933120} + \frac{346329617\sqrt{3x^2-x+2}\,x}{78382080} + \frac{43020329\sqrt{3x^2-x+2}}{17418240} + \frac{3564931\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{26873856}$$

input

```
int((1+2*x)^2*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

output

```
(13544423424*sqrt(3*x**2 - x + 2)*x**9 + 16052649984*sqrt(3*x**2 - x + 2)*
x**8 + 26566299648*sqrt(3*x**2 - x + 2)*x**7 + 34029766656*sqrt(3*x**2 - x
 + 2)*x**6 + 29980813824*sqrt(3*x**2 - x + 2)*x**5 + 25029478656*sqrt(3*x*
*2 - x + 2)*x**4 + 18576629856*sqrt(3*x**2 - x + 2)*x**3 + 10162594512*sqr
t(3*x**2 - x + 2)*x**2 + 4155955404*sqrt(3*x**2 - x + 2)*x + 2323097766*sq
rt(3*x**2 - x + 2) + 124772585*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3)
 + 6*x - 1)/sqrt(23)))/940584960
```

3.59 $\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=-\frac{1177025(1-6x)\sqrt{2-x+3x^2}}{5971968}-\frac{51175(1-6x)\left(2-x+3x^2\right)^{3/2}}{746496}-\frac{445(1-6x)\left(2-x+3x^2\right)^{5/2}}{15552}+\frac{2}{27}(1+2x)^2\left(2-x+3x^2\right)^{7/2}+\frac{1}{648}(137+122x)\left(2-x+3x^2\right)^{7/2}-\frac{27071575\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{11943936\sqrt{3}}$$

output
```
-1177025/5971968*(1-6*x)*(3*x^2-x+2)^(1/2)-51175/746496*(1-6*x)*(3*x^2-x+2
)^(3/2)-445/15552*(1-6*x)*(3*x^2-x+2)^(5/2)+2/27*(1+2*x)^2*(3*x^2-x+2)^(7/
2)+1/648*(137+122*x)*(3*x^2-x+2)^(7/2)-27071575/35831808*arcsinh(1/23*(1-6
*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.65

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}(1+3x+4x^2)\,dx=\frac{6\sqrt{2-x+3x^2}(10960335+19860062x+41031048x^2+58946544x^3+66969216x^4+8003404}{358}$$

input
```
Integrate[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(10960335 + 19860062*x + 41031048*x^2 + 58946544*x^
3 + 66969216*x^4 + 80034048*x^5 + 79377408*x^6 + 30357504*x^7 + 47775744*x
^8) - 27071575*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/35831808
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2184, 27, 1225, 1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\left(3x^2-x+2\right)^{5/2}\left(4x^2+3x+1\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{108}\int 4(2x+1)(61x+18)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{27}\int (2x+1)(61x+18)\left(3x^2-x+2\right)^{5/2}dx+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{27}\left(\frac{445}{16}\int \left(3x^2-x+2\right)^{5/2}dx+\frac{1}{24}(122x+137)\left(3x^2-x+2\right)^{7/2}\right)+\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\int \left(3x^2-x+2\right)^{3/2}dx-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{24}(122x+137)\left(3x^2-x+2\right)^{7/2}\right)+ \frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\int \sqrt{3x^2-x+2}dx-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1-6x)\left(3x^2-x+2\right)^{5/2}\right)+\frac{1}{24}(\frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23}{24}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36} \frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{1}{24}\sqrt{\frac{23}{3}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2 \frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{27}\left(\frac{445}{16}\left(\frac{115}{72}\left(\frac{23}{16}\left(\frac{23\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{24\sqrt{3}}-\frac{1}{12}(1-6x)\sqrt{3x^2-x+2}\right)-\frac{1}{24}(1-6x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{36}(1 \frac{2}{27}(2x+1)^2\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[(1 + 2*x)*(2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2),x]
```

output
```
(2*(1 + 2*x)^2*(2 - x + 3*x^2)^(7/2))/27 + (((137 + 122*x)*(2 - x + 3*x^2)
^(7/2))/24 + (445*(-1/36*((1 - 6*x)*(2 - x + 3*x^2)^(5/2)) + (115*(-1/24*(
(1 - 6*x)*(2 - x + 3*x^2)^(3/2)) + (23*(-1/12*((1 - 6*x)*Sqrt[2 - x + 3*x^
2]) + (23*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(24*Sqrt[3])))/16))/72))/16)/27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

method	result
risch	$\frac{(47775744x^8+30357504x^7+79377408x^6+80034048x^5+66969216x^4+58946544x^3+41031048x^2+19860062x+10960335)\sqrt{3x^2-x-}}{5971968}$
trager	$\left(8x^8+\frac{61}{12}x^7+\frac{319}{24}x^6+\frac{11579}{864}x^5+\frac{58133}{5184}x^4+\frac{409351}{41472}x^3+\frac{1709627}{248832}x^2+\frac{9930031}{2985984}x+\frac{1217815}{663552}\right)\sqrt{3x^2-x}$
default	$\frac{445(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{15552}+\frac{51175(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{746496}+\frac{1177025(6x-1)\sqrt{3x^2-x+2}}{5971968}+\frac{27071575\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}$

input
```
int((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x,method=_RETURNVERBOSE)
```

output
```
1/5971968*(47775744*x^8+30357504*x^7+79377408*x^6+80034048*x^5+66969216*x^
4+58946544*x^3+41031048*x^2+19860062*x+10960335)*(3*x^2-x+2)^(1/2)+2707157
5/35831808*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\frac{1}{5971968}\left(47775744\,x^8+30357504\,x^7+79377408\,x^6+80034048\,x^5+66969216\,x^4+5894654\right.$$
$$+\frac{27071575}{71663616}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="fricas")
```

output
```
1/5971968*(47775744*x^8 + 30357504*x^7 + 79377408*x^6 + 80034048*x^5 + 669
69216*x^4 + 58946544*x^3 + 41031048*x^2 + 19860062*x + 10960335)*sqrt(3*x^
2 - x + 2) + 27071575/71663616*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*
(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\sqrt{3x^2-x+2}\cdot\left(8x^8+\frac{61x^7}{12}+\frac{319x^6}{24}+\frac{11579x^5}{864}+\frac{58133x^4}{5184}+\frac{409351x^3}{41472}+\frac{1709627x^2}{248832}+\frac{9930031x}{2985984}+\frac{1217815}{663552}\right)+\frac{27071575\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{35831808}$$

input
```
integrate((1+2*x)*(3*x**2-x+2)**(5/2)*(4*x**2+3*x+1),x)
```

output
```
sqrt(3*x**2 - x + 2)*(8*x**8 + 61*x**7/12 + 319*x**6/24 + 11579*x**5/864 +
 58133*x**4/5184 + 409351*x**3/41472 + 1709627*x**2/248832 + 9930031*x/298
5984 + 1217815/663552) + 27071575*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3
5831808
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.08

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \frac{8}{27}\left(3x^2-x+2\right)^{\frac{7}{2}}x^2 + \frac{157}{324}\left(3x^2-x+2\right)^{\frac{7}{2}}x + \frac{185}{648}\left(3x^2-x+2\right)^{\frac{7}{2}} + \frac{445}{2592}\left(3x^2-x+2\right)^{\frac{5}{2}}x - \frac{445}{15552}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{51175}{124416}\left(3x^2-x+2\right)^{\frac{3}{2}}x - \frac{51175}{746496}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{1177025}{995328}\sqrt{3x^2-x+2}\,x + \frac{27071575}{35831808}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{1177025}{5971968}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="maxima")
```

output
```
8/27*(3*x^2 - x + 2)^(7/2)*x^2 + 157/324*(3*x^2 - x + 2)^(7/2)*x + 185/648
*(3*x^2 - x + 2)^(7/2) + 445/2592*(3*x^2 - x + 2)^(5/2)*x - 445/15552*(3*x
^2 - x + 2)^(5/2) + 51175/124416*(3*x^2 - x + 2)^(3/2)*x - 51175/746496*(3
*x^2 - x + 2)^(3/2) + 1177025/995328*sqrt(3*x^2 - x + 2)*x + 27071575/3583
1808*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 1177025/5971968*sqrt(3*x^2
 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx = \frac{1}{5971968}\left(2\left(12\left(6\left(8\left(6\left(36\left(2\left(96x+61\right)x+319\right)x+11579\right)x+58133\right)x+409351\right)x+170962\right.\right.\right. - \frac{27071575}{35831808}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x, algorithm="giac")
```

output
```
1/5971968*(2*(12*(6*(8*(6*(36*(2*(96*x + 61)*x + 319)*x + 11579)*x + 58133
)*x + 409351)*x + 1709627)*x + 9930031)*x + 10960335)*sqrt(3*x^2 - x + 2)
- 27071575/35831808*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2
)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=\int (2\,x+1)\,\left(3\,x^2-x+2\right)^{5/2}\,\left(4\,x^2+3\,x+1\right)\,dx$$

input
```
int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1),x)
```

output
```
int((2*x + 1)*(3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22

$$\int (1+2x)\left(2-x+3x^2\right)^{5/2}\left(1+3x+4x^2\right)\,dx=8\sqrt{3x^2-x+2}\,x^8+\frac{61\sqrt{3x^2-x+2}\,x^7}{12}+\frac{319\sqrt{3x^2-x+2}\,x^6}{24}+\frac{11579\sqrt{3x^2-x+2}\,x^5}{864}+\frac{58133\sqrt{3x^2-x+2}\,x^4}{5184}+\frac{409351\sqrt{3x^2-x+2}\,x^3}{41472}+\frac{1709627\sqrt{3x^2-x+2}\,x^2}{248832}+\frac{9930031\sqrt{3x^2-x+2}\,x}{2985984}+\frac{1217815\sqrt{3x^2-x+2}}{663552}+\frac{27071575\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{35831808}$$

input
```
int((1+2*x)*(3*x^2-x+2)^(5/2)*(4*x^2+3*x+1),x)
```

output

```
(286654464*sqrt(3*x**2 - x + 2)*x**8 + 182145024*sqrt(3*x**2 - x + 2)*x**7
 + 476264448*sqrt(3*x**2 - x + 2)*x**6 + 480204288*sqrt(3*x**2 - x + 2)*x*
*5 + 401815296*sqrt(3*x**2 - x + 2)*x**4 + 353679264*sqrt(3*x**2 - x + 2)*
x**3 + 246186288*sqrt(3*x**2 - x + 2)*x**2 + 119160372*sqrt(3*x**2 - x + 2
)*x + 65762010*sqrt(3*x**2 - x + 2) + 27071575*sqrt(3)*log((2*sqrt(3*x**2
- x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/35831808
```

3.60 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} dx$

Optimal result

Integrand size = 32, antiderivative size = 147

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x} dx = \frac{(221999-17850x)\sqrt{2-x+3x^2}}{82944} + \frac{(2449+2154x)(2-x+3x^2)^{3/2}}{10368} + \frac{(29+150x)(2-x+3x^2)^{5/2}}{1080} + \frac{2}{21}(2-x+3x^2)^{7/2} + \frac{944521\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{165888\sqrt{3}} - \frac{169}{128}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output
```
1/82944*(221999-17850*x)*(3*x^2-x+2)^(1/2)+1/10368*(2449+2154*x)*(3*x^2-x+
2)^(3/2)+1/1080*(29+150*x)*(3*x^2-x+2)^(5/2)+2/21*(3*x^2-x+2)^(7/2)+944521
/497664*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-169/128*13^(1/2)*arctanh(1/
26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{6\sqrt{2-x+3x^2}(11665053-2120998x+12466776x^2-3646512x^3\ldots}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(11665053 - 2120998*x + 12466776*x^2 - 3646512*x^3
+ 15700608*x^4 - 3836160*x^5 + 7464960*x^6) + 45995040*Sqrt[13]*ArcTanh[(S
qrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 33058235*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/17418240
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2184, 27, 1231, 25, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{2x+1}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{84}\int \frac{28(5x+4)(3x^2-x+2)^{5/2}}{2x+1}dx+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{3}\int \frac{(5x+4)(3x^2-x+2)^{5/2}}{2x+1}dx+\frac{2}{21}(3x^2-x+2)^{7/2}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{3}\left(\frac{1}{360}(150x+29)\left(3x^2-x+2\right)^{5/2}-\frac{1}{144}\int -\frac{(718x+1061)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx\right)+$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 25$$

$$\frac{1}{3}\left(\frac{1}{144}\int \frac{(718x+1061)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx+\frac{1}{360}(150x+29)\left(3x^2-x+2\right)^{5/2}\right)+$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{24}(2154x+2449)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{6(33529-5950x)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{1}{360}(150x+29)\,($$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\int \frac{(33529-5950x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{24}(2154x+2449)\left(3x^2-x+2\right)^{3/2}\right)+\frac{1}{360}(150x+29)\,(3x$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{12}(221999-17850x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(1902791-1889042x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{24}(2154x+2449)\,($$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\int \frac{1902791-1889042x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(221999-17850x)\right)+\frac{1}{24}(2154x+2449)\,(3x^2$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1269$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-944521\int\frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(2219\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{944521\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(2847312\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(221999\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(-5694624\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(22\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{1}{144}\left(\frac{1}{16}\left(\frac{1}{24}\left(-\frac{944521\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-219024\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(\right.\right.\right.\right.$$
$$\frac{2}{21}\left(3x^2-x+2\right)^{7/2}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x),x]
```

output
```
(2*(2 - x + 3*x^2)^(7/2))/21 + (((29 + 150*x)*(2 - x + 3*x^2)^(5/2))/360 +
 (((2449 + 2154*x)*(2 - x + 3*x^2)^(3/2))/24 + (((221999 - 17850*x)*Sqrt[2
 - x + 3*x^2])/12 + ((-944521*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt[3] - 2190
24*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/24)/16)/1
44)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.61

method	result
risch	$\frac{(7464960x^6-3836160x^5+15700608x^4-3646512x^3+12466776x^2-2120998x+11665053)\sqrt{3x^2-x+2}}{2903040}-\frac{944521\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}}{\ldots}\right.}{497664}$
trager	$\left(\frac{18}{7}x^6-\frac{37}{28}x^5+\frac{649}{120}x^4-\frac{8441}{6720}x^3+\frac{74207}{17280}x^2-\frac{1060499}{1451520}x+\frac{144013}{35840}\right)\sqrt{3x^2-x+2}-\frac{944521\,\text{RootOf}\left(\_Z^2\right.}{}$
default	$\frac{5(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{216}+\frac{575(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{10368}+\frac{13225(6x-1)\sqrt{3x^2-x+2}}{82944}-\frac{944521\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{497664}+\frac{2(3x}{}$

input

```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x,method=_RETURNVERBOSE)
```

output

```
1/2903040*(7464960*x^6-3836160*x^5+15700608*x^4-3646512*x^3+12466776*x^2-2
120998*x+11665053)*(3*x^2-x+2)^(1/2)-944521/497664*3^(1/2)*arcsinh(6/23*23
^(1/2)*(x-1/6))-169/128*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+
x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.92

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{2903040}\left(7464960\,x^6-3836160\,x^5+15700608\,x^4-3646512\,x^3+\right.$$
$$+\frac{944521}{995328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$
$$+\frac{169}{256}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)$$

input

```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="fricas")
```

output
```
1/2903040*(7464960*x^6 - 3836160*x^5 + 15700608*x^4 - 3646512*x^3 + 124667
76*x^2 - 2120998*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/995328*sqrt(3)
*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 169/2
56*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196
*x + 185)/(4*x^2 + 4*x + 1))
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{2x+1}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x),x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx &= \frac{2}{21}\left(3x^2-x+2\right)^{\frac{7}{2}} \\ &+ \frac{5}{36}\left(3x^2-x+2\right)^{\frac{5}{2}}x + \frac{29}{1080}\left(3x^2-x+2\right)^{\frac{5}{2}} \\ &+ \frac{359}{1728}\left(3x^2-x+2\right)^{\frac{3}{2}}x + \frac{2449}{10368}\left(3x^2-x+2\right)^{\frac{3}{2}} \\ &- \frac{2975}{13824}\sqrt{3x^2-x+2}x - \frac{944521}{497664}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &+ \frac{169}{128}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2x+1|} - \frac{9\sqrt{23}}{23\,|2x+1|}\right) + \frac{221999}{82944}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="maxima")
```

output
```
2/21*(3*x^2 - x + 2)^(7/2) + 5/36*(3*x^2 - x + 2)^(5/2)*x + 29/1080*(3*x^2
 - x + 2)^(5/2) + 359/1728*(3*x^2 - x + 2)^(3/2)*x + 2449/10368*(3*x^2 - x
 + 2)^(3/2) - 2975/13824*sqrt(3*x^2 - x + 2)*x - 944521/497664*sqrt(3)*arc
sinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 169/128*sqrt(13)*arcsinh(8/23*sqrt
(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 221999/82944*sqrt(3*x^
2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{1}{2903040}(2(12(18(8(30(72x-37)x+4543)x-8441)x+5194$$
$$+\frac{944521}{497664}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3x^2-x+2}\right)$$
$$+\frac{169}{128}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x, algorithm="giac")
```

output
```
1/2903040*(2*(12*(18*(8*(30*(72*x - 37)*x + 4543)*x - 8441)*x + 519449)*x
- 1060499)*x + 11665053)*sqrt(3*x^2 - x + 2) + 944521/497664*sqrt(3)*log(-
6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 169/128*sqrt(13)*log(-1/2
*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqr
t(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{2x+1}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1),x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.71

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{1+2x}\,dx = \frac{169\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6ix-i}{\sqrt{39}-4}\right)i}{128} + \frac{18\sqrt{3x^2-x+2}\,x^6}{7} - \frac{37\sqrt{3x^2-x+2}\,x^5}{28} + \frac{649\sqrt{3x^2-x+2}\,x^4}{120} - \frac{8441\sqrt{3x^2-x+2}\,x^3}{6720} + \frac{74207\sqrt{3x^2-x+2}\,x^2}{17280} - \frac{1060499\sqrt{3x^2-x+2}\,x}{1451520} + \frac{144013\sqrt{3x^2-x+2}}{35840} + \frac{169\sqrt{13}\log\left(24\sqrt{3x^2-x+2}\,\sqrt{3}\,x-4\sqrt{3x^2-x+2}\,\sqrt{3}+8\sqrt{39}+72x^2-24x-30\right)}{256} - \frac{169\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{3}+\sqrt{39}+6x+3\right)}{128} - \frac{944521\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{497664}$$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x),x)
```

output
```
(22997520*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sq
rt(39) - 4))*i + 44789760*sqrt(3*x**2 - x + 2)*x**6 - 23016960*sqrt(3*x**2
 - x + 2)*x**5 + 94203648*sqrt(3*x**2 - x + 2)*x**4 - 21879072*sqrt(3*x**2
 - x + 2)*x**3 + 74800656*sqrt(3*x**2 - x + 2)*x**2 - 12725988*sqrt(3*x**2
 - x + 2)*x + 69990318*sqrt(3*x**2 - x + 2) + 11498760*sqrt(13)*log(24*sqr
t(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39)
+ 72*x**2 - 24*x - 30) - 22997520*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt
(3) + sqrt(39) + 6*x + 3) - 33058235*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*s
qrt(3) + 6*x - 1)/sqrt(23)))/17418240
```

3.61 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 154

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = -\frac{11(4727-3090x)\sqrt{2-x+3x^2}}{6912}-\frac{11}{864}(67-78x)\left(2-x+3x^2\right)^{3/2} -\frac{11(37-60x)\left(2-x+3x^2\right)^{5/2}}{2340}-\frac{\left(2-x+3x^2\right)^{7/2}}{13(1+2x)} -\frac{315623\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{13824\sqrt{3}}+\frac{429}{128}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output

```
-11/6912*(4727-3090*x)*(3*x^2-x+2)^(1/2)-11/864*(67-78*x)*(3*x^2-x+2)^(3/2
)-11/2340*(37-60*x)*(3*x^2-x+2)^(5/2)-(3*x^2-x+2)^(7/2)/(13+26*x)-315623/4
1472*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)+429/128*13^(1/2)*arctanh(1/26*
(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}(-364257-322972x+310660x^2-115680x^3+251424x^4-65664x^5+103680x^6)}{1+2x}}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(-364257 - 322972*x + 310660*x^2 - 115680*x^3 + 25
1424*x^4 - 65664*x^5 + 103680*x^6))/(1 + 2*x) - 1389960*Sqrt[13]*ArcTanh[(
Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] - 1578115*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/207360
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2181, 27, 1231, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{(2x+1)^2}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{11(8x+1)(3x^2-x+2)^{5/2}}{2(2x+1)}dx - \frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{26}\int \frac{(8x+1)(3x^2-x+2)^{5/2}}{2x+1}dx - \frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$$\Big\downarrow \text{1231}$$

$$\frac{11}{26}\left(-\frac{1}{144}\int \frac{52(1-52x)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx-\frac{1}{90}(37-60x)\left(3x^2-x+2\right)^{5/2}\right)-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 27

$$\frac{11}{26}\left(-\frac{13}{36}\int \frac{(1-52x)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx-\frac{1}{90}(37-60x)\left(3x^2-x+2\right)^{5/2}\right)-\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 1231

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{12}(67-78x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{12(187-1030x)\sqrt{3x^2-x+2}}{2x+1}dx\right)-\frac{1}{90}(37-60x)\left(3x^2-\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 27

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\int \frac{(187-1030x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{12}(67-78x)\left(3x^2-x+2\right)^{3/2}\right)-\frac{1}{90}(37-60x)\left(3x^2-x+2\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 1231

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{12}(4727-3090x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(26063-57386x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}(67-78x)\left(3x^2-x+\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 27

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\int \frac{26063-57386x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(4727-3090x)\right)+\frac{1}{12}(67-78x)\left(3x^2-x+2\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 1269

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-28693\int\frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727-\right.\right.\right.\right.$$
$$\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 1090

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{28693\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(\right.\right.\right.$$
$$\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 222

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(54756\int\frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(4727-309\right.\right.\right.$$
$$\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 1154

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(-109512\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(472\right.\right.\right.$$
$$\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

$\Big\downarrow$ 219

$$\frac{11}{26}\left(-\frac{13}{36}\left(\frac{1}{8}\left(\frac{1}{24}\left(-\frac{28693\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-4212\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(472\right.\right.\right.$$
$$\frac{(3x^2-x+2)^{7/2}}{13(2x+1)}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^2,x]
```

output
```
-1/13*(2 - x + 3*x^2)^(7/2)/(1 + 2*x) + (11*(-1/90*((37 - 60*x)*(2 - x + 3
*x^2)^(5/2)) - (13*(((67 - 78*x)*(2 - x + 3*x^2)^(3/2))/12 + (((4727 - 309
0*x)*Sqrt[2 - x + 3*x^2])/12 + ((-28693*ArcSinh[(-1 + 6*x)/Sqrt[23]])/Sqrt
[3] - 4212*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/2
4)/8))/36))/26
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
risch	$\frac{311040x^8-300672x^7+1027296x^6-729792x^5+1550508x^4-1510936x^3-148479x^2-281687x-728514}{34560(1+2x)\sqrt{3x^2-x+2}}+\frac{315623\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}}{}\right.}{41472}$
trager	$\frac{(103680x^6-65664x^5+251424x^4-115680x^3+310660x^2-322972x-364257)\sqrt{3x^2-x+2}}{34560+69120x}+\frac{429\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\text{RootOf}}{}\right.}{}$
default	$\frac{(6x-1)(3x^2-x+2)^{\frac{5}{2}}}{36}+\frac{115(6x-1)(3x^2-x+2)^{\frac{3}{2}}}{1728}+\frac{2645(6x-1)\sqrt{3x^2-x+2}}{13824}+\frac{315623\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{41472}-\frac{33\left(3\left(\frac{1}{2}\right.\right.}{}$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/34560*(311040*x^8-300672*x^7+1027296*x^6-729792*x^5+1550508*x^4-1510936*
x^3-148479*x^2-281687*x-728514)/(1+2*x)/(3*x^2-x+2)^(1/2)+315623/41472*3^(
1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+429/128*13^(1/2)*arctanh(2/13*(9/2-4*x
)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{1578115\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+\right.}{}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/414720*(1578115*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*
x - 1) - 72*x^2 + 24*x - 25) + 694980*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*s
qrt(3*x^2 - x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) +
 12*(103680*x^6 - 65664*x^5 + 251424*x^4 - 115680*x^3 + 310660*x^2 - 32297
2*x - 364257)*sqrt(3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^2}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**2,x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx &= \frac{1}{6}\left(3x^2-x+2\right)^{\frac{5}{2}}x \\ &- \frac{7}{90}\left(3x^2-x+2\right)^{\frac{5}{2}} + \frac{143}{144}\left(3x^2-x+2\right)^{\frac{3}{2}}x \\ &- \frac{737}{864}\left(3x^2-x+2\right)^{\frac{3}{2}} - \frac{\left(3x^2-x+2\right)^{\frac{5}{2}}}{4(2x+1)} + \frac{5665}{1152}\sqrt{3x^2-x+2}x \\ &+ \frac{315623}{41472}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{429}{128}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) - \frac{51997}{6912}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/6*(3*x^2 - x + 2)^(5/2)*x - 7/90*(3*x^2 - x + 2)^(5/2) + 143/144*(3*x^2
- x + 2)^(3/2)*x - 737/864*(3*x^2 - x + 2)^(3/2) - 1/4*(3*x^2 - x + 2)^(5/
2)/(2*x + 1) + 5665/1152*sqrt(3*x^2 - x + 2)*x + 315623/41472*sqrt(3)*arcs
inh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 429/128*sqrt(13)*arcsinh(8/23*sqrt(
23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 51997/6912*sqrt(3*x^2 -
 x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(123) = 246$.

Time = 0.54 (sec) , antiderivative size = 760, normalized size of antiderivative = 4.94

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x, algorithm="giac")
```

output
```
429/128*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + s
qrt(13)/(2*x + 1)) - 4)*sgn(1/(2*x + 1)) - 315623/41472*sqrt(3)*log(1/2*ab
s(-2*sqrt(3) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x
 + 1))/(sqrt(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x
+ 1)))*sgn(1/(2*x + 1)) - 169/128*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)*
sgn(1/(2*x + 1)) + 1/34560*(5154065*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
3) + sqrt(13)/(2*x + 1))^11*sgn(1/(2*x + 1)) - 7837020*sqrt(13)*(sqrt(-8/(
2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^10*sgn(1/(2*x + 1)) +
 39468815*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^9
*sgn(1/(2*x + 1)) - 14445540*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2
+ 3) + sqrt(13)/(2*x + 1))^8*sgn(1/(2*x + 1)) + 460893402*(sqrt(-8/(2*x +
1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^7*sgn(1/(2*x + 1)) - 343084
680*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)
)^6*sgn(1/(2*x + 1)) + 944150094*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)
+ sqrt(13)/(2*x + 1))^5*sgn(1/(2*x + 1)) - 22871160*sqrt(13)*(sqrt(-8/(2*x
 + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^4*sgn(1/(2*x + 1)) + 139
7032245*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))^3*s
gn(1/(2*x + 1)) - 683367516*sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 +
 3) + sqrt(13)/(2*x + 1))^2*sgn(1/(2*x + 1)) + 392684355*(sqrt(-8/(2*x + 1
) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1))*sgn(1/(2*x + 1)) + 197538...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \int \frac{(3\,x^2-x+2)^{5/2}\,(4\,x^2+3\,x+1)}{(2\,x+1)^2}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2,x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.55

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^2}\,dx = \frac{622080\sqrt{3x^2-x+2}\,x^6 - 393984\sqrt{3x^2-x+2}\,x^5 + 1508544\sqrt{3x$$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^2,x)
```

output
```
(622080*sqrt(3*x**2 - x + 2)*x**6 - 393984*sqrt(3*x**2 - x + 2)*x**5 + 150
8544*sqrt(3*x**2 - x + 2)*x**4 - 694080*sqrt(3*x**2 - x + 2)*x**3 + 186396
0*sqrt(3*x**2 - x + 2)*x**2 - 1937832*sqrt(3*x**2 - x + 2)*x - 2185542*sqr
t(3*x**2 - x + 2) + 1389960*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13
) + 8*x - 9)*x + 694980*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) +
8*x - 9) - 1389960*sqrt(13)*log(2*x + 1)*x - 694980*sqrt(13)*log(2*x + 1)
+ 3156230*sqrt(3)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 157
8115*sqrt(3)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1))/(207360*(2*
x + 1))
```

3.62 $\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 161

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{(21317-10470x)\sqrt{2-x+3x^2}}{1536} + \frac{1}{832}(1227-838x)\left(2-x+3x^2\right)^{3/2} + \frac{(257+134x)\left(2-x+3x^2\right)^{5/2}}{520(1+2x)} - \frac{\left(2-x+3x^2\right)^{7/2}}{26(1+2x)^2} + \frac{118423\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3072\sqrt{3}} - \frac{1631}{256}\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)$$

output

```
1/1536*(21317-10470*x)*(3*x^2-x+2)^(1/2)+1/832*(1227-838*x)*(3*x^2-x+2)^(3
/2)+(257+134*x)*(3*x^2-x+2)^(5/2)/(520+1040*x)-1/26*(3*x^2-x+2)^(7/2)/(1+2
*x)^2+118423/9216*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1631/256*13^(1/2)
*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{\frac{6\sqrt{2-x+3x^2}(142057+464446x+256564x^2-76200x^3+83616x^4-22464x^5+27648x^6)}{(1+2x)^2}+5}{}$$

input
```
Integrate[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
((6*Sqrt[2 - x + 3*x^2]*(142057 + 464446*x + 256564*x^2 - 76200*x^3 + 8361
6*x^4 - 22464*x^5 + 27648*x^6))/(1 + 2*x)^2 + 587160*Sqrt[13]*ArcTanh[(Sqr
t[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]] + 592115*Sqrt[3]*Log
[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/46080
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2181, 27, 1230, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{(2x+1)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{26}\int -\frac{(134x+29)(3x^2-x+2)^{5/2}}{2(2x+1)^2}dx-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{52}\int \frac{(134x+29)(3x^2-x+2)^{5/2}}{(2x+1)^2}dx-\frac{(3x^2-x+2)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1230}$$

$$\frac{1}{52}\left(\frac{(134x+257)\left(3x^2-x+2\right)^{5/2}}{10(2x+1)}-\frac{1}{8}\int -\frac{2(793-1676x)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx\right)-\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\int \frac{(793-1676x)\left(3x^2-x+2\right)^{3/2}}{2x+1}dx+\frac{(134x+257)\left(3x^2-x+2\right)^{5/2}}{10(2x+1)}\right)-\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\downarrow$ 1231

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{1}{4}(1227-838x)\left(3x^2-x+2\right)^{3/2}-\frac{1}{96}\int -\frac{156(1517-3490x)\sqrt{3x^2-x+2}}{2x+1}dx\right)+\frac{(134x+257)\left(3x^2-\right.}{10(2x+1}\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\int \frac{(1517-3490x)\sqrt{3x^2-x+2}}{2x+1}dx+\frac{1}{4}(1227-838x)\left(3x^2-x+2\right)^{3/2}\right)+\frac{(134x+257)\left(3x^2-x+\right.}{10(2x+1)}\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\downarrow$ 1231

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{12}(21317-10470x)\sqrt{3x^2-x+2}-\frac{1}{48}\int -\frac{2(136013-236846x)}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{1}{4}(1227-838x)\left(3x^2-\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$\downarrow$ 27

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\int \frac{136013-236846x}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{1}{12}\sqrt{3x^2-x+2}(21317-10470x)\right)+\frac{1}{4}(1227-838x)\left(3x^2-x\right.\right.\right.$$
$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-118423\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21317\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{118423\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(254436\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21317-1\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(-508872\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)+\frac{1}{12}\sqrt{3x^2-x+2}(213\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{52}\left(\frac{1}{4}\left(\frac{13}{8}\left(\frac{1}{24}\left(-\frac{118423\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-19572\sqrt{13}\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)\right)+\frac{1}{12}\sqrt{3x^2-x+2}(21\right.\right.\right.\right.$$

$$\frac{\left(3x^2-x+2\right)^{7/2}}{26(2x+1)^2}$$

input
```
Int[((2 - x + 3*x^2)^(5/2)*(1 + 3*x + 4*x^2))/(1 + 2*x)^3,x]
```

output
```
-1/26*(2 - x + 3*x^2)^(7/2)/(1 + 2*x)^2 + (((257 + 134*x)*(2 - x + 3*x^2)^
(5/2))/(10*(1 + 2*x)) + (((1227 - 838*x)*(2 - x + 3*x^2)^(3/2))/4 + (13*((
(21317 - 10470*x)*Sqrt[2 - x + 3*x^2])/12 + ((-118423*ArcSinh[(-1 + 6*x)/S
qrt[23]])/Sqrt[3] - 19572*Sqrt[13]*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 -
x + 3*x^2])])/24))/8)/4)/52
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result
risch	$\frac{82944x^8-95040x^7+328608x^6-357144x^5+1013124x^4+984374x^3+474853x^2+786835x+284114}{7680(1+2x)^2\sqrt{3x^2-x+2}}-\frac{118423\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}\left(x-\right.}{23}\right.}{9216}$
trager	$\frac{(27648x^6-22464x^5+83616x^4-76200x^3+256564x^2+464446x+142057)\sqrt{3x^2-x+2}}{7680(1+2x)^2}+\frac{118423\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\text{RootOf}\right.}{}$
default	$\frac{1631\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{5}{2}}}{6760}-\frac{419(6x-1)\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}{2496}-\frac{1745(6x-1)\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}{1536}-\frac{118423\sqrt{3}\,\text{arcsinh}\left(\frac{6\sqrt{23}}{}\right.}{9216}$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7680*(82944*x^8-95040*x^7+328608*x^6-357144*x^5+1013124*x^4+984374*x^3+4
74853*x^2+786835*x+284114)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-118423/9216*3^(1/2)
*arcsinh(6/23*23^(1/2)*(x-1/6))-1631/256*13^(1/2)*arctanh(2/13*(9/2-4*x)*1
3^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.05

$$\int \frac{(2-x+3x^2)^{5/2}\,(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{592115\,\sqrt{3}(4\,x^2+4\,x+1)\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\right.}{}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="fricas")
```

output
```
1/92160*(592115*sqrt(3)*(4*x^2 + 4*x + 1)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2
)*(6*x - 1) - 72*x^2 + 24*x - 25) + 293580*sqrt(13)*(4*x^2 + 4*x + 1)*log(
-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2
 + 4*x + 1)) + 12*(27648*x^6 - 22464*x^5 + 83616*x^4 - 76200*x^3 + 256564*
x^2 + 464446*x + 142057)*sqrt(3*x^2 - x + 2))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{\frac{5}{2}}\cdot(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
integrate((3*x**2-x+2)**(5/2)*(4*x**2+3*x+1)/(1+2*x)**3,x)
```

output
```
Integral((3*x**2 - x + 2)**(5/2)*(4*x**2 + 3*x + 1)/(2*x + 1)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.07

$$\begin{aligned}\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx &= \frac{67}{520}\left(3x^2-x+2\right)^{\frac{5}{2}} \\ &- \frac{(3x^2-x+2)^{\frac{7}{2}}}{26(4x^2+4x+1)} - \frac{419}{416}\left(3x^2-x+2\right)^{\frac{3}{2}}x \\ &+ \frac{1227}{832}\left(3x^2-x+2\right)^{\frac{3}{2}} + \frac{19(3x^2-x+2)^{\frac{5}{2}}}{52(2x+1)} \\ &- \frac{1745}{256}\sqrt{3x^2-x+2}\,x - \frac{118423}{9216}\sqrt{3}\,\mathrm{arsinh}\left(\frac{6}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &+ \frac{1631}{256}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23|2x+1|} - \frac{9\sqrt{23}}{23|2x+1|}\right) + \frac{21317}{1536}\sqrt{3x^2-x+2}\end{aligned}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="maxima")
```

output
```
67/520*(3*x^2 - x + 2)^(5/2) - 1/26*(3*x^2 - x + 2)^(7/2)/(4*x^2 + 4*x + 1
) - 419/416*(3*x^2 - x + 2)^(3/2)*x + 1227/832*(3*x^2 - x + 2)^(3/2) + 19/
52*(3*x^2 - x + 2)^(5/2)/(2*x + 1) - 1745/256*sqrt(3*x^2 - x + 2)*x - 1184
23/9216*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1631/256*sqrt(1
3)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 21
317/1536*sqrt(3*x^2 - x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(130) = 260.

Time = 0.26 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.68

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{1}{7680}\left(6\left(4\left(18\left(16x-29\right)x+1321\right)x-7937\right)x+103837\right)\sqrt{3x^2-}$$

$$+\frac{118423}{9216}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

$$+\frac{1631}{256}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$

$$+\frac{13\left(574\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-101\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-2745\sqrt{3}x+1369\sqrt{3}+2745\sqrt{3x}\right.}{128\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x, algorithm="giac")
```

output
```
1/7680*(6*(4*(18*(16*x - 29)*x + 1321)*x - 7937)*x + 103837)*sqrt(3*x^2 -
x + 2) + 118423/9216*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x +
2)) + 1) + 1631/256*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sq
rt(3) + 4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(
3*x^2 - x + 2))) + 13/128*(574*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 101*s
qrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 2745*sqrt(3)*x + 1369*sqrt(3)
 + 2745*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \int \frac{(3x^2-x+2)^{5/2}(4x^2+3x+1)}{(2x+1)^3}\,dx$$

input
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3,x)
```

output
```
int(((3*x^2 - x + 2)^(5/2)*(3*x + 4*x^2 + 1))/(2*x + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.94

$$\int \frac{(2-x+3x^2)^{5/2}(1+3x+4x^2)}{(1+2x)^3}\,dx = \frac{165888\sqrt{3x^2-x+2}\,x^6-134784\sqrt{3x^2-x+2}\,x^5+501696\sqrt{3x^2}}{}$$

input
```
int((3*x^2-x+2)^(5/2)*(4*x^2+3*x+1)/(1+2*x)^3,x)
```

output
```
(165888*sqrt(3*x**2 - x + 2)*x**6 - 134784*sqrt(3*x**2 - x + 2)*x**5 + 501
696*sqrt(3*x**2 - x + 2)*x**4 - 457200*sqrt(3*x**2 - x + 2)*x**3 + 1539384
*sqrt(3*x**2 - x + 2)*x**2 + 2786676*sqrt(3*x**2 - x + 2)*x + 852342*sqrt(
3*x**2 - x + 2) + 1174320*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8
*x - 9)*x**2 + 1174320*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x
- 9)*x + 293580*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) -
1174320*sqrt(13)*log(2*x + 1)*x**2 - 1174320*sqrt(13)*log(2*x + 1)*x - 293
580*sqrt(13)*log(2*x + 1) + 2368460*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqr
t(3) - 6*x + 1)*x**2 + 2368460*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3)
- 6*x + 1)*x + 592115*sqrt(3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1
))/(46080*(4*x**2 + 4*x + 1))
```

3.63 $\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 674

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\left(60ceg-48bfg-\frac{12cfg^2}{h}+80cdh-70beh-64afh+\frac{63b^2fh}{c}\right)(g+hx)^2\sqrt{a+bx+cx^2}}{240c^2}$$

$$+ \frac{\left(10ce-9bf-\frac{2cfg}{h}\right)(g+hx)^3\sqrt{a+bx+cx^2}}{40c^2} + \frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$+ \frac{\left(945b^4fh^3-\frac{64c^4(3fg^4-5g^2h(3eg+16dh))}{h}-210b^2ch^2(14afh+5b(3fg+eh))+8c^2h(128a^2fh^2+275abh(\right.}{}$$

$$+ \frac{(256c^5dg^3-63b^5fh^3+70b^3ch^2(3bfg+beh+4afh)-128c^4g(afg^2+3ah(eg+dh)+bg(eg+3dh))}{}$$

output
```
1/240*(60*c*e*g-48*b*f*g-12*c*f*g^2/h+80*c*d*h-70*b*e*h-64*a*f*h+63*b^2*f*
h/c)*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c^2+1/40*(10*c*e-9*b*f-2*c*f*g/h)*(h*x+
g)^3*(c*x^2+b*x+a)^(1/2)/c^2+1/5*f*(h*x+g)^4*(c*x^2+b*x+a)^(1/2)/c/h+1/192
0*(945*b^4*f*h^3-64*c^4*(3*f*g^4-5*g^2*h*(16*d*h+3*e*g))/h-210*b^2*c*h^2*(
14*a*f*h+5*b*(e*h+3*f*g))+8*c^2*h*(128*a^2*f*h^2+275*a*b*h*(e*h+3*f*g)+3*b
^2*(129*f*g^2+50*h*(d*h+3*e*g)))-16*c^3*(16*a*h*(13*f*g^2+5*h*(d*h+3*e*g))
+b*g*(39*f*g^2+5*h*(54*d*h+47*e*g)))-2*c*(315*b^3*f*h^3-14*b*c*h^2*(46*a*f
*h+25*b*e*h+39*b*f*g)+16*c^3*(3*f*g^3-5*g*h*(10*d*h+3*e*g))+8*c^2*h*(21*b*
f*g^2+10*b*h*(5*d*h+8*e*g)+a*h*(45*e*h+71*f*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^
5+1/256*(256*c^5*d*g^3-63*b^5*f*h^3+70*b^3*c*h^2*(4*a*f*h+b*e*h+3*b*f*g)-1
28*c^4*g*(a*f*g^2+3*a*h*(d*h+e*g)+b*g*(3*d*h+e*g))-80*b*c^2*h*(3*a^2*f*h^2
+3*a*b*h*(e*h+3*f*g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))+96*c^3*(a^2*h^2*(e*h+3*f
*g)+b^2*g*(f*g^2+3*h*(d*h+e*g))+2*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*arctanh(
1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 4.38 (sec) , antiderivative size = 588, normalized size of antiderivative = 0.87

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^4fh^3-210b^2ch^2(5beh+14afh+3bf(5g+hx))+32c^4(10dh(18g^2+9ghx+}{}$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*f*h^3 - 210*b^2*c*h^2*(5*b*e*h +
 14*a*f*h + 3*b*f*(5*g + h*x)) + 32*c^4*(10*d*h*(18*g^2 + 9*g*h*x + 2*h^2*
x^2) + 15*e*(4*g^3 + 6*g^2*h*x + 4*g*h^2*x^2 + h^3*x^3) + 3*f*x*(10*g^3 +
20*g^2*h*x + 15*g*h^2*x^2 + 4*h^3*x^3)) + 4*c^2*h*(256*a^2*f*h^2 + 2*a*b*h
*(825*f*g + 275*e*h + 161*f*h*x) + b^2*(25*h*(36*e*g + 12*d*h + 7*e*h*x) +
 3*f*(300*g^2 + 175*g*h*x + 42*h^2*x^2))) - 16*c^3*(a*h*(5*h*(48*e*g + 16*
d*h + 9*e*h*x) + f*(240*g^2 + 135*g*h*x + 32*h^2*x^2)) + b*(3*f*(30*g^3 +
50*g^2*h*x + 35*g*h^2*x^2 + 9*h^3*x^3) + 5*h*(2*d*h*(27*g + 5*h*x) + e*(54
*g^2 + 30*g*h*x + 7*h^2*x^2))))) + 15*(-256*c^5*d*g^3 + 63*b^5*f*h^3 - 70*
b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) + 128*c^4*g*(a*f*g^2 + 3*a*h*(e*g +
d*h) + b*g*(e*g + 3*d*h)) + 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(3*f*g + e*h
) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) - 96*c^3*(a^2*h^2*(3*f*g + e*h) + b^2
*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*h))))*Log[b
 + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 709, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2184, 27, 1236, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3 \left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}} \, dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(g+hx)^3(bfg-10cdh+8afh+(2cfg-10ceh+9bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{5ch^2}+\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\int \frac{(g+hx)^3(bfg-10cdh+8afh+(2cfg-10ceh+9bfh)x)}{\sqrt{cx^2+bx+a}}dx}{10ch}$$

$$\Big\downarrow \text{1236}$$

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{\int -\frac{(g+hx)^2\left(9fghb^2+54afh^2b-2cg(3fg+5eh)b+4ch(20cdg-13afg-15aeh)+\left(-4\left(3fg^2-5h(3eg+4dh)\right)c^2-2h(24bfg+35beh+32afh)c+63b^2fh^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{(g\cdots}{\cdots}}{10ch}$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\int \frac{(g+hx)^2\left(9fghb^2+54afh^2b-2cg(3fg+5eh)b+4ch(20cdg-13afg-15aeh)+\left(-\left(\left(12fg^2-20h(3eg+4dh)\right)c^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}}}{8c}}{10ch}$$

$\downarrow$ 1236

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\int -\frac{(g+hx)\left(63fgh^2b^3+2\left(126afh^3-cgh(51fg+35eh)\right)b^2+4c\left(6cfg^3+10ch(3eg+2dh)g-5ah^2(29fg+14eh)\right)\cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c}-\frac{\int \frac{(g+hx)\left(63fg\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 1225

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c}-\frac{-\frac{15h\left(96c^3\left(a^2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 1092

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c}-\frac{-\frac{15h\left(96c^3\left(a^2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\downarrow$ 219

$$\frac{f(g+hx)^4\sqrt{a+bx+cx^2}}{5ch}-$$

$$\frac{\frac{(g+hx)^3\sqrt{a+bx+cx^2}(9bfh-10ceh+2cfg)}{4c}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}\left(-2ch(32afh+35beh+24bfg)+63b^2fh^2-4c^2\left(3fg^2-5h(4dh+3eg)\right)\right)}{3c}-\frac{-\frac{15h\text{arctan}\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*(g + h*x)^4*Sqrt[a + b*x + c*x^2])/(5*c*h) - (((2*c*f*g - 10*c*e*h + 9*
b*f*h)*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c) - (((63*b^2*f*h^2 - 2*c*h*
(24*b*f*g + 35*b*e*h + 32*a*f*h) - 4*c^2*(3*f*g^2 - 5*h*(3*e*g + 4*d*h)))*
(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4*((945*b^4*f*h^4 - 64*c^4*
(3*f*g^4 - 5*g^2*h*(3*e*g + 16*d*h)) - 210*b^2*c*h^3*(14*a*f*h + 5*b*(3*f*
g + e*h)) + 8*c^2*h^2*(128*a^2*f*h^2 + 275*a*b*h*(3*f*g + e*h) + 3*b^2*(12
9*f*g^2 + 50*h*(3*e*g + d*h))) - 16*c^3*h*(16*a*h*(13*f*g^2 + 5*h*(3*e*g +
 d*h)) + b*g*(39*f*g^2 + 5*h*(47*e*g + 54*d*h))) - 2*c*h*(315*b^3*f*h^3 -
14*b*c*h^2*(39*b*f*g + 25*b*e*h + 46*a*f*h) + c^3*(48*f*g^3 - 80*g*h*(3*e*
g + 10*d*h)) + 8*c^2*h*(21*b*f*g^2 + 10*b*h*(8*e*g + 5*d*h) + a*h*(71*f*g
 + 45*e*h)))*x)*Sqrt[a + b*x + c*x^2])/c^2 - (15*h*(256*c^5*d*g^3 - 63*b^5*
f*h^3 + 70*b^3*c*h^2*(3*b*f*g + b*e*h + 4*a*f*h) - 128*c^4*g*(a*f*g^2 + 3*
a*h*(e*g + d*h) + b*g*(e*g + 3*d*h)) - 80*b*c^2*h*(3*a^2*f*h^2 + 3*a*b*h*(
3*f*g + e*h) + b^2*(3*f*g^2 + 3*e*g*h + d*h^2)) + 96*c^3*(a^2*h^2*(3*f*g +
 e*h) + b^2*g*(f*g^2 + 3*h*(e*g + d*h)) + 2*a*b*h*(3*f*g^2 + h*(3*e*g + d*
h))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))
/(6*c))/(8*c))/(10*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.20

method	result
risch	$\frac{(384 f h^3 c^4 x^4 - 432 b c^3 f h^3 x^3 + 480 c^4 e h^3 x^3 + 1440 c^4 f g h^2 x^3 - 512 a c^3 f h^3 x^2 + 504 b^2 c^2 f h^3 x^2 - 560 b c^3 e h^3 x^2 - 1680 b c^3 f g h^2 x^2 + 64}{}$
default	Expression too large to display

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*c^4*f*h^3*x^4-432*b*c^3*f*h^3*x^3+480*c^4*e*h^3*x^3+1440*c^4*f
*g*h^2*x^3-512*a*c^3*f*h^3*x^2+504*b^2*c^2*f*h^3*x^2-560*b*c^3*e*h^3*x^2-1
680*b*c^3*f*g*h^2*x^2+640*c^4*d*h^3*x^2+1920*c^4*e*g*h^2*x^2+1920*c^4*f*g^
2*h*x^2+1288*a*b*c^2*f*h^3*x-720*a*c^3*e*h^3*x-2160*a*c^3*f*g*h^2*x-630*b^
3*c*f*h^3*x+700*b^2*c^2*e*h^3*x+2100*b^2*c^2*f*g*h^2*x-800*b*c^3*d*h^3*x-2
400*b*c^3*e*g*h^2*x-2400*b*c^3*f*g^2*h*x+2880*c^4*d*g*h^2*x+2880*c^4*e*g^2
*h*x+960*c^4*f*g^3*x+1024*a^2*c^2*f*h^3-2940*a*b^2*c*f*h^3+2200*a*b*c^2*e*
h^3+6600*a*b*c^2*f*g*h^2-1280*a*c^3*d*h^3-3840*a*c^3*e*g*h^2-3840*a*c^3*f*
g^2*h+945*b^4*f*h^3-1050*b^3*c*e*h^3-3150*b^3*c*f*g*h^2+1200*b^2*c^2*d*h^3
+3600*b^2*c^2*e*g*h^2+3600*b^2*c^2*f*g^2*h-4320*b*c^3*d*g*h^2-4320*b*c^3*e
*g^2*h-1440*b*c^3*f*g^3+5760*c^4*d*g^2*h+1920*c^4*e*g^3)*(c*x^2+b*x+a)^(1/
2)/c^5-1/256*(240*a^2*b*c^2*f*h^3-96*a^2*c^3*e*h^3-288*a^2*c^3*f*g*h^2-280
*a*b^3*c*f*h^3+240*a*b^2*c^2*e*h^3+720*a*b^2*c^2*f*g*h^2-192*a*b*c^3*d*h^3
-576*a*b*c^3*e*g*h^2-576*a*b*c^3*f*g^2*h+384*a*c^4*d*g*h^2+384*a*c^4*e*g^2
*h+128*a*c^4*f*g^3+63*b^5*f*h^3-70*b^4*c*e*h^3-210*b^4*c*f*g*h^2+80*b^3*c^
2*d*h^3+240*b^3*c^2*e*g*h^2+240*b^3*c^2*f*g^2*h-288*b^2*c^3*d*g*h^2-288*b^
2*c^3*e*g^2*h-96*b^2*c^3*f*g^3+384*b*c^4*d*g^2*h+128*b*c^4*e*g^3-256*c^5*d
*g^3)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1435, normalized size of antiderivative = 2.13

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/7680*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*c^3 - 4*a*c^4)*f)*g^3 - 48*
(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3*c^2 - 12*a*b*c^3)*f)*g^2*h
 + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2 - 12*a*b*c^3)*e + (35*b^4*
c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (16*(5*b^3*c^2 - 12*a*b*c^3)*d
- 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*e + (63*b^5 - 280*a*b^3*c + 24
0*a^2*b*c^2)*f)*h^3)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*f*h^3*x^4 + 480*(4*c
^5*e - 3*b*c^4*f)*g^3 + 240*(24*c^5*d - 18*b*c^4*e + (15*b^2*c^3 - 16*a*c^
4)*f)*g^2*h - 30*(144*b*c^4*d - 8*(15*b^2*c^3 - 16*a*c^4)*e + 5*(21*b^3*c^
2 - 44*a*b*c^3)*f)*g*h^2 + (80*(15*b^2*c^3 - 16*a*c^4)*d - 50*(21*b^3*c^2
- 44*a*b*c^3)*e + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*f)*h^3 + 48*
(30*c^5*f*g*h^2 + (10*c^5*e - 9*b*c^4*f)*h^3)*x^3 + 8*(240*c^5*f*g^2*h + 3
0*(8*c^5*e - 7*b*c^4*f)*g*h^2 + (80*c^5*d - 70*b*c^4*e + (63*b^2*c^3 - 64*
a*c^4)*f)*h^3)*x^2 + 2*(480*c^5*f*g^3 + 240*(6*c^5*e - 5*b*c^4*f)*g^2*h +
30*(48*c^5*d - 40*b*c^4*e + (35*b^2*c^3 - 36*a*c^4)*f)*g*h^2 - (400*b*c^4*
d - 10*(35*b^2*c^3 - 36*a*c^4)*e + 7*(45*b^3*c^2 - 92*a*b*c^3)*f)*h^3)*x)*
sqrt(c*x^2 + b*x + a))/c^6, -1/3840*(15*(32*(8*c^5*d - 4*b*c^4*e + (3*b^2*
c^3 - 4*a*c^4)*f)*g^3 - 48*(8*b*c^4*d - 2*(3*b^2*c^3 - 4*a*c^4)*e + (5*b^3
*c^2 - 12*a*b*c^3)*f)*g^2*h + 6*(16*(3*b^2*c^3 - 4*a*c^4)*d - 8*(5*b^3*c^2
 - 12*a*b*c^3)*e + (35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f)*g*h^2 - (...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1613 vs. $2(712) = 1424$.

Time = 1.36 (sec) , antiderivative size = 1613, normalized size of antiderivative = 2.39

$$\int \frac{(g + hx)^3 (d + ex + fx^2)}{\sqrt{a + bx + cx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**3*x**4/(5*c) + x**3*(-9*b*f*h**3/(
10*c) + e*h**3 + 3*f*g*h**2)/(4*c) + x**2*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f
*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2
*h)/(3*c) + x*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b
*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c)
 + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g
**3)/(2*c) + (-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 +
 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3*b*(-3*a*(
-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h**3/(5*c) -
 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**
2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c) + 3*d*g**2
*h + e*g**3)/c) + (-a*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*
c) - 5*b*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**
2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2
*h + f*g**3)/(2*c) - b*(-2*a*(-4*a*f*h**3/(5*c) - 7*b*(-9*b*f*h**3/(10*c)
+ e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 + 3*e*g*h**2 + 3*f*g**2*h)/(3*c) - 3
*b*(-3*a*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(4*c) - 5*b*(-4*a*f*h*
*3/(5*c) - 7*b*(-9*b*f*h**3/(10*c) + e*h**3 + 3*f*g*h**2)/(8*c) + d*h**3 +
 3*e*g*h**2 + 3*f*g**2*h)/(6*c) + 3*d*g*h**2 + 3*e*g**2*h + f*g**3)/(4*c)
+ 3*d*g**2*h + e*g**3)/(2*c) + d*g**3)*Piecewise((log(b + 2*sqrt(c)*sqr...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 798, normalized size of antiderivative = 1.18

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*f*h^3*x/c + (30*c^4*f*g*h^2 + 10*
c^4*e*h^3 - 9*b*c^3*f*h^3)/c^5)*x + (240*c^4*f*g^2*h + 240*c^4*e*g*h^2 - 2
10*b*c^3*f*g*h^2 + 80*c^4*d*h^3 - 70*b*c^3*e*h^3 + 63*b^2*c^2*f*h^3 - 64*a
*c^3*f*h^3)/c^5)*x + (480*c^4*f*g^3 + 1440*c^4*e*g^2*h - 1200*b*c^3*f*g^2*
h + 1440*c^4*d*g*h^2 - 1200*b*c^3*e*g*h^2 + 1050*b^2*c^2*f*g*h^2 - 1080*a*
c^3*f*g*h^2 - 400*b*c^3*d*h^3 + 350*b^2*c^2*e*h^3 - 360*a*c^3*e*h^3 - 315*
b^3*c*f*h^3 + 644*a*b*c^2*f*h^3)/c^5)*x + (1920*c^4*e*g^3 - 1440*b*c^3*f*g
^3 + 5760*c^4*d*g^2*h - 4320*b*c^3*e*g^2*h + 3600*b^2*c^2*f*g^2*h - 3840*a
*c^3*f*g^2*h - 4320*b*c^3*d*g*h^2 + 3600*b^2*c^2*e*g*h^2 - 3840*a*c^3*e*g*
h^2 - 3150*b^3*c*f*g*h^2 + 6600*a*b*c^2*f*g*h^2 + 1200*b^2*c^2*d*h^3 - 128
0*a*c^3*d*h^3 - 1050*b^3*c*e*h^3 + 2200*a*b*c^2*e*h^3 + 945*b^4*f*h^3 - 29
40*a*b^2*c*f*h^3 + 1024*a^2*c^2*f*h^3)/c^5) - 1/256*(256*c^5*d*g^3 - 128*b
*c^4*e*g^3 + 96*b^2*c^3*f*g^3 - 128*a*c^4*f*g^3 - 384*b*c^4*d*g^2*h + 288*
b^2*c^3*e*g^2*h - 384*a*c^4*e*g^2*h - 240*b^3*c^2*f*g^2*h + 576*a*b*c^3*f*
g^2*h + 288*b^2*c^3*d*g*h^2 - 384*a*c^4*d*g*h^2 - 240*b^3*c^2*e*g*h^2 + 57
6*a*b*c^3*e*g*h^2 + 210*b^4*c*f*g*h^2 - 720*a*b^2*c^2*f*g*h^2 + 288*a^2*c^
3*f*g*h^2 - 80*b^3*c^2*d*h^3 + 192*a*b*c^3*d*h^3 + 70*b^4*c*e*h^3 - 240*a*
b^2*c^2*e*h^3 + 96*a^2*c^3*e*h^3 - 63*b^5*f*h^3 + 280*a*b^3*c*f*h^3 - 240*
a^2*b*c^2*f*h^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b
))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(hx+g)^3 (f\,x^2+ex+d)}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.64 $\int \frac{(g+hx)^2(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 412

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{\left(8ce-7bf-\frac{2cfg}{h}\right)(g+hx)^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch} - \frac{\left(105b^3fh^2+\frac{32c^3(fg^3-4gh(eg+3dh))}{h}-20bch(11afh+6b(2fg+eh))+8c^2(11bfg^2+18bh(2eg+dh)+\right.}{} + \frac{\left(128c^4dg^2+35b^4fh^2-40b^2ch(2bfg+beh+3afh)-64c^3(afg^2+ah(2eg+dh)+bg(eg+2dh))+4\right.}{128c^{9/2}}$$

output
```
1/24*(8*c*e-7*b*f-2*c*f*g/h)*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*f*(h*x+
g)^3*(c*x^2+b*x+a)^(1/2)/c/h-1/192*(105*b^3*f*h^2+32*c^3*(f*g^3-4*g*h*(3*d
*h+e*g))/h-20*b*c*h*(11*a*f*h+6*b*(e*h+2*f*g))+8*c^2*(11*b*f*g^2+18*b*h*(d
*h+2*e*g)+16*a*h*(e*h+2*f*g))-2*c*(35*b^2*f*h^2-4*c*h*(9*a*f*h+10*b*e*h+6*
b*f*g)-8*c^2*(f*g^2-2*h*(3*d*h+2*e*g)))*x)*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(
128*c^4*d*g^2+35*b^4*f*h^2-40*b^2*c*h*(3*a*f*h+b*e*h+2*b*f*g)-64*c^3*(a*f*
g^2+a*h*(d*h+2*e*g)+b*g*(2*d*h+e*g))+48*c^2*(a^2*f*h^2+2*a*b*h*(e*h+2*f*g)
+b^2*(f*g^2+h*(d*h+2*e*g))))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.83

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}} \, dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3fh^2+10bch(22afh+b(24fg+12eh+7fhx))+16c^3(6dh(4g+hx)+4e(\cdots}{\cdots}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*f*h^2 + 10*b*c*h*(22*a*f*h + b*
(24*f*g + 12*e*h + 7*f*h*x)) + 16*c^3*(6*d*h*(4*g + h*x) + 4*e*(3*g^2 + 3*
g*h*x + h^2*x^2) + f*x*(6*g^2 + 8*g*h*x + 3*h^2*x^2)) - 8*c^2*(2*b*h*(18*e
*g + 9*d*h + 5*e*h*x) + a*h*(32*f*g + 16*e*h + 9*f*h*x) + b*f*(18*g^2 + 20
*g*h*x + 7*h^2*x^2))) + 3*(-128*c^4*d*g^2 - 35*b^4*f*h^2 + 40*b^2*c*h*(2*b
*f*g + b*e*h + 3*a*f*h) + 64*c^3*(a*f*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g +
 2*d*h)) - 48*c^2*(a^2*f*h^2 + 2*a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e
*g + d*h))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2
))
```

Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2184, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2 \left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}} \, dx$$
$$\Big\downarrow \ 2184$$
$$\frac{\int -\frac{h(g+hx)^2(bfg-8cdh+6afh+(2cfg-8ceh+7bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{4ch^2}+\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\int \frac{(g+hx)^2(bfg-8cdh+6afh+(2cfg-8ceh+7bfh)x)}{\sqrt{cx^2+bx+a}}dx}{8ch}$$

$\Big\downarrow$ 1236

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\frac{\int -\frac{(g+hx)\left(7fghb^2+4\left(7afh^2-cg(fg+2eh)\right)b+4ch(12cdg-7afg-8aeh)+\left(-8\left(fg^2-2h(2eg+3dh)\right)c^2-4h(6bfg+10beh+9afh)c+35b^2fh^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{(g+hx)^2}{}}{8ch}$$

$\Big\downarrow$ 27

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c}-\frac{\int \frac{(g+hx)\left(7fghb^2+28afh^2b-4cg(fg+2eh)b+4ch(12cdg-7afg-8aeh)+\left(-8\left(fg^2-2h(2eg+3dh)\right)c^2-4h(6bfg\right.\right.}{\sqrt{cx^2+bx+a}}}{6c}}{8ch}$$

$\Big\downarrow$ 1225

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c}-\frac{3h\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(ah(dh+2eg)+afg^2\right.\right.}{8c^2}}{}$$

$\Big\downarrow$ 1092

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c}-\frac{\frac{3h\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh+2bfg)-64c^3\left(ah(dh+2eg)+afg^2\right.\right.}{}}{4c^2}}{}$$

$\Big\downarrow$ 219

$$\frac{f(g+hx)^3\sqrt{a+bx+cx^2}}{4ch}-\frac{\frac{(g+hx)^2\sqrt{a+bx+cx^2}(7bfh-8ceh+2cfg)}{3c}-\frac{3h\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2fh^2+2abh(eh+2fg)+b^2\left(h(dh+2eg)+fg^2\right)\right)-40b^2ch(3afh+beh\right.}{8c^{5/2}}}{}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*(g + h*x)^3*Sqrt[a + b*x + c*x^2])/(4*c*h) - (((2*c*f*g - 8*c*e*h + 7*b
*f*h)*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4*((105*b^3*f*h^3 + 3
2*c^3*(f*g^3 - 4*g*h*(e*g + 3*d*h)) - 20*b*c*h^2*(11*a*f*h + 6*b*(2*f*g +
e*h)) + 8*c^2*h*(11*b*f*g^2 + 18*b*h*(2*e*g + d*h) + 16*a*h*(2*f*g + e*h))
 - 2*c*h*(35*b^2*f*h^2 - 4*c*h*(6*b*f*g + 10*b*e*h + 9*a*f*h) - 8*c^2*(f*g
^2 - 2*h*(2*e*g + 3*d*h)))*x)*Sqrt[a + b*x + c*x^2])/c^2 + (3*h*(128*c^4*d
*g^2 + 35*b^4*f*h^2 - 40*b^2*c*h*(2*b*f*g + b*e*h + 3*a*f*h) - 64*c^3*(a*f
*g^2 + a*h*(2*e*g + d*h) + b*g*(e*g + 2*d*h)) + 48*c^2*(a^2*f*h^2 + 2*a*b*
h*(2*f*g + e*h) + b^2*(f*g^2 + h*(2*e*g + d*h))))*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.06

method	result
risch	$\frac{(48fh^2c^3x^3-56bc^2fh^2x^2+64c^3eh^2x^2+128c^3fghx^2-72ac^2fh^2x+70b^2cfh^2x-80bc^2eh^2x-160bc^2fghx+96c^3dh^2x+192c^3eg\cdots}{\cdots}$
default	$\frac{dg^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}}+g(2dh+eg)\left(\frac{\sqrt{cx^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+h(eh+2fg)\Bigg(\cdots$

input

```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/192*(48*c^3*f*h^2*x^3-56*b*c^2*f*h^2*x^2+64*c^3*e*h^2*x^2+128*c^3*f*g*h*
x^2-72*a*c^2*f*h^2*x+70*b^2*c*f*h^2*x-80*b*c^2*e*h^2*x-160*b*c^2*f*g*h*x+9
6*c^3*d*h^2*x+192*c^3*e*g*h*x+96*c^3*f*g^2*x+220*a*b*c*f*h^2-128*a*c^2*e*h
^2-256*a*c^2*f*g*h-105*b^3*f*h^2+120*b^2*c*e*h^2+240*b^2*c*f*g*h-144*b*c^2
*d*h^2-288*b*c^2*e*g*h-144*b*c^2*f*g^2+384*c^3*d*g*h+192*c^3*e*g^2)*(c*x^2
+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*f*h^2-120*a*b^2*c*f*h^2+96*a*b*c^2*e*h
^2+192*a*b*c^2*f*g*h-64*a*c^3*d*h^2-128*a*c^3*e*g*h-64*a*c^3*f*g^2+35*b^4*
f*h^2-40*b^3*c*e*h^2-80*b^3*c*f*g*h+48*b^2*c^2*d*h^2+96*b^2*c^2*e*g*h+48*b
^2*c^2*f*g^2-128*b*c^3*d*g*h-64*b*c^3*e*g^2+128*c^4*d*g^2)/c^(9/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.09

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/768*(3*(16*(8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*
b*c^3*d - 2*(3*b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*
(3*b^2*c^2 - 4*a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2
*c + 48*a^2*c^2)*f)*h^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*h^2*x^3 + 48*(4
*c^4*e - 3*b*c^3*f)*g^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c
^3)*f)*g*h - (144*b*c^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44
*a*b*c^2)*f)*h^2 + 8*(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(4
8*c^4*f*g^2 + 16*(6*c^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*
b^2*c^2 - 36*a*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/384*(3*(16*(
8*c^4*d - 4*b*c^3*e + (3*b^2*c^2 - 4*a*c^3)*f)*g^2 - 16*(8*b*c^3*d - 2*(3*
b^2*c^2 - 4*a*c^3)*e + (5*b^3*c - 12*a*b*c^2)*f)*g*h + (16*(3*b^2*c^2 - 4*
a*c^3)*d - 8*(5*b^3*c - 12*a*b*c^2)*e + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2
)*f)*h^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f*h^2*x^3 + 48*(4*c^4*e - 3*b*c^3*f)*g
^2 + 16*(24*c^4*d - 18*b*c^3*e + (15*b^2*c^2 - 16*a*c^3)*f)*g*h - (144*b*c
^3*d - 8*(15*b^2*c^2 - 16*a*c^3)*e + 5*(21*b^3*c - 44*a*b*c^2)*f)*h^2 + 8*
(16*c^4*f*g*h + (8*c^4*e - 7*b*c^3*f)*h^2)*x^2 + 2*(48*c^4*f*g^2 + 16*(6*c
^4*e - 5*b*c^3*f)*g*h + (48*c^4*d - 40*b*c^3*e + (35*b^2*c^2 - 36*a*c^3)*f
)*h^2)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(423) = 846$.

Time = 1.12 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.21

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}} \, dx = \text{Too large to display}$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h**2*x**3/(4*c) + x**2*(-7*b*f*h**2/(
8*c) + e*h**2 + 2*f*g*h)/(3*c) + x*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2/(
8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) + (-2*a*
(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c) - 5*
b*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2
)/(4*c) + 2*d*g*h + e*g**2)/c) + (-a*(-3*a*f*h**2/(4*c) - 5*b*(-7*b*f*h**2
/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*g**2)/(2*c) - b*(-
2*a*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(3*c) - 3*b*(-3*a*f*h**2/(4*c)
- 5*b*(-7*b*f*h**2/(8*c) + e*h**2 + 2*f*g*h)/(6*c) + d*h**2 + 2*e*g*h + f*
g**2)/(4*c) + 2*d*g*h + e*g**2)/(2*c) + d*g**2)*Piecewise((log(b + 2*sqrt(
c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2
*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*
(f*h**2*(a + b*x)**(9/2)/(9*b**4) + (a + b*x)**(7/2)*(-4*a*f*h**2 + b*e*h*
*2 + 2*b*f*g*h)/(7*b**4) + (a + b*x)**(5/2)*(6*a**2*f*h**2 - 3*a*b*e*h**2
- 6*a*b*f*g*h + b**2*d*h**2 + 2*b**2*e*g*h + b**2*f*g**2)/(5*b**4) + (a +
b*x)**(3/2)*(-4*a**3*f*h**2 + 3*a**2*b*e*h**2 + 6*a**2*b*f*g*h - 2*a*b**2*
d*h**2 - 4*a*b**2*e*g*h - 2*a*b**2*f*g**2 + 2*b**3*d*g*h + b**3*e*g**2)/(3
*b**4) + sqrt(a + b*x)*(a**4*f*h**2 - a**3*b*e*h**2 - 2*a**3*b*f*g*h + a**
2*b**2*d*h**2 + 2*a**2*b**2*e*g*h + a**2*b**2*f*g**2 - 2*a*b**3*d*g*h - a*
b**3*e*g**2 + b**4*d*g**2)/b**4)/b, Ne(b, 0)), ((d*g**2*x + f*h**2*x**5...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.07

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,fh^2x}{c}+\frac{16\,c^3fgh+8\,c^3eh^2-7\,bc^2fh^2}{c^4}\right)x+\frac{48\,c^3fg^2+96\,c^3egh-80\,bc^2f\right.\right.\right.$$
$$-\frac{(128\,c^4dg^2-64\,bc^3eg^2+48\,b^2c^2fg^2-64\,ac^3fg^2-128\,bc^3dgh+96\,b^2c^2egh-128\,ac^3egh-80\,b^3cfgh}{}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*h^2*x/c + (16*c^3*f*g*h + 8*c^3*e*h
^2 - 7*b*c^2*f*h^2)/c^4)*x + (48*c^3*f*g^2 + 96*c^3*e*g*h - 80*b*c^2*f*g*h
 + 48*c^3*d*h^2 - 40*b*c^2*e*h^2 + 35*b^2*c*f*h^2 - 36*a*c^2*f*h^2)/c^4)*x
 + (192*c^3*e*g^2 - 144*b*c^2*f*g^2 + 384*c^3*d*g*h - 288*b*c^2*e*g*h + 24
0*b^2*c*f*g*h - 256*a*c^2*f*g*h - 144*b*c^2*d*h^2 + 120*b^2*c*e*h^2 - 128*
a*c^2*e*h^2 - 105*b^3*f*h^2 + 220*a*b*c*f*h^2)/c^4) - 1/128*(128*c^4*d*g^2
 - 64*b*c^3*e*g^2 + 48*b^2*c^2*f*g^2 - 64*a*c^3*f*g^2 - 128*b*c^3*d*g*h +
96*b^2*c^2*e*g*h - 128*a*c^3*e*g*h - 80*b^3*c*f*g*h + 192*a*b*c^2*f*g*h +
48*b^2*c^2*d*h^2 - 64*a*c^3*d*h^2 - 40*b^3*c*e*h^2 + 96*a*b*c^2*e*h^2 + 35
*b^4*f*h^2 - 120*a*b^2*c*f*h^2 + 48*a^2*c^2*f*h^2)*log(abs(2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\left(hx+g\right)^{2} \left(f\,x^{2}+ex+d\right)}{\sqrt{c\,x^{2}+bx+a}}dx$$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.65 $\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 217

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx = \frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch} + \frac{\left(15b^2fh+c^2\left(24eg-\frac{8fg^2}{h}+24dh\right)-2c(8afh+9b(fg+eh))-2c(2cfg-6ceh+5bfh)x\right)\sqrt{a+bx+cx^2}}{24c^3} + \frac{(16c^3dg-5b^3fh-8c^2(beg+afg+bdh+aeh)+6bc(bfg+beh+2afh))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}$$

output
```
1/3*f*(h*x+g)^2*(c*x^2+b*x+a)^(1/2)/c/h+1/24*(15*b^2*f*h+c^2*(24*e*g-8*f*g
^2/h+24*d*h)-2*c*(8*a*f*h+9*b*(e*h+f*g))-2*c*(5*b*f*h-6*c*e*h+2*c*f*g)*x)*
(c*x^2+b*x+a)^(1/2)/c^3+1/16*(16*c^3*d*g-5*b^3*f*h-8*c^2*(a*e*h+a*f*g+b*d*
h+b*e*g)+6*b*c*(2*a*f*h+b*e*h+b*f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.82

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2fh+4c^2(6eg+6dh+3fgx+3ehx+2fhx^2)-2c(8afh+b(9fg+9eh+5f}{}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*f*h + 4*c^2*(6*e*g + 6*d*h + 3*f*
g*x + 3*e*h*x + 2*f*h*x^2) - 2*c*(8*a*f*h + b*(9*f*g + 9*e*h + 5*f*h*x)))
+ 3*(-16*c^3*d*g + 5*b^3*f*h + 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) - 6*b
*c*(b*f*g + b*e*h + 2*a*f*h))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*
x)]])/(48*c^(7/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2184, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2184$$
$$\frac{\int -\frac{h(g+hx)(bfg-6cdh+4afh+(2cfg-6ceh+5bfh)x)}{2\sqrt{cx^2+bx+a}}dx}{3ch^2}+\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}$$
$$\Big\downarrow \ 27$$
$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-\frac{\int \frac{(g+hx)(bfg-6cdh+4afh+(2cfg-6ceh+5bfh)x)}{\sqrt{cx^2+bx+a}}dx}{6ch}$$

$$\Big\downarrow \text{1225}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-\frac{-\frac{3h\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg))+15b^2fh^2\right.}{4}}{6ch}$$

$$\Big\downarrow \text{1092}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-\frac{-\frac{3h\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}-\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+f\right.}{}}{6ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f(g+hx)^2\sqrt{a+bx+cx^2}}{3ch}-\frac{-\frac{3h\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(aeh+afg+bdh+beg)+6bc(2afh+beh+bfg)-5b^3fh+16c^3dg\right)}{8c^{5/2}}-\frac{\sqrt{a+bx+cx^2}\left(-2ch(8afh+9b(eh+fg\right.}{}}{6ch}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c*h) - (-1/4*((15*b^2*f*h^2 - 8*c
^2*(f*g^2 - 3*h*(e*g + d*h)) - 2*c*h*(8*a*f*h + 9*b*(f*g + e*h)) - 2*c*h*(
2*c*f*g - 6*c*e*h + 5*b*f*h)*x)*Sqrt[a + b*x + c*x^2])/c^2 - (3*h*(16*c^3*
d*g - 5*b^3*f*h - 8*c^2*(b*e*g + a*f*g + b*d*h + a*e*h) + 6*b*c*(b*f*g + b
*e*h + 2*a*f*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)))/(6*c*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\left(-8hf\,c^{2}x^{2}+10bcfhx-12c^{2}ehx-12c^{2}fgx+16acfh-15b^{2}fh+18bceh+18bcfg-24c^{2}dh-24c^{2}eg\right)\sqrt{c\,x^{2}+bx+a}}{24c^{3}}+\frac{(12abcfh-8\ldots}{}$
default	$\frac{dg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+(dh+eg)\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+(eh+fg)\left(\frac{x\sqrt{c\,x^{2}+\ldots}}{2c}\right.$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*c^2*f*h*x^2+10*b*c*f*h*x-12*c^2*e*h*x-12*c^2*f*g*x+16*a*c*f*h-15
*b^2*f*h+18*b*c*e*h+18*b*c*f*g-24*c^2*d*h-24*c^2*e*g)*(c*x^2+b*x+a)^(1/2)/
c^3+1/16*(12*a*b*c*f*h-8*a*c^2*e*h-8*a*c^2*f*g-5*b^3*f*h+6*b^2*c*e*h+6*b^2
*c*f*g-8*b*c^2*d*h-8*b*c^2*e*g+16*c^3*d*g)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+
(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.12

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$

$$=\left[\frac{3\left(2\left(8\,c^3d-4\,bc^2e+\left(3\,b^2c-4\,ac^2\right)f\right)g-\left(8\,bc^2d-2\left(3\,b^2c-4\,ac^2\right)e+\left(5\,b^3-12\,abc\right)f\right)h\right)\sqrt{c}\log\left(-8\ldots\right.}{}\right.$$

$$-\frac{3\left(2\left(8\,c^3d-4\,bc^2e+\left(3\,b^2c-4\,ac^2\right)f\right)g-\left(8\,bc^2d-2\left(3\,b^2c-4\,ac^2\right)e+\left(5\,b^3-12\,abc\right)f\right)h\right)\sqrt{-c}\,\mathrm{arcta}\ldots}{}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*c^2)*f)*g - (8*b*c^2*d -
 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h)*sqrt(c)*log(-8*c^2*x^2
 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) +
4*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d - 18*b*c^2*e + (1
5*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b*c^2*f)*h)*x)*sqrt
(c*x^2 + b*x + a))/c^4, -1/48*(3*(2*(8*c^3*d - 4*b*c^2*e + (3*b^2*c - 4*a*
c^2)*f)*g - (8*b*c^2*d - 2*(3*b^2*c - 4*a*c^2)*e + (5*b^3 - 12*a*b*c)*f)*h
)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
+ b*c*x + a*c)) - 2*(8*c^3*f*h*x^2 + 6*(4*c^3*e - 3*b*c^2*f)*g + (24*c^3*d
 - 18*b*c^2*e + (15*b^2*c - 16*a*c^2)*f)*h + 2*(6*c^3*f*g + (6*c^3*e - 5*b
*c^2*f)*h)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.01

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{fhx^2}{3c}+\frac{x\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}+\frac{-\frac{2afh}{3c}-\frac{3b\left(-\frac{5bfh}{6c}+eh+fg\right)}{4c}+dh+eg}{c}\right)+\left(-\frac{a\left(-\frac{5bfh}{6c}+eh+fg\right)}{2c}-\frac{b\left(-\frac{2afh}{3c}\right.}{}\right. \\ \frac{2\left(\frac{fh(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}(-3afh+beh+bfg)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\cdot\left(3a^2fh-2abeh-2abfg+b^2dh+b^2eg\right)}{3b^3}+\frac{\sqrt{a+bx}\left(-a^3fh+a^2beh+a^2bfg-ab^2dh-ab^2eg+b^3dg\right)}{b^3}\right)}{b} \\ \frac{dgx+\frac{fhx^4}{4}+\frac{x^3(eh+fg)}{3}+\frac{x^2(dh+eg)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(f*h*x**2/(3*c) + x*(-5*b*f*h/(6*c) + e*
h + f*g)/(2*c) + (-2*a*f*h/(3*c) - 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c)
+ d*h + e*g)/c) + (-a*(-5*b*f*h/(6*c) + e*h + f*g)/(2*c) - b*(-2*a*f*h/(3*
c) - 3*b*(-5*b*f*h/(6*c) + e*h + f*g)/(4*c) + d*h + e*g)/(2*c) + d*g)*Piec
ewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b
**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2),
True)), Ne(c, 0)), (2*(f*h*(a + b*x)**(7/2)/(7*b**3) + (a + b*x)**(5/2)*(-
3*a*f*h + b*e*h + b*f*g)/(5*b**3) + (a + b*x)**(3/2)*(3*a**2*f*h - 2*a*b*e
*h - 2*a*b*f*g + b**2*d*h + b**2*e*g)/(3*b**3) + sqrt(a + b*x)*(-a**3*f*h
+ a**2*b*e*h + a**2*b*f*g - a*b**2*d*h - a*b**2*e*g + b**3*d*g)/b**3)/b, N
e(b, 0)), ((d*g*x + f*h*x**4/4 + x**3*(e*h + f*g)/3 + x**2*(d*h + e*g)/2)/
sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\begin{aligned}&\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx\\&=\frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,fhx}{c}+\frac{6\,c^2fg+6\,c^2eh-5\,bcfh}{c^3}\right)x+\frac{24\,c^2eg-18\,bcfg+24\,c^2dh-18\,bceh+}{c^3}\right.\\&\quad-\frac{(16\,c^3dg-8\,bc^2eg+6\,b^2cfg-8\,ac^2fg-8\,bc^2dh+6\,b^2ceh-8\,ac^2eh-5\,b^3fh+12\,abcfh)\log\left(\left|2\left(\sqrt{\right.\right.}{16\,c^{\frac{7}{2}}}\end{aligned}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*f*h*x/c + (6*c^2*f*g + 6*c^2*e*h - 5*b*c*
f*h)/c^3)*x + (24*c^2*e*g - 18*b*c*f*g + 24*c^2*d*h - 18*b*c*e*h + 15*b^2*
f*h - 16*a*c*f*h)/c^3) - 1/16*(16*c^3*d*g - 8*b*c^2*e*g + 6*b^2*c*f*g - 8*
a*c^2*f*g - 8*b*c^2*d*h + 6*b^2*c*e*h - 8*a*c^2*e*h - 5*b^3*f*h + 12*a*b*c
*f*h)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(g+h\,x)\,(f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.80

$$\int \frac{(g+hx)(d+ex+fx^2)}{\sqrt{a+bx+cx^2}}\,dx = \frac{-32\sqrt{c\,x^2+bx+a}\,a\,c^2fh+30\sqrt{c\,x^2+bx+a}\,b^2cfh-36\sqrt{c\,x^2+bx+a}\,b\,c^2eh-36\sqrt{c\,x^2+bx+a}\,b\,c^2}{}$$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 32*sqrt(a + b*x + c*x**2)*a*c**2*f*h + 30*sqrt(a + b*x + c*x**2)*b**2*
c*f*h - 36*sqrt(a + b*x + c*x**2)*b*c**2*e*h - 36*sqrt(a + b*x + c*x**2)*b
*c**2*f*g - 20*sqrt(a + b*x + c*x**2)*b*c**2*f*h*x + 48*sqrt(a + b*x + c*x
**2)*c**3*d*h + 48*sqrt(a + b*x + c*x**2)*c**3*e*g + 24*sqrt(a + b*x + c*x
**2)*c**3*e*h*x + 24*sqrt(a + b*x + c*x**2)*c**3*f*g*x + 16*sqrt(a + b*x +
 c*x**2)*c**3*f*h*x**2 + 36*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2)
+ b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*f*h - 24*sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**2*e*h - 24*sqrt(
c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*
a*c**2*f*g - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
/sqrt(4*a*c - b**2))*b**3*f*h + 18*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*e*h + 18*sqrt(c)*log((2*sqr
t(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*f*g -
24*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*b*c**2*d*h - 24*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
+ 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*e*g + 48*sqrt(c)*log((2*sqrt(c)*sqrt(a
 + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*c**3*d*g)/(48*c**4)
```

3.66 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} \, dx = \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{(8c^2d+3b^2f-4c(be+af))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/4*(-3*b*f+4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*f*x*(c*x^2+b*x+a)^(1/2)/c+1
/8*(8*c^2*d+3*b^2*f-4*c*(a*f+b*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*
x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} \, dx = \frac{\sqrt{c}(4ce-3bf+2cfx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4c(be+af))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{4c^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[c]*(4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^
2*f - 4*c*(b*e + a*f))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x
)])])/(4*c^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{\int \frac{4cd-2af+(4ce-3bf)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2(2cd-af)+(4ce-3bf)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{(-4c(af+be)+3b^2f+8c^2d)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-4c(af+be)+3b^2f+8c^2d)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{c}}{4c}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(f*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2
])/c + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-2cfx+3bf-4ce)\sqrt{c\,x^{2}+bx+a}}{4c^{2}}-\frac{(4acf-3b^{2}f+4bce-8c^{2}d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{5}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+e\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+f\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+c}}{c}\right.}{}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*c*f*x+3*b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c^2-1/8*(4*a*c*f-3*b^2*f+4
*b*c*e-8*c^2*d)/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

$$=\left[-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)}{16\,c^3}\right.$$

$$-\frac{(8\,c^2d-4\,bce+(3\,b^2-4\,ac)f)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(2\,c^2fx+4\,c^2e-3\,bcf)\sqrt{cx^2+}}{8\,c^3}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2
*c^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d -
 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b
*c*f)*sqrt(c*x^2 + b*x + a))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(102) = 204$.

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.95

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \begin{cases} \left(\frac{fx}{2c}+\frac{-\frac{3bf}{4c}+e}{c}\right)\sqrt{a+bx+cx^2}+\left(-\frac{af}{2c}-\frac{b\left(-\frac{3bf}{4c}+e\right)}{2c}+d\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } a-\frac{b^2}{4c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((f*x/(2*c) + (-3*b*f/(4*c) + e)/c)*sqrt(a + b*x + c*x**2) + (-a
*f/(2*c) - b*(-3*b*f/(4*c) + e)/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sq
rt(a + b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) +
 x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*d*sq
rt(a + b*x) + 2*e*(-a*sqrt(a + b*x) + (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sq
rt(a + b*x) - 2*a*(a + b*x)**(3/2)/3 + (a + b*x)**(5/2)/5)/b**2)/b, Ne(b,
0)), ((d*x + e*x**2/2 + f*x**3/3)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}+\frac{4\,ce-3\,bf}{c^2}\right) - \frac{(8\,c^2d-4\,bce+3\,b^2f-4\,acf)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c + (4*c*e - 3*b*f)/c^2) - 1/8*(8*c^2*d -
 4*b*c*e + 3*b^2*f - 4*a*c*f)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a)
)*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.99

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{-6\sqrt{c\,x^2+bx+a}\,bcf+8\sqrt{c\,x^2+bx+a}\,c^2e+4\sqrt{c\,x^2+bx+a}\,c^2fx-4\sqrt{c}\log\left(\frac{2\sqrt{c}\,\sqrt{c\,x^2+bx+a}+b+2cx}{\sqrt{4ac-b^2}}\right)a}{}$$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x + c*x**2)*b*c*f + 8*sqrt(a + b*x + c*x**2)*c**2*e + 4*s
qrt(a + b*x + c*x**2)*c**2*f*x - 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*f + 3*sqrt(c)*log((2*sqrt(c)*s
qrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*f - 4*sqrt(c)*
log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b*c
*e + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a
*c - b**2))*c**2*d)/(8*c**3)
```

3.67 $\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{f\sqrt{a+bx+cx^2}}{ch} - \frac{(2cfg-2ceh+bfh)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}h^2} + \frac{(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{h^2\sqrt{cg^2-bgh+ah^2}}$$

output
```
f*(c*x^2+b*x+a)^(1/2)/c/h-1/2*(b*f*h-2*c*e*h+2*c*f*g)*arctanh(1/2*(2*c*x+b
)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/h^2+(f*g^2-h*(-d*h+e*g))*arctanh(1/
2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2)
)/h^2/(a*h^2-b*g*h+c*g^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{2fh\sqrt{a+x(b+cx)}}{c}+\frac{4\sqrt{-cg^2+bgh-ah^2}(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{cg^2+h(-bg+ah)}-\frac{(2cfg-2ceh+bfh)\text{arctanh}\left(\frac{b+2c}{2\sqrt{c}\sqrt{a+x}}\right.}{c^{3/2}}}{2h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*f*h*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*g^2) + b*g*h - a*h^2]*(f*g^
2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])
/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 + h*(-(b*g) + a*h)) - ((2*c*f*g -
 2*c*e*h + b*f*h)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
c^(3/2))/(2*h^2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{h(bfg-2cdh+(2cfg-2ceh+bfh)x)}{2(g+hx)\sqrt{cx^2+bx+a}}dx}{ch^2}+\frac{f\sqrt{a+bx+cx^2}}{ch}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\int \frac{bfg-2cdh+(2cfg-2ceh+bfh)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2ch}$$

$$\Big\downarrow \text{1269}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{(bfh-2ceh+2cfg)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \text{1092}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{2(bfh-2ceh+2cfg)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}-\frac{2c(dh^2-egh+fg^2)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h}}{2ch}$$

$$\Big\downarrow \text{1154}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{4c(dh^2-egh+fg^2)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{h}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}}{2ch}$$

$$\Big\downarrow \text{219}$$

$$\frac{f\sqrt{a+bx+cx^2}}{ch}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bfh-2ceh+2cfg)}{\sqrt{c}h}-\frac{2c(dh^2-egh+fg^2)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{h\sqrt{ah^2-bgh+cg^2}}}{2ch}$$

input

```
Int[(d + e*x + f*x^2)/((g + h*x)*Sqrt[a + b*x + c*x^2]),x]
```

output

```
(f*Sqrt[a + b*x + c*x^2])/(c*h) - (((2*c*f*g - 2*c*e*h + b*f*h)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h) - (2*c*(f*g^2 -
e*g*h + d*h^2)*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g
*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(h*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*c
*h)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.40

method	result
risch	$\frac{f\sqrt{c x^2+bx+a}}{ch}-\frac{\frac{(bfh-2che+2cfg)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{h\sqrt{c}}+\frac{2\left(d h^2-egh+f g^2\right)c\ln\left(\frac{\frac{2a h^2-2bgh+2c g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{a h^2-\cdots}}{\cdots}\right)}{h^2\sqrt{\frac{a h^2-bgh+\cdots}{h^2}}}}{2hc}\cdots$
default	$\frac{\frac{eh\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}+fh\left(\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{fg\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}}{h^2}-\frac{\left(d h^2-egh+f g^2\right)\ln\cdots}{\cdots}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f*(c*x^2+b*x+a)^(1/2)/c/h-1/2/h/c*((b*f*h-2*c*e*h+2*c*f*g)/h*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*(d*h^2-e*g*h+f*g^2)*c/h^2/((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 12390, normalized size of antiderivative = 69.22

$$\int \frac{d+ex+fx^2}{(g+hx)\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*
h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g
**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sq
rt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g
*h - 8*c**2*g**2))*b*c**2*d*h**3 + 2*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h +
c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b
**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*
sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h*
*2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4
*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2))*b*c**2*e*g*h**2 - 2*sqrt
(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g
*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(
a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*h + b*h +
2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*b*h - 8*sqrt(c)*sqrt
(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2 + 8*b*c*g*h - 8*c**
2*g**2))*b*c**2*f*g**2*h + 4*sqrt(4*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*
b*h - 8*sqrt(c)*sqrt(a*h**2 - b*g*h + c*g**2)*c*g - 4*a*c*h**2 - b**2*h**2
 + 8*b*c*g*h - 8*c**2*g**2)*sqrt(a*h**2 - b*g*h + c*g**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2)*h + b*h + 2*c*h*x)/sqrt(4*sqrt(c)*sqrt(a*h**2 - ...
```

3.68 $\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 239

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{h(cg^2-bgh+ah^2)(g+hx)}+\frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}h^2} - \frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2h^2(cg^2-bgh+ah^2)^{3/2}}$$

output
```
-(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g)+f*
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/h^2-1/2*(2*c*(-
d*g*h^2+f*g^3)-h*(3*b*f*g^2-b*h*(d*h+e*g)-2*a*h*(-e*h+2*f*g)))*arctanh(1/2
*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))
/h^2/(a*h^2-b*g*h+c*g^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.61 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{\sqrt{-cg^2+bgh-ah^2}(2c(fg^3-dgh^2)+h(-3bfg^2+bh(eg+dh)-2ah(-2fg+eh)))\arctan\left(\frac{\sqrt{c}(g+hx)-h}{\sqrt{-cg^2+}}\right.}{(cg^2+h(-bg+ah))^2}}{h^2}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g
) + a*h))*(g + h*x)) + (Sqrt[-(c*g^2) + b*g*h - a*h^2]*(2*c*(f*g^3 - d*g*h
^2) + h*(-3*b*f*g^2 + b*h*(e*g + d*h) - 2*a*h*(-2*f*g + e*h)))*ArcTan[(Sqr
t[c]*(g + h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])
/(c*g^2 + h*(-(b*g) + a*h))^2 + (f*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
 + c*x)]])/Sqrt[c])/h^2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2181, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{\int -\frac{\frac{bfg^2}{h}+2cdg-beg-2afg-bdh+2aeh-2f\left(-\frac{cg^2}{h}+bg-ah\right)x}{2(g+hx)\sqrt{cx^2+bx+a}}dx}{ah^2-bgh+cg^2}-\frac{\sqrt{a+bx+cx^2}\,(fg^2-h(eg-dh))}{h(g+hx)\,(ah^2-bgh+cg^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2cdg-2afg+2aeh-b\left(-\frac{fg^2}{h}+eg+dh\right)-2f\left(-\frac{cg^2}{h}+bg-ah\right)x}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{2f\left(ah^2-bgh+cg^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{h^2}-\frac{\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1092$$

$$\frac{\frac{4f\left(ah^2-bgh+cg^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{h^2}-\frac{\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(ah^2-bgh+cg^2\right)}{\sqrt{c}h^2}-\frac{\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{h^2}}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{2\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int \frac{1}{4\left(cg^2-bhg+ah^2\right)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{h^2}+\frac{2f\text{arctanh}}{}}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{2f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(ah^2-bgh+cg^2)}{\sqrt{c}h^2}-\frac{\operatorname{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+eg)+}{h^2\sqrt{ah^2-bgh+cg^2}}}{2\left(ah^2-bgh+cg^2\right)}$$
$$\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{h(g+hx)\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a*h^
2)*(g + h*x))) + ((2*f*(c*g^2 - b*g*h + a*h^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*h^2) - ((2*c*(f*g^3 - d*g*h^2) - h*(
3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*ArcTanh[(b*g - 2*a*h +
 (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/
(h^2*Sqrt[c*g^2 - b*g*h + a*h^2]))/(2*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(219) = 438$.

Time = 0.29 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.03

method	result
default	$\frac{f \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{h^2\sqrt{c}}-\frac{(eh-2fg)\ln\left(\frac{\frac{2a\,h^2-2bgh+2c\,g^2}{h^2}+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}+2\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}\sqrt{\left(x+\frac{g}{h}\right)^2c+\frac{(bh-2cg)\left(x+\frac{g}{h}\right)}{h}}}{x+\frac{g}{h}}\right.}{h^3\sqrt{\frac{a\,h^2-bgh+c\,g^2}{h^2}}}$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
f/h^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/h^3*(e*h-2*f*g
)/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g
)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(
x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+1/h^4*(d*h^2-e*g*h+f*g^2)*
(-1/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-
b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h
)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*
h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^2\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 2284, normalized size of antiderivative = 9.56

$$\int \frac{d+ex+fx^2}{(g+hx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
- b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c*e*g*h**3 + 2*sqrt(a
*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h +
 c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c*e*h**4*x - 4*sqrt(a*h**2 - b
*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2)
- 2*a*h + b*g - b*h*x + 2*c*g*x)*a*c*f*g**2*h**2 - 4*sqrt(a*h**2 - b*g*h +
 c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*
h + b*g - b*h*x + 2*c*g*x)*a*c*f*g*h**3*x - sqrt(a*h**2 - b*g*h + c*g**2)*
log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g -
 b*h*x + 2*c*g*x)*b*c*d*g*h**3 - sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(
a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*
c*g*x)*b*c*d*h**4*x - sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c
*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c*
e*g**2*h**2 - sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*s
qrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c*e*g*h**3
*x + 3*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h
**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c*f*g**3*h + 3*sq
rt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g
*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*b*c*f*g**2*h**2*x + 2*sqrt(a
*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*...
```

3.69 $\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 336

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = -\frac{(fg^2-h(eg-dh))\sqrt{a+bx+cx^2}}{2h(cg^2-bgh+ah^2)(g+hx)^2} + \frac{(2c(fg^3+gh(eg-3dh))-h(5bfg^2-bh(eg+3dh)-4ah(2fg-eh)))\sqrt{a+bx+cx^2}}{4h(cg^2-bgh+ah^2)^2(g+hx)} + \frac{(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)-4c(afg^2-ah(3eg-dh)+bg(eg+2dh))+b^2(3fg^2+h(eg+}{8(cg^2-bgh+ah^2)^{5/2}}$$

output
```
-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)/(h*x+g
)^2+1/4*(2*c*(f*g^3+g*h*(-3*d*h+e*g))-h*(5*b*f*g^2-b*h*(3*d*h+e*g)-4*a*h*(
-e*h+2*f*g)))*(c*x^2+b*x+a)^(1/2)/h/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)+1/8*(8*c
^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g)-4*c*(a*f*g^2-a*h*(-d*h+3*e*g)+b*g
*(2*d*h+e*g))+b^2*(3*f*g^2+h*(3*d*h+e*g)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*
c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)
^(5/2)
```

Mathematica [A] (verified)

Time = 11.83 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.40

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{4h(fg^2+h(-eg+dh))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)^2}+\frac{8h(-2fg+eh)\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}+\frac{4(2cg-bh)(2fg-eh)\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}}{}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/8*((4*h*(f*g^2 + h*(-(e*g) + d*h))*Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(
-(b*g) + a*h))*(g + h*x)^2) + (8*h*(-2*f*g + e*h)*Sqrt[a + x*(b + c*x)])/(
(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (4*(2*c*g - b*h)*(2*f*g - e*h)*Arc
Tanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sq
rt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2) - (8*f*ArcTanh[(-2
*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*g^2 + h*(-(b*g) + a*h)] + ((f*g^2 + h*(-(e*g) + d*h)
)*(6*h*(2*c*g - b*h)*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)]
- (8*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(2*b*g + a*h))*(g + h*x)*ArcTanh[(-2*a*h
+ 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
 c*x)])]))/((c*g^2 + h*(-(b*g) + a*h))^(5/2)*(g + h*x)))/h^2
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int -\frac{\frac{bfg^2}{h}+4cdg-beg-4afg-3bdh+4aeh+2\left(\frac{cfg^2}{h}+ceg-2bfg-cdh+2afh\right)x}{2(g+hx)^2\sqrt{cx^2+bx+a}}dx}{2\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{4cdg-4afg+4aeh-b\left(-\frac{fg^2}{h}+eg+3dh\right)-2\left(2bfg-2afh-c\left(\frac{fg^2}{h}+eg-dh\right)\right)x}{(g+hx)^2\sqrt{cx^2+bx+a}}dx}{4\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1228

$$\frac{\frac{\left(8a^2fh^2-4c(-ah(3eg-dh)+afg^2+bg(2dh+eg))-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{2(ah^2-bgh+cg^2)}+\frac{\sqrt{a+bx+cx^2}(2c}{}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\sqrt{a+bx+cx^2}\left(2c\left(gh(eg-3dh)+fg^3\right)-h\left(-4ah(2fg-eh)-bh(3dh+eg)+5bfg^2\right)\right)}{h(g+hx)(ah^2-bgh+cg^2)}-\frac{\left(8a^2fh^2-4c(-ah(3eg-dh)+afg^2+bg(2dh+eg))-4abh(eh-\right.}{}}{4\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

$\Big\downarrow$ 219

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(8a^2fh^2-4c(-ah(3eg-dh)+afg^2+bg(2dh+eg))-4abh(eh+2fg)+b^2\left(h(3dh+eg)+3fg^2\right)+8c^2dg^2\right)}{2(ah^2-bgh+cg^2)^{3/2}}+}{4\left(ah^2-bgh+cg^2\right)}-\frac{\sqrt{a+bx+cx^2}\left(fg^2-h(eg-dh)\right)}{2h(g+hx)^2\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2 - b*g*h + a
*h^2)*(g + h*x)^2) + (((2*c*(f*g^3 + g*h*(e*g - 3*d*h)) - h*(5*b*f*g^2 - b
*h*(e*g + 3*d*h) - 4*a*h*(2*f*g - e*h)))*Sqrt[a + b*x + c*x^2])/(h*(c*g^2
- b*g*h + a*h^2)*(g + h*x)) + ((8*c^2*d*g^2 + 8*a^2*f*h^2 - 4*a*b*h*(2*f*g
 + e*h) - 4*c*(a*f*g^2 - a*h*(3*e*g - d*h) + b*g*(e*g + 2*d*h)) + b^2*(3*f
*g^2 + h*(e*g + 3*d*h)))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c
*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*(c*g^2 - b*g*h + a*h^2)^
(3/2)))/(4*(c*g^2 - b*g*h + a*h^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. $2(318) = 636$.

Time = 0.35 (sec) , antiderivative size = 1013, normalized size of antiderivative = 3.01

method	result	size
default	Expression too large to display	1013

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-f/h^3/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-
2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g
)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h))+(e*h-2*f*g)/h^4*(-1/(
a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*(
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b
*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+(d*h^2-e*g*h+f*g^2)/h^5*(-1/2/(a*h^2-b*g
*h+c*g^2)*h^2/(x+g/h)^2*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)-3/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(-1/(a*h^2-b*g*h+c*g
^2)*h^2/(x+g/h)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2)+1/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)/((a*h^2-b*g*h+c*g^2)/h^2)^(1
/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*
g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2
)^(1/2))/(x+g/h)))+1/2*c/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^2)/h^2)
^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^2-b*g*h
+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/
h^2)^(1/2))/(x+g/h)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(318) = 636$.

Time = 31.27 (sec) , antiderivative size = 2034, normalized size of antiderivative = 6.05

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^4 - (8*b*c*d + 8*a*b*f -
 (b^2 + 12*a*c)*e)*g^3*h - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g^2*h^2
 + ((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^2*h^2 - (8*b*c*d + 8*a*b*f -
 (b^2 + 12*a*c)*e)*g*h^3 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*h^4)*x^
2 + 2*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*g^3*h - (8*b*c*d + 8*a*b*f
- (b^2 + 12*a*c)*e)*g^2*h^2 - (4*a*b*e - 8*a^2*f - (3*b^2 - 4*a*c)*d)*g*h^
3)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h^2 - (b^2 + 4*a*
c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 - 4*sqrt(c*g^2 -
b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x) - 2*(
4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 + 2*g*h*x + g^2))
 - 4*(2*a^2*d*h^5 - (4*c^2*e - 3*b*c*f)*g^5 + (8*c^2*d + 5*b*c*e - 3*(b^2
+ 2*a*c)*f)*g^4*h - (13*b*c*d - 9*a*b*f + (b^2 + 2*a*c)*e)*g^3*h^2 - (a*b*
e + 6*a^2*f - 5*(b^2 + 2*a*c)*d)*g^2*h^3 - (7*a*b*d - 2*a^2*e)*g*h^4 - (2*
c^2*f*g^5 + (2*c^2*e - 7*b*c*f)*g^4*h - (6*c^2*d + b*c*e - 5*(b^2 + 2*a*c)
*f)*g^3*h^2 + (9*b*c*d - 13*a*b*f - (b^2 + 2*a*c)*e)*g^2*h^3 + (5*a*b*e +
8*a^2*f - 3*(b^2 + 2*a*c)*d)*g*h^4 + (3*a*b*d - 4*a^2*e)*h^5)*x)*sqrt(c*x^
2 + b*x + a))/(c^3*g^8 - 3*b*c^2*g^7*h - 3*a^2*b*g^3*h^5 + a^3*g^2*h^6 + 3
*(b^2*c + a*c^2)*g^6*h^2 - (b^3 + 6*a*b*c)*g^5*h^3 + 3*(a*b^2 + a^2*c)*g^4
*h^4 + (c^3*g^6*h^2 - 3*b*c^2*g^5*h^3 - 3*a^2*b*g*h^7 + a^3*h^8 + 3*(b^2*c
 + a*c^2)*g^4*h^4 - (b^3 + 6*a*b*c)*g^3*h^5 + 3*(a*b^2 + a^2*c)*g^2*h^6...
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2}{(g+hx)^3\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2279 vs. $2(318) = 636$.

Time = 0.53 (sec) , antiderivative size = 2279, normalized size of antiderivative = 6.78

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(8*c^2*d*g^2 - 4*b*c*e*g^2 + 3*b^2*f*g^2 - 4*a*c*f*g^2 - 8*b*c*d*g*h +
 b^2*e*g*h + 12*a*c*e*g*h - 8*a*b*f*g*h + 3*b^2*d*h^2 - 4*a*c*d*h^2 - 4*a*
b*e*h^2 + 8*a^2*f*h^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sq
rt(c)*g)/sqrt(-c*g^2 + b*g*h - a*h^2))/((c^2*g^4 - 2*b*c*g^3*h + b^2*g^2*h
^2 + 2*a*c*g^2*h^2 - 2*a*b*g*h^3 + a^2*h^4)*sqrt(-c*g^2 + b*g*h - a*h^2))
+ 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*c^2*f*g^4*h - 16*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*b*c*f*g^3*h^2 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*
x + a))^3*c^2*d*g^2*h^3 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b*c*e*g^
2*h^3 + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*f*g^2*h^3 + 20*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^3*a*c*f*g^2*h^3 + 8*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^3*b*c*d*g*h^4 - (sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*b^2*e*g*h
^4 - 12*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*e*g*h^4 - 8*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^3*a*b*f*g*h^4 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^3*b^2*d*h^5 + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d*h^5 + 4*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e*h^5 + 8*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*c^(5/2)*f*g^5 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5
/2)*e*g^4*h - 16*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*f*g^4*h -
 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*c^(5/2)*d*g^3*h^2 - 4*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*b*c^(3/2)*e*g^3*h^2 - (sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^2*b^2*sqrt(c)*f*g^3*h^2 + 28*(sqrt(c)*x - sqrt(c*x^2 + b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{f\,x^2+e\,x+d}{{\left(g+h\,x\right)}^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5126, normalized size of antiderivative = 15.26

$$\int \frac{d+ex+fx^2}{(g+hx)^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
- b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*f*g**2*h**2 + 16*s
qrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*
g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*f*g*h**3*x + 8*sqrt(a*
h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h +
c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*f*h**4*x**2 - 4*sqrt(a*h**2
- b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**
2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a*b*e*g**2*h**2 - 8*sqrt(a*h**2 - b*g*
h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2
*a*h + b*g - b*h*x + 2*c*g*x)*a*b*e*g*h**3*x - 4*sqrt(a*h**2 - b*g*h + c*g
**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h +
b*g - b*h*x + 2*c*g*x)*a*b*e*h**4*x**2 - 8*sqrt(a*h**2 - b*g*h + c*g**2)*l
og(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g -
b*h*x + 2*c*g*x)*a*b*f*g**3*h - 16*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqr
t(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x +
2*c*g*x)*a*b*f*g**2*h**2*x - 8*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a
+ b*x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*
g*x)*a*b*f*g*h**3*x**2 - 4*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*
x + c*x**2)*sqrt(a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)
*a*c*d*g**2*h**2 - 8*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x +...
```

3.70 $\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 666

$$\int \frac{(g+hx)^3(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(ab^4fh^3 - ab^3ch^2(3fg+eh) - ab^2ch(4afh^2 - c(3fg^2+3egh+dh^2))}{}$$

$$+ \frac{h(57b^2fh^2 + 24c^2(3fg^2 + h(3eg+dh)) - 2ch(20afh + 21b(3fg+eh)))\sqrt{a+bx+cx^2}}{24c^4}$$

$$+ \frac{h^2(18cfg + 6ceh - 11bfh)x\sqrt{a+bx+cx^2}}{12c^3} + \frac{fh^3x^2\sqrt{a+bx+cx^2}}{3c^2}$$

$$- \frac{(35b^3fh^3 - 30bch^2(3bfg + beh + 2afh) - 16c^3(fg^3 + 3gh(eg+dh)) + 24c^2h(3bfg^2 + bh(3eg+dh) + a}{16c^{9/2}}$$

output
```
2*(a*b^4*f*h^3-a*b^3*c*h^2*(e*h+3*f*g)-a*b^2*c*h*(4*a*f*h^2-c*(d*h^2+3*e*g
*h+3*f*g^2))+2*a*c^2*(a^2*f*h^3+c^2*g^2*(3*d*h+e*g)-a*c*h*(d*h^2+3*e*g*h+3
*f*g^2))-b*c^2*(c^2*d*g^3-3*a^2*h^2*(e*h+3*f*g)+a*c*g*(f*g^2+3*h*(d*h+e*g)
))-(2*c^5*d*g^3-b^5*f*h^3+b^3*c*h^2*(5*a*f*h+b*e*h+3*b*f*g)-c^4*g*(2*a*f*g
^2+6*a*h*(d*h+e*g)+b*g*(3*d*h+e*g))-b*c^2*h*(5*a^2*f*h^2+4*a*b*h*(e*h+3*f*
g)+b^2*(d*h^2+3*e*g*h+3*f*g^2))+c^3*(2*a^2*h^2*(e*h+3*f*g)+b^2*g*(f*g^2+3*
h*(d*h+e*g))+3*a*b*h*(3*f*g^2+h*(d*h+3*e*g))))*x)/c^4/(-4*a*c+b^2)/(c*x^2+
b*x+a)^(1/2)+1/24*h*(57*b^2*f*h^2+24*c^2*(3*f*g^2+h*(d*h+3*e*g))-2*c*h*(20
*a*f*h+21*b*(e*h+3*f*g)))*(c*x^2+b*x+a)^(1/2)/c^4+1/12*h^2*(-11*b*f*h+6*c*
e*h+18*c*f*g)*x*(c*x^2+b*x+a)^(1/2)/c^3+1/3*f*h^3*x^2*(c*x^2+b*x+a)^(1/2)/
c^2-1/16*(35*b^3*f*h^3-30*b*c*h^2*(2*a*f*h+b*e*h+3*b*f*g)-16*c^3*(f*g^3+3*
g*h*(d*h+e*g))+24*c^2*h*(3*b*f*g^2+b*h*(d*h+3*e*g)+a*h*(e*h+3*f*g)))*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 7.53 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.04

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{-105b^5fh^3x-5b^4h^2(21afh+cx(-54fg-18eh+7fhx))+2b^3ch(5ah(27fg+9eh+53fhx)+cx(3h\cdots}{\cdots}$$

$$+\frac{(-35b^3fh^3+30bch^2(3bfg+beh+2afh)+16c^3(fg^3+3gh(eg+dh))-24c^2h(3bfg^2+bh(3eg+dh)+\cdots}{8c^{9/2}}$$

input
```
Integrate[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
-1/24*(-105*b^5*f*h^3*x - 5*b^4*h^2*(21*a*f*h + c*x*(-54*f*g - 18*e*h + 7*
f*h*x)) + 2*b^3*c*h*(5*a*h*(27*f*g + 9*e*h + 53*f*h*x) + c*x*(3*h*(-36*e*g
 - 12*d*h + 5*e*h*x) + f*(-108*g^2 + 45*g*h*x + 7*h^2*x^2))) + 16*c^2*(-16
*a^3*f*h^3 + 6*c^3*d*g^3*x + a*c^2*(6*d*h*(-3*g^2 - 3*g*h*x + h^2*x^2) - 3
*e*(2*g^3 + 6*g^2*h*x - 6*g*h^2*x^2 - h^3*x^3) + f*x*(-6*g^3 + 18*g^2*h*x
+ 9*g*h^2*x^2 + 2*h^3*x^3)) + a^2*c*h*(f*(36*g^2 + 27*g*h*x - 8*h^2*x^2) +
 3*h*(4*d*h + 3*e*(4*g + h*x)))) + 8*b*c^2*(-6*c^2*g^2*(-(d*g) + e*g*x + 3
*d*h*x) - a^2*h^2*(117*f*g + 39*e*h + 61*f*h*x) + a*c*(f*(6*g^3 + 90*g^2*h
*x - 45*g*h^2*x^2 - 7*h^3*x^3) + 3*h*(2*d*h*(3*g + 5*h*x) + e*(6*g^2 + 30*
g*h*x - 5*h^2*x^2)))) + 4*b^2*c*(115*a^2*f*h^3 - a*c*h*(3*h*(18*e*g + 6*d*
h + 31*e*h*x) + f*(54*g^2 + 279*g*h*x - 43*h^2*x^2)) - c^2*x*(f*(-12*g^3 +
 18*g^2*h*x + 9*g*h^2*x^2 + 2*h^3*x^3) + 3*h*(2*d*h*(-6*g + h*x) + e*(-12*
g^2 + 6*g*h*x + h^2*x^2)))))/(c^4*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) + (
(-35*b^3*f*h^3 + 30*b*c*h^2*(3*b*f*g + b*e*h + 2*a*f*h) + 16*c^3*(f*g^3 +
3*g*h*(e*g + d*h)) - 24*c^2*h*(3*b*f*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g
+ e*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(8*c^(9/
2))
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 520, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2175, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2175}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(g+hx)^2\left(fgb^2+6(cd+af)hb-4ac(fg+3eh)+c\left(\frac{7fb^2}{c}-6eb+12cd-16af\right)hx\right)}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(g+hx)^2\left(fgb^2+6(cd+af)hb-4ac(fg+3eh)+\left(7fb^2-6ceb+12c^2d-16acf\right)hx\right)}{\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}+\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1236

$$\frac{\frac{\int -\frac{(g+hx)\left(7fghb^3+\left(28afh^2-6cg(fg+eh)\right)b^2-4ch(6cdg+13afg+6aeh)b-8ac\left(8afh^2-3c\left(fg^2+3ehg+2dh^2\right)\right)-h\left(-35fhb^3+2c(17bfg+15beh+58afh)b+48c^3\cdots\right.\right.}{2\sqrt{cx^2+bx+a}}\cdots}{3c}\cdots}{c\left(b^2-4ac\right)}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 27

$$\frac{\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\int \frac{(g+hx)\left(7fghb^3+\left(28afh^2-6cg(fg+eh)\right)b^2-4ch(6cdg+13afg+6aeh)b-8ac\left(8afh^2-3c\left(fg^2+\cdots\right.\right.\right.}{\sqrt{c\cdots}}\cdots}{\cdots}}{c\left(b^2-4ac\right)}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1225

$$\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2afh+beh+3bfg)+35b^3fh^3-16\cdots\right.}{8c^2}\cdots}{\cdots}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1092

$$\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(2afh+beh+3bfg)+35b^3fh^3-16\cdots\right.}{4c^2}\cdots}{\cdots}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\downarrow$ 219

$$\frac{\frac{h(g+hx)^2\sqrt{a+bx+cx^2}\left(-16acf+7b^2f-6bce+12c^2d\right)}{3c}-\frac{\frac{3\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(24c^2h\left(ah(eh+3fg)+bh(dh+3eg)+3bfg^2\right)-30bch^2(\cdots}{8c^{5/2}}}{\cdots}}{\cdots}$$

$$\frac{2(g+hx)^3\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x)^3)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((12*c^2*d - 6*b*c*
e + 7*b^2*f - 16*a*c*f)*h*(g + h*x)^2*Sqrt[a + b*x + c*x^2])/(3*c) - (-1/4
*((192*c^4*d*g^2*h + 105*b^4*f*h^3 - 10*b^2*c*h^2*(46*a*f*h + 9*b*(3*f*g +
 e*h)) - 16*c^3*h*(3*b*g*(2*e*g + 3*d*h) + 4*a*(7*f*g^2 + 9*e*g*h + 3*d*h^
2)) + 8*c^2*h*(32*a^2*f*h^2 + 39*a*b*h*(3*f*g + e*h) + b^2*(20*f*g^2 + 9*h
*(3*e*g + d*h))) + 2*c*h^2*(48*c^3*d*g - 35*b^3*f*h - 8*c^2*(3*b*e*g + 11*
a*f*g + 3*b*d*h + 9*a*e*h) + 2*b*c*(17*b*f*g + 15*b*e*h + 58*a*f*h))*x)*Sq
rt[a + b*x + c*x^2])/c^2 + (3*(b^2 - 4*a*c)*(35*b^3*f*h^3 - 30*b*c*h^2*(3*
b*f*g + b*e*h + 2*a*f*h) - 16*c^3*(f*g^3 + 3*g*h*(e*g + d*h)) + 24*c^2*h*(
3*b*f*g^2 + b*h*(3*e*g + d*h) + a*h*(3*f*g + e*h)))*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 991, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{h\left(-8f\,h^{2}c^{2}x^{2}+22bcf\,h^{2}x-12c^{2}e\,h^{2}x-36c^{2}fghx+40acf\,h^{2}-57b^{2}f\,h^{2}+42bce\,h^{2}+126bcfgh-24c^{2}d\,h^{2}-72c^{2}egh-72c^{2}f\,g^{2}\right)\sqrt{c}}{24c^{4}}$
default	Expression too large to display

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*h*(-8*c^2*f*h^2*x^2+22*b*c*f*h^2*x-12*c^2*e*h^2*x-36*c^2*f*g*h*x+40*
a*c*f*h^2-57*b^2*f*h^2+42*b*c*e*h^2+126*b*c*f*g*h-24*c^2*d*h^2-72*c^2*e*g*
h-72*c^2*f*g^2)*(c*x^2+b*x+a)^(1/2)/c^4+1/16/c^4*(c*(60*a*b*c*f*h^3-24*a*c
^2*e*h^3-72*a*c^2*f*g*h^2-35*b^3*f*h^3+30*b^2*c*e*h^3+90*b^2*c*f*g*h^2-24*
b*c^2*d*h^3-72*b*c^2*e*g*h^2-72*b*c^2*f*g^2*h+48*c^3*d*g*h^2+48*c^3*e*g^2*
h+16*c^3*f*g^3)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2
)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(16*a^2*c^2*f*h^3+12*a*b^2*c*f*h^3+8*a*b*c^2
*e*h^3+24*a*b*c^2*f*g*h^2-16*a*c^3*d*h^3-48*a*c^3*e*g*h^2-48*a*c^3*f*g^2*h
-19*b^4*f*h^3+14*b^3*c*e*h^3+42*b^3*c*f*g*h^2-8*b^2*c^2*d*h^3-24*b^2*c^2*e
*g*h^2-24*b^2*c^2*f*g^2*h+48*c^4*d*g^2*h+16*c^4*e*g^3)*(-1/c/(c*x^2+b*x+a)
^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+32*c^4*d*g^3*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-38*a*b^3*f*h^3*(2*c*x+b)/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)-16*a^2*c^2*e*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2)-16*a*b*c^2*d*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+28*a*b^2*c*e
*h^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+56*a^2*b*c*f*h^3*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-48*a^2*c^2*f*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)-48*a*b*c^2*e*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)-48*a*b*c^2*f*g^2*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+84*a*b^
2*c*f*g*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1467 vs. $2(642) = 1284$.

Time = 6.95 (sec) , antiderivative size = 2937, normalized size of antiderivative = 4.41

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/96*(3*(16*(a*b^2*c^3 - 4*a^2*c^4)*f*g^3 + 24*(2*(a*b^2*c^3 - 4*a^2*c^4)
*e - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*f)*g^2*h + 6*(8*(a*b^2*c^3 - 4*a^2*c^4)*d
 - 12*(a*b^3*c^2 - 4*a^2*b*c^3)*e + 3*(5*a*b^4*c - 24*a^2*b^2*c^2 + 16*a^3
*c^3)*f)*g*h^2 - (24*(a*b^3*c^2 - 4*a^2*b*c^3)*d - 6*(5*a*b^4*c - 24*a^2*b
^2*c^2 + 16*a^3*c^3)*e + 5*(7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*f)*h^3
+ (16*(b^2*c^4 - 4*a*c^5)*f*g^3 + 24*(2*(b^2*c^4 - 4*a*c^5)*e - 3*(b^3*c^3
 - 4*a*b*c^4)*f)*g^2*h + 6*(8*(b^2*c^4 - 4*a*c^5)*d - 12*(b^3*c^3 - 4*a*b*
c^4)*e + 3*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*f)*g*h^2 - (24*(b^3*c^3
 - 4*a*b*c^4)*d - 6*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e + 5*(7*b^5*c
 - 40*a*b^3*c^2 + 48*a^2*b*c^3)*f)*h^3)*x^2 + (16*(b^3*c^3 - 4*a*b*c^4)*f*
g^3 + 24*(2*(b^3*c^3 - 4*a*b*c^4)*e - 3*(b^4*c^2 - 4*a*b^2*c^3)*f)*g^2*h +
 6*(8*(b^3*c^3 - 4*a*b*c^4)*d - 12*(b^4*c^2 - 4*a*b^2*c^3)*e + 3*(5*b^5*c
- 24*a*b^3*c^2 + 16*a^2*b*c^3)*f)*g*h^2 - (24*(b^4*c^2 - 4*a*b^2*c^3)*d -
6*(5*b^5*c - 24*a*b^3*c^2 + 16*a^2*b*c^3)*e + 5*(7*b^6 - 40*a*b^4*c + 48*a
^2*b^2*c^2)*f)*h^3)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x
^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*(b^2*c^4 - 4*a*c^5)*f*h^
3*x^4 - 48*(b*c^5*d - 2*a*c^5*e + a*b*c^4*f)*g^3 + 72*(4*a*c^5*d - 2*a*b*c
^4*e + (3*a*b^2*c^3 - 8*a^2*c^4)*f)*g^2*h - 18*(8*a*b*c^4*d - 4*(3*a*b^2*c
^3 - 8*a^2*c^4)*e + (15*a*b^3*c^2 - 52*a^2*b*c^3)*f)*g*h^2 + (24*(3*a*b^2*
c^3 - 8*a^2*c^4)*d - 6*(15*a*b^3*c^2 - 52*a^2*b*c^3)*e + (105*a*b^4*c -...
```

Sympy [F]

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**3*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)**3*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1028, normalized size of antiderivative = 1.54

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*(((2*(4*(b^2*c^3*f*h^3 - 4*a*c^4*f*h^3)*x/(b^2*c^4 - 4*a*c^5) + (18*b
^2*c^3*f*g*h^2 - 72*a*c^4*f*g*h^2 + 6*b^2*c^3*e*h^3 - 24*a*c^4*e*h^3 - 7*b
^3*c^2*f*h^3 + 28*a*b*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x + (72*b^2*c^3*f*g^
2*h - 288*a*c^4*f*g^2*h + 72*b^2*c^3*e*g*h^2 - 288*a*c^4*e*g*h^2 - 90*b^3*
c^2*f*g*h^2 + 360*a*b*c^3*f*g*h^2 + 24*b^2*c^3*d*h^3 - 96*a*c^4*d*h^3 - 30
*b^3*c^2*e*h^3 + 120*a*b*c^3*e*h^3 + 35*b^4*c*f*h^3 - 172*a*b^2*c^2*f*h^3
+ 128*a^2*c^3*f*h^3)/(b^2*c^4 - 4*a*c^5))*x - (96*c^5*d*g^3 - 48*b*c^4*e*g
^3 + 48*b^2*c^3*f*g^3 - 96*a*c^4*f*g^3 - 144*b*c^4*d*g^2*h + 144*b^2*c^3*e
*g^2*h - 288*a*c^4*e*g^2*h - 216*b^3*c^2*f*g^2*h + 720*a*b*c^3*f*g^2*h + 1
44*b^2*c^3*d*g*h^2 - 288*a*c^4*d*g*h^2 - 216*b^3*c^2*e*g*h^2 + 720*a*b*c^3
*e*g*h^2 + 270*b^4*c*f*g*h^2 - 1116*a*b^2*c^2*f*g*h^2 + 432*a^2*c^3*f*g*h^
2 - 72*b^3*c^2*d*h^3 + 240*a*b*c^3*d*h^3 + 90*b^4*c*e*h^3 - 372*a*b^2*c^2*
e*h^3 + 144*a^2*c^3*e*h^3 - 105*b^5*f*h^3 + 530*a*b^3*c*f*h^3 - 488*a^2*b*
c^2*f*h^3)/(b^2*c^4 - 4*a*c^5))*x - (48*b*c^4*d*g^3 - 96*a*c^4*e*g^3 + 48*
a*b*c^3*f*g^3 - 288*a*c^4*d*g^2*h + 144*a*b*c^3*e*g^2*h - 216*a*b^2*c^2*f*
g^2*h + 576*a^2*c^3*f*g^2*h + 144*a*b*c^3*d*g*h^2 - 216*a*b^2*c^2*e*g*h^2
+ 576*a^2*c^3*e*g*h^2 + 270*a*b^3*c*f*g*h^2 - 936*a^2*b*c^2*f*g*h^2 - 72*a
*b^2*c^2*d*h^3 + 192*a^2*c^3*d*h^3 + 90*a*b^3*c*e*h^3 - 312*a^2*b*c^2*e*h^
3 - 105*a*b^4*f*h^3 + 460*a^2*b^2*c*f*h^3 - 256*a^3*c^2*f*h^3)/(b^2*c^4 -
4*a*c^5))/sqrt(c*x^2 + b*x + a) - 1/16*(16*c^3*f*g^3 + 48*c^3*e*g^2*h -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx=\int \frac{(g+h\,x)^3\,(f\,x^2+e\,x+d)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^3*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(g+hx)^3 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(hx+g)^3 (f\,x^2+ex+d)}{(c\,x^2+bx+a)^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((h*x+g)^3*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.71 $\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 403

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{2(ab^3fh^2-ab^2ch(2fg+eh)-2ac^2(cg(eg+2dh)-ah(2fg+eh))+bc(c^2dg^2-3a^2fh^2+ac(fg^2+2eg}{}$$

$$+\frac{h(8cfg+4ceh-7bfh)\sqrt{a+bx+cx^2}}{4c^3}+\frac{fh^2x\sqrt{a+bx+cx^2}}{2c^2}$$

$$+\frac{(15b^2fh^2-12ch(2bfg+beh+afh)+8c^2(fg^2+h(2eg+dh)))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}}$$

output

```
(-2*a*b^3*f*h^2+2*a*b^2*c*h*(e*h+2*f*g)+4*a*c^2*(c*g*(2*d*h+e*g)-a*h*(e*h+
2*f*g))-2*b*c*(c^2*d*g^2-3*a^2*f*h^2+a*c*(d*h^2+2*e*g*h+f*g^2))-2*(2*c^4*d
*g^2+b^4*f*h^2-b^2*c*h*(4*a*f*h+b*e*h+2*b*f*g)-c^3*(b*g*(2*d*h+e*g)+2*a*(d
*h^2+2*e*g*h+f*g^2))+c^2*(2*a^2*f*h^2+3*a*b*h*(e*h+2*f*g)+b^2*(d*h^2+2*e*g
*h+f*g^2)))*x)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+1/4*h*(-7*b*f*h+4*c*e*
h+8*c*f*g)*(c*x^2+b*x+a)^(1/2)/c^3+1/2*f*h^2*x*(c*x^2+b*x+a)^(1/2)/c^2+1/8
*(15*b^2*f*h^2-12*c*h*(a*f*h+b*e*h+2*b*f*g)+8*c^2*(f*g^2+h*(d*h+2*e*g)))*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 3.88 (sec) , antiderivative size = 390, normalized size of antiderivative = 0.97

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = -\frac{\sqrt{c}(15b^4fh^2x+b^3h(15afh+cx(-24fg-12eh+5fhx))+4bc(-13a^2fh^2+2c^2g(-egx+d(g-2hx))\cdots}{\cdots}$$

input
```
Integrate[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-((Sqrt[c]*(15*b^4*f*h^2*x + b^3*h*(15*a*f*h + c*x*(-24*f*g - 12*e*h + 5*
f*h*x)) + 4*b*c*(-13*a^2*f*h^2 + 2*c^2*g*(-(e*g*x) + d*(g - 2*h*x)) + a*c*
(2*h*(2*e*g + d*h + 5*e*h*x) + f*(2*g^2 + 20*g*h*x - 5*h^2*x^2))) - 2*b^2*
c*(a*h*(12*f*g + 6*e*h + 31*f*h*x) + c*x*(2*h*(-4*e*g - 2*d*h + e*h*x) + f
*(-4*g^2 + 4*g*h*x + h^2*x^2))) + 8*c^2*(2*c^2*d*g^2*x + a^2*h*(8*f*g + 4*
e*h + 3*f*h*x) + a*c*(-2*d*h*(2*g + h*x) - 2*e*(g^2 + 2*g*h*x - h^2*x^2) +
 f*x*(-2*g^2 + 4*g*h*x + h^2*x^2)))))/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]
)) + (15*b^2*f*h^2 - 12*c*h*(2*b*f*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(
2*e*g + d*h)))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/(4
*c^(7/2))
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2175, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(g+hx)\left(fgb^2+4(cd+af)hb-4ac(fg+2eh)+c\left(\frac{5fb^2}{c}+8cd-4(be+3af)\right)hx\right)}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(g+hx)\left(fgb^2+4(cd+af)hb-4ac(fg+2eh)+\left(5fb^2-4ceb+8c^2d-12acf\right)hx\right)}{\sqrt{cx^2+bx+a}}dx}{c\left(b^2-4ac\right)}+\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1225

$$\frac{\frac{\left(b^2-4ac\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8c^2}+\frac{h\sqrt{a+bx+cx^2}\left(2chx\left(-12acf+5b^2f-4bce+8c^2d\right)-8\right.}{}}{c\left(b^2-4ac\right)}$$
$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\left(b^2-4ac\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{4c^2}+\frac{h\sqrt{a+bx+cx^2}\left(2chx\left(-12acf+5b^2f-4b\right.\right.}{}}{c\left(b^2-4ac\right)}$$
$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-12ch(afh+beh+2bfg)+15b^2fh^2+8c^2\left(h(dh+2eg)+fg^2\right)\right)}{8c^{5/2}}+\frac{h\sqrt{a+bx+cx^2}\left(2chx\left(-12acf+5b^2f-4bc\right.\right.}{}}{c\left(b^2-4ac\right)}$$
$$\frac{2(g+hx)^2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x)^2)/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + ((h*(32*c^3*d*g - 15
*b^3*f*h - 8*c^2*(2*b*e*g + 8*a*f*g + b*d*h + 4*a*e*h) + 4*b*c*(6*b*f*g +
3*b*e*h + 13*a*f*h) + 2*c*(8*c^2*d - 4*b*c*e + 5*b^2*f - 12*a*c*f)*h*x)*Sq
rt[a + b*x + c*x^2])/(4*c^2) + ((b^2 - 4*a*c)*(15*b^2*f*h^2 - 12*c*h*(2*b*
f*g + b*e*h + a*f*h) + 8*c^2*(f*g^2 + h*(2*e*g + d*h)))*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{h(-2xfhc+7bfh-4che-8cfg)\sqrt{c\,x^2+bx+a}}{4c^3}-\frac{c\left(12acf\,h^2-15b^2f\,h^2+12bce\,h^2+24bcfgh-8c^2d\,h^2-16c^2egh-8c^2f\,g^2\right)\left(-\frac{}{c\sqrt{}}\right.}{}$
default	$\frac{2d\,g^2(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+g(2dh+eg)\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)+h(eh+2fg)\left(\frac{x^2}{c\sqrt{c\,x^2+bx-}}\right.$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*h*(-2*c*f*h*x+7*b*f*h-4*c*e*h-8*c*f*g)*(c*x^2+b*x+a)^(1/2)/c^3-1/8/c^
3*(c*(12*a*c*f*h^2-15*b^2*f*h^2+12*b*c*e*h^2+24*b*c*f*g*h-8*c^2*d*h^2-16*c
^2*e*g*h-8*c^2*f*g^2)*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a
)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+(-4*a*b*c*f*h^2+8*a*c^2*e*h^2+16*a*c^2
*f*g*h-7*b^3*f*h^2+4*b^2*c*e*h^2+8*b^2*c*f*g*h-16*c^3*d*g*h-8*c^3*e*g^2)*(
-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-16
*c^3*d*g^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-14*a*b^2*f*h^2*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+8*a^2*c*f*h^2*(2*c*x+b)/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)+8*a*b*c*e*h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+
16*a*b*c*f*g*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(383) = 766$.

Time = 5.67 (sec) , antiderivative size = 1769, normalized size of antiderivative = 4.39

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e
- 3*(a*b^3*c - 4*a^2*b*c^2)*f)*g*h + (8*(a*b^2*c^2 - 4*a^2*c^3)*d - 12*(a*
b^3*c - 4*a^2*b*c^2)*e + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f)*h^2 +
(8*(b^2*c^3 - 4*a*c^4)*f*g^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*e - 3*(b^3*c^2 - 4
*a*b*c^3)*f)*g*h + (8*(b^2*c^3 - 4*a*c^4)*d - 12*(b^3*c^2 - 4*a*b*c^3)*e +
 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f)*h^2)*x^2 + (8*(b^3*c^2 - 4*a*b
*c^3)*f*g^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*e - 3*(b^4*c - 4*a*b^2*c^2)*f)*g*
h + (8*(b^3*c^2 - 4*a*b*c^3)*d - 12*(b^4*c - 4*a*b^2*c^2)*e + 3*(5*b^5 - 2
4*a*b^3*c + 16*a^2*b*c^2)*f)*h^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^
2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*(b^2*c^3 -
 4*a*c^4)*f*h^2*x^3 - 8*(b*c^4*d - 2*a*c^4*e + a*b*c^3*f)*g^2 + 8*(4*a*c^4
*d - 2*a*b*c^3*e + (3*a*b^2*c^2 - 8*a^2*c^3)*f)*g*h - (8*a*b*c^3*d - 4*(3*
a*b^2*c^2 - 8*a^2*c^3)*e + (15*a*b^3*c - 52*a^2*b*c^2)*f)*h^2 + (8*(b^2*c^
3 - 4*a*c^4)*f*g*h + (4*(b^2*c^3 - 4*a*c^4)*e - 5*(b^3*c^2 - 4*a*b*c^3)*f)
*h^2)*x^2 - (8*(2*c^5*d - b*c^4*e + (b^2*c^3 - 2*a*c^4)*f)*g^2 - 8*(2*b*c^
4*d - 2*(b^2*c^3 - 2*a*c^4)*e + (3*b^3*c^2 - 10*a*b*c^3)*f)*g*h + (8*(b^2*
c^3 - 2*a*c^4)*d - 4*(3*b^3*c^2 - 10*a*b*c^3)*e + (15*b^4*c - 62*a*b^2*c^2
 + 24*a^2*c^3)*f)*h^2)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 +
(b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 -
4*a^2*c^3)*f*g^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*e - 3*(a*b^3*c - 4*a^2*...
```

Sympy [F]

$$\int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)^2(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)**2*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)**2*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.40

$$\int \frac{(g+hx)^2 (d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,(b^2c^2fh^2-4\,ac^3fh^2)x}{b^2c^3-4\,ac^4} + \frac{8\,b^2c^2fgh-32\,ac^3fgh+4\,b^2c^2eh^2-16\,ac^3eh^2-5\,b^3cfh^2+20\,abc^2f}{b^2c^3-4\,ac^4}\right.\right.}{}$$

$$-\frac{(8\,c^2fg^2+16\,c^2egh-24\,bcfgh+8\,c^2dh^2-12\,bceh^2+15\,b^2fh^2-12\,acfh^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right.\right.\right.}{8\,c^{\frac{7}{2}}}$$

input
```
integrate((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/4*(((2*(b^2*c^2*f*h^2 - 4*a*c^3*f*h^2)*x/(b^2*c^3 - 4*a*c^4) + (8*b^2*c^
2*f*g*h - 32*a*c^3*f*g*h + 4*b^2*c^2*e*h^2 - 16*a*c^3*e*h^2 - 5*b^3*c*f*h^
2 + 20*a*b*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (16*c^4*d*g^2 - 8*b*c^3*e*g
^2 + 8*b^2*c^2*f*g^2 - 16*a*c^3*f*g^2 - 16*b*c^3*d*g*h + 16*b^2*c^2*e*g*h
- 32*a*c^3*e*g*h - 24*b^3*c*f*g*h + 80*a*b*c^2*f*g*h + 8*b^2*c^2*d*h^2 - 1
6*a*c^3*d*h^2 - 12*b^3*c*e*h^2 + 40*a*b*c^2*e*h^2 + 15*b^4*f*h^2 - 62*a*b^
2*c*f*h^2 + 24*a^2*c^2*f*h^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d*g^2 - 16
*a*c^3*e*g^2 + 8*a*b*c^2*f*g^2 - 32*a*c^3*d*g*h + 16*a*b*c^2*e*g*h - 24*a*
b^2*c*f*g*h + 64*a^2*c^2*f*g*h + 8*a*b*c^2*d*h^2 - 12*a*b^2*c*e*h^2 + 32*a
^2*c^2*e*h^2 + 15*a*b^3*f*h^2 - 52*a^2*b*c*f*h^2)/(b^2*c^3 - 4*a*c^4))/sqr
t(c*x^2 + b*x + a) - 1/8*(8*c^2*f*g^2 + 16*c^2*e*g*h - 24*b*c*f*g*h + 8*c^
2*d*h^2 - 12*b*c*e*h^2 + 15*b^2*f*h^2 - 12*a*c*f*h^2)*log(abs(2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{{\left(g+h\,x\right)}^2\,\left(f\,x^2+e\,x+d\right)}{{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)^2*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(g+hx)^2\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{\left(hx+g\right)^{2}\left(f\,x^{2}+ex+d\right)}{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((h*x+g)^2*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

3.72 $\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(ab^2fh - bc(cdg+afg+aeh) - 2ac(afh - c(eg+dh)) - (2c^3dg - b^3f}{c^2\left(b^2-4ac\right)\sqrt{a+}} + \frac{fh\sqrt{a+bx+cx^2}}{c^2} - \frac{(3bfh - 2c(fg+eh))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}}$$

output
```
2*(a*b^2*f*h-b*c*(a*e*h+a*f*g+c*d*g)-2*a*c*(a*f*h-c*(d*h+e*g))-(2*c^3*d*g-
b^3*f*h-c^2*(2*a*e*h+2*a*f*g+b*d*h+b*e*g)+b*c*(3*a*f*h+b*e*h+b*f*g))*x)/c^
2/(-4*a*c+b^2)/(c*x^2+b*x+a)^(1/2)+f*h*(c*x^2+b*x+a)^(1/2)/c^2-1/2*(3*b*f*
h-2*c*(e*h+f*g))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2
)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.93

$$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-3b^3fhx+2bc(aeh-cegx+cd(g-hx)+af(g+5hx))+b^2(-3afh+}{c^2(-b^2+4}$$
$$+\frac{(-3bfh+2c(fg+eh))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-3*b^3*f*h*x + 2*b*c*(a*e*h - c*e*g*x + c*d*(g - h*x) + a*f*(g + 5*h*x))
+ b^2*(-3*a*f*h + c*x*(2*f*g + 2*e*h - f*h*x)) + 4*c*(2*a^2*f*h + c^2*d*g*
x - a*c*(d*h + f*x*(g - h*x) + e*(g + h*x))))/(c^2*(-b^2 + 4*a*c)*Sqrt[a +
 x*(b + c*x)]) + ((-3*b*f*h + 2*c*(f*g + e*h))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[
a] + Sqrt[a + x*(b + c*x)])])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2175, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 2175$$

$$\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{fgb^2+2(cd+af)hb-4ac(fg+eh)+c\left(\frac{3fb^2}{c}-2eb+4cd-8af\right)hx}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{fgb^2+2(cd+af)hb-4ac(fg+eh)+(3fb^2-2ceb+4c^2d-8acf)hx}{\sqrt{cx^2+bx+a}}dx}{c(b^2-4ac)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1160

$$\frac{\frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c}-\frac{(b^2-4ac)(3bfh-2c(eh+fg))\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}}{c(b^2-4ac)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1092

$$\frac{\frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c}-\frac{(b^2-4ac)(3bfh-2c(eh+fg))\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}}{c(b^2-4ac)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{h\sqrt{a+bx+cx^2}(-8acf+3b^2f-2bce+4c^2d)}{c}-\frac{(b^2-4ac)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(3bfh-2c(eh+fg))}{2c^{3/2}}}{c(b^2-4ac)}+\frac{2(g+hx)\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x)*(
g + h*x))/(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (((4*c^2*d - 2*b*c*e +
 3*b^2*f - 8*a*c*f)*h*Sqrt[a + b*x + c*x^2])/c - ((b^2 - 4*a*c)*(3*b*f*h -
 2*c*(f*g + e*h))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/
(2*c^(3/2)))/(c*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.48

method	result
risch	$\frac{fh\sqrt{c\,x^2+bx+a}}{c^2}-\frac{c(3bfh-2che-2cfg)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{c^{\frac{3}{2}}}\right.}{}$
default	$\frac{2dg(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+(dh+eg)\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)+(eh+fg)\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}\right.$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
f*h*(c*x^2+b*x+a)^(1/2)/c^2-1/2/c^2*(c*(3*b*f*h-2*c*e*h-2*c*f*g)*(-x/c/(c*
x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^
2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2)))+(2*a*c*f*h+b^2*f*h-2*c^2*d*h-2*c^2*e*g)*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c
*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+2*a*b*h*f*(2*c*x+b)/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)-4*d*g*c^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(194) = 388$.

Time = 3.74 (sec) , antiderivative size = 905, normalized size of antiderivative = 4.35

$$\int \frac{(g+hx)\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((2*(a*b^2*c - 4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2
*c^2 - 4*a*c^3)*e - 3*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*
c^2)*e - 3*(a*b^3 - 4*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3
*c - 4*a*b*c^2)*e - 3*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(c)*log(-8*c^2*x^2 -
8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(
(b^2*c^2 - 4*a*c^3)*f*h*x^2 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a
*c^3*d - 2*a*b*c^2*e + (3*a*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*
e + (b^2*c^2 - 2*a*c^3)*f)*g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b
^3*c - 10*a*b*c^2)*f)*h)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4
+ (b^2*c^4 - 4*a*c^5)*x^2 + (b^3*c^3 - 4*a*b*c^4)*x), -1/2*((2*(a*b^2*c -
4*a^2*c^2)*f*g + (2*(b^2*c^2 - 4*a*c^3)*f*g + (2*(b^2*c^2 - 4*a*c^3)*e - 3
*(b^3*c - 4*a*b*c^2)*f)*h)*x^2 + (2*(a*b^2*c - 4*a^2*c^2)*e - 3*(a*b^3 - 4
*a^2*b*c)*f)*h + (2*(b^3*c - 4*a*b*c^2)*f*g + (2*(b^3*c - 4*a*b*c^2)*e - 3
*(b^4 - 4*a*b^2*c)*f)*h)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c
*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((b^2*c^2 - 4*a*c^3)*f*h*x^2
 - 2*(b*c^3*d - 2*a*c^3*e + a*b*c^2*f)*g + (4*a*c^3*d - 2*a*b*c^2*e + (3*a
*b^2*c - 8*a^2*c^2)*f)*h - (2*(2*c^4*d - b*c^3*e + (b^2*c^2 - 2*a*c^3)*f)*
g - (2*b*c^3*d - 2*(b^2*c^2 - 2*a*c^3)*e + (3*b^3*c - 10*a*b*c^2)*f)*h)*x)
*sqrt(c*x^2 + b*x + a))/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^2 +
 (b^3*c^3 - 4*a*b*c^4)*x)]
```

Sympy [F]

$$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)*(f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((g + h*x)*(d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.26

$$\int \frac{(g+hx)\,(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(\frac{(b^2cfh-4\,ac^2fh)x}{b^2c^2-4\,ac^3} - \frac{4\,c^3dg-2\,bc^2eg+2\,b^2cfg-4\,ac^2fg-2\,bc^2dh+2\,b^2ceh-4\,ac^2eh-3\,b^3fh+10\,\cdots}{b^2c^2-4\,ac^3}\cdots\right.}{\sqrt{cx^2+bx+a}} - \frac{(2\,cfg+2\,ceh-3\,bfh)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
(((b^2*c*f*h - 4*a*c^2*f*h)*x/(b^2*c^2 - 4*a*c^3) - (4*c^3*d*g - 2*b*c^2*e
*g + 2*b^2*c*f*g - 4*a*c^2*f*g - 2*b*c^2*d*h + 2*b^2*c*e*h - 4*a*c^2*e*h -
 3*b^3*f*h + 10*a*b*c*f*h)/(b^2*c^2 - 4*a*c^3))*x - (2*b*c^2*d*g - 4*a*c^2
*e*g + 2*a*b*c*f*g - 4*a*c^2*d*h + 2*a*b*c*e*h - 3*a*b^2*f*h + 8*a^2*c*f*h
)/(b^2*c^2 - 4*a*c^3))/sqrt(c*x^2 + b*x + a) - 1/2*(2*c*f*g + 2*c*e*h - 3*
b*f*h)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(g+h\,x)\,(f\,x^2+e\,x+d)}{(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
int(((g + h*x)*(d + e*x + f*x^2))/(a + b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1610, normalized size of antiderivative = 7.74

$$\int \frac{(g+hx)(d+ex+fx^2)}{(a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((h*x+g)*(f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(16*sqrt(a + b*x + c*x**2)*a**2*c**2*f*h - 6*sqrt(a + b*x + c*x**2)*a*b**2
*c*f*h + 4*sqrt(a + b*x + c*x**2)*a*b*c**2*e*h + 4*sqrt(a + b*x + c*x**2)*
a*b*c**2*f*g + 20*sqrt(a + b*x + c*x**2)*a*b*c**2*f*h*x - 8*sqrt(a + b*x +
 c*x**2)*a*c**3*d*h - 8*sqrt(a + b*x + c*x**2)*a*c**3*e*g - 8*sqrt(a + b*x
 + c*x**2)*a*c**3*e*h*x - 8*sqrt(a + b*x + c*x**2)*a*c**3*f*g*x + 8*sqrt(a
 + b*x + c*x**2)*a*c**3*f*h*x**2 - 6*sqrt(a + b*x + c*x**2)*b**3*c*f*h*x +
 4*sqrt(a + b*x + c*x**2)*b**2*c**2*e*h*x + 4*sqrt(a + b*x + c*x**2)*b**2*
c**2*f*g*x - 2*sqrt(a + b*x + c*x**2)*b**2*c**2*f*h*x**2 + 4*sqrt(a + b*x
+ c*x**2)*b*c**3*d*g - 4*sqrt(a + b*x + c*x**2)*b*c**3*d*h*x - 4*sqrt(a +
b*x + c*x**2)*b*c**3*e*g*x + 8*sqrt(a + b*x + c*x**2)*c**4*d*g*x - 12*sqrt
(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))
*a**2*b*c*f*h + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*
x)/sqrt(4*a*c - b**2))*a**2*c**2*e*h + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b
*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**2*f*g + 3*sqrt(c)*lo
g((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**
3*f*h - 2*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(
4*a*c - b**2))*a*b**2*c*e*h - 2*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x*
*2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*f*g - 12*sqrt(c)*log((2*sqrt
(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*f*h*x
 + 8*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*...
```

3.73 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 108

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(2ace-b(cd+af)-(2c^2d-bce+b^2f-2acf)\,x)}{c\,(b^2-4ac)\,\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

output
```
2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x)/c/(-4*a*c+b^2)/(c
*x^2+b*x+a)^(1/2)+f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.98

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{2(abf+2c^2dx+b^2fx+bc(d-ex)-2ac(e+fx))}{c\,(-b^2+4ac)\,\sqrt{a+x(b+cx)}} + \frac{2f\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x)))/(c*(-b
^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (2*f*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] +
Sqrt[a + x*(b + c*x)])])/c^(3/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{(b^2-4ac)f}{2c\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{f\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{2f\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}+\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/
(c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(98) = 196.

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.86

method	result
default	$\frac{2d(2cx+b)}{(4ac-b^2)\sqrt{c\,x^2+bx+a}}+e\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{c\,x^2+bx+a}}\right)+f\left(-\frac{x}{c\sqrt{c\,x^2+bx+a}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\cdots\right.}{2\cdots}\right.$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+e*(-1/c/(c*x^2+b*x+a)^(1/2)-
b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+f*(-x/c/(c*x^2+b*x+a)^(1/2)
-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(98) = 196.

Time = 0.41 (sec) , antiderivative size = 429, normalized size of antiderivative = 3.97

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \left[\frac{\left((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-\cdots\right.}{2\,(ab^2c\cdots}\right.$$

$$-\frac{\left((b^2c-4\,ac^2)fx^2+(b^3-4\,abc)fx+(ab^2-4\,a^2c)f\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)+2\,(bc^2d-2\,ac^2\cdots}{ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^2+(b^3c^2-4\,abc\cdots}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f
)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^
2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^
3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^
2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b
*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x
)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2
+ (b^3*c^2 - 4*a*b*c^3)*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{d+ex+fx^2}{(a+bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2c^2d-bce+b^2f-2acf)x}{b^2c-4ac^2}+\frac{bcd-2ace+abf}{b^2c-4ac^2}\right)}{\sqrt{cx^2+bx+a}} - \frac{f\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}}$$

input
```
integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x/(b^2*c - 4*a*c^2) + (b*c*d - 2*a
*c*e + a*b*f)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 19.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.32

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{c^{3/2}} - \frac{e\,(4\,a+2\,b\,x)}{(4\,a\,c-b^2)\,\sqrt{c\,x^2+b\,x+a}} + \frac{d\,\left(\frac{b}{2}+c\,x\right)}{\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}} + \frac{f\,\left(\frac{a\,b}{2}-x\,\left(a\,c-\frac{b^2}{2}\right)\right)}{c\,\left(a\,c-\frac{b^2}{4}\right)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x)
```

output
```
(f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/c^(3/2) - (e*(4*a +
 2*b*x))/((4*a*c - b^2)*(a + b*x + c*x^2)^(1/2)) + (d*(b/2 + c*x))/((a*c -
 b^2/4)*(a + b*x + c*x^2)^(1/2)) + (f*((a*b)/2 - x*(a*c - b^2/2)))/(c*(a*c
 - b^2/4)*(a + b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 574, normalized size of antiderivative = 5.31

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c\,x^2+bx+a}\,a\,c^2e+2\sqrt{c\,x^2+bx+a}\,b\,c^2d+4\sqrt{c\,x^2+bx+a}\,c^3dx-4\sqrt{c}\,c}{}$$

input
```
int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*a*b*c*f - 4*sqrt(a + b*x + c*x**2)*a*c**2*e - 4*
sqrt(a + b*x + c*x**2)*a*c**2*f*x + 2*sqrt(a + b*x + c*x**2)*b**2*c*f*x +
2*sqrt(a + b*x + c*x**2)*b*c**2*d - 2*sqrt(a + b*x + c*x**2)*b*c**2*e*x +
4*sqrt(a + b*x + c*x**2)*c**3*d*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c*f - sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*f + 4*sq
rt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2
))*a*b*c*f*x + 4*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x
)/sqrt(4*a*c - b**2))*a*c**2*f*x**2 - sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*f*x - sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*f*x**2 -
 4*sqrt(c)*a**2*c*f + 2*sqrt(c)*a*b**2*f - 2*sqrt(c)*a*b*c*e - 4*sqrt(c)*a
*b*c*f*x + 4*sqrt(c)*a*c**2*d - 4*sqrt(c)*a*c**2*f*x**2 + 2*sqrt(c)*b**3*f
*x - 2*sqrt(c)*b**2*c*e*x + 2*sqrt(c)*b**2*c*f*x**2 + 4*sqrt(c)*b*c**2*d*x
 - 2*sqrt(c)*b*c**2*e*x**2 + 4*sqrt(c)*c**3*d*x**2)/(c**2*(4*a**2*c - a*b*
*2 + 4*a*b*c*x + 4*a*c**2*x**2 - b**3*x - b**2*c*x**2))
```

3.74 $\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 220

$$\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)+((ce-bf)(bg \cdots}{(b^2-4ac)(cg^2-bgh+ah^2)\sqrt{a+bx+\cdots}} + \frac{(fg^2-h(eg-dh))\operatorname{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{(cg^2-bgh+ah^2)^{3/2}}$$

output
```
2*(b^2*d*h-b*(a*e*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)+((-b*f+c*e)*(-2*a
*h+b*g)-(-a*f+c*d)*(-b*h+2*c*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(c*x^
2+b*x+a)^(1/2)+(f*g^2-h*(-d*h+e*g))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)
/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2}{(g+hx)(a+bx+cx^2)^{3/2}}\,dx = 2\left(\frac{-2a^2fh+2c^2dgx+b^2(-dh+fgx)+2ac(-eg+dh-fgx+ehx)\cdots}{(b^2-4ac)(-cg^2+h(bg-ah))\sqrt{\cdots}} + \frac{\sqrt{-cg^2+bgh-ah^2}(fg^2+h(-eg+dh))\arctan\left(\frac{\sqrt{c}(g+hx)-h\sqrt{a+x(b+cx)}}{\sqrt{-cg^2+h(bg-ah)}}\right)}{(cg^2+h(-bg+ah))^2}\right)$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
2*((-2*a^2*f*h + 2*c^2*d*g*x + b^2*(-(d*h) + f*g*x) + 2*a*c*(-(e*g) + d*h
- f*g*x + e*h*x) + b*c*(-(e*g*x) + d*(g - h*x)) + a*b*(e*h + f*(g - h*x)))
/((b^2 - 4*a*c)*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) + (Sqrt[
-(c*g^2) + b*g*h - a*h^2]*(f*g^2 + h*(-(e*g) + d*h))*ArcTan[(Sqrt[c]*(g +
h*x) - h*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*g^2) + h*(b*g - a*h)]])/(c*g^2 +
h*(-(b*g) + a*h))^2)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$
$$\frac{2\int -\frac{\left(b^2-4ac\right)\left(fg^2-h(eg-dh)\right)}{2\left(cg^2-bhg+ah^2\right)(g+hx)\sqrt{cx^2+bx+a}}dx}{b^2-4ac}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx}{ah^2-bgh+cg^2}+$$
$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

$$\frac{2\left(fg^2-h(eg-dh)\right)\int \frac{1}{4(cg^2-bhg+ah^2)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d\left(-\frac{bg-2ah+(2cg-bh)x}{\sqrt{cx^2+bx+a}}\right)}{ah^2-bgh+cg^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(fg^2-h(eg-dh)\right)\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)}{\left(ah^2-bgh+cg^2\right)^{3/2}}+$$

$$\frac{2\left(-x\left(-c(-2aeh+2afg+bdh+beg)+bf(bg-ah)+2c^2dg\right)-b(aeh+afg+cdg)+2a(afh-cdh+ceg)+\right.}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(ah^2-bgh+cg^2\right)}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(b^2*d*h - b*(c*d*g + a*f*g + a*e*h) + 2*a*(c*e*g - c*d*h + a*f*h) - (2
*c^2*d*g + b*f*(b*g - a*h) - c*(b*e*g + 2*a*f*g + b*d*h - 2*a*e*h))*x))/((
b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)*Sqrt[a + b*x + c*x^2]) + ((f*g^2 - h*
(e*g - d*h))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h
 + a*h^2]*Sqrt[a + b*x + c*x^2])])/(c*g^2 - b*g*h + a*h^2)^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(210) = 420$.

Time = 0.30 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.49

method	result
default	$\frac{\frac{2eh(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}}+fh\left(-\frac{1}{c\sqrt{cx^2+bx+a}}-\frac{b(2cx+b)}{c(4ac-b^2)\sqrt{cx^2+bx+a}}\right)-\frac{2fg(2cx+b)}{(4ac-b^2)\sqrt{cx^2+bx+a}}}{h^2}+\frac{(dh^2-egh+fg^2)\left(\frac{\ldots}{(ah^2-bgh+c\ldots}\right.}{\ldots}$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/h^2*(2*e*h*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f*h*(-1/c/(c*x^2+b*
x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-2*f*g*(2*c*x+b)/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+(d*h^2-e*g*h+f*g^2)/h^3*(1/(a*h^2-b*g*h+c
*g^2)*h^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2
)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+g/h)+(b*h-2*c*g)/h)/(4*c*(a*h^
2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(
a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g^
2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h^
2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+
c*g^2)/h^2)^(1/2))/(x+g/h)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(210) = 420$.

Time = 9.64 (sec) , antiderivative size = 1905, normalized size of antiderivative = 8.66

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((a*b^2 - 4*a^2*c)*f*g^2 - (a*b^2 - 4*a^2*c)*e*g*h + (a*b^2 - 4*a^2*
c)*d*h^2 + ((b^2*c - 4*a*c^2)*f*g^2 - (b^2*c - 4*a*c^2)*e*g*h + (b^2*c - 4
*a*c^2)*d*h^2)*x^2 + ((b^3 - 4*a*b*c)*f*g^2 - (b^3 - 4*a*b*c)*e*g*h + (b^3
 - 4*a*b*c)*d*h^2)*x)*sqrt(c*g^2 - b*g*h + a*h^2)*log((8*a*b*g*h - 8*a^2*h
^2 - (b^2 + 4*a*c)*g^2 - (8*c^2*g^2 - 8*b*c*g*h + (b^2 + 4*a*c)*h^2)*x^2 -
 4*sqrt(c*g^2 - b*g*h + a*h^2)*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g
 - b*h)*x) - 2*(4*b*c*g^2 + 4*a*b*h^2 - (3*b^2 + 4*a*c)*g*h)*x)/(h^2*x^2 +
 2*g*h*x + g^2)) - 4*((b*c^2*d - 2*a*c^2*e + a*b*c*f)*g^3 + (3*a*b*c*e - 2
*(b^2*c - a*c^2)*d - (a*b^2 + 2*a^2*c)*f)*g^2*h + (3*a^2*b*f + (b^3 - a*b*
c)*d - (a*b^2 + 2*a^2*c)*e)*g*h^2 + (a^2*b*e - 2*a^3*f - (a*b^2 - 2*a^2*c)
*d)*h^3 + ((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*g^3 - (3*b*c^2*d - (b
^2*c + 2*a*c^2)*e + (b^3 - a*b*c)*f)*g^2*h - (3*a*b*c*e - (b^2*c + 2*a*c^2
)*d - 2*(a*b^2 - a^2*c)*f)*g*h^2 - (a*b*c*d - 2*a^2*c*e + a^2*b*f)*h^3)*x)
*sqrt(c*x^2 + b*x + a))/((a*b^2*c^2 - 4*a^2*c^3)*g^4 - 2*(a*b^3*c - 4*a^2*
b*c^2)*g^3*h + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*g^2*h^2 - 2*(a^2*b^3 - 4*
a^3*b*c)*g*h^3 + (a^3*b^2 - 4*a^4*c)*h^4 + ((b^2*c^3 - 4*a*c^4)*g^4 - 2*(b
^3*c^2 - 4*a*b*c^3)*g^3*h + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*g^2*h^2 - 2*
(a*b^3*c - 4*a^2*b*c^2)*g*h^3 + (a^2*b^2*c - 4*a^3*c^2)*h^4)*x^2 + ((b^3*c
^2 - 4*a*b*c^3)*g^4 - 2*(b^4*c - 4*a*b^2*c^2)*g^3*h + (b^5 - 2*a*b^3*c - 8
*a^2*b*c^2)*g^2*h^2 - 2*(a*b^4 - 4*a^2*b^2*c)*g*h^3 + (a^2*b^3 - 4*a^3*...
```

Sympy [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Integral((d + e*x + f*x**2)/((g + h*x)*(a + b*x + c*x**2)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(210) = 420$.

Time = 0.44 (sec) , antiderivative size = 708, normalized size of antiderivative = 3.22

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=$$

$$-\frac{2\left(\frac{(2\,c^3dg^3-bc^2eg^3+b^2cfg^3-2\,ac^2fg^3-3\,bc^2dg^2h+b^2ceg^2h+2\,ac^2eg^2h-b^3fg^2h+abcfg^2h+b^2cdgh^2+2\,ac^2dgh^2-3\,abcegh^2+2\,ab^2fgh^2-2\,a^2\cdots}{b^2c^2g^4-4\,ac^3g^4-2\,b^3cg^3h+8\,abc^2g^3h+b^4g^2h^2-2\,ab^2cg^2h^2-8\,a^2c^2g^2h^2-2\,ab^3gh^3+8\,a^2bcgh^3+a^2b^2h^4-4\,\cdots}\right.}{\cdots}$$

$$+\frac{2\,(fg^2-egh+dh^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)h+\sqrt{c}g}{\sqrt{-cg^2+bgh-ah^2}}\right)}{(cg^2-bgh+ah^2)\sqrt{-cg^2+bgh-ah^2}}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^3*d*g^3 - b*c^2*e*g^3 + b^2*c*f*g^3 - 2*a*c^2*f*g^3 - 3*b*c^2*d*g
^2*h + b^2*c*e*g^2*h + 2*a*c^2*e*g^2*h - b^3*f*g^2*h + a*b*c*f*g^2*h + b^2
*c*d*g*h^2 + 2*a*c^2*d*g*h^2 - 3*a*b*c*e*g*h^2 + 2*a*b^2*f*g*h^2 - 2*a^2*c
*f*g*h^2 - a*b*c*d*h^3 + 2*a^2*c*e*h^3 - a^2*b*f*h^3)*x/(b^2*c^2*g^4 - 4*a
*c^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2*c*g^2*h
^2 - 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2*h^4 - 4
*a^3*c*h^4) + (b*c^2*d*g^3 - 2*a*c^2*e*g^3 + a*b*c*f*g^3 - 2*b^2*c*d*g^2*h
 + 2*a*c^2*d*g^2*h + 3*a*b*c*e*g^2*h - a*b^2*f*g^2*h - 2*a^2*c*f*g^2*h + b
^3*d*g*h^2 - a*b*c*d*g*h^2 - a*b^2*e*g*h^2 - 2*a^2*c*e*g*h^2 + 3*a^2*b*f*g
*h^2 - a*b^2*d*h^3 + 2*a^2*c*d*h^3 + a^2*b*e*h^3 - 2*a^3*f*h^3)/(b^2*c^2*g
^4 - 4*a*c^3*g^4 - 2*b^3*c*g^3*h + 8*a*b*c^2*g^3*h + b^4*g^2*h^2 - 2*a*b^2
*c*g^2*h^2 - 8*a^2*c^2*g^2*h^2 - 2*a*b^3*g*h^3 + 8*a^2*b*c*g*h^3 + a^2*b^2
*h^4 - 4*a^3*c*h^4))/sqrt(c*x^2 + b*x + a) + 2*(f*g^2 - e*g*h + d*h^2)*arc
tan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*h + sqrt(c)*g)/sqrt(-c*g^2 + b*g
*h - a*h^2))/((c*g^2 - b*g*h + a*h^2)*sqrt(-c*g^2 + b*g*h - a*h^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+e\,x+d}{(g+h\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{d+ex+fx^2}{(g+hx)\left(a+bx+cx^2\right)^{3/2}}\,dx=\int \frac{f\,x^2+ex+d}{(hx+g)\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}dx$$

input
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
int((f*x^2+e*x+d)/(h*x+g)/(c*x^2+b*x+a)^(3/2),x)
```

3.75 $\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 411

$$\int \frac{d+ex+fx^2}{(g+hx)^2\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)+((ce-bf)(bg\ldots}{(b^2-4ac)\left(cg^2-bgh+ah^2\right)(g+hx)\sqrt{a+\ldots}} - \frac{h(4c^2dg^2+4a^2fh^2-2bcg(eg+2dh)-2abh(2fg+eh)+b^2(3fg^2-h(eg-3dh))-4ac(2fg^2-h(3eg\ldots}{(b^2-4ac)\left(cg^2-bgh+ah^2\right)^2(g+hx)} + \frac{(2c(fg^3-gh(2eg-3dh))+h(bfg^2+bh(eg-3dh)-2ah(2fg-eh)))\,\text{arctanh}\left(\frac{bg-2ah+(2cg-bh)x}{2\sqrt{cg^2-bgh+ah^2}\sqrt{a+bx+cx^2}}\right)}{2\left(cg^2-bgh+ah^2\right)^{5/2}}$$

output
```
2*(b^2*d*h-b*(a*e*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)+((-b*f+c*e)*(-2*a
*h+b*g)-(-a*f+c*d)*(-b*h+2*c*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(h*x+
g)/(c*x^2+b*x+a)^(1/2)-h*(4*c^2*d*g^2+4*a^2*f*h^2-2*b*c*g*(2*d*h+e*g)-2*a*
b*h*(e*h+2*f*g)+b^2*(3*f*g^2-h*(-3*d*h+e*g))-4*a*c*(2*f*g^2-h*(-2*d*h+3*e*
g)))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)+1/2*(2
*c*(f*g^3-g*h*(-3*d*h+2*e*g))+h*(b*f*g^2+b*h*(-3*d*h+e*g)-2*a*h*(-e*h+2*f*
g)))*arctanh(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c*g^2)^(5/2)
```

Mathematica [A] (verified)

Time = 11.60 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.26

$$\int \frac{d+ex+fx^2}{(g+hx)^2(a+bx+cx^2)^{3/2}}\,dx = -\frac{2f(b+2cx)}{(b^2-4ac)\,h^2\sqrt{a+x(b+cx)}}$$
$$+\frac{2(fg^2+h(-eg+dh))\left(-b^2h+2c(ah+cgx)+bc(g-hx)\right)}{(b^2-4ac)\,h^2\left(-cg^2+h(bg-ah)\right)(g+hx)\sqrt{a+x(b+cx)}}$$
$$-\frac{2(-2fg+eh)\left(b^2h-2c(ah+cgx)+bc(-g+hx)\right)}{(b^2-4ac)\,h^2\left(-cg^2+h(bg-ah)\right)\sqrt{a+x(b+cx)}}$$
$$-\frac{(fg^2+h(-eg+dh))\left(\frac{2(4c^2g^2+3b^2h^2-4ch(bg+2ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}-\frac{3(b^2-4ac)h(-2cg+bh)\operatorname{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}\right)}{2(b^2-4ac)\,h\,(cg^2+h(-bg+ah))}$$
$$-\frac{(2fg-eh)\operatorname{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*f*(b + 2*c*x))/((b^2 - 4*a*c)*h^2*Sqrt[a + x*(b + c*x)]) + (2*(f*g^2 +
 h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x)))/((b^2 -
 4*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) -
(2*(-2*f*g + e*h)*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*
a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - ((f*g^2 + h*(
-(e*g) + d*h))*((2*(4*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a +
x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*
(-2*c*g + b*h)*ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 +
h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2
)))/(2*(b^2 - 4*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))) - ((2*f*g - e*h)*ArcTan
h[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[
a + x*(b + c*x)]])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^2 (a+bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 2177$$

$$-\frac{2\int -\frac{(b^2-4ac)\left(-\left(\left(afg^2-b(eg-2dh)g-adh^2\right)h^2\right)-\left(cg(eg-2dh)+ah(2fg-eh)-b\left(fg^2-dh^2\right)\right)xh^2+c\left(fg^4-g^2h(2eg-3dh)\right)\right)}{2(cg^2-bhg+ah^2)^2(g+hx)^2\sqrt{cx^2+bx+a}}dx}{b^2-4ac}-$$

$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b(a^2fh\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-\left(\left(afg^2-b(eg-2dh)g-adh^2\right)h^2\right)-\left(cg(eg-2dh)+ah(2fg-eh)-b\left(fg^2-dh^2\right)\right)xh^2+c\left(fg^4-g^2h(2eg-3dh)\right)}{(g+hx)^2\sqrt{cx^2+bx+a}}dx}{(ah^2-bgh+cg^2)^2}-$$

$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b(a^2fh\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1228$$

$$\frac{\frac{1}{2}\left(h\left(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2\right)+2c\left(fg^3-gh(2eg-3dh)\right)\right)\int \frac{1}{(g+hx)\sqrt{cx^2+bx+a}}dx-\frac{h\sqrt{a+bx+cx^2}(}{g+}}{(ah^2-bgh+cg^2)^2}$$

$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b(a^2fh\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \; 1154$$

$$\frac{-\left(h\left(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2\right)+2c\left(fg^3-gh(2eg-3dh)\right)\right)\int \frac{1}{4(cg^2-bhg+ah^2)-\frac{(bg-2ah+(2cg-bh)x)^2}{cx^2+bx+a}}d}{(ah^2-bgh+cg^2)^2}$$

$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b(a^2fh\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)(h(-2ah(2fg-eh)+bh(eg-3dh)+bfg^2)+2c(fg^3-gh(2eg-3dh)))}{2\sqrt{ah^2-bgh+cg^2}}-\frac{h\sqrt{a+bx+cx^2}(dh^2-egh+fg^2)}{g+hx}}{(ah^2-bgh+cg^2)^2}$$
$$\frac{2\left(cx\left(2a^2fh^2-c\left(2a\left(dh^2-2egh+fg^2\right)+bg(2dh+eg)\right)-abh(eh+2fg)+b^2\left(dh^2+fg^2\right)+2c^2dg^2\right)+b\left(a^2fh\right.\right.}{(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[(d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b^3*d*h^2 - b^2*h*(2*c*d*g + a*e*h) - 2*a*c*(c*g*(e*g - 2*d*h) + a*h*
(2*f*g - e*h)) + b*(c^2*d*g^2 + a^2*f*h^2 + a*c*(f*g^2 + 2*e*g*h - 3*d*h^2
)) + c*(2*c^2*d*g^2 + 2*a^2*f*h^2 - a*b*h*(2*f*g + e*h) + b^2*(f*g^2 + d*h
^2) - c*(b*g*(e*g + 2*d*h) + 2*a*(f*g^2 - 2*e*g*h + d*h^2)))*x))/((b^2 - 4
*a*c)*(c*g^2 - b*g*h + a*h^2)^2*Sqrt[a + b*x + c*x^2]) + (-((h*(f*g^2 - e*
g*h + d*h^2)*Sqrt[a + b*x + c*x^2])/(g + h*x)) + ((2*c*(f*g^3 - g*h*(2*e*g
 - 3*d*h)) + h*(b*f*g^2 + b*h*(e*g - 3*d*h) - 2*a*h*(2*f*g - e*h)))*ArcTan
h[(b*g - 2*a*h + (2*c*g - b*h)*x)/(2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a +
b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g*h + a*h^2]))/(c*g^2 - b*g*h + a*h^2)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(397) = 794$.

Time = 0.29 (sec) , antiderivative size = 1115, normalized size of antiderivative = 2.71

method	result	size
default	Expression too large to display	1115

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*f/h^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/h^3*(e*h-2*f*g)*(1/(a*
h^2-b*g*h+c*g^2)*h^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2
)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+g/h)+(b*h-2*c*g)/h)
/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+(b*h-2*c*g)/
h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/
h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a
*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+1/h^4*(d*h^2-e*g*h+f*g^2)*(-1/(a*h
^2-b*g*h+c*g^2)*h^2/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*
h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/(a*h^2-b*g*h+
c*g^2)*h^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/
2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+g/h)+(b*h-2*c*g)/h)/(4*c*(a*h
^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+
(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c*g
^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a*h
^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h
+c*g^2)/h^2)^(1/2))/(x+g/h)))-4*c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+g/h)+(b*
h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+(
b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2528 vs. $2(397) = 794$.

Time = 35.17 (sec) , antiderivative size = 5098, normalized size of antiderivative = 12.40

$$\int \frac{d+ex+fx^2}{(g+hx)^2 (a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**2/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/h-(2*c*g)/h^2)^2>0)', see `as
sume?` for
```

Giac [F(-1)]

Timed out.

$$\int \frac{d + ex + fx^2}{(g + hx)^2 (a + bx + cx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^2 (a+bx+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{(g+h\,x)^2\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^2*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 12617, normalized size of antiderivative = 30.70

$$\int \frac{d+ex+fx^2}{(g+hx)^2 (a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)^2/(c*x^2+b*x+a)^(3/2),x)
```

output
```
( - 8*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a
*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*e*g*h**3 -
 8*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h*
*2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*e*h**4*x + 16
*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g**2*h**2 + 1
6*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**
2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*c*f*g*h**3*x + 2
*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*e*g*h**3 + 2
*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*e*h**4*x - 4
*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f*g**2*h**2
- 4*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*h
**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b**2*f*g*h**3*
x + 12*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(
a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*d*g*h**
3 + 12*sqrt(a*h**2 - b*g*h + c*g**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(
a*h**2 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**2*b*c*d*h*...
```

3.76 $\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 699

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx = \frac{2(b^2dh-b(cdg+afg+aeh)+2a(ceg-cdh+afh)+((ce-bf)(bg}{(b^2-4ac)(cg^2-bgh+ah^2)(g+hx)^2\sqrt{a+}}$$

$$-\frac{h(8c^2dg^2+8a^2fh^2-4abh(2fg+eh)+b^2(5fg^2-h(eg-5dh))-4c(3afg^2-ah(5eg-3dh)+bg(eg+}{2(b^2-4ac)(cg^2-bgh+ah^2)^2(g+hx)^2}$$

$$-\frac{h(16c^3dg^3-8c^2g(5afg^2-ah(11eg-13dh)+bg(eg+3dh))+2c(8a^2h^2(5fg-2eh)+b^2(7fg^3-gh(5e}{4(b^2-4ac)}$$

$$+\frac{(8c^2g^2(fg^2-3egh+6dh^2)+4ch(2bfg^3+3bgh(eg-4dh)-ah(11fg^2-9egh+3dh^2))+h^2(8a^2fh^2+}{8(cg^2-bgh+ah^2)^{7/2}}$$

output
```
2*(b^2*d*h-b*(a*e*h+a*f*g+c*d*g)+2*a*(a*f*h-c*d*h+c*e*g)+((-b*f+c*e)*(-2*a
*h+b*g)-(-a*f+c*d)*(-b*h+2*c*g))*x)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)/(h*x+
g)^2/(c*x^2+b*x+a)^(1/2)-1/2*h*(8*c^2*d*g^2+8*a^2*f*h^2-4*a*b*h*(e*h+2*f*g
)+b^2*(5*f*g^2-h*(-5*d*h+e*g))-4*c*(3*a*f*g^2-a*h*(-3*d*h+5*e*g)+b*g*(2*d*
h+e*g)))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(a*h^2-b*g*h+c*g^2)^2/(h*x+g)^2-
1/4*h*(16*c^3*d*g^3-8*c^2*g*(5*a*f*g^2-a*h*(-13*d*h+11*e*g)+b*g*(3*d*h+e*g
))+2*c*(8*a^2*h^2*(-2*e*h+5*f*g)+b^2*(7*f*g^3-g*h*(-19*d*h+5*e*g))-2*a*b*h
*(7*f*g^2+h*(-13*d*h+9*e*g)))-b*h*(8*a^2*f*h^2+4*a*b*h*(-3*e*h+2*f*g)-b^2*
(f*g^2+3*h*(-5*d*h+e*g))))*(c*x^2+b*x+a)^(1/2)/(-4*a*c+b^2)/(a*h^2-b*g*h+c
*g^2)^3/(h*x+g)+1/8*(8*c^2*g^2*(6*d*h^2-3*e*g*h+f*g^2)+4*c*h*(2*b*f*g^3+3*
b*g*h*(-4*d*h+e*g)-a*h*(3*d*h^2-9*e*g*h+11*f*g^2))+h^2*(8*a^2*f*h^2+4*a*b*
h*(-3*e*h+2*f*g)-b^2*(f*g^2+3*h*(-5*d*h+e*g))))*arctanh(1/2*(b*g-2*a*h+(-b
*h+2*c*g)*x)/(a*h^2-b*g*h+c*g^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*h^2-b*g*h+c
*g^2)^(7/2)
```

Mathematica [A] (verified)

Time = 13.11 (sec) , antiderivative size = 847, normalized size of antiderivative = 1.21

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2(fg^2+h(-eg+dh))\left(-b^2h+2c(ah+cgx)+bc(g-hx)\right)}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)(g+hx)^2\sqrt{a+x(b+cx)}}$$
$$-\frac{2f(b^2h-2c(ah+cgx)+bc(-g+hx))}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)\sqrt{a+x(b+cx)}}$$
$$-\frac{2(-2fg+eh)\left(b^2h-2c(ah+cgx)+bc(-g+hx)\right)}{\left(b^2-4ac\right)h^2\left(-cg^2+h(bg-ah)\right)(g+hx)\sqrt{a+x(b+cx)}}$$
$$-\frac{(-2fg+eh)\left(\frac{2(4c^2g^2+3b^2h^2-4ch(bg+2ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}-\frac{3(b^2-4ac)h(-2cg+bh)\text{arctanh}\left(\frac{-bg+2ah-2cgx+bhx}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}\right)}{2\left(b^2-4ac\right)h\left(cg^2+h(-bg+ah)\right)}$$
$$-\frac{(fg^2+h(-eg+dh))\left(\frac{4(8c^2g^2+5b^2h^2-4ch(2bg+3ah))\sqrt{a+x(b+cx)}}{(g+hx)^2}+\frac{2(2cg-bh)(8c^2g^2+15b^2h^2-4ch(2bg+13ah))\sqrt{a+x(b+cx)}}{(cg^2+h(-bg+ah))(g+hx)}\right.}{8\left(b^2-4ac\right)h\left(cg^2+h(-bg+ah)\right)}$$
$$+\frac{f\text{arctanh}\left(\frac{-2ah+2cgx+b(g-hx)}{2\sqrt{cg^2+h(-bg+ah)}\sqrt{a+x(b+cx)}}\right)}{(cg^2+h(-bg+ah))^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(f*g^2 + h*(-(e*g) + d*h))*(-(b^2*h) + 2*c*(a*h + c*g*x) + b*c*(g - h*x
)))/((b^2 - 4*a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*(g + h*x)^2*Sqrt[a + x*(
b + c*x)]) - (2*f*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*
a*c)*h^2*(-(c*g^2) + h*(b*g - a*h))*Sqrt[a + x*(b + c*x)]) - (2*(-2*f*g +
e*h)*(b^2*h - 2*c*(a*h + c*g*x) + b*c*(-g + h*x)))/((b^2 - 4*a*c)*h^2*(-(c
*g^2) + h*(b*g - a*h))*(g + h*x)*Sqrt[a + x*(b + c*x)]) - ((-2*f*g + e*h)*
((2*(4*c^2*g^2 + 3*b^2*h^2 - 4*c*h*(b*g + 2*a*h))*Sqrt[a + x*(b + c*x)])/(
(c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) - (3*(b^2 - 4*a*c)*h*(-2*c*g + b*h)*
ArcTanh[(-(b*g) + 2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h
)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(2*(b^2 - 4
*a*c)*h*(c*g^2 + h*(-(b*g) + a*h))) - ((f*g^2 + h*(-(e*g) + d*h))*((4*(8*c
^2*g^2 + 5*b^2*h^2 - 4*c*h*(2*b*g + 3*a*h))*Sqrt[a + x*(b + c*x)])/(g + h*
x)^2 + (2*(2*c*g - b*h)*(8*c^2*g^2 + 15*b^2*h^2 - 4*c*h*(2*b*g + 13*a*h))*
Sqrt[a + x*(b + c*x)])/((c*g^2 + h*(-(b*g) + a*h))*(g + h*x)) + (3*(b^2 -
4*a*c)*h*(16*c^2*g^2 + 5*b^2*h^2 - 4*c*h*(4*b*g + a*h))*ArcTanh[(-(b*g) +
2*a*h - 2*c*g*x + b*h*x)/(2*Sqrt[c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b +
 c*x)])])/(c*g^2 + h*(-(b*g) + a*h))^(3/2)))/(8*(b^2 - 4*a*c)*h*(c*g^2 + h
*(-(b*g) + a*h))^2) + (f*ArcTanh[(-2*a*h + 2*c*g*x + b*(g - h*x))/(2*Sqrt[
c*g^2 + h*(-(b*g) + a*h)]*Sqrt[a + x*(b + c*x)])])/(c*g^2 + h*(-(b*g) + a*
h))^(3/2)
```

Rubi [A] (verified)

Time = 5.78 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2}{(g+hx)^3\,(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2177$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.$$

$$2\int -\frac{\frac{\left(b^2-4ac\right)\left(\left(\left(fa^2-bea+b^2d\right)h^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)+bc\left(fg^3-3dgh^2\right)\right)x^2h^3}{\left(cg^2-bhg+ah^2\right)^3}-\frac{\left(b^2-4ac\right)\left(c^2(3eg-8dh)g^3-c\left(3bfg^2+bh(eg-9dh)-\right.\right.}{}}{}$$

$$\downarrow 27$$

$$\int \frac{\frac{\left(b^2-4ac\right)\left(\left(\left(fa^2-bea+b^2d\right)h^3-ac\left(3fg^2-h(3eg-dh)\right)h-c^2g^2(eg-3dh)+bc\left(fg^3-3dgh^2\right)\right)x^2h^3}{\left(cg^2-bhg+ah^2\right)^3}-\frac{\left(b^2-4ac\right)\left(c^2(3eg-8dh)g^3-c\left(3bfg^2+bh(eg-9dh)-2ah(\right.\right.}{}}{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.}$$

$$\downarrow 2181$$

$$\frac{-\frac{\int -\frac{\left(b^2-4ac\right)\left(4c^2\left(fg^2-3ehg+6dh^2\right)g^3+ch\left(bg\left(fg^2+11ehg-31dh^2\right)-8ah\left(2fg^2-ehg-dh^2\right)\right)g-h^2\left(g\left(fg^2+3ehg-11dh^2\right)b^2-ah\left(9fg^2-5ehg-7dh^2\right)b+4a\right.\right.}{2\left(cg^2-bhg+ah^2\right)^2(g+hx)^2\sqrt{cx^2+}}}{2(ah^2-bgh+cg^2)}}{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.}$$

$$\downarrow 27$$

$$\frac{\frac{(b^2-4ac)\int \frac{4c^2\left(fg^2-3ehg+6dh^2\right)g^3+ch\left(bg\left(fg^2+11ehg-31dh^2\right)-8ah\left(2fg^2-ehg-dh^2\right)\right)g-h^2\left(g\left(fg^2+3ehg-11dh^2\right)b^2-ah\left(9fg^2-5ehg-7dh^2\right)b+4a^2h^2(\right.}{(g+hx)^2\sqrt{cx^2+bx+a}}}{4(ah^2-bgh+cg^2)^3}\quad b}{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.}$$

$$\downarrow 1228$$

$$\frac{\frac{(b^2-4ac)\left(\frac{1}{2}\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg-4dh)+2bfg^3\right)+8c^2g^2\left(6dh^2-3\right.\right.\right.}{4(ah^2-bgh+cg^2)^3}}{2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.}$$

$$\downarrow 1154$$

$$\frac{(b^2-4ac)\left(-\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg-4dh)+2bfg^3\right)+8c^2g^2\left(6dh^2-\right.\right.}{4(}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.$$

$\downarrow$ 219

$$\frac{(b^2-4ac)\left(\frac{\text{arctanh}\left(\frac{-2ah+x(2cg-bh)+bg}{2\sqrt{a+bx+cx^2}\sqrt{ah^2-bgh+cg^2}}\right)\left(h^2\left(8a^2fh^2+4abh(2fg-3eh)-\left(b^2\left(3h(eg-5dh)+fg^2\right)\right)\right)+4ch\left(-ah\left(3dh^2-9egh+11fg^2\right)+3bgh(eg\right.\right.}{2\sqrt{ah^2-bgh+cg^2}}\right.}{4(ah^2-bgh+cg^2)^3}$$

$$2\left(b^2h\left(a^2fh^2+ach(3eg-4dh)+3c^2dg^2\right)-cx\left(c\left(2a^2h^2(3fg-eh)-3abh\left(h(eg-dh)+fg^2\right)+b^2\left(3dgh^2+fg\right.\right.\right.\right.$$

input

```
Int[(d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x]
```

output

```
(2*(b^4*d*h^3 - b^3*h^2*(3*c*d*g + a*e*h) + b^2*h*(3*c^2*d*g^2 + a^2*f*h^2
 + a*c*h*(3*e*g - 4*d*h)) - b*c*(c^2*d*g^3 + 3*a^2*h^2*(f*g - e*h) + a*c*g
*(f*g^2 + 3*e*g*h - 9*d*h^2)) - 2*a*c*(a^2*f*h^3 - c^2*g^2*(e*g - 3*d*h) -
 a*c*h*(3*f*g^2 - 3*e*g*h + d*h^2)) - c*(2*c^3*d*g^3 - b*(b^2*d - a*b*e +
a^2*f)*h^3 - c^2*g*(2*a*f*g^2 - 6*a*h*(e*g - d*h) + b*g*(e*g + 3*d*h)) + c
*(2*a^2*h^2*(3*f*g - e*h) + b^2*(f*g^3 + 3*d*g*h^2) - 3*a*b*h*(f*g^2 + h*(
e*g - d*h))))*x))/((b^2 - 4*a*c)*(c*g^2 - b*g*h + a*h^2)^3*Sqrt[a + b*x +
c*x^2]) + (-1/2*((b^2 - 4*a*c)*h*(f*g^2 - h*(e*g - d*h))*Sqrt[a + b*x + c*
x^2])/((c*g^2 - b*g*h + a*h^2)^2*(g + h*x)^2) + ((b^2 - 4*a*c)*(-((h*(6*c*
f*g^3 - 2*c*g*h*(5*e*g - 7*d*h) - 4*a*h^2*(2*f*g - e*h) + b*h*(f*g^2 + h*(
3*e*g - 7*d*h)))*Sqrt[a + b*x + c*x^2])/(g + h*x)) + ((8*c^2*g^2*(f*g^2 -
3*e*g*h + 6*d*h^2) + 4*c*h*(2*b*f*g^3 + 3*b*g*h*(e*g - 4*d*h) - a*h*(11*f*
g^2 - 9*e*g*h + 3*d*h^2)) + h^2*(8*a^2*f*h^2 + 4*a*b*h*(2*f*g - 3*e*h) - b
^2*(f*g^2 + 3*h*(e*g - 5*d*h))))*ArcTanh[(b*g - 2*a*h + (2*c*g - b*h)*x)/(
2*Sqrt[c*g^2 - b*g*h + a*h^2]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*g^2 - b*g
*h + a*h^2])))/(4*(c*g^2 - b*g*h + a*h^2)^3))/(b^2 - 4*a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2267 vs. $2(679) = 1358$.

Time = 0.36 (sec) , antiderivative size = 2268, normalized size of antiderivative = 3.24

method	result	size
default	Expression too large to display	2268

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
f/h^3*(1/(a*h^2-b*g*h+c*g^2)*h^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2
-b*g*h+c*g^2)/h^2)^(1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+g/h)+(b
*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+
(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)
*h^2/((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*
c*g)/h*(x+g/h)+2*((a*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/
h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))/(x+g/h)))+(e*h-2*f*g)/h^4*(-1/(a
*h^2-b*g*h+c*g^2)*h^2/(x+g/h)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*
g*h+c*g^2)/h^2)^(1/2)-3/2*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(1/(a*h^2-b*g*
h+c*g^2)*h^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(
1/2)-(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g^2)*(2*c*(x+g/h)+(b*h-2*c*g)/h)/(4*c*(a
*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h
)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-1/(a*h^2-b*g*h+c*g^2)*h^2/((a*h^2-b*g*h+c
*g^2)/h^2)^(1/2)*ln((2*(a*h^2-b*g*h+c*g^2)/h^2+(b*h-2*c*g)/h*(x+g/h)+2*((a
*h^2-b*g*h+c*g^2)/h^2)^(1/2)*((x+g/h)^2*c+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g
*h+c*g^2)/h^2)^(1/2))/(x+g/h)))-4*c/(a*h^2-b*g*h+c*g^2)*h^2*(2*c*(x+g/h)+(
b*h-2*c*g)/h)/(4*c*(a*h^2-b*g*h+c*g^2)/h^2-(b*h-2*c*g)^2/h^2)/((x+g/h)^2*c
+(b*h-2*c*g)/h*(x+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2))+(d*h^2-e*g*h+f*g^2)
/h^5*(-1/2/(a*h^2-b*g*h+c*g^2)*h^2/(x+g/h)^2/((x+g/h)^2*c+(b*h-2*c*g)/h*(x
+g/h)+(a*h^2-b*g*h+c*g^2)/h^2)^(1/2)-5/4*(b*h-2*c*g)*h/(a*h^2-b*g*h+c*g...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5149 vs. $2(679) = 1358$.

Time $= 155.06$ (sec) , antiderivative size $= 10340$, normalized size of antiderivative $=$ 14.79

$$\int \frac{d+ex+fx^2}{(g+hx)^3\left(a+bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input

```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas
")
```

output

```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((f*x**2+e*x+d)/(h*x+g)**3/(c*x**2+b*x+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a*h^2-b*g*h>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5562 vs. $2(679) = 1358$.

Time $= 0.35$ (sec) , antiderivative size $= 5562$, normalized size of antiderivative $= 7.96$

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^7*d*g^9 - b*c^6*e*g^9 + b^2*c^5*f*g^9 - 2*a*c^6*f*g^9 - 9*b*c^6*d
*g^8*h + 3*b^2*c^5*e*g^8*h + 6*a*c^6*e*g^8*h - 3*b^3*c^4*f*g^8*h + 3*a*b*c
^5*f*g^8*h + 18*b^2*c^5*d*g^7*h^2 - 3*b^3*c^4*e*g^7*h^2 - 24*a*b*c^5*e*g^7
*h^2 + 3*b^4*c^3*f*g^7*h^2 + 6*a*b^2*c^4*f*g^7*h^2 - 21*b^3*c^4*d*g^6*h^3
+ b^4*c^3*e*g^6*h^3 + 34*a*b^2*c^4*e*g^6*h^3 + 16*a^2*c^5*e*g^6*h^3 - b^5*
c^2*f*g^6*h^3 - 13*a*b^3*c^3*f*g^6*h^3 - 16*a^2*b*c^4*f*g^6*h^3 + 15*b^4*c
^3*d*g^5*h^4 + 6*a*b^2*c^4*d*g^5*h^4 - 12*a^2*c^5*d*g^5*h^4 - 21*a*b^3*c^3
*e*g^5*h^4 - 42*a^2*b*c^4*e*g^5*h^4 + 6*a*b^4*c^2*f*g^5*h^4 + 36*a^2*b^2*c
^3*f*g^5*h^4 + 12*a^3*c^4*f*g^5*h^4 - 6*b^5*c^2*d*g^4*h^5 - 15*a*b^3*c^3*d
*g^4*h^5 + 30*a^2*b*c^4*d*g^4*h^5 + 6*a*b^4*c^2*e*g^4*h^5 + 36*a^2*b^2*c^3
*e*g^4*h^5 + 12*a^3*c^4*e*g^4*h^5 - 21*a^2*b^3*c^2*f*g^4*h^5 - 42*a^3*b*c^
3*f*g^4*h^5 + b^6*c*d*g^3*h^6 + 12*a*b^4*c^2*d*g^3*h^6 - 18*a^2*b^2*c^3*d*
g^3*h^6 - 16*a^3*c^4*d*g^3*h^6 - a*b^5*c*e*g^3*h^6 - 13*a^2*b^3*c^2*e*g^3*
h^6 - 16*a^3*b*c^3*e*g^3*h^6 + a^2*b^4*c*f*g^3*h^6 + 34*a^3*b^2*c^2*f*g^3*
h^6 + 16*a^4*c^3*f*g^3*h^6 - 3*a*b^5*c*d*g^2*h^7 - 3*a^2*b^3*c^2*d*g^2*h^7
 + 24*a^3*b*c^3*d*g^2*h^7 + 3*a^2*b^4*c*e*g^2*h^7 + 6*a^3*b^2*c^2*e*g^2*h^
7 - 3*a^3*b^3*c*f*g^2*h^7 - 24*a^4*b*c^2*f*g^2*h^7 + 3*a^2*b^4*c*d*g*h^8 -
 6*a^3*b^2*c^2*d*g*h^8 - 6*a^4*c^3*d*g*h^8 - 3*a^3*b^3*c*e*g*h^8 + 3*a^4*b
*c^2*e*g*h^8 + 3*a^4*b^2*c*f*g*h^8 + 6*a^5*c^2*f*g*h^8 - a^3*b^3*c*d*h^9 +
 3*a^4*b*c^2*d*h^9 + a^4*b^2*c*e*h^9 - 2*a^5*c^2*e*h^9 - a^5*b*c*f*h^9)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2}{(g+hx)^3(a+bx+cx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e\,x+d}{{(g+h\,x)}^3\,{(c\,x^2+b\,x+a)}^{3/2}}\,dx$$

input
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((d + e*x + f*x^2)/((g + h*x)^3*(a + b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 27527, normalized size of antiderivative = 39.38

$$\int \frac{d+ex+fx^2}{(g+hx)^3 (a+bx+cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((f*x^2+e*x+d)/(h*x+g)^3/(c*x^2+b*x+a)^(3/2),x)
```

output
```
(32*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2
 - b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**4*c*f*g**2*h**4 + 6
4*sqrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 -
 b*g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**4*c*f*g*h**5*x + 32*s
qrt(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*
g*h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**4*c*f*h**6*x**2 - 8*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b**2*f*g**2*h**4 - 16*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b**2*f*g*h**5*x - 8*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b**2*f*h**6*x**2 - 48*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*e*g**2*h**4 - 96*sqr
t(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*
h + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*e*g*h**5*x - 48*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*e*h**6*x**2 + 32*sqrt
(a*h**2 - b*g*h + c*g**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*h**2 - b*g*h
 + c*g**2) - 2*a*h + b*g - b*h*x + 2*c*g*x)*a**3*b*c*f*g**3*h**3 + 96*s...
```

3.77 $\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 120

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{44}{135}(1+2x)^2\sqrt{2-x+3x^2}+\frac{19}{60}(1+2x)^3\sqrt{2-x+3x^2} + \frac{2}{15}(1+2x)^4\sqrt{2-x+3x^2} - \frac{(24897+6298x)\sqrt{2-x+3x^2}}{3240} + \frac{9211\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{1296\sqrt{3}}$$

output
```
44/135*(1+2*x)^2*(3*x^2-x+2)^(1/2)+19/60*(1+2*x)^3*(3*x^2-x+2)^(1/2)+2/15*
(1+2*x)^4*(3*x^2-x+2)^(1/2)-1/3240*(24897+6298*x)*(3*x^2-x+2)^(1/2)+9211/3
888*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{6\sqrt{2-x+3x^2}(-22383+7538x+26904x^2+22032x^3+6912x^4)+46055\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+\right.}{19440}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-22383 + 7538*x + 26904*x^2 + 22032*x^3 + 6912*x^4
) + 46055*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/19440
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2184, 27, 1236, 27, 1236, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}\,dx$$

$$\Big\downarrow \ 2184$$

$$\frac{1}{60}\int -\frac{4(16-57x)(2x+1)^3}{\sqrt{3x^2-x+2}}dx+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow \ 27$$

$$\frac{2}{15}(2x+1)^4\sqrt{3x^2-x+2}-\frac{1}{15}\int \frac{(16-57x)(2x+1)^3}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \ 1236$$

$$\frac{1}{15}\left(\frac{19}{4}(2x+1)^3\sqrt{3x^2-x+2}-\frac{1}{12}\int \frac{3(565-352x)(2x+1)^2}{2\sqrt{3x^2-x+2}}dx\right)+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow 27$$

$$\frac{1}{15}\left(\frac{19}{4}(2x+1)^3\sqrt{3x^2-x+2}-\frac{1}{8}\int\frac{(565-352x)(2x+1)^2}{\sqrt{3x^2-x+2}}dx\right)+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow 1236$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{352}{9}(2x+1)^2\sqrt{3x^2-x+2}-\frac{1}{9}\int\frac{(2x+1)(6298x+7725)}{\sqrt{3x^2-x+2}}dx\right)+\frac{19}{4}\sqrt{3x^2-x+2}(2x+1)^3\right)+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow 1225$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055}{6}\int\frac{1}{\sqrt{3x^2-x+2}}dx-\frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right)+\frac{352}{9}\sqrt{3x^2-x+2}(2x+1)^2\right)+\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow 1090$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{6\sqrt{69}}-\frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right)+\frac{352}{9}\sqrt{3x^2-x+2}(2x+1\right.\right.$$
$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

$$\Big\downarrow 222$$

$$\frac{1}{15}\left(\frac{1}{8}\left(\frac{1}{9}\left(-\frac{46055\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{1}{3}\sqrt{3x^2-x+2}(6298x+24897)\right)+\frac{352}{9}\sqrt{3x^2-x+2}(2x+1)^2\right)+\frac{19}{4}\right.$$
$$\frac{2}{15}\sqrt{3x^2-x+2}(2x+1)^4$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^4*Sqrt[2 - x + 3*x^2])/15 + ((19*(1 + 2*x)^3*Sqrt[2 - x + 3*x
^2])/4 + ((352*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + (-1/3*((24897 + 6298*x
)*Sqrt[2 - x + 3*x^2]) - (46055*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]))
/9)/8)/15
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(6912x^4+22032x^3+26904x^2+7538x-22383)\sqrt{3x^2-x+2}}{3240}-\frac{9211\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$
trager	$\left(\frac{32}{15}x^4+\frac{34}{5}x^3+\frac{1121}{135}x^2+\frac{3769}{1620}x-\frac{829}{120}\right)\sqrt{3x^2-x+2}-\frac{9211\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{Ro}\right.}{3888}$
default	$-\frac{9211\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}-\frac{829\sqrt{3x^2-x+2}}{120}+\frac{3769x\sqrt{3x^2-x+2}}{1620}+\frac{1121x^2\sqrt{3x^2-x+2}}{135}+\frac{34x^3\sqrt{3x^2-x+2}}{5}+32$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3240*(6912*x^4+22032*x^3+26904*x^2+7538*x-22383)*(3*x^2-x+2)^(1/2)-9211/
3888*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{3240}\left(6912\,x^4+22032\,x^3+26904\,x^2+7538\,x-22383\right)\sqrt{3\,x^2-x+2} + \frac{9211}{7776}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/3240*(6912*x^4 + 22032*x^3 + 26904*x^2 + 7538*x - 22383)*sqrt(3*x^2 - x
+ 2) + 9211/7776*sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*
x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2} \cdot \left(\frac{32x^4}{15}+\frac{34x^3}{5}+\frac{1121x^2}{135}+\frac{3769x}{1620}-\frac{829}{120}\right) - \frac{9211\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3888}$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(32*x**4/15 + 34*x**3/5 + 1121*x**2/135 + 3769*x/1620
 - 829/120) - 9211*sqrt(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/3888
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{32}{15}\sqrt{3x^2-x+2}x^4 + \frac{34}{5}\sqrt{3x^2-x+2}x^3 + \frac{1121}{135}\sqrt{3x^2-x+2}x^2 + \frac{3769}{1620}\sqrt{3x^2-x+2}x - \frac{9211}{3888}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) - \frac{829}{120}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
32/15*sqrt(3*x^2 - x + 2)*x^4 + 34/5*sqrt(3*x^2 - x + 2)*x^3 + 1121/135*sq
rt(3*x^2 - x + 2)*x^2 + 3769/1620*sqrt(3*x^2 - x + 2)*x - 9211/3888*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 829/120*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{3240}(2(12(18(16x+51)x+1121)x+3769)x-22383)\sqrt{3x^2-x+2} + \frac{9211}{3888}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/3240*(2*(12*(18*(16*x + 51)*x + 1121)*x + 3769)*x - 22383)*sqrt(3*x^2 -
x + 2) + 9211/3888*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)
) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{32\sqrt{3x^2-x+2}\,x^4}{15} + \frac{34\sqrt{3x^2-x+2}\,x^3}{5} + \frac{1121\sqrt{3x^2-x+2}\,x^2}{135} + \frac{3769\sqrt{3x^2-x+2}\,x}{1620} - \frac{829\sqrt{3x^2-x+2}}{120} - \frac{9211\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{3888}$$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

output
```
(41472*sqrt(3*x**2 - x + 2)*x**4 + 132192*sqrt(3*x**2 - x + 2)*x**3 + 1614
24*sqrt(3*x**2 - x + 2)*x**2 + 45228*sqrt(3*x**2 - x + 2)*x - 134298*sqrt(
3*x**2 - x + 2) - 46055*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x
- 1)/sqrt(23)))/19440
```

3.78 $\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 95

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = -\frac{143}{324}(3-2x)\sqrt{2-x+3x^2}+\frac{11}{27}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{6}(1+2x)^3\sqrt{2-x+3x^2}+\frac{4147\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{648\sqrt{3}}$$

output
```
-143/324*(3-2*x)*(3*x^2-x+2)^(1/2)+11/27*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/6*(
1+2*x)^3*(3*x^2-x+2)^(1/2)+4147/1944*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2
)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{6\sqrt{2-x+3x^2}(-243+1138x+1176x^2+432x^3)+4147\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{1944}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(-243 + 1138*x + 1176*x^2 + 432*x^3) + 4147*Sqrt[3]
*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/1944
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2184, 27, 1236, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{48}\int -\frac{44(1-4x)(2x+1)^2}{\sqrt{3x^2-x+2}}dx+\frac{1}{6}\sqrt{3x^2-x+2}(2x+1)^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\int \frac{(1-4x)(2x+1)^2}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\left(\frac{1}{9}\int \frac{13(3-2x)(2x+1)}{\sqrt{3x^2-x+2}}dx-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\left(\frac{13}{9}\int \frac{(3-2x)(2x+1)}{\sqrt{3x^2-x+2}}dx-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 1225}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-$$
$$\frac{11}{12}\left(\frac{13}{9}\left(\frac{29}{6}\int \frac{1}{\sqrt{3x^2-x+2}}dx+\frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right)-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\left(\frac{13}{9}\left(\frac{29\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{6\sqrt{69}}+\frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right)-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{6}(2x+1)^3\sqrt{3x^2-x+2}-\frac{11}{12}\left(\frac{13}{9}\left(\frac{29\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}+\frac{1}{3}\sqrt{3x^2-x+2}(3-2x)\right)-\frac{4}{9}(2x+1)^2\sqrt{3x^2-x+2}\right)$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
((1 + 2*x)^3*Sqrt[2 - x + 3*x^2])/6 - (11*((-4*(1 + 2*x)^2*Sqrt[2 - x + 3*
x^2])/9 + (13*(((3 - 2*x)*Sqrt[2 - x + 3*x^2])/3 + (29*ArcSinh[(-1 + 6*x)/
Sqrt[23]])/(6*Sqrt[3])))/9))/12
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.47

method	result
risch	$\frac{(432x^3+1176x^2+1138x-243)\sqrt{3x^2-x+2}}{324}-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1944}$
trager	$\left(\frac{4}{3}x^3+\frac{98}{27}x^2+\frac{569}{162}x-\frac{3}{4}\right)\sqrt{3x^2-x+2}-\frac{4147\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(6\operatorname{RootOf}\left(\_Z^2-3\right)x-\operatorname{RootOf}\left(\_Z^2-3\right)\right.}{1944}$
default	$-\frac{4147\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{1944}-\frac{3\sqrt{3x^2-x+2}}{4}+\frac{569x\sqrt{3x^2-x+2}}{162}+\frac{98x^2\sqrt{3x^2-x+2}}{27}+\frac{4x^3\sqrt{3x^2-x+2}}{3}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/324*(432*x^3+1176*x^2+1138*x-243)*(3*x^2-x+2)^(1/2)-4147/1944*3^(1/2)*ar
csinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx=\frac{1}{324}\left(432\,x^3+1176\,x^2+1138\,x-243\right)\sqrt{3\,x^2-x+2}+\frac{4147}{3888}\sqrt{3}\log\left(4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/324*(432*x^3 + 1176*x^2 + 1138*x - 243)*sqrt(3*x^2 - x + 2) + 4147/3888*
sqrt(3)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \sqrt{3x^2-x+2}\cdot\left(\frac{4x^3}{3}+\frac{98x^2}{27}+\frac{569x}{162}-\frac{3}{4}\right) - \frac{4147\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{1944}$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
sqrt(3*x**2 - x + 2)*(4*x**3/3 + 98*x**2/27 + 569*x/162 - 3/4) - 4147*sqrt
(3)*asinh(6*sqrt(23)*(x - 1/6)/23)/1944
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{4}{3}\sqrt{3\,x^2-x+2}x^3+\frac{98}{27}\sqrt{3\,x^2-x+2}x^2 + \frac{569}{162}\sqrt{3\,x^2-x+2}x - \frac{4147}{1944}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{3}{4}\sqrt{3\,x^2-x+2}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(3*x^2 - x + 2)*x^3 + 98/27*sqrt(3*x^2 - x + 2)*x^2 + 569/162*sqrt
(3*x^2 - x + 2)*x - 4147/1944*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 3
/4*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(2\left(12\left(18\,x+49\right)x+569\right)x-243\right)\sqrt{3\,x^2-x+2} + \frac{4147}{1944}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/324*(2*(12*(18*x + 49)*x + 569)*x - 243)*sqrt(3*x^2 - x + 2) + 4147/1944
*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{(2\,x+1)^2\,(4\,x^2+3\,x+1)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{4\sqrt{3x^2-x+2}\,x^3}{3} + \frac{98\sqrt{3x^2-x+2}\,x^2}{27} + \frac{569\sqrt{3x^2-x+2}\,x}{162} - \frac{3\sqrt{3x^2-x+2}}{4} - \frac{4147\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{1944}$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

output
```
(2592*sqrt(3*x**2 - x + 2)*x**3 + 7056*sqrt(3*x**2 - x + 2)*x**2 + 6828*sq
rt(3*x**2 - x + 2)*x - 1458*sqrt(3*x**2 - x + 2) - 4147*sqrt(3)*log((2*sqr
t(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)))/1944
```

3.79 $\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 70

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{2}{9}(1+2x)^2\sqrt{2-x+3x^2} + \frac{1}{54}(69+62x)\sqrt{2-x+3x^2} + \frac{251\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{108\sqrt{3}}$$

output
```
2/9*(1+2*x)^2*(3*x^2-x+2)^(1/2)+1/54*(69+62*x)*(3*x^2-x+2)^(1/2)+251/324*a
rcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{324}\left(6\sqrt{2-x+3x^2}(81+110x+48x^2) + 251\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)\right)$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(6*Sqrt[2 - x + 3*x^2]*(81 + 110*x + 48*x^2) + 251*Sqrt[3]*Log[1 - 6*x + 2
*Sqrt[6 - 3*x + 9*x^2]])/324
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2184, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{36}\int -\frac{4(6-31x)(2x+1)}{\sqrt{3x^2-x+2}}dx+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}(2x+1)^2\sqrt{3x^2-x+2}-\frac{1}{9}\int \frac{(6-31x)(2x+1)}{\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251}{12}\int \frac{1}{\sqrt{3x^2-x+2}}dx\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{12\sqrt{69}}\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{9}\left(\frac{1}{6}(62x+69)\sqrt{3x^2-x+2}-\frac{251\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{12\sqrt{3}}\right)+\frac{2}{9}\sqrt{3x^2-x+2}(2x+1)^2$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/Sqrt[2 - x + 3*x^2],x]
```

output
```
(2*(1 + 2*x)^2*Sqrt[2 - x + 3*x^2])/9 + (((69 + 62*x)*Sqrt[2 - x + 3*x^2])
/6 - (251*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(12*Sqrt[3]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.57

method	result
risch	$\frac{(48x^2+110x+81)\sqrt{3x^2-x+2}}{54}-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{324}$
default	$-\frac{251\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{324}+\frac{3\sqrt{3x^2-x+2}}{2}+\frac{55x\sqrt{3x^2-x+2}}{27}+\frac{8x^2\sqrt{3x^2-x+2}}{9}$
trager	$\left(\frac{8}{9}x^2+\frac{55}{27}x+\frac{3}{2}\right)\sqrt{3x^2-x+2}+\frac{251\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2-\right.\right.}{324}$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(48*x^2+110*x+81)*(3*x^2-x+2)^(1/2)-251/324*3^(1/2)*arcsinh(6/23*23^(
1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx=\frac{1}{54}\left(48\,x^2+110\,x+81\right)\sqrt{3\,x^2-x+2}+\frac{251}{648}\,\sqrt{3}\log\left(4\,\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-25\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(48*x^2 + 110*x + 81)*sqrt(3*x^2 - x + 2) + 251/648*sqrt(3)*log(4*sqr
t(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \left(\frac{8x^2}{9}+\frac{55x}{27}+\frac{3}{2}\right)\sqrt{3x^2-x+2} - \frac{251\sqrt{3}\operatorname{asinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{324}$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(1/2),x)
```

output
```
(8*x**2/9 + 55*x/27 + 3/2)*sqrt(3*x**2 - x + 2) - 251*sqrt(3)*asinh(6*sqrt
(23)*(x - 1/6)/23)/324
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)(1+3x+4x^2)}{\sqrt{2-x+3x^2}}\,dx = \frac{8}{9}\sqrt{3x^2-x+2}x^2 + \frac{55}{27}\sqrt{3x^2-x+2}x - \frac{251}{324}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6x-1)\right) + \frac{3}{2}\sqrt{3x^2-x+2}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
8/9*sqrt(3*x^2 - x + 2)*x^2 + 55/27*sqrt(3*x^2 - x + 2)*x - 251/324*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) + 3/2*sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.83

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{1}{54}\left(2\left(24\,x+55\right)x+81\right)\sqrt{3\,x^2-x+2} + \frac{251}{324}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right)$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
1/54*(2*(24*x + 55)*x + 81)*sqrt(3*x^2 - x + 2) + 251/324*sqrt(3)*log(-2*s
qrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \int \frac{\left(2\,x+1\right)\,\left(4\,x^2+3\,x+1\right)}{\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2),x)
```

output
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\sqrt{2-x+3x^2}}\,dx = \frac{8\sqrt{3x^2-x+2}\,x^2}{9} + \frac{55\sqrt{3x^2-x+2}\,x}{27} + \frac{3\sqrt{3x^2-x+2}}{2} - \frac{251\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{324}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(1/2),x)
```

output
```
(288*sqrt(3*x**2 - x + 2)*x**2 + 660*sqrt(3*x**2 - x + 2)*x + 486*sqrt(3*x
**2 - x + 2) - 251*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/
sqrt(23)))/324
```

3.80 $\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 78

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2}-\frac{5\text{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{6\sqrt{3}} - \frac{\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2\sqrt{13}}$$

output
```
2/3*(3*x^2-x+2)^(1/2)-5/18*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)-1/26*13^
(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{2}{3}\sqrt{2-x+3x^2}+\frac{\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{\sqrt{13}} - \frac{5\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{6\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2])/3 + ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x
+ 3*x^2])/Sqrt[13]]/Sqrt[13] - (5*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/
(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{12}\int \frac{4(5x+4)}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{5x+4}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{3}\left(\frac{5}{2}\int \frac{1}{\sqrt{3x^2-x+2}}dx+\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{5\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{2\sqrt{69}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{3}\left(\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-3\int\frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(\frac{5\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{2\sqrt{3}}-\frac{3\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}\right)+\frac{2}{3}\sqrt{3x^2-x+2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*Sqrt[2 - x + 3*x^2]),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2])/3 + ((5*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(2*Sqrt[3])
- (3*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13]))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77

method	result
default	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{26}$
risch	$\frac{5\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{18}+\frac{2\sqrt{3x^2-x+2}}{3}-\frac{\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{26}$
trager	$\frac{2\sqrt{3x^2-x+2}}{3}-\frac{5\,\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\operatorname{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{18}-\frac{\operatorname{RootOf}\left(\_Z^2-13\right)\ln}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/18*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))+2/3*(3*x^2-x+2)^(1/2)-1/26*13^
(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.35

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{5}{36}\sqrt{3}\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right) + \frac{1}{52}\sqrt{13}\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220x^2-196x+185}{4x^2+4x+1}\right) + \frac{2}{3}\sqrt{3x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
5/36*sqrt(3)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x
 - 25) + 1/52*sqrt(13)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220
*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 2/3*sqrt(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \frac{5}{18}\sqrt{3}\,\text{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{1}{26}\sqrt{13}\,\text{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{2}{3}\sqrt{3\,x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
5/18*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) + 1/26*sqrt(13)*arcs
inh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) + 2/3*sqrt(
3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = -\frac{5}{18}\sqrt{3}\log\left(-6\sqrt{3}x+\sqrt{3}+6\sqrt{3\,x^2-x+2}\right) + \frac{1}{26}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2}{3}\sqrt{3\,x^2-x+2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
-5/18*sqrt(3)*log(-6*sqrt(3)*x + sqrt(3) + 6*sqrt(3*x^2 - x + 2)) + 1/26*s
qrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt(3*x^2
- x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2))) + 2/
3*sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)\,\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.03

$$\begin{aligned}
&\int \frac{1+3x+4x^2}{(1+2x)\sqrt{2-x+3x^2}}\,dx \\
&= \frac{\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6ix-i}{\sqrt{39}-4}\right)i}{26} + \frac{2\sqrt{3x^2-x+2}}{3} \\
&\quad + \frac{\sqrt{13}\log\left(24\sqrt{3x^2-x+2}\,\sqrt{3}\,x-4\sqrt{3x^2-x+2}\,\sqrt{3}+8\sqrt{39}+72x^2-24x-30\right)}{52} \\
&\quad - \frac{\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{3}+\sqrt{39}+6x+3\right)}{26} + \frac{5\sqrt{3}\log\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}+6x-1}{\sqrt{23}}\right)}{18}
\end{aligned}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(1/2),x)
```

output
```
(18*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39)
 - 4))*i + 312*sqrt(3*x**2 - x + 2) + 9*sqrt(13)*log(24*sqrt(3*x**2 - x +
2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*
x - 30) - 18*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3) + sqrt(39) + 6*x
+ 3) + 130*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)
))/468
```

3.81 $\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 83

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{13(1+2x)} - \frac{\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{\sqrt{3}} + \frac{9\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{26\sqrt{13}}$$

output
```
-1/13*(3*x^2-x+2)^(1/2)/(1+2*x)-1/3*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
+9/338*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{13+26x} - \frac{9\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}} - \frac{\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-(Sqrt[2 - x + 3*x^2]/(13 + 26*x)) - (9*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2
*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13]) - Log[1 - 6*x + 2*Sqrt[6 -
3*x + 9*x^2]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{13}\int -\frac{52x+17}{2(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{26}\int \frac{52x+17}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{26}\left(26\int \frac{1}{\sqrt{3x^2-x+2}}dx-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{26}\left(\frac{26\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{26}\left(\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}-9\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{26}\left(18\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}+\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{26}\left(\frac{26\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}+\frac{9\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}\right)-\frac{\sqrt{3x^2-x+2}}{13(2x+1)}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-1/13*Sqrt[2 - x + 3*x^2]/(1 + 2*x) + ((26*ArcSinh[(-1 + 6*x)/Sqrt[23]])/S
qrt[3] + (9*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])
/26
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}-\frac{\sqrt{3\left(\frac{1}{2}+x\right)^{2}+\frac{5}{4}-4x}}{26\left(\frac{1}{2}+x\right)}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^{2}+5-16x}}\right)}{338}$
risch	$-\frac{\sqrt{3x^{2}-x+2}}{13(1+2x)}+\frac{\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{3}+\frac{9\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^{2}+5-16x}}\right)}{338}$
trager	$-\frac{\sqrt{3x^{2}-x+2}}{13(1+2x)}-\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(-6\operatorname{RootOf}\left(\_Z^{2}-3\right)x+6\sqrt{3x^{2}-x+2}+\operatorname{RootOf}\left(\_Z^{2}-3\right)\right)}{3}+\frac{9\operatorname{RootOf}\left(\_Z^{2}-13\right)}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))-1/26/(1/2+x)*(3*(1/2+x)^2+5/4-4
*x)^(1/2)+9/338*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-1
6*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{338\sqrt{3}(2x+1)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-25\right)+27\sqrt{13}(2x+1)\log\left(\frac{4\sqrt{13}\sqrt{3}}{}\right.}{2028(2x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/2028*(338*sqrt(3)*(2*x + 1)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x - 1)
 - 72*x^2 + 24*x - 25) + 27*sqrt(13)*(2*x + 1)*log((4*sqrt(13)*sqrt(3*x^2
- x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) - 156*sqrt(
3*x^2 - x + 2))/(2*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\,\text{arsinh}\left(\frac{6}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{9}{338}\sqrt{13}\,\text{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2-x+2}}{13\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*arcsinh(6/23*sqrt(23)*x - 1/23*sqrt(23)) - 9/338*sqrt(13)*arcs
inh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*x + 1)) - 1/13*sqrt
(3*x^2 - x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(66) = 132.

Time = 0.30 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.30

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{9\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)-4\right)}{338\,\text{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{2\sqrt{13}}{2\,x+1}\right|}{2\left(\sqrt{3}+\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}+\frac{\sqrt{13}}{2\,x+1}\right)}\right)}{3\,\text{sgn}\left(\frac{1}{2\,x+1}\right)} - \frac{\sqrt{-\frac{8}{2\,x+1}+\frac{13}{(2\,x+1)^2}+3}}{26\,\text{sgn}\left(\frac{1}{2\,x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
9/338*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqr
t(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) - 1/3*sqrt(3)*log(1/2*abs(-2*sqrt(3
) + 2*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + 2*sqrt(13)/(2*x + 1))/(sqr
t(3) + sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)))/sgn(
1/(2*x + 1)) - 1/26*sqrt(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3)/sgn(1/(2*x + 1
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx=\int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)^2\,\sqrt{3\,x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.75

$$\int \frac{1+3x+4x^2}{(1+2x)^2\sqrt{2-x+3x^2}}\,dx = \frac{-78\sqrt{3x^2-x+2}+54\sqrt{13}\log\left(-2\sqrt{3x^2-x+2}\,\sqrt{13}+8x-9\right)x+27\sqrt{13}\log\left(-2\sqrt{3x^2-x+2}\,\sqrt{13}\right.}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(1/2),x)
```

output
```
( - 78*sqrt(3*x**2 - x + 2) + 54*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sq
rt(13) + 8*x - 9)*x + 27*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) +
 8*x - 9) - 54*sqrt(13)*log(2*x + 1)*x - 27*sqrt(13)*log(2*x + 1) + 676*sq
rt(3)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1)*x + 338*sqrt(3)*log
( - 2*sqrt(3*x**2 - x + 2)*sqrt(3) - 6*x + 1))/(1014*(2*x + 1))
```

3.82 $\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 89

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = -\frac{\sqrt{2-x+3x^2}}{26(1+2x)^2}+\frac{7\sqrt{2-x+3x^2}}{169(1+2x)} - \frac{581\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{676\sqrt{13}}$$

output
```
-1/26*(3*x^2-x+2)^(1/2)/(1+2*x)^2+7*(3*x^2-x+2)^(1/2)/(169+338*x)-581/8788
*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.87

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{\frac{13(1+28x)\sqrt{2-x+3x^2}}{(1+2x)^2}+581\sqrt{13}\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{4394}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

output
```
((13*(1 + 28*x)*Sqrt[2 - x + 3*x^2])/(1 + 2*x)^2 + 581*Sqrt[13]*ArcTanh[(S
qrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/4394
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{26}\int -\frac{7(14x+5)}{2(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{7}{52}\int \frac{14x+5}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{7}{52}\left(\frac{83}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx+\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{7}{52}\left(\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}-\frac{166}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{7}{52}\left(\frac{4\sqrt{3x^2-x+2}}{13(2x+1)}-\frac{83\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}\right)-\frac{\sqrt{3x^2-x+2}}{26(2x+1)^2}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*Sqrt[2 - x + 3*x^2]),x]
```

output
```
-1/26*Sqrt[2 - x + 3*x^2]/(1 + 2*x)^2 + (7*((4*Sqrt[2 - x + 3*x^2])/(13*(1
 + 2*x)) - (83*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(13*Sq
rt[13])))/52
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

method	result	size
risch	$\frac{84x^3-25x^2+55x+2}{338(1+2x)^2\sqrt{3x^2-x+2}}-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{8788}$	68
default	$-\frac{581\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{8788}-\frac{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}{104\left(\frac{1}{2}+x\right)^2}+\frac{7\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}{338\left(\frac{1}{2}+x\right)}$	74
trager	$\frac{(28x+1)\sqrt{3x^2-x+2}}{338(1+2x)^2}-\frac{581\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{8788}$	77

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/338*(84*x^3-25*x^2+55*x+2)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-581/8788*13^(1/2)
*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$
$$=\frac{581\,\sqrt{13}(4\,x^2+4\,x+1)\log\left(-\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+52\,\sqrt{3\,x^2-x+2}(28\,x+1)}{17576\,(4\,x^2+4\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="fricas")
```

output
```
1/17576*(581*sqrt(13)*(4*x^2 + 4*x + 1)*log(-(4*sqrt(13)*sqrt(3*x^2 - x +
2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 52*sqrt(3*x^2 -
 x + 2)*(28*x + 1))/(4*x^2 + 4*x + 1)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(1/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*sqrt(3*x**2 - x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.92

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \frac{581}{8788}\sqrt{13}\,\text{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) - \frac{\sqrt{3\,x^2-x+2}}{26\,(4\,x^2+4\,x+1)} + \frac{7\sqrt{3\,x^2-x+2}}{169\,(2\,x+1)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="maxima")
```

output
```
581/8788*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) - 1/26*sqrt(3*x^2 - x + 2)/(4*x^2 + 4*x + 1) + 7/169*sqrt(3*x^2
 - x + 2)/(2*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(71) = 142$.

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{581}{8788}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right)$$
$$+\frac{190\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-53\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2-489\sqrt{3}x+289\sqrt{3}+489\sqrt{3x^2-x}}{338\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x, algorithm="giac")
```

output
```
581/8788*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*s
qrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x +
 2))) + 1/338*(190*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 - 53*sqrt(3)*(sqrt(
3)*x - sqrt(3*x^2 - x + 2))^2 - 489*sqrt(3)*x + 289*sqrt(3) + 489*sqrt(3*x
^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\sqrt{3x^2-x+2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\int \frac{1+3x+4x^2}{(1+2x)^3\sqrt{2-x+3x^2}}\,dx$$
$$= \frac{728\sqrt{3x^2-x+2}\,x+26\sqrt{3x^2-x+2}+2324\sqrt{13}\log\left(2\sqrt{3x^2-x+2}\,\sqrt{13}+8x-9\right)x^2+2324\sqrt{13}\log$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(1/2),x)
```

output
```
(728*sqrt(3*x**2 - x + 2)*x + 26*sqrt(3*x**2 - x + 2) + 2324*sqrt(13)*log(
2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**2 + 2324*sqrt(13)*log(2*sqrt
(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x + 581*sqrt(13)*log(2*sqrt(3*x**2 -
x + 2)*sqrt(13) + 8*x - 9) - 2324*sqrt(13)*log(2*x + 1)*x**2 - 2324*sqrt(1
3)*log(2*x + 1)*x - 581*sqrt(13)*log(2*x + 1))/(8788*(4*x**2 + 4*x + 1))
```

3.83 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(12839-3871x)}{1863\sqrt{2-x+3x^2}}+\frac{746}{81}\sqrt{2-x+3x^2} + \frac{412}{81}x\sqrt{2-x+3x^2}+\frac{32}{27}x^2\sqrt{2-x+3x^2}+\frac{353\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{81\sqrt{3}}$$

output

```
2/1863*(12839-3871*x)/(3*x^2-x+2)^(1/2)+746/81*(3*x^2-x+2)^(1/2)+412/81*x*
(3*x^2-x+2)^(1/2)+32/27*x^2*(3*x^2-x+2)^(1/2)+353/243*arcsinh(1/23*(1-6*x)
*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{\frac{6(29997-2974x+23207x^2+13110x^3+3312x^4)}{\sqrt{2-x+3x^2}}+8119\sqrt{3}\log\left(1-6x+2\sqrt{6-3x+\right.}{5589}$$

input

```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
((6*(29997 - 2974*x + 23207*x^2 + 13110*x^3 + 3312*x^4))/Sqrt[2 - x + 3*x^
2] + 8119*Sqrt[3]*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/5589
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2191, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 \left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23\left(432x^3+1116x^2+1002x+49\right)}{81\sqrt{3x^2-x+2}}dx+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\int \frac{432x^3+1116x^2+1002x+49}{\sqrt{3x^2-x+2}}dx+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2}{81}\left(\frac{1}{9}\int \frac{9\left(1236x^2+810x+49\right)}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\left(\int \frac{1236x^2+810x+49}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2}{81}\left(\frac{1}{6}\int -\frac{18(121-373x)}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{81}\left(-3\int \frac{121-373x}{\sqrt{3x^2-x+2}}dx+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$\Big\downarrow$ 1160

$$\frac{2}{81}\left(-3\left(\frac{353}{6}\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$\Big\downarrow$ 1090

$$\frac{2}{81}\left(-3\left(\frac{353\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{6\sqrt{69}}-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

$\Big\downarrow$ 222

$$\frac{2}{81}\left(-3\left(\frac{353\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{6\sqrt{3}}-\frac{373}{3}\sqrt{3x^2-x+2}\right)+48\sqrt{3x^2-x+2}x^2+206\sqrt{3x^2-x+2}x\right)+\frac{2(12839-3871x)}{1863\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(12839 - 3871*x))/(1863*Sqrt[2 - x + 3*x^2]) + (2*(206*x*Sqrt[2 - x + 3
*x^2] + 48*x^2*Sqrt[2 - x + 3*x^2] - 3*((-373*Sqrt[2 - x + 3*x^2])/3 + (35
3*ArcSinh[(-1 + 6*x)/Sqrt[23]])/(6*Sqrt[3]))))/81
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}$
trager	$\frac{\frac{32}{9}x^4+\frac{380}{27}x^3+\frac{2018}{81}x^2-\frac{5948}{1863}x+\frac{2222}{69}}{\sqrt{3x^2-x+2}}-\frac{353\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(6\,\mathrm{RootOf}\left(\_Z^2-3\right)x-\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{3x^2-x+2}\right)}{243}$
default	$-\frac{521(6x-1)}{414\sqrt{3x^2-x+2}}+\frac{557}{18\sqrt{3x^2-x+2}}+\frac{353x}{81\sqrt{3x^2-x+2}}-\frac{353\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{243}+\frac{2018x^2}{81\sqrt{3x^2-x+2}}+\frac{380x^3}{27\sqrt{3x^2-x+2}}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1863*(3312*x^4+13110*x^3+23207*x^2-2974*x+29997)/(3*x^2-x+2)^(1/2)-353/2
43*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{8119\sqrt{3}(3x^2-x+2)\log\left(4\sqrt{3}\sqrt{3x^2-x+2}(6x-1)-72x^2+24x-\right.}{11178\,(3}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/11178*(8119*sqrt(3)*(3*x^2 - x + 2)*log(4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6
*x - 1) - 72*x^2 + 24*x - 25) + 12*(3312*x^4 + 13110*x^3 + 23207*x^2 - 297
4*x + 29997)*sqrt(3*x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^3 \cdot (4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(1+2x)^3 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{32\,x^4}{9\sqrt{3\,x^2-x+2}} + \frac{380\,x^3}{27\sqrt{3\,x^2-x+2}} + \frac{2018\,x^2}{81\sqrt{3\,x^2-x+2}} - \frac{353}{243}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{5948\,x}{1863\sqrt{3\,x^2-x+2}} + \frac{2222}{69\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
32/9*x^4/sqrt(3*x^2 - x + 2) + 380/27*x^3/sqrt(3*x^2 - x + 2) + 2018/81*x^
2/sqrt(3*x^2 - x + 2) - 353/243*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) -
 5948/1863*x/sqrt(3*x^2 - x + 2) + 2222/69/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{353}{243}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\left(\left(23\left(6\left(24\,x+95\right)x+1009\right)x-2974\right)x+29997\right)}{1863\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
353/243*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/
1863*((23*(6*(24*x + 95)*x + 1009)*x - 2974)*x + 29997)/sqrt(3*x^2 - x + 2
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{\left(2\,x+1\right)^3\left(4\,x^2+3\,x+1\right)}{\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{19872\sqrt{3x^2-x+2}\,x^4+78660\sqrt{3x^2-x+2}\,x^3+139242\sqrt{3x^2-x+2}}{}$$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

output

```
(19872*sqrt(3*x**2 - x + 2)*x**4 + 78660*sqrt(3*x**2 - x + 2)*x**3 + 13924
2*sqrt(3*x**2 - x + 2)*x**2 - 17844*sqrt(3*x**2 - x + 2)*x + 179982*sqrt(3
*x**2 - x + 2) - 24357*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x -
 1)/sqrt(23))*x**2 + 8119*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*
x - 1)/sqrt(23))*x - 16238*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6
*x - 1)/sqrt(23)) - 23226*sqrt(3)*x**2 + 7742*sqrt(3)*x - 15484*sqrt(3))/(
5589*(3*x**2 - x + 2))
```

3.84 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 82

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(1249-2273x)}{621\sqrt{2-x+3x^2}} + \frac{112}{27}\sqrt{2-x+3x^2} + \frac{8}{9}x\sqrt{2-x+3x^2} - \frac{64\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

output
```
2/621*(1249-2273*x)/(3*x^2-x+2)^(1/2)+112/27*(3*x^2-x+2)^(1/2)+8/9*x*(3*x^
2-x+2)^(1/2)-64/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(1275-1003x+1196x^2+276x^3)}{207\sqrt{2-x+3x^2}} - \frac{64\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(1275 - 1003*x + 1196*x^2 + 276*x^3))/(207*Sqrt[2 - x + 3*x^2]) - (64*L
og[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2-x+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23} \int \frac{46(36x^2+75x+46)}{27\sqrt{3x^2-x+2}}dx + \frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27} \int \frac{36x^2+75x+46}{\sqrt{3x^2-x+2}}dx + \frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{4}{27}\left(\frac{1}{6}\int \frac{12(42x+17)}{\sqrt{3x^2-x+2}}dx + 6\sqrt{3x^2-x+2x}\right) + \frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{27}\left(2\int \frac{42x+17}{\sqrt{3x^2-x+2}}dx + 6\sqrt{3x^2-x+2x}\right) + \frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{4}{27}\left(2\left(24\int \frac{1}{\sqrt{3x^2-x+2}}dx + 14\sqrt{3x^2-x+2}\right) + 6\sqrt{3x^2-x+2x}\right) + \frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{4}{27}\left(2\left(8\sqrt{\frac{3}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)+14\sqrt{3x^2-x+2}\right)+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

$\Big\downarrow$ 222

$$\frac{4}{27}\left(2\left(8\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)+14\sqrt{3x^2-x+2}\right)+6\sqrt{3x^2-x+2}x\right)+\frac{2(1249-2273x)}{621\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(1249 - 2273*x))/(621*Sqrt[2 - x + 3*x^2]) + (4*(6*x*Sqrt[2 - x + 3*x^2
] + 2*(14*Sqrt[2 - x + 3*x^2] + 8*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]])))/
27
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	siz
risch	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}$	45
trager	$\frac{\frac{8}{3}x^3+\frac{104}{9}x^2-\frac{2006}{207}x+\frac{850}{69}}{\sqrt{3x^2-x+2}}-\frac{64\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{27}$	70
default	$-\frac{89(6x-1)}{207\sqrt{3x^2-x+2}}+\frac{107}{9\sqrt{3x^2-x+2}}-\frac{64x}{9\sqrt{3x^2-x+2}}+\frac{64\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{27}+\frac{104x^2}{9\sqrt{3x^2-x+2}}+\frac{8x^3}{3\sqrt{3x^2-x+2}}$	98

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/207*(276*x^3+1196*x^2-1003*x+1275)/(3*x^2-x+2)^(1/2)+64/27*3^(1/2)*arcsi
nh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2 \left(368 \sqrt{3}(3 x^2 - x + 2) \log \left(-4 \sqrt{3}\sqrt{3 x^2 - x + 2}(6 x - 1) - 72 x^2 + 24\right.\right.}{621 \left(3 x^2 - x\right.}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
2/621*(368*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 3*(276*x^3 + 1196*x^2 - 1003*x + 1275)*sqrt(
3*x^2 - x + 2))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^2 \cdot (4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{8 x^3}{3 \sqrt{3 x^2 - x + 2}} + \frac{104 x^2}{9 \sqrt{3 x^2 - x + 2}} + \frac{64}{27} \sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23} \sqrt{23}(6 x - 1)\right) - \frac{2006 x}{207 \sqrt{3 x^2 - x + 2}} + \frac{850}{69 \sqrt{3 x^2 - x + 2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^3/sqrt(3*x^2 - x + 2) + 104/9*x^2/sqrt(3*x^2 - x + 2) + 64/27*sqrt(3
)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2006/207*x/sqrt(3*x^2 - x + 2) + 850/
69/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{64}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{2\left((92(3x+13)x-1003)x+1275\right)}{207\sqrt{3x^2-x+2}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-64/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/2
07*((92*(3*x + 13)*x - 1003)*x + 1275)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2 (1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)^2 (4x^2+3x+1)}{(3x^2-x+2)^{3/2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.24

$$\int \frac{(1+2x)^2\,(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{4968\sqrt{3x^2-x+2}\,x^3+21528\sqrt{3x^2-x+2}\,x^2-18054\sqrt{3x^2-x+2}\,x}{}$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

output
```
(2*(2484*sqrt(3*x**2 - x + 2)*x**3 + 10764*sqrt(3*x**2 - x + 2)*x**2 - 902
7*sqrt(3*x**2 - x + 2)*x + 11475*sqrt(3*x**2 - x + 2) + 6624*sqrt(3)*log((
2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x**2 - 2208*sqrt(3)*lo
g((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x + 4416*sqrt(3)*lo
g((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)) - 6819*sqrt(3)*x**2
 + 2273*sqrt(3)*x - 4546*sqrt(3)))/(1863*(3*x**2 - x + 2))
```

3.85 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 63

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{2(73+367x)}{207\sqrt{2-x+3x^2}}+\frac{8}{9}\sqrt{2-x+3x^2}-\frac{14\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{3\sqrt{3}}$$

output
```
1/207*(-146-734*x)/(3*x^2-x+2)^(1/2)+8/9*(3*x^2-x+2)^(1/2)-14/9*arcsinh(1/
23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{2(37-153x+92x^2)}{69\sqrt{2-x+3x^2}}-\frac{14\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{3\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(2*(37 - 153*x + 92*x^2))/(69*Sqrt[2 - x + 3*x^2]) - (14*Log[1 - 6*x + 2*S
qrt[6 - 3*x + 9*x^2]])/(3*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int \frac{23(12x+19)}{9\sqrt{3x^2-x+2}}dx-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{9}\int \frac{12x+19}{\sqrt{3x^2-x+2}}dx-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2}{9}\left(21\int \frac{1}{\sqrt{3x^2-x+2}}dx+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2}{9}\left(7\sqrt{\frac{3}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{2}{9}\left(7\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)+4\sqrt{3x^2-x+2}\right)-\frac{2(367x+73)}{207\sqrt{3x^2-x+2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(3/2),x]
```

output
```
(-2*(73 + 367*x))/(207*Sqrt[2 - x + 3*x^2]) + (2*(4*Sqrt[2 - x + 3*x^2] +
7*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

method	result	size
risch	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{9}$	40
trager	$\frac{\frac{8}{3}x^2-\frac{102}{23}x+\frac{74}{69}}{\sqrt{3x^2-x+2}}-\frac{14\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{9}$	65
default	$\frac{\frac{16x}{69}-\frac{8}{207}}{\sqrt{3x^2-x+2}}+\frac{10}{9\sqrt{3x^2-x+2}}-\frac{14x}{3\sqrt{3x^2-x+2}}+\frac{14\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\,\left(x-\frac{1}{6}\right)}{23}\right)}{9}+\frac{8x^2}{3\sqrt{3x^2-x+2}}$	81

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/69*(92*x^2-153*x+37)/(3*x^2-x+2)^(1/2)+14/9*3^(1/2)*arcsinh(6/23*23^(1/2
)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.38

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{161\sqrt{3}(3\,x^2-x+2)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x-1)-72\,x^2+24\,x-\right.}{207\,(3\,x^2-x+2)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/207*(161*sqrt(3)*(3*x^2 - x + 2)*log(-4*sqrt(3)*sqrt(3*x^2 - x + 2)*(6*x
 - 1) - 72*x^2 + 24*x - 25) + 6*(92*x^2 - 153*x + 37)*sqrt(3*x^2 - x + 2))
/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = \frac{8\,x^2}{3\,\sqrt{3\,x^2-x+2}} + \frac{14}{9}\,\sqrt{3}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(6\,x-1)\right) - \frac{102\,x}{23\,\sqrt{3\,x^2-x+2}} + \frac{74}{69\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
8/3*x^2/sqrt(3*x^2 - x + 2) + 14/9*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)
) - 102/23*x/sqrt(3*x^2 - x + 2) + 74/69/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{3/2}}\,dx = -\frac{14}{9}\,\sqrt{3}\log\left(-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\,((92\,x-153)x+37)}{69\,\sqrt{3\,x^2-x+2}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-14/9*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/69
*((92*x - 153)*x + 37)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{\left(2\,x+1\right)\,\left(4\,x^2+3\,x+1\right)}{{\left(3\,x^2-x+2\right)}^{3/2}}\,dx$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2),x)
```

output
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.67

$$\int \frac{(1+2x)\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{1656\sqrt{3x^2-x+2}\,x^2-2754\sqrt{3x^2-x+2}\,x+666\sqrt{3x^2-x+2}+2898}{}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(3/2),x)
```

output
```
(2*(828*sqrt(3*x**2 - x + 2)*x**2 - 1377*sqrt(3*x**2 - x + 2)*x + 333*sqrt
(3*x**2 - x + 2) + 1449*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x
- 1)/sqrt(23))*x**2 - 483*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*
x - 1)/sqrt(23))*x + 966*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x
 - 1)/sqrt(23)) - 1101*sqrt(3)*x**2 + 367*sqrt(3)*x - 734*sqrt(3)))/(621*(
3*x**2 - x + 2))
```

3.86 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 62

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{2(101-77x)}{299\sqrt{2-x+3x^2}}-\frac{2\,\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{13\sqrt{13}}$$

output
```
1/299*(-202+154*x)/(3*x^2-x+2)^(1/2)-2/169*13^(1/2)*arctanh(1/26*(9-8*x)*1
3^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{2(-101+77x)}{299\sqrt{2-x+3x^2}}+\frac{4\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{13\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(-101 + 77*x))/(299*Sqrt[2 - x + 3*x^2]) + (4*ArcTanh[(Sqrt[3] + 2*Sqrt
[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(13*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{23}\int \frac{23}{13(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 1154}$$

$$-\frac{4}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}-\frac{2(101-77x)}{299\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(-2*(101 - 77*x))/(299*Sqrt[2 - x + 3*x^2]) - (2*ArcTanh[(9 - 8*x)/(2*Sqrt
[13]*Sqrt[2 - x + 3*x^2])])/(13*Sqrt[13])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

method	result	s
risch	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{169}$	5
trager	$\frac{-\frac{202}{299}+\frac{154x}{299}}{\sqrt{3x^2-x+2}}-\frac{2\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\text{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\text{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{169}$	7
default	$\frac{\frac{10x}{23}-\frac{5}{69}}{\sqrt{3x^2-x+2}}-\frac{2}{3\sqrt{3x^2-x+2}}+\frac{1}{13\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}+\frac{\frac{24x}{299}-\frac{4}{299}}{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}-\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{169}$	1

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/299*(-101+77*x)/(3*x^2-x+2)^(1/2)-2/169*13^(1/2)*arctanh(2/13*(9/2-4*x)*
13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.55

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{23\sqrt{13}(3\,x^2-x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)+220\,x^2-196\,x+185}{4\,x^2+4\,x+1}\right)+26\,\sqrt{}}{3887\,(3\,x^2-x+2)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/3887*(23*sqrt(13)*(3*x^2 - x + 2)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(
8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1)) + 26*sqrt(3*x^2 - x +
 2)*(77*x - 101))/(3*x^2 - x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2}{169}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{154\,x}{299\sqrt{3\,x^2-x+2}} - \frac{202}{299\sqrt{3\,x^2-x+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
2/169*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2*
x + 1)) + 154/299*x/sqrt(3*x^2 - x + 2) - 202/299/sqrt(3*x^2 - x + 2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{2}{169}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(77\,x-101)}{299\sqrt{3\,x^2-x+2}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
2/169*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqrt
(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2)
)) + 2/299*(77*x - 101)/sqrt(3*x^2 - x + 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)\,\left(3\,x^2-x+2\right)^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 411, normalized size of antiderivative = 6.63

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{414\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6ix-i}{\sqrt{39}-4}\right)i\,x^2-138\sqrt{13}\,atan\left(\frac{2\sqrt{3x^2-x+2}\,\sqrt{3}\,i+6i}{\sqrt{39}-4}\right.}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(3/2),x)
```

output
```
(414*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39
) - 4))*i*x**2 - 138*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i
*x - i)/(sqrt(39) - 4))*i*x + 276*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sq
rt(3)*i + 6*i*x - i)/(sqrt(39) - 4))*i + 6006*sqrt(3*x**2 - x + 2)*x - 787
8*sqrt(3*x**2 - x + 2) + 207*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*
x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30)*x**
2 - 69*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x
+ 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30)*x + 138*sqrt(13)*log(24*s
qrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39
) + 72*x**2 - 24*x - 30) - 414*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(3)
 + sqrt(39) + 6*x + 3)*x**2 + 138*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt
(3) + sqrt(39) + 6*x + 3)*x - 276*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt
(3) + sqrt(39) + 6*x + 3) + 6006*sqrt(3)*x**2 - 2002*sqrt(3)*x + 4004*sqrt
(3))/(11661*(3*x**2 - x + 2))
```

3.87 $\int \frac{1+3x+4x^2}{(1+2x)^2(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 87

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx=-\frac{2(197-837x)}{3887\sqrt{2-x+3x^2}}-\frac{4\sqrt{2-x+3x^2}}{169(1+2x)}+\frac{2\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

output
```
1/3887*(-394+1674*x)/(3*x^2-x+2)^(1/2)-4*(3*x^2-x+2)^(1/2)/(169+338*x)+2/2
197*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx=\frac{2\sqrt{2-x+3x^2}(-289+489x+1536x^2)}{3887\,(2+3x+x^2+6x^3)}-\frac{4\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2]*(-289 + 489*x + 1536*x^2))/(3887*(2 + 3*x + x^2 + 6
*x^3)) - (4*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[1
3]])/(169*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2 (3x^2-x+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23} \int \frac{46(4-5x)}{169(2x+1)^2\sqrt{3x^2-x+2}}dx - \frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{169} \int \frac{4-5x}{(2x+1)^2\sqrt{3x^2-x+2}}dx - \frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{4}{169} \left(-\frac{1}{2} \int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx - \frac{\sqrt{3x^2-x+2}}{2x+1}\right) - \frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{4}{169} \left(\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}} - \frac{\sqrt{3x^2-x+2}}{2x+1}\right) - \frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4}{169} \left(\frac{\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}} - \frac{\sqrt{3x^2-x+2}}{2x+1}\right) - \frac{2(197-837x)}{3887\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(-2*(197 - 837*x))/(3887*Sqrt[2 - x + 3*x^2]) + (4*(-(Sqrt[2 - x + 3*x^2]/
(1 + 2*x)) + ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])]/(2*Sqrt[1
3])))/169
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

method	result
risch	$\frac{\frac{3072}{3887}x^2+\frac{978}{3887}x-\frac{578}{3887}}{(1+2x)\sqrt{3x^2-x+2}}+\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{2197}$
trager	$\frac{2(1536x^2+489x-289)\sqrt{3x^2-x+2}}{3887(6x^3+x^2+3x+2)}-\frac{2\,\text{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{8\,\text{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}-9\,\text{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{2197}$
default	$\frac{\frac{12x}{23}-\frac{2}{23}}{\sqrt{3x^2-x+2}}-\frac{1}{26\left(\frac{1}{2}+x\right)\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}-\frac{1}{169\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}-\frac{82(6x-1)}{3887\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}+\frac{2\sqrt{13}\,\text{arctanh}\left(\frac{2(}{13\sqrt{12(}}\right.}{2197}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3887*(1536*x^2+489*x-289)/(1+2*x)/(3*x^2-x+2)^(1/2)+2/2197*13^(1/2)*arct
anh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.22

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{23\,\sqrt{13}(6\,x^3+x^2+3\,x+2)\log\left(\frac{4\,\sqrt{13}\sqrt{3\,x^2-x+2}(8\,x-9)-220\,x^2+196\,x-185}{4\,x^2+4\,x+1}\right)+}{50531\,(6\,x^3+x^2+3\,x+2)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/50531*(23*sqrt(13)*(6*x^3 + x^2 + 3*x + 2)*log((4*sqrt(13)*sqrt(3*x^2 -
x + 2)*(8*x - 9) - 220*x^2 + 196*x - 185)/(4*x^2 + 4*x + 1)) + 26*(1536*x^
2 + 489*x - 289)*sqrt(3*x^2 - x + 2))/(6*x^3 + x^2 + 3*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx=\int \frac{4x^2+3x+1}{(2x+1)^2\,(3x^2-x+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{3/2}}\,dx=-\frac{2}{2197}\,\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\,\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\,\sqrt{23}}{23\,|2\,x+1|}\right)+\frac{1536\,x}{3887\,\sqrt{3\,x^2-x+2}}-\frac{279}{3887\,\sqrt{3\,x^2-x+2}}-\frac{1}{13\,\left(2\,\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
-2/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(
2*x + 1)) + 1536/3887*x/sqrt(3*x^2 - x + 2) - 279/3887/sqrt(3*x^2 - x + 2)
 - 1/13/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(69) = 138$.

Time = 0.23 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.93

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{3/2}}\,dx = -\frac{2}{50531}\sqrt{13}\Big(256\sqrt{13}\sqrt{3}+23\log\left(\sqrt{13}\sqrt{3}-4\right)\Big)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{2\left(\frac{\frac{1047}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{299}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{768}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3887\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}} + \frac{2\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{2197\operatorname{sgn}\left(\frac{1}{2x+1}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
-2/50531*sqrt(13)*(256*sqrt(13)*sqrt(3) + 23*log(sqrt(13)*sqrt(3) - 4))*sg
n(1/(2*x + 1)) - 2/3887*((1047/sgn(1/(2*x + 1)) + 299/((2*x + 1)*sgn(1/(2*
x + 1))))/(2*x + 1) - 768/sgn(1/(2*x + 1)))/sqrt(-8/(2*x + 1) + 13/(2*x +
1)^2 + 3) + 2/2197*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) + 13/(2*x + 1)
^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{3/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{(2\,x+1)^2\,(3\,x^2-x+2)^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.49

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{3/2}}\,dx = \frac{39936\sqrt{3x^2-x+2}\,x^2+12714\sqrt{3x^2-x+2}\,x-7514\sqrt{3x^2-x+2}+$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(3/2),x)
```

output
```
(2*(19968*sqrt(3*x**2 - x + 2)*x**2 + 6357*sqrt(3*x**2 - x + 2)*x - 3757*s
qrt(3*x**2 - x + 2) + 138*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13)
+ 8*x - 9)*x**3 + 23*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x
 - 9)*x**2 + 69*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)
*x + 46*sqrt(13)*log( - 2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 138*s
qrt(13)*log(2*x + 1)*x**3 - 23*sqrt(13)*log(2*x + 1)*x**2 - 69*sqrt(13)*lo
g(2*x + 1)*x - 46*sqrt(13)*log(2*x + 1)))/(50531*(6*x**3 + x**2 + 3*x + 2)
)
```

3.88 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 112

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx = \frac{2(2363+3693x)}{50531\sqrt{2-x+3x^2}} - \frac{2\sqrt{2-x+3x^2}}{169(1+2x)^2} - \frac{4\sqrt{2-x+3x^2}}{2197(1+2x)} - \frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

output
```
2/50531*(2363+3693*x)/(3*x^2-x+2)^(1/2)-2/169*(3*x^2-x+2)^(1/2)/(1+2*x)^2-
4*(3*x^2-x+2)^(1/2)/(2197+4394*x)-487/28561*13^(1/2)*arctanh(1/26*(9-8*x)*
13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{3/2}}\,dx = \frac{2(1673+13306x+23281x^2+14496x^3)}{50531(1+2x)^2\sqrt{2-x+3x^2}} + \frac{974\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(1673 + 13306*x + 23281*x^2 + 14496*x^3))/(50531*(1 + 2*x)^2*Sqrt[2 - x
 + 3*x^2]) + (974*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/
Sqrt[13]])/(2197*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2-x+2)^{3/2}} \, dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23} \int \frac{23(1036x^2+906x+363)}{2197(2x+1)^3\sqrt{3x^2-x+2}}dx + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1036x^2+906x+363}{(2x+1)^3\sqrt{3x^2-x+2}}dx}{2197} + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{2\left(-\frac{1}{26}\int -\frac{13(958x+505)}{(2x+1)^2\sqrt{3x^2-x+2}}dx - \frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197} + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{2}\int \frac{958x+505}{(2x+1)^2\sqrt{3x^2-x+2}}dx - \frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197} + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{2\left(\frac{1}{2}\left(487\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(\frac{1}{2}\left(-974\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{487\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}-\frac{4\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{(2x+1)^2}\right)}{2197}+\frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(3/2)),x]
```

output
```
(2*(2363 + 3693*x))/(50531*Sqrt[2 - x + 3*x^2]) + (2*((-13*Sqrt[2 - x + 3*
x^2])/(1 + 2*x)^2 + ((-4*Sqrt[2 - x + 3*x^2])/(1 + 2*x) - (487*ArcTanh[(9
- 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/2))/2197
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

method	result
risch	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}-\frac{487\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{28561}$
trager	$\frac{\frac{28992}{50531}x^3+\frac{46562}{50531}x^2+\frac{26612}{50531}x+\frac{3346}{50531}}{(1+2x)^2\sqrt{3x^2-x+2}}-\frac{487\,\mathrm{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\,\mathrm{RootOf}\left(\_Z^2-13\right)x+26\sqrt{3x^2-x+2}+9\,\mathrm{RootOf}\left(\_Z^2-13\right)}{1+2x}\right)}{28561}$
default	$\frac{487}{4394\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}+\frac{\frac{7248x}{50531}-\frac{1208}{50531}}{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}-\frac{487\sqrt{13}\,\mathrm{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{28561}-\frac{1}{104\left(\frac{1}{2}+x\right)^2\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/50531*(14496*x^3+23281*x^2+13306*x+1673)/(1+2*x)^2/(3*x^2-x+2)^(1/2)-487
/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx=\frac{11201\sqrt{13}(12x^4+8x^3+7x^2+7x+2)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+220}{4x^2+4x+1}\right.}{1313806\,(12x^4+8}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="fricas")
```

output
```
1/1313806*(11201*sqrt(13)*(12*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)*log(-(4*sqrt(
13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x +
1)) + 52*(14496*x^3 + 23281*x^2 + 13306*x + 1673)*sqrt(3*x^2 - x + 2))/(12
*x^4 + 8*x^3 + 7*x^2 + 7*x + 2)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{487}{28561}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{7248\,x}{50531\sqrt{3\,x^2-x+2}} + \frac{8785}{101062\sqrt{3\,x^2-x+2}} - \frac{1}{26\left(4\sqrt{3\,x^2-x+2}x^2+4\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)} + \frac{3}{169\left(2\sqrt{3\,x^2-x+2}x+\sqrt{3\,x^2-x+2}\right)}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="maxima")
```

output
```
487/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/ab
s(2*x + 1)) + 7248/50531*x/sqrt(3*x^2 - x + 2) + 8785/101062/sqrt(3*x^2 -
x + 2) - 1/26/(4*sqrt(3*x^2 - x + 2)*x^2 + 4*sqrt(3*x^2 - x + 2)*x + sqrt(
3*x^2 - x + 2)) + 3/169/(2*sqrt(3*x^2 - x + 2)*x + sqrt(3*x^2 - x + 2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(90) = 180$.

Time = 0.17 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2-x+3x^2)^{3/2}}\,dx = \frac{487}{28561}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3x^2-x+2}\right)}\right) + \frac{2(3693x+2363)}{50531\sqrt{3x^2-x+2}} + \frac{2\left(62\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^3-37\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+263\sqrt{3}x-71\sqrt{3}-263\sqrt{3x^2-x+2}\right)}{2197\left(2\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x, algorithm="giac")
```

output
```
487/28561*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*
sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x
+ 2))) + 2/50531*(3693*x + 2363)/sqrt(3*x^2 - x + 2) + 2/2197*(62*(sqrt(3)
*x - sqrt(3*x^2 - x + 2))^3 - 37*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))
^2 + 263*sqrt(3)*x - 71*sqrt(3) - 263*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x -
 sqrt(3*x^2 - x + 2))^2 + 2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)
^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3 (2-x+3x^2)^{3/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3 (3x^2-x+2)^{3/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{3/2}}\,dx = \frac{376896\sqrt{3x^2-x+2}\,x^3+605306\sqrt{3x^2-x+2}\,x^2+345956\sqrt{3x^2-x}}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(3/2),x)
```

output
```
(376896*sqrt(3*x**2 - x + 2)*x**3 + 605306*sqrt(3*x**2 - x + 2)*x**2 + 345
956*sqrt(3*x**2 - x + 2)*x + 43498*sqrt(3*x**2 - x + 2) + 134412*sqrt(13)*
log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**4 + 89608*sqrt(13)*log(2
*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**3 + 78407*sqrt(13)*log(2*sqrt
(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**2 + 78407*sqrt(13)*log(2*sqrt(3*x*
*2 - x + 2)*sqrt(13) + 8*x - 9)*x + 22402*sqrt(13)*log(2*sqrt(3*x**2 - x +
 2)*sqrt(13) + 8*x - 9) - 134412*sqrt(13)*log(2*x + 1)*x**4 - 89608*sqrt(1
3)*log(2*x + 1)*x**3 - 78407*sqrt(13)*log(2*x + 1)*x**2 - 78407*sqrt(13)*l
og(2*x + 1)*x - 22402*sqrt(13)*log(2*x + 1))/(656903*(12*x**4 + 8*x**3 + 7
*x**2 + 7*x + 2))
```

3.89 $\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(12839-3871x)}{5589(2-x+3x^2)^{3/2}} - \frac{28(35809+42240x)}{128547\sqrt{2-x+3x^2}} + \frac{32}{27}\sqrt{2-x+3x^2} - \frac{296\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{27\sqrt{3}}$$

output
```
2/5589*(12839-3871*x)/(3*x^2-x+2)^(3/2)-28/128547*(35809+42240*x)/(3*x^2-x
+2)^(1/2)+32/27*(3*x^2-x+2)^(1/2)-296/81*arcsinh(1/23*(1-6*x)*23^(1/2))*3^
(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{(1+2x)^3(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(-44739-119459x+8630x^2-247904x^3+76176x^4)}{14283(2-x+3x^2)^{3/2}} - \frac{296\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{27\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-44739 - 119459*x + 8630*x^2 - 247904*x^3 + 76176*x^4))/(14283*(2 - x
+ 3*x^2)^(3/2)) - (296*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(27*Sqrt[3]
)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2191, 27, 2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^3 (4x^2+3x+1)}{(3x^2-x+2)^{5/2}} \, dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69} \int -\frac{-29808x^3-77004x^2-69138x+4361}{81 (3x^2-x+2)^{3/2}}dx + \frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2 \int \frac{-29808x^3-77004x^2-69138x+4361}{(3x^2-x+2)^{3/2}}dx}{5589}$$

$$\Big\downarrow \text{2191}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\left(\frac{2}{23}\int -\frac{9522(12x+35)}{\sqrt{3x^2-x+2}}dx + \frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}\right)}{5589}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(12839-3871x)}{5589 (3x^2-x+2)^{3/2}} - \frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}} - 828\int \frac{12x+35}{\sqrt{3x^2-x+2}}dx\right)}{5589}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}-\frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}-828\left(37\int\frac{1}{\sqrt{3x^2-x+2}}dx+4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

$\downarrow$ 1090

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}-\frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}-828\left(\frac{37\int\frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)}{\sqrt{69}}+4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

$\downarrow$ 222

$$\frac{2(12839-3871x)}{5589\left(3x^2-x+2\right)^{3/2}}-\frac{2\left(\frac{14(42240x+35809)}{23\sqrt{3x^2-x+2}}-828\left(\frac{37\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)}{\sqrt{3}}+4\sqrt{3x^2-x+2}\right)\right)}{5589}$$

input
```
Int[((1 + 2*x)^3*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(12839 - 3871*x))/(5589*(2 - x + 3*x^2)^(3/2)) - (2*((14*(35809 + 42240
*x))/(23*Sqrt[2 - x + 3*x^2]) - 828*(4*Sqrt[2 - x + 3*x^2] + (37*ArcSinh[(
-1 + 6*x)/Sqrt[23]])/Sqrt[3])))/5589
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{296\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{81}$
trager	$\frac{\frac{32}{3}x^4-\frac{495808}{14283}x^3+\frac{17260}{14283}x^2-\frac{238918}{14283}x-\frac{3314}{529}}{(3x^2-x+2)^{\frac{3}{2}}}-\frac{296\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2\right.\right.}{81}$
default	$\frac{\frac{13763x}{5589}-\frac{13763}{33534}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{130528x}{42849}-\frac{65264}{128547}}{\sqrt{3x^2-x+2}}-\frac{1727}{1458(3x^2-x+2)^{\frac{3}{2}}}-\frac{461x}{81(3x^2-x+2)^{\frac{3}{2}}}+\frac{8x^2}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{296x^3}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{}{27\sqrt{3}}$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/14283*(76176*x^4-247904*x^3+8630*x^2-119459*x-44739)/(3*x^2-x+2)^(3/2)+2
96/81*3^(1/2)*arcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\left(39146\sqrt{3}(9\,x^4-6\,x^3+13\,x^2-4\,x+4)\log\left(-4\sqrt{3}\sqrt{3\,x^2-x+2}(6\,x\right.\right.}{4284}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/42849*(39146*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*s
qrt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(76176*x^4 - 247904
*x^3 + 8630*x^2 - 119459*x - 44739)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 +
13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^3\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**3*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**3*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(69) = 138.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{32\,x^4}{3\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}} + \frac{296}{42849}\,x\left(\frac{426\,x}{\sqrt{3\,x^2-x+2}} - \frac{4761\,x^2}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}} - \frac{71}{\sqrt{3\,x^2-x+2}} + \frac{805\,x}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}} - \frac{2162}{\left(3\,x^2-x+2\right)^{\frac{3}{2}}}\right) + \frac{296}{81}\sqrt{3}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{42032}{42849}\sqrt{3\,x^2-x+2} - \frac{47072\,x}{42849\,\sqrt{3\,x^2-x+2}} + \frac{52\,x^2}{9\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}} - \frac{23104}{14283\,\sqrt{3\,x^2-x+2}} - \frac{7742\,x}{1863\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}} + \frac{1666}{1863\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
32/3*x^4/(3*x^2 - x + 2)^(3/2) + 296/42849*x*(426*x/sqrt(3*x^2 - x + 2) -
4761*x^2/(3*x^2 - x + 2)^(3/2) - 71/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x
 + 2)^(3/2) - 2162/(3*x^2 - x + 2)^(3/2)) + 296/81*sqrt(3)*arcsinh(1/23*sq
rt(23)*(6*x - 1)) - 42032/42849*sqrt(3*x^2 - x + 2) - 47072/42849*x/sqrt(3
*x^2 - x + 2) + 52/9*x^2/(3*x^2 - x + 2)^(3/2) - 23104/14283/sqrt(3*x^2 -
x + 2) - 7742/1863*x/(3*x^2 - x + 2)^(3/2) + 1666/1863/(3*x^2 - x + 2)^(3/
2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.78

$$\int \frac{(1+2x)^3\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{296}{81}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)+1\right) + \frac{2\left(\left(2\left(8\left(4761\,x-15494\right)x+4315\right)x-119459\right)x-44739\right)}{14283\,\left(3\,x^2-x+2\right)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-296/81*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/
14283*((2*(8*(4761*x - 15494)*x + 4315)*x - 119459)*x - 44739)/(3*x^2 - x
+ 2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2\,x+1)^3\,(4\,x^2+3\,x+1)}{(3\,x^2-x+2)^{5/2}}\,dx$$

input
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
int(((2*x + 1)^3*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.37

$$\int \frac{(1+2x)^3\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{457056\sqrt{3x^2-x+2}\,x^4 - 1487424\sqrt{3x^2-x+2}\,x^3 + 51780\sqrt{3x^2-x+}}{}$$

input
```
int((1+2*x)^3*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

output
```
(2*(228528*sqrt(3*x**2 - x + 2)*x**4 - 743712*sqrt(3*x**2 - x + 2)*x**3 +
25890*sqrt(3*x**2 - x + 2)*x**2 - 358377*sqrt(3*x**2 - x + 2)*x - 134217*s
qrt(3*x**2 - x + 2) + 704628*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) +
 6*x - 1)/sqrt(23))*x**4 - 469752*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt
(3) + 6*x - 1)/sqrt(23))*x**3 + 1017796*sqrt(3)*log((2*sqrt(3*x**2 - x + 2
)*sqrt(3) + 6*x - 1)/sqrt(23))*x**2 - 313168*sqrt(3)*log((2*sqrt(3*x**2 -
x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x + 313168*sqrt(3)*log((2*sqrt(3*x**2
- x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)) + 259056*sqrt(3)*x**4 - 172704*sqrt(
3)*x**3 + 374192*sqrt(3)*x**2 - 115136*sqrt(3)*x + 115136*sqrt(3)))/(42849
*(9*x**4 - 6*x**3 + 13*x**2 - 4*x + 4))
```

3.90 $\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 68

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(1249-2273x)}{1863\,(2-x+3x^2)^{3/2}} - \frac{8(23257-1473x)}{42849\sqrt{2-x+3x^2}} - \frac{16\operatorname{arcsinh}\left(\frac{1-6x}{\sqrt{23}}\right)}{9\sqrt{3}}$$

output
```
2/1863*(1249-2273*x)/(3*x^2-x+2)^(3/2)-8/42849*(23257-1473*x)/(3*x^2-x+2)^
(1/2)-16/27*arcsinh(1/23*(1-6*x)*23^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2(-17481+5837x-31664x^2+1964x^3)}{4761\,(2-x+3x^2)^{3/2}} - \frac{16\log\left(1-6x+2\sqrt{6-3x+9x^2}\right)}{9\sqrt{3}}$$

input
```
Integrate[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-17481 + 5837*x - 31664*x^2 + 1964*x^3))/(4761*(2 - x + 3*x^2)^(3/2))
- (16*Log[1 - 6*x + 2*Sqrt[6 - 3*x + 9*x^2]])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)^2 \left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{2\left(2484x^2+5175x+901\right)}{27\left(3x^2-x+2\right)^{3/2}}dx+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{2484x^2+5175x+901}{\left(3x^2-x+2\right)^{3/2}}dx}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{4\left(\frac{2}{23}\int \frac{9522}{\sqrt{3x^2-x+2}}dx-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(828\int \frac{1}{\sqrt{3x^2-x+2}}dx-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{4\left(12\sqrt{69}\int \frac{1}{\sqrt{\frac{1}{23}(6x-1)^2+1}}d(6x-1)-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{4\left(276\sqrt{3}\text{arcsinh}\left(\frac{6x-1}{\sqrt{23}}\right)-\frac{2(23257-1473x)}{23\sqrt{3x^2-x+2}}\right)}{1863}+\frac{2(1249-2273x)}{1863\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x)^2*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(1249 - 2273*x))/(1863*(2 - x + 3*x^2)^(3/2)) + (4*((-2*(23257 - 1473*x
))/(23*Sqrt[2 - x + 3*x^2]) + 276*Sqrt[3]*ArcSinh[(-1 + 6*x)/Sqrt[23]]))/1
863
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{16\sqrt{3}\,\operatorname{arcsinh}\left(\frac{6\sqrt{23}\left(x-\frac{1}{6}\right)}{23}\right)}{27}$
trager	$\frac{\frac{3928}{4761}x^3-\frac{63328}{4761}x^2+\frac{11674}{4761}x-\frac{11654}{1587}}{(3x^2-x+2)^{\frac{3}{2}}}-\frac{16\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-6\,\mathrm{RootOf}\left(\_Z^2-3\right)x+6\sqrt{3x^2-x+2}+\mathrm{RootOf}\left(\_Z^2-3\right)\right)}{27}$
default	$\frac{\frac{4585x}{1863}-\frac{4585}{11178}}{(3x^2-x+2)^{\frac{3}{2}}}+\frac{\frac{37784x}{14283}-\frac{18892}{42849}}{\sqrt{3x^2-x+2}}-\frac{2653}{486(3x^2-x+2)^{\frac{3}{2}}}-\frac{67x}{27(3x^2-x+2)^{\frac{3}{2}}}-\frac{92x^2}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x^3}{9(3x^2-x+2)^{\frac{3}{2}}}-\frac{16x}{9\sqrt{3x^2-x+}}$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/4761*(1964*x^3-31664*x^2+5837*x-17481)/(3*x^2-x+2)^(3/2)+16/27*3^(1/2)*a
rcsinh(6/23*23^(1/2)*(x-1/6))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{(1+2x)^2\left(1+3x+4x^2\right)}{\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2\left(2116\sqrt{3}(9x^4-6x^3+13x^2-4x+4)\log\left(-4\sqrt{3}\sqrt{3x^2-x+2}(6x\right.\right.}{14283\,(9x^4}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/14283*(2116*sqrt(3)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-4*sqrt(3)*sq
rt(3*x^2 - x + 2)*(6*x - 1) - 72*x^2 + 24*x - 25) + 3*(1964*x^3 - 31664*x^
2 + 5837*x - 17481)*sqrt(3*x^2 - x + 2))/(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4
)
```

Sympy [F]

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2\cdot(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)**2*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)**2*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{16}{14283}\,x\Bigg(\frac{426\,x}{\sqrt{3\,x^2-x+2}} - \frac{4761\,x^2}{(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{71}{\sqrt{3\,x^2-x+2}} + \frac{805\cdots}{(3\,x^2-\cdots} \\ + \frac{16}{27}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(6\,x-1)\right) - \frac{2272}{14283}\sqrt{3\,x^2-x+2} + \frac{28184\,x}{14283\sqrt{3\,x^2-x+2}} \\ - \frac{28\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2956}{4761\sqrt{3\,x^2-x+2}} - \frac{106\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{3394}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
16/14283*x*(426*x/sqrt(3*x^2 - x + 2) - 4761*x^2/(3*x^2 - x + 2)^(3/2) - 7
1/sqrt(3*x^2 - x + 2) + 805*x/(3*x^2 - x + 2)^(3/2) - 2162/(3*x^2 - x + 2)
^(3/2)) + 16/27*sqrt(3)*arcsinh(1/23*sqrt(23)*(6*x - 1)) - 2272/14283*sqrt
(3*x^2 - x + 2) + 28184/14283*x/sqrt(3*x^2 - x + 2) - 28/3*x^2/(3*x^2 - x
+ 2)^(3/2) - 2956/4761/sqrt(3*x^2 - x + 2) - 106/621*x/(3*x^2 - x + 2)^(3/
2) - 3394/621/(3*x^2 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{16}{27}\sqrt{3}\log\left(-2\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-x+2}\right)+1\right) + \frac{2\left((4(491x-7916)x+5837)x-17481\right)}{4761(3x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-16/27*sqrt(3)*log(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) + 1) + 2/4
761*((4*(491*x - 7916)*x + 5837)*x - 17481)/(3*x^2 - x + 2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)^2(4x^2+3x+1)}{(3x^2-x+2)^{5/2}}\,dx$$

input
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
int(((2*x + 1)^2*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 4.03

$$\int \frac{(1+2x)^2(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{35352\sqrt{3x^2-x+2}\,x^3-569952\sqrt{3x^2-x+2}\,x^2+105066\sqrt{3x^2-x+2}}{}$$

input
```
int((1+2*x)^2*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

output
```
(2*(17676*sqrt(3*x**2 - x + 2)*x**3 - 284976*sqrt(3*x**2 - x + 2)*x**2 + 5
2533*sqrt(3*x**2 - x + 2)*x - 157329*sqrt(3*x**2 - x + 2) + 114264*sqrt(3)
*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x**4 - 76176*sqr
t(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x**3 + 16504
8*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x**2 -
50784*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23))*x +
 50784*sqrt(3)*log((2*sqrt(3*x**2 - x + 2)*sqrt(3) + 6*x - 1)/sqrt(23)) +
103212*sqrt(3)*x**4 - 68808*sqrt(3)*x**3 + 149084*sqrt(3)*x**2 - 45872*sqr
t(3)*x + 45872*sqrt(3)))/(42849*(9*x**4 - 6*x**3 + 13*x**2 - 4*x + 4))
```

3.91 $\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 47

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx=-\frac{2(73+367x)}{621(2-x+3x^2)^{3/2}}-\frac{4(3889-4290x)}{14283\sqrt{2-x+3x^2}}$$

output
```
1/621*(-146-734*x)/(3*x^2-x+2)^(3/2)-4/14283*(3889-4290*x)/(3*x^2-x+2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx=\frac{2(-1915+1833x-3546x^2+2860x^3)}{1587(2-x+3x^2)^{3/2}}$$

input
```
Integrate[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(2*(-1915 + 1833*x - 3546*x^2 + 2860*x^3))/(1587*(2 - x + 3*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2191, 27, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+1)\left(4x^2+3x+1\right)}{\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int \frac{828x+577}{9\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{621}\int \frac{828x+577}{\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{4(3889-4290x)}{14283\sqrt{3x^2-x+2}}-\frac{2(367x+73)}{621\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[((1 + 2*x)*(1 + 3*x + 4*x^2))/(2 - x + 3*x^2)^(5/2),x]
```

output
```
(-2*(73 + 367*x))/(621*(2 - x + 3*x^2)^(3/2)) - (4*(3889 - 4290*x))/(14283
*Sqrt[2 - x + 3*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\frac{5720}{1587}x^{3}-\frac{2364}{529}x^{2}+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}$	30
trager	$\frac{\frac{5720}{1587}x^{3}-\frac{2364}{529}x^{2}+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}$	30
risch	$\frac{\frac{5720}{1587}x^{3}-\frac{2364}{529}x^{2}+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}$	30
orering	$\frac{\frac{5720}{1587}x^{3}-\frac{2364}{529}x^{2}+\frac{1222}{529}x-\frac{3830}{1587}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}$	30
default	$\frac{\frac{715x}{621}-\frac{715}{3726}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}+\frac{\frac{5720x}{4761}-\frac{2860}{14283}}{\sqrt{3x^{2}-x+2}}-\frac{295}{162\left(3x^{2}-x+2\right)^{\frac{3}{2}}}-\frac{13x}{9\left(3x^{2}-x+2\right)^{\frac{3}{2}}}-\frac{8x^{2}}{3\left(3x^{2}-x+2\right)^{\frac{3}{2}}}$	86

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/1587/(3*x^2-x+2)^(3/2)*(2860*x^3-3546*x^2+1833*x-1915)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\,(2860\,x^3-3546\,x^2+1833\,x-1915)\sqrt{3\,x^2-x+2}}{1587\,(9\,x^4-6\,x^3+13\,x^2-4\,x+4)}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/1587*(2860*x^3 - 3546*x^2 + 1833*x - 1915)*sqrt(3*x^2 - x + 2)/(9*x^4 -
6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \int \frac{(2x+1)(4x^2+3x+1)}{(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((1+2*x)*(4*x**2+3*x+1)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((2*x + 1)*(4*x**2 + 3*x + 1)/(3*x**2 - x + 2)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{5720\,x}{4761\,\sqrt{3\,x^2-x+2}} - \frac{8\,x^2}{3\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{2860}{14283\,\sqrt{3\,x^2-x+2}} - \frac{182\,x}{621\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1250}{621\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
5720/4761*x/sqrt(3*x^2 - x + 2) - 8/3*x^2/(3*x^2 - x + 2)^(3/2) - 2860/142
83/sqrt(3*x^2 - x + 2) - 182/621*x/(3*x^2 - x + 2)^(3/2) - 1250/621/(3*x^2
 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{2\,((2\,(1430\,x-1773)x+1833)x-1915)}{1587\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
2/1587*((2*(1430*x - 1773)*x + 1833)*x - 1915)/(3*x^2 - x + 2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 18.39 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{(1+2x)(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = -\frac{442\,x-5720\,x\,(3\,x^2-x+2)+15556\,x^2+11490}{\sqrt{3\,x^2-x+2}\,(14283\,x^2-4761\,x+9522)}$$

input
```
int(((2*x + 1)*(3*x + 4*x^2 + 1))/(3*x^2 - x + 2)^(5/2),x)
```

output
```
-(442*x - 5720*x*(3*x^2 - x + 2) + 15556*x^2 + 11490)/((3*x^2 - x + 2)^(1/
2)*(14283*x^2 - 4761*x + 9522))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.43

$$\int \frac{(1+2x)\,(1+3x+4x^2)}{(2-x+3x^2)^{5/2}}\,dx = \frac{51480\sqrt{3x^2-x+2}\,x^3-63828\sqrt{3x^2-x+2}\,x^2+32994\sqrt{3x^2-x+2}\,x}{128547x^4-85698}$$

input
```
int((1+2*x)*(4*x^2+3*x+1)/(3*x^2-x+2)^(5/2),x)
```

output
```
(2*(25740*sqrt(3*x**2 - x + 2)*x**3 - 31914*sqrt(3*x**2 - x + 2)*x**2 + 16
497*sqrt(3*x**2 - x + 2)*x - 17235*sqrt(3*x**2 - x + 2) + 9036*sqrt(3)*x**
4 - 6024*sqrt(3)*x**3 + 13052*sqrt(3)*x**2 - 4016*sqrt(3)*x + 4016*sqrt(3)
))/(14283*(9*x**4 - 6*x**3 + 13*x**2 - 4*x + 4))
```

3.92 $\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 85

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{2(101-77x)}{897\left(2-x+3x^2\right)^{3/2}} - \frac{4(691-13668x)}{268203\sqrt{2-x+3x^2}} - \frac{8\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{169\sqrt{13}}$$

output
```
1/897*(-202+154*x)/(3*x^2-x+2)^(3/2)-4/268203*(691-13668*x)/(3*x^2-x+2)^(1
/2)-8/2197*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(-32963+79077x-31482x^2+82008x^3)}{268203\left(2-x+3x^2\right)^{3/2}} + \frac{16\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{169\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(-32963 + 79077*x - 31482*x^2 + 82008*x^3))/(268203*(2 - x + 3*x^2)^(3/
2)) + (16*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]
])/(169*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2177, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int \frac{308x+223}{13(2x+1)\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{897}\int \frac{308x+223}{(2x+1)\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{2}{897}\left(\frac{2}{299}\int \frac{3174}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{897}\left(\frac{276}{13}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{2}{897}\left(-\frac{552}{13}\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2}{897}\left(-\frac{276\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{13\sqrt{13}}-\frac{2(691-13668x)}{299\sqrt{3x^2-x+2}}\right)-\frac{2(101-77x)}{897\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(101 - 77*x))/(897*(2 - x + 3*x^2)^(3/2)) + (2*((-2*(691 - 13668*x))/(
299*Sqrt[2 - x + 3*x^2]) - (276*ArcTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x +
 3*x^2])])/(13*Sqrt[13])))/897
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
trager	$\frac{\frac{54672}{89401}x^{3}-\frac{20988}{89401}x^{2}+\frac{52718}{89401}x-\frac{65926}{268203}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}+\frac{8\operatorname{RootOf}\left(\_Z^{2}-13\right)\ln\left(\frac{8\operatorname{RootOf}\left(\_Z^{2}-13\right)x+26\sqrt{3x^{2}-x+2}-9\operatorname{RootOf}\left(\_Z^{2}-13\right)}{1+2x}\right)}{2197}$
default	$\frac{\frac{10x}{69}-\frac{5}{207}}{\left(3x^{2}-x+2\right)^{\frac{3}{2}}}+\frac{\frac{80x}{529}-\frac{40}{1587}}{\sqrt{3x^{2}-x+2}}-\frac{2}{9\left(3x^{2}-x+2\right)^{\frac{3}{2}}}+\frac{1}{39\left(3\left(\frac{1}{2}+x\right)^{2}+\frac{5}{4}-4x\right)^{\frac{3}{2}}}+\frac{\frac{8x}{299}-\frac{4}{897}}{\left(3\left(\frac{1}{2}+x\right)^{2}+\frac{5}{4}-4x\right)^{\frac{3}{2}}}+\frac{\frac{4704x}{89401}-\frac{784}{89401}}{\sqrt{3\left(\frac{1}{2}+x\right)^{2}+\frac{5}{4}-4x}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/268203*(82008*x^3-31482*x^2+79077*x-32963)/(3*x^2-x+2)^(3/2)+8/2197*Root
Of(_Z^2-13)*ln((8*RootOf(_Z^2-13)*x+26*(3*x^2-x+2)^(1/2)-9*RootOf(_Z^2-13)
)/(1+2*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\left(3174\sqrt{13}(9x^4-6x^3+13x^2-4x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3x^2-x+2}(8x-9)+22}{4x^2+4x+1}\right.\right.}{3486639\,(9x^4-6}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/3486639*(3174*sqrt(13)*(9*x^4 - 6*x^3 + 13*x^2 - 4*x + 4)*log(-(4*sqrt(1
3)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^2 + 4*x + 1
)) + 13*(82008*x^3 - 31482*x^2 + 79077*x - 32963)*sqrt(3*x^2 - x + 2))/(9*
x^4 - 6*x^3 + 13*x^2 - 4*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)\left(3x^2-x+2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}}\,dx = \frac{8}{2197}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|}-\frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{18224\,x}{89401\sqrt{3\,x^2-x+2}} - \frac{2764}{268203\sqrt{3\,x^2-x+2}} + \frac{154\,x}{897\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{202}{897\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
8/2197*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs(2
*x + 1)) + 18224/89401*x/sqrt(3*x^2 - x + 2) - 2764/268203/sqrt(3*x^2 - x
+ 2) + 154/897*x/(3*x^2 - x + 2)^(3/2) - 202/897/(3*x^2 - x + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.19

$$\int \frac{1+3x+4x^2}{(1+2x)(2-x+3x^2)^{5/2}}\,dx = \frac{8}{2197}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(3\,(6\,(4556\,x-1749)x+26359)x-32963)}{268203\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
8/2197*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) + 4*sqr
t(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 - x + 2
))) + 2/268203*(3*(6*(4556*x - 1749)*x + 26359)*x - 32963)/(3*x^2 - x + 2)
^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=\int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)\,\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)*(3*x^2 - x + 2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 714, normalized size of antiderivative = 8.40

$$\int \frac{1+3x+4x^2}{(1+2x)\left(2-x+3x^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)/(3*x^2-x+2)^(5/2),x)
```

output
```
(2*(57132*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sq
rt(39) - 4))*i*x**4 - 38088*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*
i + 6*i*x - i)/(sqrt(39) - 4))*i*x**3 + 82524*sqrt(13)*atan((2*sqrt(3*x**2
 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39) - 4))*i*x**2 - 25392*sqrt(13)*a
tan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39) - 4))*i*x + 2
5392*sqrt(13)*atan((2*sqrt(3*x**2 - x + 2)*sqrt(3)*i + 6*i*x - i)/(sqrt(39
) - 4))*i + 1066104*sqrt(3*x**2 - x + 2)*x**3 - 409266*sqrt(3*x**2 - x + 2
)*x**2 + 1028001*sqrt(3*x**2 - x + 2)*x - 428519*sqrt(3*x**2 - x + 2) + 28
566*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2
)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30)*x**4 - 19044*sqrt(13)*log(24
*sqrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(
39) + 72*x**2 - 24*x - 30)*x**3 + 41262*sqrt(13)*log(24*sqrt(3*x**2 - x +
2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*
x - 30)*x**2 - 12696*sqrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sq
rt(3*x**2 - x + 2)*sqrt(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30)*x + 12696*s
qrt(13)*log(24*sqrt(3*x**2 - x + 2)*sqrt(3)*x - 4*sqrt(3*x**2 - x + 2)*sqr
t(3) + 8*sqrt(39) + 72*x**2 - 24*x - 30) - 57132*sqrt(13)*log(2*sqrt(3*x**
2 - x + 2)*sqrt(3) + sqrt(39) + 6*x + 3)*x**4 + 38088*sqrt(13)*log(2*sqrt(
3*x**2 - x + 2)*sqrt(3) + sqrt(39) + 6*x + 3)*x**3 - 82524*sqrt(13)*log(2*
sqrt(3*x**2 - x + 2)*sqrt(3) + sqrt(39) + 6*x + 3)*x**2 + 25392*sqrt(13...
```

3.93 $\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=-\frac{2(197-837x)}{11661\left(2-x+3x^2\right)^{3/2}}-\frac{24(841-6633x)}{1162213\sqrt{2-x+3x^2}}-\frac{16\sqrt{2-x+3x^2}}{2197(1+2x)}-\frac{56\,\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{2197\sqrt{13}}$$

output
```
1/11661*(-394+1674*x)/(3*x^2-x+2)^(3/2)-24/1162213*(841-6633*x)/(3*x^2-x+2
)^(1/2)-16*(3*x^2-x+2)^(1/2)/(2197+4394*x)-56/28561*13^(1/2)*arctanh(1/26*
(9-8*x)*13^(1/2)/(3*x^2-x+2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx=\frac{2(-170239+569989x+1021566x^2+133308x^3+1318464x^4)}{3486639(1+2x)\left(2-x+3x^2\right)^{3/2}}+\frac{112\,\text{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{2197\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(-170239 + 569989*x + 1021566*x^2 + 133308*x^3 + 1318464*x^4))/(3486639
*(1 + 2*x)*(2 - x + 3*x^2)^(3/2)) + (112*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x -
2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(2197*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2177, 27, 2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^2\left(3x^2-x+2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int \frac{6\left(1116x^2+1001x+371\right)}{169(2x+1)^2\left(3x^2-x+2\right)^{3/2}}dx-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{1116x^2+1001x+371}{(2x+1)^2\left(3x^2-x+2\right)^{3/2}}dx}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2177}$$

$$\frac{4\left(\frac{2}{23}\int \frac{1058(3x+8)}{13(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(\frac{92}{13}\int \frac{3x+8}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{4\left(\frac{92}{13}\left(\frac{7}{2}\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{4\left(\frac{92}{13}\left(-7\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{4\left(\frac{92}{13}\left(-\frac{7\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{2\sqrt{13}}-\frac{\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{6(841-6633x)}{299\sqrt{3x^2-x+2}}\right)}{3887}-\frac{2(197-837x)}{11661\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^2*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(-2*(197 - 837*x))/(11661*(2 - x + 3*x^2)^(3/2)) + (4*((-6*(841 - 6633*x))
/(299*Sqrt[2 - x + 3*x^2]) + (92*(-(Sqrt[2 - x + 3*x^2]/(1 + 2*x)) - (7*Ar
cTanh[(9 - 8*x)/(2*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/(2*Sqrt[13])))/13))/388
7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\frac{878976}{1162213} x^{4}+\frac{52388}{89401} x^{2}+\frac{1139978}{3486639} x -\frac{340478}{3486639}+\frac{168}{2197} x^{3}}{\left(1+2 x \right) \left(3 x^{2}-x +2\right)^{\frac{3}{2}}}-\frac{56 \sqrt{13}\, \operatorname{arctanh}\left(\frac{2 \left(\frac{9}{2}-4 x \right) \sqrt{13}}{13 \sqrt{12 \left(\frac{1}{2}+x \right)^{2}+5-16 x}}\right)}{28561}$
trager	$\frac{\frac{878976}{1162213} x^{4}+\frac{52388}{89401} x^{2}+\frac{1139978}{3486639} x -\frac{340478}{3486639}+\frac{168}{2197} x^{3}}{\left(1+2 x \right) \left(3 x^{2}-x +2\right)^{\frac{3}{2}}}+\frac{56 \operatorname{RootOf}\left(\_Z^{2}-13\right) \ln\left(\frac{8 \operatorname{RootOf}\left(\_Z^{2}-13\right) x +26 \sqrt{3 x^{2}-x +2}-9 \operatorname{RootOf}\left(\_ \right.}{1+2 x}\right.}{28561}$
default	$\frac{\frac{4 x}{23}-\frac{2}{69}}{\left(3 x^{2}-x +2\right)^{\frac{3}{2}}}+\frac{\frac{96 x}{529}-\frac{16}{529}}{\sqrt{3 x^{2}-x +2}}-\frac{1}{26 \left(\frac{1}{2}+x \right) \left(3 \left(\frac{1}{2}+x \right)^{2}+\frac{5}{4}-4 x \right)^{\frac{3}{2}}}+\frac{7}{507 \left(3 \left(\frac{1}{2}+x \right)^{2}+\frac{5}{4}-4 x \right)^{\frac{3}{2}}}-\frac{128 (6 x -1)}{11661 \left(3 \left(\frac{1}{2}+x \right)^{2}+\frac{5}{4}-4 x \right)^{\frac{3}{2}}}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3486639*(1318464*x^4+133308*x^3+1021566*x^2+569989*x-170239)/(3*x^2-x+2)
^(3/2)/(1+2*x)-56/28561*13^(1/2)*arctanh(2/13*(9/2-4*x)*13^(1/2)/(12*(1/2+
x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{5/2}}\,dx = \frac{2\left(22218\sqrt{13}(18\,x^5-3\,x^4+20\,x^3+5\,x^2+4\,x+4)\log\left(-\frac{4\sqrt{13}\sqrt{3\,x^2-x\cdots}}{}\right.\right.}{453263\cdots}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/45326307*(22218*sqrt(13)*(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)*log
(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*x + 185)/(4*x^
2 + 4*x + 1)) + 13*(1318464*x^4 + 133308*x^3 + 1021566*x^2 + 569989*x - 17
0239)*sqrt(3*x^2 - x + 2))/(18*x^5 - 3*x^4 + 20*x^3 + 5*x^2 + 4*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^2\,(2-x+3x^2)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^2\,(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**2/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**2*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{5/2}}\,dx = \frac{56}{28561}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{146496\,x}{1162213\sqrt{3\,x^2-x+2}} - \frac{9604}{1162213\sqrt{3\,x^2-x+2}} + \frac{420\,x}{3887\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1}{13\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x + (3\,x^2-x+2)^{\frac{3}{2}}\right)} - \frac{49}{11661\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
56/28561*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/abs
(2*x + 1)) + 146496/1162213*x/sqrt(3*x^2 - x + 2) - 9604/1162213/sqrt(3*x^
2 - x + 2) + 420/3887*x/(3*x^2 - x + 2)^(3/2) - 1/13/(2*(3*x^2 - x + 2)^(3
/2)*x + (3*x^2 - x + 2)^(3/2)) - 49/11661/(3*x^2 - x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.12

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = -\frac{56}{15108769}\sqrt{13}\left(872\sqrt{13}\sqrt{3}-529\log\left(\sqrt{13}\sqrt{3}-4\right)\right)\operatorname{sgn}\left(\frac{1}{2x+1}\right) - \frac{56\sqrt{13}\log\left(\sqrt{13}\left(\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}+\frac{\sqrt{13}}{2x+1}\right)-4\right)}{28561\operatorname{sgn}\left(\frac{1}{2x+1}\right)} + \frac{8\left(\frac{\frac{\frac{13\left(\frac{77756}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}+\frac{20631}{(2x+1)\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{2x+1}-\frac{1399650}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}+\frac{625905}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}}{2x+1}-\frac{164808}{\operatorname{sgn}\left(\frac{1}{2x+1}\right)}\right)}{3486639\left(\frac{8}{2x+1}-\frac{13}{(2x+1)^2}-3\right)\sqrt{-\frac{8}{2x+1}+\frac{13}{(2x+1)^2}+3}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
-56/15108769*sqrt(13)*(872*sqrt(13)*sqrt(3) - 529*log(sqrt(13)*sqrt(3) - 4
))*sgn(1/(2*x + 1)) - 56/28561*sqrt(13)*log(sqrt(13)*(sqrt(-8/(2*x + 1) +
13/(2*x + 1)^2 + 3) + sqrt(13)/(2*x + 1)) - 4)/sgn(1/(2*x + 1)) + 8/348663
9*(((13*(77756/sgn(1/(2*x + 1)) + 20631/((2*x + 1)*sgn(1/(2*x + 1))))/(2*x
 + 1) - 1399650/sgn(1/(2*x + 1)))/(2*x + 1) + 625905/sgn(1/(2*x + 1)))/(2*
x + 1) - 164808/sgn(1/(2*x + 1)))/((8/(2*x + 1) - 13/(2*x + 1)^2 - 3)*sqrt
(-8/(2*x + 1) + 13/(2*x + 1)^2 + 3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^2\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{{\left(2\,x+1\right)}^2\,{\left(3\,x^2-x+2\right)}^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^2*(3*x^2 - x + 2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.10

$$\int \frac{1+3x+4x^2}{(1+2x)^2 (2-x+3x^2)^{5/2}}\,dx = \frac{34280064\sqrt{3x^2-x+2}\,x^4+3466008\sqrt{3x^2-x+2}\,x^3+26560716\sqrt{3x}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^2/(3*x^2-x+2)^(5/2),x)
```

output
```
(2*(17140032*sqrt(3*x**2 - x + 2)*x**4 + 1733004*sqrt(3*x**2 - x + 2)*x**3
 + 13280358*sqrt(3*x**2 - x + 2)*x**2 + 7409857*sqrt(3*x**2 - x + 2)*x - 2
213107*sqrt(3*x**2 - x + 2) + 799848*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*s
qrt(13) + 8*x - 9)*x**5 - 133308*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(
13) + 8*x - 9)*x**4 + 888720*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13)
+ 8*x - 9)*x**3 + 222180*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*
x - 9)*x**2 + 177744*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x -
9)*x + 177744*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 79
9848*sqrt(13)*log(2*x + 1)*x**5 + 133308*sqrt(13)*log(2*x + 1)*x**4 - 8887
20*sqrt(13)*log(2*x + 1)*x**3 - 222180*sqrt(13)*log(2*x + 1)*x**2 - 177744
*sqrt(13)*log(2*x + 1)*x - 177744*sqrt(13)*log(2*x + 1)))/(45326307*(18*x*
*5 - 3*x**4 + 20*x**3 + 5*x**2 + 4*x + 4))
```

3.94 $\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 135

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2(2363+3693x)}{151593\left(2-x+3x^2\right)^{3/2}} + \frac{12(25771+103526x)}{15108769\sqrt{2-x+3x^2}} - \frac{8\sqrt{2-x+3x^2}}{2197(1+2x)^2} - \frac{144\sqrt{2-x+3x^2}}{28561(1+2x)} - \frac{2084\operatorname{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{2-x+3x^2}}\right)}{28561\sqrt{13}}$$

output
```
2/151593*(2363+3693*x)/(3*x^2-x+2)^(3/2)+12/15108769*(25771+103526*x)/(3*x
^2-x+2)^(1/2)-8/2197*(3*x^2-x+2)^(1/2)/(1+2*x)^2-144*(3*x^2-x+2)^(1/2)/(28
561+57122*x)-2084/371293*13^(1/2)*arctanh(1/26*(9-8*x)*13^(1/2)/(3*x^2-x+2
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{2\sqrt{2-x+3x^2}(847141+10777477x+21890266x^2+19381992x^3+20}{45326307\left(2+3x+x^2+6x^3\right)^2}$$
$$+\frac{4168\operatorname{arctanh}\left(\frac{\sqrt{3}+2\sqrt{3}x-2\sqrt{2-x+3x^2}}{\sqrt{13}}\right)}{28561\sqrt{13}}$$

input
```
Integrate[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[2 - x + 3*x^2]*(847141 + 10777477*x + 21890266*x^2 + 19381992*x^3
+ 20074356*x^4 + 20304864*x^5))/(45326307*(2 + 3*x + x^2 + 6*x^3)^2) + (41
68*ArcTanh[(Sqrt[3] + 2*Sqrt[3]*x - 2*Sqrt[2 - x + 3*x^2])/Sqrt[13]])/(285
61*Sqrt[13])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2177, 27, 2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^2+3x+1}{(2x+1)^3\left(3x^2-x+2\right)^{5/2}}dx$$
$$\Big\downarrow \ 2177$$
$$\frac{2}{69}\int \frac{3\left(19696x^3+53372x^2+35610x+10811\right)}{2197(2x+1)^3\left(3x^2-x+2\right)^{3/2}}dx+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$
$$\Big\downarrow \ 27$$
$$\frac{2\int \frac{19696x^3+53372x^2+35610x+10811}{(2x+1)^3\left(3x^2-x+2\right)^{3/2}}dx}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2177}$$

$$\frac{2\left(\frac{2}{23}\int \frac{2116\left(488x^2+527x+226\right)}{13(2x+1)^3\sqrt{3x^2-x+2}}dx+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{184}{13}\int \frac{488x^2+527x+226}{(2x+1)^3\sqrt{3x^2-x+2}}dx+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{2\left(\frac{184}{13}\left(-\frac{1}{26}\int -\frac{13(898x+683)}{2(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\int \frac{898x+683}{(2x+1)^2\sqrt{3x^2-x+2}}dx-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(521\int \frac{1}{(2x+1)\sqrt{3x^2-x+2}}dx-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(-1042\int \frac{1}{52-\frac{(9-8x)^2}{3x^2-x+2}}d\frac{9-8x}{\sqrt{3x^2-x+2}}-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\left(\frac{184}{13}\left(\frac{1}{4}\left(-\frac{521\text{arctanh}\left(\frac{9-8x}{2\sqrt{13}\sqrt{3x^2-x+2}}\right)}{\sqrt{13}}-\frac{36\sqrt{3x^2-x+2}}{2x+1}\right)-\frac{13\sqrt{3x^2-x+2}}{2(2x+1)^2}\right)+\frac{6(103526x+25771)}{299\sqrt{3x^2-x+2}}\right)}{50531}+\frac{2(3693x+2363)}{151593\left(3x^2-x+2\right)^{3/2}}$$

input
```
Int[(1 + 3*x + 4*x^2)/((1 + 2*x)^3*(2 - x + 3*x^2)^(5/2)),x]
```

output
```
(2*(2363 + 3693*x))/(151593*(2 - x + 3*x^2)^(3/2)) + (2*((6*(25771 + 10352
6*x))/(299*Sqrt[2 - x + 3*x^2]) + (184*((-13*Sqrt[2 - x + 3*x^2])/(2*(1 +
2*x)^2) + ((-36*Sqrt[2 - x + 3*x^2])/(1 + 2*x) - (521*ArcTanh[(9 - 8*x)/(2
*Sqrt[13]*Sqrt[2 - x + 3*x^2])])/Sqrt[13])/4))/13))/50531
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

method	result
risch	$\frac{\frac{13536576}{15108769}x^5+\frac{13382904}{15108769}x^4+\frac{12921328}{15108769}x^3+\frac{43780532}{45326307}x^2+\frac{21554954}{45326307}x+\frac{1694282}{45326307}}{(1+2x)^2(3x^2-x+2)^{\frac{3}{2}}}-\frac{2084\sqrt{13}\,\operatorname{arctanh}\left(\frac{2\left(\frac{9}{2}-4x\right)\sqrt{13}}{13\sqrt{12\left(\frac{1}{2}+x\right)^2+5-16x}}\right)}{371293}$
trager	$\frac{2(20304864x^5+20074356x^4+19381992x^3+21890266x^2+10777477x+847141)\sqrt{3x^2-x+2}}{45326307(6x^3+x^2+3x+2)^2}-\frac{2084\,\operatorname{RootOf}\left(\_Z^2-13\right)\ln\left(\frac{-8\operatorname{Ro}}{}\right.}{}$
default	$\frac{521}{13182\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}+\frac{\frac{1772x}{50531}-\frac{886}{151593}}{\left(3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x\right)^{\frac{3}{2}}}+\frac{\frac{1128048x}{15108769}-\frac{188008}{15108769}}{\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}+\frac{1042}{28561\sqrt{3\left(\frac{1}{2}+x\right)^2+\frac{5}{4}-4x}}-\frac{2084\sqrt{13}\,\operatorname{arctan}}{}$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/45326307*(20304864*x^5+20074356*x^4+19381992*x^3+21890266*x^2+10777477*x
+847141)/(3*x^2-x+2)^(3/2)/(1+2*x)^2-2084/371293*13^(1/2)*arctanh(2/13*(9/
2-4*x)*13^(1/2)/(12*(1/2+x)^2+5-16*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.16

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2\left(826827\sqrt{13}(36x^6+12x^5+37x^4+30x^3+13x^2+12x+4)\log\left(-\right.\right.}{}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="fricas")
```

output
```
2/589241991*(826827*sqrt(13)*(36*x^6 + 12*x^5 + 37*x^4 + 30*x^3 + 13*x^2 +
 12*x + 4)*log(-(4*sqrt(13)*sqrt(3*x^2 - x + 2)*(8*x - 9) + 220*x^2 - 196*
x + 185)/(4*x^2 + 4*x + 1)) + 13*(20304864*x^5 + 20074356*x^4 + 19381992*x
^3 + 21890266*x^2 + 10777477*x + 847141)*sqrt(3*x^2 - x + 2))/(36*x^6 + 12
*x^5 + 37*x^4 + 30*x^3 + 13*x^2 + 12*x + 4)
```

Sympy [F]

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \int \frac{4x^2+3x+1}{(2x+1)^3(3x^2-x+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((4*x**2+3*x+1)/(1+2*x)**3/(3*x**2-x+2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*x + 1)/((2*x + 1)**3*(3*x**2 - x + 2)**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.29

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2084}{371293}\sqrt{13}\,\mathrm{arsinh}\left(\frac{8\sqrt{23}x}{23\,|2\,x+1|} - \frac{9\sqrt{23}}{23\,|2\,x+1|}\right) + \frac{1128048\,x}{15108769\sqrt{3\,x^2-x+2}} + \frac{363210}{15108769\sqrt{3\,x^2-x+2}} + \frac{1772\,x}{50531\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{1}{26\left(4\,(3\,x^2-x+2)^{\frac{3}{2}}x^2+4\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)} - \frac{1}{169\left(2\,(3\,x^2-x+2)^{\frac{3}{2}}x+(3\,x^2-x+2)^{\frac{3}{2}}\right)} + \frac{10211}{303186\,(3\,x^2-x+2)^{\frac{3}{2}}}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="maxima")
```

output
```
2084/371293*sqrt(13)*arcsinh(8/23*sqrt(23)*x/abs(2*x + 1) - 9/23*sqrt(23)/
abs(2*x + 1)) + 1128048/15108769*x/sqrt(3*x^2 - x + 2) + 363210/15108769/s
qrt(3*x^2 - x + 2) + 1772/50531*x/(3*x^2 - x + 2)^(3/2) - 1/26/(4*(3*x^2 -
 x + 2)^(3/2)*x^2 + 4*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) - 1
/169/(2*(3*x^2 - x + 2)^(3/2)*x + (3*x^2 - x + 2)^(3/2)) + 10211/303186/(3
*x^2 - x + 2)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(109) = 218.

Time = 0.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.73

$$\int \frac{1+3x+4x^2}{(1+2x)^3(2-x+3x^2)^{5/2}}\,dx = \frac{2084}{371293}\sqrt{13}\log\left(-\frac{\left|-4\sqrt{3}x-2\sqrt{13}-2\sqrt{3}+4\sqrt{3\,x^2-x+2}\right|}{2\left(2\sqrt{3}x-\sqrt{13}+\sqrt{3}-2\sqrt{3\,x^2-x+2}\right)}\right) + \frac{2\,(3\,(6\,(310578\,x-26213)x+1455755)x+1634293)}{45326307\,(3\,x^2-x+2)^{\frac{3}{2}}} - \frac{8\left(66\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^3+21\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2-1015\sqrt{3}x+431\sqrt{3}+1015\sqrt{3\,x^2-\right.}{28561\left(2\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)^2+2\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2-x+2}\right)-5\right)^2}$$

input
```
integrate((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x, algorithm="giac")
```

output
```
2084/371293*sqrt(13)*log(-1/2*abs(-4*sqrt(3)*x - 2*sqrt(13) - 2*sqrt(3) +
4*sqrt(3*x^2 - x + 2))/(2*sqrt(3)*x - sqrt(13) + sqrt(3) - 2*sqrt(3*x^2 -
x + 2))) + 2/45326307*(3*(6*(310578*x - 26213)*x + 1455755)*x + 1634293)/(
3*x^2 - x + 2)^(3/2) - 8/28561*(66*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^3 + 2
1*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 - 1015*sqrt(3)*x + 431*sqrt(
3) + 1015*sqrt(3*x^2 - x + 2))/(2*(sqrt(3)*x - sqrt(3*x^2 - x + 2))^2 + 2*
sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - x + 2)) - 5)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \int \frac{4\,x^2+3\,x+1}{\left(2\,x+1\right)^3\left(3\,x^2-x+2\right)^{5/2}}\,dx$$

input
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)),x)
```

output
```
int((3*x + 4*x^2 + 1)/((2*x + 1)^3*(3*x^2 - x + 2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 403, normalized size of antiderivative = 2.99

$$\int \frac{1+3x+4x^2}{(1+2x)^3\left(2-x+3x^2\right)^{5/2}}\,dx = \frac{527926464\sqrt{3x^2-x+2}\,x^5+521933256\sqrt{3x^2-x+2}\,x^4+503931792}{}$$

input
```
int((4*x^2+3*x+1)/(1+2*x)^3/(3*x^2-x+2)^(5/2),x)
```

output

```
(2*(263963232*sqrt(3*x**2 - x + 2)*x**5 + 260966628*sqrt(3*x**2 - x + 2)*x
**4 + 251965896*sqrt(3*x**2 - x + 2)*x**3 + 284573458*sqrt(3*x**2 - x + 2)
*x**2 + 140107201*sqrt(3*x**2 - x + 2)*x + 11012833*sqrt(3*x**2 - x + 2) +
 59531544*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**6 + 1
9843848*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**5 + 611
85198*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**4 + 49609
620*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**3 + 2149750
2*sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x**2 + 19843848*
sqrt(13)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9)*x + 6614616*sqrt(1
3)*log(2*sqrt(3*x**2 - x + 2)*sqrt(13) + 8*x - 9) - 59531544*sqrt(13)*log(
2*x + 1)*x**6 - 19843848*sqrt(13)*log(2*x + 1)*x**5 - 61185198*sqrt(13)*lo
g(2*x + 1)*x**4 - 49609620*sqrt(13)*log(2*x + 1)*x**3 - 21497502*sqrt(13)*
log(2*x + 1)*x**2 - 19843848*sqrt(13)*log(2*x + 1)*x - 6614616*sqrt(13)*lo
g(2*x + 1)))/(589241991*(36*x**6 + 12*x**5 + 37*x**4 + 30*x**3 + 13*x**2 +
 12*x + 4))
```

3.95 $\int \sqrt{d+ex}\sqrt{a+bx+cx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 880

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{2\left(7c(2cd-be)(bCd-3Ace+aCe)-\frac{(2cCd-3Bce+2bCe)(8c^2d^2-4b^2e^2-ce(bd-10ae))}{e}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{315c^3e^2}$$
$$-\frac{2(d+ex)^{3/2}\left(7c(bCd-3Ace+aCe)-\frac{(4cd-be)(2cCd-3Bce+2bCe)}{e}+5c(2cCd-3Bce+2bCe)x\right)\sqrt{a+b}}{105c^2e^2}$$
$$+\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(16b^4Ce^4-8b^2ce^3(bCd+3bBe+9aCe)+2c^4(8Cd^4-3d^2e(4Bd-7Ae))+3c^2e^2(14a^2}{}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(8b^3Ce^3-3c^2e^2(bBd+2aCd-7Abe-10aBe)+3bce^2(bCd-4bB\right.}{3}$$

output
```
2/315*(7*c*(-b*e+2*c*d)*(-3*A*c*e+C*a*e+C*b*d)-(-3*B*c*e+2*C*b*e+2*C*c*d)*
(8*c^2*d^2-4*b^2*e^2-c*e*(-10*a*e+b*d))/e)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/
2)/c^3/e^2-2/105*(e*x+d)^(3/2)*(7*c*(-3*A*c*e+C*a*e+C*b*d)-(-b*e+4*c*d)*(-
3*B*c*e+2*C*b*e+2*C*c*d)/e+5*c*(-3*B*c*e+2*C*b*e+2*C*c*d)*x)*(c*x^2+b*x+a)
^(1/2)/c^2/e^2+2/9*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(3/2)/c/e-1/315*2^(1/2)*(
-4*a*c+b^2)^(1/2)*(16*b^4*C*e^4-8*b^2*c*e^3*(3*B*b*e+9*C*a*e+C*b*d)+2*c^4*
(8*C*d^4-3*d^2*e*(-7*A*e+4*B*d))+3*c^2*e^2*(14*a^2*C*e^2+a*b*e*(29*B*e+10*
C*d)-b^2*(-14*A*e^2-5*B*d*e+2*C*d^2))+c^3*e*(6*a*e*(-21*A*e^2-8*B*d*e+3*C*
d^2)-b*d*(42*A*e^2-15*B*d*e+8*C*d^2)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/
e^4/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)
-2/315*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(8*b^3*C*e^3-3*c^2*e
^2*(-7*A*b*e-10*B*a*e+B*b*d+2*C*a*d)+3*b*c*e^2*(-4*B*b*e-9*C*a*e+C*b*d)-2*
c^3*(8*C*d^3-3*d*e*(-7*A*e+4*B*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x
+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(
-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.72 (sec) , antiderivative size = 15669, normalized size of antiderivative = 17.81

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.72 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2184, 27, 1236, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{2\int -\frac{3}{2}e\sqrt{d+ex}(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cx^2+bx+a}dx}{9ce^2}+\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\int \sqrt{d+ex}(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)\sqrt{cx^2+bx+a}dx}{3ce}$$

$$\Big\downarrow \text{ 1236}$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\frac{2\int -\frac{\left(6Cdeb^2+2aCe^2b-cd(Cd+9Be)b+ce(21Acd-5aCd-3aBe)+\left(-\left(\left(2Cd^2-3e(Bd+7Ae)\right)c^2\right)-e(bCd+12bBe+7aCe)c+8b^2Ce^2\right)x\right)\sqrt{cx^2+bx+a}}{2\sqrt{d+ex}}dx}{7c}+2\sqrt{\cdots}}{3ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-\frac{\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{\int \frac{\left(6Cdeb^2+2aCe^2b-cd(Cd+9Be)b+ce(21Acd-5aCd-3aBe)+\left(-\left(\left(2Cd^2-3e(Bd+7Ae)\right)c^2\right)-e(bCd+1\cdots\right.\right.}{\sqrt{d+ex}}}{7c}}{3ce}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-$$

$$\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\,(3cex(-ce(7aCe+12bBe+bCd)-(c^2(2Cd^2-3e(7Ae+Bd)))+8b^2Ce^2)-3c^2e(-ae\ldots}{15ce\ldots}$$

$\downarrow$ 27

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-$$

$$\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\,(3cex(-ce(7aCe+12bBe+bCd)-(c^2(2Cd^2-3e(7Ae+Bd)))+8b^2Ce^2)-3c^2e(-ae\ldots}{15ce\ldots}$$

$\downarrow$ 1269

$$\frac{2C(d+ex)^{3/2}\left(a+bx+cx^2\right)^{3/2}}{9ce}-$$

$$\frac{2\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}(2bCe-3Bce+2cCd)}{7c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\,(3cex(-ce(7aCe+12bBe+bCd)-(c^2(2Cd^2-3e(7Ae+Bd)))+8b^2Ce^2)-3c^2e(-ae\ldots}{15ce\ldots}$$

$\downarrow$ 1172

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-$$

$$\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c}-\frac{2\sqrt{d+ex}(((8Cd^3-3de(4Bd-7Ae))c^3-3e(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be))c^2-3be^2(bCd+4bBe-aC\ldots}{15ce^2\ldots}$$

$\downarrow$ 321

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce}-$$

$$\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c}-\frac{2\sqrt{d+ex}(((8Cd^3-3de(4Bd-7Ae))c^3-3e(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be))c^2-3be^2(bCd+4bBe-aC\ldots}{15ce^2\ldots}$$

$\downarrow$ 327

$$\frac{2C(d+ex)^{3/2}\left(cx^2+bx+a\right)^{3/2}}{9ce} -$$

$$\frac{\frac{2(2cCd-3Bce+2bCe)\sqrt{d+ex}\left(cx^2+bx+a\right)^{3/2}}{7c} - \frac{2\sqrt{d+ex}\left(\left(8Cd^3-3de(4Bd-7Ae)\right)c^3-3e\left(bCd^2-be(2Bd+7Ae)-ae(Cd-5Be)\right)c^2-3be^2(bCd+4bBe-aC\ldots}{15ce^2}}{}$$

input
```
Int[Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2),x]
```

output
```
(2*C*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(3/2))/(9*c*e) - ((2*(2*c*C*d - 3*B
*c*e + 2*b*C*e)*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7*c) - ((2*Sqrt[d
+ e*x]*(8*b^3*C*e^3 - 3*b*c*e^2*(b*C*d + 4*b*B*e - a*C*e) + c^3*(8*C*d^3 -
 3*d*e*(4*B*d - 7*A*e)) - 3*c^2*e*(b*C*d^2 - b*e*(2*B*d + 7*A*e) - a*e*(C*
d - 5*B*e)) + 3*c*e*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e + 7*a*C*e) - c^2*
(2*C*d^2 - 3*e*(B*d + 7*A*e)))*x)*Sqrt[a + b*x + c*x^2])/(15*c*e^2) - ((Sq
rt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(6*b^2*C*d*e + 2*a*b*C*e^2 -
b*c*d*(C*d + 9*B*e) + c*e*(21*A*c*d - 5*a*C*d - 3*a*B*e)) - 2*(4*c^2*d^2 -
 b^2*e^2 - (3*c*e*(b*d - 2*a*e))/2)*(8*b^2*C*e^2 - c*e*(b*C*d + 12*b*B*e +
 7*a*C*e) - c^2*(2*C*d^2 - 3*e*(B*d + 7*A*e))))*Sqrt[d + e*x]*Sqrt[-((c*(a
 + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a
*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c
*d^2 - b*d*e + a*e^2)*(8*b^3*C*e^3 - 3*c^2*e^2*(b*B*d + 2*a*C*d - 7*A*b*e
- 10*a*B*e) + 3*b*c*e^2*(b*C*d - 4*b*B*e - 9*a*C*e) - 2*c^3*(8*C*d^3 - 3*d
*e*(4*B*d - 7*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*
a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1735 vs. $2(814) = 1628$.

Time = 5.55 (sec) , antiderivative size = 1736, normalized size of antiderivative = 1.97

method	result	size
elliptic	Expression too large to display	1736
risch	Expression too large to display	7113
default	Expression too large to display	19955

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/9*C*x^3
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/7*(B*c*e+C*b*e+C*c*d-2/
9*C*(4*b*e+4*c*d))/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)
+2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b
*e+C*c*d-2/9*C*(4*b*e+4*c*d))/c/e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*
d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B
*c*e+C*b*e+C*c*d-2/9*C*(4*b*e+4*c*d))/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b
*e+B*c*d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*C*
(4*b*e+4*c*d))/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+
c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A*a*d-2/5*(A*c*e+B*b*e+B*c*d+C*a*e+C*b*d
-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*C*(4*b*e+4*c*d))/c/e*(
3*b*e+3*c*d))/c/e*a*d-2/3*(A*b*e+A*c*d+B*a*e+B*b*d+1/3*C*a*d-2/7*(B*c*e+C*
b*e+C*c*d-2/9*C*(4*b*e+4*c*d))/c/e*(5/2*a*e+5/2*b*d)-2/5*(A*c*e+B*b*e+B*c*
d+C*a*e+C*b*d-2/9*C*(7/2*a*e+7/2*b*d)-2/7*(B*c*e+C*b*e+C*c*d-2/9*C*(4*b*e+
4*c*d))/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.16

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fr
icas")
```

output
```
2/945*((16*C*c^5*d^5 - 8*(2*C*b*c^4 + 3*B*c^5)*d^4*e - (5*C*b^2*c^3 - 42*A
*c^5 - 3*(10*C*a + 9*B*b)*c^4)*d^3*e^2 - (5*C*b^3*c^2 + 3*(22*B*a + 21*A*b
)*c^4 - 3*(7*C*a*b + 4*B*b^2)*c^3)*d^2*e^3 - (16*C*b^4*c - 378*A*a*c^4 + 3
*(22*C*a^2 + 41*B*a*b + 21*A*b^2)*c^3 - 3*(28*C*a*b^2 + 9*B*b^3)*c^2)*d*e^
4 + (16*C*b^5 - 9*(10*B*a^2 + 21*A*a*b)*c^3 + 3*(41*C*a^2*b + 41*B*a*b^2 +
 14*A*b^3)*c^2 - 24*(4*C*a*b^3 + B*b^4)*c)*e^5)*sqrt(c*e)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^
3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^
3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^5*d^4*e - 8*(C*b*c^4
+ 3*B*c^5)*d^3*e^2 - 3*(2*C*b^2*c^3 - 14*A*c^5 - (6*C*a + 5*B*b)*c^4)*d^2*
e^3 - (8*C*b^3*c^2 + 6*(8*B*a + 7*A*b)*c^4 - 15*(2*C*a*b + B*b^2)*c^3)*d*e
^4 + (16*C*b^4*c - 126*A*a*c^4 + 3*(14*C*a^2 + 29*B*a*b + 14*A*b^2)*c^3 -
24*(3*C*a*b^2 + B*b^3)*c^2)*e^5)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3
)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(35*C*c^5*e^5*x^3 + 8*C
*c^5*d^3*e^2 - 3*(C*b*c^4 + 4*B*c^5)*d^2*e^3 - (3*C*b^2*c^3 - 21*A*c^5 - 2
*(4*C*a + 3*B*b)*c^4)*d*e^4 + (8*C*b^3*c^2 + 3*(10*B*a + 7*A*b)*c^4 - 3...
```

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{d+ex}\left(A+Bx+Cx^2\right)\sqrt{a+bx+cx^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Integral(sqrt(d + e*x)*(A + B*x + C*x**2)*sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int (Cx^2+Bx+A)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Giac [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int (Cx^2+Bx+A)\sqrt{cx^2+bx+a}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{d+e\,x}\,\left(C\,x^2+B\,x+A\right)\,\sqrt{c\,x^2+b\,x+a}\,dx$$

input
```
int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)*(A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)\,dx = \int \sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}\,\left(C\,x^2+Bx+A\right)dx$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int((e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A),x)
```

3.96 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 654

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx =$$

$$-\frac{2\sqrt{d+ex}\left(5bcCd-\frac{24c^2Cd^2}{e}-35Ac^2e+4b^2Ce+5acCe+7Bc(4cd-be)+3c(6cCd-7Bce+4bCe)\right.}{105c^2e^2}$$

$$+\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(5c(2cd-be)(3bCd-7Ace+aCe)-\frac{(6cCd-7Bce+4bCe)(8c^2d^2-2b^2e^2-3ce(bd-2ae))}{e}\right)\sqrt{d+ex}\sqrt{}}{105c^3e^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(4b^2Ce^2+ce(8bCd-7bBe-10aCe)+c^2(48Cd^2-14e(4Bd-5Ae\right.}{105c^3e^4\sqrt{d+ex}\sqrt{a+}}$$

output
```
-2/105*(e*x+d)^(1/2)*(5*b*c*C*d-24*c^2*C*d^2/e-35*A*c^2*e+4*b^2*C*e+5*a*c*
C*e+7*B*c*(-b*e+4*c*d)+3*c*(-7*B*c*e+4*C*b*e+6*C*c*d)*x)*(c*x^2+b*x+a)^(1/
2)/c^2/e^2+2/7*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(3/2)/c/e+1/105*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(5*c*(-b*e+2*c*d)*(-7*A*c*e+C*a*e+3*C*b*d)-(-7*B*c*e+4*C*b*e
+6*C*c*d)*(8*c^2*d^2-2*b^2*e^2-3*c*e*(-2*a*e+b*d))/e)*(e*x+d)^(1/2)*(-c*(c
*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*
e))^(1/2))/c^3/e^3/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x
^2+b*x+a)^(1/2)+2/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(4*b^
2*C*e^2+c*e*(-7*B*b*e-10*C*a*e+8*C*b*d)+c^2*(48*C*d^2-14*e*(-5*A*e+4*B*d))
)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/
e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.16 (sec) , antiderivative size = 9965, normalized size of antiderivative = 15.24

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.09 (sec) , antiderivative size = 685, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2184, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 2184$$

$$\frac{2\int -\frac{e(3bCd-7Ace+aCe+(6cCd-7Bce+4bCe)x)\sqrt{cx^2+bx+a}}{2\sqrt{d+ex}}dx}{7ce^2}+\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-\frac{\int \frac{(3bCd-7Ace+aCe+(6cCd-7Bce+4bCe)x)\sqrt{cx^2+bx+a}}{\sqrt{d+ex}}dx}{7ce}$$

$$\Big\downarrow \; 1231$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{2\int \frac{5ce(bd-2ae)(3bCd-7Ace+aCe)-}{}}{}}{7ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\int \frac{5ce(bd-2ae)(3bCd-7Ace+aCe)-(}{}}{}}{7ce}$$

$$\Big\downarrow \; 1269$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\left(ae^2-bde+cd^2\right)\left(ce(-10aCe-7bBe+\right.}{}$$

$$\Big\downarrow \; 1172$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2\right.}{}}{}$$

$$\Big\downarrow \; 321$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{}}{}$$

$$\Big\downarrow \; 327$$

$$\frac{2C\sqrt{d+ex}\left(a+bx+cx^2\right)^{3/2}}{7ce}-$$

$$\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(5ce(aCe-7Ace+3bCd)+3cex(4bCe-7Bce+6cCd)-(4cd-be)(4bCe-7Bce+6cCd))}{15ce^2}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2\right.}{}}{}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/Sqrt[d + e*x],x]
```

output
```
(2*C*Sqrt[d + e*x]*(a + b*x + c*x^2)^(3/2))/(7*c*e) - ((2*Sqrt[d + e*x]*(5
*c*e*(3*b*C*d - 7*A*c*e + a*C*e) - (4*c*d - b*e)*(6*c*C*d - 7*B*c*e + 4*b*
C*e) + 3*c*e*(6*c*C*d - 7*B*c*e + 4*b*C*e)*x)*Sqrt[a + b*x + c*x^2])/(15*c
*e^2) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(5*c*e*(2*c*d - b*e)*(3*b*C*d - 7*A*c*
e + a*C*e) - (6*c*C*d - 7*B*c*e + 4*b*C*e)*(8*c^2*d^2 - 2*b^2*e^2 - 3*c*e*
(b*d - 2*a*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]
*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/
Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(
c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x +
 c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b^2*C*e
^2 + c*e*(8*b*C*d - 7*b*B*e - 10*a*C*e) + c^2*(48*C*d^2 - 14*e*(4*B*d - 5*
A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))
/(15*c*e^2))/(7*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1201 vs. $2(594) = 1188$.

Time = 3.56 (sec) , antiderivative size = 1202, normalized size of antiderivative = 1.84

method	result	size
elliptic	Expression too large to display	1202
risch	Expression too large to display	4505
default	Expression too large to display	12761

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/e*x
^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*c+b*C-2/7*C/e*(3
*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A*
c+B*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/
e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A*
a-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*a*d-2/3*(A*c+B*b+C*a-2/7*C/e*(5/
2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*
(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*
x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))+2*(A*b+B*a-4/7*C/e*a*d-2/5*(B*c+b*C-2/7*C/e*(3*b*e+3*c*d))/c/e*(3/
2*a*e+3/2*b*d)-2/3*(A*c+B*b+C*a-2/7*C/e*(5/2*a*e+5/2*b*d)-2/5*(B*c+b*C-2/7
*C/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+
b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/
(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 764, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

output
```
2/315*((48*C*c^4*d^4 - 8*(5*C*b*c^3 + 7*B*c^4)*d^3*e - (10*C*b^2*c^2 - 70*
A*c^4 - (62*C*a + 49*B*b)*c^3)*d^2*e^2 - (5*C*b^3*c + 14*(6*B*a + 5*A*b)*c
^3 - 2*(11*C*a*b + 7*B*b^2)*c^2)*d*e^3 - (8*C*b^4 - 210*A*a*c^3 + (30*C*a^
2 + 63*B*a*b + 35*A*b^2)*c^2 - (41*C*a*b^2 + 14*B*b^3)*c)*e^4)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^4*d^3*
e - 8*(2*C*b*c^3 + 7*B*c^4)*d^2*e^2 - (9*C*b^2*c^2 - 70*A*c^4 - (26*C*a +
21*B*b)*c^3)*d*e^3 - (8*C*b^3*c + 7*(6*B*a + 5*A*b)*c^3 - (29*C*a*b + 14*B
*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 -
3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c
^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2
*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*
d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*
(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 24*C*c^4*d^2*e^2 - (
5*C*b*c^3 + 28*B*c^4)*d*e^3 - (4*C*b^2*c^2 - 35*A*c^4 - (10*C*a + 7*B*b)*c
^3)*e^4 - 3*(6*C*c^4*d*e^3 - (C*b*c^3 + 7*B*c^4)*e^4)*x)*sqrt(c*x^2 + b*x
+ a)*sqrt(e*x + d))/(c^4*e^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{\sqrt{d+e\,x}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{\sqrt{d+ex}}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}\,(C\,x^2+Bx+A)}{\sqrt{ex+d}}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(1/2),x)
```

3.97 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 726

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx =$$

$$-\frac{2\sqrt{d+ex}\left(bCe^2(bd-ae)+c^2(24Cd^3-5de(4Bd-3Ae))+ce(ae(9Cd-5Be)-5b(5Cd^2-4Bde+3A\dots\right.}{15ce^3(cd^2-bde+ae^2)}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2b^2Ce^2+ce(8bCd-5bBe-6aCe)-c^2(48Cd^2-10e(4Bd-3Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+\dots}{b^2-4a\dots}}}{15c^2e^4\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(bCe^2(bd-ae)-2c^2d(24Cd^2-5e(4Bd-3Ae))+ce(32bCd^2-5be(5Bd-3Ae)-2ae(9\dots}{15c^2e^4\sqrt{d+ex}\sqrt{\dots}}$$

output
```
-2/15*(e*x+d)^(1/2)*(b*C*e^2*(-a*e+b*d)+c^2*(24*C*d^3-5*d*e*(-3*A*e+4*B*d)
)+c*e*(a*e*(-5*B*e+9*C*d)-5*b*(3*A*e^2-4*B*d*e+5*C*d^2))+3*c*e^2*(5*B*c*d+
C*b*d-6*c*C*d^2/e-5*A*c*e-C*a*e)*x)*(c*x^2+b*x+a)^(1/2)/c/e^3/(a*e^2-b*d*e
+c*d^2)-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(
e*x+d)^(1/2)-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*b^2*C*e^2+c*e*(-5*B*b*e-6*
C*a*e+8*C*b*d)-c^2*(48*C*d^2-10*e*(-3*A*e+4*B*d)))*(e*x+d)^(1/2)*(-c*(c*x^
2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2)
)^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))
^(1/2))/c^2/e^4/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+
b*x+a)^(1/2)+2/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(b*C*e^2*(-a*e+b*d)-2*c^2*d*(
24*C*d^2-5*e*(-3*A*e+4*B*d))+c*e*(32*b*C*d^2-5*b*e*(-3*A*e+5*B*d)-2*a*e*(-
5*B*e+9*C*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x
^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2
))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)^(1/2))/c^2/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.69 (sec) , antiderivative size = 13240, normalized size of antiderivative = 18.24

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 774, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2181, 27, 1231, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\,(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \ 2181$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+5Bcd+bCd-5Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e\sqrt{d+ex}}dx}{ae^2-bde+cd^2}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd-2Ae)+e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+5Bcd+bCd-5Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{\sqrt{d+ex}}dx}{e\,(ae^2-bde+cd^2)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \ 1231$$

$$-\frac{\frac{2\int -\frac{(cd^2-bed+ae^2)(-Cdeb^2+24cCd^2b-aCe^2b-5ce(4Bd-3Ae)b-2ace(6Cd-5Be)-(-(((48Cd^2-10e(4Bd-3Ae))c^2)+e(8bCd-5bBe-6aCe)c+2b^2Ce^2)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{15ce^2}}{e}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{(ae^2-bde+cd^2)\int \frac{-Cdeb^2+24cCd^2b-aCe^2b-5ce(4Bd-3Ae)b-2ace(6Cd-5Be)-(-(((48Cd^2-10e(4Bd-3Ae))c^2)+e(8bCd-5bBe-6aCe)c+2b^2Ce^2)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{15ce^2}}{e(}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\,(ae^2-bde+cd^2)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{(ae^2-bde+cd^2)\left(\frac{\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-ae)-2c^2d\left(24Cd^2-5e(4Bd-3Ae)\right)\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(ce(-6aCe-5b\cdots\right.}{\cdots}\cdots\right)}{15ce^2}\cdots$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{(cd^2-bed+ae^2)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(-2d\left(24Cd^2-5e(4Bd-3Ae)\right)c^2+e\left(32bCd^2-5be(5Bd-3Ae)-2ae(9Cd-5Be)\right)c+bCe^2(bd-ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c}{\cdots}}\cdots}{ce\sqrt{d+ex}\sqrt{cx^2+bx+a}\cdots}\cdots\right)}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{e\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\Big\downarrow \text{321}$$

$$\frac{(cd^2-bed+ae^2)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(-2d\left(24Cd^2-5e(4Bd-3Ae)\right)c^2+e\left(32bCd^2-5be(5Bd-3Ae)-2ae(9Cd-5Be)\right)c+bCe^2(bd-ae)\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c}{\cdots}}\cdots}{ce\sqrt{d+ex}\sqrt{cx^2+bx+a}\cdots}\cdots\right)}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{e\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$$\Big\downarrow \text{327}$$

$$\left(ae^2-bde+cd^2\right)\left(\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(ce\left(-2ae(9Cd-5Be)-5be(5Bd-3Ae)+32bCd^2\right)+bCe^2(bd-ae)-2c^2d\left(24Cd^2-5e(\right.\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\right.$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(3/2),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(e*(c*d^2 - b*d*e + a
*e^2)*Sqrt[d + e*x]) + ((-2*Sqrt[d + e*x]*(b*C*e^2*(b*d - a*e) + c^2*(24*C
*d^3 - 5*d*e*(4*B*d - 3*A*e)) + c*e*(a*e*(9*C*d - 5*B*e) - 5*b*(5*C*d^2 -
4*B*d*e + 3*A*e^2)) + 3*c*e^2*(5*B*c*d + b*C*d - (6*c*C*d^2)/e - 5*A*c*e -
 a*C*e)*x)*Sqrt[a + b*x + c*x^2])/(15*c*e^2) + ((c*d^2 - b*d*e + a*e^2)*(-
((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b^2*C*e^2 + c*e*(8*b*C*d - 5*b*B*e - 6*a*C*
e) - c^2*(48*C*d^2 - 10*e*(4*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*C*
e^2*(b*d - a*e) - 2*c^2*d*(24*C*d^2 - 5*e*(4*B*d - 3*A*e)) + c*e*(32*b*C*d
^2 - 5*b*e*(5*B*d - 3*A*e) - 2*a*e*(9*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a
*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])))/(15*c*e^2))/(e*(c*d^2 - b*
d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.67

method	result	size
elliptic	Expression too large to display	1215
risch	Expression too large to display	1864
default	Expression too large to display	8221

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)*(A*e^2-B*d*e+C*d^2)/e^4/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+
2/5*C/e^2*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(1/e^2*(B*
c*e+C*b*e-C*c*d)-2/5*C/e^2*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e
*x+b*d*x+a*d)^(1/2)+2*((A*b*e^3-A*c*d*e^2+B*a*e^3-B*b*d*e^2+B*c*d^2*e-C*a*
d*e^2+C*b*d^2*e-C*c*d^3)/e^4-(A*e^2-B*d*e+C*d^2)/e^4*(b*e-c*d)+b/e^3*(A*e^
2-B*d*e+C*d^2)-2/5*C/e^2*a*d-2/3*(1/e^2*(B*c*e+C*b*e-C*c*d)-2/5*C/e^2*(2*b
*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d
*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+2*(1/e^3*(A*c*e^2+B*b*e^2-B*c*d*e+C*a*e^2-C*b*d*e+C
*c*d^2)+(A*e^2-B*d*e+C*d^2)/e^3*c-2/5*C/e^2*(3/2*a*e+3/2*b*d)-2/3*(1/e^2*(
B*c*e+C*b*e-C*c*d)-2/5*C/e^2*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4
*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x, algorithm="fr
icas")
```

output
```
-2/45*((48*C*c^3*d^4 - 8*(4*C*b*c^2 + 5*B*c^3)*d^3*e - (7*C*b^2*c - 30*A*c
^3 - (42*C*a + 25*B*b)*c^2)*d^2*e^2 - (2*C*b^3 + 15*(2*B*a + A*b)*c^2 - (9
*C*a*b + 5*B*b^2)*c)*d*e^3 + (48*C*c^3*d^3*e - 8*(4*C*b*c^2 + 5*B*c^3)*d^2
*e^2 - (7*C*b^2*c - 30*A*c^3 - (42*C*a + 25*B*b)*c^2)*d*e^3 - (2*C*b^3 + 1
5*(2*B*a + A*b)*c^2 - (9*C*a*b + 5*B*b^2)*c)*e^4)*x)*sqrt(c*e)*weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3
)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^3*d^3*e - 8*(C*b
*c^2 + 5*B*c^3)*d^2*e^2 - (2*C*b^2*c - 30*A*c^3 - (6*C*a + 5*B*b)*c^2)*d*e
^3 + (48*C*c^3*d^2*e^2 - 8*(C*b*c^2 + 5*B*c^3)*d*e^3 - (2*C*b^2*c - 30*A*c
^3 - (6*C*a + 5*B*b)*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
 b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e
- 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstras
sPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*
c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^
3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(3*C*c^3*e^4*x^2 - 24*
C*c^3*d^2*e^2 - 15*A*c^3*e^4 + (C*b*c^2 + 20*B*c^3)*d*e^3 - (6*C*c^3*d*e^3
 - (C*b*c^2 + 5*B*c^3)*e^4)*x)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*e
^6*x + c^3*d*e^5)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{{(d+e\,x)}^{3/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2),x)
```

3.98 $\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 691

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{8cCd^3}{e}-cd(4Bd-Ae)-(bd-ae)(7Cd-3Be)-e\left(Bcd+bC\cdots\right.\right.}{3e^2\left(cd^2-bde+ae^2\right)\sqrt{d\cdots}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))\left(a+bx+cx^2\right)^{3/2}}{3e\left(cd^2-bde+ae^2\right)(d+ex)^{3/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}\left(2\left(4cd-\frac{be}{2}\right)\left(Bcd+bCd-\frac{2cCd^2}{e}-Ace-aCe\right)+6c(bd(Cd-Be)+e(Acd-aCd+aB\cdots\right.}{3ce^3\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{\cdots}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(e(8bCd-3bBe-2aCe)-2c(8Cd^2-e(4Bd-Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{I}\cdots}{3ce^4\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*(8*c*C*d^3/e-c*d*(-A*e+4*B*d)-(-a*e+b*d)*(-3*B*e+7*C*d)-e*(B*c*d+C*b*d
-2*c*C*d^2/e-A*c*e-C*a*e)*x)*(c*x^2+b*x+a)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)/(
e*x+d)^(1/2)-2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c
*d^2)/(e*x+d)^(3/2)+1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*(4*c*d-1/2*b*e)*(B*c
*d+C*b*d-2*c*C*d^2/e-A*c*e-C*a*e)+6*c*(b*d*(-B*e+C*d)+e*(A*c*d+B*a*e-C*a*d
)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(
2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e^3/(a*e^2-b*d*e+c*d^2)/(c*(e*x+d)/(2
*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/3*2^(1/2)*(-4*
a*c+b^2)^(1/2)*(e*(-3*B*b*e-2*C*a*e+8*C*b*d)-2*c*(8*C*d^2-e*(-A*e+4*B*d)))
*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e^4
/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.96 (sec) , antiderivative size = 8456, normalized size of antiderivative = 12.24

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2181, 27, 1230, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{2\int -\frac{3\left(bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{3/2}}dx}{3\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{3/2}}dx}{e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{-\frac{2\int \frac{2(2bd-ae)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-3be(bd(Cd-Be)+e(Acd-aCd+aBe))+\left((8cd-be)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-6ce(bd(Cd-Be)+e(A\cdots}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3e^2}\cdots}{e\left(ae^2-bde+c\cdots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{2(2bd-ae)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-3be(bd(Cd-Be)+e(Acd-aCd+aBe))+\left((8cd-be)\left(2cCd^2-Ce(bd-ae)-ce(Bd-Ae)\right)-6ce(bd(Cd-Be)+e(Ac\cdots}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3e^2}\cdots}{e\left(ae^2-bde+c\cdots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{-\frac{\frac{\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{\left(ae^2-bde+cd^2\right)\left(e(-2aCe-3bBe+8bCd)-2\cdots\right.}{\cdots}}{3e^2}}{e\left(ae^2\cdots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1172}$$

$$-\frac{-\frac{2\sqrt{cx^2+bx+a}\left(\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)xe^2+(bd-ae)(7Cd-3Be)e-c\left(8Cd^3-de(4Bd-Ae)\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\left(e(8bC\right.}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{3e\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$\downarrow$ 321

$$-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4a}}{2\sqrt{b^2-4ac}}}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 327

$$-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce\left(-2ae(7Cd-3Be)-be(7Bd-Ae)+16bCd^2\right)+bCe^2(bd-ae)+2c^2\left(8Cd^3-de(4Bd-Ae)\right)\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2}}{\sqrt{b^2-4a}}}}{\sqrt{2}}\right.\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input

```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(5/2),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(3*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(3/2)) + ((-2*(e*(b*d - a*e)*(7*C*d - 3*B*e) - c*(8*C*d^
3 - d*e*(4*B*d - A*e)) + e^2*(B*c*d + b*C*d - (2*c*C*d^2)/e - A*c*e - a*C*
e)*x)*Sqrt[a + b*x + c*x^2])/(3*e^2*Sqrt[d + e*x]) - ((Sqrt[2]*Sqrt[b^2 -
4*a*c]*(b*C*e^2*(b*d - a*e) + 2*c^2*(8*C*d^3 - d*e*(4*B*d - A*e)) - c*e*(1
6*b*C*d^2 - b*e*(7*B*d - A*e) - 2*a*e*(7*C*d - 3*B*e)))*Sqrt[d + e*x]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)
/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(c*d^2 - b*d*e + a*e^2)*(e*(8*b*C*d - 3*b*B*e - 2*a*C*e) - 2*c*(8*C*
d^2 - e*(4*B*d - A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a
 + b*x + c*x^2]))/(3*e^2))/(e*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. 2(629) = 1258.

Time = 6.16 (sec) , antiderivative size = 1340, normalized size of antiderivative = 1.94

method	result	size
elliptic	Expression too large to display	1340
risch	Expression too large to display	2759
default	Expression too large to display	21038

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3*(A*e
^2-B*d*e+C*d^2)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^2-2/3*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3
*B*a*e^3-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/((x+d/
e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2/3*C/e^3*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)+2*((A*c*e^2+B*b*e^2-2*B*c*d*e+C*a*e^2-2*C*b*d*e+3*C*c*d^2)/e
^4-1/3*(A*e^2-B*d*e+C*d^2)/e^4*c-1/3/e^4*(b*e-c*d)*(A*b*e^3-2*A*c*d*e^2+3*
B*a*e^3-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-
b*d*e+c*d^2)+1/3*b/e^3/(a*e^2-b*d*e+c*d^2)*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3-
4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)-2/3*C/e^3*(1/2*
a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d
)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
))+2*(1/e^3*(B*c*e+C*b*e-2*C*c*d)+1/3/e^3*c*(A*b*e^3-2*A*c*d*e^2+3*B*a*e^3
-4*B*b*d*e^2+5*B*c*d^2*e-6*C*a*d*e^2+7*C*b*d^2*e-8*C*c*d^3)/(a*e^2-b*d*e+c
*d^2)-2/3*C/e^3*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(639) = 1278$.

Time = 0.13 (sec) , antiderivative size = 1385, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(5/2),x, algorithm="fr
icas")
```

output
```
2/9*((16*C*c^3*d^6 - 8*(3*C*b*c^2 + B*c^3)*d^5*e + (6*C*b^2*c + 2*A*c^3 +
(26*C*a + 11*B*b)*c^2)*d^4*e^2 + (C*b^3 - 2*(6*B*a + A*b)*c^2 - 2*(7*C*a*b
 + B*b^2)*c)*d^3*e^3 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a*b - A*b^2)*
c)*d^2*e^4 + (16*C*c^3*d^4*e^2 - 8*(3*C*b*c^2 + B*c^3)*d^3*e^3 + (6*C*b^2*
c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^2*e^4 + (C*b^3 - 2*(6*B*a + A*b)*c^
2 - 2*(7*C*a*b + B*b^2)*c)*d*e^5 - (C*a*b^2 - 6*A*a*c^2 - (6*C*a^2 + 3*B*a
*b - A*b^2)*c)*e^6)*x^2 + 2*(16*C*c^3*d^5*e - 8*(3*C*b*c^2 + B*c^3)*d^4*e^
2 + (6*C*b^2*c + 2*A*c^3 + (26*C*a + 11*B*b)*c^2)*d^3*e^3 + (C*b^3 - 2*(6*
B*a + A*b)*c^2 - 2*(7*C*a*b + B*b^2)*c)*d^2*e^4 - (C*a*b^2 - 6*A*a*c^2 - (
6*C*a^2 + 3*B*a*b - A*b^2)*c)*d*e^5)*x)*sqrt(c*e)*weierstrassPInverse(4/3*
(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*
c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*C*c^3*d^5*e - 8*(2*C*b*c^2 + B*c^
3)*d^4*e^2 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^3*e^3 - (C*a*b*c
+ (6*B*a + A*b)*c^2)*d^2*e^4 + (16*C*c^3*d^3*e^3 - 8*(2*C*b*c^2 + B*c^3)*d
^2*e^4 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d*e^5 - (C*a*b*c + (6*B
*a + A*b)*c^2)*e^6)*x^2 + 2*(16*C*c^3*d^4*e^2 - 8*(2*C*b*c^2 + B*c^3)*d^3*
e^3 + (C*b^2*c + 2*A*c^3 + 7*(2*C*a + B*b)*c^2)*d^2*e^4 - (C*a*b*c + (6*B*
a + A*b)*c^2)*d*e^5)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
 (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(5/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(5/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(5/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(5/2),x)
```

output

```
( - 10*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*a*b*e**2 + 10*sqrt(d + e*x)*sq
rt(a + b*x + c*x**2)*a*c*d*e + 12*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*b**
2*d*e + 8*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*b**2*e**2*x - 18*sqrt(d + e
*x)*sqrt(a + b*x + c*x**2)*b*c*d**2 - 20*sqrt(d + e*x)*sqrt(a + b*x + c*x*
*2)*b*c*d*e*x + 2*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*b*c*e**2*x**2 + 12*
sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*c**2*d**2*x - 2*sqrt(d + e*x)*sqrt(a
+ b*x + c*x**2)*c**2*d*e*x**2 + 3*int((sqrt(d + e*x)*sqrt(a + b*x + c*x**2
)*x**2)/(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e**4*x**
3 - a*c*d**4 - 3*a*c*d**3*e*x - 3*a*c*d**2*e**2*x**2 - a*c*d*e**3*x**3 + b
**2*d**3*e*x + 3*b**2*d**2*e**2*x**2 + 3*b**2*d*e**3*x**3 + b**2*e**4*x**4
 - b*c*d**4*x - 2*b*c*d**3*e*x**2 + 2*b*c*d*e**3*x**4 + b*c*e**4*x**5 - c*
*2*d**4*x**2 - 3*c**2*d**3*e*x**3 - 3*c**2*d**2*e**2*x**4 - c**2*d*e**3*x*
*5),x)*b**4*d**2*e**4 + 6*int((sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*x**2)/
(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e**4*x**3 - a*c*
d**4 - 3*a*c*d**3*e*x - 3*a*c*d**2*e**2*x**2 - a*c*d*e**3*x**3 + b**2*d**3
*e*x + 3*b**2*d**2*e**2*x**2 + 3*b**2*d*e**3*x**3 + b**2*e**4*x**4 - b*c*d
**4*x - 2*b*c*d**3*e*x**2 + 2*b*c*d*e**3*x**4 + b*c*e**4*x**5 - c**2*d**4*
x**2 - 3*c**2*d**3*e*x**3 - 3*c**2*d**2*e**2*x**4 - c**2*d*e**3*x**5),x)*b
**4*d*e**5*x + 3*int((sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*x**2)/(a*b*d**3
*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e**4*x**3 - a*c*d**4 -...
```

3.99 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 976

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx =$$

$$-\frac{2(c^2(48Cd^4-2d^2e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3Bde-2Ae^2))-ce(b}{15e^3(cd^2-bde+ae^2)^2\sqrt{d+e}}$$

$$-\frac{2\left(23bCd^2-\frac{24cCd^3}{e}+cd(4Bd+Ae)-be(3Bd+2Ae)-5ae(5Cd-Be)+3e\left(Bcd+5bCd-\frac{6cCd^2}{e}-A\right.\right.}{15e^2(cd^2-bde+ae^2)(d+ex)^{3/2}}$$

$$-\frac{2(Cd^2-e(Bd-Ae))(a+bx+cx^2)^{3/2}}{5e(cd^2-bde+ae^2)(d+ex)^{5/2}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(c^2(48Cd^4-2d^2e(4Bd+Ae))+e^2(30a^2Ce^2-5abe(14Cd-Be)+b^2(38Cd^2-3Bde-2A}{15e^4(cd^2-}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(15bCe^2(bd-ae)+c^2(48Cd^3-2de(4Bd+Ae))-ce(64bCd^2-be(9Bd+Ae)-10ae(5}{15ce^4(cd^2-bde+ae^2)\sqrt{}}$$

output
```
-2/15*(c^2*(48*C*d^4-2*d^2*e*(A*e+4*B*d))+e^2*(30*a^2*C*e^2-5*a*b*e*(-B*e+
14*C*d)+b^2*(-2*A*e^2-3*B*d*e+38*C*d^2))-c*e*(b*d*(-2*A*e^2-13*B*d*e+88*C*
d^2)-2*a*e*(3*A*e^2-8*B*d*e+43*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d
*e+c*d^2)^2/(e*x+d)^(1/2)-2/15*(23*b*C*d^2-24*c*C*d^3/e+c*d*(A*e+4*B*d)-b*
e*(2*A*e+3*B*d)-5*a*e*(-B*e+5*C*d)+3*e*(B*c*d+5*C*b*d-6*c*C*d^2/e-A*c*e-5*
C*a*e)*x)*(c*x^2+b*x+a)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(3/2)-2/5*(C
*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(5/2)
+1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c^2*(48*C*d^4-2*d^2*e*(A*e+4*B*d))+e^2*(
30*a^2*C*e^2-5*a*b*e*(-B*e+14*C*d)+b^2*(-2*A*e^2-3*B*d*e+38*C*d^2))-c*e*(b
*d*(-2*A*e^2-13*B*d*e+88*C*d^2)-2*a*e*(3*A*e^2-8*B*d*e+43*C*d^2)))*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c
+b^2)^(1/2))*e))^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^2/(c*(e*x+d)/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/15*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(15*b*C*e^2*(-a*e+b*d)+c^2*(48*C*d^3-2*d*e*(A*e+4*B*d))-c*e*(64*b*C*d
^2-b*e*(A*e+9*B*d)-10*a*e*(-B*e+5*C*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(
2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e^4/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 37.08 (sec) , antiderivative size = 12997, normalized size of antiderivative = 13.32

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 3.29 (sec) , antiderivative size = 1013, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2181, 27, 1229, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+Bcd+5bCd-Ace-5aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{5/2}}dx}{5\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd+2Ae)+5e(Acd-aCd+aBe)-e\left(-\frac{6cCd^2}{e}+Bcd+5bCd-Ace-5aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{5/2}}dx}{5e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1229}$$

$$-\frac{2\int -\frac{15Cd^2e^2b^3-de\left(41cCd^2+30aCe^2-ce(6Bd-Ae)\right)b^2+\left(15a^2Ce^4+ac\left(59Cd^2-e(14Bd+Ae)\right)e^2+c^2\left(24Cd^4-d^2e(4Bd+Ae)\right)\right)b-2ace\left(6cCd^3-ce(Bd+4A\right.}{}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{15Cd^2e^2b^3-de\left(41cCd^2+30aCe^2-ce(6Bd-Ae)\right)b^2+\left(15a^2Ce^4+ac\left(59Cd^2-e(14Bd+Ae)\right)e^2+c^2\left(24Cd^4-d^2e(4Bd+Ae)\right)\right)b-2ace\left(6cCd^3-ce(Bd+4Ae)d+5a\right.}{\frac{\sqrt{d}}{3e^2}}}{}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{c\left(e^2\left(30a^2Ce^2-5abe(14Cd-Be)+b^2\left(-2Ae^2-3Bde+38Cd^2\right)\right)-ce\left(bd\left(-2Ae^2-13Bde+88Cd^2\right)-2ae\left(3Ae^2-8Bde+43Cd^2\right)\right)+c^2\left(48Cd^4-2d^2e(Ae+4Bd)\right)\right)\int \cdots}{e}}{3e^2\left(ae^2-bd\cdots\right.}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1172$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2C\cdots\right.\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2C\cdots\right.\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\left(\left(48Cd^4-2d^2e(4Bd+Ae)\right)c^2-e\left(bd\left(88Cd^2-13Bed-2Ae^2\right)-2ae\left(43Cd^2-8Bed+3Ae^2\right)\right)c+e^2\left(\left(38Cd^2-3Bed-2Ae^2\right)b^2-5ae(14Cd-Be)b+30a^2C\cdots\right.\right.}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}}{\cdots}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(7/2),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(5*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(5/2)) + ((-2*(c^2*(24*C*d^5 - d^3*e*(4*B*d + A*e)) + e^
2*(15*b^2*C*d^3 + 5*a^2*e^2*(C*d + B*e) - a*b*e*(22*C*d^2 + 3*B*d*e + 2*A*
e^2)) - c*d*e*(b*d*(41*C*d^2 - 6*B*d*e + A*e^2) - a*e*(37*C*d^2 - 7*B*d*e
+ 7*A*e^2)) - e*(3*e*(B*c*d + 5*b*C*d - (6*c*C*d^2)/e - A*c*e - 5*a*C*e)*(
c*d^2 - e*(b*d - a*e)) - (2*c*d - b*e)*(6*c*C*d^3 - c*d*e*(B*d + 4*A*e) +
5*a*e^2*(2*C*d - B*e) - b*e*(8*C*d^2 - e*(3*B*d + 2*A*e))))*x)*Sqrt[a + b*
x + c*x^2])/(3*e^2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + ((Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(c^2*(48*C*d^4 - 2*d^2*e*(4*B*d + A*e)) + e^2*(30*a^2*C*e^
2 - 5*a*b*e*(14*C*d - B*e) + b^2*(38*C*d^2 - 3*B*d*e - 2*A*e^2)) - c*e*(b*
d*(88*C*d^2 - 13*B*d*e - 2*A*e^2) - 2*a*e*(43*C*d^2 - 8*B*d*e + 3*A*e^2)))
*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*S
qrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*Sqrt[(c*(d +
e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(15*b*C*e^2*(b*d - a*e) + c
^2*(48*C*d^3 - 2*d*e*(4*B*d + A*e)) - c*e*(64*b*C*d^2 - b*e*(9*B*d + A*e)
- 10*a*e*(5*C*d - B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 4.16 (sec) , antiderivative size = 1766, normalized size of antiderivative = 1.81

method	result	size
elliptic	Expression too large to display	1766
default	Expression too large to display	48427

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5*(A*e
^2-B*d*e+C*d^2)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^3-2/15*(A*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e
^2+11*C*b*d^2*e-12*C*c*d^3)/e^5/(a*e^2-b*d*e+c*d^2)*(c*e*x^3+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2-2/15*(c*e*x^2+b*e*x+a*e)/e^4/(a*e^2-b*
d*e+c*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*
b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2
*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*
c^2*d^4)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*((B*c*e+C*b*e-3*C*c*d)/e^4-
1/15*c*(A*b*e^3-2*A*c*d*e^2+5*B*a*e^3-6*B*b*d*e^2+7*B*c*d^2*e-10*C*a*d*e^2
+11*C*b*d^2*e-12*C*c*d^3)/e^4/(a*e^2-b*d*e+c*d^2)-1/15/e^4*(b*e-c*d)*(6*A*
a*c*e^4-2*A*b^2*e^4+2*A*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e
^3-3*B*b^2*d*e^3+13*B*b*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^
3+56*C*a*c*d^2*e^2+23*C*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4)/(a*e^2-b*
d*e+c*d^2)^2+1/15*b/e^3/(a*e^2-b*d*e+c*d^2)^2*(6*A*a*c*e^4-2*A*b^2*e^4+2*A
*b*c*d*e^3-2*A*c^2*d^2*e^2+5*B*a*b*e^4-16*B*a*c*d*e^3-3*B*b^2*d*e^3+13*B*b
*c*d^2*e^2-8*B*c^2*d^3*e+15*C*a^2*e^4-40*C*a*b*d*e^3+56*C*a*c*d^2*e^2+23*C
*b^2*d^2*e^2-58*C*b*c*d^3*e+33*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))
/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2430 vs. $2(921) = 1842$.

Time = 0.30 (sec) , antiderivative size = 2430, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x, algorithm="fr
icas")
```

output

```
-2/45*((48*C*c^3*d^8 - 8*(14*C*b*c^2 + B*c^3)*d^7*e + (73*C*b^2*c - 2*A*c^
3 + (122*C*a + 17*B*b)*c^2)*d^6*e^2 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (1
61*C*a*b + 8*B*b^2)*c)*d^5*e^3 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*
C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3
 + 3*(10*B*a^2 - 3*A*a*b)*c)*d^3*e^5 + (48*C*c^3*d^5*e^3 - 8*(14*C*b*c^2 +
 B*c^3)*d^4*e^4 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^3*e^5
- (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^2*e^6 + (20
*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d*e^7
 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*e^8)*x^3
+ 3*(48*C*c^3*d^6*e^2 - 8*(14*C*b*c^2 + B*c^3)*d^5*e^3 + (73*C*b^2*c - 2*A
*c^3 + (122*C*a + 17*B*b)*c^2)*d^4*e^4 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 +
 (161*C*a*b + 8*B*b^2)*c)*d^3*e^5 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (
90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*
b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*d*e^7)*x^2 + 3*(48*C*c^3*d^7*e - 8*(14*C*b
*c^2 + B*c^3)*d^6*e^2 + (73*C*b^2*c - 2*A*c^3 + (122*C*a + 17*B*b)*c^2)*d^
5*e^3 - (7*C*b^3 + (22*B*a - 3*A*b)*c^2 + (161*C*a*b + 8*B*b^2)*c)*d^4*e^4
 + (20*C*a*b^2 - 3*B*b^3 - 18*A*a*c^2 + (90*C*a^2 + 31*B*a*b + 3*A*b^2)*c)
*d^3*e^5 - (15*C*a^2*b - 5*B*a*b^2 + 2*A*b^3 + 3*(10*B*a^2 - 3*A*a*b)*c)*d
^2*e^6)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input

```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(7/2),x)
```

output

```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{{(d+e\,x)}^{7/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{7}{2}}}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(7/2),x)
```

3.100 $\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 1342

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}} \, dx = \text{Too large to display}$$

output
```
2/105*(c^2*(48*C*d^4+2*d^2*e*(3*A*e+4*B*d))+c*e*(2*a*e*(51*C*d^2+e*(-5*A*e
+12*B*d))-b*(104*C*d^3+3*d*e*(2*A*e+5*B*d)))+e^2*(70*a^2*C*e^2-7*a*b*e*(B*
e+18*C*d)+b^2*(60*C*d^2+e*(4*A*e+3*B*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-
b*d*e+c*d^2)^2/(e*x+d)^(3/2)+2/105*(c^3*(48*C*d^5+2*d^3*e*(3*A*e+4*B*d))-b
*e^3*(35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^
2*d*e*(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*d))-b*(128*C*d^3+d*e*(9*A*e+19*B*d)
))+c*e^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2
*(103*C*d^3+d*e*(19*A*e+9*B*d))))*(c*x^2+b*x+a)^(1/2)/e^3/(a*e^2-b*d*e+c*d
^2)^3/(e*x+d)^(1/2)-2/35*(24*c*C*d^3/e+c*d*(3*A*e+4*B*d)+7*a*e*(B*e+3*C*d)
-b*(25*C*d^2+e*(4*A*e+3*B*d))+5*e*(B*c*d-7*C*b*d+6*c*C*d^2/e-A*c*e+7*C*a*e
)*x)*(c*x^2+b*x+a)^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(5/2)-2/7*(C*d^2-
e*(-A*e+B*d))*(c*x^2+b*x+a)^(3/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(7/2)-1/10
5*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c^3*(24*C*d^5+d^3*e*(3*A*e+4*B*d))-b*e^3*(
35*a^2*C*e^2-14*a*b*e*(B*e+3*C*d)+b^2*(8*A*e^2+6*B*d*e+15*C*d^2))+c^2*d*e*
(2*a*e*(69*C*d^2+e*(-29*A*e+15*B*d))-b*(128*C*d^3+d*e*(9*A*e+19*B*d)))+c*e
^2*(14*a^2*e^2*(-3*B*e+11*C*d)-a*b*e*(237*C*d^2+e*(-29*A*e+B*d))+b^2*(103*
C*d^3+d*e*(19*A*e+9*B*d))))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/e^4/(a*e^2-b*d*
e+c*d^2)^3/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 38.13 (sec) , antiderivative size = 19853, normalized size of antiderivative = 14.79

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.78 (sec) , antiderivative size = 1413, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2181, 27, 1229, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}\left(A+Bx+Cx^2\right)}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{2\int -\frac{\left(3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)+e\left(\frac{6cCd^2}{e}+Bcd-7bCd-Ace+7aCe\right)x\right)\sqrt{cx^2+bx+a}}{2e(d+ex)^{7/2}}dx}{7\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\left(3bCd^2-be(3Bd+4Ae)+7e(Acd-aCd+aBe)+e\left(\frac{6cCd^2}{e}+Bcd-7bCd-Ace+7aCe\right)x\right)\sqrt{cx^2+bx+a}}{(d+ex)^{7/2}}dx}{7e\left(ae^2-bde+cd^2\right)}-\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1229}$$

$$-\frac{2\int -\frac{e^2\left(15Cd^2+6Bed+8Ae^2\right)b^3-\left(14a(3Cd+Be)e^3+cd\left(43Cd^2+3e(2Bd+5Ae)\right)e\right)b^2+\left(35a^2Ce^4+ac\left(111Cd^2-e(6Bd+29Ae)\right)e^2+c^2\left(24Cd^4+e(4Bd+3Ae)\cdots\right.\right.}{\cdots}}{\cdots}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^2\left(15Cd^2+6Bed+8Ae^2\right)b^3-\left(14a(3Cd+Be)e^3+cd\left(43Cd^2+3e(2Bd+5Ae)\right)e\right)b^2+\left(35a^2Ce^4+ac\left(111Cd^2-e(6Bd+29Ae)\right)e^2+c^2\left(24Cd^4+e(4Bd+3Ae)d^2\right)\right)b-\cdots}{\cdots}}{\cdots}$$

$$\frac{2\left(a+bx+cx^2\right)^{3/2}\left(Cd^2-e(Bd-Ae)\right)}{7e(d+ex)^{7/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1237$$

$$\frac{2((48Cd^5+2e(4Bd+3Ae)d^3)c^3+de(2ae(69Cd^2+15Bed-29Ae^2)-bd(128Cd^2+19Bed+9Ae^2))c^2+e^2(d(103Cd^2+9Bed+19Ae^2)b^2-ae(237Cd^2+Bed-29Ae^2 \cdots}{(cd^2-bed+ae^2)\sqrt{d+ex}}$$

$$\frac{2(Cd^2-e(Bd-Ae))(cx^2+bx+a)^{3/2}}{7e(cd^2-bed+ae^2)(d+ex)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2((48Cd^5+2e(4Bd+3Ae)d^3)c^3+de(2ae(69Cd^2+15Bed-29Ae^2)-bd(128Cd^2+19Bed+9Ae^2))c^2+e^2(d(103Cd^2+9Bed+19Ae^2)b^2-ae(237Cd^2+Bed-29Ae^2 \cdots}{(cd^2-bed+ae^2)\sqrt{d+ex}}$$

$$\frac{2(Cd^2-e(Bd-Ae))(cx^2+bx+a)^{3/2}}{7e(cd^2-bed+ae^2)(d+ex)^{7/2}}$$

$$\Big\downarrow \; 1269$$

$$\frac{2((48Cd^5+2e(4Bd+3Ae)d^3)c^3+de(2ae(69Cd^2+15Bed-29Ae^2)-bd(128Cd^2+19Bed+9Ae^2))c^2+e^2(d(103Cd^2+9Bed+19Ae^2)b^2-ae(237Cd^2+Bed-29Ae^2 \cdots}{(cd^2-bed+ae^2)\sqrt{d+ex}}$$

$$\frac{2(Cd^2-e(Bd-Ae))(cx^2+bx+a)^{3/2}}{7e(cd^2-bed+ae^2)(d+ex)^{7/2}}$$

$$\Big\downarrow \; 1172$$

$$\frac{2((48Cd^5+2e(4Bd+3Ae)d^3)c^3+de(2ae(69Cd^2+15Bed-29Ae^2)-bd(128Cd^2+19Bed+9Ae^2))c^2+e^2(d(103Cd^2+9Bed+19Ae^2)b^2-ae(237Cd^2+Bed-29Ae^2 \cdots}{(cd^2-bed+ae^2)\sqrt{d+ex}}$$

$$\frac{2(Cd^2-e(Bd-Ae))(cx^2+bx+a)^{3/2}}{7e(cd^2-bed+ae^2)(d+ex)^{7/2}}$$

$\Big\downarrow$ 321

$$\frac{\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right.\right.\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

$\Big\downarrow$ 327

$$\frac{\frac{2\left(\left(48Cd^5+2e(4Bd+3Ae)d^3\right)c^3+de\left(2ae\left(69Cd^2+15Bed-29Ae^2\right)-bd\left(128Cd^2+19Bed+9Ae^2\right)\right)c^2+e^2\left(d\left(103Cd^2+9Bed+19Ae^2\right)b^2-ae\left(237Cd^2+Bed-29Ae^2\right.\right.\right.}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\left(cx^2+bx+a\right)^{3/2}}{7e\left(cd^2-bed+ae^2\right)(d+ex)^{7/2}}$$

input
```
Int[(Sqrt[a + b*x + c*x^2]*(A + B*x + C*x^2))/(d + e*x)^(9/2),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(a + b*x + c*x^2)^(3/2))/(7*e*(c*d^2 - b*d*e +
 a*e^2)*(d + e*x)^(7/2)) + ((-2*(c^2*(24*C*d^5 + d^3*e*(4*B*d + 3*A*e)) -
e^2*(7*a^2*e^2*(C*d - 3*B*e) - b^2*d*(15*C*d^2 + 6*B*d*e + 8*A*e^2) + a*b*
e*(12*C*d^2 + 23*B*d*e + 12*A*e^2)) - c*d*e*(b*d*(43*C*d^2 + 6*B*d*e + 15*
A*e^2) - a*e*(33*C*d^2 + 9*B*d*e + 19*A*e^2)) + e*(5*e*(B*c*d - 7*b*C*d +
(6*c*C*d^2)/e - A*c*e + 7*a*C*e)*(c*d^2 - e*(b*d - a*e)) + (2*c*d - b*e)*(
6*c*C*d^3 + c*d*e*(B*d - 8*A*e) + 7*a*e^2*(2*C*d - B*e) - b*e*(10*C*d^2 -
e*(3*B*d + 4*A*e))))*x)*Sqrt[a + b*x + c*x^2])/(15*e^2*(c*d^2 - b*d*e + a*
e^2)*(d + e*x)^(5/2)) + ((2*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*
e^3*(35*a^2*C*e^2 - 14*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A
*e^2)) + c^2*d*e*(2*a*e*(69*C*d^2 + 15*B*d*e - 29*A*e^2) - b*d*(128*C*d^2
+ 19*B*d*e + 9*A*e^2)) + c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C
*d^2 + B*d*e - 29*A*e^2) + b^2*d*(103*C*d^2 + 9*B*d*e + 19*A*e^2)))*Sqrt[a
 + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) - (c*((Sqrt[2]*Sq
rt[b^2 - 4*a*c]*(c^3*(48*C*d^5 + 2*d^3*e*(4*B*d + 3*A*e)) - b*e^3*(35*a^2*
C*e^2 - 14*a*b*e*(3*C*d + B*e) + b^2*(15*C*d^2 + 6*B*d*e + 8*A*e^2)) + c^2
*d*e*(2*a*e*(69*C*d^2 + e*(15*B*d - 29*A*e)) - b*(128*C*d^3 + d*e*(19*B*d
+ 9*A*e))) + c*e^2*(14*a^2*e^2*(11*C*d - 3*B*e) - a*b*e*(237*C*d^2 + e*(B*
d - 29*A*e)) + b^2*(103*C*d^3 + d*e*(9*B*d + 19*A*e))))*Sqrt[d + e*x]*Sqrt
[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqr...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 2484, normalized size of antiderivative = 1.85

method	result	size
elliptic	Expression too large to display	2484
default	Expression too large to display	88790

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/7*(A*e
^2-B*d*e+C*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e
)^4-2/35*(A*b*e^3-2*A*c*d*e^2+7*B*a*e^3-8*B*b*d*e^2+9*B*c*d^2*e-14*C*a*d*e
^2+15*C*b*d^2*e-16*C*c*d^3)/e^6/(a*e^2-b*d*e+c*d^2)*(c*e*x^3+b*e*x^2+c*d*x
^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^3-2/105*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*
c*d*e^3-6*A*c^2*d^2*e^2+7*B*a*b*e^4-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*
d^2*e^2-8*B*c^2*d^3*e+35*C*a^2*e^4-84*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b
^2*d^2*e^2-106*C*b*c*d^3*e+57*C*c^2*d^4)/e^5/(a*e^2-b*d*e+c*d^2)^2*(c*e*x^
3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/105*(c*e*x^2+b*e*x+a*
e)/e^4/(a*e^2-b*d*e+c*d^2)^3*(29*A*a*b*c*e^5-58*A*a*c^2*d*e^4-8*A*b^3*e^5+
19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e^3+6*A*c^3*d^3*e^2-42*B*a^2*c*e^5+14*B*a*b
^2*e^5-B*a*b*c*d*e^4+30*B*a*c^2*d^2*e^3-6*B*b^3*d*e^4+9*B*b^2*c*d^2*e^3-19
*B*b*c^2*d^3*e^2+8*B*c^3*d^4*e-35*C*a^2*b*e^5+154*C*a^2*c*d*e^4+42*C*a*b^2
*d*e^4-237*C*a*b*c*d^2*e^3+138*C*a*c^2*d^3*e^2-15*C*b^3*d^2*e^3+103*C*b^2*
c*d^3*e^2-128*C*b*c^2*d^4*e+48*C*c^3*d^5)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1
/2)+2*(C*c/e^4-1/105*c*(10*A*a*c*e^4-4*A*b^2*e^4+6*A*b*c*d*e^3-6*A*c^2*d^2
*e^2+7*B*a*b*e^4-24*B*a*c*d*e^3-3*B*b^2*d*e^3+15*B*b*c*d^2*e^2-8*B*c^2*d^3
*e+35*C*a^2*e^4-84*C*a*b*d*e^3+108*C*a*c*d^2*e^2+45*C*b^2*d^2*e^2-106*C*b*
c*d^3*e+57*C*c^2*d^4)/e^4/(a*e^2-b*d*e+c*d^2)^2+1/105/e^4*(b*e-c*d)*(29*A*
a*b*c*e^5-58*A*a*c^2*d*e^4-8*A*b^3*e^5+19*A*b^2*c*d*e^4-9*A*b*c^2*d^2*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4543 vs. 2(1278) = 2556.

Time = 0.78 (sec) , antiderivative size = 4543, normalized size of antiderivative = 3.39

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x, algorithm="fr
icas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(A+Bx+Cx^2)\sqrt{a+bx+cx^2}}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(9/2),x)
```

output
```
Integral((A + B*x + C*x**2)*sqrt(a + b*x + c*x**2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{cx^2+bx+a}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)/(e*x + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{(C\,x^2+B\,x+A)\,\sqrt{c\,x^2+b\,x+a}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2),x)
```

output
```
int(((A + B*x + C*x^2)*(a + b*x + c*x^2)^(1/2))/(d + e*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx+cx^2}(A+Bx+Cx^2)}{(d+ex)^{9/2}}\,dx = \int \frac{\sqrt{c\,x^2+bx+a}\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{9}{2}}}dx$$

input
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x)
```

output
```
int((c*x^2+b*x+a)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(9/2),x)
```

3.101 $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 693

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2\left(15bCd+\frac{6cCd^2}{e}-35Ace+25aCe-\frac{24b^2Ce}{c}-7B(3cd-4be)\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^2}$$

$$+\frac{2\left(7Bc-6bC-\frac{2cCd}{e}\right)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3-8bce^2(9bCd+7bBe+13aCe)+c^3(6Cd^3-7de(3Bd+20Ae))+c^2e(ae(82Cd}{105c^4e^2\sqrt{\frac{c(}{2cd-\left(b+\right.}}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(cd^2-bde+ae^2)\left(24b^2Ce^2-ce(15bCd+28bBe+25aCe)-c^2(6Cd^2-7e(3Bd+5Ae)\right)}{105c^4e^2\sqrt{d+ex}\sqrt{a+b}}$$

output
```
-2/105*(15*C*b*d+6*c*C*d^2/e-35*A*c*e+25*C*a*e-24*b^2*C*e/c-7*B*(-4*b*e+3*
c*d))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/35*(7*B*c-6*b*C-2*c*C*d/e)*(
e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/7*C*(e*x+d)^(5/2)*(c*x^2+b*x+a)^(1/
2)/c/e-1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(48*b^3*C*e^3-8*b*c*e^2*(7*B*b*e+1
3*C*a*e+9*C*b*d)+c^3*(6*C*d^3-7*d*e*(20*A*e+3*B*d))+c^2*e*(a*e*(63*B*e+82*
C*d)+b*(70*A*e^2+91*B*d*e+12*C*d^2)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*
a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/
2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e
^2/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-
2/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a*e^2-b*d*e+c*d^2)*(24*b^2*C*e^2-c*e*(28
*B*b*e+25*C*a*e+15*C*b*d)-c^2*(6*C*d^2-7*e*(5*A*e+3*B*d)))*(c*(e*x+d)/(2*c
*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*
EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b
^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^2/(e*x+d)^(1/2)
/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.22 (sec) , antiderivative size = 9972, normalized size of antiderivative = 14.39

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.84 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2184, 27, 1236, 27, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2\int -\frac{e(d+ex)^{3/2}(bCd-7Ace+5aCe+(2cCd-7Bce+6bCe)x)}{2\sqrt{cx^2+bx+a}}dx}{7ce^2}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-\frac{\int \frac{(d+ex)^{3/2}(bCd-7Ace+5aCe+(2cCd-7Bce+6bCe)x)}{\sqrt{cx^2+bx+a}}dx}{7ce}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-\frac{\frac{2\int -\frac{\sqrt{d+ex}\left(6Cdeb^2+18aCe^2b-cd(3Cd+7Be)b+ce(35Acd-19aCd-21aBe)+\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right)-e(15bCd+28bBe+25aCe)c+24b^2Ce^2\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{5c}+}{7ce}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-\frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{\int \frac{\sqrt{d+ex}\left(6Cdeb^2+18aCe^2b-cd(3Cd+7Be)b+ce(35Acd-19aCd-21aBe)+\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right.\right.\right.}{\sqrt{cx^2+bx+a}}}{5c}}{7ce}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-\frac{\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{2\int -\frac{24Cde^2b^3+\left(24aCe^3-cde(33Cd+28Be)\right)b^2+c\left(3cCd^3+7ce(6Bd+5Ae)d-2ae^2(47Cd+14Be)\right)b-ce\left(\right.}{}}{}}{}$$

$$\Big\downarrow \text{27}$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{\int\frac{24Cde^2}{}}{}$$

$\downarrow$ 1269

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{\left(c^2e\left(ae(\right.\right.}{}$$

$\downarrow$ 1172

$$\frac{2C(d+ex)^{5/2}\sqrt{cx^2+bx+a}}{7ce}-$$

$$\frac{2(2cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2\left(-\left(\left(6Cd^2-7e(3Bd+5Ae)\right)c^2\right)-e(15bCd+28bBe+25aCe)c+24b^2Ce^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{2\sqrt{2}\sqrt{b^2-}}{}$$

$\downarrow$ 321

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{\sqrt{2}\sqrt{b^2-}}{}$$

$\downarrow$ 327

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce}-$$

$$\frac{2(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+2cCd)}{5c}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(-ce(25aCe+28bBe+15bCd)-\left(c^2\left(6Cd^2-7e(5Ae+3Bd)\right)\right)+24b^2Ce^2\right)}{3c}-\frac{2\sqrt{2}\sqrt{b^2-}}{}$$

input
```
Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e) - ((2*(2*c*C*d - 7*B*c
*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - ((2*(24*b^2*C
*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*e*(3*B*d +
5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - ((Sqrt[2]*Sqrt[b^2 -
 4*a*c]*(48*b^3*C*e^3 - 8*b*c*e^2*(9*b*C*d + 7*b*B*e + 13*a*C*e) + c^3*(6*
C*d^3 - 7*d*e*(3*B*d + 20*A*e)) + c^2*e*(a*e*(82*C*d + 63*B*e) + b*(12*C*d
^2 + 91*B*d*e + 70*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^
2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^
2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sq
rt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2
)*(24*b^2*C*e^2 - c*e*(15*b*C*d + 28*b*B*e + 25*a*C*e) - c^2*(6*C*d^2 - 7*
e*(3*B*d + 5*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b
*x + c*x^2]))/(3*c))/(5*c))/(7*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. $2(627) = 1254$.

Time = 4.90 (sec) , antiderivative size = 1261, normalized size of antiderivative = 1.82

method	result	size
elliptic	Expression too large to display	1261
risch	Expression too large to display	4796
default	Expression too large to display	14084

input
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C*e/c
*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*e^2+2*C*d*e-2/
7*C*e/c*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)+2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C*e/c*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+2*C*d*e
-2/7*C*e/c*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+
a*e*x+b*d*x+a*d)^(1/2)+2*(A*d^2-2/5*(B*e^2+2*C*d*e-2/7*C*e/c*(3*b*e+3*c*d)
)/c/e*a*d-2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C*e/c*(5/2*a*e+5/2*b*d)-2/5*(B*e^2+
2*C*d*e-2/7*C*e/c*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))
*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ellip
ticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(2*A*d*e
+B*d^2-4/7*C*e/c*a*d-2/5*(B*e^2+2*C*d*e-2/7*C*e/c*(3*b*e+3*c*d))/c/e*(3/2*
a*e+3/2*b*d)-2/3*(A*e^2+2*B*d*e+C*d^2-2/7*C*e/c*(5/2*a*e+5/2*b*d)-2/5*(B*e
^2+2*C*d*e-2/7*C*e/c*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/315*((6*C*c^4*d^4 + 3*(3*C*b*c^3 - 7*B*c^4)*d^3*e + (39*C*b^2*c^2 + 175*
A*c^4 - (71*C*a + 56*B*b)*c^3)*d^2*e^2 - (96*C*b^3*c + 7*(27*B*a + 25*A*b)
*c^3 - (260*C*a*b + 119*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*
C*a^2 + 147*B*a*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*sqrt(
c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*
e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^
3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(6*C*c^4
*d^3*e + 3*(4*C*b*c^3 - 7*B*c^4)*d^2*e^2 - (72*C*b^2*c^2 + 140*A*c^4 - (82
*C*a + 91*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13*C
*a*b + 7*B*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 3*C*c^4*d^
2*e^2 - 3*(11*C*b*c^3 - 14*B*c^4)*d*e^3 + (24*C*b^2*c^2 + 35*A*c^4 - (25*C
*a + 28*B*b)*c^3)*e^4 + 3*(8*C*c^4*d*e^3 - (6*C*b*c^3 - 7*B*c^4)*e^4)*x)*s
qrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^5*e^3)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*(e*x + d)^(3/2)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+e\,x)^{3/2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx=\int \frac{(ex+d)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

3.102 $\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 529

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(5Bc-4bC-\frac{2cCd}{e}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2} + \frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$
$$- \frac{\sqrt{2}\sqrt{b^2-4ac}\left(3c(bCd-5Ace+3aCe)+\frac{(cd-2be)(2cCd-5Bce+4bCe)}{e}\right)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\Big(\arcsin\Big(}{15c^3e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$+ \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd-5Bce+4bCe)\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\Big(}{15c^3e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/15*(5*B*c-4*b*C-2*c*C*d/e)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2+2/5*C*(
e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(3*c*
(-5*A*c*e+3*C*a*e+C*b*d)+(-2*b*e+c*d)*(-5*B*c*e+4*C*b*e+2*C*c*d)/e)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2))/c^3/e/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)^(1/2)/(c*x^2+b*x+a)^(1/2)+2/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-5*B*c*e+4*C*
b*e+2*C*c*d)*(a*e^2-b*d*e+c*d^2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*
e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)
/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.40 (sec) , antiderivative size = 992, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(((2*(c*C*d + 5*B*c*e - 4*b*C*e))/(15*c^2*e) + (2*C*x)/(5*c))*Sqrt[d + e*x
]*(a + b*x + c*x^2))/Sqrt[a + x*(b + c*x)] - (2*(d + e*x)^(3/2)*Sqrt[a + b
*x + c*x^2]*((-8*b^2*C*e^2 + c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) + c^2*(2*C
*d^2 - 5*e*(B*d + 3*A*e)))*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*
x) + (a*e)/(d + e*x)))/(d + e*x)) + ((I/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d)
+ a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*
(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d +
 e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*b^2*C*e^2 - c*e*(3*b*C
*d + 10*b*B*e + 9*a*C*e) + c^2*(-2*C*d^2 + 5*e*(B*d + 3*A*e)))*EllipticE[I
*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (8*b^3*C*e^3 - b^2*e^2*(11*c*
C*d + 10*B*c*e + 8*C*Sqrt[(b^2 - 4*a*c)*e^2]) + c*(c*d*Sqrt[(b^2 - 4*a*c)*
e^2]*(2*C*d - 5*B*e) - 15*A*c*e^2*(2*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + a*e^
2*(14*c*C*d + 10*B*c*e + 9*C*Sqrt[(b^2 - 4*a*c)*e^2])) + b*c*e*(15*A*c*e^2
 - 17*a*C*e^2 + 3*C*d*Sqrt[(b^2 - 4*a*c)*e^2] + 5*B*(3*c*d*e + 2*e*Sqrt[(b
^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e
^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d +
 b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
)/(Sqrt[2]*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - ...
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2184, 27, 1236, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}\left(A+Bx+Cx^2\right)}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{2\int -\frac{e\sqrt{d+ex}(bCd-5Ace+3aCe+(2cCd-5Bce+4bCe)x)}{2\sqrt{cx^2+bx+a}}dx}{5ce^2}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\int \frac{\sqrt{d+ex}(bCd-5Ace+3aCe+(2cCd-5Bce+4bCe)x)}{\sqrt{cx^2+bx+a}}dx}{5ce}$$

$\downarrow$ 1236

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\int -\frac{4Cdeb^2+4aCe^2b-cd(Cd+5Be)b+ce(15Acd-7aCd-5aBe)+\left(-\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2\right)-e(3bCd+10bBe+9aCe)c+8b^2Ce^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3c}+2\sqrt{d+ex}\sqrt{a+bx}}{5ce}$$

$\downarrow$ 27

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\int \frac{4Cdeb^2+4aCe^2b-cd(Cd+5Be)b+ce(15Acd-7aCd-5aBe)+\left(-\left(\left(2Cd^2-5e(Bd+3Ae)\right)c^2\right)-e(3bCd+10bB}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3c}}{5ce}$$

$\downarrow$ 1269

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\frac{\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{\left(ae^2-bde+cd^2\right)(4bC}{}}{3c}}{5ce}$$

$\downarrow$ 1172

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 321

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce}-\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c}-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)\int}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow 327$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce} -$$

$$\frac{2\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+2cCd)}{3c} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCe+10bBe+3bCd)-\left(c^2\left(2Cd^2-5e(3Ae+Bd)\right)\right)+8b^2Ce^2\right)E}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e) - ((2*(2*c*C*d - 5*B*c
*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - ((Sqrt[2]*Sqrt[
b^2 - 4*a*c]*(8*b^2*C*e^2 - c*e*(3*b*C*d + 10*b*B*e + 9*a*C*e) - c^2*(2*C*
d^2 - 5*e*(B*d + 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
- 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d - 5*B*c*e + 4*b
*C*e)*(c*d^2 - b*d*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*
Sqrt[a + b*x + c*x^2]))/(3*c))/(5*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 954 vs. 2(469) = 938.

Time = 3.06 (sec) , antiderivative size = 955, normalized size of antiderivative = 1.81

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2Cx\sqrt{ce\,x^3+be\,x^2+x^2cd+aex+dxb+ad}}{5c}+\frac{2\left(Be+Cd-\frac{2C(2be+2cd)}{5c}\right)\sqrt{ce\,x^3+be\,x^2+x^2cd+aex+dxb+ad}}{3ce}+\frac{2\left(Ad-\frac{2}{}\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*C/c*x
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(B*e+C*d-2/5*C/c*(2*b
*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A*d-2/5*
C/c*a*d-2/3*(B*e+C*d-2/5*C/c*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/
2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(
1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
)))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x
+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A*e+B*d-2/5*C/c
*(3/2*a*e+3/2*b*d)-2/3*(B*e+C*d-2/5*C/c*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e
-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d
/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left((2\,Cc^3d^3 + (2\,Cbc^2 - 5\,Bc^3)d^2e + (7\,Cb^2c + 30\,Ac^3 - 2\,(6\,Ca + 5\,Bb)c^2)de^2 - (8\,Cb^3 + 15\,(Ba + Ab\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/45*((2*C*c^3*d^3 + (2*C*b*c^2 - 5*B*c^3)*d^2*e + (7*C*b^2*c + 30*A*c^3 -
 2*(6*C*a + 5*B*b)*c^2)*d*e^2 - (8*C*b^3 + 15*(B*a + A*b)*c^2 - (21*C*a*b
+ 10*B*b^2)*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e +
 (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c
 - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d +
 b*e)/(c*e)) + 3*(2*C*c^3*d^2*e + (3*C*b*c^2 - 5*B*c^3)*d*e^2 - (8*C*b^2*c
 + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*sqrt(c*e)*weierstrassZeta(4/3*(c^
2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2
*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), wei
erstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4
/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*
b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*C*c^3*e^3*x
+ C*c^3*d*e^2 - (4*C*b*c^2 - 5*B*c^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(e*x
+ d))/(c^4*e^3)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)*(A + B*x + C*x**2)/sqrt(a + b*x + c*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{ex+d}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(e*x + d)/sqrt(c*x^2 + b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{d+e\,x}\,(C\,x^2+B\,x+A)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\,(C\,x^2+Bx+A)}{\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(c*x^2+b*x+a)^(1/2),x)
```

3.103 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 450

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(a\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(-3*B*c*e+2*C*b*e+2*C*c*d)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(c*(e*x
+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/3*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2+C*e*(-a*e+b*d)-3*c*e*(-A*e+B*d))*(c*(e*x+d
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.53 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.40

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[
(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-
1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)
) + (I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1
 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(
-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt
[d + e*x] - (I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*
c)*e^2]) + c*(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*
B*e)))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d
/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 +
 d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +...
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{2\int -\frac{e(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{(2bCe-3Bce+2cCd)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}$$

$$\Big\downarrow \text{ 1172}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\cdots}}{\cdots}}{3ce}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{}}{}}{3ce}$$

$\Big\downarrow$ 327

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2bCe-3Bce+2cCd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+b}{b^2}}}{}}{3ce}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C
*e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(396) = 792$.

Time = 3.79 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2C\sqrt{ce\,x^3+be\,x^2+x^2cd+aex+dxb+ad}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b}{}}{-\frac{d}{e}-}}}{\sqrt{ce\,x}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*C/c/e
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(B-2/3*C/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}Cc^2e^2+(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2)\sqrt{}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b
*c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d*e +
 (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.104 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 486

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e(cd^2-bde+ae^2)\sqrt{d+ex}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd^2-Ce(bd-ae)-ce(Bd-Ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce^2(cd^2-bde+ae^2)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2}}{2cd-(b+\sqrt{}}\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^
(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2-C*e*(-a*e+b*d)-c*e*(-A*e+B*d))
*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c
*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b
+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*(e*x+d)/(2*c*
d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a*c+b
^2)^(1/2)*(-B*e+2*C*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2))/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.09 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(-e^2(Cd^2+e(-Bd+Ae))\,(a+x(b+cx))+\frac{e^2(2cCd^2+Ce(-bd+ae)+ce(-}{c}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2
+ C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(
d + e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/
((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) +
 2*a*c*C*d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e
*Sqrt[(b^2 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*
d + Sqrt[(b^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sq
rt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e +
 a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]
))]))/(Sqrt[2]*c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a +
 x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2181, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{2\int -\frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{2e\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{-\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1172}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}{}$$

$$e\left(ae^2-bde+\right.$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 321

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-}}{}$$

$$e\left(ae^2-bde+c\right.$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 327

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2}}{}$$

$$e\left(ae^2-bde\right.$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e
^2)*Sqrt[d + e*x]) + (-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(B*c*d + b*C*d - (2*c*C
*d^2)/e - A*c*e - a*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
+ b*x + c*x^2])) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d - B*e)*(c*d^2 - b*d
*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x
^2]))/(e*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(438) = 876$.

Time = 4.27 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(-\frac{2\left(ce\,x^2+bex+ae\right)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(a\,e^2-bde+c\,d^2\right)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex+ae\right)}}+\frac{2\left(\frac{Be-Cd}{e^2}-\frac{(be-cd)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(a\,e^2-bde+c\,d^2\right)}+\frac{b\left(A\,e^2-Bde+C\,d^2\right)}{e\left(a\,e^2-bde+c\,d^2\right)}\right)}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/((x+d/e)*(c*e*x^
2+b*e*x+a*e))^(1/2)+2*((B*e-C*d)/e^2-1/e^2*(b*e-c*d)*(A*e^2-B*d*e+C*d^2)/(
a*e^2-b*d*e+c*d^2)+b/e/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2))*(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e+c/e*(A*e^2-B*d
*e+C*d^2)/(a*e^2-b*d*e+c*d^2))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^
2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*
c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2/3*((2*C*c^2*d^4 - (2*C*b*c + B*c^2)*d^3*e - (C*b^2 + 2*A*c^2 - 2*(2*C*a
 + B*b)*c)*d^2*e^2 + (C*a*b - (3*B*a - A*b)*c)*d*e^3 + (2*C*c^2*d^3*e - (2
*C*b*c + B*c^2)*d^2*e^2 - (C*b^2 + 2*A*c^2 - 2*(2*C*a + B*b)*c)*d*e^3 + (C
*a*b - (3*B*a - A*b)*c)*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d^3*e - (C*b*c + B*c^2)*d^2*e^2 + (C
*a*c + A*c^2)*d*e^3 + (2*C*c^2*d^2*e^2 - (C*b*c + B*c^2)*d*e^3 + (C*a*c +
A*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(
3*c*e*x + c*d + b*e)/(c*e))) + 3*(C*c^2*d^2*e^2 - B*c^2*d*e^3 + A*c^2*e^4)
*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2
*d*e^5 + (c^3*d^2*e^4 - b*c^2*d*e^5 + a*c^2*e^6)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.105 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 660

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)\,(d+ex)^{3/2}}$$

$$+\frac{2(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{a+bx+cx^2}}{3e\,(cd^2-bde+ae^2)^2\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd^3+cde(Bd-4Ae)+3ae^2(2Cd-Be)-be(4Cd^2-e(Bd+2Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a\cdots}{b\cdots}}}{3e^2\,(cd^2-bde+ae^2)^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+c\cdots}}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\mathrm{arcs}\cdots\right.}{3ce^2\,(cd^2-bde+ae^2)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2/3*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(3/2)+2/3*(2*c*C*d^3+c*d*e*(-4*A*e+B*d)+3*a*e^2*(-B*e+2*C*d)-b*e*(4*C*d^
2-e*(2*A*e+B*d)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2
)-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^3+c*d*e*(-4*A*e+B*d)+3*a*e^2*(-B
*e+2*C*d)-b*e*(4*C*d^2-e*(2*A*e+B*d)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4
*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1
/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/e^2/
(a*e^2-b*d*e+c*d^2)^2/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(
c*x^2+b*x+a)^(1/2)+2/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2-3*C*e*(-a*e+b
*d)+c*e*(-A*e+B*d))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c
*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e))^(1/2))/c/e^2/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.95 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[d + e*x]*(a + b*x + c*x^2)*((-2*(C*d^2 - B*d*e + A*e^2))/(3*e*(c*d^2
 - b*d*e + a*e^2)*(d + e*x)^2) - (2*(-2*c*C*d^3 - B*c*d^2*e + 4*b*C*d^2*e
- b*B*d*e^2 + 4*A*c*d*e^2 - 6*a*C*d*e^2 - 2*A*b*e^3 + 3*a*B*e^3))/(3*e*(c*
d^2 - b*d*e + a*e^2)^2*(d + e*x))))/Sqrt[a + x*(b + c*x)] + (2*(d + e*x)^(
3/2)*Sqrt[a + b*x + c*x^2]*(-((2*c*C*d^3 + c*d*e*(B*d - 4*A*e) - 3*a*e^2*(
-2*C*d + B*e) + b*e*(-4*C*d^2 + e*(B*d + 2*A*e)))*(c*(-1 + d/(d + e*x))^2
+ (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x))) + ((I/2)*Sqrt[1
- (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*
(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/((-2*c*d + b*e + Sqrt[
(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(
c*d*(2*C*d^2 + e*(B*d - 4*A*e)) + e*(-4*b*C*d^2 + b*e*(B*d + 2*A*e) - 3*a*
e*(-2*C*d + B*e)))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^
2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] -
 (2*a*c*C*d^2*e^2 - 8*a*B*c*d*e^3 - 6*a^2*C*e^4 + 2*c*C*d^3*Sqrt[(b^2 - 4*
a*c)*e^2] + B*c*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2] + 6*a*C*d*e^2*Sqrt[(b^2 - 4*
a*c)*e^2] - 3*a*B*e^3*Sqrt[(b^2 - 4*a*c)*e^2] + 2*A*c*e^2*(-3*c*d^2 + a*e^
2 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) - b^2*e^2*(2*C*d^2 + e*(B*d + 2*A*e)) + b
*e*(2*A*e^2*(3*c*d + Sqrt[(b^2 - 4*a*c)*e^2]) + 2*C*d*(3*a*e^2 - 2*d*Sqrt[
(b^2 - 4*a*c)*e^2]) + B*e*(3*c*d^2 + 3*a*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2...
```

Rubi [A] (verified)

Time = 2.31 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2181, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x}{2e(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{3\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{bCd^2-be(Bd+2Ae)+3e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+Bcd-3bCd-Ace+3aCe\right)x}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1237$$

$$\frac{\frac{2\sqrt{a+bx+cx^2}\left(3ae^2(2Cd-Be)-be(4Cd^2-e(2Ae+Bd))+cde(Bd-4Ae)+2cCd^3\right)}{\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}-\frac{2\int -\frac{3b^2Ced^2-6abCe^2d-bc\left(Cd^2+e(2Bd+Ae)\right)d+Ace\left(3cd^2-a\right.}{}}{}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3b^2Ced^2-b\left(cCd^2+6aCe^2+ce(2Bd+Ae)\right)d+e\left(Ac\left(3cd^2-ae^2\right)+a\left(3aCe^2-cd(Cd-4Be)\right)\right)-c\left(2cCd^3+ce(Bd-4Ae)d+3ae^2(2Cd-Be)-be\left(4Cd^2-e(Bd+2Ae)\right.\right.}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{ae^2-bde+cd^2}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\frac{\left(ae^2-bde+cd^2\right)\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{c\left(3ae^2(2Cd-Be)-be\left(4Cd^2-e(2Ae+Bd)\right)+cde(Bd-4Ae)+2cCd^3\right)\int\frac{\sqrt{d+}}{\sqrt{cx^2+}}}{e}}{ae^2-bde+cd^2}}{3e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1172$$

$$\frac{\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(Bd-4Ae)d+3ae^2(2Cd-Be)-be\left(4Cd^2-e(Bd+2Ae)\right)\right)}{\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}+\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\left(2cCd^2-3Ce(bd-ae)+ce(Bd-Ae)\right.}{}}{}}{}-\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{3e\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}$$

$\Big\downarrow$ 321

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2cd-}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

$\Big\downarrow$ 327

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(ae^2-bde+cd^2\right)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\left(-3Ce(bd-ae)+ce(Bd-Ae)+2cCd^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2cd-}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{3e(d+ex)^{3/2}\left(ae^2-bde+cd^2\right)}$$

input

```
Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(3*e*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(3/2)) + ((2*(2*c*C*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2
*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d + 2*A*e)))*Sqrt[a + b*x + c*x^2])/((c*
d^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) + (-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C
*d^3 + c*d*e*(B*d - 4*A*e) + 3*a*e^2*(2*C*d - B*e) - b*e*(4*C*d^2 - e*(B*d
 + 2*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ell
ipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*Sq
rt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2
])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(2*c*C*d^2 - 3*
C*e*(b*d - a*e) + c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Ar
cSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2
*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
e*x]*Sqrt[a + b*x + c*x^2]))/(c*d^2 - b*d*e + a*e^2))/(3*e*(c*d^2 - b*d*e
+ a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1247 vs. $2(600) = 1200$.

Time = 5.91 (sec) , antiderivative size = 1248, normalized size of antiderivative = 1.89

method	result	size
elliptic	Expression too large to display	1248
default	Expression too large to display	20481

input
```
int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/3/e^3/
(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)/(x+d/e)^2+2/3*(c*e*x^2+b*e*x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)^2*
(2*A*b*e^3-4*A*c*d*e^2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2
*e+2*C*c*d^3)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(C/e^2-1/3*c/e^2*(A*e^
2-B*d*e+C*d^2)/(a*e^2-b*d*e+c*d^2)+1/3/e^2*(b*e-c*d)*(2*A*b*e^3-4*A*c*d*e^
2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*e^2-
b*d*e+c*d^2)^2-1/3*b/e/(a*e^2-b*d*e+c*d^2)^2*(2*A*b*e^3-4*A*c*d*e^2-3*B*a*
e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3))*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2/3*c/e*(2*A*b*e^3-4*A*c*
d*e^2-3*B*a*e^3+B*b*d*e^2+B*c*d^2*e+6*C*a*d*e^2-4*C*b*d^2*e+2*C*c*d^3)/(a*
e^2-b*d*e+c*d^2)^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b
+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/
e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. $2(608) = 1216$.

Time = 0.13 (sec) , antiderivative size = 1305, normalized size of antiderivative = 1.98

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{5/2}\sqrt{a + bx + cx^2}}\, dx = \text{Too large to display}$$

input

```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output

```
2/9*((2*C*c^2*d^6 - (5*C*b*c - B*c^2)*d^5*e + (5*C*b^2 + 5*A*c^2 + (3*C*a
- 4*B*b)*c)*d^4*e^2 - (12*C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d^3*e^3 + (9*
C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d^2*e^4 + (2*C*c^2*d^4*e^2 - (5*C*b*c
 - B*c^2)*d^3*e^3 + (5*C*b^2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^2*e^4 - (12*
C*a*b - B*b^2 - (9*B*a - 5*A*b)*c)*d*e^5 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 -
3*A*a*c)*e^6)*x^2 + 2*(2*C*c^2*d^5*e - (5*C*b*c - B*c^2)*d^4*e^2 + (5*C*b^
2 + 5*A*c^2 + (3*C*a - 4*B*b)*c)*d^3*e^3 - (12*C*a*b - B*b^2 - (9*B*a - 5*
A*b)*c)*d^2*e^4 + (9*C*a^2 - 3*B*a*b + 2*A*b^2 - 3*A*a*c)*d*e^5)*x)*sqrt(c
*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e
^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3
 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*
d^5*e - (3*B*a - 2*A*b)*c*d^2*e^4 - (4*C*b*c - B*c^2)*d^4*e^2 - (4*A*c^2 -
 (6*C*a + B*b)*c)*d^3*e^3 + (2*C*c^2*d^3*e^3 - (3*B*a - 2*A*b)*c*e^6 - (4*
C*b*c - B*c^2)*d^2*e^4 - (4*A*c^2 - (6*C*a + B*b)*c)*d*e^5)*x^2 + 2*(2*C*c
^2*d^4*e^2 - (3*B*a - 2*A*b)*c*d*e^5 - (4*C*b*c - B*c^2)*d^3*e^3 - (4*A*c^
2 - (6*C*a + B*b)*c)*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrass
PInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c
^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{5}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(d+e\,x\right)}^{5/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(5/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}\sqrt{a+bx+cx^2}}\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*b + int((sqrt(d + e*x)*sqrt(a +
 b*x + c*x**2)*x**2)/(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 +
 a*b*e**4*x**3 - a*c*d**4 - 3*a*c*d**3*e*x - 3*a*c*d**2*e**2*x**2 - a*c*d*
e**3*x**3 + b**2*d**3*e*x + 3*b**2*d**2*e**2*x**2 + 3*b**2*d*e**3*x**3 + b
**2*e**4*x**4 - b*c*d**4*x - 2*b*c*d**3*e*x**2 + 2*b*c*d*e**3*x**4 + b*c*e
**4*x**5 - c**2*d**4*x**2 - 3*c**2*d**3*e*x**3 - 3*c**2*d**2*e**2*x**4 - c
**2*d*e**3*x**5),x)*b**2*c*d**2*e**2 + 2*int((sqrt(d + e*x)*sqrt(a + b*x +
 c*x**2)*x**2)/(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e
**4*x**3 - a*c*d**4 - 3*a*c*d**3*e*x - 3*a*c*d**2*e**2*x**2 - a*c*d*e**3*x
**3 + b**2*d**3*e*x + 3*b**2*d**2*e**2*x**2 + 3*b**2*d*e**3*x**3 + b**2*e*
*4*x**4 - b*c*d**4*x - 2*b*c*d**3*e*x**2 + 2*b*c*d*e**3*x**4 + b*c*e**4*x*
*5 - c**2*d**4*x**2 - 3*c**2*d**3*e*x**3 - 3*c**2*d**2*e**2*x**4 - c**2*d*
e**3*x**5),x)*b**2*c*d*e**3*x + int((sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*
x**2)/(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e**4*x**3
- a*c*d**4 - 3*a*c*d**3*e*x - 3*a*c*d**2*e**2*x**2 - a*c*d*e**3*x**3 + b**
2*d**3*e*x + 3*b**2*d**2*e**2*x**2 + 3*b**2*d*e**3*x**3 + b**2*e**4*x**4 -
 b*c*d**4*x - 2*b*c*d**3*e*x**2 + 2*b*c*d*e**3*x**4 + b*c*e**4*x**5 - c**2
*d**4*x**2 - 3*c**2*d**3*e*x**3 - 3*c**2*d**2*e**2*x**4 - c**2*d*e**3*x**5
),x)*b**2*c*e**4*x**2 - 3*int((sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*x**2)/
(a*b*d**3*e + 3*a*b*d**2*e**2*x + 3*a*b*d*e**3*x**2 + a*b*e**4*x**3 - a...
```

3.106 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 922

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{5e\left(cd^2-bde+ae^2\right)(d+ex)^{5/2}}$$

$$+\frac{2(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))\sqrt{a+bx+cx^2}}{15e\left(cd^2-bde+ae^2\right)^2(d+ex)^{3/2}}$$

$$+\frac{2(c^2(2Cd^4+d^2e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae^2))-ce(bd(7\ldots}{15e\left(cd^2-bde+ae^2\right)^3\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(c^2(2Cd^4+d^2e(3Bd-23Ae))-e^2(15a^2Ce^2-10abe(Cd+Be)+b^2(3Cd^2+2Bde+8Ae\ldots}{15e^2\left(cd^2-\ldots\right.}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^3+cde(3Bd-8Ae)+5ae^2(2Cd-Be)-be(6Cd^2-e(Bd+4Ae)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4\ldots}\right.}}}{15e^2\left(cd^2-bde+ae^2\right)^2\sqrt{d+ex}\sqrt{a+b\ldots}}$$

output
```
-2/5*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d
)^(5/2)+2/15*(2*c*C*d^3+c*d*e*(-8*A*e+3*B*d)+5*a*e^2*(-B*e+2*C*d)-b*e*(6*C
*d^2-e*(4*A*e+B*d)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(
3/2)+2/15*(c^2*(2*C*d^4+d^2*e*(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*
(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d
^2)-a*e*(9*A*e^2-29*B*d*e+19*C*d^2)))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c
*d^2)^3/(e*x+d)^(1/2)-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c^2*(2*C*d^4+d^2*e*
(-23*A*e+3*B*d))-e^2*(15*a^2*C*e^2-10*a*b*e*(B*e+C*d)+b^2*(8*A*e^2+2*B*d*e
+3*C*d^2))-c*e*(b*d*(-23*A*e^2-7*B*d*e+7*C*d^2)-a*e*(9*A*e^2-29*B*d*e+19*C
*d^2)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*
(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2
*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^3/(c*(e*x+d
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/15*2^(1/2)
*(-4*a*c+b^2)^(1/2)*(2*c*C*d^3+c*d*e*(-8*A*e+3*B*d)+5*a*e^2*(-B*e+2*C*d)-b
*e*(6*C*d^2-e*(4*A*e+B*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b
^2)^(1/2))*e))^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^2/(e*x+d)^(1/2)/(c*x^2+b*x+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.92 (sec) , antiderivative size = 12295, normalized size of antiderivative = 13.34

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.54 (sec) , antiderivative size = 986, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2181, 27, 1237, 27, 1237, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{2\int -\frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+3Bcd-5bCd-3Ace+5aCe\right)x}{2e(d+ex)^{5/2}\sqrt{cx^2+bx+a}}dx}{5\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{bCd^2-be(Bd+4Ae)+5e(Acd-aCd+aBe)+e\left(\frac{2cCd^2}{e}+3Bcd-5bCd-3Ace+5aCe\right)x}{(d+ex)^{5/2}\sqrt{cx^2+bx+a}}dx}{5e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{\frac{2\sqrt{a+bx+cx^2}\left(5ae^2(2Cd-Be)-be\left(6Cd^2-e(4Ae+Bd)\right)+cde(3Bd-8Ae)+2cCd^3\right)}{3(d+ex)^{3/2}(ae^2-bde+cd^2)}-2\int -\frac{e\left(3Cd^2+2e(Bd+4Ae)\right)b^2+\left(cCd^3-ce(6Bd+19Ae)d-10a\cdots\right.}{\cdots}}{5e\left(ae^2-bde+c\cdots\right.}$$

$$-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{e\left(3Cd^2+2e(Bd+4Ae)\right)b^2+\left(cCd^3-ce(6Bd+19Ae)d-10ae^2(Cd+Be)\right)b+3e\left(Ac\left(5cd^2-3ae^2\right)+a\left(5aCe^2-cd(3Cd-8Be)\right)\right)+c\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2\cdots\right.}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}}{3(ae^2-bde+cd^2)}\cdots}{5e\left(ae^2-bde+cd\cdots\right.}$$

$$-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1237}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(-e^2\left(15a^2Ce^2-10abe(Be+Cd)+b^2\left(8Ae^2+2Bde+3Cd^2\right)\right)-ce\left(bd\left(-23Ae^2-7Bde+7Cd^2\right)-ae\left(9Ae^2-29Bde+19Cd^2\right)\right)+c^2\left(d^2e(3Bd-23Ae)+2\right.\right.}{\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$c\int \frac{de\left(9Cd^2+e(Bd+4Ae)\right)b^2-\left(cCd^4+26aCe^2d^2+ce(9Bd+11Ae)d^2+4ae^3(Bd-Ae)\right)b+e\left(Acd\left(15cd^2-17ae^2\right)-a\left(cd^2(7Cd-27Be)-5ae^2(5Cd-Be)\right)\right)-\left(\left(2Cd\right.\right.}{\sqrt{d+ex}\sqrt{cx^2+ba}} \Big/ \left(ae^2-bde+cd^2\right)$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$c\left(\frac{\left(ae^2-bde+cd^2\right)\left(5ae^2(2Cd-Be)-be\left(6Cd^2-e(4Ae+Bd)\right)+cde(3Bd-8Ae)+2cCd^3\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(-e^2\left(15a^2Ce^2-10abe(Be+Cd)+b^2\left(8Ae^2+\right.\right.\right.}{}\right) \Big/ \left(ae^2-bde+cd^2\right)$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{5e(d+ex)^{5/2}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be\left(6Cd^2-e(Bd+4Ae)\right)\right)}{3\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed\right.\right.\right.\right.}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be\left(6Cd^2-e(Bd+4Ae)\right)\right)}{3(cd^2-bed+ae^2)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed\right.\right.\right.}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

$\downarrow$ 327

$$\frac{\frac{2\sqrt{cx^2+bx+a}\left(2cCd^3+ce(3Bd-8Ae)d+5ae^2(2Cd-Be)-be\left(6Cd^2-e(Bd+4Ae)\right)\right)}{3(cd^2-bed+ae^2)(d+ex)^{3/2}}+\frac{2\sqrt{cx^2+bx+a}\left(\left(2Cd^4+e(3Bd-23Ae)d^2\right)c^2-e\left(bd\left(7Cd^2-7Bed\right.\right.\right.}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}}{5e\left(cd^2-bed+ae^2\right)(d+ex)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(5*e*(c*d^2 - b*d*e + a
*e^2)*(d + e*x)^(5/2)) + ((2*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*
(2*C*d - B*e) - b*e*(6*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[a + b*x + c*x^2])/(3
*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^(3/2)) + ((2*(c^2*(2*C*d^4 + d^2*e*(3*B
*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^2 +
2*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(19*C*
d^2 - 29*B*d*e + 9*A*e^2)))*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2
)*Sqrt[d + e*x]) + (c*(-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c^2*(2*C*d^4 + d^2*e*
(3*B*d - 23*A*e)) - e^2*(15*a^2*C*e^2 - 10*a*b*e*(C*d + B*e) + b^2*(3*C*d^
2 + 2*B*d*e + 8*A*e^2)) - c*e*(b*d*(7*C*d^2 - 7*B*d*e - 23*A*e^2) - a*e*(1
9*C*d^2 - 29*B*d*e + 9*A*e^2)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*Sqrt[a + b*x + c*x^2])) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e +
a*e^2)*(2*c*C*d^3 + c*d*e*(3*B*d - 8*A*e) + 5*a*e^2*(2*C*d - B*e) - b*e*(6
*C*d^2 - e*(B*d + 4*A*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. 2(856) = 1712.

Time = 8.02 (sec) , antiderivative size = 1757, normalized size of antiderivative = 1.91

method	result	size
elliptic	Expression too large to display	1757
default	Expression too large to display	46697

input
```
int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2/5/e^4/
(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)/(x+d/e)^3+2/15*(4*A*b*e^3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*
B*c*d^2*e+10*C*a*d*e^2-6*C*b*d^2*e+2*C*c*d^3)/e^3/(a*e^2-b*d*e+c*d^2)^2*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(x+d/e)^2+2/15*(c*e*x^2+b*e*
x+a*e)/e^2/(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^3-2
3*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e^2+
3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*e^2
-7*C*b*c*d^3*e+2*C*c^2*d^4)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2*(1/15*c*
(4*A*b*e^3-8*A*c*d*e^2-5*B*a*e^3+B*b*d*e^2+3*B*c*d^2*e+10*C*a*d*e^2-6*C*b*
d^2*e+2*C*c*d^3)/e^2/(a*e^2-b*d*e+c*d^2)^2+1/15/e^2*(b*e-c*d)*(9*A*a*c*e^4
-8*A*b^2*e^4+23*A*b*c*d*e^3-23*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2
*B*b^2*d*e^3+7*B*b*c*d^2*e^2+3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*
C*a*c*d^2*e^2-3*C*b^2*d^2*e^2-7*C*b*c*d^3*e+2*C*c^2*d^4)/(a*e^2-b*d*e+c*d^
2)^3-1/15*b/e/(a*e^2-b*d*e+c*d^2)^3*(9*A*a*c*e^4-8*A*b^2*e^4+23*A*b*c*d*e^
3-23*A*c^2*d^2*e^2+10*B*a*b*e^4-29*B*a*c*d*e^3-2*B*b^2*d*e^3+7*B*b*c*d^2*e
^2+3*B*c^2*d^3*e-15*C*a^2*e^4+10*C*a*b*d*e^3+19*C*a*c*d^2*e^2-3*C*b^2*d^2*
e^2-7*C*b*c*d^3*e+2*C*c^2*d^4))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/
2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2656 vs. $2(864) = 1728$.

Time = 0.37 (sec) , antiderivative size = 2656, normalized size of antiderivative = 2.88

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{7/2}\sqrt{a + bx + cx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/45*((2*C*c^3*d^8 - (8*C*b*c^2 - 3*B*c^3)*d^7*e + (17*C*b^2*c + 22*A*c^3
- (2*C*a + 17*B*b)*c^2)*d^6*e^2 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C
*a*b - 8*B*b^2)*c)*d^5*e^3 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^
2 - 31*B*a*b + 27*A*b^2)*c)*d^4*e^4 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 +
 3*(5*B*a^2 - 7*A*a*b)*c)*d^3*e^5 + (2*C*c^3*d^5*e^3 - (8*C*b*c^2 - 3*B*c^
3)*d^4*e^4 + (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^3*e^5 - (3*C
*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^2*e^6 + (10*C*a*b
^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d*e^7 - (1
5*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*e^8)*x^3 + 3*(
2*C*c^3*d^6*e^2 - (8*C*b*c^2 - 3*B*c^3)*d^5*e^3 + (17*C*b^2*c + 22*A*c^3 -
 (2*C*a + 17*B*b)*c^2)*d^4*e^4 - (3*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*
a*b - 8*B*b^2)*c)*d^3*e^5 + (10*C*a*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2
 - 31*B*a*b + 27*A*b^2)*c)*d^2*e^6 - (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 +
3*(5*B*a^2 - 7*A*a*b)*c)*d*e^7)*x^2 + 3*(2*C*c^3*d^7*e - (8*C*b*c^2 - 3*B*
c^3)*d^6*e^2 + (17*C*b^2*c + 22*A*c^3 - (2*C*a + 17*B*b)*c^2)*d^5*e^3 - (3
*C*b^3 - (52*B*a - 33*A*b)*c^2 + (49*C*a*b - 8*B*b^2)*c)*d^4*e^4 + (10*C*a
*b^2 - 2*B*b^3 - 42*A*a*c^2 + (60*C*a^2 - 31*B*a*b + 27*A*b^2)*c)*d^3*e^5
- (15*C*a^2*b - 10*B*a*b^2 + 8*A*b^3 + 3*(5*B*a^2 - 7*A*a*b)*c)*d^2*e^6)*x
)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{7}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(7/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{7/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(7/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}\sqrt{a+bx+cx^2}}\,dx=\text{too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)*b + 2*int((sqrt(d + e*x)*sqrt(a
 + b*x + c*x**2)*x**2)/(2*a*b*d**4*e + 8*a*b*d**3*e**2*x + 12*a*b*d**2*e**
3*x**2 + 8*a*b*d*e**4*x**3 + 2*a*b*e**5*x**4 - a*c*d**5 - 4*a*c*d**4*e*x -
 6*a*c*d**3*e**2*x**2 - 4*a*c*d**2*e**3*x**3 - a*c*d*e**4*x**4 + 2*b**2*d*
*4*e*x + 8*b**2*d**3*e**2*x**2 + 12*b**2*d**2*e**3*x**3 + 8*b**2*d*e**4*x*
*4 + 2*b**2*e**5*x**5 - b*c*d**5*x - 2*b*c*d**4*e*x**2 + 2*b*c*d**3*e**2*x
**3 + 8*b*c*d**2*e**3*x**4 + 7*b*c*d*e**4*x**5 + 2*b*c*e**5*x**6 - c**2*d*
*5*x**2 - 4*c**2*d**4*e*x**3 - 6*c**2*d**3*e**2*x**4 - 4*c**2*d**2*e**3*x*
*5 - c**2*d*e**4*x**6),x)*b**2*c*d**3*e**2 + 6*int((sqrt(d + e*x)*sqrt(a +
 b*x + c*x**2)*x**2)/(2*a*b*d**4*e + 8*a*b*d**3*e**2*x + 12*a*b*d**2*e**3*
x**2 + 8*a*b*d*e**4*x**3 + 2*a*b*e**5*x**4 - a*c*d**5 - 4*a*c*d**4*e*x - 6
*a*c*d**3*e**2*x**2 - 4*a*c*d**2*e**3*x**3 - a*c*d*e**4*x**4 + 2*b**2*d**4
*e*x + 8*b**2*d**3*e**2*x**2 + 12*b**2*d**2*e**3*x**3 + 8*b**2*d*e**4*x**4
 + 2*b**2*e**5*x**5 - b*c*d**5*x - 2*b*c*d**4*e*x**2 + 2*b*c*d**3*e**2*x**
3 + 8*b*c*d**2*e**3*x**4 + 7*b*c*d*e**4*x**5 + 2*b*c*e**5*x**6 - c**2*d**5
*x**2 - 4*c**2*d**4*e*x**3 - 6*c**2*d**3*e**2*x**4 - 4*c**2*d**2*e**3*x**5
 - c**2*d*e**4*x**6),x)*b**2*c*d**2*e**3*x + 6*int((sqrt(d + e*x)*sqrt(a +
 b*x + c*x**2)*x**2)/(2*a*b*d**4*e + 8*a*b*d**3*e**2*x + 12*a*b*d**2*e**3*
x**2 + 8*a*b*d*e**4*x**3 + 2*a*b*e**5*x**4 - a*c*d**5 - 4*a*c*d**4*e*x - 6
*a*c*d**3*e**2*x**2 - 4*a*c*d**2*e**3*x**3 - a*c*d*e**4*x**4 + 2*b**2*d...
```

3.107 $\int (g+hx)^m (a+bx+cx^2)^p (d+ex+fx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 508

$$\int (g+hx)^m (a+bx+cx^2)^p (d+ex+fx^2)\,dx = \frac{f(g+hx)^{1+m}(a+bx+cx^2)^{1+p}}{ch(3+m+2p)}$$

$$+\frac{(fh(bg-ah)(1+m)+2cfg^2(1+p)-ch(eg-dh)(3+m+2p))(g+hx)^{1+m}(a+bx+cx^2)^p\left(1-\right.}{ch^3(}$$

$$-\frac{(2cfg(1+p)+bfh(2+m+p)-ceh(3+m+2p))(g+hx)^{2+m}(a+bx+cx^2)^p\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)}\right.}{ch^3(2+m)(}$$

output

```
f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(p+1)/c/h/(3+m+2*p)+(f*h*(-a*h+b*g)*(1+m)+2*
c*f*g^2*(p+1)-c*h*(-d*h+e*g)*(3+m+2*p))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^p*Appe
llF1(1+m,-p,-p,2+m,2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h),2*c*(h*x+g
)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h))/c/h^3/(1+m)/(3+m+2*p)/((1-2*c*(h*x+g)/
(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h))^p)/((1-2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2
)^(1/2))*h))^p)-(2*c*f*g*(p+1)+b*f*h*(2+m+p)-c*e*h*(3+m+2*p))*(h*x+g)^(2+m
)*(c*x^2+b*x+a)^p*AppellF1(2+m,-p,-p,3+m,2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2
)^(1/2))*h),2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h))/c/h^3/(2+m)/(3+m
+2*p)/((1-2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h))^p)/((1-2*c*(h*x+g)
/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h))^p)
```

Mathematica [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2184, 25, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)(g+hx)^m \left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -h(g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p-}{ch^2(m+2p+3)}$$
$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)} -$$
$$\frac{\int h(g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p+}{ch^2(m+2p+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\int (g+hx)^m(afh(m+1)+bfg(p+1)-cdh(m+2p+3)+(2cfg(p+1)+bfh(m+p+2)-ceh(m+2p+3)}{ch(m+2p+3)}$$

$\downarrow$ 1269

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\int (g+hx)^{m+1}\left(cx^2+bx+a\right)^pdx}{h}-\frac{(fh(m+1)(bg-ah)-ch(m+2p+3)(eg-dh)+2cfg^2(p+1))\int (g+l}{h}}{ch(m+2p+3)}$$

$\downarrow$ 1179

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\int (g+hx)^{m+1}\left(1-\frac{2c}{2cg-\left(b\right.}\right.}{h^2}}{}$$

$\downarrow$ 150

$$\frac{f(g+hx)^{m+1}\left(a+bx+cx^2\right)^{p+1}}{ch(m+2p+3)}-\frac{\frac{(g+hx)^{m+2}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}(bfh(m+p+2)-ceh(m+2p+3)+2cfg(p+1))\operatorname{AppellF1}\left(m\right.}{h^2(m+2)}}{}$$

input

```
Int[(g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(1 + p))/(c*h*(3 + m + 2*p)) - (-((
(f*h*(b*g - a*h)*(1 + m) + 2*c*f*g^2*(1 + p) - c*h*(e*g - d*h)*(3 + m + 2*
p))*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (
2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g
 - (b + Sqrt[b^2 - 4*a*c])*h)])/(h^2*(1 + m)*(1 - (2*c*(g + h*x))/(2*c*g -
 (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h))^p)) + ((2*c*f*g*(1 + p) + b*f*h*(2 + m + p) - c*e*h*(3 + m
+ 2*p))*(g + h*x)^(2 + m)*(a + b*x + c*x^2)^p*AppellF1[2 + m, -p, -p, 3 +
m, (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2
*c*g - (b + Sqrt[b^2 - 4*a*c])*h)])/(h^2*(2 + m)*(1 - (2*c*(g + h*x))/(2*c
*g - (b - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt
[b^2 - 4*a*c])*h))^p))/(c*h*(3 + m + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int (hx+g)^m \left(c\,x^2+bx+a\right)^p \left(f\,x^2+ex+d\right) dx$$

input
```
int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)**m*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Giac [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p (hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \int (g+h\,x)^m\,\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int (g+hx)^m \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right)\,dx = \text{too large to display}$$

input
```
int((h*x+g)^m*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
( - 2*(g + h*x)**m*(a + b*x + c*x**2)**p*a**2*c*f*h**3*m**2*p - 4*(g + h*x
)**m*(a + b*x + c*x**2)**p*a**2*c*f*h**3*m*p**2 - 6*(g + h*x)**m*(a + b*x
+ c*x**2)**p*a**2*c*f*h**3*m*p - 4*(g + h*x)**m*(a + b*x + c*x**2)**p*a**2
*c*f*h**3*p**2 - 4*(g + h*x)**m*(a + b*x + c*x**2)**p*a**2*c*f*h**3*p + (g
 + h*x)**m*(a + b*x + c*x**2)**p*a*b**2*f*h**3*m**2*p + (g + h*x)**m*(a +
b*x + c*x**2)**p*a*b**2*f*h**3*m*p**2 + 3*(g + h*x)**m*(a + b*x + c*x**2)*
*p*a*b**2*f*h**3*m*p + (g + h*x)**m*(a + b*x + c*x**2)**p*a*b**2*f*h**3*p*
*2 + 2*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b**2*f*h**3*p - (g + h*x)**m*(
a + b*x + c*x**2)**p*a*b*c*e*h**3*m**2*p - 2*(g + h*x)**m*(a + b*x + c*x**
2)**p*a*b*c*e*h**3*m*p**2 - 4*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*e*h
**3*m*p - 2*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*e*h**3*p**2 - 3*(g +
h*x)**m*(a + b*x + c*x**2)**p*a*b*c*e*h**3*p - 2*(g + h*x)**m*(a + b*x + c
*x**2)**p*a*b*c*f*g*h**2*m**2*p - 2*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b
*c*f*g*h**2*m*p**2 - 6*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*f*g*h**2*m
*p - 4*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*f*g*h**2*p**3 - 12*(g + h*
x)**m*(a + b*x + c*x**2)**p*a*b*c*f*g*h**2*p**2 - 6*(g + h*x)**m*(a + b*x
+ c*x**2)**p*a*b*c*f*g*h**2*p + 2*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c
*f*h**3*m**2*p*x + 6*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*f*h**3*m*p**
2*x + 4*(g + h*x)**m*(a + b*x + c*x**2)**p*a*b*c*f*h**3*m*p*x + 4*(g + h*x
)**m*(a + b*x + c*x**2)**p*a*b*c*f*h**3*p**3*x + 4*(g + h*x)**m*(a + b*...
```

3.108 $\int (g+hx)^m \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 494

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx=\frac{f(g+hx)^{1+m}\left(a+bx+cx^2\right)^{3/2}}{ch(4+m)}$$
$$+\frac{(3cfg^2+fh(bg-ah)(1+m)-ch(eg-dh)(4+m))\,(g+hx)^{1+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(1+m,\right.}{ch^3(1+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+}{2cg-\left(b+\sqrt{b}\right.}}}$$
$$-\frac{(6cfg-2ceh(4+m)+bfh(5+2m))(g+hx)^{2+m}\sqrt{a+bx+cx^2}\,\text{AppellF1}\left(2+m,-\frac{1}{2},-\frac{1}{2},3+m,\right.}{2ch^3(2+m)(4+m)\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}}$$

output
```
f*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(3/2)/c/h/(4+m)+(3*c*f*g^2+f*h*(-a*h+b*g)*(1
+m)-c*h*(-d*h+e*g)*(4+m))*(h*x+g)^(1+m)*(c*x^2+b*x+a)^(1/2)*AppellF1(1+m,-
1/2,-1/2,2+m,2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h),2*c*(h*x+g)/(2*c
*g-(b+(-4*a*c+b^2)^(1/2))*h))/c/h^3/(1+m)/(4+m)/(1-2*c*(h*x+g)/(2*c*g-(b-(
-4*a*c+b^2)^(1/2))*h))^(1/2)/(1-2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*
h))^(1/2)-1/2*(6*c*f*g-2*c*e*h*(4+m)+b*f*h*(5+2*m))*(h*x+g)^(2+m)*(c*x^2+b
*x+a)^(1/2)*AppellF1(2+m,-1/2,-1/2,3+m,2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^
(1/2))*h),2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h))/c/h^3/(2+m)/(4+m)/
(1-2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h))^(1/2)/(1-2*c*(h*x+g)/(2*c
*g-(b+(-4*a*c+b^2)^(1/2))*h))^(1/2)
```

Mathematica [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx \\ = \int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2), x]
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2184, 27, 1269, 1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)(g+hx)^m\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{\int -\frac{1}{2}h(g+hx)^m(3bfg+2afh(m+1)-2cdh(m+4)+(6cfg-2ceh(m+4)+bfh(2m+5))x)\sqrt{cx^2+bx+a}}{ch^2(m+4)}$$
$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}$$

$$\Big\downarrow \text{27}$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)} - \\ \frac{\int (g+hx)^m(3bfg+2afh(m+1)-2cdh(m+4)+(6cfg-2ceh(m+4)+bfh(2m+5))x)\sqrt{cx^2+bx+a}dx}{2ch(m+4)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{(bfh(2m+5)-2ceh(m+4)+6cfg)\int(g+hx)^{m+1}\sqrt{cx^2+bx+a}dx}{h}-\frac{2\left(fh(m+1)(bg-ah)-ch(m+4)(eg-dh)+3cfg^2\right)\int(g+hx)^m\sqrt{cx^2+bx+a}dx}{h}}{2ch(m+4)}$$

$\Big\downarrow$ 1179

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{\sqrt{a+bx+cx^2}(bfh(2m+5)-2ceh(m+4)+6cfg)\int(g+hx)^{m+1}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}}\sqrt{1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}}d(g+hx)}{h^2\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+bx+cx^2}(fh(m}{}}{2ch(m+4)}$$

$\Big\downarrow$ 150

$$\frac{f\left(a+bx+cx^2\right)^{3/2}(g+hx)^{m+1}}{ch(m+4)}-\frac{\frac{\sqrt{a+bx+cx^2}(g+hx)^{m+2}(bfh(2m+5)-2ceh(m+4)+6cfg)\text{AppellF1}\left(m+2,-\frac{1}{2},-\frac{1}{2},m+3,\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h},\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)}{h^2(m+2)\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{a+b}}{}}{2ch(m}$$

input
```
Int[(g + h*x)^m*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

output
```
(f*(g + h*x)^(1 + m)*(a + b*x + c*x^2)^(3/2))/(c*h*(4 + m)) - ((-2*(3*c*f*
g^2 + f*h*(b*g - a*h)*(1 + m) - c*h*(e*g - d*h)*(4 + m))*(g + h*x)^(1 + m)
*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(g + h*x))/
(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^
2 - 4*a*c])*h)])/(h^2*(1 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[
b^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c]
)*h)]) + ((6*c*f*g - 2*c*e*h*(4 + m) + b*f*h*(5 + 2*m))*(g + h*x)^(2 + m)*
Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(g + h*x))/(
2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2
 - 4*a*c])*h)])/(h^2*(2 + m)*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b - Sqrt[b
^2 - 4*a*c])*h)]*Sqrt[1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 - 4*a*c])
*h)]))/(2*c*h*(4 + m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int (hx+g)^m \sqrt{c\,x^2+bx+a}\,\left(f\,x^2+ex+d\right) dx$$

input
```
int((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas
")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Sympy [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+hx)^m\,\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$

input
```
integrate((h*x+g)**m*(c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

output
```
Integral((g + h*x)**m*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

Maxima [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima
")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Giac [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \int \sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)(hx+g)^m\,dx$$

input
```
integrate((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*(h*x + g)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx = \int {\left(g+h\,x\right)}^m\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2),x)
```

output
```
int((g + h*x)^m*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int (g+hx)^m\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\,dx$$
$$=\int (hx+g)^m\sqrt{c\,x^2+bx+a}\,\left(f\,x^2+ex+d\right)dx$$

input
```
int((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^m*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

3.109 $\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$

Optimal result

Integrand size = 34, antiderivative size = 588

$$\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) dx$$

$$= -\frac{(fg^2-h(eg-dh))\,(g+hx)^{-2(1+p)} \left(a+bx+cx^2\right)^{1+p}}{2h\left(cg^2-bgh+ah^2\right)(1+p)}$$

$$- \frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p \left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)h}\right)^{-p} \left(1-\frac{2c(g+hx)}{2cg-\left(b+\sqrt{b^2-4ac}\right)h}\right)^{-p} \mathrm{AppellF1}\left(-2p, -\right.}{2h^3p}$$

$$- \frac{(2c(fg^3-dgh^2)-h(3bfg^2-bh(eg+dh)-2ah(2fg-eh)))\left(b-\sqrt{b^2-4ac}+2cx\right)\left(\frac{\left(2cg-\left(b-\sqrt{b^2-4ac}\right.\right.}{\left(2cg-\left(b+\sqrt{b^2-4ac}\right.\right.}\right.}{2h^2\left(2cg-\left(b\right.\right.}$$

output
```
-1/2*(f*g^2-h*(-d*h+e*g))*(c*x^2+b*x+a)^(p+1)/h/(a*h^2-b*g*h+c*g^2)/(p+1)/
((h*x+g)^(2*p+2))-1/2*f*(c*x^2+b*x+a)^p*AppellF1(-2*p,-p,-p,1-2*p,2*c*(h*x
+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h),2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1
/2))*h))/h^3/p/((h*x+g)^(2*p))/((1-2*c*(h*x+g)/(2*c*g-(b-(-4*a*c+b^2)^(1/2
))*h))^p)/((1-2*c*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h))^p)-1/2*(2*c*(-
d*g*h^2+f*g^3)-h*(3*b*f*g^2-b*h*(d*h+e*g)-2*a*h*(-e*h+2*f*g)))*(b-(-4*a*c+
b^2)^(1/2)+2*c*x)*(h*x+g)^(-1-2*p)*(c*x^2+b*x+a)^p*hypergeom([-p, -1-2*p],
[-2*p],-4*c*(-4*a*c+b^2)^(1/2)*(h*x+g)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h)/(b
-(-4*a*c+b^2)^(1/2)+2*c*x))/h^2/(2*c*g-(b-(-4*a*c+b^2)^(1/2))*h)/(a*h^2-b*
g*h+c*g^2)/(1+2*p)/(((2*c*g-(b-(-4*a*c+b^2)^(1/2))*h)*(b+(-4*a*c+b^2)^(1/2
)+2*c*x)/(2*c*g-(b+(-4*a*c+b^2)^(1/2))*h)/(b-(-4*a*c+b^2)^(1/2)+2*c*x))^p)
```

Mathematica [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$

input
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
Integrate[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2), x]
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2186, 25, 1179, 150, 1228, 1155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex+fx^2\right)(g+hx)^{-2p-3}\left(a+bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \; 2186$$

$$\frac{\int -(g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}+\frac{f\int (g+hx)^{-2p-1}\left(cx^2+bx+a\right)^p dx}{h^2}$$

$\downarrow$ 25

$$\frac{f\int (g+hx)^{-2p-1}\left(cx^2+bx+a\right)^p dx}{h^2}-\frac{\int (g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}$$

$\downarrow$ 1179

$$\frac{f\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\int (g+hx)^{-2p-1}\left(1-\frac{2c(g+hx)}{2cg-\left(b-\sqrt{b^2-4ac}\right)}\right.}{h^3}$$

$$\frac{\int (g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}$$

$\downarrow$ 150

$$-\frac{\int (g+hx)^{-2p-3}\left(fg^2-dh^2+h(2fg-eh)x\right)\left(cx^2+bx+a\right)^p dx}{h^2}-$$

$$\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1\right.}{2h^3p}$$

$\downarrow$ 1228

$$-\frac{\frac{\left(2c\left(fg^3-dgh^2\right)-h\left(-2ah(2fg-eh)-bh(dh+eg)+3bfg^2\right)\right)\int (g+hx)^{-2(p+1)}\left(cx^2+bx+a\right)^p dx}{2\left(ah^2-bgh+cg^2\right)}+\frac{h(g+hx)^{-2(p+1)}\left(a+bx+cx^2\right)^{p+1}\left(fg^2-h(eg\right.}{2(p+1)\left(ah^2-bgh+cg^2\right)}}{h^2}$$

$$\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p,1\right.}{2h^3p}$$

$\downarrow$ 1155

$$-\frac{f(g+hx)^{-2p}\left(a+bx+cx^2\right)^p\left(1-\frac{2c(g+hx)}{2cg-h\left(b-\sqrt{b^2-4ac}\right)}\right)^{-p}\left(1-\frac{2c(g+hx)}{2cg-h\left(\sqrt{b^2-4ac}+b\right)}\right)^{-p}\text{AppellF1}\left(-2p,-p,-p\right.}{2h^3p}$$

$$\frac{\frac{\left(-\sqrt{b^2-4ac}+b+2cx\right)(g+hx)^{-2p-1}(a+bx+cx^2)^p\left(\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)}{\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(2cg-h\left(\sqrt{b^2-4ac}+b\right)\right)}\right)^{-p}(2c(fg^3-dgh^2)-h(-2ah(2fg-eh)-bh(dh+}{2(2p+1)\left(2cg-h\left(b-\sqrt{b^2-4ac}\right)\right)(ah^2-bgh+cg^2}}{}$$

input
```
Int[(g + h*x)^(-3 - 2*p)*(a + b*x + c*x^2)^p*(d + e*x + f*x^2),x]
```

output
```
-1/2*(f*(a + b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, (2*c*(g + h*x)
)/(2*c*g - (b - Sqrt[b^2 - 4*a*c])*h), (2*c*(g + h*x))/(2*c*g - (b + Sqrt[
b^2 - 4*a*c])*h)])/(h^3*p*(g + h*x)^(2*p)*(1 - (2*c*(g + h*x))/(2*c*g - (b
 - Sqrt[b^2 - 4*a*c])*h))^p*(1 - (2*c*(g + h*x))/(2*c*g - (b + Sqrt[b^2 -
4*a*c])*h))^p) - ((h*(f*g^2 - h*(e*g - d*h))*(a + b*x + c*x^2)^(1 + p))/(2
*(c*g^2 - b*g*h + a*h^2)*(1 + p)*(g + h*x)^(2*(1 + p))) + ((2*c*(f*g^3 - d
*g*h^2) - h*(3*b*f*g^2 - b*h*(e*g + d*h) - 2*a*h*(2*f*g - e*h)))*(b - Sqrt
[b^2 - 4*a*c] + 2*c*x)*(g + h*x)^(-1 - 2*p)*(a + b*x + c*x^2)^p*Hypergeome
tric2F1[-1 - 2*p, -p, -2*p, (-4*c*Sqrt[b^2 - 4*a*c]*(g + h*x))/((2*c*g - (
b + Sqrt[b^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))])/(2*(2*c*g - (
b - Sqrt[b^2 - 4*a*c])*h)*(c*g^2 - b*g*h + a*h^2)*(1 + 2*p)*(((2*c*g - (b
- Sqrt[b^2 - 4*a*c])*h)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/((2*c*g - (b + Sq
rt[b^2 - 4*a*c])*h)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)))^p))/h^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1155
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(b - q + 2*c*x))*(d + e*x)^
(m + 1)*((a + b*x + c*x^2)^p/((m + 1)*(2*c*d - b*e + e*q)*((2*c*d - b*e + e
*q)*((b + q + 2*c*x)/((2*c*d - b*e - e*q)*(b - q + 2*c*x))))^p))*Hypergeome
tric2F1[m + 1, -p, m + 2, -4*c*q*((d + e*x)/((2*c*d - b*e - e*q)*(b - q + 2
*c*x)))], x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[m + 2*p + 2, 0]
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2186
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> With[{q = Expon[Pq, x]}, Simp[Coeff[Pq, x, q]/e^q   Int[(
d + e*x)^(m + q)*(a + b*x + c*x^2)^p, x], x] + Simp[1/e^q   Int[(d + e*x)^m
*(a + b*x + c*x^2)^p*ExpandToSum[e^q*Pq - Coeff[Pq, x, q]*(d + e*x)^q, x],
x], x]] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ[m, 0] && RationalQ[a, b,
 c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int (hx+g)^{-3-2p} \left(c\,x^2+bx+a\right)^p \left(f\,x^2+ex+d\right) dx$$

input
```
int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

Fricas [F]

$$\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \int \left(fx^2+ex+d\right) \left(cx^2+bx+a\right)^p (hx+g)^{-2\,p-3} \, dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="fri
cas")
```

output
```
integral((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p} \left(a+bx+cx^2\right)^p \left(d+ex+fx^2\right) \, dx = \text{Timed out}$$

input
```
integrate((h*x+g)**(-3-2*p)*(c*x**2+b*x+a)**p*(f*x**2+e*x+d),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="max
ima")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Giac [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \int \left(fx^2+ex+d\right)\left(cx^2+bx+a\right)^p(hx+g)^{-2\,p-3}\,dx$$

input
```
integrate((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x, algorithm="gia
c")
```

output
```
integrate((f*x^2 + e*x + d)*(c*x^2 + b*x + a)^p*(h*x + g)^(-2*p - 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx = \int \frac{\left(c\,x^2+b\,x+a\right)^p\,\left(f\,x^2+e\,x+d\right)}{\left(g+h\,x\right)^{2\,p+3}}\,dx$$

input
```
int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3),x)
```

output
```
int(((a + b*x + c*x^2)^p*(d + e*x + f*x^2))/(g + h*x)^(2*p + 3), x)
```

Reduce [F]

$$\int (g+hx)^{-3-2p}\left(a+bx+cx^2\right)^p\left(d+ex+fx^2\right)\,dx$$
$$=\left(\int \frac{(c\,x^2+bx+a)^p}{(hx+g)^{2p}\,g^3+3\,(hx+g)^{2p}\,g^2hx+3\,(hx+g)^{2p}\,g\,h^2x^2+(hx+g)^{2p}\,h^3x^3}dx\right)d$$
$$+\left(\int \frac{(c\,x^2+bx+a)^p\,x^2}{(hx+g)^{2p}\,g^3+3\,(hx+g)^{2p}\,g^2hx+3\,(hx+g)^{2p}\,g\,h^2x^2+(hx+g)^{2p}\,h^3x^3}dx\right)f$$
$$+\left(\int \frac{(c\,x^2+bx+a)^p\,x}{(hx+g)^{2p}\,g^3+3\,(hx+g)^{2p}\,g^2hx+3\,(hx+g)^{2p}\,g\,h^2x^2+(hx+g)^{2p}\,h^3x^3}dx\right)e$$

input
```
int((h*x+g)^(-3-2*p)*(c*x^2+b*x+a)^p*(f*x^2+e*x+d),x)
```

output
```
int((a + b*x + c*x**2)**p/((g + h*x)**(2*p)*g**3 + 3*(g + h*x)**(2*p)*g**2
*h*x + 3*(g + h*x)**(2*p)*g*h**2*x**2 + (g + h*x)**(2*p)*h**3*x**3),x)*d +
 int(((a + b*x + c*x**2)**p*x**2)/((g + h*x)**(2*p)*g**3 + 3*(g + h*x)**(2
*p)*g**2*h*x + 3*(g + h*x)**(2*p)*g*h**2*x**2 + (g + h*x)**(2*p)*h**3*x**3
),x)*f + int(((a + b*x + c*x**2)**p*x)/((g + h*x)**(2*p)*g**3 + 3*(g + h*x
)**(2*p)*g**2*h*x + 3*(g + h*x)**(2*p)*g*h**2*x**2 + (g + h*x)**(2*p)*h**3
*x**3),x)*e
```

3.110 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 150

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \frac{2(bBc^2-Ac^3-b^2cC+b^3D)\,x^2}{3b^2c^2\,(bx+cx^2)^{3/2}} - \frac{2A}{b^2\sqrt{bx+cx^2}} + \frac{2(2bBc^2-8Ac^3+b^2cC-4b^3D)\,x}{3b^3c^2\sqrt{bx+cx^2}} + \frac{2D\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(-A*c^3+B*b*c^2-C*b^2*c+D*b^3)*x^2/b^2/c^2/(c*x^2+b*x)^(3/2)-2*A/b^2/(
c*x^2+b*x)^(1/2)+2/3*(-8*A*c^3+2*B*b*c^2+C*b^2*c-4*D*b^3)*x/b^3/c^2/(c*x^2
+b*x)^(1/2)+2*D*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2x\left(\sqrt{c}(Ac^2(3b^2+12bcx+8c^2x^2)+bx(3b^3D-2Bc^3x+4b^2cDx-bc^2(3B+Cx)))+3b^3D\sqrt{x}(b+cx)^{3/}\right.}{3b^3c^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x*(Sqrt[c]*(A*c^2*(3*b^2 + 12*b*c*x + 8*c^2*x^2) + b*x*(3*b^3*D - 2*B*
c^3*x + 4*b^2*c*D*x - b*c^2*(3*B + C*x))) + 3*b^3*D*Sqrt[x]*(b + c*x)^(3/2
)*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]]))/(3*b^3*c^(5/2)*(x*(b + c*x))^(
3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2165}$$

$$\frac{2x\text{PolynomialRemainder}\left[A+Bx+Cx^2+Dx^3,0,x\right]}{3b\left(bx+cx^2\right)^{3/2}}-\frac{2\int -\frac{2b\text{PolynomialRemainder}\left[Dx^3+Cx^2+Bx+A,0,x\right]}{(cx^2+bx)^{3/2}}dx}{3b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4\int \frac{\text{PolynomialRemainder}\left[Dx^3+Cx^2+Bx+A,0,x\right]}{(cx^2+bx)^{3/2}}dx}{3b}+\frac{2x\text{PolynomialRemainder}\left[A+Bx+Cx^2+Dx^3,0,x\right]}{3b\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{4\sqrt{x}\sqrt{b+cx}\int \frac{\text{PolynomialRemainder}\left[Dx^3+Cx^2+Bx+A,0,x\right]}{x^{3/2}(b+cx)^{3/2}}dx}{3b\sqrt{bx+cx^2}}+\frac{2x\text{PolynomialRemainder}\left[A+Bx+Cx^2+Dx^3,0,x\right]}{3b\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 7284}$$

$$\frac{8\sqrt{x}\sqrt{b+cx}\int\frac{\text{PolynomialRemainder}[Dx^3+Cx^2+Bx+A,0,x]}{x(b+cx)^{3/2}}d\sqrt{x}}{3b\sqrt{bx+cx^2}}+\frac{2x\text{PolynomialRemainder}\left[A+Bx+Cx^2+Dx^3,0,x\right]}{3b\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 7299$$

$$\frac{8\sqrt{x}\sqrt{b+cx}\int\frac{\text{PolynomialRemainder}[Dx^3+Cx^2+Bx+A,0,x]}{x(b+cx)^{3/2}}d\sqrt{x}}{3b\sqrt{bx+cx^2}}+\frac{2x\text{PolynomialRemainder}\left[A+Bx+Cx^2+Dx^3,0,x\right]}{3b\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(b*x + c*x^2)^(5/2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2165
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a*e + c*d*x, x], R = Pol
ynomialRemainder[Pq, a*e + c*d*x, x]}, Simp[R*(2*c*d - b*e)*(d + e*x)^m*((a
 + b*x + c*x^2)^(p + 1)/(e*(p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b
^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[
d*e*(p + 1)*(b^2 - 4*a*c)*Qx - R*(2*c*d - b*e)*(m + 2*p + 2), x], x], x]] /
; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0] && GtQ[m, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7284
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-\frac{8Dc^{\frac{3}{2}}b^{3}x^{2}}{3}-2b^{2}\left(-\frac{1}{3}Cx^{2}-Bx+A\right)c^{\frac{5}{2}}-8\left(-\frac{Bx}{6}+A\right)xbc^{\frac{7}{2}}-\frac{16Ac^{\frac{9}{2}}x^{2}}{3}+2D\left(-\sqrt{c}\,bx+\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)\sqrt{x(cx+b)}\,(c}{c^{\frac{5}{2}}(cx+b)\sqrt{x(cx+b)}\,b^{3}}$
default	$A\left(-\frac{1}{3c(cx^{2}+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^{2}\left(cx^{2}+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^{4}\sqrt{cx^{2}+bx}}\right)}{2c}\right)+B\left(-\frac{x}{2c(cx^{2}+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(cx^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(\right.}{}\right.}{}\right.$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/c^(5/2)/(x*(c*x+b))^(1/2)*(-4/3*D*c^(3/2)*b^3*x^2-b^2*(-1/3*C*x^2-B*x+A)
*c^(5/2)-4*(-1/6*B*x+A)*x*b*c^(7/2)-8/3*A*c^(9/2)*x^2+D*(-c^(1/2)*b*x+arct
anh((x*(c*x+b))^(1/2)/x/c^(1/2))*(x*(c*x+b))^(1/2)*(c*x+b))*b^3)/(c*x+b)/b
^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.37

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \left[\frac{3\,(Db^3c^2x^3+2\,Db^4cx^2+Db^5x)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-}{3\,(}\right.$$

$$-\frac{2\left(3\,(Db^3c^2x^3+2\,Db^4cx^2+Db^5x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(3\,Ab^2c^3+(4\,Db^3c^2-Cb^2c^3-2\,Bbc^4+\right.}{3\,(b^3c^5x^3+2\,b^4c^4x^2+b^5c^3x)}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(D*b^3*c^2*x^3 + 2*D*b^4*c*x^2 + D*b^5*x)*sqrt(c)*log(2*c*x + b +
2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(3*A*b^2*c^3 + (4*D*b^3*c^2 - C*b^2*c^3 -
 2*B*b*c^4 + 8*A*c^5)*x^2 + 3*(D*b^4*c - B*b^2*c^3 + 4*A*b*c^4)*x)*sqrt(c*
x^2 + b*x))/(b^3*c^5*x^3 + 2*b^4*c^4*x^2 + b^5*c^3*x), -2/3*(3*(D*b^3*c^2*
x^3 + 2*D*b^4*c*x^2 + D*b^5*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/
(c*x + b)) + (3*A*b^2*c^3 + (4*D*b^3*c^2 - C*b^2*c^3 - 2*B*b*c^4 + 8*A*c^5
)*x^2 + 3*(D*b^4*c - B*b^2*c^3 + 4*A*b*c^4)*x)*sqrt(c*x^2 + b*x))/(b^3*c^5
*x^3 + 2*b^4*c^4*x^2 + b^5*c^3*x)]
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x(A+Bx+Cx^2+Dx^3)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2 + D*x**3)/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(135) = 270$.

Time = 0.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.21

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = -\frac{1}{3}\,Dx\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,x}{\sqrt{cx^2+bx}bc}-\frac{1}{\sqrt{cx^2+bx}c^2}\right) - \frac{Cx^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{4\,Bx}{3\sqrt{cx^2+bx}b^2}+\frac{2\,Ax}{3\,(cx^2+bx)^{\frac{3}{2}}b}-\frac{4\,Dx}{3\sqrt{cx^2+bx}c^2} - \frac{Cbx}{3\,(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,Bx}{3\,(cx^2+bx)^{\frac{3}{2}}c}+\frac{2\,Cx}{3\sqrt{cx^2+bx}bc} - \frac{16\,Acx}{3\sqrt{cx^2+bx}b^3}+\frac{D\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{5}{2}}} - \frac{8\,A}{3\sqrt{cx^2+bx}b^2}+\frac{C}{3\sqrt{cx^2+bx}c^2}-\frac{2\sqrt{cx^2+bx}D}{3\,bc^2}+\frac{2\,B}{3\sqrt{cx^2+bx}bc}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*D*x*(3*x^2/((c*x^2 + b*x)^(3/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2) -
2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(sqrt(c*x^2 + b*x)*c^2)) - C*x^2/((c*x^2 +
 b*x)^(3/2)*c) + 4/3*B*x/(sqrt(c*x^2 + b*x)*b^2) + 2/3*A*x/((c*x^2 + b*x)^
(3/2)*b) - 4/3*D*x/(sqrt(c*x^2 + b*x)*c^2) - 1/3*C*b*x/((c*x^2 + b*x)^(3/2
)*c^2) - 2/3*B*x/((c*x^2 + b*x)^(3/2)*c) + 2/3*C*x/(sqrt(c*x^2 + b*x)*b*c)
 - 16/3*A*c*x/(sqrt(c*x^2 + b*x)*b^3) + D*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c))/c^(5/2) - 8/3*A/(sqrt(c*x^2 + b*x)*b^2) + 1/3*C/(sqrt(c*x^2 +
 b*x)*c^2) - 2/3*sqrt(c*x^2 + b*x)*D/(b*c^2) + 2/3*B/(sqrt(c*x^2 + b*x)*b*
c)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Dx^3+Cx^2+Bx+A)x}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)*x/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.61

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^4dx + 2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^{\cdots}}{\cdots}$$

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**4*d*x + 3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/
sqrt(b))*b**3*c*d*x**2 + 8*sqrt(c)*sqrt(b + c*x)*a*b*c**3*x + 8*sqrt(c)*sq
rt(b + c*x)*a*c**4*x**2 - 2*sqrt(c)*sqrt(b + c*x)*b**4*d*x - 2*sqrt(c)*sqr
t(b + c*x)*b**3*c*d*x**2 - 3*sqrt(x)*a*b**2*c**3 - 12*sqrt(x)*a*b*c**4*x -
 8*sqrt(x)*a*c**5*x**2 - 3*sqrt(x)*b**4*c*d*x + 3*sqrt(x)*b**3*c**3*x - 4*
sqrt(x)*b**3*c**2*d*x**2 + 3*sqrt(x)*b**2*c**4*x**2))/(3*sqrt(b + c*x)*b**
3*c**3*x*(b + c*x))
```

3.111 $\int (d+ex)^4 (a+bx+cx^2)^5 (6bd+5ae+(12cd+11b$

Optimal result

Integrand size = 47, antiderivative size = 20

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right)\,dx = (d+ex)^5 \left(a+bx+cx^2\right)^6$$

output
```
(e*x+d)^5*(c*x^2+b*x+a)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(20) = 40.

Time = 0.46 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right)\,dx = x\big(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+cx)^3(d+ex)^5 + 15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5+x^5(b+cx)^6(d+ex)^5 + a^6e\big(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\big)\big)$$

input
```
Integrate[(d + e*x)^4*(a + b*x + c*x^2)^5*(6*b*d + 5*a*e + (12*c*d + 11*b*
e)*x + 17*c*e*x^2),x]
```

output
```
x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3
*x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^
4*(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 1
0*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.021, Rules used = {2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(5ae+x(11be+12cd)+6bd+17cex^2\right) \, dx$$

$$\Big\downarrow \text{2023}$$

$$(d+ex)^5 \left(a+bx+cx^2\right)^6$$

input
```
Int[(d + e*x)^4*(a + b*x + c*x^2)^5*(6*b*d + 5*a*e + (12*c*d + 11*b*e)*x +
 17*c*e*x^2),x]
```

output
```
(d + e*x)^5*(a + b*x + c*x^2)^6
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. 2(20) = 40.

Time = 0.27 (sec) , antiderivative size = 2052, normalized size of antiderivative = 102.60

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
parallelrisch	Expression too large to display	2468
orering	Expression too large to display	2515
default	Expression too large to display	7737

input
```
int((e*x+d)^4*(c*x^2+b*x+a)^5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c*e*x^2),x
,method=_RETURNVERBOSE)
```

output
```
(5*a^6*d^4*e+6*a^5*b*d^5)*x+(10*a^6*d^3*e^2+30*a^5*b*d^4*e+6*a^5*c*d^5+15*
a^4*b^2*d^5)*x^2+(10*a^6*d^2*e^3+60*a^5*b*d^3*e^2+30*a^5*c*d^4*e+75*a^4*b^
2*d^4*e+30*a^4*b*c*d^5+20*a^3*b^3*d^5)*x^3+(5*a^6*d*e^4+60*a^5*b*d^2*e^3+6
0*a^5*c*d^3*e^2+150*a^4*b^2*d^3*e^2+150*a^4*b*c*d^4*e+15*a^4*c^2*d^5+100*a
^3*b^3*d^4*e+60*a^3*b^2*c*d^5+15*a^2*b^4*d^5)*x^4+(a^6*e^5+30*a^5*b*d*e^4+
60*a^5*c*d^2*e^3+150*a^4*b^2*d^2*e^3+300*a^4*b*c*d^3*e^2+75*a^4*c^2*d^4*e+
200*a^3*b^3*d^3*e^2+300*a^3*b^2*c*d^4*e+60*a^3*b*c^2*d^5+75*a^2*b^4*d^4*e+
60*a^2*b^3*c*d^5+6*a*b^5*d^5)*x^5+(6*a^5*b*e^5+30*a^5*c*d*e^4+75*a^4*b^2*d
*e^4+300*a^4*b*c*d^2*e^3+150*a^4*c^2*d^3*e^2+200*a^3*b^3*d^2*e^3+600*a^3*b
^2*c*d^3*e^2+300*a^3*b*c^2*d^4*e+20*a^3*c^3*d^5+150*a^2*b^4*d^3*e^2+300*a^
2*b^3*c*d^4*e+90*a^2*b^2*c^2*d^5+30*a*b^5*d^4*e+30*a*b^4*c*d^5+b^6*d^5)*x^
6+(6*a^5*c*e^5+15*a^4*b^2*e^5+150*a^4*b*c*d*e^4+150*a^4*c^2*d^2*e^3+100*a^
3*b^3*d*e^4+600*a^3*b^2*c*d^2*e^3+600*a^3*b*c^2*d^3*e^2+100*a^3*c^3*d^4*e+
150*a^2*b^4*d^2*e^3+600*a^2*b^3*c*d^3*e^2+450*a^2*b^2*c^2*d^4*e+60*a^2*b*c
^3*d^5+60*a*b^5*d^3*e^2+150*a*b^4*c*d^4*e+60*a*b^3*c^2*d^5+5*b^6*d^4*e+6*b
^5*c*d^5)*x^7+(30*a^4*b*c*e^5+75*a^4*c^2*d*e^4+20*a^3*b^3*e^5+300*a^3*b^2*
c*d*e^4+600*a^3*b*c^2*d^2*e^3+200*a^3*c^3*d^3*e^2+75*a^2*b^4*d*e^4+600*a^2
*b^3*c*d^2*e^3+900*a^2*b^2*c^2*d^3*e^2+300*a^2*b*c^3*d^4*e+15*a^2*c^4*d^5+
60*a*b^5*d^2*e^3+300*a*b^4*c*d^3*e^2+300*a*b^3*c^2*d^4*e+60*a*b^2*c^3*d^5+
10*b^6*d^3*e^2+30*b^5*c*d^4*e+15*b^4*c^2*d^5)*x^8+(15*a^4*c^2*e^5+60*a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right)\,dx$$
$$= \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x+a)^5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c*e*
x^2),x, algorithm="fricas")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.

Time = 0.15 (sec) , antiderivative size = 2281, normalized size of antiderivative = 114.05

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right) \, dx$$
$$= \text{Too large to display}$$

input
```
integrate((e*x+d)**4*(c*x**2+b*x+a)**5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c
*e*x**2),x)
```

output
```
c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*
e**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(
30*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e
**4 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5
+ 60*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**
4*c**2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d
**3*e**2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e*
*4 + 60*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3
 + 60*a*c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c*
*3*d**2*e**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x
**11*(20*a**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4
+ 150*a**2*c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 60
0*a*b**2*c**3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6
*e**5 + 30*b**5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e*
*2 + 75*b**2*c**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 1
00*a**3*c**3*d*e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 60
0*a**2*b*c**3*d**2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*
b**4*c*d*e**4 + 600*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 15
0*a*b*c**4*d**4*e + 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 +
150*b**4*c**2*d**3*e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.05 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right)\,dx$$
$$= \text{Too large to display}$$

input

```
integrate((e*x+d)^4*(c*x^2+b*x+a)^5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c*e*
x^2),x, algorithm="maxima")
```

output

```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. $2(20) = 40$.

Time = 0.20 (sec) , antiderivative size = 2467, normalized size of antiderivative = 123.35

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right) \, dx$$
$$= \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x+a)^5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c*e*
x^2),x, algorithm="giac")
```

output
```
c^6*e^5*x^17 + 5*c^6*d*e^4*x^16 + 6*b*c^5*e^5*x^16 + 10*c^6*d^2*e^3*x^15 +
 30*b*c^5*d*e^4*x^15 + 15*b^2*c^4*e^5*x^15 + 6*a*c^5*e^5*x^15 + 10*c^6*d^3
*e^2*x^14 + 60*b*c^5*d^2*e^3*x^14 + 75*b^2*c^4*d*e^4*x^14 + 30*a*c^5*d*e^4
*x^14 + 20*b^3*c^3*e^5*x^14 + 30*a*b*c^4*e^5*x^14 + 5*c^6*d^4*e*x^13 + 60*
b*c^5*d^3*e^2*x^13 + 150*b^2*c^4*d^2*e^3*x^13 + 60*a*c^5*d^2*e^3*x^13 + 10
0*b^3*c^3*d*e^4*x^13 + 150*a*b*c^4*d*e^4*x^13 + 15*b^4*c^2*e^5*x^13 + 60*a
*b^2*c^3*e^5*x^13 + 15*a^2*c^4*e^5*x^13 + c^6*d^5*x^12 + 30*b*c^5*d^4*e*x^
12 + 150*b^2*c^4*d^3*e^2*x^12 + 60*a*c^5*d^3*e^2*x^12 + 200*b^3*c^3*d^2*e^
3*x^12 + 300*a*b*c^4*d^2*e^3*x^12 + 75*b^4*c^2*d*e^4*x^12 + 300*a*b^2*c^3*
d*e^4*x^12 + 75*a^2*c^4*d*e^4*x^12 + 6*b^5*c*e^5*x^12 + 60*a*b^3*c^2*e^5*x
^12 + 60*a^2*b*c^3*e^5*x^12 + 6*b*c^5*d^5*x^11 + 75*b^2*c^4*d^4*e*x^11 + 3
0*a*c^5*d^4*e*x^11 + 200*b^3*c^3*d^3*e^2*x^11 + 300*a*b*c^4*d^3*e^2*x^11 +
 150*b^4*c^2*d^2*e^3*x^11 + 600*a*b^2*c^3*d^2*e^3*x^11 + 150*a^2*c^4*d^2*e
^3*x^11 + 30*b^5*c*d*e^4*x^11 + 300*a*b^3*c^2*d*e^4*x^11 + 300*a^2*b*c^3*d
*e^4*x^11 + b^6*e^5*x^11 + 30*a*b^4*c*e^5*x^11 + 90*a^2*b^2*c^2*e^5*x^11 +
 20*a^3*c^3*e^5*x^11 + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 100*b^3*c^
3*d^4*e*x^10 + 150*a*b*c^4*d^4*e*x^10 + 150*b^4*c^2*d^3*e^2*x^10 + 600*a*b
^2*c^3*d^3*e^2*x^10 + 150*a^2*c^4*d^3*e^2*x^10 + 60*b^5*c*d^2*e^3*x^10 + 6
00*a*b^3*c^2*d^2*e^3*x^10 + 600*a^2*b*c^3*d^2*e^3*x^10 + 5*b^6*d*e^4*x^10
+ 150*a*b^4*c*d*e^4*x^10 + 450*a^2*b^2*c^2*d*e^4*x^10 + 100*a^3*c^3*d*e...
```

Mupad [B] (verification not implemented)

Time = 17.55 (sec) , antiderivative size = 2026, normalized size of antiderivative = 101.30

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right) \, dx$$
$$= \text{Too large to display}$$

input
```
int((d + e*x)^4*(a + b*x + c*x^2)^5*(5*a*e + 6*b*d + x*(11*b*e + 12*c*d) +
 17*c*e*x^2),x)
```

output
```
x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^
2*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^
2 + 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e
 + 300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^
11*(b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2
*c^2*e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3
 + 30*a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2
+ 300*a*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5
*(a^6*e^5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4
*d^4*e + 75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a
^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^
2) + x^3*(20*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^
3*e^2 + 30*a^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 6
0*a*b^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 +
 75*b^4*c^2*d*e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d
^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b
^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*
d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b
^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 +
 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 2459, normalized size of antiderivative = 122.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^5 \left(6bd+5ae+(12cd+11be)x+17cex^2\right) \, dx$$
$$= \text{Too large to display}$$

input
```
int((e*x+d)^4*(c*x^2+b*x+a)^5*(6*b*d+5*a*e+(11*b*e+12*c*d)*x+17*c*e*x^2),x
)
```

output
```
x*(5*a**6*d**4*e + 10*a**6*d**3*e**2*x + 10*a**6*d**2*e**3*x**2 + 5*a**6*d
*e**4*x**3 + a**6*e**5*x**4 + 6*a**5*b*d**5 + 30*a**5*b*d**4*e*x + 60*a**5
*b*d**3*e**2*x**2 + 60*a**5*b*d**2*e**3*x**3 + 30*a**5*b*d*e**4*x**4 + 6*a
**5*b*e**5*x**5 + 6*a**5*c*d**5*x + 30*a**5*c*d**4*e*x**2 + 60*a**5*c*d**3
*e**2*x**3 + 60*a**5*c*d**2*e**3*x**4 + 30*a**5*c*d*e**4*x**5 + 6*a**5*c*e
**5*x**6 + 15*a**4*b**2*d**5*x + 75*a**4*b**2*d**4*e*x**2 + 150*a**4*b**2*
d**3*e**2*x**3 + 150*a**4*b**2*d**2*e**3*x**4 + 75*a**4*b**2*d*e**4*x**5 +
 15*a**4*b**2*e**5*x**6 + 30*a**4*b*c*d**5*x**2 + 150*a**4*b*c*d**4*e*x**3
 + 300*a**4*b*c*d**3*e**2*x**4 + 300*a**4*b*c*d**2*e**3*x**5 + 150*a**4*b*
c*d*e**4*x**6 + 30*a**4*b*c*e**5*x**7 + 15*a**4*c**2*d**5*x**3 + 75*a**4*c
**2*d**4*e*x**4 + 150*a**4*c**2*d**3*e**2*x**5 + 150*a**4*c**2*d**2*e**3*x
**6 + 75*a**4*c**2*d*e**4*x**7 + 15*a**4*c**2*e**5*x**8 + 20*a**3*b**3*d**
5*x**2 + 100*a**3*b**3*d**4*e*x**3 + 200*a**3*b**3*d**3*e**2*x**4 + 200*a*
*3*b**3*d**2*e**3*x**5 + 100*a**3*b**3*d*e**4*x**6 + 20*a**3*b**3*e**5*x**
7 + 60*a**3*b**2*c*d**5*x**3 + 300*a**3*b**2*c*d**4*e*x**4 + 600*a**3*b**2
*c*d**3*e**2*x**5 + 600*a**3*b**2*c*d**2*e**3*x**6 + 300*a**3*b**2*c*d*e**
4*x**7 + 60*a**3*b**2*c*e**5*x**8 + 60*a**3*b*c**2*d**5*x**4 + 300*a**3*b*
c**2*d**4*e*x**5 + 600*a**3*b*c**2*d**3*e**2*x**6 + 600*a**3*b*c**2*d**2*e
**3*x**7 + 300*a**3*b*c**2*d*e**4*x**8 + 60*a**3*b*c**2*e**5*x**9 + 20*a**
3*c**3*d**5*x**5 + 100*a**3*c**3*d**4*e*x**6 + 200*a**3*c**3*d**3*e**2*...
```

3.112 $\int (d+ex)^3 (a+bx+cx^2)^5 (d(6bd+5ae)+(12cd^2+$

Optimal result

Integrand size = 75, antiderivative size = 20

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) dx = (d+ex)^5 \left(a+bx+cx^2\right)^6$$

output
```
(e*x+d)^5*(c*x^2+b*x+a)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(20) = 40.

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) dx = x\left(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5 + 20a^3x^2(b+cx)^3(d+ex)^5+15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5 + x^5(b+cx)^6(d+ex)^5+a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)\right)$$

input
```
Integrate[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 +
 17*b*d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

output
```
x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3
*x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^
4*(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 1
0*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.027, Rules used = {2155, 2023}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3\left(a+bx+cx^2\right)^5\left(x\left(5ae^2+17bde+12cd^2\right)+d(5ae+6bd)+ex^2(11be+29cd)+17ce^2x^3\right)dx$$

$$\Big\downarrow \text{2155}$$

$$\int (d+ex)^4\left(a+bx+cx^2\right)^5\left(5ae+x(11be+12cd)+6bd+17cex^2\right)dx$$

$$\Big\downarrow \text{2023}$$

$$(d+ex)^5\left(a+bx+cx^2\right)^6$$

input
```
Int[(d + e*x)^3*(a + b*x + c*x^2)^5*(d*(6*b*d + 5*a*e) + (12*c*d^2 + 17*b*
d*e + 5*a*e^2)*x + e*(29*c*d + 11*b*e)*x^2 + 17*c*e^2*x^3),x]
```

output
```
(d + e*x)^5*(a + b*x + c*x^2)^6
```

Defintions of rubi rules used

rule 2023
```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq
^(m + 1)*(Rr^(n + 1)/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]))
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

rule 2155
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. $2(20) = 40$.

Time = 0.24 (sec) , antiderivative size = 2052, normalized size of antiderivative = 102.60

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
parallelrisch	Expression too large to display	2468
orering	Expression too large to display	2550
default	Expression too large to display	8419

input
```
int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)
*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x,method=_RETURNVERBOSE)
```

output
```
(5*a^6*d^4*e+6*a^5*b*d^5)*x+(10*a^6*d^3*e^2+30*a^5*b*d^4*e+6*a^5*c*d^5+15*
a^4*b^2*d^5)*x^2+(10*a^6*d^2*e^3+60*a^5*b*d^3*e^2+30*a^5*c*d^4*e+75*a^4*b^
2*d^4*e+30*a^4*b*c*d^5+20*a^3*b^3*d^5)*x^3+(5*a^6*d*e^4+60*a^5*b*d^2*e^3+6
0*a^5*c*d^3*e^2+150*a^4*b^2*d^3*e^2+150*a^4*b*c*d^4*e+15*a^4*c^2*d^5+100*a
^3*b^3*d^4*e+60*a^3*b^2*c*d^5+15*a^2*b^4*d^5)*x^4+(a^6*e^5+30*a^5*b*d*e^4+
60*a^5*c*d^2*e^3+150*a^4*b^2*d^2*e^3+300*a^4*b*c*d^3*e^2+75*a^4*c^2*d^4*e+
200*a^3*b^3*d^3*e^2+300*a^3*b^2*c*d^4*e+60*a^3*b*c^2*d^5+75*a^2*b^4*d^4*e+
60*a^2*b^3*c*d^5+6*a*b^5*d^5)*x^5+(6*a^5*b*e^5+30*a^5*c*d*e^4+75*a^4*b^2*d
*e^4+300*a^4*b*c*d^2*e^3+150*a^4*c^2*d^3*e^2+200*a^3*b^3*d^2*e^3+600*a^3*b
^2*c*d^3*e^2+300*a^3*b*c^2*d^4*e+20*a^3*c^3*d^5+150*a^2*b^4*d^3*e^2+300*a^
2*b^3*c*d^4*e+90*a^2*b^2*c^2*d^5+30*a*b^5*d^4*e+30*a*b^4*c*d^5+b^6*d^5)*x^
6+(6*a^5*c*e^5+15*a^4*b^2*e^5+150*a^4*b*c*d*e^4+150*a^4*c^2*d^2*e^3+100*a^
3*b^3*d*e^4+600*a^3*b^2*c*d^2*e^3+600*a^3*b*c^2*d^3*e^2+100*a^3*c^3*d^4*e+
150*a^2*b^4*d^2*e^3+600*a^2*b^3*c*d^3*e^2+450*a^2*b^2*c^2*d^4*e+60*a^2*b*c
^3*d^5+60*a*b^5*d^3*e^2+150*a*b^4*c*d^4*e+60*a*b^3*c^2*d^5+5*b^6*d^4*e+6*b
^5*c*d^5)*x^7+(30*a^4*b*c*e^5+75*a^4*c^2*d*e^4+20*a^3*b^3*e^5+300*a^3*b^2*
c*d*e^4+600*a^3*b*c^2*d^2*e^3+200*a^3*c^3*d^3*e^2+75*a^2*b^4*d*e^4+600*a^2
*b^3*c*d^2*e^3+900*a^2*b^2*c^2*d^3*e^2+300*a^2*b*c^3*d^4*e+15*a^2*c^4*d^5+
60*a*b^5*d^2*e^3+300*a*b^4*c*d^3*e^2+300*a*b^3*c^2*d^4*e+60*a*b^2*c^3*d^5+
10*b^6*d^3*e^2+30*b^5*c*d^4*e+15*b^4*c^2*d^5)*x^8+(15*a^4*c^2*e^5+60*a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3 (a+bx+cx^2)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="fricas")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.

Time = 0.15 (sec) , antiderivative size = 2281, normalized size of antiderivative = 114.05

$$\int (d+ex)^3 (a+bx+cx^2)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x+a)**5*(d*(5*a*e+6*b*d)+(5*a*e**2+17*b*d*e
+12*c*d**2)*x+e*(11*b*e+29*c*d)*x**2+17*c*e**2*x**3),x)
```

output
```
c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*
e**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(
30*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e
**4 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5
+ 60*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**
4*c**2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d
**3*e**2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e*
*4 + 60*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3
 + 60*a*c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c*
*3*d**2*e**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x
**11*(20*a**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4
+ 150*a**2*c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 60
0*a*b**2*c**3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6
*e**5 + 30*b**5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e*
*2 + 75*b**2*c**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 1
00*a**3*c**3*d*e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 60
0*a**2*b*c**3*d**2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*
b**4*c*d*e**4 + 600*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 15
0*a*b*c**4*d**4*e + 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 +
150*b**4*c**2*d**3*e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="maxima")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. $2(20) = 40$.

Time = 0.20 (sec) , antiderivative size = 2467, normalized size of antiderivative = 123.35

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*
c*d^2)*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x, algorithm="giac")
```

output
```
c^6*e^5*x^17 + 5*c^6*d*e^4*x^16 + 6*b*c^5*e^5*x^16 + 10*c^6*d^2*e^3*x^15 +
 30*b*c^5*d*e^4*x^15 + 15*b^2*c^4*e^5*x^15 + 6*a*c^5*e^5*x^15 + 10*c^6*d^3
*e^2*x^14 + 60*b*c^5*d^2*e^3*x^14 + 75*b^2*c^4*d*e^4*x^14 + 30*a*c^5*d*e^4
*x^14 + 20*b^3*c^3*e^5*x^14 + 30*a*b*c^4*e^5*x^14 + 5*c^6*d^4*e*x^13 + 60*
b*c^5*d^3*e^2*x^13 + 150*b^2*c^4*d^2*e^3*x^13 + 60*a*c^5*d^2*e^3*x^13 + 10
0*b^3*c^3*d*e^4*x^13 + 150*a*b*c^4*d*e^4*x^13 + 15*b^4*c^2*e^5*x^13 + 60*a
*b^2*c^3*e^5*x^13 + 15*a^2*c^4*e^5*x^13 + c^6*d^5*x^12 + 30*b*c^5*d^4*e*x^
12 + 150*b^2*c^4*d^3*e^2*x^12 + 60*a*c^5*d^3*e^2*x^12 + 200*b^3*c^3*d^2*e^
3*x^12 + 300*a*b*c^4*d^2*e^3*x^12 + 75*b^4*c^2*d*e^4*x^12 + 300*a*b^2*c^3*
d*e^4*x^12 + 75*a^2*c^4*d*e^4*x^12 + 6*b^5*c*e^5*x^12 + 60*a*b^3*c^2*e^5*x
^12 + 60*a^2*b*c^3*e^5*x^12 + 6*b*c^5*d^5*x^11 + 75*b^2*c^4*d^4*e*x^11 + 3
0*a*c^5*d^4*e*x^11 + 200*b^3*c^3*d^3*e^2*x^11 + 300*a*b*c^4*d^3*e^2*x^11 +
 150*b^4*c^2*d^2*e^3*x^11 + 600*a*b^2*c^3*d^2*e^3*x^11 + 150*a^2*c^4*d^2*e
^3*x^11 + 30*b^5*c*d*e^4*x^11 + 300*a*b^3*c^2*d*e^4*x^11 + 300*a^2*b*c^3*d
*e^4*x^11 + b^6*e^5*x^11 + 30*a*b^4*c*e^5*x^11 + 90*a^2*b^2*c^2*e^5*x^11 +
 20*a^3*c^3*e^5*x^11 + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 100*b^3*c^
3*d^4*e*x^10 + 150*a*b*c^4*d^4*e*x^10 + 150*b^4*c^2*d^3*e^2*x^10 + 600*a*b
^2*c^3*d^3*e^2*x^10 + 150*a^2*c^4*d^3*e^2*x^10 + 60*b^5*c*d^2*e^3*x^10 + 6
00*a*b^3*c^2*d^2*e^3*x^10 + 600*a^2*b*c^3*d^2*e^3*x^10 + 5*b^6*d*e^4*x^10
+ 150*a*b^4*c*d*e^4*x^10 + 450*a^2*b^2*c^2*d*e^4*x^10 + 100*a^3*c^3*d*e...
```

Mupad [B] (verification not implemented)

Time = 17.64 (sec) , antiderivative size = 2026, normalized size of antiderivative = 101.30

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x + e(29cd+11be)x^2+17ce^2x^3\right) dx = \text{Too large to display}$$

input
```
int((d + e*x)^3*(a + b*x + c*x^2)^5*(d*(5*a*e + 6*b*d) + x*(5*a*e^2 + 12*c
*d^2 + 17*b*d*e) + e*x^2*(11*b*e + 29*c*d) + 17*c*e^2*x^3),x)
```

output
```
x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^
2*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^
2 + 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e
 + 300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^
11*(b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2
*c^2*e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3
 + 30*a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2
+ 300*a*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5
*(a^6*e^5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4
*d^4*e + 75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a
^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^
2) + x^3*(20*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^
3*e^2 + 30*a^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 6
0*a*b^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 +
 75*b^4*c^2*d*e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d
^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b
^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*
d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b
^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 +
 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 2459, normalized size of antiderivative = 122.95

$$\int (d+ex)^3 \left(a+bx+cx^2\right)^5 \left(d(6bd+5ae)+\left(12cd^2+17bde+5ae^2\right)x+e(29cd+11be)x^2+17ce^2x^3\right) dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(c*x^2+b*x+a)^5*(d*(5*a*e+6*b*d)+(5*a*e^2+17*b*d*e+12*c*d^2)
*x+e*(11*b*e+29*c*d)*x^2+17*c*e^2*x^3),x)
```

output

```
x*(5*a**6*d**4*e + 10*a**6*d**3*e**2*x + 10*a**6*d**2*e**3*x**2 + 5*a**6*d
*e**4*x**3 + a**6*e**5*x**4 + 6*a**5*b*d**5 + 30*a**5*b*d**4*e*x + 60*a**5
*b*d**3*e**2*x**2 + 60*a**5*b*d**2*e**3*x**3 + 30*a**5*b*d*e**4*x**4 + 6*a
**5*b*e**5*x**5 + 6*a**5*c*d**5*x + 30*a**5*c*d**4*e*x**2 + 60*a**5*c*d**3
*e**2*x**3 + 60*a**5*c*d**2*e**3*x**4 + 30*a**5*c*d*e**4*x**5 + 6*a**5*c*e
**5*x**6 + 15*a**4*b**2*d**5*x + 75*a**4*b**2*d**4*e*x**2 + 150*a**4*b**2*
d**3*e**2*x**3 + 150*a**4*b**2*d**2*e**3*x**4 + 75*a**4*b**2*d*e**4*x**5 +
 15*a**4*b**2*e**5*x**6 + 30*a**4*b*c*d**5*x**2 + 150*a**4*b*c*d**4*e*x**3
 + 300*a**4*b*c*d**3*e**2*x**4 + 300*a**4*b*c*d**2*e**3*x**5 + 150*a**4*b*
c*d*e**4*x**6 + 30*a**4*b*c*e**5*x**7 + 15*a**4*c**2*d**5*x**3 + 75*a**4*c
**2*d**4*e*x**4 + 150*a**4*c**2*d**3*e**2*x**5 + 150*a**4*c**2*d**2*e**3*x
**6 + 75*a**4*c**2*d*e**4*x**7 + 15*a**4*c**2*e**5*x**8 + 20*a**3*b**3*d**
5*x**2 + 100*a**3*b**3*d**4*e*x**3 + 200*a**3*b**3*d**3*e**2*x**4 + 200*a*
*3*b**3*d**2*e**3*x**5 + 100*a**3*b**3*d*e**4*x**6 + 20*a**3*b**3*e**5*x**
7 + 60*a**3*b**2*c*d**5*x**3 + 300*a**3*b**2*c*d**4*e*x**4 + 600*a**3*b**2
*c*d**3*e**2*x**5 + 600*a**3*b**2*c*d**2*e**3*x**6 + 300*a**3*b**2*c*d*e**
4*x**7 + 60*a**3*b**2*c*e**5*x**8 + 60*a**3*b*c**2*d**5*x**4 + 300*a**3*b*
c**2*d**4*e*x**5 + 600*a**3*b*c**2*d**3*e**2*x**6 + 600*a**3*b*c**2*d**2*e
**3*x**7 + 300*a**3*b*c**2*d*e**4*x**8 + 60*a**3*b*c**2*e**5*x**9 + 20*a**
3*c**3*d**5*x**5 + 100*a**3*c**3*d**4*e*x**6 + 200*a**3*c**3*d**3*e**2*...
```

3.113 $\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+\right.\right.$

Optimal result

Integrand size = 97, antiderivative size = 20

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x+\left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx=(d+ex)^5\left(a+bx+cx^2\right)^6$$

output
```
(e*x+d)^5*(c*x^2+b*x+a)^6
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 167 vs. 2(20) = 40.

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 8.35

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x+\left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx=x\left(6a^5(b+cx)(d+ex)^5+15a^4x(b+cx)^2(d+ex)^5+20a^3x^2(b+cx)^3(d+ex)^5+15a^2x^3(b+cx)^4(d+ex)^5+6ax^4(b+cx)^5(d+ex)^5+x^5(b+cx)^6(d+ex)^5+a^6e\left(5d^4+10d^3ex+10d^2e^2x^2+5de^3x^3+e^4x^4\right)\right)$$

input
```
Integrate[(d + e*x)^4*(a + b*x + c*x^2)^4*(a*(6*b*d + 5*a*e) + 2*(3*b^2*d
+ 6*a*c*d + 8*a*b*e)*x + (18*b*c*d + 11*b^2*e + 22*a*c*e)*x^2 + 4*c*(3*c*d
 + 7*b*e)*x^3 + 17*c^2*e*x^4),x]
```

output
```
x*(6*a^5*(b + c*x)*(d + e*x)^5 + 15*a^4*x*(b + c*x)^2*(d + e*x)^5 + 20*a^3
*x^2*(b + c*x)^3*(d + e*x)^5 + 15*a^2*x^3*(b + c*x)^4*(d + e*x)^5 + 6*a*x^
4*(b + c*x)^5*(d + e*x)^5 + x^5*(b + c*x)^6*(d + e*x)^5 + a^6*e*(5*d^4 + 1
0*d^3*e*x + 10*d^2*e^2*x^2 + 5*d*e^3*x^3 + e^4*x^4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 943 vs. 2(20) = 40.

Time = 5.88 (sec) , antiderivative size = 943, normalized size of antiderivative = 47.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ ex)^4 \left(a+bx+cx^2\right)^4 \left(x^2\left(22ace+11b^2e+18bcd\right)+2x\left(8abe+6acd+3b^2d\right)+a(5ae+6bd)+4cx^3(7be+3cd)\right.$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{17c^6(d+ex)^{16}}{e^{11}}-\frac{96c^5(2cd-be)(d+ex)^{15}}{e^{11}}+\frac{45c^4\left(22c^2d^2+5b^2e^2-2ce(11bd-ae)\right)(d+ex)^{14}}{e^{11}}+\frac{140c^3(2\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{c^6(d+ex)^{17}}{e^{12}}-\frac{6c^5(2cd-be)(d+ex)^{16}}{e^{12}}+\frac{3c^4\left(22c^2d^2+5b^2e^2-2ce(11bd-ae)\right)(d+ex)^{15}}{e^{12}}-\frac{10c^3(2cd-be)\left(11c^2d^2+2b^2e^2-ce(11bd-3ae)\right)(d+ex)^{14}}{e^{12}}+\frac{15c^2\left(33c^4d^4-6c^3e(11bd-3ae)d^2+b^4e^4-4b^2ce^3(3bd-ae)+c^2e^2\left(45b^2d^2-18abed+a^2e^2\right)\right)(d+ex)^{13}}{e^{12}}-\frac{6c(2cd-be)\left(66c^4d^4-12c^3e(11bd-5ae)d^2+b^4e^4-2b^2ce^3(9bd-5ae)+2c^2e^2\left(42b^2d^2-30abed+5a^2e^2\right)\right)(d-}{e^{12}}$$

$$\frac{\left(924c^6d^6-252c^5e(11bd-5ae)d^4+210c^4e^2\left(15b^2d^2-12abed+2a^2e^2\right)d^2+b^6e^6-6b^4ce^5(7bd-5ae)+30b^2c^2e}{e^{12}}$$

$$\frac{6(2cd-be)\left(cd^2-bed+ae^2\right)\left(66c^4d^4-12c^3e(11bd-5ae)d^2+b^4e^4-2b^2ce^3(9bd-5ae)+2c^2e^2\left(42b^2d^2-30ab\right.\right.}{e^{12}}$$

$$\frac{15\left(cd^2-bed+ae^2\right)^2\left(33c^4d^4-6c^3e(11bd-3ae)d^2+b^4e^4-4b^2ce^3(3bd-ae)+c^2e^2\left(45b^2d^2-18abed+a^2e^2\right)\right)}{e^{12}}$$

$$\frac{10(2cd-be)\left(cd^2-bed+ae^2\right)^3\left(11c^2d^2+2b^2e^2-ce(11bd-3ae)\right)(d+ex)^8}{e^{12}}+\frac{3\left(cd^2-bed+ae^2\right)^4\left(22c^2d^2+5b^2e^2-2ce(11bd-ae)\right)(d+ex)^7}{e^{12}}-\frac{6(2cd-be)\left(cd^2-bed+ae^2\right)^5(d+ex)^6}{e^{12}}+\frac{\left(cd^2-bed+ae^2\right)^6(d+ex)^5}{e^{12}}$$

input
```
Int[(d + e*x)^4*(a + b*x + c*x^2)^4*(a*(6*b*d + 5*a*e) + 2*(3*b^2*d + 6*a*
c*d + 8*a*b*e)*x + (18*b*c*d + 11*b^2*e + 22*a*c*e)*x^2 + 4*c*(3*c*d + 7*b
*e)*x^3 + 17*c^2*e*x^4),x]
```

output
```
((c*d^2 - b*d*e + a*e^2)^6*(d + e*x)^5)/e^12 - (6*(2*c*d - b*e)*(c*d^2 - b
*d*e + a*e^2)^5*(d + e*x)^6)/e^12 + (3*(c*d^2 - b*d*e + a*e^2)^4*(22*c^2*d
^2 + 5*b^2*e^2 - 2*c*e*(11*b*d - a*e))*(d + e*x)^7)/e^12 - (10*(2*c*d - b*
e)*(c*d^2 - b*d*e + a*e^2)^3*(11*c^2*d^2 + 2*b^2*e^2 - c*e*(11*b*d - 3*a*e
))*(d + e*x)^8)/e^12 + (15*(c*d^2 - b*d*e + a*e^2)^2*(33*c^4*d^4 + b^4*e^4
 - 6*c^3*d^2*e*(11*b*d - 3*a*e) - 4*b^2*c*e^3*(3*b*d - a*e) + c^2*e^2*(45*
b^2*d^2 - 18*a*b*d*e + a^2*e^2))*(d + e*x)^9)/e^12 - (6*(2*c*d - b*e)*(c*d
^2 - b*d*e + a*e^2)*(66*c^4*d^4 + b^4*e^4 - 2*b^2*c*e^3*(9*b*d - 5*a*e) -
12*c^3*d^2*e*(11*b*d - 5*a*e) + 2*c^2*e^2*(42*b^2*d^2 - 30*a*b*d*e + 5*a^2
*e^2))*(d + e*x)^10)/e^12 + ((924*c^6*d^6 + b^6*e^6 - 6*b^4*c*e^5*(7*b*d -
 5*a*e) - 252*c^5*d^4*e*(11*b*d - 5*a*e) + 210*c^4*d^2*e^2*(15*b^2*d^2 - 1
2*a*b*d*e + 2*a^2*e^2) + 30*b^2*c^2*e^4*(14*b^2*d^2 - 14*a*b*d*e + 3*a^2*e
^2) - 20*c^3*e^3*(84*b^3*d^3 - 84*a*b^2*d^2*e + 21*a^2*b*d*e^2 - a^3*e^3))
*(d + e*x)^11)/e^12 - (6*c*(2*c*d - b*e)*(66*c^4*d^4 + b^4*e^4 - 2*b^2*c*e
^3*(9*b*d - 5*a*e) - 12*c^3*d^2*e*(11*b*d - 5*a*e) + 2*c^2*e^2*(42*b^2*d^2
 - 30*a*b*d*e + 5*a^2*e^2))*(d + e*x)^12)/e^12 + (15*c^2*(33*c^4*d^4 + b^4
*e^4 - 6*c^3*d^2*e*(11*b*d - 3*a*e) - 4*b^2*c*e^3*(3*b*d - a*e) + c^2*e^2*
(45*b^2*d^2 - 18*a*b*d*e + a^2*e^2))*(d + e*x)^13)/e^12 - (10*c^3*(2*c*d -
 b*e)*(11*c^2*d^2 + 2*b^2*e^2 - c*e*(11*b*d - 3*a*e))*(d + e*x)^14)/e^12 +
 (3*c^4*(22*c^2*d^2 + 5*b^2*e^2 - 2*c*e*(11*b*d - a*e))*(d + e*x)^15)/e...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2051 vs. $2(20) = 40$.

Time = 0.31 (sec) , antiderivative size = 2052, normalized size of antiderivative = 102.60

method	result	size
norman	Expression too large to display	2052
gosper	Expression too large to display	2460
risch	Expression too large to display	2468
parallelrisch	Expression too large to display	2468
orering	Expression too large to display	2577
default	Expression too large to display	5359

input
```
int((e*x+d)^4*(c*x^2+b*x+a)^4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d+3*b^2*d)
*x+(22*a*c*e+11*b^2*e+18*b*c*d)*x^2+4*c*(7*b*e+3*c*d)*x^3+17*c^2*e*x^4),x,
method=_RETURNVERBOSE)
```

output
```
(5*a^6*d^4*e+6*a^5*b*d^5)*x+(10*a^6*d^3*e^2+30*a^5*b*d^4*e+6*a^5*c*d^5+15*
a^4*b^2*d^5)*x^2+(10*a^6*d^2*e^3+60*a^5*b*d^3*e^2+30*a^5*c*d^4*e+75*a^4*b^
2*d^4*e+30*a^4*b*c*d^5+20*a^3*b^3*d^5)*x^3+(5*a^6*d*e^4+60*a^5*b*d^2*e^3+6
0*a^5*c*d^3*e^2+150*a^4*b^2*d^3*e^2+150*a^4*b*c*d^4*e+15*a^4*c^2*d^5+100*a
^3*b^3*d^4*e+60*a^3*b^2*c*d^5+15*a^2*b^4*d^5)*x^4+(a^6*e^5+30*a^5*b*d*e^4+
60*a^5*c*d^2*e^3+150*a^4*b^2*d^2*e^3+300*a^4*b*c*d^3*e^2+75*a^4*c^2*d^4*e+
200*a^3*b^3*d^3*e^2+300*a^3*b^2*c*d^4*e+60*a^3*b*c^2*d^5+75*a^2*b^4*d^4*e+
60*a^2*b^3*c*d^5+6*a*b^5*d^5)*x^5+(6*a^5*b*e^5+30*a^5*c*d*e^4+75*a^4*b^2*d
*e^4+300*a^4*b*c*d^2*e^3+150*a^4*c^2*d^3*e^2+200*a^3*b^3*d^2*e^3+600*a^3*b
^2*c*d^3*e^2+300*a^3*b*c^2*d^4*e+20*a^3*c^3*d^5+150*a^2*b^4*d^3*e^2+300*a^
2*b^3*c*d^4*e+90*a^2*b^2*c^2*d^5+30*a*b^5*d^4*e+30*a*b^4*c*d^5+b^6*d^5)*x^
6+(6*a^5*c*e^5+15*a^4*b^2*e^5+150*a^4*b*c*d*e^4+150*a^4*c^2*d^2*e^3+100*a^
3*b^3*d*e^4+600*a^3*b^2*c*d^2*e^3+600*a^3*b*c^2*d^3*e^2+100*a^3*c^3*d^4*e+
150*a^2*b^4*d^2*e^3+600*a^2*b^3*c*d^3*e^2+450*a^2*b^2*c^2*d^4*e+60*a^2*b*c
^3*d^5+60*a*b^5*d^3*e^2+150*a*b^4*c*d^4*e+60*a*b^3*c^2*d^5+5*b^6*d^4*e+6*b
^5*c*d^5)*x^7+(30*a^4*b*c*e^5+75*a^4*c^2*d*e^4+20*a^3*b^3*e^5+300*a^3*b^2*
c*d*e^4+600*a^3*b*c^2*d^2*e^3+200*a^3*c^3*d^3*e^2+75*a^2*b^4*d*e^4+600*a^2
*b^3*c*d^2*e^3+900*a^2*b^2*c^2*d^3*e^2+300*a^2*b*c^3*d^4*e+15*a^2*c^4*d^5+
60*a*b^5*d^2*e^3+300*a*b^4*c*d^3*e^2+300*a*b^3*c^2*d^4*e+60*a*b^2*c^3*d^5+
10*b^6*d^3*e^2+30*b^5*c*d^4*e+15*b^4*c^2*d^5)*x^8+(15*a^4*c^2*e^5+60*a^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x+a)^4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d+3*
b^2*d)*x+(22*a*c*e+11*b^2*e+18*b*c*d)*x^2+4*c*(7*b*e+3*c*d)*x^3+17*c^2*e*x
^4),x, algorithm="fricas")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(17) = 34$.

Time = 0.15 (sec) , antiderivative size = 2281, normalized size of antiderivative = 114.05

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**4*(c*x**2+b*x+a)**4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d
+3*b**2*d)*x+(22*a*c*e+11*b**2*e+18*b*c*d)*x**2+4*c*(7*b*e+3*c*d)*x**3+17*
c**2*e*x**4),x)
```

output
```
c**6*e**5*x**17 + x**16*(6*b*c**5*e**5 + 5*c**6*d*e**4) + x**15*(6*a*c**5*
e**5 + 15*b**2*c**4*e**5 + 30*b*c**5*d*e**4 + 10*c**6*d**2*e**3) + x**14*(
30*a*b*c**4*e**5 + 30*a*c**5*d*e**4 + 20*b**3*c**3*e**5 + 75*b**2*c**4*d*e
**4 + 60*b*c**5*d**2*e**3 + 10*c**6*d**3*e**2) + x**13*(15*a**2*c**4*e**5
+ 60*a*b**2*c**3*e**5 + 150*a*b*c**4*d*e**4 + 60*a*c**5*d**2*e**3 + 15*b**
4*c**2*e**5 + 100*b**3*c**3*d*e**4 + 150*b**2*c**4*d**2*e**3 + 60*b*c**5*d
**3*e**2 + 5*c**6*d**4*e) + x**12*(60*a**2*b*c**3*e**5 + 75*a**2*c**4*d*e*
*4 + 60*a*b**3*c**2*e**5 + 300*a*b**2*c**3*d*e**4 + 300*a*b*c**4*d**2*e**3
 + 60*a*c**5*d**3*e**2 + 6*b**5*c*e**5 + 75*b**4*c**2*d*e**4 + 200*b**3*c*
*3*d**2*e**3 + 150*b**2*c**4*d**3*e**2 + 30*b*c**5*d**4*e + c**6*d**5) + x
**11*(20*a**3*c**3*e**5 + 90*a**2*b**2*c**2*e**5 + 300*a**2*b*c**3*d*e**4
+ 150*a**2*c**4*d**2*e**3 + 30*a*b**4*c*e**5 + 300*a*b**3*c**2*d*e**4 + 60
0*a*b**2*c**3*d**2*e**3 + 300*a*b*c**4*d**3*e**2 + 30*a*c**5*d**4*e + b**6
*e**5 + 30*b**5*c*d*e**4 + 150*b**4*c**2*d**2*e**3 + 200*b**3*c**3*d**3*e*
*2 + 75*b**2*c**4*d**4*e + 6*b*c**5*d**5) + x**10*(60*a**3*b*c**2*e**5 + 1
00*a**3*c**3*d*e**4 + 60*a**2*b**3*c*e**5 + 450*a**2*b**2*c**2*d*e**4 + 60
0*a**2*b*c**3*d**2*e**3 + 150*a**2*c**4*d**3*e**2 + 6*a*b**5*e**5 + 150*a*
b**4*c*d*e**4 + 600*a*b**3*c**2*d**2*e**3 + 600*a*b**2*c**3*d**3*e**2 + 15
0*a*b*c**4*d**4*e + 6*a*c**5*d**5 + 5*b**6*d*e**4 + 60*b**5*c*d**2*e**3 +
150*b**4*c**2*d**3*e**2 + 100*b**3*c**3*d**4*e + 15*b**2*c**4*d**5) + x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1779 vs. $2(20) = 40$.

Time = 0.04 (sec) , antiderivative size = 1779, normalized size of antiderivative = 88.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x+a)^4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d+3*
b^2*d)*x+(22*a*c*e+11*b^2*e+18*b*c*d)*x^2+4*c*(7*b*e+3*c*d)*x^3+17*c^2*e*x
^4),x, algorithm="maxima")
```

output
```
c^6*e^5*x^17 + (5*c^6*d*e^4 + 6*b*c^5*e^5)*x^16 + (10*c^6*d^2*e^3 + 30*b*c
^5*d*e^4 + 3*(5*b^2*c^4 + 2*a*c^5)*e^5)*x^15 + 5*(2*c^6*d^3*e^2 + 12*b*c^5
*d^2*e^3 + 3*(5*b^2*c^4 + 2*a*c^5)*d*e^4 + 2*(2*b^3*c^3 + 3*a*b*c^4)*e^5)*
x^14 + 5*(c^6*d^4*e + 12*b*c^5*d^3*e^2 + 6*(5*b^2*c^4 + 2*a*c^5)*d^2*e^3 +
 10*(2*b^3*c^3 + 3*a*b*c^4)*d*e^4 + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*e^
5)*x^13 + (c^6*d^5 + 30*b*c^5*d^4*e + 30*(5*b^2*c^4 + 2*a*c^5)*d^3*e^2 + 1
00*(2*b^3*c^3 + 3*a*b*c^4)*d^2*e^3 + 75*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*
d*e^4 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*e^5)*x^12 + (6*b*c^5*d^5 +
 15*(5*b^2*c^4 + 2*a*c^5)*d^4*e + 100*(2*b^3*c^3 + 3*a*b*c^4)*d^3*e^2 + 15
0*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^2*e^3 + 30*(b^5*c + 10*a*b^3*c^2 + 1
0*a^2*b*c^3)*d*e^4 + (b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3)*e^5)
*x^11 + (3*(5*b^2*c^4 + 2*a*c^5)*d^5 + 50*(2*b^3*c^3 + 3*a*b*c^4)*d^4*e +
150*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^3*e^2 + 60*(b^5*c + 10*a*b^3*c^2 +
 10*a^2*b*c^3)*d^2*e^3 + 5*(b^6 + 30*a*b^4*c + 90*a^2*b^2*c^2 + 20*a^3*c^3
)*d*e^4 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*e^5)*x^10 + 5*(2*(2*b^3*
c^3 + 3*a*b*c^4)*d^5 + 15*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^4*e + 12*(b^
5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^3*e^2 + 2*(b^6 + 30*a*b^4*c + 90*a^2*
b^2*c^2 + 20*a^3*c^3)*d^2*e^3 + 6*(a*b^5 + 10*a^2*b^3*c + 10*a^3*b*c^2)*d*
e^4 + 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*e^5)*x^9 + 5*(3*(b^4*c^2 + 4*a*b
^2*c^3 + a^2*c^4)*d^5 + 6*(b^5*c + 10*a*b^3*c^2 + 10*a^2*b*c^3)*d^4*e +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2467 vs. $2(20) = 40$.

Time = 0.13 (sec) , antiderivative size = 2467, normalized size of antiderivative = 123.35

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x+a)^4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d+3*
b^2*d)*x+(22*a*c*e+11*b^2*e+18*b*c*d)*x^2+4*c*(7*b*e+3*c*d)*x^3+17*c^2*e*x
^4),x, algorithm="giac")
```

output
```
c^6*e^5*x^17 + 5*c^6*d*e^4*x^16 + 6*b*c^5*e^5*x^16 + 10*c^6*d^2*e^3*x^15 +
 30*b*c^5*d*e^4*x^15 + 15*b^2*c^4*e^5*x^15 + 6*a*c^5*e^5*x^15 + 10*c^6*d^3
*e^2*x^14 + 60*b*c^5*d^2*e^3*x^14 + 75*b^2*c^4*d*e^4*x^14 + 30*a*c^5*d*e^4
*x^14 + 20*b^3*c^3*e^5*x^14 + 30*a*b*c^4*e^5*x^14 + 5*c^6*d^4*e*x^13 + 60*
b*c^5*d^3*e^2*x^13 + 150*b^2*c^4*d^2*e^3*x^13 + 60*a*c^5*d^2*e^3*x^13 + 10
0*b^3*c^3*d*e^4*x^13 + 150*a*b*c^4*d*e^4*x^13 + 15*b^4*c^2*e^5*x^13 + 60*a
*b^2*c^3*e^5*x^13 + 15*a^2*c^4*e^5*x^13 + c^6*d^5*x^12 + 30*b*c^5*d^4*e*x^
12 + 150*b^2*c^4*d^3*e^2*x^12 + 60*a*c^5*d^3*e^2*x^12 + 200*b^3*c^3*d^2*e^
3*x^12 + 300*a*b*c^4*d^2*e^3*x^12 + 75*b^4*c^2*d*e^4*x^12 + 300*a*b^2*c^3*
d*e^4*x^12 + 75*a^2*c^4*d*e^4*x^12 + 6*b^5*c*e^5*x^12 + 60*a*b^3*c^2*e^5*x
^12 + 60*a^2*b*c^3*e^5*x^12 + 6*b*c^5*d^5*x^11 + 75*b^2*c^4*d^4*e*x^11 + 3
0*a*c^5*d^4*e*x^11 + 200*b^3*c^3*d^3*e^2*x^11 + 300*a*b*c^4*d^3*e^2*x^11 +
 150*b^4*c^2*d^2*e^3*x^11 + 600*a*b^2*c^3*d^2*e^3*x^11 + 150*a^2*c^4*d^2*e
^3*x^11 + 30*b^5*c*d*e^4*x^11 + 300*a*b^3*c^2*d*e^4*x^11 + 300*a^2*b*c^3*d
*e^4*x^11 + b^6*e^5*x^11 + 30*a*b^4*c*e^5*x^11 + 90*a^2*b^2*c^2*e^5*x^11 +
 20*a^3*c^3*e^5*x^11 + 15*b^2*c^4*d^5*x^10 + 6*a*c^5*d^5*x^10 + 100*b^3*c^
3*d^4*e*x^10 + 150*a*b*c^4*d^4*e*x^10 + 150*b^4*c^2*d^3*e^2*x^10 + 600*a*b
^2*c^3*d^3*e^2*x^10 + 150*a^2*c^4*d^3*e^2*x^10 + 60*b^5*c*d^2*e^3*x^10 + 6
00*a*b^3*c^2*d^2*e^3*x^10 + 600*a^2*b*c^3*d^2*e^3*x^10 + 5*b^6*d*e^4*x^10
+ 150*a*b^4*c*d*e^4*x^10 + 450*a^2*b^2*c^2*d*e^4*x^10 + 100*a^3*c^3*d*e...
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 2026, normalized size of antiderivative = 101.30

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right)\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^4*(a + b*x + c*x^2)^4*(x^2*(11*b^2*e + 22*a*c*e + 18*b*c*d)
+ a*(5*a*e + 6*b*d) + 2*x*(3*b^2*d + 8*a*b*e + 6*a*c*d) + 4*c*x^3*(7*b*e +
 3*c*d) + 17*c^2*e*x^4),x)
```

output
```
x^6*(b^6*d^5 + 6*a^5*b*e^5 + 20*a^3*c^3*d^5 + 75*a^4*b^2*d*e^4 + 90*a^2*b^
2*c^2*d^5 + 150*a^2*b^4*d^3*e^2 + 200*a^3*b^3*d^2*e^3 + 150*a^4*c^2*d^3*e^
2 + 30*a*b^4*c*d^5 + 30*a*b^5*d^4*e + 30*a^5*c*d*e^4 + 300*a^2*b^3*c*d^4*e
 + 300*a^3*b*c^2*d^4*e + 300*a^4*b*c*d^2*e^3 + 600*a^3*b^2*c*d^3*e^2) + x^
11*(b^6*e^5 + 6*b*c^5*d^5 + 20*a^3*c^3*e^5 + 75*b^2*c^4*d^4*e + 90*a^2*b^2
*c^2*e^5 + 150*a^2*c^4*d^2*e^3 + 200*b^3*c^3*d^3*e^2 + 150*b^4*c^2*d^2*e^3
 + 30*a*b^4*c*e^5 + 30*a*c^5*d^4*e + 30*b^5*c*d*e^4 + 300*a*b*c^4*d^3*e^2
+ 300*a*b^3*c^2*d*e^4 + 300*a^2*b*c^3*d*e^4 + 600*a*b^2*c^3*d^2*e^3) + x^5
*(a^6*e^5 + 6*a*b^5*d^5 + 60*a^2*b^3*c*d^5 + 60*a^3*b*c^2*d^5 + 75*a^2*b^4
*d^4*e + 75*a^4*c^2*d^4*e + 60*a^5*c*d^2*e^3 + 200*a^3*b^3*d^3*e^2 + 150*a
^4*b^2*d^2*e^3 + 30*a^5*b*d*e^4 + 300*a^3*b^2*c*d^4*e + 300*a^4*b*c*d^3*e^
2) + x^3*(20*a^3*b^3*d^5 + 10*a^6*d^2*e^3 + 75*a^4*b^2*d^4*e + 60*a^5*b*d^
3*e^2 + 30*a^4*b*c*d^5 + 30*a^5*c*d^4*e) + x^12*(c^6*d^5 + 6*b^5*c*e^5 + 6
0*a*b^3*c^2*e^5 + 60*a^2*b*c^3*e^5 + 60*a*c^5*d^3*e^2 + 75*a^2*c^4*d*e^4 +
 75*b^4*c^2*d*e^4 + 150*b^2*c^4*d^3*e^2 + 200*b^3*c^3*d^2*e^3 + 30*b*c^5*d
^4*e + 300*a*b*c^4*d^2*e^3 + 300*a*b^2*c^3*d*e^4) + x^7*(6*a^5*c*e^5 + 6*b
^5*c*d^5 + 5*b^6*d^4*e + 15*a^4*b^2*e^5 + 60*a*b^3*c^2*d^5 + 60*a^2*b*c^3*
d^5 + 60*a*b^5*d^3*e^2 + 100*a^3*b^3*d*e^4 + 100*a^3*c^3*d^4*e + 150*a^2*b
^4*d^2*e^3 + 150*a^4*c^2*d^2*e^3 + 150*a*b^4*c*d^4*e + 150*a^4*b*c*d*e^4 +
 450*a^2*b^2*c^2*d^4*e + 600*a^2*b^3*c*d^3*e^2 + 600*a^3*b*c^2*d^3*e^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2459, normalized size of antiderivative = 122.95

$$\int (d+ex)^4 \left(a+bx+cx^2\right)^4 \left(a(6bd+5ae)+2\left(3b^2d+6acd+8abe\right)x + \left(18bcd+11b^2e+22ace\right)x^2+4c(3cd+7be)x^3+17c^2ex^4\right) dx = \text{Too large to display}$$

input
```
int((e*x+d)^4*(c*x^2+b*x+a)^4*(a*(5*a*e+6*b*d)+2*(8*a*b*e+6*a*c*d+3*b^2*d)
*x+(22*a*c*e+11*b^2*e+18*b*c*d)*x^2+4*c*(7*b*e+3*c*d)*x^3+17*c^2*e*x^4),x)
```

output
```
x*(5*a**6*d**4*e + 10*a**6*d**3*e**2*x + 10*a**6*d**2*e**3*x**2 + 5*a**6*d
*e**4*x**3 + a**6*e**5*x**4 + 6*a**5*b*d**5 + 30*a**5*b*d**4*e*x + 60*a**5
*b*d**3*e**2*x**2 + 60*a**5*b*d**2*e**3*x**3 + 30*a**5*b*d*e**4*x**4 + 6*a
**5*b*e**5*x**5 + 6*a**5*c*d**5*x + 30*a**5*c*d**4*e*x**2 + 60*a**5*c*d**3
*e**2*x**3 + 60*a**5*c*d**2*e**3*x**4 + 30*a**5*c*d*e**4*x**5 + 6*a**5*c*e
**5*x**6 + 15*a**4*b**2*d**5*x + 75*a**4*b**2*d**4*e*x**2 + 150*a**4*b**2*
d**3*e**2*x**3 + 150*a**4*b**2*d**2*e**3*x**4 + 75*a**4*b**2*d*e**4*x**5 +
 15*a**4*b**2*e**5*x**6 + 30*a**4*b*c*d**5*x**2 + 150*a**4*b*c*d**4*e*x**3
 + 300*a**4*b*c*d**3*e**2*x**4 + 300*a**4*b*c*d**2*e**3*x**5 + 150*a**4*b*
c*d*e**4*x**6 + 30*a**4*b*c*e**5*x**7 + 15*a**4*c**2*d**5*x**3 + 75*a**4*c
**2*d**4*e*x**4 + 150*a**4*c**2*d**3*e**2*x**5 + 150*a**4*c**2*d**2*e**3*x
**6 + 75*a**4*c**2*d*e**4*x**7 + 15*a**4*c**2*e**5*x**8 + 20*a**3*b**3*d**
5*x**2 + 100*a**3*b**3*d**4*e*x**3 + 200*a**3*b**3*d**3*e**2*x**4 + 200*a*
*3*b**3*d**2*e**3*x**5 + 100*a**3*b**3*d*e**4*x**6 + 20*a**3*b**3*e**5*x**
7 + 60*a**3*b**2*c*d**5*x**3 + 300*a**3*b**2*c*d**4*e*x**4 + 600*a**3*b**2
*c*d**3*e**2*x**5 + 600*a**3*b**2*c*d**2*e**3*x**6 + 300*a**3*b**2*c*d*e**
4*x**7 + 60*a**3*b**2*c*e**5*x**8 + 60*a**3*b*c**2*d**5*x**4 + 300*a**3*b*
c**2*d**4*e*x**5 + 600*a**3*b*c**2*d**3*e**2*x**6 + 600*a**3*b*c**2*d**2*e
**3*x**7 + 300*a**3*b*c**2*d*e**4*x**8 + 60*a**3*b*c**2*e**5*x**9 + 20*a**
3*c**3*d**5*x**5 + 100*a**3*c**3*d**4*e*x**6 + 200*a**3*c**3*d**3*e**2*...
```

3.114 $\int \frac{x^2+x^3}{-2+x+x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(2+x)$$

output
```
1/2*x^2+2/3*ln(1-x)+4/3*ln(2+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2+x^3}{-2+x+x^2} \, dx = \frac{x^2}{2} + \frac{2}{3}\log(1-x) + \frac{4}{3}\log(2+x)$$

input
```
Integrate[(x^2 + x^3)/(-2 + x + x^2),x]
```

output
```
x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2027, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2}{x^2+x-2}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x^2(x+1)}{x^2+x-2}dx$$
$$\Big\downarrow \text{1200}$$
$$\int \left(\frac{2x}{x^2+x-2}+x\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^2}{2}+\frac{2}{3}\log(1-x)+\frac{4}{3}\log(x+2)$$

input
```
Int[(x^2 + x^3)/(-2 + x + x^2),x]
```

output
```
x^2/2 + (2*Log[1 - x])/3 + (4*Log[2 + x])/3
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{x^2}{2}+\frac{4\ln(2+x)}{3}+\frac{2\ln(x-1)}{3}$	19
norman	$\frac{x^2}{2}+\frac{4\ln(2+x)}{3}+\frac{2\ln(x-1)}{3}$	19
risch	$\frac{x^2}{2}+\frac{4\ln(2+x)}{3}+\frac{2\ln(x-1)}{3}$	19
parallelrisch	$\frac{x^2}{2}+\frac{4\ln(2+x)}{3}+\frac{2\ln(x-1)}{3}$	19

input
```
int((x^3+x^2)/(x^2+x-2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+4/3*ln(2+x)+2/3*ln(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{4}{3}\,\log\left(x+2\right)+\frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="fricas")
```

output
```
1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{x^2}{2}+\frac{2\log{(x-1)}}{3}+\frac{4\log{(x+2)}}{3}$$

input
```
integrate((x**3+x**2)/(x**2+x-2),x)
```

output
```
x**2/2 + 2*log(x - 1)/3 + 4*log(x + 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{4}{3}\,\log\left(x+2\right)+\frac{2}{3}\,\log\left(x-1\right)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="maxima")
```

output
```
1/2*x^2 + 4/3*log(x + 2) + 2/3*log(x - 1)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{1}{2}\,x^2+\frac{4}{3}\,\log\left(|x+2|\right)+\frac{2}{3}\,\log\left(|x-1|\right)$$

input
```
integrate((x^3+x^2)/(x^2+x-2),x, algorithm="giac")
```

output
```
1/2*x^2 + 4/3*log(abs(x + 2)) + 2/3*log(abs(x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{2\,\ln\left(x-1\right)}{3}+\frac{4\,\ln\left(x+2\right)}{3}+\frac{x^2}{2}$$

input
```
int((x^2 + x^3)/(x + x^2 - 2),x)
```

output
```
(2*log(x - 1))/3 + (4*log(x + 2))/3 + x^2/2
```

Reduce [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \frac{x^2+x^3}{-2+x+x^2}\,dx=\frac{2\log(x-1)}{3}+\frac{4\log(x+2)}{3}+\frac{x^2}{2}$$

input
```
int((x^3+x^2)/(x^2+x-2),x)
```

output
```
(4*log(x - 1) + 8*log(x + 2) + 3*x**2)/6
```

3.115 $\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 346

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(80c^2e-70bcf+63b^2g-64acg)\,x^2\sqrt{a+bx+cx^2}}{240c^3}$$
$$+\frac{(10cf-9bg)x^3\sqrt{a+bx+cx^2}}{40c^2}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$
$$-\frac{(1050b^3cf+40bc^2(36cd-55af)-945b^4g-60b^2c(20ce-49ag)+256ac^2(5ce-4ag)-2c(480c^3d-}{1920c^5}$$
$$+\frac{(70b^4cf+48b^2c^2(2cd-5af)-32ac^3(4cd-3af)-63b^5g-40b^3c(2ce-7ag)+48abc^2(4ce-5ag))\,a}{256c^{11/2}}$$

output
```
1/240*(-64*a*c*g+63*b^2*g-70*b*c*f+80*c^2*e)*x^2*(c*x^2+b*x+a)^(1/2)/c^3+1
/40*(-9*b*g+10*c*f)*x^3*(c*x^2+b*x+a)^(1/2)/c^2+1/5*g*x^4*(c*x^2+b*x+a)^(1
/2)/c-1/1920*(1050*b^3*c*f+40*b*c^2*(-55*a*f+36*c*d)-945*b^4*g-60*b^2*c*(-
49*a*g+20*c*e)+256*a*c^2*(-4*a*g+5*c*e)-2*c*(480*c^3*d-40*c^2*(9*a*f+10*b*
e)-315*b^3*g+14*b*c*(46*a*g+25*b*f))*x)*(c*x^2+b*x+a)^(1/2)/c^5+1/256*(70*
b^4*c*f+48*b^2*c^2*(-5*a*f+2*c*d)-32*a*c^3*(-3*a*f+4*c*d)-63*b^5*g-40*b^3*
c*(-7*a*g+2*c*e)+48*a*b*c^2*(-5*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(945b^4g-210b^3c(5f+3gx)+4b^2c(300ce-735ag+7cx(25f+18gx))-8bc^2(-a(\cdots}{\cdots}$$

input
```
Integrate[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(945*b^4*g - 210*b^3*c*(5*f + 3*g*x) + 4*
b^2*c*(300*c*e - 735*a*g + 7*c*x*(25*f + 18*g*x)) - 8*b*c^2*(-(a*(275*f +
161*g*x)) + 2*c*(90*d + x*(50*e + 35*f*x + 27*g*x^2))) + 16*c^2*(64*a^2*g
- a*c*(80*e + x*(45*f + 32*g*x)) + 2*c^2*x*(30*d + x*(20*e + 3*x*(5*f + 4*
g*x))))) + 15*(-70*b^4*c*f - 48*b^2*c^2*(2*c*d - 5*a*f) + 32*a*c^3*(4*c*d
- 3*a*f) + 63*b^5*g + 40*b^3*c*(2*c*e - 7*a*g) + 48*a*b*c^2*(-4*c*e + 5*a*
g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(3840*c^(11/2))
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2184, 27, 2184, 27, 1236, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2184$$
$$\frac{\int \frac{x^2\left((10cf-9bg)x^2+2(5ce-4ag)x+10cd\right)}{2\sqrt{cx^2+bx+a}}dx}{5c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{x^2\left((10cf-9bg)x^2+2(5ce-4ag)x+10cd\right)}{\sqrt{cx^2+bx+a}}dx}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 2184

$$\frac{\frac{\int \frac{x^2\left(80dc^2-60afc+54abg+\left(63gb^2-70cfb+80c^2e-64acg\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{x^3\sqrt{a+bx+cx^2}(10cf-9bg)}{4c}}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{x^2\left(2\left(40dc^2-30afc+27abg\right)+\left(63gb^2-70cfb+80c^2e-64acg\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{x^3\sqrt{a+bx+cx^2}(10cf-9bg)}{4c}}{10c}+\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 1236

$$\frac{\frac{\frac{\int -\frac{x\left(4a\left(63gb^2-70cfb+80c^2e-64acg\right)-\left(-315gb^3+14c(25bf+46ag)b+480c^3d-40c^2(10be+9af)\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}}{8c}+\ldots}{10c}$$
$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\int \frac{x\left(4a\left(63gb^2-70cfb+80c^2e-64acg\right)-\left(-315gb^3+14c(25bf+46ag)b+480c^3d-40c^2(10be+9af)\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}}{8c}+\frac{x^3\ldots}{}}{10c}$$
$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 1225

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd\ldots\right.}{4c^2}}{}$$
$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd\right.}{4c^2}}{}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

$$\Big\downarrow 219$$

$$\frac{\frac{x^2\sqrt{a+bx+cx^2}\left(-64acg+63b^2g-70bcf+80c^2e\right)}{3c}-\frac{\sqrt{a+bx+cx^2}\left(-2cx\left(-40c^2(9af+10be)+14bc(46ag+25bf)-315b^3g+480c^3d\right)-60b^2c(20ce-49ag)+40bc^2(36cd\right.}{4c^2}}{}$$

$$\frac{gx^4\sqrt{a+bx+cx^2}}{5c}$$

input
```
Int[(x^2*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^4*Sqrt[a + b*x + c*x^2])/(5*c) + (((10*c*f - 9*b*g)*x^3*Sqrt[a + b*x
+ c*x^2])/(4*c) + (((80*c^2*e - 70*b*c*f + 63*b^2*g - 64*a*c*g)*x^2*Sqrt[a
 + b*x + c*x^2])/(3*c) - (((1050*b^3*c*f + 40*b*c^2*(36*c*d - 55*a*f) - 94
5*b^4*g - 60*b^2*c*(20*c*e - 49*a*g) + 256*a*c^2*(5*c*e - 4*a*g) - 2*c*(48
0*c^3*d - 40*c^2*(10*b*e + 9*a*f) - 315*b^3*g + 14*b*c*(25*b*f + 46*a*g))*
x)*Sqrt[a + b*x + c*x^2])/(4*c^2) - (15*(70*b^4*c*f + 48*b^2*c^2*(2*c*d -
5*a*f) - 32*a*c^3*(4*c*d - 3*a*f) - 63*b^5*g - 40*b^3*c*(2*c*e - 7*a*g) +
48*a*b*c^2*(4*c*e - 5*a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c))/(10*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(384g\,c^4x^4-432b\,c^3g\,x^3+480c^4f\,x^3-512a\,c^3g\,x^2+504b^2c^2g\,x^2-560b\,c^3f\,x^2+640c^4e\,x^2+1288ab\,c^2gx-720a\,c^3fx-630gc\,b^3x+700}{1}$
default	Expression too large to display

input
```
int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*c^4*g*x^4-432*b*c^3*g*x^3+480*c^4*f*x^3-512*a*c^3*g*x^2+504*b^
2*c^2*g*x^2-560*b*c^3*f*x^2+640*c^4*e*x^2+1288*a*b*c^2*g*x-720*a*c^3*f*x-6
30*b^3*c*g*x+700*b^2*c^2*f*x-800*b*c^3*e*x+960*c^4*d*x+1024*a^2*c^2*g-2940
*a*b^2*c*g+2200*a*b*c^2*f-1280*a*c^3*e+945*b^4*g-1050*b^3*c*f+1200*b^2*c^2
*e-1440*b*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^5-1/256*(240*a^2*b*c^2*g-96*a^2*c^3
*f-280*a*b^3*c*g+240*a*b^2*c^2*f-192*a*b*c^3*e+128*a*c^4*d+63*b^5*g-70*b^4
*c*f+80*b^3*c^2*e-96*b^2*c^3*d)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 701, normalized size of antiderivative = 2.03

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/7680*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5*b^3*c^2 - 12*a*b*c^3)*e +
 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (63*b^5 - 280*a*b^3*c + 240
*a^2*b*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(384*c^5*g*x^4 - 1440*b*c^4*d + 48*
(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f + (63*b^2*c^3 - 64*a
*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b^3*c^2 - 44*a*b*c^3)
*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2*(480*c^5*d - 400*b*
c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 - 92*a*b*c^3)*g)*x)*s
qrt(c*x^2 + b*x + a))/c^6, -1/3840*(15*(32*(3*b^2*c^3 - 4*a*c^4)*d - 16*(5
*b^3*c^2 - 12*a*b*c^3)*e + 2*(35*b^4*c - 120*a*b^2*c^2 + 48*a^2*c^3)*f - (
63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(384*c^5*g*x^4
- 1440*b*c^4*d + 48*(10*c^5*f - 9*b*c^4*g)*x^3 + 8*(80*c^5*e - 70*b*c^4*f
+ (63*b^2*c^3 - 64*a*c^4)*g)*x^2 + 80*(15*b^2*c^3 - 16*a*c^4)*e - 50*(21*b
^3*c^2 - 44*a*b*c^3)*f + (945*b^4*c - 2940*a*b^2*c^2 + 1024*a^2*c^3)*g + 2
*(480*c^5*d - 400*b*c^4*e + 10*(35*b^2*c^3 - 36*a*c^4)*f - 7*(45*b^3*c^2 -
 92*a*b*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^6]
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 700, normalized size of antiderivative = 2.02

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(
-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) - b*(-2*a*(-4*a*g/(5*c) - 7
*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4
*c) - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4
*c))/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/s
qrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/
(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**4/(5*c) + x**3*(-9*b
*g/(10*c) + f)/(4*c) + x**2*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c)
+ e)/(3*c) + x*(-3*a*(-9*b*g/(10*c) + f)/(4*c) - 5*b*(-4*a*g/(5*c) - 7*b*(
-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(2*c) + (-2*a*(-4*a*g/(5*c) - 7*b
*(-9*b*g/(10*c) + f)/(8*c) + e)/(3*c) - 3*b*(-3*a*(-9*b*g/(10*c) + f)/(4*c
) - 5*b*(-4*a*g/(5*c) - 7*b*(-9*b*g/(10*c) + f)/(8*c) + e)/(6*c) + d)/(4*c
))/c), Ne(c, 0)), (2*(g*(a + b*x)**(11/2)/(11*b**3) + (a + b*x)**(9/2)*(-5
*a*g + b*f)/(9*b**3) + (a + b*x)**(7/2)*(10*a**2*g - 4*a*b*f + b**2*e)/(7*
b**3) + (a + b*x)**(5/2)*(-10*a**3*g + 6*a**2*b*f - 3*a*b**2*e + b**3*d)/(
5*b**3) + (a + b*x)**(3/2)*(5*a**4*g - 4*a**3*b*f + 3*a**2*b**2*e - 2*a*b*
*3*d)/(3*b**3) + sqrt(a + b*x)*(-a**5*g + a**4*b*f - a**3*b**2*e + a**2*b*
*3*d)/b**3)/b**3, Ne(b, 0)), ((d*x**3/3 + e*x**4/4 + f*x**5/5 + g*x**6/6)/
sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(d + ex + fx^2 + gx^3)}{\sqrt{a + bx + cx^2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.93

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(\frac{8\,gx}{c}+\frac{10\,c^4f-9\,bc^3g}{c^5}\right)x+\frac{80\,c^4e-70\,bc^3f+63\,b^2c^2g-64\,ac^3g}{c^5}\right)x+\right.\right.$$
$$-\frac{(96\,b^2c^3d-128\,ac^4d-80\,b^3c^2e+192\,abc^3e+70\,b^4cf-240\,ab^2c^2f+96\,a^2c^3f-63\,b^5g+280\,ab^3cg-}{256\,c^{\frac{11}{2}}}$$

input
```
integrate(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*g*x/c + (10*c^4*f - 9*b*c^3*g)/c^
5)*x + (80*c^4*e - 70*b*c^3*f + 63*b^2*c^2*g - 64*a*c^3*g)/c^5)*x + (480*c
^4*d - 400*b*c^3*e + 350*b^2*c^2*f - 360*a*c^3*f - 315*b^3*c*g + 644*a*b*c
^2*g)/c^5)*x - (1440*b*c^3*d - 1200*b^2*c^2*e + 1280*a*c^3*e + 1050*b^3*c*
f - 2200*a*b*c^2*f - 945*b^4*g + 2940*a*b^2*c*g - 1024*a^2*c^2*g)/c^5) - 1
/256*(96*b^2*c^3*d - 128*a*c^4*d - 80*b^3*c^2*e + 192*a*b*c^3*e + 70*b^4*c
*f - 240*a*b^2*c^2*f + 96*a^2*c^3*f - 63*b^5*g + 280*a*b^3*c*g - 240*a^2*b
*c^2*g)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(11/
2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\,\left(g\,x^3+f\,x^2+e\,x+d\right)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x^2*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.56

$$\int \frac{x^2(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2048*sqrt(a + b*x + c*x**2)*a**2*c**3*g - 5880*sqrt(a + b*x + c*x**2)*a*b
**2*c**2*g + 4400*sqrt(a + b*x + c*x**2)*a*b*c**3*f + 2576*sqrt(a + b*x +
c*x**2)*a*b*c**3*g*x - 2560*sqrt(a + b*x + c*x**2)*a*c**4*e - 1440*sqrt(a
+ b*x + c*x**2)*a*c**4*f*x - 1024*sqrt(a + b*x + c*x**2)*a*c**4*g*x**2 + 1
890*sqrt(a + b*x + c*x**2)*b**4*c*g - 2100*sqrt(a + b*x + c*x**2)*b**3*c**
2*f - 1260*sqrt(a + b*x + c*x**2)*b**3*c**2*g*x + 2400*sqrt(a + b*x + c*x*
*2)*b**2*c**3*e + 1400*sqrt(a + b*x + c*x**2)*b**2*c**3*f*x + 1008*sqrt(a
+ b*x + c*x**2)*b**2*c**3*g*x**2 - 2880*sqrt(a + b*x + c*x**2)*b*c**4*d -
1600*sqrt(a + b*x + c*x**2)*b*c**4*e*x - 1120*sqrt(a + b*x + c*x**2)*b*c**
4*f*x**2 - 864*sqrt(a + b*x + c*x**2)*b*c**4*g*x**3 + 1920*sqrt(a + b*x +
c*x**2)*c**5*d*x + 1280*sqrt(a + b*x + c*x**2)*c**5*e*x**2 + 960*sqrt(a +
b*x + c*x**2)*c**5*f*x**3 + 768*sqrt(a + b*x + c*x**2)*c**5*g*x**4 - 3600*
sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b*
*2))*a**2*b*c**2*g + 1440*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
b + 2*c*x)/sqrt(4*a*c - b**2))*a**2*c**3*f + 4200*sqrt(c)*log((2*sqrt(c)*s
qrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**3*c*g - 3600*s
qrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**
2))*a*b**2*c**2*f + 2880*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b
 + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**3*e - 1920*sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c**4*d - 945*sqr...
```

3.116 $\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 245

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \frac{(8cf-7bg)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{gx^3\sqrt{a+bx+cx^2}}{4c} + \frac{(192c^3d-16c^2(9be+8af)-105b^3g+20bc(6bf+11ag)+2c(48c^2e-40bcf+35b^2g-36acg)\,x)\,\sqrt{a}}{192c^4} - \frac{(40b^3cf+32bc^2(2cd-3af)-35b^4g-24b^2c(2ce-5ag)+16ac^2(4ce-3ag))\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{9/2}}$$

output
```
1/24*(-7*b*g+8*c*f)*x^2*(c*x^2+b*x+a)^(1/2)/c^2+1/4*g*x^3*(c*x^2+b*x+a)^(1
/2)/c+1/192*(192*c^3*d-16*c^2*(8*a*f+9*b*e)-105*b^3*g+20*b*c*(11*a*g+6*b*f
)+2*c*(-36*a*c*g+35*b^2*g-40*b*c*f+48*c^2*e)*x)*(c*x^2+b*x+a)^(1/2)/c^4-1/
128*(40*b^3*c*f+32*b*c^2*(-3*a*f+2*c*d)-35*b^4*g-24*b^2*c*(-5*a*g+2*c*e)+1
6*a*c^2*(-3*a*g+4*c*e))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))
/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.81

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^3g+10bc(12bf+22ag+7bgx)-8c^2(18be+16af+10bfx+9agx+7bgx^2}{}$$

input
```
Integrate[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^3*g + 10*b*c*(12*b*f + 22*a*g + 7
*b*g*x) - 8*c^2*(18*b*e + 16*a*f + 10*b*f*x + 9*a*g*x + 7*b*g*x^2) + 16*c^
3*(12*d + x*(6*e + 4*f*x + 3*g*x^2))) + 3*(40*b^3*c*f + 32*b*c^2*(2*c*d -
3*a*f) - 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(9/2))
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {2184, 27, 2184, 27, 1225, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(d+ex+fx^2+gx^3\right)}{\sqrt{a+bx+cx^2}}dx$$
$$\Big\downarrow \text{2184}$$
$$\frac{\int \frac{x\left((8cf-7bg)x^2+2(4ce-3ag)x+8cd\right)}{2\sqrt{cx^2+bx+a}}dx}{4c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{x\left((8cf-7bg)x^2+2(4ce-3ag)x+8cd\right)}{\sqrt{cx^2+bx+a}}dx}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 2184

$$\frac{\frac{\int \frac{x\left(4\left(12dc^2-8afc+7abg\right)+\left(35gb^2-40cfb+48c^2e-36acg\right)x\right)}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{3c}}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{x\left(4\left(12dc^2-8afc+7abg\right)+\left(35gb^2-40cfb+48c^2e-36acg\right)x\right)}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{x^2\sqrt{a+bx+cx^2}(8cf-7bg)}{3c}}{8c}+\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 1225

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\left(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3a\right.}{8c^2}}{6c}}{8c}$$
$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\left(-24b^2c(2ce-5ag)+32bc^2(2cd-3af)+16ac^2(4ce-3a\right.}{4c}}{6c}}{8c}$$
$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\sqrt{a+bx+cx^2}\left(2cx\left(-36acg+35b^2g-40bcf+48c^2e\right)-16c^2(8af+9be)+20bc(11ag+6bf)-105b^3g+192c^3d\right)}{4c^2}-\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-24b^2c(2ce-5ag)+\right.}{8c^{5/}}}{6c}}{8c}$$
$$\frac{gx^3\sqrt{a+bx+cx^2}}{4c}$$

input
```
Int[(x*(d + e*x + f*x^2 + g*x^3))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^3*Sqrt[a + b*x + c*x^2])/(4*c) + (((8*c*f - 7*b*g)*x^2*Sqrt[a + b*x +
 c*x^2])/(3*c) + (((192*c^3*d - 16*c^2*(9*b*e + 8*a*f) - 105*b^3*g + 20*b*
c*(6*b*f + 11*a*g) + 2*c*(48*c^2*e - 40*b*c*f + 35*b^2*g - 36*a*c*g)*x)*Sq
rt[a + b*x + c*x^2])/(4*c^2) - (3*(40*b^3*c*f + 32*b*c^2*(2*c*d - 3*a*f) -
 35*b^4*g - 24*b^2*c*(2*c*e - 5*a*g) + 16*a*c^2*(4*c*e - 3*a*g))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)))/(6*c))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(48g\,c^3x^3-56b\,c^2g\,x^2+64c^3f\,x^2-72a\,c^2gx+70b^2cgx-80b\,c^2fx+96c^3ex+220abcg-128a\,c^2f-105b^3g+120b^2cf-144b\,c^2e+192c^3d)}{192c^4}$
default	$d\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+e\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right.$

input
```
int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*c^3*g*x^3-56*b*c^2*g*x^2+64*c^3*f*x^2-72*a*c^2*g*x+70*b^2*c*g*x-
80*b*c^2*f*x+96*c^3*e*x+220*a*b*c*g-128*a*c^2*f-105*b^3*g+120*b^2*c*f-144*
b*c^2*e+192*c^3*d)*(c*x^2+b*x+a)^(1/2)/c^4+1/128*(48*a^2*c^2*g-120*a*b^2*c
*g+96*a*b*c^2*f-64*a*c^3*e+35*b^4*g-40*b^3*c*f+48*b^2*c^2*e-64*b*c^3*d)/c^
(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.04

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{3\,(64\,bc^3d-16\,(3\,b^2c^2-4\,ac^3)e+8\,(5\,b^3c-12\,abc^2)f-(35\,b^4-120\,ab^2c+48\,a^2c^2)g)\sqrt{c}\log\left(-8\,c^2x\right.}{}\right.$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*
c^2)*f - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(c)*log(-8*c^2*x^2 - 8
*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(4
8*c^4*g*x^3 + 192*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(1
5*b^2*c^2 - 16*a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b
*c^3*f + (35*b^2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5, 1/384*(
3*(64*b*c^3*d - 16*(3*b^2*c^2 - 4*a*c^3)*e + 8*(5*b^3*c - 12*a*b*c^2)*f -
(35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*
x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*g*x^3 + 1
92*c^4*d - 144*b*c^3*e + 8*(8*c^4*f - 7*b*c^3*g)*x^2 + 8*(15*b^2*c^2 - 16*
a*c^3)*f - 5*(21*b^3*c - 44*a*b*c^2)*g + 2*(48*c^4*e - 40*b*c^3*f + (35*b^
2*c^2 - 36*a*c^3)*g)*x)*sqrt(c*x^2 + b*x + a))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.96

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \left(-\frac{a\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{2c}-\frac{b\left(-\frac{2a\left(-\frac{7bg}{8c}+f\right)}{3c}-\frac{3b\left(-\frac{3ag}{4c}-\frac{5b\left(-\frac{7bg}{8c}+f\right)}{6c}+e\right)}{4c}+d\right)}{2c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{a+bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{fo}\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{ot}\end{cases}\right. \\ \frac{2\left(\frac{g(a+bx)^{\frac{9}{2}}}{9b^3}+\frac{(a+bx)^{\frac{7}{2}}(-4ag+bf)}{7b^3}+\frac{(a+bx)^{\frac{5}{2}}\cdot\left(6a^2g-3abf+b^2e\right)}{5b^3}+\frac{(a+bx)^{\frac{3}{2}}\left(-4a^3g+3a^2bf-2ab^2e+b^3d\right)}{3b^3}+\frac{\sqrt{a+bx}\left(a^4g-a^3bf+a^2b^2e-ab^3d\right)}{b^3}\right)}{b^2} \\ \frac{\frac{dx^2}{2}+\frac{ex^3}{3}+\frac{fx^4}{4}+\frac{gx^5}{5}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise(((-a*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(2*c) - b
*(-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8*c) +
f)/(6*c) + e)/(4*c) + d)/(2*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x
+ c*x**2) + 2*c*x)/sqrt(c), Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(
2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)) + sqrt(a + b*x + c*x**2)*(g*x**
3/(4*c) + x**2*(-7*b*g/(8*c) + f)/(3*c) + x*(-3*a*g/(4*c) - 5*b*(-7*b*g/(8
*c) + f)/(6*c) + e)/(2*c) + (-2*a*(-7*b*g/(8*c) + f)/(3*c) - 3*b*(-3*a*g/(
4*c) - 5*b*(-7*b*g/(8*c) + f)/(6*c) + e)/(4*c) + d)/c), Ne(c, 0)), (2*(g*(
a + b*x)**(9/2)/(9*b**3) + (a + b*x)**(7/2)*(-4*a*g + b*f)/(7*b**3) + (a +
 b*x)**(5/2)*(6*a**2*g - 3*a*b*f + b**2*e)/(5*b**3) + (a + b*x)**(3/2)*(-4
*a**3*g + 3*a**2*b*f - 2*a*b**2*e + b**3*d)/(3*b**3) + sqrt(a + b*x)*(a**4
*g - a**3*b*f + a**2*b**2*e - a*b**3*d)/b**3)/b**2, Ne(b, 0)), ((d*x**2/2
+ e*x**3/3 + f*x**4/4 + g*x**5/5)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6\,gx}{c}+\frac{8\,c^3f-7\,bc^2g}{c^4}\right)x+\frac{48\,c^3e-40\,bc^2f+35\,b^2cg-36\,ac^2g}{c^4}\right)x+\frac{192\,c^3\right.$$
$$+\frac{(64\,bc^3d-48\,b^2c^2e+64\,ac^3e+40\,b^3cf-96\,abc^2f-35\,b^4g+120\,ab^2cg-48\,a^2c^2g)\log\left(\left|2\left(\sqrt{c}x-\sqrt{c}\right.\right.\right.}{128\,c^{\frac{9}{2}}}$$

input
```
integrate(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*g*x/c + (8*c^3*f - 7*b*c^2*g)/c^4)*x
+ (48*c^3*e - 40*b*c^2*f + 35*b^2*c*g - 36*a*c^2*g)/c^4)*x + (192*c^3*d -
144*b*c^2*e + 120*b^2*c*f - 128*a*c^2*f - 105*b^3*g + 220*a*b*c*g)/c^4) +
1/128*(64*b*c^3*d - 48*b^2*c^2*e + 64*a*c^3*e + 40*b^3*c*f - 96*a*b*c^2*f
- 35*b^4*g + 120*a*b^2*c*g - 48*a^2*c^2*g)*log(abs(2*(sqrt(c)*x - sqrt(c*x
^2 + b*x + a))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\,(g\,x^3+f\,x^2+e\,x+d)}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((x*(d + e*x + f*x^2 + g*x^3))/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 619, normalized size of antiderivative = 2.53

$$\int \frac{x(d+ex+fx^2+gx^3)}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{440\sqrt{c\,x^2+bx+a}\,ab\,c^2g - 256\sqrt{c\,x^2+bx+a}\,a\,c^3f - 144\sqrt{c\,x^2+bx+a}\,a\,c^3gx - 210\sqrt{c\,x^2+bx+a}\,b^3}{}$$

input
```
int(x*(g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output

```
(440*sqrt(a + b*x + c*x**2)*a*b*c**2*g - 256*sqrt(a + b*x + c*x**2)*a*c**3
*f - 144*sqrt(a + b*x + c*x**2)*a*c**3*g*x - 210*sqrt(a + b*x + c*x**2)*b*
*3*c*g + 240*sqrt(a + b*x + c*x**2)*b**2*c**2*f + 140*sqrt(a + b*x + c*x**
2)*b**2*c**2*g*x - 288*sqrt(a + b*x + c*x**2)*b*c**3*e - 160*sqrt(a + b*x
+ c*x**2)*b*c**3*f*x - 112*sqrt(a + b*x + c*x**2)*b*c**3*g*x**2 + 384*sqrt
(a + b*x + c*x**2)*c**4*d + 192*sqrt(a + b*x + c*x**2)*c**4*e*x + 128*sqrt
(a + b*x + c*x**2)*c**4*f*x**2 + 96*sqrt(a + b*x + c*x**2)*c**4*g*x**3 + 1
44*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*a**2*c**2*g - 360*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
b + 2*c*x)/sqrt(4*a*c - b**2))*a*b**2*c*g + 288*sqrt(c)*log((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c**2*f - 192*sqrt
(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))
*a*c**3*e + 105*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)
/sqrt(4*a*c - b**2))*b**4*g - 120*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*
x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**3*c*f + 144*sqrt(c)*log((2*sqrt(
c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c**2*e - 1
92*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c -
 b**2))*b*c**3*d)/(384*c**5)
```

3.117 $\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 177

$$\begin{aligned}
\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx &= \frac{(24c^2e-18bcf+15b^2g-16acg)\sqrt{a+bx+cx^2}}{24c^3} \\
&+ \frac{(6cf-5bg)x\sqrt{a+bx+cx^2}}{12c^2} + \frac{gx^2\sqrt{a+bx+cx^2}}{3c} \\
&+ \frac{(16c^3d-8c^2(be+af)-5b^3g+6bc(bf+2ag))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}}
\end{aligned}$$

output
```
1/24*(-16*a*c*g+15*b^2*g-18*b*c*f+24*c^2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/12*(
-5*b*g+6*c*f)*x*(c*x^2+b*x+a)^(1/2)/c^2+1/3*g*x^2*(c*x^2+b*x+a)^(1/2)/c+1/
16*(16*c^3*d-8*c^2*(a*f+b*e)-5*b^3*g+6*b*c*(2*a*g+b*f))*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c}\sqrt{a+x(b+cx)}(15b^2g-2c(9bf+8ag+5bgx)+4c^2(6e+x(3f+2gx)))+3(-16c^3d+8c^2(be+a}{48c^{7/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^2*g - 2*c*(9*b*f + 8*a*g + 5*b*g*x)
 + 4*c^2*(6*e + x*(3*f + 2*g*x))) + 3*(-16*c^3*d + 8*c^2*(b*e + a*f) + 5*b
^3*g - 6*b*c*(b*f + 2*a*g))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]])/(48*c^(7/2))
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2192, 27, 2192, 27, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2192}$$
$$\frac{\int \frac{(6cf-5bg)x^2+2(3ce-2ag)x+6cd}{2\sqrt{cx^2+bx+a}}dx}{3c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{(6cf-5bg)x^2+2(3ce-2ag)x+6cd}{\sqrt{cx^2+bx+a}}dx}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$
$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{24dc^2-12afc+10abg+\left(15gb^2-18cfb+24c^2e-16acg\right)x}{2\sqrt{cx^2+bx+a}}dx}{2c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{2\left(12dc^2-6afc+5abg\right)+\left(15gb^2-18cfb+24c^2e-16acg\right)x}{\sqrt{cx^2+bx+a}}dx}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\frac{3\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{\frac{3\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-8c^2(af+be)+6bc(2ag+bf)-5b^3g+16c^3d\right)}{2c^{3/2}}+\frac{\sqrt{a+bx+cx^2}\left(-16acg+15b^2g-18bcf+24c^2e\right)}{c}}{4c}+\frac{x\sqrt{a+bx+cx^2}(6cf-5bg)}{2c}}{6c}+\frac{gx^2\sqrt{a+bx+cx^2}}{3c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/Sqrt[a + b*x + c*x^2],x]
```

output
```
(g*x^2*Sqrt[a + b*x + c*x^2])/(3*c) + (((6*c*f - 5*b*g)*x*Sqrt[a + b*x + c
*x^2])/(2*c) + (((24*c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)*Sqrt[a + b*x
+ c*x^2])/c + (3*(16*c^3*d - 8*c^2*(b*e + a*f) - 5*b^3*g + 6*b*c*(b*f + 2*
a*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)))
/(4*c))/(6*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(-8g\,c^2x^2+10bcgx-12c^2fx+16acg-15b^2g+18bcf-24c^2e)\sqrt{c\,x^2+bx+a}}{24c^3}+\frac{(12abcg-8a\,c^2f-5b^3g+6b^2cf-8b\,c^2e+16c^3d)\ln\left(\right.}{16c^{\frac{7}{2}}}$
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+e\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+f\left(\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}\right.}{}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*c^2*g*x^2+10*b*c*g*x-12*c^2*f*x+16*a*c*g-15*b^2*g+18*b*c*f-24*c^
2*e)*(c*x^2+b*x+a)^(1/2)/c^3+1/16*(12*a*b*c*g-8*a*c^2*f-5*b^3*g+6*b^2*c*f-
8*b*c^2*e+16*c^3*d)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.93

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$=\left[\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+}\right.}{}\right.$$
$$-\frac{3\,(16\,c^3d-8\,bc^2e+2\,(3\,b^2c-4\,ac^2)f-(5\,b^3-12\,abc)g)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{-c}}{2\,(c^2x^2+bcx+ac)}\right)-2\,(8\,c^3g}{48\,c^4}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*a*b
*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*
b^2*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^
4, -1/48*(3*(16*c^3*d - 8*b*c^2*e + 2*(3*b^2*c - 4*a*c^2)*f - (5*b^3 - 12*
a*b*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(
c^2*x^2 + b*c*x + a*c)) - 2*(8*c^3*g*x^2 + 24*c^3*e - 18*b*c^2*f + (15*b^2
*c - 16*a*c^2)*g + 2*(6*c^3*f - 5*b*c^2*g)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(172) = 344$.

Time = 0.66 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx+cx^2}\left(\frac{gx^2}{3c}+\frac{x\left(-\frac{5bg}{6c}+f\right)}{2c}+\frac{-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e}{c}\right)+\left(-\frac{a\left(-\frac{5bg}{6c}+f\right)}{2c}-\frac{b\left(-\frac{2ag}{3c}-\frac{3b\left(-\frac{5bg}{6c}+f\right)}{4c}+e\right)}{2c}+d\right) \\ \frac{2d\sqrt{a+bx}+\frac{2e\left(-a\sqrt{a+bx}+\frac{(a+bx)^{\frac{3}{2}}}{3}\right)}{b}+\frac{2f\left(a^2\sqrt{a+bx}-\frac{2a(a+bx)^{\frac{3}{2}}}{3}+\frac{(a+bx)^{\frac{5}{2}}}{5}\right)}{b^2}+\frac{2g\left(-a^3\sqrt{a+bx}+a^2(a+bx)^{\frac{3}{2}}-\frac{3a(a+bx)^{\frac{5}{2}}}{5}+\frac{(a+bx)^{\frac{7}{2}}}{7}\right)}{b^3}}{b} \\ \frac{dx+\frac{ex^2}{2}+\frac{fx^3}{3}+\frac{gx^4}{4}}{\sqrt{a}} \end{cases}$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x + c*x**2)*(g*x**2/(3*c) + x*(-5*b*g/(6*c) + f)/(2*
c) + (-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/c) + (-a*(-5*b*g/(6
*c) + f)/(2*c) - b*(-2*a*g/(3*c) - 3*b*(-5*b*g/(6*c) + f)/(4*c) + e)/(2*c)
 + d)*Piecewise((log(b + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + 2*c*x)/sqrt(c)
, Ne(a - b**2/(4*c), 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
+ x)**2), True)), Ne(c, 0)), ((2*d*sqrt(a + b*x) + 2*e*(-a*sqrt(a + b*x) +
 (a + b*x)**(3/2)/3)/b + 2*f*(a**2*sqrt(a + b*x) - 2*a*(a + b*x)**(3/2)/3
+ (a + b*x)**(5/2)/5)/b**2 + 2*g*(-a**3*sqrt(a + b*x) + a**2*(a + b*x)**(3
/2) - 3*a*(a + b*x)**(5/2)/5 + (a + b*x)**(7/2)/7)/b**3)/b, Ne(b, 0)), ((d
*x + e*x**2/2 + f*x**3/3 + g*x**4/4)/sqrt(a), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.81

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{1}{24}\sqrt{cx^2+bx+a}\left(2\left(\frac{4\,gx}{c}+\frac{6\,c^2f-5\,bcg}{c^3}\right)x+\frac{24\,c^2e-18\,bcf+15\,b^2g-16\,acg}{c^3}\right) - \frac{(16\,c^3d-8\,bc^2e+6\,b^2cf-8\,ac^2f-5\,b^3g+12\,abcg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x + a)*(2*(4*g*x/c + (6*c^2*f - 5*b*c*g)/c^3)*x + (24*
c^2*e - 18*b*c*f + 15*b^2*g - 16*a*c*g)/c^3) - 1/16*(16*c^3*d - 8*b*c^2*e
+ 6*b^2*c*f - 8*a*c^2*f - 5*b^3*g + 12*a*b*c*g)*log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(a + b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.27

$$\int \frac{d+ex+fx^2+gx^3}{\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{-32\sqrt{c\,x^2+bx+a}\,a\,c^2g+30\sqrt{c\,x^2+bx+a}\,b^2cg-36\sqrt{c\,x^2+bx+a}\,b\,c^2f-20\sqrt{c\,x^2+bx+a}\,b\,c^2gx+}{}$$

input
```
int((g*x^3+f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 32*sqrt(a + b*x + c*x**2)*a*c**2*g + 30*sqrt(a + b*x + c*x**2)*b**2*c*
g - 36*sqrt(a + b*x + c*x**2)*b*c**2*f - 20*sqrt(a + b*x + c*x**2)*b*c**2*
g*x + 48*sqrt(a + b*x + c*x**2)*c**3*e + 24*sqrt(a + b*x + c*x**2)*c**3*f*
x + 16*sqrt(a + b*x + c*x**2)*c**3*g*x**2 + 36*sqrt(c)*log((2*sqrt(c)*sqrt
(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*b*c*g - 24*sqrt(c)*l
og((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*a*c*
*2*f - 15*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(
4*a*c - b**2))*b**3*g + 18*sqrt(c)*log((2*sqrt(c)*sqrt(a + b*x + c*x**2) +
 b + 2*c*x)/sqrt(4*a*c - b**2))*b**2*c*f - 24*sqrt(c)*log((2*sqrt(c)*sqrt(
a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*b*c**2*e + 48*sqrt(c)*l
og((2*sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*a*c - b**2))*c**3
*d)/(48*c**4)
```

3.118 $\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 155

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \frac{(4cf-3bg)\sqrt{a+bx+cx^2}}{4c^2}+\frac{gx\sqrt{a+bx+cx^2}}{2c} - \frac{d\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}} + \frac{(8c^2e+3b^2g-4c(bf+ag))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/4*(-3*b*g+4*c*f)*(c*x^2+b*x+a)^(1/2)/c^2+1/2*g*x*(c*x^2+b*x+a)^(1/2)/c-d
*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)+1/8*(8*c^2*e+3
*b^2*g-4*c*(a*g+b*f))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c
^(5/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.87

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{1}{8}\left(\frac{2(4cf-3bg+2cgx)\sqrt{a+x(b+cx)}}{c^2}+\frac{16d\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(-8c^2e-3b^2g+4c(bf+ag))\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{5/2}}\right)$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*(4*c*f - 3*b*g + 2*c*g*x)*Sqrt[a + x*(b + c*x)])/c^2 + (16*d*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + ((-8*c^2*e - 3*b^2*
g + 4*c*(b*f + a*g))*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])
])/c^(5/2))/8
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2184, 27, 2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2184$$
$$\frac{\int \frac{(4cf-3bg)x^2+2(2ce-ag)x+4cd}{2x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$
$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(4cf-3bg)x^2+2(2ce-ag)x+4cd}{x\sqrt{cx^2+bx+a}}dx}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 2184

$$\frac{\frac{\int \frac{8dc^2+\left(3gb^2+8c^2e-4c(bf+ag)\right)x}{2x\sqrt{cx^2+bx+a}}dx}{c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{8dc^2+\left(3gb^2+8c^2e-4c(bf+ag)\right)x}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 1269

$$\frac{\frac{\left(-4c(ag+bf)+3b^2g+8c^2e\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx+8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 1092

$$\frac{\frac{2\left(-4c(ag+bf)+3b^2g+8c^2e\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}+8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 219

$$\frac{\frac{8c^2d\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 1154

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}-16c^2d\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(ag+bf)+3b^2g+8c^2e\right)}{\sqrt{c}}-\frac{8c^2d\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}}{2c}+\frac{\sqrt{a+bx+cx^2}(4cf-3bg)}{c}}{4c}+\frac{gx\sqrt{a+bx+cx^2}}{2c}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(g*x*Sqrt[a + b*x + c*x^2])/(2*c) + (((4*c*f - 3*b*g)*Sqrt[a + b*x + c*x^2
])/c + ((-8*c^2*d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
Sqrt[a] + ((8*c^2*e + 3*b^2*g - 4*c*(b*f + a*g))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/(2*c))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.44

method	result
default	$\frac{e\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}-\frac{d\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+f\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+g\Bigg($

input
```
int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-d/a^(1/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+g*(1/2*x/c*(c*x^2+b*x+a)^(
1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.73

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(8*sqrt(a)*c^3*d*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 +
b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a
*b^2 - 4*a^2*c)*g)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*
b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/8*(4*sqrt(a)*c^3*d*log(-(8*a*b*x
+ (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)
/x^2) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/
2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*
(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3), 1/16
*(16*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/
(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f + (3*a*b^2 - 4*a^2*c)*g)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(c) - 4*a*c) + 4*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b*c*g)*sqrt(c*x^2
+ b*x + a))/(a*c^3), 1/8*(8*sqrt(-a)*c^3*d*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (8*a*c^2*e - 4*a*b*c*f +
 (3*a*b^2 - 4*a^2*c)*g)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x +
 b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*a*c^2*g*x + 4*a*c^2*f - 3*a*b
*c*g)*sqrt(c*x^2 + b*x + a))/(a*c^3)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionNot implemented, e.g. for multivariate mod/approx polynomi
alsError:
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1196, normalized size of antiderivative = 7.72

$$\int \frac{d+ex+fx^2+gx^3}{x\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)/x/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 8*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sqrt(c)*sqr
t(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2))
*b*c**3*d - 16*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c - b**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)/sqrt(4*sqrt(c)*sqrt(a)*b - 4*a*c
 - b**2))*a*c**3*d - 4*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2)*lo
g( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c
*x**2) + b + 2*c*x)*b*c**3*d + 4*sqrt(a)*sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c
+ b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a +
b*x + c*x**2) + b + 2*c*x)*b*c**3*d + 8*sqrt(c)*sqrt(4*sqrt(c)*sqrt(a)*b +
 4*a*c + b**2)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)
*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**3*d - 8*sqrt(c)*sqrt(4*sqrt(c)*s
qrt(a)*b + 4*a*c + b**2)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*
sqrt(c)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**3*d - 24*sqrt(a + b*x + c
*x**2)*a**2*b*c**2*g + 32*sqrt(a + b*x + c*x**2)*a**2*c**3*f + 16*sqrt(a +
 b*x + c*x**2)*a**2*c**3*g*x + 6*sqrt(a + b*x + c*x**2)*a*b**3*c*g - 8*sqr
t(a + b*x + c*x**2)*a*b**2*c**2*f - 4*sqrt(a + b*x + c*x**2)*a*b**2*c**2*g
*x + 16*sqrt(a)*log( - sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c
)*sqrt(a + b*x + c*x**2) + b + 2*c*x)*a*c**4*d - 4*sqrt(a)*log( - sqrt(4*s
qrt(c)*sqrt(a)*b + 4*a*c + b**2) + 2*sqrt(c)*sqrt(a + b*x + c*x**2) + b +
2*c*x)*b**2*c**3*d + 16*sqrt(a)*log(sqrt(4*sqrt(c)*sqrt(a)*b + 4*a*c + ...
```

3.119 $\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 139

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\sqrt{a+bx+cx^2}}{c} - \frac{d\sqrt{a+bx+cx^2}}{ax} + \frac{(bd-2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}} + \frac{(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}}$$

output
```
g*(c*x^2+b*x+a)^(1/2)/c-d*(c*x^2+b*x+a)^(1/2)/a/x+1/2*(-2*a*e+b*d)*arctanh
(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)+1/2*(-b*g+2*c*f)*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{(-cd+agx)\sqrt{a+x(b+cx)}}{acx} + \frac{(2cf-bg)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + \frac{(bd-2ae)\log(x)}{2a^{3/2}} + \frac{(-bd+2ae)\log\left(a\left(2a+bx-2\sqrt{a}\sqrt{a+x(b+cx)}\right)\right)}{2a^{3/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((-(c*d) + a*g*x)*Sqrt[a + x*(b + c*x)])/(a*c*x) + ((2*c*f - b*g)*ArcTanh[
(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/c^(3/2) + ((b*d - 2*a*e)*
Log[x])/(2*a^(3/2)) + ((-(b*d) + 2*a*e)*Log[a*(2*a + b*x - 2*Sqrt[a]*Sqrt[
a + x*(b + c*x)]])/(2*a^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2181, 27, 2184, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{\int \frac{-2agx^2-2afx+bd-2ae}{2x\sqrt{cx^2+bx+a}}dx}{a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2agx^2-2afx+bd-2ae}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{2184}$$

$$-\frac{\frac{\int \frac{c(bd-2ae)-a(2cf-bg)x}{x\sqrt{cx^2+bx+a}}dx}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-a(2cf-bg)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-2a(2cf-bg)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{c(bd-2ae)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{\frac{-2c(bd-2ae)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{-\frac{c(bd-2ae)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}-\frac{a(2cf-bg)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{c}-\frac{2ag\sqrt{a+bx+cx^2}}{c}}{2a}-\frac{d\sqrt{a+bx+cx^2}}{ax}$$

input

```
Int[(d + e*x + f*x^2 + g*x^3)/(x^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((d*Sqrt[a + b*x + c*x^2])/(a*x)) - ((-2*a*g*Sqrt[a + b*x + c*x^2])/c + (
-((c*(b*d - 2*a*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])
/Sqrt[a]) - (a*(2*c*f - b*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/Sqrt[c])/c)/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{d\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{-\frac{(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+\frac{2af\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+2ag\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x\right.}}{2c^{\frac{3}{2}}}\right.}{2a}$
default	$\frac{f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+d\left(-\frac{\sqrt{c\,x^{2}+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{e\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}+$

input
```
int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-d*(c*x^2+b*x+a)^(1/2)/a/x+1/2/a*(-(2*a*e-b*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)+2*a*f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))/c^(1/2)+2*a*g*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 703, normalized size of antiderivative = 5.06

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/4*((2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + (b*c^2*d - 2*a*c^2*e)
*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*
x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqrt(c*x^2 + b*x
+ a))/(a^2*c^2*x), -1/4*(2*(2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (b*c^2*
d - 2*a*c^2*e)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2
+ b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a^2*c*g*x - a*c^2*d)*sqr
t(c*x^2 + b*x + a))/(a^2*c^2*x), -1/4*(2*(b*c^2*d - 2*a*c^2*e)*sqrt(-a)*x*
arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a
^2)) + (2*a^2*c*f - a^2*b*g)*sqrt(c)*x*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(a^2*c*g*x - a*c^2*
d)*sqrt(c*x^2 + b*x + a))/(a^2*c^2*x), -1/2*((b*c^2*d - 2*a*c^2*e)*sqrt(-a
)*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x
 + a^2)) + (2*a^2*c*f - a^2*b*g)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(a^2*c*g*x - a*c^2*d)
*sqrt(c*x^2 + b*x + a))/(a^2*c^2*x)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.22

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx+a}g}{c} - \frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{(2\,cf-bg)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)bd+2\,a\sqrt{c}d}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)a}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x + a)*g/c - (b*d - 2*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))/sqrt(-a))/(sqrt(-a)*a) - 1/2*(2*c*f - b*g)*log(abs(2*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(3/2) + ((sqrt(c)*x - sqrt(c*x^
2 + b*x + a))*b*d + 2*a*sqrt(c)*d)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2
 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 20.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx = \frac{g\,\sqrt{c\,x^2+b\,x+a}}{c} - \frac{e\,\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}} + \frac{f\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{\sqrt{c}} - \frac{b\,g\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x+a}\right)}{2\,c^{3/2}} - \frac{d\,\sqrt{c\,x^2+b\,x+a}}{a\,x} + \frac{b\,d\,\mathrm{atanh}\left(\frac{a+\frac{b\,x}{2}}{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}\right)}{2\,a^{3/2}}$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^2*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
(g*(a + b*x + c*x^2)^(1/2))/c - (e*log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x
^2)^(1/2))/x))/a^(1/2) + (f*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1
/2)))/c^(1/2) - (b*g*log((b/2 + c*x)/c^(1/2) + (a + b*x + c*x^2)^(1/2)))/(
2*c^(3/2)) - (d*(a + b*x + c*x^2)^(1/2))/(a*x) + (b*d*atanh((a + (b*x)/2)/
(a^(1/2)*(a + b*x + c*x^2)^(1/2))))/(2*a^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.50

$$\int \frac{d+ex+fx^2+gx^3}{x^2\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\sqrt{c\,x^2+bx+a}\,a^2cgx-2\sqrt{c\,x^2+bx+a}\,a\,c^2d+2\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)a\,c^2ex-\sqrt{a}$$

input
```
int((g*x^3+f*x^2+e*x+d)/x^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a + b*x + c*x**2)*a**2*c*g*x - 2*sqrt(a + b*x + c*x**2)*a*c**2*d +
 2*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c**2*e*x -
sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*c**2*d*x - 2*s
qrt(a)*log(x)*a*c**2*e*x + sqrt(a)*log(x)*b*c**2*d*x - sqrt(c)*log( - 2*sq
rt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*b*g*x + 2*sqrt(c)*log( - 2*
sqrt(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*c*f*x)/(2*a**2*c**2*x)
```

3.120 $\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 159

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3bd-4ae)\sqrt{a+bx+cx^2}}{4a^2x} - \frac{(3b^2d-4acd-4abe+8a^2f)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}} + \frac{g\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

output

```
-1/2*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*a*e+3*b*d)*(c*x^2+b*x+a)^(1/2)/a^
2/x-1/8*(8*a^2*f-4*a*b*e-4*a*c*d+3*b^2*d)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c
*x^2+b*x+a)^(1/2))/a^(5/2)+g*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.07

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(3bdx-2a(d+2ex))}{x^2}+(3b^2d+8a^2f)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+4a(cd+be)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\ldots}{4a^{5/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(3*b*d*x - 2*a*(d + 2*e*x)))/x^2 + (3*b^2*
d + 8*a^2*f)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 4*a*(c
*d + b*e)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]] - (4*a^(
5/2)*g*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/(4*a^(5/
2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2181, 27, 2181, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2181$$
$$-\frac{\int \frac{-4agx^2+2(cd-2af)x+3bd-4ae}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$
$$\Big\downarrow \ 27$$
$$-\frac{\int \frac{-4agx^2+2(cd-2af)x+3bd-4ae}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{2181}$$

$$-\frac{-\frac{\int \frac{8fa^2+8gxa^2-4cda-4bea+3b^2d}{2x\sqrt{cx^2+bx+a}}dx}{a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{8fa^2+8gxa^2-4cda-4bea+3b^2d}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{-\frac{(8a^2f-4abe-4acd+3b^2d)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+8a^2g\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1092}$$

$$-\frac{-\frac{(8a^2f-4abe-4acd+3b^2d)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+16a^2g\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{(8a^2f-4abe-4acd+3b^2d)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx+\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{-\frac{\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-2(8a^2f-4abe-4acd+3b^2d)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\frac{8a^2g\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)(8a^2f-4abe-4acd+3b^2d)}{\sqrt{a}}}{2a}-\frac{\sqrt{a+bx+cx^2}(3bd-4ae)}{ax}}{4a}-\frac{d\sqrt{a+bx+cx^2}}{2ax^2}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(d*Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((3*b*d - 4*a*e)*Sqrt[a + b*x
+ c*x^2])/(a*x)) - (-(((3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a]) + (8*a^2*g*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c])/(2*a))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(4aex-3dxb+2ad)}{4a^2x^2}+\frac{-\frac{\left(8a^2f-4abe-4acd+3b^2d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}+\frac{8a^2g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}}{8a^2}$
default	$\frac{g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+d\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{}\right.}{2}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(4*a*e*x-3*b*d*x+2*a*d)/a^2/x^2+1/8/a^2*(-(8*a^2*
f-4*a*b*e-4*a*c*d+3*b^2*d)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)+8*a^2*g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 783, normalized size of antiderivative = 4.92

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/16*(8*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 +
 b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c
 - 4*a*c^2)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^
2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(2*a^2*c*d - (3*a*b*c*d
 - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/16*(16*a^3*sqrt(-c
)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b
*c*x + a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(a)*x^2
*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*s
qrt(a) + 8*a^2)/x^2) + 4*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^
2 + b*x + a))/(a^3*c*x^2), 1/8*(4*a^3*sqrt(c)*g*x^2*log(-8*c^2*x^2 - 8*b*c
*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*a*b*c
*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*a^2*c*d -
 (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*a^
3*sqrt(-c)*g*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^
2*x^2 + b*c*x + a*c)) + (4*a*b*c*e - 8*a^2*c*f - (3*b^2*c - 4*a*c^2)*d)*sq
rt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2
+ a*b*x + a^2)) + 2*(2*a^2*c*d - (3*a*b*c*d - 4*a^2*c*e)*x)*sqrt(c*x^2 + b
*x + a))/(a^3*c*x^2)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(133) = 266$.

Time = 0.24 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.18

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{g\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}} + \frac{(3\,b^2d-4\,acd-4\,abe+8\,a^2f)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2} - \frac{3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3b^2d-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3acd-4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3abe-8}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-g*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c) +
1/4*(3*b^2*d - 4*a*c*d - 4*a*b*e + 8*a^2*f)*arctan(-(sqrt(c)*x - sqrt(c*x^
2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/4*(3*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))^3*b^2*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*c*d - 4*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*a^2*sqrt(c)*e - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a*b^2*d - 4*
(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 +
b*x + a))*a^2*b*e - 8*a^2*b*sqrt(c)*d + 8*a^3*sqrt(c)*e)/(((sqrt(c)*x - sq
rt(c*x^2 + b*x + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.89

$$\int \frac{d+ex+fx^2+gx^3}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{-4\sqrt{c\,x^2+bx+a}\,a^2cd-8\sqrt{c\,x^2+bx+a}\,a^2cex+6\sqrt{c\,x^2+bx+a}\,abcdx+8\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+\right.}{}$$

input
```
int((g*x^3+f*x^2+e*x+d)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2*c*d - 8*sqrt(a + b*x + c*x**2)*a**2*c*e*
x + 6*sqrt(a + b*x + c*x**2)*a*b*c*d*x + 8*sqrt(a)*log(2*sqrt(a)*sqrt(a +
b*x + c*x**2) - 2*a - b*x)*a**2*c*f*x**2 - 4*sqrt(a)*log(2*sqrt(a)*sqrt(a
+ b*x + c*x**2) - 2*a - b*x)*a*b*c*e*x**2 - 4*sqrt(a)*log(2*sqrt(a)*sqrt(a
 + b*x + c*x**2) - 2*a - b*x)*a*c**2*d*x**2 + 3*sqrt(a)*log(2*sqrt(a)*sqrt
(a + b*x + c*x**2) - 2*a - b*x)*b**2*c*d*x**2 - 8*sqrt(a)*log(x)*a**2*c*f*
x**2 + 4*sqrt(a)*log(x)*a*b*c*e*x**2 + 4*sqrt(a)*log(x)*a*c**2*d*x**2 - 3*
sqrt(a)*log(x)*b**2*c*d*x**2 + 8*sqrt(c)*log( - 2*sqrt(c)*sqrt(a + b*x + c
*x**2) - b - 2*c*x)*a**3*g*x**2)/(8*a**3*c*x**2)
```

3.121 $\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{3ax^3}+\frac{(5bd-6ae)\sqrt{a+bx+cx^2}}{12a^2x^2}\\ &\quad-\frac{(15b^2d-16acd-18abe+24a^2f)\sqrt{a+bx+cx^2}}{24a^3x}\\ &\quad+\frac{(5b^3d-6ab^2e-4ab(3cd-2af)+8a^2(ce-2ag))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{7/2}}\end{aligned}$$

output
```
-1/3*d*(c*x^2+b*x+a)^(1/2)/a/x^3+1/12*(-6*a*e+5*b*d)*(c*x^2+b*x+a)^(1/2)/a
^2/x^2-1/24*(24*a^2*f-18*a*b*e-16*a*c*d+15*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/
x+1/16*(5*b^3*d-6*a*b^2*e-4*a*b*(-2*a*f+3*c*d)+8*a^2*(-2*a*g+c*e))*arctanh
(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(-15b^2dx^2+2ax(5bd+8cdx+9bex)-4a^2(2d+3x(e+2fx)))}{x^3}+3(-5b^3d+16a^3g)\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-}{24a^{7/2}}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-15*b^2*d*x^2 + 2*a*x*(5*b*d + 8*c*d*x +
9*b*e*x) - 4*a^2*(2*d + 3*x*(e + 2*f*x))))/x^3 + 3*(-5*b^3*d + 16*a^3*g)*A
rcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 6*a*(-6*b*c*d - 3*b^
2*e + 4*a*c*e + 4*a*b*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sq
rt[a]])/(24*a^(7/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2181$$
$$-\frac{\int \frac{-6agx^2+2(2cd-3af)x+5bd-6ae}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$
$$\Big\downarrow \ 27$$
$$-\frac{\int \frac{-6agx^2+2(2cd-3af)x+5bd-6ae}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 2181$$

$$-\frac{-\frac{\int \frac{15db^2-18aeb-8a(2cd-3af)+2\left(12ga^2-6cea+5bcd\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{24fa^2-16cda-18bea+15b^2d+2\left(12ga^2-6cea+5bcd\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 1228$$

$$-\frac{-\frac{-\frac{3\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6ae)}{2ax^2}}{6a}-\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{\frac{3\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6}{2ax^2}}{6a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{\frac{3\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(8a^2(ce-2ag)-6ab^2e-4ab(3cd-2af)+5b^3d\right)}{2a^{3/2}}-\frac{\sqrt{a+bx+cx^2}\left(-18abe-8a(2cd-3af)+15b^2d\right)}{ax}}{4a}-\frac{\sqrt{a+bx+cx^2}(5bd-6a}{2ax^2}}{6a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{3ax^3}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^4*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/3*(d*Sqrt[a + b*x + c*x^2])/(a*x^3) - (-1/2*((5*b*d - 6*a*e)*Sqrt[a + b
*x + c*x^2])/(a*x^2) - (-(((15*b^2*d - 18*a*b*e - 8*a*(2*c*d - 3*a*f))*Sqr
t[a + b*x + c*x^2])/(a*x)) + (3*(5*b^3*d - 6*a*b^2*e - 4*a*b*(3*c*d - 2*a*
f) + 8*a^2*(c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*
x^2])])/(2*a^(3/2)))/(4*a))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(24a^2 f\,x^2-18abe\,x^2-16acd\,x^2+15b^2 d\,x^2+12a^2 ex-10abdx+8a^2 d)}{24a^3 x^3}-\frac{(16a^3 g-8a^2 bf-8a^2 ce+6a\,b^2 e+12abcd-5}{16a^{\frac{7}{2}}}$
default	$d\left(-\frac{\sqrt{c\,x^2+bx+a}}{3a\,x^3}-\frac{5b\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right.}{6a}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x^2+b*x+a)^(1/2)*(24*a^2*f*x^2-18*a*b*e*x^2-16*a*c*d*x^2+15*b^2*d
*x^2+12*a^2*e*x-10*a*b*d*x+8*a^2*d)/a^3/x^3-1/16*(16*a^3*g-8*a^2*b*f-8*a^2
*c*e+6*a*b^2*e+12*a*b*c*d-5*b^3*d)/a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 1.68 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{a}x^3\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}(bx+2}{x^2}\right.}{96}\right.$$
$$-\frac{3\,(8\,a^2bf-16\,a^3g+(5\,b^3-12\,abc)d-2\,(3\,ab^2-4\,a^2c)e)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{cx^2+bx+a}(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)+2\,(8}{48\,a^4x^3}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/96*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c)*d - 2*(3*a*b^2 - 4*a^
2*c)*e)*sqrt(a)*x^3*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
 + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(8*a^3*d - (18*a^2*b*e - 24*a^
3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*sqrt(c*x^2
 + b*x + a))/(a^4*x^3), -1/48*(3*(8*a^2*b*f - 16*a^3*g + (5*b^3 - 12*a*b*c
)*d - 2*(3*a*b^2 - 4*a^2*c)*e)*sqrt(-a)*x^3*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(8*a^3*d - (18*a^2*b*
e - 24*a^3*f - (15*a*b^2 - 16*a^2*c)*d)*x^2 - 2*(5*a^2*b*d - 6*a^3*e)*x)*s
qrt(c*x^2 + b*x + a))/(a^4*x^3)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**4/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**4*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(164) = 328.

Time = 0.19 (sec) , antiderivative size = 680, normalized size of antiderivative = 3.66

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= -\frac{(5\,b^3d-12\,abcd-6\,ab^2e+8\,a^2ce+8\,a^2bf-16\,a^3g)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^3}$$
$$+\frac{15\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 b^3d-36\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 abcd-18\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^5 ab^2e}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(5*b^3*d - 12*a*b*c*d - 6*a*b^2*e + 8*a^2*c*e + 8*a^2*b*f - 16*a^3*g)
*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/
24*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*b^3*d - 36*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^5*a*b*c*d - 18*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b
^2*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*c*e + 24*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))^5*a^2*b*f + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
4*a^3*sqrt(c)*f - 40*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a*b^3*d + 96*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b*c*d + 48*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^3*a^2*b^2*e - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b*
f + 96*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*a^3*c^(3/2)*d + 48*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^2*a^3*b*sqrt(c)*e - 96*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2*a^4*sqrt(c)*f + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^2*b^
3*d + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^3*b*c*d - 30*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*a^3*b^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a
^4*c*e + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*a^4*b*f + 48*a^3*b^2*sqrt(
c)*d - 32*a^4*c^(3/2)*d - 48*a^4*b*sqrt(c)*e + 48*a^5*sqrt(c)*f)/(((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2 - a)^3*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx = \int \frac{g\,x^3+f\,x^2+e\,x+d}{x^4\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^4*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.37

$$\int \frac{d+ex+fx^2+gx^3}{x^4\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{-16\sqrt{c\,x^2+bx+a}\,a^3d-24\sqrt{c\,x^2+bx+a}\,a^3ex-48\sqrt{c\,x^2+bx+a}\,a^3f\,x^2+20\sqrt{c\,x^2+bx+a}\,a^2bdx-}{}$$

input
```
int((g*x^3+f*x^2+e*x+d)/x^4/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x + c*x**2)*a**3*d - 24*sqrt(a + b*x + c*x**2)*a**3*e*x
- 48*sqrt(a + b*x + c*x**2)*a**3*f*x**2 + 20*sqrt(a + b*x + c*x**2)*a**2*b
*d*x + 36*sqrt(a + b*x + c*x**2)*a**2*b*e*x**2 + 32*sqrt(a + b*x + c*x**2)
*a**2*c*d*x**2 - 30*sqrt(a + b*x + c*x**2)*a*b**2*d*x**2 + 48*sqrt(a)*log(
2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*g*x**3 - 24*sqrt(a)*log
(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*f*x**3 - 24*sqrt(a)*
log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*c*e*x**3 + 18*sqrt(
a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**2*e*x**3 + 36*sq
rt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*c*d*x**3 - 15*
sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*d*x**3 - 48
*sqrt(a)*log(x)*a**3*g*x**3 + 24*sqrt(a)*log(x)*a**2*b*f*x**3 + 24*sqrt(a)
*log(x)*a**2*c*e*x**3 - 18*sqrt(a)*log(x)*a*b**2*e*x**3 - 36*sqrt(a)*log(x
)*a*b*c*d*x**3 + 15*sqrt(a)*log(x)*b**3*d*x**3)/(48*a**4*x**3)
```

3.122 $\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 270

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{4ax^4}+\frac{(7bd-8ae)\sqrt{a+bx+cx^2}}{24a^2x^3} - \frac{(35b^2d-36acd-40abe+48a^2f)\sqrt{a+bx+cx^2}}{96a^3x^2} + \frac{(105b^3d-120ab^2e-4ab(55cd-36af)+64a^2(2ce-3ag))\sqrt{a+bx+cx^2}}{192a^4x} - \frac{(35b^4d-40ab^3e+16a^2c(3cd-4af)-24ab^2(5cd-2af)+32a^2b(3ce-2ag))\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{128a^{9/2}}$$

output
```
-1/4*d*(c*x^2+b*x+a)^(1/2)/a/x^4+1/24*(-8*a*e+7*b*d)*(c*x^2+b*x+a)^(1/2)/a
^2/x^3-1/96*(48*a^2*f-40*a*b*e-36*a*c*d+35*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^3/
x^2+1/192*(105*b^3*d-120*a*b^2*e-4*a*b*(-36*a*f+55*c*d)+64*a^2*(-3*a*g+2*c
*e))*(c*x^2+b*x+a)^(1/2)/a^4/x-1/128*(35*b^4*d-40*a*b^3*e+16*a^2*c*(-4*a*f
+3*c*d)-24*a*b^2*(-2*a*f+5*c*d)+32*a^2*b*(-2*a*g+3*c*e))*arctanh(1/2*(b*x+
2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}(105b^3dx^3-10abx^2(7bd+22cdx+12bex)+8a^2x(7bd+cx(9d+16ex)+2bx(5e+9fx))-16a^3(3d+4ex+6x^2(f+2gx)))}{x^4}+105b^4}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(105*b^3*d*x^3 - 10*a*b*x^2*(7*b*d + 22*c*
d*x + 12*b*e*x) + 8*a^2*x*(7*b*d + c*x*(9*d + 16*e*x) + 2*b*x*(5*e + 9*f*x
)) - 16*a^3*(3*d + 4*e*x + 6*x^2*(f + 2*g*x))))/x^4 + 105*b^4*d*ArcTanh[(S
qrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + 24*a*(5*b^3*e + 3*b^2*(5*c*d
- 2*a*f) + 2*a*c*(-3*c*d + 4*a*f) + 4*a*b*(-3*c*e + 2*a*g))*ArcTanh[(-(Sqr
t[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(192*a^(9/2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2181, 27, 2181, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2181}$$
$$-\frac{\int \frac{-8agx^2+2(3cd-4af)x+7bd-8ae}{2x^4\sqrt{cx^2+bx+a}}dx}{4a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{-8agx^2+2(3cd-4af)x+7bd-8ae}{x^4\sqrt{cx^2+bx+a}}dx}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 2181

$$-\frac{-\frac{\int \frac{35db^2-40aeb-12a(3cd-4af)+4\left(12ga^2-8cea+7bcd\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{3ax^3}}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 27

$$-\frac{-\frac{\int \frac{48fa^2-36cda-40bea+35b^2d+4\left(12ga^2-8cea+7bcd\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}(7bd-8ae)}{3ax^3}}{8a}-\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1237

$$-\frac{-\frac{-\frac{\int \frac{105db^3-120aeb^2-4a(55cd-36af)b+64a^2(2ce-3ag)+2c\left(35db^2-40aeb-12a(3cd-4af)\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}}{6a}-\sqrt{a+b\cdots}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 27

$$-\frac{-\frac{-\frac{\int \frac{-192ga^3+128cea^2+144bfa^2-220bcda-120b^2ea+105b^3d+2c\left(35db^2-40aeb-12a(3cd-4af)\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}}{6a}-\sqrt{\cdots}}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{3\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(-192a^3g+144a^2bf+128a^2ce-120ab^2e-22\cdots\right.}{ax}}{4a}\cdots}{6a}\cdots}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$\downarrow$ 1154

$$-\frac{-\frac{-\frac{\frac{3\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(-192a^3g+144a^2bf+128a^2ce-\cdots\right.}{ax}}{4a}\cdots}{6a}\cdots}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{-\frac{\sqrt{a+bx+cx^2}\left(48a^2f-40abe-36acd+35b^2d\right)}{2ax^2}-\frac{\frac{3\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)\left(32a^2b(3ce-2ag)+16a^2c(3cd-4af)-40ab^3e-24ab^2(5cd-2af)+35b^4d\right)}{2a^{3/2}}\cdots}{4a}}{6a}\cdots}{8a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{4ax^4}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^5*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/4*(d*Sqrt[a + b*x + c*x^2])/(a*x^4) - (-1/3*((7*b*d - 8*a*e)*Sqrt[a + b
*x + c*x^2])/(a*x^3) - (-1/2*((35*b^2*d - 36*a*c*d - 40*a*b*e + 48*a^2*f)*
Sqrt[a + b*x + c*x^2])/(a*x^2) - (-(((105*b^3*d - 220*a*b*c*d - 120*a*b^2*
e + 128*a^2*c*e + 144*a^2*b*f - 192*a^3*g)*Sqrt[a + b*x + c*x^2])/(a*x)) +
 (3*(35*b^4*d - 40*a*b^3*e + 16*a^2*c*(3*c*d - 4*a*f) - 24*a*b^2*(5*c*d -
2*a*f) + 32*a^2*b*(3*c*e - 2*a*g))*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*a^(3/2)))/(4*a))/(6*a))/(8*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x +a}\,\left(192 a^{3} g\,x^{3}-144 a^{2} b f\,x^{3}-128 a^{2} c e\,x^{3}+120 a\,b^{2} e\,x^{3}+220 a b c d\,x^{3}-105 b^{3} d\,x^{3}+96 a^{3} f\,x^{2}-80 a^{2} b e\,x^{2}-72 a^{2} c d\,x^{2}+70 a}{192 a^{4} x^{4}}$
default	$d\left(-\frac{\sqrt{c\,x^{2}+b x +a}}{4 a\,x^{4}}-\frac{7 b\left(-\frac{\sqrt{c\,x^{2}+b x +a}}{3 a\,x^{3}}-\frac{5 b\left(-\frac{\sqrt{c\,x^{2}+b x +a}}{2 a\,x^{2}}-\frac{3 b\left(-\frac{\sqrt{c\,x^{2}+b x +a}}{a x}+\frac{b \ln\left(\frac{2 a +b x +2 \sqrt{a}\,\sqrt{c\,x^{2}+b x +a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4 a}+\frac{c \ln\left(\frac{2 a +b x}{}\right.}{}\right)}{6 a}\right)}{8 a}\right.$

input
```
int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x^2+b*x+a)^(1/2)*(192*a^3*g*x^3-144*a^2*b*f*x^3-128*a^2*c*e*x^3+
120*a*b^2*e*x^3+220*a*b*c*d*x^3-105*b^3*d*x^3+96*a^3*f*x^2-80*a^2*b*e*x^2-
72*a^2*c*d*x^2+70*a*b^2*d*x^2+64*a^3*e*x-56*a^2*b*d*x+48*a^3*d)/a^4/x^4+1/
128*(64*a^3*b*g+64*a^3*c*f-48*a^2*b^2*f-96*a^2*b*c*e-48*a^2*c^2*d+40*a*b^3
*e+120*a*b^2*c*d-35*b^4*d)/a^(9/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)
```

Fricas [A] (verification not implemented)

Time = 4.16 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.94

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \left[\frac{3\,(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f)\sqrt{a}x^4\log\left(-\frac{8\,a\cdots}{}\right.}{}\right.$$
$$-\frac{3\,(64\,a^3bg-(35\,b^4-120\,ab^2c+48\,a^2c^2)d+8\,(5\,ab^3-12\,a^2bc)e-16\,(3\,a^2b^2-4\,a^3c)f)\sqrt{-a}x^4\arctan\cdots}{}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/768*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d + 8*(5*a*b^3
 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(a)*x^4*log(-(8*a*b*x +
 (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/
x^2) - 4*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*
d - 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*
b^2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))
/(a^5*x^4), -1/384*(3*(64*a^3*b*g - (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*d
+ 8*(5*a*b^3 - 12*a^2*b*c)*e - 16*(3*a^2*b^2 - 4*a^3*c)*f)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2
)) + 2*(48*a^4*d - (144*a^3*b*f - 192*a^4*g + 5*(21*a*b^3 - 44*a^2*b*c)*d
- 8*(15*a^2*b^2 - 16*a^3*c)*e)*x^3 - 2*(40*a^3*b*e - 48*a^4*f - (35*a^2*b^
2 - 36*a^3*c)*d)*x^2 - 8*(7*a^3*b*d - 8*a^4*e)*x)*sqrt(c*x^2 + b*x + a))/(
a^5*x^4)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**5/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**5*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1433 vs. $2(244) = 488$.

Time = 0.19 (sec) , antiderivative size = 1433, normalized size of antiderivative = 5.31

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
1/64*(35*b^4*d - 120*a*b^2*c*d + 48*a^2*c^2*d - 40*a*b^3*e + 96*a^2*b*c*e
+ 48*a^2*b^2*f - 64*a^3*c*f - 64*a^3*b*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))/sqrt(-a))/(sqrt(-a)*a^4) - 1/192*(105*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*b^4*d - 360*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a*b^2*c*d
+ 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*c^2*d - 120*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^7*a*b^3*e + 288*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^
7*a^2*b*c*e + 144*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^2*f - 192*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*c*f - 192*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^3*b*g - 384*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^4*sqrt
(c)*g - 385*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a*b^4*d + 1320*(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))^5*a^2*b^2*c*d - 528*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^5*a^3*c^2*d + 440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^2*b^3*e
 - 1056*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^3*b*c*e - 528*(sqrt(c)*x -
 sqrt(c*x^2 + b*x + a))^5*a^3*b^2*f + 192*(sqrt(c)*x - sqrt(c*x^2 + b*x +
a))^5*a^4*c*f + 576*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^5*a^4*b*g - 768*(s
qrt(c)*x - sqrt(c*x^2 + b*x + a))^4*a^4*c^(3/2)*e - 384*(sqrt(c)*x - sqrt(
c*x^2 + b*x + a))^4*a^4*b*sqrt(c)*f + 1152*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))^4*a^5*sqrt(c)*g + 511*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^2*b^4*d
 - 1752*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*a^3*b^2*c*d - 528*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))^3*a^4*c^2*d - 584*(sqrt(c)*x - sqrt(c*x^2 + b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^5\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^5*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.54

$$\int \frac{d+ex+fx^2+gx^3}{x^5\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{-288\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)a^2bce\,x^4+360\sqrt{a}\log\left(-2\sqrt{a}\,\sqrt{c\,x^2+bx+a}-2a-bx\right)}{}$$

input

```
int((g*x^3+f*x^2+e*x+d)/x^5/(c*x^2+b*x+a)^(1/2),x)
```

output

```
( - 96*sqrt(a + b*x + c*x**2)*a**4*d - 128*sqrt(a + b*x + c*x**2)*a**4*e*x
 - 192*sqrt(a + b*x + c*x**2)*a**4*f*x**2 - 384*sqrt(a + b*x + c*x**2)*a**
4*g*x**3 + 112*sqrt(a + b*x + c*x**2)*a**3*b*d*x + 160*sqrt(a + b*x + c*x*
*2)*a**3*b*e*x**2 + 288*sqrt(a + b*x + c*x**2)*a**3*b*f*x**3 + 144*sqrt(a
+ b*x + c*x**2)*a**3*c*d*x**2 + 256*sqrt(a + b*x + c*x**2)*a**3*c*e*x**3 -
 140*sqrt(a + b*x + c*x**2)*a**2*b**2*d*x**2 - 240*sqrt(a + b*x + c*x**2)*
a**2*b**2*e*x**3 - 440*sqrt(a + b*x + c*x**2)*a**2*b*c*d*x**3 + 210*sqrt(a
 + b*x + c*x**2)*a*b**3*d*x**3 + 192*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x
 + c*x**2) - 2*a - b*x)*a**3*b*g*x**4 + 192*sqrt(a)*log( - 2*sqrt(a)*sqrt(
a + b*x + c*x**2) - 2*a - b*x)*a**3*c*f*x**4 - 144*sqrt(a)*log( - 2*sqrt(a
)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b**2*f*x**4 - 288*sqrt(a)*log(
- 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*b*c*e*x**4 - 144*sqrt
(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*c**2*d*x**4
+ 120*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b**3*
e*x**4 + 360*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*
a*b**2*c*d*x**4 - 105*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*
a - b*x)*b**4*d*x**4 - 192*sqrt(a)*log(x)*a**3*b*g*x**4 - 192*sqrt(a)*log(
x)*a**3*c*f*x**4 + 144*sqrt(a)*log(x)*a**2*b**2*f*x**4 + 288*sqrt(a)*log(x
)*a**2*b*c*e*x**4 + 144*sqrt(a)*log(x)*a**2*c**2*d*x**4 - 120*sqrt(a)*log(
x)*a*b**3*e*x**4 - 360*sqrt(a)*log(x)*a*b**2*c*d*x**4 + 105*sqrt(a)*log...
```

3.123 $\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{5ax^5}+\frac{(9bd-10ae)\sqrt{a+bx+cx^2}}{40a^2x^4}$$
$$-\frac{(63b^2d-64acd-70abe+80a^2f)\sqrt{a+bx+cx^2}}{240a^3x^3}$$
$$+\frac{(315b^3d-350ab^2e-4ab(161cd-100af)+120a^2(3ce-4ag))\sqrt{a+bx+cx^2}}{960a^4x^2}$$
$$-\frac{(945b^4d-1050ab^3e-60ab^2(49cd-20af)+256a^2c(4cd-5af)+40a^2b(55ce-36ag))\sqrt{a+bx+cx}}{1920a^5x}$$
$$+\frac{(63b^5d-70ab^4e+48a^2bc(5cd-4af)-40ab^3(7cd-2af)-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag))\,a}{256a^{11/2}}$$

output
```
-1/5*d*(c*x^2+b*x+a)^(1/2)/a/x^5+1/40*(-10*a*e+9*b*d)*(c*x^2+b*x+a)^(1/2)/
a^2/x^4-1/240*(80*a^2*f-70*a*b*e-64*a*c*d+63*b^2*d)*(c*x^2+b*x+a)^(1/2)/a^
3/x^3+1/960*(315*b^3*d-350*a*b^2*e-4*a*b*(-100*a*f+161*c*d)+120*a^2*(-4*a*
g+3*c*e))*(c*x^2+b*x+a)^(1/2)/a^4/x^2-1/1920*(945*b^4*d-1050*a*b^3*e-60*a*
b^2*(-20*a*f+49*c*d)+256*a^2*c*(-5*a*f+4*c*d)+40*a^2*b*(-36*a*g+55*c*e))*(
c*x^2+b*x+a)^(1/2)/a^5/x+1/256*(63*b^5*d-70*a*b^4*e+48*a^2*b*c*(-4*a*f+5*c
*d)-40*a*b^3*(-2*a*f+7*c*d)-32*a^3*c*(-4*a*g+3*c*e)+48*a^2*b^2*(-2*a*g+5*c
*e))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 3.49 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.88

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = \frac{\frac{\sqrt{a}\sqrt{a+x(b+cx)}\left(-945b^4dx^4+210ab^2x^3(3bd+14cdx+5bex)-32a^4\left(12d+5x\left(3e+4fx+6gx^2\right)\right)-4a^2x^2\left(256c^2dx^2+2bcx(161d+275ex)+b^2(126d\cdots\right.\right.}{x^5}}{}$$

input
```
Integrate[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-945*b^4*d*x^4 + 210*a*b^2*x^3*(3*b*d + 1
4*c*d*x + 5*b*e*x) - 32*a^4*(12*d + 5*x*(3*e + 4*f*x + 6*g*x^2)) - 4*a^2*x
^2*(256*c^2*d*x^2 + 2*b*c*x*(161*d + 275*e*x) + b^2*(126*d + 25*x*(7*e + 1
2*f*x))) + 16*a^3*x*(c*x*(32*d + 5*x*(9*e + 16*f*x)) + b*(27*d + 5*x*(7*e
+ 2*x*(5*f + 9*g*x))))))/x^5 - 15*(63*b^5*d + 128*a^4*c*g)*ArcTanh[(Sqrt[c
]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - 30*a*(35*b^4*e + 48*a^2*c^2*e + 20
*b^3*(7*c*d - 2*a*f) + 24*a*b*c*(-5*c*d + 4*a*f) + 24*a*b^2*(-5*c*e + 2*a*
g))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(1920*a^(11/2
))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2181, 27, 2181, 27, 1237, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{\int \frac{-10agx^2+2(4cd-5af)x+9bd-10ae}{2x^5\sqrt{cx^2+bx+a}}dx}{5a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{-10agx^2+2(4cd-5af)x+9bd-10ae}{x^5\sqrt{cx^2+bx+a}}dx}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 2181$$

$$-\frac{-\frac{\int \frac{63db^2-70aeb-16a(4cd-5af)+2\left(40ga^2-30cea+27bcd\right)x}{2x^4\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{4ax^4}}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{80fa^2-64cda-70bea+63b^2d+2\left(40ga^2-30cea+27bcd\right)x}{x^4\sqrt{cx^2+bx+a}}dx}{8a}-\frac{\sqrt{a+bx+cx^2}(9bd-10ae)}{4ax^4}}{10a}-\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 1237$$

$$-\frac{-\frac{-\frac{\int \frac{315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)+4c\left(63db^2-70aeb-16a(4cd-5af)\right)x}{2x^3\sqrt{cx^2+bx+a}}dx}{3a}-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}}{8a}-\frac{\sqrt{a}\cdots}{\cdots}}{10a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{\int \frac{-480ga^3+360cea^2+400bfa^2-644bcda-350b^2ea+315b^3d+4c\left(63db^2-70aeb-16a(4cd-5af)\right)x}{x^3\sqrt{cx^2+bx+a}}dx}{6a}-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}}{8a}-\frac{\sqrt{\cdots}}{\cdots}}{10a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 1237$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{945db^4-1050aeb^3-60a(49cd-20af)b^2+40a^2(55ce-36ag)b+256a^2c(4cd-5af)+2c\left(315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)\right)x}{2x^2\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{\cdots}}{\cdots}}{6a}\cdots}{8a}\cdots}{10a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{-\frac{\int \frac{945db^4-1050aeb^3-60a(49cd-20af)b^2+40a^2(55ce-36ag)b+256a^2c(4cd-5af)+2c\left(315db^3-350aeb^2-4a(161cd-100af)b+120a^2(3ce-4ag)\right)x}{x^2\sqrt{cx^2+bx+a}}dx}{4a}-\frac{\sqrt{\ }}{}}{6a}}{8a}}{10a}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$\Big\downarrow$ 1228

$$-\frac{-\frac{-\frac{-\frac{15\left(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab^3(7cd-2af)+63b^5d\right)\int \frac{1}{x\sqrt{cx^2+bx+a}}dx}{2a}-\frac{\sqrt{a+bx+cx^2}\left(40a^2b(55ce-36a\right.}{}}{4a}}{6a}}{}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$\Big\downarrow$ 1154

$$-\frac{-\frac{-\frac{-\frac{15\left(-32a^3c(3ce-4ag)+48a^2b^2(5ce-2ag)+48a^2bc(5cd-4af)-70ab^4e-40ab^3(7cd-2af)+63b^5d\right)\int \frac{1}{4a-\frac{(2a+bx)^2}{cx^2+bx+a}}d\frac{2a+bx}{\sqrt{cx^2+bx+a}}}{a}-\frac{\sqrt{a+bx+cx^2}\left(40a^2\right.}{}}{4a}}{6a}}{}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

$\Big\downarrow$ 219

$$-\frac{-\frac{-\frac{\sqrt{a+bx+cx^2}\left(80a^2f-70abe-64acd+63b^2d\right)}{3ax^3}-\frac{-\frac{\sqrt{a+bx+cx^2}\left(120a^2(3ce-4ag)-350ab^2e-4ab(161cd-100af)+315b^3d\right)}{2ax^2}-\frac{15\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+c}}\right.}{}}{}}{}}{}$$

$$\frac{d\sqrt{a+bx+cx^2}}{5ax^5}$$

input
```
Int[(d + e*x + f*x^2 + g*x^3)/(x^6*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/5*(d*Sqrt[a + b*x + c*x^2])/(a*x^5) - (-1/4*((9*b*d - 10*a*e)*Sqrt[a +
b*x + c*x^2])/(a*x^4) - (-1/3*((63*b^2*d - 64*a*c*d - 70*a*b*e + 80*a^2*f)
*Sqrt[a + b*x + c*x^2])/(a*x^3) - (-1/2*((315*b^3*d - 350*a*b^2*e - 4*a*b*
(161*c*d - 100*a*f) + 120*a^2*(3*c*e - 4*a*g))*Sqrt[a + b*x + c*x^2])/(a*x
^2) - (-(((945*b^4*d - 1050*a*b^3*e - 60*a*b^2*(49*c*d - 20*a*f) + 256*a^2
*c*(4*c*d - 5*a*f) + 40*a^2*b*(55*c*e - 36*a*g))*Sqrt[a + b*x + c*x^2])/(a
*x)) + (15*(63*b^5*d - 70*a*b^4*e + 48*a^2*b*c*(5*c*d - 4*a*f) - 40*a*b^3*
(7*c*d - 2*a*f) - 32*a^3*c*(3*c*e - 4*a*g) + 48*a^2*b^2*(5*c*e - 2*a*g))*A
rcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)))/(4*a))
/(6*a))/(8*a))/(10*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(-1440a^3bg\,x^4-1280a^3cf\,x^4+1200a^2b^2f\,x^4+2200a^2bce\,x^4+1024a^2c^2d\,x^4-1050a\,b^3e\,x^4-2940a\,b^2cd\,x^4+945b^4d$
default	Expression too large to display

input
```
int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x^2+b*x+a)^(1/2)*(-1440*a^3*b*g*x^4-1280*a^3*c*f*x^4+1200*a^2*b
^2*f*x^4+2200*a^2*b*c*e*x^4+1024*a^2*c^2*d*x^4-1050*a*b^3*e*x^4-2940*a*b^2
*c*d*x^4+945*b^4*d*x^4+960*a^4*g*x^3-800*a^3*b*f*x^3-720*a^3*c*e*x^3+700*a
^2*b^2*e*x^3+1288*a^2*b*c*d*x^3-630*a*b^3*d*x^3+640*a^4*f*x^2-560*a^3*b*e*
x^2-512*a^3*c*d*x^2+504*a^2*b^2*d*x^2+480*a^4*e*x-432*a^3*b*d*x+384*a^4*d)
/a^5/x^5+1/256*(128*a^4*c*g-96*a^3*b^2*g-192*a^3*b*c*f-96*a^3*c^2*e+80*a^2
*b^3*f+240*a^2*b^2*c*e+240*a^2*b*c^2*d-70*a*b^4*e-280*a*b^3*c*d+63*b^5*d)/
a^(11/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 11.54 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.96

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/7680*(15*((63*b^5 - 280*a*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*
a^2*b^2*c + 48*a^3*c^2)*e + 16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2
- 4*a^4*c)*g)*sqrt(a)*x^5*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2
 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(384*a^5*d - (1440*a^4*b
*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^3*c^2)*d + 50*(21*a^2*b^3 - 44*a
^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*x^4 - 2*(400*a^4*b*f - 480*a^5*g
 + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(
70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a^4*c)*d)*x^2 - 48*(9*a^4*b*d - 1
0*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^5), -1/3840*(15*((63*b^5 - 280*a
*b^3*c + 240*a^2*b*c^2)*d - 2*(35*a*b^4 - 120*a^2*b^2*c + 48*a^3*c^2)*e +
16*(5*a^2*b^3 - 12*a^3*b*c)*f - 32*(3*a^3*b^2 - 4*a^4*c)*g)*sqrt(-a)*x^5*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^
2)) + 2*(384*a^5*d - (1440*a^4*b*g - (945*a*b^4 - 2940*a^2*b^2*c + 1024*a^
3*c^2)*d + 50*(21*a^2*b^3 - 44*a^3*b*c)*e - 80*(15*a^3*b^2 - 16*a^4*c)*f)*
x^4 - 2*(400*a^4*b*f - 480*a^5*g + 7*(45*a^2*b^3 - 92*a^3*b*c)*d - 10*(35*
a^3*b^2 - 36*a^4*c)*e)*x^3 - 8*(70*a^4*b*e - 80*a^5*f - (63*a^3*b^2 - 64*a
^4*c)*d)*x^2 - 48*(9*a^4*b*d - 10*a^5*e)*x)*sqrt(c*x^2 + b*x + a))/(a^6*x^
5)]
```

Sympy [F]

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx = \int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x**3+f*x**2+e*x+d)/x**6/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x + f*x**2 + g*x**3)/(x**6*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ex + fx^2 + gx^3}{x^6\sqrt{a + bx + cx^2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2155 vs. $2(341) = 682$.

Time = 0.21 (sec) , antiderivative size = 2155, normalized size of antiderivative = 5.81

$$\int \frac{d + ex + fx^2 + gx^3}{x^6\sqrt{a + bx + cx^2}}\, dx = \text{Too large to display}$$

input
```
integrate((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-1/128*(63*b^5*d - 280*a*b^3*c*d + 240*a^2*b*c^2*d - 70*a*b^4*e + 240*a^2*
b^2*c*e - 96*a^3*c^2*e + 80*a^2*b^3*f - 192*a^3*b*c*f - 96*a^3*b^2*g + 128
*a^4*c*g)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*
a^5) + 1/1920*(945*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*b^5*d - 4200*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a*b^3*c*d + 3600*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b*c^2*d - 1050*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a*
b^4*e + 3600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^2*b^2*c*e - 1440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*c^2*e + 1200*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^9*a^2*b^3*f - 2880*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*
b*c*f - 1440*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^9*a^3*b^2*g + 1920*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^9*a^4*c*g - 4410*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^7*a*b^5*d + 19600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^3*c
*d - 16800*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b*c^2*d + 4900*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))^7*a^2*b^4*e - 16800*(sqrt(c)*x - sqrt(c*x^2
+ b*x + a))^7*a^3*b^2*c*e + 6720*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4
*c^2*e - 5600*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^3*b^3*f + 13440*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^7*a^4*b*c*f + 6720*(sqrt(c)*x - sqrt(c*x^2
 + b*x + a))^7*a^4*b^2*g - 3840*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^7*a^5*
c*g + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*c^(3/2)*f + 3840*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))^6*a^5*b*sqrt(c)*g + 8064*(sqrt(c)*x - s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+ex+fx^2+gx^3}{x^6\sqrt{a+bx+cx^2}}\,dx=\int \frac{g\,x^3+f\,x^2+e\,x+d}{x^6\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((d + e*x + f*x^2 + g*x^3)/(x^6*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 982, normalized size of antiderivative = 2.65

$$\int \frac{d + ex + fx^2 + gx^3}{x^6\sqrt{a + bx + cx^2}}\,dx = \text{Too large to display}$$

input
```
int((g*x^3+f*x^2+e*x+d)/x^6/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 768*sqrt(a + b*x + c*x**2)*a**5*d - 960*sqrt(a + b*x + c*x**2)*a**5*e*
x - 1280*sqrt(a + b*x + c*x**2)*a**5*f*x**2 - 1920*sqrt(a + b*x + c*x**2)*
a**5*g*x**3 + 864*sqrt(a + b*x + c*x**2)*a**4*b*d*x + 1120*sqrt(a + b*x +
c*x**2)*a**4*b*e*x**2 + 1600*sqrt(a + b*x + c*x**2)*a**4*b*f*x**3 + 2880*s
qrt(a + b*x + c*x**2)*a**4*b*g*x**4 + 1024*sqrt(a + b*x + c*x**2)*a**4*c*d
*x**2 + 1440*sqrt(a + b*x + c*x**2)*a**4*c*e*x**3 + 2560*sqrt(a + b*x + c*
x**2)*a**4*c*f*x**4 - 1008*sqrt(a + b*x + c*x**2)*a**3*b**2*d*x**2 - 1400*
sqrt(a + b*x + c*x**2)*a**3*b**2*e*x**3 - 2400*sqrt(a + b*x + c*x**2)*a**3
*b**2*f*x**4 - 2576*sqrt(a + b*x + c*x**2)*a**3*b*c*d*x**3 - 4400*sqrt(a +
 b*x + c*x**2)*a**3*b*c*e*x**4 - 2048*sqrt(a + b*x + c*x**2)*a**3*c**2*d*x
**4 + 1260*sqrt(a + b*x + c*x**2)*a**2*b**3*d*x**3 + 2100*sqrt(a + b*x + c
*x**2)*a**2*b**3*e*x**4 + 5880*sqrt(a + b*x + c*x**2)*a**2*b**2*c*d*x**4 -
 1890*sqrt(a + b*x + c*x**2)*a*b**4*d*x**4 + 1920*sqrt(a)*log( - 2*sqrt(a)
*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**4*c*g*x**5 - 1440*sqrt(a)*log( - 2
*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b**2*g*x**5 - 2880*sqrt(
a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*b*c*f*x**5 -
1440*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**3*c**
2*e*x**5 + 1200*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*
x)*a**2*b**3*f*x**5 + 3600*sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*a**2*b**2*c*e*x**5 + 3600*sqrt(a)*log( - 2*sqrt(a)*sqrt(a...
```

3.124 $\int (d+ex)^3 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

Optimal result

Integrand size = 36, antiderivative size = 266

$$\begin{aligned}&\int (d+ex)^3 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\,dx\\&=\frac{3(d+ex)^4}{2e}+\frac{d(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)(d+ex)^4}{4e^7}\\&\quad-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^5}{5e^7}\\&\quad+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^6}{6e^7}\\&\quad-\frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^7}{7e^7}\\&\quad+\frac{(300d^2+85de+17e^2)(d+ex)^8}{8e^7}-\frac{(120d+17e)(d+ex)^9}{9e^7}+\frac{2(d+ex)^{10}}{e^7}\end{aligned}$$

output
```
3/2*(e*x+d)^4/e+1/4*d*(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5
)*(e*x+d)^4/e^7-1/5*(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5
)*(e*x+d)^5/e^7+1/6*(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d
)^6/e^7-2/7*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^7/e^7+1/8*(300*d^2+8
5*d*e+17*e^2)*(e*x+d)^8/e^7-1/9*(120*d+17*e)*(e*x+d)^9/e^7+2*(e*x+d)^10/e^
7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.80

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx$$
$$= 6d^3x+\frac{1}{2}d^2(7d+18e)x^2+d\left(7d^2+7de+6e^2\right)x^3$$
$$+\frac{1}{4}\left(-4d^3+63d^2e+21de^2+6e^3\right)x^4+\frac{1}{5}\left(17d^3-12d^2e+63de^2+7e^3\right)x^5$$
$$+\frac{1}{6}\left(-17d^3+51d^2e-12de^2+21e^3\right)x^6+\frac{1}{7}\left(20d^3-51d^2e+51de^2-4e^3\right)x^7$$
$$+\frac{1}{8}e\left(60d^2-51de+17e^2\right)x^8+\frac{1}{9}(60d-17e)e^2x^9+2e^3x^{10}$$

input
```
Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d^3*x + (d^2*(7*d + 18*e)*x^2)/2 + d*(7*d^2 + 7*d*e + 6*e^2)*x^3 + ((-4*
d^3 + 63*d^2*e + 21*d*e^2 + 6*e^3)*x^4)/4 + ((17*d^3 - 12*d^2*e + 63*d*e^2
 + 7*e^3)*x^5)/5 + ((-17*d^3 + 51*d^2*e - 12*d*e^2 + 21*e^3)*x^6)/6 + ((20
*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7)/7 + (e*(60*d^2 - 51*d*e + 17*e^2)
*x^8)/8 + ((60*d - 17*e)*e^2*x^9)/9 + 2*e^3*x^10
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right) \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^3 \, dx$$
$$\Big\downarrow \; 2159$$
$$\int \left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^7}{e^6}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^6}{e^6}+\frac{\left(300d^4+170d^3e+102d^2e\right.}{}\right.$$

$$\downarrow \text{2009}$$

$$\frac{(300d^2+85de+17e^2)(d+ex)^8}{8e^7} - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^7}{7e^7} + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^6}{6e^7} + \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^7} - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^5}{5e^7} + \frac{2(d+ex)^{10}}{e^7} - \frac{(120d+17e)(d+ex)^9}{9e^7}$$

input

```
Int[(d + e*x)^3*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output

```
((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
+ e*x)^4)/(4*e^7) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*
e^4 - 7*e^5)*(d + e*x)^5)/(5*e^7) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 +
12*d*e^3 + 21*e^4)*(d + e*x)^6)/(6*e^7) - (2*(200*d^3 + 85*d^2*e + 34*d*e^
2 + 2*e^3)*(d + e*x)^7)/(7*e^7) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^8
)/(8*e^7) - ((120*d + 17*e)*(d + e*x)^9)/(9*e^7) + (2*(d + e*x)^10)/e^7
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.75

method	result
norman	$2e^3x^{10} + \left(\frac{20}{3}d\,e^2 - \frac{17}{9}e^3\right)x^9 + \left(\frac{15}{2}d^2e - \frac{51}{8}d\,e^2 + \frac{17}{8}e^3\right)x^8 + \left(\frac{20}{7}d^3 - \frac{51}{7}d^2e + \frac{51}{7}d\,e^2 - \frac{4}{7}e^3\right)$
default	$2e^3x^{10} + \frac{(60d\,e^2-17e^3)x^9}{9} + \frac{(60d^2e-51d\,e^2+17e^3)x^8}{8} + \frac{(20d^3-51d^2e+51d\,e^2-4e^3)x^7}{7} + \frac{(-17d^3+51d^2e-12d\,e^2+}{6}$
orering	$\frac{x(5040x^9e^3+16800x^8d\,e^2-4760x^8e^3+18900x^7d^2e-16065x^7d\,e^2+5355x^7e^3+7200x^6d^3-18360x^6d^2e+18360x^6d\,e^2-1440e^3}{}$
gosper	$\frac{7}{5}x^5e^3 - x^4d^3 + 7d^3x^3 + \frac{17}{5}x^5d^3 - \frac{4}{7}x^7e^3 - \frac{17}{6}x^6d^3 + 2e^3x^{10} + 6d^3x - \frac{17}{9}x^9e^3 + \frac{17}{8}x^8e^3 + \frac{20}{7}$
risch	$\frac{7}{5}x^5e^3 - x^4d^3 + 7d^3x^3 + \frac{17}{5}x^5d^3 - \frac{4}{7}x^7e^3 - \frac{17}{6}x^6d^3 + 2e^3x^{10} + 6d^3x - \frac{17}{9}x^9e^3 + \frac{17}{8}x^8e^3 + \frac{20}{7}$
parallelrisch	$\frac{7}{5}x^5e^3 - x^4d^3 + 7d^3x^3 + \frac{17}{5}x^5d^3 - \frac{4}{7}x^7e^3 - \frac{17}{6}x^6d^3 + 2e^3x^{10} + 6d^3x - \frac{17}{9}x^9e^3 + \frac{17}{8}x^8e^3 + \frac{20}{7}$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
2*e^3*x^10+(20/3*d*e^2-17/9*e^3)*x^9+(15/2*d^2*e-51/8*d*e^2+17/8*e^3)*x^8+
(20/7*d^3-51/7*d^2*e+51/7*d*e^2-4/7*e^3)*x^7+(-17/6*d^3+17/2*d^2*e-2*d*e^2
+7/2*e^3)*x^6+(17/5*d^3-12/5*d^2*e+63/5*d*e^2+7/5*e^3)*x^5+(-d^3+63/4*d^2*
e+21/4*d*e^2+3/2*e^3)*x^4+(7*d^3+7*d^2*e+6*d*e^2)*x^3+(7/2*d^3+9*d^2*e)*x^
2+6*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

$$\begin{aligned}
&\int (d+ex)^3\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= 2\,e^3x^{10} + \frac{1}{9}\left(60\,de^2 - 17\,e^3\right)x^9 + \frac{1}{8}\left(60\,d^2e - 51\,de^2 + 17\,e^3\right)x^8 \\
&\quad + \frac{1}{7}\left(20\,d^3 - 51\,d^2e + 51\,de^2 - 4\,e^3\right)x^7 - \frac{1}{6}\left(17\,d^3 - 51\,d^2e + 12\,de^2 - 21\,e^3\right)x^6 \\
&\quad + \frac{1}{5}\left(17\,d^3 - 12\,d^2e + 63\,de^2 + 7\,e^3\right)x^5 - \frac{1}{4}\left(4\,d^3 - 63\,d^2e - 21\,de^2 - 6\,e^3\right)x^4 \\
&\quad + 6\,d^3x + \left(7\,d^3 + 7\,d^2e + 6\,de^2\right)x^3 + \frac{1}{2}\left(7\,d^3 + 18\,d^2e\right)x^2
\end{aligned}$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e
^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 5
1*d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e
^3)*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3
 + 7*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.86

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = 6d^3x + 2e^3x^{10} + x^9 \cdot \left(\frac{20de^2}{3} - \frac{17e^3}{9}\right) + x^8 \cdot \left(\frac{15d^2e}{2} - \frac{51de^2}{8} + \frac{17e^3}{8}\right) + x^7 \cdot \left(\frac{20d^3}{7} - \frac{51d^2e}{7} + \frac{51de^2}{7} - \frac{4e^3}{7}\right) + x^6\left(-\frac{17d^3}{6} + \frac{17d^2e}{2} - 2de^2 + \frac{7e^3}{2}\right) + x^5 \cdot \left(\frac{17d^3}{5} - \frac{12d^2e}{5} + \frac{63de^2}{5} + \frac{7e^3}{5}\right) + x^4\left(-d^3 + \frac{63d^2e}{4} + \frac{21de^2}{4} + \frac{3e^3}{2}\right) + x^3 \cdot \left(7d^3 + 7d^2e + 6de^2\right) + x^2 \cdot \left(\frac{7d^3}{2} + 9d^2e\right)$$

input
```
integrate((e*x+d)**3*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d**3*x + 2*e**3*x**10 + x**9*(20*d*e**2/3 - 17*e**3/9) + x**8*(15*d**2*e
/2 - 51*d*e**2/8 + 17*e**3/8) + x**7*(20*d**3/7 - 51*d**2*e/7 + 51*d*e**2/
7 - 4*e**3/7) + x**6*(-17*d**3/6 + 17*d**2*e/2 - 2*d*e**2 + 7*e**3/2) + x*
*5*(17*d**3/5 - 12*d**2*e/5 + 63*d*e**2/5 + 7*e**3/5) + x**4*(-d**3 + 63*d
**2*e/4 + 21*d*e**2/4 + 3*e**3/2) + x**3*(7*d**3 + 7*d**2*e + 6*d*e**2) +
x**2*(7*d**3/2 + 9*d**2*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.77

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 2 e^3x^{10} + \frac{1}{9} \left(60 de^2 - 17 e^3\right)x^9 + \frac{1}{8} \left(60 d^2e - 51 de^2 + 17 e^3\right)x^8 + \frac{1}{7} \left(20 d^3 - 51 d^2e + 51 de^2 - 4 e^3\right)x^7 - \frac{1}{6} \left(17 d^3 - 51 d^2e + 12 de^2 - 21 e^3\right)x^6 + \frac{1}{5} \left(17 d^3 - 12 d^2e + 63 de^2 + 7 e^3\right)x^5 - \frac{1}{4} \left(4 d^3 - 63 d^2e - 21 de^2 - 6 e^3\right)x^4 + 6 d^3x + \left(7 d^3 + 7 d^2e + 6 de^2\right)x^3 + \frac{1}{2} \left(7 d^3 + 18 d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
2*e^3*x^10 + 1/9*(60*d*e^2 - 17*e^3)*x^9 + 1/8*(60*d^2*e - 51*d*e^2 + 17*e
^3)*x^8 + 1/7*(20*d^3 - 51*d^2*e + 51*d*e^2 - 4*e^3)*x^7 - 1/6*(17*d^3 - 5
1*d^2*e + 12*d*e^2 - 21*e^3)*x^6 + 1/5*(17*d^3 - 12*d^2*e + 63*d*e^2 + 7*e
^3)*x^5 - 1/4*(4*d^3 - 63*d^2*e - 21*d*e^2 - 6*e^3)*x^4 + 6*d^3*x + (7*d^3
 + 7*d^2*e + 6*d*e^2)*x^3 + 1/2*(7*d^3 + 18*d^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.89

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 2 e^3x^{10} + \frac{20}{3} de^2x^9 - \frac{17}{9} e^3x^9 + \frac{15}{2} d^2ex^8 - \frac{51}{8} de^2x^8 + \frac{17}{8} e^3x^8 + \frac{20}{7} d^3x^7 - \frac{51}{7} d^2ex^7 + \frac{51}{7} de^2x^7 - \frac{4}{7} e^3x^7 - \frac{17}{6} d^3x^6 + \frac{17}{2} d^2ex^6 - 2 de^2x^6 + \frac{7}{2} e^3x^6 + \frac{17}{5} d^3x^5 - \frac{12}{5} d^2ex^5 + \frac{63}{5} de^2x^5 + \frac{7}{5} e^3x^5 - d^3x^4 + \frac{63}{4} d^2ex^4 + \frac{21}{4} de^2x^4 + \frac{3}{2} e^3x^4 + 7 d^3x^3 + 7 d^2ex^3 + 6 de^2x^3 + \frac{7}{2} d^3x^2 + 9 d^2ex^2 + 6 d^3x$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
2*e^3*x^10 + 20/3*d*e^2*x^9 - 17/9*e^3*x^9 + 15/2*d^2*e*x^8 - 51/8*d*e^2*x
^8 + 17/8*e^3*x^8 + 20/7*d^3*x^7 - 51/7*d^2*e*x^7 + 51/7*d*e^2*x^7 - 4/7*e
^3*x^7 - 17/6*d^3*x^6 + 17/2*d^2*e*x^6 - 2*d*e^2*x^6 + 7/2*e^3*x^6 + 17/5*
d^3*x^5 - 12/5*d^2*e*x^5 + 63/5*d*e^2*x^5 + 7/5*e^3*x^5 - d^3*x^4 + 63/4*d
^2*e*x^4 + 21/4*d*e^2*x^4 + 3/2*e^3*x^4 + 7*d^3*x^3 + 7*d^2*e*x^3 + 6*d*e^
2*x^3 + 7/2*d^3*x^2 + 9*d^2*e*x^2 + 6*d^3*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.74

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = 6\,d^3\,x + x^8\left(\frac{15\,d^2\,e}{2} - \frac{51\,d\,e^2}{8} + \frac{17\,e^3}{8}\right) - x^6\left(\frac{17\,d^3}{6} - \frac{17\,d^2\,e}{2} + 2\,d\,e^2 - \frac{7\,e^3}{2}\right) + x^4\left(-d^3 + \frac{63\,d^2\,e}{4} + \frac{21\,d\,e^2}{4} + \frac{3\,e^3}{2}\right) + x^5\left(\frac{17\,d^3}{5} - \frac{12\,d^2\,e}{5} + \frac{63\,d\,e^2}{5} + \frac{7\,e^3}{5}\right) + x^7\left(\frac{20\,d^3}{7} - \frac{51\,d^2\,e}{7} + \frac{51\,d\,e^2}{7} - \frac{4\,e^3}{7}\right) + 2\,e^3\,x^{10} + d\,x^3\left(7\,d^2 + 7\,d\,e + 6\,e^2\right) + \frac{d^2\,x^2\,(7\,d + 18\,e)}{2} + \frac{e^2\,x^9\,(60\,d - 17\,e)}{9}$$

input
```
int((d + e*x)^3*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
6*d^3*x + x^8*((15*d^2*e)/2 - (51*d*e^2)/8 + (17*e^3)/8) - x^6*(2*d*e^2 -
(17*d^2*e)/2 + (17*d^3)/6 - (7*e^3)/2) + x^4*((21*d*e^2)/4 + (63*d^2*e)/4
- d^3 + (3*e^3)/2) + x^5*((63*d*e^2)/5 - (12*d^2*e)/5 + (17*d^3)/5 + (7*e^
3)/5) + x^7*((51*d*e^2)/7 - (51*d^2*e)/7 + (20*d^3)/7 - (4*e^3)/7) + 2*e^3
*x^10 + d*x^3*(7*d*e + 7*d^2 + 6*e^2) + (d^2*x^2*(7*d + 18*e))/2 + (e^2*x^
9*(60*d - 17*e))/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.88

$$\int (d+ex)^3 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{x(5040e^3x^9 + 16800d\,e^2x^8 - 4760e^3x^8 + 18900d^2e\,x^7 - 16065d\,e^2x^7 + 5355e^3x^7 + 7200d^3x^6 - 18360d^2e}{}$$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(7200*d**3*x**6 - 7140*d**3*x**5 + 8568*d**3*x**4 - 2520*d**3*x**3 + 17
640*d**3*x**2 + 8820*d**3*x + 15120*d**3 + 18900*d**2*e*x**7 - 18360*d**2*
e*x**6 + 21420*d**2*e*x**5 - 6048*d**2*e*x**4 + 39690*d**2*e*x**3 + 17640*
d**2*e*x**2 + 22680*d**2*e*x + 16800*d*e**2*x**8 - 16065*d*e**2*x**7 + 183
60*d*e**2*x**6 - 5040*d*e**2*x**5 + 31752*d*e**2*x**4 + 13230*d*e**2*x**3
+ 15120*d*e**2*x**2 + 5040*e**3*x**9 - 4760*e**3*x**8 + 5355*e**3*x**7 - 1
440*e**3*x**6 + 8820*e**3*x**5 + 3528*e**3*x**4 + 3780*e**3*x**3))/2520
```

3.125 $\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

Optimal result

Integrand size = 36, antiderivative size = 151

$$\begin{aligned}\int (d+ex)^2 (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\, dx &= \frac{7d^2x^2}{2} + \frac{7}{3}d(3d+2e)x^3 - \frac{1}{4}(4d^2-42de-7e^2)\,x^4 \\ &+ \frac{1}{5}(17d^2-8de+21e^2)\,x^5 - \frac{1}{6}(17d^2-34de+4e^2)\,x^6 \\ &+ \frac{1}{7}(20d^2-34de+17e^2)\,x^7 + \frac{1}{8}(40d-17e)ex^8 + \frac{20e^2x^9}{9} + \frac{2(d+ex)^3}{e}\end{aligned}$$

output
```
7/2*d^2*x^2+7/3*d*(3*d+2*e)*x^3-1/4*(4*d^2-42*d*e-7*e^2)*x^4+1/5*(17*d^2-8
*d*e+21*e^2)*x^5-1/6*(17*d^2-34*d*e+4*e^2)*x^6+1/7*(20*d^2-34*d*e+17*e^2)*
x^7+1/8*(40*d-17*e)*e*x^8+20/9*e^2*x^9+2*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{e^2x^3(5040+4410x+10584x^2-1680x^3+6120x^4-5355x^5+5600x^6)}{2520} + d^2\left(6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}\right) + de\left(6x^2+\frac{14x^3}{3}+\frac{21x^4}{2}-\frac{8x^5}{5}+\frac{17x^6}{3}-\frac{34x^7}{7}+5x^8\right)$$

input
```
Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
(e^2*x^3*(5040 + 4410*x + 10584*x^2 - 1680*x^3 + 6120*x^4 - 5355*x^5 + 560
0*x^6))/2520 + d^2*(6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/
6 + (20*x^7)/7) + d*e*(6*x^2 + (14*x^3)/3 + (21*x^4)/2 - (8*x^5)/5 + (17*x
^6)/3 - (34*x^7)/7 + 5*x^8)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right) \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^2 \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^6\left(20d^2-34de+17e^2\right)-x^5\left(17d^2-34de+4e^2\right)+x^4\left(17d^2-8de+21e^2\right)-x^3\left(4d^2-42de-7e^2\right)+x^2(21$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7\left(20d^2-34de+17e^2\right)-\frac{1}{6}x^6\left(17d^2-34de+4e^2\right)+\frac{1}{5}x^5\left(17d^2-8de+21e^2\right)-\frac{1}{4}x^4\left(4d^2-42de-7e^2\right)+\frac{1}{3}x^3\left(21d^2+14de+6e^2\right)+6d^2x+\frac{1}{8}ex^8(40d-17e)+\frac{1}{2}dx^2(7d+12e)+\frac{20e^2x^9}{9}$$

input
```
Int[(d + e*x)^2*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d^2*x + (d*(7*d + 12*e)*x^2)/2 + ((21*d^2 + 14*d*e + 6*e^2)*x^3)/3 - ((4
*d^2 - 42*d*e - 7*e^2)*x^4)/4 + ((17*d^2 - 8*d*e + 21*e^2)*x^5)/5 - ((17*d
^2 - 34*d*e + 4*e^2)*x^6)/6 + ((20*d^2 - 34*d*e + 17*e^2)*x^7)/7 + ((40*d
- 17*e)*e*x^8)/8 + (20*e^2*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.92

method	result
norman	$\frac{20e^2x^9}{9}+\left(5de-\frac{17}{8}e^2\right)x^8+\left(\frac{20}{7}d^2-\frac{34}{7}de+\frac{17}{7}e^2\right)x^7+\left(-\frac{17}{6}d^2+\frac{17}{3}de-\frac{2}{3}e^2\right)x^6+\left(\frac{17}{5}d^2-\right.$
default	$\frac{20e^2x^9}{9}+\frac{(40de-17e^2)x^8}{8}+\frac{(20d^2-34de+17e^2)x^7}{7}+\frac{(-17d^2+34de-4e^2)x^6}{6}+\frac{(17d^2-8de+21e^2)x^5}{5}+\frac{(-4d^2+42d}{4}$
orering	$\frac{x(5600x^8e^2+12600x^7de-5355x^7e^2+7200x^6d^2-12240x^6de+6120x^6e^2-7140x^5d^2+14280x^5de-1680e^2x^5+8568x^4d^2-4032}{252}$
gosper	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d$
risch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d$
parallelrisch	$\frac{20}{9}e^2x^9+5x^8de-\frac{17}{8}x^8e^2+\frac{20}{7}x^7d^2-\frac{34}{7}x^7de+\frac{17}{7}x^7e^2-\frac{17}{6}x^6d^2+\frac{17}{3}x^6de-\frac{2}{3}x^6e^2+\frac{17}{5}x^5d$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
20/9*e^2*x^9+(5*d*e-17/8*e^2)*x^8+(20/7*d^2-34/7*d*e+17/7*e^2)*x^7+(-17/6*
d^2+17/3*d*e-2/3*e^2)*x^6+(17/5*d^2-8/5*d*e+21/5*e^2)*x^5+(-d^2+21/2*d*e+7
/4*e^2)*x^4+(7*d^2+14/3*d*e+2*e^2)*x^3+(7/2*d^2+6*d*e)*x^2+6*d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{20}{9}\,e^2x^9 + \frac{1}{8}\left(40\,de - 17\,e^2\right)x^8 + \frac{1}{7}\left(20\,d^2 - 34\,de + 17\,e^2\right)x^7 - \frac{1}{6}\left(17\,d^2 - 34\,de + 4\,e^2\right)x^6 + \frac{1}{5}\left(17\,d^2 - 8\,de + 21\,e^2\right)x^5 - \frac{1}{4}\left(4\,d^2 - 42\,de - 7\,e^2\right)x^4 + \frac{1}{3}\left(21\,d^2 + 14\,de + 6\,e^2\right)x^3 + 6\,d^2x + \frac{1}{2}\left(7\,d^2 + 12\,de\right)x^2$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*
x^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^
5 - 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 +
 6*d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = 6d^2x + \frac{20e^2x^9}{9} + x^8 \cdot \left(5de - \frac{17e^2}{8}\right) + x^7 \cdot \left(\frac{20d^2}{7} - \frac{34de}{7} + \frac{17e^2}{7}\right) + x^6\left(-\frac{17d^2}{6} + \frac{17de}{3} - \frac{2e^2}{3}\right) + x^5 \cdot \left(\frac{17d^2}{5} - \frac{8de}{5} + \frac{21e^2}{5}\right) + x^4\left(-d^2 + \frac{21de}{2} + \frac{7e^2}{4}\right) + x^3 \cdot \left(7d^2 + \frac{14de}{3} + 2e^2\right) + x^2 \cdot \left(\frac{7d^2}{2} + 6de\right)$$

input
```
integrate((e*x+d)**2*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d**2*x + 20*e**2*x**9/9 + x**8*(5*d*e - 17*e**2/8) + x**7*(20*d**2/7 - 3
4*d*e/7 + 17*e**2/7) + x**6*(-17*d**2/6 + 17*d*e/3 - 2*e**2/3) + x**5*(17*
d**2/5 - 8*d*e/5 + 21*e**2/5) + x**4*(-d**2 + 21*d*e/2 + 7*e**2/4) + x**3*
(7*d**2 + 14*d*e/3 + 2*e**2) + x**2*(7*d**2/2 + 6*d*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \frac{20}{9}\, e^2x^9 + \frac{1}{8}\left(40\, de - 17\, e^2\right)x^8 + \frac{1}{7}\left(20\, d^2 - 34\, de + 17\, e^2\right)x^7 - \frac{1}{6}\left(17\, d^2 - 34\, de + 4\, e^2\right)x^6 + \frac{1}{5}\left(17\, d^2 - 8\, de + 21\, e^2\right)x^5 - \frac{1}{4}\left(4\, d^2 - 42\, de - 7\, e^2\right)x^4 + \frac{1}{3}\left(21\, d^2 + 14\, de + 6\, e^2\right)x^3 + 6\, d^2x + \frac{1}{2}\left(7\, d^2 + 12\, de\right)x^2$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
20/9*e^2*x^9 + 1/8*(40*d*e - 17*e^2)*x^8 + 1/7*(20*d^2 - 34*d*e + 17*e^2)*
x^7 - 1/6*(17*d^2 - 34*d*e + 4*e^2)*x^6 + 1/5*(17*d^2 - 8*d*e + 21*e^2)*x^
5 - 1/4*(4*d^2 - 42*d*e - 7*e^2)*x^4 + 1/3*(21*d^2 + 14*d*e + 6*e^2)*x^3 +
 6*d^2*x + 1/2*(7*d^2 + 12*d*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.06

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \frac{20}{9}e^2x^9+5\,dex^8-\frac{17}{8}e^2x^8+\frac{20}{7}d^2x^7-\frac{34}{7}dex^7+\frac{17}{7}e^2x^7-\frac{17}{6}d^2x^6+\frac{17}{3}dex^6-\frac{2}{3}e^2x^6+\frac{17}{5}d^2x^5-\frac{8}{5}dex^5+\frac{21}{5}e^2x^5-d^2x^4+\frac{21}{2}dex^4+\frac{7}{4}e^2x^4+7\,d^2x^3+\frac{14}{3}dex^3+2\,e^2x^3+\frac{7}{2}d^2x^2+6\,dex^2+6\,d^2x$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
20/9*e^2*x^9 + 5*d*e*x^8 - 17/8*e^2*x^8 + 20/7*d^2*x^7 - 34/7*d*e*x^7 + 17
/7*e^2*x^7 - 17/6*d^2*x^6 + 17/3*d*e*x^6 - 2/3*e^2*x^6 + 17/5*d^2*x^5 - 8/
5*d*e*x^5 + 21/5*e^2*x^5 - d^2*x^4 + 21/2*d*e*x^4 + 7/4*e^2*x^4 + 7*d^2*x^
3 + 14/3*d*e*x^3 + 2*e^2*x^3 + 7/2*d^2*x^2 + 6*d*e*x^2 + 6*d^2*x
```

Mupad [B] (verification not implemented)

Time = 18.25 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.91

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = x^3\left(7\,d^2+\frac{14\,d\,e}{3}+2\,e^2\right)+x^4\left(-d^2+\frac{21\,d\,e}{2}+\frac{7\,e^2}{4}\right)-x^6\left(\frac{17\,d^2}{6}-\frac{17\,d\,e}{3}+\frac{2\,e^2}{3}\right)+x^5\left(\frac{17\,d^2}{5}-\frac{8\,d\,e}{5}+\frac{21\,e^2}{5}\right)+x^7\left(\frac{20\,d^2}{7}-\frac{34\,d\,e}{7}+\frac{17\,e^2}{7}\right)+6\,d^2\,x+\frac{20\,e^2\,x^9}{9}+\frac{d\,x^2\,(7\,d+12\,e)}{2}+\frac{e\,x^8\,(40\,d-17\,e)}{8}$$

input
```
int((d + e*x)^2*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^3*((14*d*e)/3 + 7*d^2 + 2*e^2) + x^4*((21*d*e)/2 - d^2 + (7*e^2)/4) - x^
6*((17*d^2)/6 - (17*d*e)/3 + (2*e^2)/3) + x^5*((17*d^2)/5 - (8*d*e)/5 + (2
1*e^2)/5) + x^7*((20*d^2)/7 - (34*d*e)/7 + (17*e^2)/7) + 6*d^2*x + (20*e^2
*x^9)/9 + (d*x^2*(7*d + 12*e))/2 + (e*x^8*(40*d - 17*e))/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int (d+ex)^2 \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{x(5600e^2x^8 + 12600de\,x^7 - 5355e^2x^7 + 7200d^2x^6 - 12240de\,x^6 + 6120e^2x^6 - 7140d^2x^5 + 14280de\,x^5 -}{}$$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(7200*d**2*x**6 - 7140*d**2*x**5 + 8568*d**2*x**4 - 2520*d**2*x**3 + 17
640*d**2*x**2 + 8820*d**2*x + 15120*d**2 + 12600*d*e*x**7 - 12240*d*e*x**6
 + 14280*d*e*x**5 - 4032*d*e*x**4 + 26460*d*e*x**3 + 11760*d*e*x**2 + 1512
0*d*e*x + 5600*e**2*x**8 - 5355*e**2*x**7 + 6120*e**2*x**6 - 1680*e**2*x**
5 + 10584*e**2*x**4 + 4410*e**2*x**3 + 5040*e**2*x**2))/2520
```

3.126 $\int (d+ex) (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)$

Optimal result

Integrand size = 34, antiderivative size = 93

$$\int (d+ex) (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\, dx = 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3-\frac{1}{4}(4d-21e)x^4 +\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}$$

output
```
6*d*x+1/2*(7*d+6*e)*x^2+7/3*(3*d+e)*x^3-1/4*(4*d-21*e)*x^4+1/5*(17*d-4*e)*
x^5-17/6*(d-e)*x^6+1/7*(20*d-17*e)*x^7+5/2*e*x^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00

$$\int (d+ex) (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4)\, dx = 6dx+\frac{1}{2}(7d+6e)x^2+\frac{7}{3}(3d+e)x^3+\frac{1}{4}(-4d+21e)x^4 +\frac{1}{5}(17d-4e)x^5-\frac{17}{6}(d-e)x^6+\frac{1}{7}(20d-17e)x^7+\frac{5ex^8}{2}$$

input
```
Integrate[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 + ((-4*d + 21*e)*x^4)/4
+ ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e
*x^8)/2
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)(d+ex)\,dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(x^6(20d-17e)-17x^5(d-e)+x^4(17d-4e)-x^3(4d-21e)+7x^2(3d+e)+x(7d+6e)+6d+20ex^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{7}x^7(20d-17e)-\frac{17}{6}x^6(d-e)+\frac{1}{5}x^5(17d-4e)-\frac{1}{4}x^4(4d-21e)+\frac{7}{3}x^3(3d+e)+\frac{1}{2}x^2(7d+6e)+6dx+\frac{5ex^8}{2}$$

input
```
Int[(d + e*x)*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*d*x + ((7*d + 6*e)*x^2)/2 + (7*(3*d + e)*x^3)/3 - ((4*d - 21*e)*x^4)/4 +
 ((17*d - 4*e)*x^5)/5 - (17*(d - e)*x^6)/6 + ((20*d - 17*e)*x^7)/7 + (5*e*
x^8)/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84

method	result
norman	$\frac{5e\,x^8}{2}+\left(\frac{20d}{7}-\frac{17e}{7}\right)x^7+\left(-\frac{17d}{6}+\frac{17e}{6}\right)x^6+\left(\frac{17d}{5}-\frac{4e}{5}\right)x^5+\left(-d+\frac{21e}{4}\right)x^4+\left(7d+\frac{7e}{3}\right)x^3+$
orering	$\frac{x(1050e\,x^7+1200d\,x^6-1020e\,x^6-1190d\,x^5+1190e\,x^5+1428d\,x^4-336e\,x^4-420d\,x^3+2205e\,x^3+2940d\,x^2+980e\,x^2+1470dx+}{420}$
gosper	$\frac{5}{2}e\,x^8+\frac{20}{7}x^7d-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3$
default	$\frac{5e\,x^8}{2}+\frac{(20d-17e)x^7}{7}+\frac{(-17d+17e)x^6}{6}+\frac{(17d-4e)x^5}{5}+\frac{(-4d+21e)x^4}{4}+\frac{(21d+7e)x^3}{3}+\frac{(7d+6e)x^2}{2}+6dx$
risch	$\frac{5}{2}e\,x^8+\frac{20}{7}x^7d-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3$
parallelrisch	$\frac{5}{2}e\,x^8+\frac{20}{7}x^7d-\frac{17}{7}e\,x^7-\frac{17}{6}d\,x^6+\frac{17}{6}e\,x^6+\frac{17}{5}d\,x^5-\frac{4}{5}e\,x^5-d\,x^4+\frac{21}{4}e\,x^4+7d\,x^3+\frac{7}{3}e\,x^3$

input
```
int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
5/2*e*x^8+(20/7*d-17/7*e)*x^7+(-17/6*d+17/6*e)*x^6+(17/5*d-4/5*e)*x^5+(-d+
21/4*e)*x^4+(7*d+7/3*e)*x^3+(7/2*d+3*e)*x^2+6*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\begin{aligned}&\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5\\&\quad-\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx\end{aligned}$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fric
as")
```

output
```
5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^
5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.94

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 6dx+\frac{5ex^8}{2}+x^7\cdot\left(\frac{20d}{7}-\frac{17e}{7}\right)+x^6\left(-\frac{17d}{6}+\frac{17e}{6}\right)+x^5\cdot\left(\frac{17d}{5}-\frac{4e}{5}\right)+x^4\left(-d+\frac{21e}{4}\right)+x^3\cdot\left(7d+\frac{7e}{3}\right)+x^2\cdot\left(\frac{7d}{2}+3e\right)$$

input
```
integrate((e*x+d)*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
6*d*x + 5*e*x**8/2 + x**7*(20*d/7 - 17*e/7) + x**6*(-17*d/6 + 17*e/6) + x*
*5*(17*d/5 - 4*e/5) + x**4*(-d + 21*e/4) + x**3*(7*d + 7*e/3) + x**2*(7*d/
2 + 3*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{5}{2}\,ex^8+\frac{1}{7}\,(20\,d-17\,e)x^7-\frac{17}{6}\,(d-e)x^6+\frac{1}{5}\,(17\,d-4\,e)x^5-\frac{1}{4}\,(4\,d-21\,e)x^4+\frac{7}{3}\,(3\,d+e)x^3+\frac{1}{2}\,(7\,d+6\,e)x^2+6\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxi
ma")
```

output
```
5/2*e*x^8 + 1/7*(20*d - 17*e)*x^7 - 17/6*(d - e)*x^6 + 1/5*(17*d - 4*e)*x^
5 - 1/4*(4*d - 21*e)*x^4 + 7/3*(3*d + e)*x^3 + 1/2*(7*d + 6*e)*x^2 + 6*d*x
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{5}{2}\,ex^8+\frac{20}{7}\,dx^7-\frac{17}{7}\,ex^7-\frac{17}{6}\,dx^6+\frac{17}{6}\,ex^6+\frac{17}{5}\,dx^5-\frac{4}{5}\,ex^5 - dx^4+\frac{21}{4}\,ex^4+7\,dx^3+\frac{7}{3}\,ex^3+\frac{7}{2}\,dx^2+3\,ex^2+6\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac
")
```

output
```
5/2*e*x^8 + 20/7*d*x^7 - 17/7*e*x^7 - 17/6*d*x^6 + 17/6*e*x^6 + 17/5*d*x^5
 - 4/5*e*x^5 - d*x^4 + 21/4*e*x^4 + 7*d*x^3 + 7/3*e*x^3 + 7/2*d*x^2 + 3*e*
x^2 + 6*d*x
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{5\,e\,x^8}{2}+\left(\frac{20\,d}{7}-\frac{17\,e}{7}\right)\,x^7+\left(\frac{17\,e}{6}-\frac{17\,d}{6}\right)\,x^6+\left(\frac{17\,d}{5}-\frac{4\,e}{5}\right)\,x^5 + \left(\frac{21\,e}{4}-d\right)\,x^4+\left(7\,d+\frac{7\,e}{3}\right)\,x^3+\left(\frac{7\,d}{2}+3\,e\right)\,x^2+6\,d\,x$$

input
```
int((d + e*x)*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^2*((7*d)/2 + 3*e) + x^3*(7*d + (7*e)/3) + x^5*((17*d)/5 - (4*e)/5) - x^6
*((17*d)/6 - (17*e)/6) + x^7*((20*d)/7 - (17*e)/7) + 6*d*x + (5*e*x^8)/2 -
 x^4*(d - (21*e)/4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{x(1050e\,x^7+1200d\,x^6-1020e\,x^6-1190d\,x^5+1190e\,x^5+1428d\,x^4-336e\,x^4-420d\,x^3+2205e\,x^3+}{420}$$

input
```
int((e*x+d)*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(1200*d*x**6 - 1190*d*x**5 + 1428*d*x**4 - 420*d*x**3 + 2940*d*x**2 + 1
470*d*x + 2520*d + 1050*e*x**7 - 1020*e*x**6 + 1190*e*x**5 - 336*e*x**4 +
2205*e*x**3 + 980*e*x**2 + 1260*e*x))/420
```

3.127 $\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 42

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

output
```
6*x+7/2*x^2+7*x^3-x^4+17/5*x^5-17/6*x^6+20/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = 6x+\frac{7x^2}{2}+7x^3-x^4+\frac{17x^5}{5}-\frac{17x^6}{6}+\frac{20x^7}{7}$$

input
```
Integrate[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int (20x^6-17x^5+17x^4-4x^3+21x^2+7x+6)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

input
```
Int[(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

method	result	size
orering	$\frac{x(600x^6-595x^5+714x^4-210x^3+1470x^2+735x+1260)}{210}$	34
gosper	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
default	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
norman	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
risch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35
parallelrisch	$6x+\frac{7}{2}x^2+7x^3-x^4+\frac{17}{5}x^5-\frac{17}{6}x^6+\frac{20}{7}x^7$	35

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/210*x*(600*x^6-595*x^5+714*x^4-210*x^3+1470*x^2+735*x+1260)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20x^7}{7}-\frac{17x^6}{6}+\frac{17x^5}{5}-x^4+7x^3+\frac{7x^2}{2}+6x$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**2/2 + 6*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx = \frac{20}{7}\,x^7-\frac{17}{6}\,x^6+\frac{17}{5}\,x^5-x^4+7\,x^3+\frac{7}{2}\,x^2+6\,x$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
20/7*x^7 - 17/6*x^6 + 17/5*x^5 - x^4 + 7*x^3 + 7/2*x^2 + 6*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = \frac{20\,x^7}{7}-\frac{17\,x^6}{6}+\frac{17\,x^5}{5}-x^4+7\,x^3+\frac{7\,x^2}{2}+6\,x$$

input
```
int((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
6*x + (7*x^2)/2 + 7*x^3 - x^4 + (17*x^5)/5 - (17*x^6)/6 + (20*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = \frac{x(600x^6-595x^5+714x^4-210x^3+1470x^2+735x+1260)}{210}$$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(600*x**6 - 595*x**5 + 714*x**4 - 210*x**3 + 1470*x**2 + 735*x + 1260))
/210
```

3.128 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 228

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx \\
&= -\frac{(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)\,x}{e^6} \\
&\quad+\frac{(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)\,x^2}{2e^5}-\frac{(20d^3+17d^2e+17de^2+4e^3)\,x^3}{3e^4} \\
&\quad+\frac{(20d^2+17de+17e^2)\,x^4}{4e^3}-\frac{(20d+17e)x^5}{5e^2}+\frac{10x^6}{3e} \\
&\quad+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}
\end{aligned}$$

output

```
-(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^5)*x/e^6+1/2*(20*d^4+1
7*d^3*e+17*d^2*e^2+4*d*e^3+21*e^4)*x^2/e^5-1/3*(20*d^3+17*d^2*e+17*d*e^2+4
*e^3)*x^3/e^4+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/5*(20*d+17*e)*x^5/e^2+1
0/3*x^6/e+(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x
+d)/e^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{ex(-1200d^5+60d^4e(-17+10x)-10d^3e^2(102-51x+40x^2)+10d^2e^3(-24+51x-34x^2+30x^3)-5}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
(e*x*(-1200*d^5 + 60*d^4*e*(-17 + 10*x) - 10*d^3*e^2*(102 - 51*x + 40*x^2)
 + 10*d^2*e^3*(-24 + 51*x - 34*x^2 + 30*x^3) - 5*d*e^4*(252 - 24*x + 68*x^
2 - 51*x^3 + 48*x^4) + e^5*(420 + 630*x - 80*x^2 + 255*x^3 - 204*x^4 + 200
*x^5)) + 60*(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d
*e^5 + 6*e^6)*Log[d + e*x])/(60*e^7)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)}{d+ex}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x^3(20d^2+17de+17e^2)}{e^3}-\frac{x^2(20d^3+17d^2e+17de^2+4e^3)}{e^4}+\frac{x(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{e^5}+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^4(20d^2+17de+17e^2)}{4e^3}-\frac{x^3(20d^3+17d^2e+17de^2+4e^3)}{3e^4}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}+\frac{x^2(20d^4+17d^3e+17d^2e^2+4de^3+21e^4)}{2e^5}-\frac{x(20d^5+17d^4e+17d^3e^2+4d^2e^3+21de^4-7e^5)}{e^6}-\frac{x^5(20d+17e)}{5e^2}+\frac{10x^6}{3e}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
-(((20*d^5 + 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6)
 + ((20*d^4 + 17*d^3*e + 17*d^2*e^2 + 4*d*e^3 + 21*e^4)*x^2)/(2*e^5) - ((2
0*d^3 + 17*d^2*e + 17*d*e^2 + 4*e^3)*x^3)/(3*e^4) + ((20*d^2 + 17*d*e + 17
*e^2)*x^4)/(4*e^3) - ((20*d + 17*e)*x^5)/(5*e^2) + (10*x^6)/(3*e) + ((5*d^
2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e
*x])/e^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96

method	result
norman	$\frac{10x^6}{3e} - \frac{(20d+17e)x^5}{5e^2} + \frac{(20d^2+17de+17e^2)x^4}{4e^3} - \frac{(20d^3+17d^2e+17d\,e^2+4e^3)x^3}{3e^4} + \frac{(20d^4+17d^3e+17d^2e^2+4d\,e^3+21e}{2e^5}$
default	$-\frac{-\frac{10}{3}x^6e^5+4x^5e^4d+\frac{17}{5}x^5e^5-5d^2e^3x^4-\frac{17}{4}x^4e^4d-\frac{17}{4}x^4e^5+\frac{20}{3}d^3e^2x^3+\frac{17}{3}d^2e^3x^3+\frac{17}{3}d\,e^4x^3+\frac{4}{3}e^5x^3-10d^4e\,x^2-\frac{17}{2}d^3e^2x^2}{e^6}$
parallelrisch	$300x^4d^2e^4+255x^4d\,e^5-400x^3d^3e^3-340x^3d^2e^4+1020\ln(ex+d)d^4e^2+240\ln(ex+d)d^3e^3+1260\ln(ex+d)d^2e^4-420\ln(ex+d)$
risch	$-\frac{17x^5}{5e}+\frac{21x^2}{2e}+\frac{7x}{e}+\frac{6\ln(ex+d)}{e}+\frac{10x^6}{3e}+\frac{17x^4}{4e}-\frac{4x^3}{3e}-\frac{4x^5d}{e^2}+\frac{5d^2x^4}{e^3}+\frac{17x^4d}{4e^2}-\frac{20d^3x^3}{3e^4}-\frac{17d^2x^3}{3e^3}$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
10/3*x^6/e-1/5*(20*d+17*e)*x^5/e^2+1/4*(20*d^2+17*d*e+17*e^2)*x^4/e^3-1/3*
(20*d^3+17*d^2*e+17*d*e^2+4*e^3)*x^3/e^4+1/2*(20*d^4+17*d^3*e+17*d^2*e^2+4
*d*e^3+21*e^4)*x^2/e^5-(20*d^5+17*d^4*e+17*d^3*e^2+4*d^2*e^3+21*d*e^4-7*e^
5)*x/e^6+(20*d^6+17*d^5*e+17*d^4*e^2+4*d^3*e^3+21*d^2*e^4-7*d*e^5+6*e^6)/e
^7*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.01

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = \frac{200\,e^6x^6 - 12\,(20\,de^5 + 17\,e^6)x^5 + 15\,(20\,d^2e^4 + 17\,de^5 + 17\,e^6)x^4 - 20\,(20\,d^3e^3 + 17\,d^2e^4 + 17\,de^5 + 4\,e^6}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fric
as")
```

output
```
1/60*(200*e^6*x^6 - 12*(20*d*e^5 + 17*e^6)*x^5 + 15*(20*d^2*e^4 + 17*d*e^5
 + 17*e^6)*x^4 - 20*(20*d^3*e^3 + 17*d^2*e^4 + 17*d*e^5 + 4*e^6)*x^3 + 30*
(20*d^4*e^2 + 17*d^3*e^3 + 17*d^2*e^4 + 4*d*e^5 + 21*e^6)*x^2 - 60*(20*d^5
*e + 17*d^4*e^2 + 17*d^3*e^3 + 4*d^2*e^4 + 21*d*e^5 - 7*e^6)*x + 60*(20*d^
6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(
e*x + d))/e^7
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx = x^5\left(-\frac{4d}{e^2}-\frac{17}{5e}\right)+x^4\cdot\left(\frac{5d^2}{e^3}+\frac{17d}{4e^2}+\frac{17}{4e}\right)+x^3\left(-\frac{20d^3}{3e^4}-\frac{17d^2}{3e^3}-\frac{17d}{3e^2}-\frac{4}{3e}\right)+x^2\cdot\left(\frac{10d^4}{e^5}+\frac{17d^3}{2e^4}+\frac{17d^2}{2e^3}+\frac{2d}{e^2}+\frac{21}{2e}\right)+x\left(-\frac{20d^5}{e^6}-\frac{17d^4}{e^5}-\frac{17d^3}{e^4}-\frac{4d^2}{e^3}-\frac{21d}{e^2}+\frac{7}{e}\right)+\frac{10x^6}{3e}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^7}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

output
```
x**5*(-4*d/e**2 - 17/(5*e)) + x**4*(5*d**2/e**3 + 17*d/(4*e**2) + 17/(4*e)
) + x**3*(-20*d**3/(3*e**4) - 17*d**2/(3*e**3) - 17*d/(3*e**2) - 4/(3*e))
+ x**2*(10*d**4/e**5 + 17*d**3/(2*e**4) + 17*d**2/(2*e**3) + 2*d/e**2 + 21
/(2*e)) + x*(-20*d**5/e**6 - 17*d**4/e**5 - 17*d**3/e**4 - 4*d**2/e**3 - 2
1*d/e**2 + 7/e) + 10*x**6/(3*e) + (5*d**2 - 2*d*e + 3*e**2)*(4*d**4 + 5*d*
*3*e + 3*d**2*e**2 - d*e**3 + 2*e**4)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^5x^6 - 12\,(20\,de^4+17\,e^5)x^5 + 15\,(20\,d^2e^3+17\,de^4+17\,e^5)x^4 - 20\,(20\,d^3e^2+17\,d^2e^3+17\,de^4+4\,e}{}$$
$$+ \frac{(20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="maxi
ma")
```

output
```
1/60*(200*e^5*x^6 - 12*(20*d*e^4 + 17*e^5)*x^5 + 15*(20*d^2*e^3 + 17*d*e^4
 + 17*e^5)*x^4 - 20*(20*d^3*e^2 + 17*d^2*e^3 + 17*d*e^4 + 4*e^5)*x^3 + 30*
(20*d^4*e + 17*d^3*e^2 + 17*d^2*e^3 + 4*d*e^4 + 21*e^5)*x^2 - 60*(20*d^5 +
 17*d^4*e + 17*d^3*e^2 + 4*d^2*e^3 + 21*d*e^4 - 7*e^5)*x)/e^6 + (20*d^6 +
17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(e*x
+ d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.09

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{200\,e^5x^6 - 240\,de^4x^5 - 204\,e^5x^5 + 300\,d^2e^3x^4 + 255\,de^4x^4 + 255\,e^5x^4 - 400\,d^3e^2x^3 - 340\,d^2e^3x^3 - 340\,d}{}$$
$$+ \frac{(20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6)\log(|ex+d|)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="giac
")
```

output
```
1/60*(200*e^5*x^6 - 240*d*e^4*x^5 - 204*e^5*x^5 + 300*d^2*e^3*x^4 + 255*d*
e^4*x^4 + 255*e^5*x^4 - 400*d^3*e^2*x^3 - 340*d^2*e^3*x^3 - 340*d*e^4*x^3
- 80*e^5*x^3 + 600*d^4*e*x^2 + 510*d^3*e^2*x^2 + 510*d^2*e^3*x^2 + 120*d*e
^4*x^2 + 630*e^5*x^2 - 1200*d^5*x - 1020*d^4*e*x - 1020*d^3*e^2*x - 240*d^
2*e^3*x - 1260*d*e^4*x + 420*e^5*x)/e^6 + (20*d^6 + 17*d^5*e + 17*d^4*e^2
+ 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6)*log(abs(e*x + d))/e^7
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.14

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= x\left(\frac{7}{e}-\frac{d\left(\frac{21}{e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)-x^5\left(\frac{4\,d}{e^2}+\frac{17}{5\,e}\right)$$

$$+x^4\left(\frac{17}{4\,e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{4\,e}\right)-x^3\left(\frac{4}{3\,e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{3\,e}\right)$$

$$+x^2\left(\frac{21}{2\,e}+\frac{d\left(\frac{4}{e}+\frac{d\left(\frac{17}{e}+\frac{d\left(\frac{20\,d}{e^2}+\frac{17}{e}\right)}{e}\right)}{e}\right)}{2\,e}\right)+\frac{10\,x^6}{3\,e}$$

$$+\frac{\ln(d+e\,x)\,\left(20\,d^6+17\,d^5\,e+17\,d^4\,e^2+4\,d^3\,e^3+21\,d^2\,e^4-7\,d\,e^5+6\,e^6\right)}{e^7}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

output
```
x*(7/e - (d*(21/e + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/e
))/e) - x^5*((4*d)/e^2 + 17/(5*e)) + x^4*(17/(4*e) + (d*((20*d)/e^2 + 17/e
))/(4*e)) - x^3*(4/(3*e) + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/(3*e)) +
 x^2*(21/(2*e) + (d*(4/e + (d*(17/e + (d*((20*d)/e^2 + 17/e))/e))/e))/(2*e
)) + (10*x^6)/(3*e) + (log(d + e*x)*(17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 +
 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4*e^2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.24

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{1200\log(ex+d)\,d^6+360\log(ex+d)\,e^6+200e^6x^6-204e^6x^5+255e^6x^4-80e^6x^3+630e^6x^2+420e^6x+\cdots}{}$$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

output
```
(1200*log(d + e*x)*d**6 + 1020*log(d + e*x)*d**5*e + 1020*log(d + e*x)*d**
4*e**2 + 240*log(d + e*x)*d**3*e**3 + 1260*log(d + e*x)*d**2*e**4 - 420*lo
g(d + e*x)*d*e**5 + 360*log(d + e*x)*e**6 - 1200*d**5*e*x + 600*d**4*e**2*
x**2 - 1020*d**4*e**2*x - 400*d**3*e**3*x**3 + 510*d**3*e**3*x**2 - 1020*d
**3*e**3*x + 300*d**2*e**4*x**4 - 340*d**2*e**4*x**3 + 510*d**2*e**4*x**2
- 240*d**2*e**4*x - 240*d*e**5*x**5 + 255*d*e**5*x**4 - 340*d*e**5*x**3 +
120*d*e**5*x**2 - 1260*d*e**5*x + 200*e**6*x**6 - 204*e**6*x**5 + 255*e**6
*x**4 - 80*e**6*x**3 + 630*e**6*x**2 + 420*e**6*x)/(60*e**7)
```

3.129 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 228

$$\begin{aligned}\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx &= \frac{(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x}{e^6}-\frac{(80d^3+51d^2e+34de^2+4e^3)\,x^2}{2e^5}\\&\quad+\frac{(60d^2+34de+17e^2)\,x^3}{3e^4}-\frac{(40d+17e)x^4}{4e^3}+\frac{4x^5}{e^2}\\&\quad-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}\\&\quad-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}\end{aligned}$$

output
```
(100*d^4+68*d^3*e+51*d^2*e^2+8*d*e^3+21*e^4)*x/e^6-1/2*(80*d^3+51*d^2*e+34
*d*e^2+4*e^3)*x^2/e^5+1/3*(60*d^2+34*d*e+17*e^2)*x^3/e^4-1/4*(40*d+17*e)*x
^4/e^3+4*x^5/e^2-(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)
/e^7/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*ln(e*
x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.98

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{12e(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)\,x-6e^2(80d^3+51d^2e+34de^2+4e^3)\,x^2+4e^3(60d^2+34de}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,
x]
```

output
```
(12*e*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x - 6*e^2*(80*d
^3 + 51*d^2*e + 34*d*e^2 + 4*e^3)*x^2 + 4*e^3*(60*d^2 + 34*d*e + 17*e^2)*x
^3 - 3*e^4*(40*d + 17*e)*x^4 + 48*e^5*x^5 - (12*(20*d^6 + 17*d^5*e + 17*d^
4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^6))/(d + e*x) - 12*(120*d^5
 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d + e*x])/(1
2*e^7)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)}{(d+ex)^2}\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{x^2(60d^2+34de+17e^2)}{e^4}-\frac{x(80d^3+51d^2e+34de^2+4e^3)}{e^5}+\frac{100d^4+68d^3e+51d^2e^2+8de^3+21e^4}{e^6}+\frac{-1}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3(60d^2+34de+17e^2)}{3e^4}-\frac{x^2(80d^3+51d^2e+34de^2+4e^3)}{2e^5}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^7(d+ex)}+\frac{x(100d^4+68d^3e+51d^2e^2+8de^3+21e^4)}{e^6}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log(d+ex)}{e^7}-\frac{x^4(40d+17e)}{4e^3}+\frac{4x^5}{e^2}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

output
```
((100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6 - ((80*d^3 +
51*d^2*e + 34*d*e^2 + 4*e^3)*x^2)/(2*e^5) + ((60*d^2 + 34*d*e + 17*e^2)*x^
3)/(3*e^4) - ((40*d + 17*e)*x^4)/(4*e^3) + (4*x^5)/e^2 - ((5*d^2 - 2*d*e +
 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^7*(d + e*x)) - (
(120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*Log[d +
e*x])/e^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

method	result
norman	$\frac{\frac{(120d^6+85d^5e+68d^4e^2+12d^3e^3+42d^2e^4-7d e^5+6e^6)x}{e^6 d}+\frac{4x^6}{e}-\frac{(24d+17e)x^5}{4e^2}+\frac{(120d^2+85de+68e^2)x^4}{12e^3}-\frac{(120d^3+85d^2e+68d e^2+12e^3)}{6e^4}}{ex+d}$
default	$\frac{4x^5e^4-10e^3x^4d-\frac{17}{4}x^4e^4+20d^2e^2x^3+\frac{34}{3}e^3x^3d+\frac{17}{3}e^4x^3-40d^3e x^2-\frac{51}{2}d^2e^2x^2-17d e^3x^2-2e^4x^2+100x d^4+68d^3ex+51d^2e^2}{e^6}$
risch	$-\frac{17x^4}{4e^2}+\frac{21x}{e^2}+\frac{4x^5}{e^2}-\frac{120\ln(ex+d)d^5}{e^7}-\frac{85\ln(ex+d)d^4}{e^6}-\frac{68\ln(ex+d)d^3}{e^5}-\frac{12\ln(ex+d)d^2}{e^4}-\frac{42\ln(ex+d)d}{e^3}-$
parallelrisch	$-\frac{1020d^5e+816d^4e^2+144d^3e^3+504d^2e^4-84d e^5+72e^6+1440\ln(ex+d)x d^5e+1020\ln(ex+d)x d^4e^2+816\ln(ex+d)x d^3e^3+1}{}$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERBOS
E)
```

output
```
((120*d^6+85*d^5*e+68*d^4*e^2+12*d^3*e^3+42*d^2*e^4-7*d*e^5+6*e^6)/e^6/d*x
+4*x^6/e-1/4*(24*d+17*e)/e^2*x^5+1/12*(120*d^2+85*d*e+68*e^2)/e^3*x^4-1/6*
(120*d^3+85*d^2*e+68*d*e^2+12*e^3)/e^4*x^3+1/2*(120*d^4+85*d^3*e+68*d^2*e^
2+12*d*e^3+42*e^4)/e^5*x^2)/(e*x+d)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^
3+42*d*e^4-7*e^5)*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.40

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$=\frac{48\,e^6x^6-240\,d^6-204\,d^5e-204\,d^4e^2-48\,d^3e^3-252\,d^2e^4+84\,de^5-72\,e^6-3\,(24\,de^5+17\,e^6)x^5+(12}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="fr
icas")
```

output
```
1/12*(48*e^6*x^6 - 240*d^6 - 204*d^5*e - 204*d^4*e^2 - 48*d^3*e^3 - 252*d^
2*e^4 + 84*d*e^5 - 72*e^6 - 3*(24*d*e^5 + 17*e^6)*x^5 + (120*d^2*e^4 + 85*
d*e^5 + 68*e^6)*x^4 - 2*(120*d^3*e^3 + 85*d^2*e^4 + 68*d*e^5 + 12*e^6)*x^3
 + 6*(120*d^4*e^2 + 85*d^3*e^3 + 68*d^2*e^4 + 12*d*e^5 + 42*e^6)*x^2 + 12*
(100*d^5*e + 68*d^4*e^2 + 51*d^3*e^3 + 8*d^2*e^4 + 21*d*e^5)*x - 12*(120*d
^6 + 85*d^5*e + 68*d^4*e^2 + 12*d^3*e^3 + 42*d^2*e^4 - 7*d*e^5 + (120*d^5*
e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5 - 7*e^6)*x)*log(e*x +
d))/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx &= x^4\left(-\frac{10d}{e^3}-\frac{17}{4e^2}\right)+x^3\cdot\left(\frac{20d^2}{e^4}+\frac{34d}{3e^3}+\frac{17}{3e^2}\right)\\&+x^2\left(-\frac{40d^3}{e^5}-\frac{51d^2}{2e^4}-\frac{17d}{e^3}-\frac{2}{e^2}\right)+x\left(\frac{100d^4}{e^6}+\frac{68d^3}{e^5}+\frac{51d^2}{e^4}+\frac{8d}{e^3}+\frac{21}{e^2}\right)\\&+\frac{-20d^6-17d^5e-17d^4e^2-4d^3e^3-21d^2e^4+7de^5-6e^6}{de^7+e^8x}+\frac{4x^5}{e^2}\\&-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)\log{(d+ex)}}{e^7}\end{aligned}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

output
```
x**4*(-10*d/e**3 - 17/(4*e**2)) + x**3*(20*d**2/e**4 + 34*d/(3*e**3) + 17/
(3*e**2)) + x**2*(-40*d**3/e**5 - 51*d**2/(2*e**4) - 17*d/e**3 - 2/e**2) +
 x*(100*d**4/e**6 + 68*d**3/e**5 + 51*d**2/e**4 + 8*d/e**3 + 21/e**2) + (-
20*d**6 - 17*d**5*e - 17*d**4*e**2 - 4*d**3*e**3 - 21*d**2*e**4 + 7*d*e**5
 - 6*e**6)/(d*e**7 + e**8*x) + 4*x**5/e**2 - (120*d**5 + 85*d**4*e + 68*d*
*3*e**2 + 12*d**2*e**3 + 42*d*e**4 - 7*e**5)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= -\frac{20\,d^6+17\,d^5e+17\,d^4e^2+4\,d^3e^3+21\,d^2e^4-7\,de^5+6\,e^6}{e^8x+de^7}$$
$$+\frac{48\,e^4x^5-3\,(40\,de^3+17\,e^4)x^4+4\,(60\,d^2e^2+34\,de^3+17\,e^4)x^3-6\,(80\,d^3e+51\,d^2e^2+34\,de^3+4\,e^4)x^2}{12\,e^6}$$
$$-\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="ma
xima")
```

output
```
-(20*d^6 + 17*d^5*e + 17*d^4*e^2 + 4*d^3*e^3 + 21*d^2*e^4 - 7*d*e^5 + 6*e^
6)/(e^8*x + d*e^7) + 1/12*(48*e^4*x^5 - 3*(40*d*e^3 + 17*e^4)*x^4 + 4*(60*
d^2*e^2 + 34*d*e^3 + 17*e^4)*x^3 - 6*(80*d^3*e + 51*d^2*e^2 + 34*d*e^3 + 4
*e^4)*x^2 + 12*(100*d^4 + 68*d^3*e + 51*d^2*e^2 + 8*d*e^3 + 21*e^4)*x)/e^6
 - (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(e
*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.43

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$
$$-\frac{(ex+d)^5\left(\frac{3\,(120\,de+17\,e^2)}{(ex+d)e}-\frac{4\,(300\,d^2e^2+85\,de^3+17\,e^4)}{(ex+d)^2e^2}+\frac{12\,(200\,d^3e^3+85\,d^2e^4+34\,de^5+2\,e^6)}{(ex+d)^3e^3}-\frac{12\,(300\,d^4e^4+170\,d^3e^5+102\,d}{(ex+d)^4e^4}\right.}{12\,e^7}$$
$$+\frac{(120\,d^5+85\,d^4e+68\,d^3e^2+12\,d^2e^3+42\,de^4-7\,e^5)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$
$$-\frac{\frac{20\,d^6e^5}{ex+d}+\frac{17\,d^5e^6}{ex+d}+\frac{17\,d^4e^7}{ex+d}+\frac{4\,d^3e^8}{ex+d}+\frac{21\,d^2e^9}{ex+d}-\frac{7\,de^{10}}{ex+d}+\frac{6\,e^{11}}{ex+d}}{e^{12}}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="gi
ac")
```

output
```
-1/12*(e*x + d)^5*(3*(120*d*e + 17*e^2)/((e*x + d)*e) - 4*(300*d^2*e^2 + 8
5*d*e^3 + 17*e^4)/((e*x + d)^2*e^2) + 12*(200*d^3*e^3 + 85*d^2*e^4 + 34*d*
e^5 + 2*e^6)/((e*x + d)^3*e^3) - 12*(300*d^4*e^4 + 170*d^3*e^5 + 102*d^2*e
^6 + 12*d*e^7 + 21*e^8)/((e*x + d)^4*e^4) - 48)/e^7 + (120*d^5 + 85*d^4*e
+ 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7*e^5)*log(abs(e*x + d)/((e*x + d)^
2*abs(e)))/e^7 - (20*d^6*e^5/(e*x + d) + 17*d^5*e^6/(e*x + d) + 17*d^4*e^7
/(e*x + d) + 4*d^3*e^8/(e*x + d) + 21*d^2*e^9/(e*x + d) - 7*d*e^10/(e*x +
d) + 6*e^11/(e*x + d))/e^12
```

Mupad [B] (verification not implemented)

Time = 18.36 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.59

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= x^3\left(\frac{17}{3e^2}-\frac{20d^2}{3e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{3e}\right) - x^2\left(\frac{2}{e^2}+\frac{d\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{2e^2}\right) - x^4\left(\frac{10d}{e^3}+\frac{17}{4e^2}\right) + x\left(\frac{21}{e^2}+\frac{2d\left(\frac{4}{e^2}+\frac{2d\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e}-\frac{d^2\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e^2}\right)}{e} - \frac{d^2\left(\frac{17}{e^2}-\frac{20d^2}{e^4}+\frac{2d\left(\frac{40d}{e^3}+\frac{17}{e^2}\right)}{e}\right)}{e^2}\right) + \frac{4x^5}{e^2} - \frac{\ln(d+ex)\,(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)}{e^7} - \frac{20d^6+17d^5e+17d^4e^2+4d^3e^3+21d^2e^4-7de^5+6e^6}{e\,(xe^7+de^6)}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

output
```
x^3*(17/(3*e^2) - (20*d^2)/(3*e^4) + (2*d*((40*d)/e^3 + 17/e^2))/(3*e)) -
x^2*(2/e^2 + (d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e
 - (d^2*((40*d)/e^3 + 17/e^2))/(2*e^2)) - x^4*((10*d)/e^3 + 17/(4*e^2)) +
x*(21/e^2 + (2*d*(4/e^2 + (2*d*(17/e^2 - (20*d^2)/e^4 + (2*d*((40*d)/e^3 +
 17/e^2))/e))/e - (d^2*((40*d)/e^3 + 17/e^2))/e^2))/e - (d^2*(17/e^2 - (20
*d^2)/e^4 + (2*d*((40*d)/e^3 + 17/e^2))/e))/e^2) + (4*x^5)/e^2 - (log(d +
e*x)*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12*d^2*e^3 + 68*d^3*e^2))/e^
7 - (17*d^5*e - 7*d*e^5 + 20*d^6 + 6*e^6 + 21*d^2*e^4 + 4*d^3*e^3 + 17*d^4
*e^2)/(e*(d*e^6 + e^7*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.72

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{-1440\log(ex+d)\,d^7+72e^7x-1020\log(ex+d)\,d^6e-816\log(ex+d)\,d^5e^2-144\log(ex+d)\,d^4e^3-50}{}$$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

output
```
( - 1440*log(d + e*x)*d**7 - 1440*log(d + e*x)*d**6*e*x - 1020*log(d + e*x
)*d**6*e - 1020*log(d + e*x)*d**5*e**2*x - 816*log(d + e*x)*d**5*e**2 - 81
6*log(d + e*x)*d**4*e**3*x - 144*log(d + e*x)*d**4*e**3 - 144*log(d + e*x)
*d**3*e**4*x - 504*log(d + e*x)*d**3*e**4 - 504*log(d + e*x)*d**2*e**5*x +
 84*log(d + e*x)*d**2*e**5 + 84*log(d + e*x)*d*e**6*x + 1440*d**6*e*x + 72
0*d**5*e**2*x**2 + 1020*d**5*e**2*x - 240*d**4*e**3*x**3 + 510*d**4*e**3*x
**2 + 816*d**4*e**3*x + 120*d**3*e**4*x**4 - 170*d**3*e**4*x**3 + 408*d**3
*e**4*x**2 + 144*d**3*e**4*x - 72*d**2*e**5*x**5 + 85*d**2*e**5*x**4 - 136
*d**2*e**5*x**3 + 72*d**2*e**5*x**2 + 504*d**2*e**5*x + 48*d*e**6*x**6 - 5
1*d*e**6*x**5 + 68*d*e**6*x**4 - 24*d*e**6*x**3 + 252*d*e**6*x**2 - 84*d*e
**6*x + 72*e**7*x)/(12*d*e**7*(d + e*x))
```

3.130 $\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 231

$$\begin{aligned}\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx &= -\frac{(200d^3+102d^2e+51de^2+4e^3)\,x}{e^6}+\frac{(120d^2+51de+17e^2)\,x^2}{2e^5}\\ &\quad -\frac{(60d+17e)x^3}{3e^4}+\frac{5x^4}{e^3}-\frac{(5d^2-2de+3e^2)\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2}\\ &\quad +\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)}\\ &\quad +\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}\end{aligned}$$

output
```
-(200*d^3+102*d^2*e+51*d*e^2+4*e^3)*x/e^6+1/2*(120*d^2+51*d*e+17*e^2)*x^2/
e^5-1/3*(60*d+17*e)*x^3/e^4+5*x^4/e^3-1/2*(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3
*e+3*d^2*e^2-d*e^3+2*e^4)/e^7/(e*x+d)^2+(120*d^5+85*d^4*e+68*d^3*e^2+12*d^
2*e^3+42*d*e^4-7*e^5)/e^7/(e*x+d)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+
21*e^4)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{660d^6+d^5e(459-480x)-51d^4e^2(-7+2x+40x^2)-3d^3e^3(-20-34x+357x^2+200x^3)+d^2e^4(189}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,
x]
```

output
```
(660*d^6 + d^5*e*(459 - 480*x) - 51*d^4*e^2*(-7 + 2*x + 40*x^2) - 3*d^3*e^
3*(-20 - 34*x + 357*x^2 + 200*x^3) + d^2*e^4*(189 + 48*x - 561*x^2 - 340*x
^3 + 150*x^4) - d*e^5*(21 - 252*x + 48*x^2 + 204*x^3 - 85*x^4 + 60*x^5) +
e^6*(-18 - 42*x - 24*x^3 + 51*x^4 - 34*x^5 + 30*x^6) + 6*(300*d^4 + 170*d^
3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2*Log[d + e*x])/(6*e^7*(d
 + e*x)^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)(4x^4-5x^3+3x^2+x+2)}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x(120d^2+51de+17e^2)}{e^5}+\frac{-200d^3-102d^2e-51de^2-4e^3}{e^6}+\frac{300d^4+170d^3e+102d^2e^2+12de^3+21e^4}{e^6(d+ex)}+\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{x^2(120d^2+51de+17e^2)}{2e^5}-\frac{x(200d^3+102d^2e+51de^2+4e^3)}{e^6}-\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^7(d+ex)^2}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}+\frac{120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5}{e^7(d+ex)}-\frac{x^3(60d+17e)}{3e^4}+\frac{5x^4}{e^3}$$

input
```
Int[((3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

output
```
-(((200*d^3 + 102*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6) + ((120*d^2 + 51*d*e +
 17*e^2)*x^2)/(2*e^5) - ((60*d + 17*e)*x^3)/(3*e^4) + (5*x^4)/e^3 - ((5*d^
2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(2*e^7*(
d + e*x)^2) + (120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3 + 42*d*e^4 - 7
*e^5)/(e^7*(d + e*x)) + ((300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 2
1*e^4)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{\left(600d^5+340d^4e+204d^3e^2+24d^2e^3+42d\,e^4-7e^5\right)x}{e^6}+\frac{5x^6}{e}+\frac{900d^6+510d^5e+306d^4e^2+36d^3e^3+63d^2e^4-7d\,e^5-6e^6}{2e^7}-\frac{(30d+17e)x^5}{3e^2}+\frac{(150}{}}{(ex+d)^2}$
default	$-\frac{-5e^3x^4+20d\,e^2x^3+\frac{17}{3}e^3x^3-60x^2d^2e-\frac{51}{2}d\,e^2x^2-\frac{17}{2}e^3x^2+200d^3x+102d^2ex+51d\,e^2x+4e^3x}{e^6}-\frac{20d^6+17d^5e+17d^4e^2+}{2e^7(}$
risch	$\frac{5x^4}{e^3}-\frac{20d\,x^3}{e^4}-\frac{17x^3}{3e^3}+\frac{60x^2d^2}{e^5}+\frac{51d\,x^2}{2e^4}+\frac{17x^2}{2e^3}-\frac{200d^3x}{e^6}-\frac{102d^2x}{e^5}-\frac{51dx}{e^4}-\frac{4x}{e^3}+\frac{(120d^5+85d^4e+68d^3}{}$
parallelrisch	$\frac{1530d^5e+918d^4e^2+108d^3e^3+189d^2e^4-21d\,e^5+126\ln(ex+d)x^2e^6-18e^6+1800\ln(ex+d)x^2d^4e^2+1020\ln(ex+d)x^2d^3e^3+612}{}$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERBOS
E)
```

output
```
((600*d^5+340*d^4*e+204*d^3*e^2+24*d^2*e^3+42*d*e^4-7*e^5)/e^6*x+5*x^6/e+1
/2*(900*d^6+510*d^5*e+306*d^4*e^2+36*d^3*e^3+63*d^2*e^4-7*d*e^5-6*e^6)/e^7
-1/3*(30*d+17*e)/e^2*x^5+1/6*(150*d^2+85*d*e+51*e^2)/e^3*x^4-2/3*(150*d^3+
85*d^2*e+51*d*e^2+6*e^3)/e^4*x^3)/(e*x+d)^2+(300*d^4+170*d^3*e+102*d^2*e^2
+12*d*e^3+21*e^4)*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.56

$$\int \frac{(3+2x+5x^2)\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{30\,e^6x^6+660\,d^6+459\,d^5e+357\,d^4e^2+60\,d^3e^3+189\,d^2e^4-21\,de^5-18\,e^6-2\,(30\,de^5+17\,e^6)x^5+(15}{}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="fr
icas")
```

output
```
1/6*(30*e^6*x^6 + 660*d^6 + 459*d^5*e + 357*d^4*e^2 + 60*d^3*e^3 + 189*d^2
*e^4 - 21*d*e^5 - 18*e^6 - 2*(30*d*e^5 + 17*e^6)*x^5 + (150*d^2*e^4 + 85*d
*e^5 + 51*e^6)*x^4 - 4*(150*d^3*e^3 + 85*d^2*e^4 + 51*d*e^5 + 6*e^6)*x^3 -
 3*(680*d^4*e^2 + 357*d^3*e^3 + 187*d^2*e^4 + 16*d*e^5)*x^2 - 6*(80*d^5*e
+ 17*d^4*e^2 - 17*d^3*e^3 - 8*d^2*e^4 - 42*d*e^5 + 7*e^6)*x + 6*(300*d^6 +
 170*d^5*e + 102*d^4*e^2 + 12*d^3*e^3 + 21*d^2*e^4 + (300*d^4*e^2 + 170*d^
3*e^3 + 102*d^2*e^4 + 12*d*e^5 + 21*e^6)*x^2 + 2*(300*d^5*e + 170*d^4*e^2
+ 102*d^3*e^3 + 12*d^2*e^4 + 21*d*e^5)*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8
*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^3\left(-\frac{20d}{e^4}-\frac{17}{3e^3}\right)+x^2\cdot\left(\frac{60d^2}{e^5}+\frac{51d}{2e^4}+\frac{17}{2e^3}\right)+x\left(-\frac{200d^3}{e^6}-\frac{102d^2}{e^5}-\frac{51d}{e^4}-\frac{4}{e^3}\right)$$
$$+\frac{220d^6+153d^5e+119d^4e^2+20d^3e^3+63d^2e^4-7de^5-6e^6+x(240d^5e+170d^4e^2+136d^3e^3+24d^2e^4}{2d^2e^7+4de^8x+2e^9x^2}$$
$$+\frac{5x^4}{e^3}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)\log(d+ex)}{e^7}$$

input
```
integrate((5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

output
```
x**3*(-20*d/e**4 - 17/(3*e**3)) + x**2*(60*d**2/e**5 + 51*d/(2*e**4) + 17/
(2*e**3)) + x*(-200*d**3/e**6 - 102*d**2/e**5 - 51*d/e**4 - 4/e**3) + (220
*d**6 + 153*d**5*e + 119*d**4*e**2 + 20*d**3*e**3 + 63*d**2*e**4 - 7*d*e**
5 - 6*e**6 + x*(240*d**5*e + 170*d**4*e**2 + 136*d**3*e**3 + 24*d**2*e**4
+ 84*d*e**5 - 14*e**6))/(2*d**2*e**7 + 4*d*e**8*x + 2*e**9*x**2) + 5*x**4/
e**3 + (300*d**4 + 170*d**3*e + 102*d**2*e**2 + 12*d*e**3 + 21*e**4)*log(d
 + e*x)/e**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2e^4}{2\,(e^9x^2+2\,de^8x+d^2e^7)}$$
$$+\frac{30\,e^3x^4-2\,(60\,de^2+17\,e^3)x^3+3\,(120\,d^2e+51\,de^2+17\,e^3)x^2-6\,(200\,d^3+102\,d^2e+51\,de^2+4\,e^3)x}{6\,e^6}$$
$$+\frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log(ex+d)}{e^7}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="ma
xima")
```

output
```
1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7*d*e^5
 - 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*d*e^5
- 7*e^6)*x)/(e^9*x^2 + 2*d*e^8*x + d^2*e^7) + 1/6*(30*e^3*x^4 - 2*(60*d*e^
2 + 17*e^3)*x^3 + 3*(120*d^2*e + 51*d*e^2 + 17*e^3)*x^2 - 6*(200*d^3 + 102
*d^2*e + 51*d*e^2 + 4*e^3)*x)/e^6 + (300*d^4 + 170*d^3*e + 102*d^2*e^2 + 1
2*d*e^3 + 21*e^4)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.03

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{(300\,d^4+170\,d^3e+102\,d^2e^2+12\,de^3+21\,e^4)\log(|ex+d|)}{e^7}$$
$$+\frac{220\,d^6+153\,d^5e+119\,d^4e^2+20\,d^3e^3+63\,d^2e^4-7\,de^5-6\,e^6+2\,(120\,d^5e+85\,d^4e^2+68\,d^3e^3+12\,d^2}{2\,(ex+d)^2e^7}$$
$$+\frac{30\,e^9x^4-120\,de^8x^3-34\,e^9x^3+360\,d^2e^7x^2+153\,de^8x^2+51\,e^9x^2-1200\,d^3e^6x-612\,d^2e^7x-306\,de^8}{6\,e^{12}}$$

input
```
integrate((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="gi
ac")
```

output
```
(300*d^4 + 170*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*log(abs(e*x + d))/
e^7 + 1/2*(220*d^6 + 153*d^5*e + 119*d^4*e^2 + 20*d^3*e^3 + 63*d^2*e^4 - 7
*d*e^5 - 6*e^6 + 2*(120*d^5*e + 85*d^4*e^2 + 68*d^3*e^3 + 12*d^2*e^4 + 42*
d*e^5 - 7*e^6)*x)/((e*x + d)^2*e^7) + 1/6*(30*e^9*x^4 - 120*d*e^8*x^3 - 34
*e^9*x^3 + 360*d^2*e^7*x^2 + 153*d*e^8*x^2 + 51*e^9*x^2 - 1200*d^3*e^6*x -
 612*d^2*e^7*x - 306*d*e^8*x - 24*e^9*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.29

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^2\left(\frac{17}{2\,e^3}-\frac{30\,d^2}{e^5}+\frac{3\,d\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{2\,e}\right)-x^3\left(\frac{20\,d}{e^4}+\frac{17}{3\,e^3}\right)$$
$$+\frac{x\,(120\,d^5+85\,d^4\,e+68\,d^3\,e^2+12\,d^2\,e^3+42\,d\,e^4-7\,e^5)+\frac{220\,d^6+153\,d^5\,e+119\,d^4\,e^2+20\,d^3\,e^3+63\,d^2\,e^4-7\,d\,e^5-6\,e^6}{2\,e}}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2}$$
$$-x\left(\frac{4}{e^3}+\frac{20\,d^3}{e^6}+\frac{3\,d\left(\frac{17}{e^3}-\frac{60\,d^2}{e^5}+\frac{3\,d\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{60\,d}{e^4}+\frac{17}{e^3}\right)}{e^2}\right)$$
$$+\frac{5\,x^4}{e^3}+\frac{\ln(d+e\,x)\,(300\,d^4+170\,d^3\,e+102\,d^2\,e^2+12\,d\,e^3+21\,e^4)}{e^7}$$

input
```
int(((2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

output
```
x^2*(17/(2*e^3) - (30*d^2)/e^5 + (3*d*((60*d)/e^4 + 17/e^3))/(2*e)) - x^3*
((20*d)/e^4 + 17/(3*e^3)) + (x*(42*d*e^4 + 85*d^4*e + 120*d^5 - 7*e^5 + 12
*d^2*e^3 + 68*d^3*e^2) + (153*d^5*e - 7*d*e^5 + 220*d^6 - 6*e^6 + 63*d^2*e
^4 + 20*d^3*e^3 + 119*d^4*e^2)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^7*x) - x*
(4/e^3 + (20*d^3)/e^6 + (3*d*(17/e^3 - (60*d^2)/e^5 + (3*d*((60*d)/e^4 + 1
7/e^3))/e))/e - (3*d^2*((60*d)/e^4 + 17/e^3))/e^2) + (5*x^4)/e^3 + (log(d
+ e*x)*(12*d*e^3 + 170*d^3*e + 300*d^4 + 21*e^4 + 102*d^2*e^2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.96

$$\int \frac{(3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{510d^6e+306d^5e^2+36d^4e^3+63d^3e^4-18d\,e^6+21e^7x^2+1800\log(ex+d)\,d^7+1020\log(ex+d)\,d^6e+\cdots}{}$$

input
```
int((5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

output
```
(1800*log(d + e*x)*d**7 + 3600*log(d + e*x)*d**6*e*x + 1020*log(d + e*x)*d
**6*e + 1800*log(d + e*x)*d**5*e**2*x**2 + 2040*log(d + e*x)*d**5*e**2*x +
 612*log(d + e*x)*d**5*e**2 + 1020*log(d + e*x)*d**4*e**3*x**2 + 1224*log(
d + e*x)*d**4*e**3*x + 72*log(d + e*x)*d**4*e**3 + 612*log(d + e*x)*d**3*e
**4*x**2 + 144*log(d + e*x)*d**3*e**4*x + 126*log(d + e*x)*d**3*e**4 + 72*
log(d + e*x)*d**2*e**5*x**2 + 252*log(d + e*x)*d**2*e**5*x + 126*log(d + e
*x)*d*e**6*x**2 + 900*d**7 + 510*d**6*e - 1800*d**5*e**2*x**2 + 306*d**5*e
**2 - 600*d**4*e**3*x**3 - 1020*d**4*e**3*x**2 + 36*d**4*e**3 + 150*d**3*e
**4*x**4 - 340*d**3*e**4*x**3 - 612*d**3*e**4*x**2 + 63*d**3*e**4 - 60*d**
2*e**5*x**5 + 85*d**2*e**5*x**4 - 204*d**2*e**5*x**3 - 72*d**2*e**5*x**2 +
 30*d*e**6*x**6 - 34*d*e**6*x**5 + 51*d*e**6*x**4 - 24*d*e**6*x**3 - 126*d
*e**6*x**2 - 18*d*e**6 + 21*e**7*x**2)/(6*d*e**7*(d**2 + 2*d*e*x + e**2*x*
*2))
```

3.131 $\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 265

$$\begin{aligned}
&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{33d^3x^2}{2}+\frac{1}{3}d^2(107d+99e)x^3+\frac{1}{4}d\left(65d^2+321de+99e^2\right)x^4 \\
&+\frac{1}{5}\left(148d^3+195d^2e+321de^2+33e^3\right)x^5-\frac{1}{6}\left(37d^3-444d^2e-195de^2-107e^3\right)x^6 \\
&+\frac{1}{7}\left(111d^3-111d^2e+444de^2+65e^3\right)x^7-\frac{1}{8}\left(45d^3-333d^2e+111de^2-148e^3\right)x^8 \\
&+\frac{1}{9}\left(100d^3-135d^2e+333de^2-37e^3\right)x^9+\frac{3}{10}e\left(100d^2-45de+37e^2\right)x^{10} \\
&+\frac{15}{11}(20d-3e)e^2x^{11}+\frac{25e^3x^{12}}{3}+\frac{9(d+ex)^4}{2e}
\end{aligned}$$

output
```
33/2*d^3*x^2+1/3*d^2*(107*d+99*e)*x^3+1/4*d*(65*d^2+321*d*e+99*e^2)*x^4+1/
5*(148*d^3+195*d^2*e+321*d*e^2+33*e^3)*x^5-1/6*(37*d^3-444*d^2*e-195*d*e^2
-107*e^3)*x^6+1/7*(111*d^3-111*d^2*e+444*d*e^2+65*e^3)*x^7-1/8*(45*d^3-333
*d^2*e+111*d*e^2-148*e^3)*x^8+1/9*(100*d^3-135*d^2*e+333*d*e^2-37*e^3)*x^9
+3/10*e*(100*d^2-45*d*e+37*e^2)*x^10+15/11*(20*d-3*e)*e^2*x^11+25/3*e^3*x^
12+9/2*(e*x+d)^4/e
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= 18d^3x+\frac{3}{2}d^2(11d+18e)x^2+\frac{1}{3}d\left(107d^2+99de+54e^2\right)x^3 \\
&\quad +\frac{1}{4}\left(65d^3+321d^2e+99de^2+18e^3\right)x^4+\frac{1}{5}\left(148d^3+195d^2e+321de^2+33e^3\right)x^5 \\
&\quad +\frac{1}{6}\left(-37d^3+444d^2e+195de^2+107e^3\right)x^6+\frac{1}{7}\left(111d^3-111d^2e+444de^2+65e^3\right)x^7 \\
&\quad +\frac{1}{8}\left(-45d^3+333d^2e-111de^2+148e^3\right)x^8+\frac{1}{9}\left(100d^3-135d^2e+333de^2-37e^3\right)x^9 \\
&\quad +\frac{3}{10}e\left(100d^2-45de+37e^2\right)x^{10}+\frac{15}{11}(20d-3e)e^2x^{11}+\frac{25e^3x^{12}}{3}
\end{aligned}$$

input
```
Integrate[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
18*d^3*x + (3*d^2*(11*d + 18*e)*x^2)/2 + (d*(107*d^2 + 99*d*e + 54*e^2)*x^
3)/3 + ((65*d^3 + 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4)/4 + ((148*d^3 + 195*
d^2*e + 321*d*e^2 + 33*e^3)*x^5)/5 + ((-37*d^3 + 444*d^2*e + 195*d*e^2 + 1
07*e^3)*x^6)/6 + ((111*d^3 - 111*d^2*e + 444*d*e^2 + 65*e^3)*x^7)/7 + ((-4
5*d^3 + 333*d^2*e - 111*d*e^2 + 148*e^3)*x^8)/8 + ((100*d^3 - 135*d^2*e +
333*d*e^2 - 37*e^3)*x^9)/9 + (3*e*(100*d^2 - 45*d*e + 37*e^2)*x^10)/10 + (
15*(20*d - 3*e)*e^2*x^11)/11 + (25*e^3*x^12)/3
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^3\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{(2800d^2+315de+111e^2)(d+ex)^9}{e^8} + \frac{(-5600d^3-945d^2e-666de^2-37e^3)(d+ex)^8}{e^8} + \frac{(7000d^4+1575d\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{(2800d^2+315de+111e^2)(d+ex)^{10}}{10e^9} - \frac{(5600d^3+945d^2e+666de^2+37e^3)(d+ex)^9}{9e^9} + \frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)(d+ex)^8}{8e^9} + \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^4}{4e^9} - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)(d+ex)^7}{7e^9} - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)^5}{5e^9} + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)(d+ex)^6}{6e^9} + \frac{25(d+ex)^{12}}{3e^9} - \frac{5(160d+9e)(d+ex)^{11}}{11e^9}$$

input

```
Int[(d + e*x)^3*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output

```
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^4)/(4*e^9) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d
^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^5)/(5*e^9) + ((2800*d^6
+ 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e
^6)*(d + e*x)^6)/(6*e^9) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^
2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^7)/(7*e^9) + ((7000*d^4 + 1575*d^3*e
 + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^8)/(8*e^9) - ((5600*d^3 +
 945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^9)/(9*e^9) + ((2800*d^2 + 315*d
*e + 111*e^2)*(d + e*x)^10)/(10*e^9) - (5*(160*d + 9*e)*(d + e*x)^11)/(11*
e^9) + (25*(d + e*x)^12)/(3*e^9)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.96

method	result
norman	$\frac{25e^3x^{12}}{3}+\left(\frac{300}{11}d\,e^2-\frac{45}{11}e^3\right)x^{11}+\left(30d^2e-\frac{27}{2}d\,e^2+\frac{111}{10}e^3\right)x^{10}+\left(\frac{100}{9}d^3-15d^2e+37d\,e^2-\right.$
default	$\frac{25e^3x^{12}}{3}+\frac{(300d\,e^2-45e^3)x^{11}}{11}+\frac{(300d^2e-135d\,e^2+111e^3)x^{10}}{10}+\frac{(100d^3-135d^2e+333d\,e^2-37e^3)x^9}{9}+\frac{(-45d^3+333}{}$
orering	$x(231000x^{11}e^3+756000x^{10}d\,e^2-113400e^3x^{10}+831600x^9d^2e-374220x^9d\,e^2+307692x^9e^3+308000x^8d^3-415800x^8d^2e+102$
gosper	$\frac{33}{5}x^5e^3+\frac{65}{4}x^4d^3+\frac{107}{3}d^3x^3+\frac{148}{5}x^5d^3+\frac{65}{7}x^7e^3-\frac{37}{6}x^6d^3+\frac{111}{10}e^3x^{10}+18d^3x-\frac{37}{9}x^9e^3+\frac{37}{2}$
risch	$\frac{33}{5}x^5e^3+\frac{65}{4}x^4d^3+\frac{107}{3}d^3x^3+\frac{148}{5}x^5d^3+\frac{65}{7}x^7e^3-\frac{37}{6}x^6d^3+\frac{111}{10}e^3x^{10}+18d^3x-\frac{37}{9}x^9e^3+\frac{37}{2}$
parallelrisch	$\frac{33}{5}x^5e^3+\frac{65}{4}x^4d^3+\frac{107}{3}d^3x^3+\frac{148}{5}x^5d^3+\frac{65}{7}x^7e^3-\frac{37}{6}x^6d^3+\frac{111}{10}e^3x^{10}+18d^3x-\frac{37}{9}x^9e^3+\frac{37}{2}$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
25/3*e^3*x^12+(300/11*d*e^2-45/11*e^3)*x^11+(30*d^2*e-27/2*d*e^2+111/10*e^
3)*x^10+(100/9*d^3-15*d^2*e+37*d*e^2-37/9*e^3)*x^9+(-45/8*d^3+333/8*d^2*e-
111/8*d*e^2+37/2*e^3)*x^8+(111/7*d^3-111/7*d^2*e+444/7*d*e^2+65/7*e^3)*x^7
+(-37/6*d^3+74*d^2*e+65/2*d*e^2+107/6*e^3)*x^6+(148/5*d^3+39*d^2*e+321/5*d
*e^2+33/5*e^3)*x^5+(65/4*d^3+321/4*d^2*e+99/4*d*e^2+9/2*e^3)*x^4+(107/3*d^
3+33*d^2*e+18*d*e^2)*x^3+(33/2*d^3+27*d^2*e)*x^2+18*d^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{25}{3} e^3x^{12} + \frac{15}{11} \left(20 de^2 - 3 e^3\right)x^{11} + \frac{3}{10} \left(100 d^2e - 45 de^2 + 37 e^3\right)x^{10} + \frac{1}{9} \left(100 d^3 - 135 d^2e + 333 de^2 - 37 e^3\right)x^9 - \frac{1}{8} \left(45 d^3 - 333 d^2e + 111 de^2 - 148 e^3\right)x^8 + \frac{1}{7} \left(111 d^3 - 111 d^2e + 444 de^2 + 65 e^3\right)x^7 - \frac{1}{6} \left(37 d^3 - 444 d^2e - 195 de^2 - 107 e^3\right)x^6 + \frac{1}{5} \left(148 d^3 + 195 d^2e + 321 de^2 + 33 e^3\right)x^5 + \frac{1}{4} \left(65 d^3 + 321 d^2e + 99 de^2 + 18 e^3\right)x^4 + 18 d^3x + \frac{1}{3} \left(107 d^3 + 99 d^2e + 54 de^2\right)x^3 + \frac{3}{2} \left(11 d^3 + 18 d^2e\right)x^2$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
 + 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8
*(45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e
 + 444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3
)*x^6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 +
 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e +
 54*d*e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= 18d^3x + \frac{25e^3x^{12}}{3} + x^{11}\cdot\left(\frac{300de^2}{11} - \frac{45e^3}{11}\right) + x^{10}\cdot\left(30d^2e - \frac{27de^2}{2} + \frac{111e^3}{10}\right) + x^9 \cdot\left(\frac{100d^3}{9} - 15d^2e + 37de^2 - \frac{37e^3}{9}\right) + x^8\left(-\frac{45d^3}{8} + \frac{333d^2e}{8} - \frac{111de^2}{8} + \frac{37e^3}{2}\right) + x^7 \cdot\left(\frac{111d^3}{7} - \frac{111d^2e}{7} + \frac{444de^2}{7} + \frac{65e^3}{7}\right) + x^6\left(-\frac{37d^3}{6} + 74d^2e + \frac{65de^2}{2} + \frac{107e^3}{6}\right) + x^5\cdot\left(\frac{148d^3}{5} + 39d^2e + \frac{321de^2}{5} + \frac{33e^3}{5}\right) + x^4\cdot\left(\frac{65d^3}{4} + \frac{321d^2e}{4} + \frac{99de^2}{4} + \frac{9e^3}{2}\right) + x^3\cdot\left(\frac{107d^3}{3} + 33d^2e + 18de^2\right) + x^2\cdot\left(\frac{33d^3}{2} + 27d^2e\right)$$

input
```
integrate((e*x+d)**3*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d**3*x + 25*e**3*x**12/3 + x**11*(300*d*e**2/11 - 45*e**3/11) + x**10*(
30*d**2*e - 27*d*e**2/2 + 111*e**3/10) + x**9*(100*d**3/9 - 15*d**2*e + 37
*d*e**2 - 37*e**3/9) + x**8*(-45*d**3/8 + 333*d**2*e/8 - 111*d*e**2/8 + 37
*e**3/2) + x**7*(111*d**3/7 - 111*d**2*e/7 + 444*d*e**2/7 + 65*e**3/7) + x
**6*(-37*d**3/6 + 74*d**2*e + 65*d*e**2/2 + 107*e**3/6) + x**5*(148*d**3/5
 + 39*d**2*e + 321*d*e**2/5 + 33*e**3/5) + x**4*(65*d**3/4 + 321*d**2*e/4
+ 99*d*e**2/4 + 9*e**3/2) + x**3*(107*d**3/3 + 33*d**2*e + 18*d*e**2) + x*
*2*(33*d**3/2 + 27*d**2*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.99

$$\int (d+ex)^3 (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx$$
$$= \frac{25}{3} e^3 x^{12} + \frac{15}{11} (20 \, de^2 - 3 \, e^3) x^{11} + \frac{3}{10} (100 \, d^2 e - 45 \, de^2 + 37 \, e^3) x^{10}$$
$$+ \frac{1}{9} (100 \, d^3 - 135 \, d^2 e + 333 \, de^2 - 37 \, e^3) x^9$$
$$- \frac{1}{8} (45 \, d^3 - 333 \, d^2 e + 111 \, de^2 - 148 \, e^3) x^8$$
$$+ \frac{1}{7} (111 \, d^3 - 111 \, d^2 e + 444 \, de^2 + 65 \, e^3) x^7$$
$$- \frac{1}{6} (37 \, d^3 - 444 \, d^2 e - 195 \, de^2 - 107 \, e^3) x^6$$
$$+ \frac{1}{5} (148 \, d^3 + 195 \, d^2 e + 321 \, de^2 + 33 \, e^3) x^5 + \frac{1}{4} (65 \, d^3 + 321 \, d^2 e + 99 \, de^2 + 18 \, e^3) x^4$$
$$+ 18 \, d^3 x + \frac{1}{3} (107 \, d^3 + 99 \, d^2 e + 54 \, de^2) x^3 + \frac{3}{2} (11 \, d^3 + 18 \, d^2 e) x^2$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
25/3*e^3*x^12 + 15/11*(20*d*e^2 - 3*e^3)*x^11 + 3/10*(100*d^2*e - 45*d*e^2
 + 37*e^3)*x^10 + 1/9*(100*d^3 - 135*d^2*e + 333*d*e^2 - 37*e^3)*x^9 - 1/8
*(45*d^3 - 333*d^2*e + 111*d*e^2 - 148*e^3)*x^8 + 1/7*(111*d^3 - 111*d^2*e
 + 444*d*e^2 + 65*e^3)*x^7 - 1/6*(37*d^3 - 444*d^2*e - 195*d*e^2 - 107*e^3
)*x^6 + 1/5*(148*d^3 + 195*d^2*e + 321*d*e^2 + 33*e^3)*x^5 + 1/4*(65*d^3 +
 321*d^2*e + 99*d*e^2 + 18*e^3)*x^4 + 18*d^3*x + 1/3*(107*d^3 + 99*d^2*e +
 54*d*e^2)*x^3 + 3/2*(11*d^3 + 18*d^2*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.15

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{25}{3}e^3x^{12} + \frac{300}{11}de^2x^{11} - \frac{45}{11}e^3x^{11} + 30\,d^2ex^{10} - \frac{27}{2}de^2x^{10} + \frac{111}{10}e^3x^{10} + \frac{100}{9}d^3x^9 - 15\,d^2ex^9 + 37\,de^2x^9 - \frac{37}{9}e^3x^9 - \frac{45}{8}d^3x^8 + \frac{333}{8}d^2ex^8 - \frac{111}{8}de^2x^8 + \frac{37}{2}e^3x^8 + \frac{111}{7}d^3x^7 - \frac{111}{7}d^2ex^7 + \frac{444}{7}de^2x^7 + \frac{65}{7}e^3x^7 - \frac{37}{6}d^3x^6 + 74\,d^2ex^6 + \frac{65}{2}de^2x^6 + \frac{107}{6}e^3x^6 + \frac{148}{5}d^3x^5 + 39\,d^2ex^5 + \frac{321}{5}de^2x^5 + \frac{33}{5}e^3x^5 + \frac{65}{4}d^3x^4 + \frac{321}{4}d^2ex^4 + \frac{99}{4}de^2x^4 + \frac{9}{2}e^3x^4 + \frac{107}{3}d^3x^3 + 33\,d^2ex^3 + 18\,de^2x^3 + \frac{33}{2}d^3x^2 + 27\,d^2ex^2 + 18\,d^3x$$

input
```
integrate((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
25/3*e^3*x^12 + 300/11*d*e^2*x^11 - 45/11*e^3*x^11 + 30*d^2*e*x^10 - 27/2*
d*e^2*x^10 + 111/10*e^3*x^10 + 100/9*d^3*x^9 - 15*d^2*e*x^9 + 37*d*e^2*x^9
 - 37/9*e^3*x^9 - 45/8*d^3*x^8 + 333/8*d^2*e*x^8 - 111/8*d*e^2*x^8 + 37/2*
e^3*x^8 + 111/7*d^3*x^7 - 111/7*d^2*e*x^7 + 444/7*d*e^2*x^7 + 65/7*e^3*x^7
 - 37/6*d^3*x^6 + 74*d^2*e*x^6 + 65/2*d*e^2*x^6 + 107/6*e^3*x^6 + 148/5*d^
3*x^5 + 39*d^2*e*x^5 + 321/5*d*e^2*x^5 + 33/5*e^3*x^5 + 65/4*d^3*x^4 + 321
/4*d^2*e*x^4 + 99/4*d*e^2*x^4 + 9/2*e^3*x^4 + 107/3*d^3*x^3 + 33*d^2*e*x^3
 + 18*d*e^2*x^3 + 33/2*d^3*x^2 + 27*d^2*e*x^2 + 18*d^3*x
```

Mupad [B] (verification not implemented)

Time = 18.48 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.95

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= 18\,d^3\,x + x^3\left(\frac{107\,d^3}{3} + 33\,d^2\,e + 18\,d\,e^2\right) + x^9\left(\frac{100\,d^3}{9} - 15\,d^2\,e + 37\,d\,e^2 - \frac{37\,e^3}{9}\right) + x^6\left(-\frac{37\,d^3}{6} + 74\,d^2\,e + \frac{65\,d\,e^2}{2} + \frac{107\,e^3}{6}\right) + x^4\left(\frac{65\,d^3}{4} + \frac{321\,d^2\,e}{4} + \frac{99\,d\,e^2}{4} + \frac{9\,e^3}{2}\right) - x^8\left(\frac{45\,d^3}{8} - \frac{333\,d^2\,e}{8} + \frac{111\,d\,e^2}{8} - \frac{37\,e^3}{2}\right) + x^5\left(\frac{148\,d^3}{5} + 39\,d^2\,e + \frac{321\,d\,e^2}{5} + \frac{33\,e^3}{5}\right) + x^7\left(\frac{111\,d^3}{7} - \frac{111\,d^2\,e}{7} + \frac{444\,d\,e^2}{7} + \frac{65\,e^3}{7}\right) + \frac{25\,e^3\,x^{12}}{3} + \frac{3\,e\,x^{10}\,(100\,d^2 - 45\,d\,e + 37\,e^2)}{10} + \frac{3\,d^2\,x^2\,(11\,d + 18\,e)}{2} + \frac{15\,e^2\,x^{11}\,(20\,d - 3\,e)}{11}$$

input
```
int((d + e*x)^3*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
18*d^3*x + x^3*(18*d*e^2 + 33*d^2*e + (107*d^3)/3) + x^9*(37*d*e^2 - 15*d^
2*e + (100*d^3)/9 - (37*e^3)/9) + x^6*((65*d*e^2)/2 + 74*d^2*e - (37*d^3)/
6 + (107*e^3)/6) + x^4*((99*d*e^2)/4 + (321*d^2*e)/4 + (65*d^3)/4 + (9*e^3
)/2) - x^8*((111*d*e^2)/8 - (333*d^2*e)/8 + (45*d^3)/8 - (37*e^3)/2) + x^5
*((321*d*e^2)/5 + 39*d^2*e + (148*d^3)/5 + (33*e^3)/5) + x^7*((444*d*e^2)/
7 - (111*d^2*e)/7 + (111*d^3)/7 + (65*e^3)/7) + (25*e^3*x^12)/3 + (3*e*x^1
0*(100*d^2 - 45*d*e + 37*e^2))/10 + (3*d^2*x^2*(11*d + 18*e))/2 + (15*e^2*
x^11*(20*d - 3*e))/11
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.14

$$\int (d+ex)^3 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{x(231000e^3x^{11} + 756000d\,e^2x^{10} - 113400e^3x^{10} + 831600d^2e\,x^9 - 374220d\,e^2x^9 + 307692e^3x^9 + 308000d$$

input
```
int((e*x+d)^3*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(308000*d**3*x**8 - 155925*d**3*x**7 + 439560*d**3*x**6 - 170940*d**3*x
**5 + 820512*d**3*x**4 + 450450*d**3*x**3 + 988680*d**3*x**2 + 457380*d**3
*x + 498960*d**3 + 831600*d**2*e*x**9 - 415800*d**2*e*x**8 + 1153845*d**2*
e*x**7 - 439560*d**2*e*x**6 + 2051280*d**2*e*x**5 + 1081080*d**2*e*x**4 +
2224530*d**2*e*x**3 + 914760*d**2*e*x**2 + 748440*d**2*e*x + 756000*d*e**2
*x**10 - 374220*d*e**2*x**9 + 1025640*d*e**2*x**8 - 384615*d*e**2*x**7 + 1
758240*d*e**2*x**6 + 900900*d*e**2*x**5 + 1779624*d*e**2*x**4 + 686070*d*e
**2*x**3 + 498960*d*e**2*x**2 + 231000*e**3*x**11 - 113400*e**3*x**10 + 30
7692*e**3*x**9 - 113960*e**3*x**8 + 512820*e**3*x**7 + 257400*e**3*x**6 +
494340*e**3*x**5 + 182952*e**3*x**4 + 124740*e**3*x**3))/27720
```

3.132 $\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 195

$$\begin{aligned}\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \frac{33d^2x^2}{2}+\frac{1}{3}d(107d+66e)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4\\ &+\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5-\frac{1}{6}\left(37d^2-296de-65e^2\right)x^6\\ &+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7-\frac{1}{8}\left(45d^2-222de+37e^2\right)x^8\\ &+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}+\frac{6(d+ex)^3}{e}\end{aligned}$$

output
```
33/2*d^2*x^2+1/3*d*(107*d+66*e)*x^3+1/4*(65*d^2+214*d*e+33*e^2)*x^4+1/5*(1
48*d^2+130*d*e+107*e^2)*x^5-1/6*(37*d^2-296*d*e-65*e^2)*x^6+37/7*(3*d^2-2*
d*e+4*e^2)*x^7-1/8*(45*d^2-222*d*e+37*e^2)*x^8+1/9*(100*d^2-90*d*e+111*e^2
)*x^9+1/2*(40*d-9*e)*e*x^10+100/11*e^2*x^11+6*(e*x+d)^3/e
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=18d^2x+\frac{3}{2}d(11d+12e)x^2+\frac{1}{3}\left(107d^2+66de+18e^2\right)x^3+\frac{1}{4}\left(65d^2+214de+33e^2\right)x^4$$
$$+\frac{1}{5}\left(148d^2+130de+107e^2\right)x^5+\frac{1}{6}\left(-37d^2+296de+65e^2\right)x^6$$
$$+\frac{37}{7}\left(3d^2-2de+4e^2\right)x^7+\frac{1}{8}\left(-45d^2+222de-37e^2\right)x^8$$
$$+\frac{1}{9}\left(100d^2-90de+111e^2\right)x^9+\frac{1}{2}(40d-9e)ex^{10}+\frac{100e^2x^{11}}{11}$$

input
```
Integrate[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
 + ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^
5)/5 + ((-37*d^2 + 296*d*e + 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*
x^7)/7 + ((-45*d^2 + 222*d*e - 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e
^2)*x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right)(d+ex)^2\,dx$$
$$\Big\downarrow \; 2159$$

$$\int \left(x^8\left(100d^2-90de+111e^2\right)-x^7\left(45d^2-222de+37e^2\right)+37x^6\left(3d^2-2de+4e^2\right)-x^5\left(37d^2-296de-65e^2\right)\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{9}x^9\left(100d^2-90de+111e^2\right)-\frac{1}{8}x^8\left(45d^2-222de+37e^2\right)+\frac{37}{7}x^7\left(3d^2-2de+4e^2\right)-\frac{1}{6}x^6\left(37d^2-296de-65e^2\right)+\frac{1}{5}x^5\left(148d^2+130de+107e^2\right)+\frac{1}{4}x^4\left(65d^2+214de+33e^2\right)+\frac{1}{3}x^3\left(107d^2+66de+18e^2\right)+18d^2x+\frac{1}{2}ex^{10}(40d-9e)+\frac{3}{2}dx^2(11d+12e)+\frac{100e^2x^{11}}{11}$$

input
```
Int[(d + e*x)^2*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d^2*x + (3*d*(11*d + 12*e)*x^2)/2 + ((107*d^2 + 66*d*e + 18*e^2)*x^3)/3
 + ((65*d^2 + 214*d*e + 33*e^2)*x^4)/4 + ((148*d^2 + 130*d*e + 107*e^2)*x^
5)/5 - ((37*d^2 - 296*d*e - 65*e^2)*x^6)/6 + (37*(3*d^2 - 2*d*e + 4*e^2)*x
^7)/7 - ((45*d^2 - 222*d*e + 37*e^2)*x^8)/8 + ((100*d^2 - 90*d*e + 111*e^2
)*x^9)/9 + ((40*d - 9*e)*e*x^10)/2 + (100*e^2*x^11)/11
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

method	result
norman	$\frac{100e^2x^{11}}{11}+\left(20de-\frac{9}{2}e^2\right)x^{10}+\left(\frac{100}{9}d^2-10de+\frac{37}{3}e^2\right)x^9+\left(-\frac{45}{8}d^2+\frac{111}{4}de-\frac{37}{8}e^2\right)x^8+\left(\frac{11}{7}\right.$
default	$\frac{100e^2x^{11}}{11}+\frac{(200de-45e^2)x^{10}}{10}+\frac{(100d^2-90de+111e^2)x^9}{9}+\frac{(-45d^2+222de-37e^2)x^8}{8}+\frac{(111d^2-74de+148e^2)x^7}{7}+$
orering	$\frac{x(252000x^{10}e^2+554400x^9de-124740e^2x^9+308000x^8d^2-277200x^8de+341880x^8e^2-155925x^7d^2+769230x^7de-128205x^7e}{}$
gosper	$-\frac{37}{6}x^6d^2+\frac{65}{6}x^6e^2+\frac{148}{5}x^5d^2+\frac{65}{4}x^4d^2+\frac{33}{4}x^4e^2+\frac{107}{3}x^3d^2+18d^2x-\frac{37}{8}x^8e^2+\frac{111}{7}x^7d^2+$
risch	$-\frac{37}{6}x^6d^2+\frac{65}{6}x^6e^2+\frac{148}{5}x^5d^2+\frac{65}{4}x^4d^2+\frac{33}{4}x^4e^2+\frac{107}{3}x^3d^2+18d^2x-\frac{37}{8}x^8e^2+\frac{111}{7}x^7d^2+$
parallelrisch	$-\frac{37}{6}x^6d^2+\frac{65}{6}x^6e^2+\frac{148}{5}x^5d^2+\frac{65}{4}x^4d^2+\frac{33}{4}x^4e^2+\frac{107}{3}x^3d^2+18d^2x-\frac{37}{8}x^8e^2+\frac{111}{7}x^7d^2+$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
100/11*e^2*x^11+(20*d*e-9/2*e^2)*x^10+(100/9*d^2-10*d*e+37/3*e^2)*x^9+(-45
/8*d^2+111/4*d*e-37/8*e^2)*x^8+(111/7*d^2-74/7*d*e+148/7*e^2)*x^7+(-37/6*d
^2+148/3*d*e+65/6*e^2)*x^6+(148/5*d^2+26*d*e+107/5*e^2)*x^5+(65/4*d^2+107/
2*d*e+33/4*e^2)*x^4+(107/3*d^2+22*d*e+6*e^2)*x^3+(33/2*d^2+18*d*e)*x^2+18*
d^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95

$$\begin{aligned}\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx &= \frac{100}{11}e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9 \\ &\quad -\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+4\,e^2\right)x^7 \\ &\quad -\frac{1}{6}\left(37\,d^2-296\,de-65\,e^2\right)x^6+\frac{1}{5}\left(148\,d^2+130\,de+107\,e^2\right)x^5 \\ &\quad +\frac{1}{4}\left(65\,d^2+214\,de+33\,e^2\right)x^4+\frac{1}{3}\left(107\,d^2+66\,de+18\,e^2\right)x^3 \\ &\quad +18\,d^2x+\frac{3}{2}\left(11\,d^2+12\,de\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*
e^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e
^2)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 1
07*e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e
+ 18*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$$
$$= 18d^2x + \frac{100e^2x^{11}}{11} + x^{10}\cdot\left(20de - \frac{9e^2}{2}\right) + x^9\cdot\left(\frac{100d^2}{9} - 10de + \frac{37e^2}{3}\right)$$
$$+ x^8\left(-\frac{45d^2}{8} + \frac{111de}{4} - \frac{37e^2}{8}\right) + x^7\cdot\left(\frac{111d^2}{7} - \frac{74de}{7} + \frac{148e^2}{7}\right)$$
$$+ x^6\left(-\frac{37d^2}{6} + \frac{148de}{3} + \frac{65e^2}{6}\right) + x^5\cdot\left(\frac{148d^2}{5} + 26de + \frac{107e^2}{5}\right) + x^4$$
$$\cdot\left(\frac{65d^2}{4} + \frac{107de}{2} + \frac{33e^2}{4}\right) + x^3\cdot\left(\frac{107d^2}{3} + 22de + 6e^2\right) + x^2\cdot\left(\frac{33d^2}{2} + 18de\right)$$

input
```
integrate((e*x+d)**2*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d**2*x + 100*e**2*x**11/11 + x**10*(20*d*e - 9*e**2/2) + x**9*(100*d**2
/9 - 10*d*e + 37*e**2/3) + x**8*(-45*d**2/8 + 111*d*e/4 - 37*e**2/8) + x**
7*(111*d**2/7 - 74*d*e/7 + 148*e**2/7) + x**6*(-37*d**2/6 + 148*d*e/3 + 65
*e**2/6) + x**5*(148*d**2/5 + 26*d*e + 107*e**2/5) + x**4*(65*d**2/4 + 107
*d*e/2 + 33*e**2/4) + x**3*(107*d**2/3 + 22*d*e + 6*e**2) + x**2*(33*d**2/
2 + 18*d*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{100}{11}e^2x^{11}+\frac{1}{2}\left(40\,de-9\,e^2\right)x^{10}+\frac{1}{9}\left(100\,d^2-90\,de+111\,e^2\right)x^9\\&\quad-\frac{1}{8}\left(45\,d^2-222\,de+37\,e^2\right)x^8+\frac{37}{7}\left(3\,d^2-2\,de+4\,e^2\right)x^7\\&\quad-\frac{1}{6}\left(37\,d^2-296\,de-65\,e^2\right)x^6+\frac{1}{5}\left(148\,d^2+130\,de+107\,e^2\right)x^5\\&\quad+\frac{1}{4}\left(65\,d^2+214\,de+33\,e^2\right)x^4+\frac{1}{3}\left(107\,d^2+66\,de+18\,e^2\right)x^3\\&\quad+18\,d^2x+\frac{3}{2}\left(11\,d^2+12\,de\right)x^2\end{aligned}$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
100/11*e^2*x^11 + 1/2*(40*d*e - 9*e^2)*x^10 + 1/9*(100*d^2 - 90*d*e + 111*
e^2)*x^9 - 1/8*(45*d^2 - 222*d*e + 37*e^2)*x^8 + 37/7*(3*d^2 - 2*d*e + 4*e
^2)*x^7 - 1/6*(37*d^2 - 296*d*e - 65*e^2)*x^6 + 1/5*(148*d^2 + 130*d*e + 1
07*e^2)*x^5 + 1/4*(65*d^2 + 214*d*e + 33*e^2)*x^4 + 1/3*(107*d^2 + 66*d*e
+ 18*e^2)*x^3 + 18*d^2*x + 3/2*(11*d^2 + 12*d*e)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.06

$$\begin{aligned}&\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx\\&=\frac{100}{11}e^2x^{11}+20\,dex^{10}-\frac{9}{2}e^2x^{10}+\frac{100}{9}d^2x^9-10\,dex^9+\frac{37}{3}e^2x^9-\frac{45}{8}d^2x^8\\&\quad+\frac{111}{4}dex^8-\frac{37}{8}e^2x^8+\frac{111}{7}d^2x^7-\frac{74}{7}dex^7+\frac{148}{7}e^2x^7-\frac{37}{6}d^2x^6\\&\quad+\frac{148}{3}dex^6+\frac{65}{6}e^2x^6+\frac{148}{5}d^2x^5+26\,dex^5+\frac{107}{5}e^2x^5+\frac{65}{4}d^2x^4+\frac{107}{2}dex^4\\&\quad+\frac{33}{4}e^2x^4+\frac{107}{3}d^2x^3+22\,dex^3+6\,e^2x^3+\frac{33}{2}d^2x^2+18\,dex^2+18\,d^2x\end{aligned}$$

input
```
integrate((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
100/11*e^2*x^11 + 20*d*e*x^10 - 9/2*e^2*x^10 + 100/9*d^2*x^9 - 10*d*e*x^9
+ 37/3*e^2*x^9 - 45/8*d^2*x^8 + 111/4*d*e*x^8 - 37/8*e^2*x^8 + 111/7*d^2*x
^7 - 74/7*d*e*x^7 + 148/7*e^2*x^7 - 37/6*d^2*x^6 + 148/3*d*e*x^6 + 65/6*e^
2*x^6 + 148/5*d^2*x^5 + 26*d*e*x^5 + 107/5*e^2*x^5 + 65/4*d^2*x^4 + 107/2*
d*e*x^4 + 33/4*e^2*x^4 + 107/3*d^2*x^3 + 22*d*e*x^3 + 6*e^2*x^3 + 33/2*d^2
*x^2 + 18*d*e*x^2 + 18*d^2*x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.90

$$\int (d+ex)^2\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= x^3\left(\frac{107\,d^2}{3}+22\,d\,e+6\,e^2\right)+x^9\left(\frac{100\,d^2}{9}-10\,d\,e+\frac{37\,e^2}{3}\right)$$
$$+x^4\left(\frac{65\,d^2}{4}+\frac{107\,d\,e}{2}+\frac{33\,e^2}{4}\right)-x^8\left(\frac{45\,d^2}{8}-\frac{111\,d\,e}{4}+\frac{37\,e^2}{8}\right)$$
$$+x^6\left(-\frac{37\,d^2}{6}+\frac{148\,d\,e}{3}+\frac{65\,e^2}{6}\right)+x^5\left(\frac{148\,d^2}{5}+26\,d\,e+\frac{107\,e^2}{5}\right)$$
$$+x^7\left(\frac{111\,d^2}{7}-\frac{74\,d\,e}{7}+\frac{148\,e^2}{7}\right)+18\,d^2\,x$$
$$+\frac{100\,e^2\,x^{11}}{11}+\frac{3\,d\,x^2\,(11\,d+12\,e)}{2}+\frac{e\,x^{10}\,(40\,d-9\,e)}{2}$$

input
```
int((d + e*x)^2*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^3*(22*d*e + (107*d^2)/3 + 6*e^2) + x^9*((100*d^2)/9 - 10*d*e + (37*e^2)/
3) + x^4*((107*d*e)/2 + (65*d^2)/4 + (33*e^2)/4) - x^8*((45*d^2)/8 - (111*
d*e)/4 + (37*e^2)/8) + x^6*((148*d*e)/3 - (37*d^2)/6 + (65*e^2)/6) + x^5*(
26*d*e + (148*d^2)/5 + (107*e^2)/5) + x^7*((111*d^2)/7 - (74*d*e)/7 + (148
*e^2)/7) + 18*d^2*x + (100*e^2*x^11)/11 + (3*d*x^2*(11*d + 12*e))/2 + (e*x
^10*(40*d - 9*e))/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.05

$$\int (d+ex)^2 \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{x(252000e^2x^{10} + 554400de\,x^9 - 124740e^2x^9 + 308000d^2x^8 - 277200de\,x^8 + 341880e^2x^8 - 155925d^2x^7 -}{}$$

input
```
int((e*x+d)^2*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(308000*d**2*x**8 - 155925*d**2*x**7 + 439560*d**2*x**6 - 170940*d**2*x
**5 + 820512*d**2*x**4 + 450450*d**2*x**3 + 988680*d**2*x**2 + 457380*d**2
*x + 498960*d**2 + 554400*d*e*x**9 - 277200*d*e*x**8 + 769230*d*e*x**7 - 2
93040*d*e*x**6 + 1367520*d*e*x**5 + 720720*d*e*x**4 + 1483020*d*e*x**3 + 6
09840*d*e*x**2 + 498960*d*e*x + 252000*e**2*x**10 - 124740*e**2*x**9 + 341
880*e**2*x**8 - 128205*e**2*x**7 + 586080*e**2*x**6 + 300300*e**2*x**5 + 5
93208*e**2*x**4 + 228690*e**2*x**3 + 166320*e**2*x**2))/27720
```

3.133 $\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)$

Optimal result

Integrand size = 36, antiderivative size = 121

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5 - \frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}$$

output
```
18*d*x+3/2*(11*d+6*e)*x^2+1/3*(107*d+33*e)*x^3+1/4*(65*d+107*e)*x^4+1/5*(1
48*d+65*e)*x^5-37/6*(d-4*e)*x^6+37/7*(3*d-e)*x^7-3/8*(15*d-37*e)*x^8+5/9*(
20*d-9*e)*x^9+10*e*x^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 18dx+\frac{3}{2}(11d+6e)x^2+\frac{1}{3}(107d+33e)x^3+\frac{1}{4}(65d+107e)x^4+\frac{1}{5}(148d+65e)x^5 - \frac{37}{6}(d-4e)x^6+\frac{37}{7}(3d-e)x^7-\frac{3}{8}(15d-37e)x^8+\frac{5}{9}(20d-9e)x^9+10ex^{10}$$

input
```
Integrate[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)
*x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^
7)/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex) \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(5x^8(20d-9e)-3x^7(15d-37e)+37x^6(3d-e)-37x^5(d-4e)+x^4(148d+65e)+x^3(65d+107e)+x^2(10$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{9}x^9(20d-9e)-\frac{3}{8}x^8(15d-37e)+\frac{37}{7}x^7(3d-e)-\frac{37}{6}x^6(d-4e)+\frac{1}{5}x^5(148d+65e)+\frac{1}{4}x^4(65d+107e)+\frac{1}{3}x^3(107d+33e)+\frac{3}{2}x^2(11d+6e)+18dx+10ex^{10}$$

input
```
Int[(d + e*x)*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*d*x + (3*(11*d + 6*e)*x^2)/2 + ((107*d + 33*e)*x^3)/3 + ((65*d + 107*e)
*x^4)/4 + ((148*d + 65*e)*x^5)/5 - (37*(d - 4*e)*x^6)/6 + (37*(3*d - e)*x^
7)/7 - (3*(15*d - 37*e)*x^8)/8 + (5*(20*d - 9*e)*x^9)/9 + 10*e*x^10
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

method	result
norman	$10e\,x^{10}+\left(\frac{100d}{9}-5e\right)x^{9}+\left(-\frac{45d}{8}+\frac{111e}{8}\right)x^{8}+\left(\frac{111d}{7}-\frac{37e}{7}\right)x^{7}+\left(-\frac{37d}{6}+\frac{74e}{3}\right)x^{6}+\left(\frac{148d}{5}+\right.$
orering	$\frac{x\left(25200x^{9}e+28000x^{8}d-12600e\,x^{8}-14175x^{7}d+34965e\,x^{7}+39960d\,x^{6}-13320e\,x^{6}-15540d\,x^{5}+62160e\,x^{5}+74592d\,x^{4}+3276\right.}{2520}$
gosper	$10e\,x^{10}+\frac{100}{9}x^{9}d-5x^{9}e-\frac{45}{8}x^{8}d+\frac{111}{8}e\,x^{8}+\frac{111}{7}x^{7}d-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}-$
default	$10e\,x^{10}+\frac{(100d-45e)x^{9}}{9}+\frac{(-45d+111e)x^{8}}{8}+\frac{(111d-37e)x^{7}}{7}+\frac{(-37d+148e)x^{6}}{6}+\frac{(148d+65e)x^{5}}{5}+\frac{(65d+107e}{4}$
risch	$10e\,x^{10}+\frac{100}{9}x^{9}d-5x^{9}e-\frac{45}{8}x^{8}d+\frac{111}{8}e\,x^{8}+\frac{111}{7}x^{7}d-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}-$
parallelrisch	$10e\,x^{10}+\frac{100}{9}x^{9}d-5x^{9}e-\frac{45}{8}x^{8}d+\frac{111}{8}e\,x^{8}+\frac{111}{7}x^{7}d-\frac{37}{7}e\,x^{7}-\frac{37}{6}d\,x^{6}+\frac{74}{3}e\,x^{6}+\frac{148}{5}d\,x^{5}-$

input
```
int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
10*e*x^10+(100/9*d-5*e)*x^9+(-45/8*d+111/8*e)*x^8+(111/7*d-37/7*e)*x^7+(-3
7/6*d+74/3*e)*x^6+(148/5*d+13*e)*x^5+(65/4*d+107/4*e)*x^4+(107/3*d+11*e)*x
^3+(33/2*d+9*e)*x^2+18*d*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7$$
$$-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4$$
$$+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*
x^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4
 + 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=18dx+10ex^{10}+x^9\cdot\left(\frac{100d}{9}-5e\right)+x^8\left(-\frac{45d}{8}+\frac{111e}{8}\right)+x^7$$
$$\cdot\left(\frac{111d}{7}-\frac{37e}{7}\right)+x^6\left(-\frac{37d}{6}+\frac{74e}{3}\right)+x^5\cdot\left(\frac{148d}{5}+13e\right)$$
$$+x^4\cdot\left(\frac{65d}{4}+\frac{107e}{4}\right)+x^3\cdot\left(\frac{107d}{3}+11e\right)+x^2\cdot\left(\frac{33d}{2}+9e\right)$$

input
```
integrate((e*x+d)*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
18*d*x + 10*e*x**10 + x**9*(100*d/9 - 5*e) + x**8*(-45*d/8 + 111*e/8) + x*
*7*(111*d/7 - 37*e/7) + x**6*(-37*d/6 + 74*e/3) + x**5*(148*d/5 + 13*e) +
x**4*(65*d/4 + 107*e/4) + x**3*(107*d/3 + 11*e) + x**2*(33*d/2 + 9*e)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=10\,ex^{10}+\frac{5}{9}\,(20\,d-9\,e)x^9-\frac{3}{8}\,(15\,d-37\,e)x^8+\frac{37}{7}\,(3\,d-e)x^7-\frac{37}{6}\,(d-4\,e)x^6+\frac{1}{5}\,(148\,d+65\,e)x^5+\frac{1}{4}\,(65\,d+107\,e)x^4+\frac{1}{3}\,(107\,d+33\,e)x^3+\frac{3}{2}\,(11\,d+6\,e)x^2+18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
10*e*x^10 + 5/9*(20*d - 9*e)*x^9 - 3/8*(15*d - 37*e)*x^8 + 37/7*(3*d - e)*
x^7 - 37/6*(d - 4*e)*x^6 + 1/5*(148*d + 65*e)*x^5 + 1/4*(65*d + 107*e)*x^4
 + 1/3*(107*d + 33*e)*x^3 + 3/2*(11*d + 6*e)*x^2 + 18*d*x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$=10\,ex^{10}+\frac{100}{9}\,dx^9-5\,ex^9-\frac{45}{8}\,dx^8+\frac{111}{8}\,ex^8+\frac{111}{7}\,dx^7-\frac{37}{7}\,ex^7-\frac{37}{6}\,dx^6+\frac{74}{3}\,ex^6+\frac{148}{5}\,dx^5+13\,ex^5+\frac{65}{4}\,dx^4+\frac{107}{4}\,ex^4+\frac{107}{3}\,dx^3+11\,ex^3+\frac{33}{2}\,dx^2+9\,ex^2+18\,dx$$

input
```
integrate((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
10*e*x^10 + 100/9*d*x^9 - 5*e*x^9 - 45/8*d*x^8 + 111/8*e*x^8 + 111/7*d*x^7
 - 37/7*e*x^7 - 37/6*d*x^6 + 74/3*e*x^6 + 148/5*d*x^5 + 13*e*x^5 + 65/4*d*
x^4 + 107/4*e*x^4 + 107/3*d*x^3 + 11*e*x^3 + 33/2*d*x^2 + 9*e*x^2 + 18*d*x
```

Mupad [B] (verification not implemented)

Time = 18.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.83

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = 10\,e\,x^{10}+\left(\frac{100\,d}{9}-5\,e\right)x^9+\left(\frac{111\,e}{8}-\frac{45\,d}{8}\right)x^8 +\left(\frac{111\,d}{7}-\frac{37\,e}{7}\right)x^7+\left(\frac{74\,e}{3}-\frac{37\,d}{6}\right)x^6+\left(\frac{148\,d}{5}+13\,e\right)x^5 +\left(\frac{65\,d}{4}+\frac{107\,e}{4}\right)x^4+\left(\frac{107\,d}{3}+11\,e\right)x^3+\left(\frac{33\,d}{2}+9\,e\right)x^2+18\,d\,x$$

input
```
int((d + e*x)*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
x^2*((33*d)/2 + 9*e) + x^9*((100*d)/9 - 5*e) + x^3*((107*d)/3 + 11*e) - x^
6*((37*d)/6 - (74*e)/3) + x^7*((111*d)/7 - (37*e)/7) + x^5*((148*d)/5 + 13
*e) - x^8*((45*d)/8 - (111*e)/8) + x^4*((65*d)/4 + (107*e)/4) + 18*d*x + 1
0*e*x^10
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(3+2x+5x^2\right)^2\left(2+x+3x^2-5x^3+4x^4\right)\,dx = \frac{x(25200e\,x^9+28000d\,x^8-12600e\,x^8-14175d\,x^7+34965e\,x^7+39960d\,x^6-13320e\,x^6-15540d\,x^5+}{}$$

input
```
int((e*x+d)*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(28000*d*x**8 - 14175*d*x**7 + 39960*d*x**6 - 15540*d*x**5 + 74592*d*x*
*4 + 40950*d*x**3 + 89880*d*x**2 + 41580*d*x + 45360*d + 25200*e*x**9 - 12
600*e*x**8 + 34965*e*x**7 - 13320*e*x**6 + 62160*e*x**5 + 32760*e*x**4 + 6
7410*e*x**3 + 27720*e*x**2 + 22680*e*x))/2520
```

3.134 $\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx$

Optimal result

Integrand size = 31, antiderivative size = 60

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

output
```
18*x+33/2*x^2+107/3*x^3+65/4*x^4+148/5*x^5-37/6*x^6+111/7*x^7-45/8*x^8+100
/9*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = 18x+\frac{33x^2}{2}+\frac{107x^3}{3}+\frac{65x^4}{4}+\frac{148x^5}{5}-\frac{37x^6}{6}+\frac{111x^7}{7}-\frac{45x^8}{8}+\frac{100x^9}{9}$$

input
```
Integrate[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) dx$$

$$\Big\downarrow \text{ 2188}$$

$$\int \left(100x^8-45x^7+111x^6-37x^5+148x^4+65x^3+107x^2+33x+18\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{100x^9}{9}-\frac{45x^8}{8}+\frac{111x^7}{7}-\frac{37x^6}{6}+\frac{148x^5}{5}+\frac{65x^4}{4}+\frac{107x^3}{3}+\frac{33x^2}{2}+18x$$

input
```
Int[(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

method	result	size
orering	$\frac{x(28000x^8-14175x^7+39960x^6-15540x^5+74592x^4+40950x^3+89880x^2+41580x+45360)}{2520}$	44
gosper	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
default	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
norman	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
risch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45
parallelrisch	$18x+\frac{33}{2}x^2+\frac{107}{3}x^3+\frac{65}{4}x^4+\frac{148}{5}x^5-\frac{37}{6}x^6+\frac{111}{7}x^7-\frac{45}{8}x^8+\frac{100}{9}x^9$	45

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2520*x*(28000*x^8-14175*x^7+39960*x^6-15540*x^5+74592*x^4+40950*x^3+8988
0*x^2+41580*x+45360)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = \frac{100}{9}x^9-\frac{45}{8}x^8+\frac{111}{7}x^7-\frac{37}{6}x^6+\frac{148}{5}x^5+\frac{65}{4}x^4+\frac{107}{3}x^3+\frac{33}{2}x^2+18\,x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = \frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**5/5 + 65*x**4/4 +
 107*x**3/3 + 33*x**2/2 + 18*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = \frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = \frac{100}{9}x^9 - \frac{45}{8}x^8 + \frac{111}{7}x^7 - \frac{37}{6}x^6 + \frac{148}{5}x^5 + \frac{65}{4}x^4 + \frac{107}{3}x^3 + \frac{33}{2}x^2 + 18x$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
100/9*x^9 - 45/8*x^8 + 111/7*x^7 - 37/6*x^6 + 148/5*x^5 + 65/4*x^4 + 107/3
*x^3 + 33/2*x^2 + 18*x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4) \, dx = \frac{100x^9}{9} - \frac{45x^8}{8} + \frac{111x^7}{7} - \frac{37x^6}{6} + \frac{148x^5}{5} + \frac{65x^4}{4} + \frac{107x^3}{3} + \frac{33x^2}{2} + 18x$$

input
```
int((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
18*x + (33*x^2)/2 + (107*x^3)/3 + (65*x^4)/4 + (148*x^5)/5 - (37*x^6)/6 +
(111*x^7)/7 - (45*x^8)/8 + (100*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

$$\int \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$$
$$= \frac{x(28000x^8 - 14175x^7 + 39960x^6 - 15540x^5 + 74592x^4 + 40950x^3 + 89880x^2 + 41580x + 45360)}{2520}$$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
(x*(28000*x**8 - 14175*x**7 + 39960*x**6 - 15540*x**5 + 74592*x**4 + 40950
*x**3 + 89880*x**2 + 41580*x + 45360))/2520
```

3.135 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex} dx$

Optimal result

Integrand size = 38, antiderivative size = 352

$$
\begin{aligned}
\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx
&= -\frac{(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)\,x}{e^8}\\
&\quad+\frac{(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)\,x^2}{2e^7}\\
&\quad-\frac{(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)\,x^3}{3e^6}\\
&\quad+\frac{(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)\,x^4}{4e^5}\\
&\quad-\frac{(100d^3+45d^2e+111de^2+37e^3)\,x^5}{5e^4}+\frac{(100d^2+45de+111e^2)\,x^6}{6e^3}-\frac{5(20d+9e)x^7}{7e^2}\\
&\quad+\frac{25x^8}{2e}+\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}
\end{aligned}
$$

output
```
-(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6
-33*e^7)*x/e^8+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+148*d^2*e^4-65
*d*e^5+107*e^6)*x^2/e^7-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^3+148*d
*e^4-65*e^5)*x^3/e^6+1/4*(100*d^4+45*d^3*e+111*d^2*e^2+37*d*e^3+148*e^4)*x
^4/e^5-1/5*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/6*(100*d^2+45*d*e
+111*e^2)*x^6/e^3-5/7*(20*d+9*e)*x^7/e^2+25/2*x^8/e+(5*d^2-2*d*e+3*e^2)^2*
(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.74

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{x(-42000d^7+2100d^6e(-9+10x)-70d^5e^2(666-135x+200x^2)+210d^4e^3(-74+111x-30x^2+50x}{}$$
$$+ \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),
x]
```

output
```
(x*(-42000*d^7 + 2100*d^6*e*(-9 + 10*x) - 70*d^5*e^2*(666 - 135*x + 200*x^
2) + 210*d^4*e^3*(-74 + 111*x - 30*x^2 + 50*x^3) - 105*d^3*e^4*(592 - 74*x
 + 148*x^2 - 45*x^3 + 80*x^4) + 35*d^2*e^5*(780 + 888*x - 148*x^2 + 333*x^
3 - 108*x^4 + 200*x^5) - d*e^6*(44940 + 13650*x + 20720*x^2 - 3885*x^3 + 9
324*x^4 - 3150*x^5 + 6000*x^6) + 2*e^7*(6930 + 11235*x + 4550*x^2 + 7770*x
^3 - 1554*x^4 + 3885*x^5 - 1350*x^6 + 2625*x^7)))/(420*e^8) + ((5*d^2 - 2*
d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])
/e^9
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2 (4x^4-5x^3+3x^2+x+2)}{d+ex} dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{x^5(100d^2+45de+111e^2)}{e^3} - \frac{x^4(100d^3+45d^2e+111de^2+37e^3)}{e^4} + \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^6(100d^2+45de+111e^2)}{6e^3} - \frac{x^5(100d^3+45d^2e+111de^2+37e^3)}{5e^4} + \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e^2-de^3+2e^4) \log(d+ex)}{e^9} + \frac{x^4(100d^4+45d^3e+111d^2e^2+37de^3+148e^4)}{4e^5} - \frac{x^3(100d^5+45d^4e+111d^3e^2+37d^2e^3+148de^4-65e^5)}{3e^6} + \frac{x^2(100d^6+45d^5e+111d^4e^2+37d^3e^3+148d^2e^4-65de^5+107e^6)}{2e^7} - \frac{x(100d^7+45d^6e+111d^5e^2+37d^4e^3+148d^3e^4-65d^2e^5+107de^6-33e^7)}{e^8} - \frac{5x^7(20d+9e)}{7e^2} + \frac{25x^8}{2e}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x),x]
```

output
```
-(((100*d^7 + 45*d^6*e + 111*d^5*e^2 + 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e
^5 + 107*d*e^6 - 33*e^7)*x)/e^8) + ((100*d^6 + 45*d^5*e + 111*d^4*e^2 + 37
*d^3*e^3 + 148*d^2*e^4 - 65*d*e^5 + 107*e^6)*x^2)/(2*e^7) - ((100*d^5 + 45
*d^4*e + 111*d^3*e^2 + 37*d^2*e^3 + 148*d*e^4 - 65*e^5)*x^3)/(3*e^6) + ((1
00*d^4 + 45*d^3*e + 111*d^2*e^2 + 37*d*e^3 + 148*e^4)*x^4)/(4*e^5) - ((100
*d^3 + 45*d^2*e + 111*d*e^2 + 37*e^3)*x^5)/(5*e^4) + ((100*d^2 + 45*d*e +
111*e^2)*x^6)/(6*e^3) - (5*(20*d + 9*e)*x^7)/(7*e^2) + (25*x^8)/(2*e) + ((
5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log
[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.01

method	result
norman	$\frac{25x^8}{2e}-\frac{5(20d+9e)x^7}{7e^2}+\frac{(100d^2+45de+111e^2)x^6}{6e^3}-\frac{(100d^3+45d^2e+111d\,e^2+37e^3)x^5}{5e^4}+\frac{(100d^4+45d^3e+111d^2e^2+37}{4e^5}$
default	$-\frac{-33e^7x+100x\,d^7-\frac{107}{2}e^7x^2-\frac{65}{3}e^7x^3+\frac{37}{5}e^7x^5-37e^7x^4-\frac{25}{2}x^8e^7-\frac{37}{2}x^6e^7+107d\,e^6x+37d^4e^3x+148d^3e^4x-65d^2e^5x-50d}{}$
parallelrisch	$7560\ln(ex+d)e^8+5250x^8e^8-2700x^7e^8+7770x^6e^8-3108x^5e^8+15540x^4e^8+9100x^3e^8+22470x^2e^8+13860x\,e^8+42000\ln(ex-$
risch	$-\frac{37x^5}{5e}+\frac{107x^2}{2e}+\frac{25x^8}{2e}+\frac{33x}{e}+\frac{18\ln(ex+d)}{e}+\frac{37x^6}{2e}+\frac{37x^4}{e}+\frac{65x^3}{3e}-\frac{45x^7}{7e}-\frac{111x^5d}{5e^2}+\frac{111d^2x^4}{4e^3}+\frac{37x}{4e}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x,method=_RETURNVERBOS
E)
```

output
```
25/2*x^8/e-5/7*(20*d+9*e)*x^7/e^2+1/6*(100*d^2+45*d*e+111*e^2)*x^6/e^3-1/5
*(100*d^3+45*d^2*e+111*d*e^2+37*e^3)*x^5/e^4+1/4*(100*d^4+45*d^3*e+111*d^2
*e^2+37*d*e^3+148*e^4)*x^4/e^5-1/3*(100*d^5+45*d^4*e+111*d^3*e^2+37*d^2*e^
3+148*d*e^4-65*e^5)*x^3/e^6+1/2*(100*d^6+45*d^5*e+111*d^4*e^2+37*d^3*e^3+1
48*d^2*e^4-65*d*e^5+107*e^6)*x^2/e^7-(100*d^7+45*d^6*e+111*d^5*e^2+37*d^4*
e^3+148*d^3*e^4-65*d^2*e^5+107*d*e^6-33*e^7)*x/e^8+(100*d^8+45*d^7*e+111*d
^6*e^2+37*d^5*e^3+148*d^4*e^4-65*d^3*e^5+107*d^2*e^6-33*d*e^7+18*e^8)/e^9*
ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{5250\,e^8x^8-300\,(20\,de^7+9\,e^8)x^7+70\,(100\,d^2e^6+45\,de^7+111\,e^8)x^6-84\,(100\,d^3e^5+45\,d^2e^6+111\,de^7}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="fr
icas")
```

output
```
1/420*(5250*e^8*x^8 - 300*(20*d*e^7 + 9*e^8)*x^7 + 70*(100*d^2*e^6 + 45*d*
e^7 + 111*e^8)*x^6 - 84*(100*d^3*e^5 + 45*d^2*e^6 + 111*d*e^7 + 37*e^8)*x^
5 + 105*(100*d^4*e^4 + 45*d^3*e^5 + 111*d^2*e^6 + 37*d*e^7 + 148*e^8)*x^4
- 140*(100*d^5*e^3 + 45*d^4*e^4 + 111*d^3*e^5 + 37*d^2*e^6 + 148*d*e^7 - 6
5*e^8)*x^3 + 210*(100*d^6*e^2 + 45*d^5*e^3 + 111*d^4*e^4 + 37*d^3*e^5 + 14
8*d^2*e^6 - 65*d*e^7 + 107*e^8)*x^2 - 420*(100*d^7*e + 45*d^6*e^2 + 111*d^
5*e^3 + 37*d^4*e^4 + 148*d^3*e^5 - 65*d^2*e^6 + 107*d*e^7 - 33*e^8)*x + 42
0*(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^
5 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d))/e^9
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.06

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= x^7\left(-\frac{100d}{7e^2}-\frac{45}{7e}\right)+x^6\cdot\left(\frac{50d^2}{3e^3}+\frac{15d}{2e^2}+\frac{37}{2e}\right)$$
$$+x^5\left(-\frac{20d^3}{e^4}-\frac{9d^2}{e^3}-\frac{111d}{5e^2}-\frac{37}{5e}\right)+x^4\cdot\left(\frac{25d^4}{e^5}+\frac{45d^3}{4e^4}+\frac{111d^2}{4e^3}+\frac{37d}{4e^2}+\frac{37}{e}\right)$$
$$+x^3\left(-\frac{100d^5}{3e^6}-\frac{15d^4}{e^5}-\frac{37d^3}{e^4}-\frac{37d^2}{3e^3}-\frac{148d}{3e^2}+\frac{65}{3e}\right)+x^2$$
$$\cdot\left(\frac{50d^6}{e^7}+\frac{45d^5}{2e^6}+\frac{111d^4}{2e^5}+\frac{37d^3}{2e^4}+\frac{74d^2}{e^3}-\frac{65d}{2e^2}+\frac{107}{2e}\right)$$
$$+x\left(-\frac{100d^7}{e^8}-\frac{45d^6}{e^7}-\frac{111d^5}{e^6}-\frac{37d^4}{e^5}-\frac{148d^3}{e^4}+\frac{65d^2}{e^3}-\frac{107d}{e^2}+\frac{33}{e}\right)$$
$$+\frac{25x^8}{2e}+\frac{(5d^2-2de+3e^2)^2\cdot(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d),x)
```

output
```
x**7*(-100*d/(7*e**2) - 45/(7*e)) + x**6*(50*d**2/(3*e**3) + 15*d/(2*e**2)
 + 37/(2*e)) + x**5*(-20*d**3/e**4 - 9*d**2/e**3 - 111*d/(5*e**2) - 37/(5*
e)) + x**4*(25*d**4/e**5 + 45*d**3/(4*e**4) + 111*d**2/(4*e**3) + 37*d/(4*
e**2) + 37/e) + x**3*(-100*d**5/(3*e**6) - 15*d**4/e**5 - 37*d**3/e**4 - 3
7*d**2/(3*e**3) - 148*d/(3*e**2) + 65/(3*e)) + x**2*(50*d**6/e**7 + 45*d**
5/(2*e**6) + 111*d**4/(2*e**5) + 37*d**3/(2*e**4) + 74*d**2/e**3 - 65*d/(2
*e**2) + 107/(2*e)) + x*(-100*d**7/e**8 - 45*d**6/e**7 - 111*d**5/e**6 - 3
7*d**4/e**5 - 148*d**3/e**4 + 65*d**2/e**3 - 107*d/e**2 + 33/e) + 25*x**8/
(2*e) + (5*d**2 - 2*d*e + 3*e**2)**2*(4*d**4 + 5*d**3*e + 3*d**2*e**2 - d*
e**3 + 2*e**4)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{5250\,e^7x^8 - 300\,(20\,de^6+9\,e^7)x^7 + 70\,(100\,d^2e^5+45\,de^6+111\,e^7)x^6 - 84\,(100\,d^3e^4+45\,d^2e^5+111\,de^6}{}$$
$$+ \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="ma
xima")
```

output
```
1/420*(5250*e^7*x^8 - 300*(20*d*e^6 + 9*e^7)*x^7 + 70*(100*d^2*e^5 + 45*d*
e^6 + 111*e^7)*x^6 - 84*(100*d^3*e^4 + 45*d^2*e^5 + 111*d*e^6 + 37*e^7)*x^
5 + 105*(100*d^4*e^3 + 45*d^3*e^4 + 111*d^2*e^5 + 37*d*e^6 + 148*e^7)*x^4
- 140*(100*d^5*e^2 + 45*d^4*e^3 + 111*d^3*e^4 + 37*d^2*e^5 + 148*d*e^6 - 6
5*e^7)*x^3 + 210*(100*d^6*e + 45*d^5*e^2 + 111*d^4*e^3 + 37*d^3*e^4 + 148*
d^2*e^5 - 65*d*e^6 + 107*e^7)*x^2 - 420*(100*d^7 + 45*d^6*e + 111*d^5*e^2
+ 37*d^4*e^3 + 148*d^3*e^4 - 65*d^2*e^5 + 107*d*e^6 - 33*e^7)*x)/e^8 + (10
0*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 1
07*d^2*e^6 - 33*d*e^7 + 18*e^8)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.18

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{5250\,e^7x^8 - 6000\,de^6x^7 - 2700\,e^7x^7 + 7000\,d^2e^5x^6 + 3150\,de^6x^6 + 7770\,e^7x^6 - 8400\,d^3e^4x^5 - 3780\,d^2e^5x}{}$$
$$+ \frac{(100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8)\log(|ex+d|)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x, algorithm="gi
ac")
```

output

```
1/420*(5250*e^7*x^8 - 6000*d*e^6*x^7 - 2700*e^7*x^7 + 7000*d^2*e^5*x^6 + 3
150*d*e^6*x^6 + 7770*e^7*x^6 - 8400*d^3*e^4*x^5 - 3780*d^2*e^5*x^5 - 9324*
d*e^6*x^5 - 3108*e^7*x^5 + 10500*d^4*e^3*x^4 + 4725*d^3*e^4*x^4 + 11655*d^
2*e^5*x^4 + 3885*d*e^6*x^4 + 15540*e^7*x^4 - 14000*d^5*e^2*x^3 - 6300*d^4*
e^3*x^3 - 15540*d^3*e^4*x^3 - 5180*d^2*e^5*x^3 - 20720*d*e^6*x^3 + 9100*e^
7*x^3 + 21000*d^6*e*x^2 + 9450*d^5*e^2*x^2 + 23310*d^4*e^3*x^2 + 7770*d^3*
e^4*x^2 + 31080*d^2*e^5*x^2 - 13650*d*e^6*x^2 + 22470*e^7*x^2 - 42000*d^7*
x - 18900*d^6*e*x - 46620*d^5*e^2*x - 15540*d^4*e^3*x - 62160*d^3*e^4*x +
27300*d^2*e^5*x - 44940*d*e^6*x + 13860*e^7*x)/e^8 + (100*d^8 + 45*d^7*e +
 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5 + 107*d^2*e^6 - 33*d*
e^7 + 18*e^8)*log(abs(e*x + d))/e^9
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.23

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$

$$= x\left(\frac{33}{e}-\frac{d\left(\frac{107}{e}-\frac{d\left(\frac{65}{e}-\frac{d\left(\frac{148}{e}+\frac{d\left(\frac{37}{e}+\frac{d\left(\frac{111}{e}+\frac{d\left(\frac{100\,d}{e^2}+\frac{45}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right)}{e}\right.$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x),x)
```

output
```
x*(33/e - (d*(107/e - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((1
00*d)/e^2 + 45/e))/e))/e))/e))/e))/e))/e) - x^7*((100*d)/(7*e^2) + 45/(7*e
)) + x^6*(37/(2*e) + (d*((100*d)/e^2 + 45/e))/(6*e)) - x^5*(37/(5*e) + (d*
(111/e + (d*((100*d)/e^2 + 45/e))/e))/(5*e)) + x^4*(37/e + (d*(37/e + (d*(
111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/(4*e)) + x^3*(65/(3*e) - (d*(148/
e + (d*(37/e + (d*(111/e + (d*((100*d)/e^2 + 45/e))/e))/e))/e))/(3*e)) + x
^2*(107/(2*e) - (d*(65/e - (d*(148/e + (d*(37/e + (d*(111/e + (d*((100*d)/
e^2 + 45/e))/e))/e))/e))/e))/(2*e)) + (25*x^8)/(2*e) + (log(d + e*x)*(45*d
^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2*e^6 - 65*d^3*e^5 + 148*d^4*e^
4 + 37*d^5*e^3 + 111*d^6*e^2))/e^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.31

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{d+ex}\,dx$$
$$= \frac{-15540d^4e^4x - 8400d^3e^5x^5 + 4725d^3e^5x^4 - 15540d^3e^5x^3 + 7770d^3e^5x^2 - 62160d^3e^5x + 7000d^2e^6x^6 - 3}{}$$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d),x)
```

output

```
(42000*log(d + e*x)*d**8 + 18900*log(d + e*x)*d**7*e + 46620*log(d + e*x)*
d**6*e**2 + 15540*log(d + e*x)*d**5*e**3 + 62160*log(d + e*x)*d**4*e**4 -
27300*log(d + e*x)*d**3*e**5 + 44940*log(d + e*x)*d**2*e**6 - 13860*log(d
+ e*x)*d*e**7 + 7560*log(d + e*x)*e**8 - 42000*d**7*e*x + 21000*d**6*e**2*
x**2 - 18900*d**6*e**2*x - 14000*d**5*e**3*x**3 + 9450*d**5*e**3*x**2 - 46
620*d**5*e**3*x + 10500*d**4*e**4*x**4 - 6300*d**4*e**4*x**3 + 23310*d**4*
e**4*x**2 - 15540*d**4*e**4*x - 8400*d**3*e**5*x**5 + 4725*d**3*e**5*x**4
- 15540*d**3*e**5*x**3 + 7770*d**3*e**5*x**2 - 62160*d**3*e**5*x + 7000*d*
*2*e**6*x**6 - 3780*d**2*e**6*x**5 + 11655*d**2*e**6*x**4 - 5180*d**2*e**6
*x**3 + 31080*d**2*e**6*x**2 + 27300*d**2*e**6*x - 6000*d*e**7*x**7 + 3150
*d*e**7*x**6 - 9324*d*e**7*x**5 + 3885*d*e**7*x**4 - 20720*d*e**7*x**3 - 1
3650*d*e**7*x**2 - 44940*d*e**7*x + 5250*e**8*x**8 - 2700*e**8*x**7 + 7770
*e**8*x**6 - 3108*e**8*x**5 + 15540*e**8*x**4 + 9100*e**8*x**3 + 22470*e**
8*x**2 + 13860*e**8*x)/(420*e**9)
```

3.136 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 353

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx \\
&= \frac{(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)\,x}{e^8} \\
&\quad - \frac{(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)\,x^2}{2e^7} \\
&\quad + \frac{(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)\,x^3}{3e^6} \\
&\quad - \frac{(400d^3+135d^2e+222de^2+37e^3)\,x^4}{4e^5} + \frac{3(100d^2+30de+37e^2)\,x^5}{5e^4} \\
&\quad - \frac{5(40d+9e)x^6}{6e^3} + \frac{100x^7}{7e^2} - \frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)} \\
&\quad - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
(700*d^6+270*d^5*e+555*d^4*e^2+148*d^3*e^3+444*d^2*e^4-130*d*e^5+107*e^6)*
x/e^8-1/2*(600*d^5+225*d^4*e+444*d^3*e^2+111*d^2*e^3+296*d*e^4-65*e^5)*x^2
/e^7+1/3*(500*d^4+180*d^3*e+333*d^2*e^2+74*d*e^3+148*e^4)*x^3/e^6-1/4*(400
*d^3+135*d^2*e+222*d*e^2+37*e^3)*x^4/e^5+3/5*(100*d^2+30*d*e+37*e^2)*x^5/e
^4-5/6*(40*d+9*e)*x^6/e^3+100/7*x^7/e^2-(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3
*e+3*d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4
*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.97

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{420e(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)\,x-210e^2(600d^5+225d^4e+44}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
2,x]
```

output
```
(420*e*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 13
0*d*e^5 + 107*e^6)*x - 210*e^2*(600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^
2*e^3 + 296*d*e^4 - 65*e^5)*x^2 + 140*e^3*(500*d^4 + 180*d^3*e + 333*d^2*e
^2 + 74*d*e^3 + 148*e^4)*x^3 - 105*e^4*(400*d^3 + 135*d^2*e + 222*d*e^2 +
37*e^3)*x^4 + 252*e^5*(100*d^2 + 30*d*e + 37*e^2)*x^5 - 350*e^6*(40*d + 9*
e)*x^6 + 6000*e^7*x^7 - (420*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e +
3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x) - 420*(800*d^7 + 315*d^6*e + 666*d^5
*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*Log[d
 + e*x])/(420*e^9)
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2(4x^4-5x^3+3x^2+x+2)}{(d+ex)^2}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{3x^4(100d^2+30de+37e^2)}{e^4}-\frac{x^3(400d^3+135d^2e+222de^2+37e^3)}{e^5}+\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d}{e^8(d+ex)^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^5(100d^2+30de+37e^2)}{5e^4}-\frac{x^4(400d^3+135d^2e+222de^2+37e^3)}{4e^5}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^9(d+ex)}+\frac{x^3(500d^4+180d^3e+333d^2e^2+74de^3+148e^4)}{3e^6}-\frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}-\frac{x^2(600d^5+225d^4e+444d^3e^2+111d^2e^3+296de^4-65e^5)}{2e^7}+\frac{x(700d^6+270d^5e+555d^4e^2+148d^3e^3+444d^2e^4-130de^5+107e^6)}{e^8}-\frac{5x^6(40d+9e)}{6e^3}+\frac{100x^7}{7e^2}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^2,x]
```

output
```
((700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^
5 + 107*e^6)*x)/e^8 - ((600*d^5 + 225*d^4*e + 444*d^3*e^2 + 111*d^2*e^3 +
296*d*e^4 - 65*e^5)*x^2)/(2*e^7) + ((500*d^4 + 180*d^3*e + 333*d^2*e^2 + 7
4*d*e^3 + 148*e^4)*x^3)/(3*e^6) - ((400*d^3 + 135*d^2*e + 222*d*e^2 + 37*e
^3)*x^4)/(4*e^5) + (3*(100*d^2 + 30*d*e + 37*e^2)*x^5)/(5*e^4) - (5*(40*d
+ 9*e)*x^6)/(6*e^3) + (100*x^7)/(7*e^2) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^
4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e^9*(d + e*x)) - ((5*d^2 - 2*d*
e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e
^5)*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.04

method	result
norman	$\frac{\frac{\left(800d^8+315d^7e+666d^6e^2+185d^5e^3+592d^4e^4-195d^3e^5+214d^2e^6-33d\,e^7+18e^8\right)x}{e^8d}+\frac{100x^8}{7e}-\frac{5(160d+63e)x^7}{42e^2}+\frac{\left(800d^2+315de+666e^2\right)x^6}{30e^3}}{}$
default	$\frac{700x\,d^6+\frac{100}{7}x^7e^6-\frac{135}{4}x^4d^2e^4-\frac{111}{2}x^4d\,e^5+60x^3d^3e^3+111x^3d^2e^4+\frac{74}{3}x^3d\,e^5-\frac{225}{2}x^2d^4e^2-222x^2d^3e^3-\frac{111}{2}x^2d^2e^4-148x^2d}{}$
risch	$-\frac{37x^4}{4e^2}+\frac{107x}{e^2}+\frac{100x^7}{7e^2}+\frac{111x^5}{5e^2}-\frac{15x^6}{2e^2}-\frac{666\ln(ex+d)d^5}{e^7}-\frac{185\ln(ex+d)d^4}{e^6}-\frac{592\ln(ex+d)d^3}{e^5}+\frac{195\ln(ex+}{e^4}$
parallelrisch	$-\frac{336000\ln(ex+d)x\,d^7e+132300\ln(ex+d)x\,d^6e^2+279720\ln(ex+d)x\,d^5e^3+77700\ln(ex+d)x\,d^4e^4+248640\ln(ex+d)x\,d^3e^5-}{}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x,method=_RETURNVERB
OSE)
```

output
```
((800*d^8+315*d^7*e+666*d^6*e^2+185*d^5*e^3+592*d^4*e^4-195*d^3*e^5+214*d^
2*e^6-33*d*e^7+18*e^8)/e^8/d*x+100/7*x^8/e-5/42*(160*d+63*e)/e^2*x^7+1/30*
(800*d^2+315*d*e+666*e^2)/e^3*x^6-1/20*(800*d^3+315*d^2*e+666*d*e^2+185*e^
3)/e^4*x^5+1/12*(800*d^4+315*d^3*e+666*d^2*e^2+185*d*e^3+592*e^4)/e^5*x^4-
1/6*(800*d^5+315*d^4*e+666*d^3*e^2+185*d^2*e^3+592*d*e^4-195*e^5)/e^6*x^3+
1/2*(800*d^6+315*d^5*e+666*d^4*e^2+185*d^3*e^3+592*d^2*e^4-195*d*e^5+214*e
^6)/e^7*x^2)/(e*x+d)-(800*d^7+315*d^6*e+666*d^5*e^2+185*d^4*e^3+592*d^3*e^
4-195*d^2*e^5+214*d*e^6-33*e^7)/e^9*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \frac{6000\,e^8x^8 - 42000\,d^8 - 18900\,d^7e - 46620\,d^6e^2 - 15540\,d^5e^3 - 62160\,d^4e^4 + 27300\,d^3e^5 - 44940\,d^2e^6 +}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
fricas")
```

output
```
1/420*(6000*e^8*x^8 - 42000*d^8 - 18900*d^7*e - 46620*d^6*e^2 - 15540*d^5*
e^3 - 62160*d^4*e^4 + 27300*d^3*e^5 - 44940*d^2*e^6 + 13860*d*e^7 - 7560*e
^8 - 50*(160*d*e^7 + 63*e^8)*x^7 + 14*(800*d^2*e^6 + 315*d*e^7 + 666*e^8)*
x^6 - 21*(800*d^3*e^5 + 315*d^2*e^6 + 666*d*e^7 + 185*e^8)*x^5 + 35*(800*d
^4*e^4 + 315*d^3*e^5 + 666*d^2*e^6 + 185*d*e^7 + 592*e^8)*x^4 - 70*(800*d^
5*e^3 + 315*d^4*e^4 + 666*d^3*e^5 + 185*d^2*e^6 + 592*d*e^7 - 195*e^8)*x^3
 + 210*(800*d^6*e^2 + 315*d^5*e^3 + 666*d^4*e^4 + 185*d^3*e^5 + 592*d^2*e^
6 - 195*d*e^7 + 214*e^8)*x^2 + 420*(700*d^7*e + 270*d^6*e^2 + 555*d^5*e^3
+ 148*d^4*e^4 + 444*d^3*e^5 - 130*d^2*e^6 + 107*d*e^7)*x - 420*(800*d^8 +
315*d^7*e + 666*d^6*e^2 + 185*d^5*e^3 + 592*d^4*e^4 - 195*d^3*e^5 + 214*d^
2*e^6 - 33*d*e^7 + (800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 +
592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)*log(e*x + d))/(e^10*x +
 d*e^9)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= x^6\left(-\frac{100d}{3e^3}-\frac{15}{2e^2}\right)+x^5\cdot\left(\frac{60d^2}{e^4}+\frac{18d}{e^3}+\frac{111}{5e^2}\right)$$
$$+x^4\left(-\frac{100d^3}{e^5}-\frac{135d^2}{4e^4}-\frac{111d}{2e^3}-\frac{37}{4e^2}\right)+x^3\cdot\left(\frac{500d^4}{3e^6}+\frac{60d^3}{e^5}+\frac{111d^2}{e^4}+\frac{74d}{3e^3}+\frac{148}{3e^2}\right)$$
$$+x^2\left(-\frac{300d^5}{e^7}-\frac{225d^4}{2e^6}-\frac{222d^3}{e^5}-\frac{111d^2}{2e^4}-\frac{148d}{e^3}+\frac{65}{2e^2}\right)$$
$$+x\left(\frac{700d^6}{e^8}+\frac{270d^5}{e^7}+\frac{555d^4}{e^6}+\frac{148d^3}{e^5}+\frac{444d^2}{e^4}-\frac{130d}{e^3}+\frac{107}{e^2}\right)$$
$$+\frac{-100d^8-45d^7e-111d^6e^2-37d^5e^3-148d^4e^4+65d^3e^5-107d^2e^6+33de^7-18e^8}{de^9+e^{10}x}$$
$$+\frac{100x^7}{7e^2}$$
$$-\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2,x)
```

output
```
x**6*(-100*d/(3*e**3) - 15/(2*e**2)) + x**5*(60*d**2/e**4 + 18*d/e**3 + 11
1/(5*e**2)) + x**4*(-100*d**3/e**5 - 135*d**2/(4*e**4) - 111*d/(2*e**3) -
37/(4*e**2)) + x**3*(500*d**4/(3*e**6) + 60*d**3/e**5 + 111*d**2/e**4 + 74
*d/(3*e**3) + 148/(3*e**2)) + x**2*(-300*d**5/e**7 - 225*d**4/(2*e**6) - 2
22*d**3/e**5 - 111*d**2/(2*e**4) - 148*d/e**3 + 65/(2*e**2)) + x*(700*d**6
/e**8 + 270*d**5/e**7 + 555*d**4/e**6 + 148*d**3/e**5 + 444*d**2/e**4 - 13
0*d/e**3 + 107/e**2) + (-100*d**8 - 45*d**7*e - 111*d**6*e**2 - 37*d**5*e*
*3 - 148*d**4*e**4 + 65*d**3*e**5 - 107*d**2*e**6 + 33*d*e**7 - 18*e**8)/(
d*e**9 + e**10*x) + 100*x**7/(7*e**2) - (5*d**2 - 2*d*e + 3*e**2)*(160*d**
5 + 127*d**4*e + 88*d**3*e**2 - 4*d**2*e**3 + 64*d*e**4 - 11*e**5)*log(d +
 e*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.05

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$
$$-\frac{100\,d^8+45\,d^7e+111\,d^6e^2+37\,d^5e^3+148\,d^4e^4-65\,d^3e^5+107\,d^2e^6-33\,de^7+18\,e^8}{e^{10}x+de^9}$$
$$+\frac{6000\,e^6x^7-350\,(40\,de^5+9\,e^6)x^6+252\,(100\,d^2e^4+30\,de^5+37\,e^6)x^5-105\,(400\,d^3e^3+135\,d^2e^4+22}{}$$
$$-\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
maxima")
```

output
```
-(100*d^8 + 45*d^7*e + 111*d^6*e^2 + 37*d^5*e^3 + 148*d^4*e^4 - 65*d^3*e^5
 + 107*d^2*e^6 - 33*d*e^7 + 18*e^8)/(e^10*x + d*e^9) + 1/420*(6000*e^6*x^7
 - 350*(40*d*e^5 + 9*e^6)*x^6 + 252*(100*d^2*e^4 + 30*d*e^5 + 37*e^6)*x^5
- 105*(400*d^3*e^3 + 135*d^2*e^4 + 222*d*e^5 + 37*e^6)*x^4 + 140*(500*d^4*
e^2 + 180*d^3*e^3 + 333*d^2*e^4 + 74*d*e^5 + 148*e^6)*x^3 - 210*(600*d^5*e
 + 225*d^4*e^2 + 444*d^3*e^3 + 111*d^2*e^4 + 296*d*e^5 - 65*e^6)*x^2 + 420
*(700*d^6 + 270*d^5*e + 555*d^4*e^2 + 148*d^3*e^3 + 444*d^2*e^4 - 130*d*e^
5 + 107*e^6)*x)/e^8 - (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 5
92*d^3*e^4 - 195*d^2*e^5 + 214*d*e^6 - 33*e^7)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.39

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx =$$
$$-\frac{(ex+d)^7\left(\frac{350\,(160\,de+9\,e^2)}{(ex+d)e}-\frac{84\,(2800\,d^2e^2+315\,de^3+111\,e^4)}{(ex+d)^2e^2}+\frac{105\,(5600\,d^3e^3+945\,d^2e^4+666\,de^5+37\,e^6)}{(ex+d)^3e^3}-\frac{140\,(7000\,d^4e^4+1}{}\right.}{}$$
$$+\frac{(800\,d^7+315\,d^6e+666\,d^5e^2+185\,d^4e^3+592\,d^3e^4-195\,d^2e^5+214\,de^6-33\,e^7)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^9}$$
$$-\frac{\frac{100\,d^8e^7}{ex+d}+\frac{45\,d^7e^8}{ex+d}+\frac{111\,d^6e^9}{ex+d}+\frac{37\,d^5e^{10}}{ex+d}+\frac{148\,d^4e^{11}}{ex+d}-\frac{65\,d^3e^{12}}{ex+d}+\frac{107\,d^2e^{13}}{ex+d}-\frac{33\,de^{14}}{ex+d}+\frac{18\,e^{15}}{ex+d}}{e^{16}}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x, algorithm="
giac")
```

output
```
-1/420*(e*x + d)^7*(350*(160*d*e + 9*e^2)/((e*x + d)*e) - 84*(2800*d^2*e^2
 + 315*d*e^3 + 111*e^4)/((e*x + d)^2*e^2) + 105*(5600*d^3*e^3 + 945*d^2*e^
4 + 666*d*e^5 + 37*e^6)/((e*x + d)^3*e^3) - 140*(7000*d^4*e^4 + 1575*d^3*e
^5 + 1665*d^2*e^6 + 185*d*e^7 + 148*e^8)/((e*x + d)^4*e^4) + 210*(5600*d^5
*e^5 + 1575*d^4*e^6 + 2220*d^3*e^7 + 370*d^2*e^8 + 592*d*e^9 - 65*e^10)/((
e*x + d)^5*e^5) - 420*(2800*d^6*e^6 + 945*d^5*e^7 + 1665*d^4*e^8 + 370*d^3
*e^9 + 888*d^2*e^10 - 195*d*e^11 + 107*e^12)/((e*x + d)^6*e^6) - 6000)/e^9
 + (800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^
2*e^5 + 214*d*e^6 - 33*e^7)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^9 - (
100*d^8*e^7/(e*x + d) + 45*d^7*e^8/(e*x + d) + 111*d^6*e^9/(e*x + d) + 37*
d^5*e^10/(e*x + d) + 148*d^4*e^11/(e*x + d) - 65*d^3*e^12/(e*x + d) + 107*
d^2*e^13/(e*x + d) - 33*d*e^14/(e*x + d) + 18*e^15/(e*x + d))/e^16
```

Mupad [B] (verification not implemented)

Time = 18.13 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.66

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^2,x)
```

output
```
x^2*(65/(2*e^2) - (d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e
^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2
))/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^
2))/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 +
 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(2*e^2)) + x^3*(148/(
3*e^2) + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3
+ 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/(3*e) - (d^2*(111/e^
2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(3*e^2)) - x^4*(37/(4
*e^2) + (d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/(2*
e) - (d^2*((200*d)/e^3 + 45/e^2))/(4*e^2)) + x^5*(111/(5*e^2) - (20*d^2)/e
^4 + (2*d*((200*d)/e^3 + 45/e^2))/(5*e)) - x^6*((100*d)/(3*e^3) + 15/(2*e^
2)) - x*((2*d*(65/e^2 - (2*d*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (10
0*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e
^2))/e^2))/e - (d^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2)
)/e))/e^2))/e + (d^2*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*
d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e^2))/e - 107
/e^2 + (d^2*(148/e^2 + (2*d*(37/e^2 + (2*d*(111/e^2 - (100*d^2)/e^4 + (2*d
*((200*d)/e^3 + 45/e^2))/e))/e - (d^2*((200*d)/e^3 + 45/e^2))/e^2))/e - (d
^2*(111/e^2 - (100*d^2)/e^4 + (2*d*((200*d)/e^3 + 45/e^2))/e))/e^2))/e^2)
+ (100*x^7)/(7*e^2) - (45*d^7*e - 33*d*e^7 + 100*d^8 + 18*e^8 + 107*d^2...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 607, normalized size of antiderivative = 1.72

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^2}\,dx$$
$$= \frac{-13986d^2e^7x^5+6475d^2e^7x^4-41440d^2e^7x^3-40950d^2e^7x^2+89880d^2e^7x+6000d\,e^8x^8-3150d\,e^8x^7+}{}$$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2,x)
```

output
```
( - 336000*log(d + e*x)*d**9 - 336000*log(d + e*x)*d**8*e*x - 132300*log(d
 + e*x)*d**8*e - 132300*log(d + e*x)*d**7*e**2*x - 279720*log(d + e*x)*d**
7*e**2 - 279720*log(d + e*x)*d**6*e**3*x - 77700*log(d + e*x)*d**6*e**3 -
77700*log(d + e*x)*d**5*e**4*x - 248640*log(d + e*x)*d**5*e**4 - 248640*lo
g(d + e*x)*d**4*e**5*x + 81900*log(d + e*x)*d**4*e**5 + 81900*log(d + e*x)
*d**3*e**6*x - 89880*log(d + e*x)*d**3*e**6 - 89880*log(d + e*x)*d**2*e**7
*x + 13860*log(d + e*x)*d**2*e**7 + 13860*log(d + e*x)*d*e**8*x + 336000*d
**8*e*x + 168000*d**7*e**2*x**2 + 132300*d**7*e**2*x - 56000*d**6*e**3*x**
3 + 66150*d**6*e**3*x**2 + 279720*d**6*e**3*x + 28000*d**5*e**4*x**4 - 220
50*d**5*e**4*x**3 + 139860*d**5*e**4*x**2 + 77700*d**5*e**4*x - 16800*d**4
*e**5*x**5 + 11025*d**4*e**5*x**4 - 46620*d**4*e**5*x**3 + 38850*d**4*e**5
*x**2 + 248640*d**4*e**5*x + 11200*d**3*e**6*x**6 - 6615*d**3*e**6*x**5 +
23310*d**3*e**6*x**4 - 12950*d**3*e**6*x**3 + 124320*d**3*e**6*x**2 - 8190
0*d**3*e**6*x - 8000*d**2*e**7*x**7 + 4410*d**2*e**7*x**6 - 13986*d**2*e**
7*x**5 + 6475*d**2*e**7*x**4 - 41440*d**2*e**7*x**3 - 40950*d**2*e**7*x**2
 + 89880*d**2*e**7*x + 6000*d*e**8*x**8 - 3150*d*e**8*x**7 + 9324*d*e**8*x
**6 - 3885*d*e**8*x**5 + 20720*d*e**8*x**4 + 13650*d*e**8*x**3 + 44940*d*e
**8*x**2 - 13860*d*e**8*x + 7560*e**9*x)/(420*d*e**9*(d + e*x))
```

3.137 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 354

$$\begin{aligned}
\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx
&= -\frac{(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)\,x}{e^8} \\
&\quad+\frac{(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)\,x^2}{2e^7} \\
&\quad-\frac{(1000d^3+270d^2e+333de^2+37e^3)\,x^3}{3e^6}+\frac{3(200d^2+45de+37e^2)\,x^4}{4e^5} \\
&\quad-\frac{3(20d+3e)x^5}{e^4}+\frac{50x^6}{3e^3}-\frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2} \\
&\quad+\frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)} \\
&\quad+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
-(2100*d^5+675*d^4*e+1110*d^3*e^2+222*d^2*e^3+444*d*e^4-65*e^5)*x/e^8+1/2*
(1500*d^4+450*d^3*e+666*d^2*e^2+111*d*e^3+148*e^4)*x^2/e^7-1/3*(1000*d^3+2
70*d^2*e+333*d*e^2+37*e^3)*x^3/e^6+3/4*(200*d^2+45*d*e+37*e^2)*x^4/e^5-3*(
20*d+3*e)*x^5/e^4+50/3*x^6/e^3-1/2*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*
d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^2+(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+
88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)+(2800*d^6+945*d^5*e+1665
*d^4*e^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{9000d^8-390d^7e(-9+40x)-18d^6e^2(-407+240x+2300x^2)-2d^5e^3(-999+2664x+6750x^2+5600}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
3,x]
```

output
```
(9000*d^8 - 390*d^7*e*(-9 + 40*x) - 18*d^6*e^2*(-407 + 240*x + 2300*x^2) -
 2*d^5*e^3*(-999 + 2664*x + 6750*x^2 + 5600*x^3) + 4*d^4*e^4*(1554 - 111*x
 - 5661*x^2 - 945*x^3 + 700*x^4) - d^3*e^5*(1950 - 1776*x + 4662*x^2 + 666
0*x^3 - 945*x^4 + 1120*x^5) + d^2*e^6*(1926 - 1560*x - 9768*x^2 - 1480*x^3
 + 1665*x^4 - 378*x^5 + 560*x^6) + d*e^7*(-198 + 2568*x + 1560*x^2 - 3552*
x^3 + 370*x^4 - 666*x^5 + 189*x^6 - 320*x^7) + e^8*(-108 - 396*x + 780*x^3
 + 888*x^4 - 148*x^5 + 333*x^6 - 108*x^7 + 200*x^8) + 12*(2800*d^6 + 945*d
^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*(d
+ e*x)^2*Log[d + e*x])/(12*e^9*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2 (4x^4-5x^3+3x^2+x+2)}{(d+ex)^3} \, dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{3x^3(200d^2+45de+37e^2)}{e^5} - \frac{x^2(1000d^3+270d^2e+333de^2+37e^3)}{e^6} + \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3\ldots}{e^8(d+ex)^3}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{3x^4(200d^2+45de+37e^2)}{4e^5} - \frac{x^3(1000d^3+270d^2e+333de^2+37e^3)}{3e^6} - \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{2e^9(d+ex)^2} + \frac{x^2(1500d^4+450d^3e+666d^2e^2+111de^3+148e^4)}{2e^7} + \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{e^9(d+ex)} - \frac{x(2100d^5+675d^4e+1110d^3e^2+222d^2e^3+444de^4-65e^5)}{e^8} + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9} - \frac{3x^5(20d+3e)}{e^4} + \frac{50x^6}{3e^3}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^3,x]
```

output
```
-(((2100*d^5 + 675*d^4*e + 1110*d^3*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5
)*x)/e^8) + ((1500*d^4 + 450*d^3*e + 666*d^2*e^2 + 111*d*e^3 + 148*e^4)*x^
2)/(2*e^7) - ((1000*d^3 + 270*d^2*e + 333*d*e^2 + 37*e^3)*x^3)/(3*e^6) + (
3*(200*d^2 + 45*d*e + 37*e^2)*x^4)/(4*e^5) - (3*(20*d + 3*e)*x^5)/e^4 + (5
0*x^6)/(3*e^3) - ((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 -
 d*e^3 + 2*e^4))/(2*e^9*(d + e*x)^2) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 +
 127*d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(e^9*(d + e*x))
+ ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*
d*e^5 + 107*e^6)*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.01

method	result
norman	$\frac{\frac{\left(5600d^7+1890d^6e+3330d^5e^2+740d^4e^3+1776d^3e^4-390d^2e^5+214d\,e^6-33e^7\right)x}{e^8}+\frac{50x^8}{3e}+\frac{8400d^8+2835d^7e+4995d^6e^2+1110d^5e^3+2664d^4e}{2e^9}$
default	$-\frac{-\frac{50}{3}x^6e^5+60x^5e^4d+9x^5e^5-150d^2e^3x^4-\frac{135}{4}x^4e^4d-\frac{111}{4}x^4e^5+\frac{1000}{3}d^3e^2x^3+90d^2e^3x^3+111d\,e^4x^3+\frac{37}{3}e^5x^3-750d^4e\,x^2-}{e^8}$
risch	$-\frac{37x^3}{3e^3}-\frac{1110d^3x}{e^6}-\frac{222d^2x}{e^5}-\frac{444dx}{e^4}+\frac{1665\ln(ex+d)d^4}{e^7}+\frac{370\ln(ex+d)d^3}{e^6}+\frac{888\ln(ex+d)d^2}{e^5}-\frac{195\ln(ex+d)d}{e^4}$
parallelrisch	$\frac{33600\ln(ex+d)x^2d^6e^2+11340\ln(ex+d)x^2d^5e^3+19980\ln(ex+d)x^2d^4e^4+4440\ln(ex+d)x^2d^3e^5+10656\ln(ex+d)x^2d^2e^6-23}{}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x,method=_RETURNVERB
OSE)
```

output
```
((5600*d^7+1890*d^6*e+3330*d^5*e^2+740*d^4*e^3+1776*d^3*e^4-390*d^2*e^5+21
4*d*e^6-33*e^7)/e^8*x+50/3*x^8/e+1/2*(8400*d^8+2835*d^7*e+4995*d^6*e^2+111
0*d^5*e^3+2664*d^4*e^4-585*d^3*e^5+321*d^2*e^6-33*d*e^7-18*e^8)/e^9-1/3*(8
0*d+27*e)/e^2*x^7+1/12*(560*d^2+189*d*e+333*e^2)/e^3*x^6-1/6*(560*d^3+189*
d^2*e+333*d*e^2+74*e^3)/e^4*x^5+1/12*(2800*d^4+945*d^3*e+1665*d^2*e^2+370*
d*e^3+888*e^4)/e^5*x^4-1/3*(2800*d^5+945*d^4*e+1665*d^3*e^2+370*d^2*e^3+88
8*d*e^4-195*e^5)/e^6*x^3)/(e*x+d)^2+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*d
^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*ln(e*x+d)/e^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.54

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{200\,e^8x^8+9000\,d^8+3510\,d^7e+7326\,d^6e^2+1998\,d^5e^3+6216\,d^4e^4-1950\,d^3e^5+1926\,d^2e^6-198\,de^7-}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
fricas")
```

output
```
1/12*(200*e^8*x^8 + 9000*d^8 + 3510*d^7*e + 7326*d^6*e^2 + 1998*d^5*e^3 +
6216*d^4*e^4 - 1950*d^3*e^5 + 1926*d^2*e^6 - 198*d*e^7 - 108*e^8 - 4*(80*d
*e^7 + 27*e^8)*x^7 + (560*d^2*e^6 + 189*d*e^7 + 333*e^8)*x^6 - 2*(560*d^3*
e^5 + 189*d^2*e^6 + 333*d*e^7 + 74*e^8)*x^5 + (2800*d^4*e^4 + 945*d^3*e^5
+ 1665*d^2*e^6 + 370*d*e^7 + 888*e^8)*x^4 - 4*(2800*d^5*e^3 + 945*d^4*e^4
+ 1665*d^3*e^5 + 370*d^2*e^6 + 888*d*e^7 - 195*e^8)*x^3 - 6*(6900*d^6*e^2
+ 2250*d^5*e^3 + 3774*d^4*e^4 + 777*d^3*e^5 + 1628*d^2*e^6 - 260*d*e^7)*x^
2 - 12*(1300*d^7*e + 360*d^6*e^2 + 444*d^5*e^3 + 37*d^4*e^4 - 148*d^3*e^5
+ 130*d^2*e^6 - 214*d*e^7 + 33*e^8)*x + 12*(2800*d^8 + 945*d^7*e + 1665*d^
6*e^2 + 370*d^5*e^3 + 888*d^4*e^4 - 195*d^3*e^5 + 107*d^2*e^6 + (2800*d^6*
e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 +
 107*e^8)*x^2 + 2*(2800*d^7*e + 945*d^6*e^2 + 1665*d^5*e^3 + 370*d^4*e^4 +
 888*d^3*e^5 - 195*d^2*e^6 + 107*d*e^7)*x)*log(e*x + d))/(e^11*x^2 + 2*d*e
^10*x + d^2*e^9)
```

Sympy [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= x^5\left(-\frac{60d}{e^4}-\frac{9}{e^3}\right)+x^4\cdot\left(\frac{150d^2}{e^5}+\frac{135d}{4e^4}+\frac{111}{4e^3}\right)$$
$$+x^3\left(-\frac{1000d^3}{3e^6}-\frac{90d^2}{e^5}-\frac{111d}{e^4}-\frac{37}{3e^3}\right)+x^2\cdot\left(\frac{750d^4}{e^7}+\frac{225d^3}{e^6}+\frac{333d^2}{e^5}+\frac{111d}{2e^4}+\frac{74}{e^3}\right)$$
$$+x\left(-\frac{2100d^5}{e^8}-\frac{675d^4}{e^7}-\frac{1110d^3}{e^6}-\frac{222d^2}{e^5}-\frac{444d}{e^4}+\frac{65}{e^3}\right)$$
$$+\frac{1500d^8+585d^7e+1221d^6e^2+333d^5e^3+1036d^4e^4-325d^3e^5+321d^2e^6-33de^7-18e^8+x(1600d^7e}{2d^2e^9+4de^{10}x+2e^{11}x^2}$$
$$+\frac{50x^6}{3e^3}$$
$$+\frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3,x)
```

output
```
x**5*(-60*d/e**4 - 9/e**3) + x**4*(150*d**2/e**5 + 135*d/(4*e**4) + 111/(4
*e**3)) + x**3*(-1000*d**3/(3*e**6) - 90*d**2/e**5 - 111*d/e**4 - 37/(3*e*
*3)) + x**2*(750*d**4/e**7 + 225*d**3/e**6 + 333*d**2/e**5 + 111*d/(2*e**4
) + 74/e**3) + x*(-2100*d**5/e**8 - 675*d**4/e**7 - 1110*d**3/e**6 - 222*d
**2/e**5 - 444*d/e**4 + 65/e**3) + (1500*d**8 + 585*d**7*e + 1221*d**6*e**
2 + 333*d**5*e**3 + 1036*d**4*e**4 - 325*d**3*e**5 + 321*d**2*e**6 - 33*d*
e**7 - 18*e**8 + x*(1600*d**7*e + 630*d**6*e**2 + 1332*d**5*e**3 + 370*d**
4*e**4 + 1184*d**3*e**5 - 390*d**2*e**6 + 428*d*e**7 - 66*e**8))/(2*d**2*e
**9 + 4*d*e**10*x + 2*e**11*x**2) + 50*x**6/(3*e**3) + (2800*d**6 + 945*d*
*5*e + 1665*d**4*e**2 + 370*d**3*e**3 + 888*d**2*e**4 - 195*d*e**5 + 107*e
**6)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.07

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$$
$$= \frac{1500 d^8 + 585 d^7 e + 1221 d^6 e^2 + 333 d^5 e^3 + 1036 d^4 e^4 - 325 d^3 e^5 + 321 d^2 e^6 - 33 de^7 - 18 e^8 + 2 (800 d^7}{2 (e^{11} x^2 + 2 de^{10} x + d^2 e^9)}$$
$$+ \frac{200 e^5 x^6 - 36 (20 de^4 + 3 e^5) x^5 + 9 (200 d^2 e^3 + 45 de^4 + 37 e^5) x^4 - 4 (1000 d^3 e^2 + 270 d^2 e^3 + 333 de^4}{}$$
$$+ \frac{(2800 d^6 + 945 d^5 e + 1665 d^4 e^2 + 370 d^3 e^3 + 888 d^2 e^4 - 195 de^5 + 107 e^6) \log(ex + d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
maxima")
```

output
```
1/2*(1500*d^8 + 585*d^7*e + 1221*d^6*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 32
5*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 - 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 +
 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^
8)*x)/(e^11*x^2 + 2*d*e^10*x + d^2*e^9) + 1/12*(200*e^5*x^6 - 36*(20*d*e^4
 + 3*e^5)*x^5 + 9*(200*d^2*e^3 + 45*d*e^4 + 37*e^5)*x^4 - 4*(1000*d^3*e^2
+ 270*d^2*e^3 + 333*d*e^4 + 37*e^5)*x^3 + 6*(1500*d^4*e + 450*d^3*e^2 + 66
6*d^2*e^3 + 111*d*e^4 + 148*e^5)*x^2 - 12*(2100*d^5 + 675*d^4*e + 1110*d^3
*e^2 + 222*d^2*e^3 + 444*d*e^4 - 65*e^5)*x)/e^8 + (2800*d^6 + 945*d^5*e +
1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6)*log(e*x +
d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} dx$$
$$= \frac{(2800 d^6 + 945 d^5 e + 1665 d^4 e^2 + 370 d^3 e^3 + 888 d^2 e^4 - 195 de^5 + 107 e^6) \log(|ex + d|)}{e^9}$$
$$+ \frac{1500 d^8 + 585 d^7 e + 1221 d^6 e^2 + 333 d^5 e^3 + 1036 d^4 e^4 - 325 d^3 e^5 + 321 d^2 e^6 - 33 de^7 - 18 e^8 + 2 (800}{2 (ex + d)^2 e^9}$$
$$+ \frac{200 e^{15} x^6 - 720 de^{14} x^5 - 108 e^{15} x^5 + 1800 d^2 e^{13} x^4 + 405 de^{14} x^4 + 333 e^{15} x^4 - 4000 d^3 e^{12} x^3 - 1080 d^2}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x, algorithm="
giac")
```

output
```
(2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e
^5 + 107*e^6)*log(abs(e*x + d))/e^9 + 1/2*(1500*d^8 + 585*d^7*e + 1221*d^6
*e^2 + 333*d^5*e^3 + 1036*d^4*e^4 - 325*d^3*e^5 + 321*d^2*e^6 - 33*d*e^7 -
 18*e^8 + 2*(800*d^7*e + 315*d^6*e^2 + 666*d^5*e^3 + 185*d^4*e^4 + 592*d^3
*e^5 - 195*d^2*e^6 + 214*d*e^7 - 33*e^8)*x)/((e*x + d)^2*e^9) + 1/12*(200*
e^15*x^6 - 720*d*e^14*x^5 - 108*e^15*x^5 + 1800*d^2*e^13*x^4 + 405*d*e^14*
x^4 + 333*e^15*x^4 - 4000*d^3*e^12*x^3 - 1080*d^2*e^13*x^3 - 1332*d*e^14*x
^3 - 148*e^15*x^3 + 9000*d^4*e^11*x^2 + 2700*d^3*e^12*x^2 + 3996*d^2*e^13*
x^2 + 666*d*e^14*x^2 + 888*e^15*x^2 - 25200*d^5*e^10*x - 8100*d^4*e^11*x -
 13320*d^3*e^12*x - 2664*d^2*e^13*x - 5328*d*e^14*x + 780*e^15*x)/e^18
```

Mupad [B] (verification not implemented)

Time = 18.71 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.18

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^3} \, dx = \text{Too large to display}$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^3,x)
```

output
```
x^4*(111/(4*e^3) - (75*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/(4*e)) - x^
3*(37/(3*e^3) + (100*d^3)/(3*e^6) + (d*(111/e^3 - (300*d^2)/e^5 + (3*d*((3
00*d)/e^4 + 45/e^3))/e))/e - (d^2*((300*d)/e^4 + 45/e^3))/e^2) - x^5*((60*
d)/e^4 + 9/e^3) + x*(65/e^3 - (3*d*(148/e^3 + (3*d*(37/e^3 + (100*d^3)/e^6
 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e - (3
*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e - (3*d^2*(111/e^3 - (300*d^2)/e^5 + (
3*d*((300*d)/e^4 + 45/e^3))/e))/e^2 + (d^3*((300*d)/e^4 + 45/e^3))/e^3))/e
 + (3*d^2*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*(
(300*d)/e^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/e^2 -
(d^3*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/e^3) + (x
*(214*d*e^6 + 315*d^6*e + 800*d^7 - 33*e^7 - 195*d^2*e^5 + 592*d^3*e^4 + 1
85*d^4*e^3 + 666*d^5*e^2) + (585*d^7*e - 33*d*e^7 + 1500*d^8 - 18*e^8 + 32
1*d^2*e^6 - 325*d^3*e^5 + 1036*d^4*e^4 + 333*d^5*e^3 + 1221*d^6*e^2)/(2*e)
)/(d^2*e^8 + e^10*x^2 + 2*d*e^9*x) + (50*x^6)/(3*e^3) + x^2*(74/e^3 + (3*d
*(37/e^3 + (100*d^3)/e^6 + (3*d*(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e
^4 + 45/e^3))/e))/e - (3*d^2*((300*d)/e^4 + 45/e^3))/e^2))/(2*e) - (3*d^2*
(111/e^3 - (300*d^2)/e^5 + (3*d*((300*d)/e^4 + 45/e^3))/e))/(2*e^2) + (d^3
*((300*d)/e^4 + 45/e^3))/(2*e^3)) + (log(d + e*x)*(945*d^5*e - 195*d*e^5 +
 2800*d^6 + 107*e^6 + 888*d^2*e^4 + 370*d^3*e^3 + 1665*d^4*e^2))/e^9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.97

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^3}\,dx$$
$$= \frac{-666d^2e^7x^5+370d^2e^7x^4-3552d^2e^7x^3+2340d^2e^7x^2+200d\,e^8x^8-108d\,e^8x^7+333d\,e^8x^6-148d\,e^8x^5}{}$$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3,x)
```

output
```
(33600*log(d + e*x)*d**9 + 67200*log(d + e*x)*d**8*e*x + 11340*log(d + e*x
)*d**8*e + 33600*log(d + e*x)*d**7*e**2*x**2 + 22680*log(d + e*x)*d**7*e**
2*x + 19980*log(d + e*x)*d**7*e**2 + 11340*log(d + e*x)*d**6*e**3*x**2 + 3
9960*log(d + e*x)*d**6*e**3*x + 4440*log(d + e*x)*d**6*e**3 + 19980*log(d
+ e*x)*d**5*e**4*x**2 + 8880*log(d + e*x)*d**5*e**4*x + 10656*log(d + e*x)
*d**5*e**4 + 4440*log(d + e*x)*d**4*e**5*x**2 + 21312*log(d + e*x)*d**4*e*
*5*x - 2340*log(d + e*x)*d**4*e**5 + 10656*log(d + e*x)*d**3*e**6*x**2 - 4
680*log(d + e*x)*d**3*e**6*x + 1284*log(d + e*x)*d**3*e**6 - 2340*log(d +
e*x)*d**2*e**7*x**2 + 2568*log(d + e*x)*d**2*e**7*x + 1284*log(d + e*x)*d*
e**8*x**2 + 16800*d**9 + 5670*d**8*e - 33600*d**7*e**2*x**2 + 9990*d**7*e*
*2 - 11200*d**6*e**3*x**3 - 11340*d**6*e**3*x**2 + 2220*d**6*e**3 + 2800*d
**5*e**4*x**4 - 3780*d**5*e**4*x**3 - 19980*d**5*e**4*x**2 + 5328*d**5*e**
4 - 1120*d**4*e**5*x**5 + 945*d**4*e**5*x**4 - 6660*d**4*e**5*x**3 - 4440*
d**4*e**5*x**2 - 1170*d**4*e**5 + 560*d**3*e**6*x**6 - 378*d**3*e**6*x**5
+ 1665*d**3*e**6*x**4 - 1480*d**3*e**6*x**3 - 10656*d**3*e**6*x**2 + 642*d
**3*e**6 - 320*d**2*e**7*x**7 + 189*d**2*e**7*x**6 - 666*d**2*e**7*x**5 +
370*d**2*e**7*x**4 - 3552*d**2*e**7*x**3 + 2340*d**2*e**7*x**2 + 200*d*e**
8*x**8 - 108*d*e**8*x**7 + 333*d*e**8*x**6 - 148*d*e**8*x**5 + 888*d*e**8*
x**4 + 780*d*e**8*x**3 - 1284*d*e**8*x**2 - 108*d*e**8 + 198*e**9*x**2)/(1
2*d*e**9*(d**2 + 2*d*e*x + e**2*x**2))
```

3.138 $\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 360

$$\begin{aligned}
&\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx \\
&= \frac{2(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)\,x}{e^8} \\
&- \frac{(2000d^3+450d^2e+444de^2+37e^3)\,x^2}{2e^7}+\frac{(1000d^2+180de+111e^2)\,x^3}{3e^6} \\
&- \frac{5(80d+9e)x^4}{4e^5}+\frac{20x^5}{e^4}-\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3} \\
&+ \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2} \\
&- \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)} \\
&- \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}
\end{aligned}$$

output
```
2*(1750*d^4+450*d^3*e+555*d^2*e^2+74*d*e^3+74*e^4)*x/e^8-1/2*(2000*d^3+450
*d^2*e+444*d*e^2+37*e^3)*x^2/e^7+1/3*(1000*d^2+180*d*e+111*e^2)*x^3/e^6-5/
4*(80*d+9*e)*x^4/e^5+20*x^5/e^4-1/3*(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3
*d^2*e^2-d*e^3+2*e^4)/e^9/(e*x+d)^3+1/2*(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d
^4*e+88*d^3*e^2-4*d^2*e^3+64*d*e^4-11*e^5)/e^9/(e*x+d)^2-(2800*d^6+945*d^5
*e+1665*d^4*e^2+370*d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)/e^9/(e*x+d)-(56
00*d^5+1575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.96

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} dx$$

$$= \frac{24e(1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)x-6e^2(2000d^3+450d^2e+444de^2+37e^3)x^2+4e^3(10}{}$$

input
```
Integrate[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^
4,x]
```

output
```
(24*e*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x - 6*e^2*(
2000*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2 + 4*e^3*(1000*d^2 + 180*d*e
 + 111*e^2)*x^3 - 15*e^4*(80*d + 9*e)*x^4 + 240*e^5*x^5 - (4*(5*d^2 - 2*d*
e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(d + e*x)^3 +
(6*(800*d^7 + 315*d^6*e + 666*d^5*e^2 + 185*d^4*e^3 + 592*d^3*e^4 - 195*d^
2*e^5 + 214*d*e^6 - 33*e^7))/(d + e*x)^2 - (12*(2800*d^6 + 945*d^5*e + 166
5*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^6))/(d + e*x) -
12*(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^
5)*Log[d + e*x])/(12*e^9)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(5x^2+2x+3)^2 (4x^4-5x^3+3x^2+x+2)}{(d+ex)^4} \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{x^2 (1000d^2+180de+111e^2)}{e^6} - \frac{x (2000d^3+450d^2e+444de^2+37e^3)}{e^7} + \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+\ldots}{e^8(d+ex)^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3 (1000d^2+180de+111e^2)}{3e^6} - \frac{x^2 (2000d^3+450d^2e+444de^2+37e^3)}{2e^7} - \frac{(5d^2-2de+3e^2)^2 (4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{3e^9(d+ex)^3} + \frac{2x (1750d^4+450d^3e+555d^2e^2+74de^3+74e^4)}{e^8} + \frac{(5d^2-2de+3e^2) (160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)}{2e^9(d+ex)^2} - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5) \log(d+ex)}{e^9} - \frac{2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6}{e^9(d+ex)} - \frac{5x^4(80d+9e)}{4e^5} + \frac{20x^5}{e^4}$$

input
```
Int[((3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(d + e*x)^4,x]
```

output
```
(2*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3 + 74*e^4)*x)/e^8 - ((200
0*d^3 + 450*d^2*e + 444*d*e^2 + 37*e^3)*x^2)/(2*e^7) + ((1000*d^2 + 180*d*
e + 111*e^2)*x^3)/(3*e^6) - (5*(80*d + 9*e)*x^4)/(4*e^5) + (20*x^5)/e^4 -
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/
(3*e^9*(d + e*x)^3) + ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*e + 88*d
^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5))/(2*e^9*(d + e*x)^2) - (2800*d^6 +
 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 + 107*e^
6)/(e^9*(d + e*x)) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3
+ 592*d*e^4 - 65*e^5)*Log[d + e*x])/e^9
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.00

method	result
norman	$\frac{\frac{20x^8}{e}-\frac{61600d^8+17325d^7e+24420d^6e^2+4070d^5e^3+6512d^4e^4-715d^3e^5+214d^2e^6+33d\,e^7+36e^8}{6e^9}-\frac{5(32d+9e)x^7}{4e^2}+\frac{\left(1120d^2+315de+444e^2\right.}{12e^3}}{}$
default	$\frac{20x^5e^4-100e^3x^4d-\frac{45}{4}x^4e^4+\frac{1000}{3}d^2e^2x^3+60e^3x^3d+37e^4x^3-1000d^3e\,x^2-225d^2e^2x^2-222d\,e^3x^2-\frac{37}{2}e^4x^2+3500x\,d^4+900d}{e^8}$
risch	$\frac{20x^5}{e^4}-\frac{100x^4d}{e^5}-\frac{45x^4}{4e^4}+\frac{1000d^2x^3}{3e^6}+\frac{60x^3d}{e^5}+\frac{37x^3}{e^4}-\frac{1000d^3x^2}{e^7}-\frac{225d^2x^2}{e^6}-\frac{222d\,x^2}{e^5}-\frac{37x^2}{2e^4}+\frac{3500x\,d^4}{e^8}$
parallelrisch	$-\frac{67200\ln(ex+d)x^3d^5e^3+18900\ln(ex+d)x^3d^4e^4+26640\ln(ex+d)x^3d^3e^5+4440\ln(ex+d)x^3d^2e^6+7104\ln(ex+d)x^3d\,e^7+20}{}$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x,method=_RETURNVERB
OSE)
```

output
```
(20*x^8/e-1/6*(61600*d^8+17325*d^7*e+24420*d^6*e^2+4070*d^5*e^3+6512*d^4*e
^4-715*d^3*e^5+214*d^2*e^6+33*d*e^7+36*e^8)/e^9-5/4*(32*d+9*e)/e^2*x^7+1/1
2*(1120*d^2+315*d*e+444*e^2)/e^3*x^6-1/4*(1120*d^3+315*d^2*e+444*d*e^2+74*
e^3)/e^4*x^5+1/4*(5600*d^4+1575*d^3*e+2220*d^2*e^2+370*d*e^3+592*e^4)/e^5*
x^4-(16800*d^6+4725*d^5*e+6660*d^4*e^2+1110*d^3*e^3+1776*d^2*e^4-195*d*e^5
+107*e^6)/e^7*x^2-1/2*(50400*d^7+14175*d^6*e+19980*d^5*e^2+3330*d^4*e^3+53
28*d^3*e^4-585*d^2*e^5+214*d*e^6+33*e^7)/e^8*x)/(e*x+d)^3-(5600*d^5+1575*d
^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*ln(e*x+d)/e^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.63

$$\int \frac{(3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x^4)}{(d+ex)^4} \, dx$$
$$= \frac{240 e^8 x^8 - 29200 d^8 - 9630 d^7 e - 16428 d^6 e^2 - 3478 d^5 e^3 - 7696 d^4 e^4 + 1430 d^3 e^5 - 428 d^2 e^6 - 66 d e^7 -}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
fricas")
```

output
```
1/12*(240*e^8*x^8 - 29200*d^8 - 9630*d^7*e - 16428*d^6*e^2 - 3478*d^5*e^3
- 7696*d^4*e^4 + 1430*d^3*e^5 - 428*d^2*e^6 - 66*d*e^7 - 72*e^8 - 15*(32*d
*e^7 + 9*e^8)*x^7 + (1120*d^2*e^6 + 315*d*e^7 + 444*e^8)*x^6 - 3*(1120*d^3
*e^5 + 315*d^2*e^6 + 444*d*e^7 + 74*e^8)*x^5 + 3*(5600*d^4*e^4 + 1575*d^3*
e^5 + 2220*d^2*e^6 + 370*d*e^7 + 592*e^8)*x^4 + 2*(47000*d^5*e^3 + 12510*d
^4*e^4 + 16206*d^3*e^5 + 2331*d^2*e^6 + 2664*d*e^7)*x^3 + 6*(13400*d^6*e^2
 + 3060*d^5*e^3 + 2886*d^4*e^4 + 111*d^3*e^5 - 888*d^2*e^6 + 390*d*e^7 - 2
14*e^8)*x^2 - 6*(3400*d^7*e + 1665*d^6*e^2 + 3774*d^5*e^3 + 999*d^4*e^4 +
2664*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x - 12*(5600*d^8 + 1575*d
^7*e + 2220*d^6*e^2 + 370*d^5*e^3 + 592*d^4*e^4 - 65*d^3*e^5 + (5600*d^5*e
^3 + 1575*d^4*e^4 + 2220*d^3*e^5 + 370*d^2*e^6 + 592*d*e^7 - 65*e^8)*x^3 +
 3*(5600*d^6*e^2 + 1575*d^5*e^3 + 2220*d^4*e^4 + 370*d^3*e^5 + 592*d^2*e^6
 - 65*d*e^7)*x^2 + 3*(5600*d^7*e + 1575*d^6*e^2 + 2220*d^5*e^3 + 370*d^4*e
^4 + 592*d^3*e^5 - 65*d^2*e^6)*x)*log(e*x + d))/(e^12*x^3 + 3*d*e^11*x^2 +
 3*d^2*e^10*x + d^3*e^9)
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.11

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$
$$= x^4\left(-\frac{100d}{e^5}-\frac{45}{4e^4}\right)+x^3\cdot\left(\frac{1000d^2}{3e^6}+\frac{60d}{e^5}+\frac{37}{e^4}\right)$$
$$+x^2\left(-\frac{1000d^3}{e^7}-\frac{225d^2}{e^6}-\frac{222d}{e^5}-\frac{37}{2e^4}\right)+x\left(\frac{3500d^4}{e^8}+\frac{900d^3}{e^7}+\frac{1110d^2}{e^6}+\frac{148d}{e^5}+\frac{148}{e^4}\right)$$
$$+\frac{-14600d^8-4815d^7e-8214d^6e^2-1739d^5e^3-3848d^4e^4+715d^3e^5-214d^2e^6-33de^7-36e^8+x^2(-$$
$$+\frac{20x^5}{e^4}-\frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\log(d+ex)}{e^9}$$

input
```
integrate((5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**4,x)
```

output
```
x**4*(-100*d/e**5 - 45/(4*e**4)) + x**3*(1000*d**2/(3*e**6) + 60*d/e**5 +
37/e**4) + x**2*(-1000*d**3/e**7 - 225*d**2/e**6 - 222*d/e**5 - 37/(2*e**4
)) + x*(3500*d**4/e**8 + 900*d**3/e**7 + 1110*d**2/e**6 + 148*d/e**5 + 148
/e**4) + (-14600*d**8 - 4815*d**7*e - 8214*d**6*e**2 - 1739*d**5*e**3 - 38
48*d**4*e**4 + 715*d**3*e**5 - 214*d**2*e**6 - 33*d*e**7 - 36*e**8 + x**2*
(-16800*d**6*e**2 - 5670*d**5*e**3 - 9990*d**4*e**4 - 2220*d**3*e**5 - 532
8*d**2*e**6 + 1170*d*e**7 - 642*e**8) + x*(-31200*d**7*e - 10395*d**6*e**2
 - 17982*d**5*e**3 - 3885*d**4*e**4 - 8880*d**3*e**5 + 1755*d**2*e**6 - 64
2*d*e**7 - 99*e**8))/(6*d**3*e**9 + 18*d**2*e**10*x + 18*d*e**11*x**2 + 6*
e**12*x**3) + 20*x**5/e**4 - (5600*d**5 + 1575*d**4*e + 2220*d**3*e**2 + 3
70*d**2*e**3 + 592*d*e**4 - 65*e**5)*log(d + e*x)/e**9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.08

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx =$$
$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(\cdots}{}$$
$$+\frac{240\,e^4x^5-15\,(80\,de^3+9\,e^4)x^4+4\,(1000\,d^2e^2+180\,de^3+111\,e^4)x^3-6\,(2000\,d^3e+450\,d^2e^2+444\,de\cdots}{12\,e^8}$$
$$-\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(ex+d)}{e^9}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
maxima")
```

output
```
-1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739*d^5*e^3 + 3848*d^4*e^4
- 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 + 6*(2800*d^6*e^2 + 945*d^
5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 - 195*d*e^7 + 107*e^8)*x^
2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 + 1295*d^4*e^4 + 2960*d^3
*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/(e^12*x^3 + 3*d*e^11*x^2 + 3*d
^2*e^10*x + d^3*e^9) + 1/12*(240*e^4*x^5 - 15*(80*d*e^3 + 9*e^4)*x^4 + 4*(
1000*d^2*e^2 + 180*d*e^3 + 111*e^4)*x^3 - 6*(2000*d^3*e + 450*d^2*e^2 + 44
4*d*e^3 + 37*e^4)*x^2 + 24*(1750*d^4 + 450*d^3*e + 555*d^2*e^2 + 74*d*e^3
+ 74*e^4)*x)/e^8 - (5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 5
92*d*e^4 - 65*e^5)*log(e*x + d)/e^9
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.06

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx$$
$$=-\frac{(5600\,d^5+1575\,d^4e+2220\,d^3e^2+370\,d^2e^3+592\,de^4-65\,e^5)\log(|ex+d|)}{e^9}$$
$$-\frac{14600\,d^8+4815\,d^7e+8214\,d^6e^2+1739\,d^5e^3+3848\,d^4e^4-715\,d^3e^5+214\,d^2e^6+33\,de^7+36\,e^8+6\,(\cdots}{}$$
$$+\frac{240\,e^{16}x^5-1200\,de^{15}x^4-135\,e^{16}x^4+4000\,d^2e^{14}x^3+720\,de^{15}x^3+444\,e^{16}x^3-12000\,d^3e^{13}x^2-2700\cdots}{}$$

input
```
integrate((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x, algorithm="
giac")
```

output
```
-(5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)
*log(abs(e*x + d))/e^9 - 1/6*(14600*d^8 + 4815*d^7*e + 8214*d^6*e^2 + 1739
*d^5*e^3 + 3848*d^4*e^4 - 715*d^3*e^5 + 214*d^2*e^6 + 33*d*e^7 + 36*e^8 +
6*(2800*d^6*e^2 + 945*d^5*e^3 + 1665*d^4*e^4 + 370*d^3*e^5 + 888*d^2*e^6 -
 195*d*e^7 + 107*e^8)*x^2 + 3*(10400*d^7*e + 3465*d^6*e^2 + 5994*d^5*e^3 +
 1295*d^4*e^4 + 2960*d^3*e^5 - 585*d^2*e^6 + 214*d*e^7 + 33*e^8)*x)/((e*x
+ d)^3*e^9) + 1/12*(240*e^16*x^5 - 1200*d*e^15*x^4 - 135*e^16*x^4 + 4000*d
^2*e^14*x^3 + 720*d*e^15*x^3 + 444*e^16*x^3 - 12000*d^3*e^13*x^2 - 2700*d^
2*e^14*x^2 - 2664*d*e^15*x^2 - 222*e^16*x^2 + 42000*d^4*e^12*x + 10800*d^3
*e^13*x + 13320*d^2*e^14*x + 1776*d*e^15*x + 1776*e^16*x)/e^20
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.56

$$\int \frac{(3+2x+5x^2)^2(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx = x^3\left(\frac{37}{e^4}-\frac{200\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{3\,e}\right)$$

$$-x^2\left(\frac{37}{2\,e^4}+\frac{200\,d^3}{e^7}+\frac{2\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{3\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)$$

$$-\frac{x\left(5200\,d^7+\frac{3465\,d^6\,e}{2}+2997\,d^5\,e^2+\frac{1295\,d^4\,e^3}{2}+1480\,d^3\,e^4-\frac{585\,d^2\,e^5}{2}+107\,d\,e^6+\frac{33\,e^7}{2}\right)+\frac{14600\,d^8+4815\,d^7}{}}{}$$

$$-x^4\left(\frac{100\,d}{e^5}+\frac{45}{4\,e^4}\right)$$

$$+x\left(\frac{148}{e^4}-\frac{100\,d^4}{e^8}+\frac{4\,d\left(\frac{37}{e^4}+\frac{400\,d^3}{e^7}+\frac{4\,d\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e}-\frac{6\,d^2\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^2}\right)}{e}\right.$$

$$\left.-\frac{6\,d^2\left(\frac{111}{e^4}-\frac{600\,d^2}{e^6}+\frac{4\,d\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e}\right)}{e^2}+\frac{4\,d^3\left(\frac{400\,d}{e^5}+\frac{45}{e^4}\right)}{e^3}\right)+\frac{20\,x^5}{e^4}$$

$$-\frac{\ln(d+e\,x)\left(5600\,d^5+1575\,d^4\,e+2220\,d^3\,e^2+370\,d^2\,e^3+592\,d\,e^4-65\,e^5\right)}{e^9}$$

input
```
int(((2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(d + e*x)^4,x)
```

output
```
x^3*(37/e^4 - (200*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/(3*e)) - x^2*(3
7/(2*e^4) + (200*d^3)/e^7 + (2*d*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/
e^5 + 45/e^4))/e))/e - (3*d^2*((400*d)/e^5 + 45/e^4))/e^2) - (x*(107*d*e^6
 + (3465*d^6*e)/2 + 5200*d^7 + (33*e^7)/2 - (585*d^2*e^5)/2 + 1480*d^3*e^4
 + (1295*d^4*e^3)/2 + 2997*d^5*e^2) + (33*d*e^7 + 4815*d^7*e + 14600*d^8 +
 36*e^8 + 214*d^2*e^6 - 715*d^3*e^5 + 3848*d^4*e^4 + 1739*d^5*e^3 + 8214*d
^6*e^2)/(6*e) + x^2*(2800*d^6*e - 195*d*e^6 + 107*e^7 + 888*d^2*e^5 + 370*
d^3*e^4 + 1665*d^4*e^3 + 945*d^5*e^2))/(d^3*e^8 + e^11*x^3 + 3*d^2*e^9*x +
 3*d*e^10*x^2) - x^4*((100*d)/e^5 + 45/(4*e^4)) + x*(148/e^4 - (100*d^4)/e
^8 + (4*d*(37/e^4 + (400*d^3)/e^7 + (4*d*(111/e^4 - (600*d^2)/e^6 + (4*d*(
(400*d)/e^5 + 45/e^4))/e))/e - (6*d^2*((400*d)/e^5 + 45/e^4))/e^2))/e - (6
*d^2*(111/e^4 - (600*d^2)/e^6 + (4*d*((400*d)/e^5 + 45/e^4))/e))/e^2 + (4*
d^3*((400*d)/e^5 + 45/e^4))/e^3) + (20*x^5)/e^4 - (log(d + e*x)*(592*d*e^4
 + 1575*d^4*e + 5600*d^5 - 65*e^5 + 370*d^2*e^3 + 2220*d^3*e^2))/e^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 750, normalized size of antiderivative = 2.08

$$\int \frac{(3+2x+5x^2)^2\,(2+x+3x^2-5x^3+4x^4)}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
int((5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^4,x)
```

output
```
( - 67200*log(d + e*x)*d**9 - 201600*log(d + e*x)*d**8*e*x - 18900*log(d +
 e*x)*d**8*e - 201600*log(d + e*x)*d**7*e**2*x**2 - 56700*log(d + e*x)*d**
7*e**2*x - 26640*log(d + e*x)*d**7*e**2 - 67200*log(d + e*x)*d**6*e**3*x**
3 - 56700*log(d + e*x)*d**6*e**3*x**2 - 79920*log(d + e*x)*d**6*e**3*x - 4
440*log(d + e*x)*d**6*e**3 - 18900*log(d + e*x)*d**5*e**4*x**3 - 79920*log
(d + e*x)*d**5*e**4*x**2 - 13320*log(d + e*x)*d**5*e**4*x - 7104*log(d + e
*x)*d**5*e**4 - 26640*log(d + e*x)*d**4*e**5*x**3 - 13320*log(d + e*x)*d**
4*e**5*x**2 - 21312*log(d + e*x)*d**4*e**5*x + 780*log(d + e*x)*d**4*e**5
- 4440*log(d + e*x)*d**3*e**6*x**3 - 21312*log(d + e*x)*d**3*e**6*x**2 + 2
340*log(d + e*x)*d**3*e**6*x - 7104*log(d + e*x)*d**2*e**7*x**3 + 2340*log
(d + e*x)*d**2*e**7*x**2 + 780*log(d + e*x)*d*e**8*x**3 - 56000*d**9 - 100
800*d**8*e*x - 15750*d**8*e - 28350*d**7*e**2*x - 22200*d**7*e**2 + 67200*
d**6*e**3*x**3 - 39960*d**6*e**3*x - 3700*d**6*e**3 + 16800*d**5*e**4*x**4
 + 18900*d**5*e**4*x**3 - 6660*d**5*e**4*x - 5920*d**5*e**4 - 3360*d**4*e*
*5*x**5 + 4725*d**4*e**5*x**4 + 26640*d**4*e**5*x**3 - 10656*d**4*e**5*x +
 650*d**4*e**5 + 1120*d**3*e**6*x**6 - 945*d**3*e**6*x**5 + 6660*d**3*e**6
*x**4 + 4440*d**3*e**6*x**3 + 1170*d**3*e**6*x - 480*d**2*e**7*x**7 + 315*
d**2*e**7*x**6 - 1332*d**2*e**7*x**5 + 1110*d**2*e**7*x**4 + 7104*d**2*e**
7*x**3 - 66*d**2*e**7 + 240*d*e**8*x**8 - 135*d*e**8*x**7 + 444*d*e**8*x**
6 - 222*d*e**8*x**5 + 1776*d*e**8*x**4 - 780*d*e**8*x**3 - 198*d*e**8*x...
```

3.139 $\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 221

$$\begin{aligned}\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx &= \frac{(10125d^3+34350d^2e-13215de^2-5108e^3)\,x}{15625} \\ &\quad - \frac{(4125d^3-6075d^2e-6870de^2+881e^3)\,x^2}{6250} \\ &\quad + \frac{(500d^3-2475d^2e+1215de^2+458e^3)\,x^3}{1875} \\ &\quad + \frac{3}{500}e\left(100d^2-165de+27e^2\right)x^4 + \frac{3}{125}(20d-11e)e^2x^5 + \frac{2e^3x^6}{15} \\ &\quad - \frac{(52875d^3+449175d^2e-274845de^2-53189e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{78125\sqrt{14}} \\ &\quad + \frac{(57250d^3-66075d^2e-76620de^2+23431e^3)\log\left(3+2x+5x^2\right)}{156250}\end{aligned}$$

output
```
1/15625*(10125*d^3+34350*d^2*e-13215*d*e^2-5108*e^3)*x-1/6250*(4125*d^3-60
75*d^2*e-6870*d*e^2+881*e^3)*x^2+1/1875*(500*d^3-2475*d^2*e+1215*d*e^2+458
*e^3)*x^3+3/500*e*(100*d^2-165*d*e+27*e^2)*x^4+3/125*(20*d-11*e)*e^2*x^5+2
/15*e^3*x^6-1/1093750*(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*arct
an(1/14*(1+5*x)*14^(1/2))*14^(1/2)+1/156250*(57250*d^3-66075*d^2*e-76620*d
*e^2+23431*e^3)*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} \, dx$$
$$= \frac{35x(250d^3(486-495x+200x^2)+450d^2e(916+405x-550x^2+250x^3)+45de^2(-3524+4580x+270}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
(35*x*(250*d^3*(486 - 495*x + 200*x^2) + 450*d^2*e*(916 + 405*x - 550*x^2
+ 250*x^3) + 45*d*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4) +
e^3*(-61296 - 26430*x + 45800*x^2 + 30375*x^3 - 49500*x^4 + 25000*x^5)) -
6*Sqrt[14]*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1
 + 5*x)/Sqrt[14]] + 42*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)
*Log[3 + 2*x + 5*x^2])/6562500
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^3}{5x^2+2x+3} \, dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{3}{125}ex^3(100d^2-165de+27e^2)+\frac{1}{625}x^2(500d^3-2475d^2e+1215de^2+458e^3)+\frac{875d^3-103050d^2e+x(5}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(52875d^3+449175d^2e-274845de^2-53189e^3\right)}{78125\sqrt{14}}+\frac{3}{500}ex^4\left(100d^2-165de+27e^2\right)+\frac{x^3\left(500d^3-2475d^2e+1215de^2+458e^3\right)}{1875}-\frac{x^2\left(4125d^3-6075d^2e-6870de^2+881e^3\right)}{6250}+\frac{\left(57250d^3-66075d^2e-76620de^2+23431e^3\right)\log\left(5x^2+2x+3\right)}{156250}+\frac{x\left(10125d^3+34350d^2e-13215de^2-5108e^3\right)}{15625}+\frac{3}{125}e^2x^5(20d-11e)+\frac{2e^3x^6}{15}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 5108*e^3)*x)/15625 - ((4125*d^3
- 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2)/6250 + ((500*d^3 - 2475*d^2*e +
1215*d*e^2 + 458*e^3)*x^3)/1875 + (3*e*(100*d^2 - 165*d*e + 27*e^2)*x^4)/5
00 + (3*(20*d - 11*e)*e^2*x^5)/125 + (2*e^3*x^6)/15 - ((52875*d^3 + 449175
*d^2*e - 274845*d*e^2 - 53189*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(78125*Sqrt
[14]) + ((57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*Log[3 + 2*x +
 5*x^2])/156250
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.00

method	result
default	$\frac{2e^3x^6}{15}+\frac{12x^5de^2}{25}-\frac{33x^5e^3}{125}+\frac{3x^4d^2e}{5}-\frac{99x^4de^2}{100}+\frac{81e^3x^4}{500}+\frac{4d^3x^3}{15}-\frac{33d^2e\,x^3}{25}+\frac{81d\,e^2x^3}{125}+\frac{458e^3x^3}{1875}-\frac{33d^3x^2}{50}$
risch	$-\frac{33x^5e^3}{125}+\frac{4d^3x^3}{15}+\frac{81d^3x}{125}-\frac{2643d^2e\ln(350x^2+140x+210)}{6250}-\frac{7662d\,e^2\ln(350x^2+140x+210)}{15625}-\frac{423\sqrt{14}\,d^3\arctan\left(\frac{5\sqrt{1}}{1}\right.}{8750}$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
2/15*e^3*x^6+12/25*x^5*d*e^2-33/125*x^5*e^3+3/5*x^4*d^2*e-99/100*x^4*d*e^2
+81/500*e^3*x^4+4/15*d^3*x^3-33/25*d^2*e*x^3+81/125*d*e^2*x^3+458/1875*e^3
*x^3-33/50*d^3*x^2+243/250*x^2*d^2*e+687/625*d*e^2*x^2-881/6250*e^3*x^2+81
/125*d^3*x+1374/625*d^2*e*x-2643/3125*d*e^2*x-5108/15625*e^3*x+1/156250*(5
7250*d^3-66075*d^2*e-76620*d*e^2+23431*e^3)*ln(5*x^2+2*x+3)+1/218750*(-105
75*d^3-89835*d^2*e+54969*d*e^2+53189/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*
14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \frac{2}{15}\,e^3x^6+\frac{3}{125}\left(20\,de^2-11\,e^3\right)x^5 + \frac{3}{500}\left(100\,d^2e-165\,de^2+27\,e^3\right)x^4+\frac{1}{1875}\left(500\,d^3-2475\,d^2e+1215\,de^2+458\,e^3\right)x^3 - \frac{1}{6250}\left(4125\,d^3-6075\,d^2e-6870\,de^2+881\,e^3\right)x^2 - \frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{15625}\left(10125\,d^3+34350\,d^2e-13215\,de^2-5108\,e^3\right)x + \frac{1}{156250}\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^
2 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3
 - 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*s
qrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*
sqrt(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 510
8*e^3)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*lo
g(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 450, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{2e^3x^6}{15} + x^5\cdot\left(\frac{12de^2}{25} - \frac{33e^3}{125}\right) + x^4\cdot\left(\frac{3d^2e}{5} - \frac{99de^2}{100} + \frac{81e^3}{500}\right) + x^3 \cdot\left(\frac{4d^3}{15} - \frac{33d^2e}{25} + \frac{81de^2}{125} + \frac{458e^3}{1875}\right) + x^2\left(-\frac{33d^3}{50} + \frac{243d^2e}{250} + \frac{687de^2}{625} - \frac{881e^3}{6250}\right) + x\left(\frac{81d^3}{125} + \frac{1374d^2e}{625} - \frac{2643de^2}{3125} - \frac{5108e^3}{15625}\right) + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250} - \frac{\sqrt{14}i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \text{}}{52875d^3 + 449175}\right. + \left(\frac{229d^3}{625} - \frac{2643d^2e}{6250} - \frac{7662de^2}{15625} + \frac{23431e^3}{156250} + \frac{\sqrt{14}i(52875d^3 + 449175d^2e - 274845de^2 - 53189e^3)}{2187500}\right)\log\left(x + \frac{10575d^3 + 89835d^2e - 54969de^2 - \text{}}{52875d^3 + 449175}\right.$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
2*e**3*x**6/15 + x**5*(12*d*e**2/25 - 33*e**3/125) + x**4*(3*d**2*e/5 - 99
*d*e**2/100 + 81*e**3/500) + x**3*(4*d**3/15 - 33*d**2*e/25 + 81*d*e**2/12
5 + 458*e**3/1875) + x**2*(-33*d**3/50 + 243*d**2*e/250 + 687*d*e**2/625 -
 881*e**3/6250) + x*(81*d**3/125 + 1374*d**2*e/625 - 2643*d*e**2/3125 - 51
08*e**3/15625) + (229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23
431*e**3/156250 - sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 -
 53189*e**3)/2187500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 -
53189*e**3/5 + sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53
189*e**3)/5)/(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)) +
(229*d**3/625 - 2643*d**2*e/6250 - 7662*d*e**2/15625 + 23431*e**3/156250 +
 sqrt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/2187
500)*log(x + (10575*d**3 + 89835*d**2*e - 54969*d*e**2 - 53189*e**3/5 - sq
rt(14)*I*(52875*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3)/5)/(528
75*d**3 + 449175*d**2*e - 274845*d*e**2 - 53189*e**3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{2}{15}\,e^3x^6 + \frac{3}{125}\,(20\,de^2 - 11\,e^3)x^5 + \frac{3}{500}\,(100\,d^2e - 165\,de^2 + 27\,e^3)x^4 + \frac{1}{1875}\,(500\,d^3 - 2475\,d^2e + 1215\,de^2 + 458\,e^3)x^3 - \frac{1}{6250}\,(4125\,d^3 - 6075\,d^2e - 6870\,de^2 + 881\,e^3)x^2 - \frac{1}{1093750}\,\sqrt{14}(52875\,d^3 + 449175\,d^2e - 274845\,de^2 - 53189\,e^3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{15625}\,(10125\,d^3 + 34350\,d^2e - 13215\,de^2 - 5108\,e^3)x + \frac{1}{156250}\,(57250\,d^3 - 66075\,d^2e - 76620\,de^2 + 23431\,e^3)\log\left(5\,x^2 + 2\,x + 3\right)$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
2/15*e^3*x^6 + 3/125*(20*d*e^2 - 11*e^3)*x^5 + 3/500*(100*d^2*e - 165*d*e^
2 + 27*e^3)*x^4 + 1/1875*(500*d^3 - 2475*d^2*e + 1215*d*e^2 + 458*e^3)*x^3
 - 1/6250*(4125*d^3 - 6075*d^2*e - 6870*d*e^2 + 881*e^3)*x^2 - 1/1093750*s
qrt(14)*(52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53189*e^3)*arctan(1/14*
sqrt(14)*(5*x + 1)) + 1/15625*(10125*d^3 + 34350*d^2*e - 13215*d*e^2 - 510
8*e^3)*x + 1/156250*(57250*d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*lo
g(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{2}{15}\,e^3x^6+\frac{12}{25}\,de^2x^5-\frac{33}{125}\,e^3x^5+\frac{3}{5}\,d^2ex^4 - \frac{99}{100}\,de^2x^4+\frac{81}{500}\,e^3x^4+\frac{4}{15}\,d^3x^3-\frac{33}{25}\,d^2ex^3+\frac{81}{125}\,de^2x^3+\frac{458}{1875}\,e^3x^3-\frac{33}{50}\,d^3x^2 + \frac{243}{250}\,d^2ex^2+\frac{687}{625}\,de^2x^2-\frac{881}{6250}\,e^3x^2+\frac{81}{125}\,d^3x+\frac{1374}{625}\,d^2ex-\frac{2643}{3125}\,de^2x-\frac{5108}{15625}\,e^3x - \frac{1}{1093750}\,\sqrt{14}\left(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{156250}\left(57250\,d^3-66075\,d^2e-76620\,de^2+23431\,e^3\right)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
2/15*e^3*x^6 + 12/25*d*e^2*x^5 - 33/125*e^3*x^5 + 3/5*d^2*e*x^4 - 99/100*d
*e^2*x^4 + 81/500*e^3*x^4 + 4/15*d^3*x^3 - 33/25*d^2*e*x^3 + 81/125*d*e^2*
x^3 + 458/1875*e^3*x^3 - 33/50*d^3*x^2 + 243/250*d^2*e*x^2 + 687/625*d*e^2
*x^2 - 881/6250*e^3*x^2 + 81/125*d^3*x + 1374/625*d^2*e*x - 2643/3125*d*e^
2*x - 5108/15625*e^3*x - 1/1093750*sqrt(14)*(52875*d^3 + 449175*d^2*e - 27
4845*d*e^2 - 53189*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/156250*(57250*
d^3 - 66075*d^2*e - 76620*d*e^2 + 23431*e^3)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 17.21 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} \, dx$$
$$= x^2 \left(\frac{26\,e^2\,(12\,d-5\,e)}{625} - \frac{33\,e\,(4\,d^2-5\,d\,e+e^2)}{250} - \frac{3\,d\,e^2}{50} + \frac{3\,d^2\,e}{2} - \frac{33\,d^3}{50} + \frac{622\,e^3}{3125}\right)$$
$$- x^3 \left(\frac{11\,e^2\,(12\,d-5\,e)}{375} + \frac{2\,e\,(4\,d^2-5\,d\,e+e^2)}{25} - \frac{3\,d\,e^2}{5} + d^2\,e - \frac{4\,d^3}{15} - \frac{111\,e^3}{625}\right)$$
$$+ x^5 \left(\frac{e^2\,(12\,d-5\,e)}{25} - \frac{8\,e^3}{125}\right)$$
$$- \ln\left(5\,x^2+2\,x+3\right) \left(-\frac{229\,d^3}{625} + \frac{2643\,d^2\,e}{6250} + \frac{7662\,d\,e^2}{15625} - \frac{23431\,e^3}{156250}\right)$$
$$- x^4 \left(\frac{e^2\,(12\,d-5\,e)}{50} - \frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{20} + \frac{11\,e^3}{125}\right) + \frac{2\,e^3\,x^6}{15} + x \left(\frac{61\,e^2\,(12\,d-5\,e)}{3125}\right.$$
$$\left. + \frac{3\,d\,(d^2+d\,e+2\,e^2)}{5} + \frac{156\,e\,(4\,d^2-5\,d\,e+e^2)}{625} - \frac{129\,d\,e^2}{125} + \frac{3\,d^2\,e}{5} + \frac{6\,d^3}{125} - \frac{7483\,e^3}{15625}\right)$$
$$+ \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{1093750} + \frac{\sqrt{14}\,x\,\left(-52875\,d^3-449175\,d^2\,e+274845\,d\,e^2+53189\,e^3\right)}{218750}}{-\frac{423\,d^3}{625} - \frac{17967\,d^2\,e}{3125} + \frac{54969\,d\,e^2}{15625} + \frac{53189\,e^3}{78125}}\right)\,(-52875\,d^3 -}{1093750}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x^2*((26*e^2*(12*d - 5*e))/625 - (33*e*(4*d^2 - 5*d*e + e^2))/250 - (3*d*e
^2)/50 + (3*d^2*e)/2 - (33*d^3)/50 + (622*e^3)/3125) - x^3*((11*e^2*(12*d
- 5*e))/375 + (2*e*(4*d^2 - 5*d*e + e^2))/25 - (3*d*e^2)/5 + d^2*e - (4*d^
3)/15 - (111*e^3)/625) + x^5*((e^2*(12*d - 5*e))/25 - (8*e^3)/125) - log(2
*x + 5*x^2 + 3)*((7662*d*e^2)/15625 + (2643*d^2*e)/6250 - (229*d^3)/625 -
(23431*e^3)/156250) - x^4*((e^2*(12*d - 5*e))/50 - (3*e*(4*d^2 - 5*d*e + e
^2))/20 + (11*e^3)/125) + (2*e^3*x^6)/15 + x*((61*e^2*(12*d - 5*e))/3125 +
 (3*d*(d*e + d^2 + 2*e^2))/5 + (156*e*(4*d^2 - 5*d*e + e^2))/625 - (129*d*
e^2)/125 + (3*d^2*e)/5 + (6*d^3)/125 - (7483*e^3)/15625) + (14^(1/2)*atan(
((14^(1/2)*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
+ (14^(1/2)*x*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/21875
0)/((54969*d*e^2)/15625 - (17967*d^2*e)/3125 - (423*d^3)/625 + (53189*e^3)
/78125))*(274845*d*e^2 - 449175*d^2*e - 52875*d^3 + 53189*e^3))/1093750
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{229\log(5x^2+2x+3)\,d^3}{625} + \frac{23431\log(5x^2+2x+3)\,e^3}{156250} - \frac{17967\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^2e}{43750} + \frac{54969\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,e^2}{218750} - \frac{881e^3x^2}{6250} - \frac{5108e^3x}{15625} + \frac{3d^2e\,x^4}{5} - \frac{33d^2e\,x^3}{25} + \frac{243d^2e\,x^2}{250} + \frac{1374d^2ex}{625} + \frac{12d\,e^2x^5}{25} - \frac{99d\,e^2x^4}{100} + \frac{81d\,e^2x^3}{125} + \frac{687d\,e^2x^2}{625} - \frac{423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^3}{8750} + \frac{53189\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)e^3}{1093750} - \frac{2643\log(5x^2+2x+3)\,d^2e}{6250} - \frac{7662\log(5x^2+2x+3)\,d\,e^2}{15625} - \frac{2643d\,e^2x}{3125} + \frac{4d^3x^3}{15} - \frac{33d^3x^2}{50} + \frac{81d^3x}{125} + \frac{2e^3x^6}{15} - \frac{33e^3x^5}{125} + \frac{81e^3x^4}{500} + \frac{458e^3x^3}{1875}$$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
( - 317250*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3 - 2695050*sqrt(14)*atan(
(5*x + 1)/sqrt(14))*d**2*e + 1649070*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e
**2 + 319134*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3 + 2404500*log(5*x**2 +
 2*x + 3)*d**3 - 2775150*log(5*x**2 + 2*x + 3)*d**2*e - 3218040*log(5*x**2
 + 2*x + 3)*d*e**2 + 984102*log(5*x**2 + 2*x + 3)*e**3 + 1750000*d**3*x**3
 - 4331250*d**3*x**2 + 4252500*d**3*x + 3937500*d**2*e*x**4 - 8662500*d**2
*e*x**3 + 6378750*d**2*e*x**2 + 14427000*d**2*e*x + 3150000*d*e**2*x**5 -
6496875*d*e**2*x**4 + 4252500*d*e**2*x**3 + 7213500*d*e**2*x**2 - 5550300*
d*e**2*x + 875000*e**3*x**6 - 1732500*e**3*x**5 + 1063125*e**3*x**4 + 1603
000*e**3*x**3 - 925050*e**3*x**2 - 2145360*e**3*x)/6562500
```

3.140 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 156

$$\begin{aligned}
&\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx\\
&=\frac{(2025d^2+4580de-881e^2)\,x}{3125}-\frac{(825d^2-810de-458e^2)\,x^2}{1250}\\
&\quad+\frac{1}{375}\left(100d^2-330de+81e^2\right)x^3+\frac{1}{100}(40d-33e)ex^4\\
&\quad+\frac{4e^2x^5}{25}-\frac{(10575d^2+59890de-18323e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{15625\sqrt{14}}\\
&\quad+\frac{(5725d^2-4405de-2554e^2)\log\left(3+2x+5x^2\right)}{15625}
\end{aligned}$$

output
```
1/3125*(2025*d^2+4580*d*e-881*e^2)*x-1/1250*(825*d^2-810*d*e-458*e^2)*x^2+
1/375*(100*d^2-330*d*e+81*e^2)*x^3+1/100*(40*d-33*e)*e*x^4+4/25*e^2*x^5-1/
218750*(10575*d^2+59890*d*e-18323*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1
/2)+1/15625*(5725*d^2-4405*d*e-2554*e^2)*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2} dx$$
$$= \frac{35x(50d^2(486-495x+200x^2)+60de(916+405x-550x^2+250x^3)+3e^2(-3524+4580x+2700x^2-$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
(35*x*(50*d^2*(486 - 495*x + 200*x^2) + 60*d*e*(916 + 405*x - 550*x^2 + 25
0*x^3) + 3*e^2*(-3524 + 4580*x + 2700*x^2 - 4125*x^3 + 2000*x^4)) - 6*Sqrt
[14]*(10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 84*(
5725*d^2 - 4405*d*e - 2554*e^2)*Log[3 + 2*x + 5*x^2])/1312500
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^2}{5x^2+2x+3} dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{1}{125}x^2(100d^2-330de+81e^2)+\frac{2x(5725d^2-4405de-2554e^2)+175d^2-13740de+2643e^2}{3125(5x^2+2x+3)}-\frac{1}{625}x(825$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(10575d^2+59890de-18323e^2)}{15625\sqrt{14}}+\frac{1}{375}x^3(100d^2-330de+81e^2)-\frac{x^2(825d^2-810de-458e^2)}{1250}+\frac{(5725d^2-4405de-2554e^2)\log(5x^2+2x+3)}{15625}+\frac{x(2025d^2+4580de-881e^2)}{3125}+\frac{1}{100}ex^4(40d-33e)+\frac{4e^2x^5}{25}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((2025*d^2 + 4580*d*e - 881*e^2)*x)/3125 - ((825*d^2 - 810*d*e - 458*e^2)*
x^2)/1250 + ((100*d^2 - 330*d*e + 81*e^2)*x^3)/375 + ((40*d - 33*e)*e*x^4)
/100 + (4*e^2*x^5)/25 - ((10575*d^2 + 59890*d*e - 18323*e^2)*ArcTan[(1 + 5
*x)/Sqrt[14]])/(15625*Sqrt[14]) + ((5725*d^2 - 4405*d*e - 2554*e^2)*Log[3
+ 2*x + 5*x^2])/15625
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94

method	result
default	$\frac{4e^2x^5}{25}+\frac{2x^4de}{5}-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}-\frac{22x^3de}{25}+\frac{27e^2x^3}{125}-\frac{33d^2x^2}{50}+\frac{81x^2de}{125}+\frac{229e^2x^2}{625}+\frac{81d^2x}{125}+\frac{916dex}{625}-\frac{881}{31}$
risch	$-\frac{33x^4e^2}{100}+\frac{4x^3d^2}{15}+\frac{81d^2x}{125}-\frac{881e^2x}{3125}+\frac{27e^2x^3}{125}+\frac{4e^2x^5}{25}-\frac{33d^2x^2}{50}-\frac{22x^3de}{25}+\frac{81x^2de}{125}+\frac{2x^4de}{5}-\frac{5989\sqrt{14}\,de\,\mathrm{ar}}{}$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
4/25*e^2*x^5+2/5*x^4*d*e-33/100*x^4*e^2+4/15*x^3*d^2-22/25*x^3*d*e+27/125*
e^2*x^3-33/50*d^2*x^2+81/125*x^2*d*e+229/625*e^2*x^2+81/125*d^2*x+916/625*
d*e*x-881/3125*e^2*x+1/31250*(11450*d^2-8810*d*e-5108*e^2)*ln(5*x^2+2*x+3)
+1/43750*(-2115*d^2-11978*d*e+18323/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*1
4^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= \frac{4}{25}e^2x^5 + \frac{1}{100}(40\,de-33\,e^2)x^4 + \frac{1}{375}(100\,d^2-330\,de+81\,e^2)x^3 - \frac{1}{1250}(825\,d^2-810\,de-458\,e^2)x^2 - \frac{1}{218750}\sqrt{14}(10575\,d^2+59890\,de-18323\,e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) + \frac{1}{3125}(2025\,d^2+4580\,de-881\,e^2)x + \frac{1}{15625}(5725\,d^2-4405\,de-2554\,e^2)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81
*e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(
10575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/312
5*(2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*
e^2)*log(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.94

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx$$
$$= \frac{4e^2x^5}{25}+x^4\cdot\left(\frac{2de}{5}-\frac{33e^2}{100}\right)+x^3\cdot\left(\frac{4d^2}{15}-\frac{22de}{25}+\frac{27e^2}{125}\right)+x^2\left(-\frac{33d^2}{50}+\frac{81de}{125}+\frac{229e^2}{625}\right)$$
$$+x\left(\frac{81d^2}{125}+\frac{916de}{625}-\frac{881e^2}{3125}\right)+\left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625}\right.$$
$$\left.-\frac{\sqrt{14}i(10575d^2+59890de-18323e^2)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}+\frac{\sqrt{14}i(10575d^2+59890de-18\ldots}{5}}{10575d^2+59890de-18323e^2}\right.$$
$$+\left(\frac{229d^2}{625}-\frac{881de}{3125}-\frac{2554e^2}{15625}\right.$$
$$\left.+\frac{\sqrt{14}i(10575d^2+59890de-18323e^2)}{437500}\right)\log\left(x+\frac{2115d^2+11978de-\frac{18323e^2}{5}-\frac{\sqrt{14}i(10575d^2+59890de-18\ldots}{5}}{10575d^2+59890de-18323e^2}\right.$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
4*e**2*x**5/25 + x**4*(2*d*e/5 - 33*e**2/100) + x**3*(4*d**2/15 - 22*d*e/2
5 + 27*e**2/125) + x**2*(-33*d**2/50 + 81*d*e/125 + 229*e**2/625) + x*(81*
d**2/125 + 916*d*e/625 - 881*e**2/3125) + (229*d**2/625 - 881*d*e/3125 - 2
554*e**2/15625 - sqrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/437500)*
log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 + sqrt(14)*I*(10575*d**2 + 5
9890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e - 18323*e**2)) + (229*d*
*2/625 - 881*d*e/3125 - 2554*e**2/15625 + sqrt(14)*I*(10575*d**2 + 59890*d
*e - 18323*e**2)/437500)*log(x + (2115*d**2 + 11978*d*e - 18323*e**2/5 - s
qrt(14)*I*(10575*d**2 + 59890*d*e - 18323*e**2)/5)/(10575*d**2 + 59890*d*e
 - 18323*e**2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{4}{25}\,e^2x^5 + \frac{1}{100}\,(40\,de-33\,e^2)x^4 + \frac{1}{375}\,(100\,d^2-330\,de+81\,e^2)x^3 - \frac{1}{1250}\,(825\,d^2-810\,de-458\,e^2)x^2 - \frac{1}{218750}\,\sqrt{14}(10575\,d^2+59890\,de-18323\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{3125}\,(2025\,d^2+4580\,de-881\,e^2)x + \frac{1}{15625}\,(5725\,d^2-4405\,de-2554\,e^2)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
4/25*e^2*x^5 + 1/100*(40*d*e - 33*e^2)*x^4 + 1/375*(100*d^2 - 330*d*e + 81
*e^2)*x^3 - 1/1250*(825*d^2 - 810*d*e - 458*e^2)*x^2 - 1/218750*sqrt(14)*(
10575*d^2 + 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/312
5*(2025*d^2 + 4580*d*e - 881*e^2)*x + 1/15625*(5725*d^2 - 4405*d*e - 2554*
e^2)*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{4}{25}\,e^2x^5 + \frac{2}{5}\,dex^4 - \frac{33}{100}\,e^2x^4 + \frac{4}{15}\,d^2x^3 - \frac{22}{25}\,dex^3 + \frac{27}{125}\,e^2x^3 - \frac{33}{50}\,d^2x^2 + \frac{81}{125}\,dex^2 + \frac{229}{625}\,e^2x^2 + \frac{81}{125}\,d^2x + \frac{916}{625}\,dex - \frac{881}{3125}\,e^2x - \frac{1}{218750}\,\sqrt{14}(10575\,d^2+59890\,de-18323\,e^2)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{15625}\,(5725\,d^2-4405\,de-2554\,e^2)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
4/25*e^2*x^5 + 2/5*d*e*x^4 - 33/100*e^2*x^4 + 4/15*d^2*x^3 - 22/25*d*e*x^3
 + 27/125*e^2*x^3 - 33/50*d^2*x^2 + 81/125*d*e*x^2 + 229/625*e^2*x^2 + 81/
125*d^2*x + 916/625*d*e*x - 881/3125*e^2*x - 1/218750*sqrt(14)*(10575*d^2
+ 59890*d*e - 18323*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/15625*(5725*d
^2 - 4405*d*e - 2554*e^2)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = x\left(\frac{4\,d\,e}{5}+\frac{52\,e\,(8\,d-5\,e)}{625}+\frac{81\,d^2}{125}+\frac{419\,e^2}{3125}\right) - \ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{229\,d^2}{625}+\frac{881\,d\,e}{3125}+\frac{2554\,e^2}{15625}\right) + x^4\left(\frac{e\,(8\,d-5\,e)}{20}-\frac{2\,e^2}{25}\right) - x^3\left(\frac{2\,d\,e}{3}+\frac{2\,e\,(8\,d-5\,e)}{75}-\frac{4\,d^2}{15}-\frac{31\,e^2}{375}\right) + x^2\left(d\,e-\frac{11\,e\,(8\,d-5\,e)}{250}-\frac{33\,d^2}{50}+\frac{183\,e^2}{1250}\right)+\frac{4\,e^2\,x^5}{25} - \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}+\frac{\sqrt{14}\,x\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{43750}}{\frac{423\,d^2}{625}+\frac{11978\,d\,e}{3125}-\frac{18323\,e^2}{15625}}\right)\,\left(10575\,d^2+59890\,d\,e-18323\,e^2\right)}{218750}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x*((4*d*e)/5 + (52*e*(8*d - 5*e))/625 + (81*d^2)/125 + (419*e^2)/3125) - l
og(2*x + 5*x^2 + 3)*((881*d*e)/3125 - (229*d^2)/625 + (2554*e^2)/15625) +
x^4*((e*(8*d - 5*e))/20 - (2*e^2)/25) - x^3*((2*d*e)/3 + (2*e*(8*d - 5*e))
/75 - (4*d^2)/15 - (31*e^2)/375) + x^2*(d*e - (11*e*(8*d - 5*e))/250 - (33
*d^2)/50 + (183*e^2)/1250) + (4*e^2*x^5)/25 - (14^(1/2)*atan(((14^(1/2)*(5
9890*d*e + 10575*d^2 - 18323*e^2))/218750 + (14^(1/2)*x*(59890*d*e + 10575
*d^2 - 18323*e^2))/43750)/((11978*d*e)/3125 + (423*d^2)/625 - (18323*e^2)/
15625))*(59890*d*e + 10575*d^2 - 18323*e^2))/218750
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$
$$= -\frac{423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^2}{8750} - \frac{5989\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)de}{21875} + \frac{18323\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)e^2}{218750}$$
$$+ \frac{229\log(5x^2+2x+3)\,d^2}{625} - \frac{881\log(5x^2+2x+3)\,de}{3125}$$
$$- \frac{2554\log(5x^2+2x+3)\,e^2}{15625} + \frac{4d^2x^3}{15} - \frac{33d^2x^2}{50} + \frac{81d^2x}{125} + \frac{2de\,x^4}{5} - \frac{22de\,x^3}{25}$$
$$+ \frac{81de\,x^2}{125} + \frac{916dex}{625} + \frac{4e^2x^5}{25} - \frac{33e^2x^4}{100} + \frac{27e^2x^3}{125} + \frac{229e^2x^2}{625} - \frac{881e^2x}{3125}$$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
( - 63450*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2 - 359340*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d*e + 109938*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**2 + 48
0900*log(5*x**2 + 2*x + 3)*d**2 - 370020*log(5*x**2 + 2*x + 3)*d*e - 21453
6*log(5*x**2 + 2*x + 3)*e**2 + 350000*d**2*x**3 - 866250*d**2*x**2 + 85050
0*d**2*x + 525000*d*e*x**4 - 1155000*d*e*x**3 + 850500*d*e*x**2 + 1923600*
d*e*x + 210000*e**2*x**5 - 433125*e**2*x**4 + 283500*e**2*x**3 + 480900*e*
*2*x**2 - 370020*e**2*x)/1312500
```

3.141 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 99

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \frac{1}{625}(405d+458e)x-\frac{3}{250}(55d-27e)x^2 + \frac{1}{75}(20d-33e)x^3+\frac{ex^4}{5} - \frac{(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3125\sqrt{14}} + \frac{(2290d-881e)\log\left(3+2x+5x^2\right)}{6250}$$

output
```
1/625*(405*d+458*e)*x-3/250*(55*d-27*e)*x^2+1/75*(20*d-33*e)*x^3+1/5*e*x^4
-1/43750*(2115*d+5989*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)+1/6250*(22
90*d-881*e)*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \frac{35x(5d(486-495x+200x^2)+3e(916+405x-550x^2+250x^3))-3\sqrt{14}(2115d+5989e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+\ldots}{131250}$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
(35*x*(5*d*(486 - 495*x + 200*x^2) + 3*e*(916 + 405*x - 550*x^2 + 250*x^3)
) - 3*Sqrt[14]*(2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]] + 21*(2290*d -
 881*e)*Log[3 + 2*x + 5*x^2])/131250
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{5x^2+2x+3}\,dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(\frac{1}{25}x^2(20d-33e)+\frac{x(2290d-881e)+35d-1374e}{625\left(5x^2+2x+3\right)}-\frac{3}{125}x(55d-27e)+\frac{1}{625}(405d+458e)+\frac{4ex^3}{5}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(2115d+5989e)}{3125\sqrt{14}}+\frac{1}{75}x^3(20d-33e)-\frac{3}{250}x^2(55d-27e)+\frac{(2290d-881e)\log\left(5x^2+2x+3\right)}{6250}+\frac{1}{625}x(405d+458e)+\frac{ex^4}{5}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output
```
((405*d + 458*e)*x)/625 - (3*(55*d - 27*e)*x^2)/250 + ((20*d - 33*e)*x^3)/
75 + (e*x^4)/5 - ((2115*d + 5989*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(3125*Sqrt
[14]) + ((2290*d - 881*e)*Log[3 + 2*x + 5*x^2])/6250
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

method	result
default	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{(2290d-881e)\ln(5x^2+2x+3)}{6250}+\frac{\left(-423d-\frac{5989e}{5}\right)\sqrt{14}\ \mathrm{ar}}{8750}$
risch	$\frac{e\,x^4}{5}+\frac{4d\,x^3}{15}-\frac{11e\,x^3}{25}-\frac{33d\,x^2}{50}+\frac{81e\,x^2}{250}+\frac{81dx}{125}+\frac{458ex}{625}+\frac{229d\ln(350x^2+140x+210)}{625}-\frac{881e\ln(350x^2+140x+210}{6250}$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
1/5*e*x^4+4/15*d*x^3-11/25*e*x^3-33/50*d*x^2+81/250*e*x^2+81/125*d*x+458/6
25*e*x+1/6250*(2290*d-881*e)*ln(5*x^2+2*x+3)+1/8750*(-423*d-5989/5*e)*14^(
1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{1}{5}\,ex^4 + \frac{1}{75}\,(20\,d-33\,e)x^3 - \frac{3}{250}\,(55\,d-27\,e)x^2 - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\,(405\,d+458\,e)x + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fric
as")
```

output
```
1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqr
t(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 4
58*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.65

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{ex^4}{5} + x^3\cdot\left(\frac{4d}{15}-\frac{11e}{25}\right) + x^2\left(-\frac{33d}{50}+\frac{81e}{250}\right) + x\left(\frac{81d}{125}+\frac{458e}{625}\right) + \left(\frac{229d}{625}-\frac{881e}{6250} - \frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}+\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right) + \left(\frac{229d}{625} - \frac{881e}{6250}+\frac{\sqrt{14}i(2115d+5989e)}{87500}\right)\log\left(x+\frac{423d+\frac{5989e}{5}-\frac{\sqrt{14}i(2115d+5989e)}{5}}{2115d+5989e}\right)$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
e*x**4/5 + x**3*(4*d/15 - 11*e/25) + x**2*(-33*d/50 + 81*e/250) + x*(81*d/
125 + 458*e/625) + (229*d/625 - 881*e/6250 - sqrt(14)*I*(2115*d + 5989*e)/
87500)*log(x + (423*d + 5989*e/5 + sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d
 + 5989*e)) + (229*d/625 - 881*e/6250 + sqrt(14)*I*(2115*d + 5989*e)/87500
)*log(x + (423*d + 5989*e/5 - sqrt(14)*I*(2115*d + 5989*e)/5)/(2115*d + 59
89*e))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{1}{5}\,ex^4 + \frac{1}{75}\,(20\,d-33\,e)x^3 - \frac{3}{250}\,(55\,d-27\,e)x^2 - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\,(405\,d+458\,e)x + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxi
ma")
```

output
```
1/5*e*x^4 + 1/75*(20*d - 33*e)*x^3 - 3/250*(55*d - 27*e)*x^2 - 1/43750*sqr
t(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(405*d + 4
58*e)*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{1}{5}\,ex^4 + \frac{4}{15}\,dx^3 - \frac{11}{25}\,ex^3 - \frac{33}{50}\,dx^2 + \frac{81}{250}\,ex^2 - \frac{1}{43750}\,\sqrt{14}(2115\,d+5989\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,dx + \frac{458}{625}\,ex + \frac{1}{6250}\,(2290\,d-881\,e)\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac
")
```

output
```
1/5*e*x^4 + 4/15*d*x^3 - 11/25*e*x^3 - 33/50*d*x^2 + 81/250*e*x^2 - 1/4375
0*sqrt(14)*(2115*d + 5989*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 81/125*d*x
+ 458/625*e*x + 1/6250*(2290*d - 881*e)*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = x^3\left(\frac{4\,d}{15}-\frac{11\,e}{25}\right)-x^2\left(\frac{33\,d}{50}-\frac{81\,e}{250}\right)+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{229\,d}{625}-\frac{881\,e}{6250}\right)+\frac{e\,x^4}{5}+x\left(\frac{81\,d}{125}+\frac{458\,e}{625}\right)-\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(2115\,d+5989\,e)}{43750}+\frac{\sqrt{14}\,x\,(2115\,d+5989\,e)}{8750}}{\frac{423\,d}{625}+\frac{5989\,e}{3125}}\right)\,(2115\,d+5989\,e)}{43750}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
x^3*((4*d)/15 - (11*e)/25) - x^2*((33*d)/50 - (81*e)/250) + log(2*x + 5*x^
2 + 3)*((229*d)/625 - (881*e)/6250) + (e*x^4)/5 + x*((81*d)/125 + (458*e)/
625) - (14^(1/2)*atan(((14^(1/2)*(2115*d + 5989*e))/43750 + (14^(1/2)*x*(2
115*d + 5989*e))/8750)/((423*d)/625 + (5989*e)/3125))*(2115*d + 5989*e))/4
3750
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = -\frac{423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d}{8750} - \frac{5989\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)e}{43750} + \frac{229\log\left(5x^2+2x+3\right)d}{625} - \frac{881\log\left(5x^2+2x+3\right)e}{6250} + \frac{4d\,x^3}{15} - \frac{33d\,x^2}{50} + \frac{81dx}{125} + \frac{e\,x^4}{5} - \frac{11e\,x^3}{25} + \frac{81e\,x^2}{250} + \frac{458ex}{625}$$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
( - 6345*sqrt(14)*atan((5*x + 1)/sqrt(14))*d - 17967*sqrt(14)*atan((5*x +
1)/sqrt(14))*e + 48090*log(5*x**2 + 2*x + 3)*d - 18501*log(5*x**2 + 2*x +
3)*e + 35000*d*x**3 - 86625*d*x**2 + 85050*d*x + 26250*e*x**4 - 57750*e*x*
*3 + 42525*e*x**2 + 96180*e*x)/131250
```

3.142 $\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{81x}{125}-\frac{33x^2}{50}+\frac{4x^3}{15}-\frac{423\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{229}{625}\log\left(3+2x+5x^2\right)$$

output
```
81/125*x-33/50*x^2+4/15*x^3-423/8750*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2
)+229/625*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{35x(486-495x+200x^2)-1269\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+9618\log\left(3+2x+5x^2\right)}{26250}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

output
```
(35*x*(486 - 495*x + 200*x^2) - 1269*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]] +
 9618*Log[3 + 2*x + 5*x^2])/26250
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{5x^2+2x+3}\,dx$$

$$\Big\downarrow \text{2188}$$

$$\int \left(\frac{4x^2}{5}+\frac{458x+7}{125\left(5x^2+2x+3\right)}-\frac{33x}{25}+\frac{81}{125}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{423\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{625\sqrt{14}}+\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{229}{625}\log\left(5x^2+2x+3\right)+\frac{81x}{125}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2),x]
```

output
```
(81*x)/125 - (33*x^2)/50 + (4*x^3)/15 - (423*ArcTan[(1 + 5*x)/Sqrt[14]])/(
625*Sqrt[14]) + (229*Log[3 + 2*x + 5*x^2])/625
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(5x^2+2x+3)}{625}-\frac{423\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{8750}$	44
risch	$\frac{4x^3}{15}-\frac{33x^2}{50}+\frac{81x}{125}+\frac{229\ln(25x^2+10x+15)}{625}-\frac{423\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{8750}$	44

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
4/15*x^3-33/50*x^2+81/125*x+229/625*ln(5*x^2+2*x+3)-423/8750*14^(1/2)*arct
an(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx=\frac{4}{15}\,x^3-\frac{33}{50}\,x^2-\frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+\frac{81}{125}\,x+\frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fricas")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125} + \frac{229\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{625} - \frac{423\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{8750}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
4*x**3/15 - 33*x**2/50 + 81*x/125 + 229*log(x**2 + 2*x/5 + 3/5)/625 - 423*
sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/8750
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{33}{50}\,x^2 - \frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x + \frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="maxima")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{4}{15}\,x^3 - \frac{33}{50}\,x^2 - \frac{423}{8750}\,\sqrt{14}\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{81}{125}\,x + \frac{229}{625}\,\log\left(5\,x^2+2\,x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="giac")
```

output
```
4/15*x^3 - 33/50*x^2 - 423/8750*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) +
 81/125*x + 229/625*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = \frac{81\,x}{125} + \frac{229\,\ln\left(5\,x^2+2\,x+3\right)}{625} - \frac{423\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{8750} - \frac{33\,x^2}{50} + \frac{4\,x^3}{15}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3),x)
```

output
```
(81*x)/125 + (229*log(2*x + 5*x^2 + 3))/625 - (423*14^(1/2)*atan((5*14^(1/
2)*x)/14 + 14^(1/2)/14))/8750 - (33*x^2)/50 + (4*x^3)/15
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.75

$$\int \frac{2+x+3x^2-5x^3+4x^4}{3+2x+5x^2}\,dx = -\frac{423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)}{8750} + \frac{229\log(5x^2+2x+3)}{625} + \frac{4x^3}{15} - \frac{33x^2}{50} + \frac{81x}{125}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
( - 1269*sqrt(14)*atan((5*x + 1)/sqrt(14)) + 9618*log(5*x**2 + 2*x + 3) +
7000*x**3 - 17325*x**2 + 17010*x)/26250
```

3.143 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)} dx$

Optimal result

Integrand size = 38, antiderivative size = 168

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)} dx = -\frac{(20d+33e)x}{25e^2}+\frac{2x^2}{5e}-\frac{(423d-1367e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{125\sqrt{14}(5d^2-2de+3e^2)} + \frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3(5d^2-2de+3e^2)} + \frac{(458d-7e)\log(3+2x+5x^2)}{250(5d^2-2de+3e^2)}$$

output
```
-1/25*(20*d+33*e)*x/e^2+2/5*x^2/e-1/1750*(423*d-1367*e)*arctan(1/14*(1+5*x
)*14^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*
e^4)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)+(458*d-7*e)*ln(5*x^2+2*x+3)/(1250*d
^2-500*d*e+750*e^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\,(3+2x+5x^2)}\,dx$$
$$= \frac{70e(5d^2-2de+3e^2)\,x(-20d+e(-33+10x))-\sqrt{14}(423d-1367e)e^3\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+1750(4d^4+5d}{1750e^3\,(5d^2-2de+3e^2)}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

output
```
(70*e*(5*d^2 - 2*d*e + 3*e^2)*x*(-20*d + e*(-33 + 10*x)) - Sqrt[14]*(423*d
 - 1367*e)*e^3*ArcTan[(1 + 5*x)/Sqrt[14]] + 1750*(4*d^4 + 5*d^3*e + 3*d^2*
e^2 - d*e^3 + 2*e^4)*Log[d + e*x] + 7*(458*d - 7*e)*e^3*Log[3 + 2*x + 5*x^
2])/(1750*e^3*(5*d^2 - 2*d*e + 3*e^2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)\,(d+ex)}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{x(458d-7e)+7d+272e}{25\,(5x^2+2x+3)\,(5d^2-2de+3e^2)}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2\,(5d^2-2de+3e^2)\,(d+ex)}+\frac{-20d-33e}{25e^2}+\frac{4x}{5e}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d-1367e)}{125\sqrt{14}\,(5d^2-2de+3e^2)}+\frac{(458d-7e)\log\left(5x^2+2x+3\right)}{250\,(5d^2-2de+3e^2)}+\frac{(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{e^3\,(5d^2-2de+3e^2)}-\frac{x(20d+33e)}{25e^2}+\frac{2x^2}{5e}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)),x]
```

output
```
-1/25*((20*d + 33*e)*x)/e^2 + (2*x^2)/(5*e) - ((423*d - 1367*e)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(125*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + ((4*d^4 + 5*d^3
*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)
) + ((458*d - 7*e)*Log[3 + 2*x + 5*x^2])/(250*(5*d^2 - 2*d*e + 3*e^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-10e\,x^2+20dx+33ex}{25e^2}+\frac{\frac{(458d-7e)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423d}{5}+\frac{1367e}{5}\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{125d^2-50de+75e^2}+\frac{(4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{e^3(5d^2-2de+3e^2)}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-1/25/e^2*(-10*e*x^2+20*d*x+33*e*x)+1/(125*d^2-50*d*e+75*e^2)*(1/10*(458*d
-7*e)*ln(5*x^2+2*x+3)+1/14*(-423/5*d+1367/5*e)*14^(1/2)*arctan(1/28*(10*x+
2)*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/e^3/(5*d^2-2
*d*e+3*e^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = \frac{700\,(5\,d^2e^2-2\,de^3+3\,e^4)x^2-\sqrt{14}(423\,de^3-1367\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-70\,(100\,d^3e+125\,d^2e \cdots}{1750\,(5\,d \cdots}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="fric
as")
```

output
```
1/1750*(700*(5*d^2*e^2 - 2*d*e^3 + 3*e^4)*x^2 - sqrt(14)*(423*d*e^3 - 1367
*e^4)*arctan(1/14*sqrt(14)*(5*x + 1)) - 70*(100*d^3*e + 125*d^2*e^2 - 6*d*
e^3 + 99*e^4)*x + 1750*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e
*x + d) + 7*(458*d*e^3 - 7*e^4)*log(5*x^2 + 2*x + 3))/(5*d^2*e^3 - 2*d*e^4
 + 3*e^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(ex+d)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{(458\,d-7\,e)\log(5\,x^2+2\,x+3)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{10\,ex^2-(20\,d+33\,e)x}{25\,e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="maxi
ma")
```

output
```
-1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
 2*d*e + 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x +
d)/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3
)/(5*d^2 - 2*d*e + 3*e^2) + 1/25*(10*e*x^2 - (20*d + 33*e)*x)/e^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)}\,dx = -\frac{\sqrt{14}(423\,d-1367\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)}{1750\,(5\,d^2-2\,de+3\,e^2)} + \frac{(458\,d-7\,e)\log(5\,x^2+2\,x+3)}{250\,(5\,d^2-2\,de+3\,e^2)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(|ex+d|)}{5\,d^2e^3-2\,de^4+3\,e^5} + \frac{10\,ex^2-20\,dx-33\,ex}{25\,e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x, algorithm="giac
")
```

output
```
-1/1750*sqrt(14)*(423*d - 1367*e)*arctan(1/14*sqrt(14)*(5*x + 1))/(5*d^2 -
 2*d*e + 3*e^2) + 1/250*(458*d - 7*e)*log(5*x^2 + 2*x + 3)/(5*d^2 - 2*d*e
+ 3*e^2) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(abs(e*x + d))
/(5*d^2*e^3 - 2*d*e^4 + 3*e^5) + 1/25*(10*e*x^2 - 20*d*x - 33*e*x)/e^2
```

Mupad [B] (verification not implemented)

Time = 18.76 (sec) , antiderivative size = 713, normalized size of antiderivative = 4.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\,(3+2x+5x^2)}\,dx = \text{Too large to display}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)),x)
```

output
```
(2*x^2)/(5*e) - log(d + e*x)*(((458*d)/125 - (7*e)/125)/(5*d^2 - 2*d*e + 3
*e^2) - (165*d*e + 100*d^2 + 81*e^2)/(125*e^3)) - x*((4*(5*d + 2*e))/(25*e
^2) + 1/e) - (log((1791*d*e^2 + 1053*d^2*e - 28*d^3 + 916*e^3)/(25*e^2) -
(x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/(25*e^2) + ((d*((423*14
^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/250))*((4751*d*e^3
 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/(25*e^2) + (x*(8200*d*e
^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^2))/(25*e^2) - (((750*e
^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(2500*d*e^4 + 10250*e^5 - 6
250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^
(1/2))/3500 - 7i/250)))/(d^2*5i - d*e*2i + e^2*3i)))/(d^2*5i - d*e*2i + e^
2*3i))*(d*((423*14^(1/2))/3500 - 229i/125) - e*((1367*14^(1/2))/3500 - 7i/
250)))/(d^2*5i - d*e*2i + e^2*3i) + (log((1791*d*e^2 + 1053*d^2*e - 28*d^3
 + 916*e^3)/(25*e^2) - (x*(321*d*e^2 + 2318*d^2*e + 1832*d^3 - 2249*e^3))/
(25*e^2) - ((d*((423*14^(1/2))/3500 + 229i/125) - e*((1367*14^(1/2))/3500
+ 7i/250))*((4751*d*e^3 + 4350*d^3*e - 1000*d^4 + 874*e^4 + 8490*d^2*e^2)/
(25*e^2) + (x*(8200*d*e^3 - 6250*d^3*e - 5000*d^4 + 2917*e^4 + 1850*d^2*e^
2))/(25*e^2) + (((750*e^5 - 14500*d*e^4 + 1250*d^2*e^3)/(25*e^2) - (x*(250
0*d*e^4 + 10250*e^5 - 6250*d^2*e^3))/(25*e^2))*(d*((423*14^(1/2))/3500 + 2
29i/125) - e*((1367*14^(1/2))/3500 + 7i/250)))/(d^2*5i - d*e*2i + e^2*3i))
)/(d^2*5i - d*e*2i + e^2*3i))*(d*((423*14^(1/2))/3500 + 229i/125) - e*(...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.25

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)}\,dx$$
$$= \frac{-423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,e^3+1367\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)e^4+3206\log(5x^2+2x+3)\,d\,e^3-49\log(5x^2+2x+}{}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3),x)
```

output
```
( - 423*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**3 + 1367*sqrt(14)*atan((5*x
 + 1)/sqrt(14))*e**4 + 3206*log(5*x**2 + 2*x + 3)*d*e**3 - 49*log(5*x**2 +
 2*x + 3)*e**4 + 7000*log(d + e*x)*d**4 + 8750*log(d + e*x)*d**3*e + 5250*
log(d + e*x)*d**2*e**2 - 1750*log(d + e*x)*d*e**3 + 3500*log(d + e*x)*e**4
 - 7000*d**3*e*x + 3500*d**2*e**2*x**2 - 8750*d**2*e**2*x - 1400*d*e**3*x*
*2 + 420*d*e**3*x + 2100*e**4*x**2 - 6930*e**4*x)/(1750*e**3*(5*d**2 - 2*d
*e + 3*e**2))
```

3.144 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 233

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \frac{4x}{5e^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(423d^2-2734de+293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}+\frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

output
```
4/5*x/e^2-(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e
*x+d)-1/350*(423*d^2-2734*d*e+293*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1
/2)/(5*d^2-2*d*e+3*e^2)^2-(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)
*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^2+1/25*(229*d^2-7*d*e-136*e^2)*ln(5*x^2
+2*x+3)/(5*d^2-2*d*e+3*e^2)^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$$
$$=\frac{4x}{5e^2}+\frac{-4d^4-5d^3e-3d^2e^2+de^3-2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}$$
$$+\frac{(-423d^2+2734de-293e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{14}(5d^2-2de+3e^2)^2}$$
$$+\frac{(-40d^5-d^4e-28d^3e^2-44d^2e^3+2de^4-e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}$$
$$+\frac{(229d^2-7de-136e^2)\log(3+2x+5x^2)}{25(5d^2-2de+3e^2)^2}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),
x]
```

output
```
(4*x)/(5*e^2) + (-4*d^4 - 5*d^3*e - 3*d^2*e^2 + d*e^3 - 2*e^4)/(e^3*(5*d^2
 - 2*d*e + 3*e^2)*(d + e*x)) + ((-423*d^2 + 2734*d*e - 293*e^2)*ArcTan[(1
+ 5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + ((-40*d^5 - d^
4*e - 28*d^3*e^2 - 44*d^2*e^3 + 2*d*e^4 - e^5)*Log[d + e*x])/(e^3*(5*d^2 -
 2*d*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(2
5*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)(d+ex)^2}\,dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(\frac{2x(229d^2-7de-136e^2)+7d^2+544de-113e^2}{5(5x^2+2x+3)(5d^2-2de+3e^2)^2}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^2}+\frac{-40d^5-d^4e-28d^3}{e^2(5d^2-2de}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(423d^2-2734de+293e^2)}{25\sqrt{14}(5d^2-2de+3e^2)^2}+\frac{(229d^2-7de-136e^2)\log(5x^2+2x+3)}{25(5d^2-2de+3e^2)^2}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^3(5d^2-2de+3e^2)(d+ex)}-\frac{(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)\log(d+ex)}{e^3(5d^2-2de+3e^2)^2}+\frac{4x}{5e^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)),x]
```

output
```
(4*x)/(5*e^2) - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2
- 2*d*e + 3*e^2)*(d + e*x)) - ((423*d^2 - 2734*d*e + 293*e^2)*ArcTan[(1 +
5*x)/Sqrt[14]])/(25*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) - ((40*d^5 + d^4*e
 + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*Log[d + e*x])/(e^3*(5*d^2 - 2*
d*e + 3*e^2)^2) + ((229*d^2 - 7*d*e - 136*e^2)*Log[3 + 2*x + 5*x^2])/(25*(
5*d^2 - 2*d*e + 3*e^2)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.91

method	result
default	$\frac{4x}{5e^2}+\frac{\frac{\left(458d^2-14de-272e^2\right)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423}{5}d^2+\frac{2734}{5}de-\frac{293}{5}e^2\right)\sqrt{14}\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{5\left(5d^2-2de+3e^2\right)^2}-\frac{4d^4+5d^3e+3d^2e^2-d\,e^3+2e^4}{e^3\left(5d^2-2de+3e^2\right)(ex+d)}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
4/5*x/e^2+1/5/(5*d^2-2*d*e+3*e^2)^2*(1/10*(458*d^2-14*d*e-272*e^2)*ln(5*x^
2+2*x+3)+1/14*(-423/5*d^2+2734/5*d*e-293/5*e^2)*14^(1/2)*arctan(1/28*(10*x
+2)*14^(1/2)))-(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3/(5*d^2-2*d*e+3*e^
2)/(e*x+d)+(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^4-e^5)/e^3/(5*d^2-2*
d*e+3*e^2)^2*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.79

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx =$$

$$-\frac{7000\,d^6+5950\,d^5e+5950\,d^4e^2+1400\,d^3e^3+7350\,d^2e^4-2450\,de^5+2100\,e^6-280\,(25\,d^4e^2-20\,d^3e^3}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
-1/350*(7000*d^6 + 5950*d^5*e + 5950*d^4*e^2 + 1400*d^3*e^3 + 7350*d^2*e^4
 - 2450*d*e^5 + 2100*e^6 - 280*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12*
d*e^5 + 9*e^6)*x^2 + sqrt(14)*(423*d^3*e^3 - 2734*d^2*e^4 + 293*d*e^5 + (4
23*d^2*e^4 - 2734*d*e^5 + 293*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) - 28
0*(25*d^5*e - 20*d^4*e^2 + 34*d^3*e^3 - 12*d^2*e^4 + 9*d*e^5)*x + 350*(40*
d^6 + d^5*e + 28*d^4*e^2 + 44*d^3*e^3 - 2*d^2*e^4 + d*e^5 + (40*d^5*e + d^
4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)*x)*log(e*x + d) - 14*(229
*d^3*e^3 - 7*d^2*e^4 - 136*d*e^5 + (229*d^2*e^4 - 7*d*e^5 - 136*e^6)*x)*lo
g(5*x^2 + 2*x + 3))/(25*d^5*e^3 - 20*d^4*e^4 + 34*d^3*e^5 - 12*d^2*e^6 + 9
*d*e^7 + (25*d^4*e^4 - 20*d^3*e^5 + 34*d^2*e^6 - 12*d*e^7 + 9*e^8)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx &= -\frac{\sqrt{14}(423\,d^2-2734\,de+293\,e^2)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} \\ &\quad-\frac{(40\,d^5+d^4e+28\,d^3e^2+44\,d^2e^3-2\,de^4+e^5)\log(ex+d)}{25\,d^4e^3-20\,d^3e^4+34\,d^2e^5-12\,de^6+9\,e^7} \\ &\quad+\frac{(229\,d^2-7\,de-136\,e^2)\log(5\,x^2+2\,x+3)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} \\ &\quad-\frac{4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4}{5\,d^3e^3-2\,d^2e^4+3\,de^5+(5\,d^2e^4-2\,de^5+3\,e^6)x}+\frac{4\,x}{5\,e^2}\end{aligned}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
-1/350*sqrt(14)*(423*d^2 - 2734*d*e + 293*e^2)*arctan(1/14*sqrt(14)*(5*x +
 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (40*d^5 + d^4*e
 + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5)*log(e*x + d)/(25*d^4*e^3 - 20*
d^3*e^4 + 34*d^2*e^5 - 12*d*e^6 + 9*e^7) + 1/25*(229*d^2 - 7*d*e - 136*e^2
)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)
 - (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(5*d^3*e^3 - 2*d^2*e^4 +
3*d*e^5 + (5*d^2*e^4 - 2*d*e^5 + 3*e^6)*x) + 4/5*x/e^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.53

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx = \frac{(229\,d^2-7\,de-136\,e^2)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{5\,d^2e^6-2\,de^7+3\,e^8} - \frac{\sqrt{14}(423\,d^2e^2-2734\,de^3+293\,e^4)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{350\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)e^2} + \frac{(40\,d+33\,e)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{25\,e^3} + \frac{4\,(ex+d)}{5\,e^3}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
1/25*(229*d^2 - 7*d*e - 136*e^2)*log(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 +
 2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d)^2 + 5)/(25*d^4 - 20*d
^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(
e*x + d) + 3*d^2*e^5/(e*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(5*d^2
*e^6 - 2*d*e^7 + 3*e^8) - 1/350*sqrt(14)*(423*d^2*e^2 - 2734*d*e^3 + 293*e
^4)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) + 2*d*e/(e*x + d) - e - 3*
e^2/(e*x + d))/e)/((25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*e^2
) + 1/25*(40*d + 33*e)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + 4/5*(e
*x + d)/e^3
```

Mupad [B] (verification not implemented)

Time = 17.05 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.34

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)}\,dx = \frac{4\,x}{5\,e^2} - \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}-\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}+\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}+\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} + \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{700}+\frac{229}{25}\mathrm{i}\right)\,d^2+\left(-\frac{1367\,\sqrt{14}}{350}-\frac{7}{25}\mathrm{i}\right)\,d\,e+\left(\frac{293\,\sqrt{14}}{700}-\frac{136}{25}\mathrm{i}\right)\,e^2\right)}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} - \frac{5\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,\left(5\,x\,e^3+5\,d\,e^2\right)\,\left(5\,d^2-2\,d\,e+3\,e^2\right)} - \frac{\ln\left(d+e\,x\right)\,\left(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5\right)}{e^3\,{\left(5\,d^2-2\,d\,e+3\,e^2\right)}^2}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)),x)
```

output
```
(4*x)/(5*e^2) - (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 -
 229i/25) + e^2*((293*14^(1/2))/700 + 136i/25) - d*e*((1367*14^(1/2))/350
- 7i/25)))/(d^4*25i - d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) + (log
(x + (14^(1/2)*1i)/5 + 1/5)*(d^2*((423*14^(1/2))/700 + 229i/25) + e^2*((29
3*14^(1/2))/700 - 136i/25) - d*e*((1367*14^(1/2))/350 + 7i/25)))/(d^4*25i
- d^3*e*20i - d*e^3*12i + e^4*9i + d^2*e^2*34i) - (5*(5*d^3*e - d*e^3 + 4*
d^4 + 2*e^4 + 3*d^2*e^2))/(e*(5*d*e^2 + 5*e^3*x)*(5*d^2 - 2*d*e + 3*e^2))
- (log(d + e*x)*(d^4*e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2)
)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)}\,dx$$
$$= \frac{-423\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^4e^3+2734\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^3e^4-293\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d^2e^5+3206\log(5x^2+2x}{}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3),x)
```

output
```
( - 423*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**3 - 423*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d**3*e**4*x + 2734*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
3*e**4 + 2734*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**5*x - 293*sqrt(14)
*atan((5*x + 1)/sqrt(14))*d**2*e**5 - 293*sqrt(14)*atan((5*x + 1)/sqrt(14)
)*d*e**6*x + 3206*log(5*x**2 + 2*x + 3)*d**4*e**3 + 3206*log(5*x**2 + 2*x
+ 3)*d**3*e**4*x - 98*log(5*x**2 + 2*x + 3)*d**3*e**4 - 98*log(5*x**2 + 2*
x + 3)*d**2*e**5*x - 1904*log(5*x**2 + 2*x + 3)*d**2*e**5 - 1904*log(5*x**
2 + 2*x + 3)*d*e**6*x - 14000*log(d + e*x)*d**7 - 14000*log(d + e*x)*d**6*
e*x - 350*log(d + e*x)*d**6*e - 350*log(d + e*x)*d**5*e**2*x - 9800*log(d
+ e*x)*d**5*e**2 - 9800*log(d + e*x)*d**4*e**3*x - 15400*log(d + e*x)*d**4
*e**3 - 15400*log(d + e*x)*d**3*e**4*x + 700*log(d + e*x)*d**3*e**4 + 700*
log(d + e*x)*d**2*e**5*x - 350*log(d + e*x)*d**2*e**5 - 350*log(d + e*x)*d
*e**6*x + 14000*d**6*e*x + 7000*d**5*e**2*x**2 + 350*d**5*e**2*x - 5600*d*
*4*e**3*x**2 + 15470*d**4*e**3*x + 9520*d**3*e**4*x**2 - 1960*d**3*e**4*x
- 3360*d**2*e**5*x**2 + 9870*d**2*e**5*x + 2520*d*e**6*x**2 - 2450*d*e**6*
x + 2100*e**7*x)/(350*d*e**3*(25*d**5 + 25*d**4*e*x - 20*d**4*e - 20*d**3*
e**2*x + 34*d**3*e**2 + 34*d**2*e**3*x - 12*d**2*e**3 - 12*d*e**4*x + 9*d*
e**4 + 9*e**5*x))
```

3.145 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 317

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx = -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3(5d^2-2de+3e^2)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3(5d^2-2de+3e^2)^2(d+ex)} - \frac{(423d^3-4101d^2e+879de^2+703e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{5\sqrt{14}(5d^2-2de+3e^2)^3} + \frac{(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6)\log(d+ex)}{e^3(5d^2-2de+3e^2)^3} + \frac{(458d^3-21d^2e-816de^2+113e^3)\log(3+2x+5x^2)}{10(5d^2-2de+3e^2)^3}$$

output

```
-1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3/(5*d^2-2*d*e+3*e^2)/(e*x+d)
^2+(40*d^5+d^4*e+28*d^3*e^2+44*d^2*e^3-2*d*e^4+e^5)/e^3/(5*d^2-2*d*e+3*e^2
)^2/(e*x+d)-1/70*(423*d^3-4101*d^2*e+879*d*e^2+703*e^3)*arctan(1/14*(1+5*x
)*14^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^3+(100*d^6-120*d^5*e+228*d^4*e^2-
242*d^3*e^3+141*d^2*e^4+120*d*e^5-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3
+1/10*(458*d^3-21*d^2*e-816*d*e^2+113*e^3)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*
e^2)^3
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx =$$
$$-\frac{\frac{35(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e^3(d+ex)^2}-\frac{70(5d^2-2de+3e^2)(40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5)}{e^3(d+ex)}+\sqrt{14}(423d^3-4}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),
x]
```

output
```
-1/70*((35*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4))/(e^3*(d + e*x)^2) - (70*(5*d^2 - 2*d*e + 3*e^2)*(40*d^5 + d^4*e
+ 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 + e^5))/(e^3*(d + e*x)) + Sqrt[14]*(42
3*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + (70
*(-100*d^6 + 120*d^5*e - 228*d^4*e^2 + 242*d^3*e^3 - 141*d^2*e^4 - 120*d*e
^5 + e^6)*Log[d + e*x])/e^3 - 7*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)
*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^3
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)(d+ex)^3}\,dx$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{7d^3+816d^2e+x(458d^3-21d^2e-816de^2+113e^3)-339de^2-118e^3}{(5x^2+2x+3)(5d^2-2de+3e^2)^3}+\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e^2(5d^2-2de+3e^2)(d+ex)^3}+\right.$$

$\downarrow$ 2009

$$-\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(423d^3-4101d^2e+879de^2+703e^3\right)}{5\sqrt{14}\left(5d^2-2de+3e^2\right)^3}+\frac{\left(458d^3-21d^2e-816de^2+113e^3\right)\log\left(5x^2+2x+3\right)}{10\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e^3\left(5d^2-2de+3e^2\right)(d+ex)^2}+\frac{40d^5+d^4e+28d^3e^2+44d^2e^3-2de^4+e^5}{e^3\left(5d^2-2de+3e^2\right)^2(d+ex)}+\frac{\left(100d^6-120d^5e+228d^4e^2-242d^3e^3+141d^2e^4+120de^5-e^6\right)\log(d+ex)}{e^3\left(5d^2-2de+3e^2\right)^3}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)),x]
```

output
```
-1/2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)/(e^3*(5*d^2 - 2*d*e + 3
*e^2)*(d + e*x)^2) + (40*d^5 + d^4*e + 28*d^3*e^2 + 44*d^2*e^3 - 2*d*e^4 +
 e^5)/(e^3*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) - ((423*d^3 - 4101*d^2*e +
 879*d*e^2 + 703*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d
*e + 3*e^2)^3) + ((100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3*e^3 + 141*d
^2*e^4 + 120*d*e^5 - e^6)*Log[d + e*x])/(e^3*(5*d^2 - 2*d*e + 3*e^2)^3) +
((458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*Log[3 + 2*x + 5*x^2])/(10*(5*d
^2 - 2*d*e + 3*e^2)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{\left(458d^3-21d^2e-816d\,e^2+113e^3\right)\ln\left(5x^2+2x+3\right)}{10}+\frac{\left(-\frac{423}{5}d^3+\frac{4101}{5}d^2e-\frac{879}{5}d\,e^2-\frac{703}{5}e^3\right)\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{14}}{\left(5d^2-2de+3e^2\right)^3}-\frac{4d^4+5d^3e+3d^2e^2-}{2e^3\left(5d^2-2de+3e^2\right.}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x,method=_RETURNVERBOS
E)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^3*(1/10*(458*d^3-21*d^2*e-816*d*e^2+113*e^3)*ln(5*x^
2+2*x+3)+1/14*(-423/5*d^3+4101/5*d^2*e-879/5*d*e^2-703/5*e^3)*14^(1/2)*arc
tan(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e^3
/(5*d^2-2*d*e+3*e^2)/(e*x+d)^2-(-40*d^5-d^4*e-28*d^3*e^2-44*d^2*e^3+2*d*e^
4-e^5)/e^3/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+(100*d^6-120*d^5*e+228*d^4*e^2-24
2*d^3*e^3+141*d^2*e^4+120*d*e^5-e^6)*ln(e*x+d)/e^3/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 698 vs. 2(308) = 616.

Time = 0.20 (sec) , antiderivative size = 698, normalized size of antiderivative = 2.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
1/70*(10500*d^8 - 6825*d^7*e + 14175*d^6*e^2 + 10395*d^5*e^3 - 6160*d^4*e^
4 + 12145*d^3*e^5 - 4305*d^2*e^6 + 1365*d*e^7 - 630*e^8 - sqrt(14)*(423*d^
5*e^3 - 4101*d^4*e^4 + 879*d^3*e^5 + 703*d^2*e^6 + (423*d^3*e^5 - 4101*d^2
*e^6 + 879*d*e^7 + 703*e^8)*x^2 + 2*(423*d^4*e^4 - 4101*d^3*e^5 + 879*d^2*
e^6 + 703*d*e^7)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 70*(200*d^7*e - 75*d
^6*e^2 + 258*d^5*e^3 + 167*d^4*e^4 - 14*d^3*e^5 + 141*d^2*e^6 - 8*d*e^7 +
3*e^8)*x + 70*(100*d^8 - 120*d^7*e + 228*d^6*e^2 - 242*d^5*e^3 + 141*d^4*e
^4 + 120*d^3*e^5 - d^2*e^6 + (100*d^6*e^2 - 120*d^5*e^3 + 228*d^4*e^4 - 24
2*d^3*e^5 + 141*d^2*e^6 + 120*d*e^7 - e^8)*x^2 + 2*(100*d^7*e - 120*d^6*e^
2 + 228*d^5*e^3 - 242*d^4*e^4 + 141*d^3*e^5 + 120*d^2*e^6 - d*e^7)*x)*log(
e*x + d) + 7*(458*d^5*e^3 - 21*d^4*e^4 - 816*d^3*e^5 + 113*d^2*e^6 + (458*
d^3*e^5 - 21*d^2*e^6 - 816*d*e^7 + 113*e^8)*x^2 + 2*(458*d^4*e^4 - 21*d^3*
e^5 - 816*d^2*e^6 + 113*d*e^7)*x)*log(5*x^2 + 2*x + 3))/(125*d^8*e^3 - 150
*d^7*e^4 + 285*d^6*e^5 - 188*d^5*e^6 + 171*d^4*e^7 - 54*d^3*e^8 + 27*d^2*e
^9 + (125*d^6*e^5 - 150*d^5*e^6 + 285*d^4*e^7 - 188*d^3*e^8 + 171*d^2*e^9
- 54*d*e^10 + 27*e^11)*x^2 + 2*(125*d^7*e^4 - 150*d^6*e^5 + 285*d^5*e^6 -
188*d^4*e^7 + 171*d^3*e^8 - 54*d^2*e^9 + 27*d*e^10)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log(ex+d)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}$$
$$+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log(5\,x^2+2\,x+3)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{60\,d^6-15\,d^5e+39\,d^4e^2+84\,d^3e^3-25\,d^2e^4+9\,de^5-6\,e^6+2\,(40\,d^5e+d^4e^2+28}{2\,(25\,d^6e^3-20\,d^5e^4+34\,d^4e^5-12\,d^3e^6+9\,d^2e^7+(25\,d^4e^5-20\,d^3e^6+34\,d^2e^7-12\,de^8+9\,e^9)x^2+2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
-1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sq
rt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d
^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 - 242*d^3
*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(e*x + d)/(125*d^6*e^3 - 150*d^5*
e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*e^9) + 1/10*
(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e^3)*log(5*x^2 + 2*x + 3)/(125*d^6 -
 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)
+ 1/2*(60*d^6 - 15*d^5*e + 39*d^4*e^2 + 84*d^3*e^3 - 25*d^2*e^4 + 9*d*e^5
- 6*e^6 + 2*(40*d^5*e + d^4*e^2 + 28*d^3*e^3 + 44*d^2*e^4 - 2*d*e^5 + e^6)
*x)/(25*d^6*e^3 - 20*d^5*e^4 + 34*d^4*e^5 - 12*d^3*e^6 + 9*d^2*e^7 + (25*d
^4*e^5 - 20*d^3*e^6 + 34*d^2*e^7 - 12*d*e^8 + 9*e^9)*x^2 + 2*(25*d^5*e^4 -
 20*d^4*e^5 + 34*d^3*e^6 - 12*d^2*e^7 + 9*d*e^8)*x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.38

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)}\,dx$$
$$= -\frac{\sqrt{14}(423\,d^3-4101\,d^2e+879\,de^2+703\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{70\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(458\,d^3-21\,d^2e-816\,de^2+113\,e^3)\log\left(5\,x^2+2\,x+3\right)}{10\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(100\,d^6-120\,d^5e+228\,d^4e^2-242\,d^3e^3+141\,d^2e^4+120\,de^5-e^6)\log\left(|ex+d|\right)}{125\,d^6e^3-150\,d^5e^4+285\,d^4e^5-188\,d^3e^6+171\,d^2e^7-54\,de^8+27\,e^9}$$
$$+\frac{2\,(200\,d^7-75\,d^6e+258\,d^5e^2+167\,d^4e^3-14\,d^3e^4+141\,d^2e^5-8\,de^6+3\,e^7)x+\frac{300\,d^8-195\,d^7e+405\,d^6e^2+2}{}}{2\,(5\,d^2-2\,de+3\,e^2)^3(ex+d)^2e^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
-1/70*sqrt(14)*(423*d^3 - 4101*d^2*e + 879*d*e^2 + 703*e^3)*arctan(1/14*sq
rt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d
^2*e^4 - 54*d*e^5 + 27*e^6) + 1/10*(458*d^3 - 21*d^2*e - 816*d*e^2 + 113*e
^3)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3
+ 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (100*d^6 - 120*d^5*e + 228*d^4*e^2 -
242*d^3*e^3 + 141*d^2*e^4 + 120*d*e^5 - e^6)*log(abs(e*x + d))/(125*d^6*e^
3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^2*e^7 - 54*d*e^8 + 27*
e^9) + 1/2*(2*(200*d^7 - 75*d^6*e + 258*d^5*e^2 + 167*d^4*e^3 - 14*d^3*e^4
 + 141*d^2*e^5 - 8*d*e^6 + 3*e^7)*x + (300*d^8 - 195*d^7*e + 405*d^6*e^2 +
 297*d^5*e^3 - 176*d^4*e^4 + 347*d^3*e^5 - 123*d^2*e^6 + 39*d*e^7 - 18*e^8
)/e)/((5*d^2 - 2*d*e + 3*e^2)^3*(e*x + d)^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 17.26 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx$$
$$= \frac{\frac{60\,d^6-15\,d^5\,e+39\,d^4\,e^2+84\,d^3\,e^3-25\,d^2\,e^4+9\,d\,e^5-6\,e^6}{2\,e^3\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}+\frac{x\,(40\,d^5+d^4\,e+28\,d^3\,e^2+44\,d^2\,e^3-2\,d\,e^4+e^5)}{e^2\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)}}{d^2+2\,d\,e\,x+e^2\,x^2}$$
$$-\frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{140}-\frac{229}{5}\mathrm{i}\right)\,d^3+\left(-\frac{4101\,\sqrt{14}}{140}+\frac{21}{10}\mathrm{i}\right)\,d^2\,e+\left(\frac{879\,\sqrt{14}}{140}+\frac{408}{5}\mathrm{i}\right)\,d\,e^2+\left(\frac{703\,\sqrt{14}}{140}-\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}$$
$$+\frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\,\left(\left(\frac{423\,\sqrt{14}}{140}+\frac{229}{5}\mathrm{i}\right)\,d^3+\left(-\frac{4101\,\sqrt{14}}{140}-\frac{21}{10}\mathrm{i}\right)\,d^2\,e+\left(\frac{879\,\sqrt{14}}{140}-\frac{408}{5}\mathrm{i}\right)\,d\,e^2+\left(\frac{703\,\sqrt{14}}{140}+\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,27\mathrm{i}}$$
$$+\frac{\ln(d+e\,x)\,\left(100\,d^6-120\,d^5\,e+228\,d^4\,e^2-242\,d^3\,e^3+141\,d^2\,e^4+120\,d\,e^5-e^6\right)}{e^3\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^3}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)),x)
```

output
```
((9*d*e^5 - 15*d^5*e + 60*d^6 - 6*e^6 - 25*d^2*e^4 + 84*d^3*e^3 + 39*d^4*e
^2)/(2*e^3*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x*(d^4*
e - 2*d*e^4 + 40*d^5 + e^5 + 44*d^2*e^3 + 28*d^3*e^2))/(e^2*(25*d^4 - 20*d
^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)))/(d^2 + e^2*x^2 + 2*d*e*x) - (log(x
 - (14^(1/2)*1i)/5 + 1/5)*(d^3*((423*14^(1/2))/140 - 229i/5) + e^3*((703*1
4^(1/2))/140 - 113i/10) + d*e^2*((879*14^(1/2))/140 + 408i/5) - d^2*e*((41
01*14^(1/2))/140 - 21i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i
+ d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2)*1i)/5 +
 1/5)*(d^3*((423*14^(1/2))/140 + 229i/5) + e^3*((703*14^(1/2))/140 + 113i/
10) + d*e^2*((879*14^(1/2))/140 - 408i/5) - d^2*e*((4101*14^(1/2))/140 + 2
1i/10)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3
*e^3*188i + d^4*e^2*285i) + (log(d + e*x)*(120*d*e^5 - 120*d^5*e + 100*d^6
 - e^6 + 141*d^2*e^4 - 242*d^3*e^3 + 228*d^4*e^2))/(e^3*(5*d^2 - 2*d*e + 3
*e^2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1163, normalized size of antiderivative = 3.67

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)}\,dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3),x)
```

output
```
( - 423*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**3 - 846*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d**5*e**4*x + 4101*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
5*e**4 - 423*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**5*x**2 + 8202*sqrt(
14)*atan((5*x + 1)/sqrt(14))*d**4*e**5*x - 879*sqrt(14)*atan((5*x + 1)/sqr
t(14))*d**4*e**5 + 4101*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**6*x**2 -
 1758*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**6*x - 703*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d**3*e**6 - 879*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e
**7*x**2 - 1406*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**7*x - 703*sqrt(1
4)*atan((5*x + 1)/sqrt(14))*d*e**8*x**2 + 3206*log(5*x**2 + 2*x + 3)*d**6*
e**3 + 6412*log(5*x**2 + 2*x + 3)*d**5*e**4*x - 147*log(5*x**2 + 2*x + 3)*
d**5*e**4 + 3206*log(5*x**2 + 2*x + 3)*d**4*e**5*x**2 - 294*log(5*x**2 + 2
*x + 3)*d**4*e**5*x - 5712*log(5*x**2 + 2*x + 3)*d**4*e**5 - 147*log(5*x**
2 + 2*x + 3)*d**3*e**6*x**2 - 11424*log(5*x**2 + 2*x + 3)*d**3*e**6*x + 79
1*log(5*x**2 + 2*x + 3)*d**3*e**6 - 5712*log(5*x**2 + 2*x + 3)*d**2*e**7*x
**2 + 1582*log(5*x**2 + 2*x + 3)*d**2*e**7*x + 791*log(5*x**2 + 2*x + 3)*d
*e**8*x**2 + 7000*log(d + e*x)*d**9 + 14000*log(d + e*x)*d**8*e*x - 8400*l
og(d + e*x)*d**8*e + 7000*log(d + e*x)*d**7*e**2*x**2 - 16800*log(d + e*x)
*d**7*e**2*x + 15960*log(d + e*x)*d**7*e**2 - 8400*log(d + e*x)*d**6*e**3*
x**2 + 31920*log(d + e*x)*d**6*e**3*x - 16940*log(d + e*x)*d**6*e**3 + 159
60*log(d + e*x)*d**5*e**4*x**2 - 33880*log(d + e*x)*d**5*e**4*x + 9870*...
```

3.146 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 225

$$\begin{aligned}\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx &= \frac{(500d^3-3075d^2e+1545de^2+867e^3)\,x}{3125}\\ &+\frac{e(300d^2-615de+103e^2)\,x^2}{1250}+\frac{1}{375}(60d-41e)e^2x^3+\frac{e^3x^4}{25}\\ &-\frac{170875d^3-95175d^2e-269505de^2+54969e^3+(52875d^3+449175d^2e-274845de^2-53189e^3)\,x}{437500\,(3+2x+5x^2)}\\ &+\frac{(32825d^3+317565d^2e-221643de^2-67499e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}}\\ &-\frac{(1025d^3-1545d^2e-2601de^2+832e^3)\log(3+2x+5x^2)}{6250}\end{aligned}$$

output
```
1/3125*(500*d^3-3075*d^2*e+1545*d*e^2+867*e^3)*x+1/1250*e*(300*d^2-615*d*e
+103*e^2)*x^2+1/375*(60*d-41*e)*e^2*x^3+1/25*e^3*x^4-(170875*d^3-95175*d^2
*e-269505*d*e^2+54969*e^3+(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*
x)/(2187500*x^2+875000*x+1312500)+1/1225000*(32825*d^3+317565*d^2*e-221643
*d*e^2-67499*e^3)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)-1/6250*(1025*d^3-
1545*d^2*e-2601*d*e^2+832*e^3)*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$
$$= \frac{5880(500d^3-3075d^2e+1545de^2+867e^3)\,x+14700e(300d^2-615de+103e^2)\,x^2+49000(60d-41e)e}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
(5880*(500*d^3 - 3075*d^2*e + 1545*d*e^2 + 867*e^3)*x + 14700*e*(300*d^2 -
 615*d*e + 103*e^2)*x^2 + 49000*(60*d - 41*e)*e^2*x^3 + 735000*e^3*x^4 - (
42*(e^3*(54969 - 53189*x) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 598
9*x) - 15*d*e^2*(17967 + 18323*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(32825
*d^3 + 317565*d^2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]]
 + 2940*(-1025*d^3 + 1545*d^2*e + 2601*d*e^2 - 832*e^3)*Log[3 + 2*x + 5*x^
2])/18375000
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)\,(d+ex)^3}{(5x^2+2x+3)^2} dx$$
$$\Big\downarrow \text{2175}$$
$$\frac{1}{56}\int \frac{2(d+ex)^2\,(2800ex^3+140(20d-33e)x^2-6(770d-519e)x+3(615d+1367e))}{125\,(5x^2+2x+3)}dx - \frac{(423x+1367)(d+ex)^3}{3500\,(5x^2+2x+3)}$$

$\downarrow$ 27

$$\frac{\int \frac{(d+ex)^2\left(2800ex^3+140(20d-33e)x^2-6(770d-519e)x+3(615d+1367e)\right)}{5x^2+2x+3}dx}{3500}-\frac{(423x+1367)(d+ex)^3}{3500\left(5x^2+2x+3\right)}$$

$\downarrow$ 2159

$$\frac{\int \left(560e^3x^3+28(60d-41e)e^2x^2+2e\left(840d^2-1722ed+373e^2\right)x+\frac{1}{5}\left(2800d^3-17220ed^2+9921e^2d+6053e^3\right)\right.}{3500}$$

$$\frac{(423x+1367)(d+ex)^3}{3500\left(5x^2+2x+3\right)}$$

$\downarrow$ 2009

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(32825d^3+317565d^2e-221643de^2-67499e^3\right)}{25\sqrt{14}}+ex^2\left(840d^2-1722de+373e^2\right)-\frac{14}{25}\left(1025d^3-1545d^2e-2601d\right.}{350}$$

$$\frac{(423x+1367)(d+ex)^3}{3500\left(5x^2+2x+3\right)}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x)^3)/(3 + 2*x + 5*x^2) + (((2800*d^3 - 172
20*d^2*e + 9921*d*e^2 + 6053*e^3)*x)/5 + e*(840*d^2 - 1722*d*e + 373*e^2)*
x^2 + (28*(60*d - 41*e)*e^2*x^3)/3 + 140*e^3*x^4 + ((32825*d^3 + 317565*d^
2*e - 221643*d*e^2 - 67499*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[14])
- (14*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*Log[3 + 2*x + 5*x^2])
/25)/3500
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.95

method	result
default	$\frac{e^3x^4}{25}+\frac{4d\,e^2x^3}{25}-\frac{41e^3x^3}{375}+\frac{6x^2d^2e}{25}-\frac{123d\,e^2x^2}{250}+\frac{103e^3x^2}{1250}+\frac{4d^3x}{25}-\frac{123d^2ex}{125}+\frac{309d\,e^2x}{625}+\frac{867e^3x}{3125}-\frac{(\frac{2115}{28}d^3+$
risch	$\frac{4d^3x}{25}+\frac{309d^2e\ln(350x^2+140x+210)}{1250}+\frac{2601d\,e^2\ln(350x^2+140x+210)}{6250}+\frac{1313\sqrt{14}\,d^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}-\frac{67499\sqrt{14}\,e$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
1/25*e^3*x^4+4/25*d*e^2*x^3-41/375*e^3*x^3+6/25*x^2*d^2*e-123/250*d*e^2*x^
2+103/1250*e^3*x^2+4/25*d^3*x-123/125*d^2*e*x+309/625*d*e^2*x+867/3125*e^3
*x-1/3125*((2115/28*d^3+17967/28*d^2*e-54969/140*d*e^2-53189/700*e^3)*x+68
35/28*d^3-3807/28*d^2*e-53901/140*d*e^2+54969/700*e^3)/(x^2+2/5*x+3/5)-1/1
75000*(28700*d^3-43260*d^2*e-72828*d*e^2+23296*e^3)*ln(5*x^2+2*x+3)-1/2450
00*(-6565*d^3-63513*d^2*e+221643/5*d*e^2+67499/5*e^3)*14^(1/2)*arctan(1/28
*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{3675000\,e^3x^6+1225000\,(12\,de^2-7\,e^3)x^5+122500\,(180\,d^2e-321\,de^2+47\,e^3)x^4+147000\,(100\,d^3-55}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
1/18375000*(3675000*e^3*x^6 + 1225000*(12*d*e^2 - 7*e^3)*x^5 + 122500*(180
*d^2*e - 321*d*e^2 + 47*e^3)*x^4 + 147000*(100*d^3 - 555*d^2*e + 246*d*e^2
 + 153*e^3)*x^3 - 7176750*d^3 + 3997350*d^2*e + 11319210*d*e^2 - 2308698*e
^3 + 2940*(2000*d^3 - 7800*d^2*e - 3045*d*e^2 + 5013*e^3)*x^2 + 15*sqrt(14
)*(98475*d^3 + 952695*d^2*e - 664929*d*e^2 - 202497*e^3 + 5*(32825*d^3 + 3
17565*d^2*e - 221643*d*e^2 - 67499*e^3)*x^2 + 2*(32825*d^3 + 317565*d^2*e
- 221643*d*e^2 - 67499*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(15712
5*d^3 - 1740675*d^2*e + 923745*d*e^2 + 417329*e^3)*x - 2940*(3075*d^3 - 46
35*d^2*e - 7803*d*e^2 + 2496*e^3 + 5*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 +
 832*e^3)*x^2 + 2*(1025*d^3 - 1545*d^2*e - 2601*d*e^2 + 832*e^3)*x)*log(5*
x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{e^3x^4}{25} + x^3\cdot\left(\frac{4de^2}{25} - \frac{41e^3}{375}\right) + x^2\cdot\left(\frac{6d^2e}{25} - \frac{123de^2}{250} + \frac{103e^3}{1250}\right) + x\left(\frac{4d^3}{25} - \frac{123d^2e}{125} + \frac{309de^2}{625} + \frac{867e^3}{3125}\right) + \left(-\frac{41d^3}{250} + \frac{309d^2e}{1250} + \frac{2601de^2}{6250} - \frac{416e^3}{3125} - \frac{\sqrt{14}i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\right)\log\left(x + \frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{674\dots}{5}}{32825d^3+317565d\dots}\right.$$
$$+ \left(-\frac{41d^3}{250} + \frac{309d^2e}{1250} + \frac{2601de^2}{6250} - \frac{416e^3}{3125} + \frac{\sqrt{14}i(32825d^3+317565d^2e-221643de^2-67499e^3)}{2450000}\right)\log\left(x + \frac{6565d^3+63513d^2e-\frac{221643de^2}{5}-\frac{674\dots}{5}}{32825d^3+317565d\dots}\right.$$
$$+ \frac{-170875d^3+95175d^2e+269505de^2-54969e^3+x(-52875d^3-449175d^2e+274845de^2+53189e^3)}{2187500x^2+875000x+1312500}$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
e**3*x**4/25 + x**3*(4*d*e**2/25 - 41*e**3/375) + x**2*(6*d**2*e/25 - 123*
d*e**2/250 + 103*e**3/1250) + x*(4*d**3/25 - 123*d**2*e/125 + 309*d*e**2/6
25 + 867*e**3/3125) + (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 -
 416*e**3/3125 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 -
67499*e**3)/2450000)*log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 -
 67499*e**3/5 - sqrt(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 6
7499*e**3)/5)/(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) +
 (-41*d**3/250 + 309*d**2*e/1250 + 2601*d*e**2/6250 - 416*e**3/3125 + sqrt
(14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/2450000)*
log(x + (6565*d**3 + 63513*d**2*e - 221643*d*e**2/5 - 67499*e**3/5 + sqrt(
14)*I*(32825*d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)/5)/(32825*
d**3 + 317565*d**2*e - 221643*d*e**2 - 67499*e**3)) + (-170875*d**3 + 9517
5*d**2*e + 269505*d*e**2 - 54969*e**3 + x*(-52875*d**3 - 449175*d**2*e + 2
74845*d*e**2 + 53189*e**3))/(2187500*x**2 + 875000*x + 1312500)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{1}{25}e^3x^4 + \frac{1}{375}\left(60\,de^2 - 41\,e^3\right)x^3 + \frac{1}{1250}\left(300\,d^2e - 615\,de^2 + 103\,e^3\right)x^2$$
$$+ \frac{1}{1225000}\sqrt{14}\left(32825\,d^3 + 317565\,d^2e - 221643\,de^2 - 67499\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)$$
$$+ \frac{1}{3125}\left(500\,d^3 - 3075\,d^2e + 1545\,de^2 + 867\,e^3\right)x$$
$$- \frac{1}{6250}\left(1025\,d^3 - 1545\,d^2e - 2601\,de^2 + 832\,e^3\right)\log\left(5\,x^2 + 2\,x + 3\right)$$
$$- \frac{170875\,d^3 - 95175\,d^2e - 269505\,de^2 + 54969\,e^3 + \left(52875\,d^3 + 449175\,d^2e - 274845\,de^2 - 53189\,e^3\right)x}{437500\left(5\,x^2 + 2\,x + 3\right)}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/25*e^3*x^4 + 1/375*(60*d*e^2 - 41*e^3)*x^3 + 1/1250*(300*d^2*e - 615*d*e
^2 + 103*e^3)*x^2 + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 221643*
d*e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/3125*(500*d^3 - 307
5*d^2*e + 1545*d*e^2 + 867*e^3)*x - 1/6250*(1025*d^3 - 1545*d^2*e - 2601*d
*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(170875*d^3 - 95175*d^2*e
- 269505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e - 274845*d*e^2 - 53
189*e^3)*x)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{1}{25}e^3x^4+\frac{4}{25}de^2x^3-\frac{41}{375}e^3x^3+\frac{6}{25}d^2ex^2-\frac{123}{250}de^2x^2+\frac{103}{1250}e^3x^2+\frac{4}{25}d^3x-\frac{123}{125}d^2ex+\frac{309}{625}de^2x+\frac{867}{3125}e^3x+\frac{1}{1225000}\sqrt{14}\left(32825\,d^3+317565\,d^2e-221643\,de^2-67499\,e^3\right)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)-\frac{1}{6250}\left(1025\,d^3-1545\,d^2e-2601\,de^2+832\,e^3\right)\log\left(5\,x^2+2\,x+3\right)-\frac{170875\,d^3-95175\,d^2e-269505\,de^2+54969\,e^3+(52875\,d^3+449175\,d^2e-274845\,de^2-53189\,e^3)x}{437500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
1/25*e^3*x^4 + 4/25*d*e^2*x^3 - 41/375*e^3*x^3 + 6/25*d^2*e*x^2 - 123/250*
d*e^2*x^2 + 103/1250*e^3*x^2 + 4/25*d^3*x - 123/125*d^2*e*x + 309/625*d*e^
2*x + 867/3125*e^3*x + 1/1225000*sqrt(14)*(32825*d^3 + 317565*d^2*e - 2216
43*d*e^2 - 67499*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^3 -
 1545*d^2*e - 2601*d*e^2 + 832*e^3)*log(5*x^2 + 2*x + 3) - 1/437500*(17087
5*d^3 - 95175*d^2*e - 269505*d*e^2 + 54969*e^3 + (52875*d^3 + 449175*d^2*e
 - 274845*d*e^2 - 53189*e^3)*x)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} dx$$
$$= \frac{\frac{53901\,d\,e^2}{28} + \frac{19035\,d^2\,e}{28} + x\left(-\frac{10575\,d^3}{28} - \frac{89835\,d^2\,e}{28} + \frac{54969\,d\,e^2}{28} + \frac{53189\,e^3}{140}\right) - \frac{34175\,d^3}{28} - \frac{54969\,e^3}{140}}{15625\,x^2 + 6250\,x + 9375}$$
$$+ x^3\left(\frac{e^2\,(12\,d-5\,e)}{75} - \frac{16\,e^3}{375}\right)$$
$$- x\left(\frac{18\,e^2\,(12\,d-5\,e)}{625} + \frac{12\,e\,(4\,d^2-5\,d\,e+e^2)}{125} - \frac{9\,d\,e^2}{25} + \frac{3\,d^2\,e}{5} - \frac{4\,d^3}{25} - \frac{717\,e^3}{3125}\right)$$
$$+ \ln\left(5\,x^2+2\,x+3\right)\left(-\frac{41\,d^3}{250} + \frac{309\,d^2\,e}{1250} + \frac{2601\,d\,e^2}{6250} - \frac{416\,e^3}{3125}\right)$$
$$- x^2\left(\frac{2\,e^2\,(12\,d-5\,e)}{125} - \frac{3\,e\,(4\,d^2-5\,d\,e+e^2)}{50} + \frac{36\,e^3}{625}\right) + \frac{e^3\,x^4}{25}$$
$$- \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{1225000} + \frac{\sqrt{14}\,x\left(-32825\,d^3-317565\,d^2\,e+221643\,d\,e^2+67499\,e^3\right)}{245000}}{-\frac{1313\,d^3}{3500} - \frac{63513\,d^2\,e}{17500} + \frac{221643\,d\,e^2}{87500} + \frac{67499\,e^3}{87500}}\right)\left(-32825\,d^3 - \right.}{1225000}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
((53901*d*e^2)/28 + (19035*d^2*e)/28 + x*((54969*d*e^2)/28 - (89835*d^2*e)
/28 - (10575*d^3)/28 + (53189*e^3)/140) - (34175*d^3)/28 - (54969*e^3)/140
)/(6250*x + 15625*x^2 + 9375) + x^3*((e^2*(12*d - 5*e))/75 - (16*e^3)/375)
 - x*((18*e^2*(12*d - 5*e))/625 + (12*e*(4*d^2 - 5*d*e + e^2))/125 - (9*d*
e^2)/25 + (3*d^2*e)/5 - (4*d^3)/25 - (717*e^3)/3125) + log(2*x + 5*x^2 + 3
)*((2601*d*e^2)/6250 + (309*d^2*e)/1250 - (41*d^3)/250 - (416*e^3)/3125) -
 x^2*((2*e^2*(12*d - 5*e))/125 - (3*e*(4*d^2 - 5*d*e + e^2))/50 + (36*e^3)
/625) + (e^3*x^4)/25 - (14^(1/2)*atan(((14^(1/2)*(221643*d*e^2 - 317565*d^
2*e - 32825*d^3 + 67499*e^3))/1225000 + (14^(1/2)*x*(221643*d*e^2 - 317565
*d^2*e - 32825*d^3 + 67499*e^3))/245000)/((221643*d*e^2)/87500 - (63513*d^
2*e)/17500 - (1313*d^3)/3500 + (67499*e^3)/87500))*(221643*d*e^2 - 317565*
d^2*e - 32825*d^3 + 67499*e^3))/1225000
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.70

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
(492375*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*x**2 + 196950*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d**3*x + 295425*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
3 + 4763475*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e*x**2 + 1905390*sqrt(1
4)*atan((5*x + 1)/sqrt(14))*d**2*e*x + 2858085*sqrt(14)*atan((5*x + 1)/sqr
t(14))*d**2*e - 3324645*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**2*x**2 - 13
29858*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**2*x - 1994787*sqrt(14)*atan((
5*x + 1)/sqrt(14))*d*e**2 - 1012485*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3
*x**2 - 404994*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3*x - 607491*sqrt(14)*
atan((5*x + 1)/sqrt(14))*e**3 - 3013500*log(5*x**2 + 2*x + 3)*d**3*x**2 -
1205400*log(5*x**2 + 2*x + 3)*d**3*x - 1808100*log(5*x**2 + 2*x + 3)*d**3
+ 4542300*log(5*x**2 + 2*x + 3)*d**2*e*x**2 + 1816920*log(5*x**2 + 2*x + 3
)*d**2*e*x + 2725380*log(5*x**2 + 2*x + 3)*d**2*e + 7646940*log(5*x**2 + 2
*x + 3)*d*e**2*x**2 + 3058776*log(5*x**2 + 2*x + 3)*d*e**2*x + 4588164*log
(5*x**2 + 2*x + 3)*d*e**2 - 2446080*log(5*x**2 + 2*x + 3)*e**3*x**2 - 9784
32*log(5*x**2 + 2*x + 3)*e**3*x - 1467648*log(5*x**2 + 2*x + 3)*e**3 + 294
0000*d**3*x**3 - 2123625*d**3*x**2 - 3415125*d**3 + 4410000*d**2*e*x**4 -
16317000*d**2*e*x**3 + 31967775*d**2*e*x**2 + 22731975*d**2*e + 2940000*d*
e**2*x**5 - 7864500*d*e**2*x**4 + 7232400*d*e**2*x**3 - 21189105*d*e**2*x*
*2 - 9375345*d*e**2 + 735000*e**3*x**6 - 1715000*e**3*x**5 + 1151500*e**3*
x**4 + 4498200*e**3*x**3 - 5816265*e**3*x**2 - 5720085*e**3)/(3675000*(...
```

3.147 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 160

$$\begin{aligned}
\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx &= \frac{1}{625}\left(100d^2-410de+103e^2\right)x+\frac{1}{250}(40d-41e)ex^2+\frac{4e^2x^3}{75} \\
&\quad -\frac{34175d^2-12690de-17967e^2+\left(10575d^2+59890de-18323e^2\right)x}{87500\left(3+2x+5x^2\right)} \\
&\quad +\frac{\left(32825d^2+211710de-73881e^2\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{87500\sqrt{14}} \\
&\quad -\frac{\left(1025d^2-1030de-867e^2\right)\log\left(3+2x+5x^2\right)}{6250}
\end{aligned}$$

output
```
1/625*(100*d^2-410*d*e+103*e^2)*x+1/250*(40*d-41*e)*e*x^2+4/75*e^2*x^3-(34
175*d^2-12690*d*e-17967*e^2+(10575*d^2+59890*d*e-18323*e^2)*x)/(437500*x^2
+175000*x+262500)+1/1225000*(32825*d^2+211710*d*e-73881*e^2)*arctan(1/14*(
1+5*x)*14^(1/2))*14^(1/2)-1/6250*(1025*d^2-1030*d*e-867*e^2)*ln(5*x^2+2*x+
3)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{5880(100d^2-410de+103e^2)\,x+14700(40d-41e)ex^2+196000e^2x^3-\frac{42(25d^2(1367+423x)+10de(-1269+5989x}{3+2x+5x^2}}{}$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
(5880*(100*d^2 - 410*d*e + 103*e^2)*x + 14700*(40*d - 41*e)*e*x^2 + 196000
*e^2*x^3 - (42*(25*d^2*(1367 + 423*x) + 10*d*e*(-1269 + 5989*x) - e^2*(179
67 + 18323*x)))/(3 + 2*x + 5*x^2) + 3*Sqrt[14]*(32825*d^2 + 211710*d*e - 7
3881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]] + 588*(-1025*d^2 + 1030*d*e + 867*e^2
)*Log[3 + 2*x + 5*x^2])/3675000
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)\,(d+ex)^2}{(5x^2+2x+3)^2}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{1}{56}\int \frac{2(d+ex)\,(2800ex^3+140(20d-33e)x^2-3(1540d-897e)x+1845d+2734e)}{125\,(5x^2+2x+3)}dx-\frac{(423x+1367)(d+ex)^2}{3500\,(5x^2+2x+3)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)(2800ex^3+140(20d-33e)x^2-3(1540d-897e)x+1845d+2734e)}{5x^2+2x+3}dx}{3500} - \frac{(423x+1367)(d+ex)^2}{3500\,(5x^2+2x+3)}$$

$$\Big\downarrow \; 2159$$

$$\frac{\int \left(560e^2x^2+28(40d-41e)ex+\frac{1}{5}\left(2800d^2-11480ed+3307e^2\right)+\frac{825d^2+48110ed-9921e^2-28\left(1025d^2-1030ed-867e^2\right)x}{5(5x^2+2x+3)}\right.}{3500}$$
$$\frac{(423x+1367)(d+ex)^2}{3500\,(5x^2+2x+3)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(32825d^2+211710de-73881e^2\right)}{25\sqrt{14}} - \frac{14}{25}\left(1025d^2-1030de-867e^2\right)\log\left(5x^2+2x+3\right)+\frac{1}{5}x\left(2800d^2-11480d\right.}{3500}$$
$$\frac{(423x+1367)(d+ex)^2}{3500\,(5x^2+2x+3)}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x)^2)/(3 + 2*x + 5*x^2) + (((2800*d^2 - 114
80*d*e + 3307*e^2)*x)/5 + 14*(40*d - 41*e)*e*x^2 + (560*e^2*x^3)/3 + ((328
25*d^2 + 211710*d*e - 73881*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(25*Sqrt[14])
 - (14*(1025*d^2 - 1030*d*e - 867*e^2)*Log[3 + 2*x + 5*x^2])/25)/3500
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

method	result
default	$\frac{4e^2x^3}{75}+\frac{4x^2de}{25}-\frac{41e^2x^2}{250}+\frac{4d^2x}{25}-\frac{82dex}{125}+\frac{103e^2x}{625}-\frac{\left(\frac{423}{28}d^2+\frac{5989}{70}de-\frac{18323}{700}e^2\right)x+\frac{1367d^2}{28}-\frac{1269de}{70}-\frac{17967e^2}{700}}{625\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{(287$
risch	$\frac{4d^2x}{25}+\frac{103e^2x}{625}+\frac{4e^2x^3}{75}+\frac{4x^2de}{25}+\frac{21171\sqrt{14}\,de\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{122500}-\frac{82dex}{125}-\frac{41d^2\ln\left(350x^2+140x+210\right)}{250}+\frac{867e}{}$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
4/75*e^2*x^3+4/25*x^2*d*e-41/250*e^2*x^2+4/25*d^2*x-82/125*d*e*x+103/625*e
^2*x-1/625*((423/28*d^2+5989/70*d*e-18323/700*e^2)*x+1367/28*d^2-1269/70*d
*e-17967/700*e^2)/(x^2+2/5*x+3/5)-1/175000*(28700*d^2-28840*d*e-24276*e^2)
*ln(5*x^2+2*x+3)-1/245000*(-6565*d^2-42342*d*e+73881/5*e^2)*14^(1/2)*arcta
n(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{980000\,e^2x^5+24500\,(120\,de-107\,e^2)x^4+58800\,(50\,d^2-185\,de+41\,e^2)x^3+2940\,(400\,d^2-1040\,de-}{}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
1/3675000*(980000*e^2*x^5 + 24500*(120*d*e - 107*e^2)*x^4 + 58800*(50*d^2
- 185*d*e + 41*e^2)*x^3 + 2940*(400*d^2 - 1040*d*e - 203*e^2)*x^2 + 3*sqrt
(14)*(5*(32825*d^2 + 211710*d*e - 73881*e^2)*x^2 + 98475*d^2 + 635130*d*e
- 221643*e^2 + 2*(32825*d^2 + 211710*d*e - 73881*e^2)*x)*arctan(1/14*sqrt(
14)*(5*x + 1)) - 1435350*d^2 + 532980*d*e + 754614*e^2 + 42*(31425*d^2 - 2
32090*d*e + 61583*e^2)*x - 588*(5*(1025*d^2 - 1030*d*e - 867*e^2)*x^2 + 30
75*d^2 - 3090*d*e - 2601*e^2 + 2*(1025*d^2 - 1030*d*e - 867*e^2)*x)*log(5*
x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx$$

$$= \frac{4e^2x^3}{75} + x^2 \cdot \left(\frac{4de}{25} - \frac{41e^2}{250}\right) + x\left(\frac{4d^2}{25} - \frac{82de}{125} + \frac{103e^2}{625}\right) + \left(-\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} - \frac{\sqrt{14}i(32825d^2+211710de-73881e^2)}{2450000}\right)\log\left(x + \frac{6565d^2+42342de-\frac{73881e^2}{5} - \frac{\sqrt{14}i(32825d^2+211710de-}{5}}{32825d^2+211710de-73881e^2}\right.$$

$$+ \left(-\frac{41d^2}{250} + \frac{103de}{625} + \frac{867e^2}{6250} + \frac{\sqrt{14}i(32825d^2+211710de-73881e^2)}{2450000}\right)\log\left(x + \frac{6565d^2+42342de-\frac{73881e^2}{5} + \frac{\sqrt{14}i(32825d^2+211710de-}{5}}{32825d^2+211710de-73881e^2}\right.$$

$$+ \frac{-34175d^2+12690de+17967e^2+x(-10575d^2-59890de+18323e^2)}{437500x^2+175000x+262500}$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
4*e**2*x**3/75 + x**2*(4*d*e/25 - 41*e**2/250) + x*(4*d**2/25 - 82*d*e/125
 + 103*e**2/625) + (-41*d**2/250 + 103*d*e/625 + 867*e**2/6250 - sqrt(14)*
I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000)*log(x + (6565*d**2 + 423
42*d*e - 73881*e**2/5 - sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/
5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-41*d**2/250 + 103*d*e/625 +
 867*e**2/6250 + sqrt(14)*I*(32825*d**2 + 211710*d*e - 73881*e**2)/2450000
)*log(x + (6565*d**2 + 42342*d*e - 73881*e**2/5 + sqrt(14)*I*(32825*d**2 +
 211710*d*e - 73881*e**2)/5)/(32825*d**2 + 211710*d*e - 73881*e**2)) + (-3
4175*d**2 + 12690*d*e + 17967*e**2 + x*(-10575*d**2 - 59890*d*e + 18323*e*
*2))/(437500*x**2 + 175000*x + 262500)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{4}{75}\,e^2x^3 + \frac{1}{250}\,\left(40\,de - 41\,e^2\right)x^2 + \frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{625}\,\left(100\,d^2 - 410\,de + 103\,e^2\right)x - \frac{1}{6250}\,\left(1025\,d^2 - 1030\,de - 867\,e^2\right)\log\left(5\,x^2 + 2\,x + 3\right) - \frac{34175\,d^2 - 12690\,de - 17967\,e^2 + \left(10575\,d^2 + 59890\,de - 18323\,e^2\right)x}{87500\,\left(5\,x^2 + 2\,x + 3\right)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
4/75*e^2*x^3 + 1/250*(40*d*e - 41*e^2)*x^2 + 1/1225000*sqrt(14)*(32825*d^2
 + 211710*d*e - 73881*e^2)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(100*d^
2 - 410*d*e + 103*e^2)*x - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*log(5*x^
2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d^2 + 5
9890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{4}{75}\,e^2x^3 + \frac{4}{25}\,dex^2 - \frac{41}{250}\,e^2x^2 + \frac{4}{25}\,d^2x - \frac{82}{125}\,dex + \frac{103}{625}\,e^2x + \frac{1}{1225000}\,\sqrt{14}\left(32825\,d^2 + 211710\,de - 73881\,e^2\right)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{1}{6250}\,\left(1025\,d^2 - 1030\,de - 867\,e^2\right)\log\left(5\,x^2 + 2\,x + 3\right) - \frac{34175\,d^2 - 12690\,de - 17967\,e^2 + \left(10575\,d^2 + 59890\,de - 18323\,e^2\right)x}{87500\,\left(5\,x^2 + 2\,x + 3\right)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
4/75*e^2*x^3 + 4/25*d*e*x^2 - 41/250*e^2*x^2 + 4/25*d^2*x - 82/125*d*e*x +
 103/625*e^2*x + 1/1225000*sqrt(14)*(32825*d^2 + 211710*d*e - 73881*e^2)*a
rctan(1/14*sqrt(14)*(5*x + 1)) - 1/6250*(1025*d^2 - 1030*d*e - 867*e^2)*lo
g(5*x^2 + 2*x + 3) - 1/87500*(34175*d^2 - 12690*d*e - 17967*e^2 + (10575*d
^2 + 59890*d*e - 18323*e^2)*x)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.53 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^2\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \ln\left(5\,x^2+2\,x+3\right)\,\left(-\frac{41\,d^2}{250}+\frac{103\,d\,e}{625}+\frac{867\,e^2}{6250}\right) - x\left(\frac{2\,d\,e}{5}+\frac{4\,e\,(8\,d-5\,e)}{125}-\frac{4\,d^2}{25}-\frac{3\,e^2}{625}\right)+x^2\left(\frac{e\,(8\,d-5\,e)}{50}-\frac{8\,e^2}{125}\right) + \frac{\frac{1269\,d\,e}{14}-x\left(\frac{2115\,d^2}{28}+\frac{5989\,d\,e}{14}-\frac{18323\,e^2}{140}\right)-\frac{6835\,d^2}{28}+\frac{17967\,e^2}{140}}{3125\,x^2+1250\,x+1875}+\frac{4\,e^2\,x^3}{75} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000}+\frac{\sqrt{14}\,x\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{245000}}{\frac{1313\,d^2}{3500}+\frac{21171\,d\,e}{8750}-\frac{73881\,e^2}{87500}}\right)\,\left(32825\,d^2+211710\,d\,e-73881\,e^2\right)}{1225000}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
log(2*x + 5*x^2 + 3)*((103*d*e)/625 - (41*d^2)/250 + (867*e^2)/6250) - x*(
(2*d*e)/5 + (4*e*(8*d - 5*e))/125 - (4*d^2)/25 - (3*e^2)/625) + x^2*((e*(8
*d - 5*e))/50 - (8*e^2)/125) + ((1269*d*e)/14 - x*((5989*d*e)/14 + (2115*d
^2)/28 - (18323*e^2)/140) - (6835*d^2)/28 + (17967*e^2)/140)/(1250*x + 312
5*x^2 + 1875) + (4*e^2*x^3)/75 + (14^(1/2)*atan(((14^(1/2)*(211710*d*e + 3
2825*d^2 - 73881*e^2))/1225000 + (14^(1/2)*x*(211710*d*e + 32825*d^2 - 738
81*e^2))/245000)/((21171*d*e)/8750 + (1313*d^2)/3500 - (73881*e^2)/87500))
*(211710*d*e + 32825*d^2 - 73881*e^2))/1225000
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.63

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2} \, dx$$
$$= \frac{2940000d^2x^3 + 980000e^2x^5 - 2621500e^2x^4 + 2410800e^2x^3 - 7063035e^2x^2 + 295425\sqrt{14}\, atan\left(\frac{5x+1}{\sqrt{14}}\right) d^2}{}$$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
(492375*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*x**2 + 196950*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d**2*x + 295425*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
2 + 3175650*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e*x**2 + 1270260*sqrt(14)*
atan((5*x + 1)/sqrt(14))*d*e*x + 1905390*sqrt(14)*atan((5*x + 1)/sqrt(14))
*d*e - 1108215*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**2*x**2 - 443286*sqrt(1
4)*atan((5*x + 1)/sqrt(14))*e**2*x - 664929*sqrt(14)*atan((5*x + 1)/sqrt(1
4))*e**2 - 3013500*log(5*x**2 + 2*x + 3)*d**2*x**2 - 1205400*log(5*x**2 +
2*x + 3)*d**2*x - 1808100*log(5*x**2 + 2*x + 3)*d**2 + 3028200*log(5*x**2
+ 2*x + 3)*d*e*x**2 + 1211280*log(5*x**2 + 2*x + 3)*d*e*x + 1816920*log(5*
x**2 + 2*x + 3)*d*e + 2548980*log(5*x**2 + 2*x + 3)*e**2*x**2 + 1019592*lo
g(5*x**2 + 2*x + 3)*e**2*x + 1529388*log(5*x**2 + 2*x + 3)*e**2 + 2940000*
d**2*x**3 - 2123625*d**2*x**2 - 3415125*d**2 + 2940000*d*e*x**4 - 10878000
*d*e*x**3 + 21311850*d*e*x**2 + 15154650*d*e + 980000*e**2*x**5 - 2621500*
e**2*x**4 + 2410800*e**2*x**3 - 7063035*e**2*x**2 - 3125115*e**2)/(3675000
*(5*x**2 + 2*x + 3))
```

3.148 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 103

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{(3+2x+5x^2)^2}\,dx = \frac{1}{125}(20d-41e)x+\frac{2ex^2}{25} - \frac{6835d-1269e+(2115d+5989e)x}{17500\left(3+2x+5x^2\right)} + \frac{(6565d+21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{17500\sqrt{14}} - \frac{(205d-103e)\log\left(3+2x+5x^2\right)}{1250}$$

output
```
1/125*(20*d-41*e)*x+2/25*e*x^2-(6835*d-1269*e+(2115*d+5989*e)*x)/(87500*x^
2+35000*x+52500)+1/245000*(6565*d+21171*e)*arctan(1/14*(1+5*x)*14^(1/2))*1
4^(1/2)-1/1250*(205*d-103*e)*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$=\frac{1960(20d-41e)x+19600ex^2-\frac{14(5d(1367+423x)+e(-1269+5989x))}{3+2x+5x^2}+\sqrt{14}(6565d+21171e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+}{245000}$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,
x]
```

output
```
(1960*(20*d - 41*e)*x + 19600*e*x^2 - (14*(5*d*(1367 + 423*x) + e*(-1269 +
 5989*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d + 21171*e)*ArcTan[(1 + 5*x
)/Sqrt[14]] + 196*(-205*d + 103*e)*Log[3 + 2*x + 5*x^2])/245000
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2175, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{\left(5x^2+2x+3\right)^2}\,dx$$
$$\Big\downarrow \text{2175}$$
$$\frac{1}{56}\int \frac{2\left(2800ex^3+140(20d-33e)x^2-84(55d-27e)x+1845d+1367e\right)}{125\left(5x^2+2x+3\right)}dx-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{2800ex^3+140(20d-33e)x^2-84(55d-27e)x+1845d+1367e}{5x^2+2x+3}dx}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{\int \left(28(20d-41e)+560ex+\frac{165d+4811e-28(205d-103e)x}{5x^2+2x+3}\right)dx}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d+21171e)}{5\sqrt{14}}-\frac{14}{5}(205d-103e)\log\left(5x^2+2x+3\right)+28x(20d-41e)+280ex^2}{3500}-\frac{(423x+1367)(d+ex)}{3500\left(5x^2+2x+3\right)}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2) + (28*(20*d - 41*e)*x
 + 280*e*x^2 + ((6565*d + 21171*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]
) - (14*(205*d - 103*e)*Log[3 + 2*x + 5*x^2])/5)/3500
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

method	result
default	$\frac{2e\,x^{2}}{25}+\frac{4dx}{25}-\frac{41ex}{125}-\frac{\left(\frac{423d}{140}+\frac{5989e}{700}\right)x+\frac{1367d}{140}-\frac{1269e}{700}}{125\left(x^{2}+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{\left(5740d-2884e\right)\ln\left(5x^{2}+2x+3\right)}{35000}-\frac{\left(-1313d-\frac{21171e}{5}\right)\sqrt{14}\,\arctan}{49000}$
risch	$\frac{2e\,x^{2}}{25}+\frac{4dx}{25}-\frac{41ex}{125}+\frac{\frac{\left(-\frac{423d}{140}-\frac{5989e}{700}\right)x}{125}-\frac{1367d}{17500}+\frac{1269e}{87500}}{x^{2}+\frac{2}{5}x+\frac{3}{5}}-\frac{41d\ln\left(350x^{2}+140x+210\right)}{250}+\frac{103e\ln\left(350x^{2}+140x+210\right)}{1250}+$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOS
E)
```

output
```
2/25*e*x^2+4/25*d*x-41/125*e*x-1/125*((423/140*d+5989/700*e)*x+1367/140*d-
1269/700*e)/(x^2+2/5*x+3/5)-1/35000*(5740*d-2884*e)*ln(5*x^2+2*x+3)-1/4900
0*(-1313*d-21171/5*e)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{98000\,ex^4+9800\,(20\,d-37\,e)x^3+7840\,(10\,d-13\,e)x^2+\sqrt{14}(5\,(6565\,d+21171\,e)x^2+2\,(6565\,d+21\cdots}{}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fr
icas")
```

output
```
1/245000*(98000*e*x^4 + 9800*(20*d - 37*e)*x^3 + 7840*(10*d - 13*e)*x^2 +
sqrt(14)*(5*(6565*d + 21171*e)*x^2 + 2*(6565*d + 21171*e)*x + 19695*d + 63
513*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(6285*d - 23209*e)*x - 196*(5*
(205*d - 103*e)*x^2 + 2*(205*d - 103*e)*x + 615*d - 309*e)*log(5*x^2 + 2*x
 + 3) - 95690*d + 17766*e)/(5*x^2 + 2*x + 3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{2ex^2}{25}+x\left(\frac{4d}{25}-\frac{41e}{125}\right)+\frac{-6835d+1269e+x(-2115d-5989e)}{87500x^2+35000x+52500}+\left(-\frac{41d}{250}+\frac{103e}{1250}-\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}-\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)+\left(-\frac{41d}{250}+\frac{103e}{1250}+\frac{\sqrt{14}i(6565d+21171e)}{490000}\right)\log\left(x+\frac{1313d+\frac{21171e}{5}+\frac{\sqrt{14}i(6565d+21171e)}{5}}{6565d+21171e}\right)$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
2*e*x**2/25 + x*(4*d/25 - 41*e/125) + (-6835*d + 1269*e + x*(-2115*d - 598
9*e))/(87500*x**2 + 35000*x + 52500) + (-41*d/250 + 103*e/1250 - sqrt(14)*
I*(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 - sqrt(14)*I*(656
5*d + 21171*e)/5)/(6565*d + 21171*e)) + (-41*d/250 + 103*e/1250 + sqrt(14)
*I*(6565*d + 21171*e)/490000)*log(x + (1313*d + 21171*e/5 + sqrt(14)*I*(65
65*d + 21171*e)/5)/(6565*d + 21171*e))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{2}{25}\,ex^2 + \frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{125}\,(20\,d-41\,e)x - \frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right) - \frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="ma
xima")
```

output
```
2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*
x + 1)) + 1/125*(20*d - 41*e)*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x +
 3) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{2}{25}\,ex^2 + \frac{1}{245000}\,\sqrt{14}(6565\,d+21171\,e)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{4}{25}\,dx - \frac{41}{125}\,ex - \frac{1}{1250}\,(205\,d-103\,e)\log\left(5\,x^2+2\,x+3\right) - \frac{(2115\,d+5989\,e)x+6835\,d-1269\,e}{17500\,(5\,x^2+2\,x+3)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="gi
ac")
```

output
```
2/25*e*x^2 + 1/245000*sqrt(14)*(6565*d + 21171*e)*arctan(1/14*sqrt(14)*(5*
x + 1)) + 4/25*d*x - 41/125*e*x - 1/1250*(205*d - 103*e)*log(5*x^2 + 2*x +
 3) - 1/17500*((2115*d + 5989*e)*x + 6835*d - 1269*e)/(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 16.52 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \frac{2\,e\,x^2}{25} - \ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{41\,d}{250}-\frac{103\,e}{1250}\right) + x\,\left(\frac{4\,d}{25}-\frac{41\,e}{125}\right) - \frac{\frac{1367\,d}{28}-\frac{1269\,e}{140}+x\,\left(\frac{423\,d}{28}+\frac{5989\,e}{140}\right)}{625\,x^2+250\,x+375} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(6565\,d+21171\,e)}{245000}+\frac{\sqrt{14}\,x\,(6565\,d+21171\,e)}{49000}}{\frac{1313\,d}{3500}+\frac{21171\,e}{17500}}\right)\,(6565\,d+21171\,e)}{245000}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
(2*e*x^2)/25 - log(2*x + 5*x^2 + 3)*((41*d)/250 - (103*e)/1250) + x*((4*d)
/25 - (41*e)/125) - ((1367*d)/28 - (1269*e)/140 + x*((423*d)/28 + (5989*e)
/140))/(250*x + 625*x^2 + 375) + (14^(1/2)*atan(((14^(1/2)*(6565*d + 21171
*e))/245000 + (14^(1/2)*x*(6565*d + 21171*e))/49000)/((1313*d)/3500 + (211
71*e)/17500))*(6565*d + 21171*e))/245000
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.39

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{32825\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,x^2+13130\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)dx+19695\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d+105855\sqrt{14}\,atan\left(\right.}{}$$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
(32825*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*x**2 + 13130*sqrt(14)*atan((5*x
 + 1)/sqrt(14))*d*x + 19695*sqrt(14)*atan((5*x + 1)/sqrt(14))*d + 105855*s
qrt(14)*atan((5*x + 1)/sqrt(14))*e*x**2 + 42342*sqrt(14)*atan((5*x + 1)/sq
rt(14))*e*x + 63513*sqrt(14)*atan((5*x + 1)/sqrt(14))*e - 200900*log(5*x**
2 + 2*x + 3)*d*x**2 - 80360*log(5*x**2 + 2*x + 3)*d*x - 120540*log(5*x**2
+ 2*x + 3)*d + 100940*log(5*x**2 + 2*x + 3)*e*x**2 + 40376*log(5*x**2 + 2*
x + 3)*e*x + 60564*log(5*x**2 + 2*x + 3)*e + 196000*d*x**3 - 141575*d*x**2
 - 227675*d + 98000*e*x**4 - 362600*e*x**3 + 710395*e*x**2 + 505155*e)/(24
5000*(5*x**2 + 2*x + 3))
```

3.149 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx$

Optimal result

Integrand size = 31, antiderivative size = 63

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx = \frac{4x}{25} - \frac{1367+423x}{3500(3+2x+5x^2)} + \frac{1313\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{3500\sqrt{14}} - \frac{41}{250}\log(3+2x+5x^2)$$

output
```
4/25*x-(1367+423*x)/(17500*x^2+7000*x+10500)+1313/49000*arctan(1/14*(1+5*x
)*14^(1/2))*14^(1/2)-41/250*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2} dx = \frac{7840x - \frac{14(1367+423x)}{3+2x+5x^2} + 1313\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right) - 8036\log(3+2x+5x^2)}{49000}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

output
```
(7840*x - (14*(1367 + 423*x))/(3 + 2*x + 5*x^2) + 1313*Sqrt[14]*ArcTan[(1
+ 5*x)/Sqrt[14]] - 8036*Log[3 + 2*x + 5*x^2])/49000
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2191, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^2}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{56}\int \frac{2(560x^2-924x+369)}{25(5x^2+2x+3)}dx-\frac{423x+1367}{3500(5x^2+2x+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{700}\int \frac{560x^2-924x+369}{5x^2+2x+3}dx-\frac{423x+1367}{3500(5x^2+2x+3)}$$

$$\Big\downarrow \text{2188}$$

$$\frac{1}{700}\int \left(\frac{33-1148x}{5x^2+2x+3}+112\right)dx-\frac{423x+1367}{3500(5x^2+2x+3)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{700}\left(\frac{1313\arctan\left(\frac{5x+1}{\sqrt{14}}\right)}{5\sqrt{14}}-\frac{574}{5}\log\left(5x^2+2x+3\right)+112x\right)-\frac{423x+1367}{3500(5x^2+2x+3)}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/3500*(1367 + 423*x)/(3 + 2*x + 5*x^2) + (112*x + (1313*ArcTan[(1 + 5*x)
/Sqrt[14]])/(5*Sqrt[14]) - (574*Log[3 + 2*x + 5*x^2])/5)/700
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{4x}{25}+\frac{-\frac{423x}{17500}-\frac{1367}{17500}}{x^2+\frac{2}{5}x+\frac{3}{5}}-\frac{41\ln\left(25x^2+10x+15\right)}{250}+\frac{1313\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{49000}$	50
default	$\frac{4x}{25}-\frac{\frac{423x}{700}+\frac{1367}{700}}{25\left(x^2+\frac{2}{5}x+\frac{3}{5}\right)}-\frac{41\ln\left(5x^2+2x+3\right)}{250}+\frac{1313\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{49000}$	51

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOSE)
```

output
```
4/25*x+(-423/17500*x-1367/17500)/(x^2+2/5*x+3/5)-41/250*ln(25*x^2+10*x+15)
+1313/49000*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.24

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{39200\,x^3+1313\,\sqrt{14}(5\,x^2+2\,x+3)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+15680\,x^2-8036\,(5\,x^2+2\,x+3)\log\left(5\right.}{49000\,(5\,x^2+2\,x+3)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="fricas")
```

output
```
1/49000*(39200*x^3 + 1313*sqrt(14)*(5*x^2 + 2*x + 3)*arctan(1/14*sqrt(14)*
(5*x + 1)) + 15680*x^2 - 8036*(5*x^2 + 2*x + 3)*log(5*x^2 + 2*x + 3) + 175
98*x - 19138)/(5*x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4x}{25}+\frac{-423x-1367}{17500x^2+7000x+10500}-\frac{41\log\left(x^2+\frac{2x}{5}+\frac{3}{5}\right)}{250} + \frac{1313\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
4*x/25 + (-423*x - 1367)/(17500*x**2 + 7000*x + 10500) - 41*log(x**2 + 2*x
/5 + 3/5)/250 + 1313*sqrt(14)*atan(5*sqrt(14)*x/14 + sqrt(14)/14)/49000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x - \frac{423x+1367}{3500(5x^2+2x+3)} - \frac{41}{250}\log\left(5x^2+2x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="maxima")
```

output
```
1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423
*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{1313}{49000}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right)+\frac{4}{25}x - \frac{423x+1367}{3500(5x^2+2x+3)} - \frac{41}{250}\log\left(5x^2+2x+3\right)$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="giac")
```

output
```
1313/49000*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4/25*x - 1/3500*(423
*x + 1367)/(5*x^2 + 2*x + 3) - 41/250*log(5*x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{4\,x}{25} - \frac{41\,\ln\left(5\,x^2+2\,x+3\right)}{250} - \frac{\frac{423\,x}{17500}+\frac{1367}{17500}}{x^2+\frac{2\,x}{5}+\frac{3}{5}} + \frac{1313\,\sqrt{14}\,\mathrm{atan}\left(\frac{5\,\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{49000}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^2,x)
```

output
```
(4*x)/25 - (41*log(2*x + 5*x^2 + 3))/250 - ((423*x)/17500 + 1367/17500)/((
2*x)/5 + x^2 + 3/5) + (1313*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14)
)/49000
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^2}\,dx = \frac{6565\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)x^2+2626\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)x+3939\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)-40180\log(5x^2+2x+3)\,x}{245000x^2+98000x+}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
(6565*sqrt(14)*atan((5*x + 1)/sqrt(14))*x**2 + 2626*sqrt(14)*atan((5*x + 1
)/sqrt(14))*x + 3939*sqrt(14)*atan((5*x + 1)/sqrt(14)) - 40180*log(5*x**2
+ 2*x + 3)*x**2 - 16072*log(5*x**2 + 2*x + 3)*x - 24108*log(5*x**2 + 2*x +
 3) + 39200*x**3 - 28315*x**2 - 45535)/(49000*(5*x**2 + 2*x + 3))
```

3.150 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 224

$$
\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx \\
&= -\frac{1367d-293e+(423d-1367e)x}{700\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)} \\
&\quad+\frac{\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{700\sqrt{14}\left(5d^2-2de+3e^2\right)^2} \\
&\quad+\frac{\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log(d+ex)}{e\left(5d^2-2de+3e^2\right)^2} \\
&\quad-\frac{\left(205d^3-61d^2e+23de^2+14e^3\right)\log\left(3+2x+5x^2\right)}{50\left(5d^2-2de+3e^2\right)^2}
\end{aligned}
$$

output

```
-1/700*(1367*d-293*e+(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1
/9800*(6565*d^3-26423*d^2*e+11089*d*e^2-6623*e^3)*arctan(1/14*(1+5*x)*14^(
1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^2+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)
*ln(e*x+d)/e/(5*d^2-2*d*e+3*e^2)^2-1/50*(205*d^3-61*d^2*e+23*d*e^2+14*e^3)
*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx = \frac{\frac{14(5d^2-2de+3e^2)(-d(1367+423x)+e(293+1367x))}{3+2x+5x^2}+\sqrt{14}\left(6565d^3-26423d^2e+11089de^2-6623e^3\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{9800\left(5d^2-2de\right.}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),
x]
```

output
```
((14*(5*d^2 - 2*d*e + 3*e^2)*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/(3
+ 2*x + 5*x^2) + Sqrt[14]*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3
)*ArcTan[(1 + 5*x)/Sqrt[14]] + (9800*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4)*Log[d + e*x])/e - 196*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Lo
g[3 + 2*x + 5*x^2])/(9800*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{\left(5x^2+2x+3\right)^2(d+ex)}\,dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{56}\int \frac{2\left(112x^2-\frac{(924d^2-285ed+281e^2)x}{5d^2-2ed+3e^2}+\frac{369d^2-421ed+280e^2}{5d^2-2ed+3e^2}\right)}{5(d+ex)\left(5x^2+2x+3\right)}dx-\frac{x(423d-1367e)+1367d-293e}{700\left(5x^2+2x+3\right)\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{140}\int \frac{112x^2-\frac{(924d^2-285ed+281e^2)x}{5d^2-2ed+3e^2}+\frac{369d^2-421ed+280e^2}{5d^2-2ed+3e^2}}{(d+ex)(5x^2+2x+3)}dx-\frac{x(423d-1367e)+1367d-293e}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$\big\downarrow$ 2159

$$\frac{1}{140}\int \left(\frac{140(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)}+\frac{165d^3-4943ed^2+2089e^2d-1403e^3-28(205d^3-61ed^2+2}{(5d^2-2ed+3e^2)^2(5x^2+2x+3)}\right.$$
$$\frac{x(423d-1367e)+1367d-293e}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$\big\downarrow$ 2009

$$\frac{1}{140}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d^3-26423d^2e+11089de^2-6623e^3)}{5\sqrt{14}(5d^2-2de+3e^2)^2}-\frac{14(205d^3-61d^2e+23de^2+14e^3)\log(5x^2}{5(5d^2-2de+3e^2)^2}\right.$$
$$\frac{x(423d-1367e)+1367d-293e}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/700*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3 +
 2*x + 5*x^2)) + (((6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*ArcTa
n[(1 + 5*x)/Sqrt[14]])/(5*Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^2) + (140*(4*d^
4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x])/(e*(5*d^2 - 2*d*e +
 3*e^2)^2) - (14*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*Log[3 + 2*x + 5*
x^2])/(5*(5*d^2 - 2*d*e + 3*e^2)^2))/140
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.96

method	result
default	$-\frac{\frac{\left(\frac{423}{700} d^3-\frac{7681}{3500} d^2 e+\frac{4003}{3500} d e^2-\frac{4101}{3500} e^3\right)x+\frac{1367 d^3}{700}-\frac{4199 d^2 e}{3500}+\frac{4687 d e^2}{3500}-\frac{879 e^3}{3500}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740 d^3-1708 d^2 e+644 d e^2+392 e^3\right)\ln\left(5x^2+2x+3\right)}{1400}+\frac{(-}{}}{\left(5d^2-2de+3e^2\right)^2}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x,method=_RETURNVERBOS
E)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^2*(((423/700*d^3-7681/3500*d^2*e+4003/3500*d*e^2-41
01/3500*e^3)*x+1367/700*d^3-4199/3500*d^2*e+4687/3500*d*e^2-879/3500*e^3)/
(x^2+2/5*x+3/5)+1/1400*(5740*d^3-1708*d^2*e+644*d*e^2+392*e^3)*ln(5*x^2+2*
x+3)+1/1960*(-1313*d^3+26423/5*d^2*e-11089/5*d*e^2+6623/5*e^3)*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2)))+(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x
+d)/e/(5*d^2-2*d*e+3*e^2)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. 2(215) = 430.

Time = 0.13 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.14

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = -\frac{95690\,d^3e-58786\,d^2e^2+65618\,de^3-12306\,e^4-\sqrt{14}(19695\,d^3e-79269\,d^2e^2+33267\,de^3-19869\,e}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="fr
icas")
```

output
```
-1/9800*(95690*d^3*e - 58786*d^2*e^2 + 65618*d*e^3 - 12306*e^4 - sqrt(14)*
(19695*d^3*e - 79269*d^2*e^2 + 33267*d*e^3 - 19869*e^4 + 5*(6565*d^3*e - 2
6423*d^2*e^2 + 11089*d*e^3 - 6623*e^4)*x^2 + 2*(6565*d^3*e - 26423*d^2*e^2
 + 11089*d*e^3 - 6623*e^4)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(2115*d
^3*e - 7681*d^2*e^2 + 4003*d*e^3 - 4101*e^4)*x - 9800*(12*d^4 + 15*d^3*e +
 9*d^2*e^2 - 3*d*e^3 + 6*e^4 + 5*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*
e^4)*x^2 + 2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*x)*log(e*x + d)
 + 196*(615*d^3*e - 183*d^2*e^2 + 69*d*e^3 + 42*e^4 + 5*(205*d^3*e - 61*d^
2*e^2 + 23*d*e^3 + 14*e^4)*x^2 + 2*(205*d^3*e - 61*d^2*e^2 + 23*d*e^3 + 14
*e^4)*x)*log(5*x^2 + 2*x + 3))/(75*d^4*e - 60*d^3*e^2 + 102*d^2*e^3 - 36*d
*e^4 + 27*e^5 + 5*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*
x^2 + 2*(25*d^4*e - 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.29

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^2}\,dx$$
$$=\frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}$$
$$+\frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log(ex+d)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5}$$
$$-\frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log(5\,x^2+2\,x+3)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)}$$
$$-\frac{(423\,d-1367\,e)x+1367\,d-293\,e}{700\,(5\,(5\,d^2-2\,de+3\,e^2)x^2+15\,d^2-6\,de+9\,e^2+2\,(5\,d^2-2\,de+3\,e^2)x)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="ma
xima")
```

output
```
1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1
/14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4
) + (4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*log(e*x + d)/(25*d^4*e -
 20*d^3*e^2 + 34*d^2*e^3 - 12*d*e^4 + 9*e^5) - 1/50*(205*d^3 - 61*d^2*e +
23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4) - 1/700*((423*d - 1367*e)*x + 1367*d - 293*e)/(5*(5*d^2
- 2*d*e + 3*e^2)*x^2 + 15*d^2 - 6*d*e + 9*e^2 + 2*(5*d^2 - 2*d*e + 3*e^2)*
x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.32

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = \frac{\sqrt{14}(6565\,d^3-26423\,d^2e+11089\,de^2-6623\,e^3)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{9800\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} - \frac{(205\,d^3-61\,d^2e+23\,de^2+14\,e^3)\log\left(5\,x^2+2\,x+3\right)}{50\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)} + \frac{(4\,d^4+5\,d^3e+3\,d^2e^2-de^3+2\,e^4)\log\left(|ex+d|\right)}{25\,d^4e-20\,d^3e^2+34\,d^2e^3-12\,de^4+9\,e^5} - \frac{6835\,d^3-4199\,d^2e+4687\,de^2-879\,e^3+(2115\,d^3-7681\,d^2e+4003\,de^2-4101\,e^3)x}{700\,(5\,d^2-2\,de+3\,e^2)^2(5\,x^2+2\,x+3)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x, algorithm="gi
ac")
```

output
```
1/9800*sqrt(14)*(6565*d^3 - 26423*d^2*e + 11089*d*e^2 - 6623*e^3)*arctan(1
/14*sqrt(14)*(5*x + 1))/(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4
) - 1/50*(205*d^3 - 61*d^2*e + 23*d*e^2 + 14*e^3)*log(5*x^2 + 2*x + 3)/(25
*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4) + (4*d^4 + 5*d^3*e + 3*d^
2*e^2 - d*e^3 + 2*e^4)*log(abs(e*x + d))/(25*d^4*e - 20*d^3*e^2 + 34*d^2*e
^3 - 12*d*e^4 + 9*e^5) - 1/700*(6835*d^3 - 4199*d^2*e + 4687*d*e^2 - 879*e
^3 + (2115*d^3 - 7681*d^2*e + 4003*d*e^2 - 4101*e^3)*x)/((5*d^2 - 2*d*e +
3*e^2)^2*(5*x^2 + 2*x + 3))
```

Mupad [B] (verification not implemented)

Time = 16.96 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.47

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = \frac{\ln(d+e\,x)\,\left(4\,d^4+5\,d^3\,e+3\,d^2\,e^2-d\,e^3+2\,e^4\right)}{e\,\left(5\,d^2-2\,d\,e+3\,e^2\right)^2} + \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{3920}-\frac{41}{10}\mathrm{i}\right)d^3+\left(-\frac{26423\,\sqrt{14}}{19600}+\frac{61}{50}\mathrm{i}\right)d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}-\frac{23}{50}\mathrm{i}\right)d\,e^2+\left(-\frac{6623\,\sqrt{}}{1960}\right.\right.}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} - \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{3920}+\frac{41}{10}\mathrm{i}\right)d^3+\left(-\frac{26423\,\sqrt{14}}{19600}-\frac{61}{50}\mathrm{i}\right)d^2\,e+\left(\frac{11089\,\sqrt{14}}{19600}+\frac{23}{50}\mathrm{i}\right)d\,e^2+\left(-\frac{6623\,\sqrt{}}{1960}\right.\right.}{d^4\,25\mathrm{i}-d^3\,e\,20\mathrm{i}+d^2\,e^2\,34\mathrm{i}-d\,e^3\,12\mathrm{i}+e^4\,9\mathrm{i}} - \frac{\frac{1367\,d-293\,e}{700\,(5\,d^2-2\,d\,e+3\,e^2)}+\frac{x\,(423\,d-1367\,e)}{700\,(5\,d^2-2\,d\,e+3\,e^2)}}{5\,x^2+2\,x+3}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^2),x)
```

output
```
(log(x - (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 - 41i/10) - e^3
*((6623*14^(1/2))/19600 + 7i/25) + d*e^2*((11089*14^(1/2))/19600 - 23i/50)
 - d^2*e*((26423*14^(1/2))/19600 - 61i/50)))/(d^4*25i - d^3*e*20i - d*e^3*
12i + e^4*9i + d^2*e^2*34i) - ((1367*d - 293*e)/(700*(5*d^2 - 2*d*e + 3*e^
2)) + (x*(423*d - 1367*e))/(700*(5*d^2 - 2*d*e + 3*e^2)))/(2*x + 5*x^2 + 3
) - (log(x + (14^(1/2)*1i)/5 + 1/5)*(d^3*((1313*14^(1/2))/3920 + 41i/10) -
 e^3*((6623*14^(1/2))/19600 - 7i/25) + d*e^2*((11089*14^(1/2))/19600 + 23i
/50) - d^2*e*((26423*14^(1/2))/19600 + 61i/50)))/(d^4*25i - d^3*e*20i - d*
e^3*12i + e^4*9i + d^2*e^2*34i) + (log(d + e*x)*(5*d^3*e - d*e^3 + 4*d^4 +
 2*e^4 + 3*d^2*e^2))/(e*(5*d^2 - 2*d*e + 3*e^2)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 851, normalized size of antiderivative = 3.80

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^2,x)
```

output
```
(32825*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e*x**2 + 13130*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d**3*e*x + 19695*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*
*3*e - 132115*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**2*x**2 - 52846*sqr
t(14)*atan((5*x + 1)/sqrt(14))*d**2*e**2*x - 79269*sqrt(14)*atan((5*x + 1)
/sqrt(14))*d**2*e**2 + 55445*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**3*x**2
 + 22178*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**3*x + 33267*sqrt(14)*atan(
(5*x + 1)/sqrt(14))*d*e**3 - 33115*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**4*
x**2 - 13246*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**4*x - 19869*sqrt(14)*ata
n((5*x + 1)/sqrt(14))*e**4 - 200900*log(5*x**2 + 2*x + 3)*d**3*e*x**2 - 80
360*log(5*x**2 + 2*x + 3)*d**3*e*x - 120540*log(5*x**2 + 2*x + 3)*d**3*e +
 59780*log(5*x**2 + 2*x + 3)*d**2*e**2*x**2 + 23912*log(5*x**2 + 2*x + 3)*
d**2*e**2*x + 35868*log(5*x**2 + 2*x + 3)*d**2*e**2 - 22540*log(5*x**2 + 2
*x + 3)*d*e**3*x**2 - 9016*log(5*x**2 + 2*x + 3)*d*e**3*x - 13524*log(5*x*
*2 + 2*x + 3)*d*e**3 - 13720*log(5*x**2 + 2*x + 3)*e**4*x**2 - 5488*log(5*
x**2 + 2*x + 3)*e**4*x - 8232*log(5*x**2 + 2*x + 3)*e**4 + 196000*log(d +
e*x)*d**4*x**2 + 78400*log(d + e*x)*d**4*x + 117600*log(d + e*x)*d**4 + 24
5000*log(d + e*x)*d**3*e*x**2 + 98000*log(d + e*x)*d**3*e*x + 147000*log(d
 + e*x)*d**3*e + 147000*log(d + e*x)*d**2*e**2*x**2 + 58800*log(d + e*x)*d
**2*e**2*x + 88200*log(d + e*x)*d**2*e**2 - 49000*log(d + e*x)*d*e**3*x**2
 - 19600*log(d + e*x)*d*e**3*x - 29400*log(d + e*x)*d*e**3 + 98000*log(...
```

3.151 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 313

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{e(5d^2-2de+3e^2)^2(d+ex)} \\
&\quad - \frac{1367d^2-586de-703e^2+(423d^2-2734de+293e^2)x}{140(5d^2-2de+3e^2)^2(3+2x+5x^2)} \\
&\quad + \frac{(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}(5d^2-2de+3e^2)^3} \\
&\quad + \frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(d+ex)}{(5d^2-2de+3e^2)^3} \\
&\quad - \frac{(41d^4-8d^3e-60d^2e^2+24de^3-5e^4)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^3}
\end{aligned}$$

output
```
-(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)-1/1
40*(1367*d^2-586*d*e-703*e^2+(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*
e^2)^2/(5*x^2+2*x+3)+1/392*(1313*d^4-10044*d^3*e+4290*d^2*e^2+156*d*e^3-27
1*e^4)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^3+(41*d^
4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*(
41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e
^2)^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^2}\,dx$$
$$= \frac{-\frac{1960(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e(d+ex)} - \frac{14(5d^2-2de+3e^2)(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x))}{3+2x+5x^2} + 5\sqrt{14}}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2
),x]
```

output
```
((-1960*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e
^4))/(e*(d + e*x)) - (14*(5*d^2 - 2*d*e + 3*e^2)*(e^2*(-703 + 293*x) + d^2
*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(1
313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*
x)/Sqrt[14]] + 1960*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log
[d + e*x] + 980*(-41*d^4 + 8*d^3*e + 60*d^2*e^2 - 24*d*e^3 + 5*e^4)*Log[3
+ 2*x + 5*x^2])/(1960*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^2(d+ex)^2}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{1}{56}\int \frac{2\left(\frac{(560d^4-448ed^3+677e^2d^2+278e^3d+143e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(462d^4-285ed^3+338e^2d^2-171e^3d+14e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{369d^4-842ed^3+787e^2d^2-224e^3d}{(5d^2-2ed+3e^2)^2}\right.}{(d+ex)^2(5x^2+2x+3)}$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{28}\int \frac{\frac{(560d^4-448ed^3+677e^2d^2+278e^3d+143e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(462d^4-285ed^3+338e^2d^2-171e^3d+14e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{369d^4-842ed^3+787e^2d^2-224e^3d+16}{(5d^2-2ed+3e^2)^2}}{(d+ex)^2(5x^2+2x+3)}$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{ 2159}$$

$$\frac{1}{28}\int \left(\frac{28(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)^2}-\frac{28e(-41d^4+8ed^3+60e^2d^2-24e^3d+5e^4)}{(5d^2-2ed+3e^2)^3(d+ex)}+\frac{165d^4-9820ed^3}{}\right.$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(1313d^4-10044d^3e+4290d^2e^2+156de^3-271e^4)}{\sqrt{14}(5d^2-2de+3e^2)^3}-\frac{14(41d^4-8d^3e-60d^2e^2+24de^3-5}{(5d^2-2de+3e^2}\right.$$
$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{140(5x^2+2x+3)(5d^2-2de+3e^2)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/140*(1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e^2)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)) + ((-28*(4*d^4 + 5*d^3*e + 3*
d^2*e^2 - d*e^3 + 2*e^4))/(e*(5*d^2 - 2*d*e + 3*e^2)^2*(d + e*x)) + ((1313
*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^4)*ArcTan[(1 + 5*x)/
Sqrt[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^3) + (28*(41*d^4 - 8*d^3*e -
60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^3 - (
14*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*Log[3 + 2*x + 5*x^2]
)/(5*d^2 - 2*d*e + 3*e^2)^3)/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\frac{\left(\frac{423}{140}d^4-\frac{3629}{175}d^3e+\frac{4101}{350}d^2e^2-\frac{2197}{175}d\,e^3+\frac{879}{700}e^4\right)x+\frac{1367d^4}{140}-\frac{1416d^3e}{175}+\frac{879d^2e^2}{350}-\frac{88d\,e^3}{175}-\frac{2109e^4}{700}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{\left(5740d^4-1120d^3e-8400d^2e^2+3360d\right.}{280}}{\left(5d^2-2de+3e^2\right)^3}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^3*(((423/140*d^4-3629/175*d^3*e+4101/350*d^2*e^2-21
97/175*d*e^3+879/700*e^4)*x+1367/140*d^4-1416/175*d^3*e+879/350*d^2*e^2-88
/175*d*e^3-2109/700*e^4)/(x^2+2/5*x+3/5)+1/280*(5740*d^4-1120*d^3*e-8400*d
^2*e^2+3360*d*e^3-700*e^4)*ln(5*x^2+2*x+3)+1/392*(-1313*d^4+10044*d^3*e-42
90*d^2*e^2-156*d*e^3+271*e^4)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-(4*
d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)+(41*d^4
-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(304) = 608.

Time = 0.17 (sec) , antiderivative size = 910, normalized size of antiderivative = 2.91

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
-1/1960*(117600*d^6 + 195650*d^5*e + 20664*d^4*e^2 + 48132*d^3*e^3 + 11855
2*d^2*e^4 - 70686*d*e^5 + 35280*e^6 + 14*(14000*d^6 + 11900*d^5*e + 14015*
d^4*e^2 - 11716*d^3*e^3 + 22902*d^2*e^4 - 13688*d*e^5 + 5079*e^6)*x^2 - 5*
sqrt(14)*(3939*d^5*e - 30132*d^4*e^2 + 12870*d^3*e^3 + 468*d^2*e^4 - 813*d
*e^5 + 5*(1313*d^4*e^2 - 10044*d^3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^
6)*x^3 + (6565*d^5*e - 47594*d^4*e^2 + 1362*d^3*e^3 + 9360*d^2*e^4 - 1043*
d*e^5 - 542*e^6)*x^2 + (2626*d^5*e - 16149*d^4*e^2 - 21552*d^3*e^3 + 13182
*d^2*e^4 - 74*d*e^5 - 813*e^6)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(56
00*d^6 + 6875*d^5*e - 2921*d^4*e^2 + 3658*d^3*e^3 - 1150*d^2*e^4 - 1433*d*
e^5 - 429*e^6)*x - 1960*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4
 - 15*d*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x
^3 + (205*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*
d^5*e + 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*lo
g(e*x + d) + 980*(123*d^5*e - 24*d^4*e^2 - 180*d^3*e^3 + 72*d^2*e^4 - 15*d
*e^5 + 5*(41*d^4*e^2 - 8*d^3*e^3 - 60*d^2*e^4 + 24*d*e^5 - 5*e^6)*x^3 + (2
05*d^5*e + 42*d^4*e^2 - 316*d^3*e^3 + 23*d*e^5 - 10*e^6)*x^2 + (82*d^5*e +
 107*d^4*e^2 - 144*d^3*e^3 - 132*d^2*e^4 + 62*d*e^5 - 15*e^6)*x)*log(5*x^2
 + 2*x + 3))/(375*d^7*e - 450*d^6*e^2 + 855*d^5*e^3 - 564*d^4*e^4 + 513*d^
3*e^5 - 162*d^2*e^6 + 81*d*e^7 + 5*(125*d^6*e^2 - 150*d^5*e^3 + 285*d^4*e^
4 - 188*d^3*e^5 + 171*d^2*e^6 - 54*d*e^7 + 27*e^8)*x^3 + (625*d^7*e - 5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2 (3+2x+5x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.75

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx$$
$$= \frac{\sqrt{14}(1313\,d^4-10044\,d^3e+4290\,d^2e^2+156\,de^3-271\,e^4)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(ex+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}$$
$$-\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$-\frac{1680\,d^4+3467\,d^3e+674\,d^2e^2-1123\,de^3+840\,e^4+(2800\,d^4+3500\,d^3e}{140\,(75\,d^5e-60\,d^4e^2+102\,d^3e^3-36\,d^2e^4+27\,de^5+5\,(25\,d^4e^2-20\,d^3e^3+34\,d^2e^4-12\,de^5+9\,e^6)x^3}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/392*sqrt(14)*(1313*d^4 - 10044*d^3*e + 4290*d^2*e^2 + 156*d*e^3 - 271*e^
4)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 18
8*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (41*d^4 - 8*d^3*e - 60*d^2*
e^2 + 24*d*e^3 - 5*e^4)*log(e*x + d)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 -
188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*(41*d^4 - 8*d^3*e - 6
0*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(5*x^2 + 2*x + 3)/(125*d^6 - 150*d^5*e +
285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/140*(1680
*d^4 + 3467*d^3*e + 674*d^2*e^2 - 1123*d*e^3 + 840*e^4 + (2800*d^4 + 3500*
d^3*e + 2523*d^2*e^2 - 3434*d*e^3 + 1693*e^4)*x^2 + (1120*d^4 + 1823*d^3*e
 - 527*d^2*e^2 - 573*d*e^3 - 143*e^4)*x)/(75*d^5*e - 60*d^4*e^2 + 102*d^3*
e^3 - 36*d^2*e^4 + 27*d*e^5 + 5*(25*d^4*e^2 - 20*d^3*e^3 + 34*d^2*e^4 - 12
*d*e^5 + 9*e^6)*x^3 + (125*d^5*e - 50*d^4*e^2 + 130*d^3*e^3 + 8*d^2*e^4 +
21*d*e^5 + 18*e^6)*x^2 + (50*d^5*e + 35*d^4*e^2 + 8*d^3*e^3 + 78*d^2*e^4 -
 18*d*e^5 + 27*e^6)*x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 584, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^2}\,dx = -\frac{(41\,d^4-8\,d^3e-60\,d^2e^2+24\,de^3-5\,e^4)\log\left(-\frac{10\,d}{ex+d}+\frac{5\,d^2}{(ex+d)^2}+\frac{2\,e}{ex+d}-\frac{2\,de}{(ex+d)^2}+\frac{3\,e^2}{(ex+d)^2}+5\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)} - \frac{\frac{4\,d^4e^3}{ex+d}+\frac{5\,d^3e^4}{ex+d}+\frac{3\,d^2e^5}{ex+d}-\frac{de^6}{ex+d}+\frac{2\,e^7}{ex+d}}{25\,d^4e^4-20\,d^3e^5+34\,d^2e^6-12\,de^7+9\,e^8} + \frac{\sqrt{14}(1313\,d^4e^2-10044\,d^3e^3+4290\,d^2e^4+156\,de^5-271\,e^6)\arctan\left(\frac{\sqrt{14}\left(5\,d-\frac{5\,d^2}{ex+d}+\frac{2\,de}{ex+d}-e-\frac{3\,e^2}{ex+d}\right)}{14\,e}\right)}{392\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)e^2} + \frac{\frac{423\,d^3e-4101\,d^2e^2+879\,de^3+703\,e^4}{5\,d^2-2\,de+3\,e^2}-\frac{423\,d^4e^2-5468\,d^3e^3+1758\,d^2e^4+2812\,de^5-457\,e^6}{(5\,d^2-2\,de+3\,e^2)(ex+d)e}}{28\,(5\,d^2-2\,de+3\,e^2)^2\left(\frac{10\,d}{ex+d}-\frac{5\,d^2}{(ex+d)^2}-\frac{2\,e}{ex+d}+\frac{2\,de}{(ex+d)^2}-\frac{3\,e^2}{(ex+d)^2}-5\right)}$$

input

```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output

```
-1/2*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*e^4)*log(-10*d/(e*x + d
) + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^2 + 3*e^2/(e*x + d
)^2 + 5)/(125*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 -
54*d*e^5 + 27*e^6) - (4*d^4*e^3/(e*x + d) + 5*d^3*e^4/(e*x + d) + 3*d^2*e^
5/(e*x + d) - d*e^6/(e*x + d) + 2*e^7/(e*x + d))/(25*d^4*e^4 - 20*d^3*e^5
+ 34*d^2*e^6 - 12*d*e^7 + 9*e^8) + 1/392*sqrt(14)*(1313*d^4*e^2 - 10044*d^
3*e^3 + 4290*d^2*e^4 + 156*d*e^5 - 271*e^6)*arctan(1/14*sqrt(14)*(5*d - 5*
d^2/(e*x + d) + 2*d*e/(e*x + d) - e - 3*e^2/(e*x + d))/e)/((125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6)*e^2)
+ 1/28*((423*d^3*e - 4101*d^2*e^2 + 879*d*e^3 + 703*e^4)/(5*d^2 - 2*d*e +
3*e^2) - (423*d^4*e^2 - 5468*d^3*e^3 + 1758*d^2*e^4 + 2812*d*e^5 - 457*e^6
)/((5*d^2 - 2*d*e + 3*e^2)*(e*x + d)*e))/((5*d^2 - 2*d*e + 3*e^2)^2*(10*d/
(e*x + d) - 5*d^2/(e*x + d)^2 - 2*e/(e*x + d) + 2*d*e/(e*x + d)^2 - 3*e^2/
(e*x + d)^2 - 5))
```

Mupad [B] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.92

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^2}\,dx = \ln(d+e\,x)\left(\frac{41}{25\,(5\,d^2-2\,d\,e+3\,e^2)} - \frac{4\,e^3\,(423\,d-1367\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^3} + \frac{2\,e\,(310\,d-1323\,e)}{125\,(5\,d^2-2\,d\,e+3\,e^2)^2}\right) - \frac{\frac{1680\,d^4+3467\,d^3\,e+674\,d^2\,e^2-1123\,d\,e^3+840\,e^4}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} - \frac{x\,(-1120\,d^4-1823\,d^3\,e+527\,d^2\,e^2+573\,d\,e^3+143\,e^4)}{140\,e\,(25\,d^4-20\,d^3\,e+34\,d^2\,e^2-12\,d\,e^3+9\,e^4)} + \frac{x^2\,(2800\,d^4+3500\,d^3\,e+2523\,d\ldots}{140\,e\,(25\,d^4-20\,d^3\,e+34\ldots}}{5\,e\,x^3+(5\,d+2\,e)\,x^2+(2\,d+3\,e)\,x+3\,d} + \frac{\ln\left(x+\frac{1}{5}-\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}-\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}+4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}+30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}-\ldots\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,2\ldots} - \frac{\ln\left(x+\frac{1}{5}+\frac{\sqrt{14}\,1\mathrm{i}}{5}\right)\left(\left(\frac{1313\,\sqrt{14}}{784}+\frac{41}{2}\mathrm{i}\right)d^4+\left(-\frac{2511\,\sqrt{14}}{196}-4\mathrm{i}\right)d^3\,e+\left(\frac{2145\,\sqrt{14}}{392}-30\mathrm{i}\right)d^2\,e^2+\left(\frac{39\,\sqrt{14}}{196}+\ldots\right.\right.}{d^6\,125\mathrm{i}-d^5\,e\,150\mathrm{i}+d^4\,e^2\,285\mathrm{i}-d^3\,e^3\,188\mathrm{i}+d^2\,e^4\,171\mathrm{i}-d\,e^5\,54\mathrm{i}+e^6\,2\ldots}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^2),x)
```

output
```
log(d + e*x)*(41/(25*(5*d^2 - 2*d*e + 3*e^2)) - (4*e^3*(423*d - 1367*e))/(
125*(5*d^2 - 2*d*e + 3*e^2)^3) + (2*e*(310*d - 1323*e))/(125*(5*d^2 - 2*d*
e + 3*e^2)^2)) - ((3467*d^3*e - 1123*d*e^3 + 1680*d^4 + 840*e^4 + 674*d^2*
e^2)/(140*e*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (x*(573
*d*e^3 - 1823*d^3*e - 1120*d^4 + 143*e^4 + 527*d^2*e^2))/(140*e*(25*d^4 -
20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(3500*d^3*e - 3434*d*e^3
 + 2800*d^4 + 1693*e^4 + 2523*d^2*e^2))/(140*e*(25*d^4 - 20*d^3*e - 12*d*e
^3 + 9*e^4 + 34*d^2*e^2)))/(3*d + x^2*(5*d + 2*e) + 5*e*x^3 + x*(2*d + 3*e
)) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 - 41i/2) -
e^4*((271*14^(1/2))/784 - 5i/2) + d^2*e^2*((2145*14^(1/2))/392 + 30i) + d*
e^3*((39*14^(1/2))/196 - 12i) - d^3*e*((2511*14^(1/2))/196 - 4i)))/(d^6*12
5i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*
e^2*285i) - (log(x + (14^(1/2)*1i)/5 + 1/5)*(d^4*((1313*14^(1/2))/784 + 41
i/2) - e^4*((271*14^(1/2))/784 + 5i/2) + d^2*e^2*((2145*14^(1/2))/392 - 30
i) + d*e^3*((39*14^(1/2))/196 + 12i) - d^3*e*((2511*14^(1/2))/196 + 4i)))/
(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4*171i - d^3*e^3*188i
 + d^4*e^2*285i)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1837, normalized size of antiderivative = 5.87

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^2 (3 + 2x + 5x^2)^2} \, dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^2,x)
```

output
```
(32825*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*x**2 + 13130*sqrt(14)*atan((
5*x + 1)/sqrt(14))*d**6*x + 19695*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6 +
 32825*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e*x**3 - 224840*sqrt(14)*ata
n((5*x + 1)/sqrt(14))*d**5*e*x**2 - 75493*sqrt(14)*atan((5*x + 1)/sqrt(14)
)*d**5*e*x - 142782*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e - 237970*sqrt
(14)*atan((5*x + 1)/sqrt(14))*d**4*e**2*x**3 - 88378*sqrt(14)*atan((5*x +
1)/sqrt(14))*d**4*e**2*x**2 - 140058*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
4*e**2*x + 4086*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**2 + 6810*sqrt(14
)*atan((5*x + 1)/sqrt(14))*d**3*e**3*x**3 + 49524*sqrt(14)*atan((5*x + 1)/
sqrt(14))*d**3*e**3*x**2 + 22806*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e*
*3*x + 28080*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**3 + 46800*sqrt(14)*
atan((5*x + 1)/sqrt(14))*d**2*e**4*x**3 + 13505*sqrt(14)*atan((5*x + 1)/sq
rt(14))*d**2*e**4*x**2 + 25994*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**4
*x - 3129*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**4 - 5215*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d*e**5*x**3 - 4796*sqrt(14)*atan((5*x + 1)/sqrt(14))*
d*e**5*x**2 - 4213*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**5*x - 1626*sqrt(
14)*atan((5*x + 1)/sqrt(14))*d*e**5 - 2710*sqrt(14)*atan((5*x + 1)/sqrt(14
))*e**6*x**3 - 1084*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**6*x**2 - 1626*sqr
t(14)*atan((5*x + 1)/sqrt(14))*e**6*x - 200900*log(5*x**2 + 2*x + 3)*d**6*
x**2 - 80360*log(5*x**2 + 2*x + 3)*d**6*x - 120540*log(5*x**2 + 2*x + 3...
```

3.152 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3 (3+2x+5x^2)^2} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 412

$$\begin{aligned}
&\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3 (3+2x+5x^2)^2} \, dx \\
&= -\frac{4d^4+5d^3e+3d^2e^2-de^3+2e^4}{2e(5d^2-2de+3e^2)^2 (d+ex)^2} - \frac{41d^4-8d^3e-60d^2e^2+24de^3-5e^4}{(5d^2-2de+3e^2)^3 (d+ex)} \\
&\quad - \frac{1367d^3-879d^2e-2109de^2+457e^3+(423d^3-4101d^2e+879de^2+703e^3)\,x}{28(5d^2-2de+3e^2)^3 (3+2x+5x^2)} \\
&\quad + \frac{(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{28\sqrt{14}(5d^2-2de+3e^2)^4} \\
&\quad + \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log(d+ex)}{(5d^2-2de+3e^2)^4} \\
&\quad - \frac{(205d^5-19d^4e-846d^3e^2+396d^2e^3+57de^4-21e^5)\log(3+2x+5x^2)}{2(5d^2-2de+3e^2)^4}
\end{aligned}$$

output
```
-1/2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)
^2-(41*d^4-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d
)-1/28*(1367*d^3-879*d^2*e-2109*d*e^2+457*e^3+(423*d^3-4101*d^2*e+879*d*e^
2+703*e^3)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+1/392*(6565*d^5-74017*d^
4*e+35022*d^3*e^2+42858*d^2*e^3-17247*d*e^4+579*e^5)*arctan(1/14*(1+5*x)*1
4^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^4+(205*d^5-19*d^4*e-846*d^3*e^2+396*
d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/2*(205*d^5-19*d
^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e
+3*e^2)^4
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx$$

$$= \frac{-\frac{196(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{e(d+ex)^2} + \frac{392(5d^2-2de+3e^2)(-41d^4+8d^3e+60d^2e^2-24de^3+5e^4)}{d+ex} - \frac{14(5d^2-2de+3e^2)(3de}{}}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2
),x]
```

output
```
((-196*(5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*
e^4))/(e*(d + e*x)^2) + (392*(5*d^2 - 2*d*e + 3*e^2)*(-41*d^4 + 8*d^3*e +
60*d^2*e^2 - 24*d*e^3 + 5*e^4))/(d + e*x) - (14*(5*d^2 - 2*d*e + 3*e^2)*(3
*d*e^2*(-703 + 293*x) + d^3*(1367 + 423*x) + e^3*(457 + 703*x) - 3*d^2*e*(
293 + 1367*x)))/(3 + 2*x + 5*x^2) + Sqrt[14]*(6565*d^5 - 74017*d^4*e + 350
22*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqrt[
14]] + 392*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21
*e^5)*Log[d + e*x] + 196*(-205*d^5 + 19*d^4*e + 846*d^3*e^2 - 396*d^2*e^3
- 57*d*e^4 + 21*e^5)*Log[3 + 2*x + 5*x^2])/(392*(5*d^2 - 2*d*e + 3*e^2)^4)
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^2(d+ex)^3}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{56}\int \frac{2\left(-\frac{e^3(423d^3-4101ed^2+879e^2d+703e^3)x^3}{(5d^2-2ed+3e^2)^3}+\frac{(2800d^6-3360ed^5+5115e^2d^4+5527e^3d^3+1311e^4d^2+1251e^5d-28e^6)x^2}{(5d^2-2ed+3e^2)^3}-\frac{(4620d^6-4}{}\right.}{(d+ex)^3(5x^2}\ -$$
$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{28}\int \frac{-\frac{e^3(423d^3-4101ed^2+879e^2d+703e^3)x^3}{(5d^2-2ed+3e^2)^3}+\frac{(2800d^6-3360ed^5+5115e^2d^4+5527e^3d^3+1311e^4d^2+1251e^5d-28e^6)x^2}{(5d^2-2ed+3e^2)^3}-\frac{(4620d^6-4275}{}}{(d+ex)^3(5x^2+}\ -$$
$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{28}\int \left(\frac{28(4d^4+5ed^3+3e^2d^2-e^3d+2e^4)}{(5d^2-2ed+3e^2)^2(d+ex)^3}-\frac{28e(-205d^5+19ed^4+846e^2d^3-396e^3d^2-57e^4d+21e^5)}{(5d^2-2ed+3e^2)^4(d+ex)}+\frac{3(}{}\right.$$
$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(6565d^5-74017d^4e+35022d^3e^2+42858d^2e^3-17247de^4+579e^5)}{\sqrt{14}(5d^2-2de+3e^2)^4}-\frac{28(41d^4-8d^3e-60}{(5d^2-2de+}\right.$$
$$\frac{1367d^3-879d^2e+x(423d^3-4101d^2e+879de^2+703e^3)-2109de^2+457e^3}{28(5x^2+2x+3)(5d^2-2de+3e^2)^3}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^3*(3 + 2*x + 5*x^2)^2),x]
```

output
```
-1/28*(1367*d^3 - 879*d^2*e - 2109*d*e^2 + 457*e^3 + (423*d^3 - 4101*d^2*e
 + 879*d*e^2 + 703*e^3)*x)/((5*d^2 - 2*d*e + 3*e^2)^3*(3 + 2*x + 5*x^2)) +
 ((-14*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4))/(e*(5*d^2 - 2*d*e +
3*e^2)^2*(d + e*x)^2) - (28*(41*d^4 - 8*d^3*e - 60*d^2*e^2 + 24*d*e^3 - 5*
e^4))/((5*d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) + ((6565*d^5 - 74017*d^4*e + 3
5022*d^3*e^2 + 42858*d^2*e^3 - 17247*d*e^4 + 579*e^5)*ArcTan[(1 + 5*x)/Sqr
t[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (28*(205*d^5 - 19*d^4*e - 8
46*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*e^5)*Log[d + e*x])/(5*d^2 - 2*d*e
 + 3*e^2)^4 - (14*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e
^4 - 21*e^5)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e + 3*e^2)^4)/28
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\frac{\left(\frac{423}{28}d^5-\frac{21351}{140}d^4e+\frac{6933}{70}d^3e^2-\frac{5273}{70}d^2e^3+\frac{1231}{140}d\,e^4+\frac{2109}{140}e^5\right)x+\frac{1367d^5}{28}-\frac{7129d^4e}{140}-\frac{2343d^3e^2}{70}+\frac{1933d^2e^3}{70}-\frac{7241d\,e^4}{140}+\frac{1371e^5}{140}}{x^2+\frac{2}{5}x+\frac{3}{5}}+\frac{(28700d}{}}{}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x,method=_RETURNVERB
OSE)
```

output
```
-1/(5*d^2-2*d*e+3*e^2)^4*(((423/28*d^5-21351/140*d^4*e+6933/70*d^3*e^2-527
3/70*d^2*e^3+1231/140*d*e^4+2109/140*e^5)*x+1367/28*d^5-7129/140*d^4*e-234
3/70*d^3*e^2+1933/70*d^2*e^3-7241/140*d*e^4+1371/140*e^5)/(x^2+2/5*x+3/5)+
1/280*(28700*d^5-2660*d^4*e-118440*d^3*e^2+55440*d^2*e^3+7980*d*e^4-2940*e
^5)*ln(5*x^2+2*x+3)+1/392*(-6565*d^5+74017*d^4*e-35022*d^3*e^2-42858*d^2*e
^3+17247*d*e^4-579*e^5)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))-1/2*(4*d^
4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/e/(5*d^2-2*d*e+3*e^2)^2/(e*x+d)^2-(41*d^4
-8*d^3*e-60*d^2*e^2+24*d*e^3-5*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)+(205*d^5
-19*d^4*e-846*d^3*e^2+396*d^2*e^3+57*d*e^4-21*e^5)*ln(e*x+d)/(5*d^2-2*d*e+
3*e^2)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1499 vs. $2(401) = 802$.

Time = 0.28 (sec) , antiderivative size = 1499, normalized size of antiderivative = 3.64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\,(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
-1/392*(58800*d^8 + 363230*d^7*e - 178010*d^6*e^2 - 233184*d^5*e^3 + 39516
4*d^4*e^4 - 437122*d^3*e^5 + 178542*d^2*e^6 - 37044*d*e^7 + 10584*e^8 + 14
*(28700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^
2*e^6 + 12711*d*e^7 + 9*e^8)*x^3 + 14*(7000*d^8 + 31850*d^7*e + 6400*d^6*e
^2 - 62649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*
d*e^7 + 1791*e^8)*x^2 - sqrt(14)*(19695*d^7*e - 222051*d^6*e^2 + 105066*d^
5*e^3 + 128574*d^4*e^4 - 51741*d^3*e^5 + 1737*d^2*e^6 + 5*(6565*d^5*e^3 -
74017*d^4*e^4 + 35022*d^3*e^5 + 42858*d^2*e^6 - 17247*d*e^7 + 579*e^8)*x^4
 + 2*(32825*d^6*e^2 - 363520*d^5*e^3 + 101093*d^4*e^4 + 249312*d^3*e^5 - 4
3377*d^2*e^6 - 14352*d*e^7 + 579*e^8)*x^3 + (32825*d^7*e - 343825*d^6*e^2
- 101263*d^5*e^3 + 132327*d^4*e^4 + 190263*d^3*e^5 + 62481*d^2*e^6 - 49425
*d*e^7 + 1737*e^8)*x^2 + 2*(6565*d^7*e - 54322*d^6*e^2 - 187029*d^5*e^3 +
147924*d^4*e^4 + 111327*d^3*e^5 - 51162*d^2*e^6 + 1737*d*e^7)*x)*arctan(1/
14*sqrt(14)*(5*x + 1)) + 14*(2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620
*d^5*e^3 - 17202*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 75
6*e^8)*x - 392*(615*d^7*e - 57*d^6*e^2 - 2538*d^5*e^3 + 1188*d^4*e^4 + 171
*d^3*e^5 - 63*d^2*e^6 + 5*(205*d^5*e^3 - 19*d^4*e^4 - 846*d^3*e^5 + 396*d^
2*e^6 + 57*d*e^7 - 21*e^8)*x^4 + 2*(1025*d^6*e^2 + 110*d^5*e^3 - 4249*d^4*
e^4 + 1134*d^3*e^5 + 681*d^2*e^6 - 48*d*e^7 - 21*e^8)*x^3 + (1025*d^7*e +
725*d^6*e^2 - 3691*d^5*e^3 - 1461*d^4*e^4 - 669*d^3*e^5 + 1311*d^2*e^6 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3\left(3+2x+5x^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**3/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(401) = 802$.

Time = 0.14 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.07

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^3 (3 + 2x + 5x^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 1
7247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*
e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^
6 - 216*d*e^7 + 81*e^8) + (205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3
+ 57*d*e^4 - 21*e^5)*log(e*x + d)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1
960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e
^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*e^3 + 57*d*e^4 - 21*
e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*
e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - 1/
28*(840*d^6 + 5525*d^5*e - 837*d^4*e^2 - 6981*d^3*e^3 + 3355*d^2*e^4 - 714
*d*e^5 + 252*e^6 + (5740*d^4*e^2 - 697*d^3*e^3 - 12501*d^2*e^4 + 4239*d*e^
5 + 3*e^6)*x^3 + (1400*d^6 + 6930*d^5*e + 3212*d^4*e^2 - 15403*d^3*e^3 + 2
349*d^2*e^4 - 549*d*e^5 + 597*e^6)*x^2 + (560*d^6 + 3195*d^5*e + 2105*d^4*
e^2 - 4799*d^3*e^3 - 6623*d^2*e^4 + 2454*d*e^5 - 252*e^6)*x)/(375*d^8*e -
450*d^7*e^2 + 855*d^6*e^3 - 564*d^5*e^4 + 513*d^4*e^5 - 162*d^3*e^6 + 81*d
^2*e^7 + 5*(125*d^6*e^3 - 150*d^5*e^4 + 285*d^4*e^5 - 188*d^3*e^6 + 171*d^
2*e^7 - 54*d*e^8 + 27*e^9)*x^4 + 2*(625*d^7*e^2 - 625*d^6*e^3 + 1275*d^5*e
^4 - 655*d^4*e^5 + 667*d^3*e^6 - 99*d^2*e^7 + 81*d*e^8 + 27*e^9)*x^3 + (62
5*d^8*e - 250*d^7*e^2 + 1200*d^6*e^3 - 250*d^5*e^4 + 958*d^4*e^5 - 150*d^3
*e^6 + 432*d^2*e^7 - 54*d*e^8 + 81*e^9)*x^2 + 2*(125*d^8*e + 225*d^7*e^...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.57

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3 (3+2x+5x^2)^2} dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
1/392*sqrt(14)*(6565*d^5 - 74017*d^4*e + 35022*d^3*e^2 + 42858*d^2*e^3 - 1
7247*d*e^4 + 579*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(625*d^8 - 1000*d^7*
e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^
6 - 216*d*e^7 + 81*e^8) - 1/2*(205*d^5 - 19*d^4*e - 846*d^3*e^2 + 396*d^2*
e^3 + 57*d*e^4 - 21*e^5)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e + 2100
*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*
d*e^7 + 81*e^8) + (205*d^5*e - 19*d^4*e^2 - 846*d^3*e^3 + 396*d^2*e^4 + 57
*d*e^5 - 21*e^6)*log(abs(e*x + d))/(625*d^8*e - 1000*d^7*e^2 + 2100*d^6*e^
3 - 1960*d^5*e^4 + 2086*d^4*e^5 - 1176*d^3*e^6 + 756*d^2*e^7 - 216*d*e^8 +
 81*e^9) - 1/28*(4200*d^8 + 25945*d^7*e - 12715*d^6*e^2 - 16656*d^5*e^3 +
28226*d^4*e^4 - 31223*d^3*e^5 + 12753*d^2*e^6 - 2646*d*e^7 + 756*e^8 + (28
700*d^6*e^2 - 14965*d^5*e^3 - 43891*d^4*e^4 + 44106*d^3*e^5 - 45966*d^2*e^
6 + 12711*d*e^7 + 9*e^8)*x^3 + (7000*d^8 + 31850*d^7*e + 6400*d^6*e^2 - 62
649*d^5*e^3 + 52187*d^4*e^4 - 53652*d^3*e^5 + 11130*d^2*e^6 - 2841*d*e^7 +
 1791*e^8)*x^2 + (2800*d^8 + 14855*d^7*e + 5815*d^6*e^2 - 18620*d^5*e^3 -
17202*d^4*e^4 + 11119*d^3*e^5 - 26037*d^2*e^6 + 7866*d*e^7 - 756*e^8)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^4*(e*x + d)^2*(5*x^2 + 2*x + 3)*e)
```

Mupad [B] (verification not implemented)

Time = 18.03 (sec) , antiderivative size = 887, normalized size of antiderivative = 2.15

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3 (3+2x+5x^2)^2} dx = \text{Too large to display}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^3*(2*x + 5*x^2 + 3)^2),x)
```

output
```
log(d + e*x)*(((41*d)/5 + (29*e)/5)/(5*d^2 - 2*d*e + 3*e^2)^2 + (168*e^4*(
458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) - (2*e^2*(12610*d + 1329*e))
/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) - ((5525*d^5*e - 714*d*e^5 + 840*d^6 + 2
52*e^6 + 3355*d^2*e^4 - 6981*d^3*e^3 - 837*d^4*e^2)/(28*e*(125*d^6 - 150*d
^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^
3*(4239*d*e^4 + 5740*d^4*e + 3*e^5 - 12501*d^2*e^3 - 697*d^3*e^2))/(28*(12
5*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^
4*e^2)) + (x^2*(6930*d^5*e - 549*d*e^5 + 1400*d^6 + 597*e^6 + 2349*d^2*e^4
 - 15403*d^3*e^3 + 3212*d^4*e^2))/(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 +
27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x*(2454*d*e^5 + 3195
*d^5*e + 560*d^6 - 252*e^6 - 6623*d^2*e^4 - 4799*d^3*e^3 + 2105*d^4*e^2))/
(28*e*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3
 + 285*d^4*e^2)))/(x^2*(4*d*e + 5*d^2 + 3*e^2) + x*(6*d*e + 2*d^2) + 3*d^2
 + x^3*(10*d*e + 2*e^2) + 5*e^2*x^4) + (log(x - (14^(1/2)*1i)/5 + 1/5)*(d^
5*((6565*14^(1/2))/784 - 205i/2) + e^5*((579*14^(1/2))/784 + 21i/2) + d^3*
e^2*((17511*14^(1/2))/392 + 423i) + d^2*e^3*((21429*14^(1/2))/392 - 198i)
- d*e^4*((17247*14^(1/2))/784 + 57i/2) - d^4*e*((74017*14^(1/2))/784 - 19i
/2)))/(d^8*625i - d^7*e*1000i - d*e^7*216i + e^8*81i + d^2*e^6*756i - d^3*
e^5*1176i + d^4*e^4*2086i - d^5*e^3*1960i + d^6*e^2*2100i) - (log(x + (14^
(1/2)*1i)/5 + 1/5)*(d^5*((6565*14^(1/2))/784 + 205i/2) + e^5*((579*14^(...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3116, normalized size of antiderivative = 7.56

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^3(3+2x+5x^2)^2}\,dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^3/(5*x^2+2*x+3)^2,x)
```

output
```
(164125*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**8*e*x**2 + 65650*sqrt(14)*ata
n((5*x + 1)/sqrt(14))*d**8*e*x + 98475*sqrt(14)*atan((5*x + 1)/sqrt(14))*d
**8*e + 328250*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**7*e**2*x**3 - 1686300*
sqrt(14)*atan((5*x + 1)/sqrt(14))*d**7*e**2*x**2 - 530090*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d**7*e**2*x - 1090560*sqrt(14)*atan((5*x + 1)/sqrt(14))*
d**7*e**2 + 164125*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**3*x**4 - 3569
550*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**3*x**3 - 850140*sqrt(14)*ata
n((5*x + 1)/sqrt(14))*d**6*e**3*x**2 - 1978934*sqrt(14)*atan((5*x + 1)/sqr
t(14))*d**6*e**3*x + 303279*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**3 -
1817600*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**4*x**4 + 283890*sqrt(14)
*atan((5*x + 1)/sqrt(14))*d**5*e**4*x**3 + 560372*sqrt(14)*atan((5*x + 1)/
sqrt(14))*d**5*e**4*x**2 + 1105182*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*
e**4*x + 747936*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**4 + 505465*sqrt(
14)*atan((5*x + 1)/sqrt(14))*d**4*e**5*x**4 + 2695306*sqrt(14)*atan((5*x +
 1)/sqrt(14))*d**4*e**5*x**3 + 1083642*sqrt(14)*atan((5*x + 1)/sqrt(14))*d
**4*e**5*x**2 + 1409118*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**5*x - 13
0131*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**5 + 1246560*sqrt(14)*atan((
5*x + 1)/sqrt(14))*d**3*e**6*x**4 + 64854*sqrt(14)*atan((5*x + 1)/sqrt(14)
)*d**3*e**6*x**3 + 502668*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**6*x**2
 - 288966*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**6*x - 43056*sqrt(14...
```

3.153 $\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 230

$$\int \frac{(d+ex)^3(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{1}{625}(60d-49e)e^2x+\frac{2e^3x^2}{125}$$
$$-\frac{170875d^3-95175d^2e-269505de^2+54969e^3+(52875d^3+449175d^2e-274845de^2-53189e^3)\,x}{875000\,(3+2x+5x^2)^2}$$
$$+\frac{4293375d^3-3329925d^2e-12137505de^2+2639639e^3+5(275375d^3+2726475d^2e-1941585de^2-6}{24500000\,(3+2x+5x^2)}$$
$$+\frac{3(353125d^3-855175d^2e+74085de^2+556349e^3)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{4900000\sqrt{14}}$$
$$+\frac{3e(100d^2-245de+47e^2)\log(3+2x+5x^2)}{6250}$$

output
```
1/625*(60*d-49*e)*e^2*x+2/125*e^3*x^2-1/875000*(170875*d^3-95175*d^2*e-269
505*d*e^2+54969*e^3+(52875*d^3+449175*d^2*e-274845*d*e^2-53189*e^3)*x)/(5*
x^2+2*x+3)^2+(4293375*d^3-3329925*d^2*e-12137505*d*e^2+2639639*e^3+5*(2753
75*d^3+2726475*d^2*e-1941585*d*e^2-621801*e^3)*x)/(122500000*x^2+49000000*
x+73500000)+3/68600000*(353125*d^3-855175*d^2*e+74085*d*e^2+556349*e^3)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)+3/6250*e*(100*d^2-245*d*e+47*e^2)*ln(
5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{548800(60d-49e)e^2x+5488000e^3x^2-\frac{392(e^3(54969-53189x)+125d^3(1367+423x)+75d^2e(-1269+5989x)-15de^2(17967+1\ldots}{(3+2x+5x^2)^2}}{}$$

input
```
Integrate[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
3,x]
```

output
```
(548800*(60*d - 49*e)*e^2*x + 5488000*e^3*x^2 - (392*(e^3*(54969 - 53189*x
) + 125*d^3*(1367 + 423*x) + 75*d^2*e*(-1269 + 5989*x) - 15*d*e^2*(17967 +
 18323*x)))/(3 + 2*x + 5*x^2)^2 + (14*(e^3*(2639639 - 3109005*x) + 125*d^3
*(34347 + 11015*x) + 75*d^2*e*(-44399 + 181765*x) - 15*d*e^2*(809167 + 647
195*x)))/(3 + 2*x + 5*x^2) + 15*Sqrt[14]*(353125*d^3 - 855175*d^2*e + 7408
5*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]] + 164640*e*(100*d^2 - 245
*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/343000000
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2175, 27, 2175, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^3}{(5x^2+2x+3)^3}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{1}{112}\int \frac{2(d+ex)^2 (5600ex^3+280(20d-33e)x^2-168(55d-27e)x+3(1089d+1367e))}{125(5x^2+2x+3)^2}dx - \frac{(423x+1367)(d+ex)^3}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(d+ex)^2\left(5600ex^3+280(20d-33e)x^2-168(55d-27e)x+3(1089d+1367e)\right)}{(5x^2+2x+3)^2}dx}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \ 2175$$

$$\frac{\frac{1}{56}\int \frac{10(d+ex)\left(6272e^2x^2+(10341d-22693e)ex+3\left(2825d^2-5587ed+842e^2\right)\right)}{5x^2+2x+3}dx+\frac{(x(11015d+49177e)+3(11449d-2105e))(d+ex)^2}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{5}{28}\int \frac{(d+ex)\left(6272e^2x^2+(10341d-22693e)ex+3\left(2825d^2-5587ed+842e^2\right)\right)}{5x^2+2x+3}dx+\frac{(x(11015d+49177e)+3(11449d-2105e))(d+ex)^2}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \ 2159$$

$$\frac{\frac{5}{28}\int \left(\frac{6272xe^3}{5}+\frac{1}{25}(83065d-126009e)e^2+\frac{3\left(70625d^3-139675ed^2-62015e^2d+126009e^3+1568e\left(100d^2-245ed+47e^2\right)x\right)}{25(5x^2+2x+3)}\right)dx+\ldots}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{5}{28}\left(\frac{3\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(353125d^3-855175d^2e+74085de^2+556349e^3\right)}{125\sqrt{14}}+\frac{2352}{125}e\left(100d^2-245de+47e^2\right)\log\left(5x^2+2x+3\right)+\frac{1}{25}e^2\ldots\right.}{7000}-\frac{(423x+1367)(d+ex)^3}{7000\left(5x^2+2x+3\right)^2}$$

input
```
Int[((d + e*x)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x)^3)/(3 + 2*x + 5*x^2)^2 + (((d + e*x)^2*(
3*(11449*d - 2105*e) + (11015*d + 49177*e)*x))/(28*(3 + 2*x + 5*x^2)) + (5
*(((83065*d - 126009*e)*e^2*x)/25 + (3136*e^3*x^2)/5 + (3*(353125*d^3 - 85
5175*d^2*e + 74085*d*e^2 + 556349*e^3)*ArcTan[(1 + 5*x)/Sqrt[14]])/(125*Sq
rt[14]) + (2352*e*(100*d^2 - 245*d*e + 47*e^2)*Log[3 + 2*x + 5*x^2])/125))
/28)/7000
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.91

method	result
default	$\frac{2e^3x^2}{125}+\frac{12d\,e^2x}{125}-\frac{49e^3x}{625}+\frac{\left(\frac{11015}{1568}d^3+\frac{109059}{1568}d^2e-\frac{388317}{7840}d\,e^2-\frac{621801}{39200}e^3\right)x^3+\left(\frac{38753}{1568}d^3+\frac{84921}{7840}d^2e-\frac{640827}{7840}d\,e^2+\frac{1396037}{196000}e^3\right)x}{25(5x^2+2}$
risch	$\frac{6d^2e\ln(350x^2+140x+210)}{125}-\frac{147d\,e^2\ln(350x^2+140x+210)}{1250}+\frac{339\sqrt{14}\,d^3\arctan\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}+\frac{1669047\sqrt{14}\,e^3\arctan\left(\frac{5}{}\right.}{68600000}$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
2/125*e^3*x^2+12/125*d*e^2*x-49/625*e^3*x+1/25*((11015/1568*d^3+109059/156
8*d^2*e-388317/7840*d*e^2-621801/39200*e^3)*x^3+(38753/1568*d^3+84921/7840
*d^2*e-640827/7840*d*e^2+1396037/196000*e^3)*x^2+(17979/1568*d^3+173283/78
40*d^2*e-73125/1568*d*e^2-511689/196000*e^3)*x+12953/1568*d^3-58599/7840*d
^2*e-230931/7840*d*e^2+1275957/196000*e^3)/(5*x^2+2*x+3)^2+3/9800000*(1568
00*d^2*e-384160*d*e^2+73696*e^3)*ln(5*x^2+2*x+3)+3/13720000*(70625*d^3-171
035*d^2*e+14817*d*e^2+556349/5*e^3)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(215) = 430.

Time = 0.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^3\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx$$
$$=\frac{27440000\,e^3x^6+2744000\,(60\,de^2-41\,e^3)x^5+8780800\,(15\,de^2-8\,e^3)x^4+70\,(275375\,d^3+2726475\,d^2e}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/68600000*(27440000*e^3*x^6 + 2744000*(60*d*e^2 - 41*e^3)*x^5 + 8780800*(
15*d*e^2 - 8*e^3)*x^4 + 70*(275375*d^3 + 2726475*d^2*e + 1257135*d*e^2 - 3
045929*e^3)*x^3 + 22667750*d^3 - 20509650*d^2*e - 80825850*d*e^2 + 1786339
8*e^3 + 14*(4844125*d^3 + 2123025*d^2*e - 10375875*d*e^2 - 2508283*e^3)*x^
2 + 3*sqrt(14)*(25*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*
x^4 + 20*(353125*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^3 + 3178
125*d^3 - 7696575*d^2*e + 666765*d*e^2 + 5007141*e^3 + 34*(353125*d^3 - 85
5175*d^2*e + 74085*d*e^2 + 556349*e^3)*x^2 + 12*(353125*d^3 - 855175*d^2*e
 + 74085*d*e^2 + 556349*e^3)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 42*(7491
25*d^3 + 1444025*d^2*e - 1635675*d*e^2 - 1323043*e^3)*x + 32928*(25*(100*d
^2*e - 245*d*e^2 + 47*e^3)*x^4 + 20*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^3 +
 900*d^2*e - 2205*d*e^2 + 423*e^3 + 34*(100*d^2*e - 245*d*e^2 + 47*e^3)*x^
2 + 12*(100*d^2*e - 245*d*e^2 + 47*e^3)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 +
 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.99 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{2e^3x^2}{125} + x\left(\frac{12de^2}{125} - \frac{49e^3}{625}\right) + \left(\frac{3e(100d^2-245de+47e^2)}{6250} - \frac{3\sqrt{14}i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+327139}{1}\right.$$
$$+ \left(\frac{3e(100d^2-245de+47e^2)}{6250} + \frac{3\sqrt{14}i(353125d^3-855175d^2e+74085de^2+556349e^3)}{137200000}\right)\log\left(x + \frac{211875d^3-1830225d^2e+327139}{1}\right.$$
$$+ \frac{1619125d^3-1464975d^2e-5773275de^2+1275957e^3+x^3\cdot(1376875d^3+13632375d^2e-9707925de^2}{122500000x^4+98}$$

input
```
integrate((e*x+d)**3*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
2*e**3*x**2/125 + x*(12*d*e**2/125 - 49*e**3/625) + (3*e*(100*d**2 - 245*d
*e + 47*e**2)/6250 - 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e
**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d**2*e + 3271
395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**2)/5 - 3*sq
rt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e**3)/5)/(10
59375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) + (3*e*(100*d
**2 - 245*d*e + 47*e**2)/6250 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e
+ 74085*d*e**2 + 556349*e**3)/137200000)*log(x + (211875*d**3 - 1830225*d*
*2*e + 3271395*d*e**2 - 285237*e**3 + 65856*e*(100*d**2 - 245*d*e + 47*e**
2)/5 + 3*sqrt(14)*I*(353125*d**3 - 855175*d**2*e + 74085*d*e**2 + 556349*e
**3)/5)/(1059375*d**3 - 2565525*d**2*e + 222255*d*e**2 + 1669047*e**3)) +
(1619125*d**3 - 1464975*d**2*e - 5773275*d*e**2 + 1275957*e**3 + x**3*(137
6875*d**3 + 13632375*d**2*e - 9707925*d*e**2 - 3109005*e**3) + x**2*(48441
25*d**3 + 2123025*d**2*e - 16020675*d*e**2 + 1396037*e**3) + x*(2247375*d*
*3 + 4332075*d**2*e - 9140625*d*e**2 - 511689*e**3))/(122500000*x**4 + 980
00000*x**3 + 166600000*x**2 + 58800000*x + 44100000)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx = \frac{2}{125} e^3 x^2 + \frac{3}{68600000} \sqrt{14} (353125 d^3 - 855175 d^2 e + 74085 d e^2 + 556349 e^3) \arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{1}{625}(60 d e^2 - 49 e^3) x + \frac{3}{6250}(100 d^2 e - 245 d e^2 + 47 e^3) \log(5x^2 + 2x + 3) + \frac{5(275375 d^3 + 2726475 d^2 e - 1941585 d e^2 - 621801 e^3) x^3 + 1619125 d^3 - 1464975 d^2 e - 5773275 d e^2}{4900}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
2/125*e^3*x^2 + 3/68600000*sqrt(14)*(353125*d^3 - 855175*d^2*e + 74085*d*e
^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/625*(60*d*e^2 - 49*e^
3)*x + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/49
00000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 + 1
619125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3 +
2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 144402
5*d^2*e - 3046875*d*e^2 - 170563*e^3)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x
+ 9)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2}{125}\,e^3x^2 + \frac{12}{125}\,de^2x - \frac{49}{625}\,e^3x + \frac{3}{68600000}\,\sqrt{14}\big(353125\,d^3 - 855175\,d^2e + 74085\,de^2 + 556349\,e^3\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{3}{6250}\,\big(100\,d^2e - 245\,de^2 + 47\,e^3\big)\log\big(5\,x^2 + 2\,x + 3\big) + \frac{5\,(275375\,d^3 + 2726475\,d^2e - 1941585\,de^2 - 621801\,e^3)x^3 + 1619125\,d^3 - 1464975\,d^2e - 5773275\,de^2}{}$$

input
```
integrate((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
2/125*e^3*x^2 + 12/125*d*e^2*x - 49/625*e^3*x + 3/68600000*sqrt(14)*(35312
5*d^3 - 855175*d^2*e + 74085*d*e^2 + 556349*e^3)*arctan(1/14*sqrt(14)*(5*x
 + 1)) + 3/6250*(100*d^2*e - 245*d*e^2 + 47*e^3)*log(5*x^2 + 2*x + 3) + 1/
4900000*(5*(275375*d^3 + 2726475*d^2*e - 1941585*d*e^2 - 621801*e^3)*x^3 +
 1619125*d^3 - 1464975*d^2*e - 5773275*d*e^2 + 1275957*e^3 + (4844125*d^3
+ 2123025*d^2*e - 16020675*d*e^2 + 1396037*e^3)*x^2 + 3*(749125*d^3 + 1444
025*d^2*e - 3046875*d*e^2 - 170563*e^3)*x)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = x\left(\frac{e^2\,(12\,d-5\,e)}{125}-\frac{24\,e^3}{625}\right)$$

$$-\frac{\frac{1154655\,d\,e^2}{1568}+\frac{292995\,d^2\,e}{1568}+x\left(-\frac{449475\,d^3}{1568}-\frac{866415\,d^2\,e}{1568}+\frac{1828125\,d\,e^2}{1568}+\frac{511689\,e^3}{7840}\right)-\frac{323825\,d^3}{1568}-\frac{1275957\,e^3}{7840}+x^3\,(}{15625\,x^4+12500\,x^3+212}$$

$$+\ln\left(5\,x^2+2\,x+3\right)\,\left(\frac{6\,d^2\,e}{125}-\frac{147\,d\,e^2}{1250}+\frac{141\,e^3}{6250}\right)+\frac{2\,e^3\,x^2}{125}$$

$$+\frac{3\,\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{3\,\sqrt{14}\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{68600000}+\frac{3\,\sqrt{14}\,x\,\left(353125\,d^3-855175\,d^2\,e+74085\,d\,e^2+556349\,e^3\right)}{13720000}}{\frac{339\,d^3}{1568}-\frac{102621\,d^2\,e}{196000}+\frac{44451\,d\,e^2}{980000}+\frac{1669047\,e^3}{4900000}}\right)\,(353125\,d^3}{68600000}$$

input
```
int(((d + e*x)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
x*((e^2*(12*d - 5*e))/125 - (24*e^3)/625) - ((1154655*d*e^2)/1568 + (29299
5*d^2*e)/1568 + x*((1828125*d*e^2)/1568 - (866415*d^2*e)/1568 - (449475*d^
3)/1568 + (511689*e^3)/7840) - (323825*d^3)/1568 - (1275957*e^3)/7840 + x^
3*((1941585*d*e^2)/1568 - (2726475*d^2*e)/1568 - (275375*d^3)/1568 + (6218
01*e^3)/1568) - x^2*((424605*d^2*e)/1568 - (3204135*d*e^2)/1568 + (968825*
d^3)/1568 + (1396037*e^3)/7840))/(7500*x + 21250*x^2 + 12500*x^3 + 15625*x
^4 + 5625) + log(2*x + 5*x^2 + 3)*((6*d^2*e)/125 - (147*d*e^2)/1250 + (141
*e^3)/6250) + (2*e^3*x^2)/125 + (3*14^(1/2)*atan(((3*14^(1/2)*(74085*d*e^2
 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000 + (3*14^(1/2)*x*(7408
5*d*e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/13720000)/((44451*d*e^2
)/980000 - (102621*d^2*e)/196000 + (339*d^3)/1568 + (1669047*e^3)/4900000)
)*(74085*d*e^2 - 855175*d^2*e + 353125*d^3 + 556349*e^3))/68600000
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 855, normalized size of antiderivative = 3.72

$$\int \frac{(d+ex)^3\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

output
```
(52968750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*x**4 + 42375000*sqrt(14)*
atan((5*x + 1)/sqrt(14))*d**3*x**3 + 72037500*sqrt(14)*atan((5*x + 1)/sqrt
(14))*d**3*x**2 + 25425000*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*x + 1906
8750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3 - 128276250*sqrt(14)*atan((5*x
 + 1)/sqrt(14))*d**2*e*x**4 - 102621000*sqrt(14)*atan((5*x + 1)/sqrt(14))*
d**2*e*x**3 - 174455700*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e*x**2 - 61
572600*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e*x - 46179450*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d**2*e + 11112750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d
*e**2*x**4 + 8890200*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**2*x**3 + 15113
340*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**2*x**2 + 5334120*sqrt(14)*atan(
(5*x + 1)/sqrt(14))*d*e**2*x + 4000590*sqrt(14)*atan((5*x + 1)/sqrt(14))*d
*e**2 + 83452350*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3*x**4 + 66761880*sq
rt(14)*atan((5*x + 1)/sqrt(14))*e**3*x**3 + 113495196*sqrt(14)*atan((5*x +
 1)/sqrt(14))*e**3*x**2 + 40057128*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3*
x + 30042846*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**3 + 164640000*log(5*x**2
 + 2*x + 3)*d**2*e*x**4 + 131712000*log(5*x**2 + 2*x + 3)*d**2*e*x**3 + 22
3910400*log(5*x**2 + 2*x + 3)*d**2*e*x**2 + 79027200*log(5*x**2 + 2*x + 3)
*d**2*e*x + 59270400*log(5*x**2 + 2*x + 3)*d**2*e - 403368000*log(5*x**2 +
 2*x + 3)*d*e**2*x**4 - 322694400*log(5*x**2 + 2*x + 3)*d*e**2*x**3 - 5485
80480*log(5*x**2 + 2*x + 3)*d*e**2*x**2 - 193616640*log(5*x**2 + 2*x + ...
```

3.154 $\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 165

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{4e^2x}{125} - \frac{34175d^2-12690de-17967e^2+(10575d^2+59890de-18323e^2)\,x}{175000\,(3+2x+5x^2)^2} + \frac{858675d^2-443990de-809167e^2+5(55075d^2+363530de-129439e^2)\,x}{4900000\,(3+2x+5x^2)} + \frac{(211875d^2-342070de+14817e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{980000\sqrt{14}} + \frac{(40d-49e)e\log(3+2x+5x^2)}{1250}$$

output

```
4/125*e^2*x-1/175000*(34175*d^2-12690*d*e-17967*e^2+(10575*d^2+59890*d*e-1
8323*e^2)*x)/(5*x^2+2*x+3)^2+(858675*d^2-443990*d*e-809167*e^2+5*(55075*d^
2+363530*d*e-129439*e^2)*x)/(24500000*x^2+9800000*x+14700000)+1/13720000*(
211875*d^2-342070*d*e+14817*e^2)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)+1/
1250*(40*d-49*e)*e*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{5\sqrt{14}(211875d^2-342070de+14817e^2)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)+70\left(\frac{5(5d^2(12953+17979x+38753x^2+11015x^3)+2de(-19533\cdots}{\cdots}\right.}{6\cdots}$$

input
```
Integrate[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
3,x]
```

output
```
(5*Sqrt[14]*(211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14
]] + 70*((5*(5*d^2*(12953 + 17979*x + 38753*x^2 + 11015*x^3) + 2*d*e*(-195
33 + 57761*x + 28307*x^2 + 181765*x^3) + e^2*(-76977 - 65427*x - 138345*x^
2 + 83809*x^3 + 125440*x^4 + 156800*x^5)))/(3 + 2*x + 5*x^2)^2 + 784*(40*d
 - 49*e)*e*Log[3 + 2*x + 5*x^2]))/68600000
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2175, 27, 2175, 27, 2188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^2}{(5x^2+2x+3)^3}dx$$

$$\Big\downarrow \text{ 2175}$$

$$\frac{1}{112}\int \frac{2(d+ex)(5600ex^3+280(20d-33e)x^2-3(3080d-1371e)x+3267d+2734e)}{125(5x^2+2x+3)^2}dx - \frac{(423x+1367)(d+ex)^2}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(d+ex)\left(5600ex^3+280(20d-33e)x^2-3(3080d-1371e)x+3267d+2734e\right)}{(5x^2+2x+3)^2}dx}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 2175$$

$$\frac{\frac{1}{56}\int \frac{2\left(42375d^2-55870ed+6413e^2+31360e^2x^2+1568(40d-41e)ex\right)}{5x^2+2x+3}dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{28}\int \frac{42375d^2-55870ed+6413e^2+31360e^2x^2+1568(40d-41e)ex}{5x^2+2x+3}dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 2188$$

$$\frac{\frac{1}{28}\int \left(6272e^2+\frac{42375d^2-55870ed-12403e^2+1568(40d-49e)ex}{5x^2+2x+3}\right)dx+\frac{(d+ex)(5x(2203d+8553e)+34347d-6413e)}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^2-342070de+14817e^2)}{5\sqrt{14}}+\frac{784}{5}e(40d-49e)\log\left(5x^2+2x+3\right)+6272e^2x\right)+\frac{(d+ex)(5x(2203d+85}{28(5x^2}}{7000}-\frac{(423x+1367)(d+ex)^2}{7000\left(5x^2+2x+3\right)^2}$$

input
```
Int[((d + e*x)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x)^2)/(3 + 2*x + 5*x^2)^2 + (((d + e*x)*(34
347*d - 6413*e + 5*(2203*d + 8553*e)*x))/(28*(3 + 2*x + 5*x^2)) + (6272*e^
2*x + ((211875*d^2 - 342070*d*e + 14817*e^2)*ArcTan[(1 + 5*x)/Sqrt[14]])/(
5*Sqrt[14]) + (784*(40*d - 49*e)*e*Log[3 + 2*x + 5*x^2])/5)/28)/7000
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2188
```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

method	result
default	$\frac{4e^2x}{125}+\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3+\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2+\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x+\frac{12953d^2}{7840}-\frac{19533de}{19600}-}{5(5x^2+2x+3)^2}$
risch	$\frac{4e^2x}{125}+\frac{\frac{\left(\frac{2203}{1568}d^2+\frac{36353}{3920}de-\frac{129439}{39200}e^2\right)x^3}{5}+\frac{\left(\frac{38753}{7840}d^2+\frac{28307}{19600}de-\frac{213609}{39200}e^2\right)x^2}{5}+\frac{\left(\frac{17979}{7840}d^2+\frac{57761}{19600}de-\frac{4875}{1568}e^2\right)x}{5}+\frac{12953d^2}{39200}-\frac{19533de}{98000}-}{(5x^2+2x+3)^2}$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
4/125*e^2*x+1/5*((2203/1568*d^2+36353/3920*d*e-129439/39200*e^2)*x^3+(3875
3/7840*d^2+28307/19600*d*e-213609/39200*e^2)*x^2+(17979/7840*d^2+57761/196
00*d*e-4875/1568*e^2)*x+12953/7840*d^2-19533/19600*d*e-76977/39200*e^2)/(5
*x^2+2*x+3)^2+1/1960000*(62720*d*e-76832*e^2)*ln(5*x^2+2*x+3)+1/2744000*(4
2375*d^2-68414*d*e+14817/5*e^2)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{10976000\,e^2x^5+8780800\,e^2x^4+70\,(55075\,d^2+363530\,de+83809\,e^2)x^3+70\,(193765\,d^2+56614\,de-1$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/13720000*(10976000*e^2*x^5 + 8780800*e^2*x^4 + 70*(55075*d^2 + 363530*d*
e + 83809*e^2)*x^3 + 70*(193765*d^2 + 56614*d*e - 138345*e^2)*x^2 + sqrt(1
4)*(25*(211875*d^2 - 342070*d*e + 14817*e^2)*x^4 + 20*(211875*d^2 - 342070
*d*e + 14817*e^2)*x^3 + 34*(211875*d^2 - 342070*d*e + 14817*e^2)*x^2 + 190
6875*d^2 - 3078630*d*e + 133353*e^2 + 12*(211875*d^2 - 342070*d*e + 14817*
e^2)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 4533550*d^2 - 2734620*d*e - 5388
390*e^2 + 70*(89895*d^2 + 115522*d*e - 65427*e^2)*x + 10976*(25*(40*d*e -
49*e^2)*x^4 + 20*(40*d*e - 49*e^2)*x^3 + 34*(40*d*e - 49*e^2)*x^2 + 360*d*
e - 441*e^2 + 12*(40*d*e - 49*e^2)*x)*log(5*x^2 + 2*x + 3))/(25*x^4 + 20*x
^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{4e^2x}{125} + \left(\frac{e(40d-49e)}{1250} - \frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x + \frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}}{211875d^2-342070de+1}\right.$$
$$+\left(\frac{e(40d-49e)}{1250} + \frac{\sqrt{14}i(211875d^2-342070de+14817e^2)}{27440000}\right)\log\left(x + \frac{42375d^2-244030de+218093e^2+\frac{21952e(40d-49e)}{5}}{211875d^2-342070de+1}\right.$$
$$+\frac{64765d^2-39066de-76977e^2+x^3\cdot(55075d^2+363530de-129439e^2)+x^2\cdot(193765d^2+56614de-}{4900000x^4+3920000x^3+6664000x^2+2352000x+176}$$

input
```
integrate((e*x+d)**2*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
4*e**2*x/125 + (e*(40*d - 49*e)/1250 - sqrt(14)*I*(211875*d**2 - 342070*d*
e + 14817*e**2)/27440000)*log(x + (42375*d**2 - 244030*d*e + 218093*e**2 +
 21952*e*(40*d - 49*e)/5 - sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e*
*2)/5)/(211875*d**2 - 342070*d*e + 14817*e**2)) + (e*(40*d - 49*e)/1250 +
sqrt(14)*I*(211875*d**2 - 342070*d*e + 14817*e**2)/27440000)*log(x + (4237
5*d**2 - 244030*d*e + 218093*e**2 + 21952*e*(40*d - 49*e)/5 + sqrt(14)*I*(
211875*d**2 - 342070*d*e + 14817*e**2)/5)/(211875*d**2 - 342070*d*e + 1481
7*e**2)) + (64765*d**2 - 39066*d*e - 76977*e**2 + x**3*(55075*d**2 + 36353
0*d*e - 129439*e**2) + x**2*(193765*d**2 + 56614*d*e - 213609*e**2) + x*(8
9895*d**2 + 115522*d*e - 121875*e**2))/(4900000*x**4 + 3920000*x**3 + 6664
000*x**2 + 2352000*x + 1764000)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{4}{125}\,e^2x + \frac{1}{13720000}\,\sqrt{14}\big(211875\,d^2-342070\,de+14817\,e^2\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{1250}\,\big(40\,de-49\,e^2\big)\log\big(5\,x^2+2\,x+3\big) + \frac{(55075\,d^2+363530\,de-129439\,e^2)x^3+(193765\,d^2+56614\,de-213609\,e^2)x^2+64765\,d^2-39066\,de}{196000\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*ar
ctan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x +
 3) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 +
56614*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d
^2 + 115522*d*e - 121875*e^2)*x)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^2\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{4}{125}\,e^2x + \frac{1}{13720000}\,\sqrt{14}\big(211875\,d^2-342070\,de+14817\,e^2\big)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) + \frac{1}{1250}\,\big(40\,de-49\,e^2\big)\log\big(5\,x^2+2\,x+3\big) + \frac{(55075\,d^2+363530\,de-129439\,e^2)x^3+(193765\,d^2+56614\,de-213609\,e^2)x^2+64765\,d^2-39066\,de}{196000\,(5\,x^2+2\,x+3)^2}$$

input
```
integrate((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
4/125*e^2*x + 1/13720000*sqrt(14)*(211875*d^2 - 342070*d*e + 14817*e^2)*ar
ctan(1/14*sqrt(14)*(5*x + 1)) + 1/1250*(40*d*e - 49*e^2)*log(5*x^2 + 2*x +
 3) + 1/196000*((55075*d^2 + 363530*d*e - 129439*e^2)*x^3 + (193765*d^2 +
56614*d*e - 213609*e^2)*x^2 + 64765*d^2 - 39066*d*e - 76977*e^2 + (89895*d
^2 + 115522*d*e - 121875*e^2)*x)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex)^2(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$

$$= \frac{x^3\left(\frac{55075\,d^2}{1568}+\frac{181765\,d\,e}{784}-\frac{129439\,e^2}{1568}\right)+x^2\left(\frac{193765\,d^2}{1568}+\frac{28307\,d\,e}{784}-\frac{213609\,e^2}{1568}\right)-\frac{19533\,d\,e}{784}+x\left(\frac{89895\,d^2}{1568}+\frac{57761\,d\,e}{784}\right.}{3125\,x^4+2500\,x^3+4250\,x^2+1500\,x+1125}$$

$$+\frac{4\,e^2\,x}{125}+\ln\left(5\,x^2+2\,x+3\right)\left(\frac{4\,d\,e}{125}-\frac{49\,e^2}{1250}\right)$$

$$+\frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{13720000}+\frac{\sqrt{14}\,x\left(211875\,d^2-342070\,d\,e+14817\,e^2\right)}{2744000}}{\frac{339\,d^2}{1568}-\frac{34207\,d\,e}{98000}+\frac{14817\,e^2}{980000}}\right)\left(211875\,d^2-342070\,d\,e+14817\,e\right.}{13720000}$$

input
```
int(((d + e*x)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
(x^3*((181765*d*e)/784 + (55075*d^2)/1568 - (129439*e^2)/1568) + x^2*((283
07*d*e)/784 + (193765*d^2)/1568 - (213609*e^2)/1568) - (19533*d*e)/784 + x
*((57761*d*e)/784 + (89895*d^2)/1568 - (121875*e^2)/1568) + (64765*d^2)/15
68 - (76977*e^2)/1568)/(1500*x + 4250*x^2 + 2500*x^3 + 3125*x^4 + 1125) +
(4*e^2*x)/125 + log(2*x + 5*x^2 + 3)*((4*d*e)/125 - (49*e^2)/1250) + (14^(
1/2)*atan(((14^(1/2)*(211875*d^2 - 342070*d*e + 14817*e^2))/13720000 + (14
^(1/2)*x*(211875*d^2 - 342070*d*e + 14817*e^2))/2744000)/((339*d^2)/1568 -
 (34207*d*e)/98000 + (14817*e^2)/980000))*(211875*d^2 - 342070*d*e + 14817
*e^2))/13720000
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.51

$$\int \frac{(d+ex)^2 (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

output
```
(10593750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*x**4 + 8475000*sqrt(14)*a
tan((5*x + 1)/sqrt(14))*d**2*x**3 + 14407500*sqrt(14)*atan((5*x + 1)/sqrt(
14))*d**2*x**2 + 5085000*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*x + 381375
0*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2 - 17103500*sqrt(14)*atan((5*x + 1
)/sqrt(14))*d*e*x**4 - 13682800*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e*x**3
 - 23260760*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e*x**2 - 8209680*sqrt(14)*
atan((5*x + 1)/sqrt(14))*d*e*x - 6157260*sqrt(14)*atan((5*x + 1)/sqrt(14))
*d*e + 740850*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**2*x**4 + 592680*sqrt(14
)*atan((5*x + 1)/sqrt(14))*e**2*x**3 + 1007556*sqrt(14)*atan((5*x + 1)/sqr
t(14))*e**2*x**2 + 355608*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**2*x + 26670
6*sqrt(14)*atan((5*x + 1)/sqrt(14))*e**2 + 21952000*log(5*x**2 + 2*x + 3)*
d*e*x**4 + 17561600*log(5*x**2 + 2*x + 3)*d*e*x**3 + 29854720*log(5*x**2 +
 2*x + 3)*d*e*x**2 + 10536960*log(5*x**2 + 2*x + 3)*d*e*x + 7902720*log(5*
x**2 + 2*x + 3)*d*e - 26891200*log(5*x**2 + 2*x + 3)*e**2*x**4 - 21512960*
log(5*x**2 + 2*x + 3)*e**2*x**3 - 36572032*log(5*x**2 + 2*x + 3)*e**2*x**2
 - 12907776*log(5*x**2 + 2*x + 3)*e**2*x - 9680832*log(5*x**2 + 2*x + 3)*e
**2 - 9638125*d**2*x**4 + 14019250*d**2*x**2 + 7959000*d**2*x + 5597375*d*
*2 - 63617750*d*e*x**4 - 78594180*d*e*x**2 - 14363440*d*e*x - 28371630*d*e
 + 21952000*e**2*x**5 + 2895025*e**2*x**4 - 39314842*e**2*x**2 - 16199736*
e**2*x - 16056747*e**2)/(27440000*(25*x**4 + 20*x**3 + 34*x**2 + 12*x +...
```

3.155 $\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx$

Optimal result

Integrand size = 36, antiderivative size = 110

$$\begin{aligned}\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3} dx &= -\frac{6835d-1269e+(2115d+5989e)x}{35000(3+2x+5x^2)^2}+\frac{171735d-44399e+5(11015d+36353e)x}{980000(3+2x+5x^2)} \\ &\quad +\frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}}+\frac{2}{125}e\log\left(3+2x+5x^2\right)\end{aligned}$$

output

```
-1/35000*(6835*d-1269*e+(2115*d+5989*e)*x)/(5*x^2+2*x+3)^2+(171735*d-44399
*e+5*(11015*d+36353*e)*x)/(4900000*x^2+1960000*x+2940000)+1/2744000*(42375
*d-34207*e)*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)+2/125*e*ln(5*x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)\left(2+x+3x^2-5x^3+4x^4\right)}{\left(3+2x+5x^2\right)^3}\,dx = \frac{-6835d+1269e-2115dx-5989ex}{35000\left(3+2x+5x^2\right)^2} + \frac{171735d-44399e+55075dx+181765ex}{980000\left(3+2x+5x^2\right)} + \frac{(42375d-34207e)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{196000\sqrt{14}} + \frac{2}{125}e\log\left(3+2x+5x^2\right)$$

input
```
Integrate[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,
x]
```

output
```
(-6835*d + 1269*e - 2115*d*x - 5989*e*x)/(35000*(3 + 2*x + 5*x^2)^2) + (17
1735*d - 44399*e + 55075*d*x + 181765*e*x)/(980000*(3 + 2*x + 5*x^2)) + ((
42375*d - 34207*e)*ArcTan[(1 + 5*x)/Sqrt[14]])/(196000*Sqrt[14]) + (2*e*Lo
g[3 + 2*x + 5*x^2])/125
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2175, 27, 2191, 27, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(4x^4-5x^3+3x^2+x+2\right)(d+ex)}{\left(5x^2+2x+3\right)^3}\,dx$$

$$\Big\downarrow \; 2175$$

$$\frac{1}{112}\int \frac{2\left(5600ex^3+280(20d-33e)x^2-30(308d-123e)x+3267d+1367e\right)}{125\left(5x^2+2x+3\right)^2}dx-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5600ex^3+280(20d-33e)x^2-30(308d-123e)x+3267d+1367e}{\left(5x^2+2x+3\right)^2}dx}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 2191$$

$$\frac{\frac{1}{56}\int \frac{10(8475d-5587e+6272ex)}{5x^2+2x+3}dx+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5}{28}\int \frac{8475d-5587e+6272ex}{5x^2+2x+3}dx+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{\frac{5}{28}\left(\frac{1}{5}(42375d-34207e)\int \frac{1}{5x^2+2x+3}dx+\frac{3136}{5}e\int \frac{2(5x+1)}{5x^2+2x+3}dx\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{5}{28}\left(\frac{1}{5}(42375d-34207e)\int \frac{1}{5x^2+2x+3}dx+\frac{6272}{5}e\int \frac{5x+1}{5x^2+2x+3}dx\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{5}{28}\left(\frac{6272}{5}e\int \frac{5x+1}{5x^2+2x+3}dx-\frac{2}{5}(42375d-34207e)\int \frac{1}{-(10x+2)^2-56}d(10x+2)\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{5}{28}\left(\frac{6272}{5}e\int\frac{5x+1}{5x^2+2x+3}dx+\frac{\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)(42375d-34207e)}{5\sqrt{14}}\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

$$\Big\downarrow \textbf{1103}$$

$$\frac{\frac{5}{28}\left(\frac{\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)(42375d-34207e)}{5\sqrt{14}}+\frac{3136}{5}e\log\left(5x^2+2x+3\right)\right)+\frac{x(11015d+36353e)+34347d-6511e}{28(5x^2+2x+3)}}{7000}-\frac{(423x+1367)(d+ex)}{7000\left(5x^2+2x+3\right)^2}$$

input
```
Int[((d + e*x)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*((1367 + 423*x)*(d + e*x))/(3 + 2*x + 5*x^2)^2 + ((34347*d - 6511*
e + (11015*d + 36353*e)*x)/(28*(3 + 2*x + 5*x^2)) + (5*(((42375*d - 34207*
e)*ArcTan[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[14]) + (3136*e*Log[3 + 2*x + 5
*x^2])/5))/28)/7000
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2175
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*((R*b - 2*a*S + (2*c*R - b*S)*x)/((p + 1)*(b^2 - 4*a*c))), x
] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2
)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Qx + S*(2*a*e*m + b*d
*(2*p + 3)) - R*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*R - b*S)*(m + 2*p + 3)*x
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Inte
gerQ[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] &&
 RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
default	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{(5x^2+2x+3)^2}+\frac{2e\ln(5x^2+2x+3)}{125}+\frac{(8475d-342}{}$
risch	$\frac{25\left(\frac{36353e}{980000}+\frac{2203d}{196000}\right)x^3+25\left(\frac{28307e}{4900000}+\frac{38753d}{980000}\right)x^2+25\left(\frac{57761e}{4900000}+\frac{17979d}{980000}\right)x+\frac{12953d}{39200}-\frac{19533e}{196000}}{(5x^2+2x+3)^2}+\frac{2e\ln(350x^2+140x+210)}{125}+\frac{339\sqrt{}}{}$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOS
E)
```

output
```
25*((36353/980000*e+2203/196000*d)*x^3+(28307/4900000*e+38753/980000*d)*x^
2+(57761/4900000*e+17979/980000*d)*x+12953/980000*d-19533/4900000*e)/(5*x^
2+2*x+3)^2+2/125*e*ln(5*x^2+2*x+3)+1/548800*(8475*d-34207/5*e)*14^(1/2)*ar
ctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)\,(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{70\,(11015\,d+36353\,e)x^3+14\,(193765\,d+28307\,e)x^2+\sqrt{14}(25\,(42375\,d-34207\,e)x^4+20\,(42375\,d-}{}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fr
icas")
```

output
```
1/2744000*(70*(11015*d + 36353*e)*x^3 + 14*(193765*d + 28307*e)*x^2 + sqrt
(14)*(25*(42375*d - 34207*e)*x^4 + 20*(42375*d - 34207*e)*x^3 + 34*(42375*
d - 34207*e)*x^2 + 12*(42375*d - 34207*e)*x + 381375*d - 307863*e)*arctan(
1/14*sqrt(14)*(5*x + 1)) + 14*(89895*d + 57761*e)*x + 43904*(25*e*x^4 + 20
*e*x^3 + 34*e*x^2 + 12*e*x + 9*e)*log(5*x^2 + 2*x + 3) + 906710*d - 273462
*e)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \left(\frac{2e}{125} - \frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x + \frac{8475d - \frac{34207e}{5} - \frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+ \left(\frac{2e}{125} + \frac{\sqrt{14}i(42375d-34207e)}{5488000}\right)\log\left(x + \frac{8475d - \frac{34207e}{5} + \frac{\sqrt{14}i(42375d-34207e)}{5}}{42375d-34207e}\right)$$
$$+ \frac{64765d - 19533e + x^3\cdot(55075d+181765e) + x^2\cdot(193765d+28307e) + x(89895d+57761e)}{4900000x^4 + 3920000x^3 + 6664000x^2 + 2352000x + 1764000}$$

input
```
integrate((e*x+d)*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
(2*e/125 - sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207
*e/5 - sqrt(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (2*e/125 +
 sqrt(14)*I*(42375*d - 34207*e)/5488000)*log(x + (8475*d - 34207*e/5 + sqr
t(14)*I*(42375*d - 34207*e)/5)/(42375*d - 34207*e)) + (64765*d - 19533*e +
 x**3*(55075*d + 181765*e) + x**2*(193765*d + 28307*e) + x*(89895*d + 5776
1*e))/(4900000*x**4 + 3920000*x**3 + 6664000*x**2 + 2352000*x + 1764000)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{1}{2744000}\sqrt{14}(42375\,d - 34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)$$
$$+ \frac{2}{125}\,e\log\left(5\,x^2 + 2\,x + 3\right)$$
$$+ \frac{5\,(11015\,d + 36353\,e)x^3 + (193765\,d + 28307\,e)x^2 + (89895\,d + 57761\,e)x + 64765\,d - 19533\,e}{196000\,(25\,x^4 + 20\,x^3 + 34\,x^2 + 12\,x + 9)}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="ma
xima")
```

output
```
1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2
/125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (19376
5*d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(25*x^4 +
20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{1}{2744000}\sqrt{14}(42375\,d-34207\,e)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)$$
$$+\frac{2}{125}\,e\log\left(5\,x^2+2\,x+3\right)$$
$$+\frac{5\,(11015\,d+36353\,e)x^3+(193765\,d+28307\,e)x^2+(89895\,d+57761\,e)x+64765\,d-19533\,e}{196000\,(5\,x^2+2\,x+3)^2}$$

input
```
integrate((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="gi
ac")
```

output
```
1/2744000*sqrt(14)*(42375*d - 34207*e)*arctan(1/14*sqrt(14)*(5*x + 1)) + 2
/125*e*log(5*x^2 + 2*x + 3) + 1/196000*(5*(11015*d + 36353*e)*x^3 + (19376
5*d + 28307*e)*x^2 + (89895*d + 57761*e)*x + 64765*d - 19533*e)/(5*x^2 + 2
*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 17.54 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{\left(\frac{2203\,d}{7840}+\frac{36353\,e}{39200}\right)\,x^3+\left(\frac{38753\,d}{39200}+\frac{28307\,e}{196000}\right)\,x^2+\left(\frac{17979\,d}{39200}+\frac{57761\,e}{196000}\right)\,x+\frac{12953\,d}{39200}-\frac{19533\,e}{196000}}{25\,x^4+20\,x^3+34\,x^2+12\,x+9} + \frac{2\,e\,\ln\left(5\,x^2+2\,x+3\right)}{125} + \frac{\sqrt{14}\,\mathrm{atan}\left(\frac{\frac{\sqrt{14}\,(42375\,d-34207\,e)}{2744000}+\frac{\sqrt{14}\,x\,(42375\,d-34207\,e)}{548800}}{\frac{339\,d}{1568}-\frac{34207\,e}{196000}}\right)\,(42375\,d-34207\,e)}{2744000}$$

input
```
int(((d + e*x)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^3,x)
```

output
```
((12953*d)/39200 - (19533*e)/196000 + x^3*((2203*d)/7840 + (36353*e)/39200
) + x^2*((38753*d)/39200 + (28307*e)/196000) + x*((17979*d)/39200 + (57761
*e)/196000))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + (2*e*log(2*x + 5*x^2
+ 3))/125 + (14^(1/2)*atan(((14^(1/2)*(42375*d - 34207*e))/2744000 + (14^(
1/2)*x*(42375*d - 34207*e))/548800)/((339*d)/1568 - (34207*e)/196000))*(42
375*d - 34207*e))/2744000
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.93

$$\int \frac{(d+ex)(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^3}\,dx = \frac{2118750\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,x^4+1695000\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,x^3+2881500\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)d\,x^2+101700}{}$$

input
```
int((e*x+d)*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

output
```
(2118750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*x**4 + 1695000*sqrt(14)*atan(
(5*x + 1)/sqrt(14))*d*x**3 + 2881500*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*x
**2 + 1017000*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*x + 762750*sqrt(14)*atan
((5*x + 1)/sqrt(14))*d - 1710350*sqrt(14)*atan((5*x + 1)/sqrt(14))*e*x**4
- 1368280*sqrt(14)*atan((5*x + 1)/sqrt(14))*e*x**3 - 2326076*sqrt(14)*atan
((5*x + 1)/sqrt(14))*e*x**2 - 820968*sqrt(14)*atan((5*x + 1)/sqrt(14))*e*x
 - 615726*sqrt(14)*atan((5*x + 1)/sqrt(14))*e + 2195200*log(5*x**2 + 2*x +
 3)*e*x**4 + 1756160*log(5*x**2 + 2*x + 3)*e*x**3 + 2985472*log(5*x**2 + 2
*x + 3)*e*x**2 + 1053696*log(5*x**2 + 2*x + 3)*e*x + 790272*log(5*x**2 + 2
*x + 3)*e - 1927625*d*x**4 + 2803850*d*x**2 + 1591800*d*x + 1119475*d - 63
61775*e*x**4 - 7859418*e*x**2 - 1436344*e*x - 2837163*e)/(5488000*(25*x**4
 + 20*x**3 + 34*x**2 + 12*x + 9))
```

3.156 $\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 64

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = -\frac{1367+423x}{7000\,(3+2x+5x^2)^2} + \frac{34347+11015x}{196000\,(3+2x+5x^2)} + \frac{339\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}}$$

output
```
-1/7000*(1367+423*x)/(5*x^2+2*x+3)^2+(34347+11015*x)/(980000*x^2+392000*x+
588000)+339/21952*arctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.83

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{\frac{14(12953+17979x+38753x^2+11015x^3)}{(3+2x+5x^2)^2} + 8475\sqrt{14}\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{548800}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

output
```
((14*(12953 + 17979*x + 38753*x^2 + 11015*x^3))/(3 + 2*x + 5*x^2)^2 + 8475
*Sqrt[14]*ArcTan[(1 + 5*x)/Sqrt[14]])/548800
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2191, 27, 2191, 27, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^3}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{112}\int \frac{2(5600x^2-9240x+3267)}{125(5x^2+2x+3)^2}dx-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5600x^2-9240x+3267}{(5x^2+2x+3)^2}dx}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{2191}$$

$$\frac{\frac{1}{56}\int \frac{84750}{5x^2+2x+3}dx+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{42375}{28}\int \frac{1}{5x^2+2x+3}dx+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{11015x+34347}{28(5x^2+2x+3)}-\frac{42375}{14}\int \frac{1}{-(10x+2)^2-56}d(10x+2)}{7000}-\frac{423x+1367}{7000(5x^2+2x+3)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{42375\arctan\left(\frac{10x+2}{2\sqrt{14}}\right)}{28\sqrt{14}}+\frac{11015x+34347}{28(5x^2+2x+3)}}{7000}-\frac{423x+1367}{7000\,(5x^2+2x+3)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/(3 + 2*x + 5*x^2)^3,x]
```

output
```
-1/7000*(1367 + 423*x)/(3 + 2*x + 5*x^2)^2 + ((34347 + 11015*x)/(28*(3 + 2
*x + 5*x^2)) + (42375*ArcTan[(2 + 10*x)/(2*Sqrt[14])])/(28*Sqrt[14]))/7000
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\sqrt{14}\,\arctan\left(\frac{(10x+2)\sqrt{14}}{28}\right)}{21952}$	47
risch	$\frac{\frac{2203}{7840}x^3+\frac{38753}{39200}x^2+\frac{17979}{39200}x+\frac{12953}{39200}}{(5x^2+2x+3)^2}+\frac{339\arctan\left(\frac{(1+5x)\sqrt{14}}{14}\right)\sqrt{14}}{21952}$	47

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOSE)
```

output
```
25*(2203/196000*x^3+38753/980000*x^2+17979/980000*x+12953/980000)/(5*x^2+2
*x+3)^2+339/21952*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{154210\,x^3+8475\,\sqrt{14}(25\,x^4+20\,x^3+34\,x^2+12\,x+9)\arctan\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right)+542542\,x^2+251706}{548800\,(25\,x^4+20\,x^3+34\,x^2+12\,x+9)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="fricas")
```

output
```
1/548800*(154210*x^3 + 8475*sqrt(14)*(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
*arctan(1/14*sqrt(14)*(5*x + 1)) + 542542*x^2 + 251706*x + 181342)/(25*x^4
 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{11015x^3+38753x^2+17979x+12953}{980000x^4+784000x^3+1332800x^2+470400x+352800} + \frac{339\sqrt{14}\operatorname{atan}\left(\frac{5\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{21952}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**3,x)
```

output
```
(11015*x**3 + 38753*x**2 + 17979*x + 12953)/(980000*x**4 + 784000*x**3 + 1
332800*x**2 + 470400*x + 352800) + 339*sqrt(14)*atan(5*sqrt(14)*x/14 + sqr
t(14)/14)/21952
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015x^3+38753x^2+17979x+12953}{39200\,(25x^4+20x^3+34x^2+12x+9)}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="maxima")
```

output
```
339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 +
38753*x^2 + 17979*x + 12953)/(25*x^4 + 20*x^3 + 34*x^2 + 12*x + 9)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339}{21952}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{11015x^3+38753x^2+17979x+12953}{39200(5x^2+2x+3)^2}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x, algorithm="giac")
```

output
```
339/21952*sqrt(14)*arctan(1/14*sqrt(14)*(5*x + 1)) + 1/39200*(11015*x^3 +
38753*x^2 + 17979*x + 12953)/(5*x^2 + 2*x + 3)^2
```

Mupad [B] (verification not implemented)

Time = 17.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx = \frac{339\sqrt{14}\,\mathrm{atan}\left(\frac{5\sqrt{14}\,x}{14}+\frac{\sqrt{14}}{14}\right)}{21952} + \frac{\frac{2203x^3}{196000}+\frac{38753x^2}{980000}+\frac{17979x}{980000}+\frac{12953}{980000}}{x^4+\frac{4x^3}{5}+\frac{34x^2}{25}+\frac{12x}{25}+\frac{9}{25}}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/(2*x + 5*x^2 + 3)^3,x)
```

output
```
(339*14^(1/2)*atan((5*14^(1/2)*x)/14 + 14^(1/2)/14))/21952 + ((17979*x)/98
0000 + (38753*x^2)/980000 + (2203*x^3)/196000 + 12953/980000)/((12*x)/25 +
 (34*x^2)/25 + (4*x^3)/5 + x^4 + 9/25)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(3+2x+5x^2)^3}\,dx$$
$$= \frac{84750\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)x^4+67800\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)x^3+115260\sqrt{14}\,atan\left(\frac{5x+1}{\sqrt{14}}\right)x^2+40680\sqrt{14}\,atan\left(\frac{5}{}\right.}{5488000x^4+4390400x^3+7463680x^2+2634}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^3,x)
```

output
```
(84750*sqrt(14)*atan((5*x + 1)/sqrt(14))*x**4 + 67800*sqrt(14)*atan((5*x +
 1)/sqrt(14))*x**3 + 115260*sqrt(14)*atan((5*x + 1)/sqrt(14))*x**2 + 40680
*sqrt(14)*atan((5*x + 1)/sqrt(14))*x + 30510*sqrt(14)*atan((5*x + 1)/sqrt(
14)) - 77105*x**4 + 112154*x**2 + 63672*x + 44779)/(219520*(25*x**4 + 20*x
**3 + 34*x**2 + 12*x + 9))
```

3.157 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 329

$$\begin{aligned}\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx &= -\frac{1367d-293e+(423d-1367e)x}{1400\left(5d^2-2de+3e^2\right)\left(3+2x+5x^2\right)^2}\\ &+\frac{171735d^3-92989d^2e+36207de^2+1831e^3+25(2203d^3-9033d^2e+3635de^2-1829e^3)\,x}{39200\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)}\\ &+\frac{(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5)\arctan\left(\frac{1+5x}{\sqrt{14}}\right)}{1568\sqrt{14}\left(5d^2-2de+3e^2\right)^3}\\ &+\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^3}\\ &-\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^3}\end{aligned}$$

output
```
-1/1400*(1367*d-293*e+(423*d-1367*e)*x)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)^
2+1/39200*(171735*d^3-92989*d^2*e+36207*d*e^2+1831*e^3+25*(2203*d^3-9033*d
^2*e+3635*d*e^2-1829*e^3)*x)/(5*d^2-2*d*e+3*e^2)^2/(5*x^2+2*x+3)+1/21952*(
42375*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*ar
ctan(1/14*(1+5*x)*14^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^3+e*(4*d^4+5*d^3*
e+3*d^2*e^2-d*e^3+2*e^4)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3-1/2*e*(4*d^4+5*d^
3*e+3*d^2*e^2-d*e^3+2*e^4)*ln(5*x^2+2*x+3)/(5*d^2-2*d*e+3*e^2)^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.86

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{\frac{392\left(5d^2-2de+3e^2\right)^2\left(-d(1367+423x)+e(293+1367x)\right)}{\left(3+2x+5x^2\right)^2}+\frac{14\left(5d^2-2de+3e^2\right)\left(e^3(1831-45725x)+5d^3(34347+11015x)+de^2(36207+90875x)-\right.}{3+2x+5x^2}}{}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),
x]
```

output
```
((392*(5*d^2 - 2*d*e + 3*e^2)^2*(-(d*(1367 + 423*x)) + e*(293 + 1367*x)))/
(3 + 2*x + 5*x^2)^2 + (14*(5*d^2 - 2*d*e + 3*e^2)*(e^3*(1831 - 45725*x) +
5*d^3*(34347 + 11015*x) + d*e^2*(36207 + 90875*x) - d^2*e*(92989 + 225825*
x)))/(3 + 2*x + 5*x^2) + 25*Sqrt[14]*(42375*d^5 - 16643*d^4*e + 58530*d^3*
e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]] +
 548800*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[d + e*x] - 274
400*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/
(548800*(5*d^2 - 2*d*e + 3*e^2)^3)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2177, 27, 2177, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{\left(5x^2+2x+3\right)^3(d+ex)}\,dx$$
$$\Big\downarrow \text{2177}$$

$$\frac{1}{112}\int \frac{2\left(1120x^2-\frac{3(3080d^2-809ed+481e^2)x}{5d^2-2ed+3e^2}+\frac{3267d^2-2843ed+2800e^2}{5d^2-2ed+3e^2}\right)}{25(d+ex)\left(5x^2+2x+3\right)^2}dx-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1120x^2-\frac{3\left(3080d^2-809ed+481e^2\right)x}{5d^2-2ed+3e^2}+\frac{3267d^2-2843ed+2800e^2}{5d^2-2ed+3e^2}}{(d+ex)(5x^2+2x+3)^2}dx}{1400}-\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{2177}$$

$$\frac{\frac{1}{56}\int \frac{50\left(8475d^4-1193ed^3+8339e^2d^2-3397e^3d+3136e^4+e\left(2203d^3-9033ed^2+3635e^2d-1829e^3\right)x\right)}{(5d^2-2ed+3e^2)^2(d+ex)(5x^2+2x+3)}dx+\frac{171735d^3-92989d^2e+25x\left(2203d^3-\cdots\right.}{28(5x^2+2\cdots}}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{25\int \frac{8475d^4-1193ed^3+8339e^2d^2-3397e^3d+3136e^4+e\left(2203d^3-9033ed^2+3635e^2d-1829e^3\right)x}{(d+ex)\left(5x^2+2x+3\right)}dx}{28(5d^2-2de+3e^2)^2}+\frac{171735d^3-92989d^2e+25x\left(2203d^3-9033d^2e+3635d\cdots\right.}{28(5x^2+2x+3)(5d^2-2de+\cdots}}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{1200}$$

$$\frac{\frac{25\int \left(\frac{1568\left(4d^4+5ed^3+3e^2d^2-e^3d+2e^4\right)e^2}{\left(5d^2-2ed+3e^2\right)(d+ex)}+\frac{42375d^5-22915ed^4+50690e^2d^3-60762e^3d^2+33379e^4d-11759e^5-7840e\left(4d^4+5ed^3+3e^2d^2-e^3d+2e^4\right)x}{\left(5d^2-2ed+3e^2\right)\left(5x^2+2x+3\right)}\right)dx}{28(5d^2-2de+3e^2)^2}+\cdots}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{25\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)\left(42375d^5-16643d^4e+58530d^3e^2-56058d^2e^3+31811de^4-8623e^5\right)}{\sqrt{14}\left(5d^2-2de+3e^2\right)}-\frac{784e\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)\log\left(5x^2+2x+3\right)}{5d^2-2de+3e^2}+\frac{1568e\left(4d^4+5d^3\cdots\right.}{\cdots}\right.}{28(5d^2-2de+3e^2)^2}\cdots}{1400}$$

$$\frac{x(423d-1367e)+1367d-293e}{1400\left(5x^2+2x+3\right)^2\left(5d^2-2de+3e^2\right)}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)*(3 + 2*x + 5*x^2)^3),x]
```

output
```
-1/1400*(1367*d - 293*e + (423*d - 1367*e)*x)/((5*d^2 - 2*d*e + 3*e^2)*(3
+ 2*x + 5*x^2)^2) + ((171735*d^3 - 92989*d^2*e + 36207*d*e^2 + 1831*e^3 +
25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x)/(28*(5*d^2 - 2*d*e +
 3*e^2)^2*(3 + 2*x + 5*x^2)) + (25*(((42375*d^5 - 16643*d^4*e + 58530*d^3*
e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*ArcTan[(1 + 5*x)/Sqrt[14]])/
(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)) + (1568*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2
- d*e^3 + 2*e^4)*Log[d + e*x])/(5*d^2 - 2*d*e + 3*e^2) - (784*e*(4*d^4 + 5
*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e +
 3*e^2)))/(28*(5*d^2 - 2*d*e + 3*e^2)^2))/1400
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.09

method	result
default	$\frac{25\left(\frac{2203}{1568} d^5-\frac{49571}{7840} d^4 e+\frac{4285}{784} d^3 e^2-\frac{21757}{3920} d^2 e^3+\frac{14563}{7840} d e^4-\frac{5487}{7840} e^5\right) x^3+25\left(\frac{38753}{7840} d^5-\frac{10433}{1568} d^4 e+\frac{655359}{98000} d^3 e^2-\frac{388683}{98000} d^2 e^3+\frac{250589}{196000} d e^4-\cdots\right.}{\cdots}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x,method=_RETURNVERBOS
E)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^3*(25*((2203/1568*d^5-49571/7840*d^4*e+4285/784*d^3*
e^2-21757/3920*d^2*e^3+14563/7840*d*e^4-5487/7840*e^5)*x^3+(38753/7840*d^5
-10433/1568*d^4*e+655359/98000*d^3*e^2-388683/98000*d^2*e^3+250589/196000*
d*e^4-49377/196000*e^5)*x^2+(17979/7840*d^5-33127/7840*d^4*e+380997/98000*
d^3*e^2-250449/98000*d^2*e^3+147247/196000*d*e^4-11211/196000*e^5)*x+12953
/7840*d^5-11637/7840*d^4*e+118119/98000*d^3*e^2-28843/98000*d^2*e^3-25611/
196000*d*e^4+18063/196000*e^5)/(5*x^2+2*x+3)^2+1/15680*(-31360*d^4*e-39200
*d^3*e^2-23520*d^2*e^3+7840*d*e^4-15680*e^5)*ln(5*x^2+2*x+3)+1/21952*(4237
5*d^5-16643*d^4*e+58530*d^3*e^2-56058*d^2*e^3+31811*d*e^4-8623*e^5)*14^(1/
2)*arctan(1/28*(10*x+2)*14^(1/2)))+e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)
*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. 2(318) = 636.

Time = 0.22 (sec) , antiderivative size = 1052, normalized size of antiderivative = 3.20

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="fr
icas")
```

output
```
1/109760*(4533550*d^5 - 4072950*d^4*e + 3307332*d^3*e^2 - 807604*d^2*e^3 -
 358554*d*e^4 + 252882*e^5 + 350*(11015*d^5 - 49571*d^4*e + 42850*d^3*e^2
- 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + 14*(968825*d^5 - 1304125*d
^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^5)*x^2 +
5*sqrt(14)*(381375*d^5 - 149787*d^4*e + 526770*d^3*e^2 - 504522*d^2*e^3 +
286299*d*e^4 - 77607*e^5 + 25*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 5
6058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^4 + 20*(42375*d^5 - 16643*d^4*e +
 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)*x^3 + 34*(42375*d
^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811*d*e^4 - 8623*e^5)
*x^2 + 12*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3 + 31811
*d*e^4 - 8623*e^5)*x)*arctan(1/14*sqrt(14)*(5*x + 1)) + 14*(449475*d^5 - 8
28175*d^4*e + 761994*d^3*e^2 - 500898*d^2*e^3 + 147247*d*e^4 - 11211*e^5)*
x + 109760*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 + 18*e^5 + 25*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*d^4*e + 5*d^3*e
^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3
- d*e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5
)*x)*log(e*x + d) - 54880*(36*d^4*e + 45*d^3*e^2 + 27*d^2*e^3 - 9*d*e^4 +
18*e^5 + 25*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^4 + 20*(4*
d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^3 + 34*(4*d^4*e + 5*d^3*e
^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*x^2 + 12*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)/(5*x**2+2*x+3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.74

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\left(3+2x+5x^2\right)^3}\,dx$$
$$= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(ex+d)}{125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6}$$
$$-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log(5\,x^2+2\,x+3)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{25\,(2203\,d^3-9033\,d^2e+3635\,de^2-1829\,e^3)x^3+64765\,d^3-32279\,d^2e-45}{7840\,(25\,(25\,d^4-20\,d^3e+34\,d^2e^2-12\,de^3+9\,e^4)x^4+225\,d^4-180\,d^3e+306\,d^2e^2-108\,de^3+81\,e^4}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="ma
xima")
```

output
```
1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3
+ 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) + (4*d
^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(e*x + d)/(125*d^6 - 150*
d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2
*(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(1
25*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 +
27*e^6) + 1/7840*(25*(2203*d^3 - 9033*d^2*e + 3635*d*e^2 - 1829*e^3)*x^3 +
 64765*d^3 - 32279*d^2*e - 4523*d*e^2 + 6021*e^3 + (193765*d^3 - 183319*d^
2*e + 72557*d*e^2 - 16459*e^3)*x^2 + (89895*d^3 - 129677*d^2*e + 46591*d*e
^2 - 3737*e^3)*x)/(25*(25*d^4 - 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*
x^4 + 225*d^4 - 180*d^3*e + 306*d^2*e^2 - 108*d*e^3 + 81*e^4 + 20*(25*d^4
- 20*d^3*e + 34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^3 + 34*(25*d^4 - 20*d^3*e +
34*d^2*e^2 - 12*d*e^3 + 9*e^4)*x^2 + 12*(25*d^4 - 20*d^3*e + 34*d^2*e^2 -
12*d*e^3 + 9*e^4)*x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.50

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)\,(3+2x+5x^2)^3}\,dx$$
$$= \frac{\sqrt{14}(42375\,d^5-16643\,d^4e+58530\,d^3e^2-56058\,d^2e^3+31811\,de^4-8623\,e^5)\arctan\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right)}{21952\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$-\frac{(4\,d^4e+5\,d^3e^2+3\,d^2e^3-de^4+2\,e^5)\log\left(5\,x^2+2\,x+3\right)}{2\,(125\,d^6-150\,d^5e+285\,d^4e^2-188\,d^3e^3+171\,d^2e^4-54\,de^5+27\,e^6)}$$
$$+\frac{(4\,d^4e^2+5\,d^3e^3+3\,d^2e^4-de^5+2\,e^6)\log\left(|ex+d|\right)}{125\,d^6e-150\,d^5e^2+285\,d^4e^3-188\,d^3e^4+171\,d^2e^5-54\,de^6+27\,e^7}$$
$$+\frac{323825\,d^5-290925\,d^4e+236238\,d^3e^2-57686\,d^2e^3-25611\,de^4+18063\,e^5+25\,(11015\,d^5-49571\,d^4}{}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x, algorithm="gi
ac")
```

output
```
1/21952*sqrt(14)*(42375*d^5 - 16643*d^4*e + 58530*d^3*e^2 - 56058*d^2*e^3
+ 31811*d*e^4 - 8623*e^5)*arctan(1/14*sqrt(14)*(5*x + 1))/(125*d^6 - 150*d
^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 27*e^6) - 1/2*
(4*d^4*e + 5*d^3*e^2 + 3*d^2*e^3 - d*e^4 + 2*e^5)*log(5*x^2 + 2*x + 3)/(12
5*d^6 - 150*d^5*e + 285*d^4*e^2 - 188*d^3*e^3 + 171*d^2*e^4 - 54*d*e^5 + 2
7*e^6) + (4*d^4*e^2 + 5*d^3*e^3 + 3*d^2*e^4 - d*e^5 + 2*e^6)*log(abs(e*x +
 d))/(125*d^6*e - 150*d^5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 -
54*d*e^6 + 27*e^7) + 1/7840*(323825*d^5 - 290925*d^4*e + 236238*d^3*e^2 -
57686*d^2*e^3 - 25611*d*e^4 + 18063*e^5 + 25*(11015*d^5 - 49571*d^4*e + 42
850*d^3*e^2 - 43514*d^2*e^3 + 14563*d*e^4 - 5487*e^5)*x^3 + (968825*d^5 -
1304125*d^4*e + 1310718*d^3*e^2 - 777366*d^2*e^3 + 250589*d*e^4 - 49377*e^
5)*x^2 + (449475*d^5 - 828175*d^4*e + 761994*d^3*e^2 - 500898*d^2*e^3 + 14
7247*d*e^4 - 11211*e^5)*x)/((5*d^2 - 2*d*e + 3*e^2)^3*(5*x^2 + 2*x + 3)^2)
```

Mupad [B] (verification not implemented)

Time = 17.09 (sec) , antiderivative size = 641, normalized size of antiderivative = 1.95

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)*(2*x + 5*x^2 + 3)^3),x)
```

output
```
((x*(46591*d*e^2 - 129677*d^2*e + 89895*d^3 - 3737*e^3))/(7840*(25*d^4 - 2
0*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) - (4523*d*e^2 + 32279*d^2*e - 64
765*d^3 - 6021*e^3)/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e
^2)) + (5*x^3*(3635*d*e^2 - 9033*d^2*e + 2203*d^3 - 1829*e^3))/(1568*(25*d
^4 - 20*d^3*e - 12*d*e^3 + 9*e^4 + 34*d^2*e^2)) + (x^2*(72557*d*e^2 - 1833
19*d^2*e + 193765*d^3 - 16459*e^3))/(7840*(25*d^4 - 20*d^3*e - 12*d*e^3 +
9*e^4 + 34*d^2*e^2)))/(12*x + 34*x^2 + 20*x^3 + 25*x^4 + 9) + log(d + e*x)
*((4*e)/(25*(5*d^2 - 2*d*e + 3*e^2)) + (e^2*(205*d + 21*e))/(125*(5*d^2 -
2*d*e + 3*e^2)^2) - (e^4*(458*d - 7*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3)) -
 (log(x - (14^(1/2)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 + 1i) - (4237
5*14^(1/2)*d^5)/43904 + d^2*e^3*((28029*14^(1/2))/21952 + 3i/2) - d^3*e^2*
((29265*14^(1/2))/21952 - 5i/2) + d^4*e*((16643*14^(1/2))/43904 + 2i) - d*
e^4*((31811*14^(1/2))/43904 + 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i +
 e^6*27i + d^2*e^4*171i - d^3*e^3*188i + d^4*e^2*285i) + (log(x + (14^(1/2
)*1i)/5 + 1/5)*(e^5*((8623*14^(1/2))/43904 - 1i) - (42375*14^(1/2)*d^5)/43
904 + d^2*e^3*((28029*14^(1/2))/21952 - 3i/2) - d^3*e^2*((29265*14^(1/2))/
21952 + 5i/2) + d^4*e*((16643*14^(1/2))/43904 - 2i) - d*e^4*((31811*14^(1/
2))/43904 - 1i/2)))/(d^6*125i - d^5*e*150i - d*e^5*54i + e^6*27i + d^2*e^4
*171i - d^3*e^3*188i + d^4*e^2*285i)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1994, normalized size of antiderivative = 6.06

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)/(5*x^2+2*x+3)^3,x)
```

output
```
(10593750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*x**4 + 8475000*sqrt(14)*a
tan((5*x + 1)/sqrt(14))*d**5*x**3 + 14407500*sqrt(14)*atan((5*x + 1)/sqrt(
14))*d**5*x**2 + 5085000*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*x + 381375
0*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5 - 4160750*sqrt(14)*atan((5*x + 1)
/sqrt(14))*d**4*e*x**4 - 3328600*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e*
x**3 - 5658620*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e*x**2 - 1997160*sqr
t(14)*atan((5*x + 1)/sqrt(14))*d**4*e*x - 1497870*sqrt(14)*atan((5*x + 1)/
sqrt(14))*d**4*e + 14632500*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**2*x*
*4 + 11706000*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**2*x**3 + 19900200*
sqrt(14)*atan((5*x + 1)/sqrt(14))*d**3*e**2*x**2 + 7023600*sqrt(14)*atan((
5*x + 1)/sqrt(14))*d**3*e**2*x + 5267700*sqrt(14)*atan((5*x + 1)/sqrt(14))
*d**3*e**2 - 14014500*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**3*x**4 - 1
1211600*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*e**3*x**3 - 19059720*sqrt(1
4)*atan((5*x + 1)/sqrt(14))*d**2*e**3*x**2 - 6726960*sqrt(14)*atan((5*x +
1)/sqrt(14))*d**2*e**3*x - 5045220*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**2*
e**3 + 7952750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**4*x**4 + 6362200*sqr
t(14)*atan((5*x + 1)/sqrt(14))*d*e**4*x**3 + 10815740*sqrt(14)*atan((5*x +
 1)/sqrt(14))*d*e**4*x**2 + 3817320*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e*
*4*x + 2862990*sqrt(14)*atan((5*x + 1)/sqrt(14))*d*e**4 - 2155750*sqrt(14)
*atan((5*x + 1)/sqrt(14))*e**5*x**4 - 1724600*sqrt(14)*atan((5*x + 1)/s...
```

3.158 $\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 443

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = -\frac{e(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{\left(5d^2-2de+3e^2\right)^3(d+ex)}$$
$$-\frac{1367d^2-586de-703e^2+\left(423d^2-2734de+293e^2\right)x}{280\left(5d^2-2de+3e^2\right)^2\left(3+2x+5x^2\right)^2}$$
$$+\frac{171735d^4-117284d^3e-200502d^2e^2+104428de^3-23189e^4+5(11015d^4-85924d^3e+34698d^2e^2+ \cdots}{7840\left(5d^2-2de+3e^2\right)^3\left(3+2x+5x^2\right)}$$
$$+\frac{\left(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e^6\right)\arctan\left(\frac{1+5x}{\sqrt{14}}\cdots\right.}{1568\sqrt{14}\left(5d^2-2de+3e^2\right)^4}$$
$$+\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log(d+ex)}{\left(5d^2-2de+3e^2\right)^4}$$
$$-\frac{e(40d^5+83d^4e+12d^3e^2-76d^2e^3+46de^4-9e^5)\log\left(3+2x+5x^2\right)}{2\left(5d^2-2de+3e^2\right)^4}$$

output
```
-e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e*x+d)-1/2
80*(1367*d^2-586*d*e-703*e^2+(423*d^2-2734*d*e+293*e^2)*x)/(5*d^2-2*d*e+3*
e^2)^2/(5*x^2+2*x+3)^2+1/7840*(171735*d^4-117284*d^3*e-200502*d^2*e^2+1044
28*d*e^3-23189*e^4+5*(11015*d^4-85924*d^3*e+34698*d^2*e^2+10348*d*e^3-3589
*e^4)*x)/(5*d^2-2*d*e+3*e^2)^3/(5*x^2+2*x+3)+1/21952*(211875*d^6+3070*d^5*
e+209039*d^4*e^2-921444*d^3*e^3+380621*d^2*e^4-49586*d*e^5-43695*e^6)*arct
an(1/14*(1+5*x)*14^(1/2))*14^(1/2)/(5*d^2-2*d*e+3*e^2)^4+e*(40*d^5+83*d^4*
e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e+3*e^2)^4-1/
2*e*(40*d^5+83*d^4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(5*x^2+2*x+3)
/(5*d^2-2*d*e+3*e^2)^4
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\,(3+2x+5x^2)^3}\,dx$$

$$= \frac{-\frac{109760e(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{d+ex} - \frac{392(5d^2-2de+3e^2)^2(e^2(-703+293x)+d^2(1367+423x)-2de(293+1367x))}{(3+2x+5x^2)^2} + \cdots}{\cdots}$$

input
```
Integrate[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3
),x]
```

output
```
((-109760*e*(5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 +
 2*e^4))/(d + e*x) - (392*(5*d^2 - 2*d*e + 3*e^2)^2*(e^2*(-703 + 293*x) +
d^2*(1367 + 423*x) - 2*d*e*(293 + 1367*x)))/(3 + 2*x + 5*x^2)^2 + (14*(5*d
^2 - 2*d*e + 3*e^2)*(5*d^4*(34347 + 11015*x) + 4*d*e^3*(26107 + 12935*x) -
 e^4*(23189 + 17945*x) + 6*d^2*e^2*(-33417 + 28915*x) - 4*d^3*e*(29321 + 1
07405*x)))/(3 + 2*x + 5*x^2) + 5*Sqrt[14]*(211875*d^6 + 3070*d^5*e + 20903
9*d^4*e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*Arc
Tan[(1 + 5*x)/Sqrt[14]] + 109760*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^
2*e^3 + 46*d*e^4 - 9*e^5)*Log[d + e*x] - 54880*e*(40*d^5 + 83*d^4*e + 12*d
^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(109760*(5*d
^2 - 2*d*e + 3*e^2)^4)
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2177, 27, 2177, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^4-5x^3+3x^2+x+2}{(5x^2+2x+3)^3(d+ex)^2}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{112}\int \frac{2\left(\frac{(5600d^4-4480ed^3+6347e^2d^2+5514e^3d+1137e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(4620d^4-2427ed^3+646e^2d^2-1417e^3d+140e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{3267d^4-5686ed^3+75}{(5d^2-}\right.}{5(d+ex)^2(5x^2+2x+3)^2}$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\int \frac{\frac{(5600d^4-4480ed^3+6347e^2d^2+5514e^3d+1137e^4)x^2}{(5d^2-2ed+3e^2)^2}-\frac{2(4620d^4-2427ed^3+646e^2d^2-1417e^3d+140e^4)x}{(5d^2-2ed+3e^2)^2}+\frac{3267d^4-5686ed^3+7577e}{(5d^2-2ed}}{(d+ex)^2(5x^2+2x+3)^2}$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{2177}$$

$$\frac{1}{280}\left(\frac{1}{56}\int \frac{10\left(\frac{e^2(11015d^4-85924ed^3+34698e^2d^2+10348e^3d-3589e^4)x^2}{(5d^2-2ed+3e^2)^3}+\frac{2e(11015d^5-53780ed^4+28426e^2d^3-36692e^3d^2+15227e^4d-}{(5d^2-2ed+3e^2)^3}\right.}{(d+ex)^2(5x^2+2x+3)}\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{280}\left(\frac{5}{28}\int \frac{\frac{e^2(11015d^4-85924ed^3+34698e^2d^2+10348e^3d-3589e^4)x^2}{(5d^2-2ed+3e^2)^3}+\frac{2e(11015d^5-53780ed^4+28426e^2d^3-36692e^3d^2+15227e^4d-3920}{(5d^2-2ed+3e^2)^3}}{(d+ex)^2(5x^2+2x+3)}\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{2159}$$

$$\frac{1}{280}\left(\frac{5}{28}\int \left(-\frac{1568(-40d^5-83ed^4-12e^2d^3+76e^3d^2-46e^4d+9e^5)e^2}{(5d^2-2ed+3e^2)^4(d+ex)}+\frac{1568(4d^4+5ed^3+3e^2d^2-e^3d+2}{(5d^2-2ed+3e^2)^3(d+ex)^2}\right.\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{280}\left(\frac{5}{28}\left(\frac{\arctan\left(\frac{5x+1}{\sqrt{14}}\right)(211875d^6+3070d^5e+209039d^4e^2-921444d^3e^3+380621d^2e^4-49586de^5-43695e}{\sqrt{14}(5d^2-2de+3e^2)^4}\right.\right.$$

$$\frac{x(423d^2-2734de+293e^2)+1367d^2-586de-703e^2}{280(5x^2+2x+3)^2(5d^2-2de+3e^2)^2}$$

input
```
Int[(2 + x + 3*x^2 - 5*x^3 + 4*x^4)/((d + e*x)^2*(3 + 2*x + 5*x^2)^3),x]
```

output
```
-1/280*(1367*d^2 - 586*d*e - 703*e^2 + (423*d^2 - 2734*d*e + 293*e^2)*x)/(
(5*d^2 - 2*d*e + 3*e^2)^2*(3 + 2*x + 5*x^2)^2) + ((171735*d^4 - 117284*d^3
*e - 200502*d^2*e^2 + 104428*d*e^3 - 23189*e^4 + 5*(11015*d^4 - 85924*d^3*
e + 34698*d^2*e^2 + 10348*d*e^3 - 3589*e^4)*x)/(28*(5*d^2 - 2*d*e + 3*e^2)
^3*(3 + 2*x + 5*x^2)) + (5*((-1568*e*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3
+ 2*e^4))/((5*d^2 - 2*d*e + 3*e^2)^3*(d + e*x)) + ((211875*d^6 + 3070*d^5*
e + 209039*d^4*e^2 - 921444*d^3*e^3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695
*e^6)*ArcTan[(1 + 5*x)/Sqrt[14]])/(Sqrt[14]*(5*d^2 - 2*d*e + 3*e^2)^4) + (
1568*e*(40*d^5 + 83*d^4*e + 12*d^3*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Lo
g[d + e*x])/(5*d^2 - 2*d*e + 3*e^2)^4 - (784*e*(40*d^5 + 83*d^4*e + 12*d^3
*e^2 - 76*d^2*e^3 + 46*d*e^4 - 9*e^5)*Log[3 + 2*x + 5*x^2])/(5*d^2 - 2*d*e
 + 3*e^2)^4))/28)/280
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.07

method	result
default	$\frac{25\left(\frac{11015}{1568} d^{6}-\frac{45165}{784} d^{5} e+\frac{378383}{7840} d^{4} e^{2}-\frac{68857}{1960} d^{3} e^{3}+\frac{65453}{7840} d^{2} e^{4}+\frac{19111}{3920} d e^{5}-\frac{10767}{7840} e^{6}\right) x^{3}+25\left(\frac{38753}{1568} d^{6}-\frac{183319}{3920} d^{5} e+\frac{504029}{39200} d^{4} e^{2}+\frac{5109}{9800} d^{3} e^{3}\right.}{}$
risch	Expression too large to display

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x,method=_RETURNVERB
OSE)
```

output
```
1/(5*d^2-2*d*e+3*e^2)^4*(25*((11015/1568*d^6-45165/784*d^5*e+378383/7840*d
^4*e^2-68857/1960*d^3*e^3+65453/7840*d^2*e^4+19111/3920*d*e^5-10767/7840*e
^6)*x^3+(38753/1568*d^6-183319/3920*d^5*e+504029/39200*d^4*e^2+5109/9800*d
^3*e^3-795401/39200*d^2*e^4+218053/19600*d*e^5-91101/39200*e^6)*x^2+(17979
/1568*d^6-129677/3920*d^5*e+606287/39200*d^4*e^2-3993/9800*d^3*e^3-86999/7
840*d^2*e^4+208007/19600*d*e^5-14979/7840*e^6)*x+12953/1568*d^6-32279/3920
*d^5*e-379131/39200*d^4*e^2+116869/9800*d^3*e^3-530209/39200*d^2*e^4+19809
/3920*d*e^5-6309/39200*e^6)/(5*x^2+2*x+3)^2+1/15680*(-313600*d^5*e-650720*
d^4*e^2-94080*d^3*e^3+595840*d^2*e^4-360640*d*e^5+70560*e^6)*ln(5*x^2+2*x+
3)+1/21952*(211875*d^6+3070*d^5*e+209039*d^4*e^2-921444*d^3*e^3+380621*d^2
*e^4-49586*d*e^5-43695*e^6)*14^(1/2)*arctan(1/28*(10*x+2)*14^(1/2)))+e*(40
*d^5+83*d^4*e+12*d^3*e^2-76*d^2*e^3+46*d*e^4-9*e^5)*ln(e*x+d)/(5*d^2-2*d*e
+3*e^2)^4-e*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)/(5*d^2-2*d*e+3*e^2)^3/(e
*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1734 vs. $2(432) = 864$.

Time = 0.36 (sec) , antiderivative size = 1734, normalized size of antiderivative = 3.91

$$\int \frac{2 + x + 3x^2 - 5x^3 + 4x^4}{(d + ex)^2 (3 + 2x + 5x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
fricas")
```

output
```
1/21952*(4533550*d^7 - 8470420*d^6*e - 8666490*d^5*e^2 + 3186008*d^4*e^3 -
 8213198*d^3*e^4 - 1375668*d^2*e^5 + 1294650*d*e^6 - 1185408*e^7 - 70*(101
725*d^6*e + 584930*d^5*e^2 - 245103*d^4*e^3 + 306788*d^3*e^4 + 99187*d^2*e
^5 - 93102*d*e^6 + 57807*e^7)*x^4 + 14*(275375*d^7 - 1916625*d^6*e - 47439
5*d^5*e^2 - 1406231*d^4*e^3 + 222261*d^3*e^4 - 1262851*d^2*e^5 + 601791*d*
e^6 - 279261*e^7)*x^3 + 14*(968825*d^7 - 2449955*d^6*e - 1699045*d^5*e^2 -
 279581*d^4*e^3 - 1024621*d^3*e^4 - 1118441*d^2*e^5 + 698097*d*e^6 - 39476
7*e^7)*x^2 + sqrt(14)*(1906875*d^7 + 27630*d^6*e + 1881351*d^5*e^2 - 82929
96*d^4*e^3 + 3425589*d^3*e^4 - 446274*d^2*e^5 - 393255*d*e^6 + 25*(211875*
d^6*e + 3070*d^5*e^2 + 209039*d^4*e^3 - 921444*d^3*e^4 + 380621*d^2*e^5 -
49586*d*e^6 - 43695*e^7)*x^5 + 5*(1059375*d^7 + 862850*d^6*e + 1057475*d^5
*e^2 - 3771064*d^4*e^3 - 1782671*d^3*e^4 + 1274554*d^2*e^5 - 416819*d*e^6
- 174780*e^7)*x^4 + 2*(2118750*d^7 + 3632575*d^6*e + 2142580*d^5*e^2 - 566
0777*d^4*e^3 - 11858338*d^3*e^4 + 5974697*d^2*e^5 - 1279912*d*e^6 - 742815
*e^7)*x^3 + 2*(3601875*d^7 + 1323440*d^6*e + 3572083*d^5*e^2 - 14410314*d^
4*e^3 + 941893*d^3*e^4 + 1440764*d^2*e^5 - 1040331*d*e^6 - 262170*e^7)*x^2
 + 3*(847500*d^7 + 647905*d^6*e + 845366*d^5*e^2 - 3058659*d^4*e^3 - 12418
48*d^3*e^4 + 943519*d^2*e^5 - 323538*d*e^6 - 131085*e^7)*x)*arctan(1/14*sq
rt(14)*(5*x + 1)) + 42*(149825*d^7 - 449755*d^6*e - 12125*d^5*e^2 - 238325
*d^4*e^3 - 14261*d^3*e^4 - 169777*d^2*e^5 + 84969*d*e^6 - 39735*e^7)*x ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((4*x**4-5*x**3+3*x**2+x+2)/(e*x+d)**2/(5*x**2+2*x+3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(432) = 864$.

Time = 0.14 (sec) , antiderivative size = 916, normalized size of antiderivative = 2.07

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2\left(3+2x+5x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
maxima")
```

output
```
1/21952*sqrt(14)*(211875*d^6 + 3070*d^5*e + 209039*d^4*e^2 - 921444*d^3*e^
3 + 380621*d^2*e^4 - 49586*d*e^5 - 43695*e^6)*arctan(1/14*sqrt(14)*(5*x +
1))/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1
176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) + (40*d^5*e + 83*d^4*e^2 +
 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(e*x + d)/(625*d^8 - 1000*
d^7*e + 2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^
2*e^6 - 216*d*e^7 + 81*e^8) - 1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76
*d^2*e^4 + 46*d*e^5 - 9*e^6)*log(5*x^2 + 2*x + 3)/(625*d^8 - 1000*d^7*e +
2100*d^6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 -
216*d*e^7 + 81*e^8) + 1/1568*(64765*d^5 - 95100*d^4*e - 200706*d^3*e^2 + 2
2292*d^2*e^3 + 12009*d*e^4 - 28224*e^5 - 5*(20345*d^4*e + 125124*d^3*e^2 -
 11178*d^2*e^3 - 18188*d*e^4 + 19269*e^5)*x^4 + (55075*d^5 - 361295*d^4*e
- 272442*d^3*e^2 - 173446*d^2*e^3 + 138539*d*e^4 - 93087*e^5)*x^3 + (19376
5*d^5 - 412485*d^4*e - 621062*d^3*e^2 - 56850*d^2*e^3 + 144973*d*e^4 - 131
589*e^5)*x^2 + 3*(29965*d^5 - 77965*d^4*e - 51590*d^3*e^2 - 21522*d^2*e^3
+ 19493*d*e^4 - 13245*e^5)*x)/(1125*d^7 - 1350*d^6*e + 2565*d^5*e^2 - 1692
*d^4*e^3 + 1539*d^3*e^4 - 486*d^2*e^5 + 243*d*e^6 + 25*(125*d^6*e - 150*d^
5*e^2 + 285*d^4*e^3 - 188*d^3*e^4 + 171*d^2*e^5 - 54*d*e^6 + 27*e^7)*x^5 +
 5*(625*d^7 - 250*d^6*e + 825*d^5*e^2 + 200*d^4*e^3 + 103*d^3*e^4 + 414*d^
2*e^5 - 81*d*e^6 + 108*e^7)*x^4 + 2*(1250*d^7 + 625*d^6*e + 300*d^5*e^2...
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.82

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x, algorithm="
giac")
```

output
```
-1/2*(40*d^5*e + 83*d^4*e^2 + 12*d^3*e^3 - 76*d^2*e^4 + 46*d*e^5 - 9*e^6)*
log(-10*d/(e*x + d) + 5*d^2/(e*x + d)^2 + 2*e/(e*x + d) - 2*d*e/(e*x + d)^
2 + 3*e^2/(e*x + d)^2 + 5)/(625*d^8 - 1000*d^7*e + 2100*d^6*e^2 - 1960*d^5
*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e^7 + 81*e^8) - (
4*d^4*e^7/(e*x + d) + 5*d^3*e^8/(e*x + d) + 3*d^2*e^9/(e*x + d) - d*e^10/(
e*x + d) + 2*e^11/(e*x + d))/(125*d^6*e^6 - 150*d^5*e^7 + 285*d^4*e^8 - 18
8*d^3*e^9 + 171*d^2*e^10 - 54*d*e^11 + 27*e^12) + 1/21952*sqrt(14)*(211875
*d^6*e^2 + 3070*d^5*e^3 + 209039*d^4*e^4 - 921444*d^3*e^5 + 380621*d^2*e^6
 - 49586*d*e^7 - 43695*e^8)*arctan(1/14*sqrt(14)*(5*d - 5*d^2/(e*x + d) +
2*d*e/(e*x + d) - e - 3*e^2/(e*x + d))/e)/((625*d^8 - 1000*d^7*e + 2100*d^
6*e^2 - 1960*d^5*e^3 + 2086*d^4*e^4 - 1176*d^3*e^5 + 756*d^2*e^6 - 216*d*e
^7 + 81*e^8)*e^2) + 1/1568*(275375*d^5*e - 3006775*d^4*e^2 + 1394650*d^3*e
^3 + 1835350*d^2*e^4 - 734925*d*e^5 + 17525*e^6 - 5*(165225*d^6*e^2 - 1997
830*d^5*e^3 + 1218421*d^4*e^4 + 1520564*d^3*e^5 - 947049*d^2*e^6 + 93386*d
*e^7 + 7963*e^8)/((e*x + d)*e) + (826125*d^7*e^3 - 10957975*d^6*e^4 + 8449
735*d^5*e^5 + 8211175*d^4*e^6 - 7879025*d^3*e^7 + 2996315*d^2*e^8 - 443947
*d*e^9 - 67267*e^10)/((e*x + d)^2*e^2) - (275375*d^8*e^4 - 3975600*d^7*e^5
 + 3752280*d^6*e^6 + 2119880*d^5*e^7 - 3655050*d^4*e^8 + 4008480*d^3*e^9 -
 1453312*d^2*e^10 - 197784*d*e^11 + 66483*e^12)/((e*x + d)^3*e^3))/((5*d^2
 - 2*d*e + 3*e^2)^4*(10*d/(e*x + d) - 5*d^2/(e*x + d)^2 - 2*e/(e*x + d)...
```

Mupad [B] (verification not implemented)

Time = 17.53 (sec) , antiderivative size = 965, normalized size of antiderivative = 2.18

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x + 3*x^2 - 5*x^3 + 4*x^4 + 2)/((d + e*x)^2*(2*x + 5*x^2 + 3)^3),x)
```

output
```
log(d + e*x)*((2*e^3*(620*d - 2417*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^3) - (
6*e^5*(423*d - 1367*e))/(125*(5*d^2 - 2*d*e + 3*e^2)^4) + (e*(8*d + 23*e))
/(5*(5*d^2 - 2*d*e + 3*e^2)^2)) - ((3*x*(77965*d^4*e - 19493*d*e^4 - 29965
*d^5 + 13245*e^5 + 21522*d^2*e^3 + 51590*d^3*e^2))/(1568*(125*d^6 - 150*d^
5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) - (120
09*d*e^4 - 95100*d^4*e + 64765*d^5 - 28224*e^5 + 22292*d^2*e^3 - 200706*d^
3*e^2)/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*
d^3*e^3 + 285*d^4*e^2)) + (5*x^4*(20345*d^4*e - 18188*d*e^4 + 19269*e^5 -
11178*d^2*e^3 + 125124*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 2
7*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)) + (x^3*(361295*d^4*e - 1
38539*d*e^4 - 55075*d^5 + 93087*e^5 + 173446*d^2*e^3 + 272442*d^3*e^2))/(1
568*(125*d^6 - 150*d^5*e - 54*d*e^5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 +
 285*d^4*e^2)) + (x^2*(412485*d^4*e - 144973*d*e^4 - 193765*d^5 + 131589*e
^5 + 56850*d^2*e^3 + 621062*d^3*e^2))/(1568*(125*d^6 - 150*d^5*e - 54*d*e^
5 + 27*e^6 + 171*d^2*e^4 - 188*d^3*e^3 + 285*d^4*e^2)))/(9*d + x^2*(34*d +
 12*e) + x^4*(25*d + 20*e) + x^3*(20*d + 34*e) + 25*e*x^5 + x*(12*d + 9*e)
) + (log(x - (14^(1/2)*1i)/5 + 1/5)*((211875*14^(1/2)*d^6)/43904 - e^6*((4
3695*14^(1/2))/43904 - 9i/2) - d^3*e^3*((230361*14^(1/2))/10976 + 6i) + d^
4*e^2*((209039*14^(1/2))/43904 - 83i/2) + d^2*e^4*((380621*14^(1/2))/43904
 + 38i) + d^5*e*((1535*14^(1/2))/21952 - 20i) - d*e^5*((24793*14^(1/2))...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3775, normalized size of antiderivative = 8.52

$$\int \frac{2+x+3x^2-5x^3+4x^4}{(d+ex)^2(3+2x+5x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((4*x^4-5*x^3+3*x^2+x+2)/(e*x+d)^2/(5*x^2+2*x+3)^3,x)
```

output

```
(26484375*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**8*x**4 + 21187500*sqrt(14)*
atan((5*x + 1)/sqrt(14))*d**8*x**3 + 36018750*sqrt(14)*atan((5*x + 1)/sqrt
(14))*d**8*x**2 + 12712500*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**8*x + 9534
375*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**8 + 26484375*sqrt(14)*atan((5*x +
 1)/sqrt(14))*d**7*e*x**5 + 42758750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**
7*e*x**4 + 53275750*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**7*e*x**3 + 420494
00*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**7*e*x**2 + 19888575*sqrt(14)*atan(
(5*x + 1)/sqrt(14))*d**7*e*x + 7765650*sqrt(14)*atan((5*x + 1)/sqrt(14))*d
**7*e + 21571250*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**2*x**5 + 436938
75*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**2*x**4 + 50486400*sqrt(14)*at
an((5*x + 1)/sqrt(14))*d**6*e**2*x**3 + 46308350*sqrt(14)*atan((5*x + 1)/s
qrt(14))*d**6*e**2*x**2 + 20455350*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*
e**2*x + 9517275*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**6*e**2 + 26436875*sq
rt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**3*x**5 - 73127100*sqrt(14)*atan((5
*x + 1)/sqrt(14))*d**5*e**3*x**4 - 39467130*sqrt(14)*atan((5*x + 1)/sqrt(1
4))*d**5*e**3*x**3 - 115526476*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**3
*x**2 - 35735493*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**3*x - 33939576*
sqrt(14)*atan((5*x + 1)/sqrt(14))*d**5*e**3 - 94276600*sqrt(14)*atan((5*x
+ 1)/sqrt(14))*d**4*e**4*x**5 - 119988055*sqrt(14)*atan((5*x + 1)/sqrt(14)
)*d**4*e**4*x**4 - 163869596*sqrt(14)*atan((5*x + 1)/sqrt(14))*d**4*e**...
```

3.159 $\int (5+2x)\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

Optimal result

Integrand size = 38, antiderivative size = 143

$$\begin{aligned}\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx &= -\frac{51435(1-4x)\sqrt{3-x+2x^2}}{32768}+\frac{11433(5+2x)^2\left(3-x+2x^2\right)^{3/2}}{4480}\\ &\quad -\frac{823(5+2x)^3\left(3-x+2x^2\right)^{3/2}}{1344}+\frac{5}{112}(5+2x)^4\left(3-x+2x^2\right)^{3/2}\\ &\quad -\frac{(1005757+295276x)\left(3-x+2x^2\right)^{3/2}}{71680}-\frac{1183005\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}\end{aligned}$$

output
```
-51435/32768*(1-4*x)*(2*x^2-x+3)^(1/2)+11433/4480*(5+2*x)^2*(2*x^2-x+3)^(3
/2)-823/1344*(5+2*x)^3*(2*x^2-x+3)^(3/2)+5/112*(5+2*x)^4*(2*x^2-x+3)^(3/2)
-1/71680*(1005757+295276*x)*(2*x^2-x+3)^(3/2)-1183005/131072*arcsinh(1/23*
(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.56

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{4\sqrt{3-x+2x^2}(6231117+14742332x+11357024x^2+20304768x^3+1390592x^4+12984320x^5+49152}{13762560}$$

input
```
Integrate[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(6231117 + 14742332*x + 11357024*x^2 + 20304768*x^3
 + 1390592*x^4 + 12984320*x^5 + 4915200*x^6) - 124215525*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/13762560
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2184, 25, 2184, 27, 2184, 27, 1225, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+5)\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$
$$\Big\downarrow \ 2184$$
$$\frac{1}{224}\int -\left((2x+5)\sqrt{2x^2-x+3}\left(6584x^3+10788x^2+7826x+3677\right)\right)dx+\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$
$$\Big\downarrow \ 25$$
$$\frac{5}{112}(2x+5)^4\left(2x^2-x+3\right)^{3/2}-\frac{1}{224}\int (2x+5)\sqrt{2x^2-x+3}\left(6584x^3+10788x^2+7826x+3677\right)dx$$
$$\Big\downarrow \ 2184$$

$$\frac{1}{224}\left(-\frac{1}{96}\int -24(2x+5)\sqrt{2x^2-x+3}\left(45732x^2+37828x+14097\right)dx-\frac{823}{6}\left(2x^2-x+3\right)^{3/2}(2x+5)^3\right)+$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 27

$$\frac{1}{224}\left(\frac{1}{4}\int (2x+5)\sqrt{2x^2-x+3}\left(45732x^2+37828x+14097\right)dx-\frac{823}{6}(2x+5)^3\left(2x^2-x+3\right)^{3/2}\right)+$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 2184

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{40}\int 4(38073-147638x)(2x+5)\sqrt{2x^2-x+3}dx+\frac{11433}{5}\left(2x^2-x+3\right)^{3/2}(2x+5)^2\right)-\frac{823}{6}(2x+5$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 27

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\int (38073-147638x)(2x+5)\sqrt{2x^2-x+3}dx+\frac{11433}{5}\left(2x^2-x+3\right)^{3/2}(2x+5)^2\right)-\frac{823}{6}(2x+5)$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 1225

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\int \sqrt{2x^2-x+3}dx-\frac{1}{8}(295276x+1005757)\left(2x^2-x+3\right)^{3/2}\right)+\frac{11433}{5}\left(2x^2-x+3\right)^{3/}$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 1087

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+1005757)\left(2x^2-x\right.\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\downarrow$ 1090

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+1\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

$\big\downarrow$ 222

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{1}{10}\left(\frac{1800225}{16}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{8}(295276x+1005757)\left(2x^2-x+3\right.\right.\right.\right.$$
$$\frac{5}{112}\left(2x^2-x+3\right)^{3/2}(2x+5)^4$$

input
```
Int[(5 + 2*x)*Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*(5 + 2*x)^4*(3 - x + 2*x^2)^(3/2))/112 + ((-823*(5 + 2*x)^3*(3 - x + 2*
x^2)^(3/2))/6 + ((11433*(5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))/5 + (-1/8*((100
5757 + 295276*x)*(3 - x + 2*x^2)^(3/2)) + (1800225*(-1/8*((1 - 4*x)*Sqrt[3
 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/16)/10)/
4)/224
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(4915200x^6+12984320x^5+1390592x^4+20304768x^3+11357024x^2+14742332x+6231117)\sqrt{2x^2-x+3}}{3440640}+\frac{1183005\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{}}{}\right.}{131072}$
trager	$\left(\frac{10}{7}x^6+\frac{317}{84}x^5+\frac{97}{240}x^4+\frac{52877}{8960}x^3+\frac{50701}{15360}x^2+\frac{3685583}{860160}x+\frac{2077039}{1146880}\right)\sqrt{2x^2-x+3}-\frac{1183005\,\mathrm{RootOf}\left(\right.}{}$
default	$\frac{283x^2(2x^2-x+3)^{\frac{3}{2}}}{1120}-\frac{5179x(2x^2-x+3)^{\frac{3}{2}}}{17920}+\frac{242329(2x^2-x+3)^{\frac{3}{2}}}{215040}+\frac{51435(4x-1)\sqrt{2x^2-x+3}}{32768}+\frac{1183005\sqrt{2}\,\mathrm{arcsinh}\left(\frac{4\sqrt{2}}{}\right.}{131072}$

input
```
int((5+2*x)*(2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
1/3440640*(4915200*x^6+12984320*x^5+1390592*x^4+20304768*x^3+11357024*x^2+
14742332*x+6231117)*(2*x^2-x+3)^(1/2)+1183005/131072*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.58

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{3440640}\left(4915200\,x^6+12984320\,x^5+1390592\,x^4+20304768\,x^3+11357024\,x^2+14742332\,x+623111\right.$$
$$+\frac{1183005}{262144}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
1/3440640*(4915200*x^6 + 12984320*x^5 + 1390592*x^4 + 20304768*x^3 + 11357
024*x^2 + 14742332*x + 6231117)*sqrt(2*x^2 - x + 3) + 1183005/262144*sqrt(
2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{10x^6}{7}+\frac{317x^5}{84}+\frac{97x^4}{240}+\frac{52877x^3}{8960}+\frac{50701x^2}{15360}+\frac{3685583x}{860160}+\frac{2077039}{1146880}\right) + \frac{1183005\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{131072}$$

input
```
integrate((5+2*x)*(2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(10*x**6/7 + 317*x**5/84 + 97*x**4/240 + 52877*x**3/8
960 + 50701*x**2/15360 + 3685583*x/860160 + 2077039/1146880) + 1183005*sqr
t(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/131072
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{5}{7}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{377}{168}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{283}{1120}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2 - \frac{5179}{17920}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{242329}{215040}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{51435}{8192}\sqrt{2x^2-x+3}x + \frac{1183005}{131072}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{51435}{32768}\sqrt{2x^2-x+3}$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
5/7*(2*x^2 - x + 3)^(3/2)*x^4 + 377/168*(2*x^2 - x + 3)^(3/2)*x^3 + 283/11
20*(2*x^2 - x + 3)^(3/2)*x^2 - 5179/17920*(2*x^2 - x + 3)^(3/2)*x + 242329
/215040*(2*x^2 - x + 3)^(3/2) + 51435/8192*sqrt(2*x^2 - x + 3)*x + 1183005
/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 51435/32768*sqrt(2*x^2
- x + 3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.55

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{3440640}\left(4\left(8\left(4\left(16\left(20\left(120\,x+317\right)x+679\right)x+158631\right)x+354907\right)x+3685583\right)x+6231117\right)\sqrt{2\,x^2}$$
$$-\frac{1183005}{131072}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
1/3440640*(4*(8*(4*(16*(20*(120*x + 317)*x + 679)*x + 158631)*x + 354907)*
x + 3685583)*x + 6231117)*sqrt(2*x^2 - x + 3) - 1183005/131072*sqrt(2)*log
(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.19

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{283\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{1120}+\frac{377\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{168}$$
$$+\frac{5\,x^4\,\left(2\,x^2-x+3\right)^{3/2}}{7}+\frac{4478951\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{573440}$$
$$+\frac{194737\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{17920}+\frac{242329\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{3440640}$$
$$-\frac{5179\,x\,\left(2\,x^2-x+3\right)^{3/2}}{17920}+\frac{5573567\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{4587520}$$

input
```
int((2*x + 5)*(2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
(283*x^2*(2*x^2 - x + 3)^(3/2))/1120 + (377*x^3*(2*x^2 - x + 3)^(3/2))/168
 + (5*x^4*(2*x^2 - x + 3)^(3/2))/7 + (4478951*2^(1/2)*log((2*x^2 - x + 3)^
(1/2) + (2^(1/2)*(2*x - 1/2))/2))/573440 + (194737*(x/2 - 1/8)*(2*x^2 - x
+ 3)^(1/2))/17920 + (242329*(2*x^2 - x + 3)^(1/2)*(32*x^2 - 4*x + 45))/344
0640 - (5179*x*(2*x^2 - x + 3)^(3/2))/17920 + (5573567*2^(1/2)*log(2*(2*x^
2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/4587520
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97

$$\int (5+2x)\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{10\sqrt{2x^2-x+3}\,x^6}{7}+\frac{317\sqrt{2x^2-x+3}\,x^5}{84}+\frac{97\sqrt{2x^2-x+3}\,x^4}{240} + \frac{52877\sqrt{2x^2-x+3}\,x^3}{8960}+\frac{50701\sqrt{2x^2-x+3}\,x^2}{15360}+\frac{3685583\sqrt{2x^2-x+3}\,x}{860160} + \frac{2077039\sqrt{2x^2-x+3}}{1146880}+\frac{1183005\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{131072}$$

input
```
int((5+2*x)*(2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

output
```
(19660800*sqrt(2*x**2 - x + 3)*x**6 + 51937280*sqrt(2*x**2 - x + 3)*x**5 +
 5562368*sqrt(2*x**2 - x + 3)*x**4 + 81219072*sqrt(2*x**2 - x + 3)*x**3 +
45428096*sqrt(2*x**2 - x + 3)*x**2 + 58969328*sqrt(2*x**2 - x + 3)*x + 249
24468*sqrt(2*x**2 - x + 3) + 124215525*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)
*sqrt(2) + 4*x - 1)/sqrt(23)))/13762560
```

3.160 $\int \sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)\,dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = -\frac{4609(1-4x)\sqrt{3-x+2x^2}}{16384}+\frac{287(3-x+2x^2)^{3/2}}{5120}-\frac{71x(3-x+2x^2)^{3/2}}{1280}+\frac{7}{80}x^2\left(3-x+2x^2\right)^{3/2}+\frac{5}{12}x^3\left(3-x+2x^2\right)^{3/2}-\frac{106007\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32768\sqrt{2}}$$

output
```
-4609/16384*(1-4*x)*(2*x^2-x+3)^(1/2)+287/5120*(2*x^2-x+3)^(3/2)-71/1280*x
*(2*x^2-x+3)^(3/2)+7/80*x^2*(2*x^2-x+3)^(3/2)+5/12*x^3*(2*x^2-x+3)^(3/2)-1
06007/65536*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx = \frac{4\sqrt{3-x+2x^2}(-27807+221868x+105696x^2+258432x^3-59392x^4+204800x^5)-1590105\sqrt{2}\log(}{983040}$$

input
```
Integrate[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-27807 + 221868*x + 105696*x^2 + 258432*x^3 - 5939
2*x^4 + 204800*x^5) - 1590105*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2
]])/983040
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{12}\int \frac{3}{2}\sqrt{2x^2-x+3}\left(7x^3-6x^2+8x+16\right)dx+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\int \sqrt{2x^2-x+3}\left(7x^3-6x^2+8x+16\right)dx+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{10}\int \frac{1}{2}\left(-71x^2+76x+320\right)\sqrt{2x^2-x+3}dx+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{1}{20}\int \left(-71x^2+76x+320\right)\sqrt{2x^2-x+3}dx+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{8}\int \frac{1}{2}(861x+5546)\sqrt{2x^2-x+3}dx-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\int (861x+5546)\sqrt{2x^2-x+3}dx-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}x^2\right)+\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\int \sqrt{2x^2-x+3}dx+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-x+3\right)^{3/2}\right)+\frac{7}{10}\left(2x^2-x+3\right)^{3/2}\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

$$\Big\downarrow \; 222$$

$$\frac{1}{8}\left(\frac{1}{20}\left(\frac{1}{16}\left(\frac{23045}{4}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)+\frac{287}{2}\left(2x^2-x+3\right)^{3/2}\right)-\frac{71}{8}x\left(2x^2-x-\right.\right.\right.$$
$$\frac{5}{12}\left(2x^2-x+3\right)^{3/2}x^3$$

input
```
Int[Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*x^3*(3 - x + 2*x^2)^(3/2))/12 + ((7*x^2*(3 - x + 2*x^2)^(3/2))/10 + ((-
71*x*(3 - x + 2*x^2)^(3/2))/8 + ((287*(3 - x + 2*x^2)^(3/2))/2 + (23045*(-
1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(1
6*Sqrt[2])))/4)/16)/20)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(204800x^5-59392x^4+258432x^3+105696x^2+221868x-27807)\sqrt{2x^2-x+3}}{245760}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$
trager	$\left(\frac{5}{6}x^5-\frac{29}{120}x^4+\frac{673}{640}x^3+\frac{1101}{2560}x^2+\frac{18489}{20480}x-\frac{9269}{81920}\right)\sqrt{2x^2-x+3}-\frac{106007\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootO}\right.}{}$
default	$\frac{287(2x^2-x+3)^{\frac{3}{2}}}{5120}+\frac{4609(4x-1)\sqrt{2x^2-x+3}}{16384}+\frac{106007\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}-\frac{71x(2x^2-x+3)^{\frac{3}{2}}}{1280}+\frac{7x^2(2x^2-x+3)^{\frac{3}{2}}}{80}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/245760*(204800*x^5-59392*x^4+258432*x^3+105696*x^2+221868*x-27807)*(2*x^
2-x+3)^(1/2)+106007/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{245760}\left(204800\,x^5-59392\,x^4+258432\,x^3+105696\,x^2+221868\,x-27807\right)\sqrt{2\,x^2-x+3}$$
$$+\frac{106007}{131072}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/245760*(204800*x^5 - 59392*x^4 + 258432*x^3 + 105696*x^2 + 221868*x - 27
807)*sqrt(2*x^2 - x + 3) + 106007/131072*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2
 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\sqrt{2x^2-x+3}\cdot\left(\frac{5x^5}{6}-\frac{29x^4}{120}+\frac{673x^3}{640}+\frac{1101x^2}{2560}+\frac{18489x}{20480}-\frac{9269}{81920}\right)$$
$$+\frac{106007\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{65536}$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**5/6 - 29*x**4/120 + 673*x**3/640 + 1101*x**2/25
60 + 18489*x/20480 - 9269/81920) + 106007*sqrt(2)*asinh(4*sqrt(23)*(x - 1/
4)/23)/65536
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{5}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{7}{80}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{71}{1280}\left(2x^2-x+3\right)^{\frac{3}{2}}x$$
$$+\frac{287}{5120}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4609}{4096}\sqrt{2x^2-x+3}x$$
$$+\frac{106007}{65536}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4609}{16384}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
5/12*(2*x^2 - x + 3)^(3/2)*x^3 + 7/80*(2*x^2 - x + 3)^(3/2)*x^2 - 71/1280*
(2*x^2 - x + 3)^(3/2)*x + 287/5120*(2*x^2 - x + 3)^(3/2) + 4609/4096*sqrt(
2*x^2 - x + 3)*x + 106007/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 4609/16384*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{1}{245760}\,(4\,(8\,(4\,(16\,(100\,x-29)x+2019)x+3303)x+55467)x-27807)\sqrt{2\,x^2-x+3}$$
$$-\frac{106007}{65536}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/245760*(4*(8*(4*(16*(100*x - 29)*x + 2019)*x + 3303)*x + 55467)*x - 2780
7)*sqrt(2*x^2 - x + 3) - 106007/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [B] (verification not implemented)

Time = 17.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx$$
$$=\frac{7\,x^2\,\left(2\,x^2-x+3\right)^{3/2}}{80}+\frac{5\,x^3\,\left(2\,x^2-x+3\right)^{3/2}}{12}$$
$$+\frac{63779\,\sqrt{2}\,\ln\left(\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(2\,x-\frac{1}{2}\right)}{2}\right)}{40960}+\frac{2773\,\left(\frac{x}{2}-\frac{1}{8}\right)\,\sqrt{2\,x^2-x+3}}{1280}$$
$$+\frac{287\,\sqrt{2\,x^2-x+3}\,\left(32\,x^2-4\,x+45\right)}{81920}-\frac{71\,x\,\left(2\,x^2-x+3\right)^{3/2}}{1280}$$
$$+\frac{19803\,\sqrt{2}\,\ln\left(2\,\sqrt{2\,x^2-x+3}+\frac{\sqrt{2}\,\left(4\,x-1\right)}{2}\right)}{327680}$$

input
```
int((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
(7*x^2*(2*x^2 - x + 3)^(3/2))/80 + (5*x^3*(2*x^2 - x + 3)^(3/2))/12 + (637
79*2^(1/2)*log((2*x^2 - x + 3)^(1/2) + (2^(1/2)*(2*x - 1/2))/2))/40960 + (
2773*(x/2 - 1/8)*(2*x^2 - x + 3)^(1/2))/1280 + (287*(2*x^2 - x + 3)^(1/2)*
(32*x^2 - 4*x + 45))/81920 - (71*x*(2*x^2 - x + 3)^(3/2))/1280 + (19803*2^
(1/2)*log(2*(2*x^2 - x + 3)^(1/2) + (2^(1/2)*(4*x - 1))/2))/327680
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)\,dx \\
&= \frac{5\sqrt{2x^2-x+3}\,x^5}{6}-\frac{29\sqrt{2x^2-x+3}\,x^4}{120}+\frac{673\sqrt{2x^2-x+3}\,x^3}{640} \\
&\quad+\frac{1101\sqrt{2x^2-x+3}\,x^2}{2560}+\frac{18489\sqrt{2x^2-x+3}\,x}{20480} \\
&\quad-\frac{9269\sqrt{2x^2-x+3}}{81920}+\frac{106007\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{65536}
\end{aligned}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

output
```
(819200*sqrt(2*x**2 - x + 3)*x**5 - 237568*sqrt(2*x**2 - x + 3)*x**4 + 103
3728*sqrt(2*x**2 - x + 3)*x**3 + 422784*sqrt(2*x**2 - x + 3)*x**2 + 887472
*sqrt(2*x**2 - x + 3)*x - 111228*sqrt(2*x**2 - x + 3) + 1590105*sqrt(2)*lo
g((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/983040
```

3.161 $\int \frac{\sqrt{3-x+2x^2}\left(2+x+3x^2-x^3+5x^4\right)}{5+2x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{(489587-80844x)\sqrt{3-x+2x^2}}{4096}+\frac{4535}{768}\left(3-x+2x^2\right)^{3/2}\\ &\quad-\frac{127}{128}(5+2x)\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}(5+2x)^2\left(3-x+2x^2\right)^{3/2}\\ &\quad+\frac{5627989\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}-\frac{11001\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{16\sqrt{2}}\end{aligned}$$

output
```
1/4096*(489587-80844*x)*(2*x^2-x+3)^(1/2)+4535/768*(2*x^2-x+3)^(3/2)-127/1
28*(5+2*x)*(2*x^2-x+3)^(3/2)+1/16*(5+2*x)^2*(2*x^2-x+3)^(3/2)+5627989/1638
4*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-11001/32*arctanh(1/24*(17-22*x)*2
^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$= \frac{4\sqrt{3-x+2x^2}(1561161-300404x+79840x^2-21120x^3+6144x^4)+33795072\sqrt{2}\text{arctanh}\left(\frac{1}{6}(5+2x\right.}{49152}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(1561161 - 300404*x + 79840*x^2 - 21120*x^3 + 6144*
x^4) + 33795072*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 168
83967*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/49152
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2184, 27, 2184, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{2x+5}dx$$

$$\Big\downarrow \; 2184$$

$$\frac{1}{160}\int -\frac{5\sqrt{2x^2-x+3}\left(1016x^3+1860x^2+1298x+161\right)}{2x+5}dx+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$$\Big\downarrow \; 27$$

$$\frac{1}{16}(2x+5)^2\left(2x^2-x+3\right)^{3/2}-\frac{1}{32}\int \frac{\sqrt{2x^2-x+3}\left(1016x^3+1860x^2+1298x+161\right)}{2x+5}dx$$

$$\Big\downarrow \; 2184$$

$$\frac{1}{32}\left(-\frac{1}{64}\int \frac{8\left(-18140x^2-20604x+3193\right)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(-\frac{1}{8}\int \frac{\left(-18140x^2-20604x+3193\right)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 2184

$$\frac{1}{32}\Bigg(\frac{1}{8}\Bigg(\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}-\frac{1}{24}\int -\frac{12(16289-40422x)\sqrt{2x^2-x+3}}{2x+5}dx\Bigg)-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{(16289-40422x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}\right)-\frac{127}{4}(2x+5)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 1231

$$\frac{1}{32}\Bigg(\frac{1}{8}\Bigg(\frac{1}{2}\left(\frac{1}{8}(489587-80844x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(5655127-11255978x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 27

$$\frac{1}{32}\Bigg(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\int \frac{5655127-11255978x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(489587-80844x)\right)+\frac{4535}{3}\left(2x^2-x+3\right)^{3/2}\right)-$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\Big\downarrow$ 1269

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-5627989\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x+3}(48958\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\big\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{5627989\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\big\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(33795072\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(489587-\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\big\downarrow$ 1154

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(-67590144\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(4\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

$\big\downarrow$ 219

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{1}{16}\left(-\frac{5627989\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-2816256\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2-x+3}(48\right.\right.\right.$$
$$\frac{1}{16}\left(2x^2-x+3\right)^{3/2}(2x+5)^2$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

output
```
((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2))/16 + ((-127*(5 + 2*x)*(3 - x + 2*x^2)^
(3/2))/4 + ((4535*(3 - x + 2*x^2)^(3/2))/3 + (((489587 - 80844*x)*Sqrt[3 -
 x + 2*x^2])/8 + ((-5627989*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 281625
6*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16)/2)/8)
/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$
\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx \\
&= \frac{1}{12288}\left(6144\,x^4-21120\,x^3+79840\,x^2-300404\,x+1561161\right)\sqrt{2\,x^2-x+3} \\
&\quad+\frac{5627989}{32768}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right) \\
&\quad+\frac{11001}{64}\,\sqrt{2}\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)
\end{aligned}
$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fr
icas")
```

output
```
1/12288*(6144*x^4 - 21120*x^3 + 79840*x^2 - 300404*x + 1561161)*sqrt(2*x^2
 - x + 3) + 5627989/32768*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x -
 1) - 32*x^2 + 16*x - 25) + 11001/64*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$
\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx
$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5),
x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$= \frac{1}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^2 - \frac{47}{64}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{1925}{768}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{20211}{1024}\sqrt{2\,x^2-x+3}x - \frac{5627989}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{11001}{32}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{489587}{4096}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="ma
xima")
```

output
```
1/4*(2*x^2 - x + 3)^(3/2)*x^2 - 47/64*(2*x^2 - x + 3)^(3/2)*x + 1925/768*(
2*x^2 - x + 3)^(3/2) - 20211/1024*sqrt(2*x^2 - x + 3)*x - 5627989/16384*sq
rt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 11001/32*sqrt(2)*arcsinh(
22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 489587/4096
*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$$
$$= \frac{1}{12288}\left(4\left(8\left(12\left(16\,x-55\right)x+2495\right)x-75101\right)x+1561161\right)\sqrt{2\,x^2-x+3} + \frac{5627989}{16384}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) - \frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{11001}{32}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gi
ac")
```

output
```
1/12288*(4*(8*(12*(16*x - 55)*x + 2495)*x - 75101)*x + 1561161)*sqrt(2*x^2
 - x + 3) + 5627989/16384*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^
2 - x + 3)) - 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 11001/32*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqr
t(2*x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{2\,x+5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{\sqrt{2x^2-x+3}\,x^4}{2} - \frac{55\sqrt{2x^2-x+3}\,x^3}{32} \\ &+ \frac{2495\sqrt{2x^2-x+3}\,x^2}{384} - \frac{75101\sqrt{2x^2-x+3}\,x}{3072} \\ &+ \frac{520387\sqrt{2x^2-x+3}}{4096} + \frac{11001\sqrt{2}\log\left(\frac{46\sqrt{2x^2-x+3}\,\sqrt{2}+92x-46}{\sqrt{23}}\right)}{32} \\ &- \frac{5627989\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{16384} - \frac{11001\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x+22}{\sqrt{23}}\right)}{32}\end{aligned}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

output
```
(24576*sqrt(2*x**2 - x + 3)*x**4 - 84480*sqrt(2*x**2 - x + 3)*x**3 + 31936
0*sqrt(2*x**2 - x + 3)*x**2 - 1201616*sqrt(2*x**2 - x + 3)*x + 6244644*sqr
t(2*x**2 - x + 3) + 16897536*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 92*x - 46)/sqrt(23)) - 16883967*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt
(2) + 4*x - 1)/sqrt(23)) - 16897536*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sq
rt(2) + 4*x + 22)/sqrt(23)))/49152
```

3.162 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 149

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx &= -\frac{(1996953-333380x)\sqrt{3-x+2x^2}}{18432}-\frac{541}{384}\left(3-x+2x^2\right)^{3/2}\\ &\quad-\frac{3667(3-x+2x^2)^{3/2}}{576(5+2x)}+\frac{5}{64}(5+2x)\left(3-x+2x^2\right)^{3/2}\\ &\quad-\frac{2551847\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}+\frac{239201\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{384\sqrt{2}}\end{aligned}$$

output
```
-1/18432*(1996953-333380*x)*(2*x^2-x+3)^(1/2)-541/384*(2*x^2-x+3)^(3/2)-36
67*(2*x^2-x+3)^(3/2)/(2880+1152*x)+5/64*(5+2*x)*(2*x^2-x+3)^(3/2)-2551847/
8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+239201/768*arctanh(1/24*(17-22
*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$=\frac{\frac{4\sqrt{3-x+2x^2}(-3539439-728410x+94936x^2-17344x^3+3840x^4)}{5+2x}-15308864\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-}{24576}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,
x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(-3539439 - 728410*x + 94936*x^2 - 17344*x^3 + 384
0*x^4))/(5 + 2*x) - 15308864*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x
^2])/6] - 7655541*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/24576
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.325, Rules used = {2181, 27, 2184, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{72}\int \frac{\sqrt{2x^2-x+3}(-2880x^3+7776x^2-50504x+19341)}{16(2x+5)}dx-\frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-2880x^3+7776x^2-50504x+19341)}{2x+5}dx}{1152}-\frac{3667(2x^2-x+3)^{3/2}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int \frac{128\sqrt{2x^2-x+3}(9738x^2-19762x+9333)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 27

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\int \frac{\sqrt{2x^2-x+3}(9738x^2-19762x+9333)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 2184

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{24}\int \frac{6(61677-166690x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1623}{2}\left(2x^2-x+3\right)^{3/2}\right)}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 27

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\int \frac{(61677-166690x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1623}{2}\left(2x^2-x+3\right)^{3/2}\right)}{1152}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 1231

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{1}{8}(1996953-333380x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{18(2549629-5103694x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1623}{2}\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 27

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\int \frac{2549629-5103694x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(1996953-333380x)\right)+\frac{1623}{2}(2x^2\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\downarrow$ 1269

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2551847\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x\right.\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\Big\downarrow$ 1090

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2551847\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\Big\downarrow$ 222

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(15308864\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\Big\downarrow$ 1154

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(-30617728\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

$\Big\downarrow$ 219

$$\frac{90(2x+5)\left(2x^2-x+3\right)^{3/2}-2\left(\frac{1}{4}\left(\frac{9}{16}\left(-\frac{2551847\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{3827216}{3}\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{576(2x+5)}$$

input

```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x)) + (90*(5 + 2*x)*(3 - x + 2*x
^2)^(3/2) - 2*((1623*(3 - x + 2*x^2)^(3/2))/2 + (((1996953 - 333380*x)*Sqr
t[3 - x + 2*x^2])/8 + (9*((-2551847*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2]
- (3827216*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/
3))/16)/4))/1152
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \frac{7655541\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+7654432\sqrt{2}(2x+5)\log}{4}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
fricas")
```

output
```
1/49152*(7655541*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
 - 1) - 32*x^2 + 16*x - 25) + 7654432*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x +
25)) + 8*(3840*x^4 - 17344*x^3 + 94936*x^2 - 728410*x - 3539439)*sqrt(2*x^
2 - x + 3))/(2*x + 5)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{5}{32}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{391}{384}\left(2x^2-x+3\right)^{\frac{3}{2}} + \frac{6001}{512}\sqrt{2x^2-x+3}x + \frac{2551847}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{239201}{768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{182769}{2048}\sqrt{2x^2-x+3} - \frac{3667\sqrt{2x^2-x+3}}{32(2x+5)}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
maxima")
```

output
```
5/32*(2*x^2 - x + 3)^(3/2)*x - 391/384*(2*x^2 - x + 3)^(3/2) + 6001/512*sq
rt(2*x^2 - x + 3)*x + 2551847/8192*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*
sqrt(23)) - 239201/768*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) - 182769/2048*sqrt(2*x^2 - x + 3) - 3667/32*sqrt
(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(118) = 236.

Time = 0.23 (sec) , antiderivative size = 531, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
giac")
```

output
```
1/24576*sqrt(2)*(7654432*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 7655541*log(abs(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 7655541*log(a
bs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))*sgn(1/(2*x
 + 5)) - 1408128*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)*sgn(1/(2*x + 5))
 + 2*(16367883*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^7*
sgn(1/(2*x + 5)) - 34896384*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/
(2*x + 5))^6*sgn(1/(2*x + 5)) - 93395*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 25574400*(sqrt(-11/(2*x + 5) + 3
6/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x + 5)) + 19752365*(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3*sgn(1/(2*x + 5)) - 319219
20*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x +
 5)) - 2445813*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sg
n(1/(2*x + 5)) + 7663104*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x
+ 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$$
$$= \frac{15360\sqrt{2x^2-x+3}\,x^4-69376\sqrt{2x^2-x+3}\,x^3+379744\sqrt{2x^2-x+3}\,x^2-2913640\sqrt{2x^2-x+3}\,x-}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

output
```
(15360*sqrt(2*x**2 - x + 3)*x**4 - 69376*sqrt(2*x**2 - x + 3)*x**3 + 37974
4*sqrt(2*x**2 - x + 3)*x**2 - 2913640*sqrt(2*x**2 - x + 3)*x - 14157756*sq
rt(2*x**2 - x + 3) + 15308864*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(
2) + 22*x - 17)*x + 38272160*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2
) + 22*x - 17) + 15311082*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) -
4*x + 1)*x + 38277705*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x
+ 1) - 15308864*sqrt(2)*log(2*x + 5)*x - 38272160*sqrt(2)*log(2*x + 5))/(2
4576*(2*x + 5))
```

3.163 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 151

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{5(661065-110099x)\sqrt{3-x+2x^2}}{82944}+\frac{5}{48}\left(3-x+2x^2\right)^{3/2} - \frac{3667(3-x+2x^2)^{3/2}}{1152(5+2x)^2}+\frac{357391(3-x+2x^2)^{3/2}}{82944(5+2x)} + \frac{117315\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}}-\frac{12670805\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{55296\sqrt{2}}$$

output
```
5/82944*(661065-110099*x)*(2*x^2-x+3)^(1/2)+5/48*(2*x^2-x+3)^(3/2)-3667/11
52*(2*x^2-x+3)^(3/2)/(5+2*x)^2+357391*(2*x^2-x+3)^(3/2)/(414720+165888*x)+
117315/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-12670805/110592*arctanh
(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(4880551+2959330x+272520x^2-25632x^3+3840x^4)}{(5+2x)^2}+12670805\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{55296}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(4880551 + 2959330*x + 272520*x^2 - 25632*x^3 + 3
840*x^4))/(5 + 2*x)^2 + 12670805*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x +
 4*x^2])/6] + 6335010*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/5529
6
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 27, 2184, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^3}\,dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{144}\int \frac{\sqrt{2x^2-x+3}\left(-5760x^3+15552x^2-57004x+27681\right)}{16(2x+5)^2}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}\left(-5760x^3+15552x^2-57004x+27681\right)}{(2x+5)^2}dx}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int \frac{5\sqrt{2x^2-x+3}(41472x^2-787480x+306261)}{2x+5}dx+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5}{72}\int \frac{\sqrt{2x^2-x+3}(41472x^2-787480x+306261)}{2x+5}dx+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{5}{72}\left(\frac{1}{24}\int \frac{24(332181-880792x)\sqrt{2x^2-x+3}}{2x+5}dx+3456(2x^2-x+3)^{3/2}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5}{72}\left(\int \frac{(332181-880792x)\sqrt{2x^2-x+3}}{2x+5}dx+3456(2x^2-x+3)^{3/2}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{5}{72}\left(-\frac{1}{32}\int -\frac{576(422491-844668x)}{(2x+5)\sqrt{2x^2-x+3}}dx+3456(2x^2-x+3)^{3/2}+2(661065-110099x)\sqrt{2x^2-x+3}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5}{72}\left(18\int \frac{422491-844668x}{(2x+5)\sqrt{2x^2-x+3}}dx+3456(2x^2-x+3)^{3/2}+2(661065-110099x)\sqrt{2x^2-x+3}\right)+\frac{357391(2x^2-x+3)^{3/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{5}{72}\left(18\left(2534161\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-422334\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-110099x)\sqrt{\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{5}{72}\left(18\left(2534161\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-211167\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{5}{72}\left(18\left(2534161\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-110099\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{5}{72}\left(18\left(-5068322\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5}{72}\left(18\left(-211167\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{2534161\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+3456\left(2x^2-x+3\right)^{3/2}+2(661065-11\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1152(2x+5)^2}$$

input

```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(1152*(5 + 2*x)^2) + ((357391*(3 - x + 2*x^2
)^(3/2))/(36*(5 + 2*x)) + (5*(2*(661065 - 110099*x)*Sqrt[3 - x + 2*x^2] +
3456*(3 - x + 2*x^2)^(3/2) + 18*(-211167*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[2
3]] - (2534161*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*S
qrt[2]))))/72)/2304
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$= \frac{12670020\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+12670805\sqrt{2}(4$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
fricas")
```

output
```
1/221184*(12670020*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 12670805*sqrt(2)*(4*x^2 + 20*x +
25)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x +
 1153)/(4*x^2 + 20*x + 25)) + 48*(3840*x^4 - 25632*x^3 + 272520*x^2 + 2959
330*x + 4880551)*sqrt(2*x^2 - x + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$=\int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
3, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$=\frac{5}{48}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{149}{64}\sqrt{2x^2-x+3}x-\frac{117315}{1024}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)$$
$$+\frac{12670805}{110592}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right)+\frac{3877}{144}\sqrt{2x^2-x+3}$$
$$-\frac{3667\left(2x^2-x+3\right)^{\frac{3}{2}}}{1152\left(4x^2+20x+25\right)}+\frac{357391\sqrt{2x^2-x+3}}{4608\,(2x+5)}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
maxima")
```

output
```
5/48*(2*x^2 - x + 3)^(3/2) - 149/64*sqrt(2*x^2 - x + 3)*x - 117315/1024*sq
rt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 12670805/110592*sqrt(2)*a
rcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 3877
/144*sqrt(2*x^2 - x + 3) - 3667/1152*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x +
 25) + 357391/4608*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(120) = 240$.

Time = 0.21 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx$$
$$= \frac{1}{768}\,(4\,(40\,x-467)x+19695)\sqrt{2\,x^2-x+3}$$
$$+\frac{117315}{1024}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$
$$-\frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$
$$+\frac{12670805}{110592}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\left(10693526\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^3+79895946\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2-124044603\,\sqrt{2}(\sqrt{2}\right.}{9216\left(2\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^2+10\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)-11\right.}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
giac")
```

output
```
1/768*(4*(40*x - 467)*x + 19695)*sqrt(2*x^2 - x + 3) + 117315/1024*sqrt(2)
*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 12670805/110592*s
qrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 12670805
/110592*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))
) + 1/9216*sqrt(2)*(10693526*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 +
 79895946*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 124044603*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 80334011)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^3}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{92160\sqrt{2x^2-x+3}\,x^4-615168\sqrt{2x^2-x+3}\,x^3+6540480\sqrt{2x^2-x+3}\,x^2+71023920\sqrt{2x^2-x+3}}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

output
```
(92160*sqrt(2*x**2 - x + 3)*x**4 - 615168*sqrt(2*x**2 - x + 3)*x**3 + 6540
480*sqrt(2*x**2 - x + 3)*x**2 + 71023920*sqrt(2*x**2 - x + 3)*x + 11713322
4*sqrt(2*x**2 - x + 3) + 50683220*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt
(2) + 22*x - 17)*x**2 + 253416100*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt
(2) + 22*x - 17)*x + 316770125*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2)
 + 22*x - 17) + 50680080*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x
+ 1)*x**2 + 253400400*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1
)*x + 316750500*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 50
683220*sqrt(2)*log(2*x + 5)*x**2 - 253416100*sqrt(2)*log(2*x + 5)*x - 3167
70125*sqrt(2)*log(2*x + 5))/(110592*(4*x**2 + 20*x + 25))
```

3.164 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx$

Optimal result

Integrand size = 40, antiderivative size = 158

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4} dx = -\frac{(44378877-7400779x)\sqrt{3-x+2x^2}}{5971968} - \frac{3667(3-x+2x^2)^{3/2}}{1728(5+2x)^3} + \frac{158527(3-x+2x^2)^{3/2}}{82944(5+2x)^2} - \frac{6467659(3-x+2x^2)^{3/2}}{5971968(5+2x)} - \frac{10939\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}} + \frac{170114729\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3981312\sqrt{2}}$$

output

```
-1/5971968*(44378877-7400779*x)*(2*x^2-x+3)^(1/2)-3667/1728*(2*x^2-x+3)^(3
/2)/(5+2*x)^3+158527/82944*(2*x^2-x+3)^(3/2)/(5+2*x)^2-6467659*(2*x^2-x+3)
^(3/2)/(29859840+11943936*x)-10939/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1
/2)+170114729/7962624*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^
(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-327735797-329667508x-97682900x^2-5453568x^3+414720x^4)}{(5+2x)^3} - 170114729\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2\right.\right.}{3981312}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-327735797 - 329667508*x - 97682900*x^2 - 545356
8*x^3 + 414720*x^4))/(5 + 2*x)^3 - 170114729*Sqrt[2]*ArcTanh[(5 + 2*x - Sq
rt[6 - 2*x + 4*x^2])/6] - 85061664*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/3981312
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2181, 2181, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^4}\,dx$$
$$\Big\downarrow \text{ 2181}$$
$$-\frac{1}{216}\int \frac{3\sqrt{2x^2-x+3}\left(-2880x^3+7776x^2-21168x+12007\right)}{16(2x+5)^3}dx - \frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$
$$\Big\downarrow \text{ 27}$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}\left(-2880x^3+7776x^2-21168x+12007\right)}{(2x+5)^3}dx}{1152} - \frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{\sqrt{2x^2-x+3}(207360x^2-1712380x+890709)}{(2x+5)^2}dx+\frac{158527(2x^2-x+3)^{3/2}}{72(2x+5)^2}}{1152}-\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{(22099149-59206232x)\sqrt{2x^2-x+3}}{2x+5}dx-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{158527(2x^2-x+3)^{3/2}}{72(2x+5)^2}}{1152}-\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(\frac{1}{32}\int -\frac{576(28345289-56707776x)}{(2x+5)\sqrt{2x^2-x+3}}dx-2(44378877-7400779x)\sqrt{2x^2-x+3}\right)-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{158}{}}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\int \frac{28345289-56707776x}{(2x+5)\sqrt{2x^2-x+3}}dx-2\sqrt{2x^2-x+3}(44378877-7400779x)\right)-\frac{6467659(2x^2-x+3)^{3/2}}{36(2x+5)}\right)+\frac{158527(2}{72(}}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-28353888\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)-2\sqrt{2x^2-x+3}(44378877-740077\right.\right.}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-14176944\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)-2\sqrt{2x^2-x+3}(44\right.\right.}{1152}$$
$$\frac{3667(2x^2-x+3)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(170114729\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-2\sqrt{2x^2-x+3}(44378877-740\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(-340229458\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-2\sqrt{2x^2-x+3}(443788\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-18\left(-14176944\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{170114729\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-2\sqrt{2x^2-x+3}(44378877-\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{1728(2x+5)^3}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(1728*(5 + 2*x)^3) + ((158527*(3 - x + 2*x^2
)^(3/2))/(72*(5 + 2*x)^2) + ((-6467659*(3 - x + 2*x^2)^(3/2))/(36*(5 + 2*x
)) + (-2*(44378877 - 7400779*x)*Sqrt[3 - x + 2*x^2] - 18*(-14176944*Sqrt[2
]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (170114729*ArcTanh[(17 - 22*x)/(12*Sqrt[2
]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2])))/72)/144)/1152
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{170123328\,\sqrt{2}(8\,x^3+60\,x^2+150\,x+125)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+17}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
fricas")
```

output
```
1/15925248*(170123328*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 170114729*sqrt(2)*
(8*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x -
 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(414720*x^4 - 5
453568*x^3 - 97682900*x^2 - 329667508*x - 327735797)*sqrt(2*x^2 - x + 3))/
(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx &= \frac{5}{32}\sqrt{2x^2-x+3}x + \frac{10939}{512}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &\quad - \frac{170114729}{7962624}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) \\ &\quad - \frac{693775}{165888}\sqrt{2x^2-x+3} - \frac{3667\,(2x^2-x+3)^{\frac{3}{2}}}{1728\,(8x^3+60x^2+150x+125)} \\ &\quad + \frac{158527\,(2x^2-x+3)^{\frac{3}{2}}}{82944\,(4x^2+20x+25)} - \frac{6467659\sqrt{2x^2-x+3}}{331776\,(2x+5)}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
maxima")
```

output
```
5/32*sqrt(2*x^2 - x + 3)*x + 10939/512*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1
/23*sqrt(23)) - 170114729/7962624*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x
 + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 693775/165888*sqrt(2*x^2 - x + 3) -
 3667/1728*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 158527/8
2944*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 6467659/331776*sqrt(2*x^2
 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(127) = 254$.

Time = 0.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$$
$$= \frac{1}{128}\sqrt{2\,x^2-x+3}(20\,x-413) - \frac{10939}{512}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$
$$+ \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$
$$- \frac{170114729}{7962624}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$
$$- \frac{\sqrt{2}\left(575810908\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+9206213116\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+9688786604\sqrt{2}\right.}{663552\left(2\left(\sqrt{2}x-\sqrt{2\,x^2\right.\right.}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
giac")
```

output
```
1/128*sqrt(2*x^2 - x + 3)*(20*x - 413) - 10939/512*sqrt(2)*log(-2*sqrt(2)*
(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 170114729/7962624*sqrt(2)*log(abs
(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 170114729/7962624*sqrt
(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/663552
*sqrt(2)*(575810908*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 92062131
16*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 9688786604*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^3 - 73157325092*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
49481952947*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 20269228621)/(2*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^4}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{9953280\sqrt{2x^2-x+3}\,x^4-130885632\sqrt{2x^2-x+3}\,x^3-2344389600\sqrt{2x^2-x+3}\,x^2-7912020192\sqrt{2}}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

output
```
(9953280*sqrt(2*x**2 - x + 3)*x**4 - 130885632*sqrt(2*x**2 - x + 3)*x**3 -
 2344389600*sqrt(2*x**2 - x + 3)*x**2 - 7912020192*sqrt(2*x**2 - x + 3)*x
- 7865659128*sqrt(2*x**2 - x + 3) + 1360917832*sqrt(2)*log( - 12*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 10206883740*sqrt(2)*log( - 12*sqrt
(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 25517209350*sqrt(2)*log( - 12
*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 21264341125*sqrt(2)*log( -
12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 1360986624*sqrt(2)*log( - 2
*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 10207399680*sqrt(2)*log( -
 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 25518499200*sqrt(2)*log(
 - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 21265416000*sqrt(2)*log(
- 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 1360917832*sqrt(2)*log(2*x +
 5)*x**3 - 10206883740*sqrt(2)*log(2*x + 5)*x**2 - 25517209350*sqrt(2)*log
(2*x + 5)*x - 21264341125*sqrt(2)*log(2*x + 5))/(7962624*(8*x**3 + 60*x**2
 + 150*x + 125))
```

3.165 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 165

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{7(52836655+9616196x)\sqrt{3-x+2x^2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{3/2}}{2304(5+2x)^4} + \frac{593771(3-x+2x^2)^{3/2}}{497664(5+2x)^3} - \frac{9363383(3-x+2x^2)^{3/2}}{23887872(5+2x)^2} + \frac{259\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}} - \frac{4640586097\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1146617856\sqrt{2}}$$

output
```
7*(52836655+9616196*x)*(2*x^2-x+3)^(1/2)/(477757440+191102976*x)-3667/2304
*(2*x^2-x+3)^(3/2)/(5+2*x)^4+593771/497664*(2*x^2-x+3)^(3/2)/(5+2*x)^3-936
3383/23887872*(2*x^2-x+3)^(3/2)/(5+2*x)^2+259/128*arcsinh(1/23*(1-4*x)*23^
(1/2))*2^(1/2)-4640586097/2293235712*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2
-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$=\frac{\frac{12\sqrt{3-x+2x^2}(44676885233+62847867486x+31323229164x^2+6105343976x^3+238878720x^4)}{(5+2x)^4}+4640586097\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x\right.\right.}{1146617856}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(44676885233 + 62847867486*x + 31323229164*x^2 +
6105343976*x^3 + 238878720*x^4))/(5 + 2*x)^4 + 4640586097*Sqrt[2]*ArcTanh[
(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 2320109568*Sqrt[2]*Log[1 - 4*x + 2*
Sqrt[6 - 2*x + 4*x^2]])/1146617856
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^5}\,dx$$
$$\Big\downarrow \ 2181$$
$$-\frac{1}{288}\int \frac{\sqrt{2x^2-x+3}\left(-11520x^3+31104x^2-70004x+44361\right)}{16(2x+5)^4}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$
$$\Big\downarrow \ 27$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}\left(-11520x^3+31104x^2-70004x+44361\right)}{(2x+5)^4}dx}{4608}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{216}\int \frac{3\sqrt{2x^2-x+3}\,(414720x^2-2156544x+1380023)}{(2x+5)^3}dx+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{72}\int \frac{\sqrt{2x^2-x+3}\,(414720x^2-2156544x+1380023)}{(2x+5)^3}dx+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{72}\left(-\frac{1}{144}\int \frac{7(4755675-9616196x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\int \frac{(4755675-9616196x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(-\frac{1}{8}\int \frac{2(110533831-220962816x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(9616196x+52836655)}{2(2x+5)}\right)-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(-\frac{1}{4}\int \frac{110533831-220962816x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(9616196x+52836655)}{2(2x+5)}\right)-\frac{9363383(2x^2-x+3)^{3/2}}{72(2x+5)^2}\right)+\frac{593771(2x^2-x+3)^{3/2}}{108(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{3/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(110481408\int \frac{1}{\sqrt{2x^2-x+3}}dx-662940871\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{9363}{}\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 1090

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-662940871\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{}}{2(2x+5)}\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 222

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-662940871\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{93}{}\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 1154

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(1325881742\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x}}{2(2x+5)}\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

$\downarrow$ 219

$$\frac{\frac{1}{72}\left(-\frac{7}{144}\left(\frac{1}{4}\left(55240704\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{662940871\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{(9616196x+52836655)\sqrt{2x^2-x+3}}{2(2x+5)}\right)\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2304(2x+5)^4}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(2304*(5 + 2*x)^4) + ((593771*(3 - x + 2*x^2
)^(3/2))/(108*(5 + 2*x)^3) + ((-9363383*(3 - x + 2*x^2)^(3/2))/(72*(5 + 2*
x)^2) - (7*(-1/2*((52836655 + 9616196*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) +
(55240704*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (662940871*ArcTanh[(17 -
22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4))/144)/72)/4608
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \frac{4640219136\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
fricas")
```

output
```
1/4586471424*(4640219136*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 464
0586097*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(
2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20
*x + 25)) + 48*(238878720*x^4 + 6105343976*x^3 + 31323229164*x^2 + 6284786
7486*x + 44676885233)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1
000*x + 625)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
5, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= -\frac{259}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{4640586097}{2293235712}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{16828343}{47775744}\sqrt{2\,x^2-x+3} - \frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} + \frac{593771\,(2\,x^2-x+3)^{\frac{3}{2}}}{497664\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{9363383\,(2\,x^2-x+3)^{\frac{3}{2}}}{23887872\,(4\,x^2+20\,x+25)} + \frac{201573155\sqrt{2\,x^2-x+3}}{95551488\,(2\,x+5)}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
maxima")
```

output
```
-259/128*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 4640586097/229
3235712*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs
(2*x + 5)) + 16828343/47775744*sqrt(2*x^2 - x + 3) - 3667/2304*(2*x^2 - x
+ 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 593771/497664*(2*
x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 9363383/23887872*(2*x^
2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) + 201573155/95551488*sqrt(2*x^2 - x +
 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(134) = 268.

Time = 0.22 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.98

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx =$$

$$-\frac{1}{2293235712}\sqrt{2}\left(4640586097\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)\operatorname{sgn}\left(\frac{1}{2x+5}\right)\right.$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
giac")
```

output
```
-1/2293235712*sqrt(2)*(4640586097*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)
^2 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 4640219136*log(abs(sqrt(-1
1/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 4
640219136*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) -
 1))*sgn(1/(2*x + 5)) + 12*(24*(144*(792072*sgn(1/(2*x + 5))/(2*x + 5) - 8
35793*sgn(1/(2*x + 5)))/(2*x + 5) + 57384361*sgn(1/(2*x + 5)))/(2*x + 5) -
 464569597*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 17
9159040*(11*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1
/(2*x + 5)) - 12*sgn(1/(2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
 1) + 6/(2*x + 5))^2 - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.60

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$$
$$= \frac{5733089280\sqrt{2x^2-x+3}\,x^4+146528255424\sqrt{2x^2-x+3}\,x^3+751757499936\sqrt{2x^2-x+3}\,x^2+1508}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

output
```
(5733089280*sqrt(2*x**2 - x + 3)*x**4 + 146528255424*sqrt(2*x**2 - x + 3)*
x**3 + 751757499936*sqrt(2*x**2 - x + 3)*x**2 + 1508348819664*sqrt(2*x**2
- x + 3)*x + 1072245245592*sqrt(2*x**2 - x + 3) + 74249377552*sqrt(2)*log(
12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 742493775520*sqrt(2)*l
og(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 2784351658200*sqrt(
2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 4640586097000*s
qrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 2900366310625*
sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 74243506176*sqr
t(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 742435061760*sqr
t(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 2784131481600*sq
rt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 4640219136000*s
qrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 2900136960000*sqr
t(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 74249377552*sqrt(2)*l
og(2*x + 5)*x**4 - 742493775520*sqrt(2)*log(2*x + 5)*x**3 - 2784351658200*
sqrt(2)*log(2*x + 5)*x**2 - 4640586097000*sqrt(2)*log(2*x + 5)*x - 2900366
310625*sqrt(2)*log(2*x + 5))/(2293235712*(16*x**4 + 160*x**3 + 600*x**2 +
1000*x + 625))
```

3.166 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx$

Optimal result

Integrand size = 40, antiderivative size = 165

$$\begin{aligned}
&\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx \\
&= -\frac{(4583087983+3174439702x)\sqrt{3-x+2x^2}}{6879707136(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{2880(5+2x)^5} \\
&\quad + \frac{711961(3-x+2x^2)^{3/2}}{829440(5+2x)^4} - \frac{38732321(3-x+2x^2)^{3/2}}{179159040(5+2x)^3} \\
&\quad - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}} + \frac{12895597463\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{82556485632\sqrt{2}}
\end{aligned}$$

output

```
-1/6879707136*(4583087983+3174439702*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3667/2
880*(2*x^2-x+3)^(3/2)/(5+2*x)^5+711961/829440*(2*x^2-x+3)^(3/2)/(5+2*x)^4-
38732321/179159040*(2*x^2-x+3)^(3/2)/(5+2*x)^3-5/64*arcsinh(1/23*(1-4*x)*2
3^(1/2))*2^(1/2)+12895597463/165112971264*arctanh(1/24*(17-22*x)*2^(1/2)/(
2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{-\frac{12\sqrt{3-x+2x^2}(3110673952831+5608297138216x+3919478861832x^2+1285267446304x^3+186470433136x^4)}{(5+2x)^5} - 64477987315\sqrt{2}\text{arct}}{412782428160}$$

input

```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,
x]
```

output

```
((-12*Sqrt[3 - x + 2*x^2]*(3110673952831 + 5608297138216*x + 3919478861832
*x^2 + 1285267446304*x^3 + 186470433136*x^4))/(5 + 2*x)^5 - 64477987315*Sq
rt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 32248627200*Sqrt[2]*L
og[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/412782428160
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2181, 27, 2181, 2181, 27, 1229, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{360}\int \frac{\sqrt{2x^2-x+3}(-14400x^3+38880x^2-76504x+52701)}{16(2x+5)^5}dx - \frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-14400x^3+38880x^2-76504x+52701)}{(2x+5)^5}dx}{5760} - \frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{288}\int \frac{\sqrt{2x^2-x+3}(2073600x^2-7934876x+5935131)}{(2x+5)^4}dx+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{288}\left(-\frac{1}{216}\int \frac{15(9244801-14929920x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\int \frac{(9244801-14929920x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 1229$$

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{(3174439702x+4583087983)\sqrt{2x^2-x+3}}{288(2x+5)^2}-\frac{\int -\frac{2(2146055063-4299816960x)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}\right)-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\int \frac{2146055063-4299816960x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(3174439702x+4583087983)}{288(2x+5)^2}\right)-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2149908480\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+4583087983)}{288(2x+5)^2}\right)-\frac{38732321(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)+\frac{711961(2x^2-x+3)^{3/2}}{144(2x+5)^4}}{5760}-\frac{3667(2x^2-x+3)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 1090

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1074954240\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-x+3}(317\ldots}{28\ldots}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 222

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(12895597463\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+45830\ldots}{288(2x+5)^2}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 1154

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(-25791194926\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(31744397\ldots}{288(2x+\ldots}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

$\downarrow$ 219

$$\frac{\frac{1}{288}\left(-\frac{5}{72}\left(\frac{1}{576}\left(-1074954240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{12895597463\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(3174439702x+\ldots}{288(2x+5)^2}\right.\right.}{5760}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{2880(2x+5)^5}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(2880*(5 + 2*x)^5) + ((711961*(3 - x + 2*x^2
)^(3/2))/(144*(5 + 2*x)^4) + ((-38732321*(3 - x + 2*x^2)^(3/2))/(108*(5 +
2*x)^3) - (5*(((4583087983 + 3174439702*x)*Sqrt[3 - x + 2*x^2])/(288*(5 +
2*x)^2) + (-1074954240*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (12895597463
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/576))
/72)/288)/5760
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

output
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{64497254400\sqrt{2}(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
fricas")
```

output
```
1/1651129712640*(64497254400*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x
^2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 64477987315*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(186470433136*x^4 + 12852674
46304*x^3 + 3919478861832*x^2 + 5608297138216*x + 3110673952831)*sqrt(2*x^
2 - x + 3))/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
6, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= \frac{5}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) - \frac{12895597463}{165112971264}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{46569601}{3439853568}\sqrt{2\,x^2-x+3} - \frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{2880\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)} + \frac{711961\,(2\,x^2-x+3)^{\frac{3}{2}}}{829440\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} - \frac{38732321\,(2\,x^2-x+3)^{\frac{3}{2}}}{179159040\,(8\,x^3+60\,x^2+150\,x+125)} + \frac{46569601\,(2\,x^2-x+3)^{\frac{3}{2}}}{1719926784\,(4\,x^2+20\,x+25)} - \frac{562688629\sqrt{2\,x^2-x+3}}{6879707136\,(2\,x+5)}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
maxima")
```

output
```
5/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 12895597463/165112
971264*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) - 46569601/3439853568*sqrt(2*x^2 - x + 3) - 3667/2880*(2*x^2 - x
 + 3)^(3/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 711
961/829440*(2*x^2 - x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5) - 38732321/179159040*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 12
5) + 46569601/1719926784*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 56268
8629/6879707136*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(134) = 268$.

Time = 0.19 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$= -\frac{5}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{12895597463}{165112971264}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(4368922304720\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^9+124570969998480\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^8+6378\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
giac")
```

output
```
-5/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1289
5597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
 - x + 3))) - 12895597463/165112971264*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*s
qrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/68797071360*sqrt(2)*(4368922304720*sq
rt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 124570969998480*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^8 + 637804348664160*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3))^7 + 1828845222532320*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 3763189
300187016*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 10794416351958120*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 + 25049834283305880*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^3 - 34708488692384520*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3))^2 + 10654664764755165*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 25
07056315485767)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) - 11)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$=\int \frac{\sqrt{2\,x^2-x+3}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^6}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 503, normalized size of antiderivative = 3.05

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx$$
$$=\frac{-4475290395264\sqrt{2x^2-x+3}\,x^4-30846418711296\sqrt{2x^2-x+3}\,x^3-94067492683968\sqrt{2x^2-x+3}}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

output

```
( - 4475290395264*sqrt(2*x**2 - x + 3)*x**4 - 30846418711296*sqrt(2*x**2 -
 x + 3)*x**3 - 94067492683968*sqrt(2*x**2 - x + 3)*x**2 - 134599131317184*
sqrt(2*x**2 - x + 3)*x - 74656174867944*sqrt(2*x**2 - x + 3) + 20632955940
80*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**5 + 2579
1194926000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**
4 + 128955974630000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x
- 17)*x**3 + 322389936575000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2
) + 22*x - 17)*x**2 + 402987420718750*sqrt(2)*log( - 12*sqrt(2*x**2 - x +
3)*sqrt(2) + 22*x - 17)*x + 201493710359375*sqrt(2)*log( - 12*sqrt(2*x**2
- x + 3)*sqrt(2) + 22*x - 17) + 2063912140800*sqrt(2)*log( - 2*sqrt(2*x**2
 - x + 3)*sqrt(2) - 4*x + 1)*x**5 + 25798901760000*sqrt(2)*log( - 2*sqrt(2
*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 128994508800000*sqrt(2)*log( - 2*
sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 322486272000000*sqrt(2)*log
( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 403107840000000*sqrt(
2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 201553920000000*sq
rt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 2063295594080*sqr
t(2)*log(2*x + 5)*x**5 - 25791194926000*sqrt(2)*log(2*x + 5)*x**4 - 128955
974630000*sqrt(2)*log(2*x + 5)*x**3 - 322389936575000*sqrt(2)*log(2*x + 5)
*x**2 - 402987420718750*sqrt(2)*log(2*x + 5)*x - 201493710359375*sqrt(2)*l
og(2*x + 5))/(825564856320*(32*x**5 + 400*x**4 + 2000*x**3 + 5000*x**2 ...
```

3.167 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx$

Optimal result

Integrand size = 40, antiderivative size = 169

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7} dx &= -\frac{1172725(17-22x)\sqrt{3-x+2x^2}}{330225942528(5+2x)^2} - \frac{3667(3-x+2x^2)^{3/2}}{3456(5+2x)^6} \\ &\quad + \frac{92239(3-x+2x^2)^{3/2}}{138240(5+2x)^5} - \frac{5703277(3-x+2x^2)^{3/2}}{39813120(5+2x)^4} \\ &\quad + \frac{87677717(3-x+2x^2)^{3/2}}{8599633920(5+2x)^3} - \frac{26972675\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{3962711310336\sqrt{2}}\end{aligned}$$

output
```
-1172725/330225942528*(17-22*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2-3667/3456*(2*x
^2-x+3)^(3/2)/(5+2*x)^6+92239/138240*(2*x^2-x+3)^(3/2)/(5+2*x)^5-5703277/3
9813120*(2*x^2-x+3)^(3/2)/(5+2*x)^4+87677717/8599633920*(2*x^2-x+3)^(3/2)/
(5+2*x)^3-26972675/7925422620672*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3
)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-219337079305+27245373694x+158340720344x^2+397498825328x^3+12256250416x^4+271409942624x^5)}{(5+2x)^6} + 134863375\sqrt{}}{19813556551680}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-219337079305 + 27245373694*x + 158340720344*x^2
 + 397498825328*x^3 + 12256250416*x^4 + 271409942624*x^5))/(5 + 2*x)^6 + 1
34863375*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/19813556551
680
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2181, 27, 2181, 27, 2181, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{432}\int \frac{3\sqrt{2x^2-x+3}(-5760x^3+15552x^2-27668x+20347)}{16(2x+5)^6}dx-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-5760x^3+15552x^2-27668x+20347)}{(2x+5)^6}dx}{2304}-\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{360}\int \frac{3\sqrt{2x^2-x+3}\left(345600x^2-1059208x+895257\right)}{(2x+5)^5}dx+\frac{92239\left(2x^2-x+3\right)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{120}\int \frac{\sqrt{2x^2-x+3}\left(345600x^2-1059208x+895257\right)}{(2x+5)^5}dx+\frac{92239\left(2x^2-x+3\right)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{120}\left(-\frac{1}{288}\int \frac{(20294487-26953292x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{144(2x+5)^4}\right)+\frac{92239\left(2x^2-x+3\right)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\int \frac{\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{87677717\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{144(2x+5)^4}\right)+\frac{92239\left(2x^2-x+3\right)^{3/2}}{60(2x+5)^5}}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(\frac{23}{576}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{(17-22x)\sqrt{2x^2-x+3}}{288(2x+5)^2}\right)+\frac{87677717\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277\left(2x^2-x+3\right)^{3/2}}{144(2x+5)^4}\right)\cdots}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(-\frac{23}{288}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{87677717\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277\left(2x^2-x\cdots\right.}{144(2x+5\cdots}\right.}{2304}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{120}\left(\frac{1}{288}\left(\frac{5863625}{72}\left(-\frac{23\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3456\sqrt{2}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{87677717(2x^2-x+3)^{3/2}}{108(2x+5)^3}\right)-\frac{5703277(2x^2-x+3)}{144(2x+5)^4}\right.}{2304}$$

$$\frac{3667(2x^2-x+3)^{3/2}}{3456(2x+5)^6}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(3456*(5 + 2*x)^6) + ((92239*(3 - x + 2*x^2)
^(3/2))/(60*(5 + 2*x)^5) + ((-5703277*(3 - x + 2*x^2)^(3/2))/(144*(5 + 2*x
)^4) + ((87677717*(3 - x + 2*x^2)^(3/2))/(108*(5 + 2*x)^3) + (5863625*(-1/
288*((17 - 22*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 - (23*ArcTanh[(17 - 22*x
)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(3456*Sqrt[2])))/72)/288)/120)/2304
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.52

method	result
risch	$\frac{542819885248x^7-246897441792x^6+1596971228112x^5-44048633392x^4+1088646503028x^3+9102628728x^2+301073200387x-658}{1651129712640(5+2x)^6\sqrt{2x^2-x+3}}$
trager	$\frac{(271409942624x^5+12256250416x^4+397498825328x^3+158340720344x^2+27245373694x-219337079305)\sqrt{2x^2-x+3}}{1651129712640(5+2x)^6} - \frac{2697267}{}$
default	$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{221184\left(x+\frac{5}{2}\right)^6}+\frac{92239\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{4423680\left(x+\frac{5}{2}\right)^5}-\frac{5703277\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{637009920\left(x+\frac{5}{2}\right)^4}+\frac{87677717\left(2\left(x+\frac{5}{2}\right)^2-\right.}{68797071360\left(x+\right.}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x,method=_RETURNVERB
OSE)
```

output
```
1/1651129712640*(542819885248*x^7-246897441792*x^6+1596971228112*x^5-44048
633392*x^4+1088646503028*x^3+9102628728*x^2+301073200387*x-658011237915)/(
5+2*x)^6/(2*x^2-x+3)^(1/2)-26972675/7925422620672*2^(1/2)*arctanh(1/12*(17
/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{134863375\,\sqrt{2}(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}}{}\right.}{79254226206720}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
fricas")
```

output
```
1/79254226206720*(134863375*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x
^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22
*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(2714099426
24*x^5 + 12256250416*x^4 + 397498825328*x^3 + 158340720344*x^2 + 272453736
94*x - 219337079305)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 2
0000*x^3 + 37500*x^2 + 37500*x + 15625)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
7, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx &= \frac{26972675}{7925422620672}\sqrt{2}\,\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\ &+\frac{1172725}{165112971264}\sqrt{2\,x^2-x+3}\\ &-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{3456\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}\\ &+\frac{92239\,(2\,x^2-x+3)^{\frac{3}{2}}}{138240\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}\\ &-\frac{5703277\,(2\,x^2-x+3)^{\frac{3}{2}}}{39813120\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}\\ &+\frac{87677717\,(2\,x^2-x+3)^{\frac{3}{2}}}{8599633920\,(8\,x^3+60\,x^2+150\,x+125)}\\ &-\frac{1172725\,(2\,x^2-x+3)^{\frac{3}{2}}}{82556485632\,(4\,x^2+20\,x+25)}-\frac{12899975\sqrt{2\,x^2-x+3}}{330225942528\,(2\,x+5)}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
maxima")
```

output
```
26972675/7925422620672*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) + 1172725/165112971264*sqrt(2*x^2 - x + 3) - 366
7/3456*(2*x^2 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37
500*x^2 + 37500*x + 15625) + 92239/138240*(2*x^2 - x + 3)^(3/2)/(32*x^5 +
400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) - 5703277/39813120*(2*x^2 -
 x + 3)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 87677717/85996
33920*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 1172725/82556
485632*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 12899975/330225942528*s
qrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 405 vs. $2(139) = 278$.

Time = 0.20 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.40

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$$
$$= -\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{26972675}{7925422620672}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\left(16506981498400\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11}+389429252643040\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10}+2\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
giac")
```

output
```
-26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*
x^2 - x + 3))) + 26972675/7925422620672*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*
sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 1/3302259425280*sqrt(2)*(1650698149840
0*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^11 + 389429252643040*(sqrt(2)*
x - sqrt(2*x^2 - x + 3))^10 + 226392918689840*sqrt(2)*(sqrt(2)*x - sqrt(2
*x^2 - x + 3))^9 + 11663651054548560*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 +
 902212326134736*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 84192729519
861840*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 4317200555009448*sqrt(2)*(sqr
t(2)*x - sqrt(2*x^2 - x + 3))^5 + 351543414066518760*(sqrt(2)*x - sqrt(2*x
^2 - x + 3))^4 - 376787166452923830*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^3 + 356306707647610982*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 823483531
28195465*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 15499394004553969)/(2
*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
- x + 3)) - 11)^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^7}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{6513838622976\sqrt{2x^2-x+3}\,x^5+294150009984\sqrt{2x^2-x+3}\,x^4+9539971807872\sqrt{2x^2-x+3}\,x^3+\cdots}{\cdots}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

output
```
(6513838622976*sqrt(2*x**2 - x + 3)*x**5 + 294150009984*sqrt(2*x**2 - x +
3)*x**4 + 9539971807872*sqrt(2*x**2 - x + 3)*x**3 + 3800177288256*sqrt(2*x
**2 - x + 3)*x**2 + 653888968656*sqrt(2*x**2 - x + 3)*x - 5264089903320*sq
rt(2*x**2 - x + 3) + 8631256000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2
) + 22*x - 17)*x**6 + 129468840000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqr
t(2) + 22*x - 17)*x**5 + 809180250000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*
sqrt(2) + 22*x - 17)*x**4 + 2697267500000*sqrt(2)*log(12*sqrt(2*x**2 - x +
 3)*sqrt(2) + 22*x - 17)*x**3 + 5057376562500*sqrt(2)*log(12*sqrt(2*x**2 -
 x + 3)*sqrt(2) + 22*x - 17)*x**2 + 5057376562500*sqrt(2)*log(12*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 22*x - 17)*x + 2107240234375*sqrt(2)*log(12*sqrt(2*x
**2 - x + 3)*sqrt(2) + 22*x - 17) - 8631256000*sqrt(2)*log(2*x + 5)*x**6 -
 129468840000*sqrt(2)*log(2*x + 5)*x**5 - 809180250000*sqrt(2)*log(2*x + 5
)*x**4 - 2697267500000*sqrt(2)*log(2*x + 5)*x**3 - 5057376562500*sqrt(2)*l
og(2*x + 5)*x**2 - 5057376562500*sqrt(2)*log(2*x + 5)*x - 2107240234375*sq
rt(2)*log(2*x + 5))/(39627113103360*(64*x**6 + 960*x**5 + 6000*x**4 + 2000
0*x**3 + 37500*x**2 + 37500*x + 15625))
```

3.168 $\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 194

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx &= \frac{5679449533\sqrt{3-x+2x^2}}{6934744793088(5+2x)^2}+\frac{24599098079\sqrt{3-x+2x^2}}{166433875034112(5+2x)}\\ &\quad-\frac{(982491889+2070438154x)\sqrt{3-x+2x^2}}{48157949952(5+2x)^4}\\ &\quad-\frac{3667(3-x+2x^2)^{3/2}}{4032(5+2x)^7}+\frac{948341(3-x+2x^2)^{3/2}}{1741824(5+2x)^6}\\ &\quad-\frac{1464037(3-x+2x^2)^{3/2}}{13934592(5+2x)^5}-\frac{289071245\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{285315214344192\sqrt{2}}\end{aligned}$$

output

```
5679449533/6934744793088*(2*x^2-x+3)^(1/2)/(5+2*x)^2+24599098079*(2*x^2-x+
3)^(1/2)/(832169375170560+332867750068224*x)-1/48157949952*(982491889+2070
438154*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^4-3667/4032*(2*x^2-x+3)^(3/2)/(5+2*x)^
7+948341/1741824*(2*x^2-x+3)^(3/2)/(5+2*x)^6-1464037/13934592*(2*x^2-x+3)^
(3/2)/(5+2*x)^5-289071245/570630428688384*arctanh(1/24*(17-22*x)*2^(1/2)/(
2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{\frac{12\sqrt{3-x+2x^2}(-20465234808721+590492177460x+14716683780036x^2+41058010262368x^3+4982916071952x^4+27976951397184x^5+1574342}{(5+2x)^7}}{1997206500409344}$$

input
```
Integrate[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-20465234808721 + 590492177460*x + 1471668378003
6*x^2 + 41058010262368*x^3 + 4982916071952*x^4 + 27976951397184*x^5 + 1574
342277056*x^6))/(5 + 2*x)^7 + 2023498715*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6
 - 2*x + 4*x^2])/6])/1997206500409344
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1237, 25, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2-x+3}\left(5x^4-x^3+3x^2+x+2\right)}{(2x+5)^8}\,dx$$
$$\Big\downarrow \; 2181$$
$$-\frac{1}{504}\int \frac{\sqrt{2x^2-x+3}\left(-20160x^3+54432x^2-89504x+69381\right)}{16(2x+5)^7}dx-\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$
$$\Big\downarrow \; 27$$
$$-\frac{\int \frac{\sqrt{2x^2-x+3}(-20160x^3+54432x^2-89504x+69381)}{(2x+5)^7}dx}{8064}-\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{432}\int \frac{15\sqrt{2x^2-x+3}(290304x^2-750908x+700441)}{(2x+5)^6}dx+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5}{144}\int \frac{\sqrt{2x^2-x+3}(290304x^2-750908x+700441)}{(2x+5)^6}dx+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{5}{144}\left(-\frac{1}{360}\int \frac{3(5149971-5705944x)\sqrt{2x^2-x+3}}{(2x+5)^5}dx-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{5}{144}\left(-\frac{1}{120}\int \frac{(5149971-5705944x)\sqrt{2x^2-x+3}}{(2x+5)^5}dx-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{1237}$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\int -\frac{(52011459-77659324x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx+\frac{19414831(2x^2-x+3)^{3/2}}{144(2x+5)^4}\right)-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{19414831(2x^2-x+3)^{3/2}}{144(2x+5)^4}-\frac{1}{288}\int \frac{(52011459-77659324x)\sqrt{2x^2-x+3}}{(2x+5)^4}dx\right)-\frac{1464037(2x^2-x+3)^{3/2}}{60(2x+5)^5}\right)+\frac{948341(2x^2-x+3)^{3/2}}{216(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\int \frac{\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{246159769\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831\left(2x^2-x+3\right)^{3/2}}{144(2x+5)^4}\right)-\frac{1464037\left(2x^2-x+3\right)^{3/2}}{60(2x+5)^5}\right)+}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1152$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(\frac{23}{576}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{(17-22x)\sqrt{2x^2-x+3}}{288(2x+5)^2}\right)+\frac{246159769\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831\left(2x^2-x+\right.}{144(2x+5)^4}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(-\frac{23}{288}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{246159769\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)+\frac{194148}{1}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{5}{144}\left(\frac{1}{120}\left(\frac{1}{288}\left(\frac{87978205}{72}\left(-\frac{23\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3456\sqrt{2}}-\frac{\sqrt{2x^2-x+3}(17-22x)}{288(2x+5)^2}\right)+\frac{246159769\left(2x^2-x+3\right)^{3/2}}{108(2x+5)^3}\right)+\frac{19414831(}{144(}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{3/2}}{4032(2x+5)^7}$$

input
```
Int[(Sqrt[3 - x + 2*x^2]*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(3/2))/(4032*(5 + 2*x)^7) + ((948341*(3 - x + 2*x^2
)^(3/2))/(216*(5 + 2*x)^6) + (5*((-1464037*(3 - x + 2*x^2)^(3/2))/(60*(5 +
 2*x)^5) + ((19414831*(3 - x + 2*x^2)^(3/2))/(144*(5 + 2*x)^4) + ((2461597
69*(3 - x + 2*x^2)^(3/2))/(108*(5 + 2*x)^3) + (87978205*(-1/288*((17 - 22*
x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 - (23*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*
Sqrt[3 - x + 2*x^2])])/(3456*Sqrt[2])))/72)/288)/120))/144)/8064
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.48

method	result
risch	$\frac{3148684554112x^8+54379560517312x^7-13288092422112x^6+161063958644336x^5+3324105513560x^4+109638331361988x^3+26290}{166433875034112(5+2x)^7\sqrt{2x^2-x+3}}$
trager	$\frac{(1574342277056x^6+27976951397184x^5+4982916071952x^4+41058010262368x^3+14716683780036x^2+590492177460x-20465234}{166433875034112(5+2x)^7}$
default	$-\frac{3667\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{516096\left(x+\frac{5}{2}\right)^7}+\frac{948341\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{111476736\left(x+\frac{5}{2}\right)^6}-\frac{1464037\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}{445906944\left(x+\frac{5}{2}\right)^5}+\frac{19414831\left(2\left(x+\frac{5}{2}\right)^2-\right.}{64210599936(x}$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x,method=_RETURNVERB
OSE)
```

output
```
1/166433875034112*(3148684554112*x^8+54379560517312*x^7-13288092422112*x^6
+161063958644336*x^5+3324105513560*x^4+109638331361988*x^3+2629089545206*x
^2+22236711341101*x-61395704426163)/(5+2*x)^7/(2*x^2-x+3)^(1/2)-289071245/
570630428688384*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x
-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{2023498715\,\sqrt{2}(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)\,\mathrm{l}}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
fricas")
```

output
```
1/7988826001637376*(2023498715*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 + 7
0000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-(24*sqrt(2)*sq
rt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x +
25)) + 48*(1574342277056*x^6 + 27976951397184*x^5 + 4982916071952*x^4 + 41
058010262368*x^3 + 14716683780036*x^2 + 590492177460*x - 20465234808721)*s
qrt(2*x^2 - x + 3))/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x
^3 + 262500*x^2 + 218750*x + 78125)
```

Sympy [F]

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \int \frac{\sqrt{2x^2-x+3}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

input
```
integrate((2*x**2-x+3)**(1/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

output
```
Integral(sqrt(2*x**2 - x + 3)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5)**
8, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{289071245}{570630428688384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{12568315}{11888133931008}\sqrt{2\,x^2-x+3}$$
$$-\frac{3667\,(2\,x^2-x+3)^{\frac{3}{2}}}{4032\,(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125)}$$
$$+\frac{948341\,(2\,x^2-x+3)^{\frac{3}{2}}}{1741824\,(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625)}$$
$$-\frac{1464037\,(2\,x^2-x+3)^{\frac{3}{2}}}{13934592\,(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125)}$$
$$+\frac{19414831\,(2\,x^2-x+3)^{\frac{3}{2}}}{4013162496\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)}$$
$$+\frac{246159769\,(2\,x^2-x+3)^{\frac{3}{2}}}{866843099136\,(8\,x^3+60\,x^2+150\,x+125)}$$
$$-\frac{12568315\,(2\,x^2-x+3)^{\frac{3}{2}}}{5944066965504\,(4\,x^2+20\,x+25)}-\frac{138251465\sqrt{2\,x^2-x+3}}{23776267862016\,(2\,x+5)}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
maxima")
```

output
```
289071245/570630428688384*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) -
17/23*sqrt(23)/abs(2*x + 5)) + 12568315/11888133931008*sqrt(2*x^2 - x + 3)
 - 3667/4032*(2*x^2 - x + 3)^(3/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000
*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125) + 948341/1741824*(2*x^2
 - x + 3)^(3/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 375
00*x + 15625) - 1464037/13934592*(2*x^2 - x + 3)^(3/2)/(32*x^5 + 400*x^4 +
 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 19414831/4013162496*(2*x^2 - x + 3
)^(3/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 246159769/8668430991
36*(2*x^2 - x + 3)^(3/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 12568315/5944066
965504*(2*x^2 - x + 3)^(3/2)/(4*x^2 + 20*x + 25) - 138251465/2377626786201
6*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(160) = 320$.

Time = 0.17 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.35

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= -\frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{289071245}{570630428688384}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(129503917760\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{13}-3320259746027840\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{12}-23\right.}{}$$

input
```
integrate((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
giac")
```

output
```
-289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) + 289071245/570630428688384*sqrt(2)*log(abs(-2*sqrt(2)*x
 - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/332867750068224*sqrt(2)*(12950
3917760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 - 3320259746027840*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^12 - 23966708071916736*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^11 - 186055342532355520*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3))^10 - 274256644494948976*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9
 + 796135370176031760*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 25315231391710
05408*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 - 4610393811900786336*(s
qrt(2)*x - sqrt(2*x^2 - x + 3))^6 - 7997126854300052364*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^5 + 30842713619423538868*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^4 - 21873571601855032556*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^3 + 16204706960604668100*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 319625459
3191113265*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 536799032216117911)
/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) - 11)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{\sqrt{2\,x^2-x+3}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^8}\,dx$$

input
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

output
```
int(((2*x^2 - x + 3)^(1/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.39

$$\int \frac{\sqrt{3-x+2x^2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$$
$$= \frac{37784214649344\sqrt{2x^2-x+3}\,x^6+671446833532416\sqrt{2x^2-x+3}\,x^5+119589985726848\sqrt{2x^2-x+}}{}$$

input
```
int((2*x^2-x+3)^(1/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

output
```
(37784214649344*sqrt(2*x**2 - x + 3)*x**6 + 671446833532416*sqrt(2*x**2 -
x + 3)*x**5 + 119589985726848*sqrt(2*x**2 - x + 3)*x**4 + 985392246296832*
sqrt(2*x**2 - x + 3)*x**3 + 353200410720864*sqrt(2*x**2 - x + 3)*x**2 + 14
171812259040*sqrt(2*x**2 - x + 3)*x - 491165635409304*sqrt(2*x**2 - x + 3)
 + 259007835520*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x
**7 + 4532637121600*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 1
7)*x**6 + 33994778412000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*
x - 17)*x**5 + 141644910050000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2)
 + 22*x - 17)*x**4 + 354112275125000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*s
qrt(2) + 22*x - 17)*x**3 + 531168412687500*sqrt(2)*log(12*sqrt(2*x**2 - x
+ 3)*sqrt(2) + 22*x - 17)*x**2 + 442640343906250*sqrt(2)*log(12*sqrt(2*x**
2 - x + 3)*sqrt(2) + 22*x - 17)*x + 158085837109375*sqrt(2)*log(12*sqrt(2*
x**2 - x + 3)*sqrt(2) + 22*x - 17) - 259007835520*sqrt(2)*log(2*x + 5)*x**
7 - 4532637121600*sqrt(2)*log(2*x + 5)*x**6 - 33994778412000*sqrt(2)*log(2
*x + 5)*x**5 - 141644910050000*sqrt(2)*log(2*x + 5)*x**4 - 354112275125000
*sqrt(2)*log(2*x + 5)*x**3 - 531168412687500*sqrt(2)*log(2*x + 5)*x**2 - 4
42640343906250*sqrt(2)*log(2*x + 5)*x - 158085837109375*sqrt(2)*log(2*x +
5))/(3994413000818688*(128*x**7 + 2240*x**6 + 16800*x**5 + 70000*x**4 + 17
5000*x**3 + 262500*x**2 + 218750*x + 78125))
```

3.169 $\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right)$

Optimal result

Integrand size = 38, antiderivative size = 166

$$\int (5+2x) \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx = -\frac{6398163(1-4x)\sqrt{3-x+2x^2}}{2097152} - \frac{92727(1-4x)\left(3-x+2x^2\right)^{3/2}}{131072} + \frac{69415(5+2x)^2\left(3-x+2x^2\right)^{5/2}}{32256} - \frac{1121(5+2x)^3\left(3-x+2x^2\right)^{5/2}}{2304} + \frac{5}{144}(5+2x)^4\left(3-x+2x^2\right)^{5/2} - \frac{3(661397+215900x)\left(3-x+2x^2\right)^{5/2}}{143360} - \frac{147157749\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4194304\sqrt{2}}$$

output

```
-6398163/2097152*(1-4*x)*(2*x^2-x+3)^(1/2)-92727/131072*(1-4*x)*(2*x^2-x+3
)^(3/2)+69415/32256*(5+2*x)^2*(2*x^2-x+3)^(5/2)-1121/2304*(5+2*x)^3*(2*x^2
-x+3)^(5/2)+5/144*(5+2*x)^4*(2*x^2-x+3)^(5/2)-3/143360*(661397+215900*x)*(
2*x^2-x+3)^(5/2)-147157749/8388608*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{4\sqrt{3-x+2x^2}(1592737263+12357760788x+4870637856x^2+12669290112x^3+379086848$$

input
```
Integrate[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(1592737263 + 12357760788*x + 4870637856*x^2 + 1266
9290112*x^3 + 379086848*x^4 + 12117893120*x^5 + 1033175040*x^6 + 292683776
0*x^7 + 1468006400*x^8) - 46354690935*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x
 + 4*x^2]])/2642411520
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {2184, 25, 2184, 27, 2184, 27, 1225, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\int -\left((2x+5)\left(2x^2-x+3\right)^{3/2}\left(8968x^3+15996x^2+11262x+2299\right)\right)dx+\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{144}(2x+5)^4\left(2x^2-x+3\right)^{5/2}-\frac{1}{288}\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(8968x^3+15996x^2+11262x+2299\right)dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\left(-\frac{1}{128}\int -8(2x+5)\left(2x^2-x+3\right)^{3/2}\left(277660x^2+281660x+24871\right)dx-\frac{1121}{8}\left(2x^2-x+3\right)^{5/2}(2x+5)\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{288}\left(\frac{1}{16}\int(2x+5)\left(2x^2-x+3\right)^{3/2}\left(277660x^2+281660x+24871\right)dx-\frac{1121}{8}(2x+5)^3\left(2x^2-x+3\right)^{5/2}\right)+$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{1}{56}\int 108(15467-64770x)(2x+5)\left(2x^2-x+3\right)^{3/2}dx+\frac{69415}{7}(2x+5)^2\left(2x^2-x+3\right)^{5/2}\right)-\frac{1121}{8}\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\int(15467-64770x)(2x+5)\left(2x^2-x+3\right)^{3/2}dx+\frac{69415}{7}(2x+5)^2\left(2x^2-x+3\right)^{5/2}\right)-\frac{1121}{8}(2x\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{1225}$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\int\left(2x^2-x+3\right)^{3/2}dx-\frac{1}{20}(215900x+661397)\left(2x^2-x+3\right)^{5/2}\right)+\frac{69415}{7}(2x+5)^2\left(2x\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\int\sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{1}{20}(215900x+661397)\left(2x^2\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{23}{16}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right.\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\Big\downarrow$ 1090

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x\right.\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

$\Big\downarrow$ 222

$$\frac{1}{288}\left(\frac{1}{16}\left(\frac{27}{14}\left(\frac{216363}{8}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)\right.\right.\right.\right.$$
$$\frac{5}{144}\left(2x^2-x+3\right)^{5/2}(2x+5)^4$$

input
```
Int[(5 + 2*x)*(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*(5 + 2*x)^4*(3 - x + 2*x^2)^(5/2))/144 + ((-1121*(5 + 2*x)^3*(3 - x + 2
*x^2)^(5/2))/8 + ((69415*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/7 + (27*(-1/20
*((661397 + 215900*x)*(3 - x + 2*x^2)^(5/2)) + (216363*(-1/16*((1 - 4*x)*(
3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x + 2*x^2]) + (23*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/8))/14)/16)/288
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(1468006400x^8+2926837760x^7+1033175040x^6+12117893120x^5+379086848x^4+12669290112x^3+4870637856x^2+12357760788x-}{660602880}$
trager	$\left(\frac{20}{9}x^8+\frac{319}{72}x^7+\frac{1051}{672}x^6+\frac{295847}{16128}x^5+\frac{26443}{46080}x^4+\frac{32992943}{1720320}x^3+\frac{2415991}{327680}x^2+\frac{343271133}{18350080}x+\frac{176970807}{73400320}\right)\sqrt{}$
default	$\frac{2005x^2(2x^2-x+3)^{\frac{5}{2}}}{8064}+\frac{5645x(2x^2-x+3)^{\frac{5}{2}}}{21504}+\frac{120809(2x^2-x+3)^{\frac{5}{2}}}{143360}+\frac{92727(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{131072}+\frac{6398163\sqrt{2x^2-x+3}\,(4}{2097152}$

input
```
int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
1/660602880*(1468006400*x^8+2926837760*x^7+1033175040*x^6+12117893120*x^5+
379086848*x^4+12669290112*x^3+4870637856*x^2+12357760788*x+1592737263)*(2*
x^2-x+3)^(1/2)+147157749/8388608*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int(5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{660602880}\left(1468006400\,x^8+2926837760\,x^7+1033175040\,x^6+12117893120\,x^5+379086848\right.$$
$$+\frac{147157749}{16777216}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output
```
1/660602880*(1468006400*x^8 + 2926837760*x^7 + 1033175040*x^6 + 1211789312
0*x^5 + 379086848*x^4 + 12669290112*x^3 + 4870637856*x^2 + 12357760788*x +
 1592737263)*sqrt(2*x^2 - x + 3) + 147157749/16777216*sqrt(2)*log(-4*sqrt(
2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\sqrt{2x^2-x+3}\cdot\left(\frac{20x^8}{9}+\frac{319x^7}{72}+\frac{1051x^6}{672}+\frac{295847x^5}{16128}+\frac{26443x^4}{46080}+\frac{32992943x^3}{1720320}+\frac{2415991x^2}{327680}+\frac{343271133x}{18350080}+\frac{176970807}{73400320}\right)+\frac{147157749\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8388608}$$

input
```
integrate((5+2*x)*(2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(20*x**8/9 + 319*x**7/72 + 1051*x**6/672 + 295847*x**
5/16128 + 26443*x**4/46080 + 32992943*x**3/1720320 + 2415991*x**2/327680 +
 343271133*x/18350080 + 176970807/73400320) + 147157749*sqrt(2)*asinh(4*sq
rt(23)*(x - 1/4)/23)/8388608
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.93

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{5}{9}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{479}{288}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{2005}{8064}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{5645}{21504}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{120809}{143360}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{92727}{32768}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{92727}{131072}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{6398163}{524288}\sqrt{2\,x^2-x+3}x+\frac{147157749}{8388608}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{6398163}{2097152}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
5/9*(2*x^2 - x + 3)^(5/2)*x^4 + 479/288*(2*x^2 - x + 3)^(5/2)*x^3 + 2005/8
064*(2*x^2 - x + 3)^(5/2)*x^2 + 5645/21504*(2*x^2 - x + 3)^(5/2)*x + 12080
9/143360*(2*x^2 - x + 3)^(5/2) + 92727/32768*(2*x^2 - x + 3)^(3/2)*x - 927
27/131072*(2*x^2 - x + 3)^(3/2) + 6398163/524288*sqrt(2*x^2 - x + 3)*x + 1
47157749/8388608*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6398163/209715
2*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.53

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{660602880}(4(8(4(16(20(8(28(160x+319)x+3153)x+295847)x+185101)x+98978829$$
$$-\frac{147157749}{8388608}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
1/660602880*(4*(8*(4*(16*(20*(8*(28*(160*x + 319)*x + 3153)*x + 295847)*x
+ 185101)*x + 98978829)*x + 152207433)*x + 3089440197)*x + 1592737263)*sqr
t(2*x^2 - x + 3) - 147157749/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\int (2x+5)\left(2x^2-x+3\right)^{3/2}\left(5x^4-x^3+3x^2+x+2\right)\,dx$$

input
```
int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
int((2*x + 5)*(2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.02

$$\int (5+2x)\left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{20\sqrt{2x^2-x+3}\,x^8}{9}+\frac{319\sqrt{2x^2-x+3}\,x^7}{72}+\frac{1051\sqrt{2x^2-x+3}\,x^6}{672}+\frac{295847\sqrt{2x^2-x+3}\,x^5}{16128}+\frac{26443\sqrt{2x^2-x+3}\,x^4}{46080}+\frac{32992943\sqrt{2x^2-x+3}\,x^3}{1720320}+\frac{2415991\sqrt{2x^2-x+3}\,x^2}{327680}+\frac{343271133\sqrt{2x^2-x+3}\,x}{18350080}+\frac{176970807\sqrt{2x^2-x+3}}{73400320}+\frac{147157749\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{8388608}$$

input
```
int((5+2*x)*(2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

output
```
(5872025600*sqrt(2*x**2 - x + 3)*x**8 + 11707351040*sqrt(2*x**2 - x + 3)*x
**7 + 4132700160*sqrt(2*x**2 - x + 3)*x**6 + 48471572480*sqrt(2*x**2 - x +
 3)*x**5 + 1516347392*sqrt(2*x**2 - x + 3)*x**4 + 50677160448*sqrt(2*x**2
- x + 3)*x**3 + 19482551424*sqrt(2*x**2 - x + 3)*x**2 + 49431043152*sqrt(2
*x**2 - x + 3)*x + 6370949052*sqrt(2*x**2 - x + 3) + 46354690935*sqrt(2)*l
og((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/2642411520
```

3.170 $\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)\, dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)\, dx = -\frac{593193(1-4x)\sqrt{3-x+2x^2}}{1048576} - \frac{8597(1-4x)(3-x+2x^2)^{3/2}}{65536} + \frac{1167(3-x+2x^2)^{5/2}}{14336} + \frac{125x(3-x+2x^2)^{5/2}}{3584} + \frac{23}{448}x^2(3-x+2x^2)^{5/2} + \frac{5}{16}x^3(3-x+2x^2)^{5/2} - \frac{13643439\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2097152\sqrt{2}}$$

output
```
-593193/1048576*(1-4*x)*(2*x^2-x+3)^(1/2)-8597/65536*(1-4*x)*(2*x^2-x+3)^(
3/2)+1167/14336*(2*x^2-x+3)^(5/2)+125/3584*x*(2*x^2-x+3)^(5/2)+23/448*x^2*
(2*x^2-x+3)^(5/2)+5/16*x^3*(2*x^2-x+3)^(5/2)-13643439/4194304*arcsinh(1/23
*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int (3-x+2x^2)^{3/2} (2+x+3x^2-x^3 +5x^4) \, dx = \frac{4\sqrt{3-x+2x^2}(-1663407+27845612x+3845856x^2+27023744x^3-7497728x^4+29335552}{29360128}$$

input
```
Integrate[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1663407 + 27845612*x + 3845856*x^2 + 27023744*x^3
 - 7497728*x^4 + 29335552*x^5 - 7667712*x^6 + 9175040*x^7) - 95504073*Sqrt
[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/29360128
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x^2-x+3)^{3/2} (5x^4-x^3+3x^2+x+2) \, dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{16}\int \frac{1}{2}(2x^2-x+3)^{3/2} (23x^3+6x^2+32x+64) dx+\frac{5}{16}(2x^2-x+3)^{5/2} x^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{32}\int (2x^2-x+3)^{3/2} (23x^3+6x^2+32x+64) dx+\frac{5}{16}(2x^2-x+3)^{5/2} x^3$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{32}\left(\frac{1}{14}\int \frac{1}{2}\left(2x^2-x+3\right)^{3/2}\left(375x^2+620x+1792\right)dx+\frac{23}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{28}\int \left(2x^2-x+3\right)^{3/2}\left(375x^2+620x+1792\right)dx+\frac{23}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 2192

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{12}\int \frac{3}{2}(5835x+13586)\left(2x^2-x+3\right)^{3/2}dx+\frac{125}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\int (5835x+13586)\left(2x^2-x+3\right)^{3/2}dx+\frac{125}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14}x^2\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 1160

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\int \left(2x^2-x+3\right)^{3/2}dx+\frac{1167}{2}\left(2x^2-x+3\right)^{5/2}\right)+\frac{125}{4}x\left(2x^2-x+3\right)^{5/2}\right)+\frac{23}{14}x^2\left(2x^2-x\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 1087

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\int \sqrt{2x^2-x+3}dx-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{1167}{2}\left(2x^2-x+3\right)^{5/2}\right)+\frac{125}{4}x\left(\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 1087

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{23}{16}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{1}{16}\sqrt{\frac{23}{2}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\,(2\right.\right.\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{32}\left(\frac{1}{28}\left(\frac{1}{8}\left(\frac{60179}{4}\left(\frac{69}{32}\left(\frac{23\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{2}}-\frac{1}{8}(1-4x)\sqrt{2x^2-x+3}\right)-\frac{1}{16}(1-4x)\left(2x^2-x+3\right)^{3/2}\right)+\frac{11}{}\right.\right.\right.$$
$$\frac{5}{16}\left(2x^2-x+3\right)^{5/2}x^3$$

input
```
Int[(3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4),x]
```

output
```
(5*x^3*(3 - x + 2*x^2)^(5/2))/16 + ((23*x^2*(3 - x + 2*x^2)^(5/2))/14 + ((
125*x*(3 - x + 2*x^2)^(5/2))/4 + ((1167*(3 - x + 2*x^2)^(5/2))/2 + (60179*
(-1/16*((1 - 4*x)*(3 - x + 2*x^2)^(3/2)) + (69*(-1/8*((1 - 4*x)*Sqrt[3 - x
 + 2*x^2]) + (23*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt[2])))/32))/4)/8)/2
8)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$\frac{(9175040x^7-7667712x^6+29335552x^5-7497728x^4+27023744x^3+3845856x^2+27845612x-1663407)\sqrt{2x^2-x+3}}{7340032}+\frac{13643439\sqrt{2}}{}$
trager	$\left(\frac{5}{4}x^7-\frac{117}{112}x^6+\frac{3581}{896}x^5-\frac{523}{512}x^4+\frac{211123}{57344}x^3+\frac{17169}{32768}x^2+\frac{6961403}{1835008}x-\frac{1663407}{7340032}\right)\sqrt{2x^2-x+3}+\frac{13643}{}$
default	$\frac{1167(2x^2-x+3)^{\frac{5}{2}}}{14336}+\frac{8597(4x-1)(2x^2-x+3)^{\frac{3}{2}}}{65536}+\frac{593193\sqrt{2x^2-x+3}\,(4x-1)}{1048576}+\frac{13643439\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}+\frac{125}{}$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x,method=_RETURNVERBOSE)
```

output
```
1/7340032*(9175040*x^7-7667712*x^6+29335552*x^5-7497728*x^4+27023744*x^3+3
845856*x^2+27845612*x-1663407)*(2*x^2-x+3)^(1/2)+13643439/4194304*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.60

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right)\, dx = \frac{1}{7340032} \left(9175040\, x^7 - 7667712\, x^6 + 29335552\, x^5 - 7497728\, x^4 + 27023744\, x^3 + 3845856\, x^2 \right. + \frac{13643439}{8388608} \sqrt{2} \log \left(-4\sqrt{2}\sqrt{2\, x^2 - x + 3}(4\, x - 1) - 32\, x^2 + 16\, x - 25\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="fricas")
```

output
```
1/7340032*(9175040*x^7 - 7667712*x^6 + 29335552*x^5 - 7497728*x^4 + 270237
44*x^3 + 3845856*x^2 + 27845612*x - 1663407)*sqrt(2*x^2 - x + 3) + 1364343
9/8388608*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2} \left(2+x+3x^2-x^3+5x^4\right) dx = \sqrt{2x^2-x+3} \cdot \left(\frac{5x^7}{4} - \frac{117x^6}{112} + \frac{3581x^5}{896} - \frac{523x^4}{512} + \frac{211123x^3}{57344} + \frac{17169x^2}{32768} + \frac{6961403x}{1835008} - \frac{1663407}{7340032}\right) + \frac{13643439\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4194304}$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**7/4 - 117*x**6/112 + 3581*x**5/896 - 523*x**4/5
12 + 211123*x**3/57344 + 17169*x**2/32768 + 6961403*x/1835008 - 1663407/73
40032) + 13643439*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4194304
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{5}{16}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{23}{448}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{125}{3584}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x+\frac{1167}{14336}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{8597}{16384}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x-\frac{8597}{65536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{593193}{262144}\sqrt{2\,x^2-x+3}x+\frac{13643439}{4194304}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{593193}{1048576}\sqrt{2\,x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="maxima")
```

output
```
5/16*(2*x^2 - x + 3)^(5/2)*x^3 + 23/448*(2*x^2 - x + 3)^(5/2)*x^2 + 125/35
84*(2*x^2 - x + 3)^(5/2)*x + 1167/14336*(2*x^2 - x + 3)^(5/2) + 8597/16384
*(2*x^2 - x + 3)^(3/2)*x - 8597/65536*(2*x^2 - x + 3)^(3/2) + 593193/26214
4*sqrt(2*x^2 - x + 3)*x + 13643439/4194304*sqrt(2)*arcsinh(1/23*sqrt(23)*(
4*x - 1)) - 593193/1048576*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.56

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+x+3x^2-x^3+5x^4\right)\,dx=\frac{1}{7340032}\left(4\left(8\left(4\left(16\left(4\left(8\left(140\,x-117\right)x+3581\right)x-3661\right)x+211123\right)x+120183\right)x+696140\right.\right.$$
$$-\frac{13643439}{4194304}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x, algorithm="giac")
```

output
```
1/7340032*(4*(8*(4*(16*(4*(8*(140*x - 117)*x + 3581)*x - 3661)*x + 211123)
*x + 120183)*x + 6961403)*x - 1663407)*sqrt(2*x^2 - x + 3) - 13643439/4194
304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int (3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)\,dx=\int (2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)\,dx$$

input
```
int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2),x)
```

output
```
int((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int (3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)\,dx=\frac{5\sqrt{2x^2-x+3}\,x^7}{4}-\frac{117\sqrt{2x^2-x+3}\,x^6}{112}+\frac{3581\sqrt{2x^2-x+3}\,x^5}{896}-\frac{523\sqrt{2x^2-x+3}\,x^4}{512}+\frac{211123\sqrt{2x^2-x+3}\,x^3}{57344}+\frac{17169\sqrt{2x^2-x+3}\,x^2}{32768}+\frac{6961403\sqrt{2x^2-x+3}\,x}{1835008}-\frac{1663407\sqrt{2x^2-x+3}}{7340032}+\frac{13643439\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{4194304}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2),x)
```

output
```
(36700160*sqrt(2*x**2 - x + 3)*x**7 - 30670848*sqrt(2*x**2 - x + 3)*x**6 +
 117342208*sqrt(2*x**2 - x + 3)*x**5 - 29990912*sqrt(2*x**2 - x + 3)*x**4
+ 108094976*sqrt(2*x**2 - x + 3)*x**3 + 15383424*sqrt(2*x**2 - x + 3)*x**2
 + 111382448*sqrt(2*x**2 - x + 3)*x - 6653628*sqrt(2*x**2 - x + 3) + 95504
073*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/2936
0128
```

3.171 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{(141051019-23482924x)\sqrt{3-x+2x^2}}{65536} + \frac{(500141-123060x)(3-x+2x^2)^{3/2}}{12288} + \frac{3505}{896}(3-x+2x^2)^{5/2} - \frac{311}{448}(5+2x)(3-x+2x^2)^{5/2} + \frac{5}{112}(5+2x)^2(3-x+2x^2)^{5/2} + \frac{1622009981\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{131072\sqrt{2}} - \frac{99009\text{arctanh}\left(\right.}{8\sqrt{}}$$

output
```
1/65536*(141051019-23482924*x)*(2*x^2-x+3)^(1/2)+1/12288*(500141-123060*x)
*(2*x^2-x+3)^(3/2)+3505/896*(2*x^2-x+3)^(5/2)-311/448*(5+2*x)*(2*x^2-x+3)^
(5/2)+5/112*(5+2*x)^2*(2*x^2-x+3)^(5/2)+1622009981/262144*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)-99009/16*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)
^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{4\sqrt{3-x+2x^2}(3149403255-609499532x+15997340$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),
x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(3149403255 - 609499532*x + 159973408*x^2 - 4647667
2*x^3 + 14493696*x^4 - 3710976*x^5 + 983040*x^6) + 68130865152*Sqrt[2]*Arc
Tanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 34062209601*Sqrt[2]*Log[1 - 4*
x + 2*Sqrt[6 - 2*x + 4*x^2]])/5505024
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2184, 2184, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{2x+5}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\int \frac{(2x^2-x+3)^{3/2}(-7464x^3-14508x^2-9926x+573)}{2x+5}dx+\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\left(\frac{1}{96}\int -\frac{24(-70100x^2-85940x+17923)(2x^2-x+3)^{3/2}}{2x+5}dx-\frac{311}{2}(2x+5)(2x^2-x+3)^{5/2}\right)+\frac{5}{112}(2x+5)^2(2x^2-x+3)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(-\frac{1}{4}\int \frac{\left(-70100x^2-85940x+17923\right)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx-\frac{311}{2}(2x+5)\left(2x^2-x+3\right)^{5/2}\right)+\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(3505\left(2x^2-x+3\right)^{5/2}-\frac{1}{40}\int -\frac{140(7397-20510x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx\right)-\frac{311}{2}(2x+5)\left(2x^2-x+3\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\int \frac{(7397-20510x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+3505\left(2x^2-x+3\right)^{5/2}\right)-\frac{311}{2}(2x+5)\left(2x^2-x+3\right)^{5/2}\right)$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{48}(500141-123060x)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int -\frac{2(4441417-11741462x)\sqrt{2x^2-x+3}}{2x+5}dx\right)+35\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\int \frac{(4441417-11741462x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1}{48}(500141-123060x)\left(2x^2-x+3\right)^{3/2}\right)+3505\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{1231}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{8}(141051019-23482924x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(1622930831-3244019962x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1}{48}\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\int \frac{1622930831-3244019962x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(141051019-23482924x)\right)+\frac{1}{48}(5001\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1269

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1622009981\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1090

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{1622009981\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 222

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(9732980736\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 1154

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(-19465961472\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

$\downarrow$ 219

$$\frac{1}{224}\left(\frac{1}{4}\left(\frac{7}{2}\left(\frac{1}{32}\left(\frac{1}{16}\left(-\frac{1622009981\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-811081728\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2\right.\right.\right.\right.$$
$$\frac{5}{112}(2x+5)^2\left(2x^2-x+3\right)^{5/2}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x),x]
```

output
```
(5*(5 + 2*x)^2*(3 - x + 2*x^2)^(5/2))/112 + ((-311*(5 + 2*x)*(3 - x + 2*x^
2)^(5/2))/2 + (3505*(3 - x + 2*x^2)^(5/2) + (7*(((500141 - 123060*x)*(3 -
x + 2*x^2)^(3/2))/48 + (((141051019 - 23482924*x)*Sqrt[3 - x + 2*x^2])/8 +
 ((-1622009981*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 811081728*Sqrt[2]*A
rcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16)/32))/2)/4)/224
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{1376256}\left(983040\,x^6 - 3710976\,x^5 + 14493696\,x^4 - 464\right.$$
$$+ \frac{1622009981}{524288}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1) - 32\,x^2 + 16\,x - 25\right)$$
$$+ \frac{99009}{32}\sqrt{2}\log\left(-\frac{24\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17) + 1060\,x^2 - 1036\,x + 1153}{4\,x^2+20\,x+25}\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="fr
icas")
```

output
```
1/1376256*(983040*x^6 - 3710976*x^5 + 14493696*x^4 - 46476672*x^3 + 159973
408*x^2 - 609499532*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/52428
8*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
) + 99009/32*sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 10
60*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{2x+5}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x),x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.91

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{5}{28}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2 - \frac{111}{224}\left(2x^2-x+3\right)^{\frac{5}{2}}x + \frac{1395}{896}\left(2x^2-x+3\right)^{\frac{5}{2}} - \frac{10255}{1024}\left(2x^2-x+3\right)^{\frac{3}{2}}x + \frac{500141}{12288}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{5870731}{16384}\sqrt{2x^2-x+3}x - \frac{1622009981}{262144}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{99009}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|} - \frac{17\sqrt{23}}{23|2x+5|}\right) + \frac{141051019}{65536}\sqrt{2x^2-x+3}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="ma
xima")
```

output
```
5/28*(2*x^2 - x + 3)^(5/2)*x^2 - 111/224*(2*x^2 - x + 3)^(5/2)*x + 1395/89
6*(2*x^2 - x + 3)^(5/2) - 10255/1024*(2*x^2 - x + 3)^(3/2)*x + 500141/1228
8*(2*x^2 - x + 3)^(3/2) - 5870731/16384*sqrt(2*x^2 - x + 3)*x - 1622009981
/262144*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 99009/16*sqrt(2
)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 1
41051019/65536*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx = \frac{1}{1376256}\left(4\left(8\left(12\left(16\left(4\left(40x-151\right)x+2359\right)x-1210\right.\right.\right.\right. + \frac{1622009981}{262144}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2x^2-x+3}\right) - \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{99009}{16}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x, algorithm="gi
ac")
```

output
```
1/1376256*(4*(8*(12*(16*(4*(40*x - 151)*x + 2359)*x - 121033)*x + 4999169)
*x - 152374883)*x + 3149403255)*sqrt(2*x^2 - x + 3) + 1622009981/262144*sq
rt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 99009/16*sqrt(
2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 99009/16*sqr
t(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx=\int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{2\,x+5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5),x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{5+2x}\,dx &= \frac{5\sqrt{2x^2-x+3}\,x^6}{7} \\ &-\frac{151\sqrt{2x^2-x+3}\,x^5}{56}+\frac{337\sqrt{2x^2-x+3}\,x^4}{32}-\frac{121033\sqrt{2x^2-x+3}\,x^3}{3584} \\ &+\frac{714167\sqrt{2x^2-x+3}\,x^2}{6144}-\frac{152374883\sqrt{2x^2-x+3}\,x}{344064} \\ &+\frac{1049801085\sqrt{2x^2-x+3}}{458752}+\frac{99009\sqrt{2}\log\left(\frac{46\sqrt{2x^2-x+3}\,\sqrt{2}+92x-46}{\sqrt{23}}\right)}{16} \\ &-\frac{1622009981\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{262144}-\frac{99009\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x+22}{\sqrt{23}}\right)}{16}\end{aligned}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x),x)
```

output
```
(3932160*sqrt(2*x**2 - x + 3)*x**6 - 14843904*sqrt(2*x**2 - x + 3)*x**5 +
57974784*sqrt(2*x**2 - x + 3)*x**4 - 185906688*sqrt(2*x**2 - x + 3)*x**3 +
 639893632*sqrt(2*x**2 - x + 3)*x**2 - 2437998128*sqrt(2*x**2 - x + 3)*x +
 12597613020*sqrt(2*x**2 - x + 3) + 34065432576*sqrt(2)*log((46*sqrt(2*x**
2 - x + 3)*sqrt(2) + 92*x - 46)/sqrt(23)) - 34062209601*sqrt(2)*log((2*sqr
t(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) - 34065432576*sqrt(2)*log((
2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23)))/5505024
```

3.172 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 172

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = -\frac{(85448933-14243732x)\sqrt{3-x+2x^2}}{32768}-\frac{(909513-226052x)(3-x+2x^2)^{3/2}}{18432} -\frac{839}{960}(3-x+2x^2)^{5/2}-\frac{3667(3-x+2x^2)^{5/2}}{576(5+2x)} +\frac{5}{96}(5+2x)(3-x+2x^2)^{5/2}-\frac{982669459\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{65536\sqrt{2}}+\frac{959625\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{64\sqrt{2}}$$

output

```
-1/32768*(85448933-14243732*x)*(2*x^2-x+3)^(1/2)-1/18432*(909513-226052*x)
*(2*x^2-x+3)^(3/2)-839/960*(2*x^2-x+3)^(5/2)-3667*(2*x^2-x+3)^(5/2)/(2880+
1152*x)+5/96*(5+2*x)*(2*x^2-x+3)^(5/2)-982669459/131072*arcsinh(1/23*(1-4*
x)*23^(1/2))*2^(1/2)+959625/128*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)
^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.70

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}(-6814208295-1404323114x+182033816x^2-35369408x^3+82}{5+2x}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
2,x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(-6814208295 - 1404323114*x + 182033816*x^2 - 3536
9408*x^3 + 8283904*x^4 - 1798144*x^5 + 409600*x^6))/(5 + 2*x) - 2947968000
0*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 14740041885*Sqrt[
2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/1966080
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 2184, 27, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-79840x+26675)}{16(2x+5)}dx-\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-79840x+26675)}{2x+5}dx}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{60(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{1}{96}\int \frac{96\left(2x^2-x+3\right)^{3/2}\left(20136x^2-67720x+24725\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 27

$$\frac{60(2x+5)\left(2x^2-x+3\right)^{5/2}-\int \frac{\left(2x^2-x+3\right)^{3/2}\left(20136x^2-67720x+24725\right)}{2x+5}dx}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 2184

$$\frac{-\frac{1}{40}\int \frac{80(18655-56513x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+60(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{5034}{5}\left(2x^2-x+3\right)^{5/2}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 27

$$\frac{-2\int \frac{(18655-56513x)\left(2x^2-x+3\right)^{3/2}}{2x+5}dx+60(2x+5)\left(2x^2-x+3\right)^{5/2}-\frac{5034}{5}\left(2x^2-x+3\right)^{5/2}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1231

$$\frac{-2\left(\frac{1}{32}(909513-226052x)\left(2x^2-x+3\right)^{3/2}-\frac{1}{64}\int -\frac{9(2667335-7121866x)\sqrt{2x^2-x+3}}{2x+5}dx\right)+60(2x+5)\left(2x^2-x+3\right.}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 27

$$\frac{-2\left(\frac{9}{64}\int \frac{(2667335-7121866x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{1}{32}(909513-226052x)\left(2x^2-x+3\right)^{3/2}\right)+60(2x+5)\left(2x^2-x+3\right)^{5/}}{1152}$$
$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1231

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{8}(85448933-14243732x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{2(982588705-1965338918x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{1}{32}(909513-226052x)\left(2\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 27

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\int \frac{982588705-1965338918x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-14243732x)\right)+\frac{1}{32}(909513-226052x)\left(2x^2\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1269

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-982669459\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-142437\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1090

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{982669459\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1\right.\right.}{11}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 222

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(5895936000\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{982669459\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-1424373\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 1154

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(-11791872000\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{982669459\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-\right.\right.}{11}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

$\big\downarrow$ 219

$$\frac{-2\left(\frac{9}{64}\left(\frac{1}{16}\left(-\frac{982669459\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-491328000\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{8}\sqrt{2x^2-x+3}(85448933-\right.\right.}{11}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{576(2x+5)}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^2,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(576*(5 + 2*x)) + ((-5034*(3 - x + 2*x^2)^(5
/2))/5 + 60*(5 + 2*x)*(3 - x + 2*x^2)^(5/2) - 2*(((909513 - 226052*x)*(3 -
 x + 2*x^2)^(3/2))/32 + (9*(((85448933 - 14243732*x)*Sqrt[3 - x + 2*x^2])/
8 + ((-982669459*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - 491328000*Sqrt[2]
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/16))/64))/1152
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.89

$$\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^2} \, dx = \frac{14740041885 \sqrt{2}(2\,x+5) \log \left(-4 \sqrt{2}\sqrt{2\,x^2-x+3}(4\,x\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
fricas")
```

output
```
1/3932160*(14740041885*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 14739840000*sqrt(2)*(2*x + 5)*log((24*
sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2
 + 20*x + 25)) + 8*(409600*x^6 - 1798144*x^5 + 8283904*x^4 - 35369408*x^3
+ 182033816*x^2 - 1404323114*x - 6814208295)*sqrt(2*x^2 - x + 3))/(2*x + 5
)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^2}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx &= \frac{5}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x \\ &- \frac{589}{960}\left(2x^2-x+3\right)^{\frac{5}{2}} + \frac{9059}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{185827}{6144}\left(2x^2-x+3\right)^{\frac{3}{2}} \\ &+ \frac{3560933}{8192}\sqrt{2x^2-x+3}x + \frac{982669459}{131072}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &- \frac{959625}{128}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|} - \frac{17\sqrt{23}}{23\,|2x+5|}\right) \\ &- \frac{85448933}{32768}\sqrt{2x^2-x+3} - \frac{3667\left(2x^2-x+3\right)^{\frac{3}{2}}}{32\,(2x+5)}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
maxima")
```

output
```
5/48*(2*x^2 - x + 3)^(5/2)*x - 589/960*(2*x^2 - x + 3)^(5/2) + 9059/1536*(
2*x^2 - x + 3)^(3/2)*x - 185827/6144*(2*x^2 - x + 3)^(3/2) + 3560933/8192*
sqrt(2*x^2 - x + 3)*x + 982669459/131072*sqrt(2)*arcsinh(4/23*sqrt(23)*x -
 1/23*sqrt(23)) - 959625/128*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5)
 - 17/23*sqrt(23)/abs(2*x + 5)) - 85448933/32768*sqrt(2*x^2 - x + 3) - 366
7/32*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(137) = 274$.

Time = 0.27 (sec) , antiderivative size = 707, normalized size of antiderivative = 4.11

$$\int \frac{(3 - x + 2x^2)^{3/2} (2 + x + 3x^2 - x^3 + 5x^4)}{(5 + 2x)^2} \, dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x, algorithm="
giac")
```

output
```
1/1966080*sqrt(2)*(14739840000*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 14740041885*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 147
40041885*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) -
1))*sgn(1/(2*x + 5)) - 2027704320*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
*sgn(1/(2*x + 5)) + 2*(45496763235*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 +
1) + 6/(2*x + 5))^11*sgn(1/(2*x + 5)) - 126553743360*(sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^10*sgn(1/(2*x + 5)) + 44062768335*(sqr
t(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^9*sgn(1/(2*x + 5)) +
33178982400*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^8*sgn
(1/(2*x + 5)) + 294206421582*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6
/(2*x + 5))^7*sgn(1/(2*x + 5)) - 463672074240*(sqrt(-11/(2*x + 5) + 36/(2*
x + 5)^2 + 1) + 6/(2*x + 5))^6*sgn(1/(2*x + 5)) + 35099942478*(sqrt(-11/(2
*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/(2*x + 5)) + 17132461
0560*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^4*sgn(1/(2*x
 + 5)) + 60059281615*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^3*sgn(1/(2*x + 5)) - 105051009024*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) - 5210329245*(sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) + 17058392064*sgn(1/(
2*x + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx=\int \frac{{\left(2\,x^2-x+3\right)}^{3/2}\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x+5\right)}^2}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.39

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^2}\,dx = \frac{1638400\sqrt{2x^2-x+3}\,x^6-7192576\sqrt{2x^2-x+3}\,x^5+}{}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2,x)
```

output
```
(1638400*sqrt(2*x**2 - x + 3)*x**6 - 7192576*sqrt(2*x**2 - x + 3)*x**5 + 3
3135616*sqrt(2*x**2 - x + 3)*x**4 - 141477632*sqrt(2*x**2 - x + 3)*x**3 +
728135264*sqrt(2*x**2 - x + 3)*x**2 - 5617292456*sqrt(2*x**2 - x + 3)*x -
27256833180*sqrt(2*x**2 - x + 3) + 29479680000*sqrt(2)*log( - 12*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 22*x - 17)*x + 73699200000*sqrt(2)*log( - 12*sqrt(2*
x**2 - x + 3)*sqrt(2) + 22*x - 17) + 29480083770*sqrt(2)*log( - 2*sqrt(2*x
**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 73700209425*sqrt(2)*log( - 2*sqrt(2*x*
*2 - x + 3)*sqrt(2) - 4*x + 1) - 29479680000*sqrt(2)*log(2*x + 5)*x - 7369
9200000*sqrt(2)*log(2*x + 5))/(1966080*(2*x + 5))
```

3.173 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx$

Optimal result

Integrand size = 40, antiderivative size = 174

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx = \frac{(33741483-5623292x)\sqrt{3-x+2x^2}}{24576} + \frac{(2154633-534617x)(3-x+2x^2)^{3/2}}{82944} + \frac{1}{16}(3-x+2x^2)^{5/2} - \frac{3667(3-x+2x^2)^{5/2}}{1152(5+2x)^2} + \frac{438065(3-x+2x^2)^{5/2}}{82944(5+2x)} + \frac{129342063\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}} - \frac{8083915\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1024\sqrt{2}}$$

output

```
1/24576*(33741483-5623292*x)*(2*x^2-x+3)^(1/2)+1/82944*(2154633-534617*x)*
(2*x^2-x+3)^(3/2)+1/16*(2*x^2-x+3)^(5/2)-3667/1152*(2*x^2-x+3)^(5/2)/(5+2*
x)^2+438065*(2*x^2-x+3)^(5/2)/(414720+165888*x)+129342063/32768*arcsinh(1/
23*(1-4*x)*23^(1/2))*2^(1/2)-8083915/2048*arctanh(1/24*(17-22*x)*2^(1/2)/(
2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{\frac{4\sqrt{3-x+2x^2}(298966737+181223072x+16667188x^2-1620944x^3+253312x^4}{(5+2x)^2}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
3,x]
```

output
```
((4*Sqrt[3 - x + 2*x^2]*(298966737 + 181223072*x + 16667188*x^2 - 1620944*
x^3 + 253312*x^4 - 43520*x^5 + 8192*x^6))/(5 + 2*x)^2 + 258685280*Sqrt[2]*
ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 129342063*Sqrt[2]*Log[1 - 4
*x + 2*Sqrt[6 - 2*x + 4*x^2]])/32768
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 2184, 27, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{144}\int \frac{(2x^2-x+3)^{3/2}(-5760x^3+15552x^2-86340x+35015)}{16(2x+5)^2}dx-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-5760x^3+15552x^2-86340x+35015)}{(2x+5)^2}dx}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(207360x^2-8087312x+2737465)}{2x+5}dx+\frac{438065(2x^2-x+3)^{5/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 2184

$$\frac{\frac{1}{72}\left(\frac{1}{40}\int \frac{40(2867065-8553872x)(2x^2-x+3)^{3/2}}{2x+5}dx+10368(2x^2-x+3)^{5/2}\right)+\frac{438065(2x^2-x+3)^{5/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(\int \frac{(2867065-8553872x)(2x^2-x+3)^{3/2}}{2x+5}dx+10368(2x^2-x+3)^{5/2}\right)+\frac{438065(2x^2-x+3)^{5/2}}{36(2x+5)}}{2304}-\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1231

$$\frac{\frac{1}{72}\left(-\frac{1}{64}\int -\frac{6912(527400-1405823x)\sqrt{2x^2-x+3}}{2x+5}dx+10368(2x^2-x+3)^{5/2}+2(2154633-534617x)(2x^2-x+3)^{3/2}\right.}{2304}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(108\int \frac{(527400-1405823x)\sqrt{2x^2-x+3}}{2x+5}dx+10368(2x^2-x+3)^{5/2}+2(2154633-534617x)(2x^2-x+3)^{3/2}\right)+\frac{43}{}}{2304}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1231

$$\frac{\frac{1}{72}\left(108\left(\frac{1}{16}(33741483-5623292x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{9(43115175-86228042x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+10368(2x^2-x+3)^{5/2}+2(\right.}{2304}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\int \frac{43115175-86228042x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292x)\right)+10368\left(2x^2-x+3\right)^{5/2}+2(215\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\big\downarrow$ 1269

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-43114021\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292\right.\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\big\downarrow$ 1090

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{43114021\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-56\right.\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\big\downarrow$ 222

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(258685280\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{43114021\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5623292\right.\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\big\downarrow$ 1154

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(-517370560\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{43114021\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-5\right.\right.}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{72}\left(108\left(\frac{9}{32}\left(-\frac{43114021\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{64671320}{3}\sqrt{2}\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+\frac{1}{16}\sqrt{2x^2-x+3}(33741483-56}{2304}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1152(2x+5)^2}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^3,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(1152*(5 + 2*x)^2) + ((438065*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)) + (2*(2154633 - 534617*x)*(3 - x + 2*x^2)^(3/2) +
10368*(3 - x + 2*x^2)^(5/2) + 108*(((33741483 - 5623292*x)*Sqrt[3 - x + 2*
x^2])/16 + (9*((-43114021*ArcSinh[(-1 + 4*x)/Sqrt[23]])/Sqrt[2] - (6467132
0*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/3))/32))/
72)/2304
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

$$\int \frac{(3-x+2x^2)^{3/2} (2+x+3x^2-x^3+5x^4)}{(5+2x)^3} dx = \frac{129342063 \sqrt{2}(4x^2+20x+25) \log \left(4\sqrt{2}\sqrt{2x^2-x+\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
fricas")
```

output
```
1/65536*(129342063*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 129342640*sqrt(2)*(4*x^2 + 20*x +
 25)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x
+ 1153)/(4*x^2 + 20*x + 25)) + 8*(8192*x^6 - 43520*x^5 + 253312*x^4 - 1620
944*x^3 + 16667188*x^2 + 181223072*x + 298966737)*sqrt(2*x^2 - x + 3))/(4*
x^2 + 20*x + 25)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^3}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx &= \frac{1}{16}\left(2\,x^2-x+3\right)^{\frac{5}{2}} \\ &- \frac{149}{128}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x + \frac{46691}{4608}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{1152\,(4\,x^2+20\,x+25)} \\ &- \frac{1405823}{6144}\sqrt{2\,x^2-x+3}x - \frac{129342063}{32768}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) \\ &+ \frac{8083915}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) \\ &+ \frac{11247161}{8192}\sqrt{2\,x^2-x+3} + \frac{438065\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{4608\,(2\,x+5)}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
maxima")
```

output
```
1/16*(2*x^2 - x + 3)^(5/2) - 149/128*(2*x^2 - x + 3)^(3/2)*x + 46691/4608*
(2*x^2 - x + 3)^(3/2) - 3667/1152*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25
) - 1405823/6144*sqrt(2*x^2 - x + 3)*x - 129342063/32768*sqrt(2)*arcsinh(4
/23*sqrt(23)*x - 1/23*sqrt(23)) + 8083915/2048*sqrt(2)*arcsinh(22/23*sqrt(
23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 11247161/8192*sqrt(2*x
^2 - x + 3) + 438065/4608*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.54

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{1}{8192}\left(4\left(8\left(4\left(16x-165\right)x+4879\right)x-263469\right)x+846\right.$$
$$+\frac{129342063}{32768}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$-\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{8083915}{2048}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$+\frac{\sqrt{2}\left(14243182\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+109906674\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-170996871\sqrt{2}\left(\sqrt{2}x\right.\right.}{512\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x, algorithm="
giac")
```

output
```
1/8192*(4*(8*(4*(16*x - 165)*x + 4879)*x - 263469)*x + 8460377)*sqrt(2*x^2
 - x + 3) + 129342063/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2
 - x + 3)) + 1) - 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*
sqrt(2*x^2 - x + 3))) + 8083915/2048*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqr
t(2) + 2*sqrt(2*x^2 - x + 3))) + 1/512*sqrt(2)*(14243182*sqrt(2)*(sqrt(2)*
x - sqrt(2*x^2 - x + 3))^3 + 109906674*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2
 - 170996871*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 110506087)/(2*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^3}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.80

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^3}\,dx = \frac{32768\sqrt{2x^2-x+3}\,x^6-174080\sqrt{2x^2-x+3}\,x^5+101$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3,x)
```

output
```
(32768*sqrt(2*x**2 - x + 3)*x**6 - 174080*sqrt(2*x**2 - x + 3)*x**5 + 1013
248*sqrt(2*x**2 - x + 3)*x**4 - 6483776*sqrt(2*x**2 - x + 3)*x**3 + 666687
52*sqrt(2*x**2 - x + 3)*x**2 + 724892288*sqrt(2*x**2 - x + 3)*x + 11958669
48*sqrt(2*x**2 - x + 3) + 517370560*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sq
rt(2) + 22*x - 17)*x**2 + 2586852800*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*s
qrt(2) + 22*x - 17)*x + 3233566000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqr
t(2) + 22*x - 17) + 517368252*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) -
 4*x + 1)*x**2 + 2586841260*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4
*x + 1)*x + 3233551575*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x +
1) - 517370560*sqrt(2)*log(2*x + 5)*x**2 - 2586852800*sqrt(2)*log(2*x + 5)
*x - 3233566000*sqrt(2)*log(2*x + 5))/(32768*(4*x**2 + 20*x + 25))
```

3.174 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 181

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = & -\frac{(135068604-22512089x)\sqrt{3-x+2x^2}}{331776} \\ & -\frac{(138006843-34265045x)(3-x+2x^2)^{3/2}}{17915904}-\frac{3667(3-x+2x^2)^{5/2}}{1728(5+2x)^3} \\ & +\frac{556255(3-x+2x^2)^{5/2}}{248832(5+2x)^2}-\frac{32865365(3-x+2x^2)^{5/2}}{17915904(5+2x)} \\ & -\frac{19176431\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8192\sqrt{2}}+\frac{517762327\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{221184\sqrt{2}}\end{aligned}$$

output

```
-1/331776*(135068604-22512089*x)*(2*x^2-x+3)^(1/2)-1/17915904*(138006843-3
4265045*x)*(2*x^2-x+3)^(3/2)-3667/1728*(2*x^2-x+3)^(5/2)/(5+2*x)^3+556255/
248832*(2*x^2-x+3)^(5/2)/(5+2*x)^2-32865365*(2*x^2-x+3)^(5/2)/(89579520+35
831808*x)-19176431/16384*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+517762327/
442368*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.66

$$\int \frac{(3-x+2x^2)^{3/2}\,(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-1994650739-2006873194x-594798908x^2-33595416x^3+2}{(5+2x)^3}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
4,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-1994650739 - 2006873194*x - 594798908*x^2 - 335
95416*x^3 + 2626848*x^4 - 315648*x^5 + 46080*x^6))/(5 + 2*x)^3 - 103552465
4*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 517763637*Sqrt[2]
*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/442368
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.325$, Rules used = {2181, 27, 2181, 2181, 1231, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}\,(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{1}{216}\int \frac{(2x^2-x+3)^{3/2}\,(-8640x^3+23328x^2-92840x+43355)}{16(2x+5)^3}dx-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-8640x^3+23328x^2-92840x+43355)}{(2x+5)^3}dx}{3456}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{144}\int \frac{(2x^2-x+3)^{3/2}(622080x^2-9909876x+4202675)}{(2x+5)^2}dx+\frac{556255(2x^2-x+3)^{5/2}}{72(2x+5)^2}}{3456}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 2181

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{(182685181-548240720x)(2x^2-x+3)^{3/2}}{2x+5}dx-\frac{32865365(2x^2-x+3)^{5/2}}{36(2x+5)}\right)+\frac{556255(2x^2-x+3)^{5/2}}{72(2x+5)^2}}{3456}-\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 1231

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(\frac{1}{64}\int -\frac{3456(67520547-180096712x)\sqrt{2x^2-x+3}}{2x+5}dx-2(138006843-34265045x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{32865365(2x}{36(2x}\right.}{3456}\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 27

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\int \frac{(67520547-180096712x)\sqrt{2x^2-x+3}}{2x+5}dx-2(138006843-34265045x)\left(2x^2-x+3\right)^{3/2}\right)-\frac{32865365(2x^2-x}{36(2x+5)}\right.}{3456}\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 1231

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(2(135068604-22512089x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{288(172585259-345175758x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-2(138006843-3\right.\right.}{3456}\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 27

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\int \frac{172585259-345175758x}{(2x+5)\sqrt{2x^2-x+3}}dx+2\sqrt{2x^2-x+3}(135068604-22512089x)\right)-2(138006843-3426504\right.\right.}{3456}\frac{3667(2x^2-x+3)^{5/2}}{1728(2x+5)^3}$$

$\downarrow$ 1269

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-172587879\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+2\sqrt{2x^2-x+3}(135068604-2\right.\right.\right.}{3456}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 1090

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{172587879\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+2\sqrt{2x^2-x+3}(13506860\right.\right.\right.}{34}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 222

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(1035524654\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{172587879\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(135068604-22\right.\right.\right.}{3456}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(-2071049308\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{172587879\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(1350686\right.\right.\right.}{34}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{144}\left(\frac{1}{72}\left(-54\left(9\left(-\frac{172587879\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{517762327\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{3\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(135068604-\right.\right.\right.}{3450}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{1728(2x+5)^3}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^4,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(1728*(5 + 2*x)^3) + ((556255*(3 - x + 2*x^2
)^(5/2))/(72*(5 + 2*x)^2) + ((-32865365*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*
x)) + (-2*(138006843 - 34265045*x)*(3 - x + 2*x^2)^(3/2) - 54*(2*(13506860
4 - 22512089*x)*Sqrt[3 - x + 2*x^2] + 9*((-172587879*ArcSinh[(-1 + 4*x)/Sq
rt[23]])/Sqrt[2] - (517762327*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x +
 2*x^2])])/(3*Sqrt[2]))))/72)/144)/3456
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.01

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{517763637\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
fricas")
```

output
```
1/884736*(517763637*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 517762327*sqrt(2)*(8
*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 1
7) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 24*(46080*x^6 - 3156
48*x^5 + 2626848*x^4 - 33595416*x^3 - 594798908*x^2 - 2006873194*x - 19946
50739)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{5}{64}\left(2x^2-x+3\right)^{\frac{3}{2}}x - \frac{1094743}{497664}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{1728\left(8x^3+60x^2+150x+125\right)} + \frac{556255\left(2x^2-x+3\right)^{\frac{5}{2}}}{248832\left(4x^2+20x+25\right)} + \frac{22512089}{331776}\sqrt{2x^2-x+3}x + \frac{19176431}{16384}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{517762327}{442368}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2x+5|}-\frac{17\sqrt{23}}{23\,|2x+5|}\right) - \frac{11255717}{27648}\sqrt{2x^2-x+3} - \frac{32865365\left(2x^2-x+3\right)^{\frac{3}{2}}}{995328\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
maxima")
```

output
```
5/64*(2*x^2 - x + 3)^(3/2)*x - 1094743/497664*(2*x^2 - x + 3)^(3/2) - 3667
/1728*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 556255/248832
*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 22512089/331776*sqrt(2*x^2 -
x + 3)*x + 19176431/16384*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23))
 - 517762327/442368*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*
sqrt(23)/abs(2*x + 5)) - 11255717/27648*sqrt(2*x^2 - x + 3) - 32865365/995
328*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(146) = 292$.

Time = 0.21 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.73

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{1}{4096}(4(8(20x-287)x+23341)x-1004633)\sqrt{2x^2-}$$
$$-\frac{19176431}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{517762327}{442368}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(1092794276\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^5+18284336132\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4+20314214356\sqrt{}\right.}{36864\left(2\left(\sqrt{2}x-\sqrt{2x^2}\right.\right.}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x, algorithm="
giac")
```

output
```
1/4096*(4*(8*(20*x - 287)*x + 23341)*x - 1004633)*sqrt(2*x^2 - x + 3) - 19
176431/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
 + 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) - 517762327/442368*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) +
2*sqrt(2*x^2 - x + 3))) - 1/36864*sqrt(2)*(1092794276*sqrt(2)*(sqrt(2)*x -
 sqrt(2*x^2 - x + 3))^5 + 18284336132*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4
+ 20314214356*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 151449344092*(
sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 102529692109*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3)) - 41882448755)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 +
10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^4}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.14

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^4}\,dx = \frac{552960\sqrt{2x^2-x+3}\,x^6-3787776\sqrt{2x^2-x+3}\,x^5+3$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4,x)
```

output
```
(552960*sqrt(2*x**2 - x + 3)*x**6 - 3787776*sqrt(2*x**2 - x + 3)*x**5 + 31
522176*sqrt(2*x**2 - x + 3)*x**4 - 403144992*sqrt(2*x**2 - x + 3)*x**3 - 7
137586896*sqrt(2*x**2 - x + 3)*x**2 - 24082478328*sqrt(2*x**2 - x + 3)*x -
 23935808868*sqrt(2*x**2 - x + 3) + 4142098616*sqrt(2)*log( - 12*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 31065739620*sqrt(2)*log( - 12*sqrt
(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 77664349050*sqrt(2)*log( - 12
*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 64720290875*sqrt(2)*log( -
12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 4142109096*sqrt(2)*log( - 2
*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 31065818220*sqrt(2)*log( -
 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 77664545550*sqrt(2)*log(
 - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 64720454625*sqrt(2)*log(
- 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 4142098616*sqrt(2)*log(2*x +
 5)*x**3 - 31065739620*sqrt(2)*log(2*x + 5)*x**2 - 77664349050*sqrt(2)*log
(2*x + 5)*x - 64720290875*sqrt(2)*log(2*x + 5))/(442368*(8*x**3 + 60*x**2
+ 150*x + 125))
```

3.175 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 188

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{(2339916063-389975609x)\sqrt{3-x+2x^2}}{31850496} + \frac{(762984903+67865260x)(3-x+2x^2)^{3/2}}{95551488(5+2x)} - \frac{3667(3-x+2x^2)^{5/2}}{2304(5+2x)^4} + \frac{224815(3-x+2x^2)^{5/2}}{165888(5+2x)^3} - \frac{14477995(3-x+2x^2)^{5/2}}{23887872(5+2x)^2} + \frac{432565\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}} - \frac{8969688643\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{21233664\sqrt{2}}$$

output
```
1/31850496*(2339916063-389975609*x)*(2*x^2-x+3)^(1/2)+(762984903+67865260*
x)*(2*x^2-x+3)^(3/2)/(477757440+191102976*x)-3667/2304*(2*x^2-x+3)^(5/2)/(
5+2*x)^4+224815/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^3-14477995/23887872*(2*x^
2-x+3)^(5/2)/(5+2*x)^2+432565/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-
8969688643/42467328*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1
/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(86386856771+121473790266x+60528581892x^2+117619100}{(5+2x)^4}}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
5,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(86386856771 + 121473790266*x + 60528581892*x^2 +
 11761910072*x^3 + 468043776*x^4 - 29270016*x^5 + 2949120*x^6))/(5 + 2*x)^
4 + 8969688643*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 4484
833920*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/21233664
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 27, 2181, 1230, 27, 1231, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{288}\int \frac{(2x^2-x+3)^{3/2}(-11520x^3+31104x^2-99340x+51695)}{16(2x+5)^4}dx - \frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-11520x^3+31104x^2-99340x+51695)}{(2x+5)^4}dx}{4608} - \frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 2181$$

$$\frac{\frac{1}{216}\int \frac{3(2x^2-x+3)^{3/2}(414720x^2-3955064x+1998335)}{(2x+5)^3}dx+\frac{224815(2x^2-x+3)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(414720x^2-3955064x+1998335)}{(2x+5)^3}dx+\frac{224815(2x^2-x+3)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 2181$$

$$\frac{\frac{1}{72}\left(-\frac{1}{144}\int \frac{(84332303-203595780x)(2x^2-x+3)^{3/2}}{(2x+5)^2}dx-\frac{14477995(2x^2-x+3)^{5/2}}{72(2x+5)^2}\right)+\frac{224815(2x^2-x+3)^{5/2}}{36(2x+5)^3}}{4608}-\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 1230$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{1}{8}\int \frac{6(1170176463-3119804872x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{(67865260x+762984903)(2x^2-x+3)^{3/2}}{2(2x+5)}\right)-\frac{14477995(2x^2-x+3)^{5/2}}{72(2x+5)^2}\right)+}{4608}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\int \frac{(1170176463-3119804872x)\sqrt{2x^2-x+3}}{2x+5}dx+\frac{(67865260x+762984903)(2x^2-x+3)^{3/2}}{2(2x+5)}\right)-\frac{14477995(2x^2-x+3)^{5/2}}{72(2x+5)^2}\right)+2}{4608}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 1231$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(2(2339916063-389975609x)\sqrt{2x^2-x+3}-\frac{1}{32}\int -\frac{576(1494965443-2989889280x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{(67865260x+76298}{2(2}\right.\right.}{4608}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2304(2x+5)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\int \frac{1494965443-2989889280x}{(2x+5)\sqrt{2x^2-x+3}}dx+2\sqrt{2x^2-x+3}(2339916063-389975609x)\right)+\frac{(67865260x+762984903)(2}{2(2x+5)}\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1269

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-1494944640\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+2\sqrt{2x^2-x+3}(2339916063-\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1090

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-747472320\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+2\sqrt{2x^2-x+3}\right.\right.\right.}{46}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 222

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(8969688643\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-747472320\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+2\sqrt{2x^2-x+3}(2339916063\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(-17939377286\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-747472320\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+2\sqrt{2x^2-x+3}(23\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{72}\left(\frac{1}{144}\left(\frac{3}{4}\left(18\left(-747472320\sqrt{2}\operatorname{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{8969688643\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+2\sqrt{2x^2-x+3}(2339916\right.\right.\right.}{4608}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2304(2x+5)^4}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^5,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(2304*(5 + 2*x)^4) + ((224815*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)^3) + ((-14477995*(3 - x + 2*x^2)^(5/2))/(72*(5 + 2*
x)^2) + (((762984903 + 67865260*x)*(3 - x + 2*x^2)^(3/2))/(2*(5 + 2*x)) +
(3*(2*(2339916063 - 389975609*x)*Sqrt[3 - x + 2*x^2] + 18*(-747472320*Sqrt
[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (8969688643*ArcTanh[(17 - 22*x)/(12*Sqr
t[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))))/4)/144)/72)/4608
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{8969667840\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
fricas")
```

output
```
1/84934656*(8969667840*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 89696
88643*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)*log(-(24*sqrt(2)
*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x
 + 25)) + 48*(2949120*x^6 - 29270016*x^5 + 468043776*x^4 + 11761910072*x^3
 + 60528581892*x^2 + 121473790266*x + 86386856771)*sqrt(2*x^2 - x + 3))/(1
6*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^5}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx &= \frac{16966315}{47775744}\,(2\,x^2-x+3)^{\frac{3}{2}} \\ &- \frac{3667\,(2\,x^2-x+3)^{\frac{5}{2}}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} \\ &+ \frac{224815\,(2\,x^2-x+3)^{\frac{5}{2}}}{165888\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{14477995\,(2\,x^2-x+3)^{\frac{5}{2}}}{23887872\,(4\,x^2+20\,x+25)} \\ &- \frac{389975609}{31850496}\,\sqrt{2\,x^2-x+3}x - \frac{432565}{2048}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x - \frac{1}{23}\,\sqrt{23}\right) \\ &+ \frac{8969688643}{42467328}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) \\ &+ \frac{779972021}{10616832}\,\sqrt{2\,x^2-x+3} + \frac{593321753\,(2\,x^2-x+3)^{\frac{3}{2}}}{95551488\,(2\,x+5)}\end{aligned}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
maxima")
```

output
```
16966315/47775744*(2*x^2 - x + 3)^(3/2) - 3667/2304*(2*x^2 - x + 3)^(5/2)/
(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 224815/165888*(2*x^2 - x + 3
)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 14477995/23887872*(2*x^2 - x + 3)
^(5/2)/(4*x^2 + 20*x + 25) - 389975609/31850496*sqrt(2*x^2 - x + 3)*x - 43
2565/2048*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 8969688643/42
467328*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) + 779972021/10616832*sqrt(2*x^2 - x + 3) + 593321753/95551488*(2
*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(153) = 306$.

Time = 0.25 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.68

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \text{Too large to display}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x, algorithm="
giac")
```

output
```
-1/42467328*sqrt(2)*(8969688643*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
 + 1) + 72/(2*x + 5) - 11)*sgn(1/(2*x + 5)) + 8969667840*log(abs(sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))*sgn(1/(2*x + 5)) - 896
9667840*log(abs(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1
))*sgn(1/(2*x + 5)) + 12*(24*(1296*(29336*sgn(1/(2*x + 5))/(2*x + 5) - 429
07*sgn(1/(2*x + 5)))/(2*x + 5) + 39923563*sgn(1/(2*x + 5)))/(2*x + 5) - 54
1312039*sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 13824
*(806241*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^5*sgn(1/
(2*x + 5)) - 1152288*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x +
5))^4*sgn(1/(2*x + 5)) - 957352*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
+ 6/(2*x + 5))^3*sgn(1/(2*x + 5)) + 1529280*(sqrt(-11/(2*x + 5) + 36/(2*x
+ 5)^2 + 1) + 6/(2*x + 5))^2*sgn(1/(2*x + 5)) + 394431*(sqrt(-11/(2*x + 5)
 + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))*sgn(1/(2*x + 5)) - 620352*sgn(1/(2*x
 + 5)))/((sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^3
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^5}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.45

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^5}\,dx = \frac{70778880\sqrt{2x^2-x+3}\,x^6-702480384\sqrt{2x^2-x+3}\,x}{}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5,x)
```

output

```
(70778880*sqrt(2*x**2 - x + 3)*x**6 - 702480384*sqrt(2*x**2 - x + 3)*x**5
+ 11233050624*sqrt(2*x**2 - x + 3)*x**4 + 282285841728*sqrt(2*x**2 - x + 3
)*x**3 + 1452685965408*sqrt(2*x**2 - x + 3)*x**2 + 2915370966384*sqrt(2*x*
*2 - x + 3)*x + 2073284562504*sqrt(2*x**2 - x + 3) + 143515018288*sqrt(2)*
log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 1435150182880*sqrt
(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 5381813185800*
sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 8969688643
000*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 560605540
1875*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 1435146854
40*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 1435146854
400*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 538180070
4000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 89696678
40000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 5606042400
000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 143515018288*s
qrt(2)*log(2*x + 5)*x**4 - 1435150182880*sqrt(2)*log(2*x + 5)*x**3 - 53818
13185800*sqrt(2)*log(2*x + 5)*x**2 - 8969688643000*sqrt(2)*log(2*x + 5)*x
- 5606055401875*sqrt(2)*log(2*x + 5))/(42467328*(16*x**4 + 160*x**3 + 600*
x**2 + 1000*x + 625))
```

3.176 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx$

Optimal result

Integrand size = 40, antiderivative size = 188

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6} dx = -\frac{(18519560187-3086715581x)\sqrt{3-x+2x^2}}{2293235712} - \frac{(9529279465+5098855738x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^2} - \frac{3667(3-x+2x^2)^{5/2}}{2880(5+2x)^5} + \frac{158527(3-x+2x^2)^{5/2}}{165888(5+2x)^4} - \frac{3730507(3-x+2x^2)^{5/2}}{11943936(5+2x)^3} - \frac{23775\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{512\sqrt{2}} + \frac{70991525167\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1528823808\sqrt{2}}$$

output
```
-1/2293235712*(18519560187-3086715581*x)*(2*x^2-x+3)^(1/2)-1/2293235712*(9
529279465+5098855738*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^2-3667/2880*(2*x^2-x+3)^
(5/2)/(5+2*x)^5+158527/165888*(2*x^2-x+3)^(5/2)/(5+2*x)^4-3730507/11943936
*(2*x^2-x+3)^(5/2)/(5+2*x)^3-23775/1024*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(
1/2)+70991525167/3057647616*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/
2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.64

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{12\sqrt{3-x+2x^2}(-17093312738327-30872393829992x-21590439797064x^2-}{(5+2x}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
6,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-17093312738327 - 30872393829992*x - 21590439797
064*x^2 - 7117092892448*x^3 - 1023534029552*x^4 - 30496849920*x^5 + 159252
4800*x^6))/(5 + 2*x)^5 - 354957625835*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 -
2*x + 4*x^2])/6] - 177479424000*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x
^2]])/7644119040
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.16, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 2181, 27, 1230, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{1}{360}\int \frac{5(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-21168x+12007)}{16(2x+5)^5}dx-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-2880x^3+7776x^2-21168x+12007)}{(2x+5)^5}dx}{1152}-\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{288}\int \frac{\left(2x^2-x+3\right)^{3/2}\left(414720x^2-2790652x+1622891\right)}{(2x+5)^4}dx+\frac{158527\left(2x^2-x+3\right)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 2181$$

$$\frac{\frac{1}{288}\left(-\frac{1}{216}\int \frac{3(22142555-44773976x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^3}dx-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+\frac{158527\left(2x^2-x+3\right)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{288}\left(-\frac{1}{72}\int \frac{(22142555-44773976x)\left(2x^2-x+3\right)^{3/2}}{(2x+5)^3}dx-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+\frac{158527\left(2x^2-x+3\right)^{5/2}}{144(2x+5)^4}}{1152}-\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 1230$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{32}\int \frac{4(514656291-1028823716x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx+\frac{(44773976x+246012435)\left(2x^2-x+3\right)^{3/2}}{4(2x+5)^2}\right)-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+1}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\int \frac{(514656291-1028823716x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx+\frac{(44773976x+246012435)\left(2x^2-x+3\right)^{3/2}}{4(2x+5)^2}\right)-\frac{3730507\left(2x^2-x+3\right)^{5/2}}{36(2x+5)^3}\right)+158}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 1230$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(-\frac{1}{8}\int \frac{2(11831717167-23663923200x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(1028823716x+5658774871)}{2(2x+5)}\right)+\frac{(44773976x+246012435)\left(2x^2-x+3\right.}{4(2x+5)^2}\right.\right.}{1152}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{2880(2x+5)^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(-\frac{1}{4}\int \frac{11831717167-23663923200x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{\sqrt{2x^2-x+3}(1028823716x+5658774871)}{2(2x+5)}\right)+\frac{(44773976x+246012435)(2x^2-x+3)^3}{4(2x+5)^2}\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$\big\downarrow$ 1269

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(11831961600\int \frac{1}{\sqrt{2x^2-x+3}}dx-70991525167\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+5658774871)\sqrt{2x^2-x}}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$\big\downarrow$ 1090

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-70991525167\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+5}{2}\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$\big\downarrow$ 222

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-70991525167\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{(1028823716x+5658774871)\sqrt{2x^2}}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$\big\downarrow$ 1154

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(141983050334\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{(1028823716x+565877}{2(2x+5}\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{288}\left(\frac{1}{72}\left(\frac{3}{8}\left(\frac{1}{4}\left(5915980800\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{70991525167\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{(1028823716x+5658774871)\sqrt{}}{2(2x+5)}\right.\right.\right.}{1152}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{2880(2x+5)^5}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^6,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(2880*(5 + 2*x)^5) + ((158527*(3 - x + 2*x^2
)^(5/2))/(144*(5 + 2*x)^4) + ((-3730507*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*
x)^3) + (((246012435 + 44773976*x)*(3 - x + 2*x^2)^(3/2))/(4*(5 + 2*x)^2)
+ (3*(-1/2*((5658774871 + 1028823716*x)*Sqrt[3 - x + 2*x^2])/(5 + 2*x) + (
5915980800*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (70991525167*ArcTanh[(17
 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4))/8)/72)/288)/1
152
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.13

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{354958848000\sqrt{2}(32x^5+400x^4+2000x^3+5000x^2}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
fricas")
```

output
```
1/30576476160*(354958848000*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^
2 + 6250*x + 3125)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25) + 354957625835*sqrt(2)*(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2
 + 6250*x + 3125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x
^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(1592524800*x^6 - 3049684992
0*x^5 - 1023534029552*x^4 - 7117092892448*x^3 - 21590439797064*x^2 - 30872
393829992*x - 17093312738327)*sqrt(2*x^2 - x + 3))/(32*x^5 + 400*x^4 + 200
0*x^3 + 5000*x^2 + 6250*x + 3125)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^6}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**6,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.34

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = -\frac{134077495}{3439853568}\left(2x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2x^2-x+3\right)^{\frac{5}{2}}}{2880\left(32x^5+400x^4+2000x^3+5000x^2+6250x+3125\right)} + \frac{158527\left(2x^2-x+3\right)^{\frac{5}{2}}}{165888\left(16x^4+160x^3+600x^2+1000x+625\right)} - \frac{3730507\left(2x^2-x+3\right)^{\frac{5}{2}}}{11943936\left(8x^3+60x^2+150x+125\right)} + \frac{134077495\left(2x^2-x+3\right)^{\frac{5}{2}}}{1719926784\left(4x^2+20x+25\right)} + \frac{3086715581}{2293235712}\sqrt{2x^2-x+3}x + \frac{23775}{1024}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{70991525167}{3057647616}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23|2x+5|}-\frac{17\sqrt{23}}{23|2x+5|}\right) - \frac{6173186729}{764411904}\sqrt{2x^2-x+3} - \frac{4698578717\left(2x^2-x+3\right)^{\frac{3}{2}}}{6879707136\left(2x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
maxima")
```

output
```
-134077495/3439853568*(2*x^2 - x + 3)^(3/2) - 3667/2880*(2*x^2 - x + 3)^(5
/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*x + 3125) + 158527/1658
88*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625) - 373
0507/11943936*(2*x^2 - x + 3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) + 13407
7495/1719926784*(2*x^2 - x + 3)^(5/2)/(4*x^2 + 20*x + 25) + 3086715581/229
3235712*sqrt(2*x^2 - x + 3)*x + 23775/1024*sqrt(2)*arcsinh(4/23*sqrt(23)*x
 - 1/23*sqrt(23)) - 70991525167/3057647616*sqrt(2)*arcsinh(22/23*sqrt(23)*
x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 6173186729/764411904*sqrt(
2*x^2 - x + 3) - 4698578717/6879707136*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(153) = 306$.

Time = 0.17 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.16

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{1}{256}\sqrt{2x^2-x+3}(20x-633)$$
$$-\frac{23775}{1024}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{70991525167}{3057647616}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(8281387393360\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^9+275661428628240\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^8+156038\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x, algorithm="
giac")
```

output
```
1/256*sqrt(2*x^2 - x + 3)*(20*x - 633) - 23775/1024*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 70991525167/3057647616*sqrt(2)*l
og(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 70991525167/3057
647616*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 - 1/1274019840*sqrt(2)*(8281387393360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
 + 3))^9 + 275661428628240*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 156038270
3345760*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 4938646760855520*(sq
rt(2)*x - sqrt(2*x^2 - x + 3))^6 - 9673562837036232*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^5 - 30647310393849000*(sqrt(2)*x - sqrt(2*x^2 - x + 3)
)^4 + 70060241036847960*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 9773
0658088823880*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 30180638363071845*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 7096913381268319)/(2*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11
)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^6}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.85

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^6}\,dx = \frac{-141983050334000\sqrt{2}\log(2x+5)\,x^4+113586440267}{}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^6,x)
```

output
```
(38220595200*sqrt(2*x**2 - x + 3)*x**6 - 731924398080*sqrt(2*x**2 - x + 3)
*x**5 - 24564816709248*sqrt(2*x**2 - x + 3)*x**4 - 170810229418752*sqrt(2*
x**2 - x + 3)*x**3 - 518170555129536*sqrt(2*x**2 - x + 3)*x**2 - 740937451
919808*sqrt(2*x**2 - x + 3)*x - 410239505719848*sqrt(2*x**2 - x + 3) + 113
58644026720*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x*
*5 + 141983050334000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x
 - 17)*x**4 + 709915251670000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(
2) + 22*x - 17)*x**3 + 1774788129175000*sqrt(2)*log( - 12*sqrt(2*x**2 - x
+ 3)*sqrt(2) + 22*x - 17)*x**2 + 2218485161468750*sqrt(2)*log( - 12*sqrt(2
*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 1109242580734375*sqrt(2)*log( - 12
*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 11358683136000*sqrt(2)*log( -
 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**5 + 141983539200000*sqrt(2)*
log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 709917696000000*sq
rt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 177479424000
0000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 22184
92800000000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 1
109246400000000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) -
 11358644026720*sqrt(2)*log(2*x + 5)*x**5 - 141983050334000*sqrt(2)*log(2*
x + 5)*x**4 - 709915251670000*sqrt(2)*log(2*x + 5)*x**3 - 1774788129175000
*sqrt(2)*log(2*x + 5)*x**2 - 2218485161468750*sqrt(2)*log(2*x + 5)*x - ...
```

3.177 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 195

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{(151764102421+27596573612x)\sqrt{3-x+2x^2}}{55037657088(5+2x)} - \frac{(9802984711+6793718806x)(3-x+2x^2)^{3/2}}{13759414272(5+2x)^3} - \frac{3667(3-x+2x^2)^{5/2}}{3456(5+2x)^6} + \frac{182165(3-x+2x^2)^{5/2}}{248832(5+2x)^5} - \frac{14087245(3-x+2x^2)^{5/2}}{71663616(5+2x)^4} + \frac{369\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} - \frac{1903976002333\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{660451885056\sqrt{2}}$$

output
```
(151764102421+27596573612*x)*(2*x^2-x+3)^(1/2)/(275188285440+110075314176*
x)-1/13759414272*(9802984711+6793718806*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^3-366
7/3456*(2*x^2-x+3)^(5/2)/(5+2*x)^6+182165/248832*(2*x^2-x+3)^(5/2)/(5+2*x)
^5-14087245/71663616*(2*x^2-x+3)^(5/2)/(5+2*x)^4+369/256*arcsinh(1/23*(1-4
*x)*23^(1/2))*2^(1/2)-1903976002333/1320903770112*arctanh(1/24*(17-22*x)*2
^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{12\sqrt{3-x+2x^2}(458411625354581+1011372787716826x+910256842473992x}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
7,x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(458411625354581 + 1011372787716826*x + 910256842
473992*x^2 + 422554114856528*x^3 + 103803827945872*x^4 + 11854023276320*x^
5 + 275188285440*x^6))/(5 + 2*x)^6 + 1903976002333*Sqrt[2]*ArcTanh[(5 + 2*
x - Sqrt[6 - 2*x + 4*x^2])/6] + 951979474944*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[
6 - 2*x + 4*x^2]])/660451885056
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2181, 27, 2181, 27, 2181, 1229, 27, 1230, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{432}\int \frac{(2x^2-x+3)^{3/2}(-17280x^3+46656x^2-112340x+68375)}{16(2x+5)^6}dx - \frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-17280x^3+46656x^2-112340x+68375)}{(2x+5)^6}dx}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{360}\int \frac{5(2x^2-x+3)^{3/2}(622080x^2-3234816x+2112205)}{(2x+5)^5}dx+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{72}\int \frac{(2x^2-x+3)^{3/2}(622080x^2-3234816x+2112205)}{(2x+5)^5}dx+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 2181}$$

$$\frac{\frac{1}{72}\left(-\frac{1}{288}\int \frac{(84010769-145928500x)(2x^2-x+3)^{3/2}}{(2x+5)^4}dx-\frac{14087245(2x^2-x+3)^{5/2}}{144(2x+5)^4}\right)+\frac{182165(2x^2-x+3)^{5/2}}{36(2x+5)^5}}{6912}-\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 1229}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{\int -\frac{6(13781234361-27596573612x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx}{1152}-\frac{(6793718806x+9802984711)(2x^2-x+3)^{3/2}}{96(2x+5)^3}\right)-\frac{14087245(2x^2-x+3)^{5/2}}{144(2x+5)^4}\right)+}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(-\frac{1}{192}\int \frac{(13781234361-27596573612x)\sqrt{2x^2-x+3}}{(2x+5)^2}dx-\frac{(6793718806x+9802984711)(2x^2-x+3)^{3/2}}{96(2x+5)^3}\right)-\frac{14087245(2x^2-x+3)^{5}}{144(2x+5)^4}\right.}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{8}\int \frac{2(317343544093-634652983296x)}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(27596573612x+151764102421)}{2(2x+5)}\right)-\frac{(6793718806x+9802984711)(}{96(2x+5)^3}\right.\right.}{6912}$$
$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\int\frac{317343544093-634652983296x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(27596573612x+151764102421)}{2(2x+5)}\right)-\frac{(6793718806x+9802984711)(2x}{96(2x+5)^3}\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-317326491648\int\frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(27596573612x+15}{2(2x+5)}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-158663245824\sqrt{\frac{2}{23}}\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-x}}{}\right.\right.\right.}{69}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(1903976002333\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(27596573612}{2(2x+5)}\right.\right.\right.}{6912}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(-3807952004666\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(2}{}\right.\right.\right.}{691}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{3456(2x+5)^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{72}\left(\frac{1}{288}\left(\frac{1}{192}\left(\frac{1}{4}\left(-158663245824\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{1903976002333\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(2759657}{2(2}\right.\right.\right.}{6912}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{3456(2x+5)^6}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^7,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(3456*(5 + 2*x)^6) + ((182165*(3 - x + 2*x^2
)^(5/2))/(36*(5 + 2*x)^5) + ((-14087245*(3 - x + 2*x^2)^(5/2))/(144*(5 + 2
*x)^4) + (-1/96*((9802984711 + 6793718806*x)*(3 - x + 2*x^2)^(3/2))/(5 + 2
*x)^3 + (((151764102421 + 27596573612*x)*Sqrt[3 - x + 2*x^2])/(2*(5 + 2*x)
) + (-158663245824*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (1903976002333*A
rcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/4)/192)
/288)/72)/6912
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.17

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{1903958949888\sqrt{2}(64x^6+960x^5+6000x^4+20000x$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
fricas")
```

output
```
1/2641807540224*(1903958949888*sqrt(2)*(64*x^6 + 960*x^5 + 6000*x^4 + 2000
0*x^3 + 37500*x^2 + 37500*x + 15625)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 1903976002333*sqrt(2)*(64*x^6 + 960*x^5 + 6
000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625)*log(-(24*sqrt(2)*sqrt(2
*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 + 20*x + 25))
 + 48*(275188285440*x^6 + 11854023276320*x^5 + 103803827945872*x^4 + 42255
4114856528*x^3 + 910256842473992*x^2 + 1011372787716826*x + 45841162535458
1)*sqrt(2*x^2 - x + 3))/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x
^2 + 37500*x + 15625)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx=\int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^7}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**7,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.52

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{3607708597}{1486016741376}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{3456\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)} + \frac{182165\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{248832\left(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125\right)} - \frac{14087245\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{71663616\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)} + \frac{149610673\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5159780352\left(8\,x^3+60\,x^2+150\,x+125\right)} - \frac{3607708597\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{743008370688\left(4\,x^2+20\,x+25\right)} - \frac{82772668391}{990677827584}\sqrt{2\,x^2-x+3}x - \frac{369}{256}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) + \frac{1903976002333}{1320903770112}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{165562389227}{330225942528}\sqrt{2\,x^2-x+3} + \frac{125860542215\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{2972033482752\left(2\,x+5\right)}$$

input

```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
maxima")
```

output

```
3607708597/1486016741376*(2*x^2 - x + 3)^(3/2) - 3667/3456*(2*x^2 - x + 3)
^(5/2)/(64*x^6 + 960*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15
625) + 182165/248832*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 +
5000*x^2 + 6250*x + 3125) - 14087245/71663616*(2*x^2 - x + 3)^(5/2)/(16*x^
4 + 160*x^3 + 600*x^2 + 1000*x + 625) + 149610673/5159780352*(2*x^2 - x +
3)^(5/2)/(8*x^3 + 60*x^2 + 150*x + 125) - 3607708597/743008370688*(2*x^2 -
 x + 3)^(5/2)/(4*x^2 + 20*x + 25) - 82772668391/990677827584*sqrt(2*x^2 -
x + 3)*x - 369/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1903
976002333/1320903770112*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17
/23*sqrt(23)/abs(2*x + 5)) + 165562389227/330225942528*sqrt(2*x^2 - x + 3)
 + 125860542215/2972033482752*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(160) = 320$.

Time = 0.20 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.32

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \frac{369}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{1903976002333}{1320903770112}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{5}{64}\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(159278433934432\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{11} + 6347903280912544\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{10} + 48\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x, algorithm="
giac")
```

output
```
369/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 19
03976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(
2*x^2 - x + 3))) + 1903976002333/1320903770112*sqrt(2)*log(abs(-2*sqrt(2)*
x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 5/64*sqrt(2*x^2 - x + 3) + 1/11
0075314176*sqrt(2)*(159278433934432*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
3))^11 + 6347903280912544*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^10 + 485445268
40833424*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^9 + 305716670132783088*
(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 + 8831821135911024*sqrt(2)*(sqrt(2)*x
 - sqrt(2*x^2 - x + 3))^7 - 2423668581998843376*(sqrt(2)*x - sqrt(2*x^2 -
x + 3))^6 - 3972113169703205 6*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5
 + 11708897232532299576*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 128034848607
28491138*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 1259303319786757723
4*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 3042533760672408875*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 589526263249780195)/(2*(sqrt(2)*x - sqrt(2*x
^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^7}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 609, normalized size of antiderivative = 3.12

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^7}\,dx = \text{Too large to display}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^7,x)
```

output
```
(6604518850560*sqrt(2*x**2 - x + 3)*x**6 + 284496558631680*sqrt(2*x**2 - x
 + 3)*x**5 + 2491291870700928*sqrt(2*x**2 - x + 3)*x**4 + 1014129875655667
2*sqrt(2*x**2 - x + 3)*x**3 + 21846164219375808*sqrt(2*x**2 - x + 3)*x**2
+ 24272946905203824*sqrt(2*x**2 - x + 3)*x + 11001879008509944*sqrt(2*x**2
 - x + 3) + 121854464149312*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) +
22*x - 17)*x**6 + 1827816962239680*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqr
t(2) + 22*x - 17)*x**5 + 11423856013998000*sqrt(2)*log(12*sqrt(2*x**2 - x
+ 3)*sqrt(2) + 22*x - 17)*x**4 + 38079520046660000*sqrt(2)*log(12*sqrt(2*x
**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 71399100087487500*sqrt(2)*log(12*
sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 71399100087487500*sqrt(2)
*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 29749625036453125*sq
rt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 121853372792832*s
qrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**6 + 18278005918924
80*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**5 + 1142375369
9328000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 38079
178997760000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 +
71398460620800000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x*
*2 + 71398460620800000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x +
1)*x + 29749358592000000*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x
+ 1) - 121854464149312*sqrt(2)*log(2*x + 5)*x**6 - 1827816962239680*sqr...
```

3.178 $\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 195

$$\begin{aligned}\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx= & -\frac{(146583836191+101679102454x)\sqrt{3-x+2x^2}}{440301256704(5+2x)^2} \\ & -\frac{(463558457+411822458x)(3-x+2x^2)^{3/2}}{2293235712(5+2x)^4}-\frac{3667(3-x+2x^2)^{5/2}}{4032(5+2x)^7} \\ & +\frac{114335(3-x+2x^2)^{5/2}}{193536(5+2x)^6}-\frac{1930441(3-x+2x^2)^{5/2}}{13934592(5+2x)^5} \\ & -\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}+\frac{412760561351\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5283615080448\sqrt{2}}\end{aligned}$$

output
```
-1/440301256704*(146583836191+101679102454*x)*(2*x^2-x+3)^(1/2)/(5+2*x)^2-
1/2293235712*(463558457+411822458*x)*(2*x^2-x+3)^(3/2)/(5+2*x)^4-3667/4032
*(2*x^2-x+3)^(5/2)/(5+2*x)^7+114335/193536*(2*x^2-x+3)^(5/2)/(5+2*x)^6-193
0441/13934592*(2*x^2-x+3)^(5/2)/(5+2*x)^5-5/128*arcsinh(1/23*(1-4*x)*23^(1
/2))*2^(1/2)+412760561351/10567230160896*arctanh(1/24*(17-22*x)*2^(1/2)/(2
*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.62

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{12\sqrt{3-x+2x^2}(3479517268702637+9065154700300572x+99760653674 98}{}$$

input
```
Integrate[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^
8,x]
```

output
```
((-12*Sqrt[3 - x + 2*x^2]*(3479517268702637 + 9065154700300572*x + 9976065
367498188*x^2 + 5966329646300704*x^3 + 2069947287085104*x^4 + 402255822731
712*x^5 + 38463671680832*x^6))/(5 + 2*x)^7 - 2889323929457*Sqrt[2]*ArcTanh
[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1444738498560*Sqrt[2]*Log[1 - 4*x
+ 2*Sqrt[6 - 2*x + 4*x^2]])/36985305563136
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2181, 27, 2181, 27, 2181, 27, 1229, 27, 1229, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x^2-x+3)^{3/2}(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}dx$$

$$\Big\downarrow \text{ 2181}$$

$$-\frac{1}{504}\int \frac{(2x^2-x+3)^{3/2}(-20160x^3+54432x^2-118840x+76715)}{16(2x+5)^7}dx-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(2x^2-x+3)^{3/2}(-20160x^3+54432x^2-118840x+76715)}{(2x+5)^7}dx}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{2181}$$

$$\frac{\frac{1}{432}\int \frac{9(2x^2-x+3)^{3/2}(483840x^2-2058628x+1481635)}{(2x+5)^6}dx+\frac{114335(2x^2-x+3)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{48}\int \frac{(2x^2-x+3)^{3/2}(483840x^2-2058628x+1481635)}{(2x+5)^6}dx+\frac{114335(2x^2-x+3)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{2181}$$

$$\frac{\frac{1}{48}\left(-\frac{1}{360}\int \frac{35(1640279-2488320x)(2x^2-x+3)^{3/2}}{(2x+5)^5}dx-\frac{1930441(2x^2-x+3)^{5/2}}{36(2x+5)^5}\right)+\frac{114335(2x^2-x+3)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\int \frac{(1640279-2488320x)(2x^2-x+3)^{3/2}}{(2x+5)^5}dx-\frac{1930441(2x^2-x+3)^{5/2}}{36(2x+5)^5}\right)+\frac{114335(2x^2-x+3)^{5/2}}{24(2x+5)^6}}{8064}-\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{1229}$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{(411822458x+463558457)(2x^2-x+3)^{3/2}}{576(2x+5)^4}-\frac{\int -\frac{6(300244177-477757440x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx}{2304}\right)-\frac{1930441(2x^2-x+3)^{5/2}}{36(2x+5)^5}\right)+\frac{114335\ldots}{2\ldots}}{8064} \ldots \frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\downarrow \text{27}$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\int \frac{(300244177-477757440x)\sqrt{2x^2-x+3}}{(2x+5)^3}dx+\frac{(411822458x+463558457)(2x^2-x+3)^{3/2}}{576(2x+5)^4}\right)-\frac{1930441(2x^2-x+3)^{5/2}}{36(2x+5)^5}\right)+\cdots}{8064} \ldots \frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 1229

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{(101679102454x+146583836191)\sqrt{2x^2-x+3}}{288(2x+5)^2}-\frac{\int -\frac{2(68775204551-137594142720x)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}\right)+\frac{(411822458x+463558457)(2x^2-}{576(2x+5)^4}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 27

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\int \frac{68775204551-137594142720x}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{\sqrt{2x^2-x+3}(101679102454x+146583836191)}{288(2x+5)^2}\right)+\frac{(411822458x+463558457)(2}{576(2x+5)^4}\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 1269

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-68797071360\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+\frac{\sqrt{2x^2-x+3}(101679102454x+}{288(2x+5)^2}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 1090

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-34398535680\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)+\frac{\sqrt{2x^2-}}{}\right.\right.\right.}{80}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 222

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(412760561351\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(10167910245}{288(2x+}\right.\right.\right.}{8064}$$

$$\frac{3667\left(2x^2-x+3\right)^{5/2}}{4032(2x+5)^7}$$

$\downarrow$ 1154

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(-825521122702\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)+\frac{\sqrt{2x^2-x+3}(1}{}\right.\right.\right.}{8064}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{48}\left(-\frac{7}{72}\left(\frac{1}{384}\left(\frac{1}{576}\left(-34398535680\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{412760561351\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)+\frac{\sqrt{2x^2-x+3}(101679}{288}\right.\right.\right.}{8064}$$

$$\frac{3667(2x^2-x+3)^{5/2}}{4032(2x+5)^7}$$

input
```
Int[((3 - x + 2*x^2)^(3/2)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(5 + 2*x)^8,x]
```

output
```
(-3667*(3 - x + 2*x^2)^(5/2))/(4032*(5 + 2*x)^7) + ((114335*(3 - x + 2*x^2
)^(5/2))/(24*(5 + 2*x)^6) + ((-1930441*(3 - x + 2*x^2)^(5/2))/(36*(5 + 2*x
)^5) - (7*(((463558457 + 411822458*x)*(3 - x + 2*x^2)^(3/2))/(576*(5 + 2*x
)^4) + (((146583836191 + 101679102454*x)*Sqrt[3 - x + 2*x^2])/(288*(5 + 2*
x)^2) + (-34398535680*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (412760561351
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/576)/
384))/72)/48)/8064
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

output
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.25

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \frac{2889476997120\sqrt{2}(128x^7+2240x^6+16800x^5+7000}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
fricas")
```

output
```
1/147941222252544*(2889476997120*sqrt(2)*(128*x^7 + 2240*x^6 + 16800*x^5 +
 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*x + 78125)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 2889323929457*sqrt(2)*
(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 21
8750*x + 78125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2
 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(38463671680832*x^6 + 40225582
2731712*x^5 + 2069947287085104*x^4 + 5966329646300704*x^3 + 99760653674981
88*x^2 + 9065154700300572*x + 3479517268702637)*sqrt(2*x^2 - x + 3))/(128*
x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*x^2 + 218750*
x + 78125)
```

Sympy [F]

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{(2x^2-x+3)^{\frac{3}{2}}\cdot(5x^4-x^3+3x^2+x+2)}{(2x+5)^8}\,dx$$

input
```
integrate((2*x**2-x+3)**(3/2)*(5*x**4-x**3+3*x**2+x+2)/(5+2*x)**8,x)
```

output
```
Integral((2*x**2 - x + 3)**(3/2)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x + 5
)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.78

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = -\frac{769352975}{11888133931008}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{3667\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{4032\left(128\,x^7+2240\,x^6+16800\,x^5+70000\,x^4+175000\,x^3+262500\,x^2+218750\,x+78125\right)} + \frac{114335\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{193536\left(64\,x^6+960\,x^5+6000\,x^4+20000\,x^3+37500\,x^2+37500\,x+15625\right)} - \frac{1930441\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{13934592\left(32\,x^5+400\,x^4+2000\,x^3+5000\,x^2+6250\,x+3125\right)} + \frac{7861079\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{573308928\left(16\,x^4+160\,x^3+600\,x^2+1000\,x+625\right)} - \frac{32967491\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{41278242816\left(8\,x^3+60\,x^2+150\,x+125\right)} + \frac{769352975\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5944066965504\left(4\,x^2+20\,x+25\right)} + \frac{17957520133}{7925422620672}\sqrt{2\,x^2-x+3}x + \frac{5}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{412760561351}{10567230160896}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{35893173457}{2641807540224}\sqrt{2\,x^2-x+3} - \frac{27452157541\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{23776267862016\left(2\,x+5\right)}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
maxima")
```

output
```
-769352975/11888133931008*(2*x^2 - x + 3)^(3/2) - 3667/4032*(2*x^2 - x + 3
)^(5/2)/(128*x^7 + 2240*x^6 + 16800*x^5 + 70000*x^4 + 175000*x^3 + 262500*
x^2 + 218750*x + 78125) + 114335/193536*(2*x^2 - x + 3)^(5/2)/(64*x^6 + 96
0*x^5 + 6000*x^4 + 20000*x^3 + 37500*x^2 + 37500*x + 15625) - 1930441/1393
4592*(2*x^2 - x + 3)^(5/2)/(32*x^5 + 400*x^4 + 2000*x^3 + 5000*x^2 + 6250*
x + 3125) + 7861079/573308928*(2*x^2 - x + 3)^(5/2)/(16*x^4 + 160*x^3 + 60
0*x^2 + 1000*x + 625) - 32967491/41278242816*(2*x^2 - x + 3)^(5/2)/(8*x^3
+ 60*x^2 + 150*x + 125) + 769352975/5944066965504*(2*x^2 - x + 3)^(5/2)/(4
*x^2 + 20*x + 25) + 17957520133/7925422620672*sqrt(2*x^2 - x + 3)*x + 5/12
8*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 412760561351/10567230
160896*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(
2*x + 5)) - 35893173457/2641807540224*sqrt(2*x^2 - x + 3) - 27452157541/23
776267862016*(2*x^2 - x + 3)^(3/2)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(160) = 320$.

Time = 0.27 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.51

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx =$$
$$-\frac{5}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$
$$+\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{412760561351}{10567230160896}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$
$$-\frac{\sqrt{2}\left(1121897398412224\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{13}+48260296303776704\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^{12}+\cdots\right.}{}$$

input
```
integrate((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x, algorithm="
giac")
```

output
```
-5/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 412
760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2
*x^2 - x + 3))) - 412760561351/10567230160896*sqrt(2)*log(abs(-2*sqrt(2)*x
 - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3))) - 1/6164217593856*sqrt(2)*(1121897
398412224*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^13 + 48260296303776704
*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^12 + 444673458321712704*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3))^11 + 3996455936659982656*(sqrt(2)*x - sqrt(2*x^
2 - x + 3))^10 + 6725227967167489360*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3))^9 - 17132661028483948080*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^8 - 637130
12094737246112*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^7 + 1065158801360
64432096*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^6 + 226947197958946260516*sqrt(
2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 - 856601202771483308188*(sqrt(2)*x
- sqrt(2*x^2 - x + 3))^4 + 617998258357377713732*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^3 - 467121785339763351756*(sqrt(2)*x - sqrt(2*x^2 - x + 3
))^2 + 92292080735560562227*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 15
161716093827501349)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(s
qrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \int \frac{(2\,x^2-x+3)^{3/2}\,(5\,x^4-x^3+3\,x^2+x+2)}{(2\,x+5)^8}\,dx$$

input
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8,x)
```

output
```
int(((2*x^2 - x + 3)^(3/2)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x + 5)^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.50

$$\int \frac{(3-x+2x^2)^{3/2}(2+x+3x^2-x^3+5x^4)}{(5+2x)^8}\,dx = \text{Too large to display}$$

input
```
int((2*x^2-x+3)^(3/2)*(5*x^4-x^3+3*x^2+x+2)/(5+2*x)^8,x)
```

output
```
( - 923128120339968*sqrt(2*x**2 - x + 3)*x**6 - 9654139745561088*sqrt(2*x*
*2 - x + 3)*x**5 - 49678734890042496*sqrt(2*x**2 - x + 3)*x**4 - 143191911
511216896*sqrt(2*x**2 - x + 3)*x**3 - 239425568819956512*sqrt(2*x**2 - x +
 3)*x**2 - 217563712807213728*sqrt(2*x**2 - x + 3)*x - 83508414448863288*s
qrt(2*x**2 - x + 3) + 369833462970496*sqrt(2)*log( - 12*sqrt(2*x**2 - x +
3)*sqrt(2) + 22*x - 17)*x**7 + 6472085601983680*sqrt(2)*log( - 12*sqrt(2*x
**2 - x + 3)*sqrt(2) + 22*x - 17)*x**6 + 48540642014877600*sqrt(2)*log( -
12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**5 + 202252675061990000*sqr
t(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 5056316876
54975000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3
+ 758447531482462500*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x
 - 17)*x**2 + 632039609568718750*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sq
rt(2) + 22*x - 17)*x + 225728431988828125*sqrt(2)*log( - 12*sqrt(2*x**2 -
x + 3)*sqrt(2) + 22*x - 17) + 369853055631360*sqrt(2)*log( - 2*sqrt(2*x**2
 - x + 3)*sqrt(2) - 4*x + 1)*x**7 + 6472428473548800*sqrt(2)*log( - 2*sqrt
(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**6 + 48543213551616000*sqrt(2)*log(
- 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**5 + 202263389798400000*sqrt
(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**4 + 50565847449600
0000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 75848
7711744000000*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*...
```

3.179 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 145

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = -\frac{3967(5+2x)^2\sqrt{3-x+2x^2}}{15360} + \frac{5253(5+2x)^3\sqrt{3-x+2x^2}}{2560} - \frac{619}{960}(5+2x)^4\sqrt{3-x+2x^2} + \frac{5}{96}(5+2x)^5\sqrt{3-x+2x^2} + \frac{(2911373+58268x)\sqrt{3-x+2x^2}}{122880} - \frac{1888339\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16384\sqrt{2}}$$

output

```
-3967/15360*(5+2*x)^2*(2*x^2-x+3)^(1/2)+5253/2560*(5+2*x)^3*(2*x^2-x+3)^(1
/2)-619/960*(5+2*x)^4*(2*x^2-x+3)^(1/2)+5/96*(5+2*x)^5*(2*x^2-x+3)^(1/2)+1
/122880*(2911373+58268*x)*(2*x^2-x+3)^(1/2)-1888339/32768*arcsinh(1/23*(1-
4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.52

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}} dx$$
$$= \frac{4\sqrt{3-x+2x^2}(4115973-1986852x-537504x^2+2140032x^3+1292288x^4+204800x^5)-28325085\sqrt{}}{491520}$$

input
```
Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],
x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(4115973 - 1986852*x - 537504*x^2 + 2140032*x^3 + 1
292288*x^4 + 204800*x^5) - 28325085*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
 4*x^2]])/491520
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2184, 25, 2184, 27, 2184, 27, 1236, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)^2 (5x^4-x^3+3x^2+x+2)}{\sqrt{2x^2-x+3}} dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{192}\int -\frac{(2x+5)^2 (4952x^3+6324x^2+5058x+6491)}{\sqrt{2x^2-x+3}}dx+\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{96}(2x+5)^5\sqrt{2x^2-x+3}-\frac{1}{192}\int \frac{(2x+5)^2 (4952x^3+6324x^2+5058x+6491)}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{192}\left(-\frac{1}{80}\int -\frac{24(2x+5)^2\left(21012x^2+2948x+22725\right)}{\sqrt{2x^2-x+3}}dx-\frac{619}{5}\sqrt{2x^2-x+3}(2x+5)^4\right)+\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 27$$

$$\frac{1}{192}\left(\frac{3}{10}\int \frac{(2x+5)^2\left(21012x^2+2948x+22725\right)}{\sqrt{2x^2-x+3}}dx-\frac{619}{5}(2x+5)^4\sqrt{2x^2-x+3}\right)+\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 2184$$

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{32}\int \frac{4(18957-7934x)(2x+5)^2}{\sqrt{2x^2-x+3}}dx+\frac{5253}{4}\sqrt{2x^2-x+3}(2x+5)^3\right)-\frac{619}{5}(2x+5)^4\sqrt{2x^2-x+3}\right)$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 27$$

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{8}\int \frac{(18957-7934x)(2x+5)^2}{\sqrt{2x^2-x+3}}dx+\frac{5253}{4}\sqrt{2x^2-x+3}(2x+5)^3\right)-\frac{619}{5}(2x+5)^4\sqrt{2x^2-x+3}\right)+$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 1236$$

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\int \frac{(2x+5)(29134x+644083)}{\sqrt{2x^2-x+3}}dx-\frac{3967}{3}(2x+5)^2\sqrt{2x^2-x+3}\right)+\frac{5253}{4}\sqrt{2x^2-x+3}(2x+5\right.\right.$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 1225$$

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{28325085}{8}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{1}{4}\sqrt{2x^2-x+3}(58268x+2911373)\right)-\frac{3967}{3}(2x+5)^2\sqrt{2x^2\right.\right.\right.$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{28325085\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{8\sqrt{46}}+\frac{1}{4}\sqrt{2x^2-x+3}(58268x+2911373)\right)-\frac{3967}{3}(2x+5)^2\right.\right.\right.\right.$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

$\Big\downarrow$ 222

$$\frac{1}{192}\left(\frac{3}{10}\left(\frac{1}{8}\left(\frac{1}{6}\left(\frac{28325085\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{1}{4}\sqrt{2x^2-x+3}(58268x+2911373)\right)-\frac{3967}{3}(2x+5)^2\sqrt{2x^2-x}\right.\right.\right.\right.$$
$$\frac{5}{96}\sqrt{2x^2-x+3}(2x+5)^5$$

input
```
Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

output
```
(5*(5 + 2*x)^5*Sqrt[3 - x + 2*x^2])/96 + ((-619*(5 + 2*x)^4*Sqrt[3 - x + 2
*x^2])/5 + (3*((5253*(5 + 2*x)^3*Sqrt[3 - x + 2*x^2])/4 + ((-3967*(5 + 2*x
)^2*Sqrt[3 - x + 2*x^2])/3 + (((2911373 + 58268*x)*Sqrt[3 - x + 2*x^2])/4
+ (28325085*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2]))/6)/8))/10)/192
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{(204800x^5+1292288x^4+2140032x^3-537504x^2-1986852x+4115973)\sqrt{2x^2-x+3}}{122880}+\frac{1888339\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}$
trager	$\left(\frac{5}{3}x^5+\frac{631}{60}x^4+\frac{5573}{320}x^3-\frac{5599}{1280}x^2-\frac{165571}{10240}x+\frac{1371991}{40960}\right)\sqrt{2x^2-x+3}-\frac{1888339\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\right.}{}$
default	$\frac{1888339\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{32768}+\frac{1371991\sqrt{2x^2-x+3}}{40960}-\frac{165571x\sqrt{2x^2-x+3}}{10240}-\frac{5599x^2\sqrt{2x^2-x+3}}{1280}+\frac{5573x^3\sqrt{2x^2-x}}{320}$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/122880*(204800*x^5+1292288*x^4+2140032*x^3-537504*x^2-1986852*x+4115973)
*(2*x^2-x+3)^(1/2)+1888339/32768*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.54

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$$
$$=\frac{1}{122880}\left(204800\,x^5+1292288\,x^4+2140032\,x^3-537504\,x^2-1986852\,x+4115973\right)\sqrt{2\,x^2-x+3}$$
$$+\frac{1888339}{65536}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/122880*(204800*x^5 + 1292288*x^4 + 2140032*x^3 - 537504*x^2 - 1986852*x
+ 4115973)*sqrt(2*x^2 - x + 3) + 1888339/65536*sqrt(2)*log(-4*sqrt(2)*sqrt
(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.48

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^5}{3}+\frac{631x^4}{60}+\frac{5573x^3}{320}-\frac{5599x^2}{1280}-\frac{165571x}{10240}+\frac{1371991}{40960}\right)+\frac{1888339\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}(x-\frac{1}{4})}{23}\right)}{32768}$$

input
```
integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**5/3 + 631*x**4/60 + 5573*x**3/320 - 5599*x**2/1
280 - 165571*x/10240 + 1371991/40960) + 1888339*sqrt(2)*asinh(4*sqrt(23)*(
x - 1/4)/23)/32768
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{3}\sqrt{2x^2-x+3}x^5+\frac{631}{60}\sqrt{2x^2-x+3}x^4+\frac{5573}{320}\sqrt{2x^2-x+3}x^3-\frac{5599}{1280}\sqrt{2x^2-x+3}x^2-\frac{165571}{10240}\sqrt{2x^2-x+3}x+\frac{1888339}{32768}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1371991}{40960}\sqrt{2x^2-x+3}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
5/3*sqrt(2*x^2 - x + 3)*x^5 + 631/60*sqrt(2*x^2 - x + 3)*x^4 + 5573/320*sq
rt(2*x^2 - x + 3)*x^3 - 5599/1280*sqrt(2*x^2 - x + 3)*x^2 - 165571/10240*s
qrt(2*x^2 - x + 3)*x + 1888339/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
1)) + 1371991/40960*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.50

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{1}{122880}\,(4\,(8\,(4\,(16\,(100\,x+631)x+16719)x-16797)x-496713)x+4115973)\sqrt{2\,x^2-x+3}$$
$$-\frac{1888339}{32768}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
1/122880*(4*(8*(4*(16*(100*x + 631)*x + 16719)*x - 16797)*x - 496713)*x +
4115973)*sqrt(2*x^2 - x + 3) - 1888339/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(
2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \int \frac{(2\,x+5)^2\,(5\,x^4-x^3+3\,x^2+x+2)}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.84

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{5\sqrt{2x^2-x+3}\,x^5}{3} + \frac{631\sqrt{2x^2-x+3}\,x^4}{60} + \frac{5573\sqrt{2x^2-x+3}\,x^3}{320} - \frac{5599\sqrt{2x^2-x+3}\,x^2}{1280} - \frac{165571\sqrt{2x^2-x+3}\,x}{10240} + \frac{1371991\sqrt{2x^2-x+3}}{40960} + \frac{1888339\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{32768}$$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

output
```
(819200*sqrt(2*x**2 - x + 3)*x**5 + 5169152*sqrt(2*x**2 - x + 3)*x**4 + 85
60128*sqrt(2*x**2 - x + 3)*x**3 - 2150016*sqrt(2*x**2 - x + 3)*x**2 - 7947
408*sqrt(2*x**2 - x + 3)*x + 16463892*sqrt(2*x**2 - x + 3) + 28325085*sqrt
(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/491520
```

3.180 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 120

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{761}{256}(5+2x)^2\sqrt{3-x+2x^2} - \frac{105}{128}(5+2x)^3\sqrt{3-x+2x^2} + \frac{1}{16}(5+2x)^4\sqrt{3-x+2x^2} - \frac{(19227+4676x)\sqrt{3-x+2x^2}}{2048} - \frac{85429\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4096\sqrt{2}}$$

output
```
761/256*(5+2*x)^2*(2*x^2-x+3)^(1/2)-105/128*(5+2*x)^3*(2*x^2-x+3)^(1/2)+1/
16*(5+2*x)^4*(2*x^2-x+3)^(1/2)-1/2048*(19227+4676*x)*(2*x^2-x+3)^(1/2)-854
29/8192*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(2973-6916x+352x^2+7040x^3+2048x^4)-85429\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8192}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(2973 - 6916*x + 352*x^2 + 7040*x^3 + 2048*x^4) - 8
5429*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/8192
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {2184, 27, 2184, 27, 2184, 27, 1225, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{160}\int -\frac{5(2x+5)\left(840x^3+1116x^2+878x+1011\right)}{\sqrt{2x^2-x+3}}dx+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}(2x+5)^4\sqrt{2x^2-x+3}-\frac{1}{32}\int \frac{(2x+5)\left(840x^3+1116x^2+878x+1011\right)}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{32}\left(-\frac{1}{64}\int -\frac{8(2x+5)\left(9132x^2+2636x+8187\right)}{\sqrt{2x^2-x+3}}dx-\frac{105}{4}\sqrt{2x^2-x+3}(2x+5)^3\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{8}\int \frac{(2x+5)\left(9132x^2+2636x+8187\right)}{\sqrt{2x^2-x+3}}dx-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 2184

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{24}\int \frac{12(1915-2338x)(2x+5)}{\sqrt{2x^2-x+3}}dx+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\int \frac{(1915-2338x)(2x+5)}{\sqrt{2x^2-x+3}}dx+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2x+5)^3\sqrt{2x^2-x+3}\right)+\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 1225

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429}{8}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{10}{4}\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 1090

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{8\sqrt{46}}-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right.\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

$\Big\downarrow$ 222

$$\frac{1}{32}\left(\frac{1}{8}\left(\frac{1}{2}\left(\frac{85429\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{8\sqrt{2}}-\frac{1}{4}(4676x+19227)\sqrt{2x^2-x+3}\right)+761\sqrt{2x^2-x+3}(2x+5)^2\right)-\frac{105}{4}(2\right.$$
$$\frac{1}{16}\sqrt{2x^2-x+3}(2x+5)^4$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/Sqrt[3 - x + 2*x^2],x]
```

output
```
((5 + 2*x)^4*Sqrt[3 - x + 2*x^2])/16 + ((-105*(5 + 2*x)^3*Sqrt[3 - x + 2*x
^2])/4 + (761*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2] + (-1/4*((19227 + 4676*x)*Sq
rt[3 - x + 2*x^2]) + (85429*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8*Sqrt[2]))/2)/
8)/32
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.42

method	result
risch	$\frac{(2048x^4+7040x^3+352x^2-6916x+2973)\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$
trager	$\left(x^4+\frac{55}{16}x^3+\frac{11}{64}x^2-\frac{1729}{512}x+\frac{2973}{2048}\right)\sqrt{2x^2-x+3}+\frac{85429\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootO}\right.}{8192}$
default	$\frac{11x^2\sqrt{2x^2-x+3}}{64}-\frac{1729x\sqrt{2x^2-x+3}}{512}+\frac{2973\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{8192}+\frac{55x^3\sqrt{2x^2-x+3}}{16}+x^4\sqrt{}$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
1/2048*(2048*x^4+7040*x^3+352*x^2-6916*x+2973)*(2*x^2-x+3)^(1/2)+85429/819
2*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{2048}\left(2048\,x^4+7040\,x^3+352\,x^2-6916\,x+2973\right)\sqrt{2\,x^2-x+3} + \frac{85429}{16384}\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fr
icas")
```

output
```
1/2048*(2048*x^4 + 7040*x^3 + 352*x^2 - 6916*x + 2973)*sqrt(2*x^2 - x + 3)
 + 85429/16384*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\left(x^4+\frac{55x^3}{16}+\frac{11x^2}{64}-\frac{1729x}{512}+\frac{2973}{2048}\right)+\frac{85429\sqrt{2}\,\mathrm{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8192}$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(x**4 + 55*x**3/16 + 11*x**2/64 - 1729*x/512 + 2973/2
048) + 85429*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/8192
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2\,x^2-x+3}x^4 + \frac{55}{16}\,\sqrt{2\,x^2-x+3}x^3 + \frac{11}{64}\,\sqrt{2\,x^2-x+3}x^2 - \frac{1729}{512}\,\sqrt{2\,x^2-x+3}x + \frac{85429}{8192}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{2973}{2048}\,\sqrt{2\,x^2-x+3}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="ma
xima")
```

output
```
sqrt(2*x^2 - x + 3)*x^4 + 55/16*sqrt(2*x^2 - x + 3)*x^3 + 11/64*sqrt(2*x^2
 - x + 3)*x^2 - 1729/512*sqrt(2*x^2 - x + 3)*x + 85429/8192*sqrt(2)*arcsin
h(1/23*sqrt(23)*(4*x - 1)) + 2973/2048*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{2048}\,(4\,(8\,(4\,(16\,x+55)x+11)x-1729)x+2973)\sqrt{2\,x^2-x+3} - \frac{85429}{8192}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="gi
ac")
```

output
```
1/2048*(4*(8*(4*(16*x + 55)*x + 11)*x - 1729)*x + 2973)*sqrt(2*x^2 - x + 3
) - 85429/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) +
1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx=\int \frac{(2\,x+5)\,(5\,x^4-x^3+3\,x^2+x+2)}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{\sqrt{3-x+2x^2}}\,dx=\sqrt{2x^2-x+3}\,x^4+\frac{55\sqrt{2x^2-x+3}\,x^3}{16}+\frac{11\sqrt{2x^2-x+3}\,x^2}{64}-\frac{1729\sqrt{2x^2-x+3}\,x}{512}+\frac{2973\sqrt{2x^2-x+3}}{2048}+\frac{85429\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{8192}$$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

output
```
(8192*sqrt(2*x**2 - x + 3)*x**4 + 28160*sqrt(2*x**2 - x + 3)*x**3 + 1408*s
qrt(2*x**2 - x + 3)*x**2 - 27664*sqrt(2*x**2 - x + 3)*x + 11892*sqrt(2*x**
2 - x + 3) + 85429*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/
sqrt(23)))/8192
```

3.181 $\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}} dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = -\frac{505\sqrt{3-x+2x^2}}{1024}-\frac{409}{768}x\sqrt{3-x+2x^2}+\frac{19}{96}x^2\sqrt{3-x+2x^2}+\frac{5}{8}x^3\sqrt{3-x+2x^2}-\frac{6863\,\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

output
```
-505/1024*(2*x^2-x+3)^(1/2)-409/768*x*(2*x^2-x+3)^(1/2)+19/96*x^2*(2*x^2-x
+3)^(1/2)+5/8*x^3*(2*x^2-x+3)^(1/2)-6863/4096*arcsinh(1/23*(1-4*x)*23^(1/2
))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(-1515-1636x+608x^2+1920x^3)-20589\sqrt{2}\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{12288}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(-1515 - 1636*x + 608*x^2 + 1920*x^3) - 20589*Sqrt[
2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/12288
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\int \frac{19x^3-42x^2+16x+32}{2\sqrt{2x^2-x+3}}dx+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{19x^3-42x^2+16x+32}{\sqrt{2x^2-x+3}}dx+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{6}\int \frac{-409x^2-36x+384}{2\sqrt{2x^2-x+3}}dx+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\int \frac{-409x^2-36x+384}{\sqrt{2x^2-x+3}}dx+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{1}{4}\int \frac{3(1842-505x)}{2\sqrt{2x^2-x+3}}dx-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\int\frac{1842-505x}{\sqrt{2x^2-x+3}}dx-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}\left(\frac{1}{12}\left(\frac{3}{8}\left(\frac{6863\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{505}{2}\sqrt{2x^2-x+3}\right)-\frac{409}{4}x\sqrt{2x^2-x+3}\right)+\frac{19}{6}\sqrt{2x^2-x+3}x^2\right)+\frac{5}{8}\sqrt{2x^2-x+3}x^3$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/Sqrt[3 - x + 2*x^2],x]
```

output
```
(5*x^3*Sqrt[3 - x + 2*x^2])/8 + ((19*x^2*Sqrt[3 - x + 2*x^2])/6 + ((-409*x
*Sqrt[3 - x + 2*x^2])/4 + (3*((-505*Sqrt[3 - x + 2*x^2])/2 + (6863*ArcSinh
[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/12)/16
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$\frac{(1920x^3+608x^2-1636x-1515)\sqrt{2x^2-x+3}}{3072}+\frac{6863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$
trager	$\left(\frac{5}{8}x^3+\frac{19}{96}x^2-\frac{409}{768}x-\frac{505}{1024}\right)\sqrt{2x^2-x+3}+\frac{6863\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootOf}\left(\_Z^2-\right.\right.}{4096}$
default	$-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{6863\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}-\frac{409x\sqrt{2x^2-x+3}}{768}+\frac{19x^2\sqrt{2x^2-x+3}}{96}+\frac{5x^3\sqrt{2x^2-x+3}}{8}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3072*(1920*x^3+608*x^2-1636*x-1515)*(2*x^2-x+3)^(1/2)+6863/4096*2^(1/2)*
arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(1920\,x^3+608\,x^2-1636\,x-1515\right)\sqrt{2\,x^2-x+3} + \frac{6863}{8192}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2 + 16\,x-25\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

output
```
1/3072*(1920*x^3 + 608*x^2 - 1636*x - 1515)*sqrt(2*x^2 - x + 3) + 6863/819
2*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.55

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \sqrt{2x^2-x+3}\cdot\left(\frac{5x^3}{8}+\frac{19x^2}{96}-\frac{409x}{768}-\frac{505}{1024}\right) + \frac{6863\sqrt{2}\operatorname{asinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{4096}$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(1/2),x)
```

output
```
sqrt(2*x**2 - x + 3)*(5*x**3/8 + 19*x**2/96 - 409*x/768 - 505/1024) + 6863
*sqrt(2)*asinh(4*sqrt(23)*(x - 1/4)/23)/4096
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{8}\sqrt{2\,x^2-x+3}x^3+\frac{19}{96}\sqrt{2\,x^2-x+3}x^2 - \frac{409}{768}\sqrt{2\,x^2-x+3}x + \frac{6863}{4096}\sqrt{2}\operatorname{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{505}{1024}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

output
```
5/8*sqrt(2*x^2 - x + 3)*x^3 + 19/96*sqrt(2*x^2 - x + 3)*x^2 - 409/768*sqrt
(2*x^2 - x + 3)*x + 6863/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 5
05/1024*sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx = \frac{1}{3072}\left(4\left(8\left(60\,x+19\right)x-409\right)x-1515\right)\sqrt{2\,x^2-x+3} - \frac{6863}{4096}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

output
```
1/3072*(4*(8*(60*x + 19)*x - 409)*x - 1515)*sqrt(2*x^2 - x + 3) - 6863/409
6*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \frac{2+x+3x^2-x^3+5x^4}{\sqrt{3-x+2x^2}}\,dx=\frac{5\sqrt{2x^2-x+3}\,x^3}{8}+\frac{19\sqrt{2x^2-x+3}\,x^2}{96}-\frac{409\sqrt{2x^2-x+3}\,x}{768}-\frac{505\sqrt{2x^2-x+3}}{1024}+\frac{6863\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{4096}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(1/2),x)
```

output
```
(7680*sqrt(2*x**2 - x + 3)*x**3 + 2432*sqrt(2*x**2 - x + 3)*x**2 - 6544*sq
rt(2*x**2 - x + 3)*x - 6060*sqrt(2*x**2 - x + 3) + 20589*sqrt(2)*log((2*sq
rt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)))/12288
```

3.182 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 118

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{879}{128}\sqrt{3-x+2x^2}-\frac{137}{96}x\sqrt{3-x+2x^2}+\frac{5}{12}x^2\sqrt{3-x+2x^2}+\frac{9657\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{256\sqrt{2}}-\frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{96\sqrt{2}}$$

output
```
879/128*(2*x^2-x+3)^(1/2)-137/96*x*(2*x^2-x+3)^(1/2)+5/12*x^2*(2*x^2-x+3)^
(1/2)+9657/512*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/192*arctanh(1/2
4*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{4\sqrt{3-x+2x^2}(2637-548x+160x^2)+58672\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+28971\sqrt{2}\log}{1536}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(4*Sqrt[3 - x + 2*x^2]*(2637 - 548*x + 160*x^2) + 58672*Sqrt[2]*ArcTanh[(5
 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 28971*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6
- 2*x + 4*x^2]])/1536
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2184, 25, 2184, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\int -\frac{2696x^3+4092x^2+3054x+2183}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{48}(2x+5)^2\sqrt{2x^2-x+3}-\frac{1}{96}\int \frac{2696x^3+4092x^2+3054x+2183}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{1}{96}\left(-\frac{1}{32}\int -\frac{24\left(6676x^2+5364x+1021\right)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{337}{2}\sqrt{2x^2-x+3}(2x+5)\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 27$$

$$\frac{1}{96}\left(\frac{3}{4}\int \frac{6676x^2+5364x+1021}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 2184$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{8}\int \frac{4(10387-19314x)}{(2x+5)\sqrt{2x^2-x+3}}dx+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 27$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{10387-19314x}{(2x+5)\sqrt{2x^2-x+3}}dx+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\sqrt{2x^2-x+3}\right)+\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-9657\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+ \ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{9657\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{\sqrt{46}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 222$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(58672\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2x+5)\ldots\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \ 1154$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(-117344\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{96}\left(\frac{3}{4}\left(\frac{1}{2}\left(-\frac{9657\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{\sqrt{2}}-\frac{14668}{3}\sqrt{2}\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)\right)+1669\sqrt{2x^2-x+3}\right)-\frac{337}{2}(2\right.$$
$$\frac{5}{48}\sqrt{2x^2-x+3}(2x+5)^2$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(5*(5 + 2*x)^2*Sqrt[3 - x + 2*x^2])/48 + ((-337*(5 + 2*x)*Sqrt[3 - x + 2*x
^2])/2 + (3*(1669*Sqrt[3 - x + 2*x^2] + ((-9657*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/Sqrt[2] - (14668*Sqrt[2]*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x +
2*x^2])])/3)/2))/4)/96
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{1}{384}\left(160\,x^2-548\,x+2637\right)\sqrt{2\,x^2-x+3} + \frac{9657}{1024}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right) + \frac{3667}{384}\,\sqrt{2}\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+1060\,x^2-1036\,x+1153}{4\,x^2+20\,x+25}\right)$$

input

```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="fr
icas")
```

output

```
1/384*(160*x^2 - 548*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/1024*sqrt(2)*log
(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 3667/384*
sqrt(2)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036
*x + 1153)/(4*x^2 + 20*x + 25))
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\sqrt{2x^2-x+3}}\,dx$$

input

```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(1/2),x)
```

output

```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*sqrt(2*x**2 - x + 3))
, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.84

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{5}{12}\sqrt{2\,x^2-x+3}x^2 - \frac{137}{96}\sqrt{2\,x^2-x+3}x - \frac{9657}{512}\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\sqrt{23}x - \frac{1}{23}\sqrt{23}\right) + \frac{3667}{192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{879}{128}\sqrt{2\,x^2-x+3}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="ma
xima")
```

output
```
5/12*sqrt(2*x^2 - x + 3)*x^2 - 137/96*sqrt(2*x^2 - x + 3)*x - 9657/512*sqr
t(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 3667/192*sqrt(2)*arcsinh(2
2/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) + 879/128*sqrt
(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx = \frac{1}{384}(4\,(40\,x-137)x+2637)\sqrt{2\,x^2-x+3} + \frac{9657}{512}\sqrt{2}\log\left(-4\sqrt{2}x+\sqrt{2}+4\sqrt{2\,x^2-x+3}\right) - \frac{3667}{192}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) + \frac{3667}{192}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right)$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x, algorithm="gi
ac")
```

output
```
1/384*(4*(40*x - 137)*x + 2637)*sqrt(2*x^2 - x + 3) + 9657/512*sqrt(2)*log
(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/192*sqrt(2)*log(ab
s(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 3667/192*sqrt(2)*log(
abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.14

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\sqrt{3-x+2x^2}}\,dx=\frac{5\sqrt{2x^2-x+3}\,x^2}{12}-\frac{137\sqrt{2x^2-x+3}\,x}{96}+\frac{879\sqrt{2x^2-x+3}}{128}+\frac{3667\sqrt{2}\log\left(\frac{46\sqrt{2x^2-x+3}\,\sqrt{2}+92x-46}{\sqrt{23}}\right)}{192}-\frac{9657\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x-1}{\sqrt{23}}\right)}{512}-\frac{3667\sqrt{2}\log\left(\frac{2\sqrt{2x^2-x+3}\,\sqrt{2}+4x+22}{\sqrt{23}}\right)}{192}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(1/2),x)
```

output
```
(640*sqrt(2*x**2 - x + 3)*x**2 - 2192*sqrt(2*x**2 - x + 3)*x + 10548*sqrt(
2*x**2 - x + 3) + 29336*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2) + 92*
x - 46)/sqrt(23)) - 28971*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*
x - 1)/sqrt(23)) - 29336*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x
 + 22)/sqrt(23)))/1536
```

3.183 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 122

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = -\frac{193}{64}\sqrt{3-x+2x^2}+\frac{5}{16}x\sqrt{3-x+2x^2} - \frac{3667\sqrt{3-x+2x^2}}{576(5+2x)} - \frac{2943\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}} + \frac{158527\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{6912\sqrt{2}}$$

output
```
-193/64*(2*x^2-x+3)^(1/2)+5/16*x*(2*x^2-x+3)^(1/2)-3667*(2*x^2-x+3)^(1/2)/
(2880+1152*x)-2943/256*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+158527/13824
*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.82

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \frac{\frac{24\sqrt{3-x+2x^2}(-6176-1287x+180x^2)}{5+2x}-158527\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-79461\sqrt{2}\log\left(1-4x\right.}{6912}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((24*Sqrt[3 - x + 2*x^2]*(-6176 - 1287*x + 180*x^2))/(5 + 2*x) - 158527*Sq
rt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 79461*Sqrt[2]*Log[1 -
 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/6912
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2184, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{72}\int \frac{-2880x^3+7776x^2-21168x+12007}{16(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-2880x^3+7776x^2-21168x+12007}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{180(2x+5)\sqrt{2x^2-x+3}-\frac{1}{32}\int \frac{32\left(17496x^2-13608x+15157\right)}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{180(2x+5)\sqrt{2x^2-x+3}-\int \frac{17496x^2-13608x+15157}{(2x+5)\sqrt{2x^2-x+3}}dx}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{2184}$$

$$\frac{-\frac{1}{8}\int \frac{16(13046-26487x)}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{-2\int \frac{13046-26487x}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487}{2}\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{222}$$

$$\frac{-2\left(\frac{158527}{2}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-2\left(-158527\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+}}{1152}$$
$$\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

$$\Big\downarrow 219$$

$$\frac{-2\left(-\frac{26487\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{158527\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{12\sqrt{2}}\right)+180\sqrt{2x^2-x+3}(2x+5)-4374\sqrt{2x^2-x+3}}{1152}-$$
$$\frac{3667\sqrt{2x^2-x+3}}{576(2x+5)}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(576*(5 + 2*x)) + (-4374*Sqrt[3 - x + 2*x^2] +
 180*(5 + 2*x)*Sqrt[3 - x + 2*x^2] - 2*((-26487*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/(2*Sqrt[2]) - (158527*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x
^2])])/(12*Sqrt[2])))/1152
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.09

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{158922\sqrt{2}(2x+5)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+158527\sqrt{2}(2x+5)\log\Big(}{27648\,(2x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/27648*(158922*sqrt(2)*(2*x + 5)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) + 158527*sqrt(2)*(2*x + 5)*log((24*sqrt(2)*sqrt
(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25
)) + 96*(180*x^2 - 1287*x - 6176)*sqrt(2*x^2 - x + 3))/(2*x + 5)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*sqrt(2*x**2 - x +
3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \frac{5}{16}\sqrt{2\,x^2-x+3}x + \frac{2943}{256}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{158527}{13824}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{193}{64}\sqrt{2\,x^2-x+3} - \frac{3667\sqrt{2\,x^2-x+3}}{576\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
5/16*sqrt(2*x^2 - x + 3)*x + 2943/256*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/
23*sqrt(23)) - 158527/13824*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5)
- 17/23*sqrt(23)/abs(2*x + 5)) - 193/64*sqrt(2*x^2 - x + 3) - 3667/576*sqr
t(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(95) = 190.

Time = 0.18 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.78

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx = \frac{1}{13824}\sqrt{2}\left(\frac{158527\log\left(12\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}+\frac{72}{2\,x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)} + \frac{158922\log\left(\left|\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+\right.\right.}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
1/13824*sqrt(2)*(158527*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) +
72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 158922*log(abs(sqrt(-11/(2*x + 5) +
36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 158922*log(abs(
sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x +
5)) - 44004*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)/sgn(1/(2*x + 5)) + 10
8*(3393*(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^3 - 4896*
(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 743*sqrt(-11/
(2*x + 5) + 36/(2*x + 5)^2 + 1) - 4458/(2*x + 5) + 2256)/(((sqrt(-11/(2*x
+ 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5))^2 - 1)^2*sgn(1/(2*x + 5))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^2\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.43

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\sqrt{3-x+2x^2}}\,dx$$
$$=\frac{8640\sqrt{2x^2-x+3}\,x^2-61776\sqrt{2x^2-x+3}\,x-296448\sqrt{2x^2-x+3}+317054\sqrt{2}\log\left(-12\sqrt{2x^2-x+\right.}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(1/2),x)
```

output
```
(8640*sqrt(2*x**2 - x + 3)*x**2 - 61776*sqrt(2*x**2 - x + 3)*x - 296448*sq
rt(2*x**2 - x + 3) + 317054*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2)
 + 22*x - 17)*x + 792635*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) +
22*x - 17) + 317844*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x +
1)*x + 794610*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 3
17054*sqrt(2)*log(2*x + 5)*x - 792635*sqrt(2)*log(2*x + 5))/(13824*(2*x +
5))
```

3.184 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 128

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{5}{16}\sqrt{3-x+2x^2}-\frac{3667\sqrt{3-x+2x^2}}{1152(5+2x)^2}+\frac{92239\sqrt{3-x+2x^2}}{27648(5+2x)}+\frac{149\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}-\frac{1546507\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{331776\sqrt{2}}$$

output

```
5/16*(2*x^2-x+3)^(1/2)-3667/1152*(2*x^2-x+3)^(1/2)/(5+2*x)^2+92239*(2*x^2-
x+3)^(1/2)/(138240+55296*x)+149/64*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)-
1546507/663552*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(589187+357278x+34560x^2)}{(5+2x)^2}+1546507\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)+772416\sqrt{2}\log\left(1\right.}{331776}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(589187 + 357278*x + 34560*x^2))/(5 + 2*x)^2 + 15
46507*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 772416*Sqrt[2
]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/331776
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2181, 27, 2181, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{144}\int \frac{-5760x^3+15552x^2-27668x+20347}{16(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-5760x^3+15552x^2-27668x+20347}{(2x+5)^2\sqrt{2x^2-x+3}}dx}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int \frac{3(69120x^2-359424x+215947)}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{24}\int \frac{69120x^2-359424x+215947}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 2184

$$\frac{\frac{1}{24}\left(\frac{1}{8}\int \frac{8(259147-514944x)}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{24}\left(\int \frac{259147-514944x}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1269

$$\frac{\frac{1}{24}\left(-257472\int \frac{1}{\sqrt{2x^2-x+3}}dx+1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1090

$$\frac{\frac{1}{24}\left(1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-128736\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 222

$$\frac{\frac{1}{24}\left(1546507\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{24}\left(-3093014\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{24}\left(-128736\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{1546507\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}+17280\sqrt{2x^2-x+3}\right)+\frac{92239\sqrt{2x^2-x+3}}{12(2x+5)}}{2304}-\frac{3667\sqrt{2x^2-x+3}}{1152(2x+5)^2}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(1152*(5 + 2*x)^2) + ((92239*Sqrt[3 - x + 2*x^
2])/(12*(5 + 2*x)) + (17280*Sqrt[3 - x + 2*x^2] - 128736*Sqrt[2]*ArcSinh[(
-1 + 4*x)/Sqrt[23]] - (1546507*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x
+ 2*x^2])])/(6*Sqrt[2]))/24)/2304
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{1544832\sqrt{2}(4x^2+20x+25)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+1546507\sqrt{2}(4x^2}{1327104}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/1327104*(1544832*sqrt(2)*(4*x^2 + 20*x + 25)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 1546507*sqrt(2)*(4*x^2 + 20*x + 2
5)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x +
1153)/(4*x^2 + 20*x + 25)) + 48*(34560*x^2 + 357278*x + 589187)*sqrt(2*x^2
 - x + 3))/(4*x^2 + 20*x + 25)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*sqrt(2*x**2 - x +
3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx=&-\frac{149}{64}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right)\\&+\frac{1546507}{663552}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)\\&+\frac{5}{16}\sqrt{2\,x^2-x+3}-\frac{3667\sqrt{2\,x^2-x+3}}{1152\,(4\,x^2+20\,x+25)}\\&+\frac{92239\sqrt{2\,x^2-x+3}}{27648\,(2\,x+5)}\end{aligned}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
-149/64*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) + 1546507/663552*
sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5
)) + 5/16*sqrt(2*x^2 - x + 3) - 3667/1152*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*
x + 25) + 92239/27648*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(101) = 202$.

Time = 0.22 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.94

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \frac{149}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) - \frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{1546507}{663552}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{5}{16}\sqrt{2x^2-x+3} + \frac{\sqrt{2}\left(2381290\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+16628406\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-25697445\sqrt{2}\left(\sqrt{2}x\right.\right.}{55296\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
149/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 154
6507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)
)) + 1546507/663552*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x
^2 - x + 3))) + 5/16*sqrt(2*x^2 - x + 3) + 1/55296*sqrt(2)*(2381290*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 16628406*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 - 25697445*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 16720645
)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*
x^2 - x + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\sqrt{2x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.95

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{829440\sqrt{2x^2-x+3}\,x^2+8574672\sqrt{2x^2-x+3}\,x+14140488\sqrt{2x^2-x+3}+6186028\sqrt{2}\log\left(12\sqrt{2x^2\right.}}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(1/2),x)
```

output
```
(829440*sqrt(2*x**2 - x + 3)*x**2 + 8574672*sqrt(2*x**2 - x + 3)*x + 14140
488*sqrt(2*x**2 - x + 3) + 6186028*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqr
t(2) + 22*x - 17)*x**2 + 30930140*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt
(2) + 22*x - 17)*x + 38662675*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 22*x - 17) + 6179328*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x +
1)*x**2 + 30896640*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x
 + 38620800*sqrt(2)*log(2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 618602
8*sqrt(2)*log(2*x + 5)*x**2 - 30930140*sqrt(2)*log(2*x + 5)*x - 38662675*s
qrt(2)*log(2*x + 5))/(663552*(4*x**2 + 20*x + 25))
```

3.185 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{1728(5+2x)^3}+\frac{394907\sqrt{3-x+2x^2}}{248832(5+2x)^2} - \frac{3163415\sqrt{3-x+2x^2}}{5971968(5+2x)}-\frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} + \frac{22389491\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{71663616\sqrt{2}}$$

output
```
-3667/1728*(2*x^2-x+3)^(1/2)/(5+2*x)^3+394907/248832*(2*x^2-x+3)^(1/2)/(5+
2*x)^2-3163415*(2*x^2-x+3)^(1/2)/(29859840+11943936*x)-5/32*arcsinh(1/23*(
1-4*x)*23^(1/2))*2^(1/2)+22389491/143327232*arctanh(1/24*(17-22*x)*2^(1/2)
/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{-\frac{12\sqrt{3-x+2x^2}(44369687+44312764x+12653660x^2)}{(5+2x)^3}-22389491\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)-111974}{71663616}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((-12*Sqrt[3 - x + 2*x^2]*(44369687 + 44312764*x + 12653660*x^2))/(5 + 2*x
)^3 - 22389491*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] - 1119
7440*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/71663616
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2181, 27, 2181, 2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{216}\int \frac{-8640x^3+23328x^2-34168x+28687}{16(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-8640x^3+23328x^2-34168x+28687}{(2x+5)^3\sqrt{2x^2-x+3}}dx}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{622080x^2-1655188x+1464275}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{72}\int \frac{3(3727091-7464960x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{144}\left(-\frac{1}{24}\int \frac{3727091-7464960x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(3732480\int \frac{1}{\sqrt{2x^2-x+3}}dx-22389491\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-22389491\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\cdots}{72(\cdots}}{3456}$$
$$\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{222}$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-22389491\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456}-\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(44778982\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}+1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x}}{72(2x+5)^2}}{3456}$$
$$\frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{144}\left(\frac{1}{24}\left(1866240\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+\frac{22389491\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{3163415\sqrt{2x^2-x+3}}{12(2x+5)}\right)+\frac{394907\sqrt{2x^2-x+3}}{72(2x+5)^2}}{3456} - \frac{3667\sqrt{2x^2-x+3}}{1728(2x+5)^3}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(1728*(5 + 2*x)^3) + ((394907*Sqrt[3 - x + 2*x
^2])/(72*(5 + 2*x)^2) + ((-3163415*Sqrt[3 - x + 2*x^2])/(12*(5 + 2*x)) + (
1866240*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (22389491*ArcTanh[(17 - 22*
x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/24)/144)/3456
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.21

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{22394880\sqrt{2}(8x^3+60x^2+150x+125)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)+223}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/286654464*(22394880*sqrt(2)*(8*x^3 + 60*x^2 + 150*x + 125)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 22389491*sqrt(2)*(
8*x^3 + 60*x^2 + 150*x + 125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x -
17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(12653660*x^2 +
44312764*x + 44369687)*sqrt(2*x^2 - x + 3))/(8*x^3 + 60*x^2 + 150*x + 125)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*sqrt(2*x**2 - x +
3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{5}{32}\sqrt{2}\,\text{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{22389491}{143327232}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{3667\sqrt{2\,x^2-x+3}}{1728\,(8\,x^3+60\,x^2+150\,x+125)} + \frac{394907\sqrt{2\,x^2-x+3}}{248832\,(4\,x^2+20\,x+25)} - \frac{3163415\sqrt{2\,x^2-x+3}}{5971968\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
5/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 22389491/143327232
*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x +
5)) - 3667/1728*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*x^2 + 150*x + 125) + 39490
7/248832*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x + 25) - 3163415/5971968*sqrt(2*
x^2 - x + 3)/(2*x + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(108) = 216$.

Time = 0.17 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.11

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = -\frac{5}{32}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{22389491}{143327232}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2\,x^2-x+3}\right|\right) - \frac{\sqrt{2}\left(215012404\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^5+3010410772\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)^4+2740802468\sqrt{\cdots}\right.}{11943936\left(2\left(\sqrt{2}x-\sqrt{2\,x\cdots}\right.\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
-5/32*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 2238
9491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x +
 3))) - 22389491/143327232*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*s
qrt(2*x^2 - x + 3))) - 1/11943936*sqrt(2)*(215012404*sqrt(2)*(sqrt(2)*x -
sqrt(2*x^2 - x + 3))^5 + 3010410772*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 +
2740802468*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 - 21459328844*(sqrt
(2)*x - sqrt(2*x^2 - x + 3))^2 + 14434519361*sqrt(2)*(sqrt(2)*x - sqrt(2*x
^2 - x + 3)) - 5957650879)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqr
t(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)^4\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.39

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\sqrt{3-x+2x^2}}\,dx = \frac{-303687840\sqrt{2x^2-x+3}\,x^2-1063506336\sqrt{2x^2-x+3}\,x-1064872488\sqrt{2x^2-x+3}+179115928\sqrt{}}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(1/2),x)
```

output
```
( - 303687840*sqrt(2*x**2 - x + 3)*x**2 - 1063506336*sqrt(2*x**2 - x + 3)*
x - 1064872488*sqrt(2*x**2 - x + 3) + 179115928*sqrt(2)*log( - 12*sqrt(2*x
**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 1343369460*sqrt(2)*log( - 12*sqrt
(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 3358423650*sqrt(2)*log( - 12*
sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 2798686375*sqrt(2)*log( - 12
*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) + 179159040*sqrt(2)*log( - 2*sq
rt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**3 + 1343692800*sqrt(2)*log( - 2*s
qrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x**2 + 3359232000*sqrt(2)*log( - 2*
sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1)*x + 2799360000*sqrt(2)*log( - 2*sq
rt(2*x**2 - x + 3)*sqrt(2) - 4*x + 1) - 179115928*sqrt(2)*log(2*x + 5)*x**
3 - 1343369460*sqrt(2)*log(2*x + 5)*x**2 - 3358423650*sqrt(2)*log(2*x + 5)
*x - 2798686375*sqrt(2)*log(2*x + 5))/(143327232*(8*x**3 + 60*x**2 + 150*x
 + 125))
```

3.186 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 139

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{3667\sqrt{3-x+2x^2}}{2304(5+2x)^4}+\frac{513097\sqrt{3-x+2x^2}}{497664(5+2x)^3} - \frac{16295969\sqrt{3-x+2x^2}}{71663616(5+2x)^2}+\frac{26800085\sqrt{3-x+2x^2}}{1719926784(5+2x)} + \frac{2053207\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{20639121408\sqrt{2}}$$

output

```
-3667/2304*(2*x^2-x+3)^(1/2)/(5+2*x)^4+513097/497664*(2*x^2-x+3)^(1/2)/(5+
2*x)^3-16295969/71663616*(2*x^2-x+3)^(1/2)/(5+2*x)^2+26800085*(2*x^2-x+3)^
(1/2)/(8599633920+3439853568*x)+2053207/41278242816*arctanh(1/24*(17-22*x)
*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-298655447-255525906x+43592076x^2+214400680x^3)}{(5+2x)^4}-2053207\sqrt{2}\operatorname{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20639121408}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),
x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-298655447 - 255525906*x + 43592076*x^2 + 214400
680*x^3))/(5 + 2*x)^4 - 2053207*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x +
4*x^2])/6])/20639121408
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2181, 27, 2181, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}\,dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{1}{288}\int \frac{-11520x^3+31104x^2-40668x+37027}{16(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-11520x^3+31104x^2-40668x+37027}{(2x+5)^4\sqrt{2x^2-x+3}}dx}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{216}\int \frac{1244160x^2-2364856x+2607829}{(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$\Big\downarrow$ 2181

$$\frac{\frac{1}{216}\left(-\frac{1}{144}\int \frac{13(1493165-1876588x)}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\int \frac{1493165-1876588x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1228

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(\frac{157939}{24}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$\Big\downarrow$ 1154

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(-\frac{157939}{12}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{216}\left(-\frac{13}{144}\left(-\frac{157939\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{144\sqrt{2}}-\frac{2061545\sqrt{2x^2-x+3}}{12(2x+5)}\right)-\frac{16295969\sqrt{2x^2-x+3}}{72(2x+5)^2}\right)+\frac{513097\sqrt{2x^2-x+3}}{108(2x+5)^3}}{4608}-\frac{3667\sqrt{2x^2-x+3}}{2304(2x+5)^4}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^5*Sqrt[3 - x + 2*x^2]),x]
```

output
```
(-3667*Sqrt[3 - x + 2*x^2])/(2304*(5 + 2*x)^4) + ((513097*Sqrt[3 - x + 2*x
^2])/(108*(5 + 2*x)^3) + ((-16295969*Sqrt[3 - x + 2*x^2])/(72*(5 + 2*x)^2)
 - (13*((-2061545*Sqrt[3 - x + 2*x^2])/(12*(5 + 2*x)) - (157939*ArcTanh[(1
7 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(144*Sqrt[2])))/144)/216)/460
8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.56

method	result
risch	$\frac{428801360x^5-127216528x^4+88558152x^3-211008760x^2-467922271x-895966341}{1719926784(5+2x)^4\sqrt{2x^2-x+3}}+\frac{2053207\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\frac{17}{2}-11x\right)\sqrt{2}}{12\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-}}\right.}{41278242816}$
trager	$\frac{(214400680x^3+43592076x^2-255525906x-298655447)\sqrt{2x^2-x+3}}{1719926784(5+2x)^4}+\frac{2053207\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{22\operatorname{RootOf}\left(\_Z^2-2\right)x-1}{}\right.}{41278242816}$
default	$-\frac{3667\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{36864\left(x+\frac{5}{2}\right)^4}+\frac{513097\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{3981312\left(x+\frac{5}{2}\right)^3}-\frac{16295969\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}{286654464\left(x+\frac{5}{2}\right)^2}+\frac{26800085\sqrt{2\left(x+\frac{5}{2}\right)^2-11x}}{3439853568\left(x+\frac{5}{2}\right)}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/1719926784*(428801360*x^5-127216528*x^4+88558152*x^3-211008760*x^2-46792
2271*x-895966341)/(5+2*x)^4/(2*x^2-x+3)^(1/2)+2053207/41278242816*2^(1/2)*
arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.90

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{2053207\,\sqrt{2}(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)\log\left(\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)-1060\,x^2+1036\,x-1153}{4\,x^2+20\,x+25}\right)+48}{82556485632\,(16\,x^4+160\,x^3+600\,x^2+10}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
fricas")
```

output
```
1/82556485632*(2053207*sqrt(2)*(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 625)
*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1036*x - 115
3)/(4*x^2 + 20*x + 25)) + 48*(214400680*x^3 + 43592076*x^2 - 255525906*x -
 298655447)*sqrt(2*x^2 - x + 3))/(16*x^4 + 160*x^3 + 600*x^2 + 1000*x + 62
5)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^5\sqrt{2x^2-x+3}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**5/(2*x**2-x+3)**(1/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**5*sqrt(2*x**2 - x +
3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = -\frac{2053207}{41278242816}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{3667\sqrt{2\,x^2-x+3}}{2304\,(16\,x^4+160\,x^3+600\,x^2+1000\,x+625)} + \frac{513097\sqrt{2\,x^2-x+3}}{497664\,(8\,x^3+60\,x^2+150\,x+125)} - \frac{16295969\sqrt{2\,x^2-x+3}}{71663616\,(4\,x^2+20\,x+25)} + \frac{26800085\sqrt{2\,x^2-x+3}}{1719926784\,(2\,x+5)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
maxima")
```

output
```
-2053207/41278242816*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) - 3667/2304*sqrt(2*x^2 - x + 3)/(16*x^4 + 160*x^3
+ 600*x^2 + 1000*x + 625) + 513097/497664*sqrt(2*x^2 - x + 3)/(8*x^3 + 60*
x^2 + 150*x + 125) - 16295969/71663616*sqrt(2*x^2 - x + 3)/(4*x^2 + 20*x +
 25) + 26800085/1719926784*sqrt(2*x^2 - x + 3)/(2*x + 5)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.18

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1}{41278242816}\sqrt{2}\left(12\left(\frac{24\left(\frac{144\left(\frac{513097}{\text{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{792072}{(2\,x+5)\text{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}-\frac{16295969}{\text{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{2\,x+5}+\frac{26800085}{\text{sgn}\left(\frac{1}{2\,x+5}\right)}\right)\sqrt{-\frac{11}{2\,x+5}+\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x, algorithm="
giac")
```

output
```
1/41278242816*sqrt(2)*(12*(24*(144*(513097/sgn(1/(2*x + 5)) - 792072/((2*x
 + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 16295969/sgn(1/(2*x + 5)))/(2*x + 5)
+ 26800085/sgn(1/(2*x + 5)))*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 20
53207*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 72/(2*x + 5) - 11)
/sgn(1/(2*x + 5)) - 321601020*sgn(1/(2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^5\,\sqrt{2\,x^2-x+3}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^5*(2*x^2 - x + 3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.00

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^5\sqrt{3-x+2x^2}}\,dx = \frac{5145616320\sqrt{2x^2-x+3}\,x^3+1046209824\sqrt{2x^2-x+3}\,x^2-6132621744\sqrt{2x^2-x+3}\,x-716773072}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^5/(2*x^2-x+3)^(1/2),x)
```

output
```
(5145616320*sqrt(2*x**2 - x + 3)*x**3 + 1046209824*sqrt(2*x**2 - x + 3)*x*
*2 - 6132621744*sqrt(2*x**2 - x + 3)*x - 7167730728*sqrt(2*x**2 - x + 3) +
 32851312*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4
 + 328513120*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x
**3 + 1231924200*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 1
7)*x**2 + 2053207000*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x
 - 17)*x + 1283254375*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*
x - 17) - 32851312*sqrt(2)*log(2*x + 5)*x**4 - 328513120*sqrt(2)*log(2*x +
 5)*x**3 - 1231924200*sqrt(2)*log(2*x + 5)*x**2 - 2053207000*sqrt(2)*log(2
*x + 5)*x - 1283254375*sqrt(2)*log(2*x + 5))/(41278242816*(16*x**4 + 160*x
**3 + 600*x**2 + 1000*x + 625))
```

3.187 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 124

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = -\frac{4(346-533x)}{23\sqrt{3-x+2x^2}} - \frac{13153}{512}\sqrt{3-x+2x^2} + \frac{2645}{128}x\sqrt{3-x+2x^2} + \frac{153}{16}x^2\sqrt{3-x+2x^2} + \frac{5}{4}x^3\sqrt{3-x+2x^2} + \frac{144217\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

output

```
1/23*(-1384+2132*x)/(2*x^2-x+3)^(1/2)-13153/512*(2*x^2-x+3)^(1/2)+2645/128
*x*(2*x^2-x+3)^(1/2)+153/16*x^2*(2*x^2-x+3)^(1/2)+5/4*x^3*(2*x^2-x+3)^(1/2
)+144217/2048*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx = \frac{\frac{4(-1616165+2124123x-510554x^2+418232x^3+210496x^4+29440x^5)}{\sqrt{3-x+2x^2}} + 3316991}{47104}$$

input
```
Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2
),x]
```

output
```
((4*(-1616165 + 2124123*x - 510554*x^2 + 418232*x^3 + 210496*x^4 + 29440*x
^5))/Sqrt[3 - x + 2*x^2] + 3316991*Sqrt[2]*Log[1 - 4*x + 2*Sqrt[6 - 2*x +
4*x^2]])/47104
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2191, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)^2 (5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

$$\Big\downarrow \text{ 2191}$$

$$\frac{2}{23}\int -\frac{23(-10x^4-53x^3-70x^2+25x+66)}{2\sqrt{2x^2-x+3}}dx - \frac{4(346-533x)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$-\int \frac{-10x^4-53x^3-70x^2+25x+66}{\sqrt{2x^2-x+3}}dx - \frac{4(346-533x)}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 2192}$$

$$-\frac{1}{8}\int \frac{-459x^3-470x^2+200x+528}{\sqrt{2x^2-x+3}}dx - \frac{4(346-533x)}{23\sqrt{2x^2-x+3}} + \frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{8}\left(\frac{153}{2}x^2\sqrt{2x^2-x+3}-\frac{1}{6}\int \frac{3\left(-2645x^2+2636x+2112\right)}{2\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 27$$

$$\frac{1}{8}\left(\frac{153}{2}x^2\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{-2645x^2+2636x+2112}{\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{2645}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{13153x+32766}{2\sqrt{2x^2-x+3}}dx\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 27$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{2645}{4}x\sqrt{2x^2-x+3}-\frac{1}{8}\int \frac{13153x+32766}{\sqrt{2x^2-x+3}}dx\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)-\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-x+3x}\right.$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-}\right.$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

$$\Big\downarrow \ 222$$

$$\frac{1}{8}\left(\frac{1}{4}\left(\frac{1}{8}\left(-\frac{144217\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{13153}{2}\sqrt{2x^2-x+3}\right)+\frac{2645}{4}\sqrt{2x^2-x+3}x\right)+\frac{153}{2}\sqrt{2x^2-x+3}x^2\right)$$
$$\frac{4(346-533x)}{23\sqrt{2x^2-x+3}}+\frac{5}{4}\sqrt{2x^2-x+3}x^3$$

input
```
Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(-4*(346 - 533*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*x^3*Sqrt[3 - x + 2*x^2])/
4 + ((153*x^2*Sqrt[3 - x + 2*x^2])/2 + ((2645*x*Sqrt[3 - x + 2*x^2])/4 + (
(-13153*Sqrt[3 - x + 2*x^2])/2 - (144217*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*
Sqrt[2]))/8)/4)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165}{11776\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}$
trager	$\frac{29440x^5+210496x^4+418232x^3-510554x^2+2124123x-1616165}{11776\sqrt{2x^2-x+3}}+\frac{144217\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-4\operatorname{RootOf}\left(\_Z^2-2\right)x+\operatorname{Root}\right.}{2048}$
default	$\frac{\frac{931255x}{23552}-\frac{931255}{94208}}{\sqrt{2x^2-x+3}}-\frac{521655}{4096\sqrt{2x^2-x+3}}+\frac{144217x}{1024\sqrt{2x^2-x+3}}-\frac{144217\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{2048}-\frac{11099x^2}{256\sqrt{2x^2-x+3}}+\frac{227}{64\sqrt{2x}}$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
1/11776*(29440*x^5+210496*x^4+418232*x^3-510554*x^2+2124123*x-1616165)/(2*
x^2-x+3)^(1/2)-144217/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{3316991\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-\right.}{}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/94208*(3316991*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)
*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(29440*x^5 + 210496*x^4 + 418232*x^3
- 510554*x^2 + 2124123*x - 1616165)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{(2x+5)^2\cdot(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(
3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\begin{aligned}\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx &= \frac{5\,x^5}{2\,\sqrt{2\,x^2-x+3}}+\frac{143\,x^4}{8\,\sqrt{2\,x^2-x+3}}\\ &+\frac{2273\,x^3}{64\,\sqrt{2\,x^2-x+3}}-\frac{11099\,x^2}{256\,\sqrt{2\,x^2-x+3}}-\frac{144217}{2048}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)\\ &+\frac{2124123\,x}{11776\,\sqrt{2\,x^2-x+3}}-\frac{1616165}{11776\,\sqrt{2\,x^2-x+3}}\end{aligned}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
5/2*x^5/sqrt(2*x^2 - x + 3) + 143/8*x^4/sqrt(2*x^2 - x + 3) + 2273/64*x^3/
sqrt(2*x^2 - x + 3) - 11099/256*x^2/sqrt(2*x^2 - x + 3) - 144217/2048*sqrt
(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2124123/11776*x/sqrt(2*x^2 - x + 3)
 - 1616165/11776/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx = \frac{144217}{2048} \sqrt{2} \log \left(-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2-x+3}\right) + 1\right) + \frac{(46(4(8(20x+143)x+2273)x-11099)x+2124123)x-1616165}{11776\sqrt{2x^2-x+3}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
144217/2048*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/11776*((46*(4*(8*(20*x + 143)*x + 2273)*x - 11099)*x + 2124123)*x - 16
16165)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx = \int \frac{(2x+5)^2 (5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{3/2}} dx$$

input
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.73

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}}\,dx = \frac{117760\sqrt{2x^2-x+3}\,x^5+841984\sqrt{2x^2-x+3}\,x^4+1672928\cdots}{}$$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

output
```
(117760*sqrt(2*x**2 - x + 3)*x**5 + 841984*sqrt(2*x**2 - x + 3)*x**4 + 167
2928*sqrt(2*x**2 - x + 3)*x**3 - 2042216*sqrt(2*x**2 - x + 3)*x**2 + 84964
92*sqrt(2*x**2 - x + 3)*x - 6464660*sqrt(2*x**2 - x + 3) - 6633982*sqrt(2)
*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 + 3316991*s
qrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x - 995097
3*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) + 52053
30*sqrt(2)*x**2 - 2602665*sqrt(2)*x + 7807995*sqrt(2))/(47104*(2*x**2 - x
+ 3))
```

3.188 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx$

Optimal result

Integrand size = 38, antiderivative size = 103

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx = \frac{-53+373x}{23\sqrt{3-x+2x^2}} + \frac{33}{64}\sqrt{3-x+2x^2} + \frac{193}{48}x\sqrt{3-x+2x^2} + \frac{5}{6}x^2\sqrt{3-x+2x^2} + \frac{3111\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

output
```
1/23*(-53+373*x)/(2*x^2-x+3)^(1/2)+33/64*(2*x^2-x+3)^(1/2)+193/48*x*(2*x^2
-x+3)^(1/2)+5/6*x^2*(2*x^2-x+3)^(1/2)+3111/256*arcsinh(1/23*(1-4*x)*23^(1/
2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{3/2}} dx = \frac{-3345+122607x-2162x^2+31832x^3+7360x^4}{4416\sqrt{3-x+2x^2}} + \frac{3111\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{128\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),
x]
```

output
```
(-3345 + 122607*x - 2162*x^2 + 31832*x^3 + 7360*x^4)/(4416*Sqrt[3 - x + 2*
x^2]) + (3111*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(128*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2191, 27, 2192, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23\left(-10x^3-28x^2+25\right)}{4\sqrt{2x^2-x+3}}dx-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{-10x^3-28x^2+25}{\sqrt{2x^2-x+3}}dx-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2}\left(\frac{5}{3}x^2\sqrt{2x^2-x+3}-\frac{1}{6}\int \frac{-193x^2+60x+150}{\sqrt{2x^2-x+3}}dx\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{9(262-11x)}{2\sqrt{2x^2-x+3}}dx\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\int \frac{262-11x}{\sqrt{2x^2-x+3}}dx\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037}{4}\int\frac{1}{\sqrt{2x^2-x+3}}dx-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{193}{4}x\sqrt{2x^2-x+3}-\frac{9}{8}\left(\frac{1037\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{11}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{5}{3}\sqrt{2x^2-x+3}x^2\right)-\frac{53-373x}{23\sqrt{2x^2-x+3}}$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(3/2),x]
```

output
```
-1/23*(53 - 373*x)/Sqrt[3 - x + 2*x^2] + ((5*x^2*Sqrt[3 - x + 2*x^2])/3 +
((193*x*Sqrt[3 - x + 2*x^2])/4 - (9*((-11*Sqrt[3 - x + 2*x^2])/2 + (1037*A
rcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/8)/6)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

method	result
risch	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{256}$
trager	$\frac{7360x^4+31832x^3-2162x^2+122607x-3345}{4416\sqrt{2x^2-x+3}}+\frac{3111\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-}\right.}{256}$
default	$-\frac{47x^2}{96\sqrt{2x^2-x+3}}+\frac{3111x}{128\sqrt{2x^2-x+3}}+\frac{55}{512\sqrt{2x^2-x+3}}+\frac{\frac{10185x}{2944}-\frac{10185}{11776}}{\sqrt{2x^2-x+3}}-\frac{3111\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\,\left(x-\frac{1}{4}\right)}{23}\right)}{256}+\frac{173x^3}{24\sqrt{2x^2-x+}}$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOS
E)
```

output
```
1/4416*(7360*x^4+31832*x^3-2162*x^2+122607*x-3345)/(2*x^2-x+3)^(1/2)-3111/
256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{214659\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\right.}{}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fr
icas")
```

output
```
1/35328*(214659*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(7360*x^4 + 31832*x^3 - 2162*x^2 + 122
607*x - 3345)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2
), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.94

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{5\,x^4}{3\,\sqrt{2\,x^2-x+3}}+\frac{173\,x^3}{24\,\sqrt{2\,x^2-x+3}}-\frac{47\,x^2}{96\,\sqrt{2\,x^2-x+3}}-\frac{3111}{256}\,\sqrt{2}\,\operatorname{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{40869\,x}{1472\,\sqrt{2\,x^2-x+3}}-\frac{1115}{1472\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="ma
xima")
```

output
```
5/3*x^4/sqrt(2*x^2 - x + 3) + 173/24*x^3/sqrt(2*x^2 - x + 3) - 47/96*x^2/s
qrt(2*x^2 - x + 3) - 3111/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 4
0869/1472*x/sqrt(2*x^2 - x + 3) - 1115/1472/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.65

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{3111}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(46\,(4\,(40\,x+173)x-47)x+122607)x-3345}{4416\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="gi
ac")
```

output
```
3111/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/4416*((46*(4*(40*x + 173)*x - 47)*x + 122607)*x - 3345)/sqrt(2*x^2 - x +
3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{(2\,x+5)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.93

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{29440\sqrt{2x^2-x+3}\,x^4+127328\sqrt{2x^2-x+3}\,x^3-8648\sqrt{2x^2}}{}$$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

output
```
(29440*sqrt(2*x**2 - x + 3)*x**4 + 127328*sqrt(2*x**2 - x + 3)*x**3 - 8648
*sqrt(2*x**2 - x + 3)*x**2 + 490428*sqrt(2*x**2 - x + 3)*x - 13380*sqrt(2*
x**2 - x + 3) - 429318*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x -
 1)/sqrt(23))*x**2 + 214659*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) +
4*x - 1)/sqrt(23))*x - 643977*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 4*x - 1)/sqrt(23)) + 286464*sqrt(2)*x**2 - 143232*sqrt(2)*x + 429696*sqr
t(2))/(17664*(2*x**2 - x + 3))
```

3.189 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{89+219x}{92\sqrt{3-x+2x^2}} + \frac{27}{32}\sqrt{3-x+2x^2} + \frac{5}{8}x\sqrt{3-x+2x^2} + \frac{213\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

output
```
1/92*(89+219*x)/(2*x^2-x+3)^(1/2)+27/32*(2*x^2-x+3)^(1/2)+5/8*x*(2*x^2-x+3
)^(1/2)+213/128*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{2575+2511x+782x^2+920x^3}{736\sqrt{3-x+2x^2}} + \frac{213\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{64\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(2575 + 2511*x + 782*x^2 + 920*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (213*Log[1
 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(64*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{23}\int -\frac{23(-20x^2-6x+15)}{16\sqrt{2x^2-x+3}}dx+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{219x+89}{92\sqrt{2x^2-x+3}}-\frac{1}{8}\int \frac{-20x^2-6x+15}{\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{1}{4}\int \frac{6(20-9x)}{\sqrt{2x^2-x+3}}dx\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\int \frac{20-9x}{\sqrt{2x^2-x+3}}dx\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{8}\left(5x\sqrt{2x^2-x+3}-\frac{3}{2}\left(\frac{71\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}-\frac{9}{2}\sqrt{2x^2-x+3}\right)\right)+\frac{219x+89}{92\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(3/2),x]
```

output
```
(89 + 219*x)/(92*Sqrt[3 - x + 2*x^2]) + (5*x*Sqrt[3 - x + 2*x^2] - (3*((-9
*Sqrt[3 - x + 2*x^2])/2 + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])))/
2)/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result	siz
risch	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}$	45
trager	$\frac{920x^3+782x^2+2511x+2575}{736\sqrt{2x^2-x+3}}+\frac{213\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-4\,\mathrm{RootOf}\left(\_Z^2-2\right)x+\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{128}$	70
default	$\frac{901}{256\sqrt{2x^2-x+3}}+\frac{\frac{123x}{1472}-\frac{123}{5888}}{\sqrt{2x^2-x+3}}+\frac{213x}{64\sqrt{2x^2-x+3}}-\frac{213\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{128}+\frac{17x^2}{16\sqrt{2x^2-x+3}}+\frac{5x^3}{4\sqrt{2x^2-x+3}}$	98

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/736*(920*x^3+782*x^2+2511*x+2575)/(2*x^2-x+3)^(1/2)-213/128*2^(1/2)*arcs
inh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{4899\sqrt{2}(2x^2-x+3)\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-\right.}{5888\,(2x^2-x+3}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

output
```
1/5888*(4899*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(920*x^3 + 782*x^2 + 2511*x + 2575)*sqrt(
2*x^2 - x + 3))/(2*x^2 - x + 3)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{5x^3}{4\sqrt{2x^2-x+3}} + \frac{17x^2}{16\sqrt{2x^2-x+3}} - \frac{213}{128}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{2511x}{736\sqrt{2x^2-x+3}} + \frac{2575}{736\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

output
```
5/4*x^3/sqrt(2*x^2 - x + 3) + 17/16*x^2/sqrt(2*x^2 - x + 3) - 213/128*sqrt
(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 2511/736*x/sqrt(2*x^2 - x + 3) + 25
75/736/sqrt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{213}{128}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{(46(20x+17)x+2511)x+2575}{736\sqrt{2x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

output
```
213/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((46*(20*x + 17)*x + 2511)*x + 2575)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.23

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{3/2}}\,dx = \frac{3680\sqrt{2x^2-x+3}\,x^3+3128\sqrt{2x^2-x+3}\,x^2+10044\sqrt{2x^2-x+3}\,x+1$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(3/2),x)
```

output
```
(3680*sqrt(2*x**2 - x + 3)*x**3 + 3128*sqrt(2*x**2 - x + 3)*x**2 + 10044*s
qrt(2*x**2 - x + 3)*x + 10300*sqrt(2*x**2 - x + 3) - 9798*sqrt(2)*log((2*s
qrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 + 4899*sqrt(2)*log((
2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x - 14697*sqrt(2)*log(
(2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) + 7008*sqrt(2)*x**2 -
 3504*sqrt(2)*x + 10512*sqrt(2))/(2944*(2*x**2 - x + 3))
```

3.190 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 101

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{1191+917x}{3312\sqrt{3-x+2x^2}} + \frac{5}{8}\sqrt{3-x+2x^2} + \frac{39\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{2}} - \frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1728\sqrt{2}}$$

output
```
1/3312*(1191+917*x)/(2*x^2-x+3)^(1/2)+5/8*(2*x^2-x+3)^(1/2)+39/32*arcsinh(
1/23*(1-4*x)*23^(1/2))*2^(1/2)-3667/3456*arctanh(1/24*(17-22*x)*2^(1/2)/(2
*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{\frac{12(7401-1153x+4140x^2)}{\sqrt{3-x+2x^2}} + 84341\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right) +}{39744}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),
x]
```

output
```
((12*(7401 - 1153*x + 4140*x^2))/Sqrt[3 - x + 2*x^2] + 84341*Sqrt[2]*ArcTa
nh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6] + 48438*Sqrt[2]*Log[1 - 4*x + 2*Sq
rt[6 - 2*x + 4*x^2]])/39744
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {2177, 27, 2184, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{23}\int -\frac{23\left(-720x^2-216x+293\right)}{576(2x+5)\sqrt{2x^2-x+3}}dx+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{917x+1191}{3312\sqrt{2x^2-x+3}}-\frac{1}{288}\int \frac{-720x^2-216x+293}{(2x+5)\sqrt{2x^2-x+3}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{1}{288}\left(180\sqrt{2x^2-x+3}-\frac{1}{8}\int -\frac{8(157-1404x)}{(2x+5)\sqrt{2x^2-x+3}}dx\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{288}\left(\int \frac{157-1404x}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{288}\left(-702\int \frac{1}{\sqrt{2x^2-x+3}}dx+3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{288}\left(3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-351\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{288}\left(3667\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{288}\left(-7334\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{288}\left(-351\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}+180\sqrt{2x^2-x+3}\right)+\frac{917x+1191}{3312\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(1191 + 917*x)/(3312*Sqrt[3 - x + 2*x^2]) + (180*Sqrt[3 - x + 2*x^2] - 351
*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (3667*ArcTanh[(17 - 22*x)/(12*Sqrt
[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2]))/288
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{96876\,\sqrt{2}(2\,x^2-x+3)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="fr
icas")
```

output
```
1/158976*(96876*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*
(4*x - 1) - 32*x^2 + 16*x - 25) + 84341*sqrt(2)*(2*x^2 - x + 3)*log(-(24*s
qrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2
+ 20*x + 25)) + 48*(4140*x^2 - 1153*x + 7401)*sqrt(2*x^2 - x + 3))/(2*x^2
- x + 3)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(3/
2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{5\,x^2}{4\,\sqrt{2\,x^2-x+3}} - \frac{39}{32}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{4}{23}\,\sqrt{23}x - \frac{1}{23}\,\sqrt{23}\right) + \frac{3667}{3456}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\,\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\,\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{1153\,x}{3312\,\sqrt{2\,x^2-x+3}} + \frac{2467}{1104\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="ma
xima")
```

output
```
5/4*x^2/sqrt(2*x^2 - x + 3) - 39/32*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23
*sqrt(23)) + 3667/3456*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/
23*sqrt(23)/abs(2*x + 5)) - 1153/3312*x/sqrt(2*x^2 - x + 3) + 2467/1104/sq
rt(2*x^2 - x + 3)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.17

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)(3-x+2x^2)^{3/2}}\,dx = \frac{39}{32}\,\sqrt{2}\log\left(-4\,\sqrt{2}x+\sqrt{2}+4\,\sqrt{2\,x^2-x+3}\right) - \frac{3667}{3456}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x+\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{3667}{3456}\,\sqrt{2}\log\left(\left|-2\,\sqrt{2}x-11\,\sqrt{2}+2\,\sqrt{2\,x^2-x+3}\right|\right) + \frac{(4140\,x-1153)x+7401}{3312\,\sqrt{2\,x^2-x+3}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x, algorithm="gi
ac")
```

output
```
39/32*sqrt(2)*log(-4*sqrt(2)*x + sqrt(2) + 4*sqrt(2*x^2 - x + 3)) - 3667/3
456*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3))) + 366
7/3456*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 - x + 3)))
 + 1/3312*((4140*x - 1153)*x + 7401)/sqrt(2*x^2 - x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.51

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{99360\sqrt{2x^2-x+3}\,x^2-27672\sqrt{2x^2-x+3}\,x+177624\sqrt{2x^2-x+3}}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(3/2),x)
```

output
```
(99360*sqrt(2*x**2 - x + 3)*x**2 - 27672*sqrt(2*x**2 - x + 3)*x + 177624*s
qrt(2*x**2 - x + 3) + 168682*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 92*x - 46)/sqrt(23))*x**2 - 84341*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*s
qrt(2) + 92*x - 46)/sqrt(23))*x + 253023*sqrt(2)*log((46*sqrt(2*x**2 - x +
 3)*sqrt(2) + 92*x - 46)/sqrt(23)) - 193752*sqrt(2)*log((2*sqrt(2*x**2 - x
 + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 + 96876*sqrt(2)*log((2*sqrt(2*x**2
 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x - 290628*sqrt(2)*log((2*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) - 168682*sqrt(2)*log((2*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23))*x**2 + 84341*sqrt(2)*log((2*sqrt
(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23))*x - 253023*sqrt(2)*log((2*s
qrt(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23)) + 22008*sqrt(2)*x**2 - 1
1004*sqrt(2)*x + 33012*sqrt(2))/(79488*(2*x**2 - x + 3))
```

3.191 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 108

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx=\frac{9897+2203x}{119232\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{10368(5+2x)}-\frac{5\text{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}+\frac{25951\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{41472\sqrt{2}}$$

output
```
1/119232*(9897+2203*x)/(2*x^2-x+3)^(1/2)-3667*(2*x^2-x+3)^(1/2)/(51840+207
36*x)-5/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)+25951/82944*arctanh(1/24
*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx=-\frac{\sqrt{3-x+2x^2}(51351-48653x+53290x^2)}{79488(15+x+8x^2+4x^3)}-\frac{25951\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{20736\sqrt{2}}-\frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
-1/79488*(Sqrt[3 - x + 2*x^2]*(51351 - 48653*x + 53290*x^2))/(15 + x + 8*x
^2 + 4*x^3) - (25951*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(20736*
Sqrt[2]) - (5*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2181, 27, 1269, 1090, 222, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int -\frac{23\left(-25920x^2-11410x+1463\right)}{20736(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}-\frac{\int \frac{-25920x^2-11410x+1463}{(2x+5)^2\sqrt{2x^2-x+3}}dx}{10368}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{72}\int -\frac{108(4351-8640x)}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{3}{2}\int \frac{4351-8640x}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-4320\int \frac{1}{\sqrt{2x^2-x+3}}dx\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2160\sqrt{\frac{2}{23}}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{222}$$

$$\frac{-\frac{3}{2}\left(25951\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-\frac{3}{2}\left(-51902\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{3}{2}\left(-2160\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{25951\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{6\sqrt{2}}\right)-\frac{3667\sqrt{2x^2-x+3}}{2x+5}}{10368}+\frac{2203x+9897}{119232\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(9897 + 2203*x)/(119232*Sqrt[3 - x + 2*x^2]) + ((-3667*Sqrt[3 - x + 2*x^2]
)/(5 + 2*x) - (3*(-2160*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] - (25951*ArcT
anh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(6*Sqrt[2])))/2)/10368
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

output
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.45

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{596160\,\sqrt{2}(4\,x^3+8\,x^2+x+15)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-3\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/3815424*(596160*sqrt(2)*(4*x^3 + 8*x^2 + x + 15)*log(-4*sqrt(2)*sqrt(2*x
^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 596873*sqrt(2)*(4*x^3 + 8*x^
2 + x + 15)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060*x^2 + 1
036*x - 1153)/(4*x^2 + 20*x + 25)) - 48*(53290*x^2 - 48653*x + 51351)*sqrt
(2*x^2 - x + 3))/(4*x^3 + 8*x^2 + x + 15)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^2\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**
(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.07

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{5}{16}\sqrt{2}\operatorname{arsinh}\left(\frac{4}{23}\sqrt{23}x-\frac{1}{23}\sqrt{23}\right) - \frac{25951}{82944}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{26645\,x}{79488\sqrt{2\,x^2-x+3}} + \frac{30313}{26496\sqrt{2\,x^2-x+3}} - \frac{3667}{576\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
5/16*sqrt(2)*arcsinh(4/23*sqrt(23)*x - 1/23*sqrt(23)) - 25951/82944*sqrt(2
)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/abs(2*x + 5)) - 2
6645/79488*x/sqrt(2*x^2 - x + 3) + 30313/26496/sqrt(2*x^2 - x + 3) - 3667/
576/(2*sqrt(2*x^2 - x + 3)*x + 5*sqrt(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(85) = 170$.

Time = 0.21 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.08

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{3/2}}\,dx = \frac{1}{1907712}\sqrt{2}\left(\frac{12\left(\frac{\frac{315103}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}-\frac{1012092}{(2\,x+5)\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}}{2\,x+5}-\frac{26645}{\operatorname{sgn}\left(\frac{1}{2\,x+5}\right)}\right)}{\sqrt{-\frac{11}{2\,x+5}+\frac{36}{(2\,x+5)^2}+1}} + \frac{596873\log}{}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
1/1907712*sqrt(2)*(12*((315103/sgn(1/(2*x + 5)) - 1012092/((2*x + 5)*sgn(1
/(2*x + 5))))/(2*x + 5) - 26645/sgn(1/(2*x + 5)))/sqrt(-11/(2*x + 5) + 36/
(2*x + 5)^2 + 1) + 596873*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1)
+ 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 596160*log(abs(sqrt(-11/(2*x + 5)
+ 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) + 1))/sgn(1/(2*x + 5)) - 596160*log(ab
s(sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2 + 1) + 6/(2*x + 5) - 1))/sgn(1/(2*x
+ 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)^2\,\left(2\,x^2-x+3\right)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.99

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{-1278960\sqrt{2x^2-x+3}\,x^2+1167672\sqrt{2x^2-x+3}\,x-1232424\sqrt{2x^2}}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(3/2),x)
```

output
```
( - 1278960*sqrt(2*x**2 - x + 3)*x**2 + 1167672*sqrt(2*x**2 - x + 3)*x - 1
232424*sqrt(2*x**2 - x + 3) + 2387492*sqrt(2)*log( - 12*sqrt(2*x**2 - x +
3)*sqrt(2) + 22*x - 17)*x**3 + 4774984*sqrt(2)*log( - 12*sqrt(2*x**2 - x +
 3)*sqrt(2) + 22*x - 17)*x**2 + 596873*sqrt(2)*log( - 12*sqrt(2*x**2 - x +
 3)*sqrt(2) + 22*x - 17)*x + 8953095*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3
)*sqrt(2) + 22*x - 17) + 2384640*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqr
t(2) - 4*x + 1)*x**3 + 4769280*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(
2) - 4*x + 1)*x**2 + 596160*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2)
- 4*x + 1)*x + 8942400*sqrt(2)*log( - 2*sqrt(2*x**2 - x + 3)*sqrt(2) - 4*x
 + 1) - 2387492*sqrt(2)*log(2*x + 5)*x**3 - 4774984*sqrt(2)*log(2*x + 5)*x
**2 - 596873*sqrt(2)*log(2*x + 5)*x - 8953095*sqrt(2)*log(2*x + 5))/(19077
12*(4*x**3 + 8*x**2 + x + 15))
```

3.192 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 112

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{65991-8779x}{4292352\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{20736(5+2x)^2}+\frac{115369\sqrt{3-x+2x^2}}{1492992(5+2x)}-\frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{5971968\sqrt{2}}$$

output
```
1/4292352*(65991-8779*x)/(2*x^2-x+3)^(1/2)-3667/20736*(2*x^2-x+3)^(1/2)/(5
+2*x)^2+115369*(2*x^2-x+3)^(1/2)/(7464960+2985984*x)-52631/11943936*arctan
h(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{12\left(11594283+5842933x+3263288x^2+3444340x^3\right)}{(5+2x)^2\sqrt{3-x+2x^2}}+1210513\sqrt{2}\text{arctanh}\left(\frac{1}{6}\left(5+2x-\right.\right.}{137355264}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
((12*(11594283 + 5842933*x + 3263288*x^2 + 3444340*x^3))/((5 + 2*x)^2*Sqrt
[3 - x + 2*x^2]) + 1210513*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2
])/6])/137355264
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int \frac{23\left(977500x^2+632660x+224707\right)}{746496(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{977500x^2+632660x+224707}{(2x+5)^3\sqrt{2x^2-x+3}}dx}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{-\frac{1}{144}\int \frac{288(73238-178369x)}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-2\int \frac{73238-178369x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{-2\left(-\frac{157893}{16}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{-2\left(\frac{157893}{8}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \; 219$$

$$\frac{-2\left(\frac{52631\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{32\sqrt{2}}-\frac{115369\sqrt{2x^2-x+3}}{8(2x+5)}\right)-\frac{66006\sqrt{2x^2-x+3}}{(2x+5)^2}}{373248}+\frac{65991-8779x}{4292352\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(65991 - 8779*x)/(4292352*Sqrt[3 - x + 2*x^2]) + ((-66006*Sqrt[3 - x + 2*x
^2])/(5 + 2*x)^2 - 2*((-115369*Sqrt[3 - x + 2*x^2])/(8*(5 + 2*x)) + (52631
*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(32*Sqrt[2])))/373
248
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

method	result
risch	$\frac{3444340x^3+3263288x^2+5842933x+11594283}{11446272(5+2x)^2\sqrt{2x^2-x+3}}-\frac{52631\sqrt{2}\,\text{arctanh}\left(\frac{\left(\frac{17}{2}-11x\right)\sqrt{2}}{12\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}\right)}{11943936}$
trager	$\frac{3444340x^3+3263288x^2+5842933x+11594283}{11446272(5+2x)^2\sqrt{2x^2-x+3}}-\frac{52631\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{22\,\text{RootOf}\left(\_Z^2-2\right)x-17\,\text{RootOf}\left(\_Z^2-2\right)-24}{5+2x}\right.}{11943936}$
default	$-\frac{149(4x-1)}{368\sqrt{2x^2-x+3}}-\frac{5}{16\sqrt{2x^2-x+3}}-\frac{3667}{4608\left(x+\frac{5}{2}\right)^2\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}+\frac{196043}{165888\left(x+\frac{5}{2}\right)\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}+\frac{}{1990656}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
1/11446272*(3444340*x^3+3263288*x^2+5842933*x+11594283)/(5+2*x)^2/(2*x^2-x
+3)^(1/2)-52631/11943936*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/
2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{1210513\sqrt{2}(8x^4+36x^3+42x^2+35x+75)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x+3}(22x-}{4x^2+}\right.}{549421056}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/549421056*(1210513*sqrt(2)*(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)*log(-(2
4*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x
^2 + 20*x + 25)) + 48*(3444340*x^3 + 3263288*x^2 + 5842933*x + 11594283)*s
qrt(2*x^2 - x + 3))/(8*x^4 + 36*x^3 + 42*x^2 + 35*x + 75)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3(2x^2-x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**
(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{3/2}}\,dx = \frac{52631}{11943936}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|} - \frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{861085\,x}{11446272\sqrt{2\,x^2-x+3}} - \frac{1163201}{3815424\sqrt{2\,x^2-x+3}} - \frac{3667}{1152\left(4\sqrt{2\,x^2-x+3}x^2+20\sqrt{2\,x^2-x+3}x+25\sqrt{2\,x^2-x+3}\right)} + \frac{196043}{82944\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output
```
52631/11943936*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(
23)/abs(2*x + 5)) + 861085/11446272*x/sqrt(2*x^2 - x + 3) - 1163201/381542
4/sqrt(2*x^2 - x + 3) - 3667/1152/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x
^2 - x + 3)*x + 25*sqrt(2*x^2 - x + 3)) + 196043/82944/(2*sqrt(2*x^2 - x +
 3)*x + 5*sqrt(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(90) = 180$.

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{3/2}}\,dx = -\frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{52631}{11943936}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{8779x-65991}{4292352\sqrt{2x^2-x+3}} + \frac{\sqrt{2}\left(3594214\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3+19874490\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2-30140067\sqrt{2}\left(\sqrt{2}x-\right.\right.}{2985984\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
-52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
+ 3))) + 52631/11943936*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt
(2*x^2 - x + 3))) - 1/4292352*(8779*x - 65991)/sqrt(2*x^2 - x + 3) + 1/298
5984*sqrt(2)*(3594214*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 198744
90*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 - 30140067*sqrt(2)*(sqrt(2)*x - sqr
t(2*x^2 - x + 3)) + 19989859)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*
sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\,(2x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3 (3-x+2x^2)^{3/2}} \, dx = \frac{82664160\sqrt{2x^2-x+3}\,x^3+78318912\sqrt{2x^2-x+3}\,x^2+140230392\sqrt{}}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(3/2),x)
```

output
```
(82664160*sqrt(2*x**2 - x + 3)*x**3 + 78318912*sqrt(2*x**2 - x + 3)*x**2 +
 140230392*sqrt(2*x**2 - x + 3)*x + 278262792*sqrt(2*x**2 - x + 3) + 96841
04*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 4357846
8*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 50841546
*sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 42367955*
sqrt(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 90788475*sqrt
(2)*log(12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) - 9684104*sqrt(2)*log
(2*x + 5)*x**4 - 43578468*sqrt(2)*log(2*x + 5)*x**3 - 50841546*sqrt(2)*log
(2*x + 5)*x**2 - 42367955*sqrt(2)*log(2*x + 5)*x - 90788475*sqrt(2)*log(2*
x + 5))/(274710528*(8*x**4 + 36*x**3 + 42*x**2 + 35*x + 75))
```

3.193 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 137

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{369609-175877x}{154524672\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{31104(5+2x)^3}+\frac{152885\sqrt{3-x+2x^2}}{4478976(5+2x)^2}+\frac{430799\sqrt{3-x+2x^2}}{107495424(5+2x)}-\frac{3505819\,\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{1289945088\sqrt{2}}$$

output
```
1/154524672*(369609-175877*x)/(2*x^2-x+3)^(1/2)-3667/31104*(2*x^2-x+3)^(1/
2)/(5+2*x)^3+152885/4478976*(2*x^2-x+3)^(1/2)/(5+2*x)^2+430799*(2*x^2-x+3)
^(1/2)/(537477120+214990848*x)-3505819/2579890176*arctanh(1/24*(17-22*x)*2
^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx=\frac{\frac{12\left(1873786587+1257975811x+441046842x^2+572739684x^3+56754760x^4\right)}{(5+2x)^3\sqrt{3-x+2x^2}}+80633837\sqrt{2}a}{29668737024}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)
),x]
```

output
```
((12*(1873786587 + 1257975811*x + 441046842*x^2 + 572739684*x^3 + 56754760
*x^4))/((5 + 2*x)^3*Sqrt[3 - x + 2*x^2]) + 80633837*Sqrt[2]*ArcTanh[(5 + 2
*x - Sqrt[6 - 2*x + 4*x^2])/6])/29668737024
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4\left(2x^2-x+3\right)^{3/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{23}\int \frac{23\left(453064x^3+38587980x^2+31270710x+15168577\right)}{26873856(2x+5)^4\sqrt{2x^2-x+3}}dx+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{453064x^3+38587980x^2+31270710x+15168577}{(2x+5)^4\sqrt{2x^2-x+3}}dx}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{-\frac{1}{216}\int \frac{216\left(-226532x^2-12391084x+3461275\right)}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\int \frac{-226532x^2-12391084x+3461275}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{2181}$$

$$\frac{\frac{1}{144}\int \frac{72(2061152x+1275689)}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{2061152x+1275689}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{1}{2}\left(\frac{3505819}{8}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{1}{2}\left(\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}-\frac{3505819}{4}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{430799\sqrt{2x^2-x+3}}{4(2x+5)}-\frac{3505819\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{48\sqrt{2}}\right)+\frac{458655\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1584144\sqrt{2x^2-x+3}}{(2x+5)^3}}{13436928}+\frac{369609-175877x}{154524672\sqrt{2x^2-x+3}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(3/2)),x]
```

output
```
(369609 - 175877*x)/(154524672*Sqrt[3 - x + 2*x^2]) + ((-1584144*Sqrt[3 -
x + 2*x^2])/(5 + 2*x)^3 + (458655*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2 + ((430
799*Sqrt[3 - x + 2*x^2])/(4*(5 + 2*x)) - (3505819*ArcTanh[(17 - 22*x)/(12*
Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(48*Sqrt[2]))/2)/13436928
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.53

method	result
risch	$\frac{56754760x^4+572739684x^3+441046842x^2+1257975811x+1873786587}{2472394752(5+2x)^3\sqrt{2x^2-x+3}}-\frac{3505819\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\frac{17}{2}-11x\right)\sqrt{2}}{12\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}\right)}{2579890176}$
trager	$\frac{56754760x^4+572739684x^3+441046842x^2+1257975811x+1873786587}{2472394752(5+2x)^3\sqrt{2x^2-x+3}}-\frac{3505819\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{22\operatorname{RootOf}\left(\_Z^2-2\right)\cdots}{\cdots}\right.}{2579890176}$
default	$\frac{\frac{5x}{46}-\frac{5}{184}}{\sqrt{2x^2-x+3}}-\frac{3667}{13824\left(x+\frac{5}{2}\right)^3\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}+\frac{314233}{995328\left(x+\frac{5}{2}\right)^2\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}-\frac{3127169}{35831808\left(x+\frac{5}{2}\right)\sqrt{2\left(x+\frac{5}{2}\right)^2-1\cdots}}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
1/2472394752*(56754760*x^4+572739684*x^3+441046842*x^2+1257975811*x+187378
6587)/(5+2*x)^3/(2*x^2-x+3)^(1/2)-3505819/2579890176*2^(1/2)*arctanh(1/12*
(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{80633837\,\sqrt{2}(16\,x^5+112\,x^4+264\,x^3+280\,x^2+325\,x+375)\log\left(-24\right.}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
fricas")
```

output
```
1/118674948096*(80633837*sqrt(2)*(16*x^5 + 112*x^4 + 264*x^3 + 280*x^2 + 3
25*x + 375)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 -
1036*x + 1153)/(4*x^2 + 20*x + 25)) + 48*(56754760*x^4 + 572739684*x^3 + 4
41046842*x^2 + 1257975811*x + 1873786587)*sqrt(2*x^2 - x + 3))/(16*x^5 + 1
12*x^4 + 264*x^3 + 280*x^2 + 325*x + 375)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^4\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(3/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**
(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{3/2}}\,dx = \frac{3505819}{2579890176}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) + \frac{7094345\,x}{2472394752\sqrt{2\,x^2-x+3}} + \frac{6128291}{824131584\sqrt{2\,x^2-x+3}} - \frac{3667}{1728\,\left(8\sqrt{2\,x^2-x+3}x^3+60\sqrt{2\,x^2-x+3}x^2+150\sqrt{2\,x^2-x+3}x+125\sqrt{2\,x^2-x+3}\right)} + \frac{314233}{248832\,\left(4\sqrt{2\,x^2-x+3}x^2+20\sqrt{2\,x^2-x+3}x+25\sqrt{2\,x^2-x+3}\right)} - \frac{3127169}{17915904\,\left(2\sqrt{2\,x^2-x+3}x+5\sqrt{2\,x^2-x+3}\right)}$$

input

```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
maxima")
```

output

```
3505819/2579890176*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*s
qrt(23)/abs(2*x + 5)) + 7094345/2472394752*x/sqrt(2*x^2 - x + 3) + 6128291
/824131584/sqrt(2*x^2 - x + 3) - 3667/1728/(8*sqrt(2*x^2 - x + 3)*x^3 + 60
*sqrt(2*x^2 - x + 3)*x^2 + 150*sqrt(2*x^2 - x + 3)*x + 125*sqrt(2*x^2 - x
+ 3)) + 314233/248832/(4*sqrt(2*x^2 - x + 3)*x^2 + 20*sqrt(2*x^2 - x + 3)*
x + 25*sqrt(2*x^2 - x + 3)) - 3127169/17915904/(2*sqrt(2*x^2 - x + 3)*x +
5*sqrt(2*x^2 - x + 3))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(111) = 222$.

Time = 0.22 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{3/2}}\,dx =$$

$$-\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$+\frac{3505819}{2579890176}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right)$$

$$-\frac{175877x-369609}{154524672\sqrt{2x^2-x+3}}$$

$$-\frac{\sqrt{2}\Big(10398764\sqrt{2}\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^5-303070900\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^4-529738052\sqrt{2}(\sqrt{2}x}{214990848\Big(2\left(\sqrt{2}x-\sqrt{2x^2-x\right.}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x, algorithm="
giac")
```

output
```
-3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) + 3505819/2579890176*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
+ 2*sqrt(2*x^2 - x + 3))) - 1/154524672*(175877*x - 369609)/sqrt(2*x^2 - x
 + 3) - 1/214990848*sqrt(2)*(10398764*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x
+ 3))^5 - 303070900*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^4 - 529738052*sqrt(2
)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^3 + 3644644652*(sqrt(2)*x - sqrt(2*x^2
 - x + 3))^2 - 2612608649*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1052
284471)/(2*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^4\,(2\,x^2-x+3)^{3/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{3/2}}\,dx = \frac{1362114240\sqrt{2x^2-x+3}\,x^4+13745752416\sqrt{2x^2-x+3}\,x^3+105851}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(3/2),x)
```

output
```
(1362114240*sqrt(2*x**2 - x + 3)*x**4 + 13745752416*sqrt(2*x**2 - x + 3)*x
**3 + 10585124208*sqrt(2*x**2 - x + 3)*x**2 + 30191419464*sqrt(2*x**2 - x
+ 3)*x + 44970878088*sqrt(2*x**2 - x + 3) + 1290141392*sqrt(2)*log(12*sqrt
(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**5 + 9030989744*sqrt(2)*log(12*sqr
t(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 21287332968*sqrt(2)*log(12*s
qrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**3 + 22577474360*sqrt(2)*log(12
*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**2 + 26205997025*sqrt(2)*log(
12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x + 30237688875*sqrt(2)*log(1
2*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17) - 1290141392*sqrt(2)*log(2*x +
 5)*x**5 - 9030989744*sqrt(2)*log(2*x + 5)*x**4 - 21287332968*sqrt(2)*log(
2*x + 5)*x**3 - 22577474360*sqrt(2)*log(2*x + 5)*x**2 - 26205997025*sqrt(2
)*log(2*x + 5)*x - 30237688875*sqrt(2)*log(2*x + 5))/(59337474048*(16*x**5
 + 112*x**4 + 264*x**3 + 280*x**2 + 325*x + 375))
```

3.194 $\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 105

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{4(346-533x)}{69(3-x+2x^2)^{3/2}} + \frac{4(18982-20383x)}{1587\sqrt{3-x+2x^2}} + \frac{247}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{1471\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

output
```
1/69*(-1384+2132*x)/(2*x^2-x+3)^(3/2)+4/1587*(18982-20383*x)/(2*x^2-x+3)^(
1/2)+247/16*(2*x^2-x+3)^(1/2)+5/4*x*(2*x^2-x+3)^(1/2)-1471/64*arcsinh(1/23
*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

$$\int \frac{(5+2x)^2(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{6663133-6410082x+8639625x^2-3764360x^3+1440996x^4}{25392(3-x+2x^2)^{3/2}} - \frac{1471\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{32\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2
),x]
```

output
```
(6663133 - 6410082*x + 8639625*x^2 - 3764360*x^3 + 1440996*x^4 + 126960*x^
5)/(25392*(3 - x + 2*x^2)^(3/2)) - (1471*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*
x^2]])/(32*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2191, 27, 2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)^2 \left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69} \int -\frac{-690x^4-3657x^3-4830x^2+1725x+290}{2\left(2x^2-x+3\right)^{3/2}}dx-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{69} \int \frac{-690x^4-3657x^3-4830x^2+1725x+290}{\left(2x^2-x+3\right)^{3/2}}dx-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{69} \left(\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}-\frac{2}{23} \int -\frac{1587\left(5x^2+29x+42\right)}{2\sqrt{2x^2-x+3}}dx\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{69} \left(69 \int \frac{5x^2+29x+42}{\sqrt{2x^2-x+3}}dx+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{69}\left(69\left(\frac{1}{4}\int \frac{247x+306}{2\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\int \frac{247x+306}{\sqrt{2x^2-x+3}}dx+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \ 1160$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471}{4}\int \frac{1}{\sqrt{2x^2-x+3}}dx+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \ 1090$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{4\sqrt{46}}+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \ 222$$

$$\frac{1}{69}\left(69\left(\frac{1}{8}\left(\frac{1471\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}}+\frac{247}{2}\sqrt{2x^2-x+3}\right)+\frac{5}{4}\sqrt{2x^2-x+3}x\right)+\frac{4(18982-20383x)}{23\sqrt{2x^2-x+3}}\right)-\frac{4(346-533x)}{69\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[((5 + 2*x)^2*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-4*(346 - 533*x))/(69*(3 - x + 2*x^2)^(3/2)) + ((4*(18982 - 20383*x))/(23
*Sqrt[3 - x + 2*x^2]) + 69*((5*x*Sqrt[3 - x + 2*x^2])/4 + ((247*Sqrt[3 - x
 + 2*x^2])/2 + (1471*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2]))/8))/69
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.52

method	result
risch	$\frac{126960x^5+1440996x^4-3764360x^3+8639625x^2-6410082x+6663133}{25392(2x^2-x+3)^{\frac{3}{2}}}+\frac{1471\sqrt{2}\,\text{arcsinh}\left(\frac{4\sqrt{23}\,(x-\frac{1}{4})}{23}\right)}{64}$
trager	$\frac{126960x^5+1440996x^4-3764360x^3+8639625x^2-6410082x+6663133}{25392(2x^2-x+3)^{\frac{3}{2}}}+\frac{1471\,\text{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\text{RootOf}\left(\_Z^2-2\right)x-\text{Root}\right.}{64}$
default	$-\frac{753223(4x-1)}{141312(2x^2-x+3)^{\frac{3}{2}}}-\frac{162931(4x-1)}{50784\sqrt{2x^2-x+3}}+\frac{577397}{2048(2x^2-x+3)^{\frac{3}{2}}}-\frac{32257x}{512(2x^2-x+3)^{\frac{3}{2}}}+\frac{19073x^2}{64(2x^2-x+3)^{\frac{3}{2}}}-\frac{1471x^3}{48(2x^2-x+3)}$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
1/25392*(126960*x^5+1440996*x^4-3764360*x^3+8639625*x^2-6410082*x+6663133)
/(2*x^2-x+3)^(3/2)+1471/64*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(5+2x)^2\,(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx=\frac{2334477\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-\right.}{}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/203136*(2334477*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(126960*x^5 + 14
40996*x^4 - 3764360*x^3 + 8639625*x^2 - 6410082*x + 6663133)*sqrt(2*x^2 -
x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{(2x+5)^2\cdot(5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5+2*x)**2*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((2*x + 5)**2*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(
5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(84) = 168$.

Time $= 0.11$ (sec) , antiderivative size $= 219$, normalized size of antiderivative $= 2.09$

$$\begin{aligned}\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx &= \frac{5\,x^5}{(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{227\,x^4}{4\,(2\,x^2-x+3)^{\frac{3}{2}}}\\ &+ \frac{1471}{50784}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right)\\ &+ \frac{1471}{64}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) - \frac{104441}{25392}\,\sqrt{2\,x^2-x+3} - \frac{383581\,x}{12696\,\sqrt{2\,x^2-x+3}}\\ &+ \frac{321\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{15965}{4232\,\sqrt{2\,x^2-x+3}} - \frac{4147\,x}{46\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{42883}{138\,(2\,x^2-x+3)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
5*x^5/(2*x^2 - x + 3)^(3/2) + 227/4*x^4/(2*x^2 - x + 3)^(3/2) + 1471/50784
*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2
*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2))
+ 1471/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 104441/25392*sqrt(2*x
^2 - x + 3) - 383581/12696*x/sqrt(2*x^2 - x + 3) + 321*x^2/(2*x^2 - x + 3)
^(3/2) - 15965/4232/sqrt(2*x^2 - x + 3) - 4147/46*x/(2*x^2 - x + 3)^(3/2)
+ 42883/138/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.68

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx = -\frac{1471}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right) + \frac{((4(1587(20x+227)x-941090)x+8639625)x-6410082)x+6663133}{25392(2x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
-1471/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/25392*(((4*(1587*(20*x + 227)*x - 941090)*x + 8639625)*x - 6410082)*x + 6
663133)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx = \int \frac{(2x+5)^2 (5x^4-x^3+3x^2+x+2)}{(2x^2-x+3)^{5/2}} \, dx$$

input
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int(((2*x + 5)^2*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.90

$$\int \frac{(5+2x)^2 (2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\, dx = \frac{1015680\sqrt{2x^2-x+3}\, x^5 + 11527968\sqrt{2x^2-x+3}\, x^4 - 3011}{}$$

input
```
int((5+2*x)^2*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
(1015680*sqrt(2*x**2 - x + 3)*x**5 + 11527968*sqrt(2*x**2 - x + 3)*x**4 -
30114880*sqrt(2*x**2 - x + 3)*x**3 + 69117000*sqrt(2*x**2 - x + 3)*x**2 -
51280656*sqrt(2*x**2 - x + 3)*x + 53305064*sqrt(2*x**2 - x + 3) + 18675816
*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**4 - 1
8675816*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x
**3 + 60696402*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt
(23))*x**2 - 28013724*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x -
1)/sqrt(23))*x + 42020586*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*
x - 1)/sqrt(23)) + 892276*sqrt(2)*x**4 - 892276*sqrt(2)*x**3 + 2899897*sqr
t(2)*x**2 - 1338414*sqrt(2)*x + 2007621*sqrt(2))/(203136*(4*x**4 - 4*x**3
+ 13*x**2 - 6*x + 9))
```

3.195 $\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 86

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx = \frac{-53+373x}{69(3-x+2x^2)^{3/2}} + \frac{6055-28981x}{3174\sqrt{3-x+2x^2}} + \frac{5}{4}\sqrt{3-x+2x^2} - \frac{71\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{8\sqrt{2}}$$

output
```
1/69*(-53+373*x)/(2*x^2-x+3)^(3/2)+1/3174*(6055-28981*x)/(2*x^2-x+3)^(1/2)
+5/4*(2*x^2-x+3)^(1/2)-71/16*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}} \, dx = \frac{102869-199290x+185337x^2-147664x^3+31740x^4}{6348(3-x+2x^2)^{3/2}} - \frac{71\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{8\sqrt{2}}$$

input
```
Integrate[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),
x]
```

output
```
(102869 - 199290*x + 185337*x^2 - 147664*x^3 + 31740*x^4)/(6348*(3 - x + 2
*x^2)^(3/2)) - (71*Log[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(8*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2191, 27, 2191, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int -\frac{-690x^3-1932x^2+233}{4\left(2x^2-x+3\right)^{3/2}}dx-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{138}\int \frac{-690x^3-1932x^2+233}{\left(2x^2-x+3\right)^{3/2}}dx-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{138}\left(\frac{6055-28981x}{23\sqrt{2x^2-x+3}}-\frac{2}{23}\int -\frac{1587(10x+33)}{4\sqrt{2x^2-x+3}}dx\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{138}\left(\frac{69}{2}\int \frac{10x+33}{\sqrt{2x^2-x+3}}dx+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71}{2}\int\frac{1}{\sqrt{2x^2-x+3}}dx+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71\int\frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)}{2\sqrt{46}}+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{138}\left(\frac{69}{2}\left(\frac{71\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}+5\sqrt{2x^2-x+3}\right)+\frac{6055-28981x}{23\sqrt{2x^2-x+3}}\right)-\frac{53-373x}{69\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[((5 + 2*x)*(2 + x + 3*x^2 - x^3 + 5*x^4))/(3 - x + 2*x^2)^(5/2),x]
```

output
```
-1/69*(53 - 373*x)/(3 - x + 2*x^2)^(3/2) + ((6055 - 28981*x)/(23*Sqrt[3 -
x + 2*x^2]) + (69*(5*Sqrt[3 - x + 2*x^2] + (71*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/(2*Sqrt[2])))/2)/138
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.58

method	result
risch	$\frac{31740x^4-147664x^3+185337x^2-199290x+102869}{6348(2x^2-x+3)^{\frac{3}{2}}}+\frac{71\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{16}$
trager	$\frac{31740x^4-147664x^3+185337x^2-199290x+102869}{6348(2x^2-x+3)^{\frac{3}{2}}}+\frac{71\,\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(4\operatorname{RootOf}\left(\_Z^2-2\right)x-\operatorname{RootOf}\left(\_Z^2-2\right)+4\sqrt{2}\right.}{16}$
default	$\frac{401x^2}{16(2x^2-x+3)^{\frac{3}{2}}}-\frac{945x}{128(2x^2-x+3)^{\frac{3}{2}}}+\frac{11749}{512(2x^2-x+3)^{\frac{3}{2}}}-\frac{2327(4x-1)}{35328(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{643x}{3174}-\frac{643}{12696}}{\sqrt{2x^2-x+3}}-\frac{71x^3}{12(2x^2-x+3)^{\frac{3}{2}}}-\frac{}{8\sqrt{2}}$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
1/6348*(31740*x^4-147664*x^3+185337*x^2-199290*x+102869)/(2*x^2-x+3)^(3/2)
+71/16*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.36

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{112677\,\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x\right.}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fr
icas")
```

output
```
1/50784*(112677*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*
sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(31740*x^4 - 14766
4*x^3 + 185337*x^2 - 199290*x + 102869)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{(2x+5)\left(5x^4-x^3+3x^2+x+2\right)}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5+2*x)*(5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((2*x + 5)*(5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2
), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(69) = 138$.

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.35

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = \frac{5\,x^4}{(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{71}{12696}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}} - \frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2\,x^2-x+3}} + \frac{805\,x}{(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2\,x^2-x+3)^{\frac{3}{2}}}\right) + \frac{71}{16}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{5041}{6348}\sqrt{2\,x^2-x+3} - \frac{10007\,x}{3174\sqrt{2\,x^2-x+3}} + \frac{59\,x^2}{2\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{2959}{2116\sqrt{2\,x^2-x+3}} - \frac{807\,x}{92\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{7603}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="ma
xima")
```

output
```
5*x^4/(2*x^2 - x + 3)^(3/2) + 71/12696*x*(284*x/sqrt(2*x^2 - x + 3) - 3174
*x^2/(2*x^2 - x + 3)^(3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3
)^(3/2) - 3243/(2*x^2 - x + 3)^(3/2)) + 71/16*sqrt(2)*arcsinh(1/23*sqrt(23
)*(4*x - 1)) - 5041/6348*sqrt(2*x^2 - x + 3) - 10007/3174*x/sqrt(2*x^2 - x
 + 3) + 59/2*x^2/(2*x^2 - x + 3)^(3/2) - 2959/2116/sqrt(2*x^2 - x + 3) - 8
07/92*x/(2*x^2 - x + 3)^(3/2) + 7603/276/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \frac{(5+2x)(2+x+3x^2-x^3+5x^4)}{(3-x+2x^2)^{5/2}}\,dx = -\frac{71}{16}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) + \frac{((4\,(7935\,x-36916)x+185337)x-199290)x+102869}{6348\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="gi
ac")
```

output
```
-71/16*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/6
348*(((4*(7935*x - 36916)*x + 185337)*x - 199290)*x + 102869)/(2*x^2 - x +
 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{\left(2\,x+5\right)\,\left(5\,x^4-x^3+3\,x^2+x+2\right)}{{\left(2\,x^2-x+3\right)}^{5/2}} \,d x$$

input
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int(((2*x + 5)*(x + 3*x^2 - x^3 + 5*x^4 + 2))/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.36

$$\int \frac{(5+2x)\left(2+x+3x^2-x^3+5x^4\right)}{\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{126960\sqrt{2x^2-x+3}\,x^4-590656\sqrt{2x^2-x+3}\,x^3+741348\sqrt{2}}{}$$

input
```
int((5+2*x)*(5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
(126960*sqrt(2*x**2 - x + 3)*x**4 - 590656*sqrt(2*x**2 - x + 3)*x**3 + 741
348*sqrt(2*x**2 - x + 3)*x**2 - 797160*sqrt(2*x**2 - x + 3)*x + 411476*sqr
t(2*x**2 - x + 3) + 450708*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4
*x - 1)/sqrt(23))*x**4 - 450708*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2
) + 4*x - 1)/sqrt(23))*x**3 + 1464801*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*
sqrt(2) + 4*x - 1)/sqrt(23))*x**2 - 676062*sqrt(2)*log((2*sqrt(2*x**2 - x
+ 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x + 1014093*sqrt(2)*log((2*sqrt(2*x**2 -
 x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) + 27248*sqrt(2)*x**4 - 27248*sqrt(2)*
x**3 + 88556*sqrt(2)*x**2 - 40872*sqrt(2)*x + 61308*sqrt(2))/(25392*(4*x**
4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.196 $\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 68

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{89+219x}{276(3-x+2x^2)^{3/2}} - \frac{1465+2604x}{2116\sqrt{3-x+2x^2}} - \frac{5\operatorname{arcsinh}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

output
```
1/276*(89+219*x)/(2*x^2-x+3)^(3/2)-1/2116*(1465+2604*x)/(2*x^2-x+3)^(1/2)-
5/8*arcsinh(1/23*(1-4*x)*23^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{-5569-7002x-489x^2-7812x^3}{3174(3-x+2x^2)^{3/2}} - \frac{5\log\left(1-4x+2\sqrt{6-2x+4x^2}\right)}{4\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(-5569 - 7002*x - 489*x^2 - 7812*x^3)/(3174*(3 - x + 2*x^2)^(3/2)) - (5*Lo
g[1 - 4*x + 2*Sqrt[6 - 2*x + 4*x^2]])/(4*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2191, 27, 2191, 27, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{5/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{2}{69}\int -\frac{3(-460x^2-138x+53)}{16(2x^2-x+3)^{3/2}}dx+\frac{219x+89}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{219x+89}{276(2x^2-x+3)^{3/2}}-\frac{1}{184}\int \frac{-460x^2-138x+53}{(2x^2-x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{184}\left(-\frac{2}{23}\int -\frac{2645}{\sqrt{2x^2-x+3}}dx-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{184}\left(230\int \frac{1}{\sqrt{2x^2-x+3}}dx-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{184}\left(5\sqrt{46}\int \frac{1}{\sqrt{\frac{1}{23}(4x-1)^2+1}}d(4x-1)-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{184}\left(115\sqrt{2}\text{arcsinh}\left(\frac{4x-1}{\sqrt{23}}\right)-\frac{2(2604x+1465)}{23\sqrt{2x^2-x+3}}\right)+\frac{219x+89}{276(2x^2-x+3)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/(3 - x + 2*x^2)^(5/2),x]
```

output
```
(89 + 219*x)/(276*(3 - x + 2*x^2)^(3/2)) + ((-2*(1465 + 2604*x))/(23*Sqrt[
3 - x + 2*x^2]) + 115*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]])/184
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{7812x^3+489x^2+7002x+5569}{3174(2x^2-x+3)^{\frac{3}{2}}}+\frac{5\sqrt{2}\,\operatorname{arcsinh}\left(\frac{4\sqrt{23}\left(x-\frac{1}{4}\right)}{23}\right)}{8}$
trager	$-\frac{7812x^3+489x^2+7002x+5569}{3174(2x^2-x+3)^{\frac{3}{2}}}+\frac{5\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(4\,\mathrm{RootOf}\left(\_Z^2-2\right)x-\mathrm{RootOf}\left(\_Z^2-2\right)+4\sqrt{2x^2-x+3}\right)}{8}$
default	$-\frac{271}{768(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{2423x}{4416}-\frac{2423}{17664}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{692x}{1587}-\frac{173}{1587}}{\sqrt{2x^2-x+3}}-\frac{47x}{64(2x^2-x+3)^{\frac{3}{2}}}-\frac{x^2}{8(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x^3}{6(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x}{4\sqrt{2x^2-x+}}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3174*(7812*x^3+489*x^2+7002*x+5569)/(2*x^2-x+3)^(3/2)+5/8*2^(1/2)*arcsi
nh(4/23*23^(1/2)*(x-1/4))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(55) = 110.

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.65

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx=\frac{7935\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-4\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)\right.}{25392\,(4\,x^4-4\,x}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

output
```
1/25392*(7935*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 8*(7812*x^3 + 489*x^2
+ 7002*x + 5569)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(55) = 110.

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.72

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx=\frac{5}{6348}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{(2\,x^2-x-\cdots}\right.$$
$$+\frac{5}{8}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{355}{3174}\sqrt{2\,x^2-x+3}-\frac{58\,x}{1587\sqrt{2\,x^2-x+3}}$$
$$+\frac{x^2}{2\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{1897}{6348\sqrt{2\,x^2-x+3}}-\frac{95\,x}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}+\frac{41}{276\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

output
```
5/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(
3/2)) + 5/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 355/3174*sqrt(2*x^2
 - x + 3) - 58/1587*x/sqrt(2*x^2 - x + 3) + 1/2*x^2/(2*x^2 - x + 3)^(3/2)
- 1897/6348/sqrt(2*x^2 - x + 3) - 95/276*x/(2*x^2 - x + 3)^(3/2) + 41/276/
(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = -\frac{5}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right) - \frac{3\,((2604\,x+163)x+2334)x+5569}{3174\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 1/317
4*(3*((2604*x + 163)*x + 2334)*x + 5569)/(2*x^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/(2*x^2 - x + 3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 4.01

$$\int \frac{2+x+3x^2-x^3+5x^4}{(3-x+2x^2)^{5/2}}\,dx = \frac{-31248\sqrt{2x^2-x+3}\,x^3-1956\sqrt{2x^2-x+3}\,x^2-28008\sqrt{2x^2-x+3}\,x}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(2*x^2-x+3)^(5/2),x)
```

output
```
( - 31248*sqrt(2*x**2 - x + 3)*x**3 - 1956*sqrt(2*x**2 - x + 3)*x**2 - 280
08*sqrt(2*x**2 - x + 3)*x - 22276*sqrt(2*x**2 - x + 3) + 31740*sqrt(2)*log
((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**4 - 31740*sqrt(2)
*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**3 + 103155*sq
rt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x**2 - 4761
0*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23))*x + 714
15*sqrt(2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x - 1)/sqrt(23)) + 1072
*sqrt(2)*x**4 - 1072*sqrt(2)*x**3 + 3484*sqrt(2)*x**2 - 1608*sqrt(2)*x + 2
412*sqrt(2))/(12696*(4*x**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.197 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 85

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{1191+917x}{9936\left(3-x+2x^2\right)^{3/2}}-\frac{335337+146729x}{1371168\sqrt{3-x+2x^2}}-\frac{3667\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{31104\sqrt{2}}$$

output
```
1/9936*(1191+917*x)/(2*x^2-x+3)^(3/2)-1/1371168*(335337+146729*x)/(2*x^2-x
+3)^(1/2)-3667/62208*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(
1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{-841653+21696x-523945x^2-293458x^3}{1371168\left(3-x+2x^2\right)^{3/2}}+\frac{3667\text{arctanh}\left(\frac{1}{6}\left(5+2x-\sqrt{6-2x+4x^2}\right)\right)}{15552\sqrt{2}}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),
x]
```

output
```
(-841653 + 21696*x - 523945*x^2 - 293458*x^3)/(1371168*(3 - x + 2*x^2)^(3/
2)) + (3667*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/(15552*Sqrt[2])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2177, 27, 2177, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int -\frac{-49680x^2-22240x+1877}{576(2x+5)\left(2x^2-x+3\right)^{3/2}}dx+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}-\frac{\int \frac{-49680x^2-22240x+1877}{(2x+5)\left(2x^2-x+3\right)^{3/2}}dx}{19872}$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{-\frac{2}{23}\int -\frac{1939843}{12(2x+5)\sqrt{2x^2-x+3}}dx-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{84341}{6}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{-\frac{84341}{3}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{84341\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{36\sqrt{2}}-\frac{146729x+335337}{69\sqrt{2x^2-x+3}}}{19872}+\frac{917x+1191}{9936\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(1191 + 917*x)/(9936*(3 - x + 2*x^2)^(3/2)) + (-1/69*(335337 + 146729*x)/S
qrt[3 - x + 2*x^2] - (84341*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2
*x^2])])/(36*Sqrt[2]))/19872
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

method	result
trager	$-\frac{293458x^3+523945x^2-21696x+841653}{1371168(2x^2-x+3)^{\frac{3}{2}}}-\frac{3667\,\mathrm{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{22\,\mathrm{RootOf}\left(\_Z^2-2\right)x-17\,\mathrm{RootOf}\left(\_Z^2-2\right)-24\sqrt{2x^2-}}{5+2x}\right.}{62208}$
default	$-\frac{3817(4x-1)}{2944(2x^2-x+3)^{\frac{3}{2}}}-\frac{3817(4x-1)}{4232\sqrt{2x^2-x+3}}-\frac{1597}{384(2x^2-x+3)^{\frac{3}{2}}}+\frac{59x}{32(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x^2}{4(2x^2-x+3)^{\frac{3}{2}}}+\frac{3667}{1728\left(2\left(x+\frac{5}{2}\right)^2-11x-\right.}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/1371168*(293458*x^3+523945*x^2-21696*x+841653)/(2*x^2-x+3)^(3/2)-3667/6
2208*RootOf(_Z^2-2)*ln(-(22*RootOf(_Z^2-2)*x-17*RootOf(_Z^2-2)-24*(2*x^2-x
+3)^(1/2))/(5+2*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.48

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{1939843\sqrt{2}(4\,x^4-4\,x^3+13\,x^2-6\,x+9)\log\left(-\frac{24\,\sqrt{2}\sqrt{2\,x^2-x+3}(22\,x-17)+}{4\,x^2+20\,x+}\right.}{65816064\,(4\,x^4}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="fr
icas")
```

output
```
1/65816064*(1939843*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-(24*sq
rt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1153)/(4*x^2 +
 20*x + 25)) - 48*(293458*x^3 + 523945*x^2 - 21696*x + 841653)*sqrt(2*x^2
- x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)*(2*x**2 - x + 3)**(5/
2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{3667}{62208}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right) - \frac{146729\,x}{1371168\sqrt{2\,x^2-x+3}} - \frac{5\,x^2}{4\,(2\,x^2-x+3)^{\frac{3}{2}}} + \frac{173881}{457056\sqrt{2\,x^2-x+3}} + \frac{7127\,x}{9936\,(2\,x^2-x+3)^{\frac{3}{2}}} - \frac{5813}{3312\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="ma
xima")
```

output
```
3667/62208*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23)/
abs(2*x + 5)) - 146729/1371168*x/sqrt(2*x^2 - x + 3) - 5/4*x^2/(2*x^2 - x
+ 3)^(3/2) + 173881/457056/sqrt(2*x^2 - x + 3) + 7127/9936*x/(2*x^2 - x +
3)^(3/2) - 5813/3312/(2*x^2 - x + 3)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{3667}{62208}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{((293458x+523945)x-21696)x+841653}{1371168\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x, algorithm="gi
ac")
```

output
```
-3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x + 3)
)) + 3667/62208*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*sqrt(2*x^2 -
 x + 3))) - 1/1371168*(((293458*x + 523945)*x - 21696)*x + 841653)/(2*x^2
- x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{\left(2\,x+5\right)\,{\left(2\,x^2-x+3\right)}^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)*(2*x^2 - x + 3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 433, normalized size of antiderivative = 5.09

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{-7042992\sqrt{2x^2-x+3}\,x^3-12574680\sqrt{2x^2-x+3}\,x^2+520704\sqrt{2x^2}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)/(2*x^2-x+3)^(5/2),x)
```

output
```
( - 7042992*sqrt(2*x**2 - x + 3)*x**3 - 12574680*sqrt(2*x**2 - x + 3)*x**2
 + 520704*sqrt(2*x**2 - x + 3)*x - 20199672*sqrt(2*x**2 - x + 3) + 7759372
*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2) + 92*x - 46)/sqrt(23))*x**4
- 7759372*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2) + 92*x - 46)/sqrt(2
3))*x**3 + 25217959*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2) + 92*x -
46)/sqrt(23))*x**2 - 11639058*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*sqrt(2)
 + 92*x - 46)/sqrt(23))*x + 17458587*sqrt(2)*log((46*sqrt(2*x**2 - x + 3)*
sqrt(2) + 92*x - 46)/sqrt(23)) - 7759372*sqrt(2)*log((2*sqrt(2*x**2 - x +
3)*sqrt(2) + 4*x + 22)/sqrt(23))*x**4 + 7759372*sqrt(2)*log((2*sqrt(2*x**2
 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23))*x**3 - 25217959*sqrt(2)*log((2*sqr
t(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23))*x**2 + 11639058*sqrt(2)*lo
g((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23))*x - 17458587*sqrt(
2)*log((2*sqrt(2*x**2 - x + 3)*sqrt(2) + 4*x + 22)/sqrt(23)) - 469360*sqrt
(2)*x**4 + 469360*sqrt(2)*x**3 - 1525420*sqrt(2)*x**2 + 704040*sqrt(2)*x -
 1056060*sqrt(2))/(32908032*(4*x**4 - 4*x**3 + 13*x**2 - 6*x + 9))
```

3.198 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 110

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{9897+2203x}{357696\left(3-x+2x^2\right)^{3/2}}-\frac{1255878-62021x}{24681024\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{186624(5+2x)}-\frac{2821\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{2239488\sqrt{2}}$$

output
```
1/357696*(9897+2203*x)/(2*x^2-x+3)^(3/2)-1/24681024*(1255878-62021*x)/(2*x
^2-x+3)^(1/2)-3667*(2*x^2-x+3)^(1/2)/(933120+373248*x)-2821/4478976*arctan
h(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{-\frac{12\left(79153407-18840090x+63941915x^2+10350004x^3+6767036x^4\right)}{(5+2x)\left(3-x+2x^2\right)^{3/2}}+1492309\sqrt{2}\text{arctan}}{1184689152}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((-12*(79153407 - 18840090*x + 63941915*x^2 + 10350004*x^3 + 6767036*x^4))
/((5 + 2*x)*(3 - x + 2*x^2)^(3/2)) + 1492309*Sqrt[2]*ArcTanh[(5 + 2*x - Sq
rt[6 - 2*x + 4*x^2])/6])/1184689152
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2177, 27, 2177, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{2}{69}\int \frac{1823728x^2+963530x+119353}{20736(2x+5)^2\left(2x^2-x+3\right)^{3/2}}dx+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1823728x^2+963530x+119353}{(2x+5)^2(2x^2-x+3)^{3/2}}dx}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 2177}$$

$$\frac{\frac{2}{23}\int \frac{529(19111-18758x)}{6(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{23}{3}\int \frac{19111-18758x}{(2x+5)^2\sqrt{2x^2-x+3}}dx-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{\frac{23}{3}\left(\frac{2821}{4}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{23}{3}\left(-\frac{2821}{2}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{23}{3}\left(-\frac{2821\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{24\sqrt{2}}-\frac{3667\sqrt{2x^2-x+3}}{2(2x+5)}\right)-\frac{2(1255878-62021x)}{69\sqrt{2x^2-x+3}}}{715392}+\frac{2203x+9897}{357696\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^2*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(9897 + 2203*x)/(357696*(3 - x + 2*x^2)^(3/2)) + ((-2*(1255878 - 62021*x))
/(69*Sqrt[3 - x + 2*x^2]) + (23*((-3667*Sqrt[3 - x + 2*x^2])/(2*(5 + 2*x))
 - (2821*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(24*Sqrt[2
])))/3)/715392
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{6767036x^4+10350004x^3+63941915x^2-18840090x+79153407}{98724096(2x^2-x+3)^{\frac{3}{2}}(5+2x)}-\frac{2821\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\frac{17}{2}-11x\right)\sqrt{2}}{12\sqrt{2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}}}\right)}{4478976}$
trager	$-\frac{6767036x^4+10350004x^3+63941915x^2-18840090x+79153407}{98724096(2x^2-x+3)^{\frac{3}{2}}(5+2x)}-\frac{2821\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(-\frac{22\operatorname{RootOf}\left(\_Z^2-2\right)x-17\operatorname{Root}}{5+2}\right.}{4478976}$
default	$\frac{\frac{3173x}{1104}-\frac{3173}{4416}}{(2x^2-x+3)^{\frac{3}{2}}}+\frac{\frac{3173x}{1587}-\frac{3173}{6348}}{\sqrt{2x^2-x+3}}+\frac{203}{192(2x^2-x+3)^{\frac{3}{2}}}-\frac{5x}{16(2x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{1152\left(x+\frac{5}{2}\right)\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}+\frac{}{124416\left(2(x\right.}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
-1/98724096*(6767036*x^4+10350004*x^3+63941915*x^2-18840090*x+79153407)/(2
*x^2-x+3)^(3/2)/(5+2*x)-2821/4478976*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1
/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \frac{1492309\sqrt{2}(8x^5+12x^4+6x^3+53x^2-12x+45)\log\left(-\frac{24\sqrt{2}\sqrt{2x^2-x}}{}\right.}{473}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/4738756608*(1492309*sqrt(2)*(8*x^5 + 12*x^4 + 6*x^3 + 53*x^2 - 12*x + 45
)*log(-(24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) + 1060*x^2 - 1036*x + 1
153)/(4*x^2 + 20*x + 25)) - 48*(6767036*x^4 + 10350004*x^3 + 63941915*x^2
- 18840090*x + 79153407)*sqrt(2*x^2 - x + 3))/(8*x^5 + 12*x^4 + 6*x^3 + 53
*x^2 - 12*x + 45)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^2(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**2/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**2*(2*x**2 - x + 3)**
(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2(3-x+2x^2)^{5/2}}\,dx = \frac{2821}{4478976}\sqrt{2}\,\mathrm{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)-\frac{1691759\,x}{98724096\sqrt{2\,x^2-x+3}}+\frac{265339}{32908032\sqrt{2\,x^2-x+3}}-\frac{248617\,x}{715392\,(2\,x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{576\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}+\frac{259621}{238464\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
2821/4478976*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sqrt(23
)/abs(2*x + 5)) - 1691759/98724096*x/sqrt(2*x^2 - x + 3) + 265339/32908032
/sqrt(2*x^2 - x + 3) - 248617/715392*x/(2*x^2 - x + 3)^(3/2) - 3667/576/(2
*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x^2 - x + 3)^(3/2)) + 259621/238464/(2*x^2
 - x + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(88) = 176$.

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2\,(3-x+2x^2)^{5/2}}\,dx =$$

$$-\frac{1}{2369378304}\sqrt{2}\left(\frac{1492309\,\log\left(12\sqrt{-\frac{11}{2x+5}+\frac{36}{(2x+5)^2}+1}+\frac{72}{2x+5}-11\right)}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}+\frac{12\left(\frac{\frac{\frac{48\left(\frac{23642785}{\operatorname{sgn}\left(\frac{1}{2x+5}\right)}-\frac{52375761}{(2x+5)\operatorname{sgn}\left(\cdots\right.}\right.}{2x+5}\cdots}{2x+5}\cdots}{2x\cdots}\right.}{\left(\frac{11}{2x+5}-\frac{36}{(2x+5)^2}\cdots\right.}\right.$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
-1/2369378304*sqrt(2)*(1492309*log(12*sqrt(-11/(2*x + 5) + 36/(2*x + 5)^2
+ 1) + 72/(2*x + 5) - 11)/sgn(1/(2*x + 5)) + 12*(((48*(23642785/sgn(1/(2*x
 + 5)) - 52375761/((2*x + 5)*sgn(1/(2*x + 5))))/(2*x + 5) - 240080735/sgn(
1/(2*x + 5)))/(2*x + 5) + 28660178/sgn(1/(2*x + 5)))/(2*x + 5) - 1691759/s
gn(1/(2*x + 5)))/((11/(2*x + 5) - 36/(2*x + 5)^2 - 1)*sqrt(-11/(2*x + 5) +
 36/(2*x + 5)^2 + 1)) - 20301108*sgn(1/(2*x + 5)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2 (3-x+2x^2)^{5/2}}\,dx = \int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^2\,(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^2*(2*x^2 - x + 3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.09

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^2 (3-x+2x^2)^{5/2}}\,dx = \frac{-162408864\sqrt{2x^2-x+3}\,x^4-248400096\sqrt{2x^2-x+3}\,x^3-1534605\ldots}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^2/(2*x^2-x+3)^(5/2),x)
```

output
```
( - 162408864*sqrt(2*x**2 - x + 3)*x**4 - 248400096*sqrt(2*x**2 - x + 3)*x
**3 - 1534605960*sqrt(2*x**2 - x + 3)*x**2 + 452162160*sqrt(2*x**2 - x + 3
)*x - 1899681768*sqrt(2*x**2 - x + 3) + 11938472*sqrt(2)*log(12*sqrt(2*x**
2 - x + 3)*sqrt(2) + 22*x - 17)*x**5 + 17907708*sqrt(2)*log(12*sqrt(2*x**2
 - x + 3)*sqrt(2) + 22*x - 17)*x**4 + 8953854*sqrt(2)*log(12*sqrt(2*x**2 -
 x + 3)*sqrt(2) + 22*x - 17)*x**3 + 79092377*sqrt(2)*log(12*sqrt(2*x**2 -
x + 3)*sqrt(2) + 22*x - 17)*x**2 - 17907708*sqrt(2)*log(12*sqrt(2*x**2 - x
 + 3)*sqrt(2) + 22*x - 17)*x + 67153905*sqrt(2)*log(12*sqrt(2*x**2 - x + 3
)*sqrt(2) + 22*x - 17) - 11938472*sqrt(2)*log(2*x + 5)*x**5 - 17907708*sqr
t(2)*log(2*x + 5)*x**4 - 8953854*sqrt(2)*log(2*x + 5)*x**3 - 79092377*sqrt
(2)*log(2*x + 5)*x**2 + 17907708*sqrt(2)*log(2*x + 5)*x - 67153905*sqrt(2)
*log(2*x + 5))/(2369378304*(8*x**5 + 12*x**4 + 6*x**3 + 53*x**2 - 12*x + 4
5))
```

3.199 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 135

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx=\frac{65991-8779x}{12877056\left(3-x+2x^2\right)^{3/2}}-\frac{4679797-2148263x}{592344576\sqrt{3-x+2x^2}}-\frac{3667\sqrt{3-x+2x^2}}{373248(5+2x)^2}-\frac{45979\sqrt{3-x+2x^2}}{26873856(5+2x)}+\frac{774079\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{322486272\sqrt{2}}$$

output
```
1/12877056*(65991-8779*x)/(2*x^2-x+3)^(3/2)-1/592344576*(4679797-2148263*x
)/(2*x^2-x+3)^(1/2)-3667/373248*(2*x^2-x+3)^(1/2)/(5+2*x)^2-45979*(2*x^2-x
+3)^(1/2)/(134369280+53747712*x)+774079/644972544*arctanh(1/24*(17-22*x)*2
^(1/2)/(2*x^2-x+3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{\frac{12\sqrt{3-x+2x^2}(-8953831359+2280511668x-5919924791x^2-1503926130x^3+107028732x^4+2178}{(15+x+8x^2+4x^3)^2}}{1705952378}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((12*Sqrt[3 - x + 2*x^2]*(-8953831359 + 2280511668*x - 5919924791*x^2 - 15
03926130*x^3 + 107028732*x^4 + 217883368*x^5))/(15 + x + 8*x^2 + 4*x^3)^2
- 409487791*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2])/6])/17059523
7888
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2177, 27, 2177, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3\left(2x^2-x+3\right)^{5/2}}dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int \frac{-280928x^3+65340540x^2+38386140x+11115283}{746496(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-280928x^3+65340540x^2+38386140x+11115283}{(2x+5)^3\left(2x^2-x+3\right)^{3/2}}dx}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \text{2177}$$

$$\frac{\frac{2}{23}\int -\frac{529(125300x^2+1076692x+324461)}{4(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{23}{2}\int \frac{125300x^2+1076692x+324461}{(2x+5)^3\sqrt{2x^2-x+3}}dx-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 2181

$$\frac{-\frac{23}{2}\left(\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}-\frac{1}{144}\int -\frac{288(53327x+64349)}{(2x+5)^2\sqrt{2x^2-x+3}}dx\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{-\frac{23}{2}\left(2\int \frac{53327x+64349}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1228

$$\frac{-\frac{23}{2}\left(2\left(\frac{774079}{48}\int \frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx+\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 1154

$$\frac{-\frac{23}{2}\left(2\left(\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}-\frac{774079}{24}\int \frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{-\frac{23}{2}\left(2\left(\frac{45979\sqrt{2x^2-x+3}}{24(2x+5)}-\frac{774079\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{288\sqrt{2}}\right)+\frac{22002\sqrt{2x^2-x+3}}{(2x+5)^2}\right)-\frac{4679797-2148263x}{23\sqrt{2x^2-x+3}}}{25754112}+\frac{65991-8779x}{12877056\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^3*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(65991 - 8779*x)/(12877056*(3 - x + 2*x^2)^(3/2)) + (-1/23*(4679797 - 2148
263*x)/Sqrt[3 - x + 2*x^2] - (23*((22002*Sqrt[3 - x + 2*x^2])/(5 + 2*x)^2
+ 2*((45979*Sqrt[3 - x + 2*x^2])/(24*(5 + 2*x)) - (774079*ArcTanh[(17 - 22
*x)/(12*Sqrt[2]*Sqrt[3 - x + 2*x^2])])/(288*Sqrt[2]))))/2)/25754112
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.58

method	result
risch	$\frac{217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-8953831359}{14216269824(2x^2-x+3)^{\frac{3}{2}}(5+2x)^2}+\frac{774079\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\frac{17}{2}-11x\right)\sqrt{}}{12\sqrt{2\left(x+\frac{5}{2}\right)^2-1}}\right.}{644972544}$
trager	$\frac{(217883368x^5+107028732x^4-1503926130x^3-5919924791x^2+2280511668x-8953831359)\sqrt{2x^2-x+3}}{14216269824(4x^3+8x^2+x+15)^2}+\frac{774079\,\operatorname{RootOf}\left(\_Z^2\right.}{}$
default	$-\frac{149(4x-1)}{1104(2x^2-x+3)^{\frac{3}{2}}}-\frac{149(4x-1)}{1587\sqrt{2x^2-x+3}}-\frac{5}{48(2x^2-x+3)^{\frac{3}{2}}}-\frac{3667}{4608\left(x+\frac{5}{2}\right)^2\left(2\left(x+\frac{5}{2}\right)^2-11x-\frac{19}{2}\right)^{\frac{3}{2}}}+\frac{11536}{165888\left(x+\frac{5}{2}\right)\left(2(x+\right.}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
1/14216269824*(217883368*x^5+107028732*x^4-1503926130*x^3-5919924791*x^2+2
280511668*x-8953831359)/(2*x^2-x+3)^(3/2)/(5+2*x)^2+774079/644972544*2^(1/
2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{409487791\,\sqrt{2}(16\,x^6+64\,x^5+72\,x^4+136\,x^3+241\,x^2+30\,x+225)\,\mathrm{lo}}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/682380951552*(409487791*sqrt(2)*(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 24
1*x^2 + 30*x + 225)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)*(22*x - 17) - 1060
*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(217883368*x^5 + 107028732
*x^4 - 1503926130*x^3 - 5919924791*x^2 + 2280511668*x - 8953831359)*sqrt(2
*x^2 - x + 3))/(16*x^6 + 64*x^5 + 72*x^4 + 136*x^3 + 241*x^2 + 30*x + 225)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\left(3-x+2x^2\right)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{\left(2x+5\right)^3\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**3/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**3*(2*x**2 - x + 3)**
(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.32

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3 (3-x+2x^2)^{5/2}}\,dx =$$
$$-\frac{774079}{644972544}\sqrt{2}\,\text{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)$$
$$+\frac{27235421\,x}{14216269824\sqrt{2\,x^2-x+3}}$$
$$-\frac{36393601}{4738756608\sqrt{2\,x^2-x+3}}+\frac{2323723\,x}{103016448\,(2\,x^2-x+3)^{\frac{3}{2}}}$$
$$-\frac{3667}{1152\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}$$
$$+\frac{115369}{82944\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{5254255}{34338816\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
-774079/644972544*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23*sq
rt(23)/abs(2*x + 5)) + 27235421/14216269824*x/sqrt(2*x^2 - x + 3) - 363936
01/4738756608/sqrt(2*x^2 - x + 3) + 2323723/103016448*x/(2*x^2 - x + 3)^(3
/2) - 3667/1152/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x
+ 25*(2*x^2 - x + 3)^(3/2)) + 115369/82944/(2*(2*x^2 - x + 3)^(3/2)*x + 5*
(2*x^2 - x + 3)^(3/2)) - 5254255/34338816/(2*x^2 - x + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. 2(109) = 218.

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.69

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx = \frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{774079}{644972544}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{\sqrt{2}\left(44558\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^3-10136238\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+16812201\sqrt{2}\left(\sqrt{2}x-\sqrt{2\right.}{53747712\left(2\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)^2+10\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)-11\right)^2} + \frac{((4296526x-11507857)x+10720752)x-11003805}{592344576(2x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2 - x
 + 3))) - 774079/644972544*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2) + 2*s
qrt(2*x^2 - x + 3))) + 1/53747712*sqrt(2)*(44558*sqrt(2)*(sqrt(2)*x - sqrt
(2*x^2 - x + 3))^3 - 10136238*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 168122
01*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 10182217)/(2*(sqrt(2)*x - s
qrt(2*x^2 - x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)
^2 + 1/592344576*(((4296526*x - 11507857)*x + 10720752)*x - 11003805)/(2*x
^2 - x + 3)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3(3-x+2x^2)^{5/2}}\,dx = \int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^3(2x^2-x+3)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^3*(2*x^2 - x + 3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 402, normalized size of antiderivative = 2.98

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^3\,(3-x+2x^2)^{5/2}}\,dx = \frac{5229200832\sqrt{2x^2-x+3}\,x^5+2568689568\sqrt{2x^2-x+3}\,x^4-3609422$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^3/(2*x^2-x+3)^(5/2),x)
```

output
```
(5229200832*sqrt(2*x**2 - x + 3)*x**5 + 2568689568*sqrt(2*x**2 - x + 3)*x*
*4 - 36094227120*sqrt(2*x**2 - x + 3)*x**3 - 142078194984*sqrt(2*x**2 - x
+ 3)*x**2 + 54732280032*sqrt(2*x**2 - x + 3)*x - 214891952616*sqrt(2*x**2
- x + 3) + 6551804656*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*
x - 17)*x**6 + 26207218624*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2)
+ 22*x - 17)*x**5 + 29483120952*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqr
t(2) + 22*x - 17)*x**4 + 55690339576*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3
)*sqrt(2) + 22*x - 17)*x**3 + 98686557631*sqrt(2)*log( - 12*sqrt(2*x**2 -
x + 3)*sqrt(2) + 22*x - 17)*x**2 + 12284633730*sqrt(2)*log( - 12*sqrt(2*x*
*2 - x + 3)*sqrt(2) + 22*x - 17)*x + 92134752975*sqrt(2)*log( - 12*sqrt(2*
x**2 - x + 3)*sqrt(2) + 22*x - 17) - 6551804656*sqrt(2)*log(2*x + 5)*x**6
- 26207218624*sqrt(2)*log(2*x + 5)*x**5 - 29483120952*sqrt(2)*log(2*x + 5)
*x**4 - 55690339576*sqrt(2)*log(2*x + 5)*x**3 - 98686557631*sqrt(2)*log(2*
x + 5)*x**2 - 12284633730*sqrt(2)*log(2*x + 5)*x - 92134752975*sqrt(2)*log
(2*x + 5))/(341190475776*(16*x**6 + 64*x**5 + 72*x**4 + 136*x**3 + 241*x**
2 + 30*x + 225))
```

3.200 $\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 160

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\left(3-x+2x^2\right)^{5/2}}\,dx = \frac{369609-175877x}{463574016\left(3-x+2x^2\right)^{3/2}} - \frac{27754539-31190998x}{31986607104\sqrt{3-x+2x^2}} - \frac{3667\sqrt{3-x+2x^2}}{559872(5+2x)^3} - \frac{89137\sqrt{3-x+2x^2}}{80621568(5+2x)^2} + \frac{475357\sqrt{3-x+2x^2}}{1934917632(5+2x)} + \frac{4778789\operatorname{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{3-x+2x^2}}\right)}{7739670528\sqrt{2}}$$

output
```
1/463574016*(369609-175877*x)/(2*x^2-x+3)^(3/2)-1/31986607104*(27754539-31
190998*x)/(2*x^2-x+3)^(1/2)-3667/559872*(2*x^2-x+3)^(1/2)/(5+2*x)^3-89137/
80621568*(2*x^2-x+3)^(1/2)/(5+2*x)^2+475357*(2*x^2-x+3)^(1/2)/(9674588160+
3869835264*x)+4778789/15479341056*arctanh(1/24*(17-22*x)*2^(1/2)/(2*x^2-x+
3)^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = \frac{\frac{12(-95241881529+73621973154x-6702882569x^2+27484986184x^3+46210466520x^4+348728108}{(5+2x)^3(3-x+2x^2)^{3/2}}}{409428}$$

input
```
Integrate[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)
),x]
```

output
```
((12*(-95241881529 + 73621973154*x - 6702882569*x^2 + 27484986184*x^3 + 46
210466520*x^4 + 34872810880*x^5 + 6664404208*x^6))/((5 + 2*x)^3*(3 - x + 2
*x^2)^(3/2)) - 2527979381*Sqrt[2]*ArcTanh[(5 + 2*x - Sqrt[6 - 2*x + 4*x^2]
)/6])/4094285709312
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2177, 27, 2177, 27, 2181, 27, 2181, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4(2x^2-x+3)^{5/2}}\,dx$$

$$\Big\downarrow \text{2177}$$

$$\frac{2}{69}\int \frac{-11256128x^4-81299864x^3+2240465820x^2+1454170990x+606939313}{26873856(2x+5)^4(2x^2-x+3)^{3/2}}dx+\frac{369609-175877x}{463574016(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-11256128x^4-81299864x^3+2240465820x^2+1454170990x+606939313}{(2x+5)^4(2x^2-x+3)^{3/2}}dx}{927148032}+\frac{369609-175877x}{463574016(2x^2-x+3)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2177}$$

$$\frac{\frac{2}{23}\int -\frac{1058(301804x^3+3955080x^2+20194167x+9095911)}{3(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{-\frac{92}{3}\int \frac{301804x^3+3955080x^2+20194167x+9095911}{(2x+5)^4\sqrt{2x^2-x+3}}dx-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2181}$$

$$\frac{-\frac{92}{3}\left(\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}-\frac{1}{216}\int -\frac{216(150902x^2+2392357x+2631056)}{(2x+5)^3\sqrt{2x^2-x+3}}dx\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{-\frac{92}{3}\left(\int \frac{150902x^2+2392357x+2631056}{(2x+5)^3\sqrt{2x^2-x+3}}dx+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{2181}$$

$$\frac{-\frac{92}{3}\left(-\frac{1}{144}\int -\frac{9(2276860x+9970363)}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{27}$$

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\int \frac{2276860x+9970363}{(2x+5)^2\sqrt{2x^2-x+3}}dx+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190998x)}{69\sqrt{2x^2-x+3}}}{927148032}+\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$$\Big\downarrow \textcolor{blue}{1228}$$

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(\frac{14336367}{8}\int\frac{1}{(2x+5)\sqrt{2x^2-x+3}}dx-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539-31190}{69\sqrt{2x^2-x+}}}{927148032}$$
$$\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\big\downarrow$ 1154

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(-\frac{14336367}{4}\int\frac{1}{288-\frac{(17-22x)^2}{2x^2-x+3}}d\frac{17-22x}{\sqrt{2x^2-x+3}}-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(2775}{6}}{927148032}$$
$$\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

$\big\downarrow$ 219

$$\frac{-\frac{92}{3}\left(\frac{1}{16}\left(-\frac{4778789\text{arctanh}\left(\frac{17-22x}{12\sqrt{2}\sqrt{2x^2-x+3}}\right)}{16\sqrt{2}}-\frac{475357\sqrt{2x^2-x+3}}{4(2x+5)}\right)+\frac{267411\sqrt{2x^2-x+3}}{8(2x+5)^2}+\frac{198018\sqrt{2x^2-x+3}}{(2x+5)^3}\right)-\frac{2(27754539}{69\sqrt{2}}}{927148032}$$
$$\frac{369609-175877x}{463574016\left(2x^2-x+3\right)^{3/2}}$$

input
```
Int[(2 + x + 3*x^2 - x^3 + 5*x^4)/((5 + 2*x)^4*(3 - x + 2*x^2)^(5/2)),x]
```

output
```
(369609 - 175877*x)/(463574016*(3 - x + 2*x^2)^(3/2)) + ((-2*(27754539 - 3
1190998*x))/(69*Sqrt[3 - x + 2*x^2]) - (92*((198018*Sqrt[3 - x + 2*x^2])/(
5 + 2*x)^3 + (267411*Sqrt[3 - x + 2*x^2])/(8*(5 + 2*x)^2) + ((-475357*Sqrt
[3 - x + 2*x^2])/(4*(5 + 2*x)) - (4778789*ArcTanh[(17 - 22*x)/(12*Sqrt[2]*
Sqrt[3 - x + 2*x^2])])/(16*Sqrt[2]))/16))/3)/927148032
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2177
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*
x^2, x], R = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
 x, 0], S = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 1]}, Simp[(b*R - 2*a*S + (2*c*R - b*S)*x)*((a + b*x + c*x^2)^(p + 1)/((p
 + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^
m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Qx)/(d + e*x
)^m - ((2*p + 3)*(2*c*R - b*S))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c,
 d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*
e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result
risch	$\frac{6664404208x^{6}+34872810880x^{5}+46210466520x^{4}+27484986184x^{3}-6702882569x^{2}+73621973154x-95241881529}{341190475776(5+2x)^{3}(2x^{2}-x+3)^{\frac{3}{2}}}+\frac{4778789\sqrt{2}}{}$
trager	$\frac{6664404208x^{6}+34872810880x^{5}+46210466520x^{4}+27484986184x^{3}-6702882569x^{2}+73621973154x-95241881529}{341190475776(5+2x)^{3}(2x^{2}-x+3)^{\frac{3}{2}}}+\frac{4778789\,\mathrm{Roo}}{}$
default	$\frac{\frac{5x}{138}-\frac{5}{552}}{(2x^{2}-x+3)^{\frac{3}{2}}}+\frac{\frac{40x}{1587}-\frac{10}{1587}}{\sqrt{2x^{2}-x+3}}-\frac{3667}{13824\left(x+\frac{5}{2}\right)^{3}\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}+\frac{25951}{110592\left(x+\frac{5}{2}\right)^{2}\left(2\left(x+\frac{5}{2}\right)^{2}-11x-\frac{19}{2}\right)^{\frac{3}{2}}}-\frac{}{3981312}$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
1/341190475776*(6664404208*x^6+34872810880*x^5+46210466520*x^4+27484986184
*x^3-6702882569*x^2+73621973154*x-95241881529)/(5+2*x)^3/(2*x^2-x+3)^(3/2)
+4778789/15479341056*2^(1/2)*arctanh(1/12*(17/2-11*x)*2^(1/2)/(2*(x+5/2)^2
-11*x-19/2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4\,(3-x+2x^2)^{5/2}}\,dx = \frac{2527979381\,\sqrt{2}(32\,x^7+208\,x^6+464\,x^5+632\,x^4+1162\,x^3+1265\,x^2}{}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
fricas")
```

output
```
1/16377142837248*(2527979381*sqrt(2)*(32*x^7 + 208*x^6 + 464*x^5 + 632*x^4
 + 1162*x^3 + 1265*x^2 + 600*x + 1125)*log((24*sqrt(2)*sqrt(2*x^2 - x + 3)
*(22*x - 17) - 1060*x^2 + 1036*x - 1153)/(4*x^2 + 20*x + 25)) + 48*(666440
4208*x^6 + 34872810880*x^5 + 46210466520*x^4 + 27484986184*x^3 - 670288256
9*x^2 + 73621973154*x - 95241881529)*sqrt(2*x^2 - x + 3))/(32*x^7 + 208*x^
6 + 464*x^5 + 632*x^4 + 1162*x^3 + 1265*x^2 + 600*x + 1125)
```

Sympy [F]

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx=\int \frac{5x^4-x^3+3x^2+x+2}{(2x+5)^4(2x^2-x+3)^{\frac{5}{2}}}\,dx$$

input
```
integrate((5*x**4-x**3+3*x**2+x+2)/(5+2*x)**4/(2*x**2-x+3)**(5/2),x)
```

output
```
Integral((5*x**4 - x**3 + 3*x**2 + x + 2)/((2*x + 5)**4*(2*x**2 - x + 3)**
(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.54

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx=$$
$$-\frac{4778789}{15479341056}\sqrt{2}\operatorname{arsinh}\left(\frac{22\sqrt{23}x}{23\,|2\,x+5|}-\frac{17\sqrt{23}}{23\,|2\,x+5|}\right)+\frac{416525263\,x}{341190475776\sqrt{2\,x^2-x+3}}$$
$$-\frac{245375387}{113730158592\sqrt{2\,x^2-x+3}}+\frac{16932905\,x}{2472394752\,(2\,x^2-x+3)^{\frac{3}{2}}}$$
$$-\frac{3667}{1728\left(8\,(2\,x^2-x+3)^{\frac{3}{2}}x^3+60\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+150\,(2\,x^2-x+3)^{\frac{3}{2}}x+125\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}$$
$$+\frac{25951}{27648\left(4\,(2\,x^2-x+3)^{\frac{3}{2}}x^2+20\,(2\,x^2-x+3)^{\frac{3}{2}}x+25\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}$$
$$-\frac{34861}{1990656\left(2\,(2\,x^2-x+3)^{\frac{3}{2}}x+5\,(2\,x^2-x+3)^{\frac{3}{2}}\right)}-\frac{10570421}{824131584\,(2\,x^2-x+3)^{\frac{3}{2}}}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
maxima")
```

output
```
-4778789/15479341056*sqrt(2)*arcsinh(22/23*sqrt(23)*x/abs(2*x + 5) - 17/23
*sqrt(23)/abs(2*x + 5)) + 416525263/341190475776*x/sqrt(2*x^2 - x + 3) - 2
45375387/113730158592/sqrt(2*x^2 - x + 3) + 16932905/2472394752*x/(2*x^2 -
 x + 3)^(3/2) - 3667/1728/(8*(2*x^2 - x + 3)^(3/2)*x^3 + 60*(2*x^2 - x + 3
)^(3/2)*x^2 + 150*(2*x^2 - x + 3)^(3/2)*x + 125*(2*x^2 - x + 3)^(3/2)) + 2
5951/27648/(4*(2*x^2 - x + 3)^(3/2)*x^2 + 20*(2*x^2 - x + 3)^(3/2)*x + 25*
(2*x^2 - x + 3)^(3/2)) - 34861/1990656/(2*(2*x^2 - x + 3)^(3/2)*x + 5*(2*x
^2 - x + 3)^(3/2)) - 10570421/824131584/(2*x^2 - x + 3)^(3/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(130) = 260$.

Time = 0.20 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.74

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx = \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x+\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) - \frac{4778789}{15479341056}\sqrt{2}\log\left(\left|-2\sqrt{2}x-11\sqrt{2}+2\sqrt{2x^2-x+3}\right|\right) + \frac{((15595499x-21675019)x+27298005)x-14440149}{7996651776\,(2x^2-x+3)^{\frac{3}{2}}} + \frac{\sqrt{2}\Big(38030012\sqrt{2}\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^5+734231900\big(\sqrt{2}x-\sqrt{2x^2-x+3}\big)^4+122834956\sqrt{2}\big(\sqrt{2}x\cdots}{3869835264\Big(2\big(\sqrt{2}x-\sqrt{2x^2-x\cdots}$$

input
```
integrate((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x, algorithm="
giac")
```

output
```
4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x + sqrt(2) + 2*sqrt(2*x^2
- x + 3))) - 4778789/15479341056*sqrt(2)*log(abs(-2*sqrt(2)*x - 11*sqrt(2)
 + 2*sqrt(2*x^2 - x + 3))) + 1/7996651776*(((15595499*x - 21675019)*x + 27
298005)*x - 14440149)/(2*x^2 - x + 3)^(3/2) + 1/3869835264*sqrt(2)*(380300
12*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^5 + 734231900*(sqrt(2)*x - sq
rt(2*x^2 - x + 3))^4 + 122834956*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
^3 - 2154595396*(sqrt(2)*x - sqrt(2*x^2 - x + 3))^2 + 1659431083*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 760577429)/(2*(sqrt(2)*x - sqrt(2*x^2 -
 x + 3))^2 + 10*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) - 11)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx=\int \frac{5\,x^4-x^3+3\,x^2+x+2}{(2\,x+5)^4\,(2\,x^2-x+3)^{5/2}}\,dx$$

input
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)),x)
```

output
```
int((x + 3*x^2 - x^3 + 5*x^4 + 2)/((2*x + 5)^4*(2*x^2 - x + 3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.90

$$\int \frac{2+x+3x^2-x^3+5x^4}{(5+2x)^4(3-x+2x^2)^{5/2}}\,dx=\frac{159945700992\sqrt{2x^2-x+3}\,x^6+836947461120\sqrt{2x^2-x+3}\,x^5+110}{}$$

input
```
int((5*x^4-x^3+3*x^2+x+2)/(5+2*x)^4/(2*x^2-x+3)^(5/2),x)
```

output
```
(159945700992*sqrt(2*x**2 - x + 3)*x**6 + 836947461120*sqrt(2*x**2 - x + 3
)*x**5 + 1109051196480*sqrt(2*x**2 - x + 3)*x**4 + 659639668416*sqrt(2*x**
2 - x + 3)*x**3 - 160869181656*sqrt(2*x**2 - x + 3)*x**2 + 1766927355696*s
qrt(2*x**2 - x + 3)*x - 2285805156696*sqrt(2*x**2 - x + 3) + 80895340192*s
qrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**7 + 52581971
1248*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x**6 + 11
72982432784*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x - 17)*x*
*5 + 1597682968792*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) + 22*x -
 17)*x**4 + 2937512040722*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sqrt(2) +
 22*x - 17)*x**3 + 3197893916965*sqrt(2)*log( - 12*sqrt(2*x**2 - x + 3)*sq
rt(2) + 22*x - 17)*x**2 + 1516787628600*sqrt(2)*log( - 12*sqrt(2*x**2 - x
+ 3)*sqrt(2) + 22*x - 17)*x + 2843976803625*sqrt(2)*log( - 12*sqrt(2*x**2
- x + 3)*sqrt(2) + 22*x - 17) - 80895340192*sqrt(2)*log(2*x + 5)*x**7 - 52
5819711248*sqrt(2)*log(2*x + 5)*x**6 - 1172982432784*sqrt(2)*log(2*x + 5)*
x**5 - 1597682968792*sqrt(2)*log(2*x + 5)*x**4 - 2937512040722*sqrt(2)*log
(2*x + 5)*x**3 - 3197893916965*sqrt(2)*log(2*x + 5)*x**2 - 1516787628600*s
qrt(2)*log(2*x + 5)*x - 2843976803625*sqrt(2)*log(2*x + 5))/(8188571418624
*(32*x**7 + 208*x**6 + 464*x**5 + 632*x**4 + 1162*x**3 + 1265*x**2 + 600*x
 + 1125))
```

3.201 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 354

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(ab^2ci+2ac^2(cg-ai)-ab^3j-bc(c^2f+ach-3a^2j)-(2c^4f-c^3(bg}{3c^3(b^2-4ac)(a+bx}$$
$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg-3ai)-b^5j-b^3c(ch-10aj)-4bc^2(2c^2f+ach+8a^2j)-c(16c^4f-c^3(8}{3c^3(b^2-4ac)^2\sqrt{a+bx+cx^2}}$$
$$+\frac{j\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{5/2}}$$

output

```
2/3*(a*b^2*c*i+2*a*c^2*(-a*i+c*g)-a*b^3*j-b*c*(-3*a^2*j+a*c*h+c^2*f)-(2*c^
4*f-c^3*(2*a*h+b*g)+b^4*j-b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x
)/c^3/(-4*a*c+b^2)/(c*x^2+b*x+a)^(3/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2
*(-3*a*i+2*c*g)-b^5*j-b^3*c*(-10*a*j+c*h)-4*b*c^2*(8*a^2*j+a*c*h+2*c^2*f)-
c*(16*c^4*f-c^3*(-8*a*h+8*b*g)-4*b^4*j+b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j
-6*a*b*i+b^2*h))*x)/c^3/(-4*a*c+b^2)^2/(c*x^2+b*x+a)^(1/2)+j*arctanh(1/2*(
2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 3.11 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2(-3b^5jx^2-2b^4jx(3a+2cx^2)+b^3(-3a^2j+18acjx^2+c^2(-f-3gx+\cdots}{\cdots} + \frac{2j\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-3*b^5*j*x^2 - 2*b^4*j*x*(3*a + 2*c*x^2) + b^3*(-3*a^2*j + 18*a*c*j*x^
2 + c^2*(-f - 3*g*x + 3*h*x^2 + i*x^3)) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f
 - 6*g*x + h*x^2) - a*c*(g - 6*h*x + 3*i*x^2 - 14*j*x^3)) - 8*c^2*(-2*c^3*
f*x^3 + a^3*(2*i + 3*j*x) - a*c^2*x*(3*f + h*x^2) + a^2*c*(g + 3*i*x^2 + 4
*j*x^3)) + 4*b*c*(5*a^3*j - 2*c^3*x^2*(-3*f + g*x) + 2*a^2*c*(h - 3*i*x) +
 3*a*c^2*(f - x*(g - h*x + i*x^2)))))/(3*c^2*(b^2 - 4*a*c)^2*(a + x*(b + c
*x))^(3/2)) + (2*j*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])]
)/c^(5/2)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2191, 27, 2191, 27, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 2191$$

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+\cdots\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{2\int\frac{3\left(4a-\frac{b^2}{c}\right)jx^2-\frac{3\left(b^2-4ac\right)(ci-bj)x}{c^2}+\frac{jb^4-c(bi+aj)b^2+8c^4f-c^3(4bg-4ah)+c^2\left(b^2h-4a^2j\right)}{c^3}}{2(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 27

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+\cdots\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{\int\frac{\frac{jb^4}{c^3}-\frac{(bi+aj)b^2}{c^2}-4gb+3\left(4a-\frac{b^2}{c}\right)jx^2+8cf+4ah+\frac{b^2h-4a^2j}{c}-\frac{3\left(b^2-4ac\right)(ci-bj)x}{c^2}}{(cx^2+bx+a)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 2191

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+\cdots\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c^3i-b^3c(ch-10aj)+b^2\left(4c^3g-6ac^2\cdots\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}\cdots}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 27

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+\cdots\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c^3i-b^3c(ch-10aj)+b^2\left(4c^3g-6ac^2\cdots\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}\cdots}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 1092

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+\cdots\right.}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$
$$\frac{\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c^3i-b^3c(ch-10aj)+b^2\left(4c^3g-6ac^2\cdots\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}\cdots}{3\left(b^2-4ac\right)}$$

$\Big\downarrow$ 219

$$\frac{2\left(-x\left(c^2\left(2a^2j+3abi+b^2h\right)-b^2c(4aj+bi)-c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j+ach+c^2f\right)-ab^3j+a}{3c^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}$$

$$\frac{\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)+b^2c(28aj+bi)-c^3(8bg-8ah)-4b^4j+16c^4f\right)-4bc^2\left(8a^2j+ach+2c^2f\right)+24a^2c^3i-b^3c(ch-10aj)+b^2\left(4c^3g-6ac^2\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}}{3\left(b^2-4ac\right)}$$

input

```
Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x]
```

output

```
(2*(a*b^2*c*i + 2*a*c^2*(c*g - a*i) - a*b^3*j - b*c*(c^2*f + a*c*h - 3*a^2
*j) - (2*c^4*f - c^3*(b*g + 2*a*h) + b^4*j - b^2*c*(b*i + 4*a*j) + c^2*(b^
2*h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)
) - ((2*(b^4*c*i + 24*a^2*c^3*i + b^2*(4*c^3*g - 6*a*c^2*i) - b^5*j - b^3*
c*(c*h - 10*a*j) - 4*b*c^2*(2*c^2*f + a*c*h + 8*a^2*j) - c*(16*c^4*f - c^3
*(8*b*g - 8*a*h) - 4*b^4*j + b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i
 - 16*a^2*j))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) - (3*(b^2 - 4*
a*c)*j*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2))/(3
*(b^2 - 4*a*c))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1113 vs. $2(338) = 676$.

Time = 0.43 (sec) , antiderivative size = 1114, normalized size of antiderivative = 3.15

method	result	size
default	Expression too large to display	1114

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
f*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c
*x+b)/(c*x^2+b*x+a)^(1/2))+g*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c
*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^
2+b*x+a)^(1/2)))+h*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/(c*x^2+b*
x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(
4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+b)/(4*a*c
-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/
2)))+i*(-x^2/c/(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1
/4*b/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x
^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a
/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*
c*x+b)/(c*x^2+b*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/
3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)
/(c*x^2+b*x+a)^(1/2))))+j*(-1/3*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(-x^2/c/
(c*x^2+b*x+a)^(3/2)+1/2*b/c*(-1/2*x/c/(c*x^2+b*x+a)^(3/2)-1/4*b/c*(-1/3/c/
(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+a)^(1/2)))+1/2*a/c*(2/3*(2*c*x+
b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b
*x+a)^(1/2)))+2*a/c*(-1/3/c/(c*x^2+b*x+a)^(3/2)-1/2*b/c*(2/3*(2*c*x+b)/(4*
a*c-b^2)/(c*x^2+b*x+a)^(3/2)+16/3*c/(4*a*c-b^2)^2*(2*c*x+b)/(c*x^2+b*x+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(337) = 674$.

Time = 22.00 (sec) , antiderivative size = 1373, normalized size of antiderivative = 3.88

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="fric
as")
```

output
```
[1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*j*x^2 + 2*(a*b^5
 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*
j)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i + (16*c^6*f - 8*b
*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - 12*a*b*c^4)*i - 4*(b^4*c^2 -
 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f - 4*b^2*c^4*g + (b^3*c^3 +
 4*a*b*c^4)*h - 2*(a*b^2*c^3 + 4*a^2*c^4)*i - (b^5*c - 6*a*b^3*c^2)*j)*x^2
 - (b^3*c^3 - 12*a*b*c^4)*f - 2*(a*b^2*c^3 + 4*a^2*c^4)*g - (3*a^2*b^3*c -
 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i + 2*(b^2*c^4 + 4*a*c^5
)*f - (b^3*c^3 + 4*a*b*c^4)*g - 2*(a*b^4*c - 7*a^2*b^2*c^2 + 4*a^3*c^3)*j)
*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*
b*c^6)*x^3 + (b^6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a
^2*b^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*j*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 - 6*a*b^4*c
 + 32*a^3*c^3)*j*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b
^4 - 8*a^3*b^2*c + 16*a^4*c^2)*j)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a
)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*h - 16*a^
3*c^3*i + (16*c^6*f - 8*b*c^5*g + 2*(b^2*c^4 + 4*a*c^5)*h + (b^3*c^3 - ...
```

Sympy [F]

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}}\, dx = \int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{\frac{5}{2}}}\, dx$$

input
```
integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
Integral((f + g*x + h*x**2 + i*x**3 + j*x**4)/(a + b*x + c*x**2)**(5/2), x
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.31

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx + cx^2)^{5/2}}\, dx = \frac{2\left(\left(\left(\frac{(16\,c^5f - 8\,bc^4g + 2\,b^2c^3h + 8\,ac^4h + b^3c^2i - 12\,abc^3i - 4\,b^4cj + 28\,ab^2c^2j - 32\,a^2c^3j)x}{b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4} + \frac{3\,(8}{}\right.\right.\right.}{}$$
$$- \frac{j\log\left(\left|2\left(\sqrt{c}x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} + b\right|\right)}{c^{\frac{5}{2}}}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac
")
```

output
```
2/3*((((16*c^5*f - 8*b*c^4*g + 2*b^2*c^3*h + 8*a*c^4*h + b^3*c^2*i - 12*a*
b*c^3*i - 4*b^4*c*j + 28*a*b^2*c^2*j - 32*a^2*c^3*j)*x/(b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4) + 3*(8*b*c^4*f - 4*b^2*c^3*g + b^3*c^2*h + 4*a*b*c^3*h -
 2*a*b^2*c^2*i - 8*a^2*c^3*i - b^5*j + 6*a*b^3*c*j)/(b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f + 8*a*c^4*f - b^3*c^2*g - 4*a*b*c^3*g +
 4*a*b^2*c^2*h - 8*a^2*b*c^2*i - 2*a*b^4*j + 14*a^2*b^2*c*j - 8*a^3*c^2*j)
/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*c^2*f - 12*a*b*c^3*f + 2*a
*b^2*c^2*g + 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*c^2*i + 3*a^2*b^3*j - 20
*a^3*b*c*j)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(3/2)
- j*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx=\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2),x)
```

output
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2208, normalized size of antiderivative = 6.24

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx+cx^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
(40*sqrt(a + b*x + c*x**2)*a**3*b*c**2*j - 32*sqrt(a + b*x + c*x**2)*a**3*
c**3*i - 48*sqrt(a + b*x + c*x**2)*a**3*c**3*j*x - 6*sqrt(a + b*x + c*x**2
)*a**2*b**3*c*j + 84*sqrt(a + b*x + c*x**2)*a**2*b**2*c**2*j*x + 16*sqrt(a
 + b*x + c*x**2)*a**2*b*c**3*h - 48*sqrt(a + b*x + c*x**2)*a**2*b*c**3*i*x
 - 16*sqrt(a + b*x + c*x**2)*a**2*c**4*g - 48*sqrt(a + b*x + c*x**2)*a**2*
c**4*i*x**2 - 64*sqrt(a + b*x + c*x**2)*a**2*c**4*j*x**3 - 12*sqrt(a + b*x
 + c*x**2)*a*b**4*c*j*x + 36*sqrt(a + b*x + c*x**2)*a*b**3*c**2*j*x**2 - 4
*sqrt(a + b*x + c*x**2)*a*b**2*c**3*g + 24*sqrt(a + b*x + c*x**2)*a*b**2*c
**3*h*x - 12*sqrt(a + b*x + c*x**2)*a*b**2*c**3*i*x**2 + 56*sqrt(a + b*x +
 c*x**2)*a*b**2*c**3*j*x**3 + 24*sqrt(a + b*x + c*x**2)*a*b*c**4*f - 24*sq
rt(a + b*x + c*x**2)*a*b*c**4*g*x + 24*sqrt(a + b*x + c*x**2)*a*b*c**4*h*x
**2 - 24*sqrt(a + b*x + c*x**2)*a*b*c**4*i*x**3 + 48*sqrt(a + b*x + c*x**2
)*a*c**5*f*x + 16*sqrt(a + b*x + c*x**2)*a*c**5*h*x**3 - 6*sqrt(a + b*x +
c*x**2)*b**5*c*j*x**2 - 8*sqrt(a + b*x + c*x**2)*b**4*c**2*j*x**3 - 2*sqrt
(a + b*x + c*x**2)*b**3*c**3*f - 6*sqrt(a + b*x + c*x**2)*b**3*c**3*g*x +
6*sqrt(a + b*x + c*x**2)*b**3*c**3*h*x**2 + 2*sqrt(a + b*x + c*x**2)*b**3*
c**3*i*x**3 + 12*sqrt(a + b*x + c*x**2)*b**2*c**4*f*x - 24*sqrt(a + b*x +
c*x**2)*b**2*c**4*g*x**2 + 4*sqrt(a + b*x + c*x**2)*b**2*c**4*h*x**3 + 48*
sqrt(a + b*x + c*x**2)*b*c**5*f*x**2 - 16*sqrt(a + b*x + c*x**2)*b*c**5*g*
x**3 + 32*sqrt(a + b*x + c*x**2)*c**6*f*x**3 + 48*sqrt(c)*log((2*sqrt(c...
```

3.202 $\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 353

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx = \frac{2(ab^2ci+2ac^2(cg+ai)+ab^3j-bc(c^2f-ach-3a^2j)+(2c^4f+c^3(bg\cdots}{3c^3(b^2+4ac)(a+bx\cdots}$$
$$-\frac{2(b^4ci+24a^2c^3i+2b^2c^2(2cg+3ai)+b^5j+b^3c(ch+10aj)+4bc^2(2c^2f-ach+8a^2j)-c(16c^4f+8c^3(\cdots}{3c^3(b^2+4ac)^2\sqrt{a+bx-cx^2}}$$
$$-\frac{j\arctan\left(\frac{b-2cx}{2\sqrt{c}\sqrt{a+bx-cx^2}}\right)}{c^{5/2}}$$

output
```
2/3*(a*b^2*c*i+2*a*c^2*(a*i+c*g)+a*b^3*j-b*c*(-3*a^2*j-a*c*h+c^2*f)+(2*c^4
*f+c^3*(2*a*h+b*g)+b^4*j+b^2*c*(4*a*j+b*i)+c^2*(2*a^2*j+3*a*b*i+b^2*h))*x)
/c^3/(4*a*c+b^2)/(-c*x^2+b*x+a)^(3/2)-2/3*(b^4*c*i+24*a^2*c^3*i+2*b^2*c^2*
(3*a*i+2*c*g)+b^5*j+b^3*c*(10*a*j+c*h)+4*b*c^2*(8*a^2*j-a*c*h+2*c^2*f)-c*(
16*c^4*f+8*c^3*(-a*h+b*g)-4*b^4*j-b^2*c*(28*a*j+b*i)+2*c^2*(-16*a^2*j-6*a*
b*i+b^2*h))*x)/c^3/(4*a*c+b^2)^2/(-c*x^2+b*x+a)^(1/2)-j*arctan(1/2*(-2*c*x
+b)/c^(1/2)/(-c*x^2+b*x+a)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx =$$
$$-\frac{2(3b^5jx^2+b^4(6ajx-4cjx^3)+b^3(3a^2j+18acjx^2+c^2(f+3gx-x^2(3h+ix)))+8c^2(2c^3fx^3+a^3(2i+}{}$$
$$+\frac{2j\arctan\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b-cx)}}\right)}{c^{5/2}}$$

input
```
Integrate[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

output
```
(-2*(3*b^5*j*x^2 + b^4*(6*a*j*x - 4*c*j*x^3) + b^3*(3*a^2*j + 18*a*c*j*x^2
 + c^2*(f + 3*g*x - x^2*(3*h + i*x))) + 8*c^2*(2*c^3*f*x^3 + a^3*(2*i + 3*
j*x) - a*c^2*x*(3*f + h*x^2) - a^2*c*(g + x^2*(3*i + 4*j*x))) + 4*b*c*(5*a
^3*j + 2*c^3*x^2*(-3*f + g*x) - 2*a^2*c*(h - 3*i*x) + 3*a*c^2*(f - x*(g -
h*x + i*x^2))) + 2*b^2*c*(21*a^2*j*x + c^2*x*(3*f + x*(-6*g + h*x)) + a*c*
(g + x*(-6*h + 3*i*x - 14*j*x^2)))))/(3*c^2*(b^2 + 4*a*c)^2*(a + x*(b - c*
x))^(3/2)) + (2*j*ArcTan[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b - c*x)])])/
c^(5/2)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2191, 27, 2191, 27, 1092, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx$$
$$\Big\downarrow \; 2191$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

$$\frac{2\int -\frac{-\frac{3\left(b^2+4ac\right)jx^2}{c}-\frac{3\left(b^2+4ac\right)(ci+bj)x}{c^2}+\frac{jb^4+c(bi+aj)b^2+8c^4f+4c^3(bg-ah)+c^2\left(b^2h-4a^2j\right)}{c^3}}{2(-cx^2+bx+a)^{3/2}}dx}{3\left(4ac+b^2\right)}$$

↓ 27

$$\frac{\int \frac{\frac{jb^4}{c^3}+\frac{(bi+aj)b^2}{c^2}+4gb-\frac{3\left(b^2+4ac\right)jx^2}{c}+8cf-4ah+\frac{b^2h-4a^2j}{c}-\frac{3\left(b^2+4ac\right)(ci+bj)x}{c^2}}{(-cx^2+bx+a)^{3/2}}dx}{3\left(4ac+b^2\right)}+$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

↓ 2191

$$\frac{-\frac{2\int -\frac{3\left(b^2+4ac\right)^2j}{2c^2\sqrt{-cx^2+bx+a}}dx}{4ac+b^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3i+\right.}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

↓ 27

$$\frac{\frac{3j\left(4ac+b^2\right)\int \frac{1}{\sqrt{-cx^2+bx+a}}dx}{c^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3\right.}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

↓ 1092

$$\frac{\frac{6j\left(4ac+b^2\right)\int \frac{1}{-\frac{(b-2cx)^2}{-cx^2+bx+a}-4c}d\frac{b-2cx}{\sqrt{-cx^2+bx+a}}}{c^2}-\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach\right.\right.}{c^3\left(4ac+b^2\right)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab^2\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

↓ 217

$$-\frac{\frac{2\left(-cx\left(2c^2\left(-16a^2j-6abi+b^2h\right)-b^2c(28aj+bi)+8c^3(bg-ah)-4b^4j+16c^4f\right)+4bc^2\left(8a^2j-ach+2c^2f\right)+24a^2c^3i+b^3c(10aj+ch)+2b^2c^2(3ai+2c\ldots}{c^3(4ac+b^2)\sqrt{a+bx-cx^2}}}{3\left(4ac+b^2\right)}$$

$$\frac{2\left(x\left(c^2\left(2a^2j+3abi+b^2h\right)+b^2c(4aj+bi)+c^3(2ah+bg)+b^4j+2c^4f\right)-bc\left(-3a^2j-ach+c^2f\right)+ab^3j+ab\ldots\right.}{3c^3\left(4ac+b^2\right)\left(a+bx-cx^2\right)^{3/2}}$$

input
```
Int[(f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x]
```

output
```
(2*(a*b^2*c*i + 2*a*c^2*(c*g + a*i) + a*b^3*j - b*c*(c^2*f - a*c*h - 3*a^2
*j) + (2*c^4*f + c^3*(b*g + 2*a*h) + b^4*j + b^2*c*(b*i + 4*a*j) + c^2*(b^
2*h + 3*a*b*i + 2*a^2*j))*x))/(3*c^3*(b^2 + 4*a*c)*(a + b*x - c*x^2)^(3/2)
) + ((-2*(b^4*c*i + 24*a^2*c^3*i + 2*b^2*c^2*(2*c*g + 3*a*i) + b^5*j + b^3
*c*(c*h + 10*a*j) + 4*b*c^2*(2*c^2*f - a*c*h + 8*a^2*j) - c*(16*c^4*f + 8*
c^3*(b*g - a*h) - 4*b^4*j - b^2*c*(b*i + 28*a*j) + 2*c^2*(b^2*h - 6*a*b*i
- 16*a^2*j))*x))/(c^3*(b^2 + 4*a*c)*Sqrt[a + b*x - c*x^2]) - (3*(b^2 + 4*a
*c)*j*ArcTan[(b - 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x - c*x^2])])/c^(5/2))/(3*(
b^2 + 4*a*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1146 vs. $2(337) = 674$.

Time = 0.42 (sec) , antiderivative size = 1147, normalized size of antiderivative = 3.25

method	result	size
default	Expression too large to display	1147

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
f*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*
(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))+g*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/
3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c
*x+b)/(-c*x^2+b*x+a)^(1/2)))+h*(1/2*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/
c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)
^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/
3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c
*x+b)/(-c*x^2+b*x+a)^(1/2)))+i*(x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2*x/
c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-
2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b
)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+
a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*(1/
3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+
a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2))))+j*(1/3*x
^3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(x^2/c/(-c*x^2+b*x+a)^(3/2)-1/2*b/c*(1/2
*x/c/(-c*x^2+b*x+a)^(3/2)+1/4*b/c*(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3
*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*
x+b)/(-c*x^2+b*x+a)^(1/2)))-1/2*a/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^2+b
*x+a)^(3/2)-16/3*c/(-4*a*c-b^2)^2*(-2*c*x+b)/(-c*x^2+b*x+a)^(1/2)))-2*a/c*
(1/3/c/(-c*x^2+b*x+a)^(3/2)+1/2*b/c*(2/3*(-2*c*x+b)/(-4*a*c-b^2)/(-c*x^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(339) = 678$.

Time = 22.07 (sec) , antiderivative size = 1385, normalized size of antiderivative = 3.92

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="fri
cas")
```

output
```
[-1/6*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*
c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^
5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)
*j)*sqrt(-c)*log(8*c^2*x^2 - 8*b*c*x + b^2 - 4*sqrt(-c*x^2 + b*x + a)*(2*c
*x - b)*sqrt(-c) - 4*a*c) - 4*(8*a^2*b*c^3*h - 16*a^3*c^3*i - (16*c^6*f +
8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3 + 12*a*b*c^4)*i - 4*(b^4*c^
2 + 7*a*b^2*c^3 + 8*a^2*c^4)*j)*x^3 + 3*(8*b*c^5*f + 4*b^2*c^4*g + (b^3*c^
3 - 4*a*b*c^4)*h - 2*(a*b^2*c^3 - 4*a^2*c^4)*i - (b^5*c + 6*a*b^3*c^2)*j)*
x^2 - (b^3*c^3 + 12*a*b*c^4)*f - 2*(a*b^2*c^3 - 4*a^2*c^4)*g - (3*a^2*b^3*
c + 20*a^3*b*c^2)*j + 3*(4*a*b^2*c^3*h - 8*a^2*b*c^3*i - 2*(b^2*c^4 - 4*a*
c^5)*f - (b^3*c^3 - 4*a*b*c^4)*g - 2*(a*b^4*c + 7*a^2*b^2*c^2 + 4*a^3*c^3)
*j)*x)*sqrt(-c*x^2 + b*x + a))/(a^2*b^4*c^3 + 8*a^3*b^2*c^4 + 16*a^4*c^5 +
 (b^4*c^5 + 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 - 2*(b^5*c^4 + 8*a*b^3*c^5 + 16*
a^2*b*c^6)*x^3 + (b^6*c^3 + 6*a*b^4*c^4 - 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 +
 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x), -1/3*(3*((b^4*c^2 + 8*a*b^2*c^3 + 16*a^
2*c^4)*j*x^4 - 2*(b^5*c + 8*a*b^3*c^2 + 16*a^2*b*c^3)*j*x^3 + (b^6 + 6*a*b
^4*c - 32*a^3*c^3)*j*x^2 + 2*(a*b^5 + 8*a^2*b^3*c + 16*a^3*b*c^2)*j*x + (a
^2*b^4 + 8*a^3*b^2*c + 16*a^4*c^2)*j)*sqrt(c)*arctan(1/2*sqrt(-c*x^2 + b*x
 + a)*(2*c*x - b)*sqrt(c)/(c^2*x^2 - b*c*x - a*c)) - 2*(8*a^2*b*c^3*h - 16
*a^3*c^3*i - (16*c^6*f + 8*b*c^5*g + 2*(b^2*c^4 - 4*a*c^5)*h - (b^3*c^3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((j*x**4+i*x**3+h*x**2+g*x+f)/(-c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}}\, dx = \int \frac{jx^4 + ix^3 + hx^2 + gx + f}{(-cx^2 + bx + a)^{\frac{5}{2}}}\, dx$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="max
ima")
```

output
```
-1/3*i*(32*a*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 16*a*b^2/(sqrt
(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + b^3*x/((-c*x^2 + b*x + a)^(3/2)*(b
^2 + 4*a*c)*c^2) + 2*(b^2 - 4*a*c)*b*x/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*
c)^2*c) + 6*a*b*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - 3*x^2/((-c*
x^2 + b*x + a)^(3/2)*c) - (b^2 - 4*a*c)*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 +
 4*a*c)^2*c^2) - a*b^2/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c^2) + 2*a/
((-c*x^2 + b*x + a)^(3/2)*c^2)) + 1/3*g*(16*b*c*x/(sqrt(-c*x^2 + b*x + a)*
(b^2 + 4*a*c)^2) - 8*b^2/(sqrt(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) + 2*b*x/
((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b^2/((-c*x^2 + b*x + a)^(3/2)*(
b^2 + 4*a*c)*c) + 1/((-c*x^2 + b*x + a)^(3/2)*c)) + 2/3*f*(16*c^2*x/(sqrt(
-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2) - 8*b*c/(sqrt(-c*x^2 + b*x + a)*(b^2 +
4*a*c)^2) + 2*c*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)) - b/((-c*x^2 +
b*x + a)^(3/2)*(b^2 + 4*a*c))) + 2/3*h*(2*(b^2 - 4*a*c)*x/(sqrt(-c*x^2 + b
*x + a)*(b^2 + 4*a*c)^2) + 2*a*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c))
+ b^2*x/((-c*x^2 + b*x + a)^(3/2)*(b^2 + 4*a*c)*c) - (b^2 - 4*a*c)*b/(sqrt
(-c*x^2 + b*x + a)*(b^2 + 4*a*c)^2*c) + a*b/((-c*x^2 + b*x + a)^(3/2)*(b^2
 + 4*a*c)*c)) + j*integrate(x^4/((c^2*x^4 - 2*b*c*x^3 + 2*a*b*x + (b^2 - 2
*a*c)*x^2 + a^2)*sqrt(-c*x^2 + b*x + a)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.38

$$\int \frac{f + gx + hx^2 + ix^3 + jx^4}{(a + bx - cx^2)^{5/2}}\, dx =$$

$$-\frac{2\sqrt{-cx^2 + bx + a}\left(\left(\left(\frac{(16\,c^5 f + 8\,bc^4 g + 2\,b^2c^3h - 8\,ac^4h - b^3c^2 i - 12\,abc^3 i - 4\,b^4cj - 28\,ab^2c^2 j - 32\,a^2c^3 j)x}{b^4c^2 + 8\,ab^2c^3 + 16\,a^2c^4} - \frac{3\,(8\,bc^4 f + 4\,b^2c^3 g + b^3c^2 h -}{b^4c^{}}\right.\right.\right.}{}$$

$$-\frac{j\log\left(\left|2\left(\sqrt{-c}x - \sqrt{-cx^2 + bx + a}\right)\sqrt{-c} + b\right|\right)}{\sqrt{-c}c^2}$$

input
```
integrate((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x, algorithm="gia
c")
```

output
```
-2/3*sqrt(-c*x^2 + b*x + a)*((((16*c^5*f + 8*b*c^4*g + 2*b^2*c^3*h - 8*a*c
^4*h - b^3*c^2*i - 12*a*b*c^3*i - 4*b^4*c*j - 28*a*b^2*c^2*j - 32*a^2*c^3*
j)*x/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4) - 3*(8*b*c^4*f + 4*b^2*c^3*g + b
^3*c^2*h - 4*a*b*c^3*h - 2*a*b^2*c^2*i + 8*a^2*c^3*i - b^5*j - 6*a*b^3*c*j
)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c^3*f - 8*a*c^4*f + b
^3*c^2*g - 4*a*b*c^3*g - 4*a*b^2*c^2*h + 8*a^2*b*c^2*i + 2*a*b^4*j + 14*a^
2*b^2*c*j + 8*a^3*c^2*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))*x + (b^3*c^
2*f + 12*a*b*c^3*f + 2*a*b^2*c^2*g - 8*a^2*c^3*g - 8*a^2*b*c^2*h + 16*a^3*
c^2*i + 3*a^2*b^3*j + 20*a^3*b*c*j)/(b^4*c^2 + 8*a*b^2*c^3 + 16*a^2*c^4))/
(c*x^2 - b*x - a)^2 - j*log(abs(2*(sqrt(-c)*x - sqrt(-c*x^2 + b*x + a))*sq
rt(-c) + b))/(sqrt(-c)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=\int \frac{j\,x^4+i\,x^3+h\,x^2+g\,x+f}{(-c\,x^2+b\,x+a)^{5/2}}\,dx$$

input
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2),x)
```

output
```
int((f + g*x + h*x^2 + i*x^3 + j*x^4)/(a + b*x - c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2215, normalized size of antiderivative = 6.27

$$\int \frac{f+gx+hx^2+ix^3+jx^4}{(a+bx-cx^2)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int((j*x^4+i*x^3+h*x^2+g*x+f)/(-c*x^2+b*x+a)^(5/2),x)
```

output
```
( - 192*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2)
)*a**4*c**3*j - 144*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4
*a*c + b**2))*a**3*b**2*c**2*j - 192*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((
b - 2*c*x)/sqrt(4*a*c + b**2))*a**3*b*c**3*j*x + 192*sqrt(c)*sqrt(a + b*x
- c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2))*a**3*c**4*j*x**2 - 36*sqrt(
c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2))*a**2*b**4*c
*j - 144*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2
))*a**2*b**3*c**2*j*x + 144*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x
)/sqrt(4*a*c + b**2))*a**2*b**2*c**3*j*x**2 - 3*sqrt(c)*sqrt(a + b*x - c*x
**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2))*a*b**6*j - 36*sqrt(c)*sqrt(a + b
*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2))*a*b**5*c*j*x + 36*sqrt(c
)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b**2))*a*b**4*c**2*
j*x**2 - 3*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a*c + b*
*2))*b**7*j*x + 3*sqrt(c)*sqrt(a + b*x - c*x**2)*asin((b - 2*c*x)/sqrt(4*a
*c + b**2))*b**6*c*j*x**2 + 128*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c
+ b**2)*a**3*b*c**2*j + 64*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c + b**
2)*a**3*c**3*i + 48*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c + b**2)*a**2
*b**3*c*j + 8*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c + b**2)*a**2*b**2*
c**2*i + 128*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c + b**2)*a**2*b**2*c
**2*j*x - 32*sqrt(c)*sqrt(a + b*x - c*x**2)*sqrt(4*a*c + b**2)*a**2*b*c...
```

3.203 $\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x\right.$

Optimal result

Integrand size = 38, antiderivative size = 588

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx$$
$$= \frac{(5d^2-2de+3e^2)^3 (4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{1+m}}{e^{11}(1+m)}$$
$$-\frac{(5d^2-2de+3e^2)^2 (200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5)(d+ex)^{2+m}}{e^{11}(2+m)}$$
$$+\frac{3(5d^2-2de+3e^2)(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6)(d+ex)^{3+m}}{e^{11}(3+m)}$$
$$-\frac{2(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7)(d+ex)^{4+}}{e^{11}(4+m)}$$
$$+\frac{(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6)(d+ex)^{5+m}}{e^{11}(5+m)}$$
$$-\frac{6(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5)(d+ex)^{6+m}}{e^{11}(6+m)}$$
$$+\frac{(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4)(d+ex)^{7+m}}{e^{11}(7+m)}$$
$$-\frac{2(30000d^3+450d^2e+3060de^2+49e^3)(d+ex)^{8+m}}{e^{11}(8+m)}$$
$$+\frac{45(500d^2+5de+17e^2)(d+ex)^{9+m}}{e^{11}(9+m)}$$
$$-\frac{25(200d+e)(d+ex)^{10+m}}{e^{11}(10+m)}+\frac{500(d+ex)^{11+m}}{e^{11}(11+m)}$$

output
```
(5*d^2-2*d*e+3*e^2)^3*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/
e^11/(1+m)-(5*d^2-2*d*e+3*e^2)^2*(200*d^5+169*d^4*e+108*d^3*e^2-20*d^2*e^3
+86*d*e^4-15*e^5)*(e*x+d)^(2+m)/e^11/(2+m)+3*(5*d^2-2*d*e+3*e^2)*(1500*d^6
+660*d^5*e+792*d^4*e^2+58*d^3*e^3+547*d^2*e^4-156*d*e^5+53*e^6)*(e*x+d)^(3
+m)/e^11/(3+m)-2*(30000*d^7+1050*d^6*e+21420*d^5*e^2+1715*d^4*e^3+9990*d^3
*e^4-2550*d^2*e^5+2218*d*e^6-287*e^7)*(e*x+d)^(4+m)/e^11/(4+m)+(105000*d^6
+3150*d^5*e+53550*d^4*e^2+3430*d^3*e^3+14985*d^2*e^4-2550*d*e^5+1109*e^6)*
(e*x+d)^(5+m)/e^11/(5+m)-6*(21000*d^5+525*d^4*e+7140*d^3*e^2+343*d^2*e^3+9
99*d*e^4-85*e^5)*(e*x+d)^(6+m)/e^11/(6+m)+(105000*d^4+2100*d^3*e+21420*d^2
*e^2+686*d*e^3+999*e^4)*(e*x+d)^(7+m)/e^11/(7+m)-2*(30000*d^3+450*d^2*e+30
60*d*e^2+49*e^3)*(e*x+d)^(8+m)/e^11/(8+m)+45*(500*d^2+5*d*e+17*e^2)*(e*x+d
)^(9+m)/e^11/(9+m)-25*(200*d+e)*(e*x+d)^(10+m)/e^11/(10+m)+500*(e*x+d)^(11
+m)/e^11/(11+m)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)^3(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(5d^2-2de+3e^2)^2(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5)(d+}{2+m}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 169*d^
4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x))/(2 + m) + (
3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*d^4*e^2 + 58*d^3*e^3
 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^2)/(3 + m) - (2*(30000*d^7
+ 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e^3 + 9990*d^3*e^4 - 2550*d^2*e^5
+ 2218*d*e^6 - 287*e^7)*(d + e*x)^3)/(4 + m) + ((105000*d^6 + 3150*d^5*e +
 53550*d^4*e^2 + 3430*d^3*e^3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d
+ e*x)^4)/(5 + m) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3
 + 999*d*e^4 - 85*e^5)*(d + e*x)^5)/(6 + m) + ((105000*d^4 + 2100*d^3*e +
21420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^6)/(7 + m) - (2*(30000*d^3
+ 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^7)/(8 + m) + (45*(500*d^2 + 5
*d*e + 17*e^2)*(d + e*x)^8)/(9 + m) - (25*(200*d + e)*(d + e*x)^9)/(10 + m
) + (500*(d + e*x)^10)/(11 + m)))/e^11
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^3 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^m \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{45\left(500d^2+5de+17e^2\right)(d+ex)^{m+8}}{e^{10}}-\frac{2\left(30000d^3+450d^2e+3060de^2+49e^3\right)(d+ex)^{m+7}}{e^{10}}+\frac{\left(5d^2-2de\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$
\frac{45(500d^2+5de+17e^2)(d+ex)^{m+9}}{e^{11}(m+9)}-\frac{2(30000d^3+450d^2e+3060de^2+49e^3)(d+ex)^{m+8}}{e^{11}(m+8)}+\frac{(5d^2-2de+3e^2)^3(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{m+1}}{e^{11}(m+1)}+\frac{(105000d^4+2100d^3e+21420d^2e^2+686de^3+999e^4)(d+ex)^{m+7}}{e^{11}(m+7)}-\frac{(5d^2-2de+3e^2)^2(200d^5+169d^4e+108d^3e^2-20d^2e^3+86de^4-15e^5)(d+ex)^{m+2}}{e^{11}(m+2)}-\frac{6(21000d^5+525d^4e+7140d^3e^2+343d^2e^3+999de^4-85e^5)(d+ex)^{m+6}}{e^{11}(m+6)}+\frac{3(5d^2-2de+3e^2)(1500d^6+660d^5e+792d^4e^2+58d^3e^3+547d^2e^4-156de^5+53e^6)(d+ex)^{m+3}}{e^{11}(m+3)}+\frac{(105000d^6+3150d^5e+53550d^4e^2+3430d^3e^3+14985d^2e^4-2550de^5+1109e^6)(d+ex)^{m+5}}{e^{11}(m+5)}-\frac{2(30000d^7+1050d^6e+21420d^5e^2+1715d^4e^3+9990d^3e^4-2550d^2e^5+2218de^6-287e^7)(d+ex)^{m+4}}{e^{11}(m+4)}-\frac{25(200d+e)(d+ex)^{m+10}}{e^{11}(m+10)}+\frac{500(d+ex)^{m+11}}{e^{11}(m+11)}
$$

input

```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^3*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output

```
((5*d^2 - 2*d*e + 3*e^2)^3*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^(1 + m))/(e^11*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)^2*(200*d^5 + 1
69*d^4*e + 108*d^3*e^2 - 20*d^2*e^3 + 86*d*e^4 - 15*e^5)*(d + e*x)^(2 + m)
)/(e^11*(2 + m)) + (3*(5*d^2 - 2*d*e + 3*e^2)*(1500*d^6 + 660*d^5*e + 792*
d^4*e^2 + 58*d^3*e^3 + 547*d^2*e^4 - 156*d*e^5 + 53*e^6)*(d + e*x)^(3 + m)
)/(e^11*(3 + m)) - (2*(30000*d^7 + 1050*d^6*e + 21420*d^5*e^2 + 1715*d^4*e
^3 + 9990*d^3*e^4 - 2550*d^2*e^5 + 2218*d*e^6 - 287*e^7)*(d + e*x)^(4 + m)
)/(e^11*(4 + m)) + ((105000*d^6 + 3150*d^5*e + 53550*d^4*e^2 + 3430*d^3*e^
3 + 14985*d^2*e^4 - 2550*d*e^5 + 1109*e^6)*(d + e*x)^(5 + m))/(e^11*(5 + m
)) - (6*(21000*d^5 + 525*d^4*e + 7140*d^3*e^2 + 343*d^2*e^3 + 999*d*e^4 -
85*e^5)*(d + e*x)^(6 + m))/(e^11*(6 + m)) + ((105000*d^4 + 2100*d^3*e + 21
420*d^2*e^2 + 686*d*e^3 + 999*e^4)*(d + e*x)^(7 + m))/(e^11*(7 + m)) - (2*
(30000*d^3 + 450*d^2*e + 3060*d*e^2 + 49*e^3)*(d + e*x)^(8 + m))/(e^11*(8
+ m)) + (45*(500*d^2 + 5*d*e + 17*e^2)*(d + e*x)^(9 + m))/(e^11*(9 + m)) -
 (25*(200*d + e)*(d + e*x)^(10 + m))/(e^11*(10 + m)) + (500*(d + e*x)^(11
+ m))/(e^11*(11 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 5923 vs. $2(588) = 1176$.

Time = 0.31 (sec) , antiderivative size = 5924, normalized size of antiderivative = 10.07

method	result	size
gosper	Expression too large to display	5924
orering	Expression too large to display	5927
risch	Expression too large to display	6934
parallelrisch	Expression too large to display	11277

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4795 vs. $2(588) = 1176$.

Time = 0.12 (sec) , antiderivative size = 4795, normalized size of antiderivative = 8.15

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136733 vs. 2(564) = 1128.

Time = 32.57 (sec) , antiderivative size = 136733, normalized size of antiderivative = 232.54

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)**3*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(500*x**11/11 - 5*x**10/2 + 85*x**9 - 49*x**8/4 + 999*x**7
/7 + 85*x**6 + 1109*x**5/5 + 287*x**4/2 + 159*x**3 + 135*x**2/2 + 54*x), E
q(e, 0)), (1260000*d**10*log(d/e + x)/(2520*d**10*e**11 + 25200*d**9*e**12
*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x
**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*
x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 3
690500*d**10/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x*
*2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x
**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*
x**8 + 25200*d*e**20*x**9 + 2520*e**21*x**10) + 12600000*d**9*e*x*log(d/e
+ x)/(2520*d**10*e**11 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302
400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 52
9200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 2
5200*d*e**20*x**9 + 2520*e**21*x**10) + 35645000*d**9*e*x/(2520*d**10*e**1
1 + 25200*d**9*e**12*x + 113400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 +
 529200*d**6*e**15*x**4 + 635040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6
+ 302400*d**3*e**18*x**7 + 113400*d**2*e**19*x**8 + 25200*d*e**20*x**9 + 2
520*e**21*x**10) + 6300*d**9*e/(2520*d**10*e**11 + 25200*d**9*e**12*x + 11
3400*d**8*e**13*x**2 + 302400*d**7*e**14*x**3 + 529200*d**6*e**15*x**4 + 6
35040*d**5*e**16*x**5 + 529200*d**4*e**17*x**6 + 302400*d**3*e**18*x**7...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2292 vs. $2(588) = 1176$.

Time = 0.09 (sec) , antiderivative size = 2292, normalized size of antiderivative = 3.90

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
135*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) +
54*(e*x + d)^(m + 1)/(e*(m + 1)) + 477*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m
)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e
^3) + 574*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^
3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*
m^3 + 35*m^2 + 50*m + 24)*e^4) + 1109*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*
d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^
m/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 510*((m^5 + 15*m
^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50
*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(
m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x
- 120*d^6)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 176
4*m + 720)*e^6) + 999*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764
*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*
e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4
+ 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3
+ 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 +
 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
 - 98*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 1306...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10965 vs. 2(588) = 1176.

Time = 0.22 (sec) , antiderivative size = 10965, normalized size of antiderivative = 18.65

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
(500*(e*x + d)^m*e^11*m^10*x^11 + 500*(e*x + d)^m*d*e^10*m^10*x^10 - 25*(e
*x + d)^m*e^11*m^10*x^10 + 27500*(e*x + d)^m*e^11*m^9*x^11 - 25*(e*x + d)^
m*d*e^10*m^10*x^9 + 765*(e*x + d)^m*e^11*m^10*x^9 + 22500*(e*x + d)^m*d*e^
10*m^9*x^10 - 1400*(e*x + d)^m*e^11*m^9*x^10 + 660000*(e*x + d)^m*e^11*m^8
*x^11 + 765*(e*x + d)^m*d*e^10*m^10*x^8 - 98*(e*x + d)^m*e^11*m^10*x^8 - 5
000*(e*x + d)^m*d^2*e^9*m^9*x^9 - 1175*(e*x + d)^m*d*e^10*m^9*x^9 + 43605*
(e*x + d)^m*e^11*m^9*x^9 + 435000*(e*x + d)^m*d*e^10*m^8*x^10 - 34125*(e*x
 + d)^m*e^11*m^8*x^10 + 9075000*(e*x + d)^m*e^11*m^7*x^11 - 98*(e*x + d)^m
*d*e^10*m^10*x^7 + 999*(e*x + d)^m*e^11*m^10*x^7 + 225*(e*x + d)^m*d^2*e^9
*m^9*x^8 + 37485*(e*x + d)^m*d*e^10*m^9*x^8 - 5684*(e*x + d)^m*e^11*m^9*x^
8 - 180000*(e*x + d)^m*d^2*e^9*m^8*x^9 - 23550*(e*x + d)^m*d*e^10*m^8*x^9
+ 1080180*(e*x + d)^m*e^11*m^8*x^9 + 4725000*(e*x + d)^m*d*e^10*m^7*x^10 -
 475500*(e*x + d)^m*e^11*m^7*x^10 + 78886500*(e*x + d)^m*e^11*m^6*x^11 + 9
99*(e*x + d)^m*d*e^10*m^10*x^6 + 510*(e*x + d)^m*e^11*m^10*x^6 - 6120*(e*x
 + d)^m*d^2*e^9*m^9*x^7 - 4998*(e*x + d)^m*d*e^10*m^9*x^7 + 58941*(e*x + d
)^m*e^11*m^9*x^7 + 45000*(e*x + d)^m*d^3*e^8*m^8*x^8 + 8775*(e*x + d)^m*d^
2*e^9*m^8*x^8 + 780300*(e*x + d)^m*d*e^10*m^8*x^8 - 143178*(e*x + d)^m*e^1
1*m^8*x^8 - 2730000*(e*x + d)^m*d^2*e^9*m^7*x^9 - 263550*(e*x + d)^m*d*e^1
0*m^7*x^9 + 15270930*(e*x + d)^m*e^11*m^7*x^9 + 31636500*(e*x + d)^m*d*e^1
0*m^6*x^10 - 4180575*(e*x + d)^m*e^11*m^6*x^10 + 451027500*(e*x + d)^m*...
```

Mupad [B] (verification not implemented)

Time = 24.51 (sec) , antiderivative size = 4341, normalized size of antiderivative = 7.38

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)^3*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
(500*x^11*(d + e*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m
^4 + 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628
800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133395
35*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
+ 39916800) + ((d + e*x)^m*(2155507200*d*e^10 + 99792000*d^10*e + 18144000
00*d^11 - 2694384000*d^2*e^9 + 6346771200*d^3*e^8 - 5728060800*d^4*e^7 + 8
853546240*d^5*e^6 - 3392928000*d^6*e^5 + 5696697600*d^7*e^4 + 488980800*d^
8*e^3 + 3392928000*d^9*e^2 - 4095133200*d^2*e^9*m + 7530723360*d^3*e^8*m -
 5364581040*d^4*e^7*m + 6521026464*d^5*e^6*m - 1933552800*d^6*e^5*m + 2432
604960*d^7*e^4*m + 147682080*d^8*e^3*m + 647740800*d^9*e^2*m + 3795710544*
d*e^10*m^2 + 1888225560*d*e^10*m^3 + 595543860*d*e^10*m^4 + 124791030*d*e^
10*m^5 + 17637102*d*e^10*m^6 + 1663740*d*e^10*m^7 + 100440*d*e^10*m^8 + 35
10*d*e^10*m^9 + 54*d*e^10*m^10 - 2697071580*d^2*e^9*m^2 + 3842860824*d^3*e
^8*m^2 - 2127097056*d^4*e^7*m^2 + 1983530784*d^5*e^6*m^2 - 437886000*d^6*e
^5*m^2 + 387691920*d^7*e^4*m^2 + 14817600*d^8*e^3*m^2 + 30844800*d^9*e^2*m
^2 - 1011746160*d^2*e^9*m^3 + 1102270680*d^3*e^8*m^3 - 463042356*d^4*e^7*m
^3 + 318992760*d^5*e^6*m^3 - 49266000*d^6*e^5*m^3 + 27332640*d^7*e^4*m^3 +
 493920*d^8*e^3*m^3 - 238556745*d^2*e^9*m^4 + 194510106*d^3*e^8*m^4 - 5978
7840*d^4*e^7*m^4 + 28612200*d^5*e^6*m^4 - 2754000*d^6*e^5*m^4 + 719280*d^7
*e^4*m^4 - 36710415*d^2*e^9*m^5 + 21636720*d^3*e^8*m^5 - 4580520*d^4*e^...
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6933, normalized size of antiderivative = 11.79

$$\int (d+ex)^m \left(3+2x+5x^2\right)^3 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^3*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
((d + e*x)**m*(1814400000*d**11 - 1814400000*d**10*e*m*x + 9072000*d**10*e
*m + 99792000*d**10*e + 907200000*d**9*e**2*m**2*x**2 - 9072000*d**9*e**2*
m**2*x + 30844800*d**9*e**2*m**2 + 907200000*d**9*e**2*m*x**2 - 99792000*d
**9*e**2*m*x + 647740800*d**9*e**2*m + 3392928000*d**9*e**2 - 302400000*d*
*8*e**3*m**3*x**3 + 4536000*d**8*e**3*m**3*x**2 - 30844800*d**8*e**3*m**3*
x + 493920*d**8*e**3*m**3 - 907200000*d**8*e**3*m**2*x**3 + 54432000*d**8*
e**3*m**2*x**2 - 647740800*d**8*e**3*m**2*x + 14817600*d**8*e**3*m**2 - 60
4800000*d**8*e**3*m*x**3 + 49896000*d**8*e**3*m*x**2 - 3392928000*d**8*e**
3*m*x + 147682080*d**8*e**3*m + 488980800*d**8*e**3 + 75600000*d**7*e**4*m
**4*x**4 - 1512000*d**7*e**4*m**4*x**3 + 15422400*d**7*e**4*m**4*x**2 - 49
3920*d**7*e**4*m**4*x + 719280*d**7*e**4*m**4 + 453600000*d**7*e**4*m**3*x
**4 - 21168000*d**7*e**4*m**3*x**3 + 339292800*d**7*e**4*m**3*x**2 - 14817
600*d**7*e**4*m**3*x + 27332640*d**7*e**4*m**3 + 831600000*d**7*e**4*m**2*
x**4 - 52920000*d**7*e**4*m**2*x**3 + 2020334400*d**7*e**4*m**2*x**2 - 147
682080*d**7*e**4*m**2*x + 387691920*d**7*e**4*m**2 + 453600000*d**7*e**4*m
*x**4 - 33264000*d**7*e**4*m*x**3 + 1696464000*d**7*e**4*m*x**2 - 48898080
0*d**7*e**4*m*x + 2432604960*d**7*e**4*m + 5696697600*d**7*e**4 - 15120000
*d**6*e**5*m**5*x**5 + 378000*d**6*e**5*m**5*x**4 - 5140800*d**6*e**5*m**5
*x**3 + 246960*d**6*e**5*m**5*x**2 - 719280*d**6*e**5*m**5*x - 61200*d**6*
e**5*m**5 - 151200000*d**6*e**5*m**4*x**5 + 6426000*d**6*e**5*m**4*x**4...
```

3.204 $\int (d+ex)^m (3+2x+5x^2)^2 (2+x+3x^2-5x^3+4x$

Optimal result

Integrand size = 38, antiderivative size = 432

$$\begin{aligned}
&\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx \\
&= \frac{(5d^2-2de+3e^2)^2\,(4d^4+5d^3e+3d^2e^2-de^3+2e^4)\,(d+ex)^{1+m}}{e^9(1+m)} \\
&\quad - \frac{(5d^2-2de+3e^2)\,(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)\,(d+ex)^{2+m}}{e^9(2+m)} \\
&\quad + \frac{(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6)\,(d+ex)^{3+m}}{e^9(3+m)} \\
&\quad - \frac{(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5)\,(d+ex)^{4+m}}{e^9(4+m)} \\
&\quad + \frac{(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4)\,(d+ex)^{5+m}}{e^9(5+m)} \\
&\quad - \frac{(5600d^3+945d^2e+666de^2+37e^3)\,(d+ex)^{6+m}}{e^9(6+m)} \\
&\quad + \frac{(2800d^2+315de+111e^2)\,(d+ex)^{7+m}}{e^9(7+m)} \\
&\quad - \frac{5(160d+9e)(d+ex)^{8+m}}{e^9(8+m)} + \frac{100(d+ex)^{9+m}}{e^9(9+m)}
\end{aligned}$$

output
```
(5*d^2-2*d*e+3*e^2)^2*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/
e^9/(1+m)-(5*d^2-2*d*e+3*e^2)*(160*d^5+127*d^4*e+88*d^3*e^2-4*d^2*e^3+64*d
*e^4-11*e^5)*(e*x+d)^(2+m)/e^9/(2+m)+(2800*d^6+945*d^5*e+1665*d^4*e^2+370*
d^3*e^3+888*d^2*e^4-195*d*e^5+107*e^6)*(e*x+d)^(3+m)/e^9/(3+m)-(5600*d^5+1
575*d^4*e+2220*d^3*e^2+370*d^2*e^3+592*d*e^4-65*e^5)*(e*x+d)^(4+m)/e^9/(4+
m)+(7000*d^4+1575*d^3*e+1665*d^2*e^2+185*d*e^3+148*e^4)*(e*x+d)^(5+m)/e^9/
(5+m)-(5600*d^3+945*d^2*e+666*d*e^2+37*e^3)*(e*x+d)^(6+m)/e^9/(6+m)+(2800*
d^2+315*d*e+111*e^2)*(e*x+d)^(7+m)/e^9/(7+m)-5*(160*d+9*e)*(e*x+d)^(8+m)/e
^9/(8+m)+100*(e*x+d)^(9+m)/e^9/(9+m)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)^2(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m} - \frac{(5d^2-2de+3e^2)(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5)(d+ex)}{2+m}\right.}{}$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),
x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^
2 - d*e^3 + 2*e^4))/(1 + m) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*d^4*
e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x))/(2 + m) + ((280
0*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2*e^4 - 195*d*e^5 +
 107*e^6)*(d + e*x)^2)/(3 + m) - ((5600*d^5 + 1575*d^4*e + 2220*d^3*e^2 +
370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^3)/(4 + m) + ((7000*d^4 + 1575
*d^3*e + 1665*d^2*e^2 + 185*d*e^3 + 148*e^4)*(d + e*x)^4)/(5 + m) - ((5600
*d^3 + 945*d^2*e + 666*d*e^2 + 37*e^3)*(d + e*x)^5)/(6 + m) + ((2800*d^2 +
 315*d*e + 111*e^2)*(d + e*x)^6)/(7 + m) - (5*(160*d + 9*e)*(d + e*x)^7)/(
8 + m) + (100*(d + e*x)^8)/(9 + m)))/e^9
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right)^2 \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^m \, dx$$

$$\Big\downarrow \ 2159$$

$$\int \left(\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+6}}{e^8}+\frac{\left(-5600d^3-945d^2e-666de^2-37e^3\right)(d+ex)^{m+5}}{e^8}+\frac{\left(5d^2-2de\right.}{}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(2800d^2+315de+111e^2\right)(d+ex)^{m+7}}{e^9(m+7)}-\frac{\left(5600d^3+945d^2e+666de^2+37e^3\right)(d+ex)^{m+6}}{e^9(m+6)}+\frac{\left(5d^2-2de+3e^2\right)^2\left(4d^4+5d^3e+3d^2e^2-de^3+2e^4\right)(d+ex)^{m+1}}{e^9(m+1)}+\frac{\left(7000d^4+1575d^3e+1665d^2e^2+185de^3+148e^4\right)(d+ex)^{m+5}}{e^9(m+5)}-\frac{\left(5d^2-2de+3e^2\right)\left(160d^5+127d^4e+88d^3e^2-4d^2e^3+64de^4-11e^5\right)(d+ex)^{m+2}}{e^9(m+2)}-\frac{\left(5600d^5+1575d^4e+2220d^3e^2+370d^2e^3+592de^4-65e^5\right)(d+ex)^{m+4}}{e^9(m+4)}+\frac{\left(2800d^6+945d^5e+1665d^4e^2+370d^3e^3+888d^2e^4-195de^5+107e^6\right)(d+ex)^{m+3}}{e^9(m+3)}-\frac{5(160d+9e)(d+ex)^{m+8}}{e^9(m+8)}+\frac{100(d+ex)^{m+9}}{e^9(m+9)}$$

input
```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)^2*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((5*d^2 - 2*d*e + 3*e^2)^2*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(
d + e*x)^(1 + m))/(e^9*(1 + m)) - ((5*d^2 - 2*d*e + 3*e^2)*(160*d^5 + 127*
d^4*e + 88*d^3*e^2 - 4*d^2*e^3 + 64*d*e^4 - 11*e^5)*(d + e*x)^(2 + m))/(e^
9*(2 + m)) + ((2800*d^6 + 945*d^5*e + 1665*d^4*e^2 + 370*d^3*e^3 + 888*d^2
*e^4 - 195*d*e^5 + 107*e^6)*(d + e*x)^(3 + m))/(e^9*(3 + m)) - ((5600*d^5
+ 1575*d^4*e + 2220*d^3*e^2 + 370*d^2*e^3 + 592*d*e^4 - 65*e^5)*(d + e*x)^
(4 + m))/(e^9*(4 + m)) + ((7000*d^4 + 1575*d^3*e + 1665*d^2*e^2 + 185*d*e^
3 + 148*e^4)*(d + e*x)^(5 + m))/(e^9*(5 + m)) - ((5600*d^3 + 945*d^2*e + 6
66*d*e^2 + 37*e^3)*(d + e*x)^(6 + m))/(e^9*(6 + m)) + ((2800*d^2 + 315*d*e
 + 111*e^2)*(d + e*x)^(7 + m))/(e^9*(7 + m)) - (5*(160*d + 9*e)*(d + e*x)^
(8 + m))/(e^9*(8 + m)) + (100*(d + e*x)^(9 + m))/(e^9*(9 + m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3221 vs. $2(432) = 864$.

Time = 0.18 (sec) , antiderivative size = 3222, normalized size of antiderivative = 7.46

method	result	size
gosper	Expression too large to display	3222
orering	Expression too large to display	3225
risch	Expression too large to display	3895
parallelrisch	Expression too large to display	6428

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERB
OSE)
```

output
```
1/e^9*(e*x+d)^(1+m)/(m^9+45*m^8+870*m^7+9450*m^6+63273*m^5+269325*m^4+7236
80*m^3+1172700*m^2+1026576*m+362880)*(100*e^8*m^8*x^8-45*e^8*m^8*x^7+3600*
e^8*m^7*x^8-800*d*e^7*m^7*x^7+111*e^8*m^8*x^6-1665*e^8*m^7*x^7+54600*e^8*m
^6*x^8+315*d*e^7*m^7*x^6-22400*d*e^7*m^6*x^7-37*e^8*m^8*x^5+4218*e^8*m^7*x
^6-25830*e^8*m^6*x^7+453600*e^8*m^5*x^8+5600*d^2*e^6*m^6*x^6-666*d*e^7*m^7
*x^5+9450*d*e^7*m^6*x^6-257600*d*e^7*m^5*x^7+148*e^8*m^8*x^4-1443*e^8*m^7*
x^5+67044*e^8*m^6*x^6-218610*e^8*m^5*x^7+2244900*e^8*m^4*x^8-1890*d^2*e^6*
m^6*x^5+117600*d^2*e^6*m^5*x^6+185*d*e^7*m^7*x^4-21312*d*e^7*m^6*x^5+11466
0*d*e^7*m^5*x^6-1568000*d*e^7*m^4*x^7+65*e^8*m^8*x^3+5920*e^8*m^7*x^4-2353
2*e^8*m^6*x^5+579642*e^8*m^5*x^6-1098405*e^8*m^4*x^7+6728400*e^8*m^3*x^8-3
3600*d^3*e^5*m^5*x^5+3330*d^2*e^6*m^6*x^4-45360*d^2*e^6*m^5*x^5+980000*d^2
*e^6*m^4*x^6-592*d*e^7*m^7*x^3+6290*d*e^7*m^6*x^4-274392*d*e^7*m^5*x^5+727
650*d*e^7*m^4*x^6-5415200*d*e^7*m^3*x^7+107*e^8*m^8*x^2+2665*e^8*m^7*x^3+9
9160*e^8*m^6*x^4-208458*e^8*m^5*x^5+2965809*e^8*m^4*x^6-3332385*e^8*m^3*x^
7+11812400*e^8*m^2*x^8+9450*d^3*e^5*m^5*x^4-504000*d^3*e^5*m^4*x^5-740*d^2
*e^6*m^6*x^3+89910*d^2*e^6*m^5*x^4-415800*d^2*e^6*m^4*x^5+4116000*d^2*e^6*
m^3*x^6-195*d*e^7*m^7*x^2-21312*d*e^7*m^6*x^3+86210*d*e^7*m^5*x^4-1831500*
d*e^7*m^4*x^5+2595285*d*e^7*m^3*x^6-10505600*d*e^7*m^2*x^7+33*e^8*m^8*x+44
94*e^8*m^7*x^2+45890*e^8*m^6*x^3+902800*e^8*m^5*x^4-1090353*e^8*m^4*x^5+91
34412*e^8*m^3*x^6-5906520*e^8*m^2*x^7+10958400*e^8*m*x^8+168000*d^4*e^4...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2796 vs. $2(432) = 864$.

Time = 0.10 (sec) , antiderivative size = 2796, normalized size of antiderivative = 6.47

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
fricas")
```

output
```
(18*d*e^8*m^8 + 100*(e^9*m^8 + 36*e^9*m^7 + 546*e^9*m^6 + 4536*e^9*m^5 + 2
2449*e^9*m^4 + 67284*e^9*m^3 + 118124*e^9*m^2 + 109584*e^9*m + 40320*e^9)*
x^9 + 4032000*d^9 + 2041200*d^8*e + 5754240*d^7*e^2 + 2237760*d^6*e^3 + 10
741248*d^5*e^4 - 5896800*d^4*e^5 + 12942720*d^3*e^6 - 5987520*d^2*e^7 + 65
31840*d*e^8 - 5*(408240*e^9 - (20*d*e^8 - 9*e^9)*m^8 - (560*d*e^8 - 333*e^
9)*m^7 - 14*(460*d*e^8 - 369*e^9)*m^6 - 14*(2800*d*e^8 - 3123*e^9)*m^5 - 7
*(19340*d*e^8 - 31383*e^9)*m^4 - 7*(37520*d*e^8 - 95211*e^9)*m^3 - 216*(12
10*d*e^8 - 5469*e^9)*m^2 - 36*(2800*d*e^8 - 30663*e^9)*m)*x^8 - 33*(d^2*e^
7 - 24*d*e^8)*m^7 + (5754240*e^9 - 3*(15*d*e^8 - 37*e^9)*m^8 - 2*(400*d^2*
e^7 + 675*d*e^8 - 2109*e^9)*m^7 - 12*(1400*d^2*e^7 + 1365*d*e^8 - 5587*e^9
)*m^6 - 14*(10000*d^2*e^7 + 7425*d*e^8 - 41403*e^9)*m^5 - 21*(28000*d^2*e^
7 + 17655*d*e^8 - 141229*e^9)*m^4 - 28*(46400*d^2*e^7 + 26325*d*e^8 - 3262
29*e^9)*m^3 - 36*(39200*d^2*e^7 + 20745*d*e^8 - 455211*e^9)*m^2 - 144*(400
0*d^2*e^7 + 2025*d*e^8 - 107337*e^9)*m)*x^7 + 2*(107*d^3*e^6 - 693*d^2*e^7
 + 7434*d*e^8)*m^6 - (2237760*e^9 - 37*(3*d*e^8 - e^9)*m^8 - 3*(105*d^2*e^
7 + 1184*d*e^8 - 481*e^9)*m^7 - 4*(1400*d^3*e^6 + 1890*d^2*e^7 + 11433*d*e
^8 - 5883*e^9)*m^6 - 6*(14000*d^3*e^6 + 11550*d^2*e^7 + 50875*d*e^8 - 3474
3*e^9)*m^5 - (476000*d^3*e^6 + 311850*d^2*e^7 + 1134309*d*e^8 - 1090353*e^
9)*m^4 - 3*(420000*d^3*e^6 + 241395*d^2*e^7 + 776186*d*e^8 - 1140969*e^9)*
m^3 - 2*(767200*d^3*e^6 + 407295*d^2*e^7 + 1208124*d*e^8 - 3119359*e^9)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65193 vs. $2(410) = 820$.

Time = 13.52 (sec) , antiderivative size = 65193, normalized size of antiderivative = 150.91

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)**2*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(100*x**9/9 - 45*x**8/8 + 111*x**7/7 - 37*x**6/6 + 148*x**
5/5 + 65*x**4/4 + 107*x**3/3 + 33*x**2/2 + 18*x), Eq(e, 0)), (84000*d**8*l
og(d/e + x)/(840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 4
7040*d**5*e**12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 235
20*d**2*e**15*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 228300*d**8/(84
0*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12
*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x
**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 672000*d**7*e*x*log(d/e + x)/(
840*d**8*e**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**
12*x**3 + 58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15
*x**6 + 6720*d*e**16*x**7 + 840*e**17*x**8) + 1742400*d**7*e*x/(840*d**8*e
**9 + 6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 +
58800*d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 67
20*d*e**16*x**7 + 840*e**17*x**8) + 4725*d**7*e/(840*d**8*e**9 + 6720*d**7
*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*d**4*e**1
3*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e**16*x**7
 + 840*e**17*x**8) + 2352000*d**6*e**2*x**2*log(d/e + x)/(840*d**8*e**9 +
6720*d**7*e**10*x + 23520*d**6*e**11*x**2 + 47040*d**5*e**12*x**3 + 58800*
d**4*e**13*x**4 + 47040*d**3*e**14*x**5 + 23520*d**2*e**15*x**6 + 6720*d*e
**16*x**7 + 840*e**17*x**8) + 5762400*d**6*e**2*x**2/(840*d**8*e**9 + 6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1414 vs. $2(432) = 864$.

Time = 0.07 (sec) , antiderivative size = 1414, normalized size of antiderivative = 3.27

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
maxima")
```

output
```
33*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 1
8*(e*x + d)^(m + 1)/(e*(m + 1)) + 107*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)
*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^
3) + 65*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3
- 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^4) + 148*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^
5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2
*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/(
(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 37*((m^5 + 15*m^4 +
 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2
 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3
+ 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 12
0*d^6)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
+ 720)*e^6) + 111*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m +
 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*
x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*
m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 36
0*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*
m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7) - 4
5*((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6226 vs. $2(432) = 864$.

Time = 0.22 (sec) , antiderivative size = 6226, normalized size of antiderivative = 14.41

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="
giac")
```

output
```
(100*(e*x + d)^m*e^9*m^8*x^9 + 100*(e*x + d)^m*d*e^8*m^8*x^8 - 45*(e*x + d
)^m*e^9*m^8*x^8 + 3600*(e*x + d)^m*e^9*m^7*x^9 - 45*(e*x + d)^m*d*e^8*m^8*
x^7 + 111*(e*x + d)^m*e^9*m^8*x^7 + 2800*(e*x + d)^m*d*e^8*m^7*x^8 - 1665*
(e*x + d)^m*e^9*m^7*x^8 + 54600*(e*x + d)^m*e^9*m^6*x^9 + 111*(e*x + d)^m*
d*e^8*m^8*x^6 - 37*(e*x + d)^m*e^9*m^8*x^6 - 800*(e*x + d)^m*d^2*e^7*m^7*x
^7 - 1350*(e*x + d)^m*d*e^8*m^7*x^7 + 4218*(e*x + d)^m*e^9*m^7*x^7 + 32200
*(e*x + d)^m*d*e^8*m^6*x^8 - 25830*(e*x + d)^m*e^9*m^6*x^8 + 453600*(e*x +
 d)^m*e^9*m^5*x^9 - 37*(e*x + d)^m*d*e^8*m^8*x^5 + 148*(e*x + d)^m*e^9*m^8
*x^5 + 315*(e*x + d)^m*d^2*e^7*m^7*x^6 + 3552*(e*x + d)^m*d*e^8*m^7*x^6 -
1443*(e*x + d)^m*e^9*m^7*x^6 - 16800*(e*x + d)^m*d^2*e^7*m^6*x^7 - 16380*(
e*x + d)^m*d*e^8*m^6*x^7 + 67044*(e*x + d)^m*e^9*m^6*x^7 + 196000*(e*x + d
)^m*d*e^8*m^5*x^8 - 218610*(e*x + d)^m*e^9*m^5*x^8 + 2244900*(e*x + d)^m*e
^9*m^4*x^9 + 148*(e*x + d)^m*d*e^8*m^8*x^4 + 65*(e*x + d)^m*e^9*m^8*x^4 -
666*(e*x + d)^m*d^2*e^7*m^7*x^5 - 1258*(e*x + d)^m*d*e^8*m^7*x^5 + 5920*(e
*x + d)^m*e^9*m^7*x^5 + 5600*(e*x + d)^m*d^3*e^6*m^6*x^6 + 7560*(e*x + d)^
m*d^2*e^7*m^6*x^6 + 45732*(e*x + d)^m*d*e^8*m^6*x^6 - 23532*(e*x + d)^m*e^
9*m^6*x^6 - 140000*(e*x + d)^m*d^2*e^7*m^5*x^7 - 103950*(e*x + d)^m*d*e^8*
m^5*x^7 + 579642*(e*x + d)^m*e^9*m^5*x^7 + 676900*(e*x + d)^m*d*e^8*m^4*x^
8 - 1098405*(e*x + d)^m*e^9*m^4*x^8 + 6728400*(e*x + d)^m*e^9*m^3*x^9 + 65
*(e*x + d)^m*d*e^8*m^8*x^3 + 107*(e*x + d)^m*e^9*m^8*x^3 + 185*(e*x + d...
```

Mupad [B] (verification not implemented)

Time = 19.92 (sec) , antiderivative size = 2625, normalized size of antiderivative = 6.08

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right)\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)^2*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
((d + e*x)^m*(6531840*d*e^8 + 2041200*d^8*e + 4032000*d^9 - 5987520*d^2*e^
7 + 12942720*d^3*e^6 - 5896800*d^4*e^5 + 10741248*d^5*e^4 + 2237760*d^6*e^
3 + 5754240*d^7*e^2 - 7957224*d^2*e^7*m + 12886224*d^3*e^6*m - 4396860*d^4
*e^5*m + 5860800*d^5*e^4*m + 848040*d^6*e^3*m + 1358640*d^7*e^2*m + 916207
2*d*e^8*m^2 + 3864168*d*e^8*m^3 + 983682*d*e^8*m^4 + 155232*d*e^8*m^5 + 14
868*d*e^8*m^6 + 792*d*e^8*m^7 + 18*d*e^8*m^8 - 4419954*d^2*e^7*m^2 + 52588
36*d^3*e^6*m^2 - 1296750*d^4*e^5*m^2 + 1189920*d^5*e^4*m^2 + 106560*d^6*e^
3*m^2 + 79920*d^7*e^2*m^2 - 1332177*d^2*e^7*m^3 + 1126710*d^3*e^6*m^3 - 18
9150*d^4*e^5*m^3 + 106560*d^5*e^4*m^3 + 4440*d^6*e^3*m^3 - 235620*d^2*e^7*
m^4 + 133750*d^3*e^6*m^4 - 13650*d^4*e^5*m^4 + 3552*d^5*e^4*m^4 - 24486*d^
2*e^7*m^5 + 8346*d^3*e^6*m^5 - 390*d^4*e^5*m^5 - 1386*d^2*e^7*m^6 + 214*d^
3*e^6*m^6 - 33*d^2*e^7*m^7 + 11946528*d*e^8*m + 226800*d^8*e*m))/(e^9*(102
6576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6 + 87
0*m^7 + 45*m^8 + m^9 + 362880)) + (100*x^9*(d + e*x)^m*(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320))/
(1026576*m + 1172700*m^2 + 723680*m^3 + 269325*m^4 + 63273*m^5 + 9450*m^6
+ 870*m^7 + 45*m^8 + m^9 + 362880) + (x*(d + e*x)^m*(11946528*e^9*m + 6531
840*e^9 + 9162072*e^9*m^2 + 3864168*e^9*m^3 + 983682*e^9*m^4 + 155232*e^9*
m^5 + 14868*e^9*m^6 + 792*e^9*m^7 + 18*e^9*m^8 - 12942720*d^2*e^7*m + 5896
800*d^3*e^6*m - 10741248*d^4*e^5*m - 2237760*d^5*e^4*m - 5754240*d^6*e^...
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 3894, normalized size of antiderivative = 9.01

$$\int (d+ex)^m \left(3+2x+5x^2\right)^2 \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(5*x^2+2*x+3)^2*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output

```
((d + e*x)**m*(4032000*d**9 - 4032000*d**8*e*m*x + 226800*d**8*e*m + 20412
00*d**8*e + 2016000*d**7*e**2*m**2*x**2 - 226800*d**7*e**2*m**2*x + 79920*
d**7*e**2*m**2 + 2016000*d**7*e**2*m*x**2 - 2041200*d**7*e**2*m*x + 135864
0*d**7*e**2*m + 5754240*d**7*e**2 - 672000*d**6*e**3*m**3*x**3 + 113400*d*
*6*e**3*m**3*x**2 - 79920*d**6*e**3*m**3*x + 4440*d**6*e**3*m**3 - 2016000
*d**6*e**3*m**2*x**3 + 1134000*d**6*e**3*m**2*x**2 - 1358640*d**6*e**3*m**
2*x + 106560*d**6*e**3*m**2 - 1344000*d**6*e**3*m*x**3 + 1020600*d**6*e**3
*m*x**2 - 5754240*d**6*e**3*m*x + 848040*d**6*e**3*m + 2237760*d**6*e**3 +
 168000*d**5*e**4*m**4*x**4 - 37800*d**5*e**4*m**4*x**3 + 39960*d**5*e**4*
m**4*x**2 - 4440*d**5*e**4*m**4*x + 3552*d**5*e**4*m**4 + 1008000*d**5*e**
4*m**3*x**4 - 453600*d**5*e**4*m**3*x**3 + 719280*d**5*e**4*m**3*x**2 - 10
6560*d**5*e**4*m**3*x + 106560*d**5*e**4*m**3 + 1848000*d**5*e**4*m**2*x**
4 - 1096200*d**5*e**4*m**2*x**3 + 3556440*d**5*e**4*m**2*x**2 - 848040*d**
5*e**4*m**2*x + 1189920*d**5*e**4*m**2 + 1008000*d**5*e**4*m*x**4 - 680400
*d**5*e**4*m*x**3 + 2877120*d**5*e**4*m*x**2 - 2237760*d**5*e**4*m*x + 586
0800*d**5*e**4*m + 10741248*d**5*e**4 - 33600*d**4*e**5*m**5*x**5 + 9450*d
**4*e**5*m**5*x**4 - 13320*d**4*e**5*m**5*x**3 + 2220*d**4*e**5*m**5*x**2
- 3552*d**4*e**5*m**5*x - 390*d**4*e**5*m**5 - 336000*d**4*e**5*m**4*x**5
+ 141750*d**4*e**5*m**4*x**4 - 266400*d**4*e**5*m**4*x**3 + 55500*d**4*e**
5*m**4*x**2 - 106560*d**4*e**5*m**4*x - 13650*d**4*e**5*m**4 - 1176000*...
```

3.205 $\int (d+ex)^m (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4$

Optimal result

Integrand size = 36, antiderivative size = 292

$$\begin{aligned}\int (d+ex)^m (3+2x+5x^2)(2+x+3x^2-5x^3+4x^4)\,dx &= \frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{1+m}}{e^7(1+m)} \\ &\quad - \frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^{2+m}}{e^7(2+m)} \\ &\quad + \frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{3+m}}{e^7(3+m)} \\ &\quad - \frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{4+m}}{e^7(4+m)} \\ &\quad + \frac{(300d^2+85de+17e^2)(d+ex)^{5+m}}{e^7(5+m)} - \frac{(120d+17e)(d+ex)^{6+m}}{e^7(6+m)} + \frac{20(d+ex)^{7+m}}{e^7(7+m)}\end{aligned}$$

output
```
(5*d^2-2*d*e+3*e^2)*(4*d^4+5*d^3*e+3*d^2*e^2-d*e^3+2*e^4)*(e*x+d)^(1+m)/e^
7/(1+m)-(120*d^5+85*d^4*e+68*d^3*e^2+12*d^2*e^3+42*d*e^4-7*e^5)*(e*x+d)^(2
+m)/e^7/(2+m)+(300*d^4+170*d^3*e+102*d^2*e^2+12*d*e^3+21*e^4)*(e*x+d)^(3+m
)/e^7/(3+m)-2*(200*d^3+85*d^2*e+34*d*e^2+2*e^3)*(e*x+d)^(4+m)/e^7/(4+m)+(3
00*d^2+85*d*e+17*e^2)*(e*x+d)^(5+m)/e^7/(5+m)-(120*d+17*e)*(e*x+d)^(6+m)/e
^7/(6+m)+20*(e*x+d)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.89

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx$$
$$= \frac{(d+ex)^{1+m}\left(\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)}{1+m}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)}{2+m}+\frac{(300d^4+170d$$

input
```
Integrate[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output
```
((d + e*x)^(1 + m)*(((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2
- d*e^3 + 2*e^4))/(1 + m) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2*e^3
 + 42*d*e^4 - 7*e^5)*(d + e*x))/(2 + m) + ((300*d^4 + 170*d^3*e + 102*d^2*
e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^2)/(3 + m) - (2*(200*d^3 + 85*d^2*e + 3
4*d*e^2 + 2*e^3)*(d + e*x)^3)/(4 + m) + ((300*d^2 + 85*d*e + 17*e^2)*(d +
e*x)^4)/(5 + m) - ((120*d + 17*e)*(d + e*x)^5)/(6 + m) + (20*(d + e*x)^6)/
(7 + m)))/e^7
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(5x^2+2x+3\right) \left(4x^4-5x^3+3x^2+x+2\right) (d+ex)^m \, dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{\left(300d^2+85de+17e^2\right)(d+ex)^{m+4}}{e^6}-\frac{2\left(200d^3+85d^2e+34de^2+2e^3\right)(d+ex)^{m+3}}{e^6}+\frac{\left(300d^4+170d^3e+\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(300d^2+85de+17e^2)(d+ex)^{m+5}}{e^7(m+5)}-\frac{2(200d^3+85d^2e+34de^2+2e^3)(d+ex)^{m+4}}{e^7(m+4)}+\frac{(5d^2-2de+3e^2)(4d^4+5d^3e+3d^2e^2-de^3+2e^4)(d+ex)^{m+1}}{e^7(m+1)}+\frac{(300d^4+170d^3e+102d^2e^2+12de^3+21e^4)(d+ex)^{m+3}}{e^7(m+3)}-\frac{(120d^5+85d^4e+68d^3e^2+12d^2e^3+42de^4-7e^5)(d+ex)^{m+2}}{e^7(m+2)}-\frac{(120d+17e)(d+ex)^{m+6}}{e^7(m+6)}+\frac{20(d+ex)^{m+7}}{e^7(m+7)}$$

input

```
Int[(d + e*x)^m*(3 + 2*x + 5*x^2)*(2 + x + 3*x^2 - 5*x^3 + 4*x^4),x]
```

output

```
((5*d^2 - 2*d*e + 3*e^2)*(4*d^4 + 5*d^3*e + 3*d^2*e^2 - d*e^3 + 2*e^4)*(d
+ e*x)^(1 + m))/(e^7*(1 + m)) - ((120*d^5 + 85*d^4*e + 68*d^3*e^2 + 12*d^2
*e^3 + 42*d*e^4 - 7*e^5)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + ((300*d^4 + 17
0*d^3*e + 102*d^2*e^2 + 12*d*e^3 + 21*e^4)*(d + e*x)^(3 + m))/(e^7*(3 + m)
) - (2*(200*d^3 + 85*d^2*e + 34*d*e^2 + 2*e^3)*(d + e*x)^(4 + m))/(e^7*(4
+ m)) + ((300*d^2 + 85*d*e + 17*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) - ((
120*d + 17*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (20*(d + e*x)^(7 + m))/(e
^7*(7 + m))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. $2(292) = 584$.

Time = 0.13 (sec) , antiderivative size = 1220, normalized size of antiderivative = 4.18

method	result	size
norman	Expression too large to display	1220
gosper	Expression too large to display	1504
orering	Expression too large to display	1507
risch	Expression too large to display	1908
parallelrisch	Expression too large to display	3215

input
```
int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x,method=_RETURNVERBOS
E)
```

output
```
d*(6*e^6*m^6-7*d*e^5*m^5+162*e^6*m^5+42*d^2*e^4*m^4-175*d*e^5*m^4+1770*e^6
*m^4+24*d^3*e^3*m^3+924*d^2*e^4*m^3-1715*d*e^5*m^3+9990*e^6*m^3+408*d^4*e^
2*m^2+432*d^3*e^3*m^2+7518*d^2*e^4*m^2-8225*d*e^5*m^2+30624*e^6*m^2+2040*d
^5*e*m+5304*d^4*e^2*m+2568*d^3*e^3*m+26796*d^2*e^4*m-19278*d*e^5*m+48168*e
^6*m+14400*d^6+14280*d^5*e+17136*d^4*e^2+5040*d^3*e^3+35280*d^2*e^4-17640*
d*e^5+30240*e^6)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+13132*m^2+13068
*m+5040)*exp(m*ln(e*x+d))+(20*d*m-17*e*m-119*e)/e/(m^2+13*m+42)*x^6*exp(m*
ln(e*x+d))+(17*d*e^2*m^3-4*e^3*m^3+85*d^2*e*m^2+221*d*e^2*m^2-72*e^3*m^2+6
00*d^3*m+595*d^2*e*m+714*d*e^2*m-428*e^3*m-840*e^3)/e^3/(m^4+22*m^3+179*m^
2+638*m+840)*x^4*exp(m*ln(e*x+d))+(21*d*e^4*m^5+7*e^5*m^5+12*d^2*e^3*m^4+4
62*d*e^4*m^4+175*e^5*m^4+204*d^3*e^2*m^3+216*d^2*e^3*m^3+3759*d*e^4*m^3+17
15*e^5*m^3+1020*d^4*e*m^2+2652*d^3*e^2*m^2+1284*d^2*e^3*m^2+13398*d*e^4*m^
2+8225*e^5*m^2+7200*d^5*m+7140*d^4*e*m+8568*d^3*e^2*m+2520*d^2*e^3*m+17640
*d*e^4*m+19278*e^5*m+17640*e^5)/e^5/(m^6+27*m^5+295*m^4+1665*m^3+5104*m^2+
8028*m+5040)*x^2*exp(m*ln(e*x+d))+20/(7+m)*x^7*exp(m*ln(e*x+d))-(17*d*e*m^
2-17*e^2*m^2+120*d^2*m+119*d*e*m-221*e^2*m-714*e^2)/e^2/(m^3+18*m^2+107*m+
210)*x^5*exp(m*ln(e*x+d))-(4*d*e^3*m^4-21*e^4*m^4+68*d^2*e^2*m^3+72*d*e^3*
m^3-462*e^4*m^3+340*d^3*e*m^2+884*d^2*e^2*m^2+428*d*e^3*m^2-3759*e^4*m^2+2
400*d^4*m+2380*d^3*e*m+2856*d^2*e^2*m+840*d*e^3*m-13398*e^4*m-17640*e^4)/e
^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*exp(m*ln(e*x+d))-(-7*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1448 vs. $2(292) = 584$.

Time = 0.09 (sec) , antiderivative size = 1448, normalized size of antiderivative = 4.96

$$\int (d+ex)^m (3+2x+5x^2) (2+x+3x^2-5x^3+4x^4) \, dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="fr
icas")
```

output

```
(6*d*e^6*m^6 + 20*(e^7*m^6 + 21*e^7*m^5 + 175*e^7*m^4 + 735*e^7*m^3 + 1624
*e^7*m^2 + 1764*e^7*m + 720*e^7)*x^7 + 14400*d^7 + 14280*d^6*e + 17136*d^5
*e^2 + 5040*d^4*e^3 + 35280*d^3*e^4 - 17640*d^2*e^5 + 30240*d*e^6 - (14280
*e^7 - (20*d*e^6 - 17*e^7)*m^6 - 2*(150*d*e^6 - 187*e^7)*m^5 - 170*(10*d*e
^6 - 19*e^7)*m^4 - 20*(225*d*e^6 - 697*e^7)*m^3 - (5480*d*e^6 - 31433*e^7)
*m^2 - 2*(1200*d*e^6 - 17323*e^7)*m)*x^6 - (7*d^2*e^5 - 162*d*e^6)*m^5 + (
17136*e^7 - 17*(d*e^6 - e^7)*m^6 - (120*d^2*e^5 + 289*d*e^6 - 391*e^7)*m^5
 - 3*(400*d^2*e^5 + 595*d*e^6 - 1173*e^7)*m^4 - 5*(840*d^2*e^5 + 1003*d*e^
6 - 3145*e^7)*m^3 - 2*(3000*d^2*e^5 + 3179*d*e^6 - 18224*e^7)*m^2 - 12*(24
0*d^2*e^5 + 238*d*e^6 - 3417*e^7)*m)*x^5 + (42*d^3*e^4 - 175*d^2*e^5 + 177
0*d*e^6)*m^4 - (5040*e^7 - (17*d*e^6 - 4*e^7)*m^6 - (85*d^2*e^5 + 323*d*e^
6 - 96*e^7)*m^5 - (600*d^3*e^4 + 1105*d^2*e^5 + 2227*d*e^6 - 904*e^7)*m^4
- (3600*d^3*e^4 + 4505*d^2*e^5 + 6817*d*e^6 - 4224*e^7)*m^3 - 5*(1320*d^3*
e^4 + 1411*d^2*e^5 + 1836*d*e^6 - 2036*e^7)*m^2 - 6*(600*d^3*e^4 + 595*d^2
*e^5 + 714*d*e^6 - 1968*e^7)*m)*x^4 + (24*d^4*e^3 + 924*d^3*e^4 - 1715*d^2
*e^5 + 9990*d*e^6)*m^3 + (35280*e^7 - (4*d*e^6 - 21*e^7)*m^6 - (68*d^2*e^5
 + 84*d*e^6 - 525*e^7)*m^5 - (340*d^3*e^4 + 1088*d^2*e^5 + 652*d*e^6 - 518
7*e^7)*m^4 - (2400*d^4*e^3 + 3400*d^3*e^4 + 5644*d^2*e^5 + 2268*d*e^6 - 25
599*e^7)*m^3 - 4*(1800*d^4*e^3 + 1955*d^3*e^4 + 2584*d^2*e^5 + 844*d*e^6 -
 16338*e^7)*m^2 - 4*(1200*d^4*e^3 + 1190*d^3*e^4 + 1428*d^2*e^5 + 420*d...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26165 vs. $2(272) = 544$.

Time = 4.87 (sec) , antiderivative size = 26165, normalized size of antiderivative = 89.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(5*x**2+2*x+3)*(4*x**4-5*x**3+3*x**2+x+2),x)
```

output
```
Piecewise((d**m*(20*x**7/7 - 17*x**6/6 + 17*x**5/5 - x**4 + 7*x**3 + 7*x**
2/2 + 6*x), Eq(e, 0)), (1200*d**6*log(d/e + x)/(60*d**6*e**7 + 360*d**5*e*
*8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 3
60*d*e**12*x**5 + 60*e**13*x**6) + 2940*d**6/(60*d**6*e**7 + 360*d**5*e**8
*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360
*d*e**12*x**5 + 60*e**13*x**6) + 7200*d**5*e*x*log(d/e + x)/(60*d**6*e**7
+ 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e
**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 16440*d**5*e*x/(60*d**6*e*
*7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**
2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 170*d**5*e/(60*d**6*e**
7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2
*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 18000*d**4*e**2*x**2*log
(d/e + x)/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3
*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) + 37
500*d**4*e**2*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 +
1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x
**6) + 1020*d**4*e**2*x/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x*
*2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e*
*13*x**6) - 34*d**4*e**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x
**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 6...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(292) = 584$.

Time = 0.07 (sec) , antiderivative size = 788, normalized size of antiderivative = 2.70

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="ma
xima")
```

output
```
7*(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m/((m^2 + 3*m + 2)*e^2) + 6*
(e*x + d)^(m + 1)/(e*(m + 1)) + 21*((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*
e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x + d)^m/((m^3 + 6*m^2 + 11*m + 6)*e^3)
 - 4*((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*
(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m/((m^4 + 10*m^3 +
35*m^2 + 50*m + 24)*e^4) + 17*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*
x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m/((m^5
+ 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) - 17*((m^5 + 15*m^4 + 85*m
^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24
*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 + 3*m
^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120*d^6
)*(e*x + d)^m/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720
)*e^6) + 20*((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m + 720)*
e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6*x^6 -
6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6*m^3 +
11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 360*(m^2
 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m/((m^7 + 28*m^6 +
322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3099 vs. $2(292) = 584$.

Time = 0.22 (sec) , antiderivative size = 3099, normalized size of antiderivative = 10.61

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x, algorithm="gi
ac")
```

output
```
(20*(e*x + d)^m*e^7*m^6*x^7 + 20*(e*x + d)^m*d*e^6*m^6*x^6 - 17*(e*x + d)^
m*e^7*m^6*x^6 + 420*(e*x + d)^m*e^7*m^5*x^7 - 17*(e*x + d)^m*d*e^6*m^6*x^5
 + 17*(e*x + d)^m*e^7*m^6*x^5 + 300*(e*x + d)^m*d*e^6*m^5*x^6 - 374*(e*x +
 d)^m*e^7*m^5*x^6 + 3500*(e*x + d)^m*e^7*m^4*x^7 + 17*(e*x + d)^m*d*e^6*m^
6*x^4 - 4*(e*x + d)^m*e^7*m^6*x^4 - 120*(e*x + d)^m*d^2*e^5*m^5*x^5 - 289*
(e*x + d)^m*d*e^6*m^5*x^5 + 391*(e*x + d)^m*e^7*m^5*x^5 + 1700*(e*x + d)^m
*d*e^6*m^4*x^6 - 3230*(e*x + d)^m*e^7*m^4*x^6 + 14700*(e*x + d)^m*e^7*m^3*
x^7 - 4*(e*x + d)^m*d*e^6*m^6*x^3 + 21*(e*x + d)^m*e^7*m^6*x^3 + 85*(e*x +
 d)^m*d^2*e^5*m^5*x^4 + 323*(e*x + d)^m*d*e^6*m^5*x^4 - 96*(e*x + d)^m*e^7
*m^5*x^4 - 1200*(e*x + d)^m*d^2*e^5*m^4*x^5 - 1785*(e*x + d)^m*d*e^6*m^4*x
^5 + 3519*(e*x + d)^m*e^7*m^4*x^5 + 4500*(e*x + d)^m*d*e^6*m^3*x^6 - 13940
*(e*x + d)^m*e^7*m^3*x^6 + 32480*(e*x + d)^m*e^7*m^2*x^7 + 21*(e*x + d)^m*
d*e^6*m^6*x^2 + 7*(e*x + d)^m*e^7*m^6*x^2 - 68*(e*x + d)^m*d^2*e^5*m^5*x^3
 - 84*(e*x + d)^m*d*e^6*m^5*x^3 + 525*(e*x + d)^m*e^7*m^5*x^3 + 600*(e*x +
 d)^m*d^3*e^4*m^4*x^4 + 1105*(e*x + d)^m*d^2*e^5*m^4*x^4 + 2227*(e*x + d)^
m*d*e^6*m^4*x^4 - 904*(e*x + d)^m*e^7*m^4*x^4 - 4200*(e*x + d)^m*d^2*e^5*m
^3*x^5 - 5015*(e*x + d)^m*d*e^6*m^3*x^5 + 15725*(e*x + d)^m*e^7*m^3*x^5 +
5480*(e*x + d)^m*d*e^6*m^2*x^6 - 31433*(e*x + d)^m*e^7*m^2*x^6 + 35280*(e*
x + d)^m*e^7*m*x^7 + 7*(e*x + d)^m*d*e^6*m^6*x + 6*(e*x + d)^m*e^7*m^6*x +
 12*(e*x + d)^m*d^2*e^5*m^5*x^2 + 483*(e*x + d)^m*d*e^6*m^5*x^2 + 182*(...
```

Mupad [B] (verification not implemented)

Time = 17.73 (sec) , antiderivative size = 1425, normalized size of antiderivative = 4.88

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
int((d + e*x)^m*(2*x + 5*x^2 + 3)*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2),x)
```

output
```
((d + e*x)^m*(30240*d*e^6 + 14280*d^6*e + 14400*d^7 - 17640*d^2*e^5 + 3528
0*d^3*e^4 + 5040*d^4*e^3 + 17136*d^5*e^2 - 19278*d^2*e^5*m + 26796*d^3*e^4
*m + 2568*d^4*e^3*m + 5304*d^5*e^2*m + 30624*d*e^6*m^2 + 9990*d*e^6*m^3 +
1770*d*e^6*m^4 + 162*d*e^6*m^5 + 6*d*e^6*m^6 - 8225*d^2*e^5*m^2 + 7518*d^3
*e^4*m^2 + 432*d^4*e^3*m^2 + 408*d^5*e^2*m^2 - 1715*d^2*e^5*m^3 + 924*d^3*
e^4*m^3 + 24*d^4*e^3*m^3 - 175*d^2*e^5*m^4 + 42*d^3*e^4*m^4 - 7*d^2*e^5*m^
5 + 48168*d*e^6*m + 2040*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 +
1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (20*x^7*(d + e*x)^m*(1764*m +
 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068*m + 13132*m^2
+ 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) - (x*(d + e*x)^m*(3
5280*d^2*e^5*m - 30240*e^7 - 30624*e^7*m^2 - 9990*e^7*m^3 - 1770*e^7*m^4 -
 162*e^7*m^5 - 6*e^7*m^6 - 48168*e^7*m + 5040*d^3*e^4*m + 17136*d^4*e^3*m
+ 14280*d^5*e^2*m - 19278*d*e^6*m^2 - 8225*d*e^6*m^3 - 1715*d*e^6*m^4 - 17
5*d*e^6*m^5 - 7*d*e^6*m^6 + 26796*d^2*e^5*m^2 + 2568*d^3*e^4*m^2 + 5304*d^
4*e^3*m^2 + 2040*d^5*e^2*m^2 + 7518*d^2*e^5*m^3 + 432*d^3*e^4*m^3 + 408*d^
4*e^3*m^3 + 924*d^2*e^5*m^4 + 24*d^3*e^4*m^4 + 42*d^2*e^5*m^5 - 17640*d*e^
6*m + 14400*d^6*e*m))/(e^7*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 32
2*m^5 + 28*m^6 + m^7 + 5040)) - (x^3*(d + e*x)^m*(3*m + m^2 + 2)*(2400*d^4
*m - 13398*e^4*m - 17640*e^4 - 3759*e^4*m^2 - 462*e^4*m^3 - 21*e^4*m^4 + 2
856*d^2*e^2*m + 428*d*e^3*m^2 + 340*d^3*e*m^2 + 72*d*e^3*m^3 + 4*d*e^3*...
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1907, normalized size of antiderivative = 6.53

$$\int (d+ex)^m \left(3+2x+5x^2\right) \left(2+x+3x^2-5x^3+4x^4\right) dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(5*x^2+2*x+3)*(4*x^4-5*x^3+3*x^2+x+2),x)
```

output
```
((d + e*x)**m*(14400*d**7 - 14400*d**6*e*m*x + 2040*d**6*e*m + 14280*d**6*
e + 7200*d**5*e**2*m**2*x**2 - 2040*d**5*e**2*m**2*x + 408*d**5*e**2*m**2
+ 7200*d**5*e**2*m*x**2 - 14280*d**5*e**2*m*x + 5304*d**5*e**2*m + 17136*d
**5*e**2 - 2400*d**4*e**3*m**3*x**3 + 1020*d**4*e**3*m**3*x**2 - 408*d**4*
e**3*m**3*x + 24*d**4*e**3*m**3 - 7200*d**4*e**3*m**2*x**3 + 8160*d**4*e**
3*m**2*x**2 - 5304*d**4*e**3*m**2*x + 432*d**4*e**3*m**2 - 4800*d**4*e**3*
m*x**3 + 7140*d**4*e**3*m*x**2 - 17136*d**4*e**3*m*x + 2568*d**4*e**3*m +
5040*d**4*e**3 + 600*d**3*e**4*m**4*x**4 - 340*d**3*e**4*m**4*x**3 + 204*d
**3*e**4*m**4*x**2 - 24*d**3*e**4*m**4*x + 42*d**3*e**4*m**4 + 3600*d**3*e
**4*m**3*x**4 - 3400*d**3*e**4*m**3*x**3 + 2856*d**3*e**4*m**3*x**2 - 432*
d**3*e**4*m**3*x + 924*d**3*e**4*m**3 + 6600*d**3*e**4*m**2*x**4 - 7820*d*
*3*e**4*m**2*x**3 + 11220*d**3*e**4*m**2*x**2 - 2568*d**3*e**4*m**2*x + 75
18*d**3*e**4*m**2 + 3600*d**3*e**4*m*x**4 - 4760*d**3*e**4*m*x**3 + 8568*d
**3*e**4*m*x**2 - 5040*d**3*e**4*m*x + 26796*d**3*e**4*m + 35280*d**3*e**4
 - 120*d**2*e**5*m**5*x**5 + 85*d**2*e**5*m**5*x**4 - 68*d**2*e**5*m**5*x*
*3 + 12*d**2*e**5*m**5*x**2 - 42*d**2*e**5*m**5*x - 7*d**2*e**5*m**5 - 120
0*d**2*e**5*m**4*x**5 + 1105*d**2*e**5*m**4*x**4 - 1088*d**2*e**5*m**4*x**
3 + 228*d**2*e**5*m**4*x**2 - 924*d**2*e**5*m**4*x - 175*d**2*e**5*m**4 -
4200*d**2*e**5*m**3*x**5 + 4505*d**2*e**5*m**3*x**4 - 5644*d**2*e**5*m**3*
x**3 + 1500*d**2*e**5*m**3*x**2 - 7518*d**2*e**5*m**3*x - 1715*d**2*e**...
```

3.206 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 254

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \frac{(100d^2+165de+81e^2)(d+ex)^{1+m}}{125e^3(1+m)} - \frac{(40d+33e)(d+ex)^{2+m}}{25e^3(2+m)} + \frac{4(d+ex)^{3+m}}{5e^3(3+m)} - \frac{(6412i+423\sqrt{14})(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-e-i\sqrt{14}e}\right)}{3500\left(5id-(i-\sqrt{14})e\right)(1+m)} - \frac{(6412i-423\sqrt{14})(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d-e+i\sqrt{14}e}\right)}{3500\left(5id-(i+\sqrt{14})e\right)(1+m)}$$

output

```
1/125*(100*d^2+165*d*e+81*e^2)*(e*x+d)^(1+m)/e^3/(1+m)-1/25*(40*d+33*e)*(e
*x+d)^(2+m)/e^3/(2+m)+4/5*(e*x+d)^(3+m)/e^3/(3+m)-1/3500*(6412*I+423*14^(1
/2))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e-I*14^(1/2)*e)
)/(5*I*d-(I-14^(1/2))*e)/(1+m)-1/3500*(6412*I-423*14^(1/2))*(e*x+d)^(1+m)*
hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e+I*14^(1/2)*e))/(5*I*d-(I+14^(1/2
))*e)/(1+m)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{28(100d^2+165de+81e^2)}{e^3(1+m)} - \frac{140(40d+33e)(d+ex)}{e^3(2+m)} + \frac{2800(d+ex)^2}{e^3(3+m)} - \frac{\left(6412i+423\sqrt{14}\right)\text{Hypergeometric2F1}\left(1,1+m,2\right.}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m}\right.}{3500}$$

input
```
Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),
x]
```

output
```
((d + e*x)^(1 + m)*((28*(100*d^2 + 165*d*e + 81*e^2))/(e^3*(1 + m)) - (140
*(40*d + 33*e)*(d + e*x))/(e^3*(2 + m)) + (2800*(d + e*x)^2)/(e^3*(3 + m))
 - ((6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x
))/(5*d + (-1 - I*Sqrt[14])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) -
 ((-6412*I + 423*Sqrt[14])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x)
)/(5*d + I*(I + Sqrt[14])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m))))/3
500
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^m}{5x^2+2x+3}\,dx$$

$$\Big\downarrow \; 2159$$

$$\int \left(\frac{(100d^2+165de+81e^2)(d+ex)^m}{125e^2}+\frac{(-40d-33e)(d+ex)^{m+1}}{25e^2}+\frac{4(d+ex)^{m+2}}{5e^2}+\frac{\left(\frac{458}{125}+\frac{423i}{125\sqrt{14}}\right)(d+ex}{10x-2i\sqrt{14}+2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(100d^2+165de+81e^2)(d+ex)^{m+1}}{125e^3(m+1)}-\frac{(40d+33e)(d+ex)^{m+2}}{25e^3(m+2)}+\frac{4(d+ex)^{m+3}}{5e^3(m+3)}-\frac{(-423\sqrt{14}+6412i)(d+ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{5(d+ex)}{5d+i\sqrt{14}e-e}\right)}{3500(m+1)\left(5id-\left(\sqrt{14}+i\right)e\right)}-\frac{(423\sqrt{14}+6412i)(d+ex)^{m+1}\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{5(d+ex)}{5d-\left(1+i\sqrt{14}\right)e}\right)}{3500(m+1)\left(5id-\left(-\sqrt{14}+i\right)e\right)}$$

input

```
Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2),x]
```

output

```
((100*d^2 + 165*d*e + 81*e^2)*(d + e*x)^(1 + m))/(125*e^3*(1 + m)) - ((40*
d + 33*e)*(d + e*x)^(2 + m))/(25*e^3*(2 + m)) + (4*(d + e*x)^(3 + m))/(5*e
^3*(3 + m)) - ((6412*I - 423*Sqrt[14])*(d + e*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - e + I*Sqrt[14]*e)])/(3500*((5*I)*d
- (I + Sqrt[14])*e)*(1 + m)) - ((6412*I + 423*Sqrt[14])*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e
)])/(3500*((5*I)*d - (I - Sqrt[14])*e)*(1 + m))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [F]

$$\int \frac{(ex+d)^m (4x^4-5x^3+3x^2+x+2)}{5x^2+2x+3}dx$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x
)
```

Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{3+2x+5x^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{5\,x^2+2\,x+3}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \int \frac{{\left(d+e\,x\right)}^m\,\left(4\,x^4-5\,x^3+3\,x^2+x+2\right)}{5\,x^2+2\,x+3}\,dx$$

input
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3),x)
```

output
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m \left(2+x+3x^2-5x^3+4x^4\right)}{3+2x+5x^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3),x)
```

output
```
(400*(d + e*x)**m*d**3*m - 400*(d + e*x)**m*d**2*e*m**2*x + 330*(d + e*x)*
*m*d**2*e*m**2 + 990*(d + e*x)**m*d**2*e*m + 200*(d + e*x)**m*d*e**2*m**3*
x**2 - 330*(d + e*x)**m*d*e**2*m**3*x + 620*(d + e*x)**m*d*e**2*m**3 + 200
*(d + e*x)**m*d*e**2*m**2*x**2 - 990*(d + e*x)**m*d*e**2*m**2*x + 3558*(d
+ e*x)**m*d*e**2*m**2 + 6010*(d + e*x)**m*d*e**2*m + 2748*(d + e*x)**m*d*e
**2 + 200*(d + e*x)**m*e**3*m**3*x**3 - 330*(d + e*x)**m*e**3*m**3*x**2 +
162*(d + e*x)**m*e**3*m**3*x + 7*(d + e*x)**m*e**3*m**3 + 600*(d + e*x)**m
*e**3*m**2*x**3 - 1320*(d + e*x)**m*e**3*m**2*x**2 + 810*(d + e*x)**m*e**3
*m**2*x + 42*(d + e*x)**m*e**3*m**2 + 400*(d + e*x)**m*e**3*m*x**3 - 990*(
d + e*x)**m*e**3*m*x**2 + 972*(d + e*x)**m*e**3*m*x + 77*(d + e*x)**m*e**3
*m + 42*(d + e*x)**m*e**3 - 1360*int((d + e*x)**m/(5*d*x**2 + 2*d*x + 3*d
+ 5*e*x**3 + 2*e*x**2 + 3*e*x),x)*d*e**3*m**4 - 8160*int((d + e*x)**m/(5*d
*x**2 + 2*d*x + 3*d + 5*e*x**3 + 2*e*x**2 + 3*e*x),x)*d*e**3*m**3 - 14960*
int((d + e*x)**m/(5*d*x**2 + 2*d*x + 3*d + 5*e*x**3 + 2*e*x**2 + 3*e*x),x)
*d*e**3*m**2 - 8160*int((d + e*x)**m/(5*d*x**2 + 2*d*x + 3*d + 5*e*x**3 +
2*e*x**2 + 3*e*x),x)*d*e**3*m - 21*int((d + e*x)**m/(5*d*x**2 + 2*d*x + 3*
d + 5*e*x**3 + 2*e*x**2 + 3*e*x),x)*e**4*m**4 - 126*int((d + e*x)**m/(5*d*
x**2 + 2*d*x + 3*d + 5*e*x**3 + 2*e*x**2 + 3*e*x),x)*e**4*m**3 - 231*int((
d + e*x)**m/(5*d*x**2 + 2*d*x + 3*d + 5*e*x**3 + 2*e*x**2 + 3*e*x),x)*e**4
*m**2 - 126*int((d + e*x)**m/(5*d*x**2 + 2*d*x + 3*d + 5*e*x**3 + 2*e*x...
```

3.207 $\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 376

$$\int \frac{(d+ex)^m(2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$
$$= \frac{4(d+ex)^{1+m}}{25e(1+m)} - \frac{(1367d-293e+(423d-1367e)x)(d+ex)^{1+m}}{700(5d^2-2de+3e^2)(3+2x+5x^2)}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2-i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem\right.}{19600\left(5d-\left(1+i\sqrt{14}\right)e\right)(5d^2-2de}$$
$$+ \frac{\left(80360d^2-32144de+48216e^2+i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))-5922dem\right.}{19600\left(5d+i\left(i+\sqrt{14}\right)e\right)(5d^2-2de}$$

output
```
4/25*(e*x+d)^(1+m)/e/(1+m)-1/700*(1367*d-293*e+(423*d-1367*e)*x)*(e*x+d)^(
1+m)/(5*d^2-2*d*e+3*e^2)/(5*x^2+2*x+3)+1/19600*(80360*d^2-32144*d*e+48216*
e^2-I*14^(1/2)*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-98*m))-5922*d*e*m+1
9138*e^2*m)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],5*(e*x+d)/(5*d-e-I*14^(
1/2)*e))/(5*d-(I*14^(1/2)+1)*e)/(5*d^2-2*d*e+3*e^2)/(1+m)+1/19600*(80360*d
^2-32144*d*e+48216*e^2+I*14^(1/2)*(6565*d^2-2*d*e*(1313-3206*m)+e^2*(3939-
98*m))-5922*d*e*m+19138*e^2*m)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],5*(e
*x+d)/(5*d-e+I*14^(1/2)*e))/(5*d+I*(I+14^(1/2))*e)/(5*d^2-2*d*e+3*e^2)/(1+
m)
```

Mathematica [A] (verified)

Time = 2.15 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx$$

$$= \frac{(d+ex)^{1+m}\left(\frac{3136}{e+em}-\frac{28(d(1367+423x)-e(293+1367x))}{(5d^2-2de+3e^2)(3+2x+5x^2)}+\frac{56\left(287i+31\sqrt{14}\right)\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{5(d+ex)}{5d+\left(-1-i\sqrt{14}\right)e}\right)}{\left(5id+\left(-i+\sqrt{14}\right)e\right)(1+m)}\right.}{}$$

input
```
Integrate[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^
2,x]
```

output
```
((d + e*x)^(1 + m)*(3136/(e + e*m) - (28*(d*(1367 + 423*x) - e*(293 + 1367
*x)))/((5*d^2 - 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + (56*(287*I + 31*Sqrt[1
4])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I*Sqrt[1
4])*e)])/(((5*I)*d + (-I + Sqrt[14])*e)*(1 + m)) + (56*(-287*I + 31*Sqrt[1
4])*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14
])*e)])/(((-5*I)*d + (I + Sqrt[14])*e)*(1 + m)) - (Sqrt[14]*(((2115*d^2 +
d*e*(-846 + (-6412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 - (1367*I)*Sqr
t[14])*m))*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + (-1 - I
*Sqrt[14])*e)])/((5*I)*d + (-I + Sqrt[14])*e) - ((2115*d^2 - d*e*(846 + (6
412 + (423*I)*Sqrt[14])*m) + e^2*(1269 + (98 + (1367*I)*Sqrt[14])*m))*Hype
rgeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d + I*(I + Sqrt[14])*e)])/
((5*I)*d - (I + Sqrt[14])*e)))/((5*d^2 - 2*d*e + 3*e^2)*(1 + m))))/19600
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2179, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^4-5x^3+3x^2+x+2)(d+ex)^m}{(5x^2+2x+3)^2}dx$$

$$\Big\downarrow \; 2179$$

$$\frac{\int \frac{2(d+ex)^m(1845d^2-e(738-1367m)d+560(5d^2-2ed+3e^2)x^2+e^2(1107-293m)-(4620d^2-3e(141m+616)d+e^2(1367m+2772))x)}{25(5x^2+2x+3)}dx}{56(5d^2-2de+3e^2)}-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)^m(1845d^2-e(738-1367m)d+560(5d^2-2ed+3e^2)x^2+e^2(1107-293m)-(4620d^2-3e(141m+616)d+e^2(1367m+2772))x)}{5x^2+2x+3}dx}{700(5d^2-2de+3e^2)}-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}$$

$$\Big\downarrow \; 2159$$

$$-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}+$$

$$\frac{\int \left(112(5d^2-2ed+3e^2)(d+ex)^m+\frac{\left(-5740d^2+2296ed+423emd-3444e^2-1367e^2m-\frac{i(6565d^2-2626ed+6412emd+3939e^2-98e^2m)}{\sqrt{14}}\right.}{10x-2i\sqrt{14}+2}\right.}{700(5d^2-2de+}$$

$$\Big\downarrow \; 2009$$

$$-\frac{(x(423d-1367e)+1367d-293e)(d+ex)^{m+1}}{700(5x^2+2x+3)(5d^2-2de+3e^2)}+$$

$$\frac{\frac{\left(i\sqrt{14}(6565d^2-2de(1313-3206m)+e^2(3939-98m))+80360d^2-5922dem-32144de+19138e^2m+48216e^2\right)(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m\right.}{28(m+1)\left(5d+i\left(\sqrt{14}+i\right)e\right)}}{}$$

input
```
Int[((d + e*x)^m*(2 + x + 3*x^2 - 5*x^3 + 4*x^4))/(3 + 2*x + 5*x^2)^2,x]
```

output
```
-1/700*((1367*d - 293*e + (423*d - 1367*e)*x)*(d + e*x)^(1 + m))/((5*d^2 -
 2*d*e + 3*e^2)*(3 + 2*x + 5*x^2)) + ((112*(5*d^2 - 2*d*e + 3*e^2)*(d + e*
x)^(1 + m))/(e*(1 + m)) + ((80360*d^2 - 32144*d*e + 48216*e^2 + I*Sqrt[14]
*(6565*d^2 - 2*d*e*(1313 - 3206*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 191
38*e^2*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x
))/(5*d - e + I*Sqrt[14]*e)])/(28*(5*d + I*(I + Sqrt[14])*e)*(1 + m)) + ((
80360*d^2 - 32144*d*e + 48216*e^2 - I*Sqrt[14]*(6565*d^2 - 2*d*e*(1313 - 3
206*m) + e^2*(3939 - 98*m)) - 5922*d*e*m + 19138*e^2*m)*(d + e*x)^(1 + m)*
Hypergeometric2F1[1, 1 + m, 2 + m, (5*(d + e*x))/(5*d - (1 + I*Sqrt[14])*e
)])/(28*(5*d - (1 + I*Sqrt[14])*e)*(1 + m)))/(700*(5*d^2 - 2*d*e + 3*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2179
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, a + b*x + c*x^2, x], R =
 Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], S = Coeff[Polyno
mialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p + 1)*((R*(b*c*d - b^2*e + 2*a*c*e) - a*S*(2*c*d - b*e) + c*
(R*(2*c*d - b*e) - S*(b*d - 2*a*e))*x)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*
e + a*e^2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))
 Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c
)*(c*d^2 - b*d*e + a*e^2)*Qx + R*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - S*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(S*(b*d - 2*a*e) - R
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, m}, x
] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &
& LtQ[p, -1] &&  !(IGtQ[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] |
| ILtQ[p + 1/2, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m (4x^4-5x^3+3x^2+x+2)}{(5x^2+2x+3)^2}dx$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
fricas")
```

output
```
integral((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(25*x^4 + 20*x^3 + 34
*x^2 + 12*x + 9), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**m*(4*x**4-5*x**3+3*x**2+x+2)/(5*x**2+2*x+3)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
maxima")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
 x)
```

Giac [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(4\,x^4-5\,x^3+3\,x^2+x+2)(ex+d)^m}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
integrate((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x, algorithm="
giac")
```

output
```
integrate((4*x^4 - 5*x^3 + 3*x^2 + x + 2)*(e*x + d)^m/(5*x^2 + 2*x + 3)^2,
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \int \frac{(d+e\,x)^m\,(4\,x^4-5\,x^3+3\,x^2+x+2)}{(5\,x^2+2\,x+3)^2}\,dx$$

input
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2,x)
```

output
```
int(((d + e*x)^m*(x + 3*x^2 - 5*x^3 + 4*x^4 + 2))/(2*x + 5*x^2 + 3)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m (2+x+3x^2-5x^3+4x^4)}{(3+2x+5x^2)^2}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(4*x^4-5*x^3+3*x^2+x+2)/(5*x^2+2*x+3)^2,x)
```

output

```
(1000*(d + e*x)**m*d**2*m**2*x**2 - 1650*(d + e*x)**m*d**2*m**2*x + 600*(d
 + e*x)**m*d**2*m**2 - 1000*(d + e*x)**m*d**2*m*x**2 - 2450*(d + e*x)**m*d
**2*m*x - 600*(d + e*x)**m*d**2*m - 200*(d + e*x)**m*d*e*m**3*x**2 + 330*(
d + e*x)**m*d*e*m**3*x - 620*(d + e*x)**m*d*e*m**3 + 1000*(d + e*x)**m*d*e
*m**2*x**3 - 1250*(d + e*x)**m*d*e*m**2*x**2 + 970*(d + e*x)**m*d*e*m**2*x
 - 2220*(d + e*x)**m*d*e*m**2 - 1000*(d + e*x)**m*d*e*m*x**3 - 600*(d + e*
x)**m*d*e*m*x**2 - 60*(d + e*x)**m*d*e*m*x - 850*(d + e*x)**m*d*e*m + 2050
*(d + e*x)**m*d*e*x**2 + 820*(d + e*x)**m*d*e*x + 1230*(d + e*x)**m*d*e -
200*(d + e*x)**m*e**2*m**3*x**3 + 330*(d + e*x)**m*e**2*m**3*x**2 - 162*(d
 + e*x)**m*e**2*m**3*x - 7*(d + e*x)**m*e**2*m**3 + 400*(d + e*x)**m*e**2*
m**2*x**3 - 250*(d + e*x)**m*e**2*m**2*x**2 + 76*(d + e*x)**m*e**2*m**2*x
+ 372*(d + e*x)**m*e**2*m**2 - 200*(d + e*x)**m*e**2*m*x**3 - 490*(d + e*x
)**m*e**2*m*x**2 - 78*(d + e*x)**m*e**2*m*x + 625*(d + e*x)**m*e**2*m + 41
0*(d + e*x)**m*e**2*x**2 + 164*(d + e*x)**m*e**2*x + 246*(d + e*x)**m*e**2
 + 153750*int((d + e*x)**m/(125*d**2*m*x**4 + 100*d**2*m*x**3 + 170*d**2*m
*x**2 + 60*d**2*m*x + 45*d**2*m - 125*d**2*x**4 - 100*d**2*x**3 - 170*d**2
*x**2 - 60*d**2*x - 45*d**2 - 25*d*e*m**2*x**4 - 20*d*e*m**2*x**3 - 34*d*e
*m**2*x**2 - 12*d*e*m**2*x - 9*d*e*m**2 + 125*d*e*m*x**5 + 150*d*e*m*x**4
+ 210*d*e*m*x**3 + 128*d*e*m*x**2 + 69*d*e*m*x + 18*d*e*m - 125*d*e*x**5 -
 125*d*e*x**4 - 190*d*e*x**3 - 94*d*e*x**2 - 57*d*e*x - 9*d*e - 25*e**2...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file