Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.8/110-1.2.1.8-a

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**260**]. This is test number [110].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (260)	0.00 (0)
Maple	99.62 (259)	0.38 (1)
Rubi	91.15 (237)	8.85 (23)
Fricas	64.23 (167)	35.77 (93)
Maxima	49.23 (128)	50.77 (132)
Reduce	47.69 (124)	52.31 (136)
Giac	37.69 (98)	62.31 (162)
Sympy	33.08 (86)	66.92 (174)
Mupad	1.15 (3)	98.85 (257)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

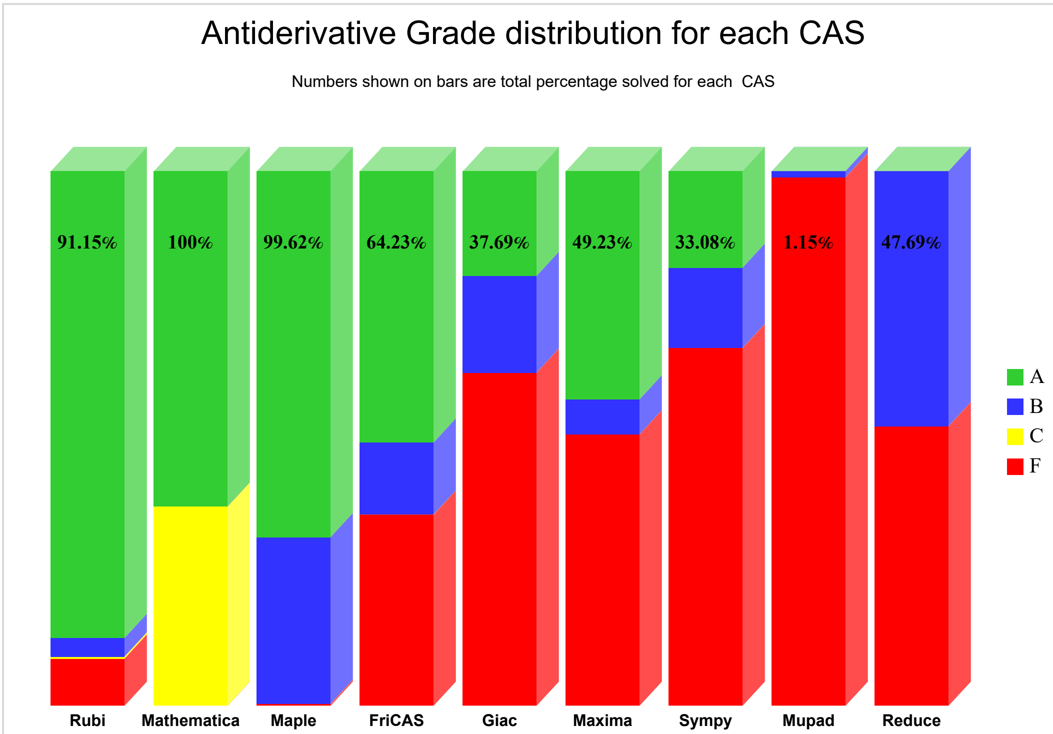
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

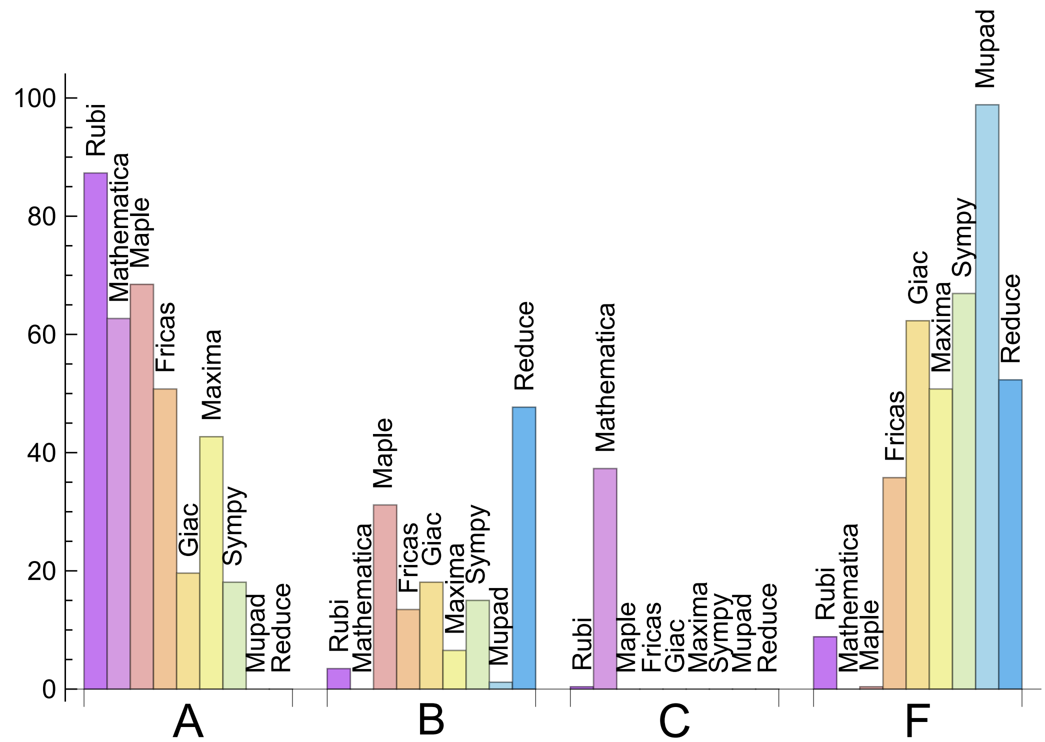
System	% A grade	% B grade	% C grade	% F grade
Rubi	87.308	3.462	0.385	8.846
Maple	68.462	31.154	0.000	0.385
Mathematica	62.692	0.000	37.308	0.000
Fricas	50.769	13.462	0.000	35.769
Maxima	42.692	6.538	0.000	50.769
Giac	19.615	18.077	0.000	62.308
Sympy	18.077	15.000	0.000	66.923
Mupad	0.000	1.154	0.000	98.846
Reduce	0.000	47.692	0.000	52.308

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	1	100.00	0.00	0.00
Rubi	23	100.00	0.00	0.00
Fricas	93	18.28	81.72	0.00
Maxima	132	100.00	0.00	0.00
Reduce	136	100.00	0.00	0.00
Giac	162	58.64	17.90	23.46
Sympy	174	82.76	17.24	0.00
Mupad	257	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Giac	0.19
Reduce	1.95
Rubi	2.10
Maple	2.75
Fricas	4.09
Sympy	10.67
Mathematica	11.85
Mupad	17.80

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	138.00	1.39	151.00	1.32
Maxima	404.64	1.51	329.00	1.18
Rubi	443.20	1.20	353.00	1.07
Maple	632.05	1.56	479.00	1.52
Giac	694.14	2.52	461.00	1.79
Mathematica	701.97	1.44	331.50	1.05
Sympy	749.87	2.85	618.50	2.56
Reduce	1037.17	4.07	677.50	2.68
Fricas	1106.72	3.69	779.00	2.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

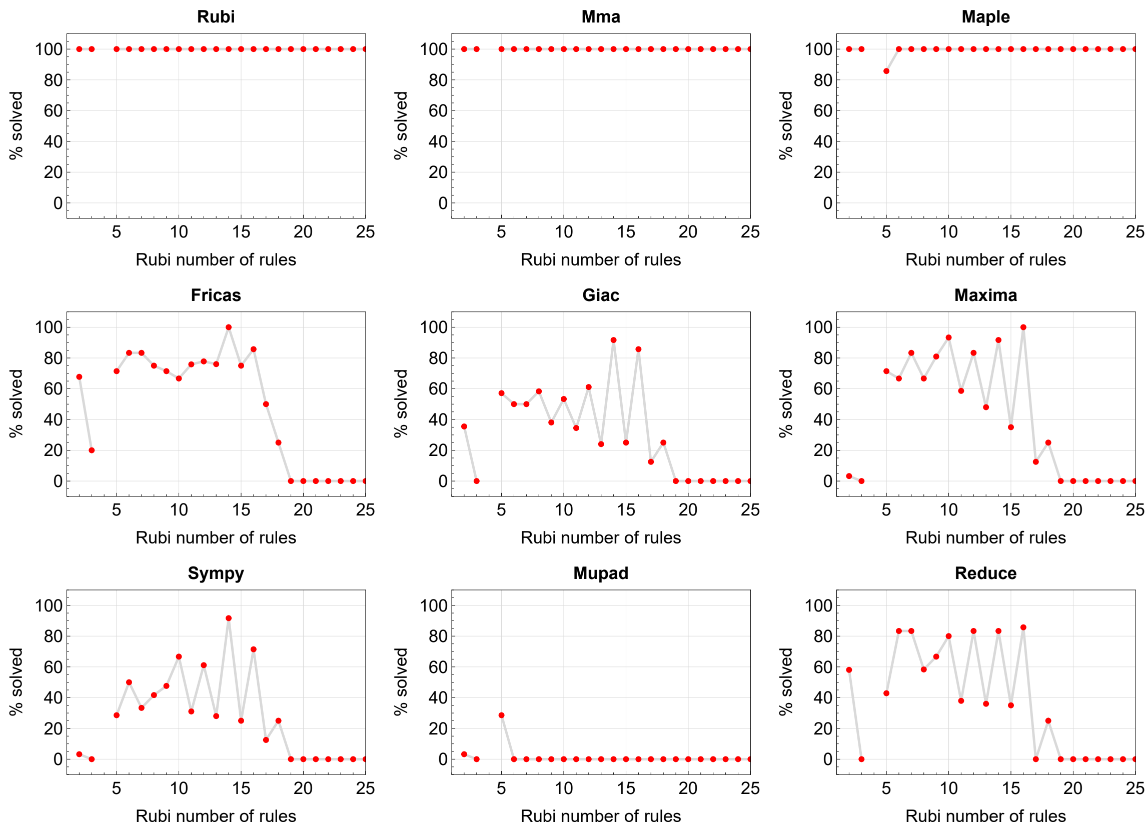


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

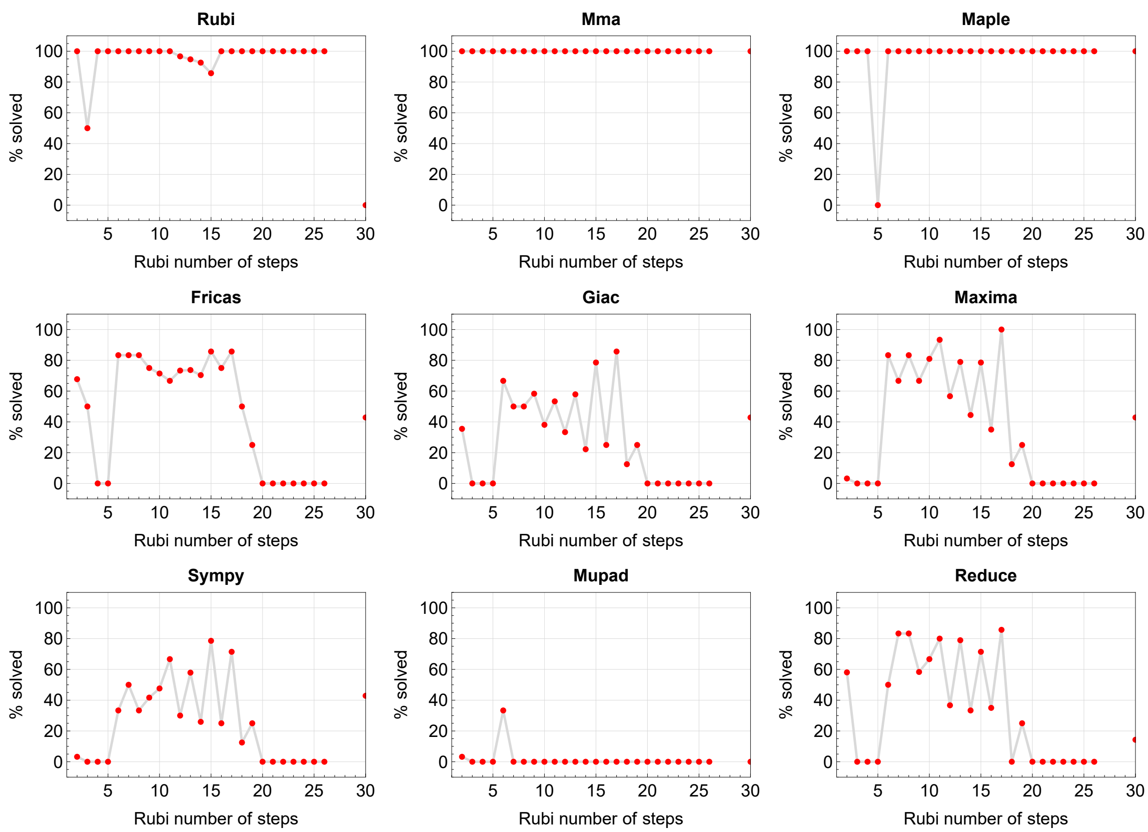


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

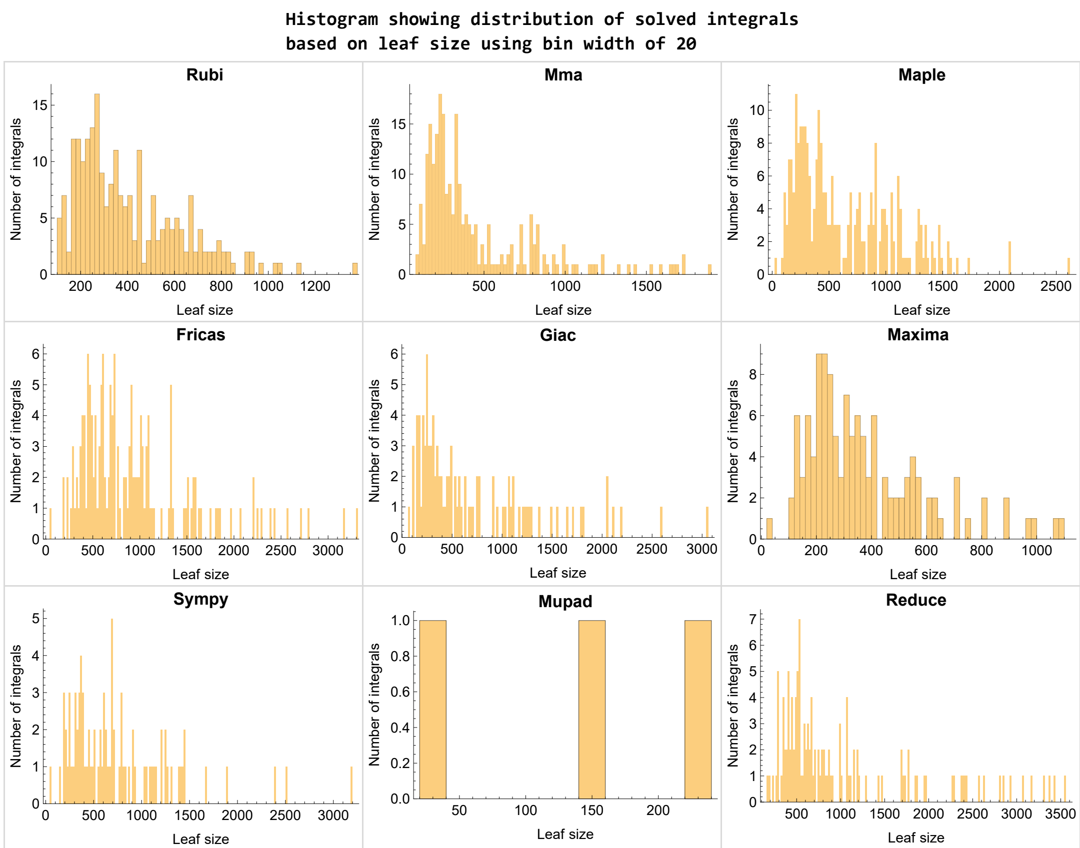


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

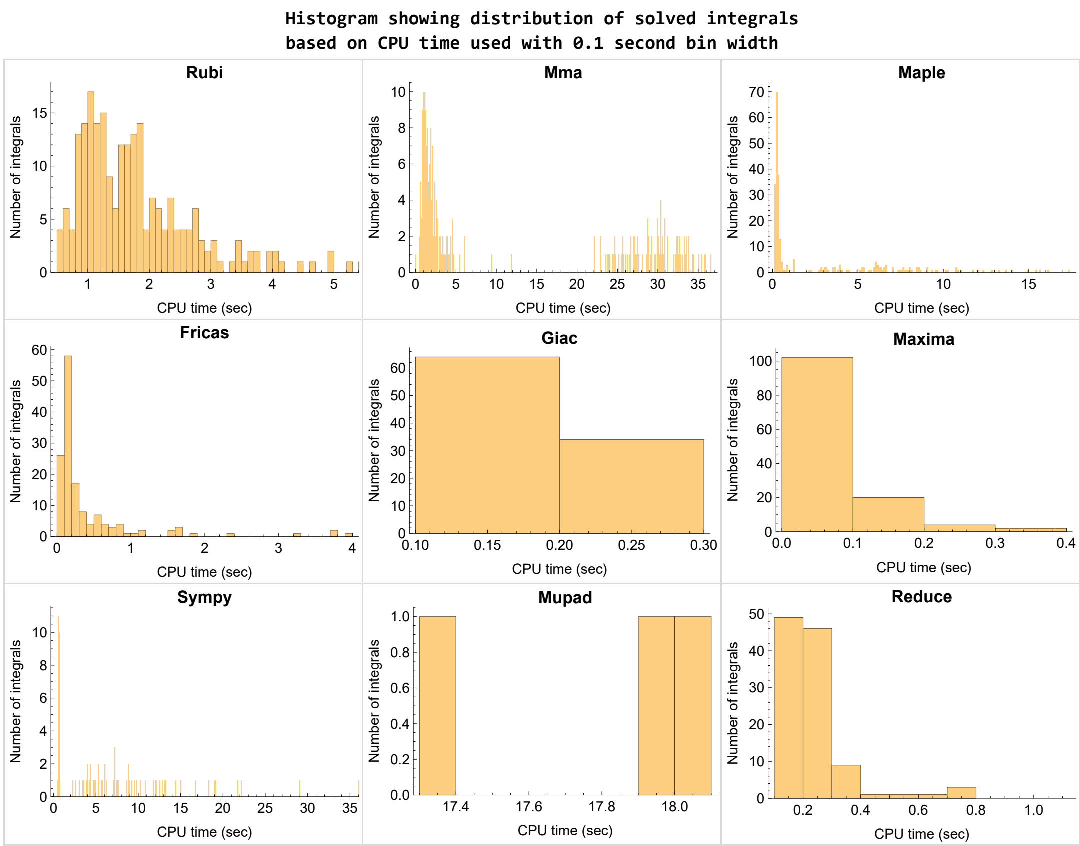


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

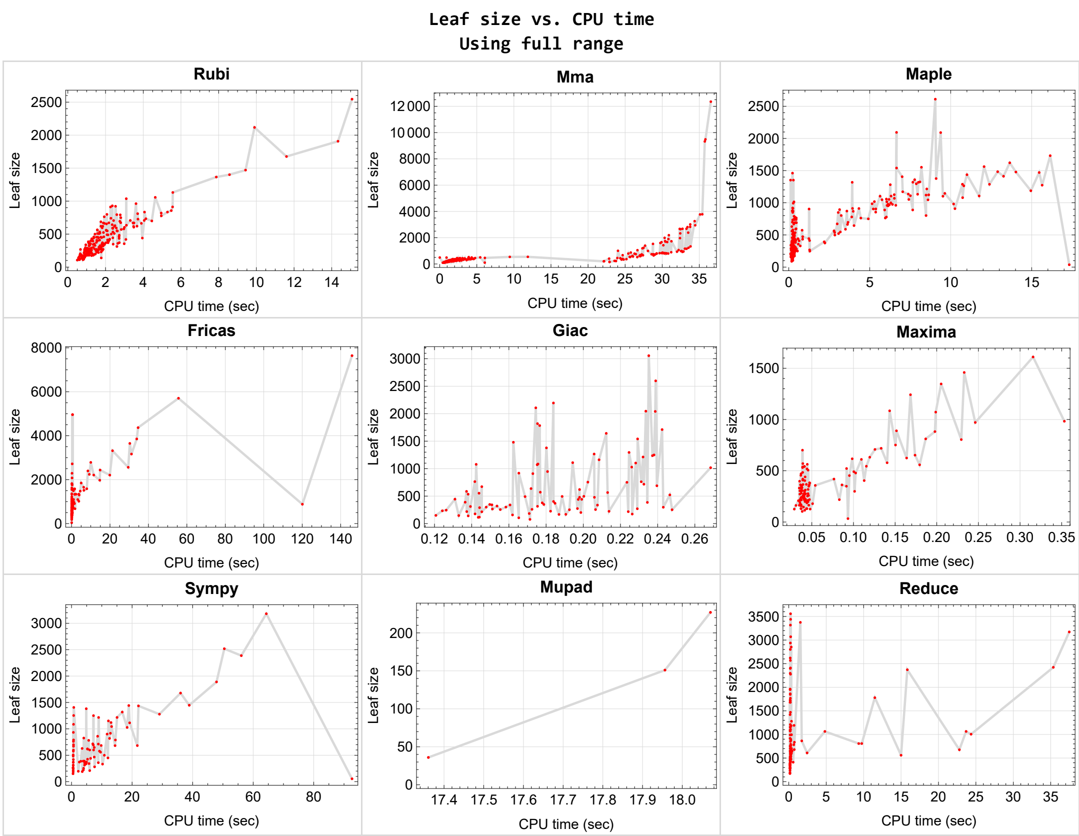


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{223\}$

Mathematica $\{258, 260\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

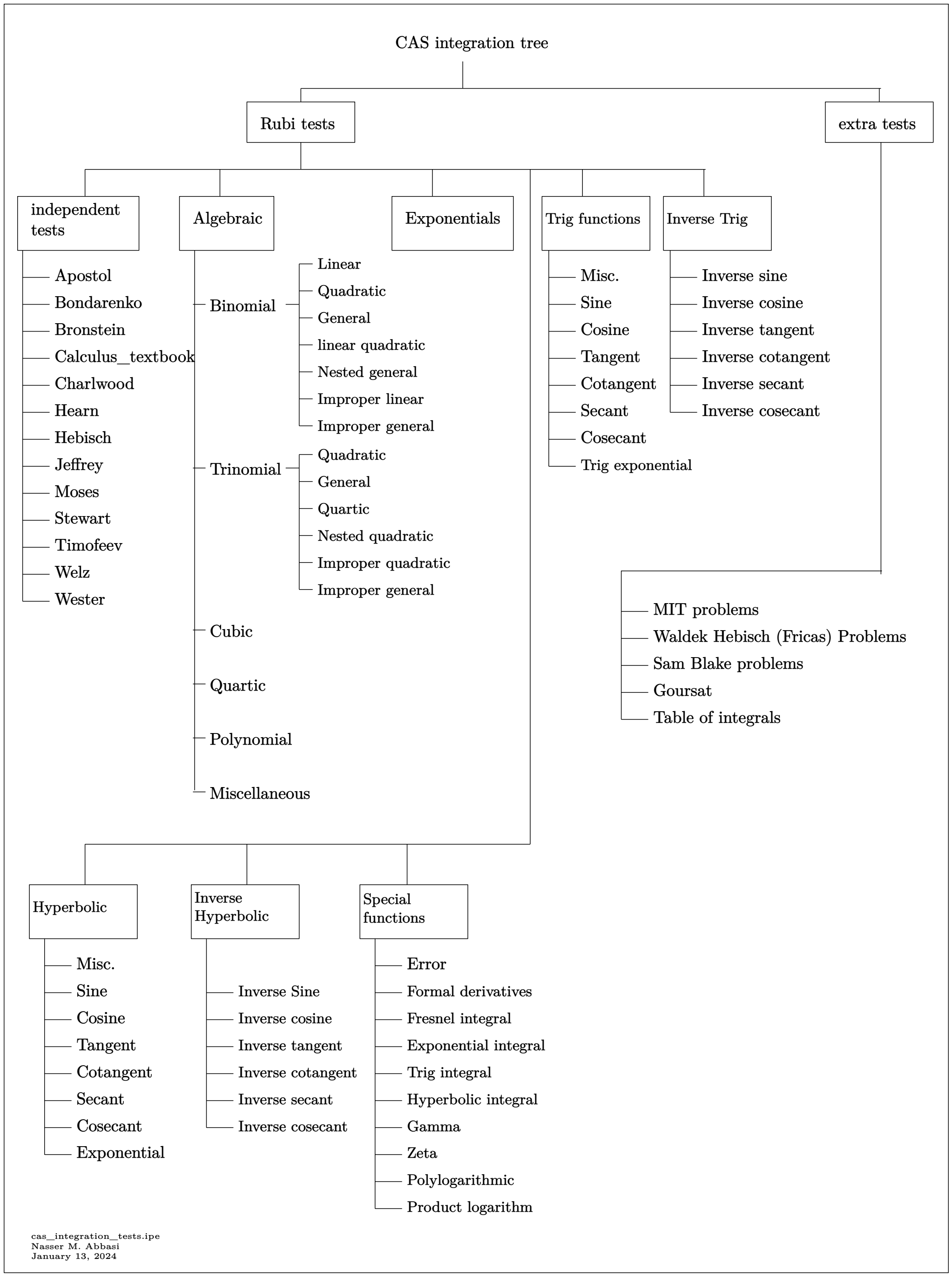
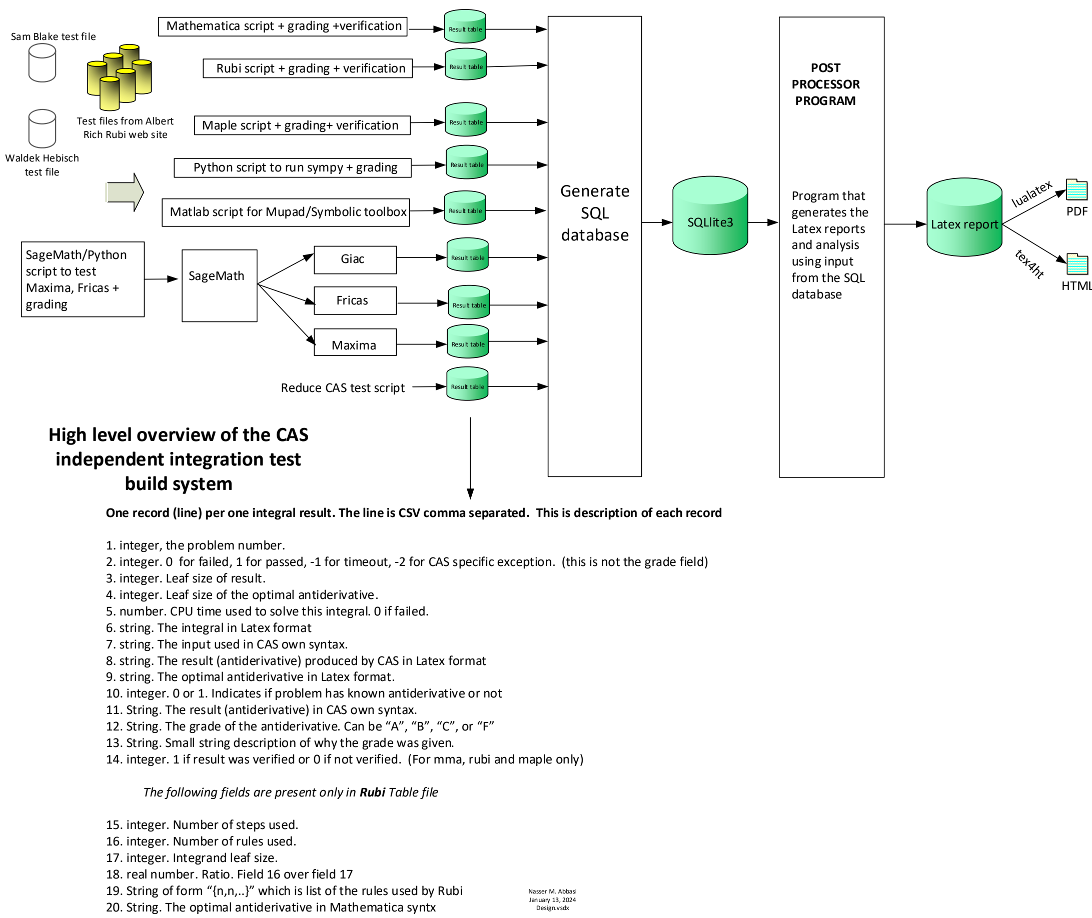


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 167, 168, 169, 170, 171, 176, 177, 178, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 206, 207, 208, 209, 210, 211, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 236, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 260 }

B grade { 165, 172, 173, 174, 175, 197, 198, 204, 205 }

C grade { 259 }

F normal fail { 16, 17, 62, 166, 179, 180, 181, 182, 183, 184, 212, 213, 214, 220, 221, 222, 228, 229, 230, 237, 238, 239, 258 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 260 }

B grade { }

C grade { 1, 2, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 38, 39, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 164, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 190, 199, 200, 201, 202, 204, 205, 207, 223, 224, 229, 230, 231, 232, 239, 240, 241, 242, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259 }

B grade { 1, 2, 33, 40, 41, 42, 43, 44, 79, 80, 88, 89, 90, 126, 127, 128, 129, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 176, 177, 178, 179, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 203, 206, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 243, 248 }

C grade { }

F normal fail { 260 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 36, 37, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 118, 120, 121, 122, 123, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 167, 168, 169, 170, 176, 177, 178, 185, 186, 187, 188, 192, 193, 194, 195, 201, 202, 206, 207, 208, 209, 210, 211, 235, 240, 241, 242, 259 }

B grade { 1, 2, 44, 45, 46, 128, 129, 130, 131, 153, 154, 155, 156, 160, 161, 162, 163, 164, 199, 200, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 231, 232, 233, 234, 236 }

C grade { }

F normal fail { 165, 166, 174, 175, 180, 181, 182, 190, 191, 196, 212, 213, 220, 229, 237, 245, 260 }

F(-1) timedout fail { 30, 31, 32, 33, 34, 35, 38, 39, 40, 41, 42, 43, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 115, 116, 117, 119, 124, 125, 126, 127, 149, 150, 151, 152, 157, 158, 159, 171, 172, 173, 179, 183, 184, 189, 197, 198, 203, 204, 205, 214, 221, 222, 228, 230, 238, 239, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258 }

F(-2) exception fail { }

Maxima

A grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 86, 87, 88, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 259 }

B grade { 39, 40, 41, 124, 125, 126, 127, 149, 150, 151, 152, 153, 157, 158, 159, 160, 161 }

C grade { }

F normal fail { 1, 2, 34, 35, 36, 37, 42, 43, 44, 45, 46, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 119, 120, 121, 122, 123, 128, 129, 130, 131, 154, 155, 156, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 3, 4, 5, 6, 8, 9, 16, 17, 18, 19, 21, 22, 47, 48, 49, 50, 52, 53, 61, 62, 63, 64, 66, 67, 68, 94, 95, 96, 97, 99, 105, 106, 107, 109, 110, 120, 121, 132, 133, 134, 135, 137, 138, 141, 142, 143, 145, 146, 155, 156, 259 }

B grade { 10, 11, 12, 13, 14, 15, 23, 24, 25, 26, 27, 28, 29, 36, 37, 54, 55, 56, 57, 58, 59, 60, 69, 70, 71, 72, 73, 74, 75, 84, 85, 100, 101, 102, 103, 104, 111, 112, 113, 114, 122, 123, 139, 140, 147, 148, 153 }

C grade { }

F normal fail { 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260 }

F(-1) timedout fail { 1, 2, 38, 39, 40, 41, 42, 44, 45, 46, 86, 87, 88, 89, 93, 124, 125, 126, 128, 129, 130, 131, 157, 158, 160, 161, 162, 163, 164 }

F(-2) exception fail { 7, 20, 30, 31, 32, 33, 34, 35, 43, 51, 65, 76, 77, 78, 79, 80, 81, 82, 83, 90, 91, 92, 98, 108, 115, 116, 117, 118, 119, 127, 136, 144, 149, 150, 151, 152, 154, 159 }

Mupad

A grade { }

B grade { 97, 135, 259 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260 }

F(-2) exception fail { }

Sympy

A grade { 3, 4, 5, 6, 7, 8, 9, 16, 17, 19, 20, 21, 22, 23, 51, 52, 53, 54, 55, 65, 66, 67, 68, 69, 70, 94, 95, 96, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 132, 133, 134, 135, 136, 137, 259 }

B grade { 10, 11, 12, 13, 14, 15, 18, 24, 25, 26, 27, 28, 29, 47, 48, 49, 50, 56, 57, 58, 59, 60, 61, 62, 63, 64, 71, 72, 73, 74, 102, 103, 104, 112, 113, 114, 138, 139, 140 }

C grade { }

F normal fail { 1, 2, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 209, 210, 211, 212, 219, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260 }

F(-1) timedout fail { 75, 164, 207, 208, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 65, 67, 69, 71, 72, 73, 74, 75, 79, 81, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 144, 145, 146, 147, 148, 154, 156, 160, 161, 162 }

C grade { }

F normal fail { 1, 2, 17, 19, 30, 31, 34, 36, 62, 64, 66, 68, 70, 76, 77, 78, 80, 82, 84, 85, 86, 105, 107, 115, 116, 120, 122, 141, 143, 149, 150, 151, 152, 153, 155, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	220	489	1352	0	4961	0	0	102	0
N.S.	1	0.96	2.13	5.88	0.00	21.57	0.00	0.00	0.44	0.00
time (sec)	N/A	1.253	0.916	0.286	0.000	0.512	0.000	0.000	0.234	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	220	489	1352	0	4961	0	0	102	0
N.S.	1	0.96	2.13	5.88	0.00	21.57	0.00	0.00	0.44	0.00
time (sec)	N/A	1.182	0.003	0.110	0.000	0.525	0.000	0.000	0.220	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	462	222	264	304	574	418	297	481	0
N.S.	1	1.67	0.80	0.96	1.10	2.08	1.51	1.08	1.74	0.00
time (sec)	N/A	2.716	1.142	0.195	0.036	0.099	0.602	0.243	0.214	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	349	197	233	255	472	367	252	413	0
N.S.	1	1.49	0.84	0.99	1.09	2.01	1.56	1.07	1.76	0.00
time (sec)	N/A	1.808	1.022	0.185	0.039	0.101	0.578	0.248	0.200	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	253	175	186	208	419	309	217	358	0
N.S.	1	1.27	0.88	0.93	1.05	2.11	1.55	1.09	1.80	0.00
time (sec)	N/A	1.071	0.802	0.139	0.034	0.094	0.568	0.224	0.184	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	180	145	155	169	325	258	173	290	0
N.S.	1	1.13	0.91	0.97	1.06	2.04	1.62	1.09	1.82	0.00
time (sec)	N/A	0.636	0.644	0.142	0.037	0.089	0.506	0.226	0.182	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	156	211	159	693	389	0	297	0
N.S.	1	1.00	0.94	1.27	0.96	4.17	2.34	0.00	1.79	0.00
time (sec)	N/A	0.920	0.819	0.129	0.037	0.468	3.870	0.000	0.166	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	167	162	202	130	603	386	166	344	0
N.S.	1	0.99	0.96	1.20	0.77	3.59	2.30	0.99	2.05	0.00
time (sec)	N/A	0.918	0.922	0.120	0.037	0.285	3.075	0.187	0.168	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	182	174	206	170	683	389	255	384	0
N.S.	1	0.90	0.86	1.02	0.84	3.38	1.93	1.26	1.90	0.00
time (sec)	N/A	1.036	0.964	0.111	0.037	0.483	4.226	0.207	0.184	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	176	181	159	163	618	350	385	623	0
N.S.	1	0.98	1.01	0.88	0.91	3.43	1.94	2.14	3.46	0.00
time (sec)	N/A	0.915	0.985	0.134	0.035	0.181	4.356	0.234	0.204	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	171	194	159	221	734	371	716	408	0
N.S.	1	0.97	1.10	0.90	1.26	4.17	2.11	4.07	2.32	0.00
time (sec)	N/A	0.905	1.599	0.139	0.036	0.189	4.963	0.233	0.197	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	160	192	171	234	350	440	751	453	0
N.S.	1	0.94	1.13	1.01	1.38	2.06	2.59	4.42	2.66	0.00
time (sec)	N/A	0.848	1.644	0.203	0.041	0.117	6.291	0.223	0.188	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	203	214	203	271	443	558	947	494	0
N.S.	1	0.98	1.03	0.98	1.31	2.14	2.70	4.57	2.39	0.00
time (sec)	N/A	0.986	1.925	0.187	0.040	0.158	9.582	0.181	0.196	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	243	244	249	318	498	916	1028	580	0
N.S.	1	0.98	0.98	1.00	1.28	2.01	3.69	4.15	2.34	0.00
time (sec)	N/A	1.108	2.311	0.275	0.046	0.190	12.842	0.226	0.232	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	290	265	282	367	593	1278	1266	630	0
N.S.	1	1.01	0.92	0.98	1.27	2.06	4.44	4.40	2.19	0.00
time (sec)	N/A	1.249	2.714	0.262	0.043	0.286	29.038	0.206	0.268	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	0	366	406	484	922	762	494	807	0
N.S.	1	0.00	0.90	1.00	1.19	2.27	1.88	1.22	1.99	0.00
time (sec)	N/A	0.000	2.057	0.203	0.039	0.118	0.625	0.189	9.344	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	0	370	382	440	832	670	439	32	0
N.S.	1	0.00	0.98	1.01	1.16	2.20	1.77	1.16	0.08	0.00
time (sec)	N/A	0.000	3.345	0.207	0.042	0.117	0.650	0.197	200.026	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	351	284	290	340	681	576	369	609	0
N.S.	1	1.17	0.95	0.97	1.13	2.27	1.92	1.23	2.03	0.00
time (sec)	N/A	1.404	1.484	0.147	0.037	0.104	0.603	0.185	2.446	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	269	246	268	305	593	484	314	29	0
N.S.	1	1.02	0.93	1.02	1.16	2.25	1.83	1.19	0.11	0.00
time (sec)	N/A	0.886	1.244	0.147	0.037	0.103	0.547	0.175	200.035	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	249	240	340	259	1049	620	0	477	0
N.S.	1	1.05	1.01	1.43	1.09	4.41	2.61	0.00	2.00	0.00
time (sec)	N/A	1.489	1.178	0.128	0.040	0.819	5.247	0.000	0.291	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	261	241	331	239	1015	629	270	559	0
N.S.	1	1.02	0.94	1.29	0.93	3.95	2.45	1.05	2.18	0.00
time (sec)	N/A	1.618	1.407	0.160	0.037	3.789	3.498	0.229	15.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	261	243	291	238	975	617	336	578	0
N.S.	1	1.04	0.97	1.16	0.95	3.90	2.47	1.34	2.31	0.00
time (sec)	N/A	1.627	1.257	0.138	0.037	1.150	5.308	0.208	0.293	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	269	213	259	248	975	600	522	676	0
N.S.	1	0.95	0.75	0.92	0.88	3.45	2.12	1.84	2.39	0.00
time (sec)	N/A	1.534	1.127	0.164	0.039	1.175	6.002	0.247	22.791	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	288	258	262	312	1030	600	1017	668	0
N.S.	1	1.20	1.07	1.09	1.29	4.27	2.49	4.22	2.77	0.00
time (sec)	N/A	1.675	1.798	0.159	0.042	0.704	7.289	0.268	0.255	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	256	249	281	340	1094	648	1212	694	0
N.S.	1	1.02	0.99	1.12	1.35	4.34	2.57	4.81	2.75	0.00
time (sec)	N/A	1.545	2.042	0.239	0.038	0.296	7.508	0.232	0.244	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	248	293	309	409	618	819	1541	765	0
N.S.	1	0.91	1.07	1.13	1.50	2.26	3.00	5.64	2.80	0.00
time (sec)	N/A	1.556	2.596	0.255	0.040	0.236	12.556	0.229	0.270	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	302	303	357	419	702	1216	1642	864	0
N.S.	1	0.96	0.97	1.14	1.34	2.24	3.88	5.25	2.76	0.00
time (sec)	N/A	1.756	2.958	0.589	0.039	0.333	15.031	0.212	1.732	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	369	381	441	542	853	1678	2046	994	0
N.S.	1	0.96	0.99	1.14	1.40	2.21	4.35	5.30	2.58	0.00
time (sec)	N/A	2.131	4.596	0.570	0.041	0.522	36.012	0.234	0.344	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	423	387	473	563	942	2518	2041	1064	0
N.S.	1	0.99	0.90	1.11	1.32	2.20	5.88	4.77	2.49	0.00
time (sec)	N/A	2.221	3.999	0.860	0.045	0.719	50.465	0.239	4.825	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	391	342	520	624	0	0	0	32	0
N.S.	1	1.07	0.93	1.42	1.70	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	2.162	1.811	0.274	0.164	0.000	0.000	0.000	200.022	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	299	273	399	491	0	0	0	30	0
N.S.	1	1.07	0.98	1.42	1.75	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.344	1.344	0.217	0.100	0.000	0.000	0.000	200.021	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	217	212	312	362	0	0	0	3439	0
N.S.	1	1.06	1.03	1.52	1.77	0.00	0.00	0.00	16.78	0.00
time (sec)	N/A	0.859	0.909	0.188	0.086	0.000	0.000	0.000	0.245	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	280	228	365	349	0	0	0	527	0
N.S.	1	1.36	1.11	1.77	1.69	0.00	0.00	0.00	2.56	0.00
time (sec)	N/A	1.156	1.372	0.183	0.092	0.000	0.000	0.000	0.194	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	267	200	306	0	0	0	0	32	0
N.S.	1	1.44	1.08	1.65	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.960	0.850	0.251	0.000	0.000	0.000	0.000	200.027	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	284	245	300	0	0	0	0	539	0
N.S.	1	1.35	1.16	1.42	0.00	0.00	0.00	0.00	2.55	0.00
time (sec)	N/A	1.012	1.851	0.267	0.000	0.000	0.000	0.000	0.208	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	246	258	334	0	1094	0	571	32	0
N.S.	1	1.02	1.08	1.39	0.00	4.56	0.00	2.38	0.13	0.00
time (sec)	N/A	0.998	1.573	0.331	0.000	3.249	0.000	0.177	200.025	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	354	319	424	0	1514	0	1249	853	0
N.S.	1	1.11	1.00	1.33	0.00	4.76	0.00	3.93	2.68	0.00
time (sec)	N/A	1.204	2.259	0.344	0.000	6.432	0.000	0.238	0.220	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	605	421	790	882	0	0	0	3172	0
N.S.	1	1.23	0.86	1.61	1.79	0.00	0.00	0.00	6.45	0.00
time (sec)	N/A	3.499	3.341	0.325	0.198	0.000	0.000	0.000	37.473	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	501	355	669	751	0	0	0	2801	0
N.S.	1	1.27	0.90	1.69	1.90	0.00	0.00	0.00	7.07	0.00
time (sec)	N/A	2.302	4.142	0.306	0.150	0.000	0.000	0.000	0.204	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	399	280	579	617	0	0	0	2296	0
N.S.	1	1.29	0.91	1.87	2.00	0.00	0.00	0.00	7.43	0.00
time (sec)	N/A	1.578	1.824	0.250	0.098	0.000	0.000	0.000	0.189	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	322	217	512	478	0	0	0	1847	0
N.S.	1	1.38	0.93	2.19	2.04	0.00	0.00	0.00	7.89	0.00
time (sec)	N/A	1.128	1.451	0.225	0.103	0.000	0.000	0.000	0.193	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	271	241	876	0	0	0	0	1420	0
N.S.	1	1.25	1.11	4.04	0.00	0.00	0.00	0.00	6.54	0.00
time (sec)	N/A	1.207	1.457	0.223	0.000	0.000	0.000	0.000	0.174	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	357	265	514	0	0	0	0	1470	0
N.S.	1	1.49	1.11	2.15	0.00	0.00	0.00	0.00	6.15	0.00
time (sec)	N/A	1.178	2.057	0.305	0.000	0.000	0.000	0.000	0.222	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	450	259	528	0	2240	0	0	1862	0
N.S.	1	1.64	0.95	1.93	0.00	8.18	0.00	0.00	6.80	0.00
time (sec)	N/A	1.380	2.188	0.332	0.000	9.243	0.000	0.000	0.195	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	500	327	589	0	2786	0	0	2409	0
N.S.	1	1.43	0.94	1.69	0.00	7.98	0.00	0.00	6.90	0.00
time (sec)	N/A	1.439	2.690	0.426	0.000	10.010	0.000	0.000	0.228	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	619	404	690	0	3650	0	0	2853	0
N.S.	1	1.42	0.93	1.58	0.00	8.37	0.00	0.00	6.54	0.00
time (sec)	N/A	1.753	4.571	0.473	0.000	30.321	0.000	0.000	0.329	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	483	268	332	349	716	790	377	604	0
N.S.	1	1.55	0.86	1.07	1.12	2.30	2.54	1.21	1.94	0.00
time (sec)	N/A	2.726	1.407	0.263	0.046	0.122	0.668	0.178	0.360	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	370	244	284	300	632	699	339	536	0
N.S.	1	1.35	0.89	1.03	1.09	2.30	2.54	1.23	1.95	0.00
time (sec)	N/A	1.834	1.119	0.265	0.037	0.113	0.653	0.161	0.277	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	277	223	240	253	573	598	299	481	0
N.S.	1	1.14	0.92	0.99	1.05	2.37	2.47	1.24	1.99	0.00
time (sec)	N/A	1.209	1.136	0.189	0.045	0.103	0.630	0.148	0.237	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	202	197	194	211	473	507	255	413	0
N.S.	1	1.03	1.00	0.98	1.07	2.40	2.57	1.29	2.10	0.00
time (sec)	N/A	0.658	1.034	0.188	0.036	0.107	0.552	0.155	0.215	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	211	210	273	215	987	901	0	424	0
N.S.	1	0.95	0.95	1.24	0.97	4.47	4.08	0.00	1.92	0.00
time (sec)	N/A	1.123	1.219	0.174	0.037	0.627	11.750	0.000	0.216	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	218	202	264	188	891	826	259	479	0
N.S.	1	0.98	0.91	1.19	0.85	4.01	3.72	1.17	2.16	0.00
time (sec)	N/A	1.045	1.065	0.229	0.036	0.310	4.045	0.172	0.189	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	253	222	278	242	907	782	348	513	0
N.S.	1	0.95	0.83	1.04	0.91	3.40	2.93	1.30	1.92	0.00
time (sec)	N/A	1.201	1.349	0.207	0.038	0.535	5.669	0.179	0.216	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	253	223	307	272	795	716	491	500	0
N.S.	1	0.91	0.81	1.11	0.98	2.87	2.58	1.77	1.81	0.00
time (sec)	N/A	1.201	1.394	0.266	0.036	0.634	7.012	0.169	0.205	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	257	229	269	331	906	712	754	514	0
N.S.	1	0.91	0.81	0.95	1.17	3.21	2.52	2.67	1.82	0.00
time (sec)	N/A	1.257	1.705	0.259	0.035	0.610	8.850	0.202	0.236	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	228	236	235	307	906	685	761	532	0
N.S.	1	0.94	0.97	0.97	1.26	3.73	2.82	3.13	2.19	0.00
time (sec)	N/A	1.050	2.065	0.367	0.039	0.221	9.790	0.231	0.225	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	216	249	233	357	1032	682	1084	535	0
N.S.	1	0.92	1.06	0.99	1.51	4.37	2.89	4.59	2.27	0.00
time (sec)	N/A	1.050	2.659	0.348	0.038	0.348	14.375	0.176	0.214	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	184	235	249	328	498	1027	1109	580	0
N.S.	1	0.88	1.12	1.19	1.56	2.37	4.89	5.28	2.76	0.00
time (sec)	N/A	0.893	1.279	0.536	0.037	0.228	18.362	0.194	0.257	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	229	266	282	363	593	1447	1295	630	0
N.S.	1	0.92	1.06	1.13	1.45	2.37	5.79	5.18	2.52	0.00
time (sec)	N/A	1.040	2.931	0.475	0.036	0.308	38.879	0.225	0.306	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	271	295	329	410	654	2387	1378	703	0
N.S.	1	0.94	1.02	1.14	1.42	2.27	8.29	4.78	2.44	0.00
time (sec)	N/A	1.198	3.622	0.710	0.039	0.494	56.081	0.180	0.359	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	616	448	438	542	1152	1406	620	1005	0
N.S.	1	1.35	0.98	0.96	1.19	2.53	3.09	1.36	2.21	0.00
time (sec)	N/A	3.487	2.332	0.366	0.041	0.132	0.740	0.198	24.357	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	B	A	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	0	451	430	518	1068	1253	565	32	0
N.S.	1	0.00	1.03	0.98	1.19	2.44	2.87	1.29	0.07	0.00
time (sec)	N/A	0.000	4.771	0.390	0.044	0.138	0.695	0.213	200.029	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	374	368	322	398	923	1086	496	807	0
N.S.	1	1.07	1.05	0.92	1.14	2.64	3.11	1.42	2.31	0.00
time (sec)	N/A	1.515	1.719	0.260	0.035	0.158	0.651	0.200	9.750	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	295	333	316	380	833	933	441	29	0
N.S.	1	0.89	1.01	0.95	1.15	2.52	2.82	1.33	0.09	0.00
time (sec)	N/A	0.931	1.730	0.274	0.048	0.153	0.603	0.199	200.029	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	324	418	330	1519	1442	0	679	0
N.S.	1	1.00	1.06	1.37	1.08	4.98	4.73	0.00	2.23	0.00
time (sec)	N/A	1.674	2.107	0.238	0.037	0.880	18.916	0.000	0.735	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	337	325	425	334	1479	1382	402	32	0
N.S.	1	0.99	0.95	1.25	0.98	4.34	4.05	1.18	0.09	0.00
time (sec)	N/A	1.884	3.211	0.324	0.038	4.524	4.865	0.184	200.026	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	353	331	367	334	1331	1251	478	783	0
N.S.	1	1.06	0.99	1.10	1.00	4.00	3.76	1.44	2.35	0.00
time (sec)	N/A	1.892	1.991	0.273	0.037	1.670	7.250	0.206	0.672	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	412	328	402	404	1339	1216	688	32	0
N.S.	1	0.97	0.77	0.94	0.95	3.14	2.85	1.62	0.08	0.00
time (sec)	N/A	2.203	2.300	0.314	0.042	3.766	8.935	0.240	200.027	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	412	324	413	471	1338	1151	1102	835	0
N.S.	1	1.12	0.88	1.13	1.28	3.65	3.14	3.00	2.28	0.00
time (sec)	N/A	2.107	2.067	0.312	0.037	2.336	12.183	0.228	0.753	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	362	302	415	455	1330	1120	1236	32	0
N.S.	1	0.95	0.79	1.08	1.19	3.47	2.92	3.23	0.08	0.00
time (sec)	N/A	1.849	2.094	0.437	0.037	1.661	13.037	0.237	200.027	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	368	348	382	531	1494	1114	1711	876	0
N.S.	1	1.12	1.05	1.16	1.61	4.53	3.38	5.18	2.65	0.00
time (sec)	N/A	2.013	3.861	0.416	0.039	0.969	19.196	0.242	0.275	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	312	337	409	506	1568	1435	1784	898	0
N.S.	1	0.97	1.04	1.27	1.57	4.85	4.44	5.52	2.78	0.00
time (sec)	N/A	1.706	3.573	0.638	0.046	0.517	22.108	0.177	0.268	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	275	383	441	570	855	1889	2107	994	0
N.S.	1	0.81	1.13	1.30	1.69	2.53	5.59	6.23	2.94	0.00
time (sec)	N/A	1.656	4.310	0.570	0.039	0.654	47.890	0.174	0.353	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	328	389	433	537	944	3181	2195	1064	0
N.S.	1	0.90	1.07	1.19	1.47	2.59	8.72	6.01	2.92	0.00
time (sec)	N/A	1.850	4.514	0.856	0.039	0.796	64.337	0.184	23.692	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	392	462	573	701	1097	0	2598	1192	0
N.S.	1	0.87	1.03	1.27	1.56	2.44	0.00	5.77	2.65	0.00
time (sec)	N/A	2.119	6.083	0.822	0.039	1.064	0.000	0.239	0.701	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	548	522	849	971	0	0	0	32	0
N.S.	1	1.04	0.99	1.60	1.84	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.631	3.907	0.243	0.246	0.000	0.000	0.000	200.013	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	447	426	699	811	0	0	0	30	0
N.S.	1	0.99	0.94	1.55	1.79	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.795	2.660	0.222	0.187	0.000	0.000	0.000	200.021	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	343	340	559	632	0	0	0	29	0
N.S.	1	0.98	0.97	1.60	1.81	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.344	1.875	0.195	0.120	0.000	0.000	0.000	200.023	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	453	332	630	578	0	0	0	1157	0
N.S.	1	1.31	0.96	1.83	1.68	0.00	0.00	0.00	3.35	0.00
time (sec)	N/A	1.747	2.187	0.187	0.140	0.000	0.000	0.000	0.191	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	394	280	551	0	0	0	0	32	0
N.S.	1	1.33	0.95	1.86	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.333	1.238	0.253	0.000	0.000	0.000	0.000	200.019	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	441	305	491	0	0	0	0	1074	0
N.S.	1	1.54	1.06	1.71	0.00	0.00	0.00	0.00	3.74	0.00
time (sec)	N/A	1.370	1.470	0.282	0.000	0.000	0.000	0.000	0.197	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	561	283	449	0	0	0	0	32	0
N.S.	1	1.90	0.96	1.52	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.571	1.334	0.338	0.000	0.000	0.000	0.000	200.032	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	687	360	493	0	0	0	0	1183	0
N.S.	1	1.99	1.04	1.42	0.00	0.00	0.00	0.00	3.42	0.00
time (sec)	N/A	1.752	2.566	0.354	0.000	0.000	0.000	0.000	0.307	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	763	449	596	0	1974	0	1819	32	0
N.S.	1	1.89	1.11	1.48	0.00	4.89	0.00	4.50	0.08	0.00
time (sec)	N/A	1.932	2.736	0.450	0.000	14.677	0.000	0.175	200.015	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	912	501	739	0	2562	0	3056	32	0
N.S.	1	1.79	0.98	1.45	0.00	5.02	0.00	5.99	0.06	0.00
time (sec)	N/A	2.265	5.461	0.454	0.000	29.494	0.000	0.235	200.025	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	614	673	538	1014	1072	0	0	0	32	0
N.S.	1	1.10	0.88	1.65	1.75	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	2.804	9.447	0.320	0.199	0.000	0.000	0.000	200.020	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	549	435	870	890	0	0	0	2398	0
N.S.	1	1.11	0.88	1.75	1.79	0.00	0.00	0.00	4.83	0.00
time (sec)	N/A	2.091	3.052	0.264	0.152	0.000	0.000	0.000	0.196	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	454	347	733	708	0	0	0	1949	0
N.S.	1	1.16	0.89	1.87	1.81	0.00	0.00	0.00	4.98	0.00
time (sec)	N/A	1.590	2.292	0.240	0.126	0.000	0.000	0.000	0.183	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	524	342	1461	0	0	0	0	1681	0
N.S.	1	1.50	0.98	4.17	0.00	0.00	0.00	0.00	4.80	0.00
time (sec)	N/A	1.889	1.849	0.230	0.000	0.000	0.000	0.000	0.170	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	533	335	744	0	0	0	0	1680	0
N.S.	1	1.48	0.93	2.07	0.00	0.00	0.00	0.00	4.68	0.00
time (sec)	N/A	1.689	2.021	0.331	0.000	0.000	0.000	0.000	0.214	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	669	350	690	0	0	0	0	1727	0
N.S.	1	1.66	0.87	1.72	0.00	0.00	0.00	0.00	4.30	0.00
time (sec)	N/A	1.882	1.691	0.361	0.000	0.000	0.000	0.000	0.180	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	466	799	408	692	0	0	0	0	1779	0
N.S.	1	1.71	0.88	1.48	0.00	0.00	0.00	0.00	3.82	0.00
time (sec)	N/A	2.073	2.489	0.421	0.000	0.000	0.000	0.000	11.495	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	929	410	765	0	2437	0	0	1962	0
N.S.	1	1.76	0.78	1.45	0.00	4.62	0.00	0.00	3.72	0.00
time (sec)	N/A	2.360	4.019	0.461	0.000	14.859	0.000	0.000	0.177	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	435	173	186	259	432	248	223	358	0
N.S.	1	1.86	0.74	0.79	1.11	1.85	1.06	0.95	1.53	0.00
time (sec)	N/A	2.719	0.844	0.261	0.046	0.112	0.575	0.214	0.180	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	325	155	155	210	338	214	183	290	0
N.S.	1	1.69	0.81	0.81	1.09	1.76	1.11	0.95	1.51	0.00
time (sec)	N/A	1.707	1.240	0.268	0.042	0.097	0.542	0.171	0.168	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	226	124	112	162	285	184	146	235	0
N.S.	1	1.42	0.78	0.70	1.02	1.79	1.16	0.92	1.48	0.00
time (sec)	N/A	1.093	0.580	0.187	0.031	0.125	0.545	0.133	0.165	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	152	96	89	126	195	150	105	171	227
N.S.	1	1.27	0.80	0.74	1.05	1.62	1.25	0.88	1.42	1.89
time (sec)	N/A	0.589	0.493	0.178	0.048	0.083	0.489	0.165	0.157	18.071

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	119	112	147	114	486	253	0	186	0
N.S.	1	1.04	0.98	1.29	1.00	4.26	2.22	0.00	1.63	0.00
time (sec)	N/A	0.843	0.580	0.162	0.041	0.476	3.976	0.000	0.161	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	110	115	103	443	196	117	370	0
N.S.	1	1.09	1.07	1.12	1.00	4.30	1.90	1.14	3.59	0.00
time (sec)	N/A	0.803	0.554	0.211	0.038	0.128	2.237	0.144	0.155	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	123	145	106	131	527	190	225	280	0
N.S.	1	1.03	1.22	0.89	1.10	4.43	1.60	1.89	2.35	0.00
time (sec)	N/A	0.837	0.918	0.188	0.044	0.166	3.537	0.182	0.160	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	136	138	103	146	220	221	348	326	0
N.S.	1	1.03	1.05	0.78	1.11	1.67	1.67	2.64	2.47	0.00
time (sec)	N/A	0.857	0.849	0.253	0.040	0.129	3.909	0.150	0.166	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	181	162	130	179	304	342	637	379	0
N.S.	1	1.06	0.95	0.76	1.05	1.79	2.01	3.75	2.23	0.00
time (sec)	N/A	0.969	1.207	0.246	0.051	0.105	5.758	0.172	0.171	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	223	194	173	226	360	694	672	453	0
N.S.	1	1.07	0.93	0.83	1.09	1.73	3.34	3.23	2.18	0.00
time (sec)	N/A	1.204	1.466	0.355	0.041	0.126	7.200	0.145	0.183	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	266	214	204	275	451	1044	918	505	0
N.S.	1	1.07	0.86	0.82	1.11	1.82	4.21	3.70	2.04	0.00
time (sec)	N/A	1.252	1.653	0.345	0.043	0.156	13.229	0.165	0.188	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	444	254	284	362	610	367	319	32	0
N.S.	1	1.43	0.82	0.92	1.17	1.97	1.18	1.03	0.10	0.00
time (sec)	N/A	2.385	1.754	0.395	0.047	0.101	0.562	0.154	200.027	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	328	203	224	281	463	313	252	411	0
N.S.	1	1.32	0.82	0.90	1.13	1.87	1.26	1.02	1.66	0.00
time (sec)	N/A	1.443	1.033	0.263	0.037	0.131	0.551	0.193	0.585	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	243	166	166	230	383	265	201	29	0
N.S.	1	1.18	0.81	0.81	1.12	1.86	1.29	0.98	0.14	0.00
time (sec)	N/A	0.882	0.700	0.267	0.041	0.094	0.503	0.199	200.023	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	191	159	257	199	717	425	0	292	0
N.S.	1	1.09	0.91	1.47	1.14	4.10	2.43	0.00	1.67	0.00
time (sec)	N/A	1.399	0.708	0.227	0.042	0.849	5.283	0.000	0.201	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	174	147	209	186	675	371	168	494	0
N.S.	1	1.09	0.92	1.31	1.17	4.25	2.33	1.06	3.11	0.00
time (sec)	N/A	1.259	0.665	0.303	0.044	1.805	2.566	0.191	0.190	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	179	182	166	198	695	313	273	410	0
N.S.	1	1.10	1.12	1.02	1.22	4.29	1.93	1.69	2.53	0.00
time (sec)	N/A	1.348	1.092	0.254	0.045	0.383	4.311	0.197	0.198	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	198	162	167	221	762	338	478	509	0
N.S.	1	1.06	0.87	0.90	1.19	4.10	1.82	2.57	2.74	0.00
time (sec)	N/A	1.461	0.928	0.300	0.043	0.236	4.800	0.198	0.199	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	226	207	189	254	404	466	909	588	0
N.S.	1	1.03	0.95	0.86	1.16	1.84	2.13	4.15	2.68	0.00
time (sec)	N/A	1.530	1.828	0.292	0.041	0.159	7.495	0.173	0.205	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	280	217	250	301	485	860	1071	661	0
N.S.	1	1.05	0.81	0.94	1.13	1.82	3.22	4.01	2.48	0.00
time (sec)	N/A	1.768	1.849	0.410	0.041	0.172	8.621	0.175	0.390	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	343	295	309	383	627	1318	1480	792	0
N.S.	1	1.05	0.90	0.94	1.17	1.91	4.02	4.51	2.41	0.00
time (sec)	N/A	1.948	2.353	0.409	0.047	0.260	16.749	0.162	0.254	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	370	275	372	558	0	0	0	32	0
N.S.	1	1.13	0.84	1.14	1.71	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.732	1.556	0.285	0.179	0.000	0.000	0.000	200.031	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	273	226	288	404	0	0	0	32	0
N.S.	1	1.11	0.92	1.18	1.65	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.782	1.686	0.232	0.113	0.000	0.000	0.000	200.027	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	193	171	236	299	0	0	0	3558	0
N.S.	1	1.07	0.94	1.30	1.65	0.00	0.00	0.00	19.66	0.00
time (sec)	N/A	1.049	0.836	0.214	0.101	0.000	0.000	0.000	0.235	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	135	137	197	218	881	0	0	3317	0
N.S.	1	1.04	1.05	1.52	1.68	6.78	0.00	0.00	25.52	0.00
time (sec)	N/A	0.592	0.556	0.193	0.083	120.084	0.000	0.000	0.218	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	182	168	202	0	0	0	0	435	0
N.S.	1	1.36	1.25	1.51	0.00	0.00	0.00	0.00	3.25	0.00
time (sec)	N/A	0.886	0.804	0.184	0.000	0.000	0.000	0.000	0.242	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	149	206	0	910	0	157	32	0
N.S.	1	1.00	1.12	1.55	0.00	6.84	0.00	1.18	0.24	0.00
time (sec)	N/A	0.715	0.653	0.244	0.000	1.558	0.000	0.162	200.026	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	205	212	246	0	1301	0	300	746	0
N.S.	1	1.10	1.13	1.32	0.00	6.96	0.00	1.60	3.99	0.00
time (sec)	N/A	0.871	1.174	0.260	0.000	3.979	0.000	0.158	0.210	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	281	260	305	0	1653	0	553	32	0
N.S.	1	1.12	1.04	1.22	0.00	6.59	0.00	2.20	0.13	0.00
time (sec)	N/A	0.990	1.671	0.311	0.000	6.313	0.000	0.144	209.917	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	398	323	398	0	2207	0	1161	1280	0
N.S.	1	1.21	0.98	1.21	0.00	6.69	0.00	3.52	3.88	0.00
time (sec)	N/A	1.214	2.718	0.339	0.000	19.907	0.000	0.208	0.362	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	419	350	525	719	0	0	0	2937	0
N.S.	1	1.25	1.04	1.57	2.15	0.00	0.00	0.00	8.77	0.00
time (sec)	N/A	3.080	2.315	0.330	0.133	0.000	0.000	0.000	0.214	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	324	286	473	611	0	0	0	2568	0
N.S.	1	1.21	1.07	1.76	2.28	0.00	0.00	0.00	9.58	0.00
time (sec)	N/A	1.882	2.547	0.296	0.110	0.000	0.000	0.000	0.212	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	245	215	430	522	0	0	0	1717	0
N.S.	1	1.20	1.05	2.11	2.56	0.00	0.00	0.00	8.42	0.00
time (sec)	N/A	1.171	1.256	0.242	0.092	0.000	0.000	0.000	0.205	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	186	177	390	419	0	0	0	1148	0
N.S.	1	1.11	1.05	2.32	2.49	0.00	0.00	0.00	6.83	0.00
time (sec)	N/A	0.718	1.170	0.192	0.077	0.000	0.000	0.000	0.208	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	285	183	408	0	1585	0	0	1093	0
N.S.	1	1.71	1.10	2.44	0.00	9.49	0.00	0.00	6.54	0.00
time (sec)	N/A	1.050	1.107	0.223	0.000	7.932	0.000	0.000	0.177	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	252	226	430	0	2210	0	0	1767	0
N.S.	1	1.22	1.10	2.09	0.00	10.73	0.00	0.00	8.58	0.00
time (sec)	N/A	0.944	1.345	0.309	0.000	11.544	0.000	0.000	0.214	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	335	334	479	0	3165	0	0	2620	0
N.S.	1	1.24	1.24	1.77	0.00	11.72	0.00	0.00	9.70	0.00
time (sec)	N/A	1.167	2.549	0.345	0.000	31.181	0.000	0.000	0.193	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	418	403	536	0	3853	0	0	3070	0
N.S.	1	1.24	1.19	1.59	0.00	11.40	0.00	0.00	9.08	0.00
time (sec)	N/A	1.287	3.418	0.396	0.000	33.924	0.000	0.000	0.254	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	220	170	207	256	517	449	218	531	0
N.S.	1	1.00	0.77	0.94	1.16	2.34	2.03	0.99	2.40	0.00
time (sec)	N/A	1.626	0.964	0.296	0.040	0.112	12.029	0.137	0.187	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	182	146	171	205	411	359	176	456	0
N.S.	1	0.99	0.79	0.93	1.11	2.23	1.95	0.96	2.48	0.00
time (sec)	N/A	1.136	1.037	0.287	0.043	0.101	8.847	0.142	0.187	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	136	115	130	158	360	280	140	395	0
N.S.	1	0.91	0.77	0.87	1.05	2.40	1.87	0.93	2.63	0.00
time (sec)	N/A	0.740	0.696	0.212	0.037	0.106	7.605	0.138	0.180	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	106	104	112	126	276	209	113	275	151
N.S.	1	0.93	0.91	0.98	1.11	2.42	1.83	0.99	2.41	1.32
time (sec)	N/A	0.500	0.600	0.204	0.029	0.096	6.001	0.144	0.177	17.956

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	114	114	150	131	706	332	0	344	0
N.S.	1	0.94	0.94	1.24	1.08	5.83	2.74	0.00	2.84	0.00
time (sec)	N/A	0.626	0.753	0.185	0.037	0.222	10.241	0.000	0.174	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	112	118	137	130	292	500	149	454	0
N.S.	1	0.93	0.98	1.14	1.08	2.43	4.17	1.24	3.78	0.00
time (sec)	N/A	0.610	0.665	0.240	0.044	0.147	10.837	0.121	0.174	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	149	153	158	164	379	578	265	650	0
N.S.	1	0.94	0.97	1.00	1.04	2.40	3.66	1.68	4.11	0.00
time (sec)	N/A	0.898	1.048	0.214	0.046	0.312	9.222	0.151	0.177	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	195	186	205	212	440	683	444	703	0
N.S.	1	0.98	0.94	1.04	1.07	2.22	3.45	2.24	3.55	0.00
time (sec)	N/A	1.310	1.248	0.273	0.045	0.290	21.732	0.131	0.192	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	230	225	225	263	539	789	764	754	0
N.S.	1	0.97	0.95	0.95	1.11	2.28	3.34	3.24	3.19	0.00
time (sec)	N/A	1.769	1.776	0.286	0.040	0.292	14.490	0.142	0.184	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	281	299	357	739	0	311	32	0
N.S.	1	1.00	0.93	0.99	1.19	2.46	0.00	1.03	0.11	0.00
time (sec)	N/A	1.648	1.803	0.418	0.054	0.124	0.000	0.139	200.054	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	203	235	229	277	564	0	244	645	0
N.S.	1	0.84	0.98	0.95	1.15	2.34	0.00	1.01	2.68	0.00
time (sec)	N/A	1.027	1.047	0.285	0.045	0.110	0.000	0.126	0.431	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	168	165	178	227	538	0	215	29	0
N.S.	1	0.84	0.83	0.89	1.14	2.70	0.00	1.08	0.15	0.00
time (sec)	N/A	0.692	0.903	0.290	0.041	0.109	0.000	0.145	200.035	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	178	163	257	217	1003	0	0	535	0
N.S.	1	0.98	0.90	1.42	1.20	5.54	0.00	0.00	2.96	0.00
time (sec)	N/A	1.022	1.039	0.255	0.041	0.535	0.000	0.000	0.258	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	181	169	223	219	1034	0	233	990	0
N.S.	1	0.97	0.91	1.20	1.18	5.56	0.00	1.25	5.32	0.00
time (sec)	N/A	1.015	1.103	0.327	0.047	0.543	0.000	0.124	0.180	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	204	223	216	234	557	0	341	917	0
N.S.	1	0.99	1.08	1.04	1.13	2.69	0.00	1.65	4.43	0.00
time (sec)	N/A	1.126	1.517	0.278	0.040	0.293	0.000	0.151	0.177	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	252	225	262	284	586	0	586	1071	0
N.S.	1	0.99	0.89	1.03	1.12	2.31	0.00	2.31	4.22	0.00
time (sec)	N/A	1.832	1.396	0.316	0.041	0.285	0.000	0.137	0.183	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	312	305	323	366	764	0	1078	1210	0
N.S.	1	1.00	0.97	1.03	1.17	2.44	0.00	3.44	3.87	0.00
time (sec)	N/A	2.604	2.113	0.336	0.039	0.296	0.000	0.142	0.202	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	354	383	508	983	0	0	0	32	0
N.S.	1	1.14	1.23	1.63	3.16	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	2.990	3.133	0.340	0.353	0.000	0.000	0.000	200.033	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	269	275	472	804	0	0	0	32	0
N.S.	1	1.11	1.13	1.94	3.31	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	1.873	2.106	0.311	0.229	0.000	0.000	0.000	200.038	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	212	221	417	652	0	0	0	32	0
N.S.	1	1.08	1.13	2.13	3.33	0.00	0.00	0.00	0.16	0.00
time (sec)	N/A	1.189	1.447	0.290	0.174	0.000	0.000	0.000	200.023	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	168	177	457	544	0	0	0	30	0
N.S.	1	1.04	1.09	2.82	3.36	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.782	0.911	0.208	0.115	0.000	0.000	0.000	200.026	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	147	386	453	721	0	290	29	0
N.S.	1	1.00	1.07	2.80	3.28	5.22	0.00	2.10	0.21	0.00
time (sec)	N/A	0.562	0.841	0.194	0.095	0.863	0.000	0.144	200.022	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	264	181	401	0	1847	0	0	1103	0
N.S.	1	1.64	1.12	2.49	0.00	11.47	0.00	0.00	6.85	0.00
time (sec)	N/A	1.091	0.871	0.201	0.000	5.014	0.000	0.000	0.236	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	239	228	429	0	2398	0	389	32	0
N.S.	1	1.11	1.06	2.00	0.00	11.15	0.00	1.81	0.15	0.00
time (sec)	N/A	0.938	1.152	0.269	0.000	8.515	0.000	0.136	200.026	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	350	326	474	0	3317	0	535	2277	0
N.S.	1	1.19	1.11	1.62	0.00	11.32	0.00	1.83	7.77	0.00
time (sec)	N/A	1.137	2.428	0.305	0.000	21.320	0.000	0.138	0.232	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	440	446	751	1610	0	0	0	32	0
N.S.	1	1.20	1.21	2.04	4.38	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	3.953	3.854	0.448	0.316	0.000	0.000	0.000	200.026	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	359	543	737	1459	0	0	0	32	0
N.S.	1	1.18	1.79	2.43	4.82	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	2.651	11.879	0.349	0.233	0.000	0.000	0.000	200.030	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	316	304	993	1347	0	0	0	32	0
N.S.	1	1.14	1.09	3.57	4.85	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.832	2.183	0.275	0.205	0.000	0.000	0.000	200.026	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	233	255	919	1242	1571	0	0	2422	0
N.S.	1	1.01	1.10	3.98	5.38	6.80	0.00	0.00	10.48	0.00
time (sec)	N/A	1.065	2.180	0.236	0.168	1.559	0.000	0.000	35.345	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	239	246	872	1085	1583	0	0	2371	0
N.S.	1	1.01	1.04	3.69	4.60	6.71	0.00	0.00	10.05	0.00
time (sec)	N/A	0.919	1.984	0.216	0.143	1.628	0.000	0.000	15.838	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	515	320	913	0	4367	0	0	3375	0
N.S.	1	1.92	1.19	3.41	0.00	16.29	0.00	0.00	12.59	0.00
time (sec)	N/A	1.686	1.905	0.256	0.000	34.619	0.000	0.000	1.549	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	430	359	747	0	5706	0	0	32	0
N.S.	1	1.23	1.02	2.13	0.00	16.26	0.00	0.00	0.09	0.00
time (sec)	N/A	1.268	3.099	0.334	0.000	55.645	0.000	0.000	200.031	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	456	565	495	777	0	7645	0	0	32	0
N.S.	1	1.24	1.09	1.70	0.00	16.77	0.00	0.00	0.07	0.00
time (sec)	N/A	1.562	4.308	0.381	0.000	145.908	0.000	0.000	200.029	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	1057	1192	888	0	0	0	0	28	0
N.S.	1	2.12	2.39	1.78	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	4.637	27.332	5.919	0.000	0.000	0.000	0.000	200.036	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	0	1049	958	0	0	0	0	136	0
N.S.	1	0.00	2.29	2.09	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.000	26.662	6.069	0.000	0.000	0.000	0.000	104.868	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	834	982	2091	0	739	0	0	33	0
N.S.	1	1.00	1.18	2.52	0.00	0.89	0.00	0.00	0.04	0.00
time (sec)	N/A	4.068	32.625	9.378	0.000	0.098	0.000	0.000	200.015	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	700	694	830	1405	0	609	0	0	33	0
N.S.	1	0.99	1.19	2.01	0.00	0.87	0.00	0.00	0.05	0.00
time (sec)	N/A	2.755	29.972	7.000	0.000	0.087	0.000	0.000	200.017	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	577	560	726	1018	0	468	0	0	1048	0
N.S.	1	0.97	1.26	1.76	0.00	0.81	0.00	0.00	1.82	0.00
time (sec)	N/A	2.033	28.710	5.286	0.000	0.096	0.000	0.000	3.906	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	457	597	802	0	381	0	0	30	0
N.S.	1	0.98	1.28	1.71	0.00	0.81	0.00	0.00	0.06	0.00
time (sec)	N/A	1.377	27.064	3.238	0.000	0.083	0.000	0.000	200.015	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	792	750	903	0	0	0	0	33	0
N.S.	1	1.53	1.45	1.74	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.781	26.432	1.253	0.000	0.000	0.000	0.000	200.014	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	1365	994	867	0	0	0	0	33	0
N.S.	1	2.70	1.97	1.72	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	7.875	28.076	3.978	0.000	0.000	0.000	0.000	200.031	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	1470	1530	849	0	0	0	0	33	0
N.S.	1	2.76	2.88	1.60	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	9.427	28.755	3.053	0.000	0.000	0.000	0.000	200.018	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	617	1676	1895	999	0	0	0	0	33	0
N.S.	1	2.72	3.07	1.62	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	11.599	30.832	6.302	0.000	0.000	0.000	0.000	200.016	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	738	1907	2679	1132	0	0	0	0	33	0
N.S.	1	2.58	3.63	1.53	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	14.331	32.737	7.956	0.000	0.000	0.000	0.000	200.024	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	990	962	1335	2610	0	916	0	0	33	0
N.S.	1	0.97	1.35	2.64	0.00	0.93	0.00	0.00	0.03	0.00
time (sec)	N/A	3.616	33.820	9.048	0.000	0.104	0.000	0.000	200.018	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	831	795	980	2094	0	738	0	0	31	0
N.S.	1	0.96	1.18	2.52	0.00	0.89	0.00	0.00	0.04	0.00
time (sec)	N/A	2.597	32.543	6.653	0.000	0.090	0.000	0.000	200.020	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	662	831	1540	0	608	0	0	30	0
N.S.	1	0.96	1.20	2.23	0.00	0.88	0.00	0.00	0.04	0.00
time (sec)	N/A	1.943	30.357	6.655	0.000	0.085	0.000	0.000	200.017	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	709	0	1669	1318	0	0	0	0	33	0
N.S.	1	0.00	2.35	1.86	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	30.670	3.916	0.000	0.000	0.000	0.000	200.012	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	0	1658	1119	0	0	0	0	33	0
N.S.	1	0.00	2.49	1.68	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	29.912	8.476	0.000	0.000	0.000	0.000	200.017	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	0	1990	1047	0	0	0	0	33	0
N.S.	1	0.00	3.00	1.58	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	30.324	5.756	0.000	0.000	0.000	0.000	200.021	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	693	0	2192	1052	0	0	0	0	33	0
N.S.	1	0.00	3.16	1.52	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	30.897	6.189	0.000	0.000	0.000	0.000	200.022	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	757	0	2808	1135	0	0	0	0	33	0
N.S.	1	0.00	3.71	1.50	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	33.407	7.380	0.000	0.000	0.000	0.000	200.032	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	893	0	3014	1288	0	0	0	0	33	0
N.S.	1	0.00	3.38	1.44	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	33.782	10.721	0.000	0.000	0.000	0.000	200.024	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	620	738	974	0	467	0	0	1048	0
N.S.	1	1.04	1.24	1.64	0.00	0.79	0.00	0.00	1.76	0.00
time (sec)	N/A	3.668	28.565	6.402	0.000	0.091	0.000	0.000	5.592	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	509	615	778	0	381	0	0	33	0
N.S.	1	1.03	1.25	1.58	0.00	0.77	0.00	0.00	0.07	0.00
time (sec)	N/A	2.464	26.949	6.027	0.000	0.089	0.000	0.000	200.017	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	407	536	663	0	283	0	0	437	0
N.S.	1	1.02	1.35	1.67	0.00	0.71	0.00	0.00	1.10	0.00
time (sec)	N/A	1.533	25.615	3.625	0.000	0.091	0.000	0.000	2.371	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	338	454	588	0	237	0	0	30	0
N.S.	1	1.02	1.36	1.77	0.00	0.71	0.00	0.00	0.09	0.00
time (sec)	N/A	0.951	23.952	3.501	0.000	0.086	0.000	0.000	200.013	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	443	669	672	0	0	0	0	33	0
N.S.	1	1.14	1.72	1.73	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.622	24.664	2.990	0.000	0.000	0.000	0.000	200.014	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	491	870	645	0	0	0	0	33	0
N.S.	1	1.14	2.03	1.50	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.078	25.763	4.028	0.000	0.000	0.000	0.000	200.036	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	570	1392	903	0	0	0	0	33	0
N.S.	1	1.11	2.70	1.75	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.833	27.690	5.217	0.000	0.000	0.000	0.000	200.033	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	597	663	791	1101	0	878	0	0	33	0
N.S.	1	1.11	1.32	1.84	0.00	1.47	0.00	0.00	0.06	0.00
time (sec)	N/A	3.901	30.543	9.518	0.000	0.109	0.000	0.000	200.012	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	549	728	888	0	662	0	0	33	0
N.S.	1	1.08	1.44	1.75	0.00	1.31	0.00	0.00	0.07	0.00
time (sec)	N/A	2.602	27.242	7.652	0.000	0.089	0.000	0.000	200.016	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	459	611	763	0	537	0	0	31	0
N.S.	1	1.06	1.41	1.77	0.00	1.24	0.00	0.00	0.07	0.00
time (sec)	N/A	1.909	25.972	5.959	0.000	0.090	0.000	0.000	200.018	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	372	533	696	0	399	0	0	30	0
N.S.	1	1.01	1.45	1.90	0.00	1.09	0.00	0.00	0.08	0.00
time (sec)	N/A	1.099	25.690	3.658	0.000	0.084	0.000	0.000	200.020	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	706	435	912	0	0	0	0	33	0
N.S.	1	1.49	0.92	1.93	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.427	24.346	3.936	0.000	0.000	0.000	0.000	200.014	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	524	1131	1699	960	0	0	0	0	633	0
N.S.	1	2.16	3.24	1.83	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	5.568	30.434	6.609	0.000	0.000	0.000	0.000	52.975	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	1401	2647	1051	0	0	0	0	33	0
N.S.	1	2.28	4.30	1.71	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	8.576	32.463	7.412	0.000	0.000	0.000	0.000	200.020	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	733	921	1106	0	1336	0	0	33	0
N.S.	1	1.08	1.35	1.62	0.00	1.96	0.00	0.00	0.05	0.00
time (sec)	N/A	4.138	32.742	11.754	0.000	0.145	0.000	0.000	200.021	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	603	634	794	981	0	1125	0	0	33	0
N.S.	1	1.05	1.32	1.63	0.00	1.87	0.00	0.00	0.05	0.00
time (sec)	N/A	3.108	30.810	10.179	0.000	0.132	0.000	0.000	200.016	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	553	718	904	0	892	0	0	31	0
N.S.	1	1.03	1.33	1.68	0.00	1.66	0.00	0.00	0.06	0.00
time (sec)	N/A	2.048	29.033	5.375	0.000	0.115	0.000	0.000	200.018	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	509	632	854	0	779	0	0	30	0
N.S.	1	1.03	1.28	1.73	0.00	1.57	0.00	0.00	0.06	0.00
time (sec)	N/A	1.576	27.119	5.367	0.000	0.107	0.000	0.000	200.015	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	619	1037	1728	1102	0	0	0	0	281	0
N.S.	1	1.68	2.79	1.78	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	3.100	31.242	5.636	0.000	0.000	0.000	0.000	75.314	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	2117	2682	1145	0	0	0	0	33	0
N.S.	1	3.11	3.94	1.68	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	9.903	33.273	9.635	0.000	0.000	0.000	0.000	200.019	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	786	2545	3781	1263	0	0	0	0	33	0
N.S.	1	3.24	4.81	1.61	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	15.074	35.406	10.809	0.000	0.000	0.000	0.000	200.033	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	268	405	498	0	182	0	0	90	0
N.S.	1	0.99	1.50	1.84	0.00	0.67	0.00	0.00	0.33	0.00
time (sec)	N/A	0.671	22.838	2.845	0.000	0.092	0.000	0.000	22.089	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	615	700	818	1079	0	981	0	0	33	0
N.S.	1	1.14	1.33	1.75	0.00	1.60	0.00	0.00	0.05	0.00
time (sec)	N/A	4.455	30.209	10.734	0.000	0.110	0.000	0.000	200.028	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	579	729	908	0	736	0	0	33	0
N.S.	1	1.11	1.40	1.75	0.00	1.42	0.00	0.00	0.06	0.00
time (sec)	N/A	2.971	28.854	10.260	0.000	0.103	0.000	0.000	200.022	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	489	643	801	0	581	0	0	33	0
N.S.	1	1.09	1.43	1.78	0.00	1.29	0.00	0.00	0.07	0.00
time (sec)	N/A	2.131	27.123	8.476	0.000	0.099	0.000	0.000	200.045	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	401	567	750	0	441	0	0	31	0
N.S.	1	1.05	1.48	1.96	0.00	1.15	0.00	0.00	0.08	0.00
time (sec)	N/A	1.247	26.057	4.910	0.000	0.094	0.000	0.000	200.021	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	385	520	762	0	451	0	0	30	0
N.S.	1	1.06	1.43	2.10	0.00	1.24	0.00	0.00	0.08	0.00
time (sec)	N/A	1.088	26.638	4.448	0.000	0.093	0.000	0.000	200.018	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	0	1223	985	0	0	0	0	33	0
N.S.	1	0.00	2.57	2.07	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.000	27.477	5.144	0.000	0.000	0.000	0.000	200.024	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	0	1739	1045	0	0	0	0	33	0
N.S.	1	0.00	3.19	1.92	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	0.000	30.392	8.596	0.000	0.000	0.000	0.000	200.027	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	623	0	2670	1123	0	0	0	0	33	0
N.S.	1	0.00	4.29	1.80	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	32.356	8.641	0.000	0.000	0.000	0.000	200.014	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	713	777	935	1483	0	1822	0	0	33	0
N.S.	1	1.09	1.31	2.08	0.00	2.56	0.00	0.00	0.05	0.00
time (sec)	N/A	4.958	31.994	12.898	0.000	0.169	0.000	0.000	200.018	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	669	834	1287	0	1548	0	0	33	0
N.S.	1	1.07	1.33	2.06	0.00	2.48	0.00	0.00	0.05	0.00
time (sec)	N/A	3.509	29.947	12.392	0.000	0.125	0.000	0.000	200.020	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	586	733	1058	0	1220	0	0	33	0
N.S.	1	1.05	1.32	1.90	0.00	2.19	0.00	0.00	0.06	0.00
time (sec)	N/A	2.385	28.771	6.026	0.000	0.126	0.000	0.000	200.035	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	516	660	968	0	1067	0	0	31	0
N.S.	1	1.04	1.33	1.94	0.00	2.14	0.00	0.00	0.06	0.00
time (sec)	N/A	1.704	27.404	6.211	0.000	0.114	0.000	0.000	200.021	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	506	665	952	0	1006	0	0	1552	0
N.S.	1	1.04	1.37	1.95	0.00	2.07	0.00	0.00	3.19	0.00
time (sec)	N/A	1.650	27.020	6.133	0.000	0.112	0.000	0.000	22.653	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	611	0	1424	1281	0	0	0	0	33	0
N.S.	1	0.00	2.33	2.10	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	30.152	6.268	0.000	0.000	0.000	0.000	200.050	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	719	0	2824	1436	0	0	0	0	33	0
N.S.	1	0.00	3.93	2.00	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	33.775	11.002	0.000	0.000	0.000	0.000	200.029	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	803	0	3770	1562	0	0	0	0	33	0
N.S.	1	0.00	4.69	1.95	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	35.099	12.059	0.000	0.000	0.000	0.000	200.027	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	854	912	1232	1470	0	2719	0	0	33	0
N.S.	1	1.07	1.44	1.72	0.00	3.18	0.00	0.00	0.04	0.00
time (sec)	N/A	5.557	33.548	15.461	0.000	0.342	0.000	0.000	200.021	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	767	810	1167	1376	0	2290	0	0	33	0
N.S.	1	1.06	1.52	1.79	0.00	2.99	0.00	0.00	0.04	0.00
time (sec)	N/A	3.704	33.062	9.098	0.000	0.241	0.000	0.000	200.031	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	714	749	891	1358	0	2078	0	0	33	0
N.S.	1	1.05	1.25	1.90	0.00	2.91	0.00	0.00	0.05	0.00
time (sec)	N/A	2.810	31.210	7.773	0.000	0.257	0.000	0.000	200.019	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	671	866	1318	0	1759	0	0	31	0
N.S.	1	1.05	1.35	2.05	0.00	2.74	0.00	0.00	0.05	0.00
time (sec)	N/A	2.176	30.059	8.003	0.000	0.308	0.000	0.000	200.018	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	627	655	832	1295	0	1804	0	0	30	0
N.S.	1	1.04	1.33	2.07	0.00	2.88	0.00	0.00	0.05	0.00
time (sec)	N/A	2.052	30.013	7.873	0.000	0.208	0.000	0.000	200.024	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	0	2384	1552	0	0	0	0	33	0
N.S.	1	0.00	3.05	1.98	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	33.506	8.191	0.000	0.000	0.000	0.000	200.031	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	919	0	9484	1621	0	0	0	0	33	0
N.S.	1	0.00	10.32	1.76	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	35.830	13.644	0.000	0.000	0.000	0.000	200.034	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1012	0	12354	1732	0	0	0	0	33	0
N.S.	1	0.00	12.21	1.71	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	36.556	16.141	0.000	0.000	0.000	0.000	200.026	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	748	819	1155	1273	0	1621	0	0	33	0
N.S.	1	1.09	1.54	1.70	0.00	2.17	0.00	0.00	0.04	0.00
time (sec)	N/A	4.952	33.284	15.644	0.000	0.148	0.000	0.000	200.017	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	681	727	1061	1187	0	1357	0	0	33	0
N.S.	1	1.07	1.56	1.74	0.00	1.99	0.00	0.00	0.05	0.00
time (sec)	N/A	3.719	32.211	14.947	0.000	0.132	0.000	0.000	200.018	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	608	629	813	1110	0	1116	0	0	33	0
N.S.	1	1.03	1.34	1.83	0.00	1.84	0.00	0.00	0.05	0.00
time (sec)	N/A	2.500	30.586	7.510	0.000	0.124	0.000	0.000	200.029	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	617	821	1126	0	1093	0	0	33	0
N.S.	1	1.03	1.37	1.87	0.00	1.82	0.00	0.00	0.05	0.00
time (sec)	N/A	2.470	29.657	6.512	0.000	0.126	0.000	0.000	200.015	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	595	782	1069	0	1006	0	0	31	0
N.S.	1	1.04	1.37	1.87	0.00	1.76	0.00	0.00	0.05	0.00
time (sec)	N/A	1.821	29.236	6.520	0.000	0.116	0.000	0.000	200.025	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	620	802	1104	0	1079	0	0	30	0
N.S.	1	1.05	1.35	1.86	0.00	1.82	0.00	0.00	0.05	0.00
time (sec)	N/A	1.851	29.250	6.314	0.000	0.116	0.000	0.000	200.016	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	0	1968	1319	0	0	0	0	33	0
N.S.	1	0.00	2.87	1.93	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	32.397	7.661	0.000	0.000	0.000	0.000	200.022	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	783	0	2943	1412	0	0	0	0	33	0
N.S.	1	0.00	3.76	1.80	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	34.450	13.260	0.000	0.000	0.000	0.000	200.031	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	845	0	9327	1478	0	0	0	0	33	0
N.S.	1	0.00	11.04	1.75	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	35.743	14.023	0.000	0.000	0.000	0.000	200.019	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	800	834	949	1324	0	687	0	0	1354	0
N.S.	1	1.04	1.19	1.66	0.00	0.86	0.00	0.00	1.69	0.00
time (sec)	N/A	5.286	32.867	8.099	0.000	0.122	0.000	0.000	37.011	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	669	797	978	0	514	0	0	970	0
N.S.	1	1.04	1.24	1.52	0.00	0.80	0.00	0.00	1.51	0.00
time (sec)	N/A	3.460	30.314	6.132	0.000	0.101	0.000	0.000	5.135	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	517	532	645	780	0	404	0	0	723	0
N.S.	1	1.03	1.25	1.51	0.00	0.78	0.00	0.00	1.40	0.00
time (sec)	N/A	2.390	28.844	3.888	0.000	0.099	0.000	0.000	3.931	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	529	591	792	894	0	0	0	0	487	0
N.S.	1	1.12	1.50	1.69	0.00	0.00	0.00	0.00	0.92	0.00
time (sec)	N/A	2.372	25.430	3.137	0.000	0.000	0.000	0.000	75.107	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	592	784	871	0	0	0	0	43	0
N.S.	1	1.13	1.50	1.67	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.572	26.263	3.587	0.000	0.000	0.000	0.000	200.031	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	603	1598	911	0	0	0	0	43	0
N.S.	1	1.10	2.92	1.66	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	3.001	29.753	4.331	0.000	0.000	0.000	0.000	200.026	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	707	2328	1015	0	0	0	0	43	0
N.S.	1	1.10	3.63	1.58	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	4.051	32.431	6.056	0.000	0.000	0.000	0.000	200.032	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	782	856	3260	1171	0	0	0	0	43	0
N.S.	1	1.09	4.17	1.50	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	5.488	34.374	7.034	0.000	0.000	0.000	0.000	200.032	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	347	333	521	0	0	0	0	208	0
N.S.	1	1.05	1.01	1.58	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	2.405	24.512	3.073	0.000	0.000	0.000	0.000	2.835	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	271	199	413	0	0	0	0	85	0
N.S.	1	1.01	0.74	1.54	0.00	0.00	0.00	0.00	0.32	0.00
time (sec)	N/A	1.717	22.122	1.285	0.000	0.000	0.000	0.000	0.469	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	179	152	248	0	0	0	0	90	0
N.S.	1	1.01	0.86	1.40	0.00	0.00	0.00	0.00	0.51	0.00
time (sec)	N/A	1.121	22.902	1.299	0.000	0.000	0.000	0.000	2.152	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	277	221	375	0	0	0	0	222	0
N.S.	1	0.99	0.79	1.34	0.00	0.00	0.00	0.00	0.79	0.00
time (sec)	N/A	1.416	23.763	2.230	0.000	0.000	0.000	0.000	0.338	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	409	366	539	0	0	0	0	30	0
N.S.	1	1.15	1.03	1.51	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.686	25.175	2.831	0.000	0.000	0.000	0.000	200.019	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	626	707	336	546	0	0	0	0	197	0
N.S.	1	1.13	0.54	0.87	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	3.312	24.795	3.184	0.000	0.000	0.000	0.000	3.275	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	643	212	443	0	0	0	0	79	0
N.S.	1	1.14	0.38	0.79	0.00	0.00	0.00	0.00	0.14	0.00
time (sec)	N/A	2.352	22.160	1.240	0.000	0.000	0.000	0.000	0.736	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	453	158	267	0	0	0	0	84	0
N.S.	1	1.11	0.39	0.66	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	1.519	22.891	1.284	0.000	0.000	0.000	0.000	2.503	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	598	740	235	393	0	0	0	0	207	0
N.S.	1	1.24	0.39	0.66	0.00	0.00	0.00	0.00	0.35	0.00
time (sec)	N/A	2.287	23.568	2.188	0.000	0.000	0.000	0.000	0.536	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	682	922	384	563	0	0	0	0	29	0
N.S.	1	1.35	0.56	0.83	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.531	24.658	2.795	0.000	0.000	0.000	0.000	200.018	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1090	0	1004	1215	0	0	0	0	34	0
N.S.	1	0.00	0.92	1.11	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	24.036	8.520	0.000	0.000	0.000	0.000	200.016	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	139	95	37	33	40	54	76	169	36
N.S.	1	3.66	2.50	0.97	0.87	1.05	1.42	2.00	4.45	0.95
time (sec)	N/A	1.773	0.407	17.318	0.093	0.079	92.643	0.171	0.195	17.361

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	176	102	0	0	0	0	0	122	0
N.S.	1	0.71	0.41	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.879	6.071	0.000	0.000	0.000	0.000	0.000	0.487	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [171] had the largest ratio of [.7142859999999999976]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	0.96	37	0.054
2	A	3	3	0.96	37	0.081
3	A	14	13	1.67	30	0.433
4	A	11	10	1.49	30	0.333
5	A	10	9	1.27	28	0.321
6	A	7	6	1.13	27	0.222
7	A	11	10	1.00	30	0.333
8	A	12	11	0.99	30	0.367
9	A	14	13	0.90	30	0.433
10	A	13	12	0.98	30	0.400
11	A	13	12	0.97	30	0.400
12	A	10	9	0.94	30	0.300
13	A	11	10	0.98	30	0.333
14	A	14	13	0.98	30	0.433
15	A	16	15	1.01	30	0.500
16	F	0	0	N/A	0.000	N/A
17	F	0	0	N/A	0.000	N/A
18	A	10	9	1.17	30	0.300
19	A	9	8	1.02	29	0.276
20	A	12	11	1.05	32	0.344
21	A	13	12	1.02	32	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	15	14	1.04	32	0.438
23	A	16	15	0.95	32	0.469
24	A	15	14	1.20	32	0.438
25	A	15	14	1.02	32	0.438
26	A	12	11	0.91	32	0.344
27	A	14	13	0.96	32	0.406
28	A	17	16	0.96	32	0.500
29	A	19	18	0.99	32	0.562
30	A	14	13	1.07	32	0.406
31	A	12	11	1.07	30	0.367
32	A	10	9	1.06	29	0.310
33	A	14	13	1.36	32	0.406
34	A	2	2	1.44	32	0.062
35	A	2	2	1.35	32	0.062
36	A	2	2	1.02	32	0.062
37	A	2	2	1.11	32	0.062
38	A	16	15	1.23	32	0.469
39	A	12	11	1.27	32	0.344
40	A	11	10	1.29	30	0.333
41	A	11	10	1.38	29	0.345
42	A	13	12	1.25	32	0.375
43	A	2	2	1.49	32	0.062
44	A	2	2	1.64	32	0.062
45	A	2	2	1.43	32	0.062
46	A	2	2	1.42	32	0.062
47	A	14	13	1.55	30	0.433
48	A	12	11	1.35	30	0.367
49	A	11	10	1.14	28	0.357
50	A	8	7	1.03	27	0.259
51	A	13	12	0.95	30	0.400
52	A	13	12	0.98	30	0.400
53	A	15	14	0.95	30	0.467

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	15	14	0.91	30	0.467
55	A	16	15	0.91	30	0.500
56	A	15	14	0.94	30	0.467
57	A	15	14	0.92	30	0.467
58	A	11	10	0.88	30	0.333
59	A	12	11	0.92	30	0.367
60	A	14	13	0.94	30	0.433
61	A	15	14	1.35	32	0.438
62	F	0	0	N/A	0.000	N/A
63	A	12	11	1.07	30	0.367
64	A	10	9	0.89	29	0.310
65	A	14	13	1.00	32	0.406
66	A	15	14	0.99	32	0.438
67	A	16	15	1.06	32	0.469
68	A	17	16	0.97	32	0.500
69	A	17	16	1.12	32	0.500
70	A	18	17	0.95	32	0.531
71	A	17	16	1.12	32	0.500
72	A	17	16	0.97	32	0.500
73	A	13	12	0.81	32	0.375
74	A	15	14	0.90	32	0.438
75	A	17	16	0.87	32	0.500
76	A	16	15	1.04	32	0.469
77	A	14	13	0.99	30	0.433
78	A	12	11	0.98	29	0.379
79	A	17	16	1.31	32	0.500
80	A	2	2	1.33	32	0.062
81	A	2	2	1.54	32	0.062
82	A	2	2	1.90	32	0.062
83	A	2	2	1.99	32	0.062
84	A	2	2	1.89	32	0.062
85	A	2	2	1.79	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	14	13	1.10	32	0.406
87	A	13	12	1.11	30	0.400
88	A	13	12	1.16	29	0.414
89	A	16	15	1.50	32	0.469
90	A	2	2	1.48	32	0.062
91	A	2	2	1.66	32	0.062
92	A	2	2	1.71	32	0.062
93	A	2	2	1.76	32	0.062
94	A	13	12	1.86	30	0.400
95	A	10	9	1.69	30	0.300
96	A	9	8	1.42	28	0.286
97	A	6	5	1.27	27	0.185
98	A	10	9	1.04	30	0.300
99	A	11	10	1.09	30	0.333
100	A	11	10	1.03	30	0.333
101	A	9	8	1.03	30	0.267
102	A	10	9	1.06	30	0.300
103	A	13	12	1.07	30	0.400
104	A	15	14	1.07	30	0.467
105	A	12	11	1.43	32	0.344
106	A	10	9	1.32	30	0.300
107	A	8	7	1.18	29	0.241
108	A	11	10	1.09	32	0.312
109	A	12	11	1.09	32	0.344
110	A	13	12	1.10	32	0.375
111	A	13	12	1.06	32	0.375
112	A	11	10	1.03	32	0.312
113	A	13	12	1.05	32	0.375
114	A	16	15	1.05	32	0.469
115	A	13	12	1.13	32	0.375
116	A	12	11	1.11	32	0.344
117	A	10	9	1.07	30	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	8	7	1.04	29	0.241
119	A	9	8	1.36	32	0.250
120	A	2	2	1.00	32	0.062
121	A	2	2	1.10	32	0.062
122	A	2	2	1.12	32	0.062
123	A	2	2	1.21	32	0.062
124	A	14	13	1.25	32	0.406
125	A	10	9	1.21	32	0.281
126	A	10	9	1.20	30	0.300
127	A	8	7	1.11	29	0.241
128	A	7	6	1.71	32	0.188
129	A	2	2	1.22	32	0.062
130	A	2	2	1.24	32	0.062
131	A	2	2	1.24	32	0.062
132	A	10	9	1.00	30	0.300
133	A	9	8	0.99	30	0.267
134	A	7	6	0.91	28	0.214
135	A	6	5	0.93	27	0.185
136	A	10	9	0.94	30	0.300
137	A	7	6	0.93	30	0.200
138	A	9	8	0.94	30	0.267
139	A	11	10	0.98	30	0.333
140	A	12	11	0.97	30	0.367
141	A	9	8	1.00	32	0.250
142	A	8	7	0.84	30	0.233
143	A	6	5	0.84	29	0.172
144	A	10	9	0.98	32	0.281
145	A	12	11	0.97	32	0.344
146	A	9	8	0.99	32	0.250
147	A	12	11	0.99	32	0.344
148	A	13	12	1.00	32	0.375
149	A	12	11	1.14	32	0.344

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	10	9	1.11	32	0.281
151	A	10	9	1.08	32	0.281
152	A	9	8	1.04	30	0.267
153	A	6	5	1.00	29	0.172
154	A	8	7	1.64	32	0.219
155	A	2	2	1.11	32	0.062
156	A	2	2	1.19	32	0.062
157	A	12	11	1.20	32	0.344
158	A	11	10	1.18	32	0.312
159	A	11	10	1.14	32	0.312
160	A	7	6	1.01	30	0.200
161	A	6	5	1.01	29	0.172
162	A	9	8	1.92	32	0.250
163	A	2	2	1.23	32	0.062
164	A	2	2	1.24	32	0.062
165	B	20	19	2.12	30	0.633
166	F	0	0	N/A	0.000	N/A
167	A	18	17	1.00	35	0.486
168	A	16	15	0.99	35	0.429
169	A	14	13	0.97	33	0.394
170	A	12	11	0.98	32	0.344
171	A	26	25	1.53	35	0.714
172	B	19	18	2.70	35	0.514
173	B	21	20	2.76	35	0.571
174	B	22	21	2.72	35	0.600
175	B	23	22	2.58	35	0.629
176	A	18	17	0.97	35	0.486
177	A	16	15	0.96	33	0.455
178	A	14	13	0.96	32	0.406
179	F	0	0	N/A	0.000	N/A
180	F	0	0	N/A	0.000	N/A
181	F	0	0	N/A	0.000	N/A

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	F	0	0	N/A	0.000	N/A
183	F	0	0	N/A	0.000	N/A
184	F	0	0	N/A	0.000	N/A
185	A	16	15	1.04	35	0.429
186	A	14	13	1.03	35	0.371
187	A	12	11	1.02	33	0.333
188	A	10	9	1.02	32	0.281
189	A	14	13	1.14	35	0.371
190	A	16	15	1.14	35	0.429
191	A	18	17	1.11	35	0.486
192	A	16	15	1.11	35	0.429
193	A	14	13	1.08	35	0.371
194	A	13	12	1.06	33	0.364
195	A	10	9	1.01	32	0.281
196	A	24	23	1.49	35	0.657
197	B	19	18	2.16	35	0.514
198	B	20	19	2.28	35	0.543
199	A	16	15	1.08	35	0.429
200	A	15	14	1.05	35	0.400
201	A	13	12	1.03	33	0.364
202	A	12	11	1.03	32	0.344
203	A	19	18	1.68	35	0.514
204	B	25	24	3.11	35	0.686
205	B	24	23	3.24	35	0.657
206	A	9	8	0.99	34	0.235
207	A	16	15	1.14	35	0.429
208	A	14	13	1.11	35	0.371
209	A	12	11	1.09	35	0.314
210	A	10	9	1.05	33	0.273
211	A	10	9	1.06	32	0.281
212	F	0	0	N/A	0.000	N/A
213	F	0	0	N/A	0.000	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	F	0	0	N/A	0.000	N/A
215	A	16	15	1.09	35	0.429
216	A	14	13	1.07	35	0.371
217	A	12	11	1.05	35	0.314
218	A	12	11	1.04	33	0.333
219	A	12	11	1.04	32	0.344
220	F	0	0	N/A	0.000	N/A
221	F	0	0	N/A	0.000	N/A
222	F	0	0	N/A	0.000	N/A
223	A	16	15	1.07	35	0.429
224	A	14	13	1.06	35	0.371
225	A	14	13	1.05	35	0.371
226	A	14	13	1.05	33	0.394
227	A	14	13	1.04	32	0.406
228	F	0	0	N/A	0.000	N/A
229	F	0	0	N/A	0.000	N/A
230	F	0	0	N/A	0.000	N/A
231	A	16	15	1.09	35	0.429
232	A	14	13	1.07	35	0.371
233	A	12	11	1.03	35	0.314
234	A	12	11	1.03	35	0.314
235	A	12	11	1.04	33	0.333
236	A	12	11	1.05	32	0.344
237	F	0	0	N/A	0.000	N/A
238	F	0	0	N/A	0.000	N/A
239	F	0	0	N/A	0.000	N/A
240	A	18	17	1.04	45	0.378
241	A	16	15	1.04	43	0.349
242	A	14	13	1.03	42	0.310
243	A	18	17	1.12	45	0.378
244	A	18	17	1.13	45	0.378

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	18	17	1.10	45	0.378
246	A	20	19	1.10	45	0.422
247	A	22	21	1.09	45	0.467
248	A	16	15	1.05	32	0.469
249	A	12	11	1.01	32	0.344
250	A	7	6	1.01	32	0.188
251	A	4	3	0.99	32	0.094
252	A	4	3	1.15	32	0.094
253	A	11	10	1.13	31	0.323
254	A	9	8	1.14	31	0.258
255	A	6	5	1.11	31	0.161
256	A	4	3	1.24	31	0.097
257	A	4	3	1.35	31	0.097
258	F	0	0	N/A	0.000	N/A
259	C	2	2	3.66	43	0.047
260	A	5	5	0.71	36	0.139

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}(a+bx^2)} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 230

$$\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)} \, dx = -\frac{\left(\sqrt{b}BC-\sqrt{-a}(A+BD)\right)\arctan\left(\frac{\sqrt{\sqrt{b}c-\sqrt{-a}d}\sqrt{ex}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{b}\sqrt{\sqrt{b}c-\sqrt{-a}d}\sqrt{e}} - \frac{\left(\sqrt{b}BC+\sqrt{-a}(A+BD)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{ex}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{b}\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{e}}$$

output
```
-(b^(1/2)*B*C-(-a)^(1/2)*(B*D+A))*arctan((b^(1/2)*c-(-a)^(1/2)*d)^(1/2)*(e
*x)^(1/2)/(-a)^(1/4)/e^(1/2)/(d*x+c)^(1/2))/(-a)^(3/4)/b^(1/2)/(b^(1/2)*c-
(-a)^(1/2)*d)^(1/2)/e^(1/2)-(b^(1/2)*B*C+(-a)^(1/2)*(B*D+A))*arctanh((b^(1
/2)*c+(-a)^(1/2)*d)^(1/2)*(e*x)^(1/2)/(-a)^(1/4)/e^(1/2)/(d*x+c)^(1/2))/(-
a)^(3/4)/b^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)/e^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.92 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.13

$$\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx$$
$$= \frac{\sqrt{x}\text{RootSum}\left[ad^4-4ad^3\#1^2+16bc^2\#1^4+6ad^2\#1^4-4ad\#1^6+a\#1^8\&,\ \frac{BCd^3\log(x)-2BCd^3\log\left(-\sqrt{c}+\sqrt{c+d\right.}}{}\right.}{}$$

input
```
Integrate[(B*C + (A + B*D)*x)/(Sqrt[e*x]*Sqrt[c + d*x]*(a + b*x^2)),x]
```

output
```
(Sqrt[x]*RootSum[a*d^4 - 4*a*d^3*#1^2 + 16*b*c^2*#1^4 + 6*a*d^2*#1^4 - 4*a
*d*#1^6 + a*#1^8 & , (B*C*d^3*Log[x] - 2*B*C*d^3*Log[-Sqrt[c] + Sqrt[c + d
*x] - Sqrt[x]*#1] + 4*A*c*d*Log[x]*#1^2 - 3*B*C*d^2*Log[x]*#1^2 + 4*B*c*d*
D*Log[x]*#1^2 - 8*A*c*d*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 +
6*B*C*d^2*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 - 8*B*c*d*D*Log[
-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 - 4*A*c*Log[x]*#1^4 + 3*B*C*d*
Log[x]*#1^4 - 4*B*c*D*Log[x]*#1^4 + 8*A*c*Log[-Sqrt[c] + Sqrt[c + d*x] - S
qrt[x]*#1]*#1^4 - 6*B*C*d*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^4
+ 8*B*c*D*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^4 - B*C*Log[x]*#1^
6 + 2*B*C*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^6)/(a*d^3*#1 - 8*b
*c^2*#1^3 - 3*a*d^2*#1^3 + 3*a*d*#1^5 - a*#1^7) & ])/(4*Sqrt[e*x])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+BD)+BC}{\sqrt{ex}\,(a+bx^2)\,\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 2353$$

$$\int \left(\frac{\sqrt{-a}BC-\frac{a(A+BD)}{\sqrt{b}}}{2a\sqrt{ex}\left(\sqrt{-a}-\sqrt{b}x\right)\sqrt{c+dx}}+\frac{\frac{a(A+BD)}{\sqrt{b}}+\sqrt{-a}BC}{2a\sqrt{ex}\left(\sqrt{-a}+\sqrt{b}x\right)\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\left(BC-\frac{\sqrt{-a}(A+BD)}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt{ex}\sqrt{\sqrt{b}c-\sqrt{-a}d}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{e}\sqrt{\sqrt{b}c-\sqrt{-a}d}}-\frac{\left(\frac{\sqrt{-a}(A+BD)}{\sqrt{b}}+BC\right)\text{arctanh}\left(\frac{\sqrt{ex}\sqrt{\sqrt{-a}d+\sqrt{b}c}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{e}\sqrt{\sqrt{-a}d+\sqrt{b}c}}$$

input
```
Int[(B*C + (A + B*D)*x)/(Sqrt[e*x]*Sqrt[c + d*x]*(a + b*x^2)),x]
```

output
```
-(((B*C - (Sqrt[-a]*(A + B*D))/Sqrt[b])*ArcTan[(Sqrt[Sqrt[b]*c - Sqrt[-a]*
d]*Sqrt[e*x])/((-a)^(1/4)*Sqrt[e]*Sqrt[c + d*x])])/((-a)^(3/4)*Sqrt[Sqrt[b
]*c - Sqrt[-a]*d]*Sqrt[e])) - ((B*C + (Sqrt[-a]*(A + B*D))/Sqrt[b])*ArcTan
h[(Sqrt[Sqrt[b]*c + Sqrt[-a]*d]*Sqrt[e*x])/((-a)^(1/4)*Sqrt[e]*Sqrt[c + d*
x])])/((-a)^(3/4)*Sqrt[Sqrt[b]*c + Sqrt[-a]*d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(170) = 340$.

Time = 0.29 (sec) , antiderivative size = 1352, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	1352

input
```
int((B*C+(B*D+A)*x)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(d*x+c)^(1/2)*x*e*(-B*C*ln((2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-a*
b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*b)
^(1/2)))*a*d^2*(-a*b)^(1/2)*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)-B*C*ln((2*(-
a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^
(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*b)^(1/2)))*b*c^2*(-a*b)^(1/2)*(-e*(c*(-
a*b)^(1/2)+d*a)/b)^(1/2)+B*C*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-
a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*
b)^(1/2)))*a*d^2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(-a*b)^(1/2)+B*C*ln((-2*
(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c
))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*b)^(1/2)))*b*c^2*(e*(c*(-a*b)^(1/2)-
d*a)/b)^(1/2)*(-a*b)^(1/2)+B*D*ln((2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-
a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*
b)^(1/2)))*a^2*d^2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)+B*D*ln((2*(-a*b)^(1/2
)*d*e*x+b*c*e*x+2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c
*e*(-a*b)^(1/2))/(b*x-(-a*b)^(1/2)))*a*b*c^2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(
1/2)+B*D*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(
1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*b)^(1/2)))*a^2*d^2*(
e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)+B*D*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-
e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b
*x+(-a*b)^(1/2)))*a*b*c^2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)+A*ln((2*(-a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4961 vs. $2(170) = 340$.

Time = 0.51 (sec) , antiderivative size = 4961, normalized size of antiderivative = 21.57

$$\int \frac{BC + (A + BD)x}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*C+(B*D+A)*x)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{BC + (A + BD)x}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \int \frac{Ax + BC + BDx}{\sqrt{ex}\,(a + bx^2)\,\sqrt{c + dx}}\,dx$$

input
```
integrate((B*C+(B*D+A)*x)/(e*x)**(1/2)/(d*x+c)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((A*x + B*C + B*D*x)/(sqrt(e*x)*(a + b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{BC + (A + BD)x}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \int \frac{BC + (BD + A)x}{(bx^2 + a)\sqrt{dx + c}\sqrt{ex}}\,dx$$

input
```
integrate((B*C+(B*D+A)*x)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="maxima")
```

output
```
integrate((B*C + (B*D + A)*x)/((b*x^2 + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*C+(B*D+A)*x)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx = \int \frac{x\,(A+B\,D)+B\,C}{\sqrt{e\,x}\,\left(b\,x^2+a\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(A + B*D) + B*C)/((e*x)^(1/2)*(a + b*x^2)*(c + d*x)^(1/2)),x)
```

output
```
int((x*(A + B*D) + B*C)/((e*x)^(1/2)*(a + b*x^2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{BC+(A+BD)x}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx = \frac{\left(\int \frac{x}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)a+\left(\int \frac{x}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)bd+\left(\int \frac{1}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)bc}{\sqrt{e}}$$

input
```
int((B*C+(B*D+A)*x)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x)
```

output

```
(int(x/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*a + int
(x/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*b*d + int(1
/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*b*c)/sqrt(e)
```

3.2 $\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}(a+bx^2)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 230

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx = -\frac{\left(\sqrt{b}BC-\sqrt{-a}(A+BD)\right)\arctan\left(\frac{\sqrt{\sqrt{b}c-\sqrt{-a}d}\sqrt{ex}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{b}\sqrt{\sqrt{b}c-\sqrt{-a}d}\sqrt{e}} - \frac{\left(\sqrt{b}BC+\sqrt{-a}(A+BD)\right)\operatorname{arctanh}\left(\frac{\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{ex}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{b}\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{e}}$$

output
```
-(b^(1/2)*B*C-(-a)^(1/2)*(B*D+A))*arctan((b^(1/2)*c-(-a)^(1/2)*d)^(1/2)*(e
*x)^(1/2)/(-a)^(1/4)/e^(1/2)/(d*x+c)^(1/2))/(-a)^(3/4)/b^(1/2)/(b^(1/2)*c-
(-a)^(1/2)*d)^(1/2)/e^(1/2)-(b^(1/2)*B*C+(-a)^(1/2)*(B*D+A))*arctanh((b^(1
/2)*c+(-a)^(1/2)*d)^(1/2)*(e*x)^(1/2)/(-a)^(1/4)/e^(1/2)/(d*x+c)^(1/2))/(-
a)^(3/4)/b^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)/e^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.13

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx$$
$$= \frac{\sqrt{x}\text{RootSum}\Big[ad^4-4ad^3\#1^2+16bc^2\#1^4+6ad^2\#1^4-4ad\#1^6+a\#1^8\&,\frac{BCd^3\log(x)-2BCd^3\log\left(-\sqrt{c}+\sqrt{c+d}\right.}{}}{}$$

input
```
Integrate[(A*x + B*(C + D*x))/(Sqrt[e*x]*Sqrt[c + d*x]*(a + b*x^2)),x]
```

output
```
(Sqrt[x]*RootSum[a*d^4 - 4*a*d^3*#1^2 + 16*b*c^2*#1^4 + 6*a*d^2*#1^4 - 4*a
*d*#1^6 + a*#1^8 & , (B*C*d^3*Log[x] - 2*B*C*d^3*Log[-Sqrt[c] + Sqrt[c + d
*x] - Sqrt[x]*#1] + 4*A*c*d*Log[x]*#1^2 - 3*B*C*d^2*Log[x]*#1^2 + 4*B*c*d*
D*Log[x]*#1^2 - 8*A*c*d*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 +
6*B*C*d^2*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 - 8*B*c*d*D*Log[
-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^2 - 4*A*c*Log[x]*#1^4 + 3*B*C*d*
Log[x]*#1^4 - 4*B*c*D*Log[x]*#1^4 + 8*A*c*Log[-Sqrt[c] + Sqrt[c + d*x] - S
qrt[x]*#1]*#1^4 - 6*B*C*d*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^4
+ 8*B*c*D*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^4 - B*C*Log[x]*#1^
6 + 2*B*C*Log[-Sqrt[c] + Sqrt[c + d*x] - Sqrt[x]*#1]*#1^6)/(a*d^3*#1 - 8*b
*c^2*#1^3 - 3*a*d^2*#1^3 + 3*a*d*#1^5 - a*#1^7) & ])/(4*Sqrt[e*x])
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2092, 2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\,(a+bx^2)\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2092}$$

$$\int \frac{x(A+BD)+BC}{\sqrt{ex}\,(a+bx^2)\,\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{\sqrt{-a}BC-\frac{a(A+BD)}{\sqrt{b}}}{2a\sqrt{ex}\left(\sqrt{-a}-\sqrt{b}x\right)\sqrt{c+dx}}+\frac{\frac{a(A+BD)}{\sqrt{b}}+\sqrt{-a}BC}{2a\sqrt{ex}\left(\sqrt{-a}+\sqrt{b}x\right)\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(BC-\frac{\sqrt{-a}(A+BD)}{\sqrt{b}}\right)\arctan\left(\frac{\sqrt{ex}\sqrt{\sqrt{b}c-\sqrt{-a}d}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{e}\sqrt{\sqrt{b}c-\sqrt{-a}d}}-\frac{\left(\frac{\sqrt{-a}(A+BD)}{\sqrt{b}}+BC\right)\operatorname{arctanh}\left(\frac{\sqrt{ex}\sqrt{\sqrt{-a}d+\sqrt{b}c}}{\sqrt[4]{-a}\sqrt{e}\sqrt{c+dx}}\right)}{(-a)^{3/4}\sqrt{e}\sqrt{\sqrt{-a}d+\sqrt{b}c}}$$

input
```
Int[(A*x + B*(C + D*x))/(Sqrt[e*x]*Sqrt[c + d*x]*(a + b*x^2)),x]
```

output
```
-(((B*C - (Sqrt[-a]*(A + B*D))/Sqrt[b])*ArcTan[(Sqrt[Sqrt[b]*c - Sqrt[-a]*
d]*Sqrt[e*x])/((-a)^(1/4)*Sqrt[e]*Sqrt[c + d*x])])/((-a)^(3/4)*Sqrt[Sqrt[b
]*c - Sqrt[-a]*d]*Sqrt[e])) - ((B*C + (Sqrt[-a]*(A + B*D))/Sqrt[b])*ArcTan
h[(Sqrt[Sqrt[b]*c + Sqrt[-a]*d]*Sqrt[e*x])/((-a)^(1/4)*Sqrt[e]*Sqrt[c + d*
x])])/((-a)^(3/4)*Sqrt[Sqrt[b]*c + Sqrt[-a]*d]*Sqrt[e])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2092
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[z, x] && BinomialQ[u
, x] && !(BinomialMatchQ[z, x] && BinomialMatchQ[u, x])
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1351 vs. $2(170) = 340$.

Time = 0.11 (sec) , antiderivative size = 1352, normalized size of antiderivative = 5.88

method	result	size
default	Expression too large to display	1352

input
```
int((A*x+B*(D*x+C))/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*(d*x+c)^(1/2)*x*e*(-B*C*ln((2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-a*
b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*b)
^(1/2)))*a*d^2*(-a*b)^(1/2)*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)-B*C*ln((2*(-
a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^
(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*b)^(1/2)))*b*c^2*(-a*b)^(1/2)*(-e*(c*(-
a*b)^(1/2)+d*a)/b)^(1/2)+B*C*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-
a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*
b)^(1/2)))*a*d^2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(-a*b)^(1/2)+B*C*ln((-2*
(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c
))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*b)^(1/2)))*b*c^2*(e*(c*(-a*b)^(1/2)-
d*a)/b)^(1/2)*(-a*b)^(1/2)+B*D*ln((2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(e*(c*(-
a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c*e*(-a*b)^(1/2))/(b*x-(-a*
b)^(1/2)))*a^2*d^2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)+B*D*ln((2*(-a*b)^(1/2
)*d*e*x+b*c*e*x+2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b+c
*e*(-a*b)^(1/2))/(b*x-(-a*b)^(1/2)))*a*b*c^2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(
1/2)+B*D*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-e*(c*(-a*b)^(1/2)+d*a)/b)^(
1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b*x+(-a*b)^(1/2)))*a^2*d^2*(
e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)+B*D*ln((-2*(-a*b)^(1/2)*d*e*x+b*c*e*x+2*(-
e*(c*(-a*b)^(1/2)+d*a)/b)^(1/2)*(x*e*(d*x+c))^(1/2)*b-c*e*(-a*b)^(1/2))/(b
*x+(-a*b)^(1/2)))*a*b*c^2*(e*(c*(-a*b)^(1/2)-d*a)/b)^(1/2)+A*ln((2*(-a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4961 vs. $2(170) = 340$.

Time = 0.52 (sec) , antiderivative size = 4961, normalized size of antiderivative = 21.57

$$\int \frac{Ax + B(C + Dx)}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((A*x+B*(D*x+C))/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{Ax + B(C + Dx)}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \int \frac{Ax + BC + BDx}{\sqrt{ex}\,(a + bx^2)\,\sqrt{c + dx}}\,dx$$

input
```
integrate((A*x+B*(D*x+C))/(e*x)**(1/2)/(d*x+c)**(1/2)/(b*x**2+a),x)
```

output
```
Integral((A*x + B*C + B*D*x)/(sqrt(e*x)*(a + b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{Ax + B(C + Dx)}{\sqrt{ex}\sqrt{c + dx}\,(a + bx^2)}\,dx = \int \frac{(Dx + C)B + Ax}{(bx^2 + a)\sqrt{dx + c}\sqrt{ex}}\,dx$$

input
```
integrate((A*x+B*(D*x+C))/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="maxima")
```

output
```
integrate(((D*x + C)*B + A*x)/((b*x^2 + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx=\text{Timed out}$$

input
```
integrate((A*x+B*(D*x+C))/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx=\int \frac{A\,x+B\,(C+x\,D)}{\sqrt{e\,x}\,\left(b\,x^2+a\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A*x + B*(C + x*D))/((e*x)^(1/2)*(a + b*x^2)*(c + d*x)^(1/2)),x)
```

output
```
int((A*x + B*(C + x*D))/((e*x)^(1/2)*(a + b*x^2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{Ax+B(C+Dx)}{\sqrt{ex}\sqrt{c+dx}\,(a+bx^2)}\,dx$$
$$=\frac{\left(\int \frac{x}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)a+\left(\int \frac{x}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)bd+\left(\int \frac{1}{\sqrt{x}\,\sqrt{dx+c}\,a+\sqrt{x}\,\sqrt{dx+c}\,b\,x^2}dx\right)bc}{\sqrt{e}}$$

input
```
int((A*x+B*(D*x+C))/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a),x)
```

output
```
(int(x/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*a + int
(x/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*b*d + int(1
/(sqrt(x)*sqrt(c + d*x)*a + sqrt(x)*sqrt(c + d*x)*b*x**2),x)*b*c)/sqrt(e)
```

3.3 $\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 276

$$\begin{aligned}\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx&=\frac{a^2(5aCd-8b(Bc+Ad))x\sqrt{a+bx^2}}{128b^3}-\frac{a(5aCd-8b(Bc+Ad))x^3\sqrt{a+bx^2}}{192b^2}\\&-\frac{(5aCd-8b(Bc+Ad))x^5\sqrt{a+bx^2}}{48b}-\frac{a(Abc-acC-aBd)\left(a+bx^2\right)^{3/2}}{3b^3}\\&+\frac{Cdx^5\left(a+bx^2\right)^{3/2}}{8b}+\frac{(Abc-2a(cC+Bd))\left(a+bx^2\right)^{5/2}}{5b^3}\\&+\frac{(cC+Bd)\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{a^3(5aCd-8b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
1/128*a^2*(5*a*C*d-8*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b^3-1/192*a*(5*a*C*d-8
*b*(A*d+B*c))*x^3*(b*x^2+a)^(1/2)/b^2-1/48*(5*a*C*d-8*b*(A*d+B*c))*x^5*(b*
x^2+a)^(1/2)/b-1/3*a*(A*b*c-B*a*d-C*a*c)*(b*x^2+a)^(3/2)/b^3+1/8*C*d*x^5*(
b*x^2+a)^(3/2)/b+1/5*(A*b*c-2*a*(B*d+C*c))*(b*x^2+a)^(5/2)/b^3+1/7*(B*d+C*
c)*(b*x^2+a)^(7/2)/b^3-1/128*a^3*(5*a*C*d-8*b*(A*d+B*c))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.80

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(a^3(1024cC+1024Bd+525Cdx)+8ab^2x^2(14A(8c+5dx)+x(70Bc+48cCx+48Bdx+\ldots}{}$$

input
```
Integrate[x^3*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^3*(1024*c*C + 1024*B*d + 525*C*d*x) + 8*a*b^2*
x^2*(14*A*(8*c + 5*d*x) + x*(70*B*c + 48*c*C*x + 48*B*d*x + 35*C*d*x^2)) -
 2*a^2*b*(28*A*(32*c + 15*d*x) + x*(420*B*c + 256*c*C*x + 256*B*d*x + 175*
C*d*x^2)) + 16*b^3*x^4*(28*A*(6*c + 5*d*x) + 5*x*(4*B*(7*c + 6*d*x) + 3*C*
x*(8*c + 7*d*x)))) + 105*a^3*(5*a*C*d - 8*b*(B*c + A*d))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/(13440*b^(7/2))
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.67, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2185, 25, 2185, 25, 2185, 27, 2185, 25, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\left((c+dx)\sqrt{bx^2+a}\left(bd^4(27cC-8Bd)x^4+d^3\left(33bCc^2-8Abd^2+5aCd^2\right)x^3+cCd^2\left(17bc^2+15ad^2\right)x^2+3c\ldots\right.\right.}{8bd^5}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\int (c+dx)\sqrt{bx^2+a}\left(bd^4(27cC-8Bd)x^4+d^3\left(33bCc^2-8Abd^2+5aCd^2\right)x^3+cCd^2\left(17bc^2+15ad^2\right)x^2+3c^2Ca\ldots}{8bd^5}$$

$\big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{\int -\left((c+dx)\sqrt{bx^2+a}\left(-b\left(35aCd^2-4b\left(57Cc^2-34Bdc+14Ad^2\right)\right)x^3d^7+b\left(8b(29cC-13Bd)c^2+ad^2(3cC-32Bd)\right)x^2d^6+abc^2(73cC-32Bd)d^6+bc\left(\ldots\right.\right.\right.}{7bd^4}}{8bd^5}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd)-\frac{\int (c+dx)\sqrt{bx^2+a}\left(-b\left(35aCd^2-4b\left(57Cc^2-34Bdc+14Ad^2\right)\right)x^3d^7+b\left(8b(29cC-13Bd)c^2+\ldots\right.\right.}{\ldots}}{8bd^5}$$

$\big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd)-\frac{\frac{\int 3(c+dx)\sqrt{bx^2+a}\left(b^2\left(ad^2(111cC-64Bd)-4bc\left(55Cc^2-50Bdc+42Ad^2\right)\right)x^2d^9+abc\left(35aCd^2-b\left(82C\ldots\right.\right.\right.}{\ldots}}{\ldots}}{\ldots}$$

$\big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd)-\frac{\frac{\int (c+dx)\sqrt{bx^2+a}\left(b^2\left(ad^2(111cC-64Bd)-4bc\left(55Cc^2-50Bdc+42Ad^2\right)\right)x^2d^9+abc\left(35aCd^2-b\left(82C\ldots\right.\right.\right.}{\ldots}}{\ldots}}{\ldots}$$

$\big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd)-\frac{\frac{\frac{\int -b^2d^{10}(c+dx)\left(ad\left(ad^2(47cC-128Bd)-2bc\left(15Cc^2-20Bdc+28Ad^2\right)\right)-\left(175a^2Cd^4-4ab\left(47Cc^2-\ldots\right.\right.\right.}{5bd^2}}{\ldots}}{\ldots}}{\ldots}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4} - \frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd) - \frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right) - \frac{\int b^2d^{10}(c+dx)\left(ad\left(\ldots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4} - \frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd) - \frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right) - \frac{1}{5}bd^8\int(c+dx)\left(ad\left(c\ldots\right.\right.}{\cdots}}{\cdots}$$

$\Big\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4} - \frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd) - \frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right) - \frac{1}{5}bd^8\left(\frac{35a^2d^4(5aC\ldots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\Big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4} - \frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd) - \frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right) - \frac{1}{5}bd^8\left(\frac{35a^2d^4(5aC\ldots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\Big\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4} - \frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd) - \frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right) - \frac{1}{5}bd^8\left(\frac{35a^2d^4(5aC\ldots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\Big\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^4}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(27cC-8Bd)-\frac{\frac{1}{5}bd^8\left(a+bx^2\right)^{3/2}(c+dx)^2\left(ad^2(111cC-64Bd)-4bc\left(42Ad^2-50Bcd+55c^2C\right)\right)-\frac{1}{5}bd^8\left(\frac{35a^2d^4\left(\frac{a\ldots}{\ldots}\right)}{\ldots}\right)}{\ldots}}{\ldots}$$

input
```
Int[x^3*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^5*(a + b*x^2)^(3/2))/(8*b*d^4) - ((d*(27*c*C - 8*B*d)*(c + d*
x)^4*(a + b*x^2)^(3/2))/7 - ((d^5*(228*b*c^2*C - 136*b*B*c*d + 56*A*b*d^2
- 35*a*C*d^2)*(c + d*x)^3*(a + b*x^2)^(3/2))/6 + ((b*d^8*(a*d^2*(111*c*C -
 64*B*d) - 4*b*c*(55*c^2*C - 50*B*c*d + 42*A*d^2))*(c + d*x)^2*(a + b*x^2)
^(3/2))/5 - (b*d^8*((-2*(64*a^2*d^4*(c*C + B*d) + 4*b^2*c^3*(15*c^2*C - 20
*B*c*d + 28*A*d^2) - a*b*c*d^2*(79*c^2*C - 96*B*c*d + 112*A*d^2))*(a + b*x
^2)^(3/2))/(3*b) - (d*(175*a^2*C*d^4 + 8*b^2*c^2*(15*c^2*C - 20*B*c*d + 28
*A*d^2) - 4*a*b*d^2*(47*c^2*C - 58*B*c*d + 70*A*d^2))*x*(a + b*x^2)^(3/2))
/(4*b) + (35*a^2*d^4*(5*a*C*d - 8*b*(B*c + A*d))*((x*Sqrt[a + b*x^2])/2 +
(a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b)))/5)/(2*b*d^3
))/(7*b*d^4))/(8*b*d^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{(-1680dCb^3x^7-1920Bb^3dx^6-1920Cb^3cx^6-2240Ab^3dx^5-2240Bb^3cx^5-280Cab^2dx^5-2688Ab^3cx^4-384Bab^2dx^4-384C}{}$
default	$(Ad+Bc)\left(\frac{x^3(bx^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+(Bd+Cc)\left(\frac{x^4(bx^2+a)}{7b}\right.$

input
```
int(x^3*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/13440/b^3*(-1680*C*b^3*d*x^7-1920*B*b^3*d*x^6-1920*C*b^3*c*x^6-2240*A*b
^3*d*x^5-2240*B*b^3*c*x^5-280*C*a*b^2*d*x^5-2688*A*b^3*c*x^4-384*B*a*b^2*d
*x^4-384*C*a*b^2*c*x^4-560*A*a*b^2*d*x^3-560*B*a*b^2*c*x^3+350*C*a^2*b*d*x
^3-896*A*a*b^2*c*x^2+512*B*a^2*b*d*x^2+512*C*a^2*b*c*x^2+840*A*a^2*b*d*x+8
40*B*a^2*b*c*x-525*C*a^3*d*x+1792*A*a^2*b*c-1024*B*a^3*d-1024*C*a^3*c)*(b*
x^2+a)^(1/2)+1/128*a^3*(8*A*b*d+8*B*b*c-5*C*a*d)/b^(7/2)*ln(b^(1/2)*x+(b*x
^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.08

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$=\left[\frac{105\,(8\,Ba^3bc-(5\,Ca^4-8\,Aa^3b)d)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(1680\,Cb^4dx^7+1920\,(C}{}\right.$$

$$-\frac{105\,(8\,Ba^3bc-(5\,Ca^4-8\,Aa^3b)d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(1680\,Cb^4dx^7+1920\,(Cb^4c+Bb^4d)x^6+}{}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/26880*(105*(8*B*a^3*b*c - (5*C*a^4 - 8*A*a^3*b)*d)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(1680*C*b^4*d*x^7 + 1920*(C*b^4*c
+ B*b^4*d)*x^6 + 1024*B*a^3*b*d + 280*(8*B*b^4*c + (C*a*b^3 + 8*A*b^4)*d)*
x^5 + 384*(B*a*b^3*d + (C*a*b^3 + 7*A*b^4)*c)*x^4 + 70*(8*B*a*b^3*c - (5*C
*a^2*b^2 - 8*A*a*b^3)*d)*x^3 - 128*(4*B*a^2*b^2*d + (4*C*a^2*b^2 - 7*A*a*b
^3)*c)*x^2 + 256*(4*C*a^3*b - 7*A*a^2*b^2)*c - 105*(8*B*a^2*b^2*c - (5*C*a
^3*b - 8*A*a^2*b^2)*d)*x)*sqrt(b*x^2 + a))/b^4, -1/13440*(105*(8*B*a^3*b*c
 - (5*C*a^4 - 8*A*a^3*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(1680*C*b^4*d*x^7 + 1920*(C*b^4*c + B*b^4*d)*x^6 + 1024*B*a^3*b*d + 280*(8
*B*b^4*c + (C*a*b^3 + 8*A*b^4)*d)*x^5 + 384*(B*a*b^3*d + (C*a*b^3 + 7*A*b^
4)*c)*x^4 + 70*(8*B*a*b^3*c - (5*C*a^2*b^2 - 8*A*a*b^3)*d)*x^3 - 128*(4*B*
a^2*b^2*d + (4*C*a^2*b^2 - 7*A*a*b^3)*c)*x^2 + 256*(4*C*a^3*b - 7*A*a^2*b^
2)*c - 105*(8*B*a^2*b^2*c - (5*C*a^3*b - 8*A*a^2*b^2)*d)*x)*sqrt(b*x^2 + a
))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.51

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \begin{cases} \frac{3a^2\left(Aad+Bac-\frac{5a\left(Abd+Bbc+\frac{Cad}{8}\right)}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2} + \sqrt{a+bx^2}\left(\frac{Cdx^7}{8} - \frac{3ax\left(Aad+Bac-\frac{5a(\right.}{}\right.}{8b^2}\right. \\ \sqrt{a}\left(\frac{Acx^4}{4}+\frac{Cdx^7}{7}+\frac{x^6(Bd+Cc)}{6}+\frac{x^5(Ad+Bc)}{5}\right) \end{cases}$$

input
```
integrate(x**3*(d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((3*a**2*(A*a*d + B*a*c - 5*a*(A*b*d + B*b*c + C*a*d/8)/(6*b))*Pi
ecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(C*d*x**7/8 - 3*a*x*
(A*a*d + B*a*c - 5*a*(A*b*d + B*b*c + C*a*d/8)/(6*b))/(8*b**2) - 2*a*(A*a*
c - 4*a*(A*b*c + B*a*d + C*a*c - 6*a*(B*b*d + C*b*c)/(7*b))/(5*b))/(3*b**2
) + x**6*(B*b*d + C*b*c)/(7*b) + x**5*(A*b*d + B*b*c + C*a*d/8)/(6*b) + x*
*4*(A*b*c + B*a*d + C*a*c - 6*a*(B*b*d + C*b*c)/(7*b))/(5*b) + x**3*(A*a*d
 + B*a*c - 5*a*(A*b*d + B*b*c + C*a*d/8)/(6*b))/(4*b) + x**2*(A*a*c - 4*a*
(A*b*c + B*a*d + C*a*c - 6*a*(B*b*d + C*b*c)/(7*b))/(5*b))/(3*b)), Ne(b, 0
)), (sqrt(a)*(A*c*x**4/4 + C*d*x**7/7 + x**6*(B*d + C*c)/6 + x**5*(A*d + B
*c)/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.10

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Cdx^5}{8\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Cadx^3}{48\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)x^4}{7\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Acx^2}{5\,b} + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)x^3}{6\,b} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ca^2dx}{64\,b^3} - \frac{5\,\sqrt{bx^2+a}Ca^3dx}{128\,b^3} - \frac{4\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)ax^2}{35\,b^2} - \frac{5\,Ca^4d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Aac}{15\,b^2} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)ax}{8\,b^2} + \frac{\sqrt{bx^2+a}(Bc+Ad)a^2x}{16\,b^2} + \frac{(Bc+Ad)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{8\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)a^2}{105\,b^3}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(3/2)*C*d*x^5/b - 5/48*(b*x^2 + a)^(3/2)*C*a*d*x^3/b^2 + 1
/7*(b*x^2 + a)^(3/2)*(C*c + B*d)*x^4/b + 1/5*(b*x^2 + a)^(3/2)*A*c*x^2/b +
 1/6*(b*x^2 + a)^(3/2)*(B*c + A*d)*x^3/b + 5/64*(b*x^2 + a)^(3/2)*C*a^2*d*
x/b^3 - 5/128*sqrt(b*x^2 + a)*C*a^3*d*x/b^3 - 4/35*(b*x^2 + a)^(3/2)*(C*c
+ B*d)*a*x^2/b^2 - 5/128*C*a^4*d*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 2/15*(b*
x^2 + a)^(3/2)*A*a*c/b^2 - 1/8*(b*x^2 + a)^(3/2)*(B*c + A*d)*a*x/b^2 + 1/1
6*sqrt(b*x^2 + a)*(B*c + A*d)*a^2*x/b^2 + 1/16*(B*c + A*d)*a^3*arcsinh(b*x
/sqrt(a*b))/b^(5/2) + 8/105*(b*x^2 + a)^(3/2)*(C*c + B*d)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.08

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{13440}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(6\left(7\,Cdx+\frac{8\,(Cb^6c+Bb^6d)}{b^6}\right)x+\frac{7\,(8\,Bb^6c+Cab^5d+8\,Ab^6d)}{b^6}\right)x+\frac{48}{}\right.\right.\right.\right.\right.\right.$$
$$-\frac{(8\,Ba^3bc-5\,Ca^4d+8\,Aa^3bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*C*d*x + 8*(C*b^6*c + B*b^6*d)/b^6
)*x + 7*(8*B*b^6*c + C*a*b^5*d + 8*A*b^6*d)/b^6)*x + 48*(C*a*b^5*c + 7*A*b
^6*c + B*a*b^5*d)/b^6)*x + 35*(8*B*a*b^5*c - 5*C*a^2*b^4*d + 8*A*a*b^5*d)/
b^6)*x - 64*(4*C*a^2*b^4*c - 7*A*a*b^5*c + 4*B*a^2*b^4*d)/b^6)*x - 105*(8*
B*a^2*b^4*c - 5*C*a^3*b^3*d + 8*A*a^2*b^4*d)/b^6)*x + 256*(4*C*a^3*b^3*c -
 7*A*a^2*b^4*c + 4*B*a^3*b^3*d)/b^6) - 1/128*(8*B*a^3*b*c - 5*C*a^4*d + 8*
A*a^3*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int x^3\,\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x^3*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.74

$$\int x^3(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{-1792\sqrt{b\,x^2+a}\,a^3b^2c-840\sqrt{b\,x^2+a}\,a^3b^2dx+1024\sqrt{b\,x^2+a}\,a^3b^2d+1024\sqrt{b\,x^2+a}\,a^3b\,c^2+525\sqrt{b\,x^2}}{}$$

input
```
int(x^3*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
( - 1792*sqrt(a + b*x**2)*a**3*b**2*c - 840*sqrt(a + b*x**2)*a**3*b**2*d*x
 + 1024*sqrt(a + b*x**2)*a**3*b**2*d + 1024*sqrt(a + b*x**2)*a**3*b*c**2 +
 525*sqrt(a + b*x**2)*a**3*b*c*d*x + 896*sqrt(a + b*x**2)*a**2*b**3*c*x**2
 - 840*sqrt(a + b*x**2)*a**2*b**3*c*x + 560*sqrt(a + b*x**2)*a**2*b**3*d*x
**3 - 512*sqrt(a + b*x**2)*a**2*b**3*d*x**2 - 512*sqrt(a + b*x**2)*a**2*b*
*2*c**2*x**2 - 350*sqrt(a + b*x**2)*a**2*b**2*c*d*x**3 + 2688*sqrt(a + b*x
**2)*a*b**4*c*x**4 + 560*sqrt(a + b*x**2)*a*b**4*c*x**3 + 2240*sqrt(a + b*
x**2)*a*b**4*d*x**5 + 384*sqrt(a + b*x**2)*a*b**4*d*x**4 + 384*sqrt(a + b*
x**2)*a*b**3*c**2*x**4 + 280*sqrt(a + b*x**2)*a*b**3*c*d*x**5 + 2240*sqrt(
a + b*x**2)*b**5*c*x**5 + 1920*sqrt(a + b*x**2)*b**5*d*x**6 + 1920*sqrt(a
+ b*x**2)*b**4*c**2*x**6 + 1680*sqrt(a + b*x**2)*b**4*c*d*x**7 + 840*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d - 525*sqrt(b)*log(
(sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c*d + 840*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c)/(13440*b**4)
```

3.4 $\int x^2(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 235

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{a(2Abc-acC-aBd)x\sqrt{a+bx^2}}{16b^2}+\frac{(2Abc-acC-aBd)x^3\sqrt{a+bx^2}}{8b} + \frac{a(aCd-b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{(cC+Bd)x^3\left(a+bx^2\right)^{3/2}}{6b} + \frac{(bBc+Abd-2aCd)\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{Cd\left(a+bx^2\right)^{7/2}}{7b^3} - \frac{a^2(2Abc-acC-aBd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/16*a*(2*A*b*c-B*a*d-C*a*c)*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*A*b*c-B*a*d-C*a*
c)*x^3*(b*x^2+a)^(1/2)/b+1/3*a*(a*C*d-b*(A*d+B*c))*(b*x^2+a)^(3/2)/b^3+1/6
*(B*d+C*c)*x^3*(b*x^2+a)^(3/2)/b+1/5*(A*b*d+B*b*c-2*C*a*d)*(b*x^2+a)^(5/2)
/b^3+1/7*C*d*(b*x^2+a)^(7/2)/b^3-1/16*a^2*(2*A*b*c-B*a*d-C*a*c)*arctanh(b^
(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.84

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{a+bx^2}(128a^3Cd - a^2b(224Ad+7B(32c+15dx)+Cx(105c+64dx))+4b^3x^3(21A(5c+4dx)+2x(}{}$$

input
```
Integrate[x^2*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^3*C*d - a^2*b*(224*A*d + 7*B*(32*c + 15*d*x) + C*x
*(105*c + 64*d*x)) + 4*b^3*x^3*(21*A*(5*c + 4*d*x) + 2*x*(7*B*(6*c + 5*d*x
) + 5*C*x*(7*c + 6*d*x))) + 2*a*b^2*x*(7*A*(15*c + 8*d*x) + x*(7*B*(8*c +
5*d*x) + C*x*(35*c + 24*d*x)))) - 105*a^2*Sqrt[b]*(-2*A*b*c + a*c*C + a*B*
d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1680*b^3)
```

Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2185, 25, 2185, 27, 2185, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\left((c+dx)\sqrt{bx^2+a}\left(bd^3(17cC-7Bd)x^3+d^2\left(13bCc^2-7Abd^2+4aCd^2\right)x^2+cCd\left(3bc^2+8ad^2\right)x+4ac^2C\right.\right.}{7bd^4}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\int (c+dx)\sqrt{bx^2+a}\left(bd^3(17cC-7Bd)x^3+d^2\left(13bCc^2-7Abd^2+4aCd^2\right)x^2+cCd\left(3bc^2+8ad^2\right)x+4ac^2Cd^2\right) \cdots}{7bd^4}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{\int -3(c+dx)\sqrt{bx^2+a}\left(-b\left(8aCd^2-b\left(25Cc^2-21Bdc+14Ad^2\right)\right)x^2d^5+abc(9cC-7Bd)d^5+b\left(b(11cC-7Bd)c^2+ad^2(cC-7Bd)\right)xd^4\right)dx}{6bd^3}+\frac{1}{6}d\left(a+\cdots\right.}{7bd^4}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\int (c+dx)\sqrt{bx^2+a}\left(-b\left(8aCd^2-b\left(25Cc^2-21Bdc+14Ad^2\right)\right)x^2d^5+abc(9cC-7Bd)d^5+b(b(11\cdots\right.}{2bd^3}}{7bd^4}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{\int bd^6(c+dx)\left(ad\left(16aCd^2-b\left(5Cc^2-7Bdc+28Ad^2\right)\right)+b\left(ad^2(29cC-35Bd)-2bc\left(10Cc^2-14Bdc+21A\cdots\right.\right.\right.}{5bd^2}\cdots}{2bd^3}}{7bd^4}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{1}{5}d^4\int (c+dx)\left(ad\left(16aCd^2-b\left(5Cc^2-7Bdc+28Ad^2\right)\right)+b\left(ad^2(29cC-35Bd)-2bc\left(10Cc^2-1\cdots\right.\right.\right.}{2\cdots}}{7bd^4}$$

$\Big\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{1}{5}d^4\left(-\frac{35}{4}ad^3(-aBd-acC+2Abc)\int \sqrt{bx^2+a}dx+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2Cd^4+2abd^2\left(-7Ad^2-7B\cdots\right.\right.}{3\cdots}\right.}{\cdots}}{\cdots}$$

$\Big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{1}{5}d^4\left(-\frac{35}{4}ad^3(-aBd-acC+2Abc)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2Cd\right.}{}\right.}{}}{}$$

$\Big\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{1}{5}d^4\left(-\frac{35}{4}ad^3(-aBd-acC+2Abc)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2\left(a+bx^2\right)^{3}}{}\right.}{}}{}$$

$\Big\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(17cC-7Bd)-\frac{\frac{1}{5}d^4\left(\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2Cd^4+2abd^2\left(-7Ad^2-7Bcd+6c^2C\right)-b^2c^2\left(21Ad^2-14Bcd+10c^2C\right)\right)}{3b}-\frac{35}{4}\right.}{}}{}$$

input
```
Int[x^2*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^4*(a + b*x^2)^(3/2))/(7*b*d^3) - ((d*(17*c*C - 7*B*d)*(c + d*
x)^3*(a + b*x^2)^(3/2))/6 - (-1/5*(d^4*(8*a*C*d^2 - b*(25*c^2*C - 21*B*c*d
 + 14*A*d^2))*(c + d*x)^2*(a + b*x^2)^(3/2)) + (d^4*((2*(8*a^2*C*d^4 + 2*a
*b*d^2*(6*c^2*C - 7*B*c*d - 7*A*d^2) - b^2*c^2*(10*c^2*C - 14*B*c*d + 21*A
*d^2))*(a + b*x^2)^(3/2))/(3*b) + (d*(a*d^2*(29*c*C - 35*B*d) - 2*b*c*(10*
c^2*C - 14*B*c*d + 21*A*d^2))*x*(a + b*x^2)^(3/2))/4 - (35*a*d^3*(2*A*b*c
 - a*c*C - a*B*d)*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(2*Sqrt[b])))/4))/5)/(2*b*d^3))/(7*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{(-240C\,b^3 d\,x^6-280B\,b^3 d\,x^5-280C\,b^3 c\,x^5-336A\,b^3 d\,x^4-336B\,b^3 c\,x^4-48Ca\,b^2 d\,x^4-420A\,b^3 c\,x^3-70Ba\,b^2 d\,x^3-70Ca\,b^2 c\,x^3-}{168}$
default	$(Ad+Bc)\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)+(Bd+Cc)\left(\frac{x^3(b\,x^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln(}{}\right)}{4b}\right)}{2b}\right.$

input
```
int(x^2*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(-240*C*b^3*d*x^6-280*B*b^3*d*x^5-280*C*b^3*c*x^5-336*A*b^3*d*x^4-
336*B*b^3*c*x^4-48*C*a*b^2*d*x^4-420*A*b^3*c*x^3-70*B*a*b^2*d*x^3-70*C*a*b
^2*c*x^3-112*A*a*b^2*d*x^2-112*B*a*b^2*c*x^2+64*C*a^2*b*d*x^2-210*A*a*b^2*
c*x+105*B*a^2*b*d*x+105*C*a^2*b*c*x+224*A*a^2*b*d+224*B*a^2*b*c-128*C*a^3*
d)*(b*x^2+a)^(1/2)/b^3-1/16*a^2/b^(5/2)*(2*A*b*c-B*a*d-C*a*c)*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 472, normalized size of antiderivative = 2.01

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$=\left[\frac{105\,(Ba^3d+(Ca^3-2\,Aa^2b)c)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(240\,Cb^3dx^6+280\,(Cb^3c+}{}\right.$$

$$-\frac{105\,(Ba^3d+(Ca^3-2\,Aa^2b)c)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(240\,Cb^3dx^6+280\,(Cb^3c+Bb^3d)x^5-224\,Ba}{}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/3360*(105*(B*a^3*d + (C*a^3 - 2*A*a^2*b)*c)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 2*(240*C*b^3*d*x^6 + 280*(C*b^3*c + B*b^3*d
)*x^5 - 224*B*a^2*b*c + 48*(7*B*b^3*c + (C*a*b^2 + 7*A*b^3)*d)*x^4 + 70*(B
*a*b^2*d + (C*a*b^2 + 6*A*b^3)*c)*x^3 + 16*(7*B*a*b^2*c - (4*C*a^2*b - 7*A
*a*b^2)*d)*x^2 + 32*(4*C*a^3 - 7*A*a^2*b)*d - 105*(B*a^2*b*d + (C*a^2*b -
2*A*a*b^2)*c)*x)*sqrt(b*x^2 + a))/b^3, -1/1680*(105*(B*a^3*d + (C*a^3 - 2*
A*a^2*b)*c)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (240*C*b^3*d*x^6
 + 280*(C*b^3*c + B*b^3*d)*x^5 - 224*B*a^2*b*c + 48*(7*B*b^3*c + (C*a*b^2
+ 7*A*b^3)*d)*x^4 + 70*(B*a*b^2*d + (C*a*b^2 + 6*A*b^3)*c)*x^3 + 16*(7*B*a
*b^2*c - (4*C*a^2*b - 7*A*a*b^2)*d)*x^2 + 32*(4*C*a^3 - 7*A*a^2*b)*d - 105
*(B*a^2*b*d + (C*a^2*b - 2*A*a*b^2)*c)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.56

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \begin{cases} -\frac{a\left(Aac-\frac{3a\left(Abc+Bad+Cac-\frac{5a(Bbd+Cbc)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Cdx^6}{7} - \frac{2a\left(Aad+Bc\right.}{} \right. \\ \sqrt{a}\left(\frac{Acx^3}{3}+\frac{Cdx^6}{6}+\frac{x^5(Bd+Cc)}{5}+\frac{x^4(Ad+Bc)}{4}\right) \end{cases}$$

input
```
integrate(x**2*(d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a*c - 3*a*(A*b*c + B*a*d + C*a*c - 5*a*(B*b*d + C*b*c)/(6
*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne
(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*d*x**6
/7 - 2*a*(A*a*d + B*a*c - 4*a*(A*b*d + B*b*c + C*a*d/7)/(5*b))/(3*b**2) +
x**5*(B*b*d + C*b*c)/(6*b) + x**4*(A*b*d + B*b*c + C*a*d/7)/(5*b) + x**3*(
A*b*c + B*a*d + C*a*c - 5*a*(B*b*d + C*b*c)/(6*b))/(4*b) + x**2*(A*a*d + B
*a*c - 4*a*(A*b*d + B*b*c + C*a*d/7)/(5*b))/(3*b) + x*(A*a*c - 3*a*(A*b*c
+ B*a*d + C*a*c - 5*a*(B*b*d + C*b*c)/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (sq
rt(a)*(A*c*x**3/3 + C*d*x**6/6 + x**5*(B*d + C*c)/5 + x**4*(A*d + B*c)/4),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.09

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(bx^2+a)^{\frac{3}{2}}Cdx^4}{7\,b}-\frac{4\,(bx^2+a)^{\frac{3}{2}}Cadx^2}{35\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)x^3}{6\,b}+\frac{(bx^2+a)^{\frac{3}{2}}Acx}{4\,b}-\frac{\sqrt{bx^2+a}Aacx}{8\,b}+\frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)x^2}{5\,b}-\frac{Aa^2c\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{8\,(bx^2+a)^{\frac{3}{2}}Ca^2d}{105\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)ax}{8\,b^2}+\frac{\sqrt{bx^2+a}(Cc+Bd)a^2x}{16\,b^2}+\frac{(Cc+Bd)a^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{3}{2}}(Bc+Ad)a}{15\,b^2}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(3/2)*C*d*x^4/b - 4/35*(b*x^2 + a)^(3/2)*C*a*d*x^2/b^2 + 1
/6*(b*x^2 + a)^(3/2)*(C*c + B*d)*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*c*x/b - 1
/8*sqrt(b*x^2 + a)*A*a*c*x/b + 1/5*(b*x^2 + a)^(3/2)*(B*c + A*d)*x^2/b - 1
/8*A*a^2*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 8/105*(b*x^2 + a)^(3/2)*C*a^2*
d/b^3 - 1/8*(b*x^2 + a)^(3/2)*(C*c + B*d)*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*(
C*c + B*d)*a^2*x/b^2 + 1/16*(C*c + B*d)*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
 - 2/15*(b*x^2 + a)^(3/2)*(B*c + A*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.07

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{1680}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,Cdx+\frac{7\,(Cb^5c+Bb^5d)}{b^5}\right)x+\frac{6\,(7\,Bb^5c+Cab^4d+7\,Ab^5d)}{b^5}\right)x+\frac{35\,(Ca\right.\right.\right.\right.$$
$$-\frac{(Ca^3c-2\,Aa^2bc+Ba^3d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*C*d*x + 7*(C*b^5*c + B*b^5*d)/b^5)*x
+ 6*(7*B*b^5*c + C*a*b^4*d + 7*A*b^5*d)/b^5)*x + 35*(C*a*b^4*c + 6*A*b^5*c
 + B*a*b^4*d)/b^5)*x + 8*(7*B*a*b^4*c - 4*C*a^2*b^3*d + 7*A*a*b^4*d)/b^5)*
x - 105*(C*a^2*b^3*c - 2*A*a*b^4*c + B*a^2*b^3*d)/b^5)*x - 32*(7*B*a^2*b^3
*c - 4*C*a^3*b^2*d + 7*A*a^2*b^3*d)/b^5) - 1/16*(C*a^3*c - 2*A*a^2*b*c + B
*a^3*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int x^2\,\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.76

$$\int x^2(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{-224\sqrt{b\,x^2+a}\,a^3bd+128\sqrt{b\,x^2+a}\,a^3cd+210\sqrt{b\,x^2+a}\,a^2b^2cx-224\sqrt{b\,x^2+a}\,a^2b^2c+112\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^2*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
( - 224*sqrt(a + b*x**2)*a**3*b*d + 128*sqrt(a + b*x**2)*a**3*c*d + 210*sq
rt(a + b*x**2)*a**2*b**2*c*x - 224*sqrt(a + b*x**2)*a**2*b**2*c + 112*sqrt
(a + b*x**2)*a**2*b**2*d*x**2 - 105*sqrt(a + b*x**2)*a**2*b**2*d*x - 105*s
qrt(a + b*x**2)*a**2*b*c**2*x - 64*sqrt(a + b*x**2)*a**2*b*c*d*x**2 + 420*
sqrt(a + b*x**2)*a*b**3*c*x**3 + 112*sqrt(a + b*x**2)*a*b**3*c*x**2 + 336*
sqrt(a + b*x**2)*a*b**3*d*x**4 + 70*sqrt(a + b*x**2)*a*b**3*d*x**3 + 70*sq
rt(a + b*x**2)*a*b**2*c**2*x**3 + 48*sqrt(a + b*x**2)*a*b**2*c*d*x**4 + 33
6*sqrt(a + b*x**2)*b**4*c*x**4 + 280*sqrt(a + b*x**2)*b**4*d*x**5 + 280*sq
rt(a + b*x**2)*b**3*c**2*x**5 + 240*sqrt(a + b*x**2)*b**3*c*d*x**6 - 210*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c + 105*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d + 105*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c**2)/(1680*b**3)
```

3.5 $\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 28, antiderivative size = 199

$$\begin{aligned}\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx &= -\frac{a(aCd-2b(Bc+Ad))x\sqrt{a+bx^2}}{16b^2}-\frac{(aCd-2b(Bc+Ad))x^3\sqrt{a+bx^2}}{8b}\\ &\quad+\frac{(Abc-a(cC+Bd))\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{Cdx^3\left(a+bx^2\right)^{3/2}}{6b}\\ &\quad+\frac{(cC+Bd)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{a^2(aCd-2b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}\end{aligned}$$

output
```
-1/16*a*(a*C*d-2*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b^2-1/8*(a*C*d-2*b*(A*d+B*
c))*x^3*(b*x^2+a)^(1/2)/b+1/3*(A*b*c-a*(B*d+C*c))*(b*x^2+a)^(3/2)/b^2+1/6*
C*d*x^3*(b*x^2+a)^(3/2)/b+1/5*(B*d+C*c)*(b*x^2+a)^(5/2)/b^2+1/16*a^2*(a*C*
d-2*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.88

$$\int x(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(-a^2(32cC+32Bd+15Cdx)+2ab(5A(8c+3dx)+x(15Bc+8cCx+8Bdx+5Cdx^2))}{}$$

input
```
Integrate[x*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-(a^2*(32*c*C + 32*B*d + 15*C*d*x)) + 2*a*b*(5*A
*(8*c + 3*d*x) + x*(15*B*c + 8*c*C*x + 8*B*d*x + 5*C*d*x^2)) + 4*b^2*x^2*(
5*A*(4*c + 3*d*x) + x*(3*B*(5*c + 4*d*x) + 2*C*x*(6*c + 5*d*x)))) + 15*a^2
*(-(a*C*d) + 2*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(240*b^
(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2185, 27, 2185, 25, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int -3(c+dx)\sqrt{bx^2+a}\left(b(3cC-2Bd)x^2d^2+acCd^2+\left(bCc^2-2Abd^2+aCd^2\right)xd\right)dx}{6bd^3}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}$$
$$\Big\downarrow \; 27$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\int (c+dx)\sqrt{bx^2+a}\left(b(3cC-2Bd)x^2d^2+acCd^2+\left(bCc^2-2Abd^2+aCd^2\right)xd\right)dx}{2bd^3}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\frac{\int -bd^3(c+dx)\left(ad(cC-4Bd)-\left(5aCd^2-2b\left(2Cc^2-3Bdc+5Ad^2\right)\right)x\right)\sqrt{bx^2+a}dx}{5bd^2}+\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(3cC-2Bd)}{2bd^3}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{\int bd^3(c+dx)\left(ad(cC-4Bd)-\left(5aCd^2-2b\left(2Cc^2-3Bdc+5Ad^2\right)\right)x\right)\sqrt{bx^2+a}dx}{5bd^2}}{2bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{1}{5}d\int (c+dx)\left(ad(cC-4Bd)-\left(5aCd^2-2b\left(2Cc^2-3Bdc+5Ad^2\right)\right)x\right)}{2bd^3}$$

$\Big\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{1}{5}d\left(\frac{5ad^2(aCd-2b(Ad+Bc))\int\sqrt{bx^2+a}dx}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(-5aCd^2+10Abd^2-6bBcd+\right.}{4b}\right.}{2bd^3}$$

$\Big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd^2}-\frac{\frac{1}{5}d\left(a+bx^2\right)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{1}{5}d\left(\frac{5ad^2(aCd-2b(Ad+Bc))\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx\left(a+bx^2\right)^{3/2}\left(-5aCd\right.}{}\right.}{2bd^3}$$

$\Big\downarrow$ 224

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^3}{6bd^2}-}{\frac{\frac{1}{5}d(a+bx^2)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{1}{5}d\left(\frac{5ad^2(aCd-2b(Ad+Bc))\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}+\frac{dx(a+bx^2)^{3/2}}{\cdots}\right.}{2bd^3}}$$

$\Big\downarrow$ 219

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^3}{6bd^2}-}{\frac{\frac{1}{5}d(a+bx^2)^{3/2}(c+dx)^2(3cC-2Bd)-\frac{1}{5}d\left(\frac{5ad^2\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(aCd-2b(Ad+Bc))}{4b}+\frac{dx(a+bx^2)^{3/2}(\cdots}{\cdots}\right.}{2bd^3}}$$

input
```
Int[x*(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^3*(a + b*x^2)^(3/2))/(6*b*d^2) - ((d*(3*c*C - 2*B*d)*(c + d*x
)^2*(a + b*x^2)^(3/2))/5 - (d*((-2*(2*a*d^2*(c*C + B*d) - b*c*(2*c^2*C - 3
*B*c*d + 5*A*d^2))*(a + b*x^2)^(3/2))/(3*b) + (d*(4*b*c^2*C - 6*b*B*c*d +
10*A*b*d^2 - 5*a*C*d^2)*x*(a + b*x^2)^(3/2))/(4*b) + (5*a*d^2*(a*C*d - 2*b
*(B*c + A*d))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*Sqrt[b])))/(4*b)))/5)/(2*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.93

method	result
risch	$\frac{(40dC\,b^2x^5+48B\,b^2d\,x^4+48C\,b^2c\,x^4+60A\,b^2d\,x^3+60B\,b^2c\,x^3+10aCdb\,x^3+80A\,b^2c\,x^2+16Badb\,x^2+16Cacb\,x^2+30Axadb+30Bac}{240b^2}$
default	$(Ad+Bc)\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+(Bd+Cc)\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right.$

input
```
int(x*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/240/b^2*(40*C*b^2*d*x^5+48*B*b^2*d*x^4+48*C*b^2*c*x^4+60*A*b^2*d*x^3+60*
B*b^2*c*x^3+10*C*a*b*d*x^3+80*A*b^2*c*x^2+16*B*a*b*d*x^2+16*C*a*b*c*x^2+30
*A*a*b*d*x+30*B*a*b*c*x-15*C*a^2*d*x+80*A*a*b*c-32*B*a^2*d-32*C*a^2*c)*(b*
x^2+a)^(1/2)-1/16*a^2*(2*A*b*d+2*B*b*c-C*a*d)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.11

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$=\left[\frac{15\left(2\,Ba^2bc-(Ca^3-2\,Aa^2b)d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(40\,Cb^3dx^5-32\,Ba^2bd+4\right.}{}\right.$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/480*(15*(2*B*a^2*b*c - (C*a^3 - 2*A*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 + 2*
sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*C*b^3*d*x^5 - 32*B*a^2*b*d + 48*(C*
b^3*c + B*b^3*d)*x^4 + 10*(6*B*b^3*c + (C*a*b^2 + 6*A*b^3)*d)*x^3 + 16*(B*
a*b^2*d + (C*a*b^2 + 5*A*b^3)*c)*x^2 - 16*(2*C*a^2*b - 5*A*a*b^2)*c + 15*(
2*B*a*b^2*c - (C*a^2*b - 2*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/240*(15*
(2*B*a^2*b*c - (C*a^3 - 2*A*a^2*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) + (40*C*b^3*d*x^5 - 32*B*a^2*b*d + 48*(C*b^3*c + B*b^3*d)*x^4 + 10
*(6*B*b^3*c + (C*a*b^2 + 6*A*b^3)*d)*x^3 + 16*(B*a*b^2*d + (C*a*b^2 + 5*A*
b^3)*c)*x^2 - 16*(2*C*a^2*b - 5*A*a*b^2)*c + 15*(2*B*a*b^2*c - (C*a^2*b -
2*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.55

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \begin{cases} -\frac{a\left(Aad+Bac-\frac{3a\left(Abd+Bbc+\frac{Cad}{6}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Cdx^5}{6}+\frac{x^4(Bbd+Cbc)}{5b}+\frac{x^3}{}\right. \\ \sqrt{a}\left(\frac{Acx^2}{2}+\frac{Cdx^5}{5}+\frac{x^4(Bd+Cc)}{4}+\frac{x^3(Ad+Bc)}{3}\right) \end{cases}$$

input
```
integrate(x*(d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a*d + B*a*c - 3*a*(A*b*d + B*b*c + C*a*d/6)/(4*b))*Piecew
ise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)
/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*d*x**5/6 + x**4*(B*b*d +
 C*b*c)/(5*b) + x**3*(A*b*d + B*b*c + C*a*d/6)/(4*b) + x**2*(A*b*c + B*a*d
 + C*a*c - 4*a*(B*b*d + C*b*c)/(5*b))/(3*b) + x*(A*a*d + B*a*c - 3*a*(A*b*
d + B*b*c + C*a*d/6)/(4*b))/(2*b) + (A*a*c - 2*a*(A*b*c + B*a*d + C*a*c -
4*a*(B*b*d + C*b*c)/(5*b))/(3*b))/b), Ne(b, 0)), (sqrt(a)*(A*c*x**2/2 + C*
d*x**5/5 + x**4*(B*d + C*c)/4 + x**3*(A*d + B*c)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.05

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Cdx^3}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Cadx}{8\,b^2} + \frac{\sqrt{bx^2+a}Ca^2dx}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)x^2}{5\,b} + \frac{Ca^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Ac}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)x}{4\,b} - \frac{\sqrt{bx^2+a}(Bc+Ad)ax}{8\,b} - \frac{(Bc+Ad)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)a}{15\,b^2}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*C*d*x^3/b - 1/8*(b*x^2 + a)^(3/2)*C*a*d*x/b^2 + 1/16
*sqrt(b*x^2 + a)*C*a^2*d*x/b^2 + 1/5*(b*x^2 + a)^(3/2)*(C*c + B*d)*x^2/b +
 1/16*C*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 1/3*(b*x^2 + a)^(3/2)*A*c/b
 + 1/4*(b*x^2 + a)^(3/2)*(B*c + A*d)*x/b - 1/8*sqrt(b*x^2 + a)*(B*c + A*d)
*a*x/b - 1/8*(B*c + A*d)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2
+ a)^(3/2)*(C*c + B*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.09

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\,Cdx + \frac{6\,(Cb^4c+Bb^4d)}{b^4}\right)x + \frac{5\,(6\,Bb^4c+Cab^3d+6\,Ab^4d)}{b^4}\right)x + \frac{8\,(Cab^3c+\right.\right.\right.\right.$$
$$+ \frac{(2\,Ba^2bc - Ca^3d + 2\,Aa^2bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*C*d*x + 6*(C*b^4*c + B*b^4*d)/b^4)*x + 5*
(6*B*b^4*c + C*a*b^3*d + 6*A*b^4*d)/b^4)*x + 8*(C*a*b^3*c + 5*A*b^4*c + B*
a*b^3*d)/b^4)*x + 15*(2*B*a*b^3*c - C*a^2*b^2*d + 2*A*a*b^3*d)/b^4)*x - 16
*(2*C*a^2*b^2*c - 5*A*a*b^3*c + 2*B*a^2*b^2*d)/b^4) + 1/16*(2*B*a^2*b*c -
C*a^3*d + 2*A*a^2*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int x\,\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x*(a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.80

$$\int x(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{80\sqrt{b\,x^2+a}\,a^2b^2c+30\sqrt{b\,x^2+a}\,a^2b^2dx-32\sqrt{b\,x^2+a}\,a^2b^2d-32\sqrt{b\,x^2+a}\,a^2b\,c^2-15\sqrt{b\,x^2+a}\,a^2bcd}{}$$

input
```
int(x*(d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
(80*sqrt(a + b*x**2)*a**2*b**2*c + 30*sqrt(a + b*x**2)*a**2*b**2*d*x - 32*
sqrt(a + b*x**2)*a**2*b**2*d - 32*sqrt(a + b*x**2)*a**2*b*c**2 - 15*sqrt(a
 + b*x**2)*a**2*b*c*d*x + 80*sqrt(a + b*x**2)*a*b**3*c*x**2 + 30*sqrt(a +
b*x**2)*a*b**3*c*x + 60*sqrt(a + b*x**2)*a*b**3*d*x**3 + 16*sqrt(a + b*x**
2)*a*b**3*d*x**2 + 16*sqrt(a + b*x**2)*a*b**2*c**2*x**2 + 10*sqrt(a + b*x*
*2)*a*b**2*c*d*x**3 + 60*sqrt(a + b*x**2)*b**4*c*x**3 + 48*sqrt(a + b*x**2
)*b**4*d*x**4 + 48*sqrt(a + b*x**2)*b**3*c**2*x**4 + 40*sqrt(a + b*x**2)*b
**3*c*d*x**5 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3
*b*d + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c*d - 3
0*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c)/(240*b*
*3)
```

3.6 $\int (c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 159

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\frac{(4Abc-acC-aBd)x\sqrt{a+bx^2}}{8b}+\frac{(bBc+Abd-aCd)\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{(cC+Bd)x\left(a+bx^2\right)^{3/2}}{4b}+\frac{Cd\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{a(4Abc-acC-aBd)\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output

```
1/8*(4*A*b*c-B*a*d-C*a*c)*x*(b*x^2+a)^(1/2)/b+1/3*(A*b*d+B*b*c-C*a*d)*(b*x
^2+a)^(3/2)/b^2+1/4*(B*d+C*c)*x*(b*x^2+a)^(3/2)/b+1/5*C*d*(b*x^2+a)^(5/2)/
b^2+1/8*a*(4*A*b*c-B*a*d-C*a*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-16a^2Cd+ab(40Ad+5B(8c+3dx)+Cx(15c+8dx))+2b^2x(10A(3c+2dx)+x(5B(4c+}{120b^2}$$

input
```
Integrate[(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*C*d + a*b*(40*A*d + 5*B*(8*c + 3*d*x) + C*x*(15*
c + 8*d*x)) + 2*b^2*x*(10*A*(3*c + 2*d*x) + x*(5*B*(4*c + 3*d*x) + 3*C*x*(
5*c + 4*d*x)))) + 15*a*Sqrt[b]*(-4*A*b*c + a*c*C + a*B*d)*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(120*b^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2185, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int d(c+dx)((5Ab-2aC)d-b(3cC-5Bd)x)\sqrt{bx^2+a}dx}{5bd^2}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (c+dx)((5Ab-2aC)d-b(3cC-5Bd)x)\sqrt{bx^2+a}dx}{5bd}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd}$$
$$\Big\downarrow \text{676}$$

$$\frac{\frac{5}{4}d(-aBd-acC+4Abc)\int\sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}(2aCd^2+b(-5Ad^2-5Bcd+3c^2C))}{3b}-\frac{1}{4}dx(a+bx^2)^{3/2}(3cC-5Bd}{5bd}$$
$$\frac{C(a+bx^2)^{3/2}(c+dx)^2}{5bd}$$

$\Big\downarrow$ 211

$$\frac{\frac{5}{4}d(-aBd-acC+4Abc)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}(2aCd^2+b(-5Ad^2-5Bcd+3c^2C))}{3b}-\frac{1}{4}dx(a+b}{5bd}$$
$$\frac{C(a+bx^2)^{3/2}(c+dx)^2}{5bd}$$

$\Big\downarrow$ 224

$$\frac{\frac{5}{4}d(-aBd-acC+4Abc)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}(2aCd^2+b(-5Ad^2-5Bcd+3c^2C))}{3b}-\frac{1}{4}dx}{5bd}$$
$$\frac{C(a+bx^2)^{3/2}(c+dx)^2}{5bd}$$

$\Big\downarrow$ 219

$$\frac{\frac{5}{4}d\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(-aBd-acC+4Abc)-\frac{(a+bx^2)^{3/2}(2aCd^2+b(-5Ad^2-5Bcd+3c^2C))}{3b}-\frac{1}{4}dx(}{5bd}$$
$$\frac{C(a+bx^2)^{3/2}(c+dx)^2}{5bd}$$

input

```
Int[(c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output

```
(C*(c + d*x)^2*(a + b*x^2)^(3/2))/(5*b*d) + (-1/3*((2*a*C*d^2 + b*(3*c^2*C
 - 5*B*c*d - 5*A*d^2))*(a + b*x^2)^(3/2))/b - (d*(3*c*C - 5*B*d)*x*(a + b*
x^2)^(3/2))/4 + (5*d*(4*A*b*c - a*c*C - a*B*d)*((x*Sqrt[a + b*x^2])/2 + (a
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97

method	result
risch	$\frac{(24C\,b^2 d\,x^4+30B\,b^2 d\,x^3+30C\,b^2 c\,x^3+40A\,b^2 d\,x^2+40B\,b^2 c\,x^2+8Cabd\,x^2+60A\,b^2 cx+15Badxb+15Cacxb+40Aabd+40Babc-16C}{120b^2}$
default	$Ac\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{(Ad+Bc)(b\,x^2+a)^{\frac{3}{2}}}{3b}+(Bd+Cc)\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln(}{}\right.}{4b}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*C*b^2*d*x^4+30*B*b^2*d*x^3+30*C*b^2*c*x^3+40*A*b^2*d*x^2+40*B*b^
2*c*x^2+8*C*a*b*d*x^2+60*A*b^2*c*x+15*B*a*b*d*x+15*C*a*b*c*x+40*A*a*b*d+40
*B*a*b*c-16*C*a^2*d)*(b*x^2+a)^(1/2)/b^2+1/8*a/b^(3/2)*(4*A*b*c-B*a*d-C*a*
c)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.04

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$=\left[\frac{15\,(Ba^2d+(Ca^2-4\,Aab)c)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(24\,Cb^2dx^4+40\,Babc+30\,(C}{}\right.$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/240*(15*(B*a^2*d + (C*a^2 - 4*A*a*b)*c)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(24*C*b^2*d*x^4 + 40*B*a*b*c + 30*(C*b^2*c +
B*b^2*d)*x^3 + 8*(5*B*b^2*c + (C*a*b + 5*A*b^2)*d)*x^2 - 8*(2*C*a^2 - 5*A*
a*b)*d + 15*(B*a*b*d + (C*a*b + 4*A*b^2)*c)*x)*sqrt(b*x^2 + a))/b^2, 1/120
*(15*(B*a^2*d + (C*a^2 - 4*A*a*b)*c)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + (24*C*b^2*d*x^4 + 40*B*a*b*c + 30*(C*b^2*c + B*b^2*d)*x^3 + 8*(5*
B*b^2*c + (C*a*b + 5*A*b^2)*d)*x^2 - 8*(2*C*a^2 - 5*A*a*b)*d + 15*(B*a*b*d
 + (C*a*b + 4*A*b^2)*c)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.62

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Cdx^4}{5}+\frac{x^3(Bbd+Cbc)}{4b}+\frac{x^2\left(Abd+Bbc+\frac{Cad}{5}\right)}{3b}+\frac{x\left(Abc+Bad+Cac-\frac{3a(Bbd+Cbc)}{4b}\right)}{2b}+\frac{Aad+Bac-\frac{2a\left(Abd+Bbc+\frac{Cad}{5}\right.}{3b}}{b}\right. \\ \sqrt{a}\left(Acx+\frac{Cdx^4}{4}+\frac{x^3(Bd+Cc)}{3}+\frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*d*x**4/5 + x**3*(B*b*d + C*b*c)/(4*b) + x**
2*(A*b*d + B*b*c + C*a*d/5)/(3*b) + x*(A*b*c + B*a*d + C*a*c - 3*a*(B*b*d
+ C*b*c)/(4*b))/(2*b) + (A*a*d + B*a*c - 2*a*(A*b*d + B*b*c + C*a*d/5)/(3*
b))/b) + (A*a*c - a*(A*b*c + B*a*d + C*a*c - 3*a*(B*b*d + C*b*c)/(4*b))/(2
*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*c*x + C*d*x**4/4
+ x**3*(B*d + C*c)/3 + x**2*(A*d + B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Cdx^2}{5\,b}+\frac{1}{2}\,\sqrt{bx^2+a}Acx+\frac{Aac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}+\frac{(bx^2+a)^{\frac{3}{2}}Bc}{3\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Cad}{15\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ad}{3\,b}+\frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)x}{4\,b}-\frac{\sqrt{bx^2+a}(Cc+Bd)ax}{8\,b}-\frac{(Cc+Bd)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*C*d*x^2/b + 1/2*sqrt(b*x^2 + a)*A*c*x + 1/2*A*a*c*ar
csinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x^2 + a)^(3/2)*B*c/b - 2/15*(b*x^2 +
 a)^(3/2)*C*a*d/b^2 + 1/3*(b*x^2 + a)^(3/2)*A*d/b + 1/4*(b*x^2 + a)^(3/2)*
(C*c + B*d)*x/b - 1/8*sqrt(b*x^2 + a)*(C*c + B*d)*a*x/b - 1/8*(C*c + B*d)*
a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.09

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Cdx+\frac{5\,(Cb^3c+Bb^3d)}{b^3}\right)x+\frac{4\,(5\,Bb^3c+Cab^2d+5\,Ab^3d)}{b^3}\right)x+\frac{15\,(Cab^2c+\right.\right. + \frac{(Ca^2c-4\,Aabc+Ba^2d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*C*d*x + 5*(C*b^3*c + B*b^3*d)/b^3)*x + 4*(
5*B*b^3*c + C*a*b^2*d + 5*A*b^3*d)/b^3)*x + 15*(C*a*b^2*c + 4*A*b^3*c + B*
a*b^2*d)/b^3)*x + 8*(5*B*a*b^2*c - 2*C*a^2*b*d + 5*A*a*b^2*d)/b^3) + 1/8*(
C*a^2*c - 4*A*a*b*c + B*a^2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3
/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.82

$$\int (c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{40\sqrt{b\,x^2+a}\,a^2bd-16\sqrt{b\,x^2+a}\,a^2cd+60\sqrt{b\,x^2+a}\,a\,b^2cx+40\sqrt{b\,x^2+a}\,a\,b^2c+40\sqrt{b\,x^2+a}\,a\,b^2d\,x^2+}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
(40*sqrt(a + b*x**2)*a**2*b*d - 16*sqrt(a + b*x**2)*a**2*c*d + 60*sqrt(a +
 b*x**2)*a*b**2*c*x + 40*sqrt(a + b*x**2)*a*b**2*c + 40*sqrt(a + b*x**2)*a
*b**2*d*x**2 + 15*sqrt(a + b*x**2)*a*b**2*d*x + 15*sqrt(a + b*x**2)*a*b*c*
*2*x + 8*sqrt(a + b*x**2)*a*b*c*d*x**2 + 40*sqrt(a + b*x**2)*b**3*c*x**2 +
 30*sqrt(a + b*x**2)*b**3*d*x**3 + 30*sqrt(a + b*x**2)*b**2*c**2*x**3 + 24
*sqrt(a + b*x**2)*b**2*c*d*x**4 + 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a**2*b*c - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**2*b*d - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**2*c**2)/(120*b**2)
```

3.7 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = Ac\sqrt{a+bx^2} - \frac{(aCd-4b(Bc+Ad))x\sqrt{a+bx^2}}{8b} + \frac{(cC+Bd)\left(a+bx^2\right)^{3/2}}{3b} + \frac{Cdx(a+bx^2)^{3/2}}{4b} - \frac{a(aCd-4b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}} - \sqrt{a}A\operatorname{carctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
A*c*(b*x^2+a)^(1/2)-1/8*(a*C*d-4*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b+1/3*(B*d
+C*c)*(b*x^2+a)^(3/2)/b+1/4*C*d*x*(b*x^2+a)^(3/2)/b-1/8*a*(a*C*d-4*b*(A*d+
B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-a^(1/2)*A*c*arctanh((b*x^
2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{\sqrt{a+bx^2}(12Ab(2c+dx)+a(8cC+8Bd+3Cdx)+2bx(6Bc+4cCx+4Bdx+3Cdx^2))}{24b}$$
$$+ 2\sqrt{a}Ac\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$
$$+ \frac{a(aCd-4b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(12*A*b*(2*c + d*x) + a*(8*c*C + 8*B*d + 3*C*d*x) + 2*b*x
*(6*B*c + 4*c*C*x + 4*B*d*x + 3*C*d*x^2)))/(24*b) + 2*Sqrt[a]*A*c*ArcTanh[
(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (a*(a*C*d - 4*b*(B*c + A*d))*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2340, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x}\,dx$$
$$\Big\downarrow \text{2340}$$
$$\frac{\int \frac{\sqrt{bx^2+a}(4b(cC+Bd)x^2-(aCd-4b(Bc+Ad))x+4Abc)}{x}dx}{4b}+\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$
$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{3b(4Abc-(aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx}{3b}+\frac{4}{3}\left(a+bx^2\right)^{3/2}(Bd+cC)}{4b}+\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 27

$$\frac{\int \frac{(4Abc-(aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx+\frac{4}{3}\left(a+bx^2\right)^{3/2}(Bd+cC)}{4b}+\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 535

$$\frac{\frac{1}{2}a\int \frac{8Abc-(aCd-4b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)))+\frac{4}{3}\left(a+bx^2\right)^{3/2}(Bd+cC)}{4b}+\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 538

$$\frac{\frac{1}{2}a\left(8Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-(aCd-4b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)))+\frac{4}{3}(}{4b}$$
$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}a\left(8Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-(aCd-4b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)}{4b}$$
$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}a\left(8Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)))+\frac{4}{3}}{4b}$$
$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\big\downarrow$ 243

$$\frac{\frac{1}{2}a\left(4Abc\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)))+}{4b}$$

$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{2}a\left(8Ac\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc)))}{4b}$$

$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{2}a\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-4b(Ad+Bc))}{\sqrt{b}}-\frac{8Abc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8Abc-x(aCd-4b(Ad+Bc))}{4b}$$

$$\frac{Cdx\left(a+bx^2\right)^{3/2}}{4b}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x,x]
```

output
```
(C*d*x*(a + b*x^2)^(3/2))/(4*b) + (((8*A*b*c - (a*C*d - 4*b*(B*c + A*d))*x
)*Sqrt[a + b*x^2])/2 + (4*(c*C + B*d)*(a + b*x^2)^(3/2))/3 + (a*(-(((a*C*d
 - 4*b*(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]) - (8*A*
b*c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.27

method	result
default	$Ad\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+Bc\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+Ac\left(\sqrt{b\,x^2+a}-\sqrt{a}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
A*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+B*
c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+A*c*
((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+1/3*B*d/b*
(b*x^2+a)^(3/2)+1/3*C*c/b*(b*x^2+a)^(3/2)+d*C*(1/4*x*(b*x^2+a)^(3/2)/b-1/4
*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 693, normalized size of antiderivative = 4.17

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/48*(24*A*sqrt(a)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 3*(4*B*a*b*c - (C*a^2 - 4*A*a*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d*x^3 + 8*B*a*b*d + 8*(C*b^2*c + B*b^
2*d)*x^2 + 8*(C*a*b + 3*A*b^2)*c + 3*(4*B*b^2*c + (C*a*b + 4*A*b^2)*d)*x)*
sqrt(b*x^2 + a))/b^2, 1/24*(12*A*sqrt(a)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(a) + 2*a)/x^2) - 3*(4*B*a*b*c - (C*a^2 - 4*A*a*b)*d)*sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*C*b^2*d*x^3 + 8*B*a*b*d + 8*(C*b^2*c
 + B*b^2*d)*x^2 + 8*(C*a*b + 3*A*b^2)*c + 3*(4*B*b^2*c + (C*a*b + 4*A*b^2)
*d)*x)*sqrt(b*x^2 + a))/b^2, 1/48*(48*A*sqrt(-a)*b^2*c*arctan(sqrt(b*x^2 +
 a)*sqrt(-a)/a) + 3*(4*B*a*b*c - (C*a^2 - 4*A*a*b)*d)*sqrt(b)*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d*x^3 + 8*B*a*b*d + 8*(C*
b^2*c + B*b^2*d)*x^2 + 8*(C*a*b + 3*A*b^2)*c + 3*(4*B*b^2*c + (C*a*b + 4*A
*b^2)*d)*x)*sqrt(b*x^2 + a))/b^2, 1/24*(24*A*sqrt(-a)*b^2*c*arctan(sqrt(b*
x^2 + a)*sqrt(-a)/a) - 3*(4*B*a*b*c - (C*a^2 - 4*A*a*b)*d)*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*C*b^2*d*x^3 + 8*B*a*b*d + 8*(C*b^2*c + B
*b^2*d)*x^2 + 8*(C*a*b + 3*A*b^2)*c + 3*(4*B*b^2*c + (C*a*b + 4*A*b^2)*d)*
x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 3.87 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.34

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = -A\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Aac}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}cx}{\sqrt{\frac{a}{bx^2}+1}} + Ad\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bc\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bd\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Cc\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Cd\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x,x)
```

output
```
-A*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x)) + A*a*c/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + A*sqrt(b)*c*x/sqrt(a/(b*x**2) + 1) + A*d*Piecewise((a*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
 B*c*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne
(b, 0)), (sqrt(a)*x, True)) + B*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x*
*2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*c*Piecewise(
(a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x
**2/2, True)) + C*d*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x
**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a
*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x
**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Cdx}{4\,b} - \frac{\sqrt{bx^2+a}Cadx}{8\,b} - \frac{Ca^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - A\sqrt{a}c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}Ac + \frac{(bx^2+a)^{\frac{3}{2}}Cc}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Bd}{3\,b} + \frac{1}{2}\sqrt{bx^2+a}(Bc+Ad)x + \frac{(Bc+Ad)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*C*d*x/b - 1/8*sqrt(b*x^2 + a)*C*a*d*x/b - 1/8*C*a^2*
d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A*sqrt(a)*c*arcsinh(a/(sqrt(a*b)*abs(x)
)) + sqrt(b*x^2 + a)*A*c + 1/3*(b*x^2 + a)^(3/2)*C*c/b + 1/3*(b*x^2 + a)^(
3/2)*B*d/b + 1/2*sqrt(b*x^2 + a)*(B*c + A*d)*x + 1/2*(B*c + A*d)*a*arcsinh
(b*x/sqrt(a*b))/sqrt(b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{24\sqrt{b\,x^2+a}\,a\,b^2c+12\sqrt{b\,x^2+a}\,a\,b^2dx+8\sqrt{b\,x^2+a}\,a\,b^2d+8\sqrt{b\,x^2+a}\,ab\,c^2+3\sqrt{b\,x^2+a}\,abcdx+12\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x)
```

output
```
(24*sqrt(a + b*x**2)*a*b**2*c + 12*sqrt(a + b*x**2)*a*b**2*d*x + 8*sqrt(a
+ b*x**2)*a*b**2*d + 8*sqrt(a + b*x**2)*a*b*c**2 + 3*sqrt(a + b*x**2)*a*b*
c*d*x + 12*sqrt(a + b*x**2)*b**3*c*x + 8*sqrt(a + b*x**2)*b**3*d*x**2 + 8*
sqrt(a + b*x**2)*b**2*c**2*x**2 + 6*sqrt(a + b*x**2)*b**2*c*d*x**3 + 24*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c - 24*
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c + 1
2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d - 3*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d + 12*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c)/(24*b**2)
```

3.8 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = (Bc+Ad)\sqrt{a+bx^2} + \frac{(2Abc+acC+aBd)x\sqrt{a+bx^2}}{2a} + \frac{Cd(a+bx^2)^{3/2}}{3b} - \frac{Ac(a+bx^2)^{3/2}}{ax} + \frac{(2Abc+acC+aBd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} - \sqrt{a}(Bc+Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
(A*d+B*c)*(b*x^2+a)^(1/2)+1/2*(2*A*b*c+B*a*d+C*a*c)*x*(b*x^2+a)^(1/2)/a+1/
3*C*d*(b*x^2+a)^(3/2)/b-A*c*(b*x^2+a)^(3/2)/a/x+1/2*(2*A*b*c+B*a*d+C*a*c)*
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-a^(1/2)*(A*d+B*c)*arctanh((b*x^
2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(2aCdx-6Ab(c-dx)+bx(3B(2c+dx)+Cx(3c+2dx)))}{6bx} + \frac{(2Abc+acC+aBd)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \sqrt{a}(Bc+Ad)\log(x)+\sqrt{a}(Bc+Ad)\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*C*d*x - 6*A*b*(c - d*x) + b*x*(3*B*(2*c + d*x) + C*x
*(3*c + 2*d*x))))/(6*b*x) + ((2*A*b*c + a*c*C + a*B*d)*ArcTanh[(Sqrt[b]*x)
/(-Sqrt[a] + Sqrt[a + b*x^2])])/Sqrt[b] - Sqrt[a]*(B*c + A*d)*Log[x] + Sqr
t[a]*(B*c + A*d)*Log[-Sqrt[a] + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2338, 25, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^2}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(aCdx^2+(2Abc+aCc+aBd)x+a(Bc+Ad)\right)}{x}dx}{a} - \frac{Ac\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(aCdx^2+(2Abc+aCc+aBd)x+a(Bc+Ad))}{x}dx}{a} - \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\int \frac{3b(a(Bc+Ad)+(2Abc+aCc+aBd)x)\sqrt{bx^2+a}}{x}dx}{3b} + \frac{aCd(a+bx^2)^{3/2}}{3b}}{a} - \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(a(Bc+Ad)+(2Abc+aCc+aBd)x)\sqrt{bx^2+a}}{x}dx + \frac{aCd(a+bx^2)^{3/2}}{3b}}{a} - \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 535

$$\frac{\frac{1}{2}a\int \frac{2a(Bc+Ad)+(2Abc+aCc+aBd)x}{x\sqrt{bx^2+a}}dx + \frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)+2a(Ad+Bc)) + \frac{aCd(a+bx^2)^{3/2}}{3b}}{a} - \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 538

$$\frac{\frac{1}{2}a\left((aBd+acC+2Abc)\int \frac{1}{\sqrt{bx^2+a}}dx + 2a(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right) + \frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)+2a(A}{a} \\ \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 224

$$\frac{\frac{1}{2}a\left((aBd+acC+2Abc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} + 2a(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right) + \frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)}{a} \\ \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 219

$$\frac{\frac{1}{2}a\left(2a(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+2Abc)}{\sqrt{b}}\right) + \frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)+2a(A}{a} \\ \frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{2}a\left(a(Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+2Abc)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)+2a(}{a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{2}a\left(\frac{2a(Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+2Abc)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)+2a(A}{a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{ax}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+2Abc)}{\sqrt{b}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad+Bc)\right)+\frac{1}{2}\sqrt{a+bx^2}(x(aBd+acC+2Abc)}{a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{ax}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^2,x]
```

output
```
-((A*c*(a + b*x^2)^(3/2))/(a*x)) + (((2*a*(B*c + A*d) + (2*A*b*c + a*c*C +
 a*B*d)*x)*Sqrt[a + b*x^2])/2 + (a*C*d*(a + b*x^2)^(3/2))/(3*b) + (a*(((2*
A*b*c + a*c*C + a*B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - 2*S
qrt[a]*(B*c + A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.20

method	result
default	$Bd\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+Cc\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+(Ad+Bc)\left(\sqrt{b\,x^{2}+\right.$
risch	$-\frac{Ac\sqrt{b\,x^{2}+a}}{x}+A\sqrt{b}\,c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)+\frac{Bad\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}+\frac{Cac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}-A\sqrt{a}$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
B*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+C*
c*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+(A*d
+B*c)*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+A*c*
(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))))+1/3*C*d*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 603, normalized size of antiderivative = 3.59

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$

$$= \left[\frac{3\,(Bad+(Ca+2\,Ab)c)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+6\,(Bbc+Abd)\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2}}{x}\right.}{12\,bx}\right.$$

$$-\frac{3\,(Bad+(Ca+2\,Ab)c)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(Bbc+Abd)\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(2\,C}{6\,bx}$$

$$-\frac{3\,(Bad+(Ca+2\,Ab)c)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-6\,(Bbc+Abd)\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(2\,Cbd}{6\,bx}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/12*(3*(B*a*d + (C*a + 2*A*b)*c)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) + 6*(B*b*c + A*b*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*C*b*d*x^3 - 6*A*b*c + 3*(C*b*c + B*b*d)*
x^2 + 2*(3*B*b*c + (C*a + 3*A*b)*d)*x)*sqrt(b*x^2 + a))/(b*x), -1/6*(3*(B*
a*d + (C*a + 2*A*b)*c)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(
B*b*c + A*b*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - (2*C*b*d*x^3 - 6*A*b*c + 3*(C*b*c + B*b*d)*x^2 + 2*(3*B*b*c + (C*a +
3*A*b)*d)*x)*sqrt(b*x^2 + a))/(b*x), 1/12*(12*(B*b*c + A*b*d)*sqrt(-a)*x*a
rctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(B*a*d + (C*a + 2*A*b)*c)*sqrt(b)*x*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*C*b*d*x^3 - 6*A*b*c
 + 3*(C*b*c + B*b*d)*x^2 + 2*(3*B*b*c + (C*a + 3*A*b)*d)*x)*sqrt(b*x^2 + a
))/(b*x), -1/6*(3*(B*a*d + (C*a + 2*A*b)*c)*sqrt(-b)*x*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 6*(B*b*c + A*b*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt
(-a)/a) - (2*C*b*d*x^3 - 6*A*b*c + 3*(C*b*c + B*b*d)*x^2 + 2*(3*B*b*c + (C
*a + 3*A*b)*d)*x)*sqrt(b*x^2 + a))/(b*x)]
```

Sympy [A] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$

$$= -\frac{A\sqrt{a}c}{x\sqrt{1+\frac{bx^2}{a}}} - A\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Aad}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + A\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{A\sqrt{b}dx}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Abcx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - B\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Bac}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}cx}{\sqrt{\frac{a}{bx^2}+1}} + Bd\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + Cc\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) + Cd\left(\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**2,x)
```

output
```
-A*sqrt(a)*c/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x
)) + A*a*d/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c*asinh(sqrt(b)*x/
sqrt(a)) + A*sqrt(b)*d*x/sqrt(a/(b*x**2) + 1) - A*b*c*x/(sqrt(a)*sqrt(1 +
b*x**2/a)) - B*sqrt(a)*c*asinh(sqrt(a)/(sqrt(b)*x)) + B*a*c/(sqrt(b)*x*sqr
t(a/(b*x**2) + 1)) + B*sqrt(b)*c*x/sqrt(a/(b*x**2) + 1) + B*d*Piecewise((a
*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x
*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)
*x, True)) + C*c*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) +
2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b
*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*d*Piecewise((a*sqrt(a + b*x**2
)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = A\sqrt{b}c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)+\frac{(bx^2+a)^{\frac{3}{2}}Cd}{3\,b} + \frac{1}{2}\sqrt{bx^2+a}(Cc+Bd)x + \frac{(Cc+Bd)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - (Bc+Ad)\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}(Bc+Ad) - \frac{\sqrt{bx^2+a}Ac}{x}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
A*sqrt(b)*c*arcsinh(b*x/sqrt(a*b)) + 1/3*(b*x^2 + a)^(3/2)*C*d/b + 1/2*sqr
t(b*x^2 + a)*(C*c + B*d)*x + 1/2*(C*c + B*d)*a*arcsinh(b*x/sqrt(a*b))/sqrt
(b) - (B*c + A*d)*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*
(B*c + A*d) - sqrt(b*x^2 + a)*A*c/x
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$
$$= \frac{2\,Aa\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{6}\sqrt{bx^2+a}\left(\left(2\,Cdx+\frac{3\,(Cbc+Bbd)}{b}\right)x+\frac{2\,(3\,Bbc+Cad+3\,Abd)}{b}\right) + \frac{2\,(Bac+Aad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{(Cac+2\,Abc+Bad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a*sqrt(b)*c/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a
)*((2*C*d*x + 3*(C*b*c + B*b*d)/b)*x + 2*(3*B*b*c + C*a*d + 3*A*b*d)/b) +
2*(B*a*c + A*a*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
 - 1/2*(C*a*c + 2*A*b*c + B*a*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sq
rt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$
$$= \frac{-24\sqrt{b\,x^2+a}\,abc+24\sqrt{b\,x^2+a}\,abdx+8\sqrt{b\,x^2+a}\,acdx+24\sqrt{b\,x^2+a}\,b^2cx+12\sqrt{b\,x^2+a}\,b^2d\,x^2+12}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a*b*c + 24*sqrt(a + b*x**2)*a*b*d*x + 8*sqrt(a + b
*x**2)*a*c*d*x + 24*sqrt(a + b*x**2)*b**2*c*x + 12*sqrt(a + b*x**2)*b**2*d
*x**2 + 12*sqrt(a + b*x**2)*b*c**2*x**2 + 8*sqrt(a + b*x**2)*b*c*d*x**3 +
24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x +
 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x
 - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*
x - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*
c*x + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*x + 12*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x + 12*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c**2*x - 24*sqrt(b)*a*b*c*x -
 3*sqrt(b)*a*b*d*x - 3*sqrt(b)*a*c**2*x)/(24*b*x)
```

3.9 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 202

$$\begin{aligned}\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx &= \frac{(Abc+2a(cC+Bd))\sqrt{a+bx^2}}{2a} \\ &+ \frac{(aCd+2b(Bc+Ad))x\sqrt{a+bx^2}}{2a} \\ &- \frac{Ac(a+bx^2)^{3/2}}{2ax^2} - \frac{(Bc+Ad)\,(a+bx^2)^{3/2}}{ax} \\ &+ \frac{(aCd+2b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} \\ &- \frac{(Abc+2a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

output
```
1/2*(A*b*c+2*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a+1/2*(a*C*d+2*b*(A*d+B*c))*x*(b
*x^2+a)^(1/2)/a-1/2*A*c*(b*x^2+a)^(3/2)/a/x^2-(A*d+B*c)*(b*x^2+a)^(3/2)/a/
x+1/2*(a*C*d+2*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-1/2
*(A*b*c+2*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= \frac{1}{2}\left(\frac{\sqrt{a+bx^2}(-A(c+2dx)+x(-2B(c-dx)+Cx(2c+dx)))}{x^2} + \frac{2Abc\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - 4\sqrt{a}(cC+Bd)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{(aCd+2b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^3,x]
```

output
```
((Sqrt[a + b*x^2]*(-(A*(c + 2*d*x)) + x*(-2*B*(c - d*x) + C*x*(2*c + d*x))
))/x^2 + (2*A*b*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a]
- 4*Sqrt[a]*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]]
- ((a*C*d + 2*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])
/2
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2338, 25, 2338, 25, 27, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^3}\,dx$$
$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(2aCdx^2+(Abc+2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^2}dx}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2aCdx^2+(Abc+2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^2}dx}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{a(Abc+2a(cC+Bd)+2(aCd+2b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a(Abc+2a(cC+Bd)+2(aCd+2b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(Abc+2a(cC+Bd)+2(aCd+2b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}a\int \frac{2(Abc+2a(cC+Bd)+(aCd+2b(Bc+Ad))x)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))+2a(Bd+cC)+Abc)-\frac{2(a+bx^2}{\ }}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{Abc+2a(cC+Bd)+(aCd+2b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))+2a(Bd+cC)+Abc)-\frac{2\left(a+bx^2\right)^{3/2}(}{x}}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{538}$$

$$\frac{a\left((aCd+2b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx+(2a(Bd+cC)+Abc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\big\downarrow$ 224

$$\frac{a\left((2a(Bd+cC)+Abc)\int \frac{1}{x\sqrt{bx^2+a}}dx+(aCd+2b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\big\downarrow$ 219

$$\frac{a\left((2a(Bd+cC)+Abc)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\big\downarrow$ 243

$$\frac{a\left(\frac{1}{2}(2a(Bd+cC)+Abc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad+B}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\big\downarrow$ 73

$$\frac{a\left(\frac{(2a(Bd+cC)+Abc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))+2}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\big\downarrow$ 221

$$\frac{a\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+2b(Ad+Bc))}{\sqrt{b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2a(Bd+cC)+Abc)}{\sqrt{a}}\right)+\sqrt{a+bx^2}(x(aCd+2b(Ad+Bc))+}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{2ax^2}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^3,x]
```

output
```
-1/2*(A*c*(a + b*x^2)^(3/2))/(a*x^2) + ((A*b*c + 2*a*(c*C + B*d) + (a*C*d
+ 2*b*(B*c + A*d))*x)*Sqrt[a + b*x^2] - (2*(B*c + A*d)*(a + b*x^2)^(3/2))/
x + a*(((a*C*d + 2*b*(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sq
rt[b] - ((A*b*c + 2*a*(c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[
a]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(2Adx+2Bcx+Ac)}{2x^2}-\frac{(Abc+2Bad+2Cac)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2\sqrt{a}}+A\sqrt{b}\,d\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)+B$
default	$dC\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+(Ad+Bc)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)+$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(2*A*d*x+2*B*c*x+A*c)/x^2-1/2*(A*b*c+2*B*a*d+2*C*a*c)
/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+A*b^(1/2)*d*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))+B*b^(1/2)*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+a*C*d*ln(b^(1/2)*
x+(b*x^2+a)^(1/2))/b^(1/2)+B*d*(b*x^2+a)^(1/2)+C*c*(b*x^2+a)^(1/2)+C*b*d*(
1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.38

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$

$$= \left[\frac{(2\,Babc + (Ca^2 + 2\,Aab)d)\sqrt{b}x^2 \log\left(-2\,bx^2 - 2\,\sqrt{bx^2+a}\sqrt{b}x - a\right) + (2\,Babd + (2\,Cab + Ab^2)c)\sqrt{a}x\cdots}{4\,a\cdots}\right.$$

$$-\frac{2\,(2\,Babc + (Ca^2 + 2\,Aab)d)\sqrt{-b}x^2 \arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,Babd + (2\,Cab + Ab^2)c)\sqrt{a}x^2 \log\left(-\frac{bx^2-2\cdots}{\cdots}\right.\cdots}{4\,abx^2}$$

$$-\frac{(2\,Babc + (Ca^2 + 2\,Aab)d)\sqrt{-b}x^2 \arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) - (2\,Babd + (2\,Cab + Ab^2)c)\sqrt{-a}x^2 \arctan\left(\frac{\sqrt{bx\cdots}}{\cdots}\right.\cdots}{2\,abx^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[1/4*((2*B*a*b*c + (C*a^2 + 2*A*a*b)*d)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(b)*x - a) + (2*B*a*b*d + (2*C*a*b + A*b^2)*c)*sqrt(a)*x^2*
log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(C*a*b*d*x^3 - A*a
*b*c + 2*(C*a*b*c + B*a*b*d)*x^2 - 2*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2 + a
))/(a*b*x^2), -1/4*(2*(2*B*a*b*c + (C*a^2 + 2*A*a*b)*d)*sqrt(-b)*x^2*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*a*b*d + (2*C*a*b + A*b^2)*c)*sqrt(a)*
x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(C*a*b*d*x^3 -
 A*a*b*c + 2*(C*a*b*c + B*a*b*d)*x^2 - 2*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2
 + a))/(a*b*x^2), 1/4*(2*(2*B*a*b*d + (2*C*a*b + A*b^2)*c)*sqrt(-a)*x^2*ar
ctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*B*a*b*c + (C*a^2 + 2*A*a*b)*d)*sqrt(
b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a*b*d*x^3 -
A*a*b*c + 2*(C*a*b*c + B*a*b*d)*x^2 - 2*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2
+ a))/(a*b*x^2), -1/2*((2*B*a*b*c + (C*a^2 + 2*A*a*b)*d)*sqrt(-b)*x^2*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*a*b*d + (2*C*a*b + A*b^2)*c)*sqrt(-a
)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (C*a*b*d*x^3 - A*a*b*c + 2*(C*a
*b*c + B*a*b*d)*x^2 - 2*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2 + a))/(a*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= -\frac{A\sqrt{a}d}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2x} + A\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$$
$$- \frac{Abc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{Abdx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{a}c}{x\sqrt{1+\frac{bx^2}{a}}} - B\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)$$
$$+ \frac{Bad}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + B\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{B\sqrt{b}dx}{\sqrt{\frac{a}{bx^2}+1}}$$
$$- \frac{Bbcx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - C\sqrt{a}c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Cac}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{C\sqrt{b}cx}{\sqrt{\frac{a}{bx^2}+1}}$$
$$+ Cd\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b \neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**3,x)
```

output
```
-A*sqrt(a)*d/(x*sqrt(1 + b*x**2/a)) - A*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*
x) + A*sqrt(b)*d*asinh(sqrt(b)*x/sqrt(a)) - A*b*c*asinh(sqrt(a)/(sqrt(b)*x
))/(2*sqrt(a)) - A*b*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*c/(x*sqr
t(1 + b*x**2/a)) - B*sqrt(a)*d*asinh(sqrt(a)/(sqrt(b)*x)) + B*a*d/(sqrt(b)
*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*c*asinh(sqrt(b)*x/sqrt(a)) + B*sqrt(b
)*d*x/sqrt(a/(b*x**2) + 1) - B*b*c*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C*sqrt
(a)*c*asinh(sqrt(a)/(sqrt(b)*x)) + C*a*c/(sqrt(b)*x*sqrt(a/(b*x**2) + 1))
+ C*sqrt(b)*c*x/sqrt(a/(b*x**2) + 1) + C*d*Piecewise((a*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx = \frac{1}{2}\sqrt{bx^2+a}Cdx + \frac{Cad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - \frac{Abc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Abc}{2a} + (Bc+Ad)\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - (Cc+Bd)\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}(Cc+Bd) - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{2ax^2} - \frac{\sqrt{bx^2+a}(Bc+Ad)}{x}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*d*x + 1/2*C*a*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2
*A*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*A*b*c/a
 + (B*c + A*d)*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (C*c + B*d)*sqrt(a)*arcsin
h(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*(C*c + B*d) - 1/2*(b*x^2 + a)^(3
/2)*A*c/(a*x^2) - sqrt(b*x^2 + a)*(B*c + A*d)/x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= \frac{1}{2}\left(Cdx+2\,Cc+2\,Bd\right)\sqrt{bx^2+a}+\frac{\left(2\,Cac+Abc+2\,Bad\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{\left(2\,Bbc+Cad+2\,Abd\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}d+\left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/2*(C*d*x + 2*C*c + 2*B*d)*sqrt(b*x^2 + a) + (2*C*a*c + A*b*c + 2*B*a*d)*
arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*(2*B*b*c +
C*a*d + 2*A*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + ((sqrt(b
)*x - sqrt(b*x^2 + a))^3*A*b*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqr
t(b)*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*d + (sqrt(b)*x - sq
rt(b*x^2 + a))*A*a*b*c - 2*B*a^2*sqrt(b)*c - 2*A*a^2*sqrt(b)*d)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= \frac{-\sqrt{b\,x^2+a}\,abc-2\sqrt{b\,x^2+a}\,abdx-2\sqrt{b\,x^2+a}\,b^2cx+2\sqrt{b\,x^2+a}\,b^2d\,x^2+2\sqrt{b\,x^2+a}\,b\,c^2x^2+\sqrt{b\,x^2}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x)
```

output
```
( - sqrt(a + b*x**2)*a*b*c - 2*sqrt(a + b*x**2)*a*b*d*x - 2*sqrt(a + b*x**
2)*b**2*c*x + 2*sqrt(a + b*x**2)*b**2*d*x**2 + 2*sqrt(a + b*x**2)*b*c**2*x
**2 + sqrt(a + b*x**2)*b*c*d*x**3 + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*b**2*c*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sq
rt(a) + sqrt(b)*x)/sqrt(a))*b**2*d*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2*x**2 - sqrt(a)*log((sqrt(a + b*x**2
) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**2 - 2*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*d*x**2 - 2*sqrt(a)*log((sqrt(a
+ b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2*x**2 + 2*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x**2 + sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a*c*d*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*b**2*c*x**2)/(2*b*x**2)
```

3.10 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 180

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = \frac{(bBc+Abd+2aCd)\sqrt{a+bx^2}}{2a} - \frac{(cC+Bd)\sqrt{a+bx^2}}{x} - \frac{Ac(a+bx^2)^{3/2}}{3ax^3} - \frac{(Bc+Ad)\left(a+bx^2\right)^{3/2}}{2ax^2} + \sqrt{b}(cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{(bBc+Abd+2aCd)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
1/2*(A*b*d+B*b*c+2*C*a*d)*(b*x^2+a)^(1/2)/a-(B*d+C*c)*(b*x^2+a)^(1/2)/x-1/
3*A*c*(b*x^2+a)^(3/2)/a/x^3-1/2*(A*d+B*c)*(b*x^2+a)^(3/2)/a/x^2+b^(1/2)*(B
*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-1/2*(A*b*d+B*b*c+2*C*a*d)*arcta
nh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-2Abcx^2-a(A(2c+3dx)+3x(2Cx(c-dx)+B(c+2dx))))}{6ax^3}$$
$$+2\sqrt{a}Cd\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{b(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$
$$-\sqrt{b}(cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*A*b*c*x^2 - a*(A*(2*c + 3*d*x) + 3*x*(2*C*x*(c - d*x)
 + B*(c + 2*d*x)))))/(6*a*x^3) + 2*Sqrt[a]*C*d*ArcTanh[(Sqrt[b]*x - Sqrt[a
 + b*x^2])/Sqrt[a]] - (b*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^
2])/Sqrt[a]])/Sqrt[a] - Sqrt[b]*(c*C + B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]]
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 27, 2338, 25, 27, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^4}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int -\frac{3\sqrt{bx^2+a}(aCdx^2+a(cC+Bd)x+a(Bc+Ad))}{x^3}dx}{3a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{bx^2+a}(aCdx^2+a(cC+Bd)x+a(Bc+Ad))}{x^3}dx}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{a(2a(cC+Bd)+(bBc+Abd+2aCd)x)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{(a+bx^2)^{3/2}(Ad+Bc)}{2x^2}}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(2a(cC+Bd)+(bBc+Abd+2aCd)x)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{(a+bx^2)^{3/2}(Ad+Bc)}{2x^2}}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}\int \frac{(2a(cC+Bd)+(bBc+Abd+2aCd)x)\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}(Ad+Bc)}{2x^2}}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 536$$

$$\frac{\frac{1}{2}\left(\int \frac{a(bBc+Abd+2aCd)+2ab(cC+Bd)x}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}\right)-\frac{(a+bx^2)^{3/2}(Ad+Bc)}{2x^2}}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{1}{2}\left(a(2aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx+2ab(Bd+cC)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}\right)-\frac{(a\cdots}{\cdots}}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{2}\left(a(2aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx+2ab(Bd+cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}\cdots\right.}{a}-\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{2}\left(a(2aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}+2a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC\right.}{a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{2}\left(\frac{1}{2}a(2aCd+Abd+bBc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}+2a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+\right.}{a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{2}\left(\frac{a(2aCd+Abd+bBc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}+2a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)\right)}{a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{2}\left(-\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2aCd+Abd+bBc)-\frac{\sqrt{a+bx^2}(2a(Bd+cC)-x(2aCd+Abd+bBc))}{x}+2a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(\right.}{a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{3ax^3}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^4,x]
```

output
```
-1/3*(A*c*(a + b*x^2)^(3/2))/(a*x^3) + (-1/2*((B*c + A*d)*(a + b*x^2)^(3/2
))/x^2 + (-(((2*a*(c*C + B*d) - (b*B*c + A*b*d + 2*a*C*d)*x)*Sqrt[a + b*x^
2])/x) + 2*a*Sqrt[b]*(c*C + B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - Sq
rt[a]*(b*B*c + A*b*d + 2*a*C*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(2Abc\,x^{2}+6Bad\,x^{2}+6Cac\,x^{2}+3Aadx+3Bacx+2Aac\right)}{6x^{3}a}-\frac{\left(Abd+Bbc+2aCd\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2\sqrt{a}}+B\sqrt{b}\,d\ln$
default	$\left(Ad+Bc\right)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)+\left(Bd+Cc\right)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b}}{}\right.}{}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(2*A*b*c*x^2+6*B*a*d*x^2+6*C*a*c*x^2+3*A*a*d*x+3*B*a*
c*x+2*A*a*c)/x^3/a-1/2*(A*b*d+B*b*c+2*C*a*d)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)+B*b^(1/2)*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+C*b^(1/2)*c*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))+d*C*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 618, normalized size of antiderivative = 3.43

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= \left[\frac{6\,(Cac+Bad)\sqrt{b}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+3\,(Bbc+(2\,Ca+Ab)d)\sqrt{a}x^3\log\left(-\frac{bx^2-2\,\sqrt{}}{}\right.}{12\,ax^3}\right.$$
$$-\frac{12\,(Cac+Bad)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(Bbc+(2\,Ca+Ab)d)\sqrt{a}x^3\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-}{12\,ax^3}$$
$$-\frac{6\,(Cac+Bad)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,(Bbc+(2\,Ca+Ab)d)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(6\,C}{6\,ax^3}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(6*(C*a*c + B*a*d)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 3*(B*b*c + (2*C*a + A*b)*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*C*a*d*x^3 - 2*A*a*c - 2*(3*B*a*d + (
3*C*a + A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/(a*x^3), -1/12
*(12*(C*a*c + B*a*d)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(
B*b*c + (2*C*a + A*b)*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - 2*(6*C*a*d*x^3 - 2*A*a*c - 2*(3*B*a*d + (3*C*a + A*b)*c)*
x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/(a*x^3), 1/6*(3*(B*b*c + (2*C*
a + A*b)*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(C*a*c + B
*a*d)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (6*C*a
*d*x^3 - 2*A*a*c - 2*(3*B*a*d + (3*C*a + A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x
)*sqrt(b*x^2 + a))/(a*x^3), -1/6*(6*(C*a*c + B*a*d)*sqrt(-b)*x^3*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) - 3*(B*b*c + (2*C*a + A*b)*d)*sqrt(-a)*x^3*arcta
n(sqrt(b*x^2 + a)*sqrt(-a)/a) - (6*C*a*d*x^3 - 2*A*a*c - 2*(3*B*a*d + (3*C
*a + A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/(a*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(165) = 330$.

Time = 4.36 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = -\frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Abd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{B\sqrt{a}d}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2x} + B\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bbc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{Bbdx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{a}c}{x\sqrt{1+\frac{bx^2}{a}}} - C\sqrt{a}d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Cad}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + C\sqrt{b}c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{C\sqrt{b}dx}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Cbcx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**4,x)
```

output
```
-A*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*sqrt(b)*d*sqrt(a/(b*x**2) +
 1)/(2*x) - A*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(3*a) - A*b*d*asinh(sqrt(a)/
(sqrt(b)*x))/(2*sqrt(a)) - B*sqrt(a)*d/(x*sqrt(1 + b*x**2/a)) - B*sqrt(b)*
c*sqrt(a/(b*x**2) + 1)/(2*x) + B*sqrt(b)*d*asinh(sqrt(b)*x/sqrt(a)) - B*b*
c*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)) - B*b*d*x/(sqrt(a)*sqrt(1 + b*x**
2/a)) - C*sqrt(a)*c/(x*sqrt(1 + b*x**2/a)) - C*sqrt(a)*d*asinh(sqrt(a)/(sq
rt(b)*x)) + C*a*d/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*sqrt(b)*c*asinh(sqr
t(b)*x/sqrt(a)) + C*sqrt(b)*d*x/sqrt(a/(b*x**2) + 1) - C*b*c*x/(sqrt(a)*sq
rt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = -C\sqrt{a}d\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\sqrt{bx^2+a}Cd + (Cc+Bd)\sqrt{b}\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(Bc+Ad)b\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}(Bc+Ad)b}{2a} - \frac{\sqrt{bx^2+a}(Cc+Bd)}{x} - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{3ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{2ax^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
-C*sqrt(a)*d*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*C*d + (C*c +
B*d)*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*(B*c + A*d)*b*arcsinh(a/(sqrt(a*
b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*(B*c + A*d)*b/a - sqrt(b*x^2 + a
)*(C*c + B*d)/x - 1/3*(b*x^2 + a)^(3/2)*A*c/(a*x^3) - 1/2*(b*x^2 + a)^(3/2
)*(B*c + A*d)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(152) = 304$.

Time = 0.23 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= \sqrt{bx^2+a}Cd - \left(C\sqrt{b}c + B\sqrt{b}d\right)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)$$
$$+ \frac{(Bbc + 2\,Cad + Abd)\arctan\left(-\frac{\sqrt{b}x - \sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+ \frac{3\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^5 Bbc + 3\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^5 Abd + 6\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4 Ca\sqrt{b}c + 6\left(\sqrt{b}x - \right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*C*d - (C*sqrt(b)*c + B*sqrt(b)*d)*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a))) + (B*b*c + 2*C*a*d + A*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2
 + a))/sqrt(-a))/sqrt(-a) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b*c +
 3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*C*a*sqrt(b)*c + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)*c + 6*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*C*a^2*sqrt(b)*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d -
 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 + a
))*A*a^2*b*d + 6*C*a^3*sqrt(b)*c + 2*A*a^2*b^(3/2)*c + 6*B*a^3*sqrt(b)*d)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= \frac{-4\sqrt{b\,x^2+a}\,a^2c-6\sqrt{b\,x^2+a}\,a^2dx-4\sqrt{b\,x^2+a}\,abc\,x^2-6\sqrt{b\,x^2+a}\,abcx-12\sqrt{b\,x^2+a}\,abd\,x^2-12\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x**2)*a**2*c - 6*sqrt(a + b*x**2)*a**2*d*x - 4*sqrt(a + b
*x**2)*a*b*c*x**2 - 6*sqrt(a + b*x**2)*a*b*c*x - 12*sqrt(a + b*x**2)*a*b*d
*x**2 - 12*sqrt(a + b*x**2)*a*c**2*x**2 + 12*sqrt(a + b*x**2)*a*c*d*x**3 +
 3*sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x -
 sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(
a)*x))*a*b*d*x**3 + 6*sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*s
qrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x*
*2) + sqrt(b)*sqrt(a)*x))*a*c*d*x**3 + 3*sqrt(a)*log(( - sqrt(a)*sqrt(a +
b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sq
rt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*b**2*c*x**3 - 3*sqrt(a)*log((
sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x
+ a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b*d*x**3 -
 6*sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sq
rt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*
x))*a*c*d*x**3 - 3*sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a
+ b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) +
sqrt(b)*sqrt(a)*x))*b**2*c*x**3 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a*b*d*x**3 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a*c**2*x**3 - 4*sqrt(b)*a*b*c*x**3 + 4*sqrt(b)*a*b*d*x**3 + 4*sqr
t(b)*a*c**2*x**3)/(12*a*x**3)
```

3.11 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 176

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = \frac{(Abc-4a(cC+Bd))\sqrt{a+bx^2}}{8ax^2} - \frac{Cd\sqrt{a+bx^2}}{x} - \frac{Ac(a+bx^2)^{3/2}}{4ax^4} - \frac{(Bc+Ad)\,(a+bx^2)^{3/2}}{3ax^3} + \sqrt{b}Cd\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{b(Abc-4a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
1/8*(A*b*c-4*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a/x^2-C*d*(b*x^2+a)^(1/2)/x-1/4*
A*c*(b*x^2+a)^(3/2)/a/x^4-1/3*(A*d+B*c)*(b*x^2+a)^(3/2)/a/x^3+b^(1/2)*C*d*
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))+1/8*b*(A*b*c-4*a*(B*d+C*c))*arctanh((b*
x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = -\frac{\sqrt{a+bx^2}(2aA(3c+4dx)+bx^2(3Ac+8Bcx+8Adx)+4ax(3Cx(c+2dx)+B(2c+3dx)))}{24ax^4} - \frac{Ab^2c\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{b(cC+Bd)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{b}Cd\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^5,x]
```

output
```
-1/24*(Sqrt[a + b*x^2]*(2*a*A*(3*c + 4*d*x) + b*x^2*(3*A*c + 8*B*c*x + 8*A
*d*x) + 4*a*x*(3*C*x*(c + 2*d*x) + B*(2*c + 3*d*x))))/(a*x^4) - (A*b^2*c*A
rcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(3/2)) - (b*(c*C + B*d
)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - Sqrt[b]*C*d
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2338, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^5}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(4aCdx^2-(Abc-4a(cC+Bd))x+4a(Bc+Ad))}{x^4}dx}{4a} - \frac{Ac\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(4aCdx^2-(Abc-4a(cC+Bd))x+4a(Bc+Ad))}{x^4}dx}{4a}-\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{3a(Abc-4a(cC+Bd)-4aCdx)\sqrt{bx^2+a}}{x^3}dx}{3a}-\frac{4(a+bx^2)^{3/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 27

$$\frac{-\int \frac{(Abc-4a(cC+Bd)-4aCdx)\sqrt{bx^2+a}}{x^3}dx-\frac{4(a+bx^2)^{3/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 537

$$\frac{\frac{1}{2}b\int -\frac{Abc-4a(cC+Bd)-8aCdx}{x\sqrt{bx^2+a}}dx+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)^{3/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 25

$$\frac{-\frac{1}{2}b\int \frac{Abc-4a(cC+Bd)-8aCdx}{x\sqrt{bx^2+a}}dx+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)^{3/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 538

$$\frac{-\frac{1}{2}b\left((Abc-4a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-8aCd\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)^{3/2}(Ad}{3x^3}}{4a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 224

$$\frac{-\frac{1}{2}b\left((Abc-4a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-8aCd\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx}{}}{4a}$$
$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 219

$$\frac{-\frac{1}{2}b\left(\left(Abc-4a(Bd+cC)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)\cdots}{\cdots}}{4a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\downarrow$ 243

$$\frac{-\frac{1}{2}b\left(\frac{1}{2}\left(Abc-4a(Bd+cC)\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+\cdots}{\cdots}}{4a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\downarrow$ 73

$$\frac{-\frac{1}{2}b\left(\frac{(Abc-4a(Bd+cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{8aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)^{3/2}(A\cdots}{3x^3}}{4a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

$\downarrow$ 221

$$\frac{-\frac{1}{2}b\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Abc-4a(Bd+cC))}{\sqrt{a}}-\frac{8aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{\sqrt{a+bx^2}(-4a(Bd+cC)-8aCdx+Abc)}{2x^2}-\frac{4(a+bx^2)\cdots}{\cdots}}{4a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{4ax^4}$$

input

```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^5,x]
```

output

```
-1/4*(A*c*(a + b*x^2)^(3/2))/(a*x^4) + (((A*b*c - 4*a*(c*C + B*d) - 8*a*C*
d*x)*Sqrt[a + b*x^2])/(2*x^2) - (4*(B*c + A*d)*(a + b*x^2)^(3/2))/(3*x^3)
- (b*((-8*a*C*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((A*b*c -
4*a*(c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(8Abd\,x^{3}+8Bbc\,x^{3}+24Cad\,x^{3}+3Abc\,x^{2}+12Bad\,x^{2}+12Cac\,x^{2}+8Aadx+8Bacx+6Aac\right)}{24x^{4}a}-\frac{b\left(-\frac{(Abc-4Bad-4Cac)\ln(\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}$
default	$-\frac{(Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}+(Bd+Cc)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)+Ac\left(-\frac{\left(b\,x^{2}+a\cdots\right.}{4a\,x^{4}}\cdots\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(8*A*b*d*x^3+8*B*b*c*x^3+24*C*a*d*x^3+3*A*b*c*x^2+12
*B*a*d*x^2+12*C*a*c*x^2+8*A*a*d*x+8*B*a*c*x+6*A*a*c)/x^4/a-1/8*b/a*(-(A*b*
c-4*B*a*d-4*C*a*c)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-8*a*C*d*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 734, normalized size of antiderivative = 4.17

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(24*C*a^2*sqrt(b)*d*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 3*(4*B*a*b*d + (4*C*a*b - A*b^2)*c)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(6*A*a^2*c + 8*(B*a*b*c + (3*C*a^2 + A
*a*b)*d)*x^3 + 3*(4*B*a^2*d + (4*C*a^2 + A*a*b)*c)*x^2 + 8*(B*a^2*c + A*a^
2*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4), -1/48*(48*C*a^2*sqrt(-b)*d*x^4*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(4*B*a*b*d + (4*C*a*b - A*b^2)*c)*sqrt(a)*
x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*A*a^2*c + 8
*(B*a*b*c + (3*C*a^2 + A*a*b)*d)*x^3 + 3*(4*B*a^2*d + (4*C*a^2 + A*a*b)*c)
*x^2 + 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4), 1/24*(12*C*a^2
*sqrt(b)*d*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3*(4*B*a*
b*d + (4*C*a*b - A*b^2)*c)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
 - (6*A*a^2*c + 8*(B*a*b*c + (3*C*a^2 + A*a*b)*d)*x^3 + 3*(4*B*a^2*d + (4*
C*a^2 + A*a*b)*c)*x^2 + 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4
), -1/24*(24*C*a^2*sqrt(-b)*d*x^4*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(
4*B*a*b*d + (4*C*a*b - A*b^2)*c)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(
-a)/a) + (6*A*a^2*c + 8*(B*a*b*c + (3*C*a^2 + A*a*b)*d)*x^3 + 3*(4*B*a^2*d
 + (4*C*a^2 + A*a*b)*c)*x^2 + 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a
^2*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(160) = 320$.

Time = 4.96 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.11

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = -\frac{Aac}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3A\sqrt{b}c}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}c}{8ax\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a} + \frac{Ab^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{B\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Bbd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{C\sqrt{a}d}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2x} + C\sqrt{b}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Cbc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{Cbdx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**5,x)
```

output
```
-A*a*c/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)*c/(8*x**3*sqrt(
a/(b*x**2) + 1)) - A*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*
c/(8*a*x*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(3*a) +
 A*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*sqrt(b)*c*sqrt(a/(b*
x**2) + 1)/(3*x**2) - B*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(3/2)*
c*sqrt(a/(b*x**2) + 1)/(3*a) - B*b*d*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)
) - C*sqrt(a)*d/(x*sqrt(1 + b*x**2/a)) - C*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/
(2*x) + C*sqrt(b)*d*asinh(sqrt(b)*x/sqrt(a)) - C*b*c*asinh(sqrt(a)/(sqrt(b
)*x))/(2*sqrt(a)) - C*b*d*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = C\sqrt{b}d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + \frac{Ab^2c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}Ab^2c}{8\,a^2} - \frac{(Cc+Bd)b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}(Cc+Bd)b}{2\,a} - \frac{\sqrt{bx^2+a}Cd}{x} + \frac{(bx^2+a)^{\frac{3}{2}}Abc}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Bc}{3\,ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ad}{3\,ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)}{2\,ax^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{4\,ax^4}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="maxima")
```

output
```
C*sqrt(b)*d*arcsinh(b*x/sqrt(a*b)) + 1/8*A*b^2*c*arcsinh(a/(sqrt(a*b)*abs(
x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*A*b^2*c/a^2 - 1/2*(C*c + B*d)*b*arcsinh
(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*(C*c + B*d)*b/a - sqr
t(b*x^2 + a)*C*d/x + 1/8*(b*x^2 + a)^(3/2)*A*b*c/(a^2*x^2) - 1/3*(b*x^2 +
a)^(3/2)*B*c/(a*x^3) - 1/3*(b*x^2 + a)^(3/2)*A*d/(a*x^3) - 1/2*(b*x^2 + a)
^(3/2)*(C*c + B*d)/(a*x^2) - 1/4*(b*x^2 + a)^(3/2)*A*c/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(148) = 296$.

Time = 0.23 (sec) , antiderivative size = 716, normalized size of antiderivative = 4.07

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = -C\sqrt{b}d\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{(4\,Cabc-Ab^2c+4\,Babd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a} + \frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cabc+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2c+12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Babd+24\left(\sqrt{b}x\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
-C*sqrt(b)*d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/4*(4*C*a*b*c - A*b
^2*c + 4*B*a*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a
)*a) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c + 3*(sqrt(b)*x - s
qrt(b*x^2 + a))^7*A*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b*d + 2
4*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a*b^(3/2)*c + 24*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*C*a^2*sqrt(b)*d + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2
)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c + 21*(sqrt(b)*x - sqrt(
b*x^2 + a))^5*A*a*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*d - 2
4*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c - 72*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*C*a^3*sqrt(b)*d - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^
(3/2)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c + 21*(sqrt(b)*x - s
qrt(b*x^2 + a))^3*A*a^2*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b
*d + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c + 72*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*C*a^4*sqrt(b)*d + 8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^
3*b^(3/2)*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b*c + 3*(sqrt(b)*x -
sqrt(b*x^2 + a))*A*a^3*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b*d
- 8*B*a^4*b^(3/2)*c - 24*C*a^5*sqrt(b)*d - 8*A*a^4*b^(3/2)*d)/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^4*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^5}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^5,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.32

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx$$
$$=\frac{-6\sqrt{b\,x^2+a}\,a^2c-8\sqrt{b\,x^2+a}\,a^2dx-3\sqrt{b\,x^2+a}\,abc\,x^2-8\sqrt{b\,x^2+a}\,abcx-8\sqrt{b\,x^2+a}\,abd\,x^3-12\sqrt{b}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2*c - 8*sqrt(a + b*x**2)*a**2*d*x - 3*sqrt(a + b
*x**2)*a*b*c*x**2 - 8*sqrt(a + b*x**2)*a*b*c*x - 8*sqrt(a + b*x**2)*a*b*d*
x**3 - 12*sqrt(a + b*x**2)*a*b*d*x**2 - 12*sqrt(a + b*x**2)*a*c**2*x**2 -
24*sqrt(a + b*x**2)*a*c*d*x**3 - 8*sqrt(a + b*x**2)*b**2*c*x**3 - 3*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 + 12*s
qrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*d*x**4 +
 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2*x
**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2
*c*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))
*b**2*d*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqr
t(a))*b*c**2*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a*c*d*x**4 - 4*sqrt(b)*a*b*d*x**4 + 12*sqrt(b)*a*c*d*x**4 - 4*sqrt(b)*b**
2*c*x**4)/(24*a*x**4)
```

3.12 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6} dx$

Optimal result

Integrand size = 30, antiderivative size = 170

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6} dx = -\frac{(4aCd-b(Bc+Ad))\sqrt{a+bx^2}}{8ax^2} - \frac{Ac(a+bx^2)^{3/2}}{5ax^5} - \frac{(Bc+Ad)(a+bx^2)^{3/2}}{4ax^4} + \frac{(2Abc-5a(cC+Bd))(a+bx^2)^{3/2}}{15a^2x^3} - \frac{b(4aCd-b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output

```
-1/8*(4*a*C*d-b*(A*d+B*c))*(b*x^2+a)^(1/2)/a/x^2-1/5*A*c*(b*x^2+a)^(3/2)/a
/x^5-1/4*(A*d+B*c)*(b*x^2+a)^(3/2)/a/x^4+1/15*(2*A*b*c-5*a*(B*d+C*c))*(b*x
^2+a)^(3/2)/a^2/x^3-1/8*b*(4*a*C*d-b*(A*d+B*c))*arctanh((b*x^2+a)^(1/2)/a^
(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(-16Ab^2cx^4+abx^2(A(8c+15dx)+5x(3Bc+8cCx+8Bdx))+a^2(6A(4c+5dx)+10x(3}{120a^2x^5}$$
$$+\frac{bCd\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{b^2(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(-16*A*b^2*c*x^4 + a*b*x^2*(A*(8*c + 15*d*x) + 5*x
*(3*B*c + 8*c*C*x + 8*B*d*x)) + a^2*(6*A*(4*c + 5*d*x) + 10*x*(3*B*c + 4*c
*C*x + 4*B*d*x + 6*C*d*x^2))))/(a^2*x^5) + (b*C*d*ArcTanh[(Sqrt[b]*x - Sqr
t[a + b*x^2])/Sqrt[a]])/Sqrt[a] + (b^2*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) +
 Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(3/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2338, 25, 2338, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^6}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(5aCdx^2-(2Abc-5a(cC+Bd))x+5a(Bc+Ad))}{x^5}dx}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(5aCdx^2-(2Abc-5a(cC+Bd))x+5a(Bc+Ad)\right)}{x^5}dx}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(4(2Abc-5a(cC+Bd))+5(bBc+Abd-4aCd)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \frac{(4(2Abc-5a(cC+Bd))+5(bBc+Abd-4aCd)x)\sqrt{bx^2+a}}{x^4}dx-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{4}\left(\frac{4\left(a+bx^2\right)^{3/2}(2Abc-5a(Bd+cC))}{3ax^3}-5(-4aCd+Abd+bBc)\int \frac{\sqrt{bx^2+a}}{x^3}dx\right)-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{4}\left(\frac{4\left(a+bx^2\right)^{3/2}(2Abc-5a(Bd+cC))}{3ax^3}-\frac{5}{2}(-4aCd+Abd+bBc)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2\right)-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{4}\left(\frac{4\left(a+bx^2\right)^{3/2}(2Abc-5a(Bd+cC))}{3ax^3}-\frac{5}{2}(-4aCd+Abd+bBc)\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{4}\left(\frac{4\left(a+bx^2\right)^{3/2}(2Abc-5a(Bd+cC))}{3ax^3}-\frac{5}{2}(-4aCd+Abd+bBc)\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{5\left(a+bx^2\right)^{3/2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{4}\left(\frac{4(a+bx^2)^{3/2}(2Abc-5a(Bd+cC))}{3ax^3}-\frac{5}{2}\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(-4aCd+Abd+bBc)\right)-\frac{5(a+bx^2)^{3/2}(Ad+}{4x^4}}{5a}$$

$$\frac{Ac(a+bx^2)^{3/2}}{5ax^5}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/5*(A*c*(a + b*x^2)^(3/2))/(a*x^5) + ((-5*(B*c + A*d)*(a + b*x^2)^(3/2))
/(4*x^4) + ((4*(2*A*b*c - 5*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(3*a*x^3) -
(5*(b*B*c + A*b*d - 4*a*C*d)*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/4)/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-16A\,b^2c\,x^4+40Babd\,x^4+40Cabc\,x^4+15Abd\,x^3a+15Bbc\,x^3a+60C\,a^2d\,x^3+8Aabc\,x^2+40B\,a^2d\,x^2+40C\,a^2c\,x^2+30A}{120x^5a^2}$
default	$(Ad+Bc)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)-\frac{(Bd+Cc)(b\,x^2+a)^{\frac{3}{2}}}{3a\,x^3}+A$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(-16*A*b^2*c*x^4+40*B*a*b*d*x^4+40*C*a*b*c*x^4+15*A
*a*b*d*x^3+15*B*a*b*c*x^3+60*C*a^2*d*x^3+8*A*a*b*c*x^2+40*B*a^2*d*x^2+40*C
*a^2*c*x^2+30*A*a^2*d*x+30*B*a^2*c*x+24*A*a^2*c)/x^5/a^2+1/8*(A*b*d+B*b*c-
4*C*a*d)/a^(3/2)*b*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx$$
$$= \left[\frac{15\,(Bb^2c-(4\,Cab-Ab^2)d)\sqrt{a}x^5\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(8\,(5\,Babd+(5\,Cab-2\,Ab^2)c)x^4+24}{24}\right.$$
$$-\frac{15\,(Bb^2c-(4\,Cab-Ab^2)d)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(8\,(5\,Babd+(5\,Cab-2\,Ab^2)c)x^4+24\,Aa}{120}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[1/240*(15*(B*b^2*c - (4*C*a*b - A*b^2)*d)*sqrt(a)*x^5*log(-(b*x^2 + 2*sqr
t(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*(5*B*a*b*d + (5*C*a*b - 2*A*b^2)*c
)*x^4 + 24*A*a^2*c + 15*(B*a*b*c + (4*C*a^2 + A*a*b)*d)*x^3 + 8*(5*B*a^2*d
 + (5*C*a^2 + A*a*b)*c)*x^2 + 30*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(
a^2*x^5), -1/120*(15*(B*b^2*c - (4*C*a*b - A*b^2)*d)*sqrt(-a)*x^5*arctan(s
qrt(b*x^2 + a)*sqrt(-a)/a) + (8*(5*B*a*b*d + (5*C*a*b - 2*A*b^2)*c)*x^4 +
24*A*a^2*c + 15*(B*a*b*c + (4*C*a^2 + A*a*b)*d)*x^3 + 8*(5*B*a^2*d + (5*C*
a^2 + A*a*b)*c)*x^2 + 30*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^2*x^5)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 440 vs. $2(153) = 306$.

Time = 6.29 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = -\frac{Aad}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{3A\sqrt{b}d}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15ax^2} - \frac{Ab^{\frac{3}{2}}d}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{2Ab^{\frac{5}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^2} + \frac{Ab^2d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{Bac}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3B\sqrt{b}c}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{B\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}c}{8ax\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a} + \frac{Bb^2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{C\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{C\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Cb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Cbd\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

input

```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**6,x)
```

output
```
-A*a*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - A*sqrt(b)*c*sqrt(a/(b*x**2)
 + 1)/(5*x**4) - 3*A*sqrt(b)*d/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*
c*sqrt(a/(b*x**2) + 1)/(15*a*x**2) - A*b**(3/2)*d/(8*a*x*sqrt(a/(b*x**2) +
 1)) + 2*A*b**(5/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**2) + A*b**2*d*asinh(sqrt
(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*a*c/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)
) - 3*B*sqrt(b)*c/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*sqrt(b)*d*sqrt(a/(b*x*
*2) + 1)/(3*x**2) - B*b**(3/2)*c/(8*a*x*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)
*d*sqrt(a/(b*x**2) + 1)/(3*a) + B*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**
(3/2)) - C*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*sqrt(b)*d*sqrt(a/(b
*x**2) + 1)/(2*x) - C*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(3*a) - C*b*d*asinh(
sqrt(a)/(sqrt(b)*x))/(2*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = -\frac{Cbd\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Cbd}{2\,a} + \frac{(Bc+Ad)b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}(Bc+Ad)b^2}{8\,a^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cd}{2\,ax^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cc}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Abc}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}Bd}{3\,ax^3} + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{5\,ax^5} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{4\,ax^4}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-1/2*C*b*d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*C*b
*d/a + 1/8*(B*c + A*d)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqr
t(b*x^2 + a)*(B*c + A*d)*b^2/a^2 - 1/2*(b*x^2 + a)^(3/2)*C*d/(a*x^2) - 1/3
*(b*x^2 + a)^(3/2)*C*c/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b*c/(a^2*x^3) -
1/3*(b*x^2 + a)^(3/2)*B*d/(a*x^3) + 1/8*(b*x^2 + a)^(3/2)*(B*c + A*d)*b/(a
^2*x^2) - 1/5*(b*x^2 + a)^(3/2)*A*c/(a*x^5) - 1/4*(b*x^2 + a)^(3/2)*(B*c +
 A*d)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(146) = 292$.

Time = 0.22 (sec) , antiderivative size = 751, normalized size of antiderivative = 4.42

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/4*(B*b^2*c - 4*C*a*b*d + A*b^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))
/sqrt(-a))/(sqrt(-a)*a) + 1/60*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*b^2*c
 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a*b*d + 15*(sqrt(b)*x - sqrt(b*x^2
 + a))^9*A*b^2*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^(3/2)*c + 120
*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a*b^(3/2)*d + 90*(sqrt(b)*x - sqrt(b*x^
2 + a))^7*B*a*b^2*c - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^2*b*d + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b^2*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*C*a^2*b^(3/2)*c + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2)*c -
 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)*d + 160*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*C*a^3*b^(3/2)*c + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^
2*b^(5/2)*c + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)*d - 90*(sq
rt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b^2*c + 120*(sqrt(b)*x - sqrt(b*x^2 + a
))^3*C*a^4*b*d - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b^2*d - 80*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2)*c + 80*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a^3*b^(5/2)*c - 80*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2)*d
 - 15*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b^2*c - 60*(sqrt(b)*x - sqrt(b*x
^2 + a))*C*a^5*b*d - 15*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^4*b^2*d + 40*C*a
^5*b^(3/2)*c - 16*A*a^4*b^(5/2)*c + 40*B*a^5*b^(3/2)*d)/(((sqrt(b)*x - sqr
t(b*x^2 + a))^2 - a)^5*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.66

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx$$

$$= \frac{-24\sqrt{b\,x^2+a}\,a^3c - 30\sqrt{b\,x^2+a}\,a^3dx - 8\sqrt{b\,x^2+a}\,a^2bc\,x^2 - 30\sqrt{b\,x^2+a}\,a^2bcx - 15\sqrt{b\,x^2+a}\,a^2bd\,x^3}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3*c - 30*sqrt(a + b*x**2)*a**3*d*x - 8*sqrt(a +
 b*x**2)*a**2*b*c*x**2 - 30*sqrt(a + b*x**2)*a**2*b*c*x - 15*sqrt(a + b*x*
*2)*a**2*b*d*x**3 - 40*sqrt(a + b*x**2)*a**2*b*d*x**2 - 40*sqrt(a + b*x**2
)*a**2*c**2*x**2 - 60*sqrt(a + b*x**2)*a**2*c*d*x**3 + 16*sqrt(a + b*x**2)
*a*b**2*c*x**4 - 15*sqrt(a + b*x**2)*a*b**2*c*x**3 - 40*sqrt(a + b*x**2)*a
*b**2*d*x**4 - 40*sqrt(a + b*x**2)*a*b*c**2*x**4 - 15*sqrt(a)*log((sqrt(a
+ b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5 + 60*sqrt(a)*log((
sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d*x**5 - 15*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**5 + 15*sq
rt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5
- 60*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d
*x**5 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b
**3*c*x**5 - 16*sqrt(b)*a*b**2*c*x**5 - 8*sqrt(b)*a*b**2*d*x**5 - 8*sqrt(b
)*a*b*c**2*x**5)/(120*a**2*x**5)
```

3.13 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7} dx$

Optimal result

Integrand size = 30, antiderivative size = 207

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7} dx = \frac{(Abc-2a(cC+Bd))\sqrt{a+bx^2}}{8ax^4} + \frac{b(Abc-2a(cC+Bd))\sqrt{a+bx^2}}{16a^2x^2} - \frac{Ac(a+bx^2)^{3/2}}{6ax^6} - \frac{(Bc+Ad)\,(a+bx^2)^{3/2}}{5ax^5} - \frac{(5aCd-2b(Bc+Ad))\,(a+bx^2)^{3/2}}{15a^2x^3} - \frac{b^2(Abc-2a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

output

```
1/8*(A*b*c-2*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(A*b*c-2*a*(B*d+C*c
))*(b*x^2+a)^(1/2)/a^2/x^2-1/6*A*c*(b*x^2+a)^(3/2)/a/x^6-1/5*(A*d+B*c)*(b*
x^2+a)^(3/2)/a/x^5-1/15*(5*a*C*d-2*b*(A*d+B*c))*(b*x^2+a)^(3/2)/a^2/x^3-1/
16*b^2*(A*b*c-2*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(b^2x^4(15Ac+32Bcx+32Adx)-4a^2(2A(5c+6dx)+x(5Cx(3c+4dx)+3B(4c+5dx)))-2abx^2(A(5c+8dx)+x(5Cx(3c+8dx)+B(8c+1}{x^6}}{240a^{5/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^7,x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(b^2*x^4*(15*A*c + 32*B*c*x + 32*A*d*x) - 4*a^2*
(2*A*(5*c + 6*d*x) + x*(5*C*x*(3*c + 4*d*x) + 3*B*(4*c + 5*d*x))) - 2*a*b*
x^2*(A*(5*c + 8*d*x) + x*(5*C*x*(3*c + 8*d*x) + B*(8*c + 15*d*x)))))/x^6 +
 30*A*b^3*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + 60*a*b^2*(c*C
 + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(240*a^(5/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2338, 27, 2338, 27, 539, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(2aCdx^2-(Abc-2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^6}dx}{6a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2aCdx^2-(Abc-2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^6}dx}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(5(Abc-2a(cC+Bd))-2(5aCd-2b(Bc+Ad))x)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{5x^5}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{5}\int \frac{(5(Abc-2a(cC+Bd))-2(5aCd-2b(Bc+Ad))x)\sqrt{bx^2+a}}{x^5}dx-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{5x^5}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{(8a(5aCd-2b(Bc+Ad))+5b(Abc-2a(cC+Bd))x)\sqrt{bx^2+a}}{x^4}dx}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{5x^5}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{5}\left(\frac{5b(Abc-2a(Bd+cC))\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{8\left(a+bx^2\right)^{3/2}(5aCd-2b(Ad+Bc))}{3x^3}}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{5x^5}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{5}\left(\frac{\frac{5}{2}b(Abc-2a(Bd+cC))\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{8\left(a+bx^2\right)^{3/2}(5aCd-2b(Ad+Bc))}{3x^3}}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2\left(a+bx^2\right)^{3/2}(Ad+Bc)}{5x^5}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{5}\left(\frac{\frac{5}{2}b(Abc-2a(Bd+cC))\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8\left(a+bx^2\right)^{3/2}(5aCd-2b(Ad+Bc))}{3x^3}}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2(a+b}{}}{2a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{5}\left(\frac{\frac{5}{2}b(Abc-2a(Bd+cC))\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8\left(a+bx^2\right)^{3/2}(5aCd-2b(Ad+Bc))}{3x^3}}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2(a+ba}{}}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{5}\left(\frac{\frac{5}{2}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(Abc-2a(Bd+cC))-\frac{8\left(a+bx^2\right)^{3/2}(5aCd-2b(Ad+Bc))}{3x^3}}{4a}+\frac{5\left(a+bx^2\right)^{3/2}(Abc-2a(Bd+cC))}{4ax^4}\right)-\frac{2}{}}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{6ax^6}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^7,x]
```

output
```
-1/6*(A*c*(a + b*x^2)^(3/2))/(a*x^6) + ((-2*(B*c + A*d)*(a + b*x^2)^(3/2))
/(5*x^5) + ((5*(A*b*c - 2*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(4*a*x^4) + ((
-8*(5*a*C*d - 2*b*(B*c + A*d))*(a + b*x^2)^(3/2))/(3*x^3) + (5*b*(A*b*c -
2*a*(c*C + B*d))*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a]))/2)/(4*a))/5)/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-32A\,b^{2}d\,x^{5}-32B\,b^{2}c\,x^{5}+80Cabd\,x^{5}-15A\,b^{2}c\,x^{4}+30Babd\,x^{4}+30Cabc\,x^{4}+16Abd\,x^{3}a+16Bbc\,x^{3}a+80C\,a^{2}d\,x^{3}+10A}{240x^{6}a^{2}}$
default	$(Ad+Bc)\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b(b\,x^{2}+a)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)+(Bd+Cc)\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\right.\right.}{2}\right)}{4a}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(-32*A*b^2*d*x^5-32*B*b^2*c*x^5+80*C*a*b*d*x^5-15*A
*b^2*c*x^4+30*B*a*b*d*x^4+30*C*a*b*c*x^4+16*A*a*b*d*x^3+16*B*a*b*c*x^3+80*
C*a^2*d*x^3+10*A*a*b*c*x^2+60*B*a^2*d*x^2+60*C*a^2*c*x^2+48*A*a^2*d*x+48*B
*a^2*c*x+40*A*a^2*c)/x^6/a^2-1/16*(A*b*c-2*B*a*d-2*C*a*c)*b^2/a^(5/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.14

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$

$$=\left[\frac{15\,(2\,Bab^2d+(2\,Cab^2-Ab^3)c)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(16\,(2\,Bab^2c-(5\,Ca^2b-2\,Aab^2)d}{}\right.$$

$$-\frac{15\,(2\,Bab^2d+(2\,Cab^2-Ab^3)c)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(16\,(2\,Bab^2c-(5\,Ca^2b-2\,Aab^2)d)x^5}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="fricas")
```

output
```
[1/480*(15*(2*B*a*b^2*d + (2*C*a*b^2 - A*b^3)*c)*sqrt(a)*x^6*log(-(b*x^2 +
 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(2*B*a*b^2*c - (5*C*a^2*b -
 2*A*a*b^2)*d)*x^5 - 40*A*a^3*c - 15*(2*B*a^2*b*d + (2*C*a^2*b - A*a*b^2)*
c)*x^4 - 16*(B*a^2*b*c + (5*C*a^3 + A*a^2*b)*d)*x^3 - 10*(6*B*a^3*d + (6*C
*a^3 + A*a^2*b)*c)*x^2 - 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/(a^3*x
^6), -1/240*(15*(2*B*a*b^2*d + (2*C*a*b^2 - A*b^3)*c)*sqrt(-a)*x^6*arctan(
sqrt(b*x^2 + a)*sqrt(-a)/a) - (16*(2*B*a*b^2*c - (5*C*a^2*b - 2*A*a*b^2)*d
)*x^5 - 40*A*a^3*c - 15*(2*B*a^2*b*d + (2*C*a^2*b - A*a*b^2)*c)*x^4 - 16*(
B*a^2*b*c + (5*C*a^3 + A*a^2*b)*d)*x^3 - 10*(6*B*a^3*d + (6*C*a^3 + A*a^2*
b)*c)*x^2 - 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(190) = 380$.

Time = 9.58 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.70

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = -\frac{Aac}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5A\sqrt{b}c}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{5x^4} + \frac{Ab^{\frac{3}{2}}c}{48ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{Ab^{\frac{5}{2}}c}{16a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Ab^{\frac{5}{2}}d\sqrt{\frac{a}{bx^2}+1}}{15a^2} - \frac{Ab^3c\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}} - \frac{Bad}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{3B\sqrt{b}d}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15ax^2} - \frac{Bb^{\frac{3}{2}}d}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{2Bb^{\frac{5}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^2} + \frac{Bb^2d\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{Cac}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3C\sqrt{b}c}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{C\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Cb^{\frac{3}{2}}c}{8ax\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a} + \frac{Cb^2c\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a*c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)*c/(24*x**5*sqrt
(a/(b*x**2) + 1)) - A*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(5*x**4) + A*b**(3/2)
*c/(48*a*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(1
5*a*x**2) + A*b**(5/2)*c/(16*a**2*x*sqrt(a/(b*x**2) + 1)) + 2*A*b**(5/2)*d
*sqrt(a/(b*x**2) + 1)/(15*a**2) - A*b**3*c*asinh(sqrt(a)/(sqrt(b)*x))/(16*
a**(5/2)) - B*a*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - B*sqrt(b)*c*sqrt
(a/(b*x**2) + 1)/(5*x**4) - 3*B*sqrt(b)*d/(8*x**3*sqrt(a/(b*x**2) + 1)) -
B*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(15*a*x**2) - B*b**(3/2)*d/(8*a*x*sqrt(a
/(b*x**2) + 1)) + 2*B*b**(5/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**2) + B*b**2*d
*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(3/2)) - C*a*c/(4*sqrt(b)*x**5*sqrt(a/(b
*x**2) + 1)) - 3*C*sqrt(b)*c/(8*x**3*sqrt(a/(b*x**2) + 1)) - C*sqrt(b)*d*s
qrt(a/(b*x**2) + 1)/(3*x**2) - C*b**(3/2)*c/(8*a*x*sqrt(a/(b*x**2) + 1)) -
 C*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(3*a) + C*b**2*c*asinh(sqrt(a)/(sqrt(b)
*x))/(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = &-\frac{Ab^3c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}Ab^3c}{16\,a^3} \\ &+ \frac{(Cc+Bd)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} \\ &- \frac{\sqrt{bx^2+a}(Cc+Bd)b^2}{8\,a^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ab^2c}{16\,a^3x^2} \\ &- \frac{(bx^2+a)^{\frac{3}{2}}Cd}{3\,ax^3} + \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)b}{8\,a^2x^2} \\ &+ \frac{(bx^2+a)^{\frac{3}{2}}Abc}{8\,a^2x^4} + \frac{2\,(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b}{15\,a^2x^3} \\ &- \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)}{4\,ax^4} \\ &- \frac{(bx^2+a)^{\frac{3}{2}}Ac}{6\,ax^6} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{5\,ax^5}\end{aligned}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="maxima")
```

output
```
-1/16*A*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*sqrt(b*x^2 + a)
*A*b^3*c/a^3 + 1/8*(C*c + B*d)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) -
 1/8*sqrt(b*x^2 + a)*(C*c + B*d)*b^2/a^2 - 1/16*(b*x^2 + a)^(3/2)*A*b^2*c/
(a^3*x^2) - 1/3*(b*x^2 + a)^(3/2)*C*d/(a*x^3) + 1/8*(b*x^2 + a)^(3/2)*(C*c
 + B*d)*b/(a^2*x^2) + 1/8*(b*x^2 + a)^(3/2)*A*b*c/(a^2*x^4) + 2/15*(b*x^2
+ a)^(3/2)*(B*c + A*d)*b/(a^2*x^3) - 1/4*(b*x^2 + a)^(3/2)*(C*c + B*d)/(a*
x^4) - 1/6*(b*x^2 + a)^(3/2)*A*c/(a*x^6) - 1/5*(b*x^2 + a)^(3/2)*(B*c + A*
d)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(179) = 358$.

Time = 0.18 (sec) , antiderivative size = 947, normalized size of antiderivative = 4.57

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
-1/8*(2*C*a*b^2*c - A*b^3*c + 2*B*a*b^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2
 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/120*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^
11*C*a*b^2*c - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*b^3*c + 30*(sqrt(b)*x
 - sqrt(b*x^2 + a))^11*B*a*b^2*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*
a^2*b^(3/2)*d + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2*c + 85*(sqrt
(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3*c + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^9
*B*a^2*b^2*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2)*c - 720*(
sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2)*d + 480*(sqrt(b)*x - sqrt(b*x
^2 + a))^8*A*a^2*b^(5/2)*d - 180*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^3*b^2
*c + 570*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c - 180*(sqrt(b)*x - sq
rt(b*x^2 + a))^7*B*a^3*b^2*d - 320*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b
^(5/2)*c + 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(3/2)*d - 320*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(5/2)*d - 180*(sqrt(b)*x - sqrt(b*x^2 +
 a))^5*C*a^4*b^2*c + 570*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^3*b^3*c - 180
*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^4*b^2*d - 480*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*C*a^5*b^(3/2)*d + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^5*b^2*c
 + 85*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^4*b^3*c + 150*(sqrt(b)*x - sqrt(
b*x^2 + a))^3*B*a^5*b^2*d - 192*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^5*b^(5
/2)*c + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^6*b^(3/2)*d - 192*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*A*a^5*b^(5/2)*d + 30*(sqrt(b)*x - sqrt(b*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^7,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$
$$= \frac{-40\sqrt{b\,x^2+a}\,a^3c - 48\sqrt{b\,x^2+a}\,a^3dx - 10\sqrt{b\,x^2+a}\,a^2bc\,x^2 - 48\sqrt{b\,x^2+a}\,a^2bcx - 16\sqrt{b\,x^2+a}\,a^2bd\,x^3}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c - 48*sqrt(a + b*x**2)*a**3*d*x - 10*sqrt(a
+ b*x**2)*a**2*b*c*x**2 - 48*sqrt(a + b*x**2)*a**2*b*c*x - 16*sqrt(a + b*x
**2)*a**2*b*d*x**3 - 60*sqrt(a + b*x**2)*a**2*b*d*x**2 - 60*sqrt(a + b*x**
2)*a**2*c**2*x**2 - 80*sqrt(a + b*x**2)*a**2*c*d*x**3 + 15*sqrt(a + b*x**2
)*a*b**2*c*x**4 - 16*sqrt(a + b*x**2)*a*b**2*c*x**3 + 32*sqrt(a + b*x**2)*
a*b**2*d*x**5 - 30*sqrt(a + b*x**2)*a*b**2*d*x**4 - 30*sqrt(a + b*x**2)*a*
b*c**2*x**4 - 80*sqrt(a + b*x**2)*a*b*c*d*x**5 + 32*sqrt(a + b*x**2)*b**3*
c*x**5 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*
b**3*c*x**6 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt
(a))*b**3*d*x**6 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*b**2*c**2*x**6 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**3*c*x**6 + 30*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**3*d*x**6 + 30*sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**6 - 32*sqrt(b)*a*b**2*d*x**6 - 32
*sqrt(b)*b**3*c*x**6)/(240*a**2*x**6)
```

3.14 $\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 248

$$\begin{aligned}\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = &-\frac{(2aCd-b(Bc+Ad))\sqrt{a+bx^2}}{8ax^4}\\ &-\frac{b(2aCd-b(Bc+Ad))\sqrt{a+bx^2}}{16a^2x^2}\\ &-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}-\frac{(Bc+Ad)\,(a+bx^2)^{3/2}}{6ax^6}\\ &+\frac{(4Abc-7a(cC+Bd))\,(a+bx^2)^{3/2}}{35a^2x^5}\\ &-\frac{2b(4Abc-7a(cC+Bd))\,(a+bx^2)^{3/2}}{105a^3x^3}\\ &+\frac{b^2(2aCd-b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}\end{aligned}$$

output
```
-1/8*(2*a*C*d-b*(A*d+B*c))*(b*x^2+a)^(1/2)/a/x^4-1/16*b*(2*a*C*d-b*(A*d+B*
c))*(b*x^2+a)^(1/2)/a^2/x^2-1/7*A*c*(b*x^2+a)^(3/2)/a/x^7-1/6*(A*d+B*c)*(b
*x^2+a)^(3/2)/a/x^6+1/35*(4*A*b*c-7*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a^2/x^5-2
/105*b*(4*A*b*c-7*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a^3/x^3+1/16*b^2*(2*a*C*d-b
*(A*d+B*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \frac{\frac{\sqrt{a+bx^2}\left(-128Ab^3cx^6+ab^2x^4(A(64c+105dx)+7x(15Bc+32cCx+32Bdx))-2a^2bx^2\left(A(24c+35dx)+7x\left(5Bc+8cCx+8Bdx+15Cdx^2\right)\right)-4a^3(1}{x^7}}{}$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^8,x]
```

output
```
((Sqrt[a + b*x^2]*(-128*A*b^3*c*x^6 + a*b^2*x^4*(A*(64*c + 105*d*x) + 7*x*
(15*B*c + 32*c*C*x + 32*B*d*x)) - 2*a^2*b*x^2*(A*(24*c + 35*d*x) + 7*x*(5*
B*c + 8*c*C*x + 8*B*d*x + 15*C*d*x^2)) - 4*a^3*(10*A*(6*c + 7*d*x) + 7*x*(
3*C*x*(4*c + 5*d*x) + 2*B*(5*c + 6*d*x)))))/x^7 - 420*a^(3/2)*b^2*C*d*ArcT
anh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 210*Sqrt[a]*b^3*(B*c + A*d)*A
rcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(1680*a^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {2338, 25, 2338, 27, 539, 25, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^8}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(7aCdx^2-(4Abc-7a(cC+Bd))x+7a(Bc+Ad)\right)}{x^7}dx}{7a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(7aCdx^2-(4Abc-7a(cC+Bd))x+7a(Bc+Ad)\right)}{x^7}dx}{7a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 2338

$$\frac{-\frac{\int \frac{3a(2(4Abc-7a(cC+Bd))+7(bBc+Abd-2aCd)x)\sqrt{bx^2+a}}{x^6}dx}{6a}-\frac{7(a+bx^2)^{3/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 27

$$\frac{-\frac{1}{2}\int \frac{(2(4Abc-7a(cC+Bd))+7(bBc+Abd-2aCd)x)\sqrt{bx^2+a}}{x^6}dx-\frac{7(a+bx^2)^{3/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 539

$$\frac{\frac{1}{2}\left(\frac{\int -\frac{(35a(bBc+Abd-2aCd)-4b(4Abc-7a(cC+Bd))x)\sqrt{bx^2+a}}{x^5}dx}{5a}+\frac{2(a+bx^2)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}\right)-\frac{7(a+bx^2)^{3/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{\int \frac{(35a(bBc+Abd-2aCd)-4b(4Abc-7a(cC+Bd))x)\sqrt{bx^2+a}}{x^5}dx}{5a}\right)-\frac{7(a+bx^2)^{3/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 539

$$\frac{\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{\int \frac{ab(16(4Abc-7a(cC+Bd))+35(bBc+Abd-2aCd)x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35(a+bx^2)^{3/2}(-2aCd+Abd+bBc)}{4x^4}}{5a}\right)-}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\int \frac{(16(4Abc-7a(cC+Bd))+35(bBc+Abd-2aCd)x)\sqrt{bx^2+a}}{x^4}dx-\frac{35(a+bx^2)^{3/2}(-2aCd+Abd+bBc)}{4x^4}}{5a}\right)-}{7a}-\frac{Ac(a+bx^2)^{3/2}}{7ax^7}$$

$\downarrow$ 534

$$\frac{1}{2}\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\left(35(-2aCd+Abd+bBc)\int\frac{\sqrt{bx^2+a}}{x^3}dx-\frac{16\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(-2aC}{4x^4}}{5a}}{7a}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 243

$$\frac{1}{2}\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(-2aCd+Abd+bBc)\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{16\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{3ax^3}\right)-\frac{35\left(a+bx^2\right)^{3/2}(-2a}{4x^4}}{5a}}{7a}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 51

$$\frac{1}{2}\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(-2aCd+Abd+bBc)\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{3ax^3}\right)-}{5a}}{7a}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 73

$$\frac{1}{2}\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}(-2aCd+Abd+bBc)\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{16\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{3ax^3}\right)-}{5a}}{7a}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC))}{5ax^5}-\frac{-\frac{1}{4}b\left(\frac{35}{2}\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(-2aCd+Abd+bBc)-\frac{16\left(a+bx^2\right)^{3/2}(4Abc-7a(Bd+cC}{3ax^3}}{5a}\right.}{7a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{7ax^7}$$

input
```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^8,x]
```

output
```
-1/7*(A*c*(a + b*x^2)^(3/2))/(a*x^7) + ((-7*(B*c + A*d)*(a + b*x^2)^(3/2))
/(6*x^6) + ((2*(4*A*b*c - 7*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(5*a*x^5) -
((-35*(b*B*c + A*b*d - 2*a*C*d)*(a + b*x^2)^(3/2))/(4*x^4) - (b*((-16*(4*A
*b*c - 7*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(3*a*x^3) + (35*(b*B*c + A*b*d
- 2*a*C*d)*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
Sqrt[a]))/2))/4)/(5*a))/2)/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(128 A b^3 c x^6-224 B a b^2 d x^6-224 C a b^2 c x^6-105 A a b^2 d x^5-105 B a b^2 c x^5+210 C a^2 b d x^5-64 A a b^2 c x^4+112 B a^2 b d x^4+}{1}$
default	$(Ad+Bc)\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{6a x^6}-\frac{b\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{4a x^4}-\frac{b\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{2a x^2}+\frac{b\left(\sqrt{b x^2+a}-\sqrt{a}\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}\right)+(Bd+$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^2+a)^(1/2)*(128*A*b^3*c*x^6-224*B*a*b^2*d*x^6-224*C*a*b^2*c*x
^6-105*A*a*b^2*d*x^5-105*B*a*b^2*c*x^5+210*C*a^2*b*d*x^5-64*A*a*b^2*c*x^4+
112*B*a^2*b*d*x^4+112*C*a^2*b*c*x^4+70*A*a^2*b*d*x^3+70*B*a^2*b*c*x^3+420*
C*a^3*d*x^3+48*A*a^2*b*c*x^2+336*B*a^3*d*x^2+336*C*a^3*c*x^2+280*A*a^3*d*x
+280*B*a^3*c*x+240*A*a^3*c)/x^7/a^3-1/16*(A*b*d+B*b*c-2*C*a*d)/a^(5/2)*b^2
*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$$

$$= \left[\frac{105\,(Bb^3c-(2\,Cab^2-Ab^3)d)\sqrt{a}x^7\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(32\,(7\,Bab^2d+(7\,Cab^2-4\,Ab^3)c)x^6}{}\right.$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="fricas")
```

output
```
[1/3360*(105*(B*b^3*c - (2*C*a*b^2 - A*b^3)*d)*sqrt(a)*x^7*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(32*(7*B*a*b^2*d + (7*C*a*b^2 - 4
*A*b^3)*c)*x^6 + 105*(B*a*b^2*c - (2*C*a^2*b - A*a*b^2)*d)*x^5 - 240*A*a^3
*c - 16*(7*B*a^2*b*d + (7*C*a^2*b - 4*A*a*b^2)*c)*x^4 - 70*(B*a^2*b*c + (6
*C*a^3 + A*a^2*b)*d)*x^3 - 48*(7*B*a^3*d + (7*C*a^3 + A*a^2*b)*c)*x^2 - 28
0*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/(a^3*x^7), 1/1680*(105*(B*b^3*c
- (2*C*a*b^2 - A*b^3)*d)*sqrt(-a)*x^7*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) +
 (32*(7*B*a*b^2*d + (7*C*a*b^2 - 4*A*b^3)*c)*x^6 + 105*(B*a*b^2*c - (2*C*a
^2*b - A*a*b^2)*d)*x^5 - 240*A*a^3*c - 16*(7*B*a^2*b*d + (7*C*a^2*b - 4*A*
a*b^2)*c)*x^4 - 70*(B*a^2*b*c + (6*C*a^3 + A*a^2*b)*d)*x^3 - 48*(7*B*a^3*d
 + (7*C*a^3 + A*a^2*b)*c)*x^2 - 280*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a)
)/(a^3*x^7)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(230) = 460$.

Time = 12.84 (sec) , antiderivative size = 916, normalized size of antiderivative = 3.69

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**8,x)
```

output

```
-15*A*a**5*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*c*x**2*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 17*A*a**3*b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*c*x**6*sqrt(a/
(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 12*A*a*b**(17/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 2
10*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a*d/(6*sqrt(b)*x**7*sqrt(a/(b
*x**2) + 1)) - 8*A*b**(19/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x
**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 5*A*sqrt(b)*d/(24*x**5*s
qrt(a/(b*x**2) + 1)) + A*b**(3/2)*d/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + A*b
**(5/2)*d/(16*a**2*x*sqrt(a/(b*x**2) + 1)) - A*b**3*d*asinh(sqrt(a)/(sqrt(
b)*x))/(16*a**(5/2)) - B*a*c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*B*s
qrt(b)*c/(24*x**5*sqrt(a/(b*x**2) + 1)) - B*sqrt(b)*d*sqrt(a/(b*x**2) + 1)
/(5*x**4) + B*b**(3/2)*c/(48*a*x**3*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*d*s
qrt(a/(b*x**2) + 1)/(15*a*x**2) + B*b**(5/2)*c/(16*a**2*x*sqrt(a/(b*x**2)
+ 1)) + 2*B*b**(5/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**2) - B*b**3*c*asinh(sqr
t(a)/(sqrt(b)*x))/(16*a**(5/2)) - C*a*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) +
1)) - C*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*C*sqrt(b)*d/(8*x**3*sq
rt(a/(b*x**2) + 1)) - C*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(15*a*x**2) - C...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \frac{Cb^2d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}Cb^2d}{8\,a^2} - \frac{(Bc+Ad)b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}(Bc+Ad)b^3}{16\,a^3} + \frac{(bx^2+a)^{\frac{3}{2}}Cbd}{8\,a^2x^2} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2c}{105\,a^3x^3} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cd}{4\,ax^4} + \frac{2\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)b}{15\,a^2x^3} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Abc}{35\,a^2x^5} + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)}{5\,ax^5} - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{7\,ax^7} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{6\,ax^6}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="maxima")
```

output
```
1/8*C*b^2*d*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*C*
b^2*d/a^2 - 1/16*(B*c + A*d)*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1
/16*sqrt(b*x^2 + a)*(B*c + A*d)*b^3/a^3 + 1/8*(b*x^2 + a)^(3/2)*C*b*d/(a^2
*x^2) - 8/105*(b*x^2 + a)^(3/2)*A*b^2*c/(a^3*x^3) - 1/16*(b*x^2 + a)^(3/2)
*(B*c + A*d)*b^2/(a^3*x^2) - 1/4*(b*x^2 + a)^(3/2)*C*d/(a*x^4) + 2/15*(b*x
^2 + a)^(3/2)*(C*c + B*d)*b/(a^2*x^3) + 4/35*(b*x^2 + a)^(3/2)*A*b*c/(a^2*
x^5) + 1/8*(b*x^2 + a)^(3/2)*(B*c + A*d)*b/(a^2*x^4) - 1/5*(b*x^2 + a)^(3/
2)*(C*c + B*d)/(a*x^5) - 1/7*(b*x^2 + a)^(3/2)*A*c/(a*x^7) - 1/6*(b*x^2 +
a)^(3/2)*(B*c + A*d)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(216) = 432$.

Time = 0.23 (sec) , antiderivative size = 1028, normalized size of antiderivative = 4.15

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
1/8*(B*b^3*c - 2*C*a*b^2*d + A*b^3*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a)
)/sqrt(-a))/(sqrt(-a)*a^2) - 1/840*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B
*b^3*c - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a*b^2*d + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^13*A*b^3*d - 700*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^
3*c - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b^2*d - 700*(sqrt(b)*x -
sqrt(b*x^2 + a))^11*A*a*b^3*d - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^
2*b^(5/2)*c - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2)*d - 3395
*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c + 2310*(sqrt(b)*x - sqrt(b*x^
2 + a))^9*C*a^3*b^2*d - 3395*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^2*b^3*d +
 5600*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2)*c - 8960*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*A*a^2*b^(7/2)*c + 5600*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
B*a^3*b^(5/2)*d - 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(5/2)*c - 4
480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(7/2)*c - 2240*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*B*a^4*b^(5/2)*d + 3395*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*
a^4*b^3*c - 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^5*b^2*d + 3395*(sqrt(
b)*x - sqrt(b*x^2 + a))^5*A*a^4*b^3*d + 1344*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*C*a^5*b^(5/2)*c - 2688*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2)*c
+ 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2)*d + 700*(sqrt(b)*x -
sqrt(b*x^2 + a))^3*B*a^5*b^3*c + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^6
*b^2*d + 700*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^5*b^3*d - 1568*(sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.34

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$$

$$=\frac{-48\sqrt{b\,x^2+a}\,a^3bc\,x^2-280\sqrt{b\,x^2+a}\,a^3bcx-70\sqrt{b\,x^2+a}\,a^3bd\,x^3-280\sqrt{b\,x^2+a}\,a^4dx-336\sqrt{b\,x^2+a}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x)
```

output

```
( - 240*sqrt(a + b*x**2)*a**4*c - 280*sqrt(a + b*x**2)*a**4*d*x - 48*sqrt(
a + b*x**2)*a**3*b*c*x**2 - 280*sqrt(a + b*x**2)*a**3*b*c*x - 70*sqrt(a +
b*x**2)*a**3*b*d*x**3 - 336*sqrt(a + b*x**2)*a**3*b*d*x**2 - 336*sqrt(a +
b*x**2)*a**3*c**2*x**2 - 420*sqrt(a + b*x**2)*a**3*c*d*x**3 + 64*sqrt(a +
b*x**2)*a**2*b**2*c*x**4 - 70*sqrt(a + b*x**2)*a**2*b**2*c*x**3 + 105*sqrt
(a + b*x**2)*a**2*b**2*d*x**5 - 112*sqrt(a + b*x**2)*a**2*b**2*d*x**4 - 11
2*sqrt(a + b*x**2)*a**2*b*c**2*x**4 - 210*sqrt(a + b*x**2)*a**2*b*c*d*x**5
 - 128*sqrt(a + b*x**2)*a*b**3*c*x**6 + 105*sqrt(a + b*x**2)*a*b**3*c*x**5
 + 224*sqrt(a + b*x**2)*a*b**3*d*x**6 + 224*sqrt(a + b*x**2)*a*b**2*c**2*x
**6 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
b**3*d*x**7 - 210*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*a*b**2*c*d*x**7 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**4*c*x**7 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
+ sqrt(b)*x)/sqrt(a))*a*b**3*d*x**7 + 210*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**7 - 105*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c*x**7 + 128*sqrt(b)*a*b**3*c
*x**7 - 224*sqrt(b)*a*b**3*d*x**7 - 224*sqrt(b)*a*b**2*c**2*x**7)/(1680*a*
*3*x**7)
```

3.15 $\int \frac{(c+dx)\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{x^9}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \frac{(5Abc-8a(cC+Bd))\sqrt{a+bx^2}}{48ax^6} + \frac{b(5Abc-8a(cC+Bd))\sqrt{a+bx^2}}{192a^2x^4} - \frac{b^2(5Abc-8a(cC+Bd))\sqrt{a+bx^2}}{128a^3x^2} - \frac{Ac(a+bx^2)^{3/2}}{8ax^8} - \frac{(Bc+Ad)\left(a+bx^2\right)^{3/2}}{7ax^7} - \frac{(7aCd-4b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{35a^2x^5} + \frac{2b(7aCd-4b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{105a^3x^3} + \frac{b^3(5Abc-8a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}$$

output
```
1/48*(5*A*b*c-8*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a/x^6+1/192*b*(5*A*b*c-8*a*(B
*d+C*c))*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b^2*(5*A*b*c-8*a*(B*d+C*c))*(b*x^2+
a)^(1/2)/a^3/x^2-1/8*A*c*(b*x^2+a)^(3/2)/a/x^8-1/7*(A*d+B*c)*(b*x^2+a)^(3/
2)/a/x^7-1/35*(7*a*C*d-4*b*(A*d+B*c))*(b*x^2+a)^(3/2)/a^2/x^5+2/105*b*(7*a
*C*d-4*b*(A*d+B*c))*(b*x^2+a)^(3/2)/a^3/x^3+1/128*b^3*(5*A*b*c-8*a*(B*d+C*
c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.71 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx$$
$$= \frac{-\frac{\sqrt{a}\sqrt{a+bx^2}(b^3x^6(525Ac+1024Bcx+1024Adx)+16a^3(15A(7c+8dx)+4x(7Cx(5c+6dx)+5B(6c+7dx)))+8a^2bx^2(A(35c+48dx)+2x(7Cx(5c}{x^8}}{}$$

input
```
Integrate[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^9,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(b^3*x^6*(525*A*c + 1024*B*c*x + 1024*A*d*x) +
 16*a^3*(15*A*(7*c + 8*d*x) + 4*x*(7*C*x*(5*c + 6*d*x) + 5*B*(6*c + 7*d*x)
)) + 8*a^2*b*x^2*(A*(35*c + 48*d*x) + 2*x*(7*C*x*(5*c + 8*d*x) + B*(24*c +
 35*d*x))) - 2*a*b^2*x^4*(A*(175*c + 256*d*x) + 4*x*(7*C*x*(15*c + 32*d*x)
 + B*(64*c + 105*d*x)))))/x^8) - 1050*A*b^4*c*ArcTanh[(Sqrt[b]*x - Sqrt[a
+ b*x^2])/Sqrt[a]] - 1680*a*b^3*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a
 + b*x^2])/Sqrt[a]])/(13440*a^(7/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 27, 539, 27, 539, 25, 27, 539, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)\left(A+Bx+Cx^2\right)}{x^9}\,dx$$

$\Big\downarrow$ 2338

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(8aCdx^2-(5Abc-8a(cC+Bd))x+8a(Bc+Ad)\right)}{x^8}dx}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\sqrt{bx^2+a}\left(8aCdx^2-(5Abc-8a(cC+Bd))x+8a(Bc+Ad)\right)}{x^8}dx}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(7(5Abc-8a(cC+Bd))-8(7aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x^7}dx}{7a}-\frac{8\left(a+bx^2\right)^{3/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{-\frac{1}{7}\int \frac{(7(5Abc-8a(cC+Bd))-8(7aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x^7}dx-\frac{8\left(a+bx^2\right)^{3/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{\int \frac{3(16a(7aCd-4b(Bc+Ad))+7b(5Abc-8a(cC+Bd))x)\sqrt{bx^2+a}}{x^6}dx}{6a}+\frac{7\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8\left(a+bx^2\right)^{3/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{\int \frac{(16a(7aCd-4b(Bc+Ad))+7b(5Abc-8a(cC+Bd))x)\sqrt{bx^2+a}}{x^6}dx}{2a}+\frac{7\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8\left(a+bx^2\right)^{3/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{-\frac{\int -\frac{ab(35(5Abc-8a(cC+Bd))-32(7aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{5x^5}}{2a}+\frac{7\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{6ax^6}\right)}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{7}\left(\frac{\frac{\int \frac{ab(35(5Abc-8a(cC+Bd))-32(7aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{5x^5}}{2a}+\frac{7\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{6ax^6}\right)-}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\int \frac{(35(5Abc-8a(cC+Bd))-32(7aCd-4b(Bc+Ad))x)\sqrt{bx^2+a}}{x^5}dx-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{5x^5}}{2a}+\frac{7\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{6ax^6}\right)-}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{\int \frac{(128a(7aCd-4b(Bc+Ad))+35b(5Abc-8a(cC+Bd))x)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{5x^5}}{2a}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{35b(5Abc-8a(Bd+cC))\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{128\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(A\cdots}{5x^5}}{2a}\cdots\right.}{8a}\cdots$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{\frac{35}{2}b(5Abc-8a(Bd+cC))\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{128\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-\frac{16\left(a+bx^2\right)^{3/2}(7aCd-4b(\cdots}{5x^5}}{2a}\cdots\right.}{8a}\cdots$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 51

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{\frac{35}{2}b(5Abc-8a(Bd+cC))\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-\frac{16\left(a+\cdots\right.}{\cdots}}{2a}\cdots\right.}{8a}\cdots$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 73

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{\frac{35}{2}b(5Abc-8a(Bd+cC))\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-\frac{16\left(a+b\right.}{}}{2a}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{5}b\left(-\frac{\frac{35}{2}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(5Abc-8a(Bd+cC))-\frac{128\left(a+bx^2\right)^{3/2}(7aCd-4b(Ad+Bc))}{3x^3}}{4a}-\frac{35\left(a+bx^2\right)^{3/2}(5Abc-8a(Bd+cC))}{4ax^4}\right)-}{2a}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{3/2}}{8ax^8}$$

input

```
Int[((c + d*x)*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^9,x]
```

output

```
-1/8*(A*c*(a + b*x^2)^(3/2))/(a*x^8) + ((-8*(B*c + A*d)*(a + b*x^2)^(3/2))
/(7*x^7) + ((7*(5*A*b*c - 8*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(6*a*x^6) +
((-16*(7*a*C*d - 4*b*(B*c + A*d))*(a + b*x^2)^(3/2))/(5*x^5) + (b*((-35*(5
*A*b*c - 8*a*(c*C + B*d))*(a + b*x^2)^(3/2))/(4*a*x^4) - ((-128*(7*a*C*d -
 4*b*(B*c + A*d))*(a + b*x^2)^(3/2))/(3*x^3) + (35*b*(5*A*b*c - 8*a*(c*C +
 B*d))*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt
[a]))/2)/(4*a)))/5)/(2*a))/7)/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(1024 A\,b^{3} d\,x^{7}+1024 B\,b^{3} c\,x^{7}-1792 C a\,b^{2} d\,x^{7}+525 A\,b^{3} c\,x^{6}-840 B a\,b^{2} d\,x^{6}-840 C a\,b^{2} c\,x^{6}-512 A a\,b^{2} d\,x^{5}-512 B a\,b^{2} c\,x}{}$
default	$(Ad+Bc)\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b(b\,x^{2}+a)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}\right)+(Bd+Cc)\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{6a\,x^{6}}-\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{4a\,x^{4}}\right.}{}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/13440*(b*x^2+a)^(1/2)*(1024*A*b^3*d*x^7+1024*B*b^3*c*x^7-1792*C*a*b^2*d
*x^7+525*A*b^3*c*x^6-840*B*a*b^2*d*x^6-840*C*a*b^2*c*x^6-512*A*a*b^2*d*x^5
-512*B*a*b^2*c*x^5+896*C*a^2*b*d*x^5-350*A*a*b^2*c*x^4+560*B*a^2*b*d*x^4+5
60*C*a^2*b*c*x^4+384*A*a^2*b*d*x^3+384*B*a^2*b*c*x^3+2688*C*a^3*d*x^3+280*
A*a^2*b*c*x^2+2240*B*a^3*d*x^2+2240*C*a^3*c*x^2+1920*A*a^3*d*x+1920*B*a^3*
c*x+1680*A*a^3*c)/x^8/a^3+1/128*(5*A*b*c-8*B*a*d-8*C*a*c)*b^3/a^(7/2)*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.06

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx$$
$$= \left[\frac{105\,(8\,Bab^3d+(8\,Cab^3-5\,Ab^4)c)\sqrt{a}x^8\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(256\,(4\,Bab^3c-(7\,Ca^2b^2-4\,A\right.$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="fricas")
```

output
```
[1/26880*(105*(8*B*a*b^3*d + (8*C*a*b^3 - 5*A*b^4)*c)*sqrt(a)*x^8*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(256*(4*B*a*b^3*c - (7*C*a
^2*b^2 - 4*A*a*b^3)*d)*x^7 - 105*(8*B*a^2*b^2*d + (8*C*a^2*b^2 - 5*A*a*b^3
)*c)*x^6 + 1680*A*a^4*c - 128*(4*B*a^2*b^2*c - (7*C*a^3*b - 4*A*a^2*b^2)*d
)*x^5 + 70*(8*B*a^3*b*d + (8*C*a^3*b - 5*A*a^2*b^2)*c)*x^4 + 384*(B*a^3*b*
c + (7*C*a^4 + A*a^3*b)*d)*x^3 + 280*(8*B*a^4*d + (8*C*a^4 + A*a^3*b)*c)*x
^2 + 1920*(B*a^4*c + A*a^4*d)*x)*sqrt(b*x^2 + a))/(a^4*x^8), 1/13440*(105*
(8*B*a*b^3*d + (8*C*a*b^3 - 5*A*b^4)*c)*sqrt(-a)*x^8*arctan(sqrt(b*x^2 + a
)*sqrt(-a)/a) - (256*(4*B*a*b^3*c - (7*C*a^2*b^2 - 4*A*a*b^3)*d)*x^7 - 105
*(8*B*a^2*b^2*d + (8*C*a^2*b^2 - 5*A*a*b^3)*c)*x^6 + 1680*A*a^4*c - 128*(4
*B*a^2*b^2*c - (7*C*a^3*b - 4*A*a^2*b^2)*d)*x^5 + 70*(8*B*a^3*b*d + (8*C*a
^3*b - 5*A*a^2*b^2)*c)*x^4 + 384*(B*a^3*b*c + (7*C*a^4 + A*a^3*b)*d)*x^3 +
 280*(8*B*a^4*d + (8*C*a^4 + A*a^3*b)*c)*x^2 + 1920*(B*a^4*c + A*a^4*d)*x)
*sqrt(b*x^2 + a))/(a^4*x^8)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1278 vs. 2(274) = 548.

Time = 29.04 (sec) , antiderivative size = 1278, normalized size of antiderivative = 4.44

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**9,x)
```

output

```
-15*A*a**5*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*d*x**2*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 17*A*a**3*b**(13/2)*d*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*d*x**6*sqrt(a/
(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 12*A*a*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 2
10*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a*c/(8*sqrt(b)*x**9*sqrt(a/(b
*x**2) + 1)) - 8*A*b**(19/2)*d*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x
**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 7*A*sqrt(b)*c/(48*x**7*s
qrt(a/(b*x**2) + 1)) + A*b**(3/2)*c/(192*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*
A*b**(5/2)*c/(384*a**2*x**3*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)*c/(128*a*
*3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a*
*(7/2)) - 15*B*a**5*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 +
210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**4*b**(11/2)*c*x**2*sqr
t(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6
*x**10) - 17*B*a**3*b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x
**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**2*b**(15/2)*c*x**
6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3
*b**6*x**10) - 12*B*a*b**(17/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \frac{5\,Ab^4c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}} - \frac{5\,\sqrt{bx^2+a}Ab^4c}{128\,a^4} - \frac{(Cc+Bd)b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}(Cc+Bd)b^3}{16\,a^3} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3c}{128\,a^4x^2} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Cbd}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)b^2}{16\,a^3x^2} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2c}{64\,a^3x^4} - \frac{8\,(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b^2}{105\,a^3x^3} - \frac{(bx^2+a)^{\frac{3}{2}}Cd}{5\,ax^5} + \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)b}{8\,a^2x^4} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Abc}{48\,a^2x^6} + \frac{4\,(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)}{6\,ax^6} - \frac{(bx^2+a)^{\frac{3}{2}}Ac}{8\,ax^8} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{7\,ax^7}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="maxima")
```

output
```
5/128*A*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/128*sqrt(b*x^2 + a
)*A*b^4*c/a^4 - 1/16*(C*c + B*d)*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2)
 + 1/16*sqrt(b*x^2 + a)*(C*c + B*d)*b^3/a^3 + 5/128*(b*x^2 + a)^(3/2)*A*b^
3*c/(a^4*x^2) + 2/15*(b*x^2 + a)^(3/2)*C*b*d/(a^2*x^3) - 1/16*(b*x^2 + a)^
(3/2)*(C*c + B*d)*b^2/(a^3*x^2) - 5/64*(b*x^2 + a)^(3/2)*A*b^2*c/(a^3*x^4)
 - 8/105*(b*x^2 + a)^(3/2)*(B*c + A*d)*b^2/(a^3*x^3) - 1/5*(b*x^2 + a)^(3/
2)*C*d/(a*x^5) + 1/8*(b*x^2 + a)^(3/2)*(C*c + B*d)*b/(a^2*x^4) + 5/48*(b*x
^2 + a)^(3/2)*A*b*c/(a^2*x^6) + 4/35*(b*x^2 + a)^(3/2)*(B*c + A*d)*b/(a^2*
x^5) - 1/6*(b*x^2 + a)^(3/2)*(C*c + B*d)/(a*x^6) - 1/8*(b*x^2 + a)^(3/2)*A
*c/(a*x^8) - 1/7*(b*x^2 + a)^(3/2)*(B*c + A*d)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(252) = 504$.

Time = 0.21 (sec) , antiderivative size = 1266, normalized size of antiderivative = 4.40

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
1/64*(8*C*a*b^3*c - 5*A*b^4*c + 8*B*a*b^3*d)*arctan(-(sqrt(b)*x - sqrt(b*x
^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 +
a))^15*C*a*b^3*c - 525*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*b^4*c + 840*(sqr
t(b)*x - sqrt(b*x^2 + a))^15*B*a*b^3*d - 6440*(sqrt(b)*x - sqrt(b*x^2 + a)
)^13*C*a^2*b^3*c + 4025*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4*c - 6440*
(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^2*b^3*d - 26880*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^3*b^(5/2)*d - 21560*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^3
*b^3*c - 13405*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4*c - 21560*(sqrt(
b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^3*d - 71680*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*B*a^3*b^(7/2)*c + 71680*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*b^(5/
2)*d - 71680*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(7/2)*d + 27160*(sqr
t(b)*x - sqrt(b*x^2 + a))^9*C*a^4*b^3*c - 97615*(sqrt(b)*x - sqrt(b*x^2 +
a))^9*A*a^3*b^4*c + 27160*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^4*b^3*d + 35
840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^4*b^(7/2)*c - 62720*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*C*a^5*b^(5/2)*d + 35840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
A*a^4*b^(7/2)*d + 27160*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^5*b^3*c - 9761
5*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^4*b^4*c + 27160*(sqrt(b)*x - sqrt(b*
x^2 + a))^7*B*a^5*b^3*d + 14336*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7
/2)*c + 28672*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^6*b^(5/2)*d + 14336*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(7/2)*d - 21560*(sqrt(b)*x - sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^9}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^9,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)*(A + B*x + C*x^2))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.19

$$\int \frac{(c+dx)\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx$$

$$= \frac{1024\sqrt{b}\,b^4c\,x^8 + 840\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3c^2x^8 + 525\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^4c\,x^8 - 840\sqrt{a}\log\left(\right.}{}$$

input

```
int((d*x+c)*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x)
```

output

```
( - 1680*sqrt(a + b*x**2)*a**4*c - 1920*sqrt(a + b*x**2)*a**4*d*x - 280*sq
rt(a + b*x**2)*a**3*b*c*x**2 - 1920*sqrt(a + b*x**2)*a**3*b*c*x - 384*sqrt
(a + b*x**2)*a**3*b*d*x**3 - 2240*sqrt(a + b*x**2)*a**3*b*d*x**2 - 2240*sq
rt(a + b*x**2)*a**3*c**2*x**2 - 2688*sqrt(a + b*x**2)*a**3*c*d*x**3 + 350*
sqrt(a + b*x**2)*a**2*b**2*c*x**4 - 384*sqrt(a + b*x**2)*a**2*b**2*c*x**3
+ 512*sqrt(a + b*x**2)*a**2*b**2*d*x**5 - 560*sqrt(a + b*x**2)*a**2*b**2*d
*x**4 - 560*sqrt(a + b*x**2)*a**2*b*c**2*x**4 - 896*sqrt(a + b*x**2)*a**2*
b*c*d*x**5 - 525*sqrt(a + b*x**2)*a*b**3*c*x**6 + 512*sqrt(a + b*x**2)*a*b
**3*c*x**5 - 1024*sqrt(a + b*x**2)*a*b**3*d*x**7 + 840*sqrt(a + b*x**2)*a*
b**3*d*x**6 + 840*sqrt(a + b*x**2)*a*b**2*c**2*x**6 + 1792*sqrt(a + b*x**2
)*a*b**2*c*d*x**7 - 1024*sqrt(a + b*x**2)*b**4*c*x**7 - 525*sqrt(a)*log((s
qrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c*x**8 + 840*sqrt(a)*
log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*d*x**8 + 840*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**8
 + 525*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*
c*x**8 - 840*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))
*b**4*d*x**8 - 840*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*b**3*c**2*x**8 + 1024*sqrt(b)*a*b**3*d*x**8 - 1792*sqrt(b)*a*b**2*c
*d*x**8 + 1024*sqrt(b)*b**4*c*x**8)/(13440*a**3*x**8)
```

3.16 $\int x^3(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 406

$$\begin{aligned}
&\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx \\
&= -\frac{a^2(8bc(Bc+2Ad)-5ad(2cC+Bd))x\sqrt{a+bx^2}}{128b^3} \\
&\quad+\frac{a(8bc(Bc+2Ad)-5ad(2cC+Bd))x^3\sqrt{a+bx^2}}{192b^2} \\
&\quad+\frac{(8bc(Bc+2Ad)-5ad(2cC+Bd))x^5\sqrt{a+bx^2}}{48b} \\
&\quad-\frac{a(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\left(a+bx^2\right)^{3/2}}{3b^4} \\
&\quad+\frac{d(2cC+Bd)x^5\left(a+bx^2\right)^{3/2}}{8b} \\
&\quad+\frac{(Ab(bc^2-2ad^2)+a(3aCd^2-2bc(cC+2Bd)))\left(a+bx^2\right)^{5/2}}{5b^4} \\
&\quad-\frac{(3aCd^2-b(c^2C+2Bcd+Ad^2))\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{Cd^2\left(a+bx^2\right)^{9/2}}{9b^4} \\
&\quad+\frac{a^3(8bc(Bc+2Ad)-5ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}
\end{aligned}$$

output
```
-1/128*a^2*(8*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/b^3+1/1
92*a*(8*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*x^3*(b*x^2+a)^(1/2)/b^2+1/48*(8
*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*x^5*(b*x^2+a)^(1/2)/b-1/3*a*(A*b*(-a*d
^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(3/2)/b^4+1/8*d*(B*d+2*C*
c)*x^5*(b*x^2+a)^(3/2)/b+1/5*(A*b*(-2*a*d^2+b*c^2)+a*(3*a*C*d^2-2*b*c*(2*B
*d+C*c)))*(b*x^2+a)^(5/2)/b^4-1/7*(3*a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x
^2+a)^(7/2)/b^4+1/9*C*d^2*(b*x^2+a)^(9/2)/b^4+1/128*a^3*(8*b*c*(2*A*d+B*c)
-5*a*d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.90

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$= \frac{\sqrt{a+bx^2}(-2048a^4Cd^2+a^3b(3072c^2C+6cd(1024B+525Cx)+d^2(3072A+1575Bx+1024Cx^2))+ \cdots}{}$$

input
```
Integrate[x^3*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(-2048*a^4*C*d^2 + a^3*b*(3072*c^2*C + 6*c*d*(1024*B + 52
5*C*x) + d^2*(3072*A + 1575*B*x + 1024*C*x^2)) + 16*b^4*x^4*(24*A*(21*c^2
+ 35*c*d*x + 15*d^2*x^2) + 5*x*(3*B*(28*c^2 + 48*c*d*x + 21*d^2*x^2) + 2*C
*x*(36*c^2 + 63*c*d*x + 28*d^2*x^2))) + 8*a*b^3*x^2*(12*A*(28*c^2 + 35*c*d
*x + 12*d^2*x^2) + x*(3*B*(70*c^2 + 96*c*d*x + 35*d^2*x^2) + 2*C*x*(72*c^2
 + 105*c*d*x + 40*d^2*x^2))) - 6*a^2*b^2*(8*A*(112*c^2 + 105*c*d*x + 32*d^
2*x^2) + x*(2*C*x*(128*c^2 + 175*c*d*x + 64*d^2*x^2) + B*(420*c^2 + 512*c*
d*x + 175*d^2*x^2)))) + 315*a^3*Sqrt[b]*(-8*b*c*(B*c + 2*A*d) + 5*a*d*(2*c
*C + B*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40320*b^4)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)\,dx$$

$\Big\downarrow$ 2185

$$\frac{\int -3(c+dx)^2\sqrt{bx^2+a}\left(bd^4(10cC-3Bd)x^4+d^3\left(12bCc^2-3Abd^2+2aCd^2\right)x^3+6cCd^2\left(bc^2+ad^2\right)x^2+c^2Cd\ldots\right.}{9bd^5}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-$$
$$\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(bd^4(10cC-3Bd)x^4+d^3\left(12bCc^2-3Abd^2+2aCd^2\right)x^3+6cCd^2\left(bc^2+ad^2\right)x^2+c^2Cd(bc\ldots\right.}{3bd^5}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-$$
$$\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b\left(16aCd^2-b\left(94Cc^2-57Bdc+24Ad^2\right)\right)x^3d^7+b\left(2b(46cC-21Bd)c^2+ad^2(2cC-15Bd)\right)x^2d^6+abc^2(34cC-15Bd)d^6+bc(b\ldots\right.}{8bd^4}}{3bd^5}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-$$
$$\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(-b\left(16aCd^2-b\left(94Cc^2-57Bdc+24Ad^2\right)\right)x^3d^7+b\left(2b(46cC-21Bd)c^2+\ldots\right.}{\ldots}}{3bd^5}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-$$
$$\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\ldots\right.\right.}{\ldots}}{\ldots}}{\ldots}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-}{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\right.}{}}{}}{}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4} -$$
$$\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd) - \frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(138bCc^2-1\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^6}{9bd^4}-$$

$$\frac{\frac{1}{8}d\left(a+bx^2\right)^{3/2}(c+dx)^5(10cC-3Bd)-\frac{-\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b^2\left(3ad^2(58cC-35Bd)-4bc\left(74Cc^2-69Bdc+60Ad^2\right)\right)x^2d^9+abc\left(64aCd^2-3 \cdots\right.}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[x^3*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

method	result
default	$c(2Ad+Bc)\left(\frac{x^3(bx^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+d(Bd+2Cc)\left(\frac{x^5(b\right.$
risch	$\frac{(4480Cd^2b^4x^8+5040Bb^4d^2x^7+10080Cb^4cdx^7+5760Ab^4d^2x^6+11520Bb^4cdx^6+640Cab^3d^2x^6+5760Cb^4c^2x^6+13440Ab^4cdx^5}{}$

input
```
int(x^3*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-
1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
)))+d*(B*d+2*C*c)*(1/8*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3
/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*
a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+(A*d^2+2*B*c*d+C*c^2)*(1/7*x^4
*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)
^(3/2)))+A*c^2*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+C*d^
2*(1/9*x^6*(b*x^2+a)^(3/2)/b-2/3*a/b*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1
/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.27

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[-1/80640*(315*(8*B*a^3*b*c^2 - 5*B*a^4*d^2 - 2*(5*C*a^4 - 8*A*a^3*b)*c*d)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4480*C*b^4*d
^2*x^8 + 5040*(2*C*b^4*c*d + B*b^4*d^2)*x^7 + 6144*B*a^3*b*c*d + 640*(9*C*
b^4*c^2 + 18*B*b^4*c*d + (C*a*b^3 + 9*A*b^4)*d^2)*x^6 + 840*(8*B*b^4*c^2 +
 B*a*b^3*d^2 + 2*(C*a*b^3 + 8*A*b^4)*c*d)*x^5 + 384*(6*B*a*b^3*c*d + 3*(C*
a*b^3 + 7*A*b^4)*c^2 - (2*C*a^2*b^2 - 3*A*a*b^3)*d^2)*x^4 + 210*(8*B*a*b^3
*c^2 - 5*B*a^2*b^2*d^2 - 2*(5*C*a^2*b^2 - 8*A*a*b^3)*c*d)*x^3 + 768*(4*C*a
^3*b - 7*A*a^2*b^2)*c^2 - 1024*(2*C*a^4 - 3*A*a^3*b)*d^2 - 128*(24*B*a^2*b
^2*c*d + 3*(4*C*a^2*b^2 - 7*A*a*b^3)*c^2 - 4*(2*C*a^3*b - 3*A*a^2*b^2)*d^2
)*x^2 - 315*(8*B*a^2*b^2*c^2 - 5*B*a^3*b*d^2 - 2*(5*C*a^3*b - 8*A*a^2*b^2)
*c*d)*x)*sqrt(b*x^2 + a))/b^4, -1/40320*(315*(8*B*a^3*b*c^2 - 5*B*a^4*d^2
- 2*(5*C*a^4 - 8*A*a^3*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (4480*C*b^4*d^2*x^8 + 5040*(2*C*b^4*c*d + B*b^4*d^2)*x^7 + 6144*B*a^3*b
*c*d + 640*(9*C*b^4*c^2 + 18*B*b^4*c*d + (C*a*b^3 + 9*A*b^4)*d^2)*x^6 + 84
0*(8*B*b^4*c^2 + B*a*b^3*d^2 + 2*(C*a*b^3 + 8*A*b^4)*c*d)*x^5 + 384*(6*B*a
*b^3*c*d + 3*(C*a*b^3 + 7*A*b^4)*c^2 - (2*C*a^2*b^2 - 3*A*a*b^3)*d^2)*x^4
+ 210*(8*B*a*b^3*c^2 - 5*B*a^2*b^2*d^2 - 2*(5*C*a^2*b^2 - 8*A*a*b^3)*c*d)*
x^3 + 768*(4*C*a^3*b - 7*A*a^2*b^2)*c^2 - 1024*(2*C*a^4 - 3*A*a^3*b)*d^2 -
 128*(24*B*a^2*b^2*c*d + 3*(4*C*a^2*b^2 - 7*A*a*b^3)*c^2 - 4*(2*C*a^3*b -
3*A*a^2*b^2)*d^2)*x^2 - 315*(8*B*a^2*b^2*c^2 - 5*B*a^3*b*d^2 - 2*(5*C*a...
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.88

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output

```
Piecewise((3*a**2*(2*A*a*c*d + B*a*c**2 - 5*a*(2*A*b*c*d + B*a*d**2 + B*b*
c**2 + 2*C*a*c*d - 7*a*(B*b*d**2 + 2*C*b*c*d)/(8*b))/(6*b))*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*
x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(C*d**2*x**8/9 - 3*a*x*(2*A*a*c*
d + B*a*c**2 - 5*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 7*a*(B*b
*d**2 + 2*C*b*c*d)/(8*b))/(6*b))/(8*b**2) - 2*a*(A*a*c**2 - 4*a*(A*a*d**2
+ A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 6*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/9
 + C*b*c**2)/(7*b))/(5*b))/(3*b**2) + x**7*(B*b*d**2 + 2*C*b*c*d)/(8*b) +
x**6*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/9 + C*b*c**2)/(7*b) + x**5*(2*A*b*c*
d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 7*a*(B*b*d**2 + 2*C*b*c*d)/(8*b))/(6
*b) + x**4*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 6*a*(A*b*d**2 + 2
*B*b*c*d + C*a*d**2/9 + C*b*c**2)/(7*b))/(5*b) + x**3*(2*A*a*c*d + B*a*c**
2 - 5*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 7*a*(B*b*d**2 + 2*C
*b*c*d)/(8*b))/(6*b))/(4*b) + x**2*(A*a*c**2 - 4*a*(A*a*d**2 + A*b*c**2 +
2*B*a*c*d + C*a*c**2 - 6*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/9 + C*b*c**2)/
(7*b))/(5*b))/(3*b)), Ne(b, 0)), (sqrt(a)*(A*c**2*x**4/4 + C*d**2*x**8/8 +
 x**7*(B*d**2 + 2*C*c*d)/7 + x**6*(A*d**2 + 2*B*c*d + C*c**2)/6 + x**5*(2*
A*c*d + B*c**2)/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.19

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{(bx^2+a)^{\frac{3}{2}}Cd^2x^6}{9\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Cad^2x^4}{21\,b^2}+\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x^5}{8\,b}+\frac{(bx^2+a)^{\frac{3}{2}}Ac^2x^2}{5\,b}+\frac{8\,(bx^2+a)^{\frac{3}{2}}Ca^2d^2x^2}{105\,b^3}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x^4}{7\,b}-\frac{5\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}ax^3}{48\,b^2}+\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Aac^2}{15\,b^2}-\frac{16\,(bx^2+a)^{\frac{3}{2}}Ca^3d^2}{315\,b^4}-\frac{4\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}ax^2}{35\,b^2}+\frac{5\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^3}-\frac{5\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}a^3x}{128\,b^3}-\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2}+\frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}a^2x}{16\,b^2}-\frac{5\,(2\,Ccd+Bd^2)a^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}+\frac{(Bc^2+2\,Acd)a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}+\frac{8\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}$$

input

```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/9*(b*x^2 + a)^(3/2)*C*d^2*x^6/b - 2/21*(b*x^2 + a)^(3/2)*C*a*d^2*x^4/b^2
 + 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x^5/b + 1/5*(b*x^2 + a)^(3/2)*A
*c^2*x^2/b + 8/105*(b*x^2 + a)^(3/2)*C*a^2*d^2*x^2/b^3 + 1/7*(C*c^2 + 2*B*
c*d + A*d^2)*(b*x^2 + a)^(3/2)*x^4/b - 5/48*(2*C*c*d + B*d^2)*(b*x^2 + a)^
(3/2)*a*x^3/b^2 + 1/6*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*x^3/b - 2/15*(b*
x^2 + a)^(3/2)*A*a*c^2/b^2 - 16/315*(b*x^2 + a)^(3/2)*C*a^3*d^2/b^4 - 4/35
*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 5/64*(2*C*c*d + B
*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b^3 - 5/128*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a
)*a^3*x/b^3 - 1/8*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*(B*c^
2 + 2*A*c*d)*sqrt(b*x^2 + a)*a^2*x/b^2 - 5/128*(2*C*c*d + B*d^2)*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(7/2) + 1/16*(B*c^2 + 2*A*c*d)*a^3*arcsinh(b*x/sqrt(a*
b))/b^(5/2) + 8/105*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 494, normalized size of antiderivative = 1.22

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{40320}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(2\left(7\left(8\,Cd^2x+\frac{9\,(2\,Cb^7cd+Bb^7d^2)}{b^7}\right)x+\frac{8\,(9\,Cb^7c^2+18\,Bb^7cd+Cab^{\cdots}}{b^7}\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{(8\,Ba^3bc^2-10\,Ca^4cd+16\,Aa^3bcd-5\,Ba^4d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/40320*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*(8*C*d^2*x + 9*(2*C*b^7*c*d + B*b
^7*d^2)/b^7)*x + 8*(9*C*b^7*c^2 + 18*B*b^7*c*d + C*a*b^6*d^2 + 9*A*b^7*d^2
)/b^7)*x + 21*(8*B*b^7*c^2 + 2*C*a*b^6*c*d + 16*A*b^7*c*d + B*a*b^6*d^2)/b
^7)*x + 48*(3*C*a*b^6*c^2 + 21*A*b^7*c^2 + 6*B*a*b^6*c*d - 2*C*a^2*b^5*d^2
 + 3*A*a*b^6*d^2)/b^7)*x + 105*(8*B*a*b^6*c^2 - 10*C*a^2*b^5*c*d + 16*A*a*
b^6*c*d - 5*B*a^2*b^5*d^2)/b^7)*x - 64*(12*C*a^2*b^5*c^2 - 21*A*a*b^6*c^2
+ 24*B*a^2*b^5*c*d - 8*C*a^3*b^4*d^2 + 12*A*a^2*b^5*d^2)/b^7)*x - 315*(8*B
*a^2*b^5*c^2 - 10*C*a^3*b^4*c*d + 16*A*a^2*b^5*c*d - 5*B*a^3*b^4*d^2)/b^7)
*x + 256*(12*C*a^3*b^4*c^2 - 21*A*a^2*b^5*c^2 + 24*B*a^3*b^4*c*d - 8*C*a^4
*b^3*d^2 + 12*A*a^3*b^4*d^2)/b^7) - 1/128*(8*B*a^3*b*c^2 - 10*C*a^4*c*d +
16*A*a^3*b*c*d - 5*B*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/
2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int x^3\,\sqrt{b\,x^2+a}\,(c+d\,x)^2\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x^3*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.99

$$\int x^3(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int(x^3*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output

```
(3072*sqrt(a + b*x**2)*a**4*b*d**2 - 2048*sqrt(a + b*x**2)*a**4*c*d**2 - 5
376*sqrt(a + b*x**2)*a**3*b**2*c**2 - 5040*sqrt(a + b*x**2)*a**3*b**2*c*d*
x + 6144*sqrt(a + b*x**2)*a**3*b**2*c*d - 1536*sqrt(a + b*x**2)*a**3*b**2*
d**2*x**2 + 1575*sqrt(a + b*x**2)*a**3*b**2*d**2*x + 3072*sqrt(a + b*x**2)
*a**3*b*c**3 + 3150*sqrt(a + b*x**2)*a**3*b*c**2*d*x + 1024*sqrt(a + b*x**
2)*a**3*b*c*d**2*x**2 + 2688*sqrt(a + b*x**2)*a**2*b**3*c**2*x**2 - 2520*s
qrt(a + b*x**2)*a**2*b**3*c**2*x + 3360*sqrt(a + b*x**2)*a**2*b**3*c*d*x**
3 - 3072*sqrt(a + b*x**2)*a**2*b**3*c*d*x**2 + 1152*sqrt(a + b*x**2)*a**2*
b**3*d**2*x**4 - 1050*sqrt(a + b*x**2)*a**2*b**3*d**2*x**3 - 1536*sqrt(a +
 b*x**2)*a**2*b**2*c**3*x**2 - 2100*sqrt(a + b*x**2)*a**2*b**2*c**2*d*x**3
 - 768*sqrt(a + b*x**2)*a**2*b**2*c*d**2*x**4 + 8064*sqrt(a + b*x**2)*a*b*
*4*c**2*x**4 + 1680*sqrt(a + b*x**2)*a*b**4*c**2*x**3 + 13440*sqrt(a + b*x
**2)*a*b**4*c*d*x**5 + 2304*sqrt(a + b*x**2)*a*b**4*c*d*x**4 + 5760*sqrt(a
 + b*x**2)*a*b**4*d**2*x**6 + 840*sqrt(a + b*x**2)*a*b**4*d**2*x**5 + 1152
*sqrt(a + b*x**2)*a*b**3*c**3*x**4 + 1680*sqrt(a + b*x**2)*a*b**3*c**2*d*x
**5 + 640*sqrt(a + b*x**2)*a*b**3*c*d**2*x**6 + 6720*sqrt(a + b*x**2)*b**5
*c**2*x**5 + 11520*sqrt(a + b*x**2)*b**5*c*d*x**6 + 5040*sqrt(a + b*x**2)*
b**5*d**2*x**7 + 5760*sqrt(a + b*x**2)*b**4*c**3*x**6 + 10080*sqrt(a + b*x
**2)*b**4*c**2*d*x**7 + 4480*sqrt(a + b*x**2)*b**4*c*d**2*x**8 + 5040*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*d - 1575*sqrt(...
```

3.17 $\int x^2(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 32, antiderivative size = 378

$$\begin{aligned}\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx &= \frac{a(8Ab(2bc^2-ad^2)+a(5aCd^2-8bc(cC+2Bd)))\,x\sqrt{a+bx^2}}{128b^3}\\
&+\frac{(8Ab(2bc^2-ad^2)+a(5aCd^2-8bc(cC+2Bd)))\,x^3\sqrt{a+bx^2}}{64b^2}\\
&-\frac{a(bc(Bc+2Ad)-ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{3b^3}\\
&-\frac{(5aCd^2-8b(c^2C+2Bcd+Ad^2))\,x^3\left(a+bx^2\right)^{3/2}}{48b^2}+\frac{Cd^2x^5\left(a+bx^2\right)^{3/2}}{8b}\\
&+\frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{d(2cC+Bd)\left(a+bx^2\right)^{7/2}}{7b^3}\\
&-\frac{a^2(8Ab(2bc^2-ad^2)+a(5aCd^2-8bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{7/2}}\end{aligned}$$

output
```
1/128*a*(8*A*b*(-a*d^2+2*b*c^2)+a*(5*a*C*d^2-8*b*c*(2*B*d+C*c)))*x*(b*x^2+
a)^(1/2)/b^3+1/64*(8*A*b*(-a*d^2+2*b*c^2)+a*(5*a*C*d^2-8*b*c*(2*B*d+C*c)))
*x^3*(b*x^2+a)^(1/2)/b^2-1/3*a*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*(b*x^2+a)
^(3/2)/b^3-1/48*(5*a*C*d^2-8*b*(A*d^2+2*B*c*d+C*c^2))*x^3*(b*x^2+a)^(3/2)/
b^2+1/8*C*d^2*x^5*(b*x^2+a)^(3/2)/b+1/5*(b*c*(2*A*d+B*c)-2*a*d*(B*d+2*C*c)
)*(b*x^2+a)^(5/2)/b^3+1/7*d*(B*d+2*C*c)*(b*x^2+a)^(7/2)/b^3-1/128*a^2*(8*A
*b*(-a*d^2+2*b*c^2)+a*(5*a*C*d^2-8*b*c*(2*B*d+C*c)))*arctanh(b^(1/2)*x/(b*
x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.98

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{b}\sqrt{a+bx^2}(a^3d(2048cC+1024Bd+525Cdx)-2a^2b(28Ad(64c+15dx)+8B(112c^2+105cdx+32d}{}$$

input
```
Integrate[x^2*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^3*d*(2048*c*C + 1024*B*d + 525*C*d*x) - 2*a^2*
b*(28*A*d*(64*c + 15*d*x) + 8*B*(112*c^2 + 105*c*d*x + 32*d^2*x^2) + C*x*(
420*c^2 + 512*c*d*x + 175*d^2*x^2)) + 16*b^3*x^3*(14*A*(15*c^2 + 24*c*d*x
+ 10*d^2*x^2) + x*(8*B*(21*c^2 + 35*c*d*x + 15*d^2*x^2) + 5*C*x*(28*c^2 +
48*c*d*x + 21*d^2*x^2))) + 8*a*b^2*x*(14*A*(15*c^2 + 16*c*d*x + 5*d^2*x^2)
 + x*(4*B*(28*c^2 + 35*c*d*x + 12*d^2*x^2) + C*x*(70*c^2 + 96*c*d*x + 35*d
^2*x^2)))) + 210*a^2*(16*A*b^2*c^2 + 5*a^2*C*d^2)*ArcTanh[(Sqrt[b]*x)/(Sqr
t[a] - Sqrt[a + b*x^2])] + 1680*a^3*b*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(S
qrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(13440*b^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(bd^3(19cC-8Bd)x^3+d^2\left(14bCc^2-8Abd^2+5aCd^2\right)x^2+cCd\left(3bc^2+10ad^2\right)x+5ac^2C\right.}{8bd^4}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$

$$\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(bd^3(19cC-8Bd)x^3+d^2\left(14bCc^2-8Abd^2+5aCd^2\right)x^2+cCd\left(3bc^2+10ad^2\right)x+5ac^2Cd^2\right.}{8bd^4}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$

$$\frac{\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(-b\left(35aCd^2-b\left(92Cc^2-80Bdc+56Ad^2\right)\right)x^2d^5+abc(41cC-32Bd)d^5+2b\left(6b(3cC-2Bd)c^2+ad^2(3cC-16Bd)\right)xd^4\right)dx}{7bd^3}+\frac{1}{7}}{8bd^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$

$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(-b\left(35aCd^2-b\left(92Cc^2-80Bdc+56Ad^2\right)\right)x^2d^5+abc(41cC-32Bd)d^5+2\right.}{7bd^3}}{8bd^4}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$

$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{\int -3bd^6(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc\left(5Cc^2-8Bdc+\right.\right.\right.}{6bd^2}}{7bd^3}}{8bd^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right)\right)}{\cdots}}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\right.\right.}{8bd^4}$$

$$\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc(5C\cdots\right.\right.}{\cdots}}{8bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-$$
$$\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int-(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc(5Cc\cdots\right.\right.}{\cdots}}{8bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc\left(5C\right.\right.\right.}{}}{8bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc\left(5Cc\right.\right.\right.}{}}{8bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc\left(5C\right.\right.\right.}{}}{8bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(10bCc^2-16bBdc+56Abd^2-35aCd^2\right)-b\left(ad^2(47cC-64Bd)-4bc\left(5Cc\right.\right.\right.}{}}{8bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^5}{8bd^3}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{3/2}(c+dx)^4(19cC-8Bd)-\frac{-\frac{1}{2}d^4\int -(c+dx)^2\left(ad\left(35aCd^2-2b\left(5Cc^2-8Bdc+28Ad^2\right)\right)+b\left(ad^2(47cC-64Bd)-4bc\left(5C\right.\right.\right.}{}}{8bd^4}$$

input
```
Int[x^2*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.01

method	result
default	$c(2Ad+Bc)\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)^{\frac{3}{2}}}{15b^2}\right)+d(Bd+2Cc)\left(\frac{x^4(b\,x^2+a)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{3}{2}}}{5b}-\frac{2a(b\,x^2+a)}{15b^2}\right.}{7b}\right.$
risch	$-\frac{(-1680C\,d^2b^3x^7-1920B\,b^3d^2x^6-3840C\,b^3cd\,x^6-2240A\,b^3d^2x^5-4480B\,b^3cd\,x^5-280Ca\,b^2d^2x^5-2240C\,b^3c^2x^5-5376A\,b^3cd\,x^4}{}$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+d*(B*
d+2*C*c)*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/1
5*a/b^2*(b*x^2+a)^(3/2)))+(A*d^2+2*B*c*d+C*c^2)*(1/6*x^3*(b*x^2+a)^(3/2)/b
-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(
1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+A*c^2*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a
/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+C*
d^2*(1/8*x^5*(b*x^2+a)^(3/2)/b-5/8*a/b*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*
(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 832, normalized size of antiderivative = 2.20

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[1/26880*(105*(16*B*a^3*b*c*d + 8*(C*a^3*b - 2*A*a^2*b^2)*c^2 - (5*C*a^4 -
 8*A*a^3*b)*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1680*C*b^4*d^2*x^7 - 1792*B*a^2*b^2*c^2 + 1024*B*a^3*b*d^2 + 1920*(2*C
*b^4*c*d + B*b^4*d^2)*x^6 + 280*(8*C*b^4*c^2 + 16*B*b^4*c*d + (C*a*b^3 + 8
*A*b^4)*d^2)*x^5 + 384*(7*B*b^4*c^2 + B*a*b^3*d^2 + 2*(C*a*b^3 + 7*A*b^4)*
c*d)*x^4 + 70*(16*B*a*b^3*c*d + 8*(C*a*b^3 + 6*A*b^4)*c^2 - (5*C*a^2*b^2 -
 8*A*a*b^3)*d^2)*x^3 + 512*(4*C*a^3*b - 7*A*a^2*b^2)*c*d + 128*(7*B*a*b^3*
c^2 - 4*B*a^2*b^2*d^2 - 2*(4*C*a^2*b^2 - 7*A*a*b^3)*c*d)*x^2 - 105*(16*B*a
^2*b^2*c*d + 8*(C*a^2*b^2 - 2*A*a*b^3)*c^2 - (5*C*a^3*b - 8*A*a^2*b^2)*d^2
)*x)*sqrt(b*x^2 + a))/b^4, -1/13440*(105*(16*B*a^3*b*c*d + 8*(C*a^3*b - 2*
A*a^2*b^2)*c^2 - (5*C*a^4 - 8*A*a^3*b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (1680*C*b^4*d^2*x^7 - 1792*B*a^2*b^2*c^2 + 1024*B*a^3*b*d^
2 + 1920*(2*C*b^4*c*d + B*b^4*d^2)*x^6 + 280*(8*C*b^4*c^2 + 16*B*b^4*c*d +
 (C*a*b^3 + 8*A*b^4)*d^2)*x^5 + 384*(7*B*b^4*c^2 + B*a*b^3*d^2 + 2*(C*a*b^
3 + 7*A*b^4)*c*d)*x^4 + 70*(16*B*a*b^3*c*d + 8*(C*a*b^3 + 6*A*b^4)*c^2 - (
5*C*a^2*b^2 - 8*A*a*b^3)*d^2)*x^3 + 512*(4*C*a^3*b - 7*A*a^2*b^2)*c*d + 12
8*(7*B*a*b^3*c^2 - 4*B*a^2*b^2*d^2 - 2*(4*C*a^2*b^2 - 7*A*a*b^3)*c*d)*x^2
- 105*(16*B*a^2*b^2*c*d + 8*(C*a^2*b^2 - 2*A*a*b^3)*c^2 - (5*C*a^3*b - 8*A
*a^2*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.77

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \begin{cases} -\frac{a\left(Aac^2-\frac{3a\left(Aad^2+Abc^2+2Bacd+Cac^2-\frac{5a\left(Abd^2+2Bbcd+\frac{Cad^2}{8}+Cbc^2\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+b}\cdots \\ \sqrt{a}\left(\frac{Ac^2x^3}{3}+\frac{Cd^2x^7}{7}+\frac{x^6\left(Bd^2+2Ccd\right)}{6}+\frac{x^5\left(Ad^2+2Bcd+Cc^2\right)}{5}+\frac{x^4\cdot\left(2Acd+Bc^2\right)}{4}\right) \end{cases}$$

input
```
integrate(x**2*(d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a*c**2 - 3*a*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2
- 5*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/8 + C*b*c**2)/(6*b))/(4*b))*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*d**2*x**7/8 - 2*a*(2*A*a*
c*d + B*a*c**2 - 4*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 6*a*(B
*b*d**2 + 2*C*b*c*d)/(7*b))/(5*b))/(3*b**2) + x**6*(B*b*d**2 + 2*C*b*c*d)/
(7*b) + x**5*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/8 + C*b*c**2)/(6*b) + x**4*(
2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 6*a*(B*b*d**2 + 2*C*b*c*d)/(
7*b))/(5*b) + x**3*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 5*a*(A*b*
d**2 + 2*B*b*c*d + C*a*d**2/8 + C*b*c**2)/(6*b))/(4*b) + x**2*(2*A*a*c*d +
 B*a*c**2 - 4*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 6*a*(B*b*d*
*2 + 2*C*b*c*d)/(7*b))/(5*b))/(3*b) + x*(A*a*c**2 - 3*a*(A*a*d**2 + A*b*c*
*2 + 2*B*a*c*d + C*a*c**2 - 5*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/8 + C*b*c
**2)/(6*b))/(4*b))/(2*b)), Ne(b, 0)), (sqrt(a)*(A*c**2*x**3/3 + C*d**2*x**
7/7 + x**6*(B*d**2 + 2*C*c*d)/6 + x**5*(A*d**2 + 2*B*c*d + C*c**2)/5 + x**
4*(2*A*c*d + B*c**2)/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.16

$$\begin{aligned}
&\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx\\
&=\frac{(bx^2+a)^{\frac{3}{2}}Cd^2x^5}{8\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Cad^2x^3}{48\,b^2}+\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x^4}{7\,b}\\
&+\frac{(bx^2+a)^{\frac{3}{2}}Ac^2x}{4\,b}-\frac{\sqrt{bx^2+a}Aac^2x}{8\,b}+\frac{5\,(bx^2+a)^{\frac{3}{2}}Ca^2d^2x}{64\,b^3}\\
&-\frac{5\,\sqrt{bx^2+a}Ca^3d^2x}{128\,b^3}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b}\\
&-\frac{Aa^2c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}-\frac{5\,Ca^4d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{7}{2}}}-\frac{4\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}ax^2}{35\,b^2}\\
&+\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}x^2}{5\,b}-\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2}\\
&+\frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}a^2x}{16\,b^2}+\frac{(Cc^2+2\,Bcd+Ad^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}\\
&+\frac{8\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}-\frac{2\,(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}
\end{aligned}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/8*(b*x^2 + a)^(3/2)*C*d^2*x^5/b - 5/48*(b*x^2 + a)^(3/2)*C*a*d^2*x^3/b^2
 + 1/7*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x^4/b + 1/4*(b*x^2 + a)^(3/2)*A
*c^2*x/b - 1/8*sqrt(b*x^2 + a)*A*a*c^2*x/b + 5/64*(b*x^2 + a)^(3/2)*C*a^2*
d^2*x/b^3 - 5/128*sqrt(b*x^2 + a)*C*a^3*d^2*x/b^3 + 1/6*(C*c^2 + 2*B*c*d +
 A*d^2)*(b*x^2 + a)^(3/2)*x^3/b - 1/8*A*a^2*c^2*arcsinh(b*x/sqrt(a*b))/b^(
3/2) - 5/128*C*a^4*d^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 4/35*(2*C*c*d + B*
d^2)*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 1/5*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)
*x^2/b - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*(C
*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/16*(C*c^2 + 2*B*c*d
+ A*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/105*(2*C*c*d + B*d^2)*(b*x
^2 + a)^(3/2)*a^2/b^3 - 2/15*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.16

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{13440}\sqrt{bx^2+a}\left(\left(\left(2\left(\left(\left(4\left(5\left(6\left(7\,Cd^2x+\frac{8\,(2\,Cb^6cd+Bb^6d^2)}{b^6}\right)x+\frac{7\,(8\,Cb^6c^2+16\,Bb^6cd+Cab^5d^2}{b^6}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{(8\,Ca^3bc^2-16\,Aa^2b^2c^2+16\,Ba^3bcd-5\,Ca^4d^2+8\,Aa^3bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{7}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*C*d^2*x + 8*(2*C*b^6*c*d + B*b^6*
d^2)/b^6)*x + 7*(8*C*b^6*c^2 + 16*B*b^6*c*d + C*a*b^5*d^2 + 8*A*b^6*d^2)/b
^6)*x + 48*(7*B*b^6*c^2 + 2*C*a*b^5*c*d + 14*A*b^6*c*d + B*a*b^5*d^2)/b^6)
*x + 35*(8*C*a*b^5*c^2 + 48*A*b^6*c^2 + 16*B*a*b^5*c*d - 5*C*a^2*b^4*d^2 +
 8*A*a*b^5*d^2)/b^6)*x + 64*(7*B*a*b^5*c^2 - 8*C*a^2*b^4*c*d + 14*A*a*b^5*
c*d - 4*B*a^2*b^4*d^2)/b^6)*x - 105*(8*C*a^2*b^4*c^2 - 16*A*a*b^5*c^2 + 16
*B*a^2*b^4*c*d - 5*C*a^3*b^3*d^2 + 8*A*a^2*b^4*d^2)/b^6)*x - 256*(7*B*a^2*
b^4*c^2 - 8*C*a^3*b^3*c*d + 14*A*a^2*b^4*c*d - 4*B*a^3*b^3*d^2)/b^6) - 1/1
28*(8*C*a^3*b*c^2 - 16*A*a^2*b^2*c^2 + 16*B*a^3*b*c*d - 5*C*a^4*d^2 + 8*A*
a^3*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int x^2\,\sqrt{b\,x^2+a}\,(c+d\,x)^2\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int x^2(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\int x^2(dx+c)^2\,\sqrt{b\,x^2+a}\,\left(C\,x^2+Bx+A\right)dx$$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

3.18 $\int x(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 300

$$\begin{aligned}\int x(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx &= \frac{a(2bc(Bc+2Ad)-ad(2cC+Bd))x\sqrt{a+bx^2}}{16b^2}\\
&+\frac{(2bc(Bc+2Ad)-ad(2cC+Bd))x^3\sqrt{a+bx^2}}{8b}\\
&+\frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))(a+bx^2)^{3/2}}{3b^3}\\
&+\frac{d(2cC+Bd)x^3(a+bx^2)^{3/2}}{6b}-\frac{(2aCd^2-b(c^2C+2Bcd+Ad^2))(a+bx^2)^{5/2}}{5b^3}\\
&+\frac{Cd^2(a+bx^2)^{7/2}}{7b^3}-\frac{a^2(2bc(Bc+2Ad)-ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}\end{aligned}$$

output
```
1/16*a*(2*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/b^2+1/8*(2*b*
c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x^3*(b*x^2+a)^(1/2)/b+1/3*(A*b*(-a*d^2+b*c^
2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(3/2)/b^3+1/6*d*(B*d+2*C*c)*x^3*
(b*x^2+a)^(3/2)/b-1/5*(2*a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(5/2)/
b^3+1/7*C*d^2*(b*x^2+a)^(7/2)/b^3-1/16*a^2*(2*b*c*(2*A*d+B*c)-a*d*(B*d+2*C
*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(128a^3Cd^2-a^2b(224c^2C+14cd(32B+15Cx)+d^2(224A+105Bx+64Cx^2))+2ab^2(14A(2}{}$$

input
```
Integrate[x*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^3*C*d^2 - a^2*b*(224*c^2*C + 14*c*d*(32*B + 15*C*x
) + d^2*(224*A + 105*B*x + 64*C*x^2)) + 2*a*b^2*(14*A*(20*c^2 + 15*c*d*x +
 4*d^2*x^2) + x*(7*B*(15*c^2 + 16*c*d*x + 5*d^2*x^2) + 2*C*x*(28*c^2 + 35*
c*d*x + 12*d^2*x^2))) + 4*b^3*x^2*(14*A*(10*c^2 + 15*c*d*x + 6*d^2*x^2) +
x*(7*B*(15*c^2 + 24*c*d*x + 10*d^2*x^2) + 4*C*x*(21*c^2 + 35*c*d*x + 15*d^
2*x^2)))) - 105*a^2*Sqrt[b]*(-2*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1680*b^3)
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {2185, 25, 2185, 27, 687, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -(c+dx)^2\sqrt{bx^2+a}\left(b(10cC-7Bd)x^2d^2+4acCd^2+\left(3bCc^2-7Abd^2+4aCd^2\right)xd\right)dx}{7bd^3}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\int (c+dx)^2\sqrt{bx^2+a}\left(b(10cC-7Bd)x^2d^2+4acCd^2+\left(3bCc^2-7Abd^2+4aCd^2\right)xd\right)dx}{7bd^3}$$

$\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\frac{\int -3bd^3(c+dx)^2\left(ad(2cC-7Bd)-\left(8aCd^2-b\left(4Cc^2-7Bdc+14Ad^2\right)\right)x\right)\sqrt{bx^2+a}dx}{6bd^2}+\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)}{7bd^3}$$

$\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\int (c+dx)^2\left(ad(2cC-7Bd)-\left(8aCd^2-b\left(4Cc^2-7Bdc+14Ad^2\right)\right.\right.}{7bd^3}$$

$\downarrow$ 687

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\left(\frac{\int (c+dx)\left(ad\left(16aCd^2+b\left(2Cc^2-21Bdc-28Ad^2\right)\right)-b\left(ad^2(6cC+35Bd)-2bc\left(4Cc^2-7\right.\right.\right.}{5b}\right.}{7bd^3}$$

$\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\left(\frac{-\frac{35}{4}ad^2(2bc(2Ad+Bc)-ad(Bd+2cC))\int \sqrt{bx^2+a}dx+\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2Cd^4-2abd^2\left(\right.\right.}{}\right.}{}$$

$\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\left(\frac{-\frac{35}{4}ad^2(2bc(2Ad+Bc)-ad(Bd+2cC))\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{2\left(a+bx^2\right)}{}}{}\right.}{}$$

$\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-$$
$$\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\left(\frac{-\frac{35}{4}ad^2(2bc(2Ad+Bc)-ad(Bd+2cC))\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\cdots}{\cdots}\right.}{\cdots}$$

$\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^4}{7bd^2}-$$
$$\frac{\frac{1}{6}d\left(a+bx^2\right)^{3/2}(c+dx)^3(10cC-7Bd)-\frac{1}{2}d\left(\frac{\frac{2\left(a+bx^2\right)^{3/2}\left(8a^2Cd^4-2abd^2\left(7Ad^2+14Bcd+c^2C\right)+b^2c^2\left(14Ad^2-7Bcd+4c^2C\right)\right)}{3b}-\frac{35}{4}ad^2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Int[x*(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^4*(a + b*x^2)^(3/2))/(7*b*d^2) - ((d*(10*c*C - 7*B*d)*(c + d*
x)^3*(a + b*x^2)^(3/2))/6 - (d*(-1/5*((8*a*C*d^2 - b*(4*c^2*C - 7*B*c*d +
14*A*d^2))*(c + d*x)^2*(a + b*x^2)^(3/2))/b + ((2*(8*a^2*C*d^4 - 2*a*b*d^2
*(c^2*C + 14*B*c*d + 7*A*d^2) + b^2*c^2*(4*c^2*C - 7*B*c*d + 14*A*d^2))*(a
 + b*x^2)^(3/2))/(3*b) - (d*(a*d^2*(6*c*C + 35*B*d) - 2*b*c*(4*c^2*C - 7*B
*c*d + 14*A*d^2))*x*(a + b*x^2)^(3/2))/4 - (35*a*d^2*(2*b*c*(B*c + 2*A*d)
- a*d*(2*c*C + B*d))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(2*Sqrt[b])))/4)/(5*b)))/2)/(7*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.97

method	result
default	$c(2Ad+Bc)\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+d(Bd+2Cc)\left(\frac{x^3(b\,x^2+a)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2 \cdots}{\cdots}\right.}{\cdots}\right.$
risch	$-\frac{(-240C\,d^2b^3x^6-280B\,b^3d^2x^5-560C\,b^3cd\,x^5-336A\,b^3d^2x^4-672B\,b^3cd\,x^4-48Ca\,b^2d^2x^4-336C\,b^3c^2x^4-840A\,b^3cd\,x^3-70Ba\,b \cdots}{\cdots}$

input
```
int(x*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*
a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+d*(B*d+2*C*c)*(1/6*x^3*(b*x^2+a)
^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1
/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(A*d^2+2*B*c*d+C*c^2)*(1/5*x
^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+1/3*A*c^2/b*(b*x^2+a)^(3/
2)+C*d^2*(1/7*x^4*(b*x^2+a)^(3/2)/b-4/7*a/b*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/1
5*a/b^2*(b*x^2+a)^(3/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.27

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[-\frac{105\left(2\,Ba^2bc^2-Ba^3d^2-2\left(Ca^3-2\,Aa^2b\right)cd\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(240\,Cb^3d^2 \cdots\right.}{\cdots}\right.$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/3360*(105*(2*B*a^2*b*c^2 - B*a^3*d^2 - 2*(C*a^3 - 2*A*a^2*b)*c*d)*sqrt
(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(240*C*b^3*d^2*x^6
 - 448*B*a^2*b*c*d + 280*(2*C*b^3*c*d + B*b^3*d^2)*x^5 + 48*(7*C*b^3*c^2 +
 14*B*b^3*c*d + (C*a*b^2 + 7*A*b^3)*d^2)*x^4 + 70*(6*B*b^3*c^2 + B*a*b^2*d
^2 + 2*(C*a*b^2 + 6*A*b^3)*c*d)*x^3 - 112*(2*C*a^2*b - 5*A*a*b^2)*c^2 + 32
*(4*C*a^3 - 7*A*a^2*b)*d^2 + 16*(14*B*a*b^2*c*d + 7*(C*a*b^2 + 5*A*b^3)*c^
2 - (4*C*a^2*b - 7*A*a*b^2)*d^2)*x^2 + 105*(2*B*a*b^2*c^2 - B*a^2*b*d^2 -
2*(C*a^2*b - 2*A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/b^3, 1/1680*(105*(2*B*a^2
*b*c^2 - B*a^3*d^2 - 2*(C*a^3 - 2*A*a^2*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (240*C*b^3*d^2*x^6 - 448*B*a^2*b*c*d + 280*(2*C*b^3*c*
d + B*b^3*d^2)*x^5 + 48*(7*C*b^3*c^2 + 14*B*b^3*c*d + (C*a*b^2 + 7*A*b^3)*
d^2)*x^4 + 70*(6*B*b^3*c^2 + B*a*b^2*d^2 + 2*(C*a*b^2 + 6*A*b^3)*c*d)*x^3
- 112*(2*C*a^2*b - 5*A*a*b^2)*c^2 + 32*(4*C*a^3 - 7*A*a^2*b)*d^2 + 16*(14*
B*a*b^2*c*d + 7*(C*a*b^2 + 5*A*b^3)*c^2 - (4*C*a^2*b - 7*A*a*b^2)*d^2)*x^2
 + 105*(2*B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a^2*b - 2*A*a*b^2)*c*d)*x)*sqrt
(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 576 vs. $2(282) = 564$.

Time = 0.60 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.92

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$= \begin{cases} -\frac{a\left(2Aacd+Bac^2-\frac{3a\left(2Abcd+Bad^2+Bbc^2+2Cacd-\frac{5a\left(Bbd^2+2Cbcd\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2} \\ \sqrt{a}\left(\frac{Ac^2x^2}{2}+\frac{Cd^2x^6}{6}+\frac{x^5\left(Bd^2+2Ccd\right)}{5}+\frac{x^4\left(Ad^2+2Bcd+Cc^2\right)}{4}+\frac{x^3\cdot\left(2Acd+Bc^2\right)}{3}\right) \end{cases}$$

input
```
integrate(x*(d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(2*A*a*c*d + B*a*c**2 - 3*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2
 + 2*C*a*c*d - 5*a*(B*b*d**2 + 2*C*b*c*d)/(6*b))/(4*b))*Piecewise((log(2*s
qrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2
), True))/(2*b) + sqrt(a + b*x**2)*(C*d**2*x**6/7 + x**5*(B*b*d**2 + 2*C*b
*c*d)/(6*b) + x**4*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/7 + C*b*c**2)/(5*b) +
x**3*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 5*a*(B*b*d**2 + 2*C*b*
c*d)/(6*b))/(4*b) + x**2*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 4*a
*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/7 + C*b*c**2)/(5*b))/(3*b) + x*(2*A*a*c*
d + B*a*c**2 - 3*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 5*a*(B*b
*d**2 + 2*C*b*c*d)/(6*b))/(4*b))/(2*b) + (A*a*c**2 - 2*a*(A*a*d**2 + A*b*c
**2 + 2*B*a*c*d + C*a*c**2 - 4*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/7 + C*b*
c**2)/(5*b))/(3*b))/b), Ne(b, 0)), (sqrt(a)*(A*c**2*x**2/2 + C*d**2*x**6/6
 + x**5*(B*d**2 + 2*C*c*d)/5 + x**4*(A*d**2 + 2*B*c*d + C*c**2)/4 + x**3*(
2*A*c*d + B*c**2)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx \\
&= \frac{(bx^2+a)^{\frac{3}{2}}Cd^2x^4}{7\,b} - \frac{4\,(bx^2+a)^{\frac{3}{2}}Cad^2x^2}{35\,b^2} \\
&\quad + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x^3}{6\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{3\,b} + \frac{8\,(bx^2+a)^{\frac{3}{2}}Ca^2d^2}{105\,b^3} \\
&\quad + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x^2}{5\,b} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} \\
&\quad + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}\,a^2x}{16\,b^2} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}x}{4\,b} \\
&\quad - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}\,ax}{8\,b} + \frac{(2\,Ccd+Bd^2)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} \\
&\quad - \frac{(Bc^2+2\,Acd)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}
\end{aligned}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(3/2)*C*d^2*x^4/b - 4/35*(b*x^2 + a)^(3/2)*C*a*d^2*x^2/b^2
 + 1/6*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x^3/b + 1/3*(b*x^2 + a)^(3/2)*A
*c^2/b + 8/105*(b*x^2 + a)^(3/2)*C*a^2*d^2/b^3 + 1/5*(C*c^2 + 2*B*c*d + A*
d^2)*(b*x^2 + a)^(3/2)*x^2/b - 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*a*x
/b^2 + 1/16*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a^2*x/b^2 + 1/4*(B*c^2 + 2*A
*c*d)*(b*x^2 + a)^(3/2)*x/b - 1/8*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*a*x/b
+ 1/16*(2*C*c*d + B*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 1/8*(B*c^2 +
 2*A*c*d)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(C*c^2 + 2*B*c*d + A*d
^2)*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.23

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{1680}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,Cd^2x+\frac{7\,(2\,Cb^5cd+Bb^5d^2)}{b^5}\right)x+\frac{6\,(7\,Cb^5c^2+14\,Bb^5cd+Cab^4d^2+7\,\ldots}{b^5}\right.\right.\right.\right.\right.$$
$$+\frac{(2\,Ba^2bc^2-2\,Ca^3cd+4\,Aa^2bcd-Ba^3d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*C*d^2*x + 7*(2*C*b^5*c*d + B*b^5*d^2)
/b^5)*x + 6*(7*C*b^5*c^2 + 14*B*b^5*c*d + C*a*b^4*d^2 + 7*A*b^5*d^2)/b^5)*
x + 35*(6*B*b^5*c^2 + 2*C*a*b^4*c*d + 12*A*b^5*c*d + B*a*b^4*d^2)/b^5)*x +
 8*(7*C*a*b^4*c^2 + 35*A*b^5*c^2 + 14*B*a*b^4*c*d - 4*C*a^2*b^3*d^2 + 7*A*
a*b^4*d^2)/b^5)*x + 105*(2*B*a*b^4*c^2 - 2*C*a^2*b^3*c*d + 4*A*a*b^4*c*d -
 B*a^2*b^3*d^2)/b^5)*x - 16*(14*C*a^2*b^3*c^2 - 35*A*a*b^4*c^2 + 28*B*a^2*
b^3*c*d - 8*C*a^3*b^2*d^2 + 14*A*a^2*b^3*d^2)/b^5) + 1/16*(2*B*a^2*b*c^2 -
 2*C*a^3*c*d + 4*A*a^2*b*c*d - B*a^3*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int x\,\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x*(a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 609, normalized size of antiderivative = 2.03

$$\int x(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$

$$=\frac{-448\sqrt{b\,x^2+a}\,a^2b^2cd+112\sqrt{b\,x^2+a}\,a^2b^2d^2x^2-105\sqrt{b\,x^2+a}\,a^2b^2d^2x+560\sqrt{b\,x^2+a}\,a\,b^3c^2x^2+210\sqrt{}}{}$$

input
```
int(x*(d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
( - 224*sqrt(a + b*x**2)*a**3*b*d**2 + 128*sqrt(a + b*x**2)*a**3*c*d**2 +
560*sqrt(a + b*x**2)*a**2*b**2*c**2 + 420*sqrt(a + b*x**2)*a**2*b**2*c*d*x
 - 448*sqrt(a + b*x**2)*a**2*b**2*c*d + 112*sqrt(a + b*x**2)*a**2*b**2*d**
2*x**2 - 105*sqrt(a + b*x**2)*a**2*b**2*d**2*x - 224*sqrt(a + b*x**2)*a**2
*b*c**3 - 210*sqrt(a + b*x**2)*a**2*b*c**2*d*x - 64*sqrt(a + b*x**2)*a**2*
b*c*d**2*x**2 + 560*sqrt(a + b*x**2)*a*b**3*c**2*x**2 + 210*sqrt(a + b*x**
2)*a*b**3*c**2*x + 840*sqrt(a + b*x**2)*a*b**3*c*d*x**3 + 224*sqrt(a + b*x
**2)*a*b**3*c*d*x**2 + 336*sqrt(a + b*x**2)*a*b**3*d**2*x**4 + 70*sqrt(a +
 b*x**2)*a*b**3*d**2*x**3 + 112*sqrt(a + b*x**2)*a*b**2*c**3*x**2 + 140*sq
rt(a + b*x**2)*a*b**2*c**2*d*x**3 + 48*sqrt(a + b*x**2)*a*b**2*c*d**2*x**4
 + 420*sqrt(a + b*x**2)*b**4*c**2*x**3 + 672*sqrt(a + b*x**2)*b**4*c*d*x**
4 + 280*sqrt(a + b*x**2)*b**4*d**2*x**5 + 336*sqrt(a + b*x**2)*b**3*c**3*x
**4 + 560*sqrt(a + b*x**2)*b**3*c**2*d*x**5 + 240*sqrt(a + b*x**2)*b**3*c*
d**2*x**6 - 420*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b
*c*d + 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d**2
 + 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c**2*d - 2
10*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c**2)/(16
80*b**3)
```

3.19 $\int (c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 264

$$\begin{aligned}\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx &= \frac{\left(8Ab^2c^2+a\left(aCd^2-2b\left(c^2C+2Bcd+Ad^2\right)\right)\right)x\sqrt{a+bx^2}}{16b^2} \\ &+ \frac{\left(bc(Bc+2Ad)-ad(2cC+Bd)\right)\left(a+bx^2\right)^{3/2}}{3b^2} \\ &- \frac{\left(aCd^2-2b\left(c^2C+2Bcd+Ad^2\right)\right)x\left(a+bx^2\right)^{3/2}}{8b^2} \\ &+ \frac{Cd^2x^3\left(a+bx^2\right)^{3/2}}{6b}+\frac{d(2cC+Bd)\left(a+bx^2\right)^{5/2}}{5b^2} \\ &+ \frac{a\left(8Ab^2c^2+a\left(aCd^2-2b\left(c^2C+2Bcd+Ad^2\right)\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}\end{aligned}$$

output
```
1/16*(8*A*b^2*c^2+a*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2)))*x*(b*x^2+a)^(1/2)
/b^2+1/3*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/b^2-1/8*(a*C*d^
2-2*b*(A*d^2+2*B*c*d+C*c^2))*x*(b*x^2+a)^(3/2)/b^2+1/6*C*d^2*x^3*(b*x^2+a)
^(3/2)/b+1/5*d*(B*d+2*C*c)*(b*x^2+a)^(5/2)/b^2+1/16*a*(8*A*b^2*c^2+a*(a*C*
d^2-2*b*(A*d^2+2*B*c*d+C*c^2)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.93

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{\sqrt{a+bx^2}(-a^2d(64cC+32Bd+15Cdx)+2ab(5Ad(16c+3dx)+Cx(15c^2+16cdx+5d^2x^2)+B(40c^2\ldots}{}$$
$$-\frac{a(2Ab(4bc^2-ad^2)+a(aCd^2-2bc(cC+2Bd)))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

input
```
Integrate[(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(-(a^2*d*(64*c*C + 32*B*d + 15*C*d*x)) + 2*a*b*(5*A*d*(16
*c + 3*d*x) + C*x*(15*c^2 + 16*c*d*x + 5*d^2*x^2) + B*(40*c^2 + 30*c*d*x +
 8*d^2*x^2)) + 4*b^2*x*(5*A*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + x*(2*B*(10*c^2
 + 15*c*d*x + 6*d^2*x^2) + C*x*(15*c^2 + 24*c*d*x + 10*d^2*x^2)))))/(240*b
^2) - (a*(2*A*b*(4*b*c^2 - a*d^2) + a*(a*C*d^2 - 2*b*c*(c*C + 2*B*d)))*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {2185, 27, 687, 27, 676, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int 3d(c+dx)^2((2Ab-aC)d-b(cC-2Bd)x)\sqrt{bx^2+a}dx}{6bd^2}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (c+dx)^2((2Ab-aC)d-b(cC-2Bd)x)\sqrt{bx^2+a}dx}{2bd}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow 687$$

$$\frac{\frac{\int b(c+dx)\left(d(10Abc-3aCc-4aBd)+\left(5(2Ab-aC)d^2-2bc(cC-2Bd)\right)x\right)\sqrt{bx^2+a}dx}{5b}-\frac{1}{5}\left(a+bx^2\right)^{3/2}(c+dx)^2(cC-2Bd)}{2bd}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{5}\int (c+dx)\left(d(10Abc-3aCc-4aBd)+\left(5(2Ab-aC)d^2-2bc(cC-2Bd)\right)x\right)\sqrt{bx^2+a}dx-\frac{1}{5}\left(a+bx^2\right)^{3/2}(c}{2bd}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow 676$$

$$\frac{\frac{1}{5}\left(\frac{5d\left(2Ab\left(4bc^2-ad^2\right)+a\left(aCd^2-2bc(2Bd+cC)\right)\right)\int\sqrt{bx^2+a}dx}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Bd+2cC)+bc\left(-10Ad^2-2Bcd+c^2C\right)\right)}{3b}+\frac{dx\left(a+bx^2\right)^3}{}\right.}{2bd}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow 211$$

$$\frac{\frac{1}{5}\left(\frac{5d\left(2Ab\left(4bc^2-ad^2\right)+a\left(aCd^2-2bc(2Bd+cC)\right)\right)\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Bd+2cC)+bc\left(-10Ad^2-2Bcd+c^2C\right.\right.}{3b}\right.}{2bd}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow 224$$

$$\frac{\frac{1}{5}\left(\frac{5d\left(2Ab\left(4bc^2-ad^2\right)+a\left(aCd^2-2bc(2Bd+cC)\right)\right)\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)}{4b}-\frac{2\left(a+bx^2\right)^{3/2}\left(2ad^2(Bd+2cC)+bc\left(-10Ad^2-2\right.\right.}{3b}\right.}{2bd}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^3}{6bd}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{5}\left(\frac{5d\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)(2Ab(4bc^2-ad^2)+a(aCd^2-2bc(2Bd+cC)))}{4b}-\frac{2(a+bx^2)^{3/2}(2ad^2(Bd+2cC)+bc(-10Ad^2-2B}{3b}\right.}{2bd}$$

$$\frac{C(a+bx^2)^{3/2}(c+dx)^3}{6bd}$$

input
```
Int[(c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^3*(a + b*x^2)^(3/2))/(6*b*d) + (-1/5*((c*C - 2*B*d)*(c + d*x)
^2*(a + b*x^2)^(3/2)) + ((-2*(2*a*d^2*(2*c*C + B*d) + b*c*(c^2*C - 2*B*c*d
 - 10*A*d^2))*(a + b*x^2)^(3/2))/(3*b) + (d*(5*(2*A*b - a*C)*d^2 - 2*b*c*(
c*C - 2*B*d))*x*(a + b*x^2)^(3/2))/(4*b) + (5*d*(2*A*b*(4*b*c^2 - a*d^2) +
 a*(a*C*d^2 - 2*b*c*(c*C + 2*B*d)))*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4*b))/5)/(2*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.02

method	result
default	$A c^2 \left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{c(2Ad+Bc)\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+d(Bd+2Cc)\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^2+a\right)}{15b^2}\right.$
risch	$\frac{(40C\,d^2b^2x^5+48B\,b^2d^2x^4+96C\,b^2cd\,x^4+60A\,b^2d^2x^3+120B\,b^2cd\,x^3+10aC\,d^2b\,x^3+60C\,b^2c^2x^3+160A\,b^2cd\,x^2+16Ba\,d^2b\,x^2+80B}{}$

input

```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output

```
A*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
1/3*c*(2*A*d+B*c)/b*(b*x^2+a)^(3/2)+d*(B*d+2*C*c)*(1/5*x^2*(b*x^2+a)^(3/2)
/b-2/15*a/b^2*(b*x^2+a)^(3/2))+(A*d^2+2*B*c*d+C*c^2)*(1/4*x*(b*x^2+a)^(3/2
)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))))+C*d^2*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1
/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.25

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\left[\frac{15\left(4\,Ba^2bcd+2\,(Ca^2b-4\,Aab^2)c^2-(Ca^3-2\,Aa^2b)d^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(4\right.}{}\right.$$

input

```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/480*(15*(4*B*a^2*b*c*d + 2*(C*a^2*b - 4*A*a*b^2)*c^2 - (C*a^3 - 2*A*a^2
*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*C
*b^3*d^2*x^5 + 80*B*a*b^2*c^2 - 32*B*a^2*b*d^2 + 48*(2*C*b^3*c*d + B*b^3*d
^2)*x^4 + 10*(6*C*b^3*c^2 + 12*B*b^3*c*d + (C*a*b^2 + 6*A*b^3)*d^2)*x^3 -
32*(2*C*a^2*b - 5*A*a*b^2)*c*d + 16*(5*B*b^3*c^2 + B*a*b^2*d^2 + 2*(C*a*b^
2 + 5*A*b^3)*c*d)*x^2 + 15*(4*B*a*b^2*c*d + 2*(C*a*b^2 + 4*A*b^3)*c^2 - (C
*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1/240*(15*(4*B*a^2*b*c*d
 + 2*(C*a^2*b - 4*A*a*b^2)*c^2 - (C*a^3 - 2*A*a^2*b)*d^2)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*C*b^3*d^2*x^5 + 80*B*a*b^2*c^2 - 32*B*a^
2*b*d^2 + 48*(2*C*b^3*c*d + B*b^3*d^2)*x^4 + 10*(6*C*b^3*c^2 + 12*B*b^3*c*
d + (C*a*b^2 + 6*A*b^3)*d^2)*x^3 - 32*(2*C*a^2*b - 5*A*a*b^2)*c*d + 16*(5*
B*b^3*c^2 + B*a*b^2*d^2 + 2*(C*a*b^2 + 5*A*b^3)*c*d)*x^2 + 15*(4*B*a*b^2*c
*d + 2*(C*a*b^2 + 4*A*b^3)*c^2 - (C*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2
+ a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.83

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\begin{cases}\sqrt{a+bx^2}\left(\frac{Cd^2x^5}{6}+\frac{x^4(Bbd^2+2Cbcd)}{5b}+\frac{x^3\left(Abd^2+2Bbcd+\frac{Cad^2}{6}+Cbc^2\right)}{4b}+\frac{x^2\cdot\left(2Abcd+Bad^2+Bbc^2+2Cacd-\frac{4a\left(Bbd^2+2Cbcd\right)}{5b}\right.}{3b}\right.\\ \sqrt{a}\left(Ac^2x+\frac{Cd^2x^5}{5}+\frac{x^4(Bd^2+2Ccd)}{4}+\frac{x^3(Ad^2+2Bcd+Cc^2)}{3}+\frac{x^2\cdot(2Acd+Bc^2)}{2}\right)\end{cases}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*d**2*x**5/6 + x**4*(B*b*d**2 + 2*C*b*c*d)/(
5*b) + x**3*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/6 + C*b*c**2)/(4*b) + x**2*(2
*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 4*a*(B*b*d**2 + 2*C*b*c*d)/(5
*b))/(3*b) + x*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d + C*a*c**2 - 3*a*(A*b*d**2
 + 2*B*b*c*d + C*a*d**2/6 + C*b*c**2)/(4*b))/(2*b) + (2*A*a*c*d + B*a*c**2
 - 2*a*(2*A*b*c*d + B*a*d**2 + B*b*c**2 + 2*C*a*c*d - 4*a*(B*b*d**2 + 2*C*
b*c*d)/(5*b))/(3*b))/b) + (A*a*c**2 - a*(A*a*d**2 + A*b*c**2 + 2*B*a*c*d +
 C*a*c**2 - 3*a*(A*b*d**2 + 2*B*b*c*d + C*a*d**2/6 + C*b*c**2)/(4*b))/(2*b
))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
(x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (sqrt(a)*(A*c**2*x + C*d**2*x**
5/5 + x**4*(B*d**2 + 2*C*c*d)/4 + x**3*(A*d**2 + 2*B*c*d + C*c**2)/3 + x**
2*(2*A*c*d + B*c**2)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.16

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Cd^2x^3}{6\,b} + \frac{1}{2}\,\sqrt{bx^2+a}Ac^2x - \frac{(bx^2+a)^{\frac{3}{2}}Cad^2x}{8\,b^2} + \frac{\sqrt{bx^2+a}Ca^2d^2x}{16\,b^2} + \frac{Aac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} + \frac{Ca^3d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}Bc^2}{3\,b} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Acd}{3\,b} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x^2}{5\,b} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}ax}{8\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*C*d^2*x^3/b + 1/2*sqrt(b*x^2 + a)*A*c^2*x - 1/8*(b*x
^2 + a)^(3/2)*C*a*d^2*x/b^2 + 1/16*sqrt(b*x^2 + a)*C*a^2*d^2*x/b^2 + 1/2*A
*a*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/16*C*a^3*d^2*arcsinh(b*x/sqrt(a*
b))/b^(5/2) + 1/3*(b*x^2 + a)^(3/2)*B*c^2/b + 2/3*(b*x^2 + a)^(3/2)*A*c*d/
b + 1/5*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x^2/b + 1/4*(C*c^2 + 2*B*c*d +
 A*d^2)*(b*x^2 + a)^(3/2)*x/b - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 +
 a)*a*x/b - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/
2) - 2/15*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.19

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx$$
$$=\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\,Cd^2x+\frac{6\,(2\,Cb^4cd+Bb^4d^2)}{b^4}\right)x+\frac{5\,(6\,Cb^4c^2+12\,Bb^4cd+Cab^3d^2+6\,Ab^4d}{b^4}\right.\right.\right.\right.\right.$$
$$+\frac{(2\,Ca^2bc^2-8\,Aab^2c^2+4\,Ba^2bcd-Ca^3d^2+2\,Aa^2bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*C*d^2*x + 6*(2*C*b^4*c*d + B*b^4*d^2)/b^4
)*x + 5*(6*C*b^4*c^2 + 12*B*b^4*c*d + C*a*b^3*d^2 + 6*A*b^4*d^2)/b^4)*x +
8*(5*B*b^4*c^2 + 2*C*a*b^3*c*d + 10*A*b^4*c*d + B*a*b^3*d^2)/b^4)*x + 15*(
2*C*a*b^3*c^2 + 8*A*b^4*c^2 + 4*B*a*b^3*c*d - C*a^2*b^2*d^2 + 2*A*a*b^3*d^
2)/b^4)*x + 16*(5*B*a*b^3*c^2 - 4*C*a^2*b^2*c*d + 10*A*a*b^3*c*d - 2*B*a^2
*b^2*d^2)/b^4) + 1/16*(2*C*a^2*b*c^2 - 8*A*a*b^2*c^2 + 4*B*a^2*b*c*d - C*a
^3*d^2 + 2*A*a^2*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int \sqrt{b\,x^2+a}\,(c+d\,x)^2\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int (c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)\,dx=\int (dx+c)^2\,\sqrt{b\,x^2+a}\,\left(C\,x^2+Bx+A\right)dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A),x)
```

3.20 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 238

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx &= Ac^2\sqrt{a+bx^2}+\frac{(4bc(Bc+2Ad)-ad(2cC+Bd))x\sqrt{a+bx^2}}{8b}\\ &\quad-\frac{(aCd^2-b(c^2C+2Bcd+Ad^2))\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{d(2cC+Bd)x(a+bx^2)^{3/2}}{4b}\\ &\quad+\frac{Cd^2(a+bx^2)^{5/2}}{5b^2}+\frac{a(4bc(Bc+2Ad)-ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\\ &\quad-\sqrt{a}Ac^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

output
```
A*c^2*(b*x^2+a)^(1/2)+1/8*(4*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x*(b*x^2+a)^
(1/2)/b-1/3*(a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(3/2)/b^2+1/4*d*(B
*d+2*C*c)*x*(b*x^2+a)^(3/2)/b+1/5*C*d^2*(b*x^2+a)^(5/2)/b^2+1/8*a*(4*b*c*(
2*A*d+B*c)-a*d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-a^(
1/2)*A*c^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.01

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-16a^2Cd^2+ab(40c^2C+10cd(8B+3Cx)+d^2x(15B+8Cx))+40Ab(ad^2+b(3c^2+3cdx+}{120b^2}$$
$$+2\sqrt{a}Ac^2\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$
$$+\frac{a(-4bc(Bc+2Ad)+ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*C*d^2 + a*b*(40*c^2*C + 10*c*d*(8*B + 3*C*x) + d
^2*x*(15*B + 8*C*x)) + 40*A*b*(a*d^2 + b*(3*c^2 + 3*c*d*x + d^2*x^2)) + 2*
b^2*x*(5*B*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + 2*C*x*(10*c^2 + 15*c*d*x + 6*d^
2*x^2))))/(120*b^2) + 2*Sqrt[a]*A*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2]
)/Sqrt[a]] + (a*(-4*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2340, 2340, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x}dx$$
$$\Big\downarrow \; 2340$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(5bd(2cC+Bd)x^3-\left(2aCd^2-5b\left(Cc^2+2Bdc+Ad^2\right)\right)x^2+5bc(Bc+2Ad)x+5Abc^2\right)}{x}dx}{5b}+\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(20Ab^2c^2-4b\left(2aCd^2-5b\left(Cc^2+2Bdc+Ad^2\right)\right)x^2+5b(4bc(Bc+2Ad)-ad(2cC+Bd))x\right)}{x}dx}{4b}+\frac{5}{4}dx\left(a+bx^2\right)^{3/2}(Bd+2cC)}{5b}+\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\frac{\int \frac{15b^2\left(4Abc^2+(4bc(Bc+2Ad)-ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx}{3b}-\frac{4}{3}\left(a+bx^2\right)^{3/2}\left(2aCd^2-5b\left(Ad^2+2Bcd+c^2C\right)\right)}{4b}+\frac{5}{4}dx\left(a+bx^2\right)^{3/2}(Bd+2cC)}{5b}+\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 27

$$\frac{\frac{5b\int \frac{\left(4Abc^2+(4bc(Bc+2Ad)-ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx-\frac{4}{3}\left(a+bx^2\right)^{3/2}\left(2aCd^2-5b\left(Ad^2+2Bcd+c^2C\right)\right)}{4b}+\frac{5}{4}dx\left(a+bx^2\right)^{3/2}(Bd+2cC)}{5b}+\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 535

$$\frac{\frac{5b\left(\frac{1}{2}a\int \frac{8Abc^2+(4bc(Bc+2Ad)-ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-\frac{4}{3}\left(a+bx^2\right)^{3/2}\left(2aCd^2-5b\left(Ad^2+2Bcd-\right.\right.}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$\Big\downarrow$ 538

$$\frac{\frac{5b\left(\frac{1}{2}a\left((4bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{\sqrt{bx^2+a}}dx+8Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-\frac{4}{3}\left(a+bx^2\right.}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{5b\left(\frac{1}{2}a\left((4bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5b\left(\frac{1}{2}a\left(8Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-\frac{4}{3}(}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{5b\left(\frac{1}{2}a\left(4Abc^2\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{5b\left(\frac{1}{2}a\left(8Ac^2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{5b\left(\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}-\frac{8Abc^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(4bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)}{4b}}{5b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{3/2}}{5b}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x,x]
```

output
```
(C*d^2*x^2*(a + b*x^2)^(3/2))/(5*b) + ((5*d*(2*c*C + B*d)*x*(a + b*x^2)^(3
/2))/4 + ((-4*(2*a*C*d^2 - 5*b*(c^2*C + 2*B*c*d + A*d^2))*(a + b*x^2)^(3/2
))/3 + 5*b*(((8*A*b*c^2 + (4*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*x)*Sqr
t[a + b*x^2])/2 + (a*(((4*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (8*A*b*c^2*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/Sqrt[a]))/2))/(4*b))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.43

method	result
default	$B\,c^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+A\,c^2\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)+\frac{A\,d^2\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
B*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
A*c^2*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+1/3*
A*d^2/b*(b*x^2+a)^(3/2)+B*d^2*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x
^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/3*C*c^2/b*(b*x
^2+a)^(3/2)+C*d^2*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+2
*A*c*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
+2/3*B*c*d/b*(b*x^2+a)^(3/2)+2*C*c*d*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2
*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 1049, normalized size of antiderivative = 4.41

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/240*(120*A*sqrt(a)*b^2*c^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 15*(4*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2 - 4*A*a*b)*c*d)*sqrt(b)*l
og(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*C*b^2*d^2*x^4 + 80*
B*a*b*c*d + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 40*(C*a*b + 3*A*b^2)*c^2 -
8*(2*C*a^2 - 5*A*a*b)*d^2 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d + (C*a*b + 5*A*b
^2)*d^2)*x^2 + 15*(4*B*b^2*c^2 + B*a*b*d^2 + 2*(C*a*b + 4*A*b^2)*c*d)*x)*s
qrt(b*x^2 + a))/b^2, 1/120*(60*A*sqrt(a)*b^2*c^2*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) - 15*(4*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2 - 4*A*
a*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (24*C*b^2*d^2*x^4
+ 80*B*a*b*c*d + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 40*(C*a*b + 3*A*b^2)*c
^2 - 8*(2*C*a^2 - 5*A*a*b)*d^2 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d + (C*a*b +
5*A*b^2)*d^2)*x^2 + 15*(4*B*b^2*c^2 + B*a*b*d^2 + 2*(C*a*b + 4*A*b^2)*c*d)
*x)*sqrt(b*x^2 + a))/b^2, 1/240*(240*A*sqrt(-a)*b^2*c^2*arctan(sqrt(b*x^2
+ a)*sqrt(-a)/a) - 15*(4*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2 - 4*A*a*b)*c*d)*
sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*C*b^2*d^2*
x^4 + 80*B*a*b*c*d + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 40*(C*a*b + 3*A*b^
2)*c^2 - 8*(2*C*a^2 - 5*A*a*b)*d^2 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d + (C*a*
b + 5*A*b^2)*d^2)*x^2 + 15*(4*B*b^2*c^2 + B*a*b*d^2 + 2*(C*a*b + 4*A*b^2)*
c*d)*x)*sqrt(b*x^2 + a))/b^2, 1/120*(120*A*sqrt(-a)*b^2*c^2*arctan(sqrt(b*
x^2 + a)*sqrt(-a)/a) - 15*(4*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2 - 4*A*a*b...
```

Sympy [A] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 620, normalized size of antiderivative = 2.61

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x,x)
```

output
```
-A*sqrt(a)*c**2*asinh(sqrt(a)/(sqrt(b)*x)) + A*a*c**2/(sqrt(b)*x*sqrt(a/(b
*x**2) + 1)) + A*sqrt(b)*c**2*x/sqrt(a/(b*x**2) + 1) + 2*A*c*d*Piecewise((
a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (
x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a
)*x, True)) + A*d**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b
*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*c**2*Piecewise((a*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True
)) + 2*B*c*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3
, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*d**2*Piecewise((-a**2*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2
)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + C*c**2*Piecewise((a*sqrt(a + b*x
**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) +
 2*C*c*d*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*
x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a +
 b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True
)) + C*d**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a
+ b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{(bx^2+a)^{\frac{3}{2}}Cd^2x^2}{5\,b} - A\sqrt{a}c^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}Ac^2 + \frac{(bx^2+a)^{\frac{3}{2}}Cc^2}{3\,b} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Bcd}{3\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Cad^2}{15\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ad^2}{3\,b} + \frac{1}{2}\left(Bc^2+2\,Acd\right)\sqrt{bx^2+a}x + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}ax}{8\,b} - \frac{(2\,Ccd+Bd^2)a^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{(Bc^2+2\,Acd)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*C*d^2*x^2/b - A*sqrt(a)*c^2*arcsinh(a/(sqrt(a*b)*abs
(x))) + sqrt(b*x^2 + a)*A*c^2 + 1/3*(b*x^2 + a)^(3/2)*C*c^2/b + 2/3*(b*x^2
 + a)^(3/2)*B*c*d/b - 2/15*(b*x^2 + a)^(3/2)*C*a*d^2/b^2 + 1/3*(b*x^2 + a)
^(3/2)*A*d^2/b + 1/2*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*x + 1/4*(2*C*c*d +
B*d^2)*(b*x^2 + a)^(3/2)*x/b - 1/8*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a*x/b
 - 1/8*(2*C*c*d + B*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/2*(B*c^2 +
 2*A*c*d)*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x}\,dx$$
$$= \frac{40\sqrt{b\,x^2+a}\,a^2b\,d^2-16\sqrt{b\,x^2+a}\,a^2c\,d^2+120\sqrt{b\,x^2+a}\,a\,b^2c^2+120\sqrt{b\,x^2+a}\,a\,b^2cdx+80\sqrt{b\,x^2+a}\,a\,b}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x,x)
```

output

```
(40*sqrt(a + b*x**2)*a**2*b*d**2 - 16*sqrt(a + b*x**2)*a**2*c*d**2 + 120*s
qrt(a + b*x**2)*a*b**2*c**2 + 120*sqrt(a + b*x**2)*a*b**2*c*d*x + 80*sqrt(
a + b*x**2)*a*b**2*c*d + 40*sqrt(a + b*x**2)*a*b**2*d**2*x**2 + 15*sqrt(a
+ b*x**2)*a*b**2*d**2*x + 40*sqrt(a + b*x**2)*a*b*c**3 + 30*sqrt(a + b*x**
2)*a*b*c**2*d*x + 8*sqrt(a + b*x**2)*a*b*c*d**2*x**2 + 60*sqrt(a + b*x**2)
*b**3*c**2*x + 80*sqrt(a + b*x**2)*b**3*c*d*x**2 + 30*sqrt(a + b*x**2)*b**
3*d**2*x**3 + 40*sqrt(a + b*x**2)*b**2*c**3*x**2 + 60*sqrt(a + b*x**2)*b**
2*c**2*d*x**3 + 24*sqrt(a + b*x**2)*b**2*c*d**2*x**4 + 120*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2 - 120*sqrt(a)*l
og((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2 + 120*sqr
t(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d - 15*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2 - 30*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c**2*d + 60*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2)/(120*b**2)
```

3.21 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 257

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx &= c(Bc+2Ad)\sqrt{a+bx^2}+\frac{(4Ab(2bc^2+ad^2)-a(aCd^2-4bc(cC+2Bd)))\,x\sqrt{a+bx^2}}{8ab}\\&\quad+\frac{d(2cC+Bd)\,(a+bx^2)^{3/2}}{3b}-\frac{Ac^2(a+bx^2)^{3/2}}{ax}+\frac{Cd^2x(a+bx^2)^{3/2}}{4b}\\&\quad+\frac{(4Ab(2bc^2+ad^2)-a(aCd^2-4bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\\&\quad-\sqrt{a}c(Bc+2Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

output

```
c*(2*A*d+B*c)*(b*x^2+a)^(1/2)+1/8*(4*A*b*(a*d^2+2*b*c^2)-a*(a*C*d^2-4*b*c*
(2*B*d+C*c)))*x*(b*x^2+a)^(1/2)/a/b+1/3*d*(B*d+2*C*c)*(b*x^2+a)^(3/2)/b-A*
c^2*(b*x^2+a)^(3/2)/a/x+1/4*C*d^2*x*(b*x^2+a)^(3/2)/b+1/8*(4*A*b*(a*d^2+2*
b*c^2)-a*(a*C*d^2-4*b*c*(2*B*d+C*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b
^(3/2)-a^(1/2)*c*(2*A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$
$$= \frac{\sqrt{a+bx^2}(12Ab(-2c^2+4cdx+d^2x^2)+x(ad(16cC+8Bd+3Cdx)+8bB(3c^2+3cdx+d^2x^2)+2bCx(}{24bx}$$
$$+\frac{(4Ab(2bc^2+ad^2)+a(-aCd^2+4bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{3/2}}$$
$$-\sqrt{a}c(Bc+2Ad)\log(x)+\sqrt{a}c(Bc+2Ad)\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(12*A*b*(-2*c^2 + 4*c*d*x + d^2*x^2) + x*(a*d*(16*c*C + 8
*B*d + 3*C*d*x) + 8*b*B*(3*c^2 + 3*c*d*x + d^2*x^2) + 2*b*C*x*(6*c^2 + 8*c
*d*x + 3*d^2*x^2))))/(24*b*x) + ((4*A*b*(2*b*c^2 + a*d^2) + a*(-(a*C*d^2)
+ 4*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])
/(4*b^(3/2)) - Sqrt[a]*c*(B*c + 2*A*d)*Log[x] + Sqrt[a]*c*(B*c + 2*A*d)*Lo
g[-Sqrt[a] + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 25, 2340, 2340, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^2}\,dx$$
$$\Big\downarrow \; 2338$$
$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(aCd^2x^3+ad(2cC+Bd)x^2+\left(ac(cC+2Bd)+A\left(2bc^2+ad^2\right)\right)x+ac(Bc+2Ad)\right)}{x}dx}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(aCd^2x^3+ad(2cC+Bd)x^2+\left(ac(cC+2Bd)+A\left(2bc^2+ad^2\right)\right)x+ac(Bc+2Ad)\right)}{x}dx}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 2340$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(4abd(2cC+Bd)x^2+\left(4Ab\left(2bc^2+ad^2\right)-a\left(aCd^2-4bc(cC+2Bd)\right)\right)x+4abc(Bc+2Ad)\right)}{x}dx}{4b}+\frac{aCd^2x\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 2340$$

$$\frac{\frac{\frac{\int \frac{3b\left(4abc(Bc+2Ad)+\left(4Ab\left(2bc^2+ad^2\right)-a\left(aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx}{3b}+\frac{4}{3}ad\left(a+bx^2\right)^{3/2}(Bd+2cC)}{4b}+\frac{aCd^2x\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\left(4abc(Bc+2Ad)+\left(4Ab\left(2bc^2+ad^2\right)-a\left(aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx+\frac{4}{3}ad\left(a+bx^2\right)^{3/2}(Bd+2cC)}{4b}+\frac{aCd^2x\left(a+bx^2\right)^{3/2}}{4b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 535$$

$$\frac{\frac{\frac{1}{2}a\int \frac{8abc(Bc+2Ad)+\left(4Ab\left(2bc^2+ad^2\right)-a\left(aCd^2-4bc(cC+2Bd)\right)\right)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)+8abc(2Ad+Bc)\right)+}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 538$$

$$\frac{\frac{\frac{1}{2}a\left(\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx+8abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2\right.\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\downarrow 224$$

$$\frac{\frac{\frac{1}{2}a\left(\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aC\right.\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\frac{1}{2}a\left(8abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-\right.\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{\frac{1}{2}a\left(4abc(2Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd\right.\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{1}{2}a\left(8ac(2Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-a\left(aC\right.\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(ad^2+2bc^2\right)-a\left(aCd^2-4bc(2Bd+cC)\right)\right)}{\sqrt{b}}-8\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2Ad+Bc)\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(ad^2+2bc^2\right)-\right.\right.}{4b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{ax}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^2,x]
```

output
```
-((A*c^2*(a + b*x^2)^(3/2))/(a*x)) + ((a*C*d^2*x*(a + b*x^2)^(3/2))/(4*b)
+ (((8*a*b*c*(B*c + 2*A*d) + (4*A*b*(2*b*c^2 + a*d^2) - a*(a*C*d^2 - 4*b*c
*(c*C + 2*B*d)))*x)*Sqrt[a + b*x^2])/2 + (4*a*d*(2*c*C + B*d)*(a + b*x^2)^
(3/2))/3 + (a*(((4*A*b*(2*b*c^2 + a*d^2) - a*(a*C*d^2 - 4*b*c*(c*C + 2*B*d
)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - 8*Sqrt[a]*b*c*(B*c + 2
*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)/(4*b))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.29

method	result
default	$A\,d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+C\,c^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\,c^{2}\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{ax}+\right.$
risch	$-\frac{A\,c^{2}\sqrt{b\,x^{2}+a}}{x}+bd(Bd+2Cc)\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+(Ab\,d^{2}+2Bbcd+aC\,d^{2}+Cb\,c^{2})\left(\frac{x\sqrt{b}}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
A*d^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
C*c^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
A*c^2*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/3*B*d^2/b*(b*x^2+a)^(3/2)+C*d^2*(1/4*x*(b
*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))))+c*(2*A*d+B*c)*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2
)*(b*x^2+a)^(1/2))/x))+2*B*c*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))+2/3*C*c*d/b*(b*x^2+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 1015, normalized size of antiderivative = 3.95

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="fricas
")
```

output
```
[1/48*(3*(8*B*a*b*c*d + 4*(C*a*b + 2*A*b^2)*c^2 - (C*a^2 - 4*A*a*b)*d^2)*s
qrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24*(B*b^2*c^2 +
 2*A*b^2*c*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(6*C*b^2*d^2*x^4 - 24*A*b^2*c^2 + 8*(2*C*b^2*c*d + B*b^2*d^2)*x^3 +
3*(4*C*b^2*c^2 + 8*B*b^2*c*d + (C*a*b + 4*A*b^2)*d^2)*x^2 + 8*(3*B*b^2*c^2
 + B*a*b*d^2 + 2*(C*a*b + 3*A*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(b^2*x), -1/24
*(3*(8*B*a*b*c*d + 4*(C*a*b + 2*A*b^2)*c^2 - (C*a^2 - 4*A*a*b)*d^2)*sqrt(-
b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*(B*b^2*c^2 + 2*A*b^2*c*d)*sqr
t(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*C*b^2*d^2*
x^4 - 24*A*b^2*c^2 + 8*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 3*(4*C*b^2*c^2 + 8*
B*b^2*c*d + (C*a*b + 4*A*b^2)*d^2)*x^2 + 8*(3*B*b^2*c^2 + B*a*b*d^2 + 2*(C
*a*b + 3*A*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(b^2*x), 1/48*(48*(B*b^2*c^2 + 2*
A*b^2*c*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(8*B*a*b*c*d
+ 4*(C*a*b + 2*A*b^2)*c^2 - (C*a^2 - 4*A*a*b)*d^2)*sqrt(b)*x*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d^2*x^4 - 24*A*b^2*c^2 + 8
*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 3*(4*C*b^2*c^2 + 8*B*b^2*c*d + (C*a*b + 4
*A*b^2)*d^2)*x^2 + 8*(3*B*b^2*c^2 + B*a*b*d^2 + 2*(C*a*b + 3*A*b^2)*c*d)*x
)*sqrt(b*x^2 + a))/(b^2*x), -1/24*(3*(8*B*a*b*c*d + 4*(C*a*b + 2*A*b^2)*c^
2 - (C*a^2 - 4*A*a*b)*d^2)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
 24*(B*b^2*c^2 + 2*A*b^2*c*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt(-a...
```

Sympy [A] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 629, normalized size of antiderivative = 2.45

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**2,x)
```

output

```
-A*sqrt(a)*c**2/(x*sqrt(1 + b*x**2/a)) - 2*A*sqrt(a)*c*d*asinh(sqrt(a)/(sq
rt(b)*x)) + 2*A*a*c*d/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c**2*as
inh(sqrt(b)*x/sqrt(a)) + 2*A*sqrt(b)*c*d*x/sqrt(a/(b*x**2) + 1) + A*d**2*P
iecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne
(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)
), (sqrt(a)*x, True)) - A*b*c**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a
)*c**2*asinh(sqrt(a)/(sqrt(b)*x)) + B*a*c**2/(sqrt(b)*x*sqrt(a/(b*x**2) +
1)) + B*sqrt(b)*c**2*x/sqrt(a/(b*x**2) + 1) + 2*B*c*d*Piecewise((a*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True
)) + B*d**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3,
 Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*c**2*Piecewise((a*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*C*
c*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0
)), (sqrt(a)*x**2/2, True)) + C*d**2*Piecewise((-a**2*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b
, 0)), (sqrt(a)*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Cd^2x}{4\,b} - \frac{\sqrt{bx^2+a}Cad^2x}{8\,b} + A\sqrt{b}c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{Ca^2d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Ccd}{3\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Bd^2}{3\,b} - \frac{\sqrt{bx^2+a}Ac^2}{x} + \frac{1}{2}\left(Cc^2+2\,Bcd+Ad^2\right)\sqrt{bx^2+a}x + \frac{(Cc^2+2\,Bcd+Ad^2)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}} - \left(Bc^2+2\,Acd\right)\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \left(Bc^2+2\,Acd\right)\sqrt{bx^2+a}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="maxima
")
```

output
```
1/4*(b*x^2 + a)^(3/2)*C*d^2*x/b - 1/8*sqrt(b*x^2 + a)*C*a*d^2*x/b + A*sqrt
(b)*c^2*arcsinh(b*x/sqrt(a*b)) - 1/8*C*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(3
/2) + 2/3*(b*x^2 + a)^(3/2)*C*c*d/b + 1/3*(b*x^2 + a)^(3/2)*B*d^2/b - sqrt
(b*x^2 + a)*A*c^2/x + 1/2*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*x + 1/
2*(C*c^2 + 2*B*c*d + A*d^2)*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - (B*c^2 + 2*
A*c*d)*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + (B*c^2 + 2*A*c*d)*sqrt(b*x^
2 + a)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \frac{2\,Aa\sqrt{b}c^2}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(3\,Cd^2x+\frac{4\,(2\,Cb^2cd+Bb^2d^2)}{b^2}\right)x+\frac{3\,(4\,Cb^2c^2+8\,Bb^2cd+Cabd^2+4\,Ab^2d^2)}{b^2}\right)x\right. + \frac{2\,(Bac^2+2\,Aacd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{(4\,Cabc^2+8\,Ab^2c^2+8\,Babcd-Ca^2d^2+4\,Aabd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a*sqrt(b)*c^2/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/24*sqrt(b*x^2
+ a)*((2*(3*C*d^2*x + 4*(2*C*b^2*c*d + B*b^2*d^2)/b^2)*x + 3*(4*C*b^2*c^2
+ 8*B*b^2*c*d + C*a*b*d^2 + 4*A*b^2*d^2)/b^2)*x + 8*(3*B*b^2*c^2 + 2*C*a*b
*c*d + 6*A*b^2*c*d + B*a*b*d^2)/b^2) + 2*(B*a*c^2 + 2*A*a*c*d)*arctan(-(sq
rt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/8*(4*C*a*b*c^2 + 8*A*b^2
*c^2 + 8*B*a*b*c*d - C*a^2*d^2 + 4*A*a*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 15.00 (sec) , antiderivative size = 559, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2}\,dx$$

$$= \frac{12\sqrt{b\,x^2+a}\,a\,b^2d^2x^2+8\sqrt{b\,x^2+a}\,a\,b^2d^2x+24\sqrt{b\,x^2+a}\,b^3cd\,x^2+16\sqrt{b\,x^2+a}\,b^2c^2d\,x^3+6\sqrt{b\,x^2+a}\,b^2}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a*b**2*c**2 + 48*sqrt(a + b*x**2)*a*b**2*c*d*x + 1
2*sqrt(a + b*x**2)*a*b**2*d**2*x**2 + 8*sqrt(a + b*x**2)*a*b**2*d**2*x + 1
6*sqrt(a + b*x**2)*a*b*c**2*d*x + 3*sqrt(a + b*x**2)*a*b*c*d**2*x**2 + 24*
sqrt(a + b*x**2)*b**3*c**2*x + 24*sqrt(a + b*x**2)*b**3*c*d*x**2 + 8*sqrt(
a + b*x**2)*b**3*d**2*x**3 + 12*sqrt(a + b*x**2)*b**2*c**3*x**2 + 16*sqrt(
a + b*x**2)*b**2*c**2*d*x**3 + 6*sqrt(a + b*x**2)*b**2*c*d**2*x**4 + 48*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x +
 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**
2*x - 48*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
**2*c*d*x - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a
))*b**3*c**2*x + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*2*b*d**2*x - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c
*d**2*x + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*
*2*x + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x
 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**3*x - 3*s
qrt(b)*a**2*b*d**2*x - 24*sqrt(b)*a*b**2*c**2*x - 6*sqrt(b)*a*b**2*c*d*x -
 3*sqrt(b)*a*b*c**3*x)/(24*b**2*x)
```

3.22 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 250

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx = \left(c^2C+2Bcd+Ad^2\right)\sqrt{a+bx^2}-\frac{Ac^2\sqrt{a+bx^2}}{2x^2}+\frac{(2bc(Bc+2Ad)+ad(2cC+Bd))x\sqrt{a+bx^2}}{2a}+\frac{Cd^2(a+bx^2)^{3/2}}{3b}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{ax}+\frac{(2bc(Bc+2Ad)+ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\frac{(2ac(cC+2Bd)+A(bc^2+2ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
(A*d^2+2*B*c*d+C*c^2)*(b*x^2+a)^(1/2)-1/2*A*c^2*(b*x^2+a)^(1/2)/x^2+1/2*(2
*b*c*(2*A*d+B*c)+a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/a+1/3*C*d^2*(b*x^2+a)^
(3/2)/b-c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x+1/2*(2*b*c*(2*A*d+B*c)+a*d*(B*d+
2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-1/2*(2*a*c*(2*B*d+C*c)+
A*(2*a*d^2+b*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-3Ab(c^2+4cdx-2d^2x^2)+x(2aCd^2x+2bCx(3c^2+3cdx+d^2x^2)+bB(-6c^2+12cdx+3d}{6bx^2}$$
$$+\frac{Abc^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$
$$-2\sqrt{a}(c^2C+2Bcd+Ad^2)\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$
$$-\frac{(2bc(Bc+2Ad)+ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*A*b*(c^2 + 4*c*d*x - 2*d^2*x^2) + x*(2*a*C*d^2*x + 2*
b*C*x*(3*c^2 + 3*c*d*x + d^2*x^2) + b*B*(-6*c^2 + 12*c*d*x + 3*d^2*x^2))))
/(6*b*x^2) + (A*b*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt
[a] - 2*Sqrt[a]*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a +
 b*x^2])/Sqrt[a]] - ((2*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2338, 25, 2338, 25, 2340, 27, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^3}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}(2aCd^2x^3+2ad(2cC+Bd)x^2+(2ac(cC+2Bd)+A(bc^2+2ad^2))x+2ac(Bc+2Ad))}{x^2}dx}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}(2aCd^2x^3+2ad(2cC+Bd)x^2+(2ac(cC+2Bd)+A(bc^2+2ad^2))x+2ac(Bc+2Ad))}{x^2}dx}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(2a^2Cd^2x^2+2a(2bc(Bc+2Ad)+ad(2cC+Bd))x+a\left(2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)\right)\right)}{x}dx}{a}-\frac{2c(a+bx^2)^{3/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(2a^2Cd^2x^2+2a(2bc(Bc+2Ad)+ad(2cC+Bd))x+a\left(2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)\right)\right)}{x}dx}{a}-\frac{2c(a+bx^2)^{3/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\frac{\int \frac{3ab\left(2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)+2(2bc(Bc+2Ad)+ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx}{3b}+\frac{2a^2Cd^2\left(a+bx^2\right)^{3/2}}{3b}}{a}-\frac{2c(a+bx^2)^{3/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{\left(2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)+2(2bc(Bc+2Ad)+ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx+\frac{2a^2Cd^2\left(a+bx^2\right)^{3/2}}{3b}}{a}-\frac{2c(a+bx^2)^{3/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2(a+bx^2)^{3/2}}{2ax^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{a\left(\frac{1}{2}a\int \frac{2\left(2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)+(2bc(Bc+2Ad)+ad(2cC+Bd))x\right)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2Ad+Bc))+A\left(2ad^2+bc^2\right)+2ac(2Bd+c\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\downarrow$ 27

$$\frac{\frac{a\left(a\int \frac{2ac(cC+2Bd)+A\left(bc^2+2ad^2\right)+(2bc(Bc+2Ad)+ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2Ad+Bc))+A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\right)}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\downarrow$ 538

$$\frac{\frac{a\left(a\left(\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+(ad(Bd+2cC)+2bc(2Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2Ad+Bc))\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\downarrow$ 224

$$\frac{\frac{a\left(a\left(\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+(ad(Bd+2cC)+2bc(2Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\downarrow$ 219

$$\frac{\frac{a\left(a\left(\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+2bc(2Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2Ad+\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$\downarrow$ 243

$$\frac{\frac{a\left(a\left(\frac{1}{2}\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+2bc(2Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2\right.\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow 73$$

$$\frac{\frac{a\left(a\left(\frac{\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+2bc(2Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x(ad(Bd+2cC)+2bc(2Ad\right.\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{2a^2Cd^2\left(a+bx^2\right)^{3/2}}{3b}+a\left(a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+2bc(2Ad+Bc))}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(A\left(2ad^2+bc^2\right)+2ac(2Bd+cC)\right)}{\sqrt{a}}\right)+\sqrt{a+bx^2}\left(x\right.\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{2ax^2}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^3,x]
```

output
```
-1/2*(A*c^2*(a + b*x^2)^(3/2))/(a*x^2) + ((-2*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/x + ((2*a^2*C*d^2*(a + b*x^2)^(3/2))/(3*b) + a*((2*a*c*(c*C + 2*B*d
) + A*(b*c^2 + 2*a*d^2) + (2*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*x)*Sqr
t[a + b*x^2] + a*(((2*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((2*a*c*(c*C + 2*B*d) + A*(b*c^2 + 2*a*
d^2))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.16

method	result
default	$B d^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\left(A\,d^2+2Bcd+C\,c^2\right)\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+}}{x}\right.\right.$
risch	$-\frac{c\sqrt{b\,x^2+a}\,(4Adx+2Bcx+Ac)}{2x^2}+bd(Bd+2Cc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{(2Ab\,d^2+4Bbcd+2aC\,d^2+}{2b}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
B*d^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
(A*d^2+2*B*c*d+C*c^2)*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x))+A*c^2*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(
1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+1/3*C*d^2*(b*x^2+a)^(3/2)/b+c
*(2*A*d+B*c)*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^
(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+2*C*c*d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a
/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.15 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.90

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="fricas
")
```

output
```
[1/12*(3*(2*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*sqrt(b)*x^2*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3*(4*B*a*b*c*d + 2*A*a*b*
d^2 + (2*C*a*b + A*b^2)*c^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(2*C*a*b*d^2*x^4 - 3*A*a*b*c^2 + 3*(2*C*a*b*c*d + B
*a*b*d^2)*x^3 + 2*(3*C*a*b*c^2 + 6*B*a*b*c*d + (C*a^2 + 3*A*a*b)*d^2)*x^2
- 6*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a*b*x^2), -1/12*(6*(2*B
*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*sqrt(-b)*x^2*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - 3*(4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b + A*b^2)*c
^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2
*C*a*b*d^2*x^4 - 3*A*a*b*c^2 + 3*(2*C*a*b*c*d + B*a*b*d^2)*x^3 + 2*(3*C*a*
b*c^2 + 6*B*a*b*c*d + (C*a^2 + 3*A*a*b)*d^2)*x^2 - 6*(B*a*b*c^2 + 2*A*a*b*
c*d)*x)*sqrt(b*x^2 + a))/(a*b*x^2), 1/12*(6*(4*B*a*b*c*d + 2*A*a*b*d^2 + (
2*C*a*b + A*b^2)*c^2)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*
(2*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*sqrt(b)*x^2*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*C*a*b*d^2*x^4 - 3*A*a*b*c^2 +
 3*(2*C*a*b*c*d + B*a*b*d^2)*x^3 + 2*(3*C*a*b*c^2 + 6*B*a*b*c*d + (C*a^2 +
 3*A*a*b)*d^2)*x^2 - 6*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a*b*
x^2), -1/6*(3*(2*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 2*A*a*b)*c*d)*sqrt(-b)
*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(4*B*a*b*c*d + 2*A*a*b*d^2 + (
2*C*a*b + A*b^2)*c^2)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) -...
```

Sympy [A] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.47

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**3,x)
```

output
```
-2*A*sqrt(a)*c*d/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*d**2*asinh(sqrt(a)/(sq
rt(b)*x)) + A*a*d**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) - A*sqrt(b)*c**2*sqr
t(a/(b*x**2) + 1)/(2*x) + 2*A*sqrt(b)*c*d*asinh(sqrt(b)*x/sqrt(a)) + A*sqr
t(b)*d**2*x/sqrt(a/(b*x**2) + 1) - A*b*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(2*
sqrt(a)) - 2*A*b*c*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*c**2/(x*sq
rt(1 + b*x**2/a)) - 2*B*sqrt(a)*c*d*asinh(sqrt(a)/(sqrt(b)*x)) + 2*B*a*c*d
/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*c**2*asinh(sqrt(b)*x/sqrt(a)
) + 2*B*sqrt(b)*c*d*x/sqrt(a/(b*x**2) + 1) + B*d**2*Piecewise((a*Piecewise
((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sq
rt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
 - B*b*c**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C*sqrt(a)*c**2*asinh(sqrt(a)/
(sqrt(b)*x)) + C*a*c**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*sqrt(b)*c**2*
x/sqrt(a/(b*x**2) + 1) + 2*C*c*d*Piecewise((a*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*d**2*Piecewise
((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*
x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= -\frac{Abc^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Abc^2}{2a} + \frac{(bx^2+a)^{\frac{3}{2}}Cd^2}{3b} + \frac{1}{2}\left(2Ccd+Bd^2\right)\sqrt{bx^2+a}x + \frac{(2Ccd+Bd^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \left(Bc^2+2Acd\right)\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \left(Cc^2+2Bcd+Ad^2\right)\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \left(Cc^2+2Bcd+Ad^2\right)\sqrt{bx^2+a} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{2ax^2} - \frac{(Bc^2+2Acd)\sqrt{bx^2+a}}{x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="maxima
")
```

output
```
-1/2*A*b*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*A
*b*c^2/a + 1/3*(b*x^2 + a)^(3/2)*C*d^2/b + 1/2*(2*C*c*d + B*d^2)*sqrt(b*x^
2 + a)*x + 1/2*(2*C*c*d + B*d^2)*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + (B*c^2
 + 2*A*c*d)*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (C*c^2 + 2*B*c*d + A*d^2)*sqr
t(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + (C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2
+ a) - 1/2*(b*x^2 + a)^(3/2)*A*c^2/(a*x^2) - (B*c^2 + 2*A*c*d)*sqrt(b*x^2
+ a)/x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$=\frac{1}{6}\sqrt{bx^2+a}\left(\left(2\,Cd^2x+\frac{3\,(2\,Cbcd+Bbd^2)}{b}\right)x+\frac{2\,(3\,Cbc^2+6\,Bbcd+Cad^2+3\,Abd^2)}{b}\right)$$
$$+\frac{(2\,Cac^2+Abc^2+4\,Bacd+2\,Aad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{(2\,Bbc^2+2\,Cacd+4\,Abcd+Bad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc^2+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c^2+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}cd+\left(\sqrt{b}x\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*C*d^2*x + 3*(2*C*b*c*d + B*b*d^2)/b)*x + 2*(3*C*b*
c^2 + 6*B*b*c*d + C*a*d^2 + 3*A*b*d^2)/b) + (2*C*a*c^2 + A*b*c^2 + 4*B*a*c
*d + 2*A*a*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) -
 1/2*(2*B*b*c^2 + 2*C*a*c*d + 4*A*b*c*d + B*a*d^2)*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/sqrt(b) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b*c^2 + 2*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*c^2 + 4*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*A*a*sqrt(b)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b*c^2 - 2*B*a^2
*sqrt(b)*c^2 - 4*A*a^2*sqrt(b)*c*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^3}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^3,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3}\,dx$$
$$= \frac{-3\sqrt{b\,x^2+a}\,ab\,c^2-12\sqrt{b\,x^2+a}\,abcdx+6\sqrt{b\,x^2+a}\,ab\,d^2x^2+2\sqrt{b\,x^2+a}\,ac\,d^2x^2-6\sqrt{b\,x^2+a}\,b^2c^2x+}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3,x)
```

output

```
( - 3*sqrt(a + b*x**2)*a*b*c**2 - 12*sqrt(a + b*x**2)*a*b*c*d*x + 6*sqrt(a
 + b*x**2)*a*b*d**2*x**2 + 2*sqrt(a + b*x**2)*a*c*d**2*x**2 - 6*sqrt(a + b
*x**2)*b**2*c**2*x + 12*sqrt(a + b*x**2)*b**2*c*d*x**2 + 3*sqrt(a + b*x**2
)*b**2*d**2*x**3 + 6*sqrt(a + b*x**2)*b*c**3*x**2 + 6*sqrt(a + b*x**2)*b*c
**2*d*x**3 + 2*sqrt(a + b*x**2)*b*c*d**2*x**4 + 6*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**2 + 3*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**2 + 12*sqrt(a)*lo
g((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**2 + 6*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*x**2 - 6*
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**
2 - 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c
**2*x**2 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a)
)*b**2*c*d*x**2 - 6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*b*c**3*x**2 + 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a*b*c*d*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
b*d**2*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c**2
*d*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c**2*
x**2)/(6*b*x**2)
```

3.23 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 283

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx &= \frac{(bc(Bc+2Ad)+2ad(2cC+Bd))\sqrt{a+bx^2}}{2a} \\ &+ \frac{(aCd^2+2b(c^2C+2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{2a} - \frac{Ac^2(a+bx^2)^{3/2}}{3ax^3} \\ &- \frac{c(Bc+2Ad)\,(a+bx^2)^{3/2}}{2ax^2} - \frac{(c^2C+2Bcd+Ad^2)\,(a+bx^2)^{3/2}}{ax} \\ &+ \frac{(aCd^2+2b(c^2C+2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}} \\ &- \frac{(bc(Bc+2Ad)+2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}\end{aligned}$$

output
```
1/2*(b*c*(2*A*d+B*c)+2*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a+1/2*(a*C*d^2+2*b
*(A*d^2+2*B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/a-1/3*A*c^2*(b*x^2+a)^(3/2)/a/x^
3-1/2*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x^2-(A*d^2+2*B*c*d+C*c^2)*(b*x^2+a)^
(3/2)/a/x+1/2*(a*C*d^2+2*b*(A*d^2+2*B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(1/2)-1/2*(b*c*(2*A*d+B*c)+2*a*d*(B*d+2*C*c))*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= \frac{1}{6}\left(\frac{\sqrt{a+bx^2}(-2Abc^2x^2-2aA(c^2+3cdx+3d^2x^2)-3ax(B(c^2+4cdx-2d^2x^2)-Cx(-2c^2+4cdx+}{ax^3}\right.$$
$$-\frac{6(bc(Bc+2Ad)+2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$
$$\left.-\frac{3(aCd^2+2b(c^2C+2Bcd+Ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^4,x]
```

output
```
((Sqrt[a + b*x^2]*(-2*A*b*c^2*x^2 - 2*a*A*(c^2 + 3*c*d*x + 3*d^2*x^2) - 3*
a*x*(B*(c^2 + 4*c*d*x - 2*d^2*x^2) - C*x*(-2*c^2 + 4*c*d*x + d^2*x^2))))/(
a*x^3) - (6*(b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x
) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (3*(a*C*d^2 + 2*b*(c^2*C + 2*B*c*
d + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/6
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2338, 27, 2338, 25, 2338, 25, 27, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^4}\,dx$$
$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(aCd^2x^3+ad(2cC+Bd)x^2+a\left(Cc^2+2Bdc+Ad^2\right)x+ac(Bc+2Ad)\right)}{x^3}dx}{3a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(aCd^2x^3+ad(2cC+Bd)x^2+a\left(Cc^2+2Bdc+Ad^2\right)x+ac(Bc+2Ad)\right)}{x^3}dx}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{\sqrt{bx^2+a}\left(2Cd^2x^2a^2+2\left(Cc^2+2Bdc+Ad^2\right)a^2+(bc(Bc+2Ad)+2ad(2cC+Bd))xa\right)}{x^2}dx}{2a}-\frac{c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{2x^2}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(2Cd^2x^2a^2+2\left(Cc^2+2Bdc+Ad^2\right)a^2+(bc(Bc+2Ad)+2ad(2cC+Bd))xa\right)}{x^2}dx}{2a}-\frac{c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{2x^2}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{-\frac{\int -\frac{a^2\left(bc(Bc+2Ad)+2ad(2cC+Bd)+2\left(aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2a\left(a+bx^2\right)^{3/2}\left(Ad^2+2Bcd+c^2C\right)}{x}}{2a}-\frac{c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{2x^2}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{a^2\left(bc(Bc+2Ad)+2ad(2cC+Bd)+2\left(aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x}dx}{a}-\frac{2a\left(a+bx^2\right)^{3/2}\left(Ad^2+2Bcd+c^2C\right)}{x}}{2a}-\frac{c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{2x^2}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{a\int \frac{\left(bc(Bc+2Ad)+2ad(2cC+Bd)+2\left(aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x}dx-\frac{2a\left(a+bx^2\right)^{3/2}\left(Ad^2+2Bcd+c^2C\right)}{x}}{2a}-\frac{c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{2x^2}}{a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 535

$$\frac{a\left(\frac{1}{2}a\int \frac{2\left(bc(Bc+2Ad)+2ad(2cC+Bd)+\left(aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+2ad(Bd+2cC)+bc(2Ad+B\right.}{2a} \quad a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 27

$$\frac{a\left(a\int \frac{bc(Bc+2Ad)+2ad(2cC+Bd)+\left(aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+2ad(Bd+2cC)+bc(2Ad+Bc)\right)\right.}{2a} \quad a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 538

$$\frac{a\left(a\left(\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx+(2ad(Bd+2cC)+bc(2Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+\right.\right.}{2a} \quad a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 224

$$\frac{a\left(a\left(\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+(2ad(Bd+2cC)+bc(2Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd\right.\right.\right.\right.}{2a} \quad a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 219

$$\frac{a\left(a\left((2ad(Bd+2cC)+bc(2Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd+\right.\right.\right.\right.}{2a} \quad a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 243

$$\frac{a\left(a\left(\frac{1}{2}(2ad(Bd+2cC)+bc(2Ad+Bc))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bc\right.\right.\right.}{2a}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 73

$$\frac{a\left(a\left(\frac{(2ad(Bd+2cC)+bc(2Ad+Bc))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bcd+\right.\right.\right.}{2a}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

$\Big\downarrow$ 221

$$\frac{a\left(a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2ad(Bd+2cC)+bc(2Ad+Bc))}{\sqrt{a}}\right)+\sqrt{a+bx^2}\left(x\left(aCd^2+2b\left(Ad^2+2Bc\right.\right.\right.}{2a}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{3ax^3}$$

input

```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^4,x]
```

output

```
-1/3*(A*c^2*(a + b*x^2)^(3/2))/(a*x^3) + (-1/2*(c*(B*c + 2*A*d)*(a + b*x^2
)^(3/2))/x^2 + ((-2*a*(c^2*C + 2*B*c*d + A*d^2)*(a + b*x^2)^(3/2))/x + a*(
(b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d) + (a*C*d^2 + 2*b*(c^2*C + 2*B*c*d
 + A*d^2))*x)*Sqrt[a + b*x^2] + a*(((a*C*d^2 + 2*b*(c^2*C + 2*B*c*d + A*d^
2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((b*c*(B*c + 2*A*d) +
2*a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])))/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

method	result
default	$C d^2\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)+\left(A d^2+2Bcd+C c^2\right)\left(-\frac{\left(b x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\right.}{2\sqrt{}}\right.}{a}\right.$
risch	$-\frac{\sqrt{b x^2+a}\,\left(6Aa d^2x^2+2Ab c^2x^2+12Bacd x^2+6Ca c^2x^2+6Aacdx+3Ba c^2x+2A c^2a\right)}{6x^3a}-\frac{\left(2Abcd+2aB d^2+bB c^2+4Cacd\right)\ln\left(\frac{2a}{}\right.}{2\sqrt{a}}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
C*d^2*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+
(A*d^2+2*B*c*d+C*c^2)*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)
+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))-1/3*A*c^2*(b*x^2+a)^(3/2)/a
/x^3+c*(2*A*d+B*c)*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+d*(B*d+2*C*c)*((b*x^2+a)^(1/
2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 975, normalized size of antiderivative = 3.45

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="fricas
")
```

output
```
[1/12*(3*(2*C*a*b*c^2 + 4*B*a*b*c*d + (C*a^2 + 2*A*a*b)*d^2)*sqrt(b)*x^3*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3*(B*b^2*c^2 + 2*B*a*b*d^
2 + 2*(2*C*a*b + A*b^2)*c*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) + 2*(3*C*a*b*d^2*x^4 - 2*A*a*b*c^2 + 6*(2*C*a*b*c*d + B
*a*b*d^2)*x^3 - 2*(6*B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b + A*b^2)*c^2)*x^2
- 3*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a*b*x^3), -1/12*(6*(2*C
*a*b*c^2 + 4*B*a*b*c*d + (C*a^2 + 2*A*a*b)*d^2)*sqrt(-b)*x^3*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - 3*(B*b^2*c^2 + 2*B*a*b*d^2 + 2*(2*C*a*b + A*b^2)*c
*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3
*C*a*b*d^2*x^4 - 2*A*a*b*c^2 + 6*(2*C*a*b*c*d + B*a*b*d^2)*x^3 - 2*(6*B*a*
b*c*d + 3*A*a*b*d^2 + (3*C*a*b + A*b^2)*c^2)*x^2 - 3*(B*a*b*c^2 + 2*A*a*b*
c*d)*x)*sqrt(b*x^2 + a))/(a*b*x^3), 1/12*(6*(B*b^2*c^2 + 2*B*a*b*d^2 + 2*(
2*C*a*b + A*b^2)*c*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*
(2*C*a*b*c^2 + 4*B*a*b*c*d + (C*a^2 + 2*A*a*b)*d^2)*sqrt(b)*x^3*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(3*C*a*b*d^2*x^4 - 2*A*a*b*c^2 +
 6*(2*C*a*b*c*d + B*a*b*d^2)*x^3 - 2*(6*B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b
 + A*b^2)*c^2)*x^2 - 3*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a*b*
x^3), -1/6*(3*(2*C*a*b*c^2 + 4*B*a*b*c*d + (C*a^2 + 2*A*a*b)*d^2)*sqrt(-b)
*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(B*b^2*c^2 + 2*B*a*b*d^2 + 2*(
2*C*a*b + A*b^2)*c*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) -...
```

Sympy [A] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.12

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx$$
$$= -\frac{A\sqrt{a}d^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{A\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{A\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{x} + A\sqrt{b}d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$$
$$- \frac{Ab^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Abcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} - \frac{Abd^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{2B\sqrt{a}cd}{x\sqrt{1+\frac{bx^2}{a}}}$$
$$- B\sqrt{a}d^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Bad^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} - \frac{B\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{2x}$$
$$+ 2B\sqrt{b}cd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{B\sqrt{b}d^2x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Bbc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{2Bbcdx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$
$$- \frac{C\sqrt{a}c^2}{x\sqrt{1+\frac{bx^2}{a}}} - 2C\sqrt{a}cd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{2Cacd}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + C\sqrt{b}c^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$$
$$+ \frac{2C\sqrt{b}cdx}{\sqrt{\frac{a}{bx^2}+1}} + Cd^2\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for}\ b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$
$$- \frac{Cbc^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**4,x)
```

output
```
-A*sqrt(a)*d**2/(x*sqrt(1 + b*x**2/a)) - A*sqrt(b)*c**2*sqrt(a/(b*x**2) +
1)/(3*x**2) - A*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/x + A*sqrt(b)*d**2*asinh(
sqrt(b)*x/sqrt(a)) - A*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(3*a) - A*b*c*d*
asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) - A*b*d**2*x/(sqrt(a)*sqrt(1 + b*x**2/a
)) - 2*B*sqrt(a)*c*d/(x*sqrt(1 + b*x**2/a)) - B*sqrt(a)*d**2*asinh(sqrt(a)
/(sqrt(b)*x)) + B*a*d**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) - B*sqrt(b)*c**2
*sqrt(a/(b*x**2) + 1)/(2*x) + 2*B*sqrt(b)*c*d*asinh(sqrt(b)*x/sqrt(a)) + B
*sqrt(b)*d**2*x/sqrt(a/(b*x**2) + 1) - B*b*c**2*asinh(sqrt(a)/(sqrt(b)*x))
/(2*sqrt(a)) - 2*B*b*c*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C*sqrt(a)*c**2/(
x*sqrt(1 + b*x**2/a)) - 2*C*sqrt(a)*c*d*asinh(sqrt(a)/(sqrt(b)*x)) + 2*C*a
*c*d/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*sqrt(b)*c**2*asinh(sqrt(b)*x/sqr
t(a)) + 2*C*sqrt(b)*c*d*x/sqrt(a/(b*x**2) + 1) + C*d**2*Piecewise((a*Piece
wise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x
)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, Tr
ue)) - C*b*c**2*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.88

$$
\begin{aligned}
\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx &= \frac{1}{2}\sqrt{bx^2+a}Cd^2x + \frac{Cad^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} \\
&\quad + \left(Cc^2+2\,Bcd+Ad^2\right)\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) \\
&\quad - \left(2\,Ccd+Bd^2\right)\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) \\
&\quad - \frac{(Bc^2+2\,Acd)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} \\
&\quad + \left(2\,Ccd+Bd^2\right)\sqrt{bx^2+a} \\
&\quad + \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}b}{2\,a} \\
&\quad - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{3\,ax^3} \\
&\quad - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}}{x} \\
&\quad - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}}{2\,ax^2}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*C*d^2*x + 1/2*C*a*d^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) +
 (C*c^2 + 2*B*c*d + A*d^2)*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (2*C*c*d + B*d
^2)*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/2*(B*c^2 + 2*A*c*d)*b*arcsin
h(a/(sqrt(a*b)*abs(x)))/sqrt(a) + (2*C*c*d + B*d^2)*sqrt(b*x^2 + a) + 1/2*
(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b/a - 1/3*(b*x^2 + a)^(3/2)*A*c^2/(a*x^3
) - (C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)/x - 1/2*(B*c^2 + 2*A*c*d)*(b
*x^2 + a)^(3/2)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(249) = 498$.

Time = 0.25 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx = \frac{1}{2}\left(Cd^2x+4\,Ccd+2\,Bd^2\right)\sqrt{bx^2+a}$$
$$+\frac{\left(Bbc^2+4\,Cacd+2\,Abcd+2\,Bad^2\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{\left(2\,Cbc^2+4\,Bbcd+Cad^2+2\,Abd^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}$$
$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bbc^2+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Abcd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}c^2+6\left(\sqrt{b}x\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/2*(C*d^2*x + 4*C*c*d + 2*B*d^2)*sqrt(b*x^2 + a) + (B*b*c^2 + 4*C*a*c*d +
 2*A*b*c*d + 2*B*a*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sq
rt(-a) - 1/2*(2*C*b*c^2 + 4*B*b*c*d + C*a*d^2 + 2*A*b*d^2)*log(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B
*b*c^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b*c*d + 6*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*C*a*sqrt(b)*c^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)
*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*c*d + 6*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*A*a*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C
*a^2*sqrt(b)*c^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c*d -
12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*sqrt(b)*d^2 - 3*(sqrt(b)*x - sqrt
(b*x^2 + a))*B*a^2*b*c^2 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*c*d + 6
*C*a^3*sqrt(b)*c^2 + 2*A*a^2*b^(3/2)*c^2 + 12*B*a^3*sqrt(b)*c*d + 6*A*a^3*
sqrt(b)*d^2)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 22.79 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4,x)
```

output

```
( - 4*sqrt(a + b*x**2)*a**2*b*c**2 - 12*sqrt(a + b*x**2)*a**2*b*c*d*x - 12
*sqrt(a + b*x**2)*a**2*b*d**2*x**2 - 4*sqrt(a + b*x**2)*a*b**2*c**2*x**2 -
 6*sqrt(a + b*x**2)*a*b**2*c**2*x - 24*sqrt(a + b*x**2)*a*b**2*c*d*x**2 +
12*sqrt(a + b*x**2)*a*b**2*d**2*x**3 - 12*sqrt(a + b*x**2)*a*b*c**3*x**2 +
 24*sqrt(a + b*x**2)*a*b*c**2*d*x**3 + 6*sqrt(a + b*x**2)*a*b*c*d**2*x**4
+ 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*
c*d*x**3 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)
)*a*b**2*d**2*x**3 + 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*
x)/sqrt(a))*a*b*c**2*d*x**3 + 6*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
sqrt(b)*x)/sqrt(a))*b**3*c**2*x**3 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**3 - 12*sqrt(a)*log((sqrt(a + b*x
**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**3 - 24*sqrt(a)*log((sq
rt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x**3 - 6*sqrt(a)
*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**3 + 12
*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2*x**3 + 6*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d**2*x**3 + 24*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**3 + 12*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**3*x**3 + 4*sqrt(
b)*a**2*b*d**2*x**3 + sqrt(b)*a**2*c*d**2*x**3 - 4*sqrt(b)*a*b**2*c**2*x**
3 + 8*sqrt(b)*a*b**2*c*d*x**3 + 4*sqrt(b)*a*b*c**3*x**3)/(12*a*b*x**3)
```

3.24 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 241

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx &= Cd^2\sqrt{a+bx^2}-\frac{Ac^2\sqrt{a+bx^2}}{4x^4}-\frac{(4ac(cC+2Bd)+A(bc^2+4ad^2))\sqrt{a+bx^2}}{8ax^2}\\ &\quad-\frac{d(2cC+Bd)\sqrt{a+bx^2}}{x}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{3ax^3}\\ &\quad+\sqrt{b}d(2cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\\ &\quad+\frac{(Ab(bc^2-4ad^2)-4a(2aCd^2+bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}\end{aligned}$$

output

```
C*d^2*(b*x^2+a)^(1/2)-1/4*A*c^2*(b*x^2+a)^(1/2)/x^4-1/8*(4*a*c*(2*B*d+C*c)
+A*(4*a*d^2+b*c^2))*(b*x^2+a)^(1/2)/a/x^2-d*(B*d+2*C*c)*(b*x^2+a)^(1/2)/x-
1/3*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x^3+b^(1/2)*d*(B*d+2*C*c)*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))+1/8*(A*b*(-4*a*d^2+b*c^2)-4*a*(2*a*C*d^2+b*c*(2*B*d
+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-bcx^2(3Ac+8Bcx+16Adx)-2a(6Cx^2(c^2+4cdx-2d^2x^2)+4Bx(c^2+3cdx+3d^2x^2)+A}{24ax^4}$$
$$+\frac{(-Ab^2c^2+8a^2Cd^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}}$$
$$-\frac{b(c^2C+2Bcd+Ad^2)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$
$$-\sqrt{b}d(2cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-(b*c*x^2*(3*A*c + 8*B*c*x + 16*A*d*x)) - 2*a*(6*C*x^2*(
c^2 + 4*c*d*x - 2*d^2*x^2) + 4*B*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + A*(3*c^2
+ 8*c*d*x + 6*d^2*x^2))))/(24*a*x^4) + ((-(A*b^2*c^2) + 8*a^2*C*d^2)*ArcTa
nh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(3/2)) - (b*(c^2*C + 2*B*c
*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - S
qrt[b]*d*(2*c*C + B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2338, 25, 2338, 27, 2338, 25, 27, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^5}\,dx$$
$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(4aCd^2x^3+4ad(2cC+Bd)x^2+\left(4ac(cC+2Bd)-A\left(bc^2-4ad^2\right)\right)x+4ac(Bc+2Ad)\right)}{x^4}dx}{4a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 25

$$\frac{\int \frac{\sqrt{bx^2+a}\left(4aCd^2x^3+4ad(2cC+Bd)x^2+\left(4ac(cC+2Bd)-A\left(bc^2-4ad^2\right)\right)x+4ac(Bc+2Ad)\right)}{x^4}dx}{4a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 2338

$$-\frac{\frac{\int -\frac{3\sqrt{bx^2+a}\left(4Cd^2x^2a^2+4d(2cC+Bd)xa^2+\left(4ac(cC+2Bd)-A\left(bc^2-4ad^2\right)\right)a\right)}{x^3}dx}{3a}-\frac{4c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(4Cd^2x^2a^2+4d(2cC+Bd)xa^2+\left(4ac(cC+2Bd)-A\left(bc^2-4ad^2\right)\right)a\right)}{x^3}dx}{a}-\frac{4c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 2338

$$\frac{\frac{-\frac{\int -\frac{a\left(8a^2d(2cC+Bd)-\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{\left(a+bx^2\right)^{3/2}\left(4ac(2Bd+cC)-A\left(bc^2-4ad^2\right)\right)}{2x^2}}{a}-\frac{4c\left(a+bx^2\right)^{3/2}(2Ad}{3x^3}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a\left(8a^2d(2cC+Bd)-\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^2}dx}{2a}-\frac{\left(a+bx^2\right)^{3/2}\left(4ac(2Bd+cC)-A\left(bc^2-4ad^2\right)\right)}{2x^2}}{a}-\frac{4c\left(a+bx^2\right)^{3/2}(2Ad+B}{3x^3}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\int \frac{\left(8a^2d(2cC+Bd)-\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(4ac(2Bd+cC)-A\left(bc^2-4ad^2\right)\right)}{2x^2}}{a}-\frac{4c\left(a+bx^2\right)^{3/2}(2Ad+B}{3x^3}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 536

$$\frac{\frac{\frac{1}{2}\left(\int \frac{8bd(2cC+Bd)xa^2+\left(4a\left(2aCd^2+bc(cC+2Bd)\right)-Ab\left(bc^2-4ad^2\right)\right)a}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}\right)-\frac{(a}{}}{a}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 538

$$\frac{\frac{\frac{1}{2}\left(8a^2bd(Bd+2cC)\int \frac{1}{\sqrt{bx^2+a}}dx-a\left(Ab(bc^2-4ad^2)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2\right.\right.\right.}{x}\right.}{a}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{1}{2}\left(8a^2bd(Bd+2cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-a\left(Ab(bc^2-4ad^2)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ac\right.\right.\right.}{x}\right.}{a}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{1}{2}\left(-a\left(Ab(bc^2-4ad^2)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+8a^2\sqrt{b}\right.}{a}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{2}\left(-\frac{1}{2}a\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+8a}{a}$$

$$4a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{2}\left(-\frac{a\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+8a^2\sqrt{b}\right.}{a}$$

$$4a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{2}\left(-\frac{\sqrt{a+bx^2}\left(8a^2d(Bd+2cC)+x\left(Ab\left(bc^2-4ad^2\right)-4a\left(2aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+8a^2\sqrt{b}d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)+\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right(}{a}$$

$$4a$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{4ax^4}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^5,x]
```

output
```
-1/4*(A*c^2*(a + b*x^2)^(3/2))/(a*x^4) + ((-4*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(3*x^3) + (-1/2*((4*a*c*(c*C + 2*B*d) - A*(b*c^2 - 4*a*d^2))*(a + b
*x^2)^(3/2))/x^2 + (-(((8*a^2*d*(2*c*C + B*d) + (A*b*(b*c^2 - 4*a*d^2) - 4
*a*(2*a*C*d^2 + b*c*(c*C + 2*B*d)))*x)*Sqrt[a + b*x^2])/x) + 8*a^2*Sqrt[b]
*d*(2*c*C + B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] + Sqrt[a]*(A*b*(b*c^
2 - 4*a*d^2) - 4*a*(2*a*C*d^2 + b*c*(c*C + 2*B*d)))*ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a]])/2)/a)/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.09

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(16Abcd\,x^3+24Ba\,d^2x^3+8Bb\,c^2x^3+48Cacd\,x^3+12Aa\,d^2x^2+3Ab\,c^2x^2+24Bacd\,x^2+12Ca\,c^2x^2+16Aacdx+8Ba\,c^2x+}{24x^4a}$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)+A\,c^2\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(16*A*b*c*d*x^3+24*B*a*d^2*x^3+8*B*b*c^2*x^3+48*C*a*
c*d*x^3+12*A*a*d^2*x^2+3*A*b*c^2*x^2+24*B*a*c*d*x^2+12*C*a*c^2*x^2+16*A*a*
c*d*x+8*B*a*c^2*x+6*A*a*c^2)/x^4/a+1/8/a*(-(4*A*a*b*d^2-A*b^2*c^2+8*B*a*b*
c*d+8*C*a^2*d^2+4*C*a*b*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
+8*B*b^(1/2)*d^2*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+16*C*b^(1/2)*c*d*a*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))+8*C*d^2*a*(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1030, normalized size of antiderivative = 4.27

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="fricas
")
```

output
```
[1/48*(24*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(b)*x^4*log(-2*b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(b)*x - a) + 3*(8*B*a*b*c*d + (4*C*a*b - A*b^2)*c^2 + 4*(2*C*a^
2 + A*a*b)*d^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(24*C*a^2*d^2*x^4 - 6*A*a^2*c^2 - 8*(B*a*b*c^2 + 3*B*a^2*d^2 + 2
*(3*C*a^2 + A*a*b)*c*d)*x^3 - 3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 + A*
a*b)*c^2)*x^2 - 8*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4),
 -1/48*(48*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(-b)*x^4*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - 3*(8*B*a*b*c*d + (4*C*a*b - A*b^2)*c^2 + 4*(2*C*a^2 + A*a*b)*
d^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(
24*C*a^2*d^2*x^4 - 6*A*a^2*c^2 - 8*(B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 +
 A*a*b)*c*d)*x^3 - 3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 + A*a*b)*c^2)*x
^2 - 8*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4), 1/24*(3*(8
*B*a*b*c*d + (4*C*a*b - A*b^2)*c^2 + 4*(2*C*a^2 + A*a*b)*d^2)*sqrt(-a)*x^4
*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 12*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(b)
*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (24*C*a^2*d^2*x^4 -
 6*A*a^2*c^2 - 8*(B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + A*a*b)*c*d)*x^3 -
 3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 + A*a*b)*c^2)*x^2 - 8*(B*a^2*c^2
+ 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^4), -1/24*(24*(2*C*a^2*c*d + B*a
^2*d^2)*sqrt(-b)*x^4*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(8*B*a*b*c*d +
 (4*C*a*b - A*b^2)*c^2 + 4*(2*C*a^2 + A*a*b)*d^2)*sqrt(-a)*x^4*arctan(s...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(224) = 448$.

Time = 7.29 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = -\frac{Aac^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3A\sqrt{b}c^2}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2A\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{A\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab^{\frac{3}{2}}c^2}{8ax\sqrt{\frac{a}{bx^2}+1}} - \frac{2Ab^{\frac{3}{2}}cd\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Abd^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} + \frac{Ab^2c^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{B\sqrt{a}d^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{B\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{x} + B\sqrt{b}d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Bb^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Bbcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} - \frac{Bbd^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{2C\sqrt{a}cd}{x\sqrt{1+\frac{bx^2}{a}}} - C\sqrt{a}d^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Cad^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} - \frac{C\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{2x} + 2C\sqrt{b}cd\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{C\sqrt{b}d^2x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Cbc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} - \frac{2Cbcdx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**5,x)
```

output
```
-A*a*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*A*sqrt(b)*c**2/(8*x**3
*sqrt(a/(b*x**2) + 1)) - 2*A*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - A
*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)*c**2/(8*a*x*sqrt(a/(
b*x**2) + 1)) - 2*A*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(3*a) - A*b*d**2*asi
nh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)) + A*b**2*c**2*asinh(sqrt(a)/(sqrt(b)*x
))/(8*a**(3/2)) - B*sqrt(a)*d**2/(x*sqrt(1 + b*x**2/a)) - B*sqrt(b)*c**2*s
qrt(a/(b*x**2) + 1)/(3*x**2) - B*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/x + B*sq
rt(b)*d**2*asinh(sqrt(b)*x/sqrt(a)) - B*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)
/(3*a) - B*b*c*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) - B*b*d**2*x/(sqrt(a)*
sqrt(1 + b*x**2/a)) - 2*C*sqrt(a)*c*d/(x*sqrt(1 + b*x**2/a)) - C*sqrt(a)*d
**2*asinh(sqrt(a)/(sqrt(b)*x)) + C*a*d**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1))
 - C*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(2*x) + 2*C*sqrt(b)*c*d*asinh(sqrt(
b)*x/sqrt(a)) + C*sqrt(b)*d**2*x/sqrt(a/(b*x**2) + 1) - C*b*c**2*asinh(sqr
t(a)/(sqrt(b)*x))/(2*sqrt(a)) - 2*C*b*c*d*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.29

$$\begin{aligned}
&\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx \\
&= \frac{Ab^2c^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - C\sqrt{a}d^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{\sqrt{bx^2+a}Ab^2c^2}{8\,a^2} \\
&\quad + \sqrt{bx^2+a}Cd^2 + \left(2\,Ccd+Bd^2\right)\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) \\
&\quad - \frac{(Cc^2+2\,Bcd+Ad^2)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b}{2\,a} \\
&\quad + \frac{(bx^2+a)^{\frac{3}{2}}Abc^2}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Bc^2}{3\,ax^3} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Acd}{3\,ax^3} \\
&\quad - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}}{x} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{4\,ax^4} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}}{2\,ax^2}
\end{aligned}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="maxima
")
```

output
```
1/8*A*b^2*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - C*sqrt(a)*d^2*arcsin
h(a/(sqrt(a*b)*abs(x))) - 1/8*sqrt(b*x^2 + a)*A*b^2*c^2/a^2 + sqrt(b*x^2 +
 a)*C*d^2 + (2*C*c*d + B*d^2)*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*(C*c^2
+ 2*B*c*d + A*d^2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*(C*c^2 +
2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*b/a + 1/8*(b*x^2 + a)^(3/2)*A*b*c^2/(a^2*
x^2) - 1/3*(b*x^2 + a)^(3/2)*B*c^2/(a*x^3) - 2/3*(b*x^2 + a)^(3/2)*A*c*d/(
a*x^3) - (2*C*c*d + B*d^2)*sqrt(b*x^2 + a)/x - 1/4*(b*x^2 + a)^(3/2)*A*c^2
/(a*x^4) - 1/2*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(211) = 422$.

Time = 0.27 (sec) , antiderivative size = 1017, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*C*d^2 - (2*C*sqrt(b)*c*d + B*sqrt(b)*d^2)*log(abs(-sqrt(b)
*x + sqrt(b*x^2 + a))) + 1/4*(4*C*a*b*c^2 - A*b^2*c^2 + 8*B*a*b*c*d + 8*C*
a^2*d^2 + 4*A*a*b*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sq
rt(-a)*a) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^2 + 3*(sqrt(b
)*x - sqrt(b*x^2 + a))^7*A*b^2*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*
a*b*c*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b*d^2 + 24*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*B*a*b^(3/2)*c^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a
^2*sqrt(b)*c*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2)*c*d + 24*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*
x^2 + a))^5*C*a^2*b*c^2 + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c^2 -
 24*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*c*d - 12*(sqrt(b)*x - sqrt(b*x
^2 + a))^5*A*a^2*b*d^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*
c^2 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*sqrt(b)*c*d - 48*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2)*c*d - 72*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*B*a^3*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c^2 + 2
1*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c^2 - 24*(sqrt(b)*x - sqrt(b*x
^2 + a))^3*B*a^3*b*c*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b*d^2 +
8*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c^2 + 144*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*C*a^4*sqrt(b)*c*d + 16*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a
^3*b^(3/2)*c*d + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*sqrt(b)*d^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^5}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^5,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.77

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5}\,dx$$
$$= \frac{-8\sqrt{b\,x^2+a}\,b^2c^2x^3-16\sqrt{b\,x^2+a}\,abcd\,x^3+12\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab\,d^2x^4+24\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{a}}\right.}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5,x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2*c**2 - 16*sqrt(a + b*x**2)*a**2*c*d*x - 12*sqr
t(a + b*x**2)*a**2*d**2*x**2 - 3*sqrt(a + b*x**2)*a*b*c**2*x**2 - 8*sqrt(a
 + b*x**2)*a*b*c**2*x - 16*sqrt(a + b*x**2)*a*b*c*d*x**3 - 24*sqrt(a + b*x
**2)*a*b*c*d*x**2 - 24*sqrt(a + b*x**2)*a*b*d**2*x**3 - 12*sqrt(a + b*x**2
)*a*c**3*x**2 - 48*sqrt(a + b*x**2)*a*c**2*d*x**3 + 24*sqrt(a + b*x**2)*a*
c*d**2*x**4 - 8*sqrt(a + b*x**2)*b**2*c**2*x**3 + 12*sqrt(a)*log((sqrt(a +
 b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**4 + 24*sqrt(a)*log((s
qrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*c*d**2*x**4 - 3*sqrt(a)*
log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 + 24*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**
4 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**
3*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*
a*b*d**2*x**4 - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*a*c*d**2*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**2*c**2*x**4 - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**2*c*d*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**4 + 48*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*a*c**2*d*x**4 - 8*sqrt(b)*a*b*c*d*x**4 + 12*sqrt(b)*a*
b*d**2*x**4 + 24*sqrt(b)*a*c**2*d*x**4 - 4*sqrt(b)*b**2*c**2*x**4)/(24*...
```

3.25 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 252

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = \frac{(bc(Bc+2Ad)-4ad(2cC+Bd))\sqrt{a+bx^2}}{8ax^2}-\frac{Cd^2\sqrt{a+bx^2}}{x}-\frac{Ac^2(a+bx^2)^{3/2}}{5ax^5}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{4ax^4}-\frac{(5ac(cC+2Bd)-A(2bc^2-5ad^2))\left(a+bx^2\right)^{3/2}}{15a^2x^3}+\sqrt{b}Cd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{b(bc(Bc+2Ad)-4ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output

```
1/8*(b*c*(2*A*d+B*c)-4*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a/x^2-C*d^2*(b*x^2
+a)^(1/2)/x-1/5*A*c^2*(b*x^2+a)^(3/2)/a/x^5-1/4*c*(2*A*d+B*c)*(b*x^2+a)^(3
/2)/a/x^4-1/15*(5*a*c*(2*B*d+C*c)-A*(-5*a*d^2+2*b*c^2))*(b*x^2+a)^(3/2)/a^
2/x^3+b^(1/2)*C*d^2*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))+1/8*b*(b*c*(2*A*d+B
*c)-4*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(-16Ab^2c^2x^4+abx^2(5cx(3Bc+8cCx+16Bdx)+A(8c^2+30cdx+40d^2x^2))+2a^2(2A(6c}{120a^2x^5}$$
$$+\frac{b(bc(Bc+2Ad)-4ad(2cC+Bd))\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{3/2}}$$
$$-\sqrt{b}Cd^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(-16*A*b^2*c^2*x^4 + a*b*x^2*(5*c*x*(3*B*c + 8*c*C
*x + 16*B*d*x) + A*(8*c^2 + 30*c*d*x + 40*d^2*x^2)) + 2*a^2*(2*A*(6*c^2 +
15*c*d*x + 10*d^2*x^2) + 5*x*(4*C*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + B*(3*c^2
 + 8*c*d*x + 6*d^2*x^2)))))/(a^2*x^5) + (b*(b*c*(B*c + 2*A*d) - 4*a*d*(2*c
*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(3/2))
- Sqrt[b]*C*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2338, 25, 2338, 25, 2338, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^6}\,dx$$
$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(5aCd^2x^3+5ad(2cC+Bd)x^2+\left(5ac(cC+2Bd)-A\left(2bc^2-5ad^2\right)\right)x+5ac(Bc+2Ad)\right)}{x^5}dx}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(5aCd^2x^3+5ad(2cC+Bd)x^2+\left(5ac(cC+2Bd)-A\left(2bc^2-5ad^2\right)\right)x+5ac(Bc+2Ad)\right)}{x^5}dx}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{\sqrt{bx^2+a}\left(20a^2Cd^2x^2-5a(bc(Bc+2Ad)-4ad(2cC+Bd))x+4a\left(5ac(cC+2Bd)-A\left(2bc^2-5ad^2\right)\right)\right)}{x^4}dx}{4a}-\frac{5c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(20a^2Cd^2x^2-5a(bc(Bc+2Ad)-4ad(2cC+Bd))x+4a\left(5ac(cC+2Bd)-A\left(2bc^2-5ad^2\right)\right)\right)}{x^4}dx}{4a}-\frac{5c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{-\frac{\int \frac{15a^2\left(-4aCxd^2-4a(2cC+Bd)d+bc(Bc+2Ad)\right)\sqrt{bx^2+a}}{x^3}dx}{3a}-\frac{4\left(a+bx^2\right)^{3/2}\left(5ac(2Bd+cC)-A\left(2bc^2-5ad^2\right)\right)}{3x^3}}{4a}-\frac{5c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-5a\int \frac{\left(-4aCxd^2-4a(2cC+Bd)d+bc(Bc+2Ad)\right)\sqrt{bx^2+a}}{x^3}dx-\frac{4\left(a+bx^2\right)^{3/2}\left(5ac(2Bd+cC)-A\left(2bc^2-5ad^2\right)\right)}{3x^3}}{4a}-\frac{5c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \ 537$$

$$\frac{-5a\left(-\frac{1}{2}b\int -\frac{-8aCxd^2-4a(2cC+Bd)d+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{4\left(a+bx^2\right)^{3/2}\left(5ac(2Bd+cC)-A\left(2bc^2-5a\right.\right.}{3x^3}}{4a} \cdots}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 25

$$\frac{-5a\left(\frac{1}{2}b\int \frac{-8aCxd^2-4a(2cC+Bd)d+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{4\left(a+bx^2\right)^{3/2}\left(5ac(2Bd+cC)-A\left(2bc^2-5ad^2\right)\right.}{3x^3}}{4a} \cdots}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 538

$$\frac{-5a\left(\frac{1}{2}b\left((bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-8aCd^2\int \frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{4\left(a+bx^2\right)^{3/2}}{} \cdots}{4a} \cdots}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 224

$$\frac{-5a\left(\frac{1}{2}b\left((bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-8aCd^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{4(\cdots}{} \cdots}{4a} \cdots}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 219

$$\frac{-5a\left(\frac{1}{2}b\left((bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{4(\cdots}{} \cdots}{4a} \cdots}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 243

$$\frac{-5a\left(\frac{\frac{1}{2}b\left(\frac{1}{2}(bc(2Ad+Bc)-4ad(Bd+2cC))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{4a}\right)-}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 73

$$\frac{-5a\left(\frac{\frac{1}{2}b\left(\frac{(bc(2Ad+Bc)-4ad(Bd+2cC))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{8aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{4a}\right)-\frac{4(a}{}}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$\Big\downarrow$ 221

$$\frac{-5a\left(\frac{\frac{1}{2}b\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bc(2Ad+Bc)-4ad(Bd+2cC))}{\sqrt{a}}-\frac{8aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-4ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{4a}\right)}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{5ax^5}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/5*(A*c^2*(a + b*x^2)^(3/2))/(a*x^5) + ((-5*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(4*x^4) + ((-4*(5*a*c*(c*C + 2*B*d) - A*(2*b*c^2 - 5*a*d^2))*(a + b
*x^2)^(3/2))/(3*x^3) - 5*a*(-1/2*((b*c*(B*c + 2*A*d) - 4*a*d*(2*c*C + B*d)
 - 8*a*C*d^2*x)*Sqrt[a + b*x^2])/x^2 + (b*((-8*a*C*d^2*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/Sqrt[b] - ((b*c*(B*c + 2*A*d) - 4*a*d*(2*c*C + B*d))*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/(4*a))/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(40Ab\,d^{2}x^{4}a-16A\,b^{2}c^{2}x^{4}+80Bbcd\,x^{4}a+120C\,a^{2}d^{2}x^{4}+40Cb\,c^{2}x^{4}a+30Abcd\,x^{3}a+60B\,a^{2}d^{2}x^{3}+15Bb\,c^{2}x^{3}a+120C\,a^{2}}{120x^{5}a^{2}}$
default	$-\frac{(A\,d^{2}+2Bcd+C\,c^{2})\,(b\,x^{2}+a)^{\frac{3}{2}}}{3a\,x^{3}}+A\,c^{2}\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b(b\,x^{2}+a)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)+C\,d^{2}\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a}{}\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(40*A*a*b*d^2*x^4-16*A*b^2*c^2*x^4+80*B*a*b*c*d*x^4
+120*C*a^2*d^2*x^4+40*C*a*b*c^2*x^4+30*A*a*b*c*d*x^3+60*B*a^2*d^2*x^3+15*B
*a*b*c^2*x^3+120*C*a^2*c*d*x^3+40*A*a^2*d^2*x^2+8*A*a*b*c^2*x^2+80*B*a^2*c
*d*x^2+40*C*a^2*c^2*x^2+60*A*a^2*c*d*x+30*B*a^2*c^2*x+24*A*a^2*c^2)/x^5/a^
2-1/8*b/a*(-(2*A*b*c*d-4*B*a*d^2+B*b*c^2-8*C*a*c*d)/a^(1/2)*ln((2*a+2*a^(1
/2)*(b*x^2+a)^(1/2))/x)-8*a*C*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1094, normalized size of antiderivative = 4.34

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="fricas
")
```

output
```
[1/240*(120*C*a^2*sqrt(b)*d^2*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 15*(B*b^2*c^2 - 4*B*a*b*d^2 - 2*(4*C*a*b - A*b^2)*c*d)*sqrt(a)*x
^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(24*A*a^2*c^2 +
 8*(10*B*a*b*c*d + (5*C*a*b - 2*A*b^2)*c^2 + 5*(3*C*a^2 + A*a*b)*d^2)*x^4
+ 15*(B*a*b*c^2 + 4*B*a^2*d^2 + 2*(4*C*a^2 + A*a*b)*c*d)*x^3 + 8*(10*B*a^2
*c*d + 5*A*a^2*d^2 + (5*C*a^2 + A*a*b)*c^2)*x^2 + 30*(B*a^2*c^2 + 2*A*a^2*
c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^5), -1/240*(240*C*a^2*sqrt(-b)*d^2*x^5*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 15*(B*b^2*c^2 - 4*B*a*b*d^2 - 2*(4*C*a*b
 - A*b^2)*c*d)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(24*A*a^2*c^2 + 8*(10*B*a*b*c*d + (5*C*a*b - 2*A*b^2)*c^2 + 5*(3*
C*a^2 + A*a*b)*d^2)*x^4 + 15*(B*a*b*c^2 + 4*B*a^2*d^2 + 2*(4*C*a^2 + A*a*b
)*c*d)*x^3 + 8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 + A*a*b)*c^2)*x^2 +
30*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^5), 1/120*(60*C*a^
2*sqrt(b)*d^2*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 15*(B*
b^2*c^2 - 4*B*a*b*d^2 - 2*(4*C*a*b - A*b^2)*c*d)*sqrt(-a)*x^5*arctan(sqrt(
b*x^2 + a)*sqrt(-a)/a) - (24*A*a^2*c^2 + 8*(10*B*a*b*c*d + (5*C*a*b - 2*A*
b^2)*c^2 + 5*(3*C*a^2 + A*a*b)*d^2)*x^4 + 15*(B*a*b*c^2 + 4*B*a^2*d^2 + 2*
(4*C*a^2 + A*a*b)*c*d)*x^3 + 8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 + A*
a*b)*c^2)*x^2 + 30*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^5)
, -1/120*(120*C*a^2*sqrt(-b)*d^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(235) = 470$.

Time = 7.51 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.57

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = -\frac{Aacd}{2\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{3A\sqrt{b}cd}{4x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ab^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{15ax^2} - \frac{Ab^{\frac{3}{2}}cd}{4ax\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{3}{2}}d^2\sqrt{\frac{a}{bx^2}+1}}{3a} + \frac{2Ab^{\frac{5}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{15a^2} + \frac{Ab^2cd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{4a^{\frac{3}{2}}} - \frac{Bac^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3B\sqrt{b}c^2}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2B\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{B\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}c^2}{8ax\sqrt{\frac{a}{bx^2}+1}} - \frac{2Bb^{\frac{3}{2}}cd\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Bbd^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}} + \frac{Bb^2c^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}} - \frac{C\sqrt{a}d^2}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{C\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{x} + C\sqrt{b}d^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Cb^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{3a} - \frac{Cbcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} - \frac{Cbd^2x}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**6,x)
```

output
```
-A*a*c*d/(2*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - A*sqrt(b)*c**2*sqrt(a/(b*
x**2) + 1)/(5*x**4) - 3*A*sqrt(b)*c*d/(4*x**3*sqrt(a/(b*x**2) + 1)) - A*sq
rt(b)*d**2*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*c**2*sqrt(a/(b*x**2)
 + 1)/(15*a*x**2) - A*b**(3/2)*c*d/(4*a*x*sqrt(a/(b*x**2) + 1)) - A*b**(3/
2)*d**2*sqrt(a/(b*x**2) + 1)/(3*a) + 2*A*b**(5/2)*c**2*sqrt(a/(b*x**2) + 1
)/(15*a**2) + A*b**2*c*d*asinh(sqrt(a)/(sqrt(b)*x))/(4*a**(3/2)) - B*a*c**
2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*B*sqrt(b)*c**2/(8*x**3*sqrt(a/
(b*x**2) + 1)) - 2*B*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*sqrt(b)
*d**2*sqrt(a/(b*x**2) + 1)/(2*x) - B*b**(3/2)*c**2/(8*a*x*sqrt(a/(b*x**2)
+ 1)) - 2*B*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(3*a) - B*b*d**2*asinh(sqrt(
a)/(sqrt(b)*x))/(2*sqrt(a)) + B*b**2*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a*
*(3/2)) - C*sqrt(a)*d**2/(x*sqrt(1 + b*x**2/a)) - C*sqrt(b)*c**2*sqrt(a/(b
*x**2) + 1)/(3*x**2) - C*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/x + C*sqrt(b)*d*
*2*asinh(sqrt(b)*x/sqrt(a)) - C*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(3*a) -
 C*b*c*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) - C*b*d**2*x/(sqrt(a)*sqrt(1 +
 b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = C\sqrt{b}d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}Cd^2}{x} - \frac{(2\,Ccd+Bd^2)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{(Bc^2+2\,Acd)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}b}{2\,a} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}b^2}{8\,a^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cc^2}{3\,ax^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}Abc^2}{15\,a^2x^3} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Bcd}{3\,ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}Ad^2}{3\,ax^3} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}}{2\,ax^2} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{5\,ax^5} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}}{4\,ax^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="maxima
")
```

output
```
C*sqrt(b)*d^2*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)*C*d^2/x - 1/2*(2*C*
c*d + B*d^2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(B*c^2 + 2*A*c*
d)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/2*(2*C*c*d + B*d^2)*sqrt(
b*x^2 + a)*b/a - 1/8*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b^2/a^2 - 1/3*(b*x^
2 + a)^(3/2)*C*c^2/(a*x^3) + 2/15*(b*x^2 + a)^(3/2)*A*b*c^2/(a^2*x^3) - 2/
3*(b*x^2 + a)^(3/2)*B*c*d/(a*x^3) - 1/3*(b*x^2 + a)^(3/2)*A*d^2/(a*x^3) -
1/2*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)/(a*x^2) + 1/8*(B*c^2 + 2*A*c*d)*(b
*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/5*(b*x^2 + a)^(3/2)*A*c^2/(a*x^5) - 1/4*(B
*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1212 vs. $2(220) = 440$.

Time = 0.23 (sec) , antiderivative size = 1212, normalized size of antiderivative = 4.81

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
-C*sqrt(b)*d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) - 1/4*(B*b^2*c^2 - 8
*C*a*b*c*d + 2*A*b^2*c*d - 4*B*a*b*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 1/60*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*b^
2*c^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a*b*c*d + 30*(sqrt(b)*x - sq
rt(b*x^2 + a))^9*A*b^2*c*d + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a*b*d^2
+ 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^(3/2)*c^2 + 240*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*B*a*b^(3/2)*c*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*
a^2*sqrt(b)*d^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^(3/2)*d^2 + 90
*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2*c^2 - 240*(sqrt(b)*x - sqrt(b*x^2
 + a))^7*C*a^2*b*c*d + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b^2*c*d - 1
20*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a^2*b*d^2 - 240*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*C*a^2*b^(3/2)*c^2 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(
5/2)*c^2 - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)*c*d - 480*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b)*d^2 - 240*(sqrt(b)*x - sqrt(b*x
^2 + a))^6*A*a^2*b^(3/2)*d^2 + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*b
^(3/2)*c^2 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)*c^2 + 320*(s
qrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)*c*d + 720*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*C*a^4*sqrt(b)*d^2 + 160*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*
b^(3/2)*d^2 - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b^2*c^2 + 240*(sqrt
(b)*x - sqrt(b*x^2 + a))^3*C*a^4*b*c*d - 180*(sqrt(b)*x - sqrt(b*x^2 + ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.75

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^6}\,dx$$
$$= \frac{-120\sqrt{b\,x^2+a}\,a^2c\,d^2x^4+16\sqrt{b\,x^2+a}\,a\,b^2c^2x^4-40\sqrt{b\,x^2+a}\,ab\,c^3x^4-15\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3c^2}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3*c**2 - 60*sqrt(a + b*x**2)*a**3*c*d*x - 40*sq
rt(a + b*x**2)*a**3*d**2*x**2 - 8*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 30*s
qrt(a + b*x**2)*a**2*b*c**2*x - 30*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 80*s
qrt(a + b*x**2)*a**2*b*c*d*x**2 - 40*sqrt(a + b*x**2)*a**2*b*d**2*x**4 - 6
0*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 40*sqrt(a + b*x**2)*a**2*c**3*x**2 -
 120*sqrt(a + b*x**2)*a**2*c**2*d*x**3 - 120*sqrt(a + b*x**2)*a**2*c*d**2*
x**4 + 16*sqrt(a + b*x**2)*a*b**2*c**2*x**4 - 15*sqrt(a + b*x**2)*a*b**2*c
**2*x**3 - 80*sqrt(a + b*x**2)*a*b**2*c*d*x**4 - 40*sqrt(a + b*x**2)*a*b*c
**3*x**4 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)
)*a*b**2*c*d*x**5 + 60*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*a*b**2*d**2*x**5 + 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
+ sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x**5 - 15*sqrt(a)*log((sqrt(a + b*x**2) -
 sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**5 + 30*sqrt(a)*log((sqrt(a + b
*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**5 - 60*sqrt(a)*log((s
qrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**5 - 120*sqr
t(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x**5
 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c
**2*x**5 + 120*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*
d**2*x**5 - 8*sqrt(b)*a**2*b*d**2*x**5 + 72*sqrt(b)*a**2*c*d**2*x**5 - 16*
sqrt(b)*a*b**2*c**2*x**5 - 16*sqrt(b)*a*b**2*c*d*x**5 - 8*sqrt(b)*a*b*c...
```

3.26 $\int \frac{(c+dx)^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{x^7}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 273

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx &= -\frac{Ac^2\sqrt{a+bx^2}}{6x^6}-\frac{(6ac(cC+2Bd)+A(bc^2+6ad^2))\sqrt{a+bx^2}}{24ax^4}\\ &\quad+\frac{(Ab(bc^2-2ad^2)-2a(4aCd^2+bc(cC+2Bd)))\sqrt{a+bx^2}}{16a^2x^2}\\ &\quad-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{5ax^5}+\frac{(2bc(Bc+2Ad)-5ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{15a^2x^3}\\ &\quad-\frac{b(Ab(bc^2-2ad^2)+2a(4aCd^2-bc(cC+2Bd)))\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}\end{aligned}$$

output

```
-1/6*A*c^2*(b*x^2+a)^(1/2)/x^6-1/24*(6*a*c*(2*B*d+C*c)+A*(6*a*d^2+b*c^2))*
(b*x^2+a)^(1/2)/a/x^4+1/16*(A*b*(-2*a*d^2+b*c^2)-2*a*(4*a*C*d^2+b*c*(2*B*d
+C*c)))*(b*x^2+a)^(1/2)/a^2/x^2-1/5*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x^5+1/
15*(2*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a^2/x^3-1/16*b*(A
*b*(-2*a*d^2+b*c^2)+2*a*(4*a*C*d^2-b*c*(2*B*d+C*c)))*arctanh((b*x^2+a)^(1/
2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+bx^2}\left(b^2cx^4(15Ac+32Bcx+64Adx)-4a^2\left(A\left(10c^2+24cdx+15d^2x^2\right)+x\left(5Cx\left(3c^2+8cdx+6d^2x^2\right)+2B\left(6c^2+15cdx+10d^2x^2\right)\right)\right)-2abx^2\left(A\right.\right.}{x^6}}{}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^7,x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(b^2*c*x^4*(15*A*c + 32*B*c*x + 64*A*d*x) - 4*a^
2*(A*(10*c^2 + 24*c*d*x + 15*d^2*x^2) + x*(5*C*x*(3*c^2 + 8*c*d*x + 6*d^2*
x^2) + 2*B*(6*c^2 + 15*c*d*x + 10*d^2*x^2))) - 2*a*b*x^2*(A*(5*c^2 + 16*c*
d*x + 15*d^2*x^2) + x*(5*c*C*x*(3*c + 16*d*x) + B*(8*c^2 + 30*c*d*x + 40*d
^2*x^2)))))/x^6 + 30*b*(A*b^2*c^2 + 8*a^2*C*d^2)*ArcTanh[(Sqrt[b]*x - Sqrt
[a + b*x^2])/Sqrt[a]] + 60*a*b^2*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt
[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(240*a^(5/2))
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2338, 27, 2338, 25, 2338, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^7}\,dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^6}dx}{6a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^6}dx}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{\sqrt{bx^2+a}\left(10a^2Cd^2x^2-2a(2bc(Bc+2Ad)-5ad(2cC+Bd))x+5a\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)\right)}{x^5}dx}{5a}-\frac{2c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{5x^5}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(10a^2Cd^2x^2-2a(2bc(Bc+2Ad)-5ad(2cC+Bd))x+5a\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)\right)}{x^5}dx}{5a}-\frac{2c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{5x^5}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 2338

$$\frac{\frac{-\frac{\int \frac{a\left(8a(2bc(Bc+2Ad)-5ad(2cC+Bd))-5\left(8a^2Cd^2-2abc(cC+2Bd)+Ab\left(bc^2-2ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{5\left(a+bx^2\right)^{3/2}\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)}{4x^4}}{5a}-\frac{2c(}{}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{1}{4}\int \frac{\left(8a(2bc(Bc+2Ad)-5ad(2cC+Bd))-5\left(8a^2Cd^2-2abc(cC+2Bd)+Ab\left(bc^2-2ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x^4}dx-\frac{5\left(a+bx^2\right)^{3/2}\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)}{4x^4}}{5a}-\frac{2c(}{}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{1}{4}\left(5\left(8a^2Cd^2+Ab\left(bc^2-2ad^2\right)-2abc(2Bd+cC)\right)\int \frac{\sqrt{bx^2+a}}{x^3}dx+\frac{8\left(a+bx^2\right)^{3/2}(2bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\left(a+bx^2\right)^{3/2}\left(2ac(2Bd+cC)-A\left(bc^2-\right.\right.}{4x^4}}{5a}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{1}{4}\left(\frac{5}{2}\left(8a^2Cd^2+Ab\left(bc^2-2ad^2\right)-2abc(2Bd+cC)\right)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2+\frac{8\left(a+bx^2\right)^{3/2}(2bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\left(a+bx^2\right)^{3/2}\left(2ac(2Bd+cC)-A\left(bc^2\right.\right.}{4x^4}}{5a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\big\downarrow$ 51

$$\frac{\frac{\frac{1}{4}\left(\frac{5}{2}\left(8a^2Cd^2+Ab\left(bc^2-2ad^2\right)-2abc(2Bd+cC)\right)\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)+\frac{8\left(a+bx^2\right)^{3/2}(2bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\left(a+bx^2\right)^{3/2}\left(2\right.}{}}{5a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\big\downarrow$ 73

$$\frac{\frac{\frac{1}{4}\left(\frac{5}{2}\left(8a^2Cd^2+Ab\left(bc^2-2ad^2\right)-2abc(2Bd+cC)\right)\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)+\frac{8\left(a+bx^2\right)^{3/2}(2bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\left(a+bx^2\right)^{3/2}\left(2c\right.}{}}{5a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

$\big\downarrow$ 221

$$\frac{\frac{\frac{1}{4}\left(\frac{5}{2}\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)\left(8a^2Cd^2+Ab\left(bc^2-2ad^2\right)-2abc(2Bd+cC)\right)+\frac{8\left(a+bx^2\right)^{3/2}(2bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\left(a+bx^2\right.}{}}{5a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{6ax^6}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^7,x]
```

output
```
-1/6*(A*c^2*(a + b*x^2)^(3/2))/(a*x^6) + ((-2*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(5*x^5) + ((-5*(2*a*c*(c*C + 2*B*d) - A*(b*c^2 - 2*a*d^2))*(a + b*x
^2)^(3/2))/(4*x^4) + ((8*(2*b*c*(B*c + 2*A*d) - 5*a*d*(2*c*C + B*d))*(a +
b*x^2)^(3/2))/(3*x^3) + (5*(8*a^2*C*d^2 - 2*a*b*c*(c*C + 2*B*d) + A*b*(b*c
^2 - 2*a*d^2))*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]])/Sqrt[a]))/2)/4)/(5*a))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(-64 A b^2 c d x^5+80 B a b d^2 x^5-32 B b^2 c^2 x^5+160 C a b c d x^5+30 A b d^2 x^4 a-15 A b^2 c^2 x^4+60 B b c d x^4 a+120 C a^2 d^2 x^4+30 C b$
default	$(A d^2+2 B c d+C c^2)\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{4 a x^4}-\frac{b\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{2 a x^2}+\frac{b\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2 a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)\right)}{2 a}\right)}{4 a}\right)+A c^2\left(-\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(-64*A*b^2*c*d*x^5+80*B*a*b*d^2*x^5-32*B*b^2*c^2*x^
5+160*C*a*b*c*d*x^5+30*A*a*b*d^2*x^4-15*A*b^2*c^2*x^4+60*B*a*b*c*d*x^4+120
*C*a^2*d^2*x^4+30*C*a*b*c^2*x^4+32*A*a*b*c*d*x^3+80*B*a^2*d^2*x^3+16*B*a*b
*c^2*x^3+160*C*a^2*c*d*x^3+60*A*a^2*d^2*x^2+10*A*a*b*c^2*x^2+120*B*a^2*c*d
*x^2+60*C*a^2*c^2*x^2+96*A*a^2*c*d*x+48*B*a^2*c^2*x+40*A*a^2*c^2)/x^6/a^2+
1/16*(2*A*a*b*d^2-A*b^2*c^2+4*B*a*b*c*d-8*C*a^2*d^2+2*C*a*b*c^2)*b/a^(5/2)
*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.26

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$

$$= \left[\frac{15\,(4\,Bab^2cd + (2\,Cab^2 - Ab^3)c^2 - 2\,(4\,Ca^2b - Aab^2)d^2)\sqrt{a}x^6\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) - 2\,(40\,Aa^3c^2\right.$$

$$-\frac{15\,(4\,Bab^2cd + (2\,Cab^2 - Ab^3)c^2 - 2\,(4\,Ca^2b - Aab^2)d^2)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) + (40\,Aa^3c^2 - 1}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="fricas
")
```

output
```
[1/480*(15*(4*B*a*b^2*c*d + (2*C*a*b^2 - A*b^3)*c^2 - 2*(4*C*a^2*b - A*a*b
^2)*d^2)*sqrt(a)*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(40*A*a^3*c^2 - 16*(2*B*a*b^2*c^2 - 5*B*a^2*b*d^2 - 2*(5*C*a^2*b - 2*A*
a*b^2)*c*d)*x^5 + 15*(4*B*a^2*b*c*d + (2*C*a^2*b - A*a*b^2)*c^2 + 2*(4*C*a
^3 + A*a^2*b)*d^2)*x^4 + 16*(B*a^2*b*c^2 + 5*B*a^3*d^2 + 2*(5*C*a^3 + A*a^
2*b)*c*d)*x^3 + 10*(12*B*a^3*c*d + 6*A*a^3*d^2 + (6*C*a^3 + A*a^2*b)*c^2)*
x^2 + 48*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^6), -1/240*(
15*(4*B*a*b^2*c*d + (2*C*a*b^2 - A*b^3)*c^2 - 2*(4*C*a^2*b - A*a*b^2)*d^2)
*sqrt(-a)*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (40*A*a^3*c^2 - 16*(2*B
*a*b^2*c^2 - 5*B*a^2*b*d^2 - 2*(5*C*a^2*b - 2*A*a*b^2)*c*d)*x^5 + 15*(4*B*
a^2*b*c*d + (2*C*a^2*b - A*a*b^2)*c^2 + 2*(4*C*a^3 + A*a^2*b)*d^2)*x^4 + 1
6*(B*a^2*b*c^2 + 5*B*a^3*d^2 + 2*(5*C*a^3 + A*a^2*b)*c*d)*x^3 + 10*(12*B*a
^3*c*d + 6*A*a^3*d^2 + (6*C*a^3 + A*a^2*b)*c^2)*x^2 + 48*(B*a^3*c^2 + 2*A*
a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(258) = 516$.

Time = 12.56 (sec) , antiderivative size = 819, normalized size of antiderivative = 3.00

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a*c**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - A*a*d**2/(4*sqrt(b)*x**5
*sqrt(a/(b*x**2) + 1)) - 5*A*sqrt(b)*c**2/(24*x**5*sqrt(a/(b*x**2) + 1)) -
 2*A*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*A*sqrt(b)*d**2/(8*x**3*
sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*c**2/(48*a*x**3*sqrt(a/(b*x**2) + 1)) -
 2*A*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(15*a*x**2) - A*b**(3/2)*d**2/(8*a*
x*sqrt(a/(b*x**2) + 1)) + A*b**(5/2)*c**2/(16*a**2*x*sqrt(a/(b*x**2) + 1))
 + 4*A*b**(5/2)*c*d*sqrt(a/(b*x**2) + 1)/(15*a**2) + A*b**2*d**2*asinh(sqr
t(a)/(sqrt(b)*x))/(8*a**(3/2)) - A*b**3*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(1
6*a**(5/2)) - B*a*c*d/(2*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - B*sqrt(b)*c*
*2*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*B*sqrt(b)*c*d/(4*x**3*sqrt(a/(b*x**2)
 + 1)) - B*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*c**2*sq
rt(a/(b*x**2) + 1)/(15*a*x**2) - B*b**(3/2)*c*d/(4*a*x*sqrt(a/(b*x**2) + 1
)) - B*b**(3/2)*d**2*sqrt(a/(b*x**2) + 1)/(3*a) + 2*B*b**(5/2)*c**2*sqrt(a
/(b*x**2) + 1)/(15*a**2) + B*b**2*c*d*asinh(sqrt(a)/(sqrt(b)*x))/(4*a**(3/
2)) - C*a*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*C*sqrt(b)*c**2/(8
*x**3*sqrt(a/(b*x**2) + 1)) - 2*C*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(3*x**2
) - C*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(2*x) - C*b**(3/2)*c**2/(8*a*x*sqr
t(a/(b*x**2) + 1)) - 2*C*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(3*a) - C*b*d**
2*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt(a)) + C*b**2*c**2*asinh(sqrt(a)/(sqrt
(b)*x))/(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx$$
$$= -\frac{Ab^3c^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} - \frac{Cbd^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Ab^3c^2}{16\,a^3} + \frac{\sqrt{bx^2+a}Cbd^2}{2\,a}$$
$$+ \frac{(Cc^2+2\,Bcd+Ad^2)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b^2}{8\,a^2}$$
$$- \frac{(bx^2+a)^{\frac{3}{2}}Ab^2c^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cd^2}{2\,ax^2} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ccd}{3\,ax^3} - \frac{(bx^2+a)^{\frac{3}{2}}Bd^2}{3\,ax^3}$$
$$+ \frac{(bx^2+a)^{\frac{3}{2}}Abc^2}{8\,a^2x^4} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^2} + \frac{2\,(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b}{15\,a^2x^3}$$
$$- \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{6\,ax^6} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}}{4\,ax^4} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}}{5\,ax^5}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="maxima
")
```

output
```
-1/16*A*b^3*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 1/2*C*b*d^2*arcsin
h(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*sqrt(b*x^2 + a)*A*b^3*c^2/a^3 + 1/2
*sqrt(b*x^2 + a)*C*b*d^2/a + 1/8*(C*c^2 + 2*B*c*d + A*d^2)*b^2*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)
*b^2/a^2 - 1/16*(b*x^2 + a)^(3/2)*A*b^2*c^2/(a^3*x^2) - 1/2*(b*x^2 + a)^(3
/2)*C*d^2/(a*x^2) - 2/3*(b*x^2 + a)^(3/2)*C*c*d/(a*x^3) - 1/3*(b*x^2 + a)^
(3/2)*B*d^2/(a*x^3) + 1/8*(b*x^2 + a)^(3/2)*A*b*c^2/(a^2*x^4) + 1/8*(C*c^2
 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*b/(a^2*x^2) + 2/15*(B*c^2 + 2*A*c*d)
*(b*x^2 + a)^(3/2)*b/(a^2*x^3) - 1/6*(b*x^2 + a)^(3/2)*A*c^2/(a*x^6) - 1/4
*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/(a*x^4) - 1/5*(B*c^2 + 2*A*c*
d)*(b*x^2 + a)^(3/2)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1541 vs. $2(245) = 490$.

Time = 0.23 (sec) , antiderivative size = 1541, normalized size of antiderivative = 5.64

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
-1/8*(2*C*a*b^2*c^2 - A*b^3*c^2 + 4*B*a*b^2*c*d - 8*C*a^2*b*d^2 + 2*A*a*b^
2*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/
120*(30*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^2*c^2 - 15*(sqrt(b)*x - sqr
t(b*x^2 + a))^11*A*b^3*c^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^2*c
*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b*d^2 + 30*(sqrt(b)*x - sq
rt(b*x^2 + a))^11*A*a*b^2*d^2 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2
*b^(3/2)*c*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(3/2)*d^2 + 15
0*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2*c^2 + 85*(sqrt(b)*x - sqrt(b*x
^2 + a))^9*A*a*b^3*c^2 + 300*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^2*c*d
 - 360*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^3*b*d^2 + 150*(sqrt(b)*x - sqrt
(b*x^2 + a))^9*A*a^2*b^2*d^2 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b
^(5/2)*c^2 - 1440*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2)*c*d + 960*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(5/2)*c*d - 720*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*B*a^3*b^(3/2)*d^2 - 180*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^
3*b^2*c^2 + 570*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c^2 - 360*(sqrt(
b)*x - sqrt(b*x^2 + a))^7*B*a^3*b^2*c*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a)
)^7*C*a^4*b*d^2 - 180*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^3*b^2*d^2 - 320*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2)*c^2 + 1600*(sqrt(b)*x - sqrt
(b*x^2 + a))^6*C*a^4*b^(3/2)*c*d - 640*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a
^3*b^(5/2)*c*d + 800*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(3/2)*d^2 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^7,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 765, normalized size of antiderivative = 2.80

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^7}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c**2 - 96*sqrt(a + b*x**2)*a**3*c*d*x - 60*sq
rt(a + b*x**2)*a**3*d**2*x**2 - 10*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 48*
sqrt(a + b*x**2)*a**2*b*c**2*x - 32*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 120
*sqrt(a + b*x**2)*a**2*b*c*d*x**2 - 30*sqrt(a + b*x**2)*a**2*b*d**2*x**4 -
 80*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 60*sqrt(a + b*x**2)*a**2*c**3*x**2
 - 160*sqrt(a + b*x**2)*a**2*c**2*d*x**3 - 120*sqrt(a + b*x**2)*a**2*c*d**
2*x**4 + 15*sqrt(a + b*x**2)*a*b**2*c**2*x**4 - 16*sqrt(a + b*x**2)*a*b**2
*c**2*x**3 + 64*sqrt(a + b*x**2)*a*b**2*c*d*x**5 - 60*sqrt(a + b*x**2)*a*b
**2*c*d*x**4 - 80*sqrt(a + b*x**2)*a*b**2*d**2*x**5 - 30*sqrt(a + b*x**2)*
a*b*c**3*x**4 - 160*sqrt(a + b*x**2)*a*b*c**2*d*x**5 + 32*sqrt(a + b*x**2)
*b**3*c**2*x**5 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b**2*d**2*x**6 + 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
sqrt(b)*x)/sqrt(a))*a*b*c*d**2*x**6 + 15*sqrt(a)*log((sqrt(a + b*x**2) - s
qrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**6 - 60*sqrt(a)*log((sqrt(a + b*x
**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*d*x**6 - 30*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**3*x**6 + 30*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**6 - 120
*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2*
x**6 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*
*3*c**2*x**6 + 60*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/...
```

3.27 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 313

$$\begin{aligned}\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx &= \frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\sqrt{a+bx^2}}{8ax^4} \\ &+ \frac{b(bc(Bc+2Ad)-2ad(2cC+Bd))\sqrt{a+bx^2}}{16a^2x^2} - \frac{Ac^2(a+bx^2)^{3/2}}{7ax^7} \\ &- \frac{c(Bc+2Ad)\,(a+bx^2)^{3/2}}{6ax^6} - \frac{(7ac(cC+2Bd)-A(4bc^2-7ad^2))\,(a+bx^2)^{3/2}}{35a^2x^5} \\ &- \frac{(2Ab(4bc^2-7ad^2)+7a(5aCd^2-2bc(cC+2Bd)))\,(a+bx^2)^{3/2}}{105a^3x^3} \\ &- \frac{b^2(bc(Bc+2Ad)-2ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}\end{aligned}$$

output
```
1/8*(b*c*(2*A*d+B*c)-2*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a/x^4+1/16*b*(b*c*
(2*A*d+B*c)-2*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x^2-1/7*A*c^2*(b*x^2+a)
^(3/2)/a/x^7-1/6*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x^6-1/35*(7*a*c*(2*B*d+C*
c)-A*(-7*a*d^2+4*b*c^2))*(b*x^2+a)^(3/2)/a^2/x^5-1/105*(2*A*b*(-7*a*d^2+4*
b*c^2)+7*a*(5*a*C*d^2-2*b*c*(2*B*d+C*c)))*(b*x^2+a)^(3/2)/a^3/x^3-1/16*b^2
*(b*c*(2*A*d+B*c)-2*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5
/2)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-128Ab^3c^2x^6+ab^2x^4(7cx(15Bc+32cCx+64Bdx)+A(64c^2+210cdx+224d^2x^2))-4a^3(4\ldots}{}$$
$$+\frac{b^2(-bc(Bc+2Ad)+2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^8,x]
```

output
```
(Sqrt[a + b*x^2]*(-128*A*b^3*c^2*x^6 + a*b^2*x^4*(7*c*x*(15*B*c + 32*c*C*x
 + 64*B*d*x) + A*(64*c^2 + 210*c*d*x + 224*d^2*x^2)) - 4*a^3*(4*A*(15*c^2
+ 35*c*d*x + 21*d^2*x^2) + 7*x*(2*C*x*(6*c^2 + 15*c*d*x + 10*d^2*x^2) + B*
(10*c^2 + 24*c*d*x + 15*d^2*x^2))) - 2*a^2*b*x^2*(A*(24*c^2 + 70*c*d*x + 5
6*d^2*x^2) + 7*x*(B*(5*c^2 + 16*c*d*x + 15*d^2*x^2) + 2*C*x*(4*c^2 + 15*c*
d*x + 20*d^2*x^2)))))/(1680*a^3*x^7) + (b^2*(-(b*c*(B*c + 2*A*d)) + 2*a*d*
(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(8*a^(5/
2))
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2338, 25, 2338, 27, 2338, 27, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^8}\,dx$$
$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(7aCd^2x^3+7ad(2cC+Bd)x^2+\left(7ac(cC+2Bd)-A\left(4bc^2-7ad^2\right)\right)x+7ac(Bc+2Ad)\right)}{x^7}dx}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(7aCd^2x^3+7ad(2cC+Bd)x^2+\left(7ac(cC+2Bd)-A\left(4bc^2-7ad^2\right)\right)x+7ac(Bc+2Ad)\right)}{x^7}dx}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{3\sqrt{bx^2+a}\left(14a^2Cd^2x^2-7a(bc(Bc+2Ad)-2ad(2cC+Bd))x+2a\left(7ac(cC+2Bd)-A\left(4bc^2-7ad^2\right)\right)\right)}{x^6}dx}{6a}-\frac{7c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(14a^2Cd^2x^2-7a(bc(Bc+2Ad)-2ad(2cC+Bd))x+2a\left(7ac(cC+2Bd)-A\left(4bc^2-7ad^2\right)\right)\right)}{x^6}dx}{2a}-\frac{7c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{-\frac{\int \frac{a\left(35a(bc(Bc+2Ad)-2ad(2cC+Bd))-2\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{2\left(a+bx^2\right)^{3/2}\left(7ac(2Bd+cC)-A\left(4bc^2-7ad^2\right)\right)}{5x^5}}{2a}-}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{1}{5}\int \frac{\left(35a(bc(Bc+2Ad)-2ad(2cC+Bd))-2\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^5}dx-\frac{2\left(a+bx^2\right)^{3/2}\left(7ac(2Bd+cC)-A\left(4bc^2-7ad^2\right)\right)}{5x^5}}{2a}-}{7a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{\frac{1}{5}\left(\frac{\int \frac{a\left(8\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(cC+2Bd)\right)\right)+35b(bc(Bc+2Ad)-2ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}+\frac{35\left(a+bx^2\right)^{3/2}(bc(2Ad+Bc)-2ad(Bd+2cC))}{4x^4}\right)-}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{4}\int \frac{\left(8\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(cC+2Bd)\right)\right)+35b(bc(Bc+2Ad)-2ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^4}dx+\frac{35\left(a+bx^2\right)^{3/2}(bc(2Ad+Bc)-2ad(Bd+2cC))}{4x^4}\right)}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{4}\left(35b(bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{8\left(a+bx^2\right)^{3/2}\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(2Bd+cC)\right)\right)}{3ax^3}\right)+\frac{35\left(a+bx^2\right)^{3/2}(bc(2Ad+Bc)\cdots}{4x^4}\right.}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{35}{2}b(bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{8\left(a+bx^2\right)^{3/2}\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(2Bd+cC)\right)\right)}{3ax^3}\right)+\frac{35\left(a+bx^2\right)^{3/2}(bc(2Ad+Bc\cdots}{4x^4}\right.}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{35}{2}b(bc(2Ad+Bc)-2ad(Bd+2cC))\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8\left(a+bx^2\right)^{3/2}\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(2Bd+cC)\right)\right)}{3ax^3}\right)+35\left(a+\cdots\right.\right.}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{35}{2}b(bc(2Ad+Bc)-2ad(Bd+2cC))\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{8\left(a+bx^2\right)^{3/2}\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(2Bd+cC)\right)\right)}{3ax^3}\right)+\frac{35(a+\cdots}{\cdots}\right.}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{5}\left(\frac{1}{4}\left(\frac{35}{2}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(bc(2Ad+Bc)-2ad(Bd+2cC))-\frac{8\left(a+bx^2\right)^{3/2}\left(2Ab\left(4bc^2-7ad^2\right)+7a\left(5aCd^2-2bc(2Bd+cC)\right)\right)}{3ax^3}\right)+\cdots\right.}{2a}}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{7ax^7}$$

input

```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^8,x]
```

output

```
-1/7*(A*c^2*(a + b*x^2)^(3/2))/(a*x^7) + ((-7*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(6*x^6) + ((-2*(7*a*c*(c*C + 2*B*d) - A*(4*b*c^2 - 7*a*d^2))*(a + b
*x^2)^(3/2))/(5*x^5) + ((35*(b*c*(B*c + 2*A*d) - 2*a*d*(2*c*C + B*d))*(a +
 b*x^2)^(3/2))/(4*x^4) + ((-8*(2*A*b*(4*b*c^2 - 7*a*d^2) + 7*a*(5*a*C*d^2
- 2*b*c*(c*C + 2*B*d)))*(a + b*x^2)^(3/2))/(3*a*x^3) + (35*b*(b*c*(B*c + 2
*A*d) - 2*a*d*(2*c*C + B*d))*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/4)/5)/(2*a))/(7*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.14

method	result
default	$(A d^2+2Bcd+C c^2)\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{5a x^5}+\frac{2b(b x^2+a)^{\frac{3}{2}}}{15a^2 x^3}\right)+A c^2\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{7a x^7}-\frac{4b\left(-\frac{(b x^2+a)^{\frac{3}{2}}}{5a x^5}+\frac{2b(b x^2+a)^{\frac{3}{2}}}{15a^2 x^3}\right.}{7a}\right.$
risch	$-\frac{\sqrt{b x^2+a}\,(-224Aa b^2 d^2 x^6+128A b^3 c^2 x^6-448Ba b^2 cd x^6+560C a^2 b d^2 x^6-224Ca b^2 c^2 x^6-210Aa b^2 cd x^5+210B a^2 b d^2 x^5-105B}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
(A*d^2+2*B*c*d+C*c^2)*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/15*b/a^2/x^3*(b*x^2+a)
^(3/2))+A*c^2*(-1/7/a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3
/2)+2/15*b/a^2/x^3*(b*x^2+a)^(3/2)))-1/3*C*d^2/a/x^3*(b*x^2+a)^(3/2)+c*(2*
A*d+B*c)*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1
/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*
a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+d*(B*d+2*C*c)*(-1/4/a/x^4*(b*x^2+a)^(3
/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.24

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="fricas
")
```

output
```
[-1/3360*(105*(B*b^3*c^2 - 2*B*a*b^2*d^2 - 2*(2*C*a*b^2 - A*b^3)*c*d)*sqrt
(a)*x^7*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*(28*B*
a*b^2*c*d + 2*(7*C*a*b^2 - 4*A*b^3)*c^2 - 7*(5*C*a^2*b - 2*A*a*b^2)*d^2)*x
^6 - 240*A*a^3*c^2 + 105*(B*a*b^2*c^2 - 2*B*a^2*b*d^2 - 2*(2*C*a^2*b - A*a
*b^2)*c*d)*x^5 - 16*(14*B*a^2*b*c*d + (7*C*a^2*b - 4*A*a*b^2)*c^2 + 7*(5*C
*a^3 + A*a^2*b)*d^2)*x^4 - 70*(B*a^2*b*c^2 + 6*B*a^3*d^2 + 2*(6*C*a^3 + A*
a^2*b)*c*d)*x^3 - 48*(14*B*a^3*c*d + 7*A*a^3*d^2 + (7*C*a^3 + A*a^2*b)*c^2
)*x^2 - 280*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^7), 1/168
0*(105*(B*b^3*c^2 - 2*B*a*b^2*d^2 - 2*(2*C*a*b^2 - A*b^3)*c*d)*sqrt(-a)*x^
7*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (16*(28*B*a*b^2*c*d + 2*(7*C*a*b^2
- 4*A*b^3)*c^2 - 7*(5*C*a^2*b - 2*A*a*b^2)*d^2)*x^6 - 240*A*a^3*c^2 + 105*
(B*a*b^2*c^2 - 2*B*a^2*b*d^2 - 2*(2*C*a^2*b - A*a*b^2)*c*d)*x^5 - 16*(14*B
*a^2*b*c*d + (7*C*a^2*b - 4*A*a*b^2)*c^2 + 7*(5*C*a^3 + A*a^2*b)*d^2)*x^4
- 70*(B*a^2*b*c^2 + 6*B*a^3*d^2 + 2*(6*C*a^3 + A*a^2*b)*c*d)*x^3 - 48*(14*
B*a^3*c*d + 7*A*a^3*d^2 + (7*C*a^3 + A*a^2*b)*c^2)*x^2 - 280*(B*a^3*c^2 +
2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^7)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(299) = 598$.

Time = 15.03 (sec) , antiderivative size = 1216, normalized size of antiderivative = 3.88

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**8,x)
```

output
```
-15*A*a**5*b**(9/2)*c**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a*
*4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**4*b**(11/2)*c**2*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x*
*10) - 17*A*a**3*b**(13/2)*c**2*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x
**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**2*b**(15/2)*c**2*
x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 12*A*a*b**(17/2)*c**2*x**8*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a*c*d/(3*sqrt(
b)*x**7*sqrt(a/(b*x**2) + 1)) - 8*A*b**(19/2)*c**2*x**10*sqrt(a/(b*x**2) +
 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 5*A*
sqrt(b)*c*d/(12*x**5*sqrt(a/(b*x**2) + 1)) - A*sqrt(b)*d**2*sqrt(a/(b*x**2
) + 1)/(5*x**4) + A*b**(3/2)*c*d/(24*a*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(
3/2)*d**2*sqrt(a/(b*x**2) + 1)/(15*a*x**2) + A*b**(5/2)*c*d/(8*a**2*x*sqrt
(a/(b*x**2) + 1)) + 2*A*b**(5/2)*d**2*sqrt(a/(b*x**2) + 1)/(15*a**2) - A*b
**3*c*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*a*c**2/(6*sqrt(b)*x**7
*sqrt(a/(b*x**2) + 1)) - B*a*d**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) -
5*B*sqrt(b)*c**2/(24*x**5*sqrt(a/(b*x**2) + 1)) - 2*B*sqrt(b)*c*d*sqrt(a/(
b*x**2) + 1)/(5*x**4) - 3*B*sqrt(b)*d**2/(8*x**3*sqrt(a/(b*x**2) + 1)) + B
*b**(3/2)*c**2/(48*a*x**3*sqrt(a/(b*x**2) + 1)) - 2*B*b**(3/2)*c*d*sqrt(a/
(b*x**2) + 1)/(15*a*x**2) - B*b**(3/2)*d**2/(8*a*x*sqrt(a/(b*x**2) + 1)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx$$
$$= \frac{(2\,Ccd+Bd^2)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{(Bc^2+2\,Acd)b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}\,b^2}{8\,a^2} + \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}\,b^3}{16\,a^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}Ab^2c^2}{105\,a^3x^3} - \frac{(bx^2+a)^{\frac{3}{2}}Cd^2}{3\,ax^3} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^2} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b^2}{16\,a^3x^2} + \frac{4\,(bx^2+a)^{\frac{3}{2}}Abc^2}{35\,a^2x^5} + \frac{2\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b}{15\,a^2x^3} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}}{4\,ax^4} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{7\,ax^7} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}}{5\,ax^5} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}}{6\,ax^6}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="maxima
")
```

output
```
1/8*(2*C*c*d + B*d^2)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/16*(B*
c^2 + 2*A*c*d)*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 1/8*(2*C*c*d +
B*d^2)*sqrt(b*x^2 + a)*b^2/a^2 + 1/16*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b^
3/a^3 - 8/105*(b*x^2 + a)^(3/2)*A*b^2*c^2/(a^3*x^3) - 1/3*(b*x^2 + a)^(3/2
)*C*d^2/(a*x^3) + 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/
16*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 4/35*(b*x^2 + a)^(3
/2)*A*b*c^2/(a^2*x^5) + 2/15*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*b
/(a^2*x^3) - 1/4*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)/(a*x^4) + 1/8*(B*c^2
+ 2*A*c*d)*(b*x^2 + a)^(3/2)*b/(a^2*x^4) - 1/7*(b*x^2 + a)^(3/2)*A*c^2/(a*
x^7) - 1/5*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/(a*x^5) - 1/6*(B*c^
2 + 2*A*c*d)*(b*x^2 + a)^(3/2)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. $2(281) = 562$.

Time = 0.21 (sec) , antiderivative size = 1642, normalized size of antiderivative = 5.25

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
1/8*(B*b^3*c^2 - 4*C*a*b^2*c*d + 2*A*b^3*c*d - 2*B*a*b^2*d^2)*arctan(-(sqr
t(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/840*(105*(sqrt(b)*x
 - sqrt(b*x^2 + a))^13*B*b^3*c^2 - 420*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*
a*b^2*c*d + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*b^3*c*d - 210*(sqrt(b)*
x - sqrt(b*x^2 + a))^13*B*a*b^2*d^2 - 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^1
2*C*a^2*b^(3/2)*d^2 - 700*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^3*c^2 - 1
680*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b^2*c*d - 1400*(sqrt(b)*x - sqr
t(b*x^2 + a))^11*A*a*b^3*c*d - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^2*
b^2*d^2 - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(5/2)*c^2 - 6720*(
sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2)*c*d + 6720*(sqrt(b)*x - sqrt
(b*x^2 + a))^10*C*a^3*b^(3/2)*d^2 - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
A*a^2*b^(5/2)*d^2 - 3395*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c^2 + 4
620*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^3*b^2*c*d - 6790*(sqrt(b)*x - sqrt
(b*x^2 + a))^9*A*a^2*b^3*c*d + 2310*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^3*
b^2*d^2 + 5600*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2)*c^2 - 8960*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2)*c^2 + 11200*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*B*a^3*b^(5/2)*c*d - 10640*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*
a^4*b^(3/2)*d^2 + 5600*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^3*b^(5/2)*d^2 -
 2240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(5/2)*c^2 - 4480*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*A*a^3*b^(7/2)*c^2 - 4480*(sqrt(b)*x - sqrt(b*x^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx=\int \frac{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 1.73 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^8,x)
```

output
```
( - 240*sqrt(a + b*x**2)*a**4*c**2 - 560*sqrt(a + b*x**2)*a**4*c*d*x - 336
*sqrt(a + b*x**2)*a**4*d**2*x**2 - 48*sqrt(a + b*x**2)*a**3*b*c**2*x**2 -
280*sqrt(a + b*x**2)*a**3*b*c**2*x - 140*sqrt(a + b*x**2)*a**3*b*c*d*x**3
- 672*sqrt(a + b*x**2)*a**3*b*c*d*x**2 - 112*sqrt(a + b*x**2)*a**3*b*d**2*
x**4 - 420*sqrt(a + b*x**2)*a**3*b*d**2*x**3 - 336*sqrt(a + b*x**2)*a**3*c
**3*x**2 - 840*sqrt(a + b*x**2)*a**3*c**2*d*x**3 - 560*sqrt(a + b*x**2)*a*
*3*c*d**2*x**4 + 64*sqrt(a + b*x**2)*a**2*b**2*c**2*x**4 - 70*sqrt(a + b*x
**2)*a**2*b**2*c**2*x**3 + 210*sqrt(a + b*x**2)*a**2*b**2*c*d*x**5 - 224*s
qrt(a + b*x**2)*a**2*b**2*c*d*x**4 + 224*sqrt(a + b*x**2)*a**2*b**2*d**2*x
**6 - 210*sqrt(a + b*x**2)*a**2*b**2*d**2*x**5 - 112*sqrt(a + b*x**2)*a**2
*b*c**3*x**4 - 420*sqrt(a + b*x**2)*a**2*b*c**2*d*x**5 - 560*sqrt(a + b*x*
*2)*a**2*b*c*d**2*x**6 - 128*sqrt(a + b*x**2)*a*b**3*c**2*x**6 + 105*sqrt(
a + b*x**2)*a*b**3*c**2*x**5 + 448*sqrt(a + b*x**2)*a*b**3*c*d*x**6 + 224*
sqrt(a + b*x**2)*a*b**2*c**3*x**6 + 210*sqrt(a)*log((sqrt(a + b*x**2) - sq
rt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c*d*x**7 - 210*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*d**2*x**7 - 420*sqrt(a)*log((
sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*d*x**7 + 105*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c**2*x*
*7 - 210*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
**3*c*d*x**7 + 210*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)...
```

3.28 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 386

$$
\begin{aligned}
\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx
&= -\frac{Ac^2\sqrt{a+bx^2}}{8x^8}-\frac{(8ac(cC+2Bd)+A(bc^2+8ad^2))\sqrt{a+bx^2}}{48ax^6} \\
&\quad+\frac{(Ab(5bc^2-8ad^2)-8a(6aCd^2+bc(cC+2Bd)))\sqrt{a+bx^2}}{192a^2x^4} \\
&\quad-\frac{b(Ab(5bc^2-8ad^2)+8a(2aCd^2-bc(cC+2Bd)))\sqrt{a+bx^2}}{128a^3x^2} \\
&\quad-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{7ax^7}+\frac{(4bc(Bc+2Ad)-7ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{35a^2x^5} \\
&\quad-\frac{2b(4bc(Bc+2Ad)-7ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{105a^3x^3} \\
&\quad+\frac{b^2(Ab(5bc^2-8ad^2)+8a(2aCd^2-bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}
\end{aligned}
$$

output
```
-1/8*A*c^2*(b*x^2+a)^(1/2)/x^8-1/48*(8*a*c*(2*B*d+C*c)+A*(8*a*d^2+b*c^2))*
(b*x^2+a)^(1/2)/a/x^6+1/192*(A*b*(-8*a*d^2+5*b*c^2)-8*a*(6*a*C*d^2+b*c*(2*
B*d+C*c)))*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b*(A*b*(-8*a*d^2+5*b*c^2)+8*a*(2*
a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^3/x^2-1/7*c*(2*A*d+B*c)*(b*x^2
+a)^(3/2)/a/x^7+1/35*(4*b*c*(2*A*d+B*c)-7*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)
/a^2/x^5-2/105*b*(4*b*c*(2*A*d+B*c)-7*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a^3
/x^3+1/128*b^2*(A*b*(-8*a*d^2+5*b*c^2)+8*a*(2*a*C*d^2-b*c*(2*B*d+C*c)))*ar
ctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 4.60 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = -\frac{\sqrt{a}\sqrt{a+bx^2}\left(b^3cx^6(525Ac+1024Bcx+2048Adx)+16a^3\left(5A\left(21c^2+48cdx+28d^2x^2\right)+2x\left(7Cx\left(10c^2+24cdx+15d^2x^2\right)+4B\left(15c^2+35cdx+21d^2\right.\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^9,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(b^3*c*x^6*(525*A*c + 1024*B*c*x + 2048*A*d*x)
 + 16*a^3*(5*A*(21*c^2 + 48*c*d*x + 28*d^2*x^2) + 2*x*(7*C*x*(10*c^2 + 24*
c*d*x + 15*d^2*x^2) + 4*B*(15*c^2 + 35*c*d*x + 21*d^2*x^2))) + 8*a^2*b*x^2
*(A*(35*c^2 + 96*c*d*x + 70*d^2*x^2) + 2*x*(7*C*x*(5*c^2 + 16*c*d*x + 15*d
^2*x^2) + B*(24*c^2 + 70*c*d*x + 56*d^2*x^2))) - 2*a*b^2*x^4*(A*(175*c^2 +
 512*c*d*x + 420*d^2*x^2) + 4*x*(7*c*C*x*(15*c + 64*d*x) + B*(64*c^2 + 210
*c*d*x + 224*d^2*x^2)))))/x^8) - 210*b^2*(5*A*b^2*c^2 + 16*a^2*C*d^2)*ArcT
anh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 1680*a*b^3*(c^2*C + 2*B*c*d +
 A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(13440*a^(7/2))
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.96, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 25, 2338, 27, 539, 27, 539, 25, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^9}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\sqrt{bx^2+a}\left(8aCd^2x^3+8ad(2cC+Bd)x^2+\left(8ac(cC+2Bd)-A\left(5bc^2-8ad^2\right)\right)x+8ac(Bc+2Ad)\right)}{x^8}dx}{8a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(8aCd^2x^3+8ad(2cC+Bd)x^2+\left(8ac(cC+2Bd)-A\left(5bc^2-8ad^2\right)\right)x+8ac(Bc+2Ad)\right)}{x^8}dx}{8a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{\sqrt{bx^2+a}\left(56a^2Cd^2x^2-8a(4bc(Bc+2Ad)-7ad(2cC+Bd))x+7a\left(8ac(cC+2Bd)-A\left(5bc^2-8ad^2\right)\right)\right)}{x^7}dx}{7a}-\frac{8c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{7x^7}}{8a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(56a^2Cd^2x^2-8a(4bc(Bc+2Ad)-7ad(2cC+Bd))x+7a\left(8ac(cC+2Bd)-A\left(5bc^2-8ad^2\right)\right)\right)}{x^7}dx}{7a}-\frac{8c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{7x^7}}{8a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{3a\left(16a(4bc(Bc+2Ad)-7ad(2cC+Bd))-7\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^6}dx}{6a}-\frac{7\left(a+bx^2\right)^{3/2}\left(8ac(2Bd+cC)-A\left(5bc^2-8ad^2\right)\right)}{6x^6}}{7a} \quad - \cdots}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{1}{2}\int \frac{\left(16a(4bc(Bc+2Ad)-7ad(2cC+Bd))-7\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^6}dx-\frac{7\left(a+bx^2\right)^{3/2}\left(8ac(2Bd+cC)-A\left(5bc^2-8ad^2\right)\right)}{6x^6}}{7a} \quad - \cdots}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{\frac{1}{2}\left(\frac{\int \frac{a\left(35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)+32b(4bc(Bc+2Ad)-7ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^5}dx}{5a}+\frac{16\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{5x^5}\right) \cdots}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{2}\left(\frac{1}{5}\int \frac{\left(35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)+32b(4bc(Bc+2Ad)-7ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^5}dx+\frac{16\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{5x^5}\right) \cdots}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{\frac{1}{2}\left(\frac{1}{5}\left(-\frac{\int -\frac{b\left(128a(4bc(Bc+2Ad)-7ad(2cC+Bd))-35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2a \cdots\right.\right.}{4ax^4} \cdots\right.\right.}{7a}}{8a \cdots}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{\frac{\int \frac{b\left(128a(4bc(Bc+2Ad)-7ad(2cC+Bd))-35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2\cdots\right.\right.}{4ax^4}}{7a}\cdots\right.\right.}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\big\downarrow$ 27

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{\frac{b\int \frac{\left(128a(4bc(Bc+2Ad)-7ad(2cC+Bd))-35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{35\left(a+bx^2\right)^{3/2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd\cdots\right.\right.}{4ax^4}}{7a}\cdots\right.\right.}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\big\downarrow$ 534

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{\frac{b\left(-35\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(2Bd+cC)\right)\right)\int \frac{\sqrt{bx^2+a}}{x^3}dx-\frac{128\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{3x^3}\right)}{4a}-\frac{35\left(a+bx^2\right)^{3/2}\left(Ab\left(5bc^2-8a\cdots\right.\right.}{4\cdots}}{7a}\cdots\right.\right.}{\cdots}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\big\downarrow$ 243

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{\frac{b\left(-\frac{35}{2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(2Bd+cC)\right)\right)\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{128\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{3x^3}\right)}{4a}-\frac{35\left(a+bx^2\right)^{3/2}\left(Ab\left(5bc^2-8\cdots\right.\right.}{\cdots}}{7a}\cdots\right.\right.}{\cdots}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\big\downarrow$ 51

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{b\left(-\frac{35}{2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(2Bd+cC)\right)\right)\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{3x^3}\right)}{4a}-\frac{35\left(a+b\right.}{}\right.\right.}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\downarrow$ 73

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{b\left(-\frac{35}{2}\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(2Bd+cC)\right)\right)\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{128\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{3x^3}\right)}{4a}-\frac{35\left(a+bx\right.}{}\right.\right.}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{b\left(-\frac{35}{2}\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)\left(Ab\left(5bc^2-8ad^2\right)+8a\left(2aCd^2-bc(2Bd+cC)\right)\right)-\frac{128\left(a+bx^2\right)^{3/2}(4bc(2Ad+Bc)-7ad(Bd+2cC))}{3x^3}\right)}{4a}-\frac{3}{}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{8ax^8}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^9,x]
```

output
```
-1/8*(A*c^2*(a + b*x^2)^(3/2))/(a*x^8) + ((-8*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(7*x^7) + ((-7*(8*a*c*(c*C + 2*B*d) - A*(5*b*c^2 - 8*a*d^2))*(a + b
*x^2)^(3/2))/(6*x^6) + ((16*(4*b*c*(B*c + 2*A*d) - 7*a*d*(2*c*C + B*d))*(a
 + b*x^2)^(3/2))/(5*x^5) + ((-35*(A*b*(5*b*c^2 - 8*a*d^2) + 8*a*(2*a*C*d^2
 - b*c*(c*C + 2*B*d)))*(a + b*x^2)^(3/2))/(4*a*x^4) + (b*((-128*(4*b*c*(B*
c + 2*A*d) - 7*a*d*(2*c*C + B*d))*(a + b*x^2)^(3/2))/(3*x^3) - (35*(A*b*(5
*b*c^2 - 8*a*d^2) + 8*a*(2*a*C*d^2 - b*c*(c*C + 2*B*d)))*(-(Sqrt[a + b*x^2
]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/(4*a))/5)/2)/(
7*a))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2048 A\,b^{3} c d\,x^{7}-1792 B a\,b^{2} d^{2} x^{7}+1024 B\,b^{3} c^{2} x^{7}-3584 C a\,b^{2} c d\,x^{7}-840 A a\,b^{2} d^{2} x^{6}+525 A\,b^{3} c^{2} x^{6}-1680 B a\,b^{2} c d\,x^{6}+168}{}$
default	$(A\,d^{2}+2 B c d+C\,c^{2}) \left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{6 a\,x^{6}}-\frac{b \left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{4 a\,x^{4}}-\frac{b \left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{2 a\,x^{2}}+\frac{b \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2 a}\right)}{4 a}\right)}{2 a}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/13440*(b*x^2+a)^(1/2)*(2048*A*b^3*c*d*x^7-1792*B*a*b^2*d^2*x^7+1024*B*b
^3*c^2*x^7-3584*C*a*b^2*c*d*x^7-840*A*a*b^2*d^2*x^6+525*A*b^3*c^2*x^6-1680
*B*a*b^2*c*d*x^6+1680*C*a^2*b*d^2*x^6-840*C*a*b^2*c^2*x^6-1024*A*a*b^2*c*d
*x^5+896*B*a^2*b*d^2*x^5-512*B*a*b^2*c^2*x^5+1792*C*a^2*b*c*d*x^5+560*A*a^
2*b*d^2*x^4-350*A*a*b^2*c^2*x^4+1120*B*a^2*b*c*d*x^4+3360*C*a^3*d^2*x^4+56
0*C*a^2*b*c^2*x^4+768*A*a^2*b*c*d*x^3+2688*B*a^3*d^2*x^3+384*B*a^2*b*c^2*x
^3+5376*C*a^3*c*d*x^3+2240*A*a^3*d^2*x^2+280*A*a^2*b*c^2*x^2+4480*B*a^3*c*
d*x^2+2240*C*a^3*c^2*x^2+3840*A*a^3*c*d*x+1920*B*a^3*c^2*x+1680*A*a^3*c^2)
/x^8/a^3-1/128*(8*A*a*b*d^2-5*A*b^2*c^2+16*B*a*b*c*d-16*C*a^2*d^2+8*C*a*b*
c^2)*b^2/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 853, normalized size of antiderivative = 2.21

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="fricas
")
```

output
```
[1/26880*(105*(16*B*a*b^3*c*d + (8*C*a*b^3 - 5*A*b^4)*c^2 - 8*(2*C*a^2*b^2
 - A*a*b^3)*d^2)*sqrt(a)*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) - 2*(256*(4*B*a*b^3*c^2 - 7*B*a^2*b^2*d^2 - 2*(7*C*a^2*b^2 - 4*A*a*
b^3)*c*d)*x^7 + 1680*A*a^4*c^2 - 105*(16*B*a^2*b^2*c*d + (8*C*a^2*b^2 - 5*
A*a*b^3)*c^2 - 8*(2*C*a^3*b - A*a^2*b^2)*d^2)*x^6 - 128*(4*B*a^2*b^2*c^2 -
 7*B*a^3*b*d^2 - 2*(7*C*a^3*b - 4*A*a^2*b^2)*c*d)*x^5 + 70*(16*B*a^3*b*c*d
 + (8*C*a^3*b - 5*A*a^2*b^2)*c^2 + 8*(6*C*a^4 + A*a^3*b)*d^2)*x^4 + 384*(B
*a^3*b*c^2 + 7*B*a^4*d^2 + 2*(7*C*a^4 + A*a^3*b)*c*d)*x^3 + 280*(16*B*a^4*
c*d + 8*A*a^4*d^2 + (8*C*a^4 + A*a^3*b)*c^2)*x^2 + 1920*(B*a^4*c^2 + 2*A*a
^4*c*d)*x)*sqrt(b*x^2 + a))/(a^4*x^8), 1/13440*(105*(16*B*a*b^3*c*d + (8*C
*a*b^3 - 5*A*b^4)*c^2 - 8*(2*C*a^2*b^2 - A*a*b^3)*d^2)*sqrt(-a)*x^8*arctan
(sqrt(b*x^2 + a)*sqrt(-a)/a) - (256*(4*B*a*b^3*c^2 - 7*B*a^2*b^2*d^2 - 2*(
7*C*a^2*b^2 - 4*A*a*b^3)*c*d)*x^7 + 1680*A*a^4*c^2 - 105*(16*B*a^2*b^2*c*d
 + (8*C*a^2*b^2 - 5*A*a*b^3)*c^2 - 8*(2*C*a^3*b - A*a^2*b^2)*d^2)*x^6 - 12
8*(4*B*a^2*b^2*c^2 - 7*B*a^3*b*d^2 - 2*(7*C*a^3*b - 4*A*a^2*b^2)*c*d)*x^5
+ 70*(16*B*a^3*b*c*d + (8*C*a^3*b - 5*A*a^2*b^2)*c^2 + 8*(6*C*a^4 + A*a^3*
b)*d^2)*x^4 + 384*(B*a^3*b*c^2 + 7*B*a^4*d^2 + 2*(7*C*a^4 + A*a^3*b)*c*d)*
x^3 + 280*(16*B*a^4*c*d + 8*A*a^4*d^2 + (8*C*a^4 + A*a^3*b)*c^2)*x^2 + 192
0*(B*a^4*c^2 + 2*A*a^4*c*d)*x)*sqrt(b*x^2 + a))/(a^4*x^8)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1678 vs. 2(374) = 748.

Time = 36.01 (sec) , antiderivative size = 1678, normalized size of antiderivative = 4.35

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**9,x)
```

output
```
-30*A*a**5*b**(9/2)*c*d*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 66*A*a**4*b**(11/2)*c*d*x**2*sqrt(a/(
b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**1
0) - 34*A*a**3*b**(13/2)*c*d*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 6*A*a**2*b**(15/2)*c*d*x**6
*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*
b**6*x**10) - 24*A*a*b**(17/2)*c*d*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**
4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a*c**2/(8*sqrt(b)*x
**9*sqrt(a/(b*x**2) + 1)) - A*a*d**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1))
 - 16*A*b**(19/2)*c*d*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 7*A*sqrt(b)*c**2/(48*x**7*sqrt(a/
(b*x**2) + 1)) - 5*A*sqrt(b)*d**2/(24*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(3
/2)*c**2/(192*a*x**5*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*d**2/(48*a*x**3*sq
rt(a/(b*x**2) + 1)) - 5*A*b**(5/2)*c**2/(384*a**2*x**3*sqrt(a/(b*x**2) + 1
)) + A*b**(5/2)*d**2/(16*a**2*x*sqrt(a/(b*x**2) + 1)) - 5*A*b**(7/2)*c**2/
(128*a**3*x*sqrt(a/(b*x**2) + 1)) - A*b**3*d**2*asinh(sqrt(a)/(sqrt(b)*x))
/(16*a**(5/2)) + 5*A*b**4*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(7/2)) -
 15*B*a**5*b**(9/2)*c**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a*
*4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**4*b**(11/2)*c**2*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx$$
$$= \frac{5\,Ab^4c^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{7}{2}}} + \frac{Cb^2d^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{5\,\sqrt{bx^2+a}Ab^4c^2}{128\,a^4}$$
$$- \frac{\sqrt{bx^2+a}Cb^2d^2}{8\,a^2} - \frac{(Cc^2+2\,Bcd+Ad^2)b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}}$$
$$+ \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b^3}{16\,a^3} + \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^3c^2}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{3}{2}}Cbd^2}{8\,a^2x^2}$$
$$- \frac{5\,(bx^2+a)^{\frac{3}{2}}Ab^2c^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{3}{2}}Cd^2}{4\,ax^4} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b^2}{16\,a^3x^2}$$
$$+ \frac{2\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b}{15\,a^2x^3} - \frac{8\,(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b^2}{105\,a^3x^3}$$
$$+ \frac{5\,(bx^2+a)^{\frac{3}{2}}Abc^2}{48\,a^2x^6} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^4}$$
$$- \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}}{5\,ax^5} + \frac{4\,(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{8\,ax^8}$$
$$- \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}}{6\,ax^6} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}}{7\,ax^7}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="maxima
")
```

output
```
5/128*A*b^4*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/8*C*b^2*d^2*arcs
inh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 5/128*sqrt(b*x^2 + a)*A*b^4*c^2/a^4 -
1/8*sqrt(b*x^2 + a)*C*b^2*d^2/a^2 - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*b^3*arc
sinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b
*x^2 + a)*b^3/a^3 + 5/128*(b*x^2 + a)^(3/2)*A*b^3*c^2/(a^4*x^2) + 1/8*(b*x
^2 + a)^(3/2)*C*b*d^2/(a^2*x^2) - 5/64*(b*x^2 + a)^(3/2)*A*b^2*c^2/(a^3*x^
4) - 1/4*(b*x^2 + a)^(3/2)*C*d^2/(a*x^4) - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*
(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 2/15*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)
*b/(a^2*x^3) - 8/105*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 5
/48*(b*x^2 + a)^(3/2)*A*b*c^2/(a^2*x^6) + 1/8*(C*c^2 + 2*B*c*d + A*d^2)*(b
*x^2 + a)^(3/2)*b/(a^2*x^4) - 1/5*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)/(a*x
^5) + 4/35*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b/(a^2*x^5) - 1/8*(b*x^2 +
a)^(3/2)*A*c^2/(a*x^8) - 1/6*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/(
a*x^6) - 1/7*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2046 vs. $2(350) = 700$.

Time = 0.23 (sec) , antiderivative size = 2046, normalized size of antiderivative = 5.30

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
1/64*(8*C*a*b^3*c^2 - 5*A*b^4*c^2 + 16*B*a*b^3*c*d - 16*C*a^2*b^2*d^2 + 8*
A*a*b^3*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3
) - 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a*b^3*c^2 - 525*(sqrt(b
)*x - sqrt(b*x^2 + a))^15*A*b^4*c^2 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^1
5*B*a*b^3*c*d - 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*b^2*d^2 + 840*
(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a*b^3*d^2 - 6440*(sqrt(b)*x - sqrt(b*x^
2 + a))^13*C*a^2*b^3*c^2 + 4025*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4*c
^2 - 12880*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^2*b^3*c*d - 5040*(sqrt(b)*
x - sqrt(b*x^2 + a))^13*C*a^3*b^2*d^2 - 6440*(sqrt(b)*x - sqrt(b*x^2 + a))
^13*A*a^2*b^3*d^2 - 53760*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(5/2)*c
*d - 26880*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(5/2)*d^2 - 21560*(sqr
t(b)*x - sqrt(b*x^2 + a))^11*C*a^3*b^3*c^2 - 13405*(sqrt(b)*x - sqrt(b*x^2
 + a))^11*A*a^2*b^4*c^2 - 43120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^3
*c*d + 25200*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^4*b^2*d^2 - 21560*(sqrt(
b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^3*d^2 - 71680*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*B*a^3*b^(7/2)*c^2 + 143360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*
b^(5/2)*c*d - 143360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(7/2)*c*d +
71680*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(5/2)*d^2 + 27160*(sqrt(b)*
x - sqrt(b*x^2 + a))^9*C*a^4*b^3*c^2 - 97615*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*A*a^3*b^4*c^2 + 54320*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^4*b^3*c*d ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \int \frac{\sqrt{b\,x^2+a}\,{(c+d\,x)}^2\,\left(C\,x^2+B\,x+A\right)}{x^9}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^9,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 994, normalized size of antiderivative = 2.58

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^9,x)
```

output
```
( - 1680*sqrt(a + b*x**2)*a**4*c**2 - 3840*sqrt(a + b*x**2)*a**4*c*d*x - 2
240*sqrt(a + b*x**2)*a**4*d**2*x**2 - 280*sqrt(a + b*x**2)*a**3*b*c**2*x**
2 - 1920*sqrt(a + b*x**2)*a**3*b*c**2*x - 768*sqrt(a + b*x**2)*a**3*b*c*d*
x**3 - 4480*sqrt(a + b*x**2)*a**3*b*c*d*x**2 - 560*sqrt(a + b*x**2)*a**3*b
*d**2*x**4 - 2688*sqrt(a + b*x**2)*a**3*b*d**2*x**3 - 2240*sqrt(a + b*x**2
)*a**3*c**3*x**2 - 5376*sqrt(a + b*x**2)*a**3*c**2*d*x**3 - 3360*sqrt(a +
b*x**2)*a**3*c*d**2*x**4 + 350*sqrt(a + b*x**2)*a**2*b**2*c**2*x**4 - 384*
sqrt(a + b*x**2)*a**2*b**2*c**2*x**3 + 1024*sqrt(a + b*x**2)*a**2*b**2*c*d
*x**5 - 1120*sqrt(a + b*x**2)*a**2*b**2*c*d*x**4 + 840*sqrt(a + b*x**2)*a*
*2*b**2*d**2*x**6 - 896*sqrt(a + b*x**2)*a**2*b**2*d**2*x**5 - 560*sqrt(a
+ b*x**2)*a**2*b*c**3*x**4 - 1792*sqrt(a + b*x**2)*a**2*b*c**2*d*x**5 - 16
80*sqrt(a + b*x**2)*a**2*b*c*d**2*x**6 - 525*sqrt(a + b*x**2)*a*b**3*c**2*
x**6 + 512*sqrt(a + b*x**2)*a*b**3*c**2*x**5 - 2048*sqrt(a + b*x**2)*a*b**
3*c*d*x**7 + 1680*sqrt(a + b*x**2)*a*b**3*c*d*x**6 + 1792*sqrt(a + b*x**2)
*a*b**3*d**2*x**7 + 840*sqrt(a + b*x**2)*a*b**2*c**3*x**6 + 3584*sqrt(a +
b*x**2)*a*b**2*c**2*d*x**7 - 1024*sqrt(a + b*x**2)*b**4*c**2*x**7 + 840*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*d**2*x*
*8 - 1680*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
b**2*c*d**2*x**8 - 525*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*b**4*c**2*x**8 + 1680*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a...
```

3.29 $\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 428

$$\begin{aligned}
\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx
&= \frac{(5bc(Bc+2Ad)-8ad(2cC+Bd))\sqrt{a+bx^2}}{48ax^6} \\
&\quad + \frac{b(5bc(Bc+2Ad)-8ad(2cC+Bd))\sqrt{a+bx^2}}{192a^2x^4} \\
&\quad - \frac{b^2(5bc(Bc+2Ad)-8ad(2cC+Bd))\sqrt{a+bx^2}}{128a^3x^2} - \frac{Ac^2(a+bx^2)^{3/2}}{9ax^9} \\
&\quad - \frac{c(Bc+2Ad)\left(a+bx^2\right)^{3/2}}{8ax^8} - \frac{(3ac(cC+2Bd)-A(2bc^2-3ad^2))\left(a+bx^2\right)^{3/2}}{21a^2x^7} \\
&\quad - \frac{(4Ab(2bc^2-3ad^2)+3a(7aCd^2-4bc(cC+2Bd)))\left(a+bx^2\right)^{3/2}}{105a^3x^5} \\
&\quad + \frac{2b(4Ab(2bc^2-3ad^2)+3a(7aCd^2-4bc(cC+2Bd)))\left(a+bx^2\right)^{3/2}}{315a^4x^3} \\
&\quad + \frac{b^3(5bc(Bc+2Ad)-8ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{7/2}}
\end{aligned}$$

output
```
1/48*(5*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a/x^6+1/192*b*(
5*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x^4-1/128*b^2*(5*
b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^3/x^2-1/9*A*c^2*(b*x^
2+a)^(3/2)/a/x^9-1/8*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)/a/x^8-1/21*(3*a*c*(2*B*
d+C*c)-A*(-3*a*d^2+2*b*c^2))*(b*x^2+a)^(3/2)/a^2/x^7-1/105*(4*A*b*(-3*a*d^
2+2*b*c^2)+3*a*(7*a*C*d^2-4*b*c*(2*B*d+C*c)))*(b*x^2+a)^(3/2)/a^3/x^5+2/31
5*b*(4*A*b*(-3*a*d^2+2*b*c^2)+3*a*(7*a*C*d^2-4*b*c*(2*B*d+C*c)))*(b*x^2+a)
^(3/2)/a^4/x^3+1/128*b^3*(5*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*arctanh((b*
x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 4.00 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx =$$

$$-\frac{\sqrt{a+bx^2}(-2048Ab^4c^2x^8+ab^3x^6(3cx(525Bc+1024cCx+2048Bdx)+2A(512c^2+1575cdx+1536}{}$$

$$+\frac{b^3(5bc(Bc+2Ad)-8ad(2cC+Bd))\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{7/2}}$$

input
```
Integrate[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/40320*(Sqrt[a + b*x^2]*(-2048*A*b^4*c^2*x^8 + a*b^3*x^6*(3*c*x*(525*B*c
 + 1024*c*C*x + 2048*B*d*x) + 2*A*(512*c^2 + 1575*c*d*x + 1536*d^2*x^2)) +
 16*a^4*(10*A*(28*c^2 + 63*c*d*x + 36*d^2*x^2) + 3*x*(8*C*x*(15*c^2 + 35*c
*d*x + 21*d^2*x^2) + 5*B*(21*c^2 + 48*c*d*x + 28*d^2*x^2))) + 8*a^3*b*x^2*
(2*A*(40*c^2 + 105*c*d*x + 72*d^2*x^2) + 3*x*(4*C*x*(12*c^2 + 35*c*d*x + 2
8*d^2*x^2) + B*(35*c^2 + 96*c*d*x + 70*d^2*x^2))) - 6*a^2*b^2*x^4*(2*A*(64
*c^2 + 175*c*d*x + 128*d^2*x^2) + x*(8*C*x*(32*c^2 + 105*c*d*x + 112*d^2*x
^2) + B*(175*c^2 + 512*c*d*x + 420*d^2*x^2)))))/(a^4*x^9) + (b^3*(5*b*c*(B
*c + 2*A*d) - 8*a*d*(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2]
)/Sqrt[a]])/(64*a^(7/2))
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.99, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {2338, 27, 2338, 25, 2338, 27, 539, 27, 539, 25, 27, 539, 27, 534, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^{10}}dx$$

$$\Big\downarrow \textbf{2338}$$

$$-\frac{\int -\frac{3\sqrt{bx^2+a}\left(3aCd^2x^3+3ad(2cC+Bd)x^2+\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)x+3ac(Bc+2Ad)\right)}{x^9}dx}{9a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(3aCd^2x^3+3ad(2cC+Bd)x^2+\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)x+3ac(Bc+2Ad)\right)}{x^9}dx}{3a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{2338}$$

$$\frac{-\frac{\int -\frac{\sqrt{bx^2+a}\left(24a^2Cd^2x^2-3a(5bc(Bc+2Ad)-8ad(2cC+Bd))x+8a\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)\right)}{x^8}dx}{8a}-\frac{3c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{8x^8}}{3a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(24a^2Cd^2x^2-3a(5bc(Bc+2Ad)-8ad(2cC+Bd))x+8a\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)\right)}{x^8}dx}{8a}-\frac{3c\left(a+bx^2\right)^{3/2}(2Ad+Bc)}{8x^8}}{3a}-\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \textbf{2338}$$

$$\frac{\frac{-\frac{\int \frac{a\left(21a(5bc(Bc+2Ad)-8ad(2cC+Bd))-8\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^7}dx}{7a}-\frac{8\left(a+bx^2\right)^{3/2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{7x^7}}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{1}{7}\int \frac{\left(21a(5bc(Bc+2Ad)-8ad(2cC+Bd))-8\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^7}dx-\frac{8\left(a+bx^2\right)^{3/2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{7x^7}}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{\frac{1}{7}\left(\frac{\int \frac{3a\left(16\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)+21b(5bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^6}dx}{6a}+\frac{7\left(a+bx^2\right)^{3/2}(5bc(2Ad+Bc)-8ad(Bd+2cC))}{2x^6}\right.}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{1}{7}\left(\frac{1}{2}\int \frac{\left(16\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)+21b(5bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^6}dx+\frac{7\left(a+bx^2\right)^{3/2}(5bc(2Ad+Bc)-8ad(Bd+2cC))}{2x^6}\right.}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 539$$

$$\frac{\frac{\frac{1}{7}\left(\frac{1}{2}\left(-\frac{\int -\frac{b\left(105a(5bc(Bc+2Ad)-8ad(2cC+Bd))-32\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\right.}{5ax^5}\right.\right.}{8a}}{3}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{\int \frac{b\left(105a(5bc(Bc+2Ad)-8ad(2cC+Bd))-32\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7a\right.\right.}{5ax^5}\right.}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{b\int \frac{\left(105a(5bc(Bc+2Ad)-8ad(2cC+Bd))-32\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^5}dx}{5a}-\frac{16\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7a\right.\right.}{5ax^5}\right.}{8a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 539

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(-\frac{\int \frac{a\left(128\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)+105b(5bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^4}dx}{4a}-\frac{105\left(a+bx^2\right)^{3/2}(5bc(2Ad+Bc)-8ad}{4x^4}\right.}{5a}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 27

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(-\frac{1}{4}\int \frac{\left(128\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(cC+2Bd)\right)\right)+105b(5bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x^4}dx-\frac{105\left(a+bx^2\right)^{3/2}(5bc(2Ad+Bc)-8ad}{4x^4}\right.}{5a}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$\downarrow$ 534

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(\frac{1}{4}\left(\frac{128\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(2Bd+cC)\right)\right)}{3ax^3}-105b(5bc(2Ad+Bc)-8ad(Bd+2cC))\int\frac{\sqrt{bx^2+a}}{x^3}dx\right)-\frac{105\left(a+bx^2\right)^{3/2}(5bc(2A}{4a}}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(\frac{1}{4}\left(\frac{128\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(2Bd+cC)\right)\right)}{3ax^3}-\frac{105}{2}b(5bc(2Ad+Bc)-8ad(Bd+2cC))\int\frac{\sqrt{bx^2+a}}{x^4}dx^2\right)-\frac{105\left(a+bx^2\right)^{3/2}(5bc(2}{}}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(\frac{1}{4}\left(\frac{128\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(2Bd+cC)\right)\right)}{3ax^3}-\frac{105}{2}b(5bc(2Ad+Bc)-8ad(Bd+2cC))\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{10}{}}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(\frac{1}{4}\left(\frac{128\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(2Bd+cC)\right)\right)}{3ax^3}-\frac{105}{2}b(5bc(2Ad+Bc)-8ad(Bd+2cC))\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)\right)-\frac{105}{}}{5a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{7}\left(\frac{1}{2}\left(\frac{b\left(\frac{1}{4}\left(\frac{128\left(a+bx^2\right)^{3/2}\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(7aCd^2-4bc(2Bd+cC)\right)\right)}{3ax^3}-\frac{105}{2}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)(5bc(2Ad+Bc)-8ad(Bd+2cC)}{5a}}{}}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{3/2}}{9ax^9}$$

input
```
Int[((c + d*x)^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/9*(A*c^2*(a + b*x^2)^(3/2))/(a*x^9) + ((-3*c*(B*c + 2*A*d)*(a + b*x^2)^
(3/2))/(8*x^8) + ((-8*(3*a*c*(c*C + 2*B*d) - A*(2*b*c^2 - 3*a*d^2))*(a + b
*x^2)^(3/2))/(7*x^7) + ((7*(5*b*c*(B*c + 2*A*d) - 8*a*d*(2*c*C + B*d))*(a
+ b*x^2)^(3/2))/(2*x^6) + ((-16*(4*A*b*(2*b*c^2 - 3*a*d^2) + 3*a*(7*a*C*d^
2 - 4*b*c*(c*C + 2*B*d)))*(a + b*x^2)^(3/2))/(5*a*x^5) + (b*((-105*(5*b*c*
(B*c + 2*A*d) - 8*a*d*(2*c*C + B*d))*(a + b*x^2)^(3/2))/(4*x^4) + ((128*(4
*A*b*(2*b*c^2 - 3*a*d^2) + 3*a*(7*a*C*d^2 - 4*b*c*(c*C + 2*B*d)))*(a + b*x
^2)^(3/2))/(3*a*x^3) - (105*b*(5*b*c*(B*c + 2*A*d) - 8*a*d*(2*c*C + B*d))*
(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2
)/4))/(5*a))/2)/7)/(8*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.11

method	result
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{7a\,x^7}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{5a\,x^5}+\frac{2b(b\,x^2+a)^{\frac{3}{2}}}{15a^2x^3}\right)}{7a}\right)+A\,c^2\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{9a\,x^9}-\frac{2b\left(-\frac{(b\,x}{7}\right.}{}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,(3072Aa\,b^3d^2x^8-2048A\,b^4c^2x^8+6144Ba\,b^3cd\,x^8-5376C\,a^2b^2d^2x^8+3072Ca\,b^3c^2x^8+3150Aa\,b^3cd\,x^7-2520B\,a^2b^2d^2x}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
(A*d^2+2*B*c*d+C*c^2)*(-1/7/a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x
^2+a)^(3/2)+2/15*b/a^2/x^3*(b*x^2+a)^(3/2)))+A*c^2*(-1/9/a/x^9*(b*x^2+a)^(
3/2)-2/3*b/a*(-1/7/a/x^7*(b*x^2+a)^(3/2)-4/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(3/
2)+2/15*b/a^2/x^3*(b*x^2+a)^(3/2))))+C*d^2*(-1/5/a/x^5*(b*x^2+a)^(3/2)+2/1
5*b/a^2/x^3*(b*x^2+a)^(3/2))+c*(2*A*d+B*c)*(-1/8/a/x^8*(b*x^2+a)^(3/2)-5/8
*b/a*(-1/6/a/x^6*(b*x^2+a)^(3/2)-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b
/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*
a^(1/2)*(b*x^2+a)^(1/2))/x))))))+d*(B*d+2*C*c)*(-1/6/a/x^6*(b*x^2+a)^(3/2)
-1/2*b/a*(-1/4/a/x^4*(b*x^2+a)^(3/2)-1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1
/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Fricas [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 942, normalized size of antiderivative = 2.20

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^10,x, algorithm="frica
s")
```

output
```
[-1/80640*(315*(5*B*b^4*c^2 - 8*B*a*b^3*d^2 - 2*(8*C*a*b^3 - 5*A*b^4)*c*d)
*sqrt(a)*x^9*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(256*
(24*B*a*b^3*c*d + 4*(3*C*a*b^3 - 2*A*b^4)*c^2 - 3*(7*C*a^2*b^2 - 4*A*a*b^3
)*d^2)*x^8 + 315*(5*B*a*b^3*c^2 - 8*B*a^2*b^2*d^2 - 2*(8*C*a^2*b^2 - 5*A*a
*b^3)*c*d)*x^7 + 4480*A*a^4*c^2 - 128*(24*B*a^2*b^2*c*d + 4*(3*C*a^2*b^2 -
 2*A*a*b^3)*c^2 - 3*(7*C*a^3*b - 4*A*a^2*b^2)*d^2)*x^6 - 210*(5*B*a^2*b^2*
c^2 - 8*B*a^3*b*d^2 - 2*(8*C*a^3*b - 5*A*a^2*b^2)*c*d)*x^5 + 384*(6*B*a^3*
b*c*d + (3*C*a^3*b - 2*A*a^2*b^2)*c^2 + 3*(7*C*a^4 + A*a^3*b)*d^2)*x^4 + 8
40*(B*a^3*b*c^2 + 8*B*a^4*d^2 + 2*(8*C*a^4 + A*a^3*b)*c*d)*x^3 + 640*(18*B
*a^4*c*d + 9*A*a^4*d^2 + (9*C*a^4 + A*a^3*b)*c^2)*x^2 + 5040*(B*a^4*c^2 +
2*A*a^4*c*d)*x)*sqrt(b*x^2 + a))/(a^4*x^9), -1/40320*(315*(5*B*b^4*c^2 - 8
*B*a*b^3*d^2 - 2*(8*C*a*b^3 - 5*A*b^4)*c*d)*sqrt(-a)*x^9*arctan(sqrt(b*x^2
 + a)*sqrt(-a)/a) + (256*(24*B*a*b^3*c*d + 4*(3*C*a*b^3 - 2*A*b^4)*c^2 - 3
*(7*C*a^2*b^2 - 4*A*a*b^3)*d^2)*x^8 + 315*(5*B*a*b^3*c^2 - 8*B*a^2*b^2*d^2
 - 2*(8*C*a^2*b^2 - 5*A*a*b^3)*c*d)*x^7 + 4480*A*a^4*c^2 - 128*(24*B*a^2*b
^2*c*d + 4*(3*C*a^2*b^2 - 2*A*a*b^3)*c^2 - 3*(7*C*a^3*b - 4*A*a^2*b^2)*d^2
)*x^6 - 210*(5*B*a^2*b^2*c^2 - 8*B*a^3*b*d^2 - 2*(8*C*a^3*b - 5*A*a^2*b^2)
*c*d)*x^5 + 384*(6*B*a^3*b*c*d + (3*C*a^3*b - 2*A*a^2*b^2)*c^2 + 3*(7*C*a^
4 + A*a^3*b)*d^2)*x^4 + 840*(B*a^3*b*c^2 + 8*B*a^4*d^2 + 2*(8*C*a^4 + A*a^
3*b)*c*d)*x^3 + 640*(18*B*a^4*c*d + 9*A*a^4*d^2 + (9*C*a^4 + A*a^3*b)*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2518 vs. $2(420) = 840$.

Time = 50.47 (sec) , antiderivative size = 2518, normalized size of antiderivative = 5.88

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**10,x)
```

output
```
-35*A*a**7*b**(19/2)*c**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a
**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**
6*b**(21/2)*c**2*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*
b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**5*b*
*(23/2)*c**2*x**4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(9/2
)*d**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105
*a**3*b**6*x**10) - 40*A*a**4*b**(25/2)*c**2*x**6*sqrt(a/(b*x**2) + 1)/(31
5*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*
b**12*x**14) - 33*A*a**4*b**(11/2)*d**2*x**2*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 5*A*a**3*b**(27/
2)*c**2*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**
10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 17*A*a**3*b**(13/2)*d*
*2*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 10
5*a**3*b**6*x**10) + 30*A*a**2*b**(29/2)*c**2*x**10*sqrt(a/(b*x**2) + 1)/(
315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**
4*b**12*x**14) - 3*A*a**2*b**(15/2)*d**2*x**6*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a*b**(31/2
)*c**2*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**
10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 12*A*a*b**(17/2)*d*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^10,x, algorithm="maxim
a")
```

output
```
-1/16*(2*C*c*d + B*d^2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 5/128*
(B*c^2 + 2*A*c*d)*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*(2*C*c*
d + B*d^2)*sqrt(b*x^2 + a)*b^3/a^3 - 5/128*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 +
a)*b^4/a^4 + 16/315*(b*x^2 + a)^(3/2)*A*b^3*c^2/(a^4*x^3) + 2/15*(b*x^2 +
a)^(3/2)*C*b*d^2/(a^2*x^3) - 1/16*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b^2/
(a^3*x^2) + 5/128*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b^3/(a^4*x^2) - 8/10
5*(b*x^2 + a)^(3/2)*A*b^2*c^2/(a^3*x^5) - 1/5*(b*x^2 + a)^(3/2)*C*d^2/(a*x
^5) - 8/105*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 1/
8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b/(a^2*x^4) - 5/64*(B*c^2 + 2*A*c*d)
*(b*x^2 + a)^(3/2)*b^2/(a^3*x^4) + 2/21*(b*x^2 + a)^(3/2)*A*b*c^2/(a^2*x^7
) + 4/35*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*b/(a^2*x^5) - 1/6*(2*
C*c*d + B*d^2)*(b*x^2 + a)^(3/2)/(a*x^6) + 5/48*(B*c^2 + 2*A*c*d)*(b*x^2 +
 a)^(3/2)*b/(a^2*x^6) - 1/9*(b*x^2 + a)^(3/2)*A*c^2/(a*x^9) - 1/7*(C*c^2 +
 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/(a*x^7) - 1/8*(B*c^2 + 2*A*c*d)*(b*x^2
 + a)^(3/2)/(a*x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(388) = 776$.

Time = 0.24 (sec) , antiderivative size = 2041, normalized size of antiderivative = 4.77

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^10,x, algorithm="giac"
)
```

output
```
-1/64*(5*B*b^4*c^2 - 16*C*a*b^3*c*d + 10*A*b^4*c*d - 8*B*a*b^3*d^2)*arctan
(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/20160*(1575*(
sqrt(b)*x - sqrt(b*x^2 + a))^17*B*b^4*c^2 - 5040*(sqrt(b)*x - sqrt(b*x^2 +
 a))^17*C*a*b^3*c*d + 3150*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*b^4*c*d - 25
20*(sqrt(b)*x - sqrt(b*x^2 + a))^17*B*a*b^3*d^2 - 13650*(sqrt(b)*x - sqrt(
b*x^2 + a))^15*B*a*b^4*c^2 + 43680*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*
b^3*c*d - 27300*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a*b^4*c*d + 21840*(sqrt
(b)*x - sqrt(b*x^2 + a))^15*B*a^2*b^3*d^2 + 80640*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*C*a^3*b^(5/2)*d^2 + 52290*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^2*
b^4*c^2 + 90720*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^3*b^3*c*d + 104580*(s
qrt(b)*x - sqrt(b*x^2 + a))^13*A*a^2*b^4*c*d + 45360*(sqrt(b)*x - sqrt(b*x
^2 + a))^13*B*a^3*b^3*d^2 + 215040*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*
b^(7/2)*c^2 + 430080*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(7/2)*c*d -
295680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^4*b^(5/2)*d^2 + 215040*(sqrt(b
)*x - sqrt(b*x^2 + a))^12*A*a^3*b^(7/2)*d^2 + 252630*(sqrt(b)*x - sqrt(b*x
^2 + a))^11*B*a^3*b^4*c^2 - 292320*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^4*
b^3*c*d + 505260*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^4*c*d - 146160*(
sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^4*b^3*d^2 - 322560*(sqrt(b)*x - sqrt(b
*x^2 + a))^10*C*a^4*b^(7/2)*c^2 + 645120*(sqrt(b)*x - sqrt(b*x^2 + a))^10*
A*a^3*b^(9/2)*c^2 - 645120*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(7/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^10,x)
```

output
```
int(((a + b*x^2)^(1/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^10,x)
```

output
```
( - 4480*sqrt(a + b*x**2)*a**5*c**2 - 10080*sqrt(a + b*x**2)*a**5*c*d*x -
5760*sqrt(a + b*x**2)*a**5*d**2*x**2 - 640*sqrt(a + b*x**2)*a**4*b*c**2*x*
*2 - 5040*sqrt(a + b*x**2)*a**4*b*c**2*x - 1680*sqrt(a + b*x**2)*a**4*b*c*
d*x**3 - 11520*sqrt(a + b*x**2)*a**4*b*c*d*x**2 - 1152*sqrt(a + b*x**2)*a*
*4*b*d**2*x**4 - 6720*sqrt(a + b*x**2)*a**4*b*d**2*x**3 - 5760*sqrt(a + b*
x**2)*a**4*c**3*x**2 - 13440*sqrt(a + b*x**2)*a**4*c**2*d*x**3 - 8064*sqrt
(a + b*x**2)*a**4*c*d**2*x**4 + 768*sqrt(a + b*x**2)*a**3*b**2*c**2*x**4 -
 840*sqrt(a + b*x**2)*a**3*b**2*c**2*x**3 + 2100*sqrt(a + b*x**2)*a**3*b**
2*c*d*x**5 - 2304*sqrt(a + b*x**2)*a**3*b**2*c*d*x**4 + 1536*sqrt(a + b*x*
*2)*a**3*b**2*d**2*x**6 - 1680*sqrt(a + b*x**2)*a**3*b**2*d**2*x**5 - 1152
*sqrt(a + b*x**2)*a**3*b*c**3*x**4 - 3360*sqrt(a + b*x**2)*a**3*b*c**2*d*x
**5 - 2688*sqrt(a + b*x**2)*a**3*b*c*d**2*x**6 - 1024*sqrt(a + b*x**2)*a**
2*b**3*c**2*x**6 + 1050*sqrt(a + b*x**2)*a**2*b**3*c**2*x**5 - 3150*sqrt(a
 + b*x**2)*a**2*b**3*c*d*x**7 + 3072*sqrt(a + b*x**2)*a**2*b**3*c*d*x**6 -
 3072*sqrt(a + b*x**2)*a**2*b**3*d**2*x**8 + 2520*sqrt(a + b*x**2)*a**2*b*
*3*d**2*x**7 + 1536*sqrt(a + b*x**2)*a**2*b**2*c**3*x**6 + 5040*sqrt(a + b
*x**2)*a**2*b**2*c**2*d*x**7 + 5376*sqrt(a + b*x**2)*a**2*b**2*c*d**2*x**8
 + 2048*sqrt(a + b*x**2)*a*b**4*c**2*x**8 - 1575*sqrt(a + b*x**2)*a*b**4*c
**2*x**7 - 6144*sqrt(a + b*x**2)*a*b**4*c*d*x**8 - 3072*sqrt(a + b*x**2)*a
*b**3*c**3*x**8 - 3150*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b...
```

3.30 $\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 367

$$\begin{aligned}
&\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx \\
&= \frac{(8bc^2(c^2C-Bcd+Ad^2)+d(ad^2(cC-Bd)-4bc(c^2C-Bcd+Ad^2))x)\sqrt{a+bx^2}}{8bd^5} \\
&\quad -\frac{(8aCd^2-b(47c^2C-35Bcd+20Ad^2))(a+bx^2)^{3/2}}{60b^2d^3} \\
&\quad -\frac{(13cC-5Bd)(c+dx)(a+bx^2)^{3/2}}{20bd^3}+\frac{C(c+dx)^2(a+bx^2)^{3/2}}{5bd^3} \\
&\quad +\frac{(a^2d^4(cC-Bd)-8b^2c^3(c^2C-Bcd+Ad^2)-4abcd^2(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}d^6} \\
&\quad -\frac{c^2\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6}
\end{aligned}$$

output

```
1/8*(8*b*c^2*(A*d^2-B*c*d+C*c^2)+d*(a*d^2*(-B*d+C*c)-4*b*c*(A*d^2-B*c*d+C*
c^2))*x)*(b*x^2+a)^(1/2)/b/d^5-1/60*(8*a*C*d^2-b*(20*A*d^2-35*B*c*d+47*C*c
^2))*(b*x^2+a)^(3/2)/b^2/d^3-1/20*(-5*B*d+13*C*c)*(d*x+c)*(b*x^2+a)^(3/2)/
b/d^3+1/5*C*(d*x+c)^2*(b*x^2+a)^(3/2)/b/d^3+1/8*(a^2*d^4*(-B*d+C*c)-8*b^2*
c^3*(A*d^2-B*c*d+C*c^2)-4*a*b*c*d^2*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x
/(b*x^2+a)^(1/2))/b^(3/2)/d^6-c^2*(a*d^2+b*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)*
arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.93

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(-16a^2Cd^4+abd^2\left(40c^2C-5cd(8B+3Cx)+d^2\left(40A+15Bx+8Cx^2\right)\right)+2b^2\left(60c^4C-30c^3d(2B+Cx)+10c^2d^2(6A+x(3B+2Cx))-5cd^3\right.}{b^2}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(-16*a^2*C*d^4 + a*b*d^2*(40*c^2*C - 5*c*d*(8*B + 3*C*
x) + d^2*(40*A + 15*B*x + 8*C*x^2)) + 2*b^2*(60*c^4*C - 30*c^3*d*(2*B + C*
x) + 10*c^2*d^2*(6*A + x*(3*B + 2*C*x)) - 5*c*d^3*x*(6*A + x*(4*B + 3*C*x)
) + d^4*x^2*(20*A + 3*x*(5*B + 4*C*x)))))/b^2 + 240*c^2*Sqrt[-(b*c^2) - a*
d^2]*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2]
)/Sqrt[-(b*c^2) - a*d^2]] + (15*(a^2*d^4*(-(c*C) + B*d) + 8*b^2*c^3*(c^2*C
 - B*c*d + A*d^2) + 4*a*b*c*d^2*(c^2*C - B*c*d + A*d^2))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/b^(3/2))/(120*d^6)
```

Rubi [A] (verified)

Time = 2.16 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2185, 25, 2185, 25, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{c+dx}\,dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int -\frac{\sqrt{bx^2+a}\left(bd^3(13cC-5Bd)x^3+d^2\left(11bCc^2-5Abd^2+2aCd^2\right)x^2+cCd\left(3bc^2+4ad^2\right)x+2ac^2Cd^2\right)}{c+dx}dx}{5bd^4}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\int \frac{\sqrt{bx^2+a}\left(bd^3(13cC-5Bd)x^3+d^2\left(11bCc^2-5Abd^2+2aCd^2\right)x^2+cCd\left(3bc^2+4ad^2\right)x+2ac^2Cd^2\right)}{c+dx}dx}{5bd^4}$$

$$\Big\downarrow \ 2185$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(-b\left(8aCd^2-b\left(47Cc^2-35Bdc+20Ad^2\right)\right)x^2d^5+5abc(cC-Bd)d^5+b\left(3bc^2(9cC-5Bd)-ad^2(3cC+5Bd)\right)xd^4\right)}{c+dx}dx}{4bd^3}+\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+d\ldots}{5bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{\int \frac{\sqrt{bx^2+a}\left(-b\left(8aCd^2-b\left(47Cc^2-35Bdc+20Ad^2\right)\right)x^2d^5+5abc(cC-Bd)d^5+b\left(3bc^2(9cC-5Bd)-ad^2\ldots\right.\right.}{c+dx}}{4bd^3}}{5bd^4}$$

$$\Big\downarrow \ 2185$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{\frac{\int \frac{15b^2d^6\left(acd(cC-Bd)+\left(ad^2(cC-Bd)-4bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}-\frac{1}{3}d^4\left(a+bx^2\right)^{3/2}(8\ldots}{4bd^3}}{5bd^4}$$

$$\Big\downarrow \ 27$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\int \frac{\left(acd(cC-Bd)+\left(ad^2(cC-Bd)-4bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx-\frac{1}{3}d^4\left(a+bx^2\right)^{3/2}(8a\ldots}{4bd^3}}{5bd^4}$$

$$\Big\downarrow \ 682$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\int \frac{b\left(acd\left(a(cC-Bd)d^2+4bc\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^2(cC-Bd)d^4-4abc\left(Cc^2-Bdc+Ad^2\right)d^2-\ldots\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}\ldots\right.}{\ldots}}{\ldots}$$

$\downarrow$ 27

$$\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\int \frac{acd\left(a(cC-Bd)d^2+4bc\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^2(cC-Bd)d^4-4abc\left(Cc^2-Bdc+Ad^2\right)d^2-8b^2\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\right.}{}\quad \frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-$$

$\downarrow$ 719

$$\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\frac{\left(a^2d^4(cC-Bd)-4abcd^2\left(Ad^2-Bcd+c^2C\right)-8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{8bc^2(a}{}}{2d^2}\right.}{}\quad \frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-$$

$\downarrow$ 224

$$\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\frac{\left(a^2d^4(cC-Bd)-4abcd^2\left(Ad^2-Bcd+c^2C\right)-8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}\right.}{}\quad \frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-$$

$\downarrow$ 219

$$\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\frac{8bc^2\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^4(cC-\right.}{2d^2}\right.}{}\quad \frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-$$

$\downarrow$ 488

$$\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^4(cC-Bd)-4abcd^2\left(Ad^2-Bcd+c^2C\right)-8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}}{2d^2}\right.}{}\quad \frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-$$

$$\Big\downarrow \; 219$$

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)^2}{5bd^3}-\frac{\frac{1}{4}d\left(a+bx^2\right)^{3/2}(c+dx)(13cC-5Bd)-\frac{5bd^4\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^4(cC-Bd)-4abcd^2\left(Ad^2-Bcd+c^2C\right)-8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}}{2d^2}\right.}{}}{}$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(c + d*x)^2*(a + b*x^2)^(3/2))/(5*b*d^3) - ((d*(13*c*C - 5*B*d)*(c + d*
x)*(a + b*x^2)^(3/2))/4 - (-1/3*(d^4*(8*a*C*d^2 - b*(47*c^2*C - 35*B*c*d +
 20*A*d^2))*(a + b*x^2)^(3/2)) + 5*b*d^4*(((8*b*c^2*(c^2*C - B*c*d + A*d^2
) + d*(a*d^2*(c*C - B*d) - 4*b*c*(c^2*C - B*c*d + A*d^2))*x)*Sqrt[a + b*x^
2])/(2*d^2) + (((a^2*d^4*(c*C - B*d) - 8*b^2*c^3*(c^2*C - B*c*d + A*d^2) -
 4*a*b*c*d^2*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(Sqrt[b]*d) - (8*b*c^2*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*b
*d^3))/(5*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.42

method	result
risch	$\frac{(24C\,d^4b^2x^4+30B\,b^2d^4x^3-30C\,b^2c\,d^3x^3+40A\,b^2d^4x^2-40B\,b^2c\,d^3x^2+8Cab\,d^4x^2+40C\,b^2c^2d^2x^2-60A\,b^2c\,d^3x+15Bab\,d^4x+60B\,\cdots}{120\cdots}$
default	$\frac{c^2\left(A\,d^2-Bcd+C\,c^2\right)\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}\right)}{d}-\frac{\left(a\,d^2+b\,c^2\cdots\right.}{\cdots}\right.}{d^5}$

input
```
int(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*C*b^2*d^4*x^4+30*B*b^2*d^4*x^3-30*C*b^2*c*d^3*x^3+40*A*b^2*d^4*x
^2-40*B*b^2*c*d^3*x^2+8*C*a*b*d^4*x^2+40*C*b^2*c^2*d^2*x^2-60*A*b^2*c*d^3*
x+15*B*a*b*d^4*x+60*B*b^2*c^2*d^2*x-15*C*a*b*c*d^3*x-60*C*b^2*c^3*d*x+40*A
*a*b*d^4+120*A*b^2*c^2*d^2-40*B*a*b*c*d^3-120*B*b^2*c^3*d-16*C*a^2*d^4+40*
C*a*b*c^2*d^2+120*C*b^2*c^4)*(b*x^2+a)^(1/2)/b^2/d^5-1/8/d^5/b*((4*A*a*b*c
*d^4+8*A*b^2*c^3*d^2+B*a^2*d^5-4*B*a*b*c^2*d^3-8*B*b^2*c^4*d-C*a^2*c*d^4+4
*C*a*b*c^3*d^2+8*C*b^2*c^5)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8*c^2*
(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)*b/d^2/((a*d^
2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^
2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/
d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.70

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*C*x^2/(b*d) - 1/2*sqrt(b*x^2 + a)*C*c^3*x/d^4 + 1/2*
sqrt(b*x^2 + a)*B*c^2*x/d^3 - 1/2*sqrt(b*x^2 + a)*A*c*x/d^2 - 1/4*(b*x^2 +
 a)^(3/2)*C*c*x/(b*d^2) + 1/8*sqrt(b*x^2 + a)*C*a*c*x/(b*d^2) + 1/4*(b*x^2
 + a)^(3/2)*B*x/(b*d) - 1/8*sqrt(b*x^2 + a)*B*a*x/(b*d) - C*sqrt(b)*c^5*ar
csinh(b*x/sqrt(a*b))/d^6 + B*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 - 1/2*
C*a*c^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) - A*sqrt(b)*c^3*arcsinh(b*x/s
qrt(a*b))/d^4 + 1/2*B*a*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) + 1/8*C*a
^2*c*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) - 1/2*A*a*c*arcsinh(b*x/sqrt(a*b
))/(sqrt(b)*d^2) - 1/8*B*a^2*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) + C*sqrt(a
 + b*c^2/d^2)*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*
abs(d*x + c)))/d^5 - B*sqrt(a + b*c^2/d^2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*ab
s(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + A*sqrt(a + b*c^2/d^2)*c^
2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d
^3 + sqrt(b*x^2 + a)*C*c^4/d^5 - sqrt(b*x^2 + a)*B*c^3/d^4 + sqrt(b*x^2 +
a)*A*c^2/d^3 + 1/3*(b*x^2 + a)^(3/2)*C*c^2/(b*d^3) - 1/3*(b*x^2 + a)^(3/2)
*B*c/(b*d^2) - 2/15*(b*x^2 + a)^(3/2)*C*a/(b^2*d) + 1/3*(b*x^2 + a)^(3/2)*
A/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x^2\,\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{c+d\,x}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x^2\sqrt{b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{dx+c}dx$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

3.31 $\int \frac{x\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{c+dx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 280

$$\begin{aligned}\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx &= -\frac{\left(8bc(c^2C-Bcd+Ad^2)-d^3\left(4Ab-aC+\frac{4bc(cC-Bd)}{d^2}\right)x\right)\sqrt{a+bx^2}}{8bd^4} \\ &\quad -\frac{(7cC-4Bd)\left(a+bx^2\right)^{3/2}}{12bd^2}+\frac{C(c+dx)\left(a+bx^2\right)^{3/2}}{4bd^2} \\ &\quad +\frac{\left(2abc^2C+(2bc^2+ad^2)\left(4Ab-aC+\frac{4bc(cC-Bd)}{d^2}\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}d^3} \\ &\quad +\frac{c\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5}\end{aligned}$$

output
```
-1/8*(8*b*c*(A*d^2-B*c*d+C*c^2)-d^3*(4*A*b-a*C+4*b*c*(-B*d+C*c)/d^2)*x)*(b
*x^2+a)^(1/2)/b/d^4-1/12*(-4*B*d+7*C*c)*(b*x^2+a)^(3/2)/b/d^2+1/4*C*(d*x+c
)*(b*x^2+a)^(3/2)/b/d^2+1/8*(2*a*b*c^2*C+(a*d^2+2*b*c^2)*(4*A*b-a*C+4*b*c*
(-B*d+C*c)/d^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^3+c*(a*d^2+b
*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(
b*x^2+a)^(1/2))/d^5
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.98

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(ad^2(-8cC+8Bd+3Cdx)-2b\left(12c^3C-6c^2d(2B+Cx)+2cd^2(6A+3Bx+2Cx^2)-d^3x(6A+4Bx+3Cx^2)\right)\right)}{b}-48c\sqrt{-bc^2-ad^2}\left(\right.}{}$$

input
```
Integrate[(x*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(-8*c*C + 8*B*d + 3*C*d*x) - 2*b*(12*c^3*C - 6*
c^2*d*(2*B + C*x) + 2*c*d^2*(6*A + 3*B*x + 2*C*x^2) - d^3*x*(6*A + 4*B*x +
 3*C*x^2))))/b - 48*c*Sqrt[-(b*c^2) - a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTa
n[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (3*(-(
a^2*C*d^4) + 8*b^2*c^2*(c^2*C - B*c*d + A*d^2) + 4*a*b*d^2*(c^2*C - B*c*d
+ A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(24*d^5)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2185, 25, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int -\frac{\sqrt{bx^2+a}\left(b(7cC-4Bd)x^2d^2+acCd^2+(3bCc^2-4Abd^2+aCd^2)xd\right)}{c+dx}dx}{4bd^3}+\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{\int \frac{\sqrt{bx^2+a}\left(b(7cC-4Bd)x^2d^2+acCd^2+(3bCc^2-4Abd^2+aCd^2)xd\right)}{c+dx}dx}{4bd^3}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{\frac{\int \frac{3bd^3\left(acCd+\left(aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}+\frac{1}{3}d\left(a+bx^2\right)^{3/2}(7cC-4Bd)}{4bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\int \frac{(acCd+(aCd^2-4b(Cc^2-Bdc+Ad^2))x)\sqrt{bx^2+a}}{c+dx}dx+\frac{1}{3}d\left(a+bx^2\right)^{3/2}(7cC-4Bd)}{4bd^3}$$

$\Big\downarrow$ 682

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{\int \frac{b\left(acd\left(aCd^2+4b\left(Cc^2-Bdc+Ad^2\right)\right)-\left(2abc^2Cd^2-\left(2bc^2+ad^2\right)\left(aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(dx\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)-\right.}{2d^2}\right.}{4bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{\int \frac{acd\left(aCd^2+4b\left(Cc^2-Bdc+Ad^2\right)\right)-\left(2abc^2Cd^2-\left(2bc^2+ad^2\right)\left(aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(dx\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)+8b\right.}{2d^2}\right.}{4bd^3}$$

$\Big\downarrow$ 719

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{\frac{8bc\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(2abc^2Cd^2-\left(ad^2+2bc^2\right)\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(dx\left(aCd^2-\right.\right.}{}\right.}{4bd^3}$$

$\Big\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{\frac{8bc\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(2abc^2Cd^2-\left(ad^2+2bc^2\right)\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}(d}{}\right)}{4bd^3}$$

$\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{\frac{8bc\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2abc^2Cd^2-\left(ad^2+2bc^2\right)\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}(d}{}\right)}{4bd^3}$$

$\downarrow$ 488

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{-\frac{8bc\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2abc^2Cd^2-\left(ad^2+2bc^2\right)\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)\right)}{\sqrt{b}d}}{2d^2}\right)}{4bd^3}$$

$\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{3/2}(c+dx)}{4bd^2}-\frac{d\left(\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(2abc^2Cd^2-\left(ad^2+2bc^2\right)\left(aCd^2-4b\left(Ad^2-Bcd+c^2C\right)\right)\right)}{\sqrt{b}d}-\frac{8bc\sqrt{ad^2+bc^2}\left(Ad^2-Bcd+c^2C\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}\right)}{4bd^3}$$

input
```
Int[(x*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(c + d*x)*(a + b*x^2)^(3/2))/(4*b*d^2) - ((d*(7*c*C - 4*B*d)*(a + b*x^2
)^(3/2))/3 + d*(((8*b*c*(c^2*C - B*c*d + A*d^2) + d*(a*C*d^2 - 4*b*(c^2*C
- B*c*d + A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) + (-(((2*a*b*c^2*C*d^2 - (2*
b*c^2 + a*d^2)*(a*C*d^2 - 4*b*(c^2*C - B*c*d + A*d^2)))*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (8*b*c*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c
*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/
d)/(2*d^2)))/(4*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{(-6C\,d^3b\,x^3-8Bb\,d^3x^2+8Cbc\,d^2x^2-12Ab\,d^3x+12Bbc\,d^2x-3Ca\,d^3x-12Cb\,c^2dx+24Abc\,d^2-8Ba\,d^3-24Bb\,c^2d+8Cac\,d^2+24C}{24b\,d^4}$
default	$\frac{A\,d^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+C\,c^2\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{d(Bd-Cc)\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+C\,d^2\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a(}{}\right.}{d^3}$

input
```
int(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/24/b*(-6*C*b*d^3*x^3-8*B*b*d^3*x^2+8*C*b*c*d^2*x^2-12*A*b*d^3*x+12*B*b*
c*d^2*x-3*C*a*d^3*x-12*C*b*c^2*d*x+24*A*b*c*d^2-8*B*a*d^3-24*B*b*c^2*d+8*C
*a*c*d^2+24*C*b*c^3)*(b*x^2+a)^(1/2)/d^4+1/8/b/d^4*((4*A*a*b*d^4+8*A*b^2*c
^2*d^2-4*B*a*b*c*d^3-8*B*b^2*c^3*d-C*a^2*d^4+4*C*a*b*c^2*d^2+8*C*b^2*c^4)/
d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8*c*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3
-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)*b/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d
^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(x*sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.75

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \frac{\sqrt{bx^2+a}Cc^2x}{2\,d^3} - \frac{\sqrt{bx^2+a}Bcx}{2\,d^2} + \frac{\sqrt{bx^2+a}Ax}{2\,d} + \frac{(bx^2+a)^{\frac{3}{2}}Cx}{4\,bd} - \frac{\sqrt{bx^2+a}Cax}{8\,bd} + \frac{C\sqrt{b}c^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^5} - \frac{B\sqrt{b}c^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} + \frac{Cac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d^3} + \frac{A\sqrt{b}c^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} - \frac{Bac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d^2} - \frac{Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}d} + \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d} - \frac{C\sqrt{a+\frac{bc^2}{d^2}}c^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^4} + \frac{B\sqrt{a+\frac{bc^2}{d^2}}c^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} - \frac{A\sqrt{a+\frac{bc^2}{d^2}}c\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} - \frac{\sqrt{bx^2+a}Cc^3}{d^4} + \frac{\sqrt{bx^2+a}Bc^2}{d^3} - \frac{\sqrt{bx^2+a}Ac}{d^2} - \frac{(bx^2+a)^{\frac{3}{2}}Cc}{3\,bd^2} + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,bd}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*c^2*x/d^3 - 1/2*sqrt(b*x^2 + a)*B*c*x/d^2 + 1/2*sqrt
(b*x^2 + a)*A*x/d + 1/4*(b*x^2 + a)^(3/2)*C*x/(b*d) - 1/8*sqrt(b*x^2 + a)*
C*a*x/(b*d) + C*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 - B*sqrt(b)*c^3*arc
sinh(b*x/sqrt(a*b))/d^4 + 1/2*C*a*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3)
 + A*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 1/2*B*a*c*arcsinh(b*x/sqrt(a
*b))/(sqrt(b)*d^2) - 1/8*C*a^2*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) + 1/2*A*
a*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - C*sqrt(a + b*c^2/d^2)*c^3*arcsinh(b
*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + B*sqrt
(a + b*c^2/d^2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b
)*abs(d*x + c)))/d^3 - A*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*ab
s(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 - sqrt(b*x^2 + a)*C*c^3/d^
4 + sqrt(b*x^2 + a)*B*c^2/d^3 - sqrt(b*x^2 + a)*A*c/d^2 - 1/3*(b*x^2 + a)^
(3/2)*C*c/(b*d^2) + 1/3*(b*x^2 + a)^(3/2)*B/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x\,\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{c+d\,x}\,dx$$

input
```
int((x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int((x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x\sqrt{b\,x^2+a}\,(C\,x^2+Bx+A)}{dx+c}dx$$

input
```
int(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

3.32 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 205

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = -\frac{\left(2\left(Bc-\frac{c^2C}{d}-Ad\right)+(cC-Bd)x\right)\sqrt{a+bx^2}}{2d^2}+\frac{C(a+bx^2)^{3/2}}{3bd} - \frac{\left(2Abcd-\left(B-\frac{cC}{d}\right)(2bc^2+ad^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^3} - \frac{\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4}$$

output
```
-1/2*(2*B*c-2*c^2*C/d-2*A*d+(-B*d+C*c)*x)*(b*x^2+a)^(1/2)/d^2+1/3*C*(b*x^2
+a)^(3/2)/b/d-1/2*(2*A*b*c*d-(B-c*C/d)*(a*d^2+2*b*c^2))*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(1/2)/d^3-(a*d^2+b*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}(2aCd^2+b(6c^2C-3cd(2B+Cx)+d^2(6A+3Bx+2Cx^2)))}{b} + 12\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)}{\sqrt{-bc}}\right.}{6d^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(2*a*C*d^2 + b*(6*c^2*C - 3*c*d*(2*B + C*x) + d^2*(6*A
 + 3*B*x + 2*C*x^2))))/b + 12*Sqrt[-(b*c^2) - a*d^2]*(c^2*C - B*c*d + A*d^
2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]]
+ (3*(a*d^2*(c*C - B*d) + 2*b*c*(c^2*C - B*c*d + A*d^2))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b])/(6*d^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int \frac{3bd(Ad-(cC-Bd)x)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(Ad-(cC-Bd)x)\sqrt{bx^2+a}}{c+dx}dx}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 682

$$\frac{\frac{\int \frac{b\left(ad\left(Cc^2-Bdc+2Ad^2\right)-\left(2Abcd^2+(cC-Bd)\left(2bc^2+ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+c^2C\right)-dx(cC-Bd)\right)}{2d^2}}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{ad\left(Cc^2-Bdc+2Ad^2\right)-\left(2Abcd^2+(cC-Bd)\left(2bc^2+ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+c^2C\right)-dx(cC-Bd)\right)}{2d^2}}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 719

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)(cC-Bd)+2Abcd^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+c^2C\right)-dx(cC-Bd)\right)}{2d^2}}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)(cC-Bd)+2Abcd^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+c^2C\right)-dx(cC-\cdots\right.}{2d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(cC-Bd)+2Abcd^2\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+c^2C\right)-dx(cC-\cdots\right.}{2d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$\Big\downarrow$ 488

$$\frac{\frac{-\frac{2\left(ad^2+bc^2\right)\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(cC-Bd)+2Abcd^2\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+\cdots\right.\right.}{2d^2}}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(cC-Bd)+2Abcd^2\right)}{\sqrt{b}d}-\frac{2\sqrt{ad^2+bc^2}\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(Ad^2-Bcd+\cdots\right.\right.}{2d^2}}{d}+\frac{C\left(a+bx^2\right)^{3/2}}{3bd}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(a + b*x^2)^(3/2))/(3*b*d) + (((2*(c^2*C - B*c*d + A*d^2) - d*(c*C - B*
d)*x)*Sqrt[a + b*x^2])/(2*d^2) + (-(((2*A*b*c*d^2 + (c*C - B*d)*(2*b*c^2 +
 a*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (2*Sqrt[b*c^
2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d
^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.52

method	result
risch	$\frac{(2C d^2 b x^2+3Bb d^2 x-3Cbcdx+6Ab d^2-6Bbcd+2aC d^2+6Cb c^2)\sqrt{b x^2+a}}{6b d^3}-\frac{\frac{(2Abc d^2-Ba d^3-2Bb c^2 d+Cac d^2+2Cb c^3)\ln(\sqrt{b}\,x+\sqrt{}}{d\sqrt{b}}}{}$
default	$\frac{Bd\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)+\frac{Cd(b x^2+a)^{\frac{3}{2}}}{3b}-Cc\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{d^2}+\frac{(A d^2-Bcd+C c^2)\left(\sqrt{b(x+\frac{c}{d})}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x-3*C*b*c*d*x+6*A*b*d^2-6*B*b*c*d+2*C*a*d^2+6
*C*b*c^2)*(b*x^2+a)^(1/2)/b/d^3-1/2/d^3*((2*A*b*c*d^2-B*a*d^3-2*B*b*c^2*d+
C*a*c*d^2+2*C*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*(A*a*d^4+A*
b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/d^2/((a*d^2+b*c^2)/d^2)
^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.77

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = &-\frac{\sqrt{bx^2+a}Ccx}{2\,d^2} + \frac{\sqrt{bx^2+a}Bx}{2\,d} \\ &- \frac{C\sqrt{b}c^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} \\ &+ \frac{B\sqrt{b}c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} - \frac{Cac \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d^2} \\ &- \frac{A\sqrt{b}c \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2} + \frac{Ba \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}d} \\ &+ \frac{C\sqrt{a+\frac{bc^2}{d^2}}c^2 \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} \\ &- \frac{B\sqrt{a+\frac{bc^2}{d^2}}c \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} \\ &+ \frac{A\sqrt{a+\frac{bc^2}{d^2}} \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d} \\ &+ \frac{\sqrt{bx^2+a}Cc^2}{d^3} - \frac{\sqrt{bx^2+a}Bc}{d^2} \\ &+ \frac{\sqrt{bx^2+a}A}{d} + \frac{(bx^2+a)^{\frac{3}{2}}C}{3\,bd}\end{aligned}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*C*c*x/d^2 + 1/2*sqrt(b*x^2 + a)*B*x/d - C*sqrt(b)*c^3
*arcsinh(b*x/sqrt(a*b))/d^4 + B*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 1
/2*C*a*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - A*sqrt(b)*c*arcsinh(b*x/sq
rt(a*b))/d^2 + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) + C*sqrt(a + b*c
^2/d^2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*
x + c)))/d^3 - B*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + A*sqrt(a + b*c^2/d^2)*arcsinh(b*
c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d + sqrt(b*x^
2 + a)*C*c^2/d^3 - sqrt(b*x^2 + a)*B*c/d^2 + sqrt(b*x^2 + a)*A/d + 1/3*(b*
x^2 + a)^(3/2)*C/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3439, normalized size of antiderivative = 16.78

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
( - 6*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c*d**2 + 6*sqrt(b)*sq
rt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**2*d - 6*sqrt(b)*sqrt(2*sqrt(b)*sq
rt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*b*c**4 - 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d*
*2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b*d**4 - 6*sqrt(2*sqrt(b)*s
qrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqr
t(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b
**2*c**2*d**2 + 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*a*b**2*c*d**3 - 6*sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)
/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c**3*d**
2 + 6*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sq
rt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c ...
```

3.33 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = -\frac{(cC-Bd)\sqrt{a+bx^2}}{d^2}+\frac{Cx\sqrt{a+bx^2}}{2d} + \frac{(aCd^2+2b(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^3} + \frac{\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{cd^3} - \frac{\sqrt{a}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}$$

output
```
-(-B*d+C*c)*(b*x^2+a)^(1/2)/d^2+1/2*C*x*(b*x^2+a)^(1/2)/d+1/2*(a*C*d^2+2*b
*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^3+(a*d^
2+b*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2
)/(b*x^2+a)^(1/2))/c/d^3-a^(1/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx$$
$$=\frac{(-2cC+2Bd+Cdx)\sqrt{a+bx^2}}{2d^2} - \frac{2\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{cd^3} + \frac{(aCd^2+2b(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{\sqrt{b}d^3} - \frac{\sqrt{a}A\log(x)}{c} + \frac{\sqrt{a}A\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x*(c + d*x)),x]
```

output
```
((-2*c*C + 2*B*d + C*d*x)*Sqrt[a + b*x^2])/(2*d^2) - (2*Sqrt[-(b*c^2) - a*
d^2]*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c
 + d*x) - c*Sqrt[a + b*x^2])])/(c*d^3) + ((a*C*d^2 + 2*b*(c^2*C - B*c*d +
A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(Sqrt[b]*d^3) -
 (Sqrt[a]*A*Log[x])/c + (Sqrt[a]*A*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/c
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.36, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2351, 606, 243, 73, 221, 682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx$$
$$\Big\downarrow \; 2351$$

$$A\int \frac{\sqrt{bx^2+a}}{x(c+dx)}dx+\int \frac{(B+Cx)\sqrt{bx^2+a}}{c+dx}dx$$

$$\Big\downarrow \text{606}$$

$$A\left(\frac{a\int \frac{1}{x\sqrt{bx^2+a}}dx}{c}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{c+dx}dx$$

$$\Big\downarrow \text{243}$$

$$A\left(\frac{a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2c}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{c+dx}dx$$

$$\Big\downarrow \text{73}$$

$$A\left(\frac{a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{bc}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{c+dx}dx$$

$$\Big\downarrow \text{221}$$

$$A\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{c+dx}dx$$

$$\Big\downarrow \text{682}$$

$$A\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)+\frac{\int -\frac{b\left(ad(cC-2Bd)-\left(aCd^2+2bc(cC-Bd)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \text{25}$$

$$A\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{\int \frac{b\left(ad(cC-2Bd)-\left(aCd^2+2bc(cC-Bd)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \text{27}$$

$$A\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{\int \frac{ad(cC-2Bd)-\left(aCd^2+2bc(cC-Bd)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \ 719$$

$$A\left(-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{\frac{2\left(ad^2+bc^2\right)(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aCd^2+2bc(cC-Bd)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \ 224$$

$$A\left(-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{\frac{2\left(ad^2+bc^2\right)(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aCd^2+2bc(cC-Bd)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \ 219$$

$$A\left(-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{\frac{2\left(ad^2+bc^2\right)(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2bc(cC-Bd)\right)}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \ 488$$

$$A\left(-\frac{-\frac{\left(ad^2+bc^2\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{-\frac{2\left(ad^2+bc^2\right)(cC-Bd)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2bc(cC-Bd)\right)}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

$$\Big\downarrow \text{219}$$

$$A\left(-\frac{-\frac{\sqrt{ad^2+bc^2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)-\frac{-\frac{2\sqrt{ad^2+bc^2}(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd^2+2bc(cC-Bd))}{\sqrt{b}d}}{2d^2}-\frac{\sqrt{a+bx^2}(2(cC-Bd)-Cdx)}{2d^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x*(c + d*x)),x]
```

output
```
-1/2*((2*(c*C - B*d) - C*d*x)*Sqrt[a + b*x^2])/d^2 - (-(((a*C*d^2 + 2*b*c*
(c*C - B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (2*(c*C
- B*d)*Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt
[a + b*x^2])])/d)/(2*d^2) + A*(-((-((Sqrt[b]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/d) - (Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a
*d^2]*Sqrt[a + b*x^2])])/d)/c) - (Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(180) = 360$.

Time = 0.18 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.77

method	result
default	$\frac{C\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{d}+\frac{A\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{c}-\frac{\left(A\,d^{2}-Bcd+C\,c^{2}\right)\left(\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\right.}{d}}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
C/d*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+A/
c*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-1/d^2*(A
*d^2-B*c*d+C*c^2)/c*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*
(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2
-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \frac{Cd\left(\frac{\sqrt{bx^2+a}cx}{d^2}+\frac{2\sqrt{b}c^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4}+\frac{ac\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2}-\frac{2\sqrt{a+\frac{bc^2}{d^2}}c^2\operatorname{arsinh}\left(\frac{2\,bcx}{\sqrt{ab}|2\,dx+2\,c|}-\frac{2\,ad}{\sqrt{ab}|2\,dx+2\,c|}\right)}{d^3}-\frac{2\sqrt{bx^2+a}c^2}{d^3}\right)}{2\,c} - \frac{Bd\left(\frac{\sqrt{b}c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3}-\frac{\sqrt{a+\frac{bc^2}{d^2}}c\operatorname{arsinh}\left(\frac{2\,bcx}{\sqrt{ab}|2\,dx+2\,c|}-\frac{2\,ad}{\sqrt{ab}|2\,dx+2\,c|}\right)}{d^2}-\frac{\sqrt{bx^2+a}c}{d^2}\right)}{c} + \frac{Ad\left(\frac{\sqrt{b}c\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2}-\frac{\sqrt{a+\frac{bc^2}{d^2}}\operatorname{arsinh}\left(\frac{2\,bcx}{\sqrt{ab}|2\,dx+2\,c|}-\frac{2\,ad}{\sqrt{ab}|2\,dx+2\,c|}\right)}{d}-\frac{\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{d}\right)}{c}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*C*d*(sqrt(b*x^2 + a)*c*x/d^2 + 2*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^
4 + a*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 2*sqrt(a + b*c^2/d^2)*c^2*a
rcsinh(2*b*c*x/(sqrt(a*b)*abs(2*d*x + 2*c)) - 2*a*d/(sqrt(a*b)*abs(2*d*x +
 2*c)))/d^3 - 2*sqrt(b*x^2 + a)*c^2/d^3)/c - B*d*(sqrt(b)*c^2*arcsinh(b*x/
sqrt(a*b))/d^3 - sqrt(a + b*c^2/d^2)*c*arcsinh(2*b*c*x/(sqrt(a*b)*abs(2*d*
x + 2*c)) - 2*a*d/(sqrt(a*b)*abs(2*d*x + 2*c)))/d^2 - sqrt(b*x^2 + a)*c/d^
2)/c + A*d*(sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 - sqrt(a + b*c^2/d^2)*arc
sinh(2*b*c*x/(sqrt(a*b)*abs(2*d*x + 2*c)) - 2*a*d/(sqrt(a*b)*abs(2*d*x + 2
*c)))/d - sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))/d)/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{x\,\left(c+d\,x\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)}\,dx$$
$$= \frac{4\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)ab\,d^2-4\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c\right.}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c),x)
```

output
```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*d**2 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c*d + 4*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**3 - 4*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a*b*d**2 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*b**2*c*d - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**3 + 4*sqrt(a + b*x*
*2)*b**2*c*d**2 - 4*sqrt(a + b*x**2)*b*c**3*d + 2*sqrt(a + b*x**2)*b*c**2*
d**2*x + 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a*b*d**3 - 2*sqrt(a)*lo
g(sqrt(a + b*x**2) + sqrt(a))*a*b*d**3 - 2*sqrt(b)*log(sqrt(a + b*x**2) -
sqrt(b)*x)*a*b*c*d**2 - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*a*c**2*d
**2 + 2*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b**2*c**2*d - 2*sqrt(b)*
log(sqrt(a + b*x**2) - sqrt(b)*x)*b*c**4 + 2*sqrt(b)*log(sqrt(a + b*x**2)
+ sqrt(b)*x)*a*b*c*d**2 + sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*a*c**2
*d**2 - 2*sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*b**2*c**2*d + 2*sqrt(b
)*log(sqrt(a + b*x**2) + sqrt(b)*x)*b*c**4)/(4*b*c*d**3)
```

3.34 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 186

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx &= \frac{C\sqrt{a+bx^2}}{d}-\frac{A\sqrt{a+bx^2}}{cx}-\frac{\sqrt{b}(cC-Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2}\\ &\quad-\frac{\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2d^2}\\ &\quad-\frac{\sqrt{a}(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}\end{aligned}$$

output
```
C*(b*x^2+a)^(1/2)/d-A*(b*x^2+a)^(1/2)/c/x-b^(1/2)*(-B*d+C*c)*arctanh(b^(1/
2)*x/(b*x^2+a)^(1/2))/d^2-(a*d^2+b*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)*arctanh(
(-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2/d^2-a^(1/2)*(-A*d+B*
c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$$
$$= \frac{2\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\,x\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)+2\sqrt{a}d^2(-Bc+Ad)x\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a}}{\sqrt{a}}\right.}{c^2d^2x}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^2*(c + d*x)),x]
```

output
```
(2*Sqrt[-(b*c^2) - a*d^2]*(c^2*C - B*c*d + A*d^2)*x*ArcTan[(Sqrt[b]*(c + d
*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 2*Sqrt[a]*d^2*(-(B*c) +
 A*d)*x*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]] + c*(d*(-(A*d) +
 c*C*x)*Sqrt[a + b*x^2] + Sqrt[b]*c*(c*C - B*d)*x*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]]))/(c^2*d^2*x)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$$
$$\Big\downarrow \text{2353}$$
$$\int \left(\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^2(c+dx)}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2x}+\frac{A\sqrt{a+bx^2}}{cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{cd^2}-\frac{\sqrt{ad^2+bc^2}(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2d^2}-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{c^2}+\frac{A\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{c}+\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^2d}+\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2}-\frac{A\sqrt{a+bx^2}}{cx}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^2*(c + d*x)),x]
```

output
```
((B*c - A*d)*Sqrt[a + b*x^2])/c^2 + ((c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^
2])/(c^2*d) - (A*Sqrt[a + b*x^2])/(c*x) + (A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/c - (Sqrt[b]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(c*d^2) - (Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*A
rcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*d^2) - (
Sqrt[a]*(B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{c x}+\frac{\frac{b c \left(\frac{B d \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{C d \sqrt{b\,x^{2}+a}}{b}-\frac{C c \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}\right)}{d^{2}}-\frac{\left(A a\,d^{4}+A b\,c^{2} d^{2}-B a c\,d^{3}-c^{3} B b d+C a\,c^{2} d^{2}+c^{4} C b\right)\cdots}{\cdots}}{\cdots}$
default	$\frac{A \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{a x}+\frac{2 b \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{a}\right)}{c}-\frac{\left(A d -B c \right) \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{c^{2}}+\frac{\left(A\,d^{2}-B c d+\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^2+a)^(1/2)/c/x+1/c*(b*c/d^2*(B*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(
1/2)+C*d*(b*x^2+a)^(1/2)/b-C*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))-1/d^
3*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/c/((a*d^2+
b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)
/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)
)+a^(1/2)*(A*d-B*c)/c*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**2/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^2\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+Bx+A)}{x^2\,(dx+c)}\,dx$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x)
```

output
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x)
```

3.35 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 211

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = -\frac{A\sqrt{a+bx^2}}{2cx^2}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{c^2x}+\frac{\sqrt{b}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d} + \frac{\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3d} - \frac{(2ac(cC-Bd)+A(bc^2+2ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c^3}$$

output
```
-1/2*A*(b*x^2+a)^(1/2)/c/x^2-(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x+b^(1/2)*C*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d+(a*d^2+b*c^2)^(1/2)*(A*d^2-B*c*d+C*c^2)
*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^3/d-1/2*(2*a*
c*(-B*d+C*c)+A*(2*a*d^2+b*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c
^3
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx =$$

$$-\frac{\frac{c\sqrt{a+bx^2}(2Bcx+A(c-2dx))}{x^2}+\frac{4\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{d}-4\sqrt{a}Ad^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right.}{2c^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^3*(c + d*x)),x]
```

output
```
-1/2*((c*Sqrt[a + b*x^2]*(2*B*c*x + A*(c - 2*d*x)))/x^2 + (4*Sqrt[-(b*c^2)
 - a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b
*x^2])/Sqrt[-(b*c^2) - a*d^2]])/d - 4*Sqrt[a]*A*d^2*ArcTanh[(Sqrt[b]*x - S
qrt[a + b*x^2])/Sqrt[a]] + (2*c*(A*b*c + 2*a*c*C - 2*a*B*d)*ArcTanh[(-(Sqr
t[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + (2*Sqrt[b]*c^3*C*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/d)/c^3
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2x^2}+\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3x}-\frac{d\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3(c+dx)}+\frac{A\sqrt{a+bx^2}}{cx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{c^2d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-Ad)}{c^2}+\frac{\sqrt{ad^2+bc^2}(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{c^3}-\frac{Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c}-\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2x}-\frac{A\sqrt{a+bx^2}}{2cx^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^3*(c + d*x)),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(c*x^2) - ((B*c - A*d)*Sqrt[a + b*x^2])/(c^2*x) +
 (Sqrt[b]*(B*c - A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^2 + (Sqrt[b]
*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(c^2*d) + (
Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*
c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^3*d) - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(2*Sqrt[a]*c) - (Sqrt[a]*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-2Adx+2Bcx+Ac)}{2c^{2}x^{2}}-\frac{\frac{\left(2Aa\,d^{2}+bA\,c^{2}-2Bacd+2Ca\,c^{2}\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{c\sqrt{a}}-\frac{2\left(Aa\,d^{4}+Ab\,c^{2}d^{2}-Bac\,d^{3}-c^{3}Bbd+Ca\,c^{2}\right.}{}}{}$
default	$\frac{A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}\right)}{c^{2}}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-2*A*d*x+2*B*c*x+A*c)/c^2/x^2-1/2/c^2*((2*A*a*d^2+A*
b*c^2-2*B*a*c*d+2*C*a*c^2)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
-2/d^2*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/c/((a
*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b
*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x
+c/d))-2*C*b^(1/2)*c^2/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x^3\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**3/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)x^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^3\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.55

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx$$
$$= \frac{4\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)a\,d^2x^2-4\sqrt{a\,d^2+b\,c^2}\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\right.}{}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x)
```

output
```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*d**2*x**2 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c*d*x**2 + 4*sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*c**3*x**2 - 4
*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*d**2*x**2 + 4*sqrt(a*d**2 + b*c**2)*
log(c + d*x)*b*c*d*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*c**3*x**2 -
 2*sqrt(a + b*x**2)*a*c**2*d + 4*sqrt(a + b*x**2)*a*c*d**2*x - 4*sqrt(a +
b*x**2)*b*c**2*d*x + 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a*d**3*x**2
 + sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*b*c**2*d*x**2 - 2*sqrt(a)*log(s
qrt(a + b*x**2) - sqrt(a))*b*c*d**2*x**2 + 2*sqrt(a)*log(sqrt(a + b*x**2)
- sqrt(a))*c**3*d*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a*d**3*
x**2 - sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*b*c**2*d*x**2 + 2*sqrt(a)*l
og(sqrt(a + b*x**2) + sqrt(a))*b*c*d**2*x**2 - 2*sqrt(a)*log(sqrt(a + b*x*
*2) + sqrt(a))*c**3*d*x**2 - 2*sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*c
**4*x**2 + 2*sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*c**4*x**2)/(4*c**3*
d*x**2)
```

3.36 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 240

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx &= -\frac{A\sqrt{a+bx^2}}{3cx^3}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{2c^2x^2}-\frac{(3ac(cC-Bd)+A(bc^2+3ad^2))\sqrt{a+bx^2}}{3ac^3x}\\&\quad-\frac{\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4}\\&\quad-\frac{(bc^2(Bc-Ad)-2ad(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c^4}\end{aligned}$$

output

```
-1/3*A*(b*x^2+a)^(1/2)/c/x^3-1/2*(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x^2-1/3*(3
*a*c*(-B*d+C*c)+A*(3*a*d^2+b*c^2))*(b*x^2+a)^(1/2)/a/c^3/x-(a*d^2+b*c^2)^(
1/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a
)^(1/2))/c^4-1/2*(b*c^2*(-A*d+B*c)-2*a*d*(A*d^2-B*c*d+C*c^2))*arctanh((b*x
^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^4
```

Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx$$
$$= \frac{-\frac{c\sqrt{a+bx^2}(2Abc^2x^2+aA(2c^2-3cdx+6d^2x^2)+3acx(2cCx+B(c-2dx)))}{ax^3}+12\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(\ldots}{}\right.}{6c^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^4*(c + d*x)),x]
```

output
```
(-((c*Sqrt[a + b*x^2]*(2*A*b*c^2*x^2 + a*A*(2*c^2 - 3*c*d*x + 6*d^2*x^2) +
 3*a*c*x*(2*c*C*x + B*(c - 2*d*x))))/(a*x^3)) + 12*Sqrt[-(b*c^2) - a*d^2]*
(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqr
t[-(b*c^2) - a*d^2]] - 12*Sqrt[a]*A*d^3*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^
2])/Sqrt[a]] + (6*c*(b*c*(-(B*c) + A*d) + 2*a*d*(c*C - B*d))*ArcTanh[(-(Sq
rt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a])/(6*c^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2x^3}+\frac{d^2\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^4(c+dx)}-\frac{d\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^4x}+\frac{\sqrt{a+bx^2}(Ad^2\ldots}{c^3\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{2\sqrt{a}c^2}-\frac{\sqrt{ad^2+bc^2}(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4}+\frac{\sqrt{a}d\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{c^4}-\frac{\sqrt{a+bx^2}(Bc-Ad)}{2c^2x^2}-\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3x}-\frac{A(a+bx^2)^{3/2}}{3acx^3}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^4*(c + d*x)),x]
```

output
```
-1/2*((B*c - A*d)*Sqrt[a + b*x^2])/(c^2*x^2) - ((c^2*C - B*c*d + A*d^2)*Sq
rt[a + b*x^2])/(c^3*x) - (A*(a + b*x^2)^(3/2))/(3*a*c*x^3) - (Sqrt[b*c^2 +
 a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]
*Sqrt[a + b*x^2])])/c^4 - (b*(B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/(2*Sqrt[a]*c^2) + (Sqrt[a]*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(6Aa\,d^2x^2+2Ab\,c^2x^2-6Bacd\,x^2+6Ca\,c^2x^2-3Aacdx+3Ba\,c^2x+2A\,c^2a)}{6a\,c^3x^3}+\frac{\frac{(2Aa\,d^3+Ab\,c^2d-2Bac\,d^2-Bb\,c^3+2Ca\,c^2d)}{c\sqrt{a}}}{}$
default	$-\frac{A(b\,x^2+a)^{\frac{3}{2}}}{3ca\,x^3}-\frac{(Ad-Bc)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{c^2}+\frac{(A\,d^2-Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b}{}\right.}{c}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(6*A*a*d^2*x^2+2*A*b*c^2*x^2-6*B*a*c*d*x^2+6*C*a*c^2*
x^2-3*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/a/c^3/x^3+1/2/c^3*((2*A*a*d^3+A*b*c
^2*d-2*B*a*c*d^2-B*b*c^3+2*C*a*c^2*d)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a
)^(1/2))/x)-2*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4
)/c/d/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*
((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 1094, normalized size of antiderivative = 4.56

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="fricas")
```

output
```
[1/12*(6*(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(b*c^2 + a*d^2)*x^3*log((2*a*b*
c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a
*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 3*(B*b*c
^3 + 2*B*a*c*d^2 - 2*A*a*d^3 - (2*C*a + A*b)*c^2*d)*sqrt(a)*x^3*log(-(b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*A*a*c^3 - 2*(3*B*a*c^2*d
- 3*A*a*c*d^2 - (3*C*a + A*b)*c^3)*x^2 + 3*(B*a*c^3 - A*a*c^2*d)*x)*sqrt(b
*x^2 + a))/(a*c^4*x^3), -1/12*(12*(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(-b*c^
2 - a*d^2)*x^3*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(
a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + 3*(B*b*c^3 + 2*B*a*c*d^2 -
 2*A*a*d^3 - (2*C*a + A*b)*c^2*d)*sqrt(a)*x^3*log(-(b*x^2 + 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(2*A*a*c^3 - 2*(3*B*a*c^2*d - 3*A*a*c*d^2 - (3
*C*a + A*b)*c^3)*x^2 + 3*(B*a*c^3 - A*a*c^2*d)*x)*sqrt(b*x^2 + a))/(a*c^4*
x^3), 1/6*(3*(B*b*c^3 + 2*B*a*c*d^2 - 2*A*a*d^3 - (2*C*a + A*b)*c^2*d)*sqr
t(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 3*(C*a*c^2 - B*a*c*d + A*a*
d^2)*sqrt(b*c^2 + a*d^2)*x^3*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b
^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a
))/(d^2*x^2 + 2*c*d*x + c^2)) - (2*A*a*c^3 - 2*(3*B*a*c^2*d - 3*A*a*c*d^2
- (3*C*a + A*b)*c^3)*x^2 + 3*(B*a*c^3 - A*a*c^2*d)*x)*sqrt(b*x^2 + a))/(a*
c^4*x^3), -1/6*(6*(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(-b*c^2 - a*d^2)*x^3*a
rctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x^4\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**4/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**4*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(215) = 430$.

Time = 0.18 (sec) , antiderivative size = 571, normalized size of antiderivative = 2.38

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx$$
$$= \frac{2\left(Cbc^4 - Bbc^3d + Cac^2d^2 + Abc^2d^2 - Bacd^3 + Aad^4\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,c^4}$$
$$+ \frac{\left(Bbc^3 - 2\,Cac^2d - Abc^2d + 2\,Bacd^2 - 2\,Aad^3\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,c^4}$$
$$+ \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Bbc^2 - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Abcd + 6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 Ca\sqrt{b}c^2 + 6\left(\sqrt{b}x\right.}{}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="giac")
```

output
```
2*(C*b*c^4 - B*b*c^3*d + C*a*c^2*d^2 + A*b*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*
arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2)
)/(sqrt(-b*c^2 - a*d^2)*c^4) + (B*b*c^3 - 2*C*a*c^2*d - A*b*c^2*d + 2*B*a*
c*d^2 - 2*A*a*d^3)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-
a)*c^4) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b*c^2 - 3*(sqrt(b)*x -
sqrt(b*x^2 + a))^5*A*b*c*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a*sqrt(b)
*c^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2)*c^2 - 6*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*B*a*sqrt(b)*c*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*s
qrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b)*c^2 + 12*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c*d - 12*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a^2*sqrt(b)*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c^2
+ 3*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*c*d + 6*C*a^3*sqrt(b)*c^2 + 2*A*
a^2*b^(3/2)*c^2 - 6*B*a^3*sqrt(b)*c*d + 6*A*a^3*sqrt(b)*d^2)/(((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^3*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^4\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^4\,(dx+c)}dx$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x)
```

output
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x)
```

3.37 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 318

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx &= -\frac{A\sqrt{a+bx^2}}{4cx^4}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{3c^2x^3}-\frac{(4ac(cC-Bd)+A(bc^2+4ad^2))\sqrt{a+bx^2}}{8ac^3x^2}\\
&\quad-\frac{(bc^2(Bc-Ad)-3ad(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3ac^4x}\\
&\quad+\frac{d\sqrt{bc^2+ad^2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5}\\
&\quad-\frac{(4ac(cC-Bd)(bc^2+2ad^2)-A(b^2c^4-4abc^2d^2-8a^2d^4))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}c^5}\end{aligned}$$

output
```
-1/4*A*(b*x^2+a)^(1/2)/c/x^4-1/3*(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x^3-1/8*(4
*a*c*(-B*d+C*c)+A*(4*a*d^2+b*c^2))*(b*x^2+a)^(1/2)/a/c^3/x^2-1/3*(b*c^2*(-
A*d+B*c)-3*a*d*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/a/c^4/x+d*(a*d^2+b*c^2
)^(1/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^
2+a)^(1/2))/c^5-1/8*(4*a*c*(-B*d+C*c)*(2*a*d^2+b*c^2)-A*(-8*a^2*d^4-4*a*b*
c^2*d^2+b^2*c^4))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^5
```

Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx^2}(bc^2x^2(-3Ac-8Bcx+8Adx)-2a(A(3c^3-4c^2dx+6cd^2x^2-12d^3x^3)+2cx(3cCx(c-2dx)+B(2c^2-3cdx+6d^2x^2))))}{ax^4} - 48d\sqrt{-bc^2}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^5*(c + d*x)),x]
```

output
```
((c*Sqrt[a + b*x^2]*(b*c^2*x^2*(-3*A*c - 8*B*c*x + 8*A*d*x) - 2*a*(A*(3*c^
3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3) + 2*c*x*(3*c*C*x*(c - 2*d*x) + B
*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))))/(a*x^4) - 48*d*Sqrt[-(b*c^2) - a*d^2]*(
c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt
[-(b*c^2) - a*d^2]] + 48*Sqrt[a]*A*d^4*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2
])/Sqrt[a]] + (6*c*(A*b*c*(b*c^2 - 4*a*d^2) - 4*a*(c*C - B*d)*(b*c^2 + 2*a
*d^2))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/(24*c^5
)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-Ad)}{c^2x^4} + \frac{d^2\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^5x} - \frac{d^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^5(c+dx)} - \frac{d\sqrt{a+bx^2}(A}{c}\right.$$

$$\downarrow \text{2009}$$

$$\frac{Ab^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}c}+\frac{d\sqrt{ad^2+bc^2}\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^5}-\frac{\sqrt{a}d^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(Ad^2-Bcd+c^2C\right)}{c^5}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(Ad^2-Bcd+c^2C\right)}{2\sqrt{a}c^3}-\frac{\left(a+bx^2\right)^{3/2}(Bc-Ad)}{3ac^2x^3}+\frac{d\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c^4x}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{2c^3x^2}-\frac{Ab\sqrt{a+bx^2}}{8acx^2}-\frac{A\sqrt{a+bx^2}}{4cx^4}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^5*(c + d*x)),x]
```

output
```
-1/4*(A*Sqrt[a + b*x^2])/(c*x^4) - (A*b*Sqrt[a + b*x^2])/(8*a*c*x^2) - ((c
^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(2*c^3*x^2) + (d*(c^2*C - B*c*d + A
*d^2)*Sqrt[a + b*x^2])/(c^4*x) - ((B*c - A*d)*(a + b*x^2)^(3/2))/(3*a*c^2*
x^3) + (d*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x
)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/c^5 + (A*b^2*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(8*a^(3/2)*c) - (b*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/(2*Sqrt[a]*c^3) - (Sqrt[a]*d^2*(c^2*C - B*c*d + A*d^2)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-24 A a\,d^3 x^3-8 A b\,c^2 d\,x^3+24 B a c\,d^2 x^3+8 B b\,c^3 x^3-24 C a\,c^2 d\,x^3+12 A a c\,d^2 x^2+3 A b\,c^3 x^2-12 B a\,c^2 d\,x^2+12 C a\,c^3 x^2-8\right.}{24 a\,c^4 x^4}$
default	$\frac{A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{4 a\,x^4}-\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{2 a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2 a}\right)}{4 a}\right)}{c}+\frac{(A d-B c)\left(b\,x^2+a\right)^{\frac{3}{2}}}{3 c^2 a\,x^3}+\frac{\left(A\,d^2-B c d+C\,c^2\right)\left(\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-24*A*a*d^3*x^3-8*A*b*c^2*d*x^3+24*B*a*c*d^2*x^3+8*
B*b*c^3*x^3-24*C*a*c^2*d*x^3+12*A*a*c*d^2*x^2+3*A*b*c^3*x^2-12*B*a*c^2*d*x
^2+12*C*a*c^3*x^2-8*A*a*c^2*d*x+8*B*a*c^3*x+6*A*a*c^3)/a/c^4/x^4-1/8/c^4/a
*((8*A*a^2*d^4+4*A*a*b*c^2*d^2-A*b^2*c^4-8*B*a^2*c*d^3-4*B*a*b*c^3*d+8*C*a
^2*c^2*d^2+4*C*a*b*c^4)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-8*
a*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/c/((a*d^2+
b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)
/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)
))
```

Fricas [A] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 1514, normalized size of antiderivative = 4.76

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
[1/48*(24*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(b*c^2 + a*d^2)*x^4*
log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqr
t(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2))
 - 3*(4*B*a*b*c^3*d + 8*B*a^2*c*d^3 - 8*A*a^2*d^4 - (4*C*a*b - A*b^2)*c^4
- 4*(2*C*a^2 + A*a*b)*c^2*d^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) - 2*(6*A*a^2*c^4 + 8*(B*a*b*c^4 + 3*B*a^2*c^2*d^2 - 3
*A*a^2*c*d^3 - (3*C*a^2 + A*a*b)*c^3*d)*x^3 - 3*(4*B*a^2*c^3*d - 4*A*a^2*c
^2*d^2 - (4*C*a^2 + A*a*b)*c^4)*x^2 + 8*(B*a^2*c^4 - A*a^2*c^3*d)*x)*sqrt(
b*x^2 + a))/(a^2*c^5*x^4), 1/48*(48*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3
)*sqrt(-b*c^2 - a*d^2)*x^4*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(
b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - 3*(4*B*a*b*c^3
*d + 8*B*a^2*c*d^3 - 8*A*a^2*d^4 - (4*C*a*b - A*b^2)*c^4 - 4*(2*C*a^2 + A*
a*b)*c^2*d^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(6*A*a^2*c^4 + 8*(B*a*b*c^4 + 3*B*a^2*c^2*d^2 - 3*A*a^2*c*d^3 - (3
*C*a^2 + A*a*b)*c^3*d)*x^3 - 3*(4*B*a^2*c^3*d - 4*A*a^2*c^2*d^2 - (4*C*a^2
 + A*a*b)*c^4)*x^2 + 8*(B*a^2*c^4 - A*a^2*c^3*d)*x)*sqrt(b*x^2 + a))/(a^2*
c^5*x^4), -1/24*(3*(4*B*a*b*c^3*d + 8*B*a^2*c*d^3 - 8*A*a^2*d^4 - (4*C*a*b
 - A*b^2)*c^4 - 4*(2*C*a^2 + A*a*b)*c^2*d^2)*sqrt(-a)*x^4*arctan(sqrt(b*x^
2 + a)*sqrt(-a)/a) - 12*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(b*c^2
 + a*d^2)*x^4*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x^5\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**5/(d*x+c),x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)x^5}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1249 vs. $2(289) = 578$.

Time = 0.24 (sec) , antiderivative size = 1249, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="giac")
```

output
```
-2*(C*b*c^4*d - B*b*c^3*d^2 + C*a*c^2*d^3 + A*b*c^2*d^3 - B*a*c*d^4 + A*a*
d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a
*d^2))/(sqrt(-b*c^2 - a*d^2)*c^5) + 1/4*(4*C*a*b*c^4 - A*b^2*c^4 - 4*B*a*b
*c^3*d + 8*C*a^2*c^2*d^2 + 4*A*a*b*c^2*d^2 - 8*B*a^2*c*d^3 + 8*A*a^2*d^4)*
arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^5) + 1/12*(1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^3 + 3*(sqrt(b)*x - sqrt(b*x^2 +
a))^7*A*b^2*c^3 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b*c^2*d + 12*(sqr
t(b)*x - sqrt(b*x^2 + a))^7*A*a*b*c*d^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*B*a*b^(3/2)*c^3 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*sqrt(b)*c^2*
d - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2)*c^2*d + 24*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*B*a^2*sqrt(b)*c*d^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*A*a^2*sqrt(b)*d^3 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c^3 + 21
*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c^3 + 12*(sqrt(b)*x - sqrt(b*x^2
+ a))^5*B*a^2*b*c^2*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^2*b*c*d^2 -
 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c^3 + 72*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*C*a^3*sqrt(b)*c^2*d + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
A*a^2*b^(3/2)*c^2*d - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b)*c*d
^2 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*sqrt(b)*d^3 - 12*(sqrt(b)*x
- sqrt(b*x^2 + a))^3*C*a^3*b*c^3 + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^
2*b^2*c^3 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b*c^2*d - 12*(sqrt...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{x^5\,\left(c+d\,x\right)}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 853, normalized size of antiderivative = 2.68

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*d**3*x**4 - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**2*x**4 + 48*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
c**3*d*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**3*x**4 + 48*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**2*x**4 - 48*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a*c**3*d*x**4 - 12*sqrt(a + b*x**2)*a**2*c**4 + 16*sqrt(a +
 b*x**2)*a**2*c**3*d*x - 24*sqrt(a + b*x**2)*a**2*c**2*d**2*x**2 + 48*sqrt
(a + b*x**2)*a**2*c*d**3*x**3 - 6*sqrt(a + b*x**2)*a*b*c**4*x**2 - 16*sqrt
(a + b*x**2)*a*b*c**4*x + 16*sqrt(a + b*x**2)*a*b*c**3*d*x**3 + 24*sqrt(a
+ b*x**2)*a*b*c**3*d*x**2 - 48*sqrt(a + b*x**2)*a*b*c**2*d**2*x**3 - 24*sq
rt(a + b*x**2)*a*c**5*x**2 + 48*sqrt(a + b*x**2)*a*c**4*d*x**3 - 16*sqrt(a
 + b*x**2)*b**2*c**4*x**3 + 24*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**
2*d**4*x**4 + 12*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a*b*c**2*d**2*x**
4 - 24*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a*b*c*d**3*x**4 + 24*sqrt(a
)*log(sqrt(a + b*x**2) - sqrt(a))*a*c**3*d**2*x**4 - 3*sqrt(a)*log(sqrt(a
+ b*x**2) - sqrt(a))*b**2*c**4*x**4 - 12*sqrt(a)*log(sqrt(a + b*x**2) - sq
rt(a))*b**2*c**3*d*x**4 + 12*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*b*c**
5*x**4 - 24*sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a**2*d**4*x**4 - 12*sq
rt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a*b*c**2*d**2*x**4 + 24*sqrt(a)*l...
```

3.38 $\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 492

$$\begin{aligned}
&\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx \\
&= -\frac{(2a^2Cd^4-5abd^2(3c^2C-2Bcd+Ad^2)-15b^2c^2(5c^2C-4Bcd+3Ad^2))\sqrt{a+bx^2}}{15b^2d^6} \\
&\quad -\frac{(ad^2(2cC-Bd)+4bc(4c^2C-3Bcd+2Ad^2))\,x\sqrt{a+bx^2}}{8bd^5} \\
&\quad +\frac{(aCd^2+5b(3c^2C-2Bcd+Ad^2))\,x^2\sqrt{a+bx^2}}{15bd^4} \\
&\quad -\frac{(2cC-Bd)x^3\sqrt{a+bx^2}}{4d^3}+\frac{Cx^4\sqrt{a+bx^2}}{5d^2}+\frac{c^3(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^6(c+dx)} \\
&\quad +\frac{(a^2d^4(2cC-Bd)-4abcd^2(4c^2C-3Bcd+2Ad^2)-8b^2c^3(6c^2C-5Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}d^7} \\
&\quad -\frac{c^2(ad^2(5c^2C-4Bcd+3Ad^2)+bc^2(6c^2C-5Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7\sqrt{bc^2+ad^2}}
\end{aligned}$$

output
```
-1/15*(2*a^2*C*d^4-5*a*b*d^2*(A*d^2-2*B*c*d+3*C*c^2)-15*b^2*c^2*(3*A*d^2-4
*B*c*d+5*C*c^2))*(b*x^2+a)^(1/2)/b^2/d^6-1/8*(a*d^2*(-B*d+2*C*c)+4*b*c*(2*
A*d^2-3*B*c*d+4*C*c^2))*x*(b*x^2+a)^(1/2)/b/d^5+1/15*(a*C*d^2+5*b*(A*d^2-2
*B*c*d+3*C*c^2))*x^2*(b*x^2+a)^(1/2)/b/d^4-1/4*(-B*d+2*C*c)*x^3*(b*x^2+a)^
(1/2)/d^3+1/5*C*x^4*(b*x^2+a)^(1/2)/d^2+c^3*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^
(1/2)/d^6/(d*x+c)+1/8*(a^2*d^4*(-B*d+2*C*c)-4*a*b*c*d^2*(2*A*d^2-3*B*c*d+4
*C*c^2)-8*b^2*c^3*(4*A*d^2-5*B*c*d+6*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(
1/2))/b^(3/2)/d^7-c^2*(a*d^2*(3*A*d^2-4*B*c*d+5*C*c^2)+b*c^2*(4*A*d^2-5*B*
c*d+6*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^
7/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 3.34 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.86

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}\left(-16a^2Cd^4(c+dx)+abd^2(c+dx)\left(120c^2C-10cd(8B+3Cx)+d^2\left(40A+15Bx+8Cx^2\right)\right)+2b^2\left(360c^5C-60c^4d(5B-3Cx)+30c^3d^2(8A-x\right.\right.}{b^2(c+dx)}}{}$$

input
```
Integrate[(x^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-16*a^2*C*d^4*(c + d*x) + a*b*d^2*(c + d*x)*(120*c^2*
C - 10*c*d*(8*B + 3*C*x) + d^2*(40*A + 15*B*x + 8*C*x^2)) + 2*b^2*(360*c^5
*C - 60*c^4*d*(5*B - 3*C*x) + 30*c^3*d^2*(8*A - x*(5*B + 2*C*x)) + 10*c^2*
d^3*x*(12*A + x*(5*B + 3*C*x)) + d^5*x^3*(20*A + 3*x*(5*B + 4*C*x)) - c*d^
4*x^2*(40*A + x*(25*B + 18*C*x)))))/(b^2*(c + d*x)) - (240*c^2*(a*d^2*(5*c
^2*C - 4*B*c*d + 3*A*d^2) + b*c^2*(6*c^2*C - 5*B*c*d + 4*A*d^2))*ArcTan[(S
qrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^
2) - a*d^2] + (15*(a^2*d^4*(-2*c*C + B*d) + 4*a*b*c*d^2*(4*c^2*C - 3*B*c*d
 + 2*A*d^2) + 8*b^2*c^3*(6*c^2*C - 5*B*c*d + 4*A*d^2))*Log[-(Sqrt[b]*x) +
Sqrt[a + b*x^2]])/b^(3/2))/(120*d^7)
```

Rubi [A] (verified)

Time = 3.50 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {2182, 25, 2185, 25, 2185, 25, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}-\frac{\int -\frac{\sqrt{bx^2+a}\left(C\left(\frac{bc^2}{d}+ad\right)x^4-\frac{(cC-Bd)\left(bc^2+ad^2\right)x^3}{d^2}+\frac{\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}-\frac{c\left(3bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}\right)}{c+dx}d}{ad^2+bc^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{bx^2+a}\left(C\left(\frac{bc^2}{d}+ad\right)x^4-\frac{(cC-Bd)\left(bc^2+ad^2\right)x^3}{d^2}+\frac{\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}-\frac{c\left(3bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}\right)}{c+dx}dx}{ad^2+bc^2}$$
$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int -\frac{\sqrt{bx^2+a}(bd^2(18cC-5Bd)(bc^2+ad^2)x^3+d(bc^2+ad^2)(2aCd^2+b(6Cc^2+5Bdc-5Ad^2))x^2+c(4a^2Cd^4+ab(12Cc^2-5Bdc+5Ad^2)d^2+3b^2c^2(6Cc^2-5Bdc+5}{c+dx}}{5bd^4}}{ad^2+bc^2}$$
$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\int \frac{\sqrt{bx^2+a}(bd^2(18cC-5Bd)(bc^2+ad^2)x^3+d(bc^2+ad^2)(2aCd^2+b(6Cc^2+5Bdc-5Ad^2))x^2+c(4a^2Cd^4+ab(12Cc^2-}{c+dx}}{5bd^4}}{ad^2+bc^2}$$
$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{\int -\frac{\sqrt{bx^2+a}\left(-b\left(bc^2+ad^2\right)\left(8aCd^2-b\left(102Cc^2-55Bdc+20Ad^2\right)\right)x^2d^4+5abc\left(a(2cC-Bd)d^2+bc\left(6Cc^2-5Bdc+4Ad^2\right)\right.}{c+dx}}{4bd^3}}{\cdots}}{ad^2+bc^2}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{\int \frac{\sqrt{bx^2+a}\left(-b\left(bc^2+ad^2\right)\left(8aCd^2-b\left(102Cc^2-55Bdc+20Ad^2\right)\right)x^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{ad^2+bc^2}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 2185$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{\int \frac{15b^2d^5\left(acd\left(a(2cC-Bd)d^2+bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)+\left(a^2(2cC\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{ad^2+b\cdots}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\int \frac{\left(acd\left(a(2cC-Bd)d^2+bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)+\left(a^2(2cC-\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{ad^2+b\cdots}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 682$$

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\int \frac{b\left(bc^2+ad^2\right)\left(acd\left(a(2cC-Bd)d^2+4bc\left(6Cc^2-5Bdc+4Ad^2\cdots\right.\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\frac{(ad^2+bc^2)\int\frac{acd\left(a(2cC-Bd)d^2+4bc\left(6Cc^2-5Bdc+4Ad^2\right)\right.}{}}{}\right.}{}}{}}{}$$

$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 719

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\frac{(ad^2+bc^2)\left(\frac{\left(a^2d^4(2cC-Bd)-4abcd^2\left(2Ad^2-3Bcd+4c^2C\right.\right.}{}\right.}{}\right.}{}}{}}{}$$

$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 224

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\frac{(ad^2+bc^2)\left(\frac{\left(a^2d^4(2cC-Bd)-4abcd^2\left(2Ad^2-3Bcd+4c^2C\right.\right.}{}\right.}{}\right.}{}}{}}{}$$

$$\frac{c^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 219

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\left(ad^2+bc^2\right)\left(\frac{8bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^2\left(4Ad^2-5\right.\right.}{d}\right.\right.}{}}{}}{}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 488

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\left(ad^2+bc^2\right)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^4(2cC-Bd)-4ab\right.}{}\right.\right.}{}}{}}{}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 219

$$\frac{\frac{C(a+bx^2)^{3/2}(c+dx)^2(ad^2+bc^2)}{5bd^4}-\frac{\frac{1}{4}(a+bx^2)^{3/2}(c+dx)(ad^2+bc^2)(18cC-5Bd)-\frac{5bd^3\left(\left(ad^2+bc^2\right)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2d^4(2cC-Bd)-4ab\right.}{}\right.\right.}{}}{}}{}$$

$$\frac{c^3\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(x^3*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
(c^3*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(d^4*(b*c^2 + a*d^2)*(c +
d*x)) + ((C*(b*c^2 + a*d^2)*(c + d*x)^2*(a + b*x^2)^(3/2))/(5*b*d^4) - (((
18*c*C - 5*B*d)*(b*c^2 + a*d^2)*(c + d*x)*(a + b*x^2)^(3/2))/4 - (-1/3*(d^
3*(b*c^2 + a*d^2)*(8*a*C*d^2 - b*(102*c^2*C - 55*B*c*d + 20*A*d^2))*(a + b
*x^2)^(3/2)) + 5*b*d^3*(((8*b*c^2*(a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) + b
*c^2*(6*c^2*C - 5*B*c*d + 4*A*d^2)) + d*(a^2*d^4*(2*c*C - B*d) - 4*b^2*c^3
*(6*c^2*C - 5*B*c*d + 4*A*d^2) - a*b*c*d^2*(14*c^2*C - 11*B*c*d + 8*A*d^2)
)*x)*Sqrt[a + b*x^2])/(2*d^2) + ((b*c^2 + a*d^2)*(((a^2*d^4*(2*c*C - B*d)
- 4*a*b*c*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) - 8*b^2*c^3*(6*c^2*C - 5*B*c*d
 + 4*A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (8*b*c^2*
(a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) + b*c^2*(6*c^2*C - 5*B*c*d + 4*A*d^2)
)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*
c^2 + a*d^2])))/(2*d^2)))/(4*b*d^3))/(5*b*d^4))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 790, normalized size of antiderivative = 1.61

method	result
risch	$\frac{(24C\,d^4b^2x^4+30B\,b^2d^4x^3-60C\,b^2c\,d^3x^3+40A\,b^2d^4x^2-80B\,b^2c\,d^3x^2+8Cab\,d^4x^2+120C\,b^2c^2d^2x^2-120A\,b^2c\,d^3x+15Bab\,d^4x+18}{}$
default	Expression too large to display

input
```
int(x^3*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*C*b^2*d^4*x^4+30*B*b^2*d^4*x^3-60*C*b^2*c*d^3*x^3+40*A*b^2*d^4*x
^2-80*B*b^2*c*d^3*x^2+8*C*a*b*d^4*x^2+120*C*b^2*c^2*d^2*x^2-120*A*b^2*c*d^
3*x+15*B*a*b*d^4*x+180*B*b^2*c^2*d^2*x-30*C*a*b*c*d^3*x-240*C*b^2*c^3*d*x+
40*A*a*b*d^4+360*A*b^2*c^2*d^2-80*B*a*b*c*d^3-480*B*b^2*c^3*d-16*C*a^2*d^4
+120*C*a*b*c^2*d^2+600*C*b^2*c^4)*(b*x^2+a)^(1/2)/b^2/d^6-1/8/b/d^6*((8*A*
a*b*c*d^4+32*A*b^2*c^3*d^2+B*a^2*d^5-12*B*a*b*c^2*d^3-40*B*b^2*c^4*d-2*C*a
^2*c*d^4+16*C*a*b*c^3*d^2+48*C*b^2*c^5)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^
(1/2)+8*b*c^2/d^2*(3*A*a*d^4+5*A*b*c^2*d^2-4*B*a*c*d^3-6*B*b*c^3*d+5*C*a*c
^2*d^2+7*C*b*c^4)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/
d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2))/(x+c/d))+8*b*c^3*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3
*d+C*a*c^2*d^2+C*b*c^4)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^3\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

input
```
integrate(x**3*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(x**3*sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 882, normalized size of antiderivative = 1.79

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
sqrt(b*x^2 + a)*C*c^5/(d^7*x + c*d^6) - sqrt(b*x^2 + a)*B*c^4/(d^6*x + c*d
^5) + sqrt(b*x^2 + a)*A*c^3/(d^5*x + c*d^4) + 1/5*(b*x^2 + a)^(3/2)*C*x^2/
(b*d^2) - 2*sqrt(b*x^2 + a)*C*c^3*x/d^5 + 3/2*sqrt(b*x^2 + a)*B*c^2*x/d^4
- sqrt(b*x^2 + a)*A*c*x/d^3 - 1/2*(b*x^2 + a)^(3/2)*C*c*x/(b*d^3) + 1/4*sq
rt(b*x^2 + a)*C*a*c*x/(b*d^3) + 1/4*(b*x^2 + a)^(3/2)*B*x/(b*d^2) - 1/8*sq
rt(b*x^2 + a)*B*a*x/(b*d^2) - 6*C*sqrt(b)*c^5*arcsinh(b*x/sqrt(a*b))/d^7 +
 5*B*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^6 - 2*C*a*c^3*arcsinh(b*x/sqrt(a
*b))/(sqrt(b)*d^5) - 4*A*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 3/2*B*a*
c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) + 1/4*C*a^2*c*arcsinh(b*x/sqrt(a*
b))/(b^(3/2)*d^3) - A*a*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 1/8*B*a^2
*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) + C*b*c^6*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^8) - B
*b*c^5*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c
)))/(sqrt(a + b*c^2/d^2)*d^7) + 5*C*sqrt(a + b*c^2/d^2)*c^4*arcsinh(b*c*x/
(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^6 + A*b*c^4*arc
sinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(
a + b*c^2/d^2)*d^6) - 4*B*sqrt(a + b*c^2/d^2)*c^3*arcsinh(b*c*x/(sqrt(a*b)
*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 3*A*sqrt(a + b*c^2/d^
2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c
)))/d^4 + 5*sqrt(b*x^2 + a)*C*c^4/d^6 - 4*sqrt(b*x^2 + a)*B*c^3/d^5 + 3...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^3\,\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{{(c+d\,x)}^2}\,dx$$

input
```
int((x^3*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int((x^3*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 37.47 (sec) , antiderivative size = 3172, normalized size of antiderivative = 6.45

$$\int \frac{x^3\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(x^3*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output

```
(720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a**2*b**2*c**3*d**4 + 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**5*x + 960*sqr
t(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*a*b**3*c**5*d**2 + 960*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**3*x - 960*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4
*d**3 - 960*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a*b**3*c**3*d**4*x + 1200*sqrt(a*d**2 + b*c**2)*log(sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**6*d**2 + 1200*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a*b**2*c**5*d**3*x - 1200*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**6*d - 1200*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d
**2*x + 1440*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*b**3*c**8 + 1440*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**7*d*x - 720*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a**2*b**2*c**3*d**4 - 720*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**2*b**2*c**2*d**5*x - 960*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
*b**3*c**5*d**2 - 960*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**3*c**4*d*...
```

3.39 $\int \frac{x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 396

$$\begin{aligned}
&\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx \\
&= -\frac{(ad^2(2cC-Bd)+3bc(4c^2C-3Bcd+2Ad^2))\sqrt{a+bx^2}}{3bd^5} \\
&\quad +\frac{(aCd^2+4b(3c^2C-2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8bd^4} \\
&\quad -\frac{(2cC-Bd)x^2\sqrt{a+bx^2}}{3d^3}+\frac{Cx^3\sqrt{a+bx^2}}{4d^2}-\frac{c^2(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^5(c+dx)} \\
&\quad -\frac{(a^2Cd^4-4abd^2(3c^2C-2Bcd+Ad^2)-8b^2c^2(5c^2C-4Bcd+3Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}d^6} \\
&\quad +\frac{c(ad^2(4c^2C-3Bcd+2Ad^2)+bc^2(5c^2C-4Bcd+3Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6\sqrt{bc^2+ad^2}}
\end{aligned}$$

output
```
-1/3*(a*d^2*(-B*d+2*C*c)+3*b*c*(2*A*d^2-3*B*c*d+4*C*c^2))*(b*x^2+a)^(1/2)/
b/d^5+1/8*(a*C*d^2+4*b*(A*d^2-2*B*c*d+3*C*c^2))*x*(b*x^2+a)^(1/2)/b/d^4-1/
3*(-B*d+2*C*c)*x^2*(b*x^2+a)^(1/2)/d^3+1/4*C*x^3*(b*x^2+a)^(1/2)/d^2-c^2*(
A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^5/(d*x+c)-1/8*(a^2*C*d^4-4*a*b*d^2*(A
*d^2-2*B*c*d+3*C*c^2)-8*b^2*c^2*(3*A*d^2-4*B*c*d+5*C*c^2))*arctanh(b^(1/2)
*x/(b*x^2+a)^(1/2))/b^(3/2)/d^6+c*(a*d^2*(2*A*d^2-3*B*c*d+4*C*c^2)+b*c^2*(
3*A*d^2-4*B*c*d+5*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+
a)^(1/2))/d^6/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 4.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.90

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}\left(ad^2(c+dx)(-16cC+8Bd+3Cdx)-2b\left(60c^4C-6c^3d(8B-5Cx)+2c^2d^2\left(18A-12Bx-5Cx^2\right)-d^4x^2\left(6A+4Bx+3Cx^2\right)+cd^3x\left(18A+8B\right.\right.\right.}{b(c+dx)}}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(c + d*x)*(-16*c*C + 8*B*d + 3*C*d*x) - 2*b*(60
*c^4*C - 6*c^3*d*(8*B - 5*C*x) + 2*c^2*d^2*(18*A - 12*B*x - 5*C*x^2) - d^4
*x^2*(6*A + 4*B*x + 3*C*x^2) + c*d^3*x*(18*A + 8*B*x + 5*C*x^2))))/(b*(c +
 d*x)) + (48*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(5*c^2*C - 4*B
*c*d + 3*A*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*
Sqrt[a + b*x^2])])/Sqrt[-(b*c^2) - a*d^2] + (6*(-(a^2*C*d^4) + 4*a*b*d^2*(
3*c^2*C - 2*B*c*d + A*d^2) + 8*b^2*c^2*(5*c^2*C - 4*B*c*d + 3*A*d^2))*ArcT
anh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2))/(24*d^6)
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.27, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2182, 2185, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx^2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2182}$$

$$-\frac{\int \frac{\sqrt{bx^2+a}\left(-C\left(\frac{bc^2}{d}+ad\right)x^3+\frac{(cC-Bd)\left(bc^2+ad^2\right)x^2}{d^2}-\frac{\left(3bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}\right)}{c+dx}dx}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\frac{\int \frac{\sqrt{bx^2+a}\left(bd(11cC-4Bd)\left(bc^2+ad^2\right)x^2+\left(3bc^2+ad^2\right)\left(aCd^2-b\left(3Cc^2-4Bdc+4Ad^2\right)\right)x+acd\left(aCd^2+b\left(5Cc^2-4Bdc+4Ad^2\right)\right)\right)}{c+dx}dx}{4bd^3}-\frac{C\left(a+bx^2\right)^{3/2}(c+d}{4bd^3}}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\frac{\frac{\int \frac{3bd^2\left(acd\left(aCd^2+b\left(5Cc^2-4Bdc+4Ad^2\right)\right)+\left(a^2Cd^4-ab\left(11Cc^2-8Bdc+4Ad^2\right)d^2-4b^2c^2\left(5Cc^2-4Bdc+3Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}+\frac{1}{3}\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)}{4bd^3}}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{\left(acd\left(aCd^2+b\left(5Cc^2-4Bdc+4Ad^2\right)\right)+\left(a^2Cd^4-ab\left(11Cc^2-8Bdc+4Ad^2\right)d^2-4b^2c^2\left(5Cc^2-4Bdc+3Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx+\frac{1}{3}\left(a+bx^2\right)^{3/2}\left(ad^2+bc^2\right)(11c}{4bd^3}}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 682$$

$$-\frac{\frac{\int \frac{b\left(bc^2+ad^2\right)\left(acd\left(20bCc^2-16bBdc+12Abd^2+aCd^2\right)+\left(a^2Cd^4-4ab\left(3Cc^2-2Bdc+Ad^2\right)d^2-8b^2c^2\left(5Cc^2-4Bdc+3Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(dx\left(a^2Cd^4-abd\cdots\right.\right.}{\cdots}}{4\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{acd\left(20bCc^2-16bBdc+12Abd^2+aCd^2\right)+\left(a^2Cd^4-4ab\left(3Cc^2-2Bdc+Ad^2\right)d^2-8b^2c^2\left(5Cc^2-4Bdc+3Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(dx\left(a^2Cd^4-abd^2\left(\cdots\right.\right.\right.}{\cdots}}{4b\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$-\frac{\frac{\left(ad^2+bc^2\right)\left(\frac{\left(a^2Cd^4-4abd^2\left(Ad^2-2Bcd+3c^2C\right)-8b^2c^2\left(3Ad^2-4Bcd+5c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{8bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(3Ad^2-4Bcd+5c^2C\right)\right)\int\cdots}{d}\cdots\right.}{2d^2}\cdots}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\left(ad^2+bc^2\right)\left(\frac{\left(a^2Cd^4-4abd^2\left(Ad^2-2Bcd+3c^2C\right)-8b^2c^2\left(3Ad^2-4Bcd+5c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}+\frac{8bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(3Ad^2-4Bcd+\cdots\right.\right.}{d}\cdots\right.}{2d^2}\cdots}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{(ad^2+bc^2)\left(\frac{8bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(3Ad^2-4Bcd+5c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2Cd^4-4abd^2\left(Ad^2-2Bcd+3c^2C\right)-\right.}{\sqrt{b}d}\right.}{2d^2}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 488

$$-\frac{\frac{(ad^2+bc^2)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2Cd^4-4abd^2\left(Ad^2-2Bcd+3c^2C\right)-8b^2c^2\left(3Ad^2-4Bcd+5c^2C\right)\right)}{\sqrt{b}d}-\frac{8bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(3Ad^2-4Bcd+\right.\right.}{d}\right.}{2d^2}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 219

$$-\frac{\frac{(ad^2+bc^2)\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^2Cd^4-4abd^2\left(Ad^2-2Bcd+3c^2C\right)-8b^2c^2\left(3Ad^2-4Bcd+5c^2C\right)\right)}{\sqrt{b}d}-\frac{8bc\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(2Ad^2-\right.\right.}{d\sqrt{ad^2+b}}\right.}{2d^2}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(x^2*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
-((c^2*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(d^3*(b*c^2 + a*d^2)*(c
+ d*x))) - (-1/4*(C*(b*c^2 + a*d^2)*(c + d*x)*(a + b*x^2)^(3/2))/(b*d^3) +
 (((8*b*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(5*c^2*C - 4*B*c*d
+ 3*A*d^2)) + d*(a^2*C*d^4 - 4*b^2*c^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) - a*b
*d^2*(11*c^2*C - 8*B*c*d + 4*A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) + ((11*c*
C - 4*B*d)*(b*c^2 + a*d^2)*(a + b*x^2)^(3/2))/3 + ((b*c^2 + a*d^2)*(((a^2*
C*d^4 - 4*a*b*d^2*(3*c^2*C - 2*B*c*d + A*d^2) - 8*b^2*c^2*(5*c^2*C - 4*B*c
*d + 3*A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (8*b*c*
(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(5*c^2*C - 4*B*c*d + 3*A*d^2)
)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*
c^2 + a*d^2])))/(2*d^2))/(4*b*d^3))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{(-6C\,d^3b\,x^3-8Bb\,d^3x^2+16Cbc\,d^2x^2-12Ab\,d^3x+24Bbc\,d^2x-3Ca\,d^3x-36Cb\,c^2dx+48Abc\,d^2-8Ba\,d^3-72Bb\,c^2d+16Cac\,d^2+9}{24b\,d^5}$
default	Expression too large to display

input
```
int(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24/b*(-6*C*b*d^3*x^3-8*B*b*d^3*x^2+16*C*b*c*d^2*x^2-12*A*b*d^3*x+24*B*b
*c*d^2*x-3*C*a*d^3*x-36*C*b*c^2*d*x+48*A*b*c*d^2-8*B*a*d^3-72*B*b*c^2*d+16
*C*a*c*d^2+96*C*b*c^3)*(b*x^2+a)^(1/2)/d^5+1/8/b/d^5*((4*A*a*b*d^4+24*A*b^
2*c^2*d^2-8*B*a*b*c*d^3-32*B*b^2*c^3*d-C*a^2*d^4+12*C*a*b*c^2*d^2+40*C*b^2
*c^4)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8*b*c/d^2*(2*A*a*d^4+4*A*b*c
^2*d^2-3*B*a*c*d^3-5*B*b*c^3*d+4*C*a*c^2*d^2+6*C*b*c^4)/((a*d^2+b*c^2)/d^2
)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+8*b*c^2*
(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/d^3*(-1/(a*d
^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*
b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*
d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^2\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(x**2*sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 751 vs. $2(365) = 730$.

Time $= 0.15$ (sec) , antiderivative size $= 751$, normalized size of antiderivative $= 1.90$

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
-sqrt(b*x^2 + a)*C*c^4/(d^6*x + c*d^5) + sqrt(b*x^2 + a)*B*c^3/(d^5*x + c*
d^4) - sqrt(b*x^2 + a)*A*c^2/(d^4*x + c*d^3) + 3/2*sqrt(b*x^2 + a)*C*c^2*x
/d^4 - sqrt(b*x^2 + a)*B*c*x/d^3 + 1/2*sqrt(b*x^2 + a)*A*x/d^2 + 1/4*(b*x^
2 + a)^(3/2)*C*x/(b*d^2) - 1/8*sqrt(b*x^2 + a)*C*a*x/(b*d^2) + 5*C*sqrt(b)
*c^4*arcsinh(b*x/sqrt(a*b))/d^6 - 4*B*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d
^5 + 3/2*C*a*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) + 3*A*sqrt(b)*c^2*ar
csinh(b*x/sqrt(a*b))/d^4 - B*a*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 1/
8*C*a^2*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) + 1/2*A*a*arcsinh(b*x/sqrt(a*
b))/(sqrt(b)*d^2) - C*b*c^5*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(
sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^7) + B*b*c^4*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/
d^2)*d^6) - 4*C*sqrt(a + b*c^2/d^2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
 c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 - A*b*c^3*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5)
+ 3*B*sqrt(a + b*c^2/d^2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d
/(sqrt(a*b)*abs(d*x + c)))/d^4 - 2*A*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(
sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - 4*sqrt(b*x^2
 + a)*C*c^3/d^5 + 3*sqrt(b*x^2 + a)*B*c^2/d^4 - 2*sqrt(b*x^2 + a)*A*c/d^3
- 2/3*(b*x^2 + a)^(3/2)*C*c/(b*d^3) + 1/3*(b*x^2 + a)^(3/2)*B/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^2\,\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{(c+d\,x)^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int((x^2*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2801, normalized size of antiderivative = 7.07

$$\int \frac{x^2\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output

```
(96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*b**2*c**2*d**4 + 96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x + 144*sq
rt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a*b**3*c**4*d**2 + 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d**3*x - 144*sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
*b**3*c**3*d**3 - 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x + 192*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*
*5*d**2 + 192*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**2*c**4*d**3*x - 192*sqrt(a*d**2 + b*c**2)*log(
 - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d - 192
*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*b**4*c**4*d**2*x + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**7 + 240*sqrt(a*d**2 + b*
c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c*
*6*d*x - 96*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**2*d**4 - 96*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c*d**5*x - 144*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a*b**3*c**4*d**2 - 144*sqrt(a*d**2 + b*c**2)*log(c +...
```

3.40 $\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 309

$$\begin{aligned}\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx &= \frac{(aCd^2+3b(3c^2C-2Bcd+Ad^2))\sqrt{a+bx^2}}{3bd^4}-\frac{(2cC-Bd)x\sqrt{a+bx^2}}{2d^3}\\ &\quad+\frac{Cx^2\sqrt{a+bx^2}}{3d^2}+\frac{c(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^4(c+dx)}\\ &\quad-\frac{(ad^2(2cC-Bd)+2bc(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^5}\\ &\quad-\frac{(ad^2(3c^2C-2Bcd+Ad^2)+bc^2(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
1/3*(a*C*d^2+3*b*(A*d^2-2*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2)/b/d^4-1/2*(-B*d+
2*C*c)*x*(b*x^2+a)^(1/2)/d^3+1/3*C*x^2*(b*x^2+a)^(1/2)/d^2+c*(A*d^2-B*c*d+
C*c^2)*(b*x^2+a)^(1/2)/d^4/(d*x+c)-1/2*(a*d^2*(-B*d+2*C*c)+2*b*c*(2*A*d^2-
3*B*c*d+4*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^5-(a*d^2*(A
*d^2-2*B*c*d+3*C*c^2)+b*c^2*(2*A*d^2-3*B*c*d+4*C*c^2))*arctanh((-b*c*x+a*d
)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(2aCd^2(c+dx)+b\left(24c^3C-6c^2d(3B-2Cx)+cd^2\left(12A-9Bx-4Cx^2\right)+d^3x\left(6A+3Bx+2Cx^2\right)\right)\right)}{b(c+dx)} - \frac{12\left(ad^2\left(3c^2C-2Bcd+Ad^2\right)+bc^2\left(4\right.\right.}{}}{6d^5}$$

input
```
Integrate[(x*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(2*a*C*d^2*(c + d*x) + b*(24*c^3*C - 6*c^2*d*(3*B - 2*
C*x) + c*d^2*(12*A - 9*B*x - 4*C*x^2) + d^3*x*(6*A + 3*B*x + 2*C*x^2))))/(
b*(c + d*x)) - (12*(a*d^2*(3*c^2*C - 2*B*c*d + A*d^2) + b*c^2*(4*c^2*C - 3
*B*c*d + 2*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b
*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + (3*(a*d^2*(2*c*C - B*d) + 2*b*c*
(4*c^2*C - 3*B*c*d + 2*A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b
])/(6*d^5)
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2182, 2185, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{2182}$$

$$\frac{\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\int\frac{\sqrt{bx^2+a}\left(-C\left(\frac{bc^2}{d}+ad\right)x^2+\left(2Abc+\frac{(cC-Bd)\left(3bc^2+ad^2\right)}{d^2}\right)x+a\left(-\frac{Cc^2}{d}+Bc-Ad\right)\right)}{c+dx}dx}{ad^2+bc^2}$$

$\big\downarrow$ 2185

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\frac{\int-\frac{3b\left(ad\left(Cc^2-Bdc+Ad^2\right)-\left(a(2cC-Bd)d^2+bc\left(4Cc^2-3Bdc+2Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{3bd^2}-\frac{1}{3}C\left(a+bx^2\right)^{3/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$\big\downarrow$ 27

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\int\frac{\left(ad\left(Cc^2-Bdc+Ad^2\right)-\left(a(2cC-Bd)d^2+bc\left(4Cc^2-3Bdc+2Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{d^2}-\frac{1}{3}C\left(a+bx^2\right)^{3/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$\big\downarrow$ 682

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\frac{\int\frac{b\left(bc^2+ad^2\right)\left(ad\left(4Cc^2-3Bdc+2Ad^2\right)-\left(a(2cC-Bd)d^2+2bc\left(4Cc^2-3Bdc+2Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\cdots\right.\right.}{2d^2}}{d^2}}{ad^2+bc^2}$$

$\big\downarrow$ 27

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\frac{\left(ad^2+bc^2\right)\int\frac{ad\left(4Cc^2-3Bdc+2Ad^2\right)-\left(a(2cC-Bd)d^2+2bc\left(4Cc^2-3Bdc+2Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)-\cdots\right.}{2d^2}}{d^2}}{ad^2+bc^2}$$

$\big\downarrow$ 719

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2(2cC-Bd)+2bc\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{2d^2}+\frac{\sqrt{\cdots}}{d^2}}{ad^2+b\cdots}$$

$\downarrow$ 224

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2(2cC-Bd)+2bc\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+\cdots}}}{d}\cdots\right)}{2d^2}\cdots}{ad^2\cdots}$$

$\downarrow$ 219

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(2cC-Bd)+2bc\left(2Ad^2-3Bcd+4c^2C\right)\cdots\right)}{\sqrt{b}d}\cdots\right)}{2d^2}\cdots}{ad^2\cdots}$$

$\downarrow$ 488

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(-\frac{2\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(2cC-Bd)+2bc\left(2A\cdots\right)\right)}{\sqrt{b}d}\cdots\right)}{2d^2}\cdots}{\cdots}$$

$\downarrow$ 219

$$\frac{c\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(2cC-Bd)+2bc\left(2Ad^2-3Bcd+4c^2C\right)\right)}{\sqrt{b}d}-\frac{2\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(2Ad\cdots\right)\right)}{d\sqrt{ad^2+bc^2}}\cdots\right)}{2d^2}\cdots}{\cdots}$$

input
```
Int[(x*Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
(c*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(d^2*(b*c^2 + a*d^2)*(c + d*
x)) - (-1/3*(C*(a/b + c^2/d^2)*(a + b*x^2)^(3/2)) - (((2*(a*d^2*(3*c^2*C -
 2*B*c*d + A*d^2) + b*c^2*(4*c^2*C - 3*B*c*d + 2*A*d^2)) - d*(a*d^2*(2*c*C
 - B*d) + b*c*(4*c^2*C - 3*B*c*d + 2*A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) +
 ((b*c^2 + a*d^2)*(-(((a*d^2*(2*c*C - B*d) + 2*b*c*(4*c^2*C - 3*B*c*d + 2*
A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (2*(a*d^2*(3*
c^2*C - 2*B*c*d + A*d^2) + b*c^2*(4*c^2*C - 3*B*c*d + 2*A*d^2))*ArcTanh[(a
*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]
)))/(2*d^2))/d^2)/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. 2(281) = 562.

Time = 0.25 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.87

method	result
risch	$\frac{(2C d^2 b x^2+3Bb d^2 x-6Cbcdx+6Ab d^2-12Bbcd+2aC d^2+18Cb c^2)\sqrt{b x^2+a}}{6b d^4} - \frac{\frac{\left(4Abc d^2-Ba d^3-6Bb c^2 d+2Cac d^2+8Cb c^3\right)\ln\left(\sqrt{b}x\right.}{d\sqrt{b}}}{}$
default	$\frac{Bd\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)+\frac{Cd\left(b x^2+a\right)^{\frac{3}{2}}}{3b}-2Cc\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{d^3} + \frac{(A d^2-2Bcd+3C c^2)\left(\sqrt{b(x-}\right.}{}$

input

```
int(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x-6*C*b*c*d*x+6*A*b*d^2-12*B*b*c*d+2*C*a*d^2+
18*C*b*c^2)*(b*x^2+a)^(1/2)/b/d^4-1/2/d^4*((4*A*b*c*d^2-B*a*d^3-6*B*b*c^2*
d+2*C*a*c*d^2+8*C*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2/d^2*(A*
a*d^4+3*A*b*c^2*d^2-2*B*a*c*d^3-4*B*b*c^3*d+3*C*a*c^2*d^2+5*C*b*c^4)/((a*d
^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c
^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c
/d))+2*c*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/d^3
*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^
2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

input
```
integrate(x*(b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(x*sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(282) = 564$.

Time $= 0.10$ (sec) , antiderivative size $= 617$, normalized size of antiderivative $= 2.00$

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*C*c^3/(d^5*x + c*d^4) - sqrt(b*x^2 + a)*B*c^2/(d^4*x + c*d
^3) + sqrt(b*x^2 + a)*A*c/(d^3*x + c*d^2) - sqrt(b*x^2 + a)*C*c*x/d^3 + 1/
2*sqrt(b*x^2 + a)*B*x/d^2 - 4*C*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 3
*B*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - C*a*c*arcsinh(b*x/sqrt(a*b))/(
sqrt(b)*d^3) - 2*A*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + 1/2*B*a*arcsinh(
b*x/sqrt(a*b))/(sqrt(b)*d^2) + C*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^6) - B*b*c^3*ar
csinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt
(a + b*c^2/d^2)*d^5) + 3*C*sqrt(a + b*c^2/d^2)*c^2*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + A*b*c^2*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2
/d^2)*d^4) - 2*B*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 + A*sqrt(a + b*c^2/d^2)*arcsinh(b*
c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + 3*sqrt(
b*x^2 + a)*C*c^2/d^4 - 2*sqrt(b*x^2 + a)*B*c/d^3 + sqrt(b*x^2 + a)*A/d^2 +
 1/3*(b*x^2 + a)^(3/2)*C/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x\,\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int((x*(a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2296, normalized size of antiderivative = 7.43

$$\int \frac{x\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b*c*d**4 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5*x + 24*sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d*
*2 + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b**2*c**2*d**3*x - 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3 - 24*sqrt(a*d
**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b**2*c*d**4*x + 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
+ b*c**2) - a*d + b*c*x)*a*b*c**4*d**2 + 36*sqrt(a*d**2 + b*c**2)*log(sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**3*x - 36*sqr
t(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*b**3*c**4*d - 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*b**3*c**3*d**2*x + 48*sqrt(a*d**2 + b*c**2)*log(
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**6 + 48*sqrt(
a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*b**2*c**5*d*x - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4 - 12*
sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**5*x - 24*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a*b**2*c**3*d**2 - 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*
b**2*c**2*d**3*x + 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d*...
```

3.41 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2} dx$

Optimal result

Integrand size = 29, antiderivative size = 234

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2} dx &= -\frac{(2cC-Bd)\sqrt{a+bx^2}}{d^3}+\frac{Cx\sqrt{a+bx^2}}{2d^2}-\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^3(c+dx)} \\ &+\frac{(aCd^2+2b(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}d^4} \\ &+\frac{(ad^2(2cC-Bd)+bc(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
-(-B*d+2*C*c)*(b*x^2+a)^(1/2)/d^3+1/2*C*x*(b*x^2+a)^(1/2)/d^2-(A*d^2-B*c*d
+C*c^2)*(b*x^2+a)^(1/2)/d^3/(d*x+c)+1/2*(a*C*d^2+2*b*(A*d^2-2*B*c*d+3*C*c^
2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^4+(a*d^2*(-B*d+2*C*c)+b*c
*(A*d^2-2*B*c*d+3*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+
a)^(1/2))/d^4/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \frac{\frac{d\sqrt{a+bx^2}(-6c^2C+cd(4B-3Cx)+d^2(-2A+x(2B+Cx)))}{c+dx}+\frac{4(ad^2(2cC-Bd)+bc(3c^2C-2Bcd+Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}-\ (a}{2d^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-6*c^2*C + c*d*(4*B - 3*C*x) + d^2*(-2*A + x*(2*B + C
*x))))/(c + d*x) + (4*(a*d^2*(2*c*C - B*d) + b*c*(3*c^2*C - 2*B*c*d + A*d^
2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]]
)/Sqrt[-(b*c^2) - a*d^2] - ((a*C*d^2 + 2*b*(3*c^2*C - 2*B*c*d + A*d^2))*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(2*d^4)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {2182, 25, 682, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{\left(Abc-aCc+aBd+\left(aCd-b\left(-\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{ad^2+bc^2}-\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{d(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(Abc-aCc+aBd+\left(aCd-b\left(-\frac{3Cc^2}{d}+2Bc-2Ad\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 682

$$\frac{\frac{\int -\frac{b\left(bc^2+ad^2\right)\left(ad(3cC-2Bd)-\left(aCd^2+2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x\right)}{d(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)-dx\left(aCd^2+b\left(2Ad^2-2\cdots\right)\right)\right)}{2d^3}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{b\left(bc^2+ad^2\right)\left(ad(3cC-2Bd)-\left(aCd^2+2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x\right)}{d(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)-dx\left(aCd^2+b\left(2Ad^2-\cdots\right)\right)\right)}{2d^3}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{\left(ad^2+bc^2\right)\int \frac{ad(3cC-2Bd)-\left(aCd^2+2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^3}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)-dx\left(aCd^2+b\left(2Ad^2-2B\cdots\right)\right)\right)}{2d^3}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 719

$$\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aCd^2+2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{2d^3}-\frac{\sqrt{a+bx^2}\left(2\left(ad^2(2cC\cdots\right)\right)}{\cdots}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 224

$$-\frac{\frac{(ad^2+bc^2)\left(\frac{2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(aCd^2+2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{2d^3}-\frac{\sqrt{a+bx^2}(2(}{}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{(ad^2+bc^2)\left(\frac{2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}\right)}{2d^3}-\frac{\sqrt{a+bx^2}(2(}{}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$-\frac{\frac{(ad^2+bc^2)\left(-\frac{2\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}\right)}{2d^3}}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{(ad^2+bc^2)\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2+2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}-\frac{2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2(2cC-Bd)+bc\left(Ad^2-2Bcd+3c^2C\right)\right)}{d\sqrt{ad^2+bc^2}}\right)}{2d^3}-}{ad^2+bc^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

input

```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
-(((c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(d*(b*c^2 + a*d^2)*(c + d*x)
)) + (-1/2*((2*(a*d^2*(2*c*C - B*d) + b*c*(3*c^2*C - 2*B*c*d + A*d^2)) - d
*(a*C*d^2 + b*(3*c^2*C - 2*B*c*d + 2*A*d^2))*x)*Sqrt[a + b*x^2])/d^3 - ((b
*c^2 + a*d^2)*(-(((a*C*d^2 + 2*b*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (2*(a*d^2*(2*c*C - B*d) + b*c*(3*
c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[
a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*d^3))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(212) = 424$.

Time = 0.22 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.19

method	result
risch	$\frac{(Cxd+2Bd-4Cc)\sqrt{b\,x^2+a}}{2d^3}+\frac{\frac{\left(2Ab\,d^2-4Bbcd+aC\,d^2+6Cb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{2\left(2Abc\,d^2-Ba\,d^3-3Bb\,c^2d+2Cac\,d^2+4Cb\,c^3\right)\ln\left(\ldots\right)}{\ldots}}{\ldots}$
default	$\frac{C\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{d^2}+\frac{(Bd-2Cc)\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc}{\ldots}}\right)}{d}\ldots\right)}{\ldots}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d-4*C*c)*(b*x^2+a)^(1/2)/d^3+1/2/d^3*((2*A*b*d^2-4*B*b*c*d+
C*a*d^2+6*C*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2/d^2*(2*A*b*c*
d^2-B*a*d^3-3*B*b*c^2*d+2*C*a*c*d^2+4*C*b*c^3)/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*(A*a*d^4+A*b*c^
2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/d^3*(-1/(a*d^2+b*c^2)*d^2/(
x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+
b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2
*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(213) = 426$.

Time = 0.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.04

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = -\frac{\sqrt{bx^2+a}Cc^2}{d^4x+cd^3} + \frac{\sqrt{bx^2+a}Bc}{d^3x+cd^2} - \frac{\sqrt{bx^2+a}A}{d^2x+cd} + \frac{\sqrt{bx^2+a}Cx}{2\,d^2} + \frac{3\,C\sqrt{b}c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^4} - \frac{2\,B\sqrt{b}c \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^3} + \frac{Ca \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}d^2} + \frac{A\sqrt{b} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{d^2} - \frac{Cbc^3 \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^5} + \frac{Bbc^2 \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} - \frac{2\,C\sqrt{a+\frac{bc^2}{d^2}}c \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^3} - \frac{Abc \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} + \frac{B\sqrt{a+\frac{bc^2}{d^2}} \operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{d^2} - \frac{2\,\sqrt{bx^2+a}Cc}{d^3} + \frac{\sqrt{bx^2+a}B}{d^2}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)*C*c^2/(d^4*x + c*d^3) + sqrt(b*x^2 + a)*B*c/(d^3*x + c*d^
2) - sqrt(b*x^2 + a)*A/(d^2*x + c*d) + 1/2*sqrt(b*x^2 + a)*C*x/d^2 + 3*C*s
qrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - 2*B*sqrt(b)*c*arcsinh(b*x/sqrt(a*b
))/d^3 + 1/2*C*a*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + A*sqrt(b)*arcsinh(
b*x/sqrt(a*b))/d^2 - C*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5) + B*b*c^2*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2
/d^2)*d^4) - 2*C*sqrt(a + b*c^2/d^2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - A*b*c*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) + B
*sqrt(a + b*c^2/d^2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*
b)*abs(d*x + c)))/d^2 - 2*sqrt(b*x^2 + a)*C*c/d^3 + sqrt(b*x^2 + a)*B/d^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1847, normalized size of antiderivative = 7.89

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b**2*c**2*d**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x - 4*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b
**2*c*d**3 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a*b**2*d**4*x + 8*sqrt(a*d**2 + b*c**2)*log( - sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 + 8*sqrt(
a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c
*x)*a*b*c**2*d**3*x - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d - 8*sqrt(a*d**2 + b*c**2)*log
( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**2*d**2*x
 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*b**2*c**5 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x - 4*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a*b**2*c**2*d**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*
a*b**2*c*d**3*x + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**3 + 4*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*d**4*x - 8*sqrt(a*d**2 + b*c**2)*
log(c + d*x)*a*b*c**3*d**2 - 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2
*d**3*x + 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d + 8*sqrt(a*d**2
 + b*c**2)*log(c + d*x)*b**3*c**2*d**2*x - 12*sqrt(a*d**2 + b*c**2)*log...
```

3.42 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 217

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2} dx = \frac{C\sqrt{a+bx^2}}{d^2} + \frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^2(c+dx)} - \frac{\sqrt{b}(2cC-Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} - \frac{(bc^3(2cC-Bd)+ad^2(c^2C-Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2d^3\sqrt{bc^2+ad^2}} - \frac{\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}$$

output
```
C*(b*x^2+a)^(1/2)/d^2+(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^2/(d*x+c)-b^
(1/2)*(-B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^3-(b*c^3*(-B*d+2*C
*c)+a*d^2*(-A*d^2+C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+
a)^(1/2))/c^2/d^3/(a*d^2+b*c^2)^(1/2)-a^(1/2)*A*arctanh((b*x^2+a)^(1/2)/a^
(1/2))/c^2
```

Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx$$
$$= \frac{(2c^2C-Bcd+Ad^2+cCdx)\sqrt{a+bx^2}}{cd^2(c+dx)}$$
$$+ \frac{2\sqrt{-bc^2-ad^2}(bc^3(2cC-Bd)+ad^2(c^2C-Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{bc^4d^3+ac^2d^5}$$
$$+ \frac{2\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2} + \frac{\sqrt{b}(2cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x*(c + d*x)^2),x]
```

output
```
((2*c^2*C - B*c*d + A*d^2 + c*C*d*x)*Sqrt[a + b*x^2])/(c*d^2*(c + d*x)) +
(2*Sqrt[-(b*c^2) - a*d^2]*(b*c^3*(2*c*C - B*d) + a*d^2*(c^2*C - A*d^2))*Ar
cTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(b*c
^4*d^3 + a*c^2*d^5) + (2*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/S
qrt[a]])/c^2 + (Sqrt[b]*(2*c*C - B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/d^3
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2351, 606, 617, 677, 681, 27, 719, 224, 219, 488, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx$$
$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{\sqrt{bx^2+a}}{x(c+dx)^2}dx+\int \frac{(B+Cx)\sqrt{bx^2+a}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{606}$$

$$A\left(\frac{a\int \frac{1}{x(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\int \frac{ad-bcx}{(c+dx)^2\sqrt{bx^2+a}}dx}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{617}$$

$$A\left(\frac{a\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}-\frac{\int \frac{ad-bcx}{(c+dx)^2\sqrt{bx^2+a}}dx}{c}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{677}$$

$$A\left(\frac{a\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\int \frac{(B+Cx)\sqrt{bx^2+a}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{681}$$

$$A\left(\frac{a\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)-\frac{\int -\frac{2(aCd-b(2cC-Bd)x)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$A\left(\frac{a\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{\int \frac{aCd-b(2cC-Bd)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{719}$$

$$A\left(\frac{a\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{\frac{(aCd^2+bc(2cC-Bd))\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b(2cC-Bd)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{a\int\left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{\frac{(aCd^2+bc(2cC-Bd))\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b(2cC-Bd)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{ 219}$$

$$A\left(\frac{a\int\left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{\frac{(aCd^2+bc(2cC-Bd))\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cC-Bd)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{ 488}$$

$$A\left(\frac{a\int\left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{-\frac{(aCd^2+bc(2cC-Bd))\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cC-Bd)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{ 219}$$

$$A\left(\frac{a\int\left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)(aCd^2+bc(2cC-Bd))}{d\sqrt{ad^2+bc^2}}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cC-Bd)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(\frac{a\left(\frac{d\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{ac}}\right)}{c}+\frac{\sqrt{a+bx^2}}{c(c+dx)}\right)+\frac{-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)(aCd^2+bc(2cC-Bd))}{d\sqrt{ad^2+bc^2}}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2cC-Bd)}{d}}{d^2}+\frac{\sqrt{a+bx^2}(-Bd+2cC+Cdx)}{d^2(c+dx)}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x*(c + d*x)^2),x]
```

output
```
((2*c*C - B*d + C*d*x)*Sqrt[a + b*x^2])/(d^2*(c + d*x)) + (-((Sqrt[b]*(2*c
*C - B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - ((a*C*d^2 + b*c*(2*c*
C - B*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d
*Sqrt[b*c^2 + a*d^2]))/d^2 + A*(Sqrt[a + b*x^2]/(c*(c + d*x)) + (a*((d*Arc
Tanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c*Sqrt[b*c^2 +
 a*d^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(Sqrt[a]*c)))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 677
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] &&
 EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(195) = 390$.

Time = 0.22 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.04

method	result
default	$\frac{A\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{c^2}-\frac{(A\,d^2-C\,c^2)\left(\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}-\frac{\sqrt{b}\,c\ln\left(\frac{-\frac{bc}{d}+b\left(x+\frac{c}{d}\right)}{\sqrt{b}}+\sqrt{b\left(x+\frac{c}{d}\right.}\right.}{d}\right.}{}$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
A/c^2*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-(A*d
^2-C*c^2)/c^2/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b
^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d
^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a
*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-(A*d^2-B*c*d+C*c^2)/d^3
/c*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(3/2)-b*c*d/(a*d^2+b*c^2)*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)
^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+2*b/(a*d
^2+b*c^2)*d^2*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*l
n((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)^2x}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\left(C\,x^2+B\,x+A\right)}{x\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1420, normalized size of antiderivative = 6.54

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*c*d**4 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x - 2*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d**2
- 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*c**3*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d + 2*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3
*d**2*x - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*
c**2) - a*d + b*c*x)*b*c**6 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**5*d*x - 2*sqrt(a*d**2 + b*c*
*2)*log(c + d*x)*a**2*c*d**4 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d
**5*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**4*d**2 + 2*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a*c**3*d**3*x - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*b**2*c**4*d - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**3*d**2*x + 4*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**6 + 4*sqrt(a*d**2 + b*c**2)*log(c +
 d*x)*b*c**5*d*x + 2*sqrt(a + b*x**2)*a**2*c*d**5 + 2*sqrt(a + b*x**2)*a*b
*c**3*d**3 - 2*sqrt(a + b*x**2)*a*b*c**2*d**4 + 4*sqrt(a + b*x**2)*a*c**4*
d**3 + 2*sqrt(a + b*x**2)*a*c**3*d**4*x - 2*sqrt(a + b*x**2)*b**2*c**4*d**
2 + 4*sqrt(a + b*x**2)*b*c**6*d + 2*sqrt(a + b*x**2)*b*c**5*d**2*x + sq...
```

3.43 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 239

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx \\
&= -\frac{(c^2C-Bcd+2Ad^2)\sqrt{a+bx^2}}{c^2d^2x} \\
&\quad+\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^2x(c+dx)}+\frac{\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} \\
&\quad+\frac{(ad^3(Bc-2Ad)+b(c^4C-Ac^2d^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3d^2\sqrt{bc^2+ad^2}} \\
&\quad-\frac{\sqrt{a}(Bc-2Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^3}
\end{aligned}$$

output
```
-(2*A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c^2/d^2/x+(A*d^2-B*c*d+C*c^2)*(b*x^
2+a)^(1/2)/c/d^2/x/(d*x+c)+b^(1/2)*C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^
2+(a*d^3*(-2*A*d+B*c)+b*(-A*c^2*d^2+C*c^4))*arctanh((-b*c*x+a*d)/(a*d^2+b*
c^2)^(1/2)/(b*x^2+a)^(1/2))/c^3/d^2/(a*d^2+b*c^2)^(1/2)-a^(1/2)*(-2*A*d+B*
c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx$$
$$= -\frac{\sqrt{a+bx^2}(c(cC-Bd)x+Ad(c+2dx))}{c^2dx(c+dx)}$$
$$-\frac{2\sqrt{-bc^2-ad^2}(ad^3(Bc-2Ad)+b(c^4C-Ac^2d^2))\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{bc^5d^2+ac^3d^4}$$
$$+\frac{2\sqrt{b}C\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{d^2}-\frac{\sqrt{a}(Bc-2Ad)\log(x)}{c^3}$$
$$+\frac{\sqrt{a}(Bc-2Ad)\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c^3}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x]
```

output
```
-((Sqrt[a + b*x^2]*(c*(c*C - B*d)*x + A*d*(c + 2*d*x)))/(c^2*d*x*(c + d*x)
)) - (2*Sqrt[-(b*c^2) - a*d^2]*(a*d^3*(B*c - 2*A*d) + b*(c^4*C - A*c^2*d^2
))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a + b*x^2
])])/(b*c^5*d^2 + a*c^3*d^4) + (2*Sqrt[b]*C*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a]
+ Sqrt[a + b*x^2])])/d^2 - (Sqrt[a]*(B*c - 2*A*d)*Log[x])/c^3 + (Sqrt[a]*(
B*c - 2*A*d)*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/c^3
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx$$
$$\Big\downarrow \; 2353$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-2Ad)}{c^3x} - \frac{d\sqrt{a+bx^2}(Bc-2Ad)}{c^3(c+dx)} + \frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c^2(c+dx)^2} + \frac{A\sqrt{a+bx^2}}{c^2x^2} \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{c^3} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(Ad^2-Bcd+c^2C\right)}{c^2d^2} + \frac{b\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{cd^2\sqrt{ad^2+bc^2}} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-2Ad)}{c^2d} + \frac{\sqrt{ad^2+bc^2}(Bc-2Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3d} + \frac{A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{c^2} - \frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c^2d(c+dx)} - \frac{A\sqrt{a+bx^2}}{c^2x}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(c^2*x)) - ((c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])
/(c^2*d*(c + d*x)) + (A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^2
+ (Sqrt[b]*(B*c - 2*A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(c^2*d) + (
Sqrt[b]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(c^2
*d^2) + ((B*c - 2*A*d)*Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^3*d) + (b*(c^2*C - B*c*d + A*d^2)*ArcTan
h[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c*d^2*Sqrt[b*c^2
+ a*d^2]) - (Sqrt[a]*(B*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(217) = 434$.

Time = 0.30 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.15

method	result
risch	$-\frac{A\sqrt{b\,x^2+a}}{c^2x}-\frac{-\frac{\left(A a\,d^4+A b\,c^2d^2-B a c\,d^3-c^3Bbd+C a\,c^2d^2+c^4Cb\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2b}{}\right.}{}\right.}{d^4}}{}$ $(A\,d^2-Bcd$
default	$\frac{A\left(-\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)}{c^2}-\frac{(2Ad-Bc)\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{c^3}+$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/c^2/x*(b*x^2+a)^(1/2)-1/c^2*(-(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+
C*a*c^2*d^2+C*b*c^4)/d^4*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/
d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)
^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-a^(1/2)/
c*(2*A*d-B*c)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+1/d^3*(2*A*a*d^4-B*a*c
*d^3+B*b*c^3*d-2*C*b*c^4)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/
d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))-C*b^(1/2)*c^2/d^2*ln(b^(1/2)*x+(b*x^
2+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**2/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)^2*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^2\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1470, normalized size of antiderivative = 6.15

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x)
```

output

```
(4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*c*d**4*x + 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x**2 + 2*sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2*x
+ 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b*c**2*d**3*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**3*x - 2*sqrt(a*d**2 + b
*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**
4*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*b*c**6*x - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**5*d*x**2 - 4*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a**2*c*d**4*x - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d
**5*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2*x - 2*sqrt(a
*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**3*x**2 + 2*sqrt(a*d**2 + b*c**2)*
log(c + d*x)*a*b*c**2*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*
d**4*x**2 + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**6*x + 2*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*b*c**5*d*x**2 - 2*sqrt(a + b*x**2)*a**2*c**2*d**4 -
 4*sqrt(a + b*x**2)*a**2*c*d**5*x - 2*sqrt(a + b*x**2)*a*b*c**4*d**2 - 4*s
qrt(a + b*x**2)*a*b*c**3*d**3*x + 2*sqrt(a + b*x**2)*a*b*c**2*d**4*x - 2*s
qrt(a + b*x**2)*a*c**4*d**3*x + 2*sqrt(a + b*x**2)*b**2*c**4*d**2*x - 2...
```

3.44 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 274

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx &= -\frac{(2c^2C-2Bcd+3Ad^2)\sqrt{a+bx^2}}{2c^2d^2x^2} \\ &\quad+\frac{(c^2C-2Bcd+3Ad^2)\sqrt{a+bx^2}}{c^3dx}+\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^2x^2(c+dx)} \\ &\quad-\frac{(bc^2(Bc-2Ad)-ad(c^2C-2Bcd+3Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4\sqrt{bc^2+ad^2}} \\ &\quad-\frac{(2ac(cC-2Bd)+A(bc^2+6ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c^4}\end{aligned}$$

output
```
-1/2*(3*A*d^2-2*B*c*d+2*C*c^2)*(b*x^2+a)^(1/2)/c^2/d^2/x^2+(3*A*d^2-2*B*c*
d+C*c^2)*(b*x^2+a)^(1/2)/c^3/d/x+(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^2
/x^2/(d*x+c)-(b*c^2*(-2*A*d+B*c)-a*d*(3*A*d^2-2*B*c*d+C*c^2))*arctanh((-b*
c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^4/(a*d^2+b*c^2)^(1/2)-1/2*
(2*a*c*(-2*B*d+C*c)+A*(6*a*d^2+b*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^
(1/2)/c^4
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx$$
$$=\frac{\frac{c\sqrt{a+bx^2}(A(-c^2+3cdx+6d^2x^2)+2cx(cCx-B(c+2dx)))}{x^2(c+dx)}-\frac{4(bc^2(Bc-2Ad)-ad(c^2C-2Bcd+3Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+12}{2c^4}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x]
```

output
```
((c*Sqrt[a + b*x^2]*(A*(-c^2 + 3*c*d*x + 6*d^2*x^2) + 2*c*x*(c*C*x - B*(c
+ 2*d*x))))/(x^2*(c + d*x)) - (4*(b*c^2*(B*c - 2*A*d) - a*d*(c^2*C - 2*B*c
*d + 3*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2
) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + 12*Sqrt[a]*A*d^2*ArcTanh[(Sqrt[b]*x
- Sqrt[a + b*x^2])/Sqrt[a]] - (2*c*(A*b*c + 2*a*c*C - 4*a*B*d)*ArcTanh[(-(
Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a])/(2*c^4)
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx$$
$$\Big\downarrow \ 2353$$
$$\int \left(\frac{\sqrt{a+bx^2}(Bc-2Ad)}{c^3x^2}+\frac{\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{c^4x}-\frac{d\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{c^4(c+dx)}-\frac{d\sqrt{a+bx^2}}{c}\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-2Ad)}{c^3}-\frac{b(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2d\sqrt{ad^2+bc^2}}+\frac{\sqrt{ad^2+bc^2}(3Ad^2-2Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4d}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3Ad^2-2Bcd+c^2C)}{c^4}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{c^3d}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3Ad^2-2Bcd+c^2C)}{c^3d}-\frac{Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c^2}-\frac{\sqrt{a+bx^2}(Bc-2Ad)}{c^3x}+\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3(c+dx)}-\frac{A\sqrt{a+bx^2}}{2c^2x^2}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(c^2*x^2) - ((B*c - 2*A*d)*Sqrt[a + b*x^2])/(c^3*
x) + ((c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c^3*(c + d*x)) + (Sqrt[b]*
(B*c - 2*A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^3 - (Sqrt[b]*(c^2*C
- B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(c^3*d) + (Sqrt[b]*
(c^2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(c^3*d)
- (b*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sq
rt[a + b*x^2])])/(c^2*d*Sqrt[b*c^2 + a*d^2]) + (Sqrt[b*c^2 + a*d^2]*(c^2*C
 - 2*B*c*d + 3*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a +
b*x^2])])/(c^4*d) - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a]*c^2)
 - (Sqrt[a]*(c^2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(252) = 504$.

Time = 0.33 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.93

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-4Adx+2Bcx+Ac)}{2c^3x^2}-\frac{\frac{\left(6Aa\,d^2+bA\,c^2-4Bacd+2Ca\,c^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}+\frac{2\left(Aa\,d^4+Ab\,c^2d^2-Bac\,d^3-c^3Bbd+Ca\,c^2\right.}{}}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-4*A*d*x+2*B*c*x+A*c)/c^3/x^2-1/2/c^3*(1/c*(6*A*a*d^
2+A*b*c^2-4*B*a*c*d+2*C*a*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)+2*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)/d^3*(-1
/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d
^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-2/c*(3*A*a*d^4+A*b*c^2*d^2-2*B*a*c*d
^3+C*a*c^2*d^2-C*b*c^4)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/
d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(253) = 506$.

Time = 9.24 (sec) , antiderivative size = 2240, normalized size of antiderivative = 8.18

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
[-1/4*(2*((B*a*b*c^3*d + 2*B*a^2*c*d^3 - 3*A*a^2*d^4 - (C*a^2 + 2*A*a*b)*c
^2*d^2)*x^3 + (B*a*b*c^4 + 2*B*a^2*c^2*d^2 - 3*A*a^2*c*d^3 - (C*a^2 + 2*A*
a*b)*c^3*d)*x^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^
2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b
*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + ((4*B*a*b*c^3*d^2 + 4*B*a^2*c*d^4
- 6*A*a^2*d^5 - (2*C*a*b + A*b^2)*c^4*d - (2*C*a^2 + 7*A*a*b)*c^2*d^3)*x^3
 + (4*B*a*b*c^4*d + 4*B*a^2*c^2*d^3 - 6*A*a^2*c*d^4 - (2*C*a*b + A*b^2)*c^
5 - (2*C*a^2 + 7*A*a*b)*c^3*d^2)*x^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(A*a*b*c^5 + A*a^2*c^3*d^2 - 2*(C*a*b*c^5 - 2*
B*a*b*c^4*d - 2*B*a^2*c^2*d^3 + 3*A*a^2*c*d^4 + (C*a^2 + 3*A*a*b)*c^3*d^2)
*x^2 + (2*B*a*b*c^5 - 3*A*a*b*c^4*d + 2*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3)*x
)*sqrt(b*x^2 + a))/((a*b*c^6*d + a^2*c^4*d^3)*x^3 + (a*b*c^7 + a^2*c^5*d^2
)*x^2), -1/4*(4*((B*a*b*c^3*d + 2*B*a^2*c*d^3 - 3*A*a^2*d^4 - (C*a^2 + 2*A
*a*b)*c^2*d^2)*x^3 + (B*a*b*c^4 + 2*B*a^2*c^2*d^2 - 3*A*a^2*c*d^3 - (C*a^2
 + 2*A*a*b)*c^3*d)*x^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(
b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)
) + ((4*B*a*b*c^3*d^2 + 4*B*a^2*c*d^4 - 6*A*a^2*d^5 - (2*C*a*b + A*b^2)*c^
4*d - (2*C*a^2 + 7*A*a*b)*c^2*d^3)*x^3 + (4*B*a*b*c^4*d + 4*B*a^2*c^2*d^3
- 6*A*a^2*c*d^4 - (2*C*a*b + A*b^2)*c^5 - (2*C*a^2 + 7*A*a*b)*c^3*d^2)*x^2
)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a*...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x^3\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**3/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)^2x^3}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)^2*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^3\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1862, normalized size of antiderivative = 6.80

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x)
```

output
```
(12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*c*d**3*x**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**4*x**3 + 8*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b*c**3*d*x**2 + 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**2*x**3 - 8*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**2*x
**2 - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*a*b*c*d**3*x**3 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d*x**2 + 4*sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
*c**3*d**2*x**3 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(
 - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x**3
- 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*c*d**3*x**2 - 12*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a**2*d**4*x**3 - 8*sqrt(a*d**2 + b*c**2)*log(c + d*
x)*a*b*c**3*d*x**2 - 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2*x*
*3 + 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2*x**2 + 8*sqrt(a*d*
*2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x**3 - 4*sqrt(a*d**2 + b*c**2)*log(c
+ d*x)*a*c**4*d*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**3*d**2...
```

3.45 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 349

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx &= -\frac{(3c^2C-3Bcd+4Ad^2)\sqrt{a+bx^2}}{3c^2d^2x^3}+\frac{(2c^2C-3Bcd+4Ad^2)\sqrt{a+bx^2}}{2c^3dx^2} \\
&\quad -\frac{(Abc^2+6ac^2C-9aBcd+12aAd^2)\sqrt{a+bx^2}}{3ac^4x}+\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^2x^3(c+dx)} \\
&\quad -\frac{(bc^2(c^2C-2Bcd+3Ad^2)+ad^2(2c^2C-3Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5\sqrt{bc^2+ad^2}} \\
&\quad -\frac{(bc^2(Bc-2Ad)-2ad(2c^2C-3Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}c^5}
\end{aligned}$$

output
```
-1/3*(4*A*d^2-3*B*c*d+3*C*c^2)*(b*x^2+a)^(1/2)/c^2/d^2/x^3+1/2*(4*A*d^2-3*
B*c*d+2*C*c^2)*(b*x^2+a)^(1/2)/c^3/d/x^2-1/3*(12*A*a*d^2+A*b*c^2-9*B*a*c*d
+6*C*a*c^2)*(b*x^2+a)^(1/2)/a/c^4/x+(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/
d^2/x^3/(d*x+c)-(b*c^2*(3*A*d^2-2*B*c*d+C*c^2)+a*d^2*(4*A*d^2-3*B*c*d+2*C*
c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^5/(a*d^2
+b*c^2)^(1/2)-1/2*(b*c^2*(-2*A*d+B*c)-2*a*d*(4*A*d^2-3*B*c*d+2*C*c^2))*arc
tanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^5
```

Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx =$$

$$-\frac{\frac{c\sqrt{a+bx^2}(2Abc^2x^2(c+dx)+2aA(c^3-2c^2dx+6cd^2x^2+12d^3x^3)+3acx(2cCx(c+2dx)+B(c^2-3cdx-6d^2x^2)))}{ax^3(c+dx)}+\frac{12(bc^2(c^2C-2Bcd+3Ad^2}{}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x]
```

output
```
-1/6*((c*Sqrt[a + b*x^2]*(2*A*b*c^2*x^2*(c + d*x) + 2*a*A*(c^3 - 2*c^2*d*x
 + 6*c*d^2*x^2 + 12*d^3*x^3) + 3*a*c*x*(2*c*C*x*(c + 2*d*x) + B*(c^2 - 3*c
*d*x - 6*d^2*x^2))))/(a*x^3*(c + d*x)) + (12*(b*c^2*(c^2*C - 2*B*c*d + 3*A
*d^2) + a*d^2*(2*c^2*C - 3*B*c*d + 4*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d
*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + 48*Sqr
t[a]*A*d^3*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (6*c*(b*c*(B*c
 - 2*A*d) + 2*a*d*(-2*c*C + 3*B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2
])/Sqrt[a]])/Sqrt[a])/c^5
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.43, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-2Ad)}{c^3x^3}+\frac{d^2\sqrt{a+bx^2}(4Ad^2-3Bcd+2c^2C)}{c^5(c+dx)}-\frac{d\sqrt{a+bx^2}(4Ad^2-3Bcd+2c^2C)}{c^5x}+\frac{d^2\sqrt{a-}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{2\sqrt{a}c^3}-\frac{\sqrt{ad^2+bc^2}\,(4Ad^2-3Bcd+2c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^5}+\frac{\sqrt{a}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4Ad^2-3Bcd+2c^2C)}{c^5}+\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{c^4}+\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3Ad^2-2Bcd+c^2C)}{c^4}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4Ad^2-3Bcd+2c^2C)}{c^4}+\frac{b(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3\sqrt{ad^2+bc^2}}-\frac{\sqrt{a+bx^2}(Bc-2Ad)}{2c^3x^2}-\frac{\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{c^4x}-\frac{d\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^4(c+dx)}-\frac{A(a+bx^2)^{3/2}}{3ac^2x^3}$$

input

```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x]
```

output

```
-1/2*((B*c - 2*A*d)*Sqrt[a + b*x^2])/(c^3*x^2) - ((c^2*C - 2*B*c*d + 3*A*d
^2)*Sqrt[a + b*x^2])/(c^4*x) - (d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])
/(c^4*(c + d*x)) - (A*(a + b*x^2)^(3/2))/(3*a*c^2*x^3) + (Sqrt[b]*(c^2*C -
 B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^4 + (Sqrt[b]*(c^2*
C - 2*B*c*d + 3*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^4 - (Sqrt[b
]*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^4
+ (b*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sq
rt[a + b*x^2])])/(c^3*Sqrt[b*c^2 + a*d^2]) - (Sqrt[b*c^2 + a*d^2]*(2*c^2*C
 - 3*B*c*d + 4*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a +
b*x^2])])/c^5 - (b*(B*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt
[a]*c^3) + (Sqrt[a]*d*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a]])/c^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.69

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(18 A a\,d^{2} x^{2}+2 A b\,c^{2} x^{2}-12 B a c d\,x^{2}+6 C a\,c^{2} x^{2}-6 A a c d x+3 B a\,c^{2} x+2 A\,c^{2} a\right)}{6 a\,c^{4} x^{3}}+\frac{\left(2 A a\,d^{4}+2 A b\,c^{2} d^{2}-2 B a c\,d^{3}-2 c^{3} B b d+2C\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(18*A*a*d^2*x^2+2*A*b*c^2*x^2-12*B*a*c*d*x^2+6*C*a*c^
2*x^2-6*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/a/c^4/x^3+1/2/c^4*((2*A*a*d^4+2*A
*b*c^2*d^2-2*B*a*c*d^3-2*B*b*c^3*d+2*C*a*c^2*d^2+2*C*b*c^4)/d^2*(-1/(a*d^2
+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-
b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*
c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/c*(8*A*a*d^3+2*A*b*c^2*d-6*B*a*c*d^2-B*b*
c^3+4*C*a*c^2*d)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-2/c*(4*A*a*
d^3+2*A*b*c^2*d-3*B*a*c*d^2-B*b*c^3+2*C*a*c^2*d)/((a*d^2+b*c^2)/d^2)^(1/2)
*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(322) = 644$.

Time = 10.01 (sec) , antiderivative size = 2786, normalized size of antiderivative = 7.98

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
[1/12*(6*((C*a*b*c^4*d - 2*B*a*b*c^3*d^2 - 3*B*a^2*c*d^4 + 4*A*a^2*d^5 + (
2*C*a^2 + 3*A*a*b)*c^2*d^3)*x^4 + (C*a*b*c^5 - 2*B*a*b*c^4*d - 3*B*a^2*c^2
*d^3 + 4*A*a^2*c*d^4 + (2*C*a^2 + 3*A*a*b)*c^3*d^2)*x^3)*sqrt(b*c^2 + a*d^
2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*
sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^
2)) - 3*((B*b^2*c^5*d + 7*B*a*b*c^3*d^3 + 6*B*a^2*c*d^5 - 8*A*a^2*d^6 - 2*
(2*C*a*b + A*b^2)*c^4*d^2 - 2*(2*C*a^2 + 5*A*a*b)*c^2*d^4)*x^4 + (B*b^2*c^
6 + 7*B*a*b*c^4*d^2 + 6*B*a^2*c^2*d^4 - 8*A*a^2*c*d^5 - 2*(2*C*a*b + A*b^2
)*c^5*d - 2*(2*C*a^2 + 5*A*a*b)*c^3*d^3)*x^3)*sqrt(a)*log(-(b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*A*a*b*c^6 + 2*A*a^2*c^4*d^2 - 2*(9*
B*a*b*c^4*d^2 + 9*B*a^2*c^2*d^4 - 12*A*a^2*c*d^5 - (6*C*a*b + A*b^2)*c^5*d
 - (6*C*a^2 + 13*A*a*b)*c^3*d^3)*x^3 - (9*B*a*b*c^5*d + 9*B*a^2*c^3*d^3 -
12*A*a^2*c^2*d^4 - 2*(3*C*a*b + A*b^2)*c^6 - 2*(3*C*a^2 + 7*A*a*b)*c^4*d^2
)*x^2 + (3*B*a*b*c^6 - 4*A*a*b*c^5*d + 3*B*a^2*c^4*d^2 - 4*A*a^2*c^3*d^3)*
x)*sqrt(b*x^2 + a))/((a*b*c^7*d + a^2*c^5*d^3)*x^4 + (a*b*c^8 + a^2*c^6*d^
2)*x^3), -1/12*(12*((C*a*b*c^4*d - 2*B*a*b*c^3*d^2 - 3*B*a^2*c*d^4 + 4*A*a
^2*d^5 + (2*C*a^2 + 3*A*a*b)*c^2*d^3)*x^4 + (C*a*b*c^5 - 2*B*a*b*c^4*d - 3
*B*a^2*c^2*d^3 + 4*A*a^2*c*d^4 + (2*C*a^2 + 3*A*a*b)*c^3*d^2)*x^3)*sqrt(-b
*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a
*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + 3*((B*b^2*c^5*d + 7*B*a*...
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4\left(c+dx\right)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**4/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**4*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{\left(dx+c\right)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)^2*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^4\,{(c+d\,x)}^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2409, normalized size of antiderivative = 6.90

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**3*c*d**4*x**3 + 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*d**5*x**4 + 36*sqrt(a*d**2 + b*
c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**
3*d**2*x**3 + 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**2*b*c**2*d**3*x**4 - 36*sqrt(a*d**2 + b*c**2)*lo
g(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**2*d**3*x
**3 - 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**2*b*c*d**4*x**4 + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*c**4*d**2*x**3 + 24*sqrt
(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**2*c**3*d**3*x**4 - 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d*x**3 - 24*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3
*d**2*x**4 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a*b*c**6*x**3 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a
+ b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**5*d*x**4 - 48*sqrt(a
*d**2 + b*c**2)*log(c + d*x)*a**3*c*d**4*x**3 - 48*sqrt(a*d**2 + b*c**2)*l
og(c + d*x)*a**3*d**5*x**4 - 36*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*
c**3*d**2*x**3 - 36*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**2*d**3...
```

3.46 $\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 436

$$\begin{aligned}
&\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx \\
&= -\frac{(4c^2C-4Bcd+5Ad^2)\sqrt{a+bx^2}}{4c^2d^2x^4}+\frac{(3c^2C-4Bcd+5Ad^2)\sqrt{a+bx^2}}{3c^3dx^3} \\
&\quad -\frac{(Abc^2+12ac^2C-16aBcd+20aAd^2)\sqrt{a+bx^2}}{8ac^4x^2} \\
&\quad -\frac{(bc^2(Bc-2Ad)-3ad(3c^2C-4Bcd+5Ad^2))\sqrt{a+bx^2}}{3ac^5x} \\
&\quad +\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^2x^4(c+dx)} \\
&\quad +\frac{d(bc^2(2c^2C-3Bcd+4Ad^2)+ad^2(3c^2C-4Bcd+5Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^6\sqrt{bc^2+ad^2}} \\
&\quad +\frac{(A(b^2c^4-12abc^2d^2-40a^2d^4)-4ac(2ad^2(3cC-4Bd)+bc^2(cC-2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}c^6}
\end{aligned}$$

output
```
-1/4*(5*A*d^2-4*B*c*d+4*C*c^2)*(b*x^2+a)^(1/2)/c^2/d^2/x^4+1/3*(5*A*d^2-4*
B*c*d+3*C*c^2)*(b*x^2+a)^(1/2)/c^3/d/x^3-1/8*(20*A*a*d^2+A*b*c^2-16*B*a*c*
d+12*C*a*c^2)*(b*x^2+a)^(1/2)/a/c^4/x^2-1/3*(b*c^2*(-2*A*d+B*c)-3*a*d*(5*A
*d^2-4*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2)/a/c^5/x+(A*d^2-B*c*d+C*c^2)*(b*x^2+
a)^(1/2)/c/d^2/x^4/(d*x+c)+d*(b*c^2*(4*A*d^2-3*B*c*d+2*C*c^2)+a*d^2*(5*A*d
^2-4*B*c*d+3*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1
/2))/c^6/(a*d^2+b*c^2)^(1/2)+1/8*(A*(-40*a^2*d^4-12*a*b*c^2*d^2+b^2*c^4)-4
*a*c*(2*a*d^2*(-4*B*d+3*C*c)+b*c^2*(-2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(3/2)/c^6
```

Mathematica [A] (verified)

Time = 4.57 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx^2}(-bc^2x^2(c+dx)(3Ac+8Bcx-16Adx)-2a(A(3c^4-5c^3dx+10c^2d^2x^2-30cd^3x^3-60d^4x^4)+2cx(3cCx(c^2-3cdx-6d^2x^2)+2B(c^3-2c^2d}{ax^4(c+dx)}}{}$$

input
```
Integrate[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x]
```

output
```
((c*Sqrt[a + b*x^2]*(-(b*c^2*x^2*(c + d*x)*(3*A*c + 8*B*c*x - 16*A*d*x)) -
 2*a*(A*(3*c^4 - 5*c^3*d*x + 10*c^2*d^2*x^2 - 30*c*d^3*x^3 - 60*d^4*x^4) +
 2*c*x*(3*c*C*x*(c^2 - 3*c*d*x - 6*d^2*x^2) + 2*B*(c^3 - 2*c^2*d*x + 6*c*d
^2*x^2 + 12*d^3*x^3)))))/(a*x^4*(c + d*x)) + (48*d*(b*c^2*(2*c^2*C - 3*B*c
*d + 4*A*d^2) + a*d^2*(3*c^2*C - 4*B*c*d + 5*A*d^2))*ArcTan[(Sqrt[b]*(c +
d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2]
+ 240*Sqrt[a]*A*d^4*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (6*c*
(A*b*c*(b*c^2 - 12*a*d^2) - 4*a*(2*a*d^2*(3*c*C - 4*B*d) + b*c^2*(c*C - 2*
B*d)))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/(24*c^6
)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.42, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$$

$$\big\downarrow \text{ 2353}$$

$$\int \left(\frac{\sqrt{a+bx^2}(Bc-2Ad)}{c^3x^4}+\frac{d^2\sqrt{a+bx^2}(5Ad^2-4Bcd+3c^2C)}{c^6x}-\frac{d^3\sqrt{a+bx^2}(5Ad^2-4Bcd+3c^2C)}{c^6(c+dx)}-\frac{d\sqrt{a\ -}}{}\right.$$

$$\big\downarrow \text{ 2009}$$

$$\frac{Ab^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}c^2}+\frac{d\sqrt{ad^2+bc^2}(5Ad^2-4Bcd+3c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^6}-\frac{\sqrt{a}d^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(5Ad^2-4Bcd+3c^2C)}{c^6}-\frac{\sqrt{b}d\text{arctan}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{c^5}-\frac{\sqrt{b}d\text{arctan}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4Ad^2-3Bcd+2c^2C)}{c^5}+\frac{\sqrt{b}d\text{arctan}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(5Ad^2-4Bcd+3c^2C)}{c^5}-\frac{bd(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4\sqrt{ad^2+bc^2}}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3Ad^2-2Bcd+c^2C)}{2\sqrt{a}c^4}-\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{3ac^3x^3}+\frac{d\sqrt{a+bx^2}(4Ad^2-3Bcd+2c^2C)}{c^5x}+\frac{d^2\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^5(c+dx)}-\frac{\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{2c^4x^2}-\frac{Ab\sqrt{a+bx^2}}{8ac^2x^2}-\frac{A\sqrt{a+bx^2}}{4c^2x^4}$$

input
```
Int[(Sqrt[a + b*x^2]*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x]
```

output

```
-1/4*(A*Sqrt[a + b*x^2])/(c^2*x^4) - (A*b*Sqrt[a + b*x^2])/(8*a*c^2*x^2) -
 ((c^2*C - 2*B*c*d + 3*A*d^2)*Sqrt[a + b*x^2])/(2*c^4*x^2) + (d*(2*c^2*C -
 3*B*c*d + 4*A*d^2)*Sqrt[a + b*x^2])/(c^5*x) + (d^2*(c^2*C - B*c*d + A*d^2
)*Sqrt[a + b*x^2])/(c^5*(c + d*x)) - ((B*c - 2*A*d)*(a + b*x^2)^(3/2))/(3*
a*c^3*x^3) - (Sqrt[b]*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/c^5 - (Sqrt[b]*d*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/c^5 + (Sqrt[b]*d*(3*c^2*C - 4*B*c*d + 5*A*d^2)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^5 - (b*d*(c^2*C - B*c*d + A*d^2)*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])]])/(c^4*Sqrt[b*c^2 +
 a*d^2]) + (d*Sqrt[b*c^2 + a*d^2]*(3*c^2*C - 4*B*c*d + 5*A*d^2)*ArcTanh[(a
*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])]])/c^6 + (A*b^2*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/(8*a^(3/2)*c^2) - (b*(c^2*C - 2*B*c*d + 3*A*d^2)*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a]*c^4) - (Sqrt[a]*d^2*(3*c^2*C
- 4*B*c*d + 5*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^6
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353

```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-96Aa\,d^3x^3-16Ab\,c^2d\,x^3+72Bac\,d^2x^3+8Bb\,c^3x^3-48Ca\,c^2d\,x^3+36Aac\,d^2x^2+3Ab\,c^3x^2-24Ba\,c^2d\,x^2+12Ca\,c^3x^2-\right.}{24a\,c^5x^4}$
default	Expression too large to display

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-96*A*a*d^3*x^3-16*A*b*c^2*d*x^3+72*B*a*c*d^2*x^3+8
*B*b*c^3*x^3-48*C*a*c^2*d*x^3+36*A*a*c*d^2*x^2+3*A*b*c^3*x^2-24*B*a*c^2*d*
x^2+12*C*a*c^3*x^2-16*A*a*c^2*d*x+8*B*a*c^3*x+6*A*a*c^3)/a/c^5/x^4-1/8/c^5
/a*(1/c*(40*A*a^2*d^4+12*A*a*b*c^2*d^2-A*b^2*c^4-32*B*a^2*c*d^3-8*B*a*b*c^
3*d+24*C*a^2*c^2*d^2+4*C*a*b*c^4)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)+8/d*a*(A*a*d^4+A*b*c^2*d^2-B*a*c*d^3-B*b*c^3*d+C*a*c^2*d^2+C*b*c^4)*
(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d
^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2
)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-8*a/c*(5*A*a*d^4+3*A*b*c^2*d^2-4*
B*a*c*d^3-2*B*b*c^3*d+3*C*a*c^2*d^2+C*b*c^4)/((a*d^2+b*c^2)/d^2)^(1/2)*ln(
(2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(406) = 812$.

Time = 30.32 (sec) , antiderivative size = 3650, normalized size of antiderivative = 8.37

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx=\int \frac{\sqrt{a+bx^2}\,(A+Bx+Cx^2)}{x^5\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**5/(d*x+c)**2,x)
```

output
```
Integral(sqrt(a + b*x**2)*(A + B*x + C*x**2)/(x**5*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{bx^2+a}}{(dx+c)^2x^5}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(b*x^2 + a)/((d*x + c)^2*x^5), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx=\int \frac{\sqrt{b\,x^2+a}\,(C\,x^2+B\,x+A)}{x^5\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 2853, normalized size of antiderivative = 6.54

$$\int \frac{\sqrt{a+bx^2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x)
```

output
```
(240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**3*c*d**5*x**4 + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*d**6*x**5 + 192*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**2*b*c**3*d**3*x**4 + 192*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**2*d**4*x**5 - 192*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**2*b*c**2*d**4*x**4 - 192*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**5*x**5 + 144*sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
**2*c**4*d**3*x**4 + 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*c**3*d**4*x**5 - 144*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**
2*c**4*d**2*x**4 - 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**3*x**5 + 96*sqrt(a*d**2 + b
*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*
*6*d*x**4 + 96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a*b*c**5*d**2*x**5 - 240*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**3*c*d**5*x**4 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*d
**6*x**5 - 192*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**3*d**3*x**4...
```

3.47 $\int x^3(c+dx) \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx$

Optimal result

Integrand size = 30, antiderivative size = 311

$$\int x^3(c+dx) \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx = \frac{3a^3(aCd-2b(Bc+Ad))x\sqrt{a+bx^2}}{256b^3} - \frac{a^2(aCd-2b(Bc+Ad))x^3\sqrt{a+bx^2}}{128b^2} - \frac{a(aCd-2b(Bc+Ad))x^5\sqrt{a+bx^2}}{32b} - \frac{(aCd-2b(Bc+Ad))x^5(a+bx^2)^{3/2}}{16b} - \frac{a(Abc-acC-aBd)\left(a+bx^2\right)^{5/2}}{5b^3} + \frac{Cdx^5(a+bx^2)^{5/2}}{10b} + \frac{(Abc-2a(cC+Bd))\left(a+bx^2\right)^{7/2}}{7b^3} + \frac{(cC+Bd)\left(a+bx^2\right)^{9/2}}{9b^3} - \frac{3a^4(aCd-2b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output

```
3/256*a^3*(a*C*d-2*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b^3-1/128*a^2*(a*C*d-2*b
*(A*d+B*c))*x^3*(b*x^2+a)^(1/2)/b^2-1/32*a*(a*C*d-2*b*(A*d+B*c))*x^5*(b*x^
2+a)^(1/2)/b-1/16*(a*C*d-2*b*(A*d+B*c))*x^5*(b*x^2+a)^(3/2)/b-1/5*a*(A*b*c
-B*a*d-C*a*c)*(b*x^2+a)^(5/2)/b^3+1/10*C*d*x^5*(b*x^2+a)^(5/2)/b+1/7*(A*b*
c-2*a*(B*d+C*c))*(b*x^2+a)^(7/2)/b^3+1/9*(B*d+C*c)*(b*x^2+a)^(9/2)/b^3-3/2
56*a^4*(a*C*d-2*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.86

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(a^4(2048cC+2048Bd+945Cdx)+12a^2b^2x^2(3A(64c+35dx)+x(105Bc+64c\ldots}{}$$

input
```
Integrate[x^3*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^4*(2048*c*C + 2048*B*d + 945*C*d*x) + 12*a^2*b
^2*x^2*(3*A*(64*c + 35*d*x) + x*(105*B*c + 64*c*C*x + 64*B*d*x + 42*C*d*x^
2)) - 2*a^3*b*(9*A*(256*c + 105*d*x) + x*(945*B*c + 512*c*C*x + 512*B*d*x
+ 315*C*d*x^2)) + 16*a*b^3*x^4*(9*A*(128*c + 105*d*x) + x*(945*B*c + 800*c
*C*x + 800*B*d*x + 693*C*d*x^2)) + 32*b^4*x^6*(45*A*(8*c + 7*d*x) + 7*x*(5
*B*(9*c + 8*d*x) + 4*C*x*(10*c + 9*d*x)))) + 945*a^4*(a*C*d - 2*b*(B*c + A
*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(80640*b^(7/2))
```

Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.55, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}=0.433$, Rules used = {2185, 27, 2185, 25, 2185, 2185, 25, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -5(c+dx)\left(bx^2+a\right)^{3/2}\left(bd^4(7cC-2Bd)x^4+d^3\left(9bCc^2-2Abd^2+aCd^2\right)x^3+cCd^2\left(5bc^2+3ad^2\right)x^2+c^2Cc\ldots\right)}{10bd^5}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(bd^4(7cC-2Bd)x^4+d^3\left(9bCc^2-2Abd^2+aCd^2\right)x^3+cCd^2\left(5bc^2+3ad^2\right)x^2+c^2Cd(bc \cdots}{2bd^5}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{\int -\left((c+dx)\left(bx^2+a\right)^{3/2}\left(-b\left(9aCd^2-2b\left(40Cc^2-23Bdc+9Ad^2\right)\right)x^3d^7+b\left(2b(44cC-19Bd)c^2+ad^2(cC-8Bd)\right)x^2d^6+abc^2(19cC-8Bd)d^6+bc(2b \cdots}{9bd^4}}{2bd^5}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(-b\left(9aCd^2-2b\left(40Cc^2-23Bdc+9Ad^2\right)\right)x^3d^7+b\left(2b(44cC-19Bd)c^2+a \cdots}{9b \cdots}}{2bd^5}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(b^2\left(ad^2(125cC-64Bd)-6bc\left(56Cc^2-49Bdc+39Ad^2\right)\right)x^2d^9+abc\left(27aCd^2-b\left(88 \cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{\frac{\int -b^2d^{10}(c+dx)\left(ad\left(ad^2(61cC-128Bd)-2bc\left(28Cc^2-35Bdc+45Ad^2\right)\right)-3\left(63a^2Cd^4-2ab\left(61Cc^2-6 \cdots\right.\right.\right.}{7bd^2}}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{\int b^2d^{10}(c+dx)\left(ad\left(a \cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\int(c+dx)\left(ad\left(ad\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\left(\frac{63a^2d^4(aCd-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\left(\frac{63a^2d^4(aCd-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\left(\frac{63a^2d^4(aCd-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\left(\frac{63a^2d^4(aCd-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^4}-$$

$$\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(7cC-2Bd)-\frac{\frac{1}{7}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^2\left(ad^2(125cC-64Bd)-6bc\left(39Ad^2-49Bcd+56c^2C\right)\right)-\frac{1}{7}bd^8\left(\frac{63a^2d^4\left(\frac{3}{4}a\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Int[x^3*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^5*(a + b*x^2)^(5/2))/(10*b*d^4) - ((d*(7*c*C - 2*B*d)*(c + d*
x)^4*(a + b*x^2)^(5/2))/9 - ((d^5*(80*b*c^2*C - 46*b*B*c*d + 18*A*b*d^2 -
9*a*C*d^2)*(c + d*x)^3*(a + b*x^2)^(5/2))/8 + ((b*d^8*(a*d^2*(125*c*C - 64
*B*d) - 6*b*c*(56*c^2*C - 49*B*c*d + 39*A*d^2))*(c + d*x)^2*(a + b*x^2)^(5
/2))/7 - (b*d^8*((-2*(64*a^2*d^4*(c*C + B*d) + 6*b^2*c^3*(28*c^2*C - 35*B*
c*d + 45*A*d^2) - a*b*c*d^2*(155*c^2*C - 160*B*c*d + 144*A*d^2))*(a + b*x^
2)^(5/2))/(5*b) - (d*(63*a^2*C*d^4 + 4*b^2*c^2*(28*c^2*C - 35*B*c*d + 45*A
*d^2) - 2*a*b*d^2*(61*c^2*C - 65*B*c*d + 63*A*d^2))*x*(a + b*x^2)^(5/2))/(
2*b) + (63*a^2*d^4*(a*C*d - 2*b*(B*c + A*d))*((x*(a + b*x^2)^(3/2))/4 + (3
*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sq
rt[b])))/4))/(2*b)))/7)/(8*b*d^3))/(9*b*d^4))/(2*b*d^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.07

method	result
default	$(Ad+Bc)\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+(Bd+$
risch	$-\frac{(-8064C\,b^{4}d\,x^{9}-8960B\,b^{4}d\,x^{8}-8960C\,b^{4}c\,x^{8}-10080A\,b^{4}d\,x^{7}-10080B\,b^{4}c\,x^{7}-11088Ca\,b^{3}d\,x^{7}-11520A\,b^{4}c\,x^{6}-12800Ba\,b^{3}d\,x}{}$

input
```
int(x^3*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
(A*d+B*c)*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*
a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))))+(B*d+C*c)*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b
*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)))+A*c*(1/7*x^2*(b*x
^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+d*C*(1/10*x^5*(b*x^2+a)^(5/2)/b-
1/2*a/b*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/
b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.30

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/161280*(945*(2*B*a^4*b*c - (C*a^5 - 2*A*a^4*b)*d)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8064*C*b^5*d*x^9 + 8960*(C*b^5*c +
 B*b^5*d)*x^8 + 1008*(10*B*b^5*c + (11*C*a*b^4 + 10*A*b^5)*d)*x^7 + 2048*B
*a^4*b*d + 1280*(10*B*a*b^4*d + (10*C*a*b^4 + 9*A*b^5)*c)*x^6 + 504*(30*B*
a*b^4*c + (C*a^2*b^3 + 30*A*a*b^4)*d)*x^5 + 768*(B*a^2*b^3*d + (C*a^2*b^3
+ 24*A*a*b^4)*c)*x^4 + 630*(2*B*a^2*b^3*c - (C*a^3*b^2 - 2*A*a^2*b^3)*d)*x
^3 - 256*(4*B*a^3*b^2*d + (4*C*a^3*b^2 - 9*A*a^2*b^3)*c)*x^2 + 512*(4*C*a^
4*b - 9*A*a^3*b^2)*c - 945*(2*B*a^3*b^2*c - (C*a^4*b - 2*A*a^3*b^2)*d)*x)*
sqrt(b*x^2 + a))/b^4, -1/80640*(945*(2*B*a^4*b*c - (C*a^5 - 2*A*a^4*b)*d)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*C*b^5*d*x^9 + 8960*(C*
b^5*c + B*b^5*d)*x^8 + 1008*(10*B*b^5*c + (11*C*a*b^4 + 10*A*b^5)*d)*x^7 +
 2048*B*a^4*b*d + 1280*(10*B*a*b^4*d + (10*C*a*b^4 + 9*A*b^5)*c)*x^6 + 504
*(30*B*a*b^4*c + (C*a^2*b^3 + 30*A*a*b^4)*d)*x^5 + 768*(B*a^2*b^3*d + (C*a
^2*b^3 + 24*A*a*b^4)*c)*x^4 + 630*(2*B*a^2*b^3*c - (C*a^3*b^2 - 2*A*a^2*b^
3)*d)*x^3 - 256*(4*B*a^3*b^2*d + (4*C*a^3*b^2 - 9*A*a^2*b^3)*c)*x^2 + 512*
(4*C*a^4*b - 9*A*a^3*b^2)*c - 945*(2*B*a^3*b^2*c - (C*a^4*b - 2*A*a^3*b^2)
*d)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(294) = 588.

Time = 0.67 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.54

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(x**3*(d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output

```
Piecewise((3*a**2*(A*a**2*d + B*a**2*c - 5*a*(2*A*a*b*d + 2*B*a*b*c + C*a*
*2*d - 7*a*(A*b**2*d + B*b**2*c + 11*C*a*b*d/10)/(8*b))/(6*b))*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(C*b*d*x**9/10 - 3*a*x*(A*a**
2*d + B*a**2*c - 5*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 7*a*(A*b**2*d + B
*b**2*c + 11*C*a*b*d/10)/(8*b))/(6*b))/(8*b**2) - 2*a*(A*a**2*c - 4*a*(2*A
*a*b*c + B*a**2*d + C*a**2*c - 6*a*(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c - 8*a
*(B*b**2*d + C*b**2*c)/(9*b))/(7*b))/(5*b))/(3*b**2) + x**8*(B*b**2*d + C*
b**2*c)/(9*b) + x**7*(A*b**2*d + B*b**2*c + 11*C*a*b*d/10)/(8*b) + x**6*(A
*b**2*c + 2*B*a*b*d + 2*C*a*b*c - 8*a*(B*b**2*d + C*b**2*c)/(9*b))/(7*b) +
 x**5*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 7*a*(A*b**2*d + B*b**2*c + 11*C*
a*b*d/10)/(8*b))/(6*b) + x**4*(2*A*a*b*c + B*a**2*d + C*a**2*c - 6*a*(A*b*
*2*c + 2*B*a*b*d + 2*C*a*b*c - 8*a*(B*b**2*d + C*b**2*c)/(9*b))/(7*b))/(5*
b) + x**3*(A*a**2*d + B*a**2*c - 5*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 7
*a*(A*b**2*d + B*b**2*c + 11*C*a*b*d/10)/(8*b))/(6*b))/(4*b) + x**2*(A*a**
2*c - 4*a*(2*A*a*b*c + B*a**2*d + C*a**2*c - 6*a*(A*b**2*c + 2*B*a*b*d + 2
*C*a*b*c - 8*a*(B*b**2*d + C*b**2*c)/(9*b))/(7*b))/(5*b))/(3*b)), Ne(b, 0)
), (a**(3/2)*(A*c*x**4/4 + C*d*x**7/7 + x**6*(B*d + C*c)/6 + x**5*(A*d + B
*c)/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.12

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cdx^5}{10\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Cadx^3}{16\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}(Cc+Bd)x^4}{9\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Acx^2}{7\,b}+\frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)x^3}{8\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ca^2dx}{32\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}Ca^3dx}{128\,b^3}-\frac{3\sqrt{bx^2+a}Ca^4dx}{256\,b^3}-\frac{4\,(bx^2+a)^{\frac{5}{2}}(Cc+Bd)ax^2}{63\,b^2}-\frac{3\,Ca^5d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Aac}{35\,b^2}-\frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)ax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)a^2x}{64\,b^2}+\frac{3\sqrt{bx^2+a}(Bc+Ad)a^3x}{128\,b^2}+\frac{3\,(Bc+Ad)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}+\frac{8\,(bx^2+a)^{\frac{5}{2}}(Cc+Bd)a^2}{315\,b^3}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(5/2)*C*d*x^5/b - 1/16*(b*x^2 + a)^(5/2)*C*a*d*x^3/b^2 +
1/9*(b*x^2 + a)^(5/2)*(C*c + B*d)*x^4/b + 1/7*(b*x^2 + a)^(5/2)*A*c*x^2/b
+ 1/8*(b*x^2 + a)^(5/2)*(B*c + A*d)*x^3/b + 1/32*(b*x^2 + a)^(5/2)*C*a^2*d
*x/b^3 - 1/128*(b*x^2 + a)^(3/2)*C*a^3*d*x/b^3 - 3/256*sqrt(b*x^2 + a)*C*a
^4*d*x/b^3 - 4/63*(b*x^2 + a)^(5/2)*(C*c + B*d)*a*x^2/b^2 - 3/256*C*a^5*d*
arcsinh(b*x/sqrt(a*b))/b^(7/2) - 2/35*(b*x^2 + a)^(5/2)*A*a*c/b^2 - 1/16*(
b*x^2 + a)^(5/2)*(B*c + A*d)*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*(B*c + A*d)*
a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*(B*c + A*d)*a^3*x/b^2 + 3/128*(B*c + A*d
)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/315*(b*x^2 + a)^(5/2)*(C*c + B*d)
*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.21

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\left(2\left(7\left(8\left(9\,Cbdx+\frac{10\,(Cb^9c+Bb^9d)}{b^8}\right)x+\frac{9\,(10\,Bb^9c+11\,Ca\right.\right.\right.\right.\right.\right.\right.\right.}{b^8}$$
$$-\frac{3\,(2\,Ba^4bc-Ca^5d+2\,Aa^4bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*C*b*d*x + 10*(C*b^9*c + B*b^9
*d)/b^8)*x + 9*(10*B*b^9*c + 11*C*a*b^8*d + 10*A*b^9*d)/b^8)*x + 80*(10*C*
a*b^8*c + 9*A*b^9*c + 10*B*a*b^8*d)/b^8)*x + 63*(30*B*a*b^8*c + C*a^2*b^7*
d + 30*A*a*b^8*d)/b^8)*x + 96*(C*a^2*b^7*c + 24*A*a*b^8*c + B*a^2*b^7*d)/b
^8)*x + 315*(2*B*a^2*b^7*c - C*a^3*b^6*d + 2*A*a^2*b^7*d)/b^8)*x - 128*(4*
C*a^3*b^6*c - 9*A*a^2*b^7*c + 4*B*a^3*b^6*d)/b^8)*x - 945*(2*B*a^3*b^6*c -
 C*a^4*b^5*d + 2*A*a^3*b^6*d)/b^8)*x + 512*(4*C*a^4*b^5*c - 9*A*a^3*b^6*c
+ 4*B*a^4*b^5*d)/b^8) - 3/256*(2*B*a^4*b*c - C*a^5*d + 2*A*a^4*b*d)*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x^3\,{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x^3*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.94

$$\int x^3(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{-1890\sqrt{b\,x^2+a}\,a^3b^3cx-1024\sqrt{b\,x^2+a}\,a^3b^3d\,x^2+1260\sqrt{b\,x^2+a}\,a^2b^4c\,x^3+768\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^3*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
( - 4608*sqrt(a + b*x**2)*a**4*b**2*c - 1890*sqrt(a + b*x**2)*a**4*b**2*d*
x + 2048*sqrt(a + b*x**2)*a**4*b**2*d + 2048*sqrt(a + b*x**2)*a**4*b*c**2
+ 945*sqrt(a + b*x**2)*a**4*b*c*d*x + 2304*sqrt(a + b*x**2)*a**3*b**3*c*x*
*2 - 1890*sqrt(a + b*x**2)*a**3*b**3*c*x + 1260*sqrt(a + b*x**2)*a**3*b**3
*d*x**3 - 1024*sqrt(a + b*x**2)*a**3*b**3*d*x**2 - 1024*sqrt(a + b*x**2)*a
**3*b**2*c**2*x**2 - 630*sqrt(a + b*x**2)*a**3*b**2*c*d*x**3 + 18432*sqrt(
a + b*x**2)*a**2*b**4*c*x**4 + 1260*sqrt(a + b*x**2)*a**2*b**4*c*x**3 + 15
120*sqrt(a + b*x**2)*a**2*b**4*d*x**5 + 768*sqrt(a + b*x**2)*a**2*b**4*d*x
**4 + 768*sqrt(a + b*x**2)*a**2*b**3*c**2*x**4 + 504*sqrt(a + b*x**2)*a**2
*b**3*c*d*x**5 + 11520*sqrt(a + b*x**2)*a*b**5*c*x**6 + 15120*sqrt(a + b*x
**2)*a*b**5*c*x**5 + 10080*sqrt(a + b*x**2)*a*b**5*d*x**7 + 12800*sqrt(a +
 b*x**2)*a*b**5*d*x**6 + 12800*sqrt(a + b*x**2)*a*b**4*c**2*x**6 + 11088*s
qrt(a + b*x**2)*a*b**4*c*d*x**7 + 10080*sqrt(a + b*x**2)*b**6*c*x**7 + 896
0*sqrt(a + b*x**2)*b**6*d*x**8 + 8960*sqrt(a + b*x**2)*b**5*c**2*x**8 + 80
64*sqrt(a + b*x**2)*b**5*c*d*x**9 + 1890*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**5*b*d - 945*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**5*c*d + 1890*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqr
t(a))*a**4*b**2*c)/(80640*b**4)
```

3.48 $\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 275

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{a^2(8Abc-3a(cC+Bd))x\sqrt{a+bx^2}}{128b^2}+\frac{a(8Abc-3a(cC+Bd))x^3\sqrt{a+bx^2}}{64b}+\frac{(8Abc-3a(cC+Bd))x^3\left(a+bx^2\right)^{3/2}}{48b}+\frac{a(aCd-b(Bc+Ad))\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{(cC+Bd)x^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{(bBc+Abd-2aCd)\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{Cd\left(a+bx^2\right)^{9/2}}{9b^3}-\frac{a^3(8Abc-3a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/128*a^2*(8*A*b*c-3*a*(B*d+C*c))*x*(b*x^2+a)^(1/2)/b^2+1/64*a*(8*A*b*c-3*
a*(B*d+C*c))*x^3*(b*x^2+a)^(1/2)/b+1/48*(8*A*b*c-3*a*(B*d+C*c))*x^3*(b*x^2
+a)^(3/2)/b+1/5*a*(a*C*d-b*(A*d+B*c))*(b*x^2+a)^(5/2)/b^3+1/8*(B*d+C*c)*x^
3*(b*x^2+a)^(5/2)/b+1/7*(A*b*d+B*b*c-2*C*a*d)*(b*x^2+a)^(7/2)/b^3+1/9*C*d*
(b*x^2+a)^(9/2)/b^3-1/128*a^3*(8*A*b*c-3*a*(B*d+C*c))*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.89

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\sqrt{a+bx^2}(1024a^4Cd-a^3b(2304Bc+2304Ad+945cCx+945Bdx+512Cdx^2)+8ab^3x^3(6A}{}$$

input
```
Integrate[x^2*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(1024*a^4*C*d - a^3*b*(2304*B*c + 2304*A*d + 945*c*C*x +
945*B*d*x + 512*C*d*x^2) + 8*a*b^3*x^3*(6*A*(245*c + 192*d*x) + x*(1152*B*
c + 945*c*C*x + 945*B*d*x + 800*C*d*x^2)) + 80*b^4*x^5*(12*A*(7*c + 6*d*x)
 + x*(9*B*(8*c + 7*d*x) + 7*C*x*(9*c + 8*d*x))) + 6*a^2*b^2*x*(12*A*(35*c
+ 16*d*x) + x*(3*B*(64*c + 35*d*x) + C*x*(105*c + 64*d*x)))) - 315*a^3*Sqr
t[b]*(-8*A*b*c + 3*a*(c*C + B*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40
320*b^3)
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2185, 25, 2185, 25, 2185, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\left((c+dx)\left(bx^2+a\right)^{3/2}\left(bd^3(23cC-9Bd)x^3+d^2\left(19bCc^2-9Abd^2+4aCd^2\right)x^2+cCd\left(5bc^2+8ad^2\right)x+4ac\right.\right.}{9bd^4}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}$$

$$\Big\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(bd^3(23cC-9Bd)x^3+d^2\left(19bCc^2-9Abd^2+4aCd^2\right)x^2+cCd\left(5bc^2+8ad^2\right)x+4ac^2C\right.}{9bd^4}$$

$$\Big\downarrow 2185$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\frac{\int -\left((c+dx)\left(bx^2+a\right)^{3/2}\left(-b\left(32aCd^2-3b\left(49Cc^2-39Bdc+24Ad^2\right)\right)x^2d^5+abc(37cC-27Bd)d^5+b\left(15b(5cC-3Bd)c^2+ad^2(5cC-27Bd)\right)xd^4\right)\right)dx}{8bd^3}}{9bd^4}$$

$$\Big\downarrow 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(23cC-9Bd)-\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(-b\left(32aCd^2-3b\left(49Cc^2-39Bdc+24Ad^2\right)\right)x^2d^5+abc(37cC-27Bd)d^5\right.}{8bd^3}}{9bd^4}$$

$$\Big\downarrow 2185$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(23cC-9Bd)-\frac{\frac{\int bd^6(c+dx)\left(ad\left(64aCd^2-b\left(35Cc^2-45Bdc+144Ad^2\right)\right)+3b\left(ad^2(65cC-63Bd)-10bc\left(7Cc^2-9Bdc+\right.\right.\right.}{7bd^2}}{8}}{9bd^4}$$

$$\Big\downarrow 27$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(23cC-9Bd)-\frac{\frac{1}{7}d^4\int (c+dx)\left(ad\left(64aCd^2-b\left(35Cc^2-45Bdc+144Ad^2\right)\right)+3b\left(ad^2(65cC-63Bd)-10bc(7Cc\right.\right.}{}}{9bd^4}$$

$$\Big\downarrow 676$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^3}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(23cC-9Bd)-\frac{\frac{1}{7}d^4\left(-\frac{21}{2}ad^3(8Abc-3a(Bd+cC))\int \left(bx^2+a\right)^{3/2}dx+\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2Cd^4+8abd^2\left(-9Ad^2\right.\right.}{}\right.}{}}{}$$

$$\Big\downarrow 211$$

$$\frac{C(a+bx^2)^{5/2}(c+dx)^4}{9bd^3} - \frac{\frac{1}{8}d(a+bx^2)^{5/2}(c+dx)^3(23cC-9Bd) - \frac{\frac{1}{7}d^4\left(-\frac{21}{2}ad^3(8Abc-3a(Bd+cC))\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{2(a+bx^2)^{5/2}(32a}{}\right.}{}}{}$$

$\downarrow$ 211

$$\frac{C(a+bx^2)^{5/2}(c+dx)^4}{9bd^3} - \frac{\frac{1}{8}d(a+bx^2)^{5/2}(c+dx)^3(23cC-9Bd) - \frac{\frac{1}{7}d^4\left(-\frac{21}{2}ad^3(8Abc-3a(Bd+cC))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right.\right.}{}}{}$$

$\downarrow$ 224

$$\frac{C(a+bx^2)^{5/2}(c+dx)^4}{9bd^3} - \frac{\frac{1}{8}d(a+bx^2)^{5/2}(c+dx)^3(23cC-9Bd) - \frac{\frac{1}{7}d^4\left(-\frac{21}{2}ad^3(8Abc-3a(Bd+cC))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a\right.\right.}{}}{}$$

$\downarrow$ 219

$$\frac{C(a+bx^2)^{5/2}(c+dx)^4}{9bd^3} - \frac{\frac{1}{8}d(a+bx^2)^{5/2}(c+dx)^3(23cC-9Bd) - \frac{\frac{1}{7}d^4\left(\frac{2(a+bx^2)^{5/2}(32a^2Cd^4+8abd^2(-9Ad^2-9Bcd+10c^2C)-15b^2c^2(12Ad^2-9Bcd+7c^2C))}{5b}-\right.}{}}{}$$

input
```
Int[x^2*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^4*(a + b*x^2)^(5/2))/(9*b*d^3) - ((d*(23*c*C - 9*B*d)*(c + d*
x)^3*(a + b*x^2)^(5/2))/8 - ((d^4*(147*b*c^2*C - 117*b*B*c*d + 72*A*b*d^2
- 32*a*C*d^2)*(c + d*x)^2*(a + b*x^2)^(5/2))/7 + (d^4*((2*(32*a^2*C*d^4 +
8*a*b*d^2*(10*c^2*C - 9*B*c*d - 9*A*d^2) - 15*b^2*c^2*(7*c^2*C - 9*B*c*d +
 12*A*d^2))*(a + b*x^2)^(5/2))/(5*b) + (d*(a*d^2*(65*c*C - 63*B*d) - 10*b*
c*(7*c^2*C - 9*B*c*d + 12*A*d^2))*x*(a + b*x^2)^(5/2))/2 - (21*a*d^3*(8*A*
b*c - 3*a*(c*C + B*d))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2]
)/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/2))/7)/(8
*b*d^3))/(9*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.03

method	result
default	$(Ad+Bc)\left(\frac{x^2(bx^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(bx^2+a)^{\frac{5}{2}}}{35b^2}\right)+(Bd+Cc)\left(\frac{x^3(bx^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}\right.$
risch	$-\frac{(-4480Cb^4dx^8-5040Bb^4dx^7-5040Cb^4cx^7-5760Ab^4dx^6-5760Bb^4cx^6-6400Cab^3dx^6-6720Ab^4cx^5-7560Bab^3dx^5-75\cdots}{\cdots}$

input
```
int(x^2*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
(A*d+B*c)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+(B*d+C*c)
*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*
x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))))))+A*c*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)
^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2)))))+d*C*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b
-2/35*a/b^2*(b*x^2+a)^(5/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.30

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \left[\frac{315\,(3\,Ba^4d+(3\,Ca^4-8\,Aa^3b)c)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(4480\,Cb^4dx^8+\cdots}{\cdots}\right.$$

$$-\frac{315\,(3\,Ba^4d+(3\,Ca^4-8\,Aa^3b)c)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(4480\,Cb^4dx^8+5040\,(Cb^4c+Bb^4d)x^7+640\,\cdots}{\cdots}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/80640*(315*(3*B*a^4*d + (3*C*a^4 - 8*A*a^3*b)*c)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(4480*C*b^4*d*x^8 + 5040*(C*b^4*c +
B*b^4*d)*x^7 + 640*(9*B*b^4*c + (10*C*a*b^3 + 9*A*b^4)*d)*x^6 - 2304*B*a^3
*b*c + 840*(9*B*a*b^3*d + (9*C*a*b^3 + 8*A*b^4)*c)*x^5 + 384*(24*B*a*b^3*c
 + (C*a^2*b^2 + 24*A*a*b^3)*d)*x^4 + 210*(3*B*a^2*b^2*d + (3*C*a^2*b^2 + 5
6*A*a*b^3)*c)*x^3 + 128*(9*B*a^2*b^2*c - (4*C*a^3*b - 9*A*a^2*b^2)*d)*x^2
+ 256*(4*C*a^4 - 9*A*a^3*b)*d - 315*(3*B*a^3*b*d + (3*C*a^3*b - 8*A*a^2*b^
2)*c)*x)*sqrt(b*x^2 + a))/b^3, -1/40320*(315*(3*B*a^4*d + (3*C*a^4 - 8*A*a
^3*b)*c)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (4480*C*b^4*d*x^8 +
 5040*(C*b^4*c + B*b^4*d)*x^7 + 640*(9*B*b^4*c + (10*C*a*b^3 + 9*A*b^4)*d)
*x^6 - 2304*B*a^3*b*c + 840*(9*B*a*b^3*d + (9*C*a*b^3 + 8*A*b^4)*c)*x^5 +
384*(24*B*a*b^3*c + (C*a^2*b^2 + 24*A*a*b^3)*d)*x^4 + 210*(3*B*a^2*b^2*d +
 (3*C*a^2*b^2 + 56*A*a*b^3)*c)*x^3 + 128*(9*B*a^2*b^2*c - (4*C*a^3*b - 9*A
*a^2*b^2)*d)*x^2 + 256*(4*C*a^4 - 9*A*a^3*b)*d - 315*(3*B*a^3*b*d + (3*C*a
^3*b - 8*A*a^2*b^2)*c)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(260) = 520$.

Time = 0.65 (sec) , antiderivative size = 699, normalized size of antiderivative = 2.54

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\begin{cases}-\frac{a\left(Aa^2c-\frac{3a\left(2Aabc+Ba^2d+Ca^2c-\frac{5a\left(Ab^2c+2Babd+2Cabc-\frac{7a\left(Bb^2d+Cb^2c\right)}{8b}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherw}\end{cases}\right)}{2b} \\ a^{\frac{3}{2}}\left(\frac{Acx^3}{3}+\frac{Cdx^6}{6}+\frac{x^5(Bd+Cc)}{5}+\frac{x^4(Ad+Bc)}{4}\right)\end{cases}$$

input
```
integrate(x**2*(d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a**2*c - 3*a*(2*A*a*b*c + B*a**2*d + C*a**2*c - 5*a*(A*b*
*2*c + 2*B*a*b*d + 2*C*a*b*c - 7*a*(B*b**2*d + C*b**2*c)/(8*b))/(6*b))/(4*
b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)),
 (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*b*d*x**8/9 - 2
*a*(A*a**2*d + B*a**2*c - 4*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 6*a*(A*b
**2*d + B*b**2*c + 10*C*a*b*d/9)/(7*b))/(5*b))/(3*b**2) + x**7*(B*b**2*d +
 C*b**2*c)/(8*b) + x**6*(A*b**2*d + B*b**2*c + 10*C*a*b*d/9)/(7*b) + x**5*
(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c - 7*a*(B*b**2*d + C*b**2*c)/(8*b))/(6*b)
 + x**4*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 6*a*(A*b**2*d + B*b**2*c + 10*
C*a*b*d/9)/(7*b))/(5*b) + x**3*(2*A*a*b*c + B*a**2*d + C*a**2*c - 5*a*(A*b
**2*c + 2*B*a*b*d + 2*C*a*b*c - 7*a*(B*b**2*d + C*b**2*c)/(8*b))/(6*b))/(4
*b) + x**2*(A*a**2*d + B*a**2*c - 4*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d -
6*a*(A*b**2*d + B*b**2*c + 10*C*a*b*d/9)/(7*b))/(5*b))/(3*b) + x*(A*a**2*c
 - 3*a*(2*A*a*b*c + B*a**2*d + C*a**2*c - 5*a*(A*b**2*c + 2*B*a*b*d + 2*C*
a*b*c - 7*a*(B*b**2*d + C*b**2*c)/(8*b))/(6*b))/(4*b))/(2*b)), Ne(b, 0)),
(a**(3/2)*(A*c*x**3/3 + C*d*x**6/6 + x**5*(B*d + C*c)/5 + x**4*(A*d + B*c)
/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.09

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}Cdx^4}{9\,b}-\frac{4\left(bx^2+a\right)^{\frac{5}{2}}Cadx^2}{63\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)x^3}{8\,b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Acx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Aacx}{24\,b}-\frac{\sqrt{bx^2+a}Aa^2cx}{16\,b}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)x^2}{7\,b}-\frac{Aa^3c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{8\left(bx^2+a\right)^{\frac{5}{2}}Ca^2d}{315\,b^3}-\frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)ax}{16\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)a^2x}{64\,b^2}+\frac{3\sqrt{bx^2+a}(Cc+Bd)a^3x}{128\,b^2}+\frac{3\,(Cc+Bd)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)a}{35\,b^2}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(5/2)*C*d*x^4/b - 4/63*(b*x^2 + a)^(5/2)*C*a*d*x^2/b^2 + 1
/8*(b*x^2 + a)^(5/2)*(C*c + B*d)*x^3/b + 1/6*(b*x^2 + a)^(5/2)*A*c*x/b - 1
/24*(b*x^2 + a)^(3/2)*A*a*c*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*c*x/b + 1/7*(
b*x^2 + a)^(5/2)*(B*c + A*d)*x^2/b - 1/16*A*a^3*c*arcsinh(b*x/sqrt(a*b))/b
^(3/2) + 8/315*(b*x^2 + a)^(5/2)*C*a^2*d/b^3 - 1/16*(b*x^2 + a)^(5/2)*(C*c
 + B*d)*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*(C*c + B*d)*a^2*x/b^2 + 3/128*sqr
t(b*x^2 + a)*(C*c + B*d)*a^3*x/b^2 + 3/128*(C*c + B*d)*a^4*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 2/35*(b*x^2 + a)^(5/2)*(B*c + A*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.23

$$\int x^2(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)\;dx = \frac{1}{40320}\sqrt{bx^2+a}\Bigg(\bigg(2\bigg(\bigg(\bigg(4\bigg(5\bigg(2\bigg(7\bigg(8\,Cbdx+\frac{9\,(Cb^8c+Bb^8d)}{b^7}\bigg)x+\frac{8\,(9\,Bb^8c+10\,Cab^7d}{b^7}$$
$$-\frac{(3\,Ca^4c-8\,Aa^3bc+3\,Ba^4d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/40320*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*(8*C*b*d*x + 9*(C*b^8*c + B*b^8*d
)/b^7)*x + 8*(9*B*b^8*c + 10*C*a*b^7*d + 9*A*b^8*d)/b^7)*x + 21*(9*C*a*b^7
*c + 8*A*b^8*c + 9*B*a*b^7*d)/b^7)*x + 48*(24*B*a*b^7*c + C*a^2*b^6*d + 24
*A*a*b^7*d)/b^7)*x + 105*(3*C*a^2*b^6*c + 56*A*a*b^7*c + 3*B*a^2*b^6*d)/b^
7)*x + 64*(9*B*a^2*b^6*c - 4*C*a^3*b^5*d + 9*A*a^2*b^6*d)/b^7)*x - 315*(3*
C*a^3*b^5*c - 8*A*a^2*b^6*c + 3*B*a^3*b^5*d)/b^7)*x - 256*(9*B*a^3*b^5*c -
 4*C*a^4*b^4*d + 9*A*a^3*b^5*d)/b^7) - 1/128*(3*C*a^4*c - 8*A*a^3*b*c + 3*
B*a^4*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x^2\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.95

$$\int x^2(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{945\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^4bd-512\sqrt{b\,x^2+a}\,a^3bcd\,x^2+384\sqrt{b\,x^2+a}\,a^2b^2cd\,x^4+6400\sqrt{b\,x^2}}{}$$

input
```
int(x^2*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output

```
( - 2304*sqrt(a + b*x**2)*a**4*b*d + 1024*sqrt(a + b*x**2)*a**4*c*d + 2520
*sqrt(a + b*x**2)*a**3*b**2*c*x - 2304*sqrt(a + b*x**2)*a**3*b**2*c + 1152
*sqrt(a + b*x**2)*a**3*b**2*d*x**2 - 945*sqrt(a + b*x**2)*a**3*b**2*d*x -
945*sqrt(a + b*x**2)*a**3*b*c**2*x - 512*sqrt(a + b*x**2)*a**3*b*c*d*x**2
+ 11760*sqrt(a + b*x**2)*a**2*b**3*c*x**3 + 1152*sqrt(a + b*x**2)*a**2*b**
3*c*x**2 + 9216*sqrt(a + b*x**2)*a**2*b**3*d*x**4 + 630*sqrt(a + b*x**2)*a
**2*b**3*d*x**3 + 630*sqrt(a + b*x**2)*a**2*b**2*c**2*x**3 + 384*sqrt(a +
b*x**2)*a**2*b**2*c*d*x**4 + 6720*sqrt(a + b*x**2)*a*b**4*c*x**5 + 9216*sq
rt(a + b*x**2)*a*b**4*c*x**4 + 5760*sqrt(a + b*x**2)*a*b**4*d*x**6 + 7560*
sqrt(a + b*x**2)*a*b**4*d*x**5 + 7560*sqrt(a + b*x**2)*a*b**3*c**2*x**5 +
6400*sqrt(a + b*x**2)*a*b**3*c*d*x**6 + 5760*sqrt(a + b*x**2)*b**5*c*x**6
+ 5040*sqrt(a + b*x**2)*b**5*d*x**7 + 5040*sqrt(a + b*x**2)*b**4*c**2*x**7
 + 4480*sqrt(a + b*x**2)*b**4*c*d*x**8 - 2520*sqrt(b)*log((sqrt(a + b*x**2
) + sqrt(b)*x)/sqrt(a))*a**4*b*c + 945*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**4*b*d + 945*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)
/sqrt(a))*a**4*c**2)/(40320*b**3)
```

3.49 $\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)dx$

Optimal result

Integrand size = 28, antiderivative size = 242

$$\begin{aligned}\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)dx= & -\frac{a^2(3aCd-8b(Bc+Ad))x\sqrt{a+bx^2}}{128b^2}-\frac{a(3aCd-8b(Bc+Ad))x^3\sqrt{a+bx^2}}{64b}\\ & -\frac{(3aCd-8b(Bc+Ad))x^3(a+bx^2)^{3/2}}{48b}\\ & +\frac{(Abc-a(cC+Bd))\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{Cdx^3(a+bx^2)^{5/2}}{8b}\\ & +\frac{(cC+Bd)\left(a+bx^2\right)^{7/2}}{7b^2}+\frac{a^3(3aCd-8b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}\end{aligned}$$

output
```
-1/128*a^2*(3*a*C*d-8*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b^2-1/64*a*(3*a*C*d-8
*b*(A*d+B*c))*x^3*(b*x^2+a)^(1/2)/b-1/48*(3*a*C*d-8*b*(A*d+B*c))*x^3*(b*x^
2+a)^(3/2)/b+1/5*(A*b*c-a*(B*d+C*c))*(b*x^2+a)^(5/2)/b^2+1/8*C*d*x^3*(b*x^
2+a)^(5/2)/b+1/7*(B*d+C*c)*(b*x^2+a)^(7/2)/b^2+1/128*a^3*(3*a*C*d-8*b*(A*d
+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.92

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(-3a^3(256cC+256Bd+105Cdx)+6a^2b(28A(16c+5dx)+x(140Bc+64cCx}{}$$

input
```
Integrate[x*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-3*a^3*(256*c*C + 256*B*d + 105*C*d*x) + 6*a^2*b
*(28*A*(16*c + 5*d*x) + x*(140*B*c + 64*c*C*x + 64*B*d*x + 35*C*d*x^2)) +
8*a*b^2*x^2*(14*A*(48*c + 35*d*x) + x*(490*B*c + 384*c*C*x + 384*B*d*x + 3
15*C*d*x^2)) + 16*b^3*x^4*(28*A*(6*c + 5*d*x) + 5*x*(4*B*(7*c + 6*d*x) + 3
*C*x*(8*c + 7*d*x)))) - 105*a^3*(3*a*C*d - 8*b*(B*c + A*d))*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(13440*b^(5/2))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2185, 25, 2185, 25, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\left((c+dx)\left(bx^2+a\right)^{3/2}\left(b(13cC-8Bd)x^2d^2+3acCd^2+\left(5bCc^2-8Abd^2+3aCd^2\right)xd\right)\right)dx}{8bd^3}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\int (c+dx)\left(bx^2+a\right)^{3/2}\left(b(13cC-8Bd)x^2d^2+3acCd^2+\left(5bCc^2-8Abd^2+3aCd^2\right)xd\right)dx}{8bd^3}$$

$\Big\downarrow$ 2185

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{\int -bd^3(c+dx)\left(ad(5cC-16Bd)-\left(21aCd^2-b\left(30Cc^2-40Bdc+56Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7bd^2}+\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(13cC-8Bd)}{8bd^3}$$

$\Big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{\int bd^3(c+dx)\left(ad(5cC-16Bd)-\left(21aCd^2-b\left(30Cc^2-40Bdc+56Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7bd^2}}{8bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\int (c+dx)\left(ad(5cC-16Bd)-\left(21aCd^2-b\left(30Cc^2-40Bdc+56A\right.\right.\right.}{8bd^3}$$

$\Big\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\left(\frac{7ad^2(3aCd-8b(Ad+Bc))\int\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Bd+cC)-bc(28A\right.}{5b}\right.}{8bd^3}$$

$\Big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d\left(a+bx^2\right)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\left(\frac{7ad^2(3aCd-8b(Ad+Bc))\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad\right.}{}\right.}{8bd^3}$$

$\Big\downarrow$ 211

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d(a+bx^2)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\left(\frac{7ad^2(3aCd-8b(Ad+Bc))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}\right.}{8bd^3}$$

$\Big\downarrow$ 224

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d(a+bx^2)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\left(\frac{7ad^2(3aCd-8b(Ad+Bc))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2\right.}{6b}\right.}{8bd^3}$$

$\Big\downarrow$ 219

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd^2}-\frac{\frac{1}{7}d(a+bx^2)^{5/2}(c+dx)^2(13cC-8Bd)-\frac{1}{7}d\left(\frac{7ad^2\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(3aCd-8b(Ad\right.}{6b}\right.}{8bd^3}$$

input
```
Int[x*(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^3*(a + b*x^2)^(5/2))/(8*b*d^2) - ((d*(13*c*C - 8*B*d)*(c + d*
x)^2*(a + b*x^2)^(5/2))/7 - (d*((-2*(8*a*d^2*(c*C + B*d) - b*c*(15*c^2*C -
 20*B*c*d + 28*A*d^2))*(a + b*x^2)^(5/2))/(5*b) - (d*(21*a*C*d^2 - b*(30*c
^2*C - 40*B*c*d + 56*A*d^2))*x*(a + b*x^2)^(5/2))/(6*b) + (7*a*d^2*(3*a*C*
d - 8*b*(B*c + A*d))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/
2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b)))/7)/
(8*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99

method	result
default	$(Ad+Bc)\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+(Bd+Cc)\left(\frac{x^2(b\,x^2+a)^{\frac{5}{2}}}{7b}\right.$
risch	$\frac{(1680dC\,b^3x^7+1920B\,b^3d\,x^6+1920C\,b^3c\,x^6+2240A\,b^3d\,x^5+2240B\,b^3c\,x^5+2520Ca\,b^2d\,x^5+2688A\,b^3c\,x^4+3072Ba\,b^2d\,x^4+3072Ca}{}$

input
```
int(x*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
(A*d+B*c)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1
/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+(B*d+C
*c)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+1/5*A*c*(b*x^2+
a)^(5/2)/b+d*C*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b
-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)
*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.37

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\left[\frac{105\left(8\,Ba^3bc-(3\,Ca^4-8\,Aa^3b)d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(1680\,Cb^4dx^7}{}\right.$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/26880*(105*(8*B*a^3*b*c - (3*C*a^4 - 8*A*a^3*b)*d)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(1680*C*b^4*d*x^7 + 1920*(C*b^4*c
+ B*b^4*d)*x^6 - 768*B*a^3*b*d + 280*(8*B*b^4*c + (9*C*a*b^3 + 8*A*b^4)*d)
*x^5 + 384*(8*B*a*b^3*d + (8*C*a*b^3 + 7*A*b^4)*c)*x^4 + 70*(56*B*a*b^3*c
+ (3*C*a^2*b^2 + 56*A*a*b^3)*d)*x^3 + 384*(B*a^2*b^2*d + (C*a^2*b^2 + 14*A
*a*b^3)*c)*x^2 - 384*(2*C*a^3*b - 7*A*a^2*b^2)*c + 105*(8*B*a^2*b^2*c - (3
*C*a^3*b - 8*A*a^2*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/13440*(105*(8*B*a^3*
b*c - (3*C*a^4 - 8*A*a^3*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 + (1680*C*b^4*d*x^7 + 1920*(C*b^4*c + B*b^4*d)*x^6 - 768*B*a^3*b*d + 280*
(8*B*b^4*c + (9*C*a*b^3 + 8*A*b^4)*d)*x^5 + 384*(8*B*a*b^3*d + (8*C*a*b^3
+ 7*A*b^4)*c)*x^4 + 70*(56*B*a*b^3*c + (3*C*a^2*b^2 + 56*A*a*b^3)*d)*x^3 +
 384*(B*a^2*b^2*d + (C*a^2*b^2 + 14*A*a*b^3)*c)*x^2 - 384*(2*C*a^3*b - 7*A
*a^2*b^2)*c + 105*(8*B*a^2*b^2*c - (3*C*a^3*b - 8*A*a^2*b^2)*d)*x)*sqrt(b*
x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(224) = 448$.

Time = 0.63 (sec) , antiderivative size = 598, normalized size of antiderivative = 2.47

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\begin{cases} -\frac{a\left(Aa^2d+Ba^2c-\frac{3a\left(2Aabd+2Babc+Ca^2d-\frac{5a\left(Ab^2d+Bb^2c+\frac{9Cabd}{8}\right)}{6b}\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+ \\ a^{\frac{3}{2}}\left(\frac{Acx^2}{2}+\frac{Cdx^5}{5}+\frac{x^4(Bd+Cc)}{4}+\frac{x^3(Ad+Bc)}{3}\right) \end{cases}$$

input
```
integrate(x*(d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a**2*d + B*a**2*c - 3*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d
 - 5*a*(A*b**2*d + B*b**2*c + 9*C*a*b*d/8)/(6*b))/(4*b))*Piecewise((log(2*
sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**
2), True))/(2*b) + sqrt(a + b*x**2)*(C*b*d*x**7/8 + x**6*(B*b**2*d + C*b**
2*c)/(7*b) + x**5*(A*b**2*d + B*b**2*c + 9*C*a*b*d/8)/(6*b) + x**4*(A*b**2
*c + 2*B*a*b*d + 2*C*a*b*c - 6*a*(B*b**2*d + C*b**2*c)/(7*b))/(5*b) + x**3
*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 5*a*(A*b**2*d + B*b**2*c + 9*C*a*b*d/
8)/(6*b))/(4*b) + x**2*(2*A*a*b*c + B*a**2*d + C*a**2*c - 4*a*(A*b**2*c +
2*B*a*b*d + 2*C*a*b*c - 6*a*(B*b**2*d + C*b**2*c)/(7*b))/(5*b))/(3*b) + x*
(A*a**2*d + B*a**2*c - 3*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 5*a*(A*b**2
*d + B*b**2*c + 9*C*a*b*d/8)/(6*b))/(4*b))/(2*b) + (A*a**2*c - 2*a*(2*A*a*
b*c + B*a**2*d + C*a**2*c - 4*a*(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c - 6*a*(B
*b**2*d + C*b**2*c)/(7*b))/(5*b))/(3*b))/b), Ne(b, 0)), (a**(3/2)*(A*c*x**
2/2 + C*d*x**5/5 + x**4*(B*d + C*c)/4 + x**3*(A*d + B*c)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.05

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cdx^3}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Cadx}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ca^2dx}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}Ca^3dx}{128\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}(Cc+Bd)x^2}{7\,b}+\frac{3\,Ca^4d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}+\frac{(bx^2+a)^{\frac{5}{2}}Ac}{5\,b}+\frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)x}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)ax}{24\,b}-\frac{\sqrt{bx^2+a}(Bc+Ad)a^2x}{16\,b}-\frac{(Bc+Ad)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}(Cc+Bd)a}{35\,b^2}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*C*d*x^3/b - 1/16*(b*x^2 + a)^(5/2)*C*a*d*x/b^2 + 1/6
4*(b*x^2 + a)^(3/2)*C*a^2*d*x/b^2 + 3/128*sqrt(b*x^2 + a)*C*a^3*d*x/b^2 +
1/7*(b*x^2 + a)^(5/2)*(C*c + B*d)*x^2/b + 3/128*C*a^4*d*arcsinh(b*x/sqrt(a
*b))/b^(5/2) + 1/5*(b*x^2 + a)^(5/2)*A*c/b + 1/6*(b*x^2 + a)^(5/2)*(B*c +
A*d)*x/b - 1/24*(b*x^2 + a)^(3/2)*(B*c + A*d)*a*x/b - 1/16*sqrt(b*x^2 + a)
*(B*c + A*d)*a^2*x/b - 1/16*(B*c + A*d)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
 - 2/35*(b*x^2 + a)^(5/2)*(C*c + B*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.24

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{1}{13440}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(6\left(7\,Cbdx+\frac{8\,(Cb^7c+Bb^7d)}{b^6}\right)x+\frac{7\,(8\,Bb^7c+9\,Cab^6d+8\,A}{b^6}\right.\right.\right.\right.\right.\right.\right.$$
$$+\frac{(8\,Ba^3bc-3\,Ca^4d+8\,Aa^3bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*C*b*d*x + 8*(C*b^7*c + B*b^7*d)/b
^6)*x + 7*(8*B*b^7*c + 9*C*a*b^6*d + 8*A*b^7*d)/b^6)*x + 48*(8*C*a*b^6*c +
 7*A*b^7*c + 8*B*a*b^6*d)/b^6)*x + 35*(56*B*a*b^6*c + 3*C*a^2*b^5*d + 56*A
*a*b^6*d)/b^6)*x + 192*(C*a^2*b^5*c + 14*A*a*b^6*c + B*a^2*b^5*d)/b^6)*x +
 105*(8*B*a^2*b^5*c - 3*C*a^3*b^4*d + 8*A*a^2*b^5*d)/b^6)*x - 384*(2*C*a^3
*b^4*c - 7*A*a^2*b^5*c + 2*B*a^3*b^4*d)/b^6) + 1/128*(8*B*a^3*b*c - 3*C*a^
4*d + 8*A*a^3*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x\left(b\,x^2+a\right)^{3/2}\left(c+d\,x\right)\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int(x*(a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.99

$$\int x(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{2688\sqrt{b\,x^2+a}\,a^3b^2c+840\sqrt{b\,x^2+a}\,a^3b^2dx-768\sqrt{b\,x^2+a}\,a^3b^2d-768\sqrt{b\,x^2+a}\,a^3b\,c^2-31}{}$$

input
```
int(x*(d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
(2688*sqrt(a + b*x**2)*a**3*b**2*c + 840*sqrt(a + b*x**2)*a**3*b**2*d*x -
768*sqrt(a + b*x**2)*a**3*b**2*d - 768*sqrt(a + b*x**2)*a**3*b*c**2 - 315*
sqrt(a + b*x**2)*a**3*b*c*d*x + 5376*sqrt(a + b*x**2)*a**2*b**3*c*x**2 + 8
40*sqrt(a + b*x**2)*a**2*b**3*c*x + 3920*sqrt(a + b*x**2)*a**2*b**3*d*x**3
 + 384*sqrt(a + b*x**2)*a**2*b**3*d*x**2 + 384*sqrt(a + b*x**2)*a**2*b**2*
c**2*x**2 + 210*sqrt(a + b*x**2)*a**2*b**2*c*d*x**3 + 2688*sqrt(a + b*x**2
)*a*b**4*c*x**4 + 3920*sqrt(a + b*x**2)*a*b**4*c*x**3 + 2240*sqrt(a + b*x*
*2)*a*b**4*d*x**5 + 3072*sqrt(a + b*x**2)*a*b**4*d*x**4 + 3072*sqrt(a + b*
x**2)*a*b**3*c**2*x**4 + 2520*sqrt(a + b*x**2)*a*b**3*c*d*x**5 + 2240*sqrt
(a + b*x**2)*b**5*c*x**5 + 1920*sqrt(a + b*x**2)*b**5*d*x**6 + 1920*sqrt(a
 + b*x**2)*b**4*c**2*x**6 + 1680*sqrt(a + b*x**2)*b**4*c*d*x**7 - 840*sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*d + 315*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*c*d - 840*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b**2*c)/(13440*b**3)
```

3.50 $\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 27, antiderivative size = 197

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{a(6Abc-acC-aBd)x\sqrt{a+bx^2}}{16b}+\frac{(6Abc-acC-aBd)x(a+bx^2)^{3/2}}{24b}+\frac{(bBc+Abd-aCd)\left(a+bx^2\right)^{5/2}}{5b^2}+\frac{(cC+Bd)x(a+bx^2)^{5/2}}{6b}+\frac{Cd(a+bx^2)^{7/2}}{7b^2}+\frac{a^2(6Abc-acC-aBd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a*(6*A*b*c-B*a*d-C*a*c)*x*(b*x^2+a)^(1/2)/b+1/24*(6*A*b*c-B*a*d-C*a*c
)*x*(b*x^2+a)^(3/2)/b+1/5*(A*b*d+B*b*c-C*a*d)*(b*x^2+a)^(5/2)/b^2+1/6*(B*d
+C*c)*x*(b*x^2+a)^(5/2)/b+1/7*C*d*(b*x^2+a)^(7/2)/b^2+1/16*a^2*(6*A*b*c-B*
a*d-C*a*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\sqrt{a+bx^2}(-96a^3Cd+3a^2b(112Ad+7B(16c+5dx)+Cx(35c+16dx))+4b^3x^3(21A(5c+4$$

input
```
Integrate[(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*C*d + 3*a^2*b*(112*A*d + 7*B*(16*c + 5*d*x) + C*
x*(35*c + 16*d*x)) + 4*b^3*x^3*(21*A*(5*c + 4*d*x) + 2*x*(7*B*(6*c + 5*d*x
) + 5*C*x*(7*c + 6*d*x))) + 2*a*b^2*x*(21*A*(25*c + 16*d*x) + x*(7*B*(48*c
 + 35*d*x) + C*x*(245*c + 192*d*x)))) + 105*a^2*Sqrt[b]*(-6*A*b*c + a*c*C
+ a*B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(1680*b^2)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2185, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int d(c+dx)((7Ab-2aC)d-b(5cC-7Bd)x)\left(bx^2+a\right)^{3/2}dx}{7bd^2}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (c+dx)((7Ab-2aC)d-b(5cC-7Bd)x)\left(bx^2+a\right)^{3/2}dx}{7bd}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{7}{6}d(-aBd-acC+6Abc)\int \left(bx^2+a\right)^{3/2}dx-\frac{\left(a+bx^2\right)^{5/2}\left(2aCd^2+b\left(-7Ad^2-7Bcd+5c^2C\right)\right)}{5b}-\frac{1}{6}dx\left(a+bx^2\right)^{5/2}(5cC-7}{7bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{7}{6}d(-aBd-acC+6Abc)\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(2aCd^2+b\left(-7Ad^2-7Bcd+5c^2C\right)\right)}{5b}-\frac{1}{6}dx(}{7bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \ 211$$

$$\frac{\frac{7}{6}d(-aBd-acC+6Abc)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(2aCd^2+b\left(-7Ad^2-7\right.\right.}{5b}}{7bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{7}{6}d(-aBd-acC+6Abc)\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}\left(2aCd^2+b\left(-\right.\right.}{5b}}{7bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{7}{6}d\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)(-aBd-acC+6Abc)-\frac{\left(a+bx^2\right)^{5/2}\left(2aCd^2+b\left(-\right.\right.}{5b}}{7bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd}$$

input
```
Int[(c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^2*(a + b*x^2)^(5/2))/(7*b*d) + (-1/5*((2*a*C*d^2 + b*(5*c^2*C
 - 7*B*c*d - 7*A*d^2))*(a + b*x^2)^(5/2))/b - (d*(5*c*C - 7*B*d)*x*(a + b*
x^2)^(5/2))/6 + (7*d*(6*A*b*c - a*c*C - a*B*d)*((x*(a + b*x^2)^(3/2))/4 +
(3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*
Sqrt[b])))/4))/6)/(7*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.98

method	result
default	$Ac\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{(Ad+Bc)(bx^2+a)^{\frac{5}{2}}}{5b}+(Bd+Cc)\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\right.$
risch	$\frac{(240Cb^3dx^6+280Bb^3dx^5+280Cb^3cx^5+336Ab^3dx^4+336Bb^3cx^4+384Cab^2dx^4+420Ab^3cx^3+490Bab^2dx^3+490Cab^2cx^3+6}{1680b}$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*(A*d+B*c)*(b*x^2+a)^(5/2)/b+(B*d+C*c)*(1/6
*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)
^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d*C*(1/7*x^2*(b*x^2+
a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.40

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\left[\frac{105\,(Ba^3d+(Ca^3-6\,Aa^2b)c)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(240\,Cb^3dx^6+28}{}\right.$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/3360*(105*(B*a^3*d + (C*a^3 - 6*A*a^2*b)*c)*sqrt(b)*log(-2*b*x^2 + 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 2*(240*C*b^3*d*x^6 + 280*(C*b^3*c + B*b^3*d
)*x^5 + 336*B*a^2*b*c + 48*(7*B*b^3*c + (8*C*a*b^2 + 7*A*b^3)*d)*x^4 + 70*
(7*B*a*b^2*d + (7*C*a*b^2 + 6*A*b^3)*c)*x^3 + 48*(14*B*a*b^2*c + (C*a^2*b
+ 14*A*a*b^2)*d)*x^2 - 48*(2*C*a^3 - 7*A*a^2*b)*d + 105*(B*a^2*b*d + (C*a^
2*b + 10*A*a*b^2)*c)*x)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*(B*a^3*d + (C*a^
3 - 6*A*a^2*b)*c)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*C*b^3
*d*x^6 + 280*(C*b^3*c + B*b^3*d)*x^5 + 336*B*a^2*b*c + 48*(7*B*b^3*c + (8*
C*a*b^2 + 7*A*b^3)*d)*x^4 + 70*(7*B*a*b^2*d + (7*C*a*b^2 + 6*A*b^3)*c)*x^3
 + 48*(14*B*a*b^2*c + (C*a^2*b + 14*A*a*b^2)*d)*x^2 - 48*(2*C*a^3 - 7*A*a^
2*b)*d + 105*(B*a^2*b*d + (C*a^2*b + 10*A*a*b^2)*c)*x)*sqrt(b*x^2 + a))/b^
2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(189) = 378$.

Time = 0.55 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.57

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \begin{cases} \sqrt{a+bx^2}\left(\frac{Cbdx^6}{7} + \frac{x^5(Bb^2d+Cb^2c)}{6b} + \frac{x^4\left(Ab^2d+Bb^2c+\frac{8Cabd}{7}\right)}{5b} + \frac{x^3\left(Ab^2c+2Babd+2Cabc-\frac{5a\left(Bb^2d+Cb^2c\right)}{6b}\right)}{4b}\right. \\ a^{\frac{3}{2}}\left(Acx + \frac{Cdx^4}{4} + \frac{x^3(Bd+Cc)}{3} + \frac{x^2(Ad+Bc)}{2}\right) \end{cases}$$

input

```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output

```
Piecewise((sqrt(a + b*x**2)*(C*b*d*x**6/7 + x**5*(B*b**2*d + C*b**2*c)/(6*
b) + x**4*(A*b**2*d + B*b**2*c + 8*C*a*b*d/7)/(5*b) + x**3*(A*b**2*c + 2*B
*a*b*d + 2*C*a*b*c - 5*a*(B*b**2*d + C*b**2*c)/(6*b))/(4*b) + x**2*(2*A*a*
b*d + 2*B*a*b*c + C*a**2*d - 4*a*(A*b**2*d + B*b**2*c + 8*C*a*b*d/7)/(5*b)
)/(3*b) + x*(2*A*a*b*c + B*a**2*d + C*a**2*c - 3*a*(A*b**2*c + 2*B*a*b*d +
 2*C*a*b*c - 5*a*(B*b**2*d + C*b**2*c)/(6*b))/(4*b))/(2*b) + (A*a**2*d + B
*a**2*c - 2*a*(2*A*a*b*d + 2*B*a*b*c + C*a**2*d - 4*a*(A*b**2*d + B*b**2*c
 + 8*C*a*b*d/7)/(5*b))/(3*b))/b) + (A*a**2*c - a*(2*A*a*b*c + B*a**2*d + C
*a**2*c - 3*a*(A*b**2*c + 2*B*a*b*d + 2*C*a*b*c - 5*a*(B*b**2*d + C*b**2*c
)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a**(3/2)*(
A*c*x + C*d*x**4/4 + x**3*(B*d + C*c)/3 + x**2*(A*d + B*c)/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.07

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cdx^2}{7\,b}+\frac{1}{4}\,(bx^2+a)^{\frac{3}{2}}Acx+\frac{3}{8}\,\sqrt{bx^2+a}Aacx+\frac{3\,Aa^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{(bx^2+a)^{\frac{5}{2}}Bc}{5\,b}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Cad}{35\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ad}{5\,b}+\frac{(bx^2+a)^{\frac{5}{2}}(Cc+Bd)x}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)ax}{24\,b}-\frac{\sqrt{bx^2+a}(Cc+Bd)a^2x}{16\,b}-\frac{(Cc+Bd)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*C*d*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*c*x + 3/8*sqrt(b
*x^2 + a)*A*a*c*x + 3/8*A*a^2*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^
2 + a)^(5/2)*B*c/b - 2/35*(b*x^2 + a)^(5/2)*C*a*d/b^2 + 1/5*(b*x^2 + a)^(5
/2)*A*d/b + 1/6*(b*x^2 + a)^(5/2)*(C*c + B*d)*x/b - 1/24*(b*x^2 + a)^(3/2)
*(C*c + B*d)*a*x/b - 1/16*sqrt(b*x^2 + a)*(C*c + B*d)*a^2*x/b - 1/16*(C*c
+ B*d)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.29

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{1}{1680}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5\left(6\,Cbdx+\frac{7\,(Cb^6c+Bb^6d)}{b^5}\right)x+\frac{6\,(7\,Bb^6c+8\,Cab^5d+7\,Ab^6d)}{b^5}\right.\right.\right.\right.\right.+\frac{(Ca^3c-6\,Aa^2bc+Ba^3d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/1680*sqrt(b*x^2 + a)*((2*((4*(5*(6*C*b*d*x + 7*(C*b^6*c + B*b^6*d)/b^5)*
x + 6*(7*B*b^6*c + 8*C*a*b^5*d + 7*A*b^6*d)/b^5)*x + 35*(7*C*a*b^5*c + 6*A
*b^6*c + 7*B*a*b^5*d)/b^5)*x + 24*(14*B*a*b^5*c + C*a^2*b^4*d + 14*A*a*b^5
*d)/b^5)*x + 105*(C*a^2*b^4*c + 10*A*a*b^5*c + B*a^2*b^4*d)/b^5)*x + 48*(7
*B*a^2*b^4*c - 2*C*a^3*b^3*d + 7*A*a^2*b^4*d)/b^5) + 1/16*(C*a^3*c - 6*A*a
^2*b*c + B*a^3*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int {\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.10

$$\int (c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{336\sqrt{b\,x^2+a}\,a^3bd-96\sqrt{b\,x^2+a}\,a^3cd+1050\sqrt{b\,x^2+a}\,a^2b^2cx+336\sqrt{b\,x^2+a}\,a^2b^2c+672\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output

```
(336*sqrt(a + b*x**2)*a**3*b*d - 96*sqrt(a + b*x**2)*a**3*c*d + 1050*sqrt(
a + b*x**2)*a**2*b**2*c*x + 336*sqrt(a + b*x**2)*a**2*b**2*c + 672*sqrt(a
+ b*x**2)*a**2*b**2*d*x**2 + 105*sqrt(a + b*x**2)*a**2*b**2*d*x + 105*sqrt
(a + b*x**2)*a**2*b*c**2*x + 48*sqrt(a + b*x**2)*a**2*b*c*d*x**2 + 420*sqr
t(a + b*x**2)*a*b**3*c*x**3 + 672*sqrt(a + b*x**2)*a*b**3*c*x**2 + 336*sqr
t(a + b*x**2)*a*b**3*d*x**4 + 490*sqrt(a + b*x**2)*a*b**3*d*x**3 + 490*sqr
t(a + b*x**2)*a*b**2*c**2*x**3 + 384*sqrt(a + b*x**2)*a*b**2*c*d*x**4 + 33
6*sqrt(a + b*x**2)*b**4*c*x**4 + 280*sqrt(a + b*x**2)*b**4*d*x**5 + 280*sq
rt(a + b*x**2)*b**3*c**2*x**5 + 240*sqrt(a + b*x**2)*b**3*c*d*x**6 + 630*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*c - 105*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*d - 105*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c**2)/(1680*b**2)
```

3.51 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x} dx$

Optimal result

Integrand size = 30, antiderivative size = 221

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x} dx = aAc\sqrt{a+bx^2} - \frac{a(aCd-6b(Bc+Ad))x\sqrt{a+bx^2}}{16b} + \frac{1}{3}Ac(a+bx^2)^{3/2} - \frac{(aCd-6b(Bc+Ad))x(a+bx^2)^{3/2}}{24b} + \frac{(cC+Bd)(a+bx^2)^{5/2}}{5b} + \frac{Cdx(a+bx^2)^{5/2}}{6b} - \frac{a^2(aCd-6b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} - a^{3/2}Ac\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output

```
a*A*c*(b*x^2+a)^(1/2)-1/16*a*(a*C*d-6*b*(A*d+B*c))*x*(b*x^2+a)^(1/2)/b+1/3
*A*c*(b*x^2+a)^(3/2)-1/24*(a*C*d-6*b*(A*d+B*c))*x*(b*x^2+a)^(3/2)/b+1/5*(B
*d+C*c)*(b*x^2+a)^(5/2)/b+1/6*C*d*x*(b*x^2+a)^(5/2)/b-1/16*a^2*(a*C*d-6*b*
(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-a^(3/2)*A*c*arctanh(
(b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx = \frac{\sqrt{a+bx^2}(3a^2(16cC+16Bd+5Cdx)+2ab(5A(32c+15d$$

$$+2a^{3/2}A\text{carctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$+\frac{a^2(aCd-6b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(3*a^2*(16*c*C + 16*B*d + 5*C*d*x) + 2*a*b*(5*A*(32*c + 1
5*d*x) + x*(75*B*c + 48*c*C*x + 48*B*d*x + 35*C*d*x^2)) + 4*b^2*x^2*(5*A*(
4*c + 3*d*x) + x*(3*B*(5*c + 4*d*x) + 2*C*x*(6*c + 5*d*x)))))/(240*b) + 2*
a^(3/2)*A*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (a^2*(a*C*d -
 6*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.95, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2340, 2340, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(6b(cC+Bd)x^2-(aCd-6b(Bc+Ad))x+6Abc\right)}{x}dx}{6b}+\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{5b(6Abc-(aCd-6b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x}dx}{5b}+\frac{6}{5}\left(a+bx^2\right)^{5/2}(Bd+cC)}{6b}+\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 27

$$\frac{\int \frac{(6Abc-(aCd-6b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x}dx+\frac{6}{5}\left(a+bx^2\right)^{5/2}(Bd+cC)}{6b}+\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 535

$$\frac{\frac{1}{4}a\int \frac{3(8Abc-(aCd-6b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8Abc-x(aCd-6b(Ad+Bc)))+\frac{6}{5}\left(a+bx^2\right)^{5/2}(Bd+}{6b}$$
$$\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 27

$$\frac{\frac{3}{4}a\int \frac{(8Abc-(aCd-6b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8Abc-x(aCd-6b(Ad+Bc)))+\frac{6}{5}\left(a+bx^2\right)^{5/2}(Bd+}{6b}$$
$$\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 535

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{16Abc-(aCd-6b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad+Bc)))\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8Abc-x(}{6b}$$
$$\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 538

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-(aCd-6b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad+Bc)}{6b}$$
$$\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$\downarrow$ 224

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-(aCd-6b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ac}{6b}$$
$$\frac{Cdx\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Abc\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-6b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad+Bc)}{6b}$$

$$\frac{Cdx(a+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(8Abc\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-6b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad+Bc}{6b}$$

$$\frac{Cdx(a+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16Ac\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-6b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad+}{6b}$$

$$\frac{Cdx(a+bx^2)^{5/2}}{6b}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-6b(Ad+Bc))}{\sqrt{b}}-\frac{16Abc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16Abc-x(aCd-6b(Ad-}{6b}$$

$$\frac{Cdx(a+bx^2)^{5/2}}{6b}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x,x]
```

output
```
(C*d*x*(a + b*x^2)^(5/2))/(6*b) + (((8*A*b*c - (a*C*d - 6*b*(B*c + A*d))*x
)*(a + b*x^2)^(3/2))/4 + (6*(c*C + B*d)*(a + b*x^2)^(5/2))/5 + (3*a*(((16*
A*b*c - (a*C*d - 6*b*(B*c + A*d))*x)*Sqrt[a + b*x^2])/2 + (a*(-(((a*C*d -
6*b*(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]) - (16*A*b*
c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4)/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

method	result
default	$Ad\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+Bc\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right.}{4}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output

```
A*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+B*c*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2
+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+A*c*(1/3*(b*x^2+a)
^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+
1/5*B*d*(b*x^2+a)^(5/2)/b+1/5*C*c*(b*x^2+a)^(5/2)/b+d*C*(1/6*x*(b*x^2+a)^(
5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b
^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 987, normalized size of antiderivative = 4.47

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output

```
[1/480*(240*A*a^(3/2)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 15*(6*B*a^2*b*c - (C*a^3 - 6*A*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*
sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*C*b^3*d*x^5 + 48*B*a^2*b*d + 48*(C*
b^3*c + B*b^3*d)*x^4 + 10*(6*B*b^3*c + (7*C*a*b^2 + 6*A*b^3)*d)*x^3 + 16*(
6*B*a*b^2*d + (6*C*a*b^2 + 5*A*b^3)*c)*x^2 + 16*(3*C*a^2*b + 20*A*a*b^2)*c
 + 15*(10*B*a*b^2*c + (C*a^2*b + 10*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^2, 1
/240*(120*A*a^(3/2)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 15*(6*B*a^2*b*c - (C*a^3 - 6*A*a^2*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) + (40*C*b^3*d*x^5 + 48*B*a^2*b*d + 48*(C*b^3*c + B*b^3*d)
*x^4 + 10*(6*B*b^3*c + (7*C*a*b^2 + 6*A*b^3)*d)*x^3 + 16*(6*B*a*b^2*d + (6
*C*a*b^2 + 5*A*b^3)*c)*x^2 + 16*(3*C*a^2*b + 20*A*a*b^2)*c + 15*(10*B*a*b^
2*c + (C*a^2*b + 10*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^2, 1/480*(480*A*sqrt
(-a)*a*b^2*c*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 15*(6*B*a^2*b*c - (C*a^3
 - 6*A*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(40*C*b^3*d*x^5 + 48*B*a^2*b*d + 48*(C*b^3*c + B*b^3*d)*x^4 + 10*(6*B*b
^3*c + (7*C*a*b^2 + 6*A*b^3)*d)*x^3 + 16*(6*B*a*b^2*d + (6*C*a*b^2 + 5*A*b
^3)*c)*x^2 + 16*(3*C*a^2*b + 20*A*a*b^2)*c + 15*(10*B*a*b^2*c + (C*a^2*b +
 10*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^2, 1/240*(240*A*sqrt(-a)*a*b^2*c*arc
tan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 15*(6*B*a^2*b*c - (C*a^3 - 6*A*a^2*b)*d)
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*C*b^3*d*x^5 + 48*B*a...
```

Sympy [A] (verification not implemented)

Time = 11.75 (sec) , antiderivative size = 901, normalized size of antiderivative = 4.08

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x,x)
```

output
```
-A*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2*c/(sqrt(b)*x*sqrt(a/(b*x
**2) + 1)) + A*a*sqrt(b)*c*x/sqrt(a/(b*x**2) + 1) + A*a*d*Piecewise((a*Pie
cewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log
(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x,
True)) + A*b*c*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)
/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + A*b*d*Piecewise((-a**2*Piecewise(
(log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqr
t(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**
2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + B*a*c*Piecewise((a*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
B*a*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b,
 0)), (sqrt(a)*x**2/2, True)) + B*b*c*Piecewise((-a**2*Piecewise((log(2*sq
rt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2)
, True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(
b, 0)), (sqrt(a)*x**3/3, True)) + B*b*d*Piecewise((-2*a**2*sqrt(a + b*x**2
)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne
(b, 0)), (sqrt(a)*x**4/4, True)) + C*a*c*Piecewise((a*sqrt(a + b*x**2)/(3*
b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*a*d*P
iecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx = \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cdx}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Cadx}{24\,b} - \frac{\sqrt{bx^2+a}Ca^2dx}{16\,b} - \frac{Ca^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - Aa^{\frac{3}{2}}c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Ac + \sqrt{bx^2+a}Aac + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cc}{5\,b} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bd}{5\,b} + \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}(Bc+Ad)x + \frac{3}{8}\sqrt{bx^2+a}(Bc+Ad)ax + \frac{3\,(Bc+Ad)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*C*d*x/b - 1/24*(b*x^2 + a)^(3/2)*C*a*d*x/b - 1/16*sq
rt(b*x^2 + a)*C*a^2*d*x/b - 1/16*C*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) -
A*a^(3/2)*c*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*A*c + sq
rt(b*x^2 + a)*A*a*c + 1/5*(b*x^2 + a)^(5/2)*C*c/b + 1/5*(b*x^2 + a)^(5/2)*
B*d/b + 1/4*(b*x^2 + a)^(3/2)*(B*c + A*d)*x + 3/8*sqrt(b*x^2 + a)*(B*c + A
*d)*a*x + 3/8*(B*c + A*d)*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\left(c+d\,x\right)\left(C\,x^2+B\,x+A\right)}{x}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x}\,dx=\frac{320\sqrt{b\,x^2+a}\,a^2b^2c+150\sqrt{b\,x^2+a}\,a^2b^2dx+48\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x)
```

output
```
(320*sqrt(a + b*x**2)*a**2*b**2*c + 150*sqrt(a + b*x**2)*a**2*b**2*d*x + 4
8*sqrt(a + b*x**2)*a**2*b**2*d + 48*sqrt(a + b*x**2)*a**2*b*c**2 + 15*sqrt
(a + b*x**2)*a**2*b*c*d*x + 80*sqrt(a + b*x**2)*a*b**3*c*x**2 + 150*sqrt(a
 + b*x**2)*a*b**3*c*x + 60*sqrt(a + b*x**2)*a*b**3*d*x**3 + 96*sqrt(a + b*
x**2)*a*b**3*d*x**2 + 96*sqrt(a + b*x**2)*a*b**2*c**2*x**2 + 70*sqrt(a + b
*x**2)*a*b**2*c*d*x**3 + 60*sqrt(a + b*x**2)*b**4*c*x**3 + 48*sqrt(a + b*x
**2)*b**4*d*x**4 + 48*sqrt(a + b*x**2)*b**3*c**2*x**4 + 40*sqrt(a + b*x**2
)*b**3*c*d*x**5 + 240*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*a**2*b**2*c - 240*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*a**2*b**2*c + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a**3*b*d - 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**3*c*d + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*
b**2*c)/(240*b**2)
```

3.52 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2} dx$

Optimal result

Integrand size = 30, antiderivative size = 222

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2} dx = a(Bc+Ad)\sqrt{a+bx^2} + \frac{3}{8}(4Abc+acC+aBd)x\sqrt{a+bx^2} + \frac{1}{3}(Bc+Ad)\left(a+bx^2\right)^{3/2} + \frac{(4Abc+acC+aBd)x(a+bx^2)^{3/2}}{4a} + \frac{Cd(a+bx^2)^{5/2}}{5b} - \frac{Ac(a+bx^2)^{5/2}}{ax} + \frac{3a(4Abc+acC+aBd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - a^{3/2}(Bc+Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
a*(A*d+B*c)*(b*x^2+a)^(1/2)+3/8*(4*A*b*c+B*a*d+C*a*c)*x*(b*x^2+a)^(1/2)+1/
3*(A*d+B*c)*(b*x^2+a)^(3/2)+1/4*(4*A*b*c+B*a*d+C*a*c)*x*(b*x^2+a)^(3/2)/a+
1/5*C*d*(b*x^2+a)^(5/2)/b-A*c*(b*x^2+a)^(5/2)/a/x+3/8*a*(4*A*b*c+B*a*d+C*a
*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-a^(3/2)*(A*d+B*c)*arctanh((
b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(24a^2Cdx+ab(-40A(3c-4dx)+x(160Bc+75$$

$$-2a^{3/2}(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$-\frac{3a(4Abc+acC+aBd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(24*a^2*C*d*x + a*b*(-40*A*(3*c - 4*d*x) + x*(160*B*c + 7
5*c*C*x + 75*B*d*x + 48*C*d*x^2)) + 2*b^2*x^2*(10*A*(3*c + 2*d*x) + x*(5*B
*(4*c + 3*d*x) + 3*C*x*(5*c + 4*d*x)))))/(120*b*x) - 2*a^(3/2)*(B*c + A*d)
*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]] - (3*a*(4*A*b*c + a*c*C
 + a*B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.98, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2340, 27, 535, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^2}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(aCdx^2+(4Abc+aCc+aBd)x+a(Bc+Ad))}{x}dx}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(aCdx^2+(4Abc+aCc+aBd)x+a(Bc+Ad))}{x}dx}{a}-\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{5b(a(Bc+Ad)+(4Abc+aCc+aBd)x)(bx^2+a)^{3/2}}{x}dx}{5b}+\frac{aCd(a+bx^2)^{5/2}}{5b}}{a}-\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a(Bc+Ad)+(4Abc+aCc+aBd)x)(bx^2+a)^{3/2}}{x}dx+\frac{aCd(a+bx^2)^{5/2}}{5b}}{a}-\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{4}a\int \frac{(4a(Bc+Ad)+3(4Abc+aCc+aBd)x)\sqrt{bx^2+a}}{x}dx+\frac{1}{12}(a+bx^2)^{3/2}(3x(aBd+acC+4Abc)+4a(Ad+Bc))+\frac{aCd(a+}{5}}{a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\int \frac{8a(Bc+Ad)+3(4Abc+aCc+aBd)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+4Abc)+8a(Ad+Bc))\right)+\frac{1}{12}(a+bx^2)^{3}}{a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(3(aBd+acC+4Abc)\int \frac{1}{\sqrt{bx^2+a}}dx+8a(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+4Abc)\right.}{a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(3(aBd+acC+4Abc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8a(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+\right.}{a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{ax}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(8a(Ad+Bc)\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+4Abc)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+4Abc)}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(4a(Ad+Bc)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+4Abc)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+4Ab}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{8a(Ad+Bc)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+4Abc)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+4Abc)-}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aBd+acC+4Abc)}{\sqrt{b}}-8\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad+Bc)\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aBd+acC+}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{ax}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^2,x]
```

output
```
-((A*c*(a + b*x^2)^(5/2))/(a*x)) + (((4*a*(B*c + A*d) + 3*(4*A*b*c + a*c*C
 + a*B*d)*x)*(a + b*x^2)^(3/2))/12 + (a*C*d*(a + b*x^2)^(5/2))/(5*b) + (a*
(((8*a*(B*c + A*d) + 3*(4*A*b*c + a*c*C + a*B*d)*x)*Sqrt[a + b*x^2])/2 + (
a*((3*(4*A*b*c + a*c*C + a*B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt
[b] - 8*Sqrt[a]*(B*c + A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2))/4)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.19

method	result
default	$Bd\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+Cc\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right.$
risch	$-\frac{aAc\sqrt{b\,x^2+a}}{x}+\frac{bx\sqrt{b\,x^2+a}\,Ac}{2}+\frac{Cbd\,x^4\sqrt{b\,x^2+a}}{5}+\frac{3Aa\sqrt{b}\,c\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2}+\frac{b\,x^3\sqrt{b\,x^2+a}\,Bd}{4}+\frac{b\,x^3\sqrt{b\,x^2+a}\,C}{4}$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
B*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+C*c*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2
+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d+B*c)*(1/3*(b*
x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))+A*c*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/5*C*d*(
b*x^2+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 891, normalized size of antiderivative = 4.01

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
[1/240*(45*(B*a^2*d + (C*a^2 + 4*A*a*b)*c)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) + 120*(B*a*b*c + A*a*b*d)*sqrt(a)*x*log(-(b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(24*C*b^2*d*x^5 + 30*(C*b^2*c
 + B*b^2*d)*x^4 - 120*A*a*b*c + 8*(5*B*b^2*c + (6*C*a*b + 5*A*b^2)*d)*x^3
+ 15*(5*B*a*b*d + (5*C*a*b + 4*A*b^2)*c)*x^2 + 8*(20*B*a*b*c + (3*C*a^2 +
20*A*a*b)*d)*x)*sqrt(b*x^2 + a))/(b*x), -1/120*(45*(B*a^2*d + (C*a^2 + 4*A
*a*b)*c)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*(B*a*b*c + A*a
*b*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (24*
C*b^2*d*x^5 + 30*(C*b^2*c + B*b^2*d)*x^4 - 120*A*a*b*c + 8*(5*B*b^2*c + (6
*C*a*b + 5*A*b^2)*d)*x^3 + 15*(5*B*a*b*d + (5*C*a*b + 4*A*b^2)*c)*x^2 + 8*
(20*B*a*b*c + (3*C*a^2 + 20*A*a*b)*d)*x)*sqrt(b*x^2 + a))/(b*x), 1/240*(24
0*(B*a*b*c + A*a*b*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 45*(
B*a^2*d + (C*a^2 + 4*A*a*b)*c)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a) + 2*(24*C*b^2*d*x^5 + 30*(C*b^2*c + B*b^2*d)*x^4 - 120*A*a*
b*c + 8*(5*B*b^2*c + (6*C*a*b + 5*A*b^2)*d)*x^3 + 15*(5*B*a*b*d + (5*C*a*b
 + 4*A*b^2)*c)*x^2 + 8*(20*B*a*b*c + (3*C*a^2 + 20*A*a*b)*d)*x)*sqrt(b*x^2
 + a))/(b*x), -1/120*(45*(B*a^2*d + (C*a^2 + 4*A*a*b)*c)*sqrt(-b)*x*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*(B*a*b*c + A*a*b*d)*sqrt(-a)*x*arctan(s
qrt(b*x^2 + a)*sqrt(-a)/a) - (24*C*b^2*d*x^5 + 30*(C*b^2*c + B*b^2*d)*x^4
- 120*A*a*b*c + 8*(5*B*b^2*c + (6*C*a*b + 5*A*b^2)*d)*x^3 + 15*(5*B*a*b...
```

Sympy [A] (verification not implemented)

Time = 4.04 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.72

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**(3/2)*c/(x*sqrt(1 + b*x**2/a)) - A*a**(3/2)*d*asinh(sqrt(a)/(sqrt(b)
*x)) - A*sqrt(a)*b*c*x/sqrt(1 + b*x**2/a) + A*a**2*d/(sqrt(b)*x*sqrt(a/(b*
x**2) + 1)) + A*a*sqrt(b)*c*asinh(sqrt(b)*x/sqrt(a)) + A*a*sqrt(b)*d*x/sqr
t(a/(b*x**2) + 1) + A*b*c*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b
*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*s
qrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + A*b*d*Piecewise((a*sqrt
(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2,
True)) - B*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2*c/(sqrt(b)*x*sqr
t(a/(b*x**2) + 1)) + B*a*sqrt(b)*c*x/sqrt(a/(b*x**2) + 1) + B*a*d*Piecewis
e((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0))
, (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqr
t(a)*x, True)) + B*b*c*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a +
 b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*b*d*Piecewise((-a**2*Pi
ecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a
 + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + C*a*c*Piecewise((a*Piec
ewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(
x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, T
rue)) + C*a*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/
3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*b*c*Piecewise((-a**2*Piecewis...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx = \frac{3}{2}\sqrt{bx^2+a}Abcx + \frac{3}{2}Aa\sqrt{b}c\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cd}{5\,b} + \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)x + \frac{3}{8}\sqrt{bx^2+a}(Cc+Bd)ax + \frac{3\,(Cc+Bd)a^2\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - (Bc+Ad)a^{\frac{3}{2}}\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}(Bc+Ad) + \sqrt{bx^2+a}(Bc+Ad)a - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ac}{x}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*A*b*c*x + 3/2*A*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b)) + 1
/5*(b*x^2 + a)^(5/2)*C*d/b + 1/4*(b*x^2 + a)^(3/2)*(C*c + B*d)*x + 3/8*sqr
t(b*x^2 + a)*(C*c + B*d)*a*x + 3/8*(C*c + B*d)*a^2*arcsinh(b*x/sqrt(a*b))/
sqrt(b) - (B*c + A*d)*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 +
 a)^(3/2)*(B*c + A*d) + sqrt(b*x^2 + a)*(B*c + A*d)*a - (b*x^2 + a)^(3/2)*
A*c/x
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx = \frac{2\,Aa^2\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Cbdx+\frac{5\,(Cb^4c+Bb^4d)}{b^3}\right)x+\frac{4\,(5\,Bb^4c+6\,Cab^3d+5\,Ab^4d)}{b^3}\right)x+\frac{15\,(5\,Cab^3c\right.\right. + \frac{2\,(Ba^2c+Aa^2d)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3\,(Ca^2c+4\,Aabc+Ba^2d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a^2*sqrt(b)*c/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*C*b*d*x + 5*(C*b^4*c + B*b^4*d)/b^3)*x + 4*(5*B*b^4*c + 6*
C*a*b^3*d + 5*A*b^4*d)/b^3)*x + 15*(5*C*a*b^3*c + 4*A*b^4*c + 5*B*a*b^3*d)
/b^3)*x + 8*(20*B*a*b^3*c + 3*C*a^2*b^2*d + 20*A*a*b^3*d)/b^3) + 2*(B*a^2*
c + A*a^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/
8*(C*a^2*c + 4*A*a*b*c + B*a^2*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/s
qrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.16

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx=\frac{-120\sqrt{b\,x^2+a}\,a^2bc+160\sqrt{b\,x^2+a}\,a^2bdx+24\sqrt{b\,x^2+a}\,a}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x)
```

output
```
( - 120*sqrt(a + b*x**2)*a**2*b*c + 160*sqrt(a + b*x**2)*a**2*b*d*x + 24*s
qrt(a + b*x**2)*a**2*c*d*x + 60*sqrt(a + b*x**2)*a*b**2*c*x**2 + 160*sqrt(
a + b*x**2)*a*b**2*c*x + 40*sqrt(a + b*x**2)*a*b**2*d*x**3 + 75*sqrt(a + b
*x**2)*a*b**2*d*x**2 + 75*sqrt(a + b*x**2)*a*b*c**2*x**2 + 48*sqrt(a + b*x
**2)*a*b*c*d*x**3 + 40*sqrt(a + b*x**2)*b**3*c*x**3 + 30*sqrt(a + b*x**2)*
b**3*d*x**4 + 30*sqrt(a + b*x**2)*b**2*c**2*x**4 + 24*sqrt(a + b*x**2)*b**
2*c*d*x**5 + 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt
(a))*a**2*b*d*x + 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*a*b**2*c*x - 120*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(
b)*x)/sqrt(a))*a**2*b*d*x - 120*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
sqrt(b)*x)/sqrt(a))*a*b**2*c*x + 180*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a**2*b*c*x + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a**2*b*d*x + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**2*c**2*x - 135*sqrt(b)*a**2*b*c*x - 15*sqrt(b)*a**2*b*d*x - 15*sqrt(
b)*a**2*c**2*x)/(120*b*x)
```

3.53 $\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 267

$$\begin{aligned}\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx &= \frac{1}{2}(3Abc+2a(cC+Bd))\sqrt{a+bx^2} \\ &+ \frac{3}{8}(aCd+4b(Bc+Ad))x\sqrt{a+bx^2}+\frac{(3Abc+2a(cC+Bd))\left(a+bx^2\right)^{3/2}}{6a} \\ &+ \frac{(aCd+4b(Bc+Ad))x(a+bx^2)^{3/2}}{4a}-\frac{Ac(a+bx^2)^{5/2}}{2ax^2} \\ &- \frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{ax}+\frac{3a(aCd+4b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} \\ &- \frac{1}{2}\sqrt{a}(3Abc+2a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

output
```
1/2*(3*A*b*c+2*a*(B*d+C*c))*(b*x^2+a)^(1/2)+3/8*(a*C*d+4*b*(A*d+B*c))*x*(b
*x^2+a)^(1/2)+1/6*(3*A*b*c+2*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a+1/4*(a*C*d+4*b
*(A*d+B*c))*x*(b*x^2+a)^(3/2)/a-1/2*A*c*(b*x^2+a)^(5/2)/a/x^2-(A*d+B*c)*(b
*x^2+a)^(5/2)/a/x+3/8*a*(a*C*d+4*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(1/2)-1/2*a^(1/2)*(3*A*b*c+2*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)
/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(2bx^2(6A(2c+dx)+x(6Bc+4cCx+4Bdx+3C$$

$$+3\sqrt{a}Abc\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-2a^{3/2}(cC+Bd)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$-\frac{3a(aCd+4b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(2*b*x^2*(6*A*(2*c + d*x) + x*(6*B*c + 4*c*C*x + 4*B*d*x
+ 3*C*d*x^2)) + a*(-12*A*(c + 2*d*x) + x*(-24*B*c + 32*c*C*x + 32*B*d*x +
15*C*d*x^2))))/(24*x^2) + 3*Sqrt[a]*A*b*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]] - 2*a^(3/2)*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*
x^2])/Sqrt[a]] - (3*a*(a*C*d + 4*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.95, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 25, 2338, 25, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^3}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(2aCdx^2+(3Abc+2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^2}dx}{2a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(2aCdx^2+(3Abc+2a(cC+Bd))x+2a(Bc+Ad)\right)}{x^2}dx}{2a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 2338$$

$$\frac{-\frac{\int -\frac{a(3Abc+2a(cC+Bd)+2(aCd+4b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{5/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 25$$

$$\frac{\frac{\int \frac{a(3Abc+2a(cC+Bd)+2(aCd+4b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{5/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 27$$

$$\frac{\int \frac{(3Abc+2a(cC+Bd)+2(aCd+4b(Bc+Ad))x)(bx^2+a)^{3/2}}{x}dx-\frac{2\left(a+bx^2\right)^{5/2}(Ad+Bc)}{x}}{2a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 535$$

$$\frac{\frac{1}{4}a\int \frac{2(2(3Abc+2a(cC+Bd))+3(aCd+4b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}\left(a+bx^2\right)^{3/2}(3x(aCd+4b(Ad+Bc))+2(2a(Bd+cC}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 27$$

$$\frac{\frac{1}{2}a\int \frac{(2(3Abc+2a(cC+Bd))+3(aCd+4b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}\left(a+bx^2\right)^{3/2}(3x(aCd+4b(Ad+Bc))+2(2a(Bd+cC)}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 535$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\int \frac{4(3Abc+2a(cC+Bd))+3(aCd+4b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+4b(Ad+Bc))+4(2a(Bd+cC)+3Ab\right.}{2a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow 538$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(3(aCd+4b(Ad+Bc))\int\frac{1}{\sqrt{bx^2+a}}dx+4(2a(Bd+cC)+3Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+4b}{\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(4(2a(Bd+cC)+3Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx+3(aCd+4b(Ad+Bc))\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(a}{\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(4(2a(Bd+cC)+3Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+4}{\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(2(2a(Bd+cC)+3Abc)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+}{\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{4(2a(Bd+cC)+3Abc)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+4b(Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+4b(Ad}{\frac{Ac\left(a+bx^2\right)^{5/2}}{2ax^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd+4b(Ad+Bc))}{\sqrt{b}}-\frac{4\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2a(Bd+cC)+3Abc)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(3x(aCd+4b(A\right.}{}$$

$$\frac{Ac(a+bx^2)^{5/2}}{2ax^2}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^3,x]
```

output
```
-1/2*(A*c*(a + b*x^2)^(5/2))/(a*x^2) + (((2*(3*A*b*c + 2*a*(c*C + B*d)) +
3*(a*C*d + 4*b*(B*c + A*d))*x)*(a + b*x^2)^(3/2))/6 - (2*(B*c + A*d)*(a +
b*x^2)^(5/2))/x + (a*(((4*(3*A*b*c + 2*a*(c*C + B*d)) + 3*(a*C*d + 4*b*(B*
c + A*d))*x)*Sqrt[a + b*x^2])/2 + (a*((3*(a*C*d + 4*b*(B*c + A*d))*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (4*(3*A*b*c + 2*a*(c*C + B*d))*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/2)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.04

method	result
default	$dC\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+(Ad+Bc)\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(x\right.}{}\right.}{}\right.$
risch	$-\frac{a\sqrt{bx^2+a}\,(2Adx+2Bcx+Ac)}{2x^2}+\frac{bx^2\sqrt{bx^2+a}\,Bd}{3}+\frac{bx^2\sqrt{bx^2+a}\,Cc}{3}+\frac{4a\sqrt{bx^2+a}\,Bd}{3}+\frac{4a\sqrt{bx^2+a}\,Cc}{3}+\frac{bx\sqrt{bx^2+a}\,A}{2}$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*C*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d+B*c)*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x
*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(
b*x^2+a)^(1/2)))))+(B*d+C*c)*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/
2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+A*c*(-1/2/a/x^2*(b*x^2+a)^(5/2)
+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)
*(b*x^2+a)^(1/2))/x))))
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 907, normalized size of antiderivative = 3.40

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
[1/48*(9*(4*B*a*b*c + (C*a^2 + 4*A*a*b)*d)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 12*(2*B*a*b*d + (2*C*a*b + 3*A*b^2)*c)*sqrt
(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*C*b^2*d
*x^5 + 8*(C*b^2*c + B*b^2*d)*x^4 - 12*A*a*b*c + 3*(4*B*b^2*c + (5*C*a*b +
4*A*b^2)*d)*x^3 + 8*(4*B*a*b*d + (4*C*a*b + 3*A*b^2)*c)*x^2 - 24*(B*a*b*c
+ A*a*b*d)*x)*sqrt(b*x^2 + a))/(b*x^2), -1/24*(9*(4*B*a*b*c + (C*a^2 + 4*A
*a*b)*d)*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*(2*B*a*b*d +
(2*C*a*b + 3*A*b^2)*c)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - (6*C*b^2*d*x^5 + 8*(C*b^2*c + B*b^2*d)*x^4 - 12*A*a*b*c + 3
*(4*B*b^2*c + (5*C*a*b + 4*A*b^2)*d)*x^3 + 8*(4*B*a*b*d + (4*C*a*b + 3*A*b
^2)*c)*x^2 - 24*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2 + a))/(b*x^2), 1/48*(24*
(2*B*a*b*d + (2*C*a*b + 3*A*b^2)*c)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sq
rt(-a)/a) + 9*(4*B*a*b*c + (C*a^2 + 4*A*a*b)*d)*sqrt(b)*x^2*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d*x^5 + 8*(C*b^2*c + B*b^2*
d)*x^4 - 12*A*a*b*c + 3*(4*B*b^2*c + (5*C*a*b + 4*A*b^2)*d)*x^3 + 8*(4*B*a
*b*d + (4*C*a*b + 3*A*b^2)*c)*x^2 - 24*(B*a*b*c + A*a*b*d)*x)*sqrt(b*x^2 +
 a))/(b*x^2), -1/24*(9*(4*B*a*b*c + (C*a^2 + 4*A*a*b)*d)*sqrt(-b)*x^2*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*(2*B*a*b*d + (2*C*a*b + 3*A*b^2)*c)*sq
rt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (6*C*b^2*d*x^5 + 8*(C*b^2*
c + B*b^2*d)*x^4 - 12*A*a*b*c + 3*(4*B*b^2*c + (5*C*a*b + 4*A*b^2)*d)*x...
```

Sympy [A] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 782, normalized size of antiderivative = 2.93

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**3,x)
```

output
```
-A*a**(3/2)*d/(x*sqrt(1 + b*x**2/a)) - 3*A*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt
(b)*x))/2 - A*sqrt(a)*b*d*x/sqrt(1 + b*x**2/a) - A*a*sqrt(b)*c*sqrt(a/(b*x
**2) + 1)/(2*x) + A*a*sqrt(b)*c/(x*sqrt(a/(b*x**2) + 1)) + A*a*sqrt(b)*d*a
sinh(sqrt(b)*x/sqrt(a)) + A*b**(3/2)*c*x/sqrt(a/(b*x**2) + 1) + A*b*d*Piec
ewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)),
(sqrt(a)*x, True)) - B*a**(3/2)*c/(x*sqrt(1 + b*x**2/a)) - B*a**(3/2)*d*as
inh(sqrt(a)/(sqrt(b)*x)) - B*sqrt(a)*b*c*x/sqrt(1 + b*x**2/a) + B*a**2*d/(
sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a*sqrt(b)*c*asinh(sqrt(b)*x/sqrt(a)) +
 B*a*sqrt(b)*d*x/sqrt(a/(b*x**2) + 1) + B*b*c*Piecewise((a*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*b
*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)
), (sqrt(a)*x**2/2, True)) - C*a**(3/2)*c*asinh(sqrt(a)/(sqrt(b)*x)) + C*a
**2*c/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*a*sqrt(b)*c*x/sqrt(a/(b*x**2) +
 1) + C*a*d*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x
)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2
)/2, Ne(b, 0)), (sqrt(a)*x, True)) + C*b*c*Piecewise((a*sqrt(a + b*x**2)/(
3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + C*b*d
*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.91

$$
\begin{aligned}
\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^3}\,dx &= \frac{1}{4}\,\left(bx^2+a\right)^{\frac{3}{2}}Cdx \\
&+ \frac{3}{8}\,\sqrt{bx^2+a}Cadx + \frac{3\,Ca^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} \\
&- \frac{3}{2}\,A\sqrt{a}bc\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\,\sqrt{bx^2+a}Abc + \frac{(bx^2+a)^{\frac{3}{2}}Abc}{2\,a} \\
&+ \frac{3}{2}\,\sqrt{bx^2+a}(Bc+Ad)bx + \frac{3}{2}\,(Bc+Ad)a\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) \\
&- (Cc+Bd)a^{\frac{3}{2}}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\,\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd) \\
&+ \sqrt{bx^2+a}(Cc+Bd)a - \frac{(bx^2+a)^{\frac{5}{2}}Ac}{2\,ax^2} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)}{x}
\end{aligned}
$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*C*d*x + 3/8*sqrt(b*x^2 + a)*C*a*d*x + 3/8*C*a^2*d*ar
csinh(b*x/sqrt(a*b))/sqrt(b) - 3/2*A*sqrt(a)*b*c*arcsinh(a/(sqrt(a*b)*abs(
x))) + 3/2*sqrt(b*x^2 + a)*A*b*c + 1/2*(b*x^2 + a)^(3/2)*A*b*c/a + 3/2*sqr
t(b*x^2 + a)*(B*c + A*d)*b*x + 3/2*(B*c + A*d)*a*sqrt(b)*arcsinh(b*x/sqrt(
a*b)) - (C*c + B*d)*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a
)^(3/2)*(C*c + B*d) + sqrt(b*x^2 + a)*(C*c + B*d)*a - 1/2*(b*x^2 + a)^(5/2
)*A*c/(a*x^2) - (b*x^2 + a)^(3/2)*(B*c + A*d)/x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx=\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(3\,Cbdx+\frac{4\left(Cb^3c+Bb^3d\right)}{b^2}\right)x+\frac{3\,(4\,Bb\right.\right.\right.$$

$$+\frac{\left(2\,Ca^2c+3\,Aabc+2\,Ba^2d\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

$$-\frac{3\left(4\,Babc+Ca^2d+4\,Aabd\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aabc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}c+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa^2\sqrt{b}d+\left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*C*b*d*x + 4*(C*b^3*c + B*b^3*d)/b^2)*x + 3*(4*
B*b^3*c + 5*C*a*b^2*d + 4*A*b^3*d)/b^2)*x + 8*(4*C*a*b^2*c + 3*A*b^3*c + 4
*B*a*b^2*d)/b^2) + (2*C*a^2*c + 3*A*a*b*c + 2*B*a^2*d)*arctan(-(sqrt(b)*x
- sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*(4*B*a*b*c + C*a^2*d + 4*A*a*b
*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + ((sqrt(b)*x - sqrt(b*
x^2 + a))^3*A*a*b*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c +
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*sqrt(b)*d + (sqrt(b)*x - sqrt(b*x^
2 + a))*A*a^2*b*c - 2*B*a^3*sqrt(b)*c - 2*A*a^3*sqrt(b)*d)/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx=\frac{-12\sqrt{b\,x^2+a}\,a^2bc-24\sqrt{b\,x^2+a}\,a^2bdx+24\sqrt{b\,x^2+a}\,a\,b^2}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x)
```

output
```
( - 12*sqrt(a + b*x**2)*a**2*b*c - 24*sqrt(a + b*x**2)*a**2*b*d*x + 24*sqr
t(a + b*x**2)*a*b**2*c*x**2 - 24*sqrt(a + b*x**2)*a*b**2*c*x + 12*sqrt(a +
 b*x**2)*a*b**2*d*x**3 + 32*sqrt(a + b*x**2)*a*b**2*d*x**2 + 32*sqrt(a + b
*x**2)*a*b*c**2*x**2 + 15*sqrt(a + b*x**2)*a*b*c*d*x**3 + 12*sqrt(a + b*x*
*2)*b**3*c*x**3 + 8*sqrt(a + b*x**2)*b**3*d*x**4 + 8*sqrt(a + b*x**2)*b**2
*c**2*x**4 + 6*sqrt(a + b*x**2)*b**2*c*d*x**5 + 36*sqrt(a)*log((sqrt(a + b
*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**2 + 24*sqrt(a)*log((sqr
t(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**2 + 24*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*x**2 - 36*sq
rt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**2
- 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*
d*x**2 - 24*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*
a*b*c**2*x**2 + 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**
2*b*d*x**2 + 9*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*
d*x**2 + 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x
**2)/(24*b*x**2)
```

3.54 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 277

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \frac{1}{2}(2aCd+3b(Bc+Ad))\sqrt{a+bx^2} + \frac{b(2Abc+3a(cC+Bd))x\sqrt{a+bx^2}}{2a} + \frac{(2aCd+3b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{6a} - \frac{(2Abc+3a(cC+Bd))\left(a+bx^2\right)^{3/2}}{3ax} - \frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3} - \frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{2ax^2} + \frac{1}{2}\sqrt{b}(2Abc+3a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{1}{2}\sqrt{a}(2aCd+3b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output

```
1/2*(2*a*C*d+3*b*(A*d+B*c))*(b*x^2+a)^(1/2)+1/2*b*(2*A*b*c+3*a*(B*d+C*c))*
x*(b*x^2+a)^(1/2)/a+1/6*(2*a*C*d+3*b*(A*d+B*c))*(b*x^2+a)^(3/2)/a-1/3*(2*A
*b*c+3*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a/x-1/3*A*c*(b*x^2+a)^(5/2)/a/x^3-1/2*
(A*d+B*c)*(b*x^2+a)^(5/2)/a/x^2+1/2*b^(1/2)*(2*A*b*c+3*a*(B*d+C*c))*arctan
h(b^(1/2)*x/(b*x^2+a)^(1/2))-1/2*a^(1/2)*(2*a*C*d+3*b*(A*d+B*c))*arctanh((
b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \frac{\sqrt{a+bx^2}(-a(A(2c+3dx)+x(3Bc+6cCx+6Bdx-8C}{}$$

$$+2a^{3/2}Cd\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-3\sqrt{a}b(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$-\frac{1}{2}\sqrt{b}(2Abc+3a(cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*(A*(2*c + 3*d*x) + x*(3*B*c + 6*c*C*x + 6*B*d*x - 8*
C*d*x^2))) + b*x^2*(A*(-8*c + 6*d*x) + x*(6*B*c + 3*c*C*x + 3*B*d*x + 2*C*
d*x^2))))/(6*x^3) + 2*a^(3/2)*C*d*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sq
rt[a]] - 3*Sqrt[a]*b*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/
Sqrt[a]] - (Sqrt[b]*(2*A*b*c + 3*a*(c*C + B*d))*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/2
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2338, 25, 2338, 25, 27, 536, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^4}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(3aCdx^2+(2Abc+3a(cC+Bd))x+3a(Bc+Ad)\right)}{x^3}dx}{3a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(3aCdx^2+(2Abc+3a(cC+Bd))x+3a(Bc+Ad))}{x^3}dx}{3a}-\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{a(2(2Abc+3a(cC+Bd))+3(2aCd+3b(Bc+Ad))x)(bx^2+a)^{3/2}}{x^2}dx}{2a}-\frac{3(a+bx^2)^{5/2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a(2(2Abc+3a(cC+Bd))+3(2aCd+3b(Bc+Ad))x)(bx^2+a)^{3/2}}{x^2}dx}{2a}-\frac{3(a+bx^2)^{5/2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\int \frac{(2(2Abc+3a(cC+Bd))+3(2aCd+3b(Bc+Ad))x)(bx^2+a)^{3/2}}{x^2}dx-\frac{3(a+bx^2)^{5/2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{536}$$

$$\frac{\frac{1}{2}\left(\int \frac{(3a(2aCd+3b(Bc+Ad))+6b(2Abc+3a(cC+Bd))x)\sqrt{bx^2+a}}{x}dx-\frac{(a+bx^2)^{3/2}(2(3a(Bd+cC)+2Abc)-x(2aCd+3b(Ad+Bc)))}{x}\right)-\frac{3(a}{}}{3a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{535}$$

$$\frac{\frac{1}{2}\left(\frac{1}{2}a\int \frac{6(a(2aCd+3b(Bc+Ad))+b(2Abc+3a(cC+Bd))x)}{x\sqrt{bx^2+a}}dx-\frac{(a+bx^2)^{3/2}(2(3a(Bd+cC)+2Abc)-x(2aCd+3b(Ad+Bc)))}{x}+3\sqrt{a+bx}\right.}{3a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}\left(3a\int \frac{a(2aCd+3b(Bc+Ad))+b(2Abc+3a(cC+Bd))x}{x\sqrt{bx^2+a}}dx-\frac{(a+bx^2)^{3/2}(2(3a(Bd+cC)+2Abc)-x(2aCd+3b(Ad+Bc)))}{x}+3\sqrt{a+bx^2}(\right.}{3a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{1}{2}\left(3a\left(b(3a(Bd+cC)+2Abc)\int \frac{1}{\sqrt{bx^2+a}}dx+a(2aCd+3b(Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{\left(a+bx^2\right)^{3/2}(2(3a(Bd+cC)+2A}{x}\right.}{3}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 224

$$\frac{1}{2}\left(3a\left(a(2aCd+3b(Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx+b(3a(Bd+cC)+2Abc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(2(3a(Bd}{}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 219

$$\frac{1}{2}\left(3a\left(a(2aCd+3b(Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a(Bd+cC)+2Abc)\right)-\frac{\left(a+bx^2\right)^{3/2}(2(3a(Bc}{}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 243

$$\frac{1}{2}\left(3a\left(\frac{1}{2}a(2aCd+3b(Ad+Bc))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a(Bd+cC)+2Abc)\right)-\frac{\left(a+bx^2\right)^{3/2}(2(3a}{}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 73

$$\frac{1}{2}\left(3a\left(\frac{a(2aCd+3b(Ad+Bc))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a(Bd+cC)+2Abc)\right)-\frac{\left(a+bx^2\right)^{3/2}(2(3a(Bd+cC}{}\right.$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 221

$$\frac{\frac{1}{2}\left(3a\left(\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a(Bd+cC)+2Abc)-\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2aCd+3b(Ad+Bc))\right)-\frac{(a+bx^2)^{3/2}}{}\right.}{}$$

$$\frac{Ac(a+bx^2)^{5/2}}{3ax^3}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^4,x]
```

output
```
-1/3*(A*c*(a + b*x^2)^(5/2))/(a*x^3) + ((-3*(B*c + A*d)*(a + b*x^2)^(5/2))
/(2*x^2) + (3*(a*(2*a*C*d + 3*b*(B*c + A*d)) + b*(2*A*b*c + 3*a*(c*C + B*d
))*x)*Sqrt[a + b*x^2] - ((2*(2*A*b*c + 3*a*(c*C + B*d)) - (2*a*C*d + 3*b*(
B*c + A*d))*x)*(a + b*x^2)^(3/2))/x + 3*a*(Sqrt[b]*(2*A*b*c + 3*a*(c*C + B
*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - Sqrt[a]*(2*a*C*d + 3*b*(B*c +
A*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(8Abc\,x^{2}+6Bad\,x^{2}+6Cac\,x^{2}+3Aadx+3Bacx+2Aac\right)}{6x^{3}}-\frac{3A\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)bd}{2}-\frac{3B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.}{2}$
default	$(Ad+Bc)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)+(Bd+Cc)\left(-\frac{(b\,x^{2}\right.$

input

```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/6*(b*x^2+a)^(1/2)*(8*A*b*c*x^2+6*B*a*d*x^2+6*C*a*c*x^2+3*A*a*d*x+3*B*a*
c*x+2*A*a*c)/x^3-3/2*A*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*b*d-3
/2*B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*b*c-C*a^(3/2)*ln((2*a+2
*a^(1/2)*(b*x^2+a)^(1/2))/x)*d+A*b^(3/2)*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+A
*b*d*(b*x^2+a)^(1/2)+B*b*c*(b*x^2+a)^(1/2)+1/2*B*b*d*x*(b*x^2+a)^(1/2)+3/2
*B*a*d*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*C*b*c*x*(b*x^2+a)^(1/2)+3
/2*C*a*c*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/3*C*b*d*x^2*(b*x^2+a)^(1/
2)+4/3*a*C*d*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 795, normalized size of antiderivative = 2.87

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(3*(3*B*a*d + (3*C*a + 2*A*b)*c)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 3*(3*B*b*c + (2*C*a + 3*A*b)*d)*sqrt(a)*x^3*log
(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*C*b*d*x^5 + 3*(C*b
*c + B*b*d)*x^4 + 2*(3*B*b*c + (4*C*a + 3*A*b)*d)*x^3 - 2*A*a*c - 2*(3*B*a
*d + (3*C*a + 4*A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/x^3, -
1/12*(6*(3*B*a*d + (3*C*a + 2*A*b)*c)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) - 3*(3*B*b*c + (2*C*a + 3*A*b)*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*
sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*C*b*d*x^5 + 3*(C*b*c + B*b*d)*x
^4 + 2*(3*B*b*c + (4*C*a + 3*A*b)*d)*x^3 - 2*A*a*c - 2*(3*B*a*d + (3*C*a +
 4*A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/x^3, 1/12*(6*(3*B*b
*c + (2*C*a + 3*A*b)*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) +
3*(3*B*a*d + (3*C*a + 2*A*b)*c)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(2*C*b*d*x^5 + 3*(C*b*c + B*b*d)*x^4 + 2*(3*B*b*c +
(4*C*a + 3*A*b)*d)*x^3 - 2*A*a*c - 2*(3*B*a*d + (3*C*a + 4*A*b)*c)*x^2 - 3
*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/x^3, -1/6*(3*(3*B*a*d + (3*C*a + 2*A*
b)*c)*sqrt(-b)*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(3*B*b*c + (2*C*
a + 3*A*b)*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*C*b*d*x
^5 + 3*(C*b*c + B*b*d)*x^4 + 2*(3*B*b*c + (4*C*a + 3*A*b)*d)*x^3 - 2*A*a*c
 - 2*(3*B*a*d + (3*C*a + 4*A*b)*c)*x^2 - 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 +
 a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 7.01 (sec) , antiderivative size = 716, normalized size of antiderivative = 2.58

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**4,x)
```

output
```
-A*sqrt(a)*b*c/(x*sqrt(1 + b*x**2/a)) - 3*A*sqrt(a)*b*d*asinh(sqrt(a)/(sqr
t(b)*x))/2 - A*a*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*a*sqrt(b)*d*s
qrt(a/(b*x**2) + 1)/(2*x) + A*a*sqrt(b)*d/(x*sqrt(a/(b*x**2) + 1)) - A*b**
(3/2)*c*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*c*asinh(sqrt(b)*x/sqrt(a)) + A
*b**(3/2)*d*x/sqrt(a/(b*x**2) + 1) - A*b**2*c*x/(sqrt(a)*sqrt(1 + b*x**2/a
)) - B*a**(3/2)*d/(x*sqrt(1 + b*x**2/a)) - 3*B*sqrt(a)*b*c*asinh(sqrt(a)/(
sqrt(b)*x))/2 - B*sqrt(a)*b*d*x/sqrt(1 + b*x**2/a) - B*a*sqrt(b)*c*sqrt(a/
(b*x**2) + 1)/(2*x) + B*a*sqrt(b)*c/(x*sqrt(a/(b*x**2) + 1)) + B*a*sqrt(b)
*d*asinh(sqrt(b)*x/sqrt(a)) + B*b**(3/2)*c*x/sqrt(a/(b*x**2) + 1) + B*b*d*
Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0
)), (sqrt(a)*x, True)) - C*a**(3/2)*c/(x*sqrt(1 + b*x**2/a)) - C*a**(3/2)*
d*asinh(sqrt(a)/(sqrt(b)*x)) - C*sqrt(a)*b*c*x/sqrt(1 + b*x**2/a) + C*a**2
*d/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + C*a*sqrt(b)*c*asinh(sqrt(b)*x/sqrt(a
)) + C*a*sqrt(b)*d*x/sqrt(a/(b*x**2) + 1) + C*b*c*Piecewise((a*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
 C*b*d*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b
, 0)), (sqrt(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4}\,dx = \frac{\sqrt{bx^2+a}Ab^2cx}{a} + Ab^{\frac{3}{2}}c\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ca^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}(bx^2+a)^{\frac{3}{2}}Cd + \sqrt{bx^2+a}Cad + \frac{3}{2}\sqrt{bx^2+a}(Cc+Bd)bx + \frac{3}{2}(Cc+Bd)a\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}(Bc+Ad)\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}(Bc+Ad)b + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b}{2a} - \frac{2(bx^2+a)^{\frac{3}{2}}Abc}{3ax} - \frac{(bx^2+a)^{\frac{3}{2}}(Cc+Bd)}{x} - \frac{(bx^2+a)^{\frac{5}{2}}Ac}{3ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)}{2ax^2}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*A*b^2*c*x/a + A*b^(3/2)*c*arcsinh(b*x/sqrt(a*b)) - C*a^(3/
2)*d*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*C*d + sqrt(b*x^
2 + a)*C*a*d + 3/2*sqrt(b*x^2 + a)*(C*c + B*d)*b*x + 3/2*(C*c + B*d)*a*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*(B*c + A*d)*sqrt(a)*b*arcsinh(a/(sqrt(a*
b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*(B*c + A*d)*b + 1/2*(b*x^2 + a)^(3/2)*(B
*c + A*d)*b/a - 2/3*(b*x^2 + a)^(3/2)*A*b*c/(a*x) - (b*x^2 + a)^(3/2)*(C*c
 + B*d)/x - 1/3*(b*x^2 + a)^(5/2)*A*c/(a*x^3) - 1/2*(b*x^2 + a)^(5/2)*(B*c
 + A*d)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(237) = 474$.

Time = 0.17 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.77

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx =$$
$$-\frac{1}{2}\left(3\,Ca\sqrt{b}c+2\,Ab^{\frac{3}{2}}c+3\,Ba\sqrt{b}d\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)$$
$$+\frac{1}{6}\sqrt{bx^2+a}\left(\left(2\,Cbdx+\frac{3\left(Cb^2c+Bb^2d\right)}{b}\right)x+\frac{2\left(3\,Bb^2c+4\,Cabd+3\,Ab^2d\right)}{b}\right)$$
$$+\frac{(3\,Babc+2\,Ca^2d+3\,Aabd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Babc+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Aabd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca^2\sqrt{b}c+12\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
-1/2*(3*C*a*sqrt(b)*c + 2*A*b^(3/2)*c + 3*B*a*sqrt(b)*d)*log(abs(-sqrt(b)*
x + sqrt(b*x^2 + a))) + 1/6*sqrt(b*x^2 + a)*((2*C*b*d*x + 3*(C*b^2*c + B*b
^2*d)/b)*x + 2*(3*B*b^2*c + 4*C*a*b*d + 3*A*b^2*d)/b) + (3*B*a*b*c + 2*C*a
^2*d + 3*A*a*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
 + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a*b*c + 3*(sqrt(b)*x - sqrt(b*
x^2 + a))^5*A*a*b*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^2*sqrt(b)*c +
12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2)*c + 6*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*B*a^2*sqrt(b)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^3*sqrt
(b)*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2)*c - 12*(sqrt(b)*x
 - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b)*d - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*
a^3*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b*d + 6*C*a^4*sqrt(b)*c +
8*A*a^3*b^(3/2)*c + 6*B*a^4*sqrt(b)*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.81

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx=\frac{-4\sqrt{b\,x^2+a}\,a^2c-6\sqrt{b\,x^2+a}\,a^2dx-16\sqrt{b\,x^2+a}\,abc\,x^2 -}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x**2)*a**2*c - 6*sqrt(a + b*x**2)*a**2*d*x - 16*sqrt(a +
b*x**2)*a*b*c*x**2 - 6*sqrt(a + b*x**2)*a*b*c*x + 12*sqrt(a + b*x**2)*a*b*
d*x**3 - 12*sqrt(a + b*x**2)*a*b*d*x**2 - 12*sqrt(a + b*x**2)*a*c**2*x**2
+ 16*sqrt(a + b*x**2)*a*c*d*x**3 + 12*sqrt(a + b*x**2)*b**2*c*x**3 + 6*sqr
t(a + b*x**2)*b**2*d*x**4 + 6*sqrt(a + b*x**2)*b*c**2*x**4 + 4*sqrt(a + b*
x**2)*b*c*d*x**5 + 18*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*a*b*d*x**3 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*c*d*x**3 + 18*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**2*c*x**3 - 18*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a*b*d*x**3 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqr
t(a) + sqrt(b)*x)/sqrt(a))*a*c*d*x**3 - 18*sqrt(a)*log((sqrt(a + b*x**2) +
 sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**3 + 12*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a*b*c*x**3 + 18*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*a*b*d*x**3 + 18*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a*c**2*x**3 + 5*sqrt(b)*a*b*d*x**3 + 5*sqrt(b)*a*c**2*x**3)/
(12*x**3)
```

3.55 $\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 282

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx=\frac{3b(Abc+4a(cC+Bd))\sqrt{a+bx^2}}{8a}$$
$$+\frac{b(3aCd+2b(Bc+Ad))x\sqrt{a+bx^2}}{2a}-\frac{(Abc+4a(cC+Bd))\left(a+bx^2\right)^{3/2}}{8ax^2}$$
$$-\frac{(3aCd+2b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{3ax}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}-\frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{3ax^3}$$
$$+\frac{1}{2}\sqrt{b}(3aCd+2b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{3b(Abc+4a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
3/8*b*(A*b*c+4*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a+1/2*b*(3*a*C*d+2*b*(A*d+B*c)
)*x*(b*x^2+a)^(1/2)/a-1/8*(A*b*c+4*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a/x^2-1/3*
(3*a*C*d+2*b*(A*d+B*c))*(b*x^2+a)^(3/2)/a/x-1/4*A*c*(b*x^2+a)^(5/2)/a/x^4-
1/3*(A*d+B*c)*(b*x^2+a)^(5/2)/a/x^3+1/2*b^(1/2)*(3*a*C*d+2*b*(A*d+B*c))*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-3/8*b*(A*b*c+4*a*(B*d+C*c))*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(bx^2(A(15c+32dx)-4x(-8Bc+6cCx+6Bdx+3Cdx^2))+2a(A(3c+4dx)+2x(3Cx(c+}{24x^4}$$
$$+\frac{3Ab^2c\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}}-3\sqrt{a}b(cC+Bd)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$
$$-\frac{1}{2}\sqrt{b}(3aCd+2b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^5,x]
```

output
```
-1/24*(Sqrt[a + b*x^2]*(b*x^2*(A*(15*c + 32*d*x) - 4*x*(-8*B*c + 6*c*C*x +
 6*B*d*x + 3*C*d*x^2)) + 2*a*(A*(3*c + 4*d*x) + 2*x*(3*C*x*(c + 2*d*x) + B
*(2*c + 3*d*x)))))/x^4 + (3*A*b^2*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/(4*Sqrt[a]) - 3*Sqrt[a]*b*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sq
rt[a + b*x^2])/Sqrt[a]] - (Sqrt[b]*(3*a*C*d + 2*b*(B*c + A*d))*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/2
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.91, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 25, 27, 537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^5}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(4aCdx^2+(Abc+4a(cC+Bd))x+4a(Bc+Ad)\right)}{x^4}dx}{4a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(4aCdx^2+(Abc+4a(cC+Bd))x+4a(Bc+Ad)\right)}{x^4}dx}{4a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{a(3(Abc+4a(cC+Bd))+4(3aCd+2b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^3}dx}{3a}-\frac{4\left(a+bx^2\right)^{5/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{a(3(Abc+4a(cC+Bd))+4(3aCd+2b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^3}dx}{3a}-\frac{4\left(a+bx^2\right)^{5/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{(3(Abc+4a(cC+Bd))+4(3aCd+2b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^3}dx-\frac{4\left(a+bx^2\right)^{5/2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 537$$

$$\frac{\frac{1}{3}\left(-\frac{3}{2}b\int -\frac{(3(Abc+4a(cC+Bd))+8(3aCd+2b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}(8x(3aCd+2b(Ad+Bc))+3(4a(Bd+cC)+Abc))}{2x^2}\right)-}{4a}$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\int \frac{(3(Abc+4a(cC+Bd))+8(3aCd+2b(Bc+Ad))x)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}(8x(3aCd+2b(Ad+Bc))+3(4a(Bd+cC)+Abc))}{2x^2}\right)-\frac{4(a}{}}{4a}$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 535$$

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(\frac{1}{2}a\int \frac{2(3(Abc+4a(cC+Bd))+4(3aCd+2b(Bc+Ad))x)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(4x(3aCd+2b(Ad+Bc))+3(4a(Bd+cC)+\right.\right.}{4a}$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\int\frac{3(Abc+4a(cC+Bd))+4(3aCd+2b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(4x(3aCd+2b(Ad+Bc))+3(4a(Bd+cC)+Ab\right.\right.$$
$$4a$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 538

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\left(4(3aCd+2b(Ad+Bc))\int\frac{1}{\sqrt{bx^2+a}}dx+3(4a(Bd+cC)+Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}(4x(3aCd+2\right.\right.$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 224

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\left(3(4a(Bd+cC)+Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx+4(3aCd+2b(Ad+Bc))\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}(4x(3a\right.\right.$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\left(3(4a(Bd+cC)+Abc)\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(4x(3aCd+\right.\right.$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 243

$$\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{3}{2}(4a(Bd+cC)+Abc)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(4x(3aCd\right.\right.$$
$$\frac{Ac\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\downarrow$ 73

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{3(4a(Bd+cC)+Abc)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd+2b(Ad+Bc))}{\sqrt{b}}\right)+\sqrt{a+bx^2}(4x(3aCd+2b(A\right.\right.}{4a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{4ax^4}$$

$\big\downarrow$ 221

$$\frac{\frac{1}{3}\left(\frac{3}{2}b\left(a\left(\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd+2b(Ad+Bc))}{\sqrt{b}}-\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4a(Bd+cC)+Abc)}{\sqrt{a}}\right)+\sqrt{a+bx^2}(4x(3aCd+2b\right.\right.}{4}$$

$$\frac{Ac(a+bx^2)^{5/2}}{4ax^4}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^5,x]
```

output
```
-1/4*(A*c*(a + b*x^2)^(5/2))/(a*x^4) + ((-4*(B*c + A*d)*(a + b*x^2)^(5/2))
/(3*x^3) + (-1/2*((3*(A*b*c + 4*a*(c*C + B*d)) + 8*(3*a*C*d + 2*b*(B*c + A
*d))*x)*(a + b*x^2)^(3/2))/x^2 + (3*b*((3*(A*b*c + 4*a*(c*C + B*d)) + 4*(3
*a*C*d + 2*b*(B*c + A*d))*x)*Sqrt[a + b*x^2] + a*((4*(3*a*C*d + 2*b*(B*c +
 A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (3*(A*b*c + 4*a*(c*
C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])))/2)/3)/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(32Abd x^3+32Bbc x^3+24Cad x^3+15Abc x^2+12Bad x^2+12Cac x^2+8Aadx+8Bacx+6Aac)}{24x^4}+\frac{b\left(-\frac{(3Abc+12Bad+12C \cdots}{\cdots}\right.}{\cdots}$
default	$(Ad+Bc)\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{3a x^3}+\frac{2b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)+(Bd+ \cdots$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(32*A*b*d*x^3+32*B*b*c*x^3+24*C*a*d*x^3+15*A*b*c*x^2
+12*B*a*d*x^2+12*C*a*c*x^2+8*A*a*d*x+8*B*a*c*x+6*A*a*c)/x^4+1/8*b*(-(3*A*b
*c+12*B*a*d+12*C*a*c)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+8*A*b^
(1/2)*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+8*B*b^(1/2)*c*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))+16*a*C*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8*B*d*(b*x^2+a)^(1/
2)+8*C*c*(b*x^2+a)^(1/2)+8*C*b*d*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 906, normalized size of antiderivative = 3.21

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="fricas")
```

output
```
[1/48*(12*(2*B*a*b*c + (3*C*a^2 + 2*A*a*b)*d)*sqrt(b)*x^4*log(-2*b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 9*(4*B*a*b*d + (4*C*a*b + A*b^2)*c)*sqrt
(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(12*C*a*b*
d*x^5 + 24*(C*a*b*c + B*a*b*d)*x^4 - 6*A*a^2*c - 8*(4*B*a*b*c + (3*C*a^2 +
 4*A*a*b)*d)*x^3 - 3*(4*B*a^2*d + (4*C*a^2 + 5*A*a*b)*c)*x^2 - 8*(B*a^2*c
+ A*a^2*d)*x)*sqrt(b*x^2 + a))/(a*x^4), -1/48*(24*(2*B*a*b*c + (3*C*a^2 +
2*A*a*b)*d)*sqrt(-b)*x^4*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 9*(4*B*a*b*d
 + (4*C*a*b + A*b^2)*c)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) - 2*(12*C*a*b*d*x^5 + 24*(C*a*b*c + B*a*b*d)*x^4 - 6*A*a^2*c
 - 8*(4*B*a*b*c + (3*C*a^2 + 4*A*a*b)*d)*x^3 - 3*(4*B*a^2*d + (4*C*a^2 + 5
*A*a*b)*c)*x^2 - 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a*x^4), 1/24*(
9*(4*B*a*b*d + (4*C*a*b + A*b^2)*c)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sq
rt(-a)/a) + 6*(2*B*a*b*c + (3*C*a^2 + 2*A*a*b)*d)*sqrt(b)*x^4*log(-2*b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (12*C*a*b*d*x^5 + 24*(C*a*b*c + B*a*
b*d)*x^4 - 6*A*a^2*c - 8*(4*B*a*b*c + (3*C*a^2 + 4*A*a*b)*d)*x^3 - 3*(4*B*
a^2*d + (4*C*a^2 + 5*A*a*b)*c)*x^2 - 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 +
 a))/(a*x^4), -1/24*(12*(2*B*a*b*c + (3*C*a^2 + 2*A*a*b)*d)*sqrt(-b)*x^4*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 9*(4*B*a*b*d + (4*C*a*b + A*b^2)*c)*sq
rt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (12*C*a*b*d*x^5 + 24*(C*a*
b*c + B*a*b*d)*x^4 - 6*A*a^2*c - 8*(4*B*a*b*c + (3*C*a^2 + 4*A*a*b)*d)*...
```

Sympy [A] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 712, normalized size of antiderivative = 2.52

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}(A+Bx+Cx^2)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**5,x)
```

output
```
-A*sqrt(a)*b*d/(x*sqrt(1 + b*x**2/a)) - A*a**2*c/(4*sqrt(b)*x**5*sqrt(a/(b
*x**2) + 1)) - 3*A*a*sqrt(b)*c/(8*x**3*sqrt(a/(b*x**2) + 1)) - A*a*sqrt(b)
*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(2*x)
 - A*b**(3/2)*c/(8*x*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*d*sqrt(a/(b*x**2)
+ 1)/3 + A*b**(3/2)*d*asinh(sqrt(b)*x/sqrt(a)) - 3*A*b**2*c*asinh(sqrt(a)/
(sqrt(b)*x))/(8*sqrt(a)) - A*b**2*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqr
t(a)*b*c/(x*sqrt(1 + b*x**2/a)) - 3*B*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x
))/2 - B*a*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*sqrt(b)*d*sqrt(a/
(b*x**2) + 1)/(2*x) + B*a*sqrt(b)*d/(x*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*
c*sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*c*asinh(sqrt(b)*x/sqrt(a)) + B*b**(3
/2)*d*x/sqrt(a/(b*x**2) + 1) - B*b**2*c*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C
*a**(3/2)*d/(x*sqrt(1 + b*x**2/a)) - 3*C*sqrt(a)*b*c*asinh(sqrt(a)/(sqrt(b
)*x))/2 - C*sqrt(a)*b*d*x/sqrt(1 + b*x**2/a) - C*a*sqrt(b)*c*sqrt(a/(b*x**
2) + 1)/(2*x) + C*a*sqrt(b)*c/(x*sqrt(a/(b*x**2) + 1)) + C*a*sqrt(b)*d*asi
nh(sqrt(b)*x/sqrt(a)) + C*b**(3/2)*c*x/sqrt(a/(b*x**2) + 1) + C*b*d*Piecew
ise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (s
qrt(a)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx = \frac{3}{2}\,\sqrt{bx^2+a}Cbdx + \frac{3}{2}\,Ca\sqrt{b}d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3\,Ab^2c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab^2c}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}Ab^2c}{8\,a} + \frac{\sqrt{bx^2+a}(Bc+Ad)b^2x}{a} + (Bc+Ad)b^{\frac{3}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}\,(Cc+Bd)\sqrt{a}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\,\sqrt{bx^2+a}(Cc+Bd)b + \frac{\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)b}{2\,a} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Cd}{x} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Abc}{8\,a^2x^2} - \frac{2\,\left(bx^2+a\right)^{\frac{3}{2}}(Bc+Ad)b}{3\,ax} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)}{2\,ax^2} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ac}{4\,ax^4} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)}{3\,ax^3}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*C*b*d*x + 3/2*C*a*sqrt(b)*d*arcsinh(b*x/sqrt(a*b)) - 3
/8*A*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*A
*b^2*c/a^2 + 3/8*sqrt(b*x^2 + a)*A*b^2*c/a + sqrt(b*x^2 + a)*(B*c + A*d)*b
^2*x/a + (B*c + A*d)*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/2*(C*c + B*d)*sqrt
(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*(C*c + B*d)*b +
1/2*(b*x^2 + a)^(3/2)*(C*c + B*d)*b/a - (b*x^2 + a)^(3/2)*C*d/x - 1/8*(b*x
^2 + a)^(5/2)*A*b*c/(a^2*x^2) - 2/3*(b*x^2 + a)^(3/2)*(B*c + A*d)*b/(a*x)
 - 1/2*(b*x^2 + a)^(5/2)*(C*c + B*d)/(a*x^2) - 1/4*(b*x^2 + a)^(5/2)*A*c/(a
*x^4) - 1/3*(b*x^2 + a)^(5/2)*(B*c + A*d)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(242) = 484$.

Time = 0.20 (sec) , antiderivative size = 754, normalized size of antiderivative = 2.67

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/2*(2*B*b^(3/2)*c + 3*C*a*sqrt(b)*d + 2*A*b^(3/2)*d)*log(abs(-sqrt(b)*x
+ sqrt(b*x^2 + a))) + 1/2*(C*b*d*x + 2*C*b*c + 2*B*b*d)*sqrt(b*x^2 + a) +
3/4*(4*C*a*b*c + A*b^2*c + 4*B*a*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a)
)/sqrt(-a))/sqrt(-a) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c +
15*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 +
a))^7*B*a*b*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a*b^(3/2)*c + 24*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^2*sqrt(b)*d + 48*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*A*a*b^(3/2)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c + 9*(s
qrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))
^5*B*a^2*b*d - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c - 72*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*sqrt(b)*d - 96*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*A*a^2*b^(3/2)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c + 9
*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2
+ a))^3*B*a^3*b*d + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c + 7
2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*sqrt(b)*d + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*A*a^3*b^(3/2)*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b*c
+ 15*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^
2 + a))*B*a^4*b*d - 32*B*a^4*b^(3/2)*c - 24*C*a^5*sqrt(b)*d - 32*A*a^4*b^(
3/2)*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^5}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^5,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5}\,dx = \frac{-24\sqrt{b\,x^2+a}\,a^2c-32\sqrt{b\,x^2+a}\,a^2dx-60\sqrt{b\,x^2+a}\,abc\,x^2}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**2*c - 32*sqrt(a + b*x**2)*a**2*d*x - 60*sqrt(a
+ b*x**2)*a*b*c*x**2 - 32*sqrt(a + b*x**2)*a*b*c*x - 128*sqrt(a + b*x**2)*
a*b*d*x**3 - 48*sqrt(a + b*x**2)*a*b*d*x**2 - 48*sqrt(a + b*x**2)*a*c**2*x
**2 - 96*sqrt(a + b*x**2)*a*c*d*x**3 - 128*sqrt(a + b*x**2)*b**2*c*x**3 +
96*sqrt(a + b*x**2)*b**2*d*x**4 + 96*sqrt(a + b*x**2)*b*c**2*x**4 + 48*sqr
t(a + b*x**2)*b*c*d*x**5 + 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**2*c*x**4 + 144*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*b**2*d*x**4 + 144*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**
2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 - 144*sqrt(a)*log((sqrt(a +
 b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*d*x**4 - 144*sqrt(a)*log((sq
rt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2*x**4 + 96*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d*x**4 + 144*sqrt(b)*log((sq
rt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*d*x**4 + 96*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 + 32*sqrt(b)*a*b*d*x**4 + 63*sq
rt(b)*a*c*d*x**4 + 32*sqrt(b)*b**2*c*x**4)/(96*x**4)
```

3.56 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 243

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6} \, dx = \frac{3b(bBc+Abd+4aCd)\sqrt{a+bx^2}}{8a} - \frac{b(cC+Bd)\sqrt{a+bx^2}}{x} - \frac{(cC+Bd)(a+bx^2)^{3/2}}{3x^3} - \frac{(bBc+Abd+4aCd)(a+bx^2)^{3/2}}{8ax^2} - \frac{Ac(a+bx^2)^{5/2}}{5ax^5} - \frac{(Bc+Ad)(a+bx^2)^{5/2}}{4ax^4} + b^{3/2}(cC+Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3b(bBc+Abd+4aCd)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
3/8*b*(A*b*d+B*b*c+4*C*a*d)*(b*x^2+a)^(1/2)/a-b*(B*d+C*c)*(b*x^2+a)^(1/2)/
x-1/3*(B*d+C*c)*(b*x^2+a)^(3/2)/x^3-1/8*(A*b*d+B*b*c+4*C*a*d)*(b*x^2+a)^(3
/2)/a/x^2-1/5*A*c*(b*x^2+a)^(5/2)/a/x^5-1/4*(A*d+B*c)*(b*x^2+a)^(5/2)/a/x^
4+b^(3/2)*(B*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-3/8*b*(A*b*d+B*b*c+
4*C*a*d)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx =$$
$$-\frac{\sqrt{a+bx^2}\left(24Ab^2cx^4+a^2(6A(4c+5dx)+10x(3Bc+4cCx+4Bdx+6Cdx^2))+abx^2(A(48c+75dx)+\right.}{120ax^5}$$
$$+3\sqrt{a}bCd\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{3b^2(Bc+Ad)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-b^{3/2}(cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/120*(Sqrt[a + b*x^2]*(24*A*b^2*c*x^4 + a^2*(6*A*(4*c + 5*d*x) + 10*x*(3
*B*c + 4*c*C*x + 4*B*d*x + 6*C*d*x^2)) + a*b*x^2*(A*(48*c + 75*d*x) + 5*x*
(8*C*x*(4*c - 3*d*x) + B*(15*c + 32*d*x)))))/(a*x^5) + 3*Sqrt[a]*b*C*d*Arc
Tanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - (3*b^2*(B*c + A*d)*ArcTanh[(
-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(4*Sqrt[a]) - b^(3/2)*(c*C + B*d
)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2338, 27, 2338, 25, 27, 537, 25, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^6}\,dx$$
$$\Big\downarrow \ 2338$$
$$-\frac{\int -\frac{5\left(bx^2+a\right)^{3/2}\left(aCdx^2+a(cC+Bd)x+a(Bc+Ad)\right)}{x^5}dx}{5a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(aCdx^2+a(cC+Bd)x+a(Bc+Ad)\right)}{x^5}dx}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{a(4a(cC+Bd)+(bBc+Abd+4aCd)x)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{4x^4}}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(4a(cC+Bd)+(bBc+Abd+4aCd)x)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{4x^4}}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{4}\int \frac{(4a(cC+Bd)+(bBc+Abd+4aCd)x)\left(bx^2+a\right)^{3/2}}{x^4}dx-\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{4x^4}}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}b\int -\frac{(8a(cC+Bd)+3(bBc+Abd+4aCd)x)\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}(3x(4aCd+Abd+bBc)+8a(Bd+cC))}{6x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{4x^4}}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\int \frac{(8a(cC+Bd)+3(bBc+Abd+4aCd)x)\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}(3x(4aCd+Abd+bBc)+8a(Bd+cC))}{6x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}(Ad+Bc)}{4x^4}}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 536$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\int \frac{3a(bBc+Abd+4aCd)+8ab(cC+Bd)x}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}(3x(4aCd+Abd+bBc)+\ldots}{6x^3}\right.}{a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx+8ab(Bd+cC)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}\right.\right.}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 224

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx+8ab(Bd+cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd\right.\right.}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 219

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(3a(4aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}+8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(B\right.\right.}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 243

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\frac{3}{2}a(4aCd+Abd+bBc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}+8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(\right.\right.}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 73

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(\frac{3a(4aCd+Abd+bBc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}+8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+\right.\right.}{a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 221

$$\frac{\frac{1}{4}\left(\frac{1}{2}b\left(-3\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4aCd+Abd+bBc)-\frac{\sqrt{a+bx^2}(8a(Bd+cC)-3x(4aCd+Abd+bBc))}{x}+8a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{\ }}{\sqrt{a}}\right.\right.\right.}{a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/5*(A*c*(a + b*x^2)^(5/2))/(a*x^5) + (-1/4*((B*c + A*d)*(a + b*x^2)^(5/2
))/x^4 + (-1/6*((8*a*(c*C + B*d) + 3*(b*B*c + A*b*d + 4*a*C*d)*x)*(a + b*x
^2)^(3/2))/x^3 + (b*(-(((8*a*(c*C + B*d) - 3*(b*B*c + A*b*d + 4*a*C*d)*x)*
Sqrt[a + b*x^2])/x) + 8*a*Sqrt[b]*(c*C + B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]] - 3*Sqrt[a]*(b*B*c + A*b*d + 4*a*C*d)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]]))/2)/4)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.97

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(24A\,b^{2}c\,x^{4}+160Babd\,x^{4}+160Cabc\,x^{4}+75Abd\,x^{3}a+75Bbc\,x^{3}a+60C\,a^{2}d\,x^{3}+48Aabc\,x^{2}+40B\,a^{2}d\,x^{2}+40C\,a^{2}c\,x^{2}+30}{120x^{5}a}$
default	$(Ad+Bc)\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)+(Bd+$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(24*A*b^2*c*x^4+160*B*a*b*d*x^4+160*C*a*b*c*x^4+75*
A*a*b*d*x^3+75*B*a*b*c*x^3+60*C*a^2*d*x^3+48*A*a*b*c*x^2+40*B*a^2*d*x^2+40
*C*a^2*c*x^2+30*A*a^2*d*x+30*B*a^2*c*x+24*A*a^2*c)/x^5/a+1/8*b*(-(3*A*b*d+
3*B*b*c+12*C*a*d)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+8*B*b^(1/2
)*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+8*C*b^(1/2)*c*ln(b^(1/2)*x+(b*x^2+a)^(1/
2))+8*d*C*(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 906, normalized size of antiderivative = 3.73

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="fricas")
```

output
```
[1/240*(120*(C*a*b*c + B*a*b*d)*sqrt(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 45*(B*b^2*c + (4*C*a*b + A*b^2)*d)*sqrt(a)*x^5*log(-(b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(120*C*a*b*d*x^5 - 8*(20*
B*a*b*d + (20*C*a*b + 3*A*b^2)*c)*x^4 - 24*A*a^2*c - 15*(5*B*a*b*c + (4*C*
a^2 + 5*A*a*b)*d)*x^3 - 8*(5*B*a^2*d + (5*C*a^2 + 6*A*a*b)*c)*x^2 - 30*(B*
a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a*x^5), -1/240*(240*(C*a*b*c + B*a*b
*d)*sqrt(-b)*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 45*(B*b^2*c + (4*C*a
*b + A*b^2)*d)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) - 2*(120*C*a*b*d*x^5 - 8*(20*B*a*b*d + (20*C*a*b + 3*A*b^2)*c)*x^4 -
24*A*a^2*c - 15*(5*B*a*b*c + (4*C*a^2 + 5*A*a*b)*d)*x^3 - 8*(5*B*a^2*d + (
5*C*a^2 + 6*A*a*b)*c)*x^2 - 30*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a*
x^5), 1/120*(45*(B*b^2*c + (4*C*a*b + A*b^2)*d)*sqrt(-a)*x^5*arctan(sqrt(b
*x^2 + a)*sqrt(-a)/a) + 60*(C*a*b*c + B*a*b*d)*sqrt(b)*x^5*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (120*C*a*b*d*x^5 - 8*(20*B*a*b*d + (20*
C*a*b + 3*A*b^2)*c)*x^4 - 24*A*a^2*c - 15*(5*B*a*b*c + (4*C*a^2 + 5*A*a*b)
*d)*x^3 - 8*(5*B*a^2*d + (5*C*a^2 + 6*A*a*b)*c)*x^2 - 30*(B*a^2*c + A*a^2*
d)*x)*sqrt(b*x^2 + a))/(a*x^5), -1/120*(120*(C*a*b*c + B*a*b*d)*sqrt(-b)*x
^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 45*(B*b^2*c + (4*C*a*b + A*b^2)*d)
*sqrt(-a)*x^5*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (120*C*a*b*d*x^5 - 8*(2
0*B*a*b*d + (20*C*a*b + 3*A*b^2)*c)*x^4 - 24*A*a^2*c - 15*(5*B*a*b*c + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(233) = 466$.

Time = 9.79 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.82

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx = -\frac{Aa^2d}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{Aa\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{3Aa\sqrt{b}d}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2Ab^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Ab^{\frac{3}{2}}d}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}c\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{3Ab^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{B\sqrt{a}bd}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^2c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ba\sqrt{b}c}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ba\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Bb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}c}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3} + Bb^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{3Bb^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{Bb^2dx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - \frac{C\sqrt{a}bc}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{3C\sqrt{a}bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Ca\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Ca\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Ca\sqrt{b}d}{x\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3} + Cb^{\frac{3}{2}}c\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + \frac{Cb^{\frac{3}{2}}dx}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^2cx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**6,x)
```

output
```
-A*a**2*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - A*a*sqrt(b)*c*sqrt(a/(b*
x**2) + 1)/(5*x**4) - 3*A*a*sqrt(b)*d/(8*x**3*sqrt(a/(b*x**2) + 1)) - 2*A*
b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(5*x**2) - A*b**(3/2)*d*sqrt(a/(b*x**2) +
1)/(2*x) - A*b**(3/2)*d/(8*x*sqrt(a/(b*x**2) + 1)) - A*b**(5/2)*c*sqrt(a/(
b*x**2) + 1)/(5*a) - 3*A*b**2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - B
*sqrt(a)*b*d/(x*sqrt(1 + b*x**2/a)) - B*a**2*c/(4*sqrt(b)*x**5*sqrt(a/(b*x
**2) + 1)) - 3*B*a*sqrt(b)*c/(8*x**3*sqrt(a/(b*x**2) + 1)) - B*a*sqrt(b)*d
*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(2*x) -
 B*b**(3/2)*c/(8*x*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*d*sqrt(a/(b*x**2) +
1)/3 + B*b**(3/2)*d*asinh(sqrt(b)*x/sqrt(a)) - 3*B*b**2*c*asinh(sqrt(a)/(s
qrt(b)*x))/(8*sqrt(a)) - B*b**2*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - C*sqrt(
a)*b*c/(x*sqrt(1 + b*x**2/a)) - 3*C*sqrt(a)*b*d*asinh(sqrt(a)/(sqrt(b)*x))
/2 - C*a*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*a*sqrt(b)*d*sqrt(a/(b
*x**2) + 1)/(2*x) + C*a*sqrt(b)*d/(x*sqrt(a/(b*x**2) + 1)) - C*b**(3/2)*c*
sqrt(a/(b*x**2) + 1)/3 + C*b**(3/2)*c*asinh(sqrt(b)*x/sqrt(a)) + C*b**(3/2
)*d*x/sqrt(a/(b*x**2) + 1) - C*b**2*c*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.26

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx = -\frac{3}{2}\,C\sqrt{a}bd\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}Cbd + \frac{(bx^2+a)^{\frac{3}{2}}Cbd}{2\,a} + \frac{\sqrt{bx^2+a}(Cc+Bd)b^2x}{a} + (Cc+Bd)b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3\,(Bc+Ad)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b^2}{8\,a^2} + \frac{3\sqrt{bx^2+a}(Bc+Ad)b^2}{8\,a} - \frac{(bx^2+a)^{\frac{5}{2}}Cd}{2\,ax^2} - \frac{2\,(bx^2+a)^{\frac{3}{2}}(Cc+Bd)b}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}(Cc+Bd)}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}Ac}{5\,ax^5} - \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)}{4\,ax^4}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="maxima")
```

output
```
-3/2*C*sqrt(a)*b*d*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*C*b
*d + 1/2*(b*x^2 + a)^(3/2)*C*b*d/a + sqrt(b*x^2 + a)*(C*c + B*d)*b^2*x/a +
 (C*c + B*d)*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/8*(B*c + A*d)*b^2*arcsinh(
a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*(B*c + A*d)*b^2/a^2
+ 3/8*sqrt(b*x^2 + a)*(B*c + A*d)*b^2/a - 1/2*(b*x^2 + a)^(5/2)*C*d/(a*x^2
) - 2/3*(b*x^2 + a)^(3/2)*(C*c + B*d)*b/(a*x) - 1/8*(b*x^2 + a)^(5/2)*(B*c
 + A*d)*b/(a^2*x^2) - 1/3*(b*x^2 + a)^(5/2)*(C*c + B*d)/(a*x^3) - 1/5*(b*x
^2 + a)^(5/2)*A*c/(a*x^5) - 1/4*(b*x^2 + a)^(5/2)*(B*c + A*d)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(207) = 414$.

Time = 0.23 (sec) , antiderivative size = 761, normalized size of antiderivative = 3.13

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*C*b*d - (C*b^(3/2)*c + B*b^(3/2)*d)*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a))) + 3/4*(B*b^2*c + 4*C*a*b*d + A*b^2*d)*arctan(-(sqrt(b)*x
- sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1/60*(75*(sqrt(b)*x - sqrt(b*x^2 +
 a))^9*B*b^2*c + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a*b*d + 75*(sqrt(b)*
x - sqrt(b*x^2 + a))^9*A*b^2*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b
^(3/2)*c + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2)*c + 240*(sqrt(b)*
x - sqrt(b*x^2 + a))^8*B*a*b^(3/2)*d - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^7*
B*a*b^2*c - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^2*b*d - 30*(sqrt(b)*x
- sqrt(b*x^2 + a))^7*A*a*b^2*d - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2
*b^(3/2)*c - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)*d + 880*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*b^(3/2)*c + 240*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*a^2*b^(5/2)*c + 880*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/
2)*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b^2*c + 120*(sqrt(b)*x - s
qrt(b*x^2 + a))^3*C*a^4*b*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b^2
*d - 560*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2)*c - 560*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*B*a^4*b^(3/2)*d - 75*(sqrt(b)*x - sqrt(b*x^2 + a))*B*
a^4*b^2*c - 60*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^5*b*d - 75*(sqrt(b)*x - s
qrt(b*x^2 + a))*A*a^4*b^2*d + 160*C*a^5*b^(3/2)*c + 24*A*a^4*b^(5/2)*c + 1
60*B*a^5*b^(3/2)*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^6}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.19

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^6}\,dx = \frac{-24\sqrt{b\,x^2+a}\,a^3c-30\sqrt{b\,x^2+a}\,a^3dx-48\sqrt{b\,x^2+a}\,a^2bc\,x}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3*c - 30*sqrt(a + b*x**2)*a**3*d*x - 48*sqrt(a
+ b*x**2)*a**2*b*c*x**2 - 30*sqrt(a + b*x**2)*a**2*b*c*x - 75*sqrt(a + b*x
**2)*a**2*b*d*x**3 - 40*sqrt(a + b*x**2)*a**2*b*d*x**2 - 40*sqrt(a + b*x**
2)*a**2*c**2*x**2 - 60*sqrt(a + b*x**2)*a**2*c*d*x**3 - 24*sqrt(a + b*x**2
)*a*b**2*c*x**4 - 75*sqrt(a + b*x**2)*a*b**2*c*x**3 - 160*sqrt(a + b*x**2)
*a*b**2*d*x**4 - 160*sqrt(a + b*x**2)*a*b*c**2*x**4 + 120*sqrt(a + b*x**2)
*a*b*c*d*x**5 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a))*a*b**2*d*x**5 + 180*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*a*b*c*d*x**5 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**3*c*x**5 - 45*sqrt(a)*log((sqrt(a + b*x**2) + sqrt
(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5 - 180*sqrt(a)*log((sqrt(a + b*x**2
) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d*x**5 - 45*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**5 + 120*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5 + 120*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**2*x**5 - 24*sqrt(b)*a*b**2*c*x**5
 + 64*sqrt(b)*a*b**2*d*x**5 + 64*sqrt(b)*a*b*c**2*x**5)/(120*a*x**5)
```

3.57 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 236

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = \frac{b(Abc-6a(cC+Bd))\sqrt{a+bx^2}}{16ax^2} - \frac{bCd\sqrt{a+bx^2}}{x} + \frac{(Abc-6a(cC+Bd))\left(a+bx^2\right)^{3/2}}{24ax^4} - \frac{Cd(a+bx^2)^{3/2}}{3x^3} - \frac{Ac(a+bx^2)^{5/2}}{6ax^6} - \frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{5ax^5} + b^{3/2}Cd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{b^2(Abc-6a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output

```
1/16*b*(A*b*c-6*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a/x^2-b*C*d*(b*x^2+a)^(1/2)/x
+1/24*(A*b*c-6*a*(B*d+C*c))*(b*x^2+a)^(3/2)/a/x^4-1/3*C*d*(b*x^2+a)^(3/2)/
x^3-1/6*A*c*(b*x^2+a)^(5/2)/a/x^6-1/5*(A*d+B*c)*(b*x^2+a)^(5/2)/a/x^5+b^(3
/2)*C*d*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))+1/16*b^2*(A*b*c-6*a*(B*d+C*c))*
arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx =$$

$$-\frac{\sqrt{a+bx^2}(3b^2x^4(5Ac+16Bcx+16Adx)+2abx^2(A(35c+48dx)+x(48Bc+75cCx+75Bdx+160Cd}{240ax^6}$$

$$-\frac{Ab^3c\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{3b^2(cC+Bd)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}}$$

$$-b^{3/2}Cd\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^7,x]
```

output
```
-1/240*(Sqrt[a + b*x^2]*(3*b^2*x^4*(5*A*c + 16*B*c*x + 16*A*d*x) + 2*a*b*x
^2*(A*(35*c + 48*d*x) + x*(48*B*c + 75*c*C*x + 75*B*d*x + 160*C*d*x^2)) +
a^2*(8*A*(5*c + 6*d*x) + 4*x*(5*C*x*(3*c + 4*d*x) + 3*B*(4*c + 5*d*x)))))/
(a*x^6) - (A*b^3*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(8*a^(3
/2)) - (3*b^2*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]
])/(4*Sqrt[a]) - b^(3/2)*C*d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.92, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2338, 25, 2338, 27, 537, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^7}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(6aCdx^2-(Abc-6a(cC+Bd))x+6a(Bc+Ad)\right)}{x^6}dx}{6a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(6aCdx^2-(Abc-6a(cC+Bd))x+6a(Bc+Ad))}{x^6}dx}{6a}-\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int \frac{5a(Abc-6a(cC+Bd)-6aCdx)\left(bx^2+a\right)^{3/2}}{x^5}dx}{5a}-\frac{6(a+bx^2)^{5/2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 27$$

$$\frac{-\int \frac{(Abc-6a(cC+Bd)-6aCdx)(bx^2+a)^{3/2}}{x^5}dx-\frac{6(a+bx^2)^{5/2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 537$$

$$\frac{\frac{1}{4}b\int -\frac{3(Abc-6a(cC+Bd)-8aCdx)\sqrt{bx^2+a}}{x^3}dx+\frac{(a+bx^2)^{3/2}(-6a(Bd+cC)-8aCdx+Abc)}{4x^4}-\frac{6(a+bx^2)^{5/2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{3}{4}b\int \frac{(Abc-6a(cC+Bd)-8aCdx)\sqrt{bx^2+a}}{x^3}dx+\frac{(a+bx^2)^{3/2}(-6a(Bd+cC)-8aCdx+Abc)}{4x^4}-\frac{6(a+bx^2)^{5/2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 537$$

$$\frac{-\frac{3}{4}b\left(-\frac{1}{2}b\int -\frac{Abc-6a(cC+Bd)-16aCdx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6a(Bd+cC)-8aCdx+Abc)}{4x^4}-}{6a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$$\Big\downarrow \ 25$$

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\int \frac{Abc-6a(cC+Bd)-16aCdx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+\frac{(a+bx^2)^{3/2}(-6a(Bd+cC)-8aCdx+Abc)}{4x^4}-6(a}{6a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

$\downarrow$ 538

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Abc-6a(Bd+cC))\int\frac{1}{x\sqrt{bx^2+a}}dx-16aCd\int\frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+\frac{(a+bx^2}{}}{6a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\downarrow$ 224

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Abc-6a(Bd+cC))\int\frac{1}{x\sqrt{bx^2+a}}dx-16aCd\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)}{6a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\downarrow$ 219

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left((Abc-6a(Bd+cC))\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{16aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+}{6a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\downarrow$ 243

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(\frac{1}{2}(Abc-6a(Bd+cC))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{16aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)}{6a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\downarrow$ 73

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(\frac{(Abc-6a(Bd+cC))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{16aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+\frac{(a+}{}}{6a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$\downarrow$ 221

$$\frac{-\frac{3}{4}b\left(\frac{1}{2}b\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Abc-6a(Bd+cC))}{\sqrt{a}}-\frac{16aCd\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(-6a(Bd+cC)-16aCdx+Abc)}{2x^2}\right)+}{6a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{6ax^6}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^7,x]
```

output
```
-1/6*(A*c*(a + b*x^2)^(5/2))/(a*x^6) + (((A*b*c - 6*a*(c*C + B*d) - 8*a*C*
d*x)*(a + b*x^2)^(3/2))/(4*x^4) - (6*(B*c + A*d)*(a + b*x^2)^(5/2))/(5*x^5
) - (3*b*(-1/2*((A*b*c - 6*a*(c*C + B*d) - 16*a*C*d*x)*Sqrt[a + b*x^2])/x^
2 + (b*((-16*a*C*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((A*b*c
 - 6*a*(c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4)/(6*
a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(48A\,b^2d\,x^5+48B\,b^2c\,x^5+320Cabd\,x^5+15A\,b^2c\,x^4+150Babd\,x^4+150Cabc\,x^4+96Abd\,x^3a+96Bbc\,x^3a+80C\,a^2d\,x^3+7}{240x^6a}$
default	$-\frac{(Ad+Bc)(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}+(Bd+Cc)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.\right.\right.}{2a}\right.}{4a}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(48*A*b^2*d*x^5+48*B*b^2*c*x^5+320*C*a*b*d*x^5+15*A
*b^2*c*x^4+150*B*a*b*d*x^4+150*C*a*b*c*x^4+96*A*a*b*d*x^3+96*B*a*b*c*x^3+8
0*C*a^2*d*x^3+70*A*a*b*c*x^2+60*B*a^2*d*x^2+60*C*a^2*c*x^2+48*A*a^2*d*x+48
*B*a^2*c*x+40*A*a^2*c)/x^6/a-1/16*b^2/a*(-(A*b*c-6*B*a*d-6*C*a*c)/a^(1/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-16*a*C*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2
))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 1032, normalized size of antiderivative = 4.37

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="fricas")
```

output
```
[1/480*(240*C*a^2*b^(3/2)*d*x^6*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) + 15*(6*B*a*b^2*d + (6*C*a*b^2 - A*b^3)*c)*sqrt(a)*x^6*log(-(b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*(3*B*a*b^2*c + (20*C*a^2*b
+ 3*A*a*b^2)*d)*x^5 + 40*A*a^3*c + 15*(10*B*a^2*b*d + (10*C*a^2*b + A*a*b^
2)*c)*x^4 + 16*(6*B*a^2*b*c + (5*C*a^3 + 6*A*a^2*b)*d)*x^3 + 10*(6*B*a^3*d
 + (6*C*a^3 + 7*A*a^2*b)*c)*x^2 + 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a
))/(a^2*x^6), -1/480*(480*C*a^2*sqrt(-b)*b*d*x^6*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - 15*(6*B*a*b^2*d + (6*C*a*b^2 - A*b^3)*c)*sqrt(a)*x^6*log(-(b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(3*B*a*b^2*c + (20*C*a^
2*b + 3*A*a*b^2)*d)*x^5 + 40*A*a^3*c + 15*(10*B*a^2*b*d + (10*C*a^2*b + A*
a*b^2)*c)*x^4 + 16*(6*B*a^2*b*c + (5*C*a^3 + 6*A*a^2*b)*d)*x^3 + 10*(6*B*a
^3*d + (6*C*a^3 + 7*A*a^2*b)*c)*x^2 + 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2
 + a))/(a^2*x^6), 1/240*(120*C*a^2*b^(3/2)*d*x^6*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 15*(6*B*a*b^2*d + (6*C*a*b^2 - A*b^3)*c)*sqrt(-a)
*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (16*(3*B*a*b^2*c + (20*C*a^2*b +
 3*A*a*b^2)*d)*x^5 + 40*A*a^3*c + 15*(10*B*a^2*b*d + (10*C*a^2*b + A*a*b^2
)*c)*x^4 + 16*(6*B*a^2*b*c + (5*C*a^3 + 6*A*a^2*b)*d)*x^3 + 10*(6*B*a^3*d
+ (6*C*a^3 + 7*A*a^2*b)*c)*x^2 + 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a)
)/(a^2*x^6), -1/240*(240*C*a^2*sqrt(-b)*b*d*x^6*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) - 15*(6*B*a*b^2*d + (6*C*a*b^2 - A*b^3)*c)*sqrt(-a)*x^6*arctan...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(216) = 432$.

Time = 14.37 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.89

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = -\frac{Aa^2c}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11Aa\sqrt{b}c}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{Aa\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{17Ab^{\frac{3}{2}}c}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2Ab^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Ab^{\frac{5}{2}}c}{16ax\sqrt{\frac{a}{bx^2}+1}} - \frac{Ab^{\frac{5}{2}}d\sqrt{\frac{a}{bx^2}+1}}{5a} + \frac{Ab^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}} - \frac{Ba^2d}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{Ba\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{3Ba\sqrt{b}d}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2Bb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{Bb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Bb^{\frac{3}{2}}d}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{Bb^{\frac{5}{2}}c\sqrt{\frac{a}{bx^2}+1}}{5a} - \frac{3Bb^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{C\sqrt{a}bd}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ca^2c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ca\sqrt{b}c}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{Ca\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{Cb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{Cb^{\frac{3}{2}}c}{8x\sqrt{\frac{a}{bx^2}+1}} - \frac{Cb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3} + Cb^{\frac{3}{2}}d\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{3Cb^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}} - \frac{Cb^2dx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a**2*c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)*c/(24*x**
5*sqrt(a/(b*x**2) + 1)) - A*a*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(5*x**4) - 17
*A*b**(3/2)*c/(48*x**3*sqrt(a/(b*x**2) + 1)) - 2*A*b**(3/2)*d*sqrt(a/(b*x*
*2) + 1)/(5*x**2) - A*b**(5/2)*c/(16*a*x*sqrt(a/(b*x**2) + 1)) - A*b**(5/2
)*d*sqrt(a/(b*x**2) + 1)/(5*a) + A*b**3*c*asinh(sqrt(a)/(sqrt(b)*x))/(16*a
**(3/2)) - B*a**2*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - B*a*sqrt(b)*c*
sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*B*a*sqrt(b)*d/(8*x**3*sqrt(a/(b*x**2) +
1)) - 2*B*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(3/2)*d*sqrt(a/(
b*x**2) + 1)/(2*x) - B*b**(3/2)*d/(8*x*sqrt(a/(b*x**2) + 1)) - B*b**(5/2)*
c*sqrt(a/(b*x**2) + 1)/(5*a) - 3*B*b**2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*sq
rt(a)) - C*sqrt(a)*b*d/(x*sqrt(1 + b*x**2/a)) - C*a**2*c/(4*sqrt(b)*x**5*s
qrt(a/(b*x**2) + 1)) - 3*C*a*sqrt(b)*c/(8*x**3*sqrt(a/(b*x**2) + 1)) - C*a
*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - C*b**(3/2)*c*sqrt(a/(b*x**2) +
1)/(2*x) - C*b**(3/2)*c/(8*x*sqrt(a/(b*x**2) + 1)) - C*b**(3/2)*d*sqrt(a/(
b*x**2) + 1)/3 + C*b**(3/2)*d*asinh(sqrt(b)*x/sqrt(a)) - 3*C*b**2*c*asinh(
sqrt(a)/(sqrt(b)*x))/(8*sqrt(a)) - C*b**2*d*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = \frac{\sqrt{bx^2+a}Cb^2dx}{a} + Cb^{\frac{3}{2}}d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + \frac{Ab^3c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab^3c}{48\,a^3} - \frac{\sqrt{bx^2+a}Ab^3c}{16\,a^2} - \frac{3\,(Cc+Bd)b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)b^2}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}(Cc+Bd)b^2}{8\,a} - \frac{2\,\left(bx^2+a\right)^{\frac{3}{2}}Cbd}{3\,ax} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^2c}{48\,a^3x^2} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cd}{3\,ax^3} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)b}{8\,a^2x^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Abc}{24\,a^2x^4} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bc}{5\,ax^5} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ad}{5\,ax^5} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)}{4\,ax^4} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ac}{6\,ax^6}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*C*b^2*d*x/a + C*b^(3/2)*d*arcsinh(b*x/sqrt(a*b)) + 1/16*A*
b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*A*b^3
*c/a^3 - 1/16*sqrt(b*x^2 + a)*A*b^3*c/a^2 - 3/8*(C*c + B*d)*b^2*arcsinh(a/
(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*(C*c + B*d)*b^2/a^2 +
3/8*sqrt(b*x^2 + a)*(C*c + B*d)*b^2/a - 2/3*(b*x^2 + a)^(3/2)*C*b*d/(a*x)
+ 1/48*(b*x^2 + a)^(5/2)*A*b^2*c/(a^3*x^2) - 1/3*(b*x^2 + a)^(5/2)*C*d/(a*
x^3) - 1/8*(b*x^2 + a)^(5/2)*(C*c + B*d)*b/(a^2*x^2) + 1/24*(b*x^2 + a)^(5
/2)*A*b*c/(a^2*x^4) - 1/5*(b*x^2 + a)^(5/2)*B*c/(a*x^5) - 1/5*(b*x^2 + a)^
(5/2)*A*d/(a*x^5) - 1/4*(b*x^2 + a)^(5/2)*(C*c + B*d)/(a*x^4) - 1/6*(b*x^2
 + a)^(5/2)*A*c/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. $2(200) = 400$.

Time = 0.18 (sec) , antiderivative size = 1084, normalized size of antiderivative = 4.59

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
-C*b^(3/2)*d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/8*(6*C*a*b^2*c - A
*b^3*c + 6*B*a*b^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqr
t(-a)*a) + 1/120*(150*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^2*c + 15*(sqr
t(b)*x - sqrt(b*x^2 + a))^11*A*b^3*c + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^1
1*B*a*b^2*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a*b^(5/2)*c + 480*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(3/2)*d + 240*(sqrt(b)*x - sqrt(b*x^
2 + a))^10*A*a*b^(5/2)*d - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2*c
 + 235*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3*c - 210*(sqrt(b)*x - sqrt(b
*x^2 + a))^9*B*a^2*b^2*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/
2)*c - 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2)*d - 240*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*A*a^2*b^(5/2)*d + 60*(sqrt(b)*x - sqrt(b*x^2 + a))
^7*C*a^3*b^2*c + 390*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c + 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^7*B*a^3*b^2*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a)
)^6*B*a^3*b^(5/2)*c + 3200*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(3/2)*d
 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(5/2)*d + 60*(sqrt(b)*x - s
qrt(b*x^2 + a))^5*C*a^4*b^2*c + 390*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^3*
b^3*c + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^4*b^2*d - 480*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*B*a^4*b^(5/2)*c - 2880*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
C*a^5*b^(3/2)*d - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(5/2)*d - 21
0*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^5*b^2*c + 235*(sqrt(b)*x - sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^7,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^7}\,dx = \frac{-48\sqrt{b\,x^2+a}\,a^3dx-60\sqrt{b\,x^2+a}\,a^2c^2x^2-48\sqrt{b\,x^2+a}\,b^3}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c - 48*sqrt(a + b*x**2)*a**3*d*x - 70*sqrt(a
+ b*x**2)*a**2*b*c*x**2 - 48*sqrt(a + b*x**2)*a**2*b*c*x - 96*sqrt(a + b*x
**2)*a**2*b*d*x**3 - 60*sqrt(a + b*x**2)*a**2*b*d*x**2 - 60*sqrt(a + b*x**
2)*a**2*c**2*x**2 - 80*sqrt(a + b*x**2)*a**2*c*d*x**3 - 15*sqrt(a + b*x**2
)*a*b**2*c*x**4 - 96*sqrt(a + b*x**2)*a*b**2*c*x**3 - 48*sqrt(a + b*x**2)*
a*b**2*d*x**5 - 150*sqrt(a + b*x**2)*a*b**2*d*x**4 - 150*sqrt(a + b*x**2)*
a*b*c**2*x**4 - 320*sqrt(a + b*x**2)*a*b*c*d*x**5 - 48*sqrt(a + b*x**2)*b*
*3*c*x**5 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a
))*b**3*c*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**3*d*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**2*c**2*x**6 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**3*c*x**6 - 90*sqrt(a)*log((sqrt(a + b*x**2) + sqrt
(a) + sqrt(b)*x)/sqrt(a))*b**3*d*x**6 - 90*sqrt(a)*log((sqrt(a + b*x**2) +
 sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**6 + 240*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d*x**6 - 32*sqrt(b)*a*b**2*d*x**6 + 16
0*sqrt(b)*a*b*c*d*x**6 - 32*sqrt(b)*b**3*c*x**6)/(240*a*x**6)
```

3.58 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^8}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 210

$$\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^8}\,dx = -\frac{b(6aCd-b(Bc+Ad))\sqrt{a+bx^2}}{16ax^2} - \frac{(6aCd-b(Bc+Ad))(a+bx^2)^{3/2}}{24ax^4} - \frac{Ac(a+bx^2)^{5/2}}{7ax^7} - \frac{(Bc+Ad)(a+bx^2)^{5/2}}{6ax^6} + \frac{(2Abc-7a(cC+Bd))(a+bx^2)^{5/2}}{35a^2x^5} - \frac{b^2(6aCd-b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
-1/16*b*(6*a*C*d-b*(A*d+B*c))*(b*x^2+a)^(1/2)/a/x^2-1/24*(6*a*C*d-b*(A*d+B
*c))*(b*x^2+a)^(3/2)/a/x^4-1/7*A*c*(b*x^2+a)^(5/2)/a/x^7-1/6*(A*d+B*c)*(b*
x^2+a)^(5/2)/a/x^6+1/35*(2*A*b*c-7*a*(B*d+C*c))*(b*x^2+a)^(5/2)/a^2/x^5-1/
16*b^2*(6*a*C*d-b*(A*d+B*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx =$$
$$-\frac{\sqrt{a+bx^2}\left(-96Ab^3cx^6+3ab^2x^4(A(16c+35dx)+7x(5Bc+16cCx+16Bdx))+2a^2bx^2\left(A(192c+245dx)+7x\left(35Bc+48cCx+48Bdx+75Cdx^2\right)\right)+4a\right.}{x^7}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^8,x]
```

output
```
-1/1680*((Sqrt[a + b*x^2]*(-96*A*b^3*c*x^6 + 3*a*b^2*x^4*(A*(16*c + 35*d*x
) + 7*x*(5*B*c + 16*c*C*x + 16*B*d*x)) + 2*a^2*b*x^2*(A*(192*c + 245*d*x)
+ 7*x*(35*B*c + 48*c*C*x + 48*B*d*x + 75*C*d*x^2)) + 4*a^3*(10*A*(6*c + 7*
d*x) + 7*x*(3*C*x*(4*c + 5*d*x) + 2*B*(5*c + 6*d*x)))))/x^7 + 105*Sqrt[a]*
b^2*(6*a*C*d - b*(B*c + A*d))*Log[x] - 105*Sqrt[a]*b^2*(6*a*C*d - b*(B*c +
 A*d))*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/a^2
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.88, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2338, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^8}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(7aCdx^2-(2Abc-7a(cC+Bd))x+7a(Bc+Ad)\right)}{x^7}dx}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(7aCdx^2-(2Abc-7a(cC+Bd))x+7a(Bc+Ad)\right)}{x^7}dx}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int \frac{a(6(2Abc-7a(cC+Bd))+7(bBc+Abd-6aCd)x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{7\left(a+bx^2\right)^{5/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{1}{6}\int \frac{(6(2Abc-7a(cC+Bd))+7(bBc+Abd-6aCd)x)\left(bx^2+a\right)^{3/2}}{x^6}dx-\frac{7\left(a+bx^2\right)^{5/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-7(-6aCd+Abd+bBc)\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx\right)-\frac{7\left(a+bx^2\right)^{5/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-\frac{7}{2}(-6aCd+Abd+bBc)\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2\right)-\frac{7\left(a+bx^2\right)^{5/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-\frac{7}{2}(-6aCd+Abd+bBc)\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\right)-\frac{7\left(a+bx^2\right)^{5/2}(Ad+Bc)}{6x^6}}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 51$$

$$\frac{\frac{1}{6}\left(\frac{6\left(a+bx^2\right)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-\frac{7}{2}(-6aCd+Abd+bBc)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\right)-7}{7a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{6}\left(\frac{6(a+bx^2)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-\frac{7}{2}(-6aCd+Abd+bBc)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)\right)-}{7a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{1}{6}\left(\frac{6(a+bx^2)^{5/2}(2Abc-7a(Bd+cC))}{5ax^5}-\frac{7}{2}\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)(-6aCd+Abd+bBc)\right.}{7a}$$

$$\frac{Ac(a+bx^2)^{5/2}}{7ax^7}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^8,x]
```

output
```
-1/7*(A*c*(a + b*x^2)^(5/2))/(a*x^7) + ((-7*(B*c + A*d)*(a + b*x^2)^(5/2))
/(6*x^6) + ((6*(2*A*b*c - 7*a*(c*C + B*d))*(a + b*x^2)^(5/2))/(5*a*x^5) -
(7*(b*B*c + A*b*d - 6*a*C*d)*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a
+ b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)/6)/(
7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(-96 A b^3 c x^6+336 B a b^2 d x^6+336 C a b^2 c x^6+105 A a b^2 d x^5+105 B a b^2 c x^5+1050 C a^2 b d x^5+48 A a b^2 c x^4+672 B a^2 b d x^4}{}$
default	$(Ad+Bc)\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{6a x^6}-\frac{b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{4a x^4}+\frac{b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{2a x^2}+\frac{3b\left(\frac{(b x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)}{6a}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^2+a)^(1/2)*(-96*A*b^3*c*x^6+336*B*a*b^2*d*x^6+336*C*a*b^2*c*x
^6+105*A*a*b^2*d*x^5+105*B*a*b^2*c*x^5+1050*C*a^2*b*d*x^5+48*A*a*b^2*c*x^4
+672*B*a^2*b*d*x^4+672*C*a^2*b*c*x^4+490*A*a^2*b*d*x^3+490*B*a^2*b*c*x^3+4
20*C*a^3*d*x^3+384*A*a^2*b*c*x^2+336*B*a^3*d*x^2+336*C*a^3*c*x^2+280*A*a^3
*d*x+280*B*a^3*c*x+240*A*a^3*c)/x^7/a^2+1/16*(A*b*d+B*b*c-6*C*a*d)/a^(3/2)
*b^2*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.37

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx = \left[\frac{105\,(Bb^3c-(6\,Cab^2-Ab^3)d)\sqrt{a}x^7\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+}{x^2}\right.}{}\right.$$

$$-\frac{105\,(Bb^3c-(6\,Cab^2-Ab^3)d)\sqrt{-a}x^7\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(48\,(7\,Bab^2d+(7\,Cab^2-2\,Ab^3)c)x^6+105}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="fricas")
```

output
```
[1/3360*(105*(B*b^3*c - (6*C*a*b^2 - A*b^3)*d)*sqrt(a)*x^7*log(-(b*x^2 + 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(48*(7*B*a*b^2*d + (7*C*a*b^2 - 2
*A*b^3)*c)*x^6 + 105*(B*a*b^2*c + (10*C*a^2*b + A*a*b^2)*d)*x^5 + 240*A*a^
3*c + 48*(14*B*a^2*b*d + (14*C*a^2*b + A*a*b^2)*c)*x^4 + 70*(7*B*a^2*b*c +
 (6*C*a^3 + 7*A*a^2*b)*d)*x^3 + 48*(7*B*a^3*d + (7*C*a^3 + 8*A*a^2*b)*c)*x
^2 + 280*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/(a^2*x^7), -1/1680*(105*(
B*b^3*c - (6*C*a*b^2 - A*b^3)*d)*sqrt(-a)*x^7*arctan(sqrt(b*x^2 + a)*sqrt(
-a)/a) + (48*(7*B*a*b^2*d + (7*C*a*b^2 - 2*A*b^3)*c)*x^6 + 105*(B*a*b^2*c
+ (10*C*a^2*b + A*a*b^2)*d)*x^5 + 240*A*a^3*c + 48*(14*B*a^2*b*d + (14*C*a
^2*b + A*a*b^2)*c)*x^4 + 70*(7*B*a^2*b*c + (6*C*a^3 + 7*A*a^2*b)*d)*x^3 +
48*(7*B*a^3*d + (7*C*a^3 + 8*A*a^2*b)*c)*x^2 + 280*(B*a^3*c + A*a^3*d)*x)*
sqrt(b*x^2 + a))/(a^2*x^7)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(189) = 378$.

Time = 18.36 (sec) , antiderivative size = 1027, normalized size of antiderivative = 4.89

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**8,x)
```

output
```
-15*A*a**6*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*c*x**2*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 17*A*a**4*b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*c*x**6*sqrt(a/
(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 12*A*a**2*b**(17/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a**2*d/(6*sqrt(b)*x**7*sqr
t(a/(b*x**2) + 1)) - 8*A*a*b**(19/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 11*A*a*sqrt(b)*d
/(24*x**5*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(5*x**
4) - 17*A*b**(3/2)*d/(48*x**3*sqrt(a/(b*x**2) + 1)) - A*b**(5/2)*c*sqrt(a/
(b*x**2) + 1)/(15*a*x**2) - A*b**(5/2)*d/(16*a*x*sqrt(a/(b*x**2) + 1)) + 2
*A*b**(7/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**2) + A*b**3*d*asinh(sqrt(a)/(sqr
t(b)*x))/(16*a**(3/2)) - B*a**2*c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) -
11*B*a*sqrt(b)*c/(24*x**5*sqrt(a/(b*x**2) + 1)) - B*a*sqrt(b)*d*sqrt(a/(b*
x**2) + 1)/(5*x**4) - 17*B*b**(3/2)*c/(48*x**3*sqrt(a/(b*x**2) + 1)) - 2*B
*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(5/2)*c/(16*a*x*sqrt(a/(b
*x**2) + 1)) - B*b**(5/2)*d*sqrt(a/(b*x**2) + 1)/(5*a) + B*b**3*c*asinh(sq
rt(a)/(sqrt(b)*x))/(16*a**(3/2)) - C*a**2*d/(4*sqrt(b)*x**5*sqrt(a/(b*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^8}\,dx = -\frac{3\,Cb^2d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{(bx^2+a)^{\frac{3}{2}}Cb^2d}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}Cb^2d}{8\,a} + \frac{(Bc+Ad)b^3\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{(bx^2+a)^{\frac{3}{2}}(Bc+Ad)b^3}{48\,a^3} - \frac{\sqrt{bx^2+a}(Bc+Ad)b^3}{16\,a^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cbd}{8\,a^2x^2} + \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)b^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cd}{4\,ax^4} - \frac{(bx^2+a)^{\frac{5}{2}}Cc}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Abc}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}Bd}{5\,ax^5} + \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)b}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}Ac}{7\,ax^7} - \frac{(bx^2+a)^{\frac{5}{2}}(Bc+Ad)}{6\,ax^6}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="maxima")
```

output
```
-3/8*C*b^2*d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)
*C*b^2*d/a^2 + 3/8*sqrt(b*x^2 + a)*C*b^2*d/a + 1/16*(B*c + A*d)*b^3*arcsin
h(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*(B*c + A*d)*b^3/a
^3 - 1/16*sqrt(b*x^2 + a)*(B*c + A*d)*b^3/a^2 - 1/8*(b*x^2 + a)^(5/2)*C*b*
d/(a^2*x^2) + 1/48*(b*x^2 + a)^(5/2)*(B*c + A*d)*b^2/(a^3*x^2) - 1/4*(b*x^
2 + a)^(5/2)*C*d/(a*x^4) - 1/5*(b*x^2 + a)^(5/2)*C*c/(a*x^5) + 2/35*(b*x^2
 + a)^(5/2)*A*b*c/(a^2*x^5) - 1/5*(b*x^2 + a)^(5/2)*B*d/(a*x^5) + 1/24*(b*
x^2 + a)^(5/2)*(B*c + A*d)*b/(a^2*x^4) - 1/7*(b*x^2 + a)^(5/2)*A*c/(a*x^7)
 - 1/6*(b*x^2 + a)^(5/2)*(B*c + A*d)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(182) = 364$.

Time = 0.19 (sec) , antiderivative size = 1109, normalized size of antiderivative = 5.28

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
-1/8*(B*b^3*c - 6*C*a*b^2*d + A*b^3*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a
))/sqrt(-a))/(sqrt(-a)*a) + 1/840*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*
b^3*c + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a*b^2*d + 105*(sqrt(b)*x -
 sqrt(b*x^2 + a))^13*A*b^3*d + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a*b
^(5/2)*c + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a*b^(5/2)*d + 1540*(sqr
t(b)*x - sqrt(b*x^2 + a))^11*B*a*b^3*c - 2520*(sqrt(b)*x - sqrt(b*x^2 + a)
)^11*C*a^2*b^2*d + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a*b^3*d - 3360*
(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(5/2)*c + 3360*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*A*a*b^(7/2)*c - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2
*b^(5/2)*d + 1085*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c + 1890*(sqrt
(b)*x - sqrt(b*x^2 + a))^9*C*a^3*b^2*d + 1085*(sqrt(b)*x - sqrt(b*x^2 + a)
)^9*A*a^2*b^3*d + 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2)*c + 3
360*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2)*c + 5040*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*B*a^3*b^(5/2)*d - 6720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^4*b^(5/2)*c + 6720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(7/2)*c - 672
0*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2)*d - 1085*(sqrt(b)*x - sqrt
(b*x^2 + a))^5*B*a^4*b^3*c - 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^5*b^
2*d - 1085*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^4*b^3*d + 3696*(sqrt(b)*x -
 sqrt(b*x^2 + a))^4*C*a^5*b^(5/2)*c + 1344*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*A*a^4*b^(7/2)*c + 3696*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2)*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^8}\,dx = \frac{-384\sqrt{b\,x^2+a}\,a^3bc\,x^2-280\sqrt{b\,x^2+a}\,a^3bcx-490\sqrt{b\,x^2+}}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x)
```

output
```
( - 240*sqrt(a + b*x**2)*a**4*c - 280*sqrt(a + b*x**2)*a**4*d*x - 384*sqrt
(a + b*x**2)*a**3*b*c*x**2 - 280*sqrt(a + b*x**2)*a**3*b*c*x - 490*sqrt(a
+ b*x**2)*a**3*b*d*x**3 - 336*sqrt(a + b*x**2)*a**3*b*d*x**2 - 336*sqrt(a
+ b*x**2)*a**3*c**2*x**2 - 420*sqrt(a + b*x**2)*a**3*c*d*x**3 - 48*sqrt(a
+ b*x**2)*a**2*b**2*c*x**4 - 490*sqrt(a + b*x**2)*a**2*b**2*c*x**3 - 105*s
qrt(a + b*x**2)*a**2*b**2*d*x**5 - 672*sqrt(a + b*x**2)*a**2*b**2*d*x**4 -
 672*sqrt(a + b*x**2)*a**2*b*c**2*x**4 - 1050*sqrt(a + b*x**2)*a**2*b*c*d*
x**5 + 96*sqrt(a + b*x**2)*a*b**3*c*x**6 - 105*sqrt(a + b*x**2)*a*b**3*c*x
**5 - 336*sqrt(a + b*x**2)*a*b**3*d*x**6 - 336*sqrt(a + b*x**2)*a*b**2*c**
2*x**6 - 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))
*a*b**3*d*x**7 + 630*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b**2*c*d*x**7 - 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*b**4*c*x**7 + 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(
a) + sqrt(b)*x)/sqrt(a))*a*b**3*d*x**7 - 630*sqrt(a)*log((sqrt(a + b*x**2)
 + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**7 + 105*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c*x**7 - 96*sqrt(b)*a*b**3
*c*x**7 - 144*sqrt(b)*a*b**3*d*x**7 - 144*sqrt(b)*a*b**2*c**2*x**7)/(1680*
a**2*x**7)
```

3.59 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^9}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 250

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = \frac{b(3Abc-8a(cC+Bd))\sqrt{a+bx^2}}{64ax^4} + \frac{b^2(3Abc-8a(cC+Bd))\sqrt{a+bx^2}}{128a^2x^2} + \frac{(3Abc-8a(cC+Bd))\left(a+bx^2\right)^{3/2}}{48ax^6} - \frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8} - \frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{7ax^7} - \frac{(7aCd-2b(Bc+Ad))\left(a+bx^2\right)^{5/2}}{35a^2x^5} - \frac{b^3(3Abc-8a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output

```
1/64*b*(3*A*b*c-8*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a/x^4+1/128*b^2*(3*A*b*c-8*
a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^2/x^2+1/48*(3*A*b*c-8*a*(B*d+C*c))*(b*x^2+a
)^(3/2)/a/x^6-1/8*A*c*(b*x^2+a)^(5/2)/a/x^8-1/7*(A*d+B*c)*(b*x^2+a)^(5/2)/
a/x^7-1/35*(7*a*C*d-2*b*(A*d+B*c))*(b*x^2+a)^(5/2)/a^2/x^5-1/128*b^3*(3*A*
b*c-8*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 2.93 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^2}\left(-3b^3x^6(105Ac+256Bcx+256Adx)+16a^3(15A(7c+8dx)+4x(7Cx\right.}{}}{}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^9,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(-3*b^3*x^6*(105*A*c + 256*B*c*x + 256*A*d*x)
+ 16*a^3*(15*A*(7*c + 8*d*x) + 4*x*(7*C*x*(5*c + 6*d*x) + 5*B*(6*c + 7*d*x
))) + 6*a*b^2*x^4*(A*(35*c + 64*d*x) + 4*x*(7*C*x*(5*c + 16*d*x) + B*(16*c
 + 35*d*x))) + 8*a^2*b*x^2*(A*(315*c + 384*d*x) + 2*x*(7*C*x*(35*c + 48*d*
x) + B*(192*c + 245*d*x)))))/x^8) + 630*A*b^4*c*ArcTanh[(Sqrt[b]*x - Sqrt[
a + b*x^2])/Sqrt[a]] + 1680*a*b^3*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt
[a + b*x^2])/Sqrt[a]])/(13440*a^(5/2))
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {2338, 25, 2338, 27, 539, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^9}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(8aCdx^2-(3Abc-8a(cC+Bd))x+8a(Bc+Ad)\right)}{x^8}dx}{8a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(8aCdx^2-(3Abc-8a(cC+Bd))x+8a(Bc+Ad)\right)}{x^8}dx}{8a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(7(3Abc-8a(cC+Bd))-8(7aCd-2b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}-\frac{8\left(a+bx^2\right)^{5/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{7}\int \frac{(7(3Abc-8a(cC+Bd))-8(7aCd-2b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^7}dx-\frac{8\left(a+bx^2\right)^{5/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{7}\left(\frac{\int \frac{(48a(7aCd-2b(Bc+Ad))+7b(3Abc-8a(cC+Bd))x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8\left(a+bx^2\right)^{5/2}(Ad+Bc)}{7x^7}}{8a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{1}{7}\left(\frac{7b(3Abc-8a(Bd+cC))\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8\left(a+bx^2\right)^{5/2}(Ad+B}{7x^7}}{8a}\quad\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{7}\left(\frac{\frac{7}{2}b(3Abc-8a(Bd+cC))\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8\left(a+bx^2\right)^{5/2}(Ad+}{7x^7}}{8a}\quad\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \text{51}$$

$$\frac{\frac{1}{7}\left(\frac{\frac{7}{2}b(3Abc-8a(Bd+cC))\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a(Bd+cC))}{6ax^6}\right)-\frac{8}{}}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 51

$$\frac{\frac{1}{7}\left(\frac{\frac{7}{2}b(3Abc-8a(Bd+cC))\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a}{6ax^6}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 73

$$\frac{\frac{1}{7}\left(\frac{\frac{7}{2}b(3Abc-8a(Bd+cC))\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Abc-8a(}{6ax^6}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 221

$$\frac{\frac{1}{7}\left(\frac{\frac{7}{2}b\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)(3Abc-8a(Bd+cC))-\frac{48\left(a+bx^2\right)^{5/2}(7aCd-2b(Ad+Bc))}{5x^5}}{6a}+\frac{7\left(a+bx^2\right)^{5/2}(3Ab}{6ax}\right.}{8a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{8ax^8}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^9,x]
```

output
```
-1/8*(A*c*(a + b*x^2)^(5/2))/(a*x^8) + ((-8*(B*c + A*d)*(a + b*x^2)^(5/2))
/(7*x^7) + ((7*(3*A*b*c - 8*a*(c*C + B*d))*(a + b*x^2)^(5/2))/(6*a*x^6) +
((-48*(7*a*C*d - 2*b*(B*c + A*d))*(a + b*x^2)^(5/2))/(5*x^5) + (7*b*(3*A*b
*c - 8*a*(c*C + B*d))*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2
]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)/(6*a))/7)/(
8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-768A\,b^3d\,x^7-768B\,b^3c\,x^7+2688Ca\,b^2d\,x^7-315A\,b^3c\,x^6+840Ba\,b^2d\,x^6+840Ca\,b^2c\,x^6+384Aa\,b^2d\,x^5+384Ba\,b^2c\,x^5}{}$
default	$(Ad+Bc)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{7a\,x^7}+\frac{2b(b\,x^2+a)^{\frac{5}{2}}}{35a^2x^5}\right)+(Bd+Cc)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b}{}\right)}{}\right)}{}\right)$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/13440*(b*x^2+a)^(1/2)*(-768*A*b^3*d*x^7-768*B*b^3*c*x^7+2688*C*a*b^2*d*
x^7-315*A*b^3*c*x^6+840*B*a*b^2*d*x^6+840*C*a*b^2*c*x^6+384*A*a*b^2*d*x^5+
384*B*a*b^2*c*x^5+5376*C*a^2*b*d*x^5+210*A*a*b^2*c*x^4+3920*B*a^2*b*d*x^4+
3920*C*a^2*b*c*x^4+3072*A*a^2*b*d*x^3+3072*B*a^2*b*c*x^3+2688*C*a^3*d*x^3+
2520*A*a^2*b*c*x^2+2240*B*a^3*d*x^2+2240*C*a^3*c*x^2+1920*A*a^3*d*x+1920*B
*a^3*c*x+1680*A*a^3*c)/x^8/a^2-1/128*(3*A*b*c-8*B*a*d-8*C*a*c)*b^3/a^(5/2)
*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.37

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = \left[\frac{105\left(8\,Bab^3d+\left(8\,Cab^3-3\,Ab^4\right)c\right)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+}}{x^2}\right.}{}\right.$$

$$-\frac{105\left(8\,Bab^3d+\left(8\,Cab^3-3\,Ab^4\right)c\right)\sqrt{-a}x^8\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-\left(384\left(2\,Bab^3c-\left(7\,Ca^2b^2-2\,Aab^3\right)d\right)x\right.}{}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="fricas")
```

output
```
[1/26880*(105*(8*B*a*b^3*d + (8*C*a*b^3 - 3*A*b^4)*c)*sqrt(a)*x^8*log(-(b*
x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(384*(2*B*a*b^3*c - (7*C*a
^2*b^2 - 2*A*a*b^3)*d)*x^7 - 105*(8*B*a^2*b^2*d + (8*C*a^2*b^2 - 3*A*a*b^3
)*c)*x^6 - 1680*A*a^4*c - 384*(B*a^2*b^2*c + (14*C*a^3*b + A*a^2*b^2)*d)*x
^5 - 70*(56*B*a^3*b*d + (56*C*a^3*b + 3*A*a^2*b^2)*c)*x^4 - 384*(8*B*a^3*b
*c + (7*C*a^4 + 8*A*a^3*b)*d)*x^3 - 280*(8*B*a^4*d + (8*C*a^4 + 9*A*a^3*b)
*c)*x^2 - 1920*(B*a^4*c + A*a^4*d)*x)*sqrt(b*x^2 + a))/(a^3*x^8), -1/13440
*(105*(8*B*a*b^3*d + (8*C*a*b^3 - 3*A*b^4)*c)*sqrt(-a)*x^8*arctan(sqrt(b*x
^2 + a)*sqrt(-a)/a) - (384*(2*B*a*b^3*c - (7*C*a^2*b^2 - 2*A*a*b^3)*d)*x^7
 - 105*(8*B*a^2*b^2*d + (8*C*a^2*b^2 - 3*A*a*b^3)*c)*x^6 - 1680*A*a^4*c -
384*(B*a^2*b^2*c + (14*C*a^3*b + A*a^2*b^2)*d)*x^5 - 70*(56*B*a^3*b*d + (5
6*C*a^3*b + 3*A*a^2*b^2)*c)*x^4 - 384*(8*B*a^3*b*c + (7*C*a^4 + 8*A*a^3*b)
*d)*x^3 - 280*(8*B*a^4*d + (8*C*a^4 + 9*A*a^3*b)*c)*x^2 - 1920*(B*a^4*c +
A*a^4*d)*x)*sqrt(b*x^2 + a))/(a^3*x^8)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(231) = 462$.

Time = 38.88 (sec) , antiderivative size = 1447, normalized size of antiderivative = 5.79

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**9,x)
```

output
```
-15*A*a**6*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*
b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*d*x**2*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) -
 17*A*a**4*b**(13/2)*d*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210
*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*d*x**6*sqrt(a/
(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**
10) - 12*A*a**2*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
+ 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a**2*c/(8*sqrt(b)*x**9*sqr
t(a/(b*x**2) + 1)) - 8*A*a*b**(19/2)*d*x**10*sqrt(a/(b*x**2) + 1)/(105*a**
5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 5*A*a*sqrt(b)*c/
(16*x**7*sqrt(a/(b*x**2) + 1)) - 13*A*b**(3/2)*c/(64*x**5*sqrt(a/(b*x**2)
+ 1)) - A*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(5*x**4) + A*b**(5/2)*c/(128*a*x
**3*sqrt(a/(b*x**2) + 1)) - A*b**(5/2)*d*sqrt(a/(b*x**2) + 1)/(15*a*x**2)
+ 3*A*b**(7/2)*c/(128*a**2*x*sqrt(a/(b*x**2) + 1)) + 2*A*b**(7/2)*d*sqrt(a
/(b*x**2) + 1)/(15*a**2) - 3*A*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(
5/2)) - 15*B*a**6*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 21
0*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*B*a**5*b**(11/2)*c*x**2*sqrt(
a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x
**10) - 17*B*a**4*b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**
6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*B*a**3*b**(15/2)*c*x**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = -\frac{3\,Ab^4c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab^4c}{128\,a^4} + \frac{3\,\sqrt{bx^2+a}Ab^4c}{128\,a^3} + \frac{(Cc+Bd)b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}(Cc+Bd)b^3}{48\,a^3} - \frac{\sqrt{bx^2+a}(Cc+Bd)b^3}{16\,a^2} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^3c}{128\,a^4x^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)b^2}{48\,a^3x^2} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab^2c}{64\,a^3x^4} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cd}{5\,ax^5} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)b}{24\,a^2x^4} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Abc}{16\,a^2x^6} + \frac{2\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)b}{35\,a^2x^5} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)}{6\,ax^6} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ac}{8\,ax^8} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)}{7\,ax^7}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="maxima")
```

output
```
-3/128*A*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(
3/2)*A*b^4*c/a^4 + 3/128*sqrt(b*x^2 + a)*A*b^4*c/a^3 + 1/16*(C*c + B*d)*b^
3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*(C*c + B*
d)*b^3/a^3 - 1/16*sqrt(b*x^2 + a)*(C*c + B*d)*b^3/a^2 - 1/128*(b*x^2 + a)^
(5/2)*A*b^3*c/(a^4*x^2) + 1/48*(b*x^2 + a)^(5/2)*(C*c + B*d)*b^2/(a^3*x^2)
 - 1/64*(b*x^2 + a)^(5/2)*A*b^2*c/(a^3*x^4) - 1/5*(b*x^2 + a)^(5/2)*C*d/(a
*x^5) + 1/24*(b*x^2 + a)^(5/2)*(C*c + B*d)*b/(a^2*x^4) + 1/16*(b*x^2 + a)^
(5/2)*A*b*c/(a^2*x^6) + 2/35*(b*x^2 + a)^(5/2)*(B*c + A*d)*b/(a^2*x^5) - 1
/6*(b*x^2 + a)^(5/2)*(C*c + B*d)/(a*x^6) - 1/8*(b*x^2 + a)^(5/2)*A*c/(a*x^
8) - 1/7*(b*x^2 + a)^(5/2)*(B*c + A*d)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1295 vs. $2(218) = 436$.

Time = 0.22 (sec) , antiderivative size = 1295, normalized size of antiderivative = 5.18

$$\int \frac{(c+dx)\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^9}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
-1/64*(8*C*a*b^3*c - 3*A*b^4*c + 8*B*a*b^3*d)*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 +
 a))^15*C*a*b^3*c - 315*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*b^4*c + 840*(sq
rt(b)*x - sqrt(b*x^2 + a))^15*B*a*b^3*d + 13440*(sqrt(b)*x - sqrt(b*x^2 +
a))^14*C*a^2*b^(5/2)*d + 11480*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^2*b^3*
c + 2415*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4*c + 11480*(sqrt(b)*x - s
qrt(b*x^2 + a))^13*B*a^2*b^3*d + 26880*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*
a^2*b^(7/2)*c - 40320*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(5/2)*d + 2
6880*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(7/2)*d - 3640*(sqrt(b)*x -
sqrt(b*x^2 + a))^11*C*a^3*b^3*c + 34965*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A
*a^2*b^4*c - 3640*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^3*b^3*d + 67200*(sq
rt(b)*x - sqrt(b*x^2 + a))^10*C*a^4*b^(5/2)*d - 8680*(sqrt(b)*x - sqrt(b*x
^2 + a))^9*C*a^4*b^3*c + 70455*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^3*b^4*c
 - 8680*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^4*b^3*d + 26880*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*B*a^4*b^(7/2)*c - 94080*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
C*a^5*b^(5/2)*d + 26880*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(7/2)*d -
8680*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^5*b^3*c + 70455*(sqrt(b)*x - sqrt
(b*x^2 + a))^7*A*a^4*b^4*c - 8680*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a^5*b^
3*d - 43008*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^5*b^(7/2)*c + 83328*(sqrt(
b)*x - sqrt(b*x^2 + a))^6*C*a^6*b^(5/2)*d - 43008*(sqrt(b)*x - sqrt(b*x...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^9}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^9,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.52

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx=\frac{-768\sqrt{b}\,b^4c\,x^8-840\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3c^2x^8-315}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x)
```

output
```
( - 1680*sqrt(a + b*x**2)*a**4*c - 1920*sqrt(a + b*x**2)*a**4*d*x - 2520*s
qrt(a + b*x**2)*a**3*b*c*x**2 - 1920*sqrt(a + b*x**2)*a**3*b*c*x - 3072*sq
rt(a + b*x**2)*a**3*b*d*x**3 - 2240*sqrt(a + b*x**2)*a**3*b*d*x**2 - 2240*
sqrt(a + b*x**2)*a**3*c**2*x**2 - 2688*sqrt(a + b*x**2)*a**3*c*d*x**3 - 21
0*sqrt(a + b*x**2)*a**2*b**2*c*x**4 - 3072*sqrt(a + b*x**2)*a**2*b**2*c*x*
*3 - 384*sqrt(a + b*x**2)*a**2*b**2*d*x**5 - 3920*sqrt(a + b*x**2)*a**2*b*
*2*d*x**4 - 3920*sqrt(a + b*x**2)*a**2*b*c**2*x**4 - 5376*sqrt(a + b*x**2)
*a**2*b*c*d*x**5 + 315*sqrt(a + b*x**2)*a*b**3*c*x**6 - 384*sqrt(a + b*x**
2)*a*b**3*c*x**5 + 768*sqrt(a + b*x**2)*a*b**3*d*x**7 - 840*sqrt(a + b*x**
2)*a*b**3*d*x**6 - 840*sqrt(a + b*x**2)*a*b**2*c**2*x**6 - 2688*sqrt(a + b
*x**2)*a*b**2*c*d*x**7 + 768*sqrt(a + b*x**2)*b**4*c*x**7 + 315*sqrt(a)*lo
g((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c*x**8 - 840*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*d*x**8 - 84
0*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*
x**8 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b
**4*c*x**8 + 840*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt
(a))*b**4*d*x**8 + 840*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x
)/sqrt(a))*b**3*c**2*x**8 - 768*sqrt(b)*a*b**3*d*x**8 - 672*sqrt(b)*a*b**2
*c*d*x**8 - 768*sqrt(b)*b**4*c*x**8)/(13440*a**2*x**8)
```

3.60 $\int \frac{(c+dx)(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^{10}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 288

$$\begin{aligned}\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx =\\ &-\frac{b(8aCd-3b(Bc+Ad))\sqrt{a+bx^2}}{64ax^4}-\frac{b^2(8aCd-3b(Bc+Ad))\sqrt{a+bx^2}}{128a^2x^2}\\ &-\frac{(8aCd-3b(Bc+Ad))\left(a+bx^2\right)^{3/2}}{48ax^6}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}\\ &-\frac{(Bc+Ad)\left(a+bx^2\right)^{5/2}}{8ax^8}+\frac{(4Abc-9a(cC+Bd))\left(a+bx^2\right)^{5/2}}{63a^2x^7}\\ &-\frac{2b(4Abc-9a(cC+Bd))\left(a+bx^2\right)^{5/2}}{315a^3x^5}\\ &+\frac{b^3(8aCd-3b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}\end{aligned}$$

output
```
-1/64*b*(8*a*C*d-3*b*(A*d+B*c))*(b*x^2+a)^(1/2)/a/x^4-1/128*b^2*(8*a*C*d-3
*b*(A*d+B*c))*(b*x^2+a)^(1/2)/a^2/x^2-1/48*(8*a*C*d-3*b*(A*d+B*c))*(b*x^2+
a)^(3/2)/a/x^6-1/9*A*c*(b*x^2+a)^(5/2)/a/x^9-1/8*(A*d+B*c)*(b*x^2+a)^(5/2)
/a/x^8+1/63*(4*A*b*c-9*a*(B*d+C*c))*(b*x^2+a)^(5/2)/a^2/x^7-2/315*b*(4*A*b
*c-9*a*(B*d+C*c))*(b*x^2+a)^(5/2)/a^3/x^5+1/128*b^3*(8*a*C*d-3*b*(A*d+B*c)
)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 3.62 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx =$$
$$-\frac{\sqrt{a+bx^2}\left(1024Ab^4cx^8-ab^3x^6(A(512c+945dx)+9x(105Bc+256cCx+256Bdx))+6a^2b^2x^4(A(64c+\right.}{}$$
$$-\frac{b^3Cd\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}-\frac{3b^4(Bc+Ad)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/40320*(Sqrt[a + b*x^2]*(1024*A*b^4*c*x^8 - a*b^3*x^6*(A*(512*c + 945*d*
x) + 9*x*(105*B*c + 256*c*C*x + 256*B*d*x)) + 6*a^2*b^2*x^4*(A*(64*c + 105
*d*x) + 3*x*(35*B*c + 64*c*C*x + 64*B*d*x + 140*C*d*x^2)) + 8*a^3*b*x^2*(A
*(800*c + 945*d*x) + 3*x*(315*B*c + 384*c*C*x + 384*B*d*x + 490*C*d*x^2))
+ 80*a^4*(7*A*(8*c + 9*d*x) + 3*x*(4*C*x*(6*c + 7*d*x) + 3*B*(7*c + 8*d*x)
))))/(a^3*x^9) - (b^3*C*d*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/
(8*a^(3/2)) - (3*b^4*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/
Sqrt[a]])/(64*a^(5/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.94, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2338, 25, 2338, 27, 539, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)\left(A+Bx+Cx^2\right)}{x^{10}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(9aCdx^2-(4Abc-9a(cC+Bd))x+9a(Bc+Ad)\right)}{x^9}dx}{9a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(9aCdx^2-(4Abc-9a(cC+Bd))x+9a(Bc+Ad)\right)}{x^9}dx}{9a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{2338}$$

$$\frac{-\frac{\int \frac{a(8(4Abc-9a(cC+Bd))-9(8aCd-3b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^8}dx}{8a}-\frac{9\left(a+bx^2\right)^{5/2}(Ad+Bc)}{8x^8}}{9a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{-\frac{1}{8}\int \frac{(8(4Abc-9a(cC+Bd))-9(8aCd-3b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^8}dx-\frac{9\left(a+bx^2\right)^{5/2}(Ad+Bc)}{8x^8}}{9a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{539}$$

$$\frac{\frac{1}{8}\left(\frac{\int \frac{(63a(8aCd-3b(Bc+Ad))+16b(4Abc-9a(cC+Bd))x)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}+\frac{8\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{7ax^7}\right)-\frac{9\left(a+bx^2\right)^{5/2}(Ad+Bc)}{8x^8}}{9a}-\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{539}$$

$$\frac{\frac{1}{8}\left(\frac{-\frac{\int -\frac{3ab(32(4Abc-9a(cC+Bd))-21(8aCd-3b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{21\left(a+bx^2\right)^{5/2}(8aCd-3b(Ad+Bc))}{2x^6}}{7a}+\frac{8\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC)}{7ax^7}\right.}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{1}{8}\left(\frac{\frac{1}{2}b\int \frac{(32(4Abc-9a(cC+Bd))-21(8aCd-3b(Bc+Ad))x)\left(bx^2+a\right)^{3/2}}{x^6}dx-\frac{21\left(a+bx^2\right)^{5/2}(8aCd-3b(Ad+Bc))}{2x^6}}{7a}+\frac{8\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{7ax^7}\right)}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 534

$$\frac{\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-21(8aCd-3b(Ad+Bc))\int\frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(8aCd-3b(Ad+Bc))}{2x^6}}{7a}+\frac{8\left(a+bx^2\right)^{5/2}(4Ab}{7ax}\right.}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 243

$$\frac{\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-\frac{21}{2}(8aCd-3b(Ad+Bc))\int\frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(8aCd-3b(Ad+Bc))}{2x^6}}{7a}+\frac{8\left(a+bx^2\right)^{5/2}(4A}{7a}\right.}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-\frac{21}{2}(8aCd-3b(Ad+Bc))\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(8aCd-3b(Ad+Bc))}{2x^6}}{7a}+\right.}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-\frac{21}{2}(8aCd-3b(Ad+Bc))\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(8aC}{2x^6}}{7a}\right.}{9a}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 73

$$\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-\frac{21}{2}(8aCd-3b(Ad+Bc))\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5/2}(8aC}{2x^6}}{7a}\right.$$

$$\overline{\qquad 9a \qquad}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$\downarrow$ 221

$$\frac{1}{8}\left(\frac{\frac{1}{2}b\left(-\frac{21}{2}\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)(8aCd-3b(Ad+Bc))-\frac{32\left(a+bx^2\right)^{5/2}(4Abc-9a(Bd+cC))}{5ax^5}\right)-\frac{21\left(a+bx^2\right)^{5}}{}}{7a}\right.$$

$$\overline{\qquad 9a \qquad}$$

$$\frac{Ac\left(a+bx^2\right)^{5/2}}{9ax^9}$$

input
```
Int[((c + d*x)*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/9*(A*c*(a + b*x^2)^(5/2))/(a*x^9) + ((-9*(B*c + A*d)*(a + b*x^2)^(5/2))
/(8*x^8) + ((8*(4*A*b*c - 9*a*(c*C + B*d))*(a + b*x^2)^(5/2))/(7*a*x^7) +
((-21*(8*a*C*d - 3*b*(B*c + A*d))*(a + b*x^2)^(5/2))/(2*x^6) + (b*((-32*(4
*A*b*c - 9*a*(c*C + B*d))*(a + b*x^2)^(5/2))/(5*a*x^5) - (21*(8*a*C*d - 3*
b*(B*c + A*d))*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2)
- (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2))/2)/(7*a))/8)/(9*a
)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(1024A\,b^4c\,x^8-2304Ba\,b^3d\,x^8-2304Ca\,b^3c\,x^8-945Aa\,b^3d\,x^7-945Ba\,b^3c\,x^7+2520C\,a^2b^2d\,x^7-512Aa\,b^3c\,x^6+1152B\,a}{}$
default	$(Ad+Bc)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{8a\,x^8}-\frac{3b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2}{}\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}\right.$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/40320*(b*x^2+a)^(1/2)*(1024*A*b^4*c*x^8-2304*B*a*b^3*d*x^8-2304*C*a*b^3
*c*x^8-945*A*a*b^3*d*x^7-945*B*a*b^3*c*x^7+2520*C*a^2*b^2*d*x^7-512*A*a*b^
3*c*x^6+1152*B*a^2*b^2*d*x^6+1152*C*a^2*b^2*c*x^6+630*A*a^2*b^2*d*x^5+630*
B*a^2*b^2*c*x^5+11760*C*a^3*b*d*x^5+384*A*a^2*b^2*c*x^4+9216*B*a^3*b*d*x^4
+9216*C*a^3*b*c*x^4+7560*A*a^3*b*d*x^3+7560*B*a^3*b*c*x^3+6720*C*a^4*d*x^3
+6400*A*a^3*b*c*x^2+5760*B*a^4*d*x^2+5760*C*a^4*c*x^2+5040*A*a^4*d*x+5040*
B*a^4*c*x+4480*A*a^4*c)/x^9/a^3-1/128*(3*A*b*d+3*B*b*c-8*C*a*d)/a^(5/2)*b^
3*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.27

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="fricas"
)
```

output
```
[1/80640*(315*(3*B*b^4*c - (8*C*a*b^3 - 3*A*b^4)*d)*sqrt(a)*x^9*log(-(b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(256*(9*B*a*b^3*d + (9*C*a*b
^3 - 4*A*b^4)*c)*x^8 + 315*(3*B*a*b^3*c - (8*C*a^2*b^2 - 3*A*a*b^3)*d)*x^7
 - 128*(9*B*a^2*b^2*d + (9*C*a^2*b^2 - 4*A*a*b^3)*c)*x^6 - 4480*A*a^4*c -
210*(3*B*a^2*b^2*c + (56*C*a^3*b + 3*A*a^2*b^2)*d)*x^5 - 384*(24*B*a^3*b*d
 + (24*C*a^3*b + A*a^2*b^2)*c)*x^4 - 840*(9*B*a^3*b*c + (8*C*a^4 + 9*A*a^3
*b)*d)*x^3 - 640*(9*B*a^4*d + (9*C*a^4 + 10*A*a^3*b)*c)*x^2 - 5040*(B*a^4*
c + A*a^4*d)*x)*sqrt(b*x^2 + a))/(a^3*x^9), 1/40320*(315*(3*B*b^4*c - (8*C
*a*b^3 - 3*A*b^4)*d)*sqrt(-a)*x^9*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (25
6*(9*B*a*b^3*d + (9*C*a*b^3 - 4*A*b^4)*c)*x^8 + 315*(3*B*a*b^3*c - (8*C*a^
2*b^2 - 3*A*a*b^3)*d)*x^7 - 128*(9*B*a^2*b^2*d + (9*C*a^2*b^2 - 4*A*a*b^3)
*c)*x^6 - 4480*A*a^4*c - 210*(3*B*a^2*b^2*c + (56*C*a^3*b + 3*A*a^2*b^2)*d
)*x^5 - 384*(24*B*a^3*b*d + (24*C*a^3*b + A*a^2*b^2)*c)*x^4 - 840*(9*B*a^3
*b*c + (8*C*a^4 + 9*A*a^3*b)*d)*x^3 - 640*(9*B*a^4*d + (9*C*a^4 + 10*A*a^3
*b)*c)*x^2 - 5040*(B*a^4*c + A*a^4*d)*x)*sqrt(b*x^2 + a))/(a^3*x^9)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(270) = 540$.

Time = 56.08 (sec) , antiderivative size = 2387, normalized size of antiderivative = 8.29

$$\int \frac{(c + dx) (a + bx^2)^{3/2} (A + Bx + Cx^2)}{x^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**10,x)
```

output
```
-35*A*a**8*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6
*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**7*b
**(21/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*
x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b**(23/2
)*c*x**4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 40*A*a**5*b**(25/2)*c*x**6
*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**
5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*A*a**5*b**(11/2)*c*sqrt(a/(b*x*
*2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) +
 5*A*a**4*b**(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*
a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 33*A*a**
4*b**(13/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**
5*x**8 + 105*a**3*b**6*x**10) + 30*A*a**3*b**(29/2)*c*x**10*sqrt(a/(b*x**2
) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
 315*a**4*b**12*x**14) - 17*A*a**3*b**(15/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(
105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) + 40*A*a**2
*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**
10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 3*A*a**2*b**(17/
2)*c*x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
105*a**3*b**6*x**10) - A*a**2*d/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx = \frac{Cb^3d\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Cb^3d}{48\,a^3} - \frac{\sqrt{bx^2+a}Cb^3d}{16\,a^2} - \frac{3\,(Bc+Ad)b^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} + \frac{\left(bx^2+a\right)^{\frac{3}{2}}(Bc+Ad)b^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}(Bc+Ad)b^4}{128\,a^3} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cb^2d}{48\,a^3x^2} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)b^3}{128\,a^4x^2} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cbd}{24\,a^2x^4} - \frac{8\left(bx^2+a\right)^{\frac{5}{2}}Ab^2c}{315\,a^3x^5} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)b^2}{64\,a^3x^4} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Cd}{6\,ax^6} + \frac{2\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)b}{35\,a^2x^5} + \frac{4\left(bx^2+a\right)^{\frac{5}{2}}Abc}{63\,a^2x^7} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)b}{16\,a^2x^6} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Cc+Bd)}{7\,ax^7} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ac}{9\,ax^9} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}(Bc+Ad)}{8\,ax^8}$$

input
```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="maxima"
)
```

output
```
1/16*C*b^3*d*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2
)*C*b^3*d/a^3 - 1/16*sqrt(b*x^2 + a)*C*b^3*d/a^2 - 3/128*(B*c + A*d)*b^4*a
rcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(3/2)*(B*c + A*d)
*b^4/a^4 + 3/128*sqrt(b*x^2 + a)*(B*c + A*d)*b^4/a^3 + 1/48*(b*x^2 + a)^(5
/2)*C*b^2*d/(a^3*x^2) - 1/128*(b*x^2 + a)^(5/2)*(B*c + A*d)*b^3/(a^4*x^2)
+ 1/24*(b*x^2 + a)^(5/2)*C*b*d/(a^2*x^4) - 8/315*(b*x^2 + a)^(5/2)*A*b^2*c
/(a^3*x^5) - 1/64*(b*x^2 + a)^(5/2)*(B*c + A*d)*b^2/(a^3*x^4) - 1/6*(b*x^2
 + a)^(5/2)*C*d/(a*x^6) + 2/35*(b*x^2 + a)^(5/2)*(C*c + B*d)*b/(a^2*x^5) +
 4/63*(b*x^2 + a)^(5/2)*A*b*c/(a^2*x^7) + 1/16*(b*x^2 + a)^(5/2)*(B*c + A*
d)*b/(a^2*x^6) - 1/7*(b*x^2 + a)^(5/2)*(C*c + B*d)/(a*x^7) - 1/9*(b*x^2 +
a)^(5/2)*A*c/(a*x^9) - 1/8*(b*x^2 + a)^(5/2)*(B*c + A*d)/(a*x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1378 vs. $2(252) = 504$.

Time = 0.18 (sec) , antiderivative size = 1378, normalized size of antiderivative = 4.78

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="giac")
```

output

```
1/64*(3*B*b^4*c - 8*C*a*b^3*d + 3*A*b^4*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2
 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/20160*(945*(sqrt(b)*x - sqrt(b*x^2 + a
))^17*B*b^4*c - 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^17*C*a*b^3*d + 945*(sqr
t(b)*x - sqrt(b*x^2 + a))^17*A*b^4*d - 8190*(sqrt(b)*x - sqrt(b*x^2 + a))^
15*B*a*b^4*c - 31920*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*b^3*d - 8190*(
sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a*b^4*d - 80640*(sqrt(b)*x - sqrt(b*x^2
+ a))^14*C*a^2*b^(7/2)*c - 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^2*b^
(7/2)*d - 97650*(sqrt(b)*x - sqrt(b*x^2 + a))^13*B*a^2*b^4*c + 45360*(sqrt
(b)*x - sqrt(b*x^2 + a))^13*C*a^3*b^3*d - 97650*(sqrt(b)*x - sqrt(b*x^2 +
a))^13*A*a^2*b^4*d + 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^3*b^(7/2)*
c - 215040*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2)*c + 80640*(sqrt(
b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(7/2)*d - 106470*(sqrt(b)*x - sqrt(b*x^
2 + a))^11*B*a^3*b^4*c + 15120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^4*b^3*
d - 106470*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^4*d - 80640*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*C*a^4*b^(7/2)*c - 322560*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*A*a^3*b^(9/2)*c - 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(7
/2)*d + 209664*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^5*b^(7/2)*c - 451584*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2)*c + 209664*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*B*a^5*b^(7/2)*d + 106470*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a^
5*b^4*c - 15120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^6*b^3*d + 106470*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{x^{10}}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^10,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)*(A + B*x + C*x^2))/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 703, normalized size of antiderivative = 2.44

$$\int \frac{(c+dx)\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{10}}\,dx=\frac{-2520\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a\,b^3cd\,x^9+2520\sqrt{a}\log\left(\frac{\sqrt{b}}{}\right.}{}$$

input
```
int((d*x+c)*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x)
```

output

```
( - 4480*sqrt(a + b*x**2)*a**5*c - 5040*sqrt(a + b*x**2)*a**5*d*x - 6400*s
qrt(a + b*x**2)*a**4*b*c*x**2 - 5040*sqrt(a + b*x**2)*a**4*b*c*x - 7560*sq
rt(a + b*x**2)*a**4*b*d*x**3 - 5760*sqrt(a + b*x**2)*a**4*b*d*x**2 - 5760*
sqrt(a + b*x**2)*a**4*c**2*x**2 - 6720*sqrt(a + b*x**2)*a**4*c*d*x**3 - 38
4*sqrt(a + b*x**2)*a**3*b**2*c*x**4 - 7560*sqrt(a + b*x**2)*a**3*b**2*c*x*
*3 - 630*sqrt(a + b*x**2)*a**3*b**2*d*x**5 - 9216*sqrt(a + b*x**2)*a**3*b*
*2*d*x**4 - 9216*sqrt(a + b*x**2)*a**3*b*c**2*x**4 - 11760*sqrt(a + b*x**2
)*a**3*b*c*d*x**5 + 512*sqrt(a + b*x**2)*a**2*b**3*c*x**6 - 630*sqrt(a + b
*x**2)*a**2*b**3*c*x**5 + 945*sqrt(a + b*x**2)*a**2*b**3*d*x**7 - 1152*sqr
t(a + b*x**2)*a**2*b**3*d*x**6 - 1152*sqrt(a + b*x**2)*a**2*b**2*c**2*x**6
 - 2520*sqrt(a + b*x**2)*a**2*b**2*c*d*x**7 - 1024*sqrt(a + b*x**2)*a*b**4
*c*x**8 + 945*sqrt(a + b*x**2)*a*b**4*c*x**7 + 2304*sqrt(a + b*x**2)*a*b**
4*d*x**8 + 2304*sqrt(a + b*x**2)*a*b**3*c**2*x**8 + 945*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*d*x**9 - 2520*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c*d*x**9 + 945
*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*c*x**9
 - 945*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**
4*d*x**9 + 2520*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**3*c*d*x**9 - 945*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b
)*x)/sqrt(a))*b**5*c*x**9 + 1024*sqrt(b)*a*b**4*c*x**9 - 2304*sqrt(b)*a...
```

3.61 $\int x^3(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 455

$$\begin{aligned}
&\int x^3(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx = \\
&-\frac{3a^3(2bc(Bc+2Ad)-ad(2cC+Bd))x\sqrt{a+bx^2}}{256b^3} \\
&+\frac{a^2(2bc(Bc+2Ad)-ad(2cC+Bd))x^3\sqrt{a+bx^2}}{128b^2} \\
&+\frac{a(2bc(Bc+2Ad)-ad(2cC+Bd))x^5\sqrt{a+bx^2}}{32b} \\
&+\frac{(2bc(Bc+2Ad)-ad(2cC+Bd))x^5\left(a+bx^2\right)^{3/2}}{16b} \\
&-\frac{a\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)\left(a+bx^2\right)^{5/2}}{5b^4} \\
&+\frac{d(2cC+Bd)x^5\left(a+bx^2\right)^{5/2}}{10b} \\
&+\frac{\left(Ab\left(bc^2-2ad^2\right)+a\left(3aCd^2-2bc(cC+2Bd)\right)\right)\left(a+bx^2\right)^{7/2}}{7b^4} \\
&-\frac{\left(3aCd^2-b\left(c^2C+2Bcd+Ad^2\right)\right)\left(a+bx^2\right)^{9/2}}{9b^4}+\frac{Cd^2\left(a+bx^2\right)^{11/2}}{11b^4} \\
&+\frac{3a^4(2bc(Bc+2Ad)-ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}
\end{aligned}$$

output
```
-3/256*a^3*(2*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/b^3+1/128
*a^2*(2*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x^3*(b*x^2+a)^(1/2)/b^2+1/32*a*(2
*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x^5*(b*x^2+a)^(1/2)/b+1/16*(2*b*c*(2*A*d
+B*c)-a*d*(B*d+2*C*c))*x^5*(b*x^2+a)^(3/2)/b-1/5*a*(A*b*(-a*d^2+b*c^2)+a*(
a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(5/2)/b^4+1/10*d*(B*d+2*C*c)*x^5*(b*x^
2+a)^(5/2)/b+1/7*(A*b*(-2*a*d^2+b*c^2)+a*(3*a*C*d^2-2*b*c*(2*B*d+C*c)))*(b
*x^2+a)^(7/2)/b^4-1/9*(3*a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(9/2)/
b^4+1/11*C*d^2*(b*x^2+a)^(11/2)/b^4+3/256*a^4*(2*b*c*(2*A*d+B*c)-a*d*(B*d+
2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.98

$$\int x^3(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt{a+bx^2}(-12288a^5Cd^2+a^4b(22528c^2C+22cd(2048B+945Cx)+d^2(22528A+10395Bx}{}$$

input
```
Integrate[x^3*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(-12288*a^5*C*d^2 + a^4*b*(22528*c^2*C + 22*c*d*(2048*B +
 945*C*x) + d^2*(22528*A + 10395*B*x + 6144*C*x^2)) + 32*b^5*x^6*(110*A*(3
6*c^2 + 63*c*d*x + 28*d^2*x^2) + 7*x*(11*B*(45*c^2 + 80*c*d*x + 36*d^2*x^2
) + 8*C*x*(55*c^2 + 99*c*d*x + 45*d^2*x^2))) + 12*a^2*b^3*x^2*(22*A*(96*c^
2 + 105*c*d*x + 32*d^2*x^2) + x*(11*B*(105*c^2 + 128*c*d*x + 42*d^2*x^2) +
 4*C*x*(176*c^2 + 231*c*d*x + 80*d^2*x^2))) - 2*a^3*b^2*(22*A*(1152*c^2 +
945*c*d*x + 256*d^2*x^2) + x*(11*B*(945*c^2 + 1024*c*d*x + 315*d^2*x^2) +
2*C*x*(2816*c^2 + 3465*c*d*x + 1152*d^2*x^2))) + 16*a*b^4*x^4*(22*A*(576*c
^2 + 945*c*d*x + 400*d^2*x^2) + x*(11*B*(945*c^2 + 1600*c*d*x + 693*d^2*x^
2) + 2*C*x*(4400*c^2 + 7623*c*d*x + 3360*d^2*x^2)))) + 10395*a^4*Sqrt[b]*(
-2*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(887040*b^4)
```

Rubi [A] (verified)

Time = 3.49 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.35, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2185, 25, 2185, 27, 2185, 2185, 25, 27, 687, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -(c+dx)^2 \left(bx^2+a\right)^{3/2} \left(bd^4(38cC-11Bd)x^4+d^3\left(48bCc^2-11Abd^2+6aCd^2\right)x^3+2cCd^2\left(13bc^2+9ad^2\right)x^2\right.}{11bd^5}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{\int (c+dx)^2 \left(bx^2+a\right)^{3/2} \left(bd^4(38cC-11Bd)x^4+d^3\left(48bCc^2-11Abd^2+6aCd^2\right)x^3+2cCd^2\left(13bc^2+9ad^2\right)x^2+\right.}{11bd^5}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{\frac{\int -5(c+dx)^2\left(bx^2+a\right)^{3/2}\left(-b\left(12aCd^2-b\left(94Cc^2-55Bdc+22Ad^2\right)\right)x^3d^7+b\left(4b(25cC-11Bd)c^2+ad^2(2cC-11Bd)\right)x^2d^6+abc^2(26cC-11Bd)d^6+\right.}{10bd^4}}{11bd^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\int (c+dx)^2\left(bx^2+a\right)^{3/2}\left(-b\left(12aCd^2-b\left(94Cc^2-55Bdc+22Ad^2\right)\right)x^3d^7+b\left(4b(25cC-11B\right.\right.}{}}{11bd^5}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\int(c+dx)^2\left(bx^2+a\right)^{3/2}\left(b^2\left(3ad^2(62cC-33Bd)-2bc\left(208Cc^2-187Bdc+154Ad^2\right)\right)x^2d^9+abc\left(48aC\right.\right.}{}$$

$$\Big\downarrow \; 2185$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{\int -b^2d^{10}(c+dx)^2\left(ad\left(3ad^2(58cC-99Bd)-2bc\left(56Cc^2-77Bdc+110Ad^2\right)\right)-\left(384a^2Cd^4-ab\left(578\right.\right.\right.}{8bd^2}}{}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{\int b^2d^{10}(c+dx}{}}{}$$

$$\Big\downarrow \; 27$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\int(c+dx)}{}$$

$$\Big\downarrow \; 687$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(\frac{\int(c+d}{}\right.}{}$$

$$\Big\downarrow \; 676$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-$$

$$\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(\frac{-\frac{693}{2}}{}\right.}{}$$

$\big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-\frac{\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(-\frac{693}{2}\right.}{}}{}$$

$\big\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-\frac{\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(-\frac{693}{2}\right.}{}}{}$$

$\big\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-\frac{\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(-\frac{693}{2}\right.}{}}{}$$

$\big\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^6}{11bd^4}-\frac{\frac{1}{10}d\left(a+bx^2\right)^{5/2}(c+dx)^5(38cC-11Bd)-\frac{\frac{1}{8}bd^8\left(a+bx^2\right)^{5/2}(c+dx)^3\left(3ad^2(62cC-33Bd)-2bc\left(154Ad^2-187Bcd+208c^2C\right)\right)-\frac{1}{8}bd^8\left(-\frac{693}{2}\right.}{}}{}$$

input
```
Int[x^3*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^6*(a + b*x^2)^(5/2))/(11*b*d^4) - ((d*(38*c*C - 11*B*d)*(c +
d*x)^5*(a + b*x^2)^(5/2))/10 - (-1/9*(d^5*(12*a*C*d^2 - b*(94*c^2*C - 55*B
*c*d + 22*A*d^2))*(c + d*x)^4*(a + b*x^2)^(5/2)) + ((b*d^8*(3*a*d^2*(62*c*
C - 33*B*d) - 2*b*c*(208*c^2*C - 187*B*c*d + 154*A*d^2))*(c + d*x)^3*(a +
b*x^2)^(5/2))/8 - (b*d^8*(-1/7*((384*a^2*C*d^4 + 6*b^2*c^2*(56*c^2*C - 77*
B*c*d + 110*A*d^2) - a*b*d^2*(578*c^2*C - 671*B*c*d + 704*A*d^2))*(c + d*x
)^2*(a + b*x^2)^(5/2))/b + ((2*(384*a^3*C*d^6 + a*b^2*c^2*d^2*(130*c^2*C -
 55*B*c*d - 176*A*d^2) + 64*a^2*b*d^4*(4*c^2*C - 22*B*c*d - 11*A*d^2) - 6*
b^3*c^4*(56*c^2*C - 77*B*c*d + 110*A*d^2))*(a + b*x^2)^(5/2))/(5*b) + (d*(
3*a^2*d^4*(50*c*C - 231*B*d) + 4*a*b*c*d^2*(31*c^2*C - 22*B*c*d - 11*A*d^2
) - 4*b^2*c^3*(56*c^2*C - 77*B*c*d + 110*A*d^2))*x*(a + b*x^2)^(5/2))/2 -
(693*a^2*d^4*(2*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*((x*(a + b*x^2)^(3/
2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*Sqrt[b])))/4))/2)/(7*b)))/8)/(9*b*d^3))/(2*b*d^4))/(11*b*d^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.96

method	result
default	$c(2Ad+Bc)\left(\frac{x^3(bx^2+a)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+d(Bc$
risch	$\frac{(80640C\,b^5d^2x^{10}+88704B\,b^5d^2x^9+177408C\,b^5cd\,x^9+98560A\,b^5d^2x^8+197120B\,b^5cd\,x^8+107520Ca\,b^4d^2x^8+98560C\,b^5c^2x^8+221}{}$

input
```
int(x^3*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-
1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+d*(B*d+2*C*c)*(1/10*x^5*(b*x^2+a)^(5/2)/
b-1/2*a/b*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*
a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2)))))))+(A*d^2+2*B*c*d+C*c^2)*(1/9*x^4*(b*x^2+a)^(5
/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)))+A*c^
2*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+C*d^2*(1/11*x^6*(
b*x^2+a)^(5/2)/b-6/11*a/b*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x
^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 1152, normalized size of antiderivative = 2.53

$$\int x^3(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[-1/1774080*(10395*(2*B*a^4*b*c^2 - B*a^5*d^2 - 2*(C*a^5 - 2*A*a^4*b)*c*d)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(80640*C*b^5*
d^2*x^10 + 88704*(2*C*b^5*c*d + B*b^5*d^2)*x^9 + 8960*(11*C*b^5*c^2 + 22*B
*b^5*c*d + (12*C*a*b^4 + 11*A*b^5)*d^2)*x^8 + 45056*B*a^4*b*c*d + 11088*(1
0*B*b^5*c^2 + 11*B*a*b^4*d^2 + 2*(11*C*a*b^4 + 10*A*b^5)*c*d)*x^7 + 1280*(
220*B*a*b^4*c*d + 11*(10*C*a*b^4 + 9*A*b^5)*c^2 + (3*C*a^2*b^3 + 110*A*a*b
^4)*d^2)*x^6 + 5544*(30*B*a*b^4*c^2 + B*a^2*b^3*d^2 + 2*(C*a^2*b^3 + 30*A*
a*b^4)*c*d)*x^5 + 768*(22*B*a^2*b^3*c*d + 11*(C*a^2*b^3 + 24*A*a*b^4)*c^2
- (6*C*a^3*b^2 - 11*A*a^2*b^3)*d^2)*x^4 + 6930*(2*B*a^2*b^3*c^2 - B*a^3*b^
2*d^2 - 2*(C*a^3*b^2 - 2*A*a^2*b^3)*c*d)*x^3 + 5632*(4*C*a^4*b - 9*A*a^3*b
^2)*c^2 - 2048*(6*C*a^5 - 11*A*a^4*b)*d^2 - 256*(88*B*a^3*b^2*c*d + 11*(4*
C*a^3*b^2 - 9*A*a^2*b^3)*c^2 - 4*(6*C*a^4*b - 11*A*a^3*b^2)*d^2)*x^2 - 103
95*(2*B*a^3*b^2*c^2 - B*a^4*b*d^2 - 2*(C*a^4*b - 2*A*a^3*b^2)*c*d)*x)*sqrt
(b*x^2 + a))/b^4, -1/887040*(10395*(2*B*a^4*b*c^2 - B*a^5*d^2 - 2*(C*a^5 -
 2*A*a^4*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (80640*C*b^
5*d^2*x^10 + 88704*(2*C*b^5*c*d + B*b^5*d^2)*x^9 + 8960*(11*C*b^5*c^2 + 22
*B*b^5*c*d + (12*C*a*b^4 + 11*A*b^5)*d^2)*x^8 + 45056*B*a^4*b*c*d + 11088*
(10*B*b^5*c^2 + 11*B*a*b^4*d^2 + 2*(11*C*a*b^4 + 10*A*b^5)*c*d)*x^7 + 1280
*(220*B*a*b^4*c*d + 11*(10*C*a*b^4 + 9*A*b^5)*c^2 + (3*C*a^2*b^3 + 110*A*a
*b^4)*d^2)*x^6 + 5544*(30*B*a*b^4*c^2 + B*a^2*b^3*d^2 + 2*(C*a^2*b^3 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1406 vs. $2(434) = 868$.

Time = 0.74 (sec) , antiderivative size = 1406, normalized size of antiderivative = 3.09

$$\int x^3(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((3*a**2*(2*A*a**2*c*d + B*a**2*c**2 - 5*a*(4*A*a*b*c*d + B*a**2*
d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 7*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*
b**2*c**2 + 4*C*a*b*c*d - 9*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(10*b))/(8*b))/
(6*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(C*b*d**2*x
**10/11 - 3*a*x*(2*A*a**2*c*d + B*a**2*c**2 - 5*a*(4*A*a*b*c*d + B*a**2*d*
*2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 7*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b*
*2*c**2 + 4*C*a*b*c*d - 9*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(10*b))/(8*b))/(6
*b))/(8*b**2) - 2*a*(A*a**2*c**2 - 4*a*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a
**2*c*d + C*a**2*c**2 - 6*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*
a**2*d**2 + 2*C*a*b*c**2 - 8*a*(A*b**2*d**2 + 2*B*b**2*c*d + 12*C*a*b*d**2
/11 + C*b**2*c**2)/(9*b))/(7*b))/(5*b))/(3*b**2) + x**9*(B*b**2*d**2 + 2*C
*b**2*c*d)/(10*b) + x**8*(A*b**2*d**2 + 2*B*b**2*c*d + 12*C*a*b*d**2/11 +
C*b**2*c**2)/(9*b) + x**7*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C
*a*b*c*d - 9*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(10*b))/(8*b) + x**6*(2*A*a*b*
d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*c**2 - 8*a*(A*b**
2*d**2 + 2*B*b**2*c*d + 12*C*a*b*d**2/11 + C*b**2*c**2)/(9*b))/(7*b) + x**
5*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 7*a*(2*A*b**2
*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 9*a*(B*b**2*d**2 + 2*C*b
**2*c*d)/(10*b))/(8*b))/(6*b) + x**4*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.19

$$\int x^3(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/11*(b*x^2 + a)^(5/2)*C*d^2*x^6/b - 2/33*(b*x^2 + a)^(5/2)*C*a*d^2*x^4/b^
2 + 1/10*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*x^5/b + 1/7*(b*x^2 + a)^(5/2)
*A*c^2*x^2/b + 8/231*(b*x^2 + a)^(5/2)*C*a^2*d^2*x^2/b^3 + 1/9*(C*c^2 + 2*
B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*x^4/b - 1/16*(2*C*c*d + B*d^2)*(b*x^2 + a
)^(5/2)*a*x^3/b^2 + 1/8*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*x^3/b - 2/35*(
b*x^2 + a)^(5/2)*A*a*c^2/b^2 - 16/1155*(b*x^2 + a)^(5/2)*C*a^3*d^2/b^4 - 4
/63*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 1/32*(2*C*c*d
+ B*d^2)*(b*x^2 + a)^(5/2)*a^2*x/b^3 - 1/128*(2*C*c*d + B*d^2)*(b*x^2 + a)
^(3/2)*a^3*x/b^3 - 3/256*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a^4*x/b^3 - 1/1
6*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(B*c^2 + 2*A*c*d)*(b*
x^2 + a)^(3/2)*a^2*x/b^2 + 3/128*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*a^3*x/b
^2 - 3/256*(2*C*c*d + B*d^2)*a^5*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 3/128*(B
*c^2 + 2*A*c*d)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/315*(C*c^2 + 2*B*c*
d + A*d^2)*(b*x^2 + a)^(5/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.36

$$\int x^3(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx = \frac{1}{887040} \sqrt{bx^2+a} \left(\left(2 \left(\left(4 \left(\left(2 \left(7 \left(8 \left(9 \left(10 \, Cbd^2 x + \frac{11 \left(2 \, Cb^{10}cd + Bb^{10}d^2 \right)}{b^9} \right) x + \frac{10 (11 \, C \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. - \frac{3 \left(2 \, Ba^4bc^2 - 2 \, Ca^5cd + 4 \, Aa^4bcd - Ba^5d^2 \right) \log \left(\left| -\sqrt{b}x + \sqrt{bx^2+a} \right| \right)}{256 \, b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/887040*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*(10*C*b*d^2*x + 11*(2*C*b^10
*c*d + B*b^10*d^2)/b^9)*x + 10*(11*C*b^10*c^2 + 22*B*b^10*c*d + 12*C*a*b^9
*d^2 + 11*A*b^10*d^2)/b^9)*x + 99*(10*B*b^10*c^2 + 22*C*a*b^9*c*d + 20*A*b
^10*c*d + 11*B*a*b^9*d^2)/b^9)*x + 80*(110*C*a*b^9*c^2 + 99*A*b^10*c^2 + 2
20*B*a*b^9*c*d + 3*C*a^2*b^8*d^2 + 110*A*a*b^9*d^2)/b^9)*x + 693*(30*B*a*b
^9*c^2 + 2*C*a^2*b^8*c*d + 60*A*a*b^9*c*d + B*a^2*b^8*d^2)/b^9)*x + 96*(11
*C*a^2*b^8*c^2 + 264*A*a*b^9*c^2 + 22*B*a^2*b^8*c*d - 6*C*a^3*b^7*d^2 + 11
*A*a^2*b^8*d^2)/b^9)*x + 3465*(2*B*a^2*b^8*c^2 - 2*C*a^3*b^7*c*d + 4*A*a^2
*b^8*c*d - B*a^3*b^7*d^2)/b^9)*x - 128*(44*C*a^3*b^7*c^2 - 99*A*a^2*b^8*c^
2 + 88*B*a^3*b^7*c*d - 24*C*a^4*b^6*d^2 + 44*A*a^3*b^7*d^2)/b^9)*x - 10395
*(2*B*a^3*b^7*c^2 - 2*C*a^4*b^6*c*d + 4*A*a^3*b^7*c*d - B*a^4*b^6*d^2)/b^9
)*x + 512*(44*C*a^4*b^6*c^2 - 99*A*a^3*b^7*c^2 + 88*B*a^4*b^6*c*d - 24*C*a
^5*b^5*d^2 + 44*A*a^4*b^6*d^2)/b^9) - 3/256*(2*B*a^4*b*c^2 - 2*C*a^5*c*d +
 4*A*a^4*b*c*d - B*a^5*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^3(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx = \int x^3 \left(b x^2+a\right)^{3/2} (c+d x)^2 \left(C x^2+B x+A\right) \, dx$$

input
```
int(x^3*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x^3*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 24.36 (sec) , antiderivative size = 1005, normalized size of antiderivative = 2.21

$$\int x^3(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
int(x^3*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output

```
(22528*sqrt(a + b*x**2)*a**5*b*d**2 - 12288*sqrt(a + b*x**2)*a**5*c*d**2 -
 50688*sqrt(a + b*x**2)*a**4*b**2*c**2 - 41580*sqrt(a + b*x**2)*a**4*b**2*
c*d*x + 45056*sqrt(a + b*x**2)*a**4*b**2*c*d - 11264*sqrt(a + b*x**2)*a**4
*b**2*d**2*x**2 + 10395*sqrt(a + b*x**2)*a**4*b**2*d**2*x + 22528*sqrt(a +
 b*x**2)*a**4*b*c**3 + 20790*sqrt(a + b*x**2)*a**4*b*c**2*d*x + 6144*sqrt(
a + b*x**2)*a**4*b*c*d**2*x**2 + 25344*sqrt(a + b*x**2)*a**3*b**3*c**2*x**
2 - 20790*sqrt(a + b*x**2)*a**3*b**3*c**2*x + 27720*sqrt(a + b*x**2)*a**3*
b**3*c*d*x**3 - 22528*sqrt(a + b*x**2)*a**3*b**3*c*d*x**2 + 8448*sqrt(a +
b*x**2)*a**3*b**3*d**2*x**4 - 6930*sqrt(a + b*x**2)*a**3*b**3*d**2*x**3 -
11264*sqrt(a + b*x**2)*a**3*b**2*c**3*x**2 - 13860*sqrt(a + b*x**2)*a**3*b
**2*c**2*d*x**3 - 4608*sqrt(a + b*x**2)*a**3*b**2*c*d**2*x**4 + 202752*sqr
t(a + b*x**2)*a**2*b**4*c**2*x**4 + 13860*sqrt(a + b*x**2)*a**2*b**4*c**2*
x**3 + 332640*sqrt(a + b*x**2)*a**2*b**4*c*d*x**5 + 16896*sqrt(a + b*x**2)
*a**2*b**4*c*d*x**4 + 140800*sqrt(a + b*x**2)*a**2*b**4*d**2*x**6 + 5544*s
qrt(a + b*x**2)*a**2*b**4*d**2*x**5 + 8448*sqrt(a + b*x**2)*a**2*b**3*c**3
*x**4 + 11088*sqrt(a + b*x**2)*a**2*b**3*c**2*d*x**5 + 3840*sqrt(a + b*x**
2)*a**2*b**3*c*d**2*x**6 + 126720*sqrt(a + b*x**2)*a*b**5*c**2*x**6 + 1663
20*sqrt(a + b*x**2)*a*b**5*c**2*x**5 + 221760*sqrt(a + b*x**2)*a*b**5*c*d*
x**7 + 281600*sqrt(a + b*x**2)*a*b**5*c*d*x**6 + 98560*sqrt(a + b*x**2)*a*
b**5*d**2*x**8 + 121968*sqrt(a + b*x**2)*a*b**5*d**2*x**7 + 140800*sqrt...
```

3.62 $\int x^2(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx$

Optimal result

Integrand size = 32, antiderivative size = 437

$$\int x^2(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx = \frac{a^2(16Ab^2c^2+3a(aCd^2-2b(c^2C+2Bcd+Ad^2)))\, x\sqrt{a+bx^2}}{256b^3} + \frac{a(16Ab^2c^2+3a(aCd^2-2b(c^2C+2Bcd+Ad^2)))\, x^3\sqrt{a+bx^2}}{128b^2} + \frac{(16Ab^2c^2+3a(aCd^2-2b(c^2C+2Bcd+Ad^2)))\, x^3\left(a+bx^2\right)^{3/2}}{96b^2} - \frac{a(bc(Bc+2Ad)-ad(2cC+Bd))\left(a+bx^2\right)^{5/2}}{5b^3} - \frac{(aCd^2-2b(c^2C+2Bcd+Ad^2))\, x^3\left(a+bx^2\right)^{5/2}}{16b^2} + \frac{Cd^2x^5\left(a+bx^2\right)^{5/2}}{10b} + \frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\left(a+bx^2\right)^{7/2}}{7b^3} + \frac{d(2cC+Bd)\left(a+bx^2\right)^{9/2}}{9b^3} - \frac{a^3(16Ab^2c^2+3a(aCd^2-2b(c^2C+2Bcd+Ad^2)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{7/2}}$$

output
```
1/256*a^2*(16*A*b^2*c^2+3*a*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2)))*x*(b*x^2+
a)^(1/2)/b^3+1/128*a*(16*A*b^2*c^2+3*a*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2))
)*x^3*(b*x^2+a)^(1/2)/b^2+1/96*(16*A*b^2*c^2+3*a*(a*C*d^2-2*b*(A*d^2+2*B*c
*d+C*c^2)))*x^3*(b*x^2+a)^(3/2)/b^2-1/5*a*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c)
)*(b*x^2+a)^(5/2)/b^3-1/16*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2))*x^3*(b*x^2+
a)^(5/2)/b^2+1/10*C*d^2*x^5*(b*x^2+a)^(5/2)/b+1/7*(b*c*(2*A*d+B*c)-2*a*d*(
B*d+2*C*c))*(b*x^2+a)^(7/2)/b^3+1/9*d*(B*d+2*C*c)*(b*x^2+a)^(9/2)/b^3-1/25
6*a^3*(16*A*b^2*c^2+3*a*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2)))*arctanh(b^(1/
2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 4.77 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.03

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \frac{\sqrt{b}\sqrt{a+bx^2}(a^4d(4096cC+2048Bd+945Cdx)-2a^3b(9Ad(512c+105dx)+2B(1152c^2+9\cdots}{\cdots}$$

input
```
Integrate[x^2*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(a^4*d*(4096*c*C + 2048*B*d + 945*C*d*x) - 2*a^3*
b*(9*A*d*(512*c + 105*d*x) + 2*B*(1152*c^2 + 945*c*d*x + 256*d^2*x^2) + C*
x*(945*c^2 + 1024*c*d*x + 315*d^2*x^2)) + 32*b^4*x^5*(15*A*(28*c^2 + 48*c*
d*x + 21*d^2*x^2) + x*(10*B*(36*c^2 + 63*c*d*x + 28*d^2*x^2) + 7*C*x*(45*c
^2 + 80*c*d*x + 36*d^2*x^2))) + 12*a^2*b^2*x*(3*A*(140*c^2 + 128*c*d*x + 3
5*d^2*x^2) + x*(2*B*(96*c^2 + 105*c*d*x + 32*d^2*x^2) + C*x*(105*c^2 + 128
*c*d*x + 42*d^2*x^2))) + 16*a*b^3*x^3*(3*A*(490*c^2 + 768*c*d*x + 315*d^2*
x^2) + x*(2*B*(576*c^2 + 945*c*d*x + 400*d^2*x^2) + C*x*(945*c^2 + 1600*c*
d*x + 693*d^2*x^2)))) + 630*a^3*(16*A*b^2*c^2 + 3*a^2*C*d^2)*ArcTanh[(Sqrt
[b]*x)/(Sqrt[a] - Sqrt[a + b*x^2])] + 3780*a^4*b*(c^2*C + 2*B*c*d + A*d^2)
*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(80640*b^(7/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int -5(c+dx)^2 \left(bx^2+a\right)^{3/2} \left(bd^3(5cC-2Bd)x^3+d^2\left(4bCc^2-2Abd^2+aCd^2\right)x^2+cCd\left(bc^2+2ad^2\right)x+ac^2Cd^2}{10bd^4}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}$$

$$\Big\downarrow \ 27$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-$$
$$\frac{\int (c+dx)^2 \left(bx^2+a\right)^{3/2} \left(bd^3(5cC-2Bd)x^3+d^2\left(4bCc^2-2Abd^2+aCd^2\right)x^2+cCd\left(bc^2+2ad^2\right)x+ac^2Cd^2\right) d}{2bd^4}$$

$$\Big\downarrow \ 2185$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-$$
$$\frac{\frac{\int -(c+dx)^2\left(bx^2+a\right)^{3/2}\left(-b\left(9aCd^2-2b\left(17Cc^2-14Bdc+9Ad^2\right)\right)x^2d^5+abc(11cC-8Bd)d^5+2b\left(b(8cC-5Bd)c^2+ad^2(cC-4Bd)\right)xd^4\right)dx}{9bd^3}+\frac{1}{9}d(}{2bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-$$
$$\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\int (c+dx)^2\left(bx^2+a\right)^{3/2}\left(-b\left(9aCd^2-2b\left(17Cc^2-14Bdc+9Ad^2\right)\right)x^2d^5+abc(11cC-8Bd)d^5+2}{9bd^3}}{2bd^4}$$

$$\Big\downarrow \ 2185$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-$$
$$\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}}{\cdots}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+15\cdots\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}}{\cdots}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+15\cdots\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}}{\cdots}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+15\cdots\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$$\Big\downarrow \; 25$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\right.\right.}{}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\right.\right.}{}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\right.\right.}{}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\right.\right.\right.}{8bd^2}}{9bd^3}}{2bd^4}$$

$\big\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}\cdots}{\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\cdots\right.\right.\right.}{8bd^2}}{9bd^3}\cdots}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}\cdots}{\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\cdots\right.\right.\right.}{8bd^2}}{9bd^3}\cdots}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\cdots\right.\right.}{9bd^3}\cdots}{\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\left(ad\left(14bCc^2-20bBdc+54Abd^2-27aCd^2\right)-b\left(ad^2(61cC-64Bd)-6bc\left(7Cc^2-10Bdc+1\cdots\right.\right.\right.}{8bd^2}}{9bd^3}\cdots}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(27aCd^2-\cdots}{\cdots}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(14bCc^2-20bBdc+54Abd^2-27aCd^2\bigr)-b\bigl(ad^2(61cC-64Bd)-6bc\bigl(7Cc^2-10Bdc+1\cdots}{8bd^2}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(27aCd^2-\cdots}{\cdots}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(14bCc^2-20bBdc+54Abd^2-27aCd^2\bigr)-b\bigl(ad^2(61cC-64Bd)-6bc\bigl(7Cc^2-10Bdc+1\cdots}{8bd^2}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(27aCd^2-\cdots}{\cdots}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{\int -bd^6(c+dx)^2\bigl(ad\bigl(14bCc^2-20bBdc+54Abd^2-27aCd^2\bigr)-b\bigl(ad^2(61cC-64Bd)-6bc\bigl(7Cc^2-10Bdc+1\cdots}{8bd^2}}{9bd^3\cdots}}{2bd^4}$$

$\downarrow$ 25

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^5}{10bd^3}-$$
$$\frac{\frac{1}{9}d\left(a+bx^2\right)^{5/2}(c+dx)^4(5cC-2Bd)-\frac{\frac{1}{8}d^4\left(a+bx^2\right)^{5/2}(c+dx)^3\left(-9aCd^2+18Abd^2-28bBcd+34bc^2C\right)-\frac{\int -bd^6(c+dx)^2\left(ad\left(27aCd^2-\right.\right.}{9bd}}{}}{2bd^4}$$

input
```
Int[x^2*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 430, normalized size of antiderivative = 0.98

method	result
default	$c(2Ad+Bc)\left(\frac{x^{2}(b\,x^{2}+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^{2}+a)^{\frac{5}{2}}}{35b^{2}}\right)+d(Bd+2Cc)\left(\frac{x^{4}(b\,x^{2}+a)^{\frac{5}{2}}}{9b}-\frac{4a\left(\frac{x^{2}(b\,x^{2}+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^{2}+a)}{35b^{2}}\right.}{9b}\right.$
risch	$-\frac{(-8064C\,b^{4}d^{2}x^{9}-8960B\,b^{4}d^{2}x^{8}-17920C\,b^{4}cd\,x^{8}-10080A\,b^{4}d^{2}x^{7}-20160B\,b^{4}cd\,x^{7}-11088Ca\,b^{3}d^{2}x^{7}-10080C\,b^{4}c^{2}x^{7}-23040A}{}$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+d*(B*
d+2*C*c)*(1/9*x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/3
5*a/b^2*(b*x^2+a)^(5/2)))+(A*d^2+2*B*c*d+C*c^2)*(1/8*x^3*(b*x^2+a)^(5/2)/b
-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+A*c^2*
(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^
2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+C*d^2*(1/10*x^5*
(b*x^2+a)^(5/2)/b-1/2*a/b*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2
+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/
2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.44

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas
")
```

output
```
[1/161280*(315*(12*B*a^4*b*c*d + 2*(3*C*a^4*b - 8*A*a^3*b^2)*c^2 - 3*(C*a^
5 - 2*A*a^4*b)*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*C*b^5*d^2*x^9 + 8960*(2*C*b^5*c*d + B*b^5*d^2)*x^8 - 4608*B*a^
3*b^2*c^2 + 2048*B*a^4*b*d^2 + 1008*(10*C*b^5*c^2 + 20*B*b^5*c*d + (11*C*a
*b^4 + 10*A*b^5)*d^2)*x^7 + 1280*(9*B*b^5*c^2 + 10*B*a*b^4*d^2 + 2*(10*C*a
*b^4 + 9*A*b^5)*c*d)*x^6 + 168*(180*B*a*b^4*c*d + 10*(9*C*a*b^4 + 8*A*b^5)
*c^2 + 3*(C*a^2*b^3 + 30*A*a*b^4)*d^2)*x^5 + 768*(24*B*a*b^4*c^2 + B*a^2*b
^3*d^2 + 2*(C*a^2*b^3 + 24*A*a*b^4)*c*d)*x^4 + 210*(12*B*a^2*b^3*c*d + 2*(
3*C*a^2*b^3 + 56*A*a*b^4)*c^2 - 3*(C*a^3*b^2 - 2*A*a^2*b^3)*d^2)*x^3 + 102
4*(4*C*a^4*b - 9*A*a^3*b^2)*c*d + 256*(9*B*a^2*b^3*c^2 - 4*B*a^3*b^2*d^2 -
 2*(4*C*a^3*b^2 - 9*A*a^2*b^3)*c*d)*x^2 - 315*(12*B*a^3*b^2*c*d + 2*(3*C*a
^3*b^2 - 8*A*a^2*b^3)*c^2 - 3*(C*a^4*b - 2*A*a^3*b^2)*d^2)*x)*sqrt(b*x^2 +
 a))/b^4, -1/80640*(315*(12*B*a^4*b*c*d + 2*(3*C*a^4*b - 8*A*a^3*b^2)*c^2
- 3*(C*a^5 - 2*A*a^4*b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
 (8064*C*b^5*d^2*x^9 + 8960*(2*C*b^5*c*d + B*b^5*d^2)*x^8 - 4608*B*a^3*b^2
*c^2 + 2048*B*a^4*b*d^2 + 1008*(10*C*b^5*c^2 + 20*B*b^5*c*d + (11*C*a*b^4
+ 10*A*b^5)*d^2)*x^7 + 1280*(9*B*b^5*c^2 + 10*B*a*b^4*d^2 + 2*(10*C*a*b^4
+ 9*A*b^5)*c*d)*x^6 + 168*(180*B*a*b^4*c*d + 10*(9*C*a*b^4 + 8*A*b^5)*c^2
+ 3*(C*a^2*b^3 + 30*A*a*b^4)*d^2)*x^5 + 768*(24*B*a*b^4*c^2 + B*a^2*b^3*d^
2 + 2*(C*a^2*b^3 + 24*A*a*b^4)*c*d)*x^4 + 210*(12*B*a^2*b^3*c*d + 2*(3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(425) = 850$.

Time = 0.70 (sec) , antiderivative size = 1253, normalized size of antiderivative = 2.87

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(A*a**2*c**2 - 3*a*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*
d + C*a**2*c**2 - 5*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d
**2 + 2*C*a*b*c**2 - 7*a*(A*b**2*d**2 + 2*B*b**2*c*d + 11*C*a*b*d**2/10 +
C*b**2*c**2)/(8*b))/(6*b))/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2
) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt
(a + b*x**2)*(C*b*d**2*x**9/10 - 2*a*(2*A*a**2*c*d + B*a**2*c**2 - 4*a*(4*
A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 6*a*(2*A*b**2*c*d
+ 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 8*a*(B*b**2*d**2 + 2*C*b**2*c
*d)/(9*b))/(7*b))/(5*b))/(3*b**2) + x**8*(B*b**2*d**2 + 2*C*b**2*c*d)/(9*b
) + x**7*(A*b**2*d**2 + 2*B*b**2*c*d + 11*C*a*b*d**2/10 + C*b**2*c**2)/(8*
b) + x**6*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 8*a*(
B*b**2*d**2 + 2*C*b**2*c*d)/(9*b))/(7*b) + x**5*(2*A*a*b*d**2 + A*b**2*c**
2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*c**2 - 7*a*(A*b**2*d**2 + 2*B*b**2
*c*d + 11*C*a*b*d**2/10 + C*b**2*c**2)/(8*b))/(6*b) + x**4*(4*A*a*b*c*d +
B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 6*a*(2*A*b**2*c*d + 2*B*a*b*d*
*2 + B*b**2*c**2 + 4*C*a*b*c*d - 8*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(9*b))/(
7*b))/(5*b) + x**3*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*d + C*a**2*c**
2 - 5*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*
c**2 - 7*a*(A*b**2*d**2 + 2*B*b**2*c*d + 11*C*a*b*d**2/10 + C*b**2*c**2)/(
8*b))/(6*b))/(4*b) + x**2*(2*A*a**2*c*d + B*a**2*c**2 - 4*a*(4*A*a*b*c*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.19

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cd^2x^5}{10\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Cad^2x^3}{16\,b^2}+\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}x^4}{9\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ac^2x}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Aac^2x}{24\,b}-\frac{\sqrt{bx^2+a}Aa^2c^2x}{16\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ca^2d^2x}{32\,b^3}-\frac{(bx^2+a)^{\frac{3}{2}}Ca^3d^2x}{128\,b^3}-\frac{3\,\sqrt{bx^2+a}Ca^4d^2x}{256\,b^3}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}x^3}{8\,b}-\frac{Aa^3c^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{3\,Ca^5d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{7}{2}}}-\frac{4\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}ax^2}{63\,b^2}+\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}x^2}{7\,b}-\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}ax}{16\,b^2}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}a^3x}{128\,b^2}+\frac{3\,(Cc^2+2\,Bcd+Ad^2)a^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}+\frac{8\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}a^2}{315\,b^3}-\frac{2\,(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima
")
```

output
```
1/10*(b*x^2 + a)^(5/2)*C*d^2*x^5/b - 1/16*(b*x^2 + a)^(5/2)*C*a*d^2*x^3/b^
2 + 1/9*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*x^4/b + 1/6*(b*x^2 + a)^(5/2)*
A*c^2*x/b - 1/24*(b*x^2 + a)^(3/2)*A*a*c^2*x/b - 1/16*sqrt(b*x^2 + a)*A*a^
2*c^2*x/b + 1/32*(b*x^2 + a)^(5/2)*C*a^2*d^2*x/b^3 - 1/128*(b*x^2 + a)^(3/
2)*C*a^3*d^2*x/b^3 - 3/256*sqrt(b*x^2 + a)*C*a^4*d^2*x/b^3 + 1/8*(C*c^2 +
2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*x^3/b - 1/16*A*a^3*c^2*arcsinh(b*x/sqrt
(a*b))/b^(3/2) - 3/256*C*a^5*d^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 4/63*(2*
C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 1/7*(B*c^2 + 2*A*c*d)*(b*x^2
+ a)^(5/2)*x^2/b - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*a*x/b^
2 + 1/64*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b^2 + 3/128*(C*
c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a^3*x/b^2 + 3/128*(C*c^2 + 2*B*c*d
+ A*d^2)*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/315*(2*C*c*d + B*d^2)*(b*x
^2 + a)^(5/2)*a^2/b^3 - 2/35*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.29

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{1}{80640}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(\left(\left(2\left(7\left(8\left(9\,Cbd^2x+\frac{10\left(2\,Cb^9cd+Bb^9d^2\right)}{b^8}\right)x+\frac{9\left(10\,Cb^9c^2+2\right.}{}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{\left(6\,Ca^4bc^2-16\,Aa^3b^2c^2+12\,Ba^4bcd-3\,Ca^5d^2+6\,Aa^4bd^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{7}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/80640*sqrt(b*x^2 + a)*((2*((4*((2*(7*(8*(9*C*b*d^2*x + 10*(2*C*b^9*c*d +
 B*b^9*d^2)/b^8)*x + 9*(10*C*b^9*c^2 + 20*B*b^9*c*d + 11*C*a*b^8*d^2 + 10*
A*b^9*d^2)/b^8)*x + 80*(9*B*b^9*c^2 + 20*C*a*b^8*c*d + 18*A*b^9*c*d + 10*B
*a*b^8*d^2)/b^8)*x + 21*(90*C*a*b^8*c^2 + 80*A*b^9*c^2 + 180*B*a*b^8*c*d +
 3*C*a^2*b^7*d^2 + 90*A*a*b^8*d^2)/b^8)*x + 96*(24*B*a*b^8*c^2 + 2*C*a^2*b
^7*c*d + 48*A*a*b^8*c*d + B*a^2*b^7*d^2)/b^8)*x + 105*(6*C*a^2*b^7*c^2 + 1
12*A*a*b^8*c^2 + 12*B*a^2*b^7*c*d - 3*C*a^3*b^6*d^2 + 6*A*a^2*b^7*d^2)/b^8
)*x + 128*(9*B*a^2*b^7*c^2 - 8*C*a^3*b^6*c*d + 18*A*a^2*b^7*c*d - 4*B*a^3*
b^6*d^2)/b^8)*x - 315*(6*C*a^3*b^6*c^2 - 16*A*a^2*b^7*c^2 + 12*B*a^3*b^6*c
*d - 3*C*a^4*b^5*d^2 + 6*A*a^3*b^6*d^2)/b^8)*x - 512*(9*B*a^3*b^6*c^2 - 8*
C*a^4*b^5*c*d + 18*A*a^3*b^6*c*d - 4*B*a^4*b^5*d^2)/b^8) - 1/256*(6*C*a^4*
b*c^2 - 16*A*a^3*b^2*c^2 + 12*B*a^4*b*c*d - 3*C*a^5*d^2 + 6*A*a^4*b*d^2)*l
og(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x^2\left(b\,x^2+a\right)^{3/2}\left(c+d\,x\right)^2\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x^2*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int x^2(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x^2(dx+c)^2\left(b\,x^2+a\right)^{\frac{3}{2}}\left(C\,x^2+Bx+A\right)dx$$

input
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
int(x^2*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

3.63 $\int x(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)\,dx$

Optimal result

Integrand size = 30, antiderivative size = 349

$$\begin{aligned}\int x(c+dx)^2 \left(a+bx^2\right)^{3/2} (A+Bx &+ Cx^2)\,dx = \frac{a^2(8bc(Bc+2Ad)-3ad(2cC+Bd))x\sqrt{a+bx^2}}{128b^2} \\
&+ \frac{a(8bc(Bc+2Ad)-3ad(2cC+Bd))x^3\sqrt{a+bx^2}}{64b} \\
&+ \frac{(8bc(Bc+2Ad)-3ad(2cC+Bd))x^3\left(a+bx^2\right)^{3/2}}{48b} \\
&+ \frac{\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)\left(a+bx^2\right)^{5/2}}{5b^3} \\
&+ \frac{d(2cC+Bd)x^3\left(a+bx^2\right)^{5/2}}{8b} - \frac{\left(2aCd^2-b\left(c^2C+2Bcd+Ad^2\right)\right)\left(a+bx^2\right)^{7/2}}{7b^3} \\
&+ \frac{Cd^2\left(a+bx^2\right)^{9/2}}{9b^3} - \frac{a^3(8bc(Bc+2Ad)-3ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}\end{aligned}$$

output
```
1/128*a^2*(8*b*c*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/b^2+1/64
*a*(8*b*c*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*x^3*(b*x^2+a)^(1/2)/b+1/48*(8*b*c
*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*x^3*(b*x^2+a)^(3/2)/b+1/5*(A*b*(-a*d^2+b*c
^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(5/2)/b^3+1/8*d*(B*d+2*C*c)*x^3
*(b*x^2+a)^(5/2)/b-1/7*(2*a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(7/2)
/b^3+1/9*C*d^2*(b*x^2+a)^(9/2)/b^3-1/128*a^3*(8*b*c*(2*A*d+B*c)-3*a*d*(B*d
+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.05

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{\sqrt{a+bx^2}(1024a^4Cd^2-a^3b(2304c^2C+18cd(256B+105Cx)+d^2(2304A+945Bx+512Cx}{}$$

input
```
Integrate[x*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[a + b*x^2]*(1024*a^4*C*d^2 - a^3*b*(2304*c^2*C + 18*c*d*(256*B + 105
*C*x) + d^2*(2304*A + 945*B*x + 512*C*x^2)) + 16*b^4*x^4*(24*A*(21*c^2 + 3
5*c*d*x + 15*d^2*x^2) + 5*x*(3*B*(28*c^2 + 48*c*d*x + 21*d^2*x^2) + 2*C*x*
(36*c^2 + 63*c*d*x + 28*d^2*x^2))) + 6*a^2*b^2*(24*A*(56*c^2 + 35*c*d*x +
8*d^2*x^2) + x*(2*C*x*(96*c^2 + 105*c*d*x + 32*d^2*x^2) + 3*B*(140*c^2 + 1
28*c*d*x + 35*d^2*x^2))) + 8*a*b^3*x^2*(12*A*(168*c^2 + 245*c*d*x + 96*d^2
*x^2) + x*(3*B*(490*c^2 + 768*c*d*x + 315*d^2*x^2) + 2*C*x*(576*c^2 + 945*
c*d*x + 400*d^2*x^2)))) - 315*a^3*Sqrt[b]*(-8*b*c*(B*c + 2*A*d) + 3*a*d*(2
*c*C + B*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40320*b^3)
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2185, 25, 2185, 25, 27, 687, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -(c+dx)^2\left(bx^2+a\right)^{3/2}\left(b(14cC-9Bd)x^2d^2+4acCd^2+\left(5bCc^2-9Abd^2+4aCd^2\right)xd\right)dx}{9bd^3}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\int (c+dx)^2\left(bx^2+a\right)^{3/2}\left(b(14cC-9Bd)x^2d^2+4acCd^2+\left(5bCc^2-9Abd^2+4aCd^2\right)xd\right)dx}{9bd^3}$$

$$\Big\downarrow \text{2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{\int -bd^3(c+dx)^2\left(ad(10cC-27Bd)-\left(32aCd^2-b\left(30Cc^2-45Bdc+72Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{8bd^2}+\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd}{9bd^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{\int bd^3(c+dx)^2\left(ad(10cC-27Bd)-\left(32aCd^2-b\left(30Cc^2-45Bdc+72Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{8bd^2}}{9bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\int(c+dx)^2\left(ad(10cC-27Bd)-\left(32aCd^2-b\left(30Cc^2-45Bdc+72\cdots\right.\right.\right.}{9bd^3}$$

$\downarrow$ 687

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{\int(c+dx)\left(ad\left(64aCd^2+b\left(10Cc^2-99Bdc-144Ad^2\right)\right)+3b\left(a(2cC-63Bd)d^2+2bc\left(10C\cdots\right.\right.\right.}{7b}\cdots\right.}{9bd^3}$$

$\downarrow$ 676

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{-\frac{21}{2}ad^2(8bc(2Ad+Bc)-3ad(Bd+2cC))\int\left(bx^2+a\right)^{3/2}dx+\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2Cd^4+8c\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{-\frac{21}{2}ad^2(8bc(2Ad+Bc)-3ad(Bd+2cC))\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{2\left(a+b\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

$\downarrow$ 211

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2}-\frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{-\frac{21}{2}ad^2(8bc(2Ad+Bc)-3ad(Bd+2cC))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(\cdots\right.}{\cdots}\right.}{\cdots}$$

$\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{-\frac{21}{2}ad^2(8bc(2Ad+Bc)-3ad(Bd+2cC))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx}\right.\right.}{}\right.}{}$$

$\downarrow$ 219

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^4}{9bd^2} - \frac{\frac{1}{8}d\left(a+bx^2\right)^{5/2}(c+dx)^3(14cC-9Bd)-\frac{1}{8}d\left(\frac{\frac{2\left(a+bx^2\right)^{5/2}\left(32a^2Cd^4+8abd^2\left(-9Ad^2-18Bcd+c^2C\right)+3b^2c^2\left(24Ad^2-15Bcd+10c^2C\right)\right)}{5b}-}{}\right.}{}$$

input
```
Int[x*(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(C*(c + d*x)^4*(a + b*x^2)^(5/2))/(9*b*d^2) - ((d*(14*c*C - 9*B*d)*(c + d*
x)^3*(a + b*x^2)^(5/2))/8 - (d*(-1/7*((32*a*C*d^2 - b*(30*c^2*C - 45*B*c*d
 + 72*A*d^2))*(c + d*x)^2*(a + b*x^2)^(5/2))/b + ((2*(32*a^2*C*d^4 + 8*a*b
*d^2*(c^2*C - 18*B*c*d - 9*A*d^2) + 3*b^2*c^2*(10*c^2*C - 15*B*c*d + 24*A*
d^2))*(a + b*x^2)^(5/2))/(5*b) + (d*(a*d^2*(2*c*C - 63*B*d) + 2*b*c*(10*c^
2*C - 15*B*c*d + 24*A*d^2))*x*(a + b*x^2)^(5/2))/2 - (21*a*d^2*(8*b*c*(B*c
 + 2*A*d) - 3*a*d*(2*c*C + B*d))*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[
a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))
/2)/(7*b)))/8)/(9*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.92

method	result
default	$c(2Ad+Bc)\left(\frac{x(b\,x^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+d(Bd+2Cc)\left(\frac{x^3(b\,x\right.$
risch	$-\frac{(-4480C\,b^4d^2x^8-5040B\,b^4d^2x^7-10080C\,b^4cd\,x^7-5760A\,b^4d^2x^6-11520B\,b^4cd\,x^6-6400Ca\,b^3d^2x^6-5760C\,b^4c^2x^6-13440A\,b^4}{}$

input
```
int(x*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
c*(2*A*d+B*c)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*
a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+d*
(B*d+2*C*c)*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/
6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))))+(A*d^2+2*B*c*d+C*c^2)*(1/7*x^2*(b*x^2+a)^(
5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+1/5*A*c^2*(b*x^2+a)^(5/2)/b+C*d^2*(1/9*
x^4*(b*x^2+a)^(5/2)/b-4/9*a/b*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2
+a)^(5/2)))
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 923, normalized size of antiderivative = 2.64

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[-1/80640*(315*(8*B*a^3*b*c^2 - 3*B*a^4*d^2 - 2*(3*C*a^4 - 8*A*a^3*b)*c*d)
*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4480*C*b^4*d
^2*x^8 + 5040*(2*C*b^4*c*d + B*b^4*d^2)*x^7 - 4608*B*a^3*b*c*d + 640*(9*C*
b^4*c^2 + 18*B*b^4*c*d + (10*C*a*b^3 + 9*A*b^4)*d^2)*x^6 + 840*(8*B*b^4*c^
2 + 9*B*a*b^3*d^2 + 2*(9*C*a*b^3 + 8*A*b^4)*c*d)*x^5 + 384*(48*B*a*b^3*c*d
 + 3*(8*C*a*b^3 + 7*A*b^4)*c^2 + (C*a^2*b^2 + 24*A*a*b^3)*d^2)*x^4 + 210*(
56*B*a*b^3*c^2 + 3*B*a^2*b^2*d^2 + 2*(3*C*a^2*b^2 + 56*A*a*b^3)*c*d)*x^3 -
 1152*(2*C*a^3*b - 7*A*a^2*b^2)*c^2 + 256*(4*C*a^4 - 9*A*a^3*b)*d^2 + 128*
(18*B*a^2*b^2*c*d + 9*(C*a^2*b^2 + 14*A*a*b^3)*c^2 - (4*C*a^3*b - 9*A*a^2*
b^2)*d^2)*x^2 + 315*(8*B*a^2*b^2*c^2 - 3*B*a^3*b*d^2 - 2*(3*C*a^3*b - 8*A*
a^2*b^2)*c*d)*x)*sqrt(b*x^2 + a))/b^3, 1/40320*(315*(8*B*a^3*b*c^2 - 3*B*a
^4*d^2 - 2*(3*C*a^4 - 8*A*a^3*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^
2 + a)) + (4480*C*b^4*d^2*x^8 + 5040*(2*C*b^4*c*d + B*b^4*d^2)*x^7 - 4608*
B*a^3*b*c*d + 640*(9*C*b^4*c^2 + 18*B*b^4*c*d + (10*C*a*b^3 + 9*A*b^4)*d^2
)*x^6 + 840*(8*B*b^4*c^2 + 9*B*a*b^3*d^2 + 2*(9*C*a*b^3 + 8*A*b^4)*c*d)*x^
5 + 384*(48*B*a*b^3*c*d + 3*(8*C*a*b^3 + 7*A*b^4)*c^2 + (C*a^2*b^2 + 24*A*
a*b^3)*d^2)*x^4 + 210*(56*B*a*b^3*c^2 + 3*B*a^2*b^2*d^2 + 2*(3*C*a^2*b^2 +
 56*A*a*b^3)*c*d)*x^3 - 1152*(2*C*a^3*b - 7*A*a^2*b^2)*c^2 + 256*(4*C*a^4
- 9*A*a^3*b)*d^2 + 128*(18*B*a^2*b^2*c*d + 9*(C*a^2*b^2 + 14*A*a*b^3)*c^2
- (4*C*a^3*b - 9*A*a^2*b^2)*d^2)*x^2 + 315*(8*B*a^2*b^2*c^2 - 3*B*a^3*b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1086 vs. 2(333) = 666.

Time = 0.65 (sec) , antiderivative size = 1086, normalized size of antiderivative = 3.11

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((-a*(2*A*a**2*c*d + B*a**2*c**2 - 3*a*(4*A*a*b*c*d + B*a**2*d**2
 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 5*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2
*c**2 + 4*C*a*b*c*d - 7*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(8*b))/(6*b))/(4*b)
)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (
x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*b*d**2*x**8/9 +
x**7*(B*b**2*d**2 + 2*C*b**2*c*d)/(8*b) + x**6*(A*b**2*d**2 + 2*B*b**2*c*d
 + 10*C*a*b*d**2/9 + C*b**2*c**2)/(7*b) + x**5*(2*A*b**2*c*d + 2*B*a*b*d**
2 + B*b**2*c**2 + 4*C*a*b*c*d - 7*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(8*b))/(6
*b) + x**4*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a
*b*c**2 - 6*a*(A*b**2*d**2 + 2*B*b**2*c*d + 10*C*a*b*d**2/9 + C*b**2*c**2)
/(7*b))/(5*b) + x**3*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*
c*d - 5*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 7*a*(
B*b**2*d**2 + 2*C*b**2*c*d)/(8*b))/(6*b))/(4*b) + x**2*(A*a**2*d**2 + 2*A*
a*b*c**2 + 2*B*a**2*c*d + C*a**2*c**2 - 4*a*(2*A*a*b*d**2 + A*b**2*c**2 +
4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*c**2 - 6*a*(A*b**2*d**2 + 2*B*b**2*c*d
 + 10*C*a*b*d**2/9 + C*b**2*c**2)/(7*b))/(5*b))/(3*b) + x*(2*A*a**2*c*d +
B*a**2*c**2 - 3*a*(4*A*a*b*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d
 - 5*a*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 7*a*(B*b
**2*d**2 + 2*C*b**2*c*d)/(8*b))/(6*b))/(4*b))/(2*b) + (A*a**2*c**2 - 2*a*(
A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*d + C*a**2*c**2 - 4*a*(2*A*a*b*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.14

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Cd^2x^4}{9\,b}-\frac{4\,(bx^2+a)^{\frac{5}{2}}Cad^2x^2}{63\,b^2}+\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}x^3}{8\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{5\,b}+\frac{8\,(bx^2+a)^{\frac{5}{2}}Ca^2d^2}{315\,b^3}+\frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}x^2}{7\,b}-\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}ax}{16\,b^2}+\frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}a^2x}{64\,b^2}+\frac{3\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}a^3x}{128\,b^2}+\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}x}{6\,b}-\frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}ax}{24\,b}-\frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}a^2x}{16\,b}+\frac{3\,(2\,Ccd+Bd^2)a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{(Bc^2+2\,Acd)a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(5/2)*C*d^2*x^4/b - 4/63*(b*x^2 + a)^(5/2)*C*a*d^2*x^2/b^2
 + 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*x^3/b + 1/5*(b*x^2 + a)^(5/2)*A
*c^2/b + 8/315*(b*x^2 + a)^(5/2)*C*a^2*d^2/b^3 + 1/7*(C*c^2 + 2*B*c*d + A*
d^2)*(b*x^2 + a)^(5/2)*x^2/b - 1/16*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*a*
x/b^2 + 1/64*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*a^2*x/b^2 + 3/128*(2*C*c*
d + B*d^2)*sqrt(b*x^2 + a)*a^3*x/b^2 + 1/6*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(
5/2)*x/b - 1/24*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*a*x/b - 1/16*(B*c^2 +
2*A*c*d)*sqrt(b*x^2 + a)*a^2*x/b + 3/128*(2*C*c*d + B*d^2)*a^4*arcsinh(b*x
/sqrt(a*b))/b^(5/2) - 1/16*(B*c^2 + 2*A*c*d)*a^3*arcsinh(b*x/sqrt(a*b))/b^
(3/2) - 2/35*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.42

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\frac{1}{40320}\sqrt{bx^2+a}\left(\left(2\left(\left(\left(4\left(5\left(2\left(7\left(8\,Cbd^2x+\frac{9\,(2\,Cb^8cd+Bb^8d^2)}{b^7}\right)x+\frac{8\,(9\,Cb^8c^2+18\,B\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$+\frac{(8\,Ba^3bc^2-6\,Ca^4cd+16\,Aa^3bcd-3\,Ba^4d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/40320*sqrt(b*x^2 + a)*((2*((4*(5*(2*(7*(8*C*b*d^2*x + 9*(2*C*b^8*c*d + B
*b^8*d^2)/b^7)*x + 8*(9*C*b^8*c^2 + 18*B*b^8*c*d + 10*C*a*b^7*d^2 + 9*A*b^
8*d^2)/b^7)*x + 21*(8*B*b^8*c^2 + 18*C*a*b^7*c*d + 16*A*b^8*c*d + 9*B*a*b^
7*d^2)/b^7)*x + 48*(24*C*a*b^7*c^2 + 21*A*b^8*c^2 + 48*B*a*b^7*c*d + C*a^2
*b^6*d^2 + 24*A*a*b^7*d^2)/b^7)*x + 105*(56*B*a*b^7*c^2 + 6*C*a^2*b^6*c*d
+ 112*A*a*b^7*c*d + 3*B*a^2*b^6*d^2)/b^7)*x + 64*(9*C*a^2*b^6*c^2 + 126*A*
a*b^7*c^2 + 18*B*a^2*b^6*c*d - 4*C*a^3*b^5*d^2 + 9*A*a^2*b^6*d^2)/b^7)*x +
 315*(8*B*a^2*b^6*c^2 - 6*C*a^3*b^5*c*d + 16*A*a^2*b^6*c*d - 3*B*a^3*b^5*d
^2)/b^7)*x - 128*(18*C*a^3*b^5*c^2 - 63*A*a^2*b^6*c^2 + 36*B*a^3*b^5*c*d -
 8*C*a^4*b^4*d^2 + 18*A*a^3*b^5*d^2)/b^7) + 1/128*(8*B*a^3*b*c^2 - 6*C*a^4
*c*d + 16*A*a^3*b*c*d - 3*B*a^4*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))
)/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\int x\left(b\,x^2+a\right)^{3/2}\left(c+d\,x\right)^2\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int(x*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int(x*(a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.31

$$\int x(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)\,dx=\text{Too large to display}$$

input
```
int(x*(d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
( - 2304*sqrt(a + b*x**2)*a**4*b*d**2 + 1024*sqrt(a + b*x**2)*a**4*c*d**2
+ 8064*sqrt(a + b*x**2)*a**3*b**2*c**2 + 5040*sqrt(a + b*x**2)*a**3*b**2*c
*d*x - 4608*sqrt(a + b*x**2)*a**3*b**2*c*d + 1152*sqrt(a + b*x**2)*a**3*b*
*2*d**2*x**2 - 945*sqrt(a + b*x**2)*a**3*b**2*d**2*x - 2304*sqrt(a + b*x**
2)*a**3*b*c**3 - 1890*sqrt(a + b*x**2)*a**3*b*c**2*d*x - 512*sqrt(a + b*x*
*2)*a**3*b*c*d**2*x**2 + 16128*sqrt(a + b*x**2)*a**2*b**3*c**2*x**2 + 2520
*sqrt(a + b*x**2)*a**2*b**3*c**2*x + 23520*sqrt(a + b*x**2)*a**2*b**3*c*d*
x**3 + 2304*sqrt(a + b*x**2)*a**2*b**3*c*d*x**2 + 9216*sqrt(a + b*x**2)*a*
*2*b**3*d**2*x**4 + 630*sqrt(a + b*x**2)*a**2*b**3*d**2*x**3 + 1152*sqrt(a
 + b*x**2)*a**2*b**2*c**3*x**2 + 1260*sqrt(a + b*x**2)*a**2*b**2*c**2*d*x*
*3 + 384*sqrt(a + b*x**2)*a**2*b**2*c*d**2*x**4 + 8064*sqrt(a + b*x**2)*a*
b**4*c**2*x**4 + 11760*sqrt(a + b*x**2)*a*b**4*c**2*x**3 + 13440*sqrt(a +
b*x**2)*a*b**4*c*d*x**5 + 18432*sqrt(a + b*x**2)*a*b**4*c*d*x**4 + 5760*sq
rt(a + b*x**2)*a*b**4*d**2*x**6 + 7560*sqrt(a + b*x**2)*a*b**4*d**2*x**5 +
 9216*sqrt(a + b*x**2)*a*b**3*c**3*x**4 + 15120*sqrt(a + b*x**2)*a*b**3*c*
*2*d*x**5 + 6400*sqrt(a + b*x**2)*a*b**3*c*d**2*x**6 + 6720*sqrt(a + b*x**
2)*b**5*c**2*x**5 + 11520*sqrt(a + b*x**2)*b**5*c*d*x**6 + 5040*sqrt(a + b
*x**2)*b**5*d**2*x**7 + 5760*sqrt(a + b*x**2)*b**4*c**3*x**6 + 10080*sqrt(
a + b*x**2)*b**4*c**2*d*x**7 + 4480*sqrt(a + b*x**2)*b**4*c*d**2*x**8 - 50
40*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4*b*c*d + 945...
```

3.64 $\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx$

Optimal result

Integrand size = 29, antiderivative size = 331

$$\begin{aligned}
\int (c+dx)^2 \left(a+bx^2\right)^{3/2} &(A+Bx \\
&+Cx^2) \, dx = \frac{a(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(cC+2Bd)))\,x\sqrt{a+bx^2}}{128b^2} \\
&+ \frac{(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(cC+2Bd)))\,x(a+bx^2)^{3/2}}{192b^2} \\
&+ \frac{(bc(Bc+2Ad)-ad(2cC+Bd))\,(a+bx^2)^{5/2}}{5b^2} \\
&- \frac{(3aCd^2-8b(c^2C+2Bcd+Ad^2))\,x(a+bx^2)^{5/2}}{48b^2} \\
&+ \frac{Cd^2x^3(a+bx^2)^{5/2}}{8b} + \frac{d(2cC+Bd)\,(a+bx^2)^{7/2}}{7b^2} \\
&+ \frac{a^2(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}
\end{aligned}$$

output
```
1/128*a*(8*A*b*(-a*d^2+6*b*c^2)+a*(3*a*C*d^2-8*b*c*(2*B*d+C*c)))*x*(b*x^2+
a)^(1/2)/b^2+1/192*(8*A*b*(-a*d^2+6*b*c^2)+a*(3*a*C*d^2-8*b*c*(2*B*d+C*c))
)*x*(b*x^2+a)^(3/2)/b^2+1/5*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*(b*x^2+a)^(5
/2)/b^2-1/48*(3*a*C*d^2-8*b*(A*d^2+2*B*c*d+C*c^2))*x*(b*x^2+a)^(5/2)/b^2+1
/8*C*d^2*x^3*(b*x^2+a)^(5/2)/b+1/7*d*(B*d+2*C*c)*(b*x^2+a)^(7/2)/b^2+1/128
*a^2*(8*A*b*(-a*d^2+6*b*c^2)+a*(3*a*C*d^2-8*b*c*(2*B*d+C*c)))*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.01

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) dx = \frac{\sqrt{b}\sqrt{a+bx^2}(-3a^3d(512cC+256Bd+105Cdx)+6a^2b(28Ad(32c+5dx)+8B(56c^2+35cd\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(-3*a^3*d*(512*c*C + 256*B*d + 105*C*d*x) + 6*a^2
*b*(28*A*d*(32*c + 5*d*x) + 8*B*(56*c^2 + 35*c*d*x + 8*d^2*x^2) + C*x*(140
*c^2 + 128*c*d*x + 35*d^2*x^2)) + 16*b^3*x^3*(14*A*(15*c^2 + 24*c*d*x + 10
*d^2*x^2) + x*(8*B*(21*c^2 + 35*c*d*x + 15*d^2*x^2) + 5*C*x*(28*c^2 + 48*c
*d*x + 21*d^2*x^2))) + 8*a*b^2*x*(14*A*(75*c^2 + 96*c*d*x + 35*d^2*x^2) +
x*(4*B*(168*c^2 + 245*c*d*x + 96*d^2*x^2) + C*x*(490*c^2 + 768*c*d*x + 315
*d^2*x^2)))) - 105*a^2*(8*A*b*(6*b*c^2 - a*d^2) + a*(3*a*C*d^2 - 8*b*c*(c*
C + 2*B*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(13440*b^(5/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {2185, 27, 687, 27, 676, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right) \, dx$$

$$\Big\downarrow \; 2185$$

$$\frac{\int d(c+dx)^2((8Ab-3aC)d-b(5cC-8Bd)x)\left(bx^2+a\right)^{3/2}dx}{8bd^2}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int (c+dx)^2((8Ab-3aC)d-b(5cC-8Bd)x)\left(bx^2+a\right)^{3/2}dx}{8bd}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 687$$

$$\frac{\frac{\int b(c+dx)\left(d(56Abc-11aCc-16aBd)+\left(7(8Ab-3aC)d^2-2bc(5cC-8Bd)\right)x\right)\left(bx^2+a\right)^{3/2}dx}{7b}-\frac{1}{7}\left(a+bx^2\right)^{5/2}(c+dx)^2(5cC-8Bd)}{8bd}+$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}\int (c+dx)\left(d(56Abc-11aCc-16aBd)+\left(7(8Ab-3aC)d^2-2bc(5cC-8Bd)\right)x\right)\left(bx^2+a\right)^{3/2}dx-\frac{1}{7}\left(a+bx\right.}{8bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{1}{7}\left(\frac{7d\left(8Ab\left(6bc^2-ad^2\right)+a\left(3aCd^2-8bc(2Bd+cC)\right)\right)\int\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Bd+2cC)+bc\left(-56Ad^2-8Bcd+5c^2C\right)\right)}{5b}+\frac{dx(a+}{}\right.}{8bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}\left(\frac{7d\left(8Ab\left(6bc^2-ad^2\right)+a\left(3aCd^2-8bc(2Bd+cC)\right)\right)\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)}{6b}-\frac{2\left(a+bx^2\right)^{5/2}\left(8ad^2(Bd+2cC)+bc\left(-56Ad^2-8Bcd+\right.\right.}{5b}\right.}{8bd}$$
$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \; 211$$

$$\frac{\frac{1}{7}\left(\frac{7d(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(2Bd+cC)))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(Bd+2cC)-}{5b}\right.}{8bd}$$

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{1}{7}\left(\frac{7d(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(2Bd+cC)))\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(I}{}\right.}{8}$$

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{7}\left(\frac{7d\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)(8Ab(6bc^2-ad^2)+a(3aCd^2-8bc(2Bd+cC)))}{6b}-\frac{2(a+bx^2)^{5/2}(8ad^2(Bc}{}\right.}{8l}$$

$$\frac{C(a+bx^2)^{5/2}(c+dx)^3}{8bd}$$

input

```
Int[(c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2),x]
```

output

```
(C*(c + d*x)^3*(a + b*x^2)^(5/2))/(8*b*d) + (-1/7*((5*c*C - 8*B*d)*(c + d*
x)^2*(a + b*x^2)^(5/2)) + ((-2*(8*a*d^2*(2*c*C + B*d) + b*c*(5*c^2*C - 8*B
*c*d - 56*A*d^2))*(a + b*x^2)^(5/2))/(5*b) + (d*(7*(8*A*b - 3*a*C)*d^2 - 2
*b*c*(5*c*C - 8*B*d))*x*(a + b*x^2)^(5/2))/(6*b) + (7*d*(8*A*b*(6*b*c^2 -
a*d^2) + a*(3*a*C*d^2 - 8*b*c*(c*C + 2*B*d)))*((x*(a + b*x^2)^(3/2))/4 + (
3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*S
qrt[b])))/4))/(6*b))/7)/(8*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.95

method	result
default	$A\,c^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{c(2Ad+Bc)\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}+d(Bd+2Cc)\left(\frac{x^{2}\left(b\,x^{2}+a\right.}{7b}\right.$
risch	$\frac{(1680C\,b^{3}d^{2}x^{7}+1920B\,b^{3}d^{2}x^{6}+3840C\,b^{3}cd\,x^{6}+2240A\,b^{3}d^{2}x^{5}+4480B\,b^{3}cd\,x^{5}+2520Ca\,b^{2}d^{2}x^{5}+2240C\,b^{3}c^{2}x^{5}+5376A\,b^{3}cd\,x^{4}+3}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
A*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+1/5*c*(2*A*d+B*c)*(b*x^2+a)^(5/2)/b+d*(B*d+2
*C*c)*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+(A*d^2+2*B*c*
d+C*c^2)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/
2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+C*d^2*(
1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*
(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b
*x^2+a)^(1/2))))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 833, normalized size of antiderivative = 2.52

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
[1/26880*(105*(16*B*a^3*b*c*d + 8*(C*a^3*b - 6*A*a^2*b^2)*c^2 - (3*C*a^4 -
 8*A*a^3*b)*d^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(1680*C*b^4*d^2*x^7 + 2688*B*a^2*b^2*c^2 - 768*B*a^3*b*d^2 + 1920*(2*C*
b^4*c*d + B*b^4*d^2)*x^6 + 280*(8*C*b^4*c^2 + 16*B*b^4*c*d + (9*C*a*b^3 +
8*A*b^4)*d^2)*x^5 + 384*(7*B*b^4*c^2 + 8*B*a*b^3*d^2 + 2*(8*C*a*b^3 + 7*A*
b^4)*c*d)*x^4 + 70*(112*B*a*b^3*c*d + 8*(7*C*a*b^3 + 6*A*b^4)*c^2 + (3*C*a
^2*b^2 + 56*A*a*b^3)*d^2)*x^3 - 768*(2*C*a^3*b - 7*A*a^2*b^2)*c*d + 384*(1
4*B*a*b^3*c^2 + B*a^2*b^2*d^2 + 2*(C*a^2*b^2 + 14*A*a*b^3)*c*d)*x^2 + 105*
(16*B*a^2*b^2*c*d + 8*(C*a^2*b^2 + 10*A*a*b^3)*c^2 - (3*C*a^3*b - 8*A*a^2*
b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1/13440*(105*(16*B*a^3*b*c*d + 8*(C*a^3
*b - 6*A*a^2*b^2)*c^2 - (3*C*a^4 - 8*A*a^3*b)*d^2)*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) + (1680*C*b^4*d^2*x^7 + 2688*B*a^2*b^2*c^2 - 768*B*a^
3*b*d^2 + 1920*(2*C*b^4*c*d + B*b^4*d^2)*x^6 + 280*(8*C*b^4*c^2 + 16*B*b^4
*c*d + (9*C*a*b^3 + 8*A*b^4)*d^2)*x^5 + 384*(7*B*b^4*c^2 + 8*B*a*b^3*d^2 +
 2*(8*C*a*b^3 + 7*A*b^4)*c*d)*x^4 + 70*(112*B*a*b^3*c*d + 8*(7*C*a*b^3 + 6
*A*b^4)*c^2 + (3*C*a^2*b^2 + 56*A*a*b^3)*d^2)*x^3 - 768*(2*C*a^3*b - 7*A*a
^2*b^2)*c*d + 384*(14*B*a*b^3*c^2 + B*a^2*b^2*d^2 + 2*(C*a^2*b^2 + 14*A*a*
b^3)*c*d)*x^2 + 105*(16*B*a^2*b^2*c*d + 8*(C*a^2*b^2 + 10*A*a*b^3)*c^2 - (
3*C*a^3*b - 8*A*a^2*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(318) = 636$.

Time = 0.60 (sec) , antiderivative size = 933, normalized size of antiderivative = 2.82

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*b*d**2*x**7/8 + x**6*(B*b**2*d**2 + 2*C*b**
2*c*d)/(7*b) + x**5*(A*b**2*d**2 + 2*B*b**2*c*d + 9*C*a*b*d**2/8 + C*b**2*
c**2)/(6*b) + x**4*(2*A*b**2*c*d + 2*B*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*
d - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(7*b))/(5*b) + x**3*(2*A*a*b*d**2 + A
*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*a*b*c**2 - 5*a*(A*b**2*d**2 +
 2*B*b**2*c*d + 9*C*a*b*d**2/8 + C*b**2*c**2)/(6*b))/(4*b) + x**2*(4*A*a*b
*c*d + B*a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 4*a*(2*A*b**2*c*d + 2*B
*a*b*d**2 + B*b**2*c**2 + 4*C*a*b*c*d - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(
7*b))/(5*b))/(3*b) + x*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*d + C*a**2
*c**2 - 3*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d**2 + 2*C*
a*b*c**2 - 5*a*(A*b**2*d**2 + 2*B*b**2*c*d + 9*C*a*b*d**2/8 + C*b**2*c**2)
/(6*b))/(4*b))/(2*b) + (2*A*a**2*c*d + B*a**2*c**2 - 2*a*(4*A*a*b*c*d + B*
a**2*d**2 + 2*B*a*b*c**2 + 2*C*a**2*c*d - 4*a*(2*A*b**2*c*d + 2*B*a*b*d**2
 + B*b**2*c**2 + 4*C*a*b*c*d - 6*a*(B*b**2*d**2 + 2*C*b**2*c*d)/(7*b))/(5*
b))/(3*b))/b) + (A*a**2*c**2 - a*(A*a**2*d**2 + 2*A*a*b*c**2 + 2*B*a**2*c*
d + C*a**2*c**2 - 3*a*(2*A*a*b*d**2 + A*b**2*c**2 + 4*B*a*b*c*d + C*a**2*d
**2 + 2*C*a*b*c**2 - 5*a*(A*b**2*d**2 + 2*B*b**2*c*d + 9*C*a*b*d**2/8 + C*
b**2*c**2)/(6*b))/(4*b))/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2)
+ 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), (a
**(3/2)*(A*c**2*x + C*d**2*x**5/5 + x**4*(B*d**2 + 2*C*c*d)/4 + x**3*(A...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.15

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)\,dx = \frac{(bx^2+a)^{\frac{5}{2}}Cd^2x^3}{8\,b} + \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ac^2x + \frac{3}{8}\sqrt{bx^2+a}Aac^2x - \frac{(bx^2+a)^{\frac{5}{2}}Cad^2x}{16\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}Ca^2d^2x}{64\,b^2} + \frac{3\sqrt{bx^2+a}Ca^3d^2x}{128\,b^2} + \frac{3\,Aa^2c^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3\,Ca^4d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{5}{2}}Bc^2}{5\,b} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Acd}{5\,b} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}x^2}{7\,b} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}x}{6\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}ax}{24\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}a^2x}{16\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)a^3\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - \frac{2\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*C*d^2*x^3/b + 1/4*(b*x^2 + a)^(3/2)*A*c^2*x + 3/8*sq
rt(b*x^2 + a)*A*a*c^2*x - 1/16*(b*x^2 + a)^(5/2)*C*a*d^2*x/b^2 + 1/64*(b*x
^2 + a)^(3/2)*C*a^2*d^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*C*a^3*d^2*x/b^2 + 3/
8*A*a^2*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/128*C*a^4*d^2*arcsinh(b*x/s
qrt(a*b))/b^(5/2) + 1/5*(b*x^2 + a)^(5/2)*B*c^2/b + 2/5*(b*x^2 + a)^(5/2)*
A*c*d/b + 1/7*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*x^2/b + 1/6*(C*c^2 + 2*B
*c*d + A*d^2)*(b*x^2 + a)^(5/2)*x/b - 1/24*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^
2 + a)^(3/2)*a*x/b - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a^2*x/
b - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/
35*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.33

$$\int (c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)\, dx = \frac{1}{13440} \sqrt{bx^2+a}\Bigg(\Bigg(2\Bigg(\Bigg(4\Bigg(5\Bigg(6\left(7\,Cbd^2x + \frac{8\,(2\,Cb^7cd+Bb^7d^2)}{b^6}\right)x + \frac{7\,(8\,Cb^7c^2+16\,Bb^7c\cdots}{\cdots}$$
$$+ \frac{(8\,Ca^3bc^2 - 48\,Aa^2b^2c^2 + 16\,Ba^3bcd - 3\,Ca^4d^2 + 8\,Aa^3bd^2)\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13440*sqrt(b*x^2 + a)*((2*((4*(5*(6*(7*C*b*d^2*x + 8*(2*C*b^7*c*d + B*b^
7*d^2)/b^6)*x + 7*(8*C*b^7*c^2 + 16*B*b^7*c*d + 9*C*a*b^6*d^2 + 8*A*b^7*d^
2)/b^6)*x + 48*(7*B*b^7*c^2 + 16*C*a*b^6*c*d + 14*A*b^7*c*d + 8*B*a*b^6*d^
2)/b^6)*x + 35*(56*C*a*b^6*c^2 + 48*A*b^7*c^2 + 112*B*a*b^6*c*d + 3*C*a^2*
b^5*d^2 + 56*A*a*b^6*d^2)/b^6)*x + 192*(14*B*a*b^6*c^2 + 2*C*a^2*b^5*c*d +
 28*A*a*b^6*c*d + B*a^2*b^5*d^2)/b^6)*x + 105*(8*C*a^2*b^5*c^2 + 80*A*a*b^
6*c^2 + 16*B*a^2*b^5*c*d - 3*C*a^3*b^4*d^2 + 8*A*a^2*b^5*d^2)/b^6)*x + 384
*(7*B*a^2*b^5*c^2 - 4*C*a^3*b^4*c*d + 14*A*a^2*b^5*c*d - 2*B*a^3*b^4*d^2)/
b^6) + 1/128*(8*C*a^3*b*c^2 - 48*A*a^2*b^2*c^2 + 16*B*a^3*b*c*d - 3*C*a^4*
d^2 + 8*A*a^3*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)\, dx = \int \left(b\,x^2+a\right)^{3/2} (c+d\,x)^2 \left(C\,x^2+B\,x+A\right) dx$$

input
```
int((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2),x)
```

output
```
int((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int (c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right) \, dx = \int (dx+c)^2 \left(b\,x^2+a\right)^{\frac{3}{2}} \left(C\,x^2+Bx+A\right) dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A),x)
```

3.65 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 305

$$\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x}\,dx = aAc^2\sqrt{a+bx^2} + \frac{a(6bc(Bc+2Ad)-ad(2cC+Bd))x\sqrt{a+bx^2}}{16b} + \frac{1}{3}Ac^2(a+bx^2)^{3/2} + \frac{(6bc(Bc+2Ad)-ad(2cC+Bd))x(a+bx^2)^{3/2}}{24b} - \frac{(aCd^2-b(c^2C+2Bcd+Ad^2))(a+bx^2)^{5/2}}{5b^2} + \frac{d(2cC+Bd)x(a+bx^2)^{5/2}}{6b} + \frac{Cd^2(a+bx^2)^{7/2}}{7b^2} + \frac{a^2(6bc(Bc+2Ad)-ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}} - a^{3/2}Ac^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
a*A*c^2*(b*x^2+a)^(1/2)+1/16*a*(6*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x*(b*x^
2+a)^(1/2)/b+1/3*A*c^2*(b*x^2+a)^(3/2)+1/24*(6*b*c*(2*A*d+B*c)-a*d*(B*d+2*
C*c))*x*(b*x^2+a)^(3/2)/b-1/5*(a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^
(5/2)/b^2+1/6*d*(B*d+2*C*c)*x*(b*x^2+a)^(5/2)/b+1/7*C*d^2*(b*x^2+a)^(7/2)/
b^2+1/16*a^2*(6*b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(3/2)-a^(3/2)*A*c^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} \, dx = \frac{\sqrt{a+bx^2}(-96a^3Cd^2+3a^2b(112c^2C+14cd(16B+5Cx)$$

$$+2a^{3/2}Ac^2\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$+\frac{a^2(-6bc(Bc+2Ad)+ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x,x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*C*d^2 + 3*a^2*b*(112*c^2*C + 14*c*d*(16*B + 5*C*
x) + d^2*(112*A + 35*B*x + 16*C*x^2)) + 4*b^3*x^2*(14*A*(10*c^2 + 15*c*d*x
 + 6*d^2*x^2) + x*(7*B*(15*c^2 + 24*c*d*x + 10*d^2*x^2) + 4*C*x*(21*c^2 +
35*c*d*x + 15*d^2*x^2))) + 2*a*b^2*(14*A*(80*c^2 + 75*c*d*x + 24*d^2*x^2)
+ x*(7*B*(75*c^2 + 96*c*d*x + 35*d^2*x^2) + 2*C*x*(168*c^2 + 245*c*d*x + 9
6*d^2*x^2)))))/(1680*b^2) + 2*a^(3/2)*A*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a +
b*x^2])/Sqrt[a]] + (a^2*(-6*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2340, 2340, 2340, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2} (c+dx)^2 (A+Bx+Cx^2)}{x} \, dx$$

$$\Big\downarrow \; 2340$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(7bd(2cC+Bd)x^3-\left(2aCd^2-7b\left(Cc^2+2Bdc+Ad^2\right)\right)x^2+7bc(Bc+2Ad)x+7Abc^2\right)}{x}dx}{7b}+\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(42Ab^2c^2-6b\left(2aCd^2-7b\left(Cc^2+2Bdc+Ad^2\right)\right)x^2+7b(6bc(Bc+2Ad)-ad(2cC+Bd))x\right)}{x}dx}{6b}+\frac{7}{6}dx\left(a+bx^2\right)^{5/2}(Bd+2cC)}{7b}+\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 2340

$$\frac{\frac{\frac{\int \frac{35b^2\left(6Abc^2+(6bc(Bc+2Ad)-ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x}dx}{5b}-\frac{6}{5}\left(a+bx^2\right)^{5/2}\left(2aCd^2-7b\left(Ad^2+2Bcd+c^2C\right)\right)}{6b}+\frac{7}{6}dx\left(a+bx^2\right)^{5/2}(Bd+2cC}{7b}$$
$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{\frac{7b\int \frac{\left(6Abc^2+(6bc(Bc+2Ad)-ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x}dx-\frac{6}{5}\left(a+bx^2\right)^{5/2}\left(2aCd^2-7b\left(Ad^2+2Bcd+c^2C\right)\right)}{6b}+\frac{7}{6}dx\left(a+bx^2\right)^{5/2}(Bd+2cC}{7b}$$
$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 535

$$\frac{\frac{7b\left(\frac{1}{4}a\int \frac{3\left(8Abc^2+(6bc(Bc+2Ad)-ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-\frac{6}{5}\left(a+bx^2\right)^{5/2}\left(2aCd^2-\right.}{6b}}{7b}$$
$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$\Big\downarrow$ 27

$$\frac{\frac{7b\left(\frac{3}{4}a\int \frac{\left(8Abc^2+(6bc(Bc+2Ad)-ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+8Abc^2\right)\right)-\frac{6}{5}\left(a+bx^2\right)^{5/2}\left(2aCd^2-7\right.}{6b}}{7b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 535$$

$$\frac{\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{16Abc^2+(6bc(Bc+2Ad)-ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16Abc^2\right)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}\left(x(6bc(2Ad+Bc)-\right.}{6b}}{7b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 538$$

$$\frac{\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left((6bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{\sqrt{bx^2+a}}dx+16Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16Abc^2\right)\right)+\frac{1}{4}\right.}{6b}}{7}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 224$$

$$\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left((6bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+16Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16A\right.\right.\right.}{6b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 219$$

$$\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left(16Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16Abc^2\right)\right.\right.}{6b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \; 243$$

$$\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left(8Abc^2\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16Abc^2\right.\right.\right.}{6b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{73}$$

$$\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left(16Ac^2\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+16A\right.\right.\right.}{6b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{221}$$

$$\frac{7b\left(\frac{3}{4}a\left(\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(6bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}-\frac{16Abc^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x(6bc(2Ad+Bc)-ad(Bd+2cC))+1\right.\right.\right.}{6b}$$

$$\frac{Cd^2x^2\left(a+bx^2\right)^{5/2}}{7b}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x,x]
```

output
```
(C*d^2*x^2*(a + b*x^2)^(5/2))/(7*b) + ((7*d*(2*c*C + B*d)*x*(a + b*x^2)^(5
/2))/6 + ((-6*(2*a*C*d^2 - 7*b*(c^2*C + 2*B*c*d + A*d^2))*(a + b*x^2)^(5/2
))/5 + 7*b*(((8*A*b*c^2 + (6*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*x)*(a
+ b*x^2)^(3/2))/4 + (3*a*(((16*A*b*c^2 + (6*b*c*(B*c + 2*A*d) - a*d*(2*c*C
 + B*d))*x)*Sqrt[a + b*x^2])/2 + (a*(((6*b*c*(B*c + 2*A*d) - a*d*(2*c*C +
B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (16*A*b*c^2*ArcTanh[
Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4))/(6*b))/(7*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.37

method	result
default	$B\,c^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\,c^{2}\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a}{}\right.\right.\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
B*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+A*c^2*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2
)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+1/5*A*d^2*(b*x^2+a)^(5/2
)/b+B*d^2*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1
/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/5*C*
c^2*(b*x^2+a)^(5/2)/b+C*d^2*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a
)^(5/2))+2*A*c*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a
/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+2/5*B*c*d*(b*x^2+a)^(5/2)/b+2*C*c
*d*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b
*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 1519, normalized size of antiderivative = 4.98

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
[1/3360*(1680*A*a^(3/2)*b^2*c^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 105*(6*B*a^2*b*c^2 - B*a^3*d^2 - 2*(C*a^3 - 6*A*a^2*b)*c*d)*sq
rt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(240*C*b^3*d^2*x
^6 + 672*B*a^2*b*c*d + 280*(2*C*b^3*c*d + B*b^3*d^2)*x^5 + 48*(7*C*b^3*c^2
 + 14*B*b^3*c*d + (8*C*a*b^2 + 7*A*b^3)*d^2)*x^4 + 70*(6*B*b^3*c^2 + 7*B*a
*b^2*d^2 + 2*(7*C*a*b^2 + 6*A*b^3)*c*d)*x^3 + 112*(3*C*a^2*b + 20*A*a*b^2)
*c^2 - 48*(2*C*a^3 - 7*A*a^2*b)*d^2 + 16*(84*B*a*b^2*c*d + 7*(6*C*a*b^2 +
5*A*b^3)*c^2 + 3*(C*a^2*b + 14*A*a*b^2)*d^2)*x^2 + 105*(10*B*a*b^2*c^2 + B
*a^2*b*d^2 + 2*(C*a^2*b + 10*A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/b^2, 1/1680
*(840*A*a^(3/2)*b^2*c^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 105*(6*B*a^2*b*c^2 - B*a^3*d^2 - 2*(C*a^3 - 6*A*a^2*b)*c*d)*sqrt(-b)*a
rctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*C*b^3*d^2*x^6 + 672*B*a^2*b*c*d +
 280*(2*C*b^3*c*d + B*b^3*d^2)*x^5 + 48*(7*C*b^3*c^2 + 14*B*b^3*c*d + (8*C
*a*b^2 + 7*A*b^3)*d^2)*x^4 + 70*(6*B*b^3*c^2 + 7*B*a*b^2*d^2 + 2*(7*C*a*b^
2 + 6*A*b^3)*c*d)*x^3 + 112*(3*C*a^2*b + 20*A*a*b^2)*c^2 - 48*(2*C*a^3 - 7
*A*a^2*b)*d^2 + 16*(84*B*a*b^2*c*d + 7*(6*C*a*b^2 + 5*A*b^3)*c^2 + 3*(C*a^
2*b + 14*A*a*b^2)*d^2)*x^2 + 105*(10*B*a*b^2*c^2 + B*a^2*b*d^2 + 2*(C*a^2*
b + 10*A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/b^2, 1/3360*(3360*A*sqrt(-a)*a*b^
2*c^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 105*(6*B*a^2*b*c^2 - B*a^3*d^2
- 2*(C*a^3 - 6*A*a^2*b)*c*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s...
```

Sympy [A] (verification not implemented)

Time = 18.92 (sec) , antiderivative size = 1442, normalized size of antiderivative = 4.73

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x,x)
```

output
```
-A*a**(3/2)*c**2*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2*c**2/(sqrt(b)*x*sqrt(
a/(b*x**2) + 1)) + A*a*sqrt(b)*c**2*x/sqrt(a/(b*x**2) + 1) + 2*A*a*c*d*Pie
cewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a
, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)),
 (sqrt(a)*x, True)) + A*a*d**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*
sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + A*b*c**2*Piecewis
e((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)
*x**2/2, True)) + 2*A*b*c*d*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt
(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(
8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (s
qrt(a)*x**3/3, True)) + A*b*d**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b
**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0))
, (sqrt(a)*x**4/4, True)) + B*a*c**2*Piecewise((a*Piecewise((log(2*sqrt(b)
*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tru
e))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*B*a*c*d*Pi
ecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (s
qrt(a)*x**2/2, True)) + B*a*d**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)
*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), Tru
e))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)
), (sqrt(a)*x**3/3, True)) + B*b*c**2*Piecewise((-a**2*Piecewise((log(2...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} \, dx = \frac{(bx^2+a)^{\frac{5}{2}} C d^2 x^2}{7\, b} - A a^{\frac{3}{2}} c^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3} \left(bx^2+a\right)^{\frac{3}{2}} A c^2 + \sqrt{bx^2+a} A a c^2 + \frac{(bx^2+a)^{\frac{5}{2}} C c^2}{5\, b} + \frac{2 \left(bx^2+a\right)^{\frac{5}{2}} B c d}{5\, b} - \frac{2 \left(bx^2+a\right)^{\frac{5}{2}} C a d^2}{35\, b^2} + \frac{(bx^2+a)^{\frac{5}{2}} A d^2}{5\, b} + \frac{1}{4} \left(Bc^2+2\, Acd\right) \left(bx^2+a\right)^{\frac{3}{2}} x + \frac{3}{8} \left(Bc^2+2\, Acd\right) \sqrt{bx^2+a}\, a x + \frac{(2\, Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}} x}{6\, b} - \frac{(2\, Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}} a x}{24\, b} - \frac{(2\, Ccd+Bd^2)\sqrt{bx^2+a}\, a^2 x}{16\, b} - \frac{(2\, Ccd+Bd^2) a^3 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\, b^{\frac{3}{2}}} + \frac{3\, (Bc^2+2\, Acd) a^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\, \sqrt{b}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*C*d^2*x^2/b - A*a^(3/2)*c^2*arcsinh(a/(sqrt(a*b)*abs
(x))) + 1/3*(b*x^2 + a)^(3/2)*A*c^2 + sqrt(b*x^2 + a)*A*a*c^2 + 1/5*(b*x^2
 + a)^(5/2)*C*c^2/b + 2/5*(b*x^2 + a)^(5/2)*B*c*d/b - 2/35*(b*x^2 + a)^(5/
2)*C*a*d^2/b^2 + 1/5*(b*x^2 + a)^(5/2)*A*d^2/b + 1/4*(B*c^2 + 2*A*c*d)*(b*
x^2 + a)^(3/2)*x + 3/8*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*a*x + 1/6*(2*C*c*
d + B*d^2)*(b*x^2 + a)^(5/2)*x/b - 1/24*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2
)*a*x/b - 1/16*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a^2*x/b - 1/16*(2*C*c*d +
 B*d^2)*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*(B*c^2 + 2*A*c*d)*a^2*arc
sinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x} dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 679, normalized size of antiderivative = 2.23

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x} dx = \frac{672\sqrt{b\,x^2+a}\,a^2b^2cd + 672\sqrt{b\,x^2+a}\,a^2b^2d^2x^2 + 105\sqrt{b\,x^2}}{}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x,x)
```

output

```
(336*sqrt(a + b*x**2)*a**3*b*d**2 - 96*sqrt(a + b*x**2)*a**3*c*d**2 + 2240
*sqrt(a + b*x**2)*a**2*b**2*c**2 + 2100*sqrt(a + b*x**2)*a**2*b**2*c*d*x +
 672*sqrt(a + b*x**2)*a**2*b**2*c*d + 672*sqrt(a + b*x**2)*a**2*b**2*d**2*
x**2 + 105*sqrt(a + b*x**2)*a**2*b**2*d**2*x + 336*sqrt(a + b*x**2)*a**2*b
*c**3 + 210*sqrt(a + b*x**2)*a**2*b*c**2*d*x + 48*sqrt(a + b*x**2)*a**2*b*
c*d**2*x**2 + 560*sqrt(a + b*x**2)*a*b**3*c**2*x**2 + 1050*sqrt(a + b*x**2
)*a*b**3*c**2*x + 840*sqrt(a + b*x**2)*a*b**3*c*d*x**3 + 1344*sqrt(a + b*x
**2)*a*b**3*c*d*x**2 + 336*sqrt(a + b*x**2)*a*b**3*d**2*x**4 + 490*sqrt(a
+ b*x**2)*a*b**3*d**2*x**3 + 672*sqrt(a + b*x**2)*a*b**2*c**3*x**2 + 980*s
qrt(a + b*x**2)*a*b**2*c**2*d*x**3 + 384*sqrt(a + b*x**2)*a*b**2*c*d**2*x*
*4 + 420*sqrt(a + b*x**2)*b**4*c**2*x**3 + 672*sqrt(a + b*x**2)*b**4*c*d*x
**4 + 280*sqrt(a + b*x**2)*b**4*d**2*x**5 + 336*sqrt(a + b*x**2)*b**3*c**3
*x**4 + 560*sqrt(a + b*x**2)*b**3*c**2*d*x**5 + 240*sqrt(a + b*x**2)*b**3*
c*d**2*x**6 + 1680*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a))*a**2*b**2*c**2 - 1680*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqr
t(b)*x)/sqrt(a))*a**2*b**2*c**2 + 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqr
t(b)*x)/sqrt(a))*a**3*b*c*d - 105*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**3*b*d**2 - 210*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/s
qrt(a))*a**3*c**2*d + 630*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*b**2*c**2)/(1680*b**2)
```

3.66 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 341

$$\begin{aligned}\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx &= ac(Bc+2Ad)\sqrt{a+bx^2}\\
&+\frac{\left(6Ab(4bc^2+ad^2)-a(aCd^2-6bc(cC+2Bd))\right)x\sqrt{a+bx^2}}{16b}\\
&+\frac{1}{3}c(Bc+2Ad)\left(a+bx^2\right)^{3/2}\\
&+\frac{\left(6Ab(4bc^2+ad^2)-a(aCd^2-6bc(cC+2Bd))\right)x\left(a+bx^2\right)^{3/2}}{24ab}\\
&+\frac{d(2cC+Bd)\left(a+bx^2\right)^{5/2}}{5b}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}+\frac{Cd^2x\left(a+bx^2\right)^{5/2}}{6b}\\
&+\frac{a\left(6Ab(4bc^2+ad^2)-a(aCd^2-6bc(cC+2Bd))\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}\\
&-a^{3/2}c(Bc+2Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

output
```
a*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)+1/16*(6*A*b*(a*d^2+4*b*c^2)-a*(a*C*d^2-6*b
*c*(2*B*d+C*c)))*x*(b*x^2+a)^(1/2)/b+1/3*c*(2*A*d+B*c)*(b*x^2+a)^(3/2)+1/2
4*(6*A*b*(a*d^2+4*b*c^2)-a*(a*C*d^2-6*b*c*(2*B*d+C*c)))*x*(b*x^2+a)^(3/2)/
a/b+1/5*d*(B*d+2*C*c)*(b*x^2+a)^(5/2)/b-A*c^2*(b*x^2+a)^(5/2)/a/x+1/6*C*d^
2*x*(b*x^2+a)^(5/2)/b+1/16*a*(6*A*b*(a*d^2+4*b*c^2)-a*(a*C*d^2-6*b*c*(2*B*
d+C*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-a^(3/2)*c*(2*A*d+B*c)*
arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 3.21 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^2}\,dx = \frac{\sqrt{a+bx^2}(3a^2dx(32cC+16Bd+5Cdx)+4b^2x^2(5A(6c^2+$$

$$-\frac{a(-6Ab(4bc^2+ad^2)+a(aCd^2-6bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

$$-a^{3/2}c(Bc+2Ad)\log(x)+a^{3/2}c(Bc+2Ad)\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(3*a^2*d*x*(32*c*C + 16*B*d + 5*C*d*x) + 4*b^2*x^2*(5*A*(
6*c^2 + 8*c*d*x + 3*d^2*x^2) + x*(2*B*(10*c^2 + 15*c*d*x + 6*d^2*x^2) + C*
x*(15*c^2 + 24*c*d*x + 10*d^2*x^2))) + 2*a*b*(A*(-120*c^2 + 320*c*d*x + 75
*d^2*x^2) + x*(2*B*(80*c^2 + 75*c*d*x + 24*d^2*x^2) + C*x*(75*c^2 + 96*c*d
*x + 35*d^2*x^2)))))/(240*b*x) - (a*(-6*A*b*(4*b*c^2 + a*d^2) + a*(a*C*d^2
 - 6*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]
)/(8*b^(3/2)) - a^(3/2)*c*(B*c + 2*A*d)*Log[x] + a^(3/2)*c*(B*c + 2*A*d)*L
og[-Sqrt[a] + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2338, 25, 2340, 2340, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right)}{x^2} \, dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(aCd^2x^3+ad(2cC+Bd)x^2+\left(ac(cC+2Bd)+A\left(4bc^2+ad^2\right)\right)x+ac(Bc+2Ad)\right)}{x}dx}{a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(aCd^2x^3+ad(2cC+Bd)x^2+\left(ac(cC+2Bd)+A\left(4bc^2+ad^2\right)\right)x+ac(Bc+2Ad)\right)}{x}dx}{a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(6abd(2cC+Bd)x^2+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x+6abc(Bc+2Ad)\right)}{x}dx}{6b}+\frac{aCd^2x\left(a+bx^2\right)^{5/2}}{6b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\frac{\int \frac{5b\left(6abc(Bc+2Ad)+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x}dx}{5b}+\frac{6}{5}ad\left(a+bx^2\right)^{5/2}(Bd+2cC)}{6b}+\frac{aCd^2x\left(a+bx^2\right)^{5/2}}{6b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(6abc(Bc+2Ad)+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x}dx+\frac{6}{5}ad\left(a+bx^2\right)^{5/2}(Bd+2cC)}{6b}+\frac{aCd^2x\left(a+bx^2\right)^{5/2}}{6b}}{a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 535

$$\frac{\frac{\frac{1}{4}a\int \frac{3\left(8abc(Bc+2Ad)+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)+8ab\right.}{6b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{3}{4}a\int \frac{\left(8abc(Bc+2Ad)+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}\left(a+bx^2\right)^{3/2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)+8abc\right.}{6b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 535

$$\frac{\frac{\frac{3}{4}a\left(\frac{1}{2}a\int \frac{16abc(Bc+2Ad)+\left(6Ab\left(4bc^2+ad^2\right)-a\left(aCd^2-6bc(cC+2Bd)\right)\right)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)+16abc(2Ad+\right.\right.}{6b}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 538

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx+16abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$\Big\downarrow$ 224

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+16abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16abc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(a\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(8abc(2Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-a\left(\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(16ac(2Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4bc^2\right)-\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{3}{4}a\left(\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(6Ab\left(ad^2+4bc^2\right)-a\left(aCd^2-6bc(2Bd+cC)\right)\right)}{\sqrt{b}}-16\sqrt{a}bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2Ad+Bc)\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(6Ab\left(ad^2+4\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{ax}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^2,x]
```

output
```
-((A*c^2*(a + b*x^2)^(5/2))/(a*x)) + ((a*C*d^2*x*(a + b*x^2)^(5/2))/(6*b)
+ (((8*a*b*c*(B*c + 2*A*d) + (6*A*b*(4*b*c^2 + a*d^2) - a*(a*C*d^2 - 6*b*c
*(c*C + 2*B*d)))*x)*(a + b*x^2)^(3/2))/4 + (6*a*d*(2*c*C + B*d)*(a + b*x^2
)^(5/2))/5 + (3*a*(((16*a*b*c*(B*c + 2*A*d) + (6*A*b*(4*b*c^2 + a*d^2) - a
*(a*C*d^2 - 6*b*c*(c*C + 2*B*d)))*x)*Sqrt[a + b*x^2])/2 + (a*(((6*A*b*(4*b
*c^2 + a*d^2) - a*(a*C*d^2 - 6*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/Sqrt[b] - 16*Sqrt[a]*b*c*(B*c + 2*A*d)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]]))/2))/4)/(6*b))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.25

method	result
default	$A\,d^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+C\,c^{2}\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+\right.}}{2\sqrt{b}}\right.}{4}\right.$
risch	$\frac{3A\,c^{2}a\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2}+\frac{3C\,a^{2}c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}+\frac{3A\,a^{2}d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}-\frac{a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)C\,d^{2}}{16b^{\frac{3}{2}}}$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
A*d^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+C*c^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b
*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+A*c^2*(-1/a/x*
(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+
1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+1/5*B*d^2*(b*x^2+a)^(5/2)/b
+C*d^2*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+c*(2*A*d+
B*c)*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)))+2*B*c*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(
1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))+2/5*C*c*d*(b*x^2+a)^(5/
2)/b
```

Fricas [A] (verification not implemented)

Time = 4.52 (sec) , antiderivative size = 1479, normalized size of antiderivative = 4.34

$$\int\frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="fricas
")
```

output
```
[1/480*(15*(12*B*a^2*b*c*d + 6*(C*a^2*b + 4*A*a*b^2)*c^2 - (C*a^3 - 6*A*a^
2*b)*d^2)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 240*
(B*a*b^2*c^2 + 2*A*a*b^2*c*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(40*C*b^3*d^2*x^6 - 240*A*a*b^2*c^2 + 48*(2*C*b^3*c*
d + B*b^3*d^2)*x^5 + 10*(6*C*b^3*c^2 + 12*B*b^3*c*d + (7*C*a*b^2 + 6*A*b^3
)*d^2)*x^4 + 16*(5*B*b^3*c^2 + 6*B*a*b^2*d^2 + 2*(6*C*a*b^2 + 5*A*b^3)*c*d
)*x^3 + 15*(20*B*a*b^2*c*d + 2*(5*C*a*b^2 + 4*A*b^3)*c^2 + (C*a^2*b + 10*A
*a*b^2)*d^2)*x^2 + 16*(20*B*a*b^2*c^2 + 3*B*a^2*b*d^2 + 2*(3*C*a^2*b + 20*
A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(b^2*x), -1/240*(15*(12*B*a^2*b*c*d + 6*
(C*a^2*b + 4*A*a*b^2)*c^2 - (C*a^3 - 6*A*a^2*b)*d^2)*sqrt(-b)*x*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - 120*(B*a*b^2*c^2 + 2*A*a*b^2*c*d)*sqrt(a)*x*log
(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (40*C*b^3*d^2*x^6 - 240
*A*a*b^2*c^2 + 48*(2*C*b^3*c*d + B*b^3*d^2)*x^5 + 10*(6*C*b^3*c^2 + 12*B*b
^3*c*d + (7*C*a*b^2 + 6*A*b^3)*d^2)*x^4 + 16*(5*B*b^3*c^2 + 6*B*a*b^2*d^2
+ 2*(6*C*a*b^2 + 5*A*b^3)*c*d)*x^3 + 15*(20*B*a*b^2*c*d + 2*(5*C*a*b^2 + 4
*A*b^3)*c^2 + (C*a^2*b + 10*A*a*b^2)*d^2)*x^2 + 16*(20*B*a*b^2*c^2 + 3*B*a
^2*b*d^2 + 2*(3*C*a^2*b + 20*A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(b^2*x), 1/
480*(480*(B*a*b^2*c^2 + 2*A*a*b^2*c*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*s
qrt(-a)/a) + 15*(12*B*a^2*b*c*d + 6*(C*a^2*b + 4*A*a*b^2)*c^2 - (C*a^3 - 6
*A*a^2*b)*d^2)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a...
```

Sympy [A] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 1382, normalized size of antiderivative = 4.05

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^2} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**(3/2)*c**2/(x*sqrt(1 + b*x**2/a)) - 2*A*a**(3/2)*c*d*asinh(sqrt(a)/(
sqrt(b)*x)) - A*sqrt(a)*b*c**2*x/sqrt(1 + b*x**2/a) + 2*A*a**2*c*d/(sqrt(b
)*x*sqrt(a/(b*x**2) + 1)) + A*a*sqrt(b)*c**2*asinh(sqrt(b)*x/sqrt(a)) + 2*
A*a*sqrt(b)*c*d*x/sqrt(a/(b*x**2) + 1) + A*a*d**2*Piecewise((a*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt
(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
 A*b*c**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/
2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*A*b*c*d*Piecewise((a*sqrt(a + b*x**2)
/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + A*b
*d**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*
x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
- B*a**(3/2)*c**2*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2*c**2/(sqrt(b)*x*sqrt
(a/(b*x**2) + 1)) + B*a*sqrt(b)*c**2*x/sqrt(a/(b*x**2) + 1) + 2*B*a*c*d*Pi
ecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(
a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0))
, (sqrt(a)*x, True)) + B*a*d**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2
*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*b*c**2*Piecewi
se((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqr...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^2}\,dx = \frac{3}{2}\sqrt{bx^2+a}Abc^2x + \frac{(bx^2+a)^{\frac{5}{2}}Cd^2x}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Cad^2x}{24\,b} - \frac{\sqrt{bx^2+a}Ca^2d^2x}{16\,b} + \frac{3}{2}\,Aa\sqrt{b}c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{Ca^3d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Ccd}{5\,b} + \frac{(bx^2+a)^{\frac{5}{2}}Bd^2}{5\,b} - \frac{(bx^2+a)^{\frac{3}{2}}Ac^2}{x} + \frac{1}{4}\left(Cc^2+2\,Bcd+Ad^2\right)\left(bx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\left(Cc^2+2\,Bcd+Ad^2\right)\sqrt{bx^2+a}ax + \frac{3\,(Cc^2+2\,Bcd+Ad^2)a^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \left(Bc^2+2\,Acd\right)a^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(Bc^2+2\,Acd\right)\left(bx^2+a\right)^{\frac{3}{2}} + \left(Bc^2+2\,Acd\right)\sqrt{bx^2+a}a$$

input

```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="maxima
")
```

output

```
3/2*sqrt(b*x^2 + a)*A*b*c^2*x + 1/6*(b*x^2 + a)^(5/2)*C*d^2*x/b - 1/24*(b*
x^2 + a)^(3/2)*C*a*d^2*x/b - 1/16*sqrt(b*x^2 + a)*C*a^2*d^2*x/b + 3/2*A*a*
sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b)) - 1/16*C*a^3*d^2*arcsinh(b*x/sqrt(a*b))
/b^(3/2) + 2/5*(b*x^2 + a)^(5/2)*C*c*d/b + 1/5*(b*x^2 + a)^(5/2)*B*d^2/b -
 (b*x^2 + a)^(3/2)*A*c^2/x + 1/4*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/
2)*x + 3/8*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a*x + 3/8*(C*c^2 + 2*
B*c*d + A*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - (B*c^2 + 2*A*c*d)*a^(3
/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2
) + (B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*a
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^2} dx = \frac{2 Aa^2\sqrt{b}c^2}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$
$$+\frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(5Cbd^2x+\frac{6(2Cb^5cd+Bb^5d^2)}{b^4}\right)x+\frac{5(6Cb^5c^2+12Bb^5cd+7Cab^4d^2+6Ab^5\right.\right.\right.\right.$$
$$+\frac{2(Ba^2c^2+2Aa^2cd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{(6Ca^2bc^2+24Aab^2c^2+12Ba^2bcd-Ca^3d^2+6Aa^2bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
2*A*a^2*sqrt(b)*c^2/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*C*b*d^2*x + 6*(2*C*b^5*c*d + B*b^5*d^2)/b^4)*x + 5*(6*C
*b^5*c^2 + 12*B*b^5*c*d + 7*C*a*b^4*d^2 + 6*A*b^5*d^2)/b^4)*x + 8*(5*B*b^5
*c^2 + 12*C*a*b^4*c*d + 10*A*b^5*c*d + 6*B*a*b^4*d^2)/b^4)*x + 15*(10*C*a*
b^4*c^2 + 8*A*b^5*c^2 + 20*B*a*b^4*c*d + C*a^2*b^3*d^2 + 10*A*a*b^4*d^2)/b
^4)*x + 16*(20*B*a*b^4*c^2 + 6*C*a^2*b^3*c*d + 40*A*a*b^4*c*d + 3*B*a^2*b^
3*d^2)/b^4) + 2*(B*a^2*c^2 + 2*A*a^2*c*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2
+ a))/sqrt(-a))/sqrt(-a) - 1/16*(6*C*a^2*b*c^2 + 24*A*a*b^2*c^2 + 12*B*a^2
*b*c*d - C*a^3*d^2 + 6*A*a^2*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^2} dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^2}\,dx = \int \frac{(dx+c)^2 (b\,x^2+a)^{\frac{3}{2}} (C\,x^2+Bx+A)}{x^2}dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2,x)
```

3.67 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 333

$$\begin{aligned}\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3}\,dx &= \left(ac(cC+2Bd)\right.\\
&\quad\left.+A\left(bc^2+ad^2\right)\right)\sqrt{a+bx^2}-\frac{aAc^2\sqrt{a+bx^2}}{2x^2}\\
&\quad+\frac{3}{8}(4bc(Bc+2Ad)+ad(2cC+Bd))x\sqrt{a+bx^2}\\
&\quad+\frac{1}{3}\left(c^2C+2Bcd+Ad^2\right)\left(a+bx^2\right)^{3/2}\\
&\quad+\frac{(4bc(Bc+2Ad)+ad(2cC+Bd))x(a+bx^2)^{3/2}}{4a}\\
&\quad+\frac{Cd^2(a+bx^2)^{5/2}}{5b}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{ax}\\
&\quad+\frac{3a(4bc(Bc+2Ad)+ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}\\
&\quad-\frac{1}{2}\sqrt{a}\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\end{aligned}$$

output
```
(a*c*(2*B*d+C*c)+A*(a*d^2+b*c^2))*(b*x^2+a)^(1/2)-1/2*a*A*c^2*(b*x^2+a)^(1
/2)/x^2+3/8*(4*b*c*(2*A*d+B*c)+a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)+1/3*(A*d
^2+2*B*c*d+C*c^2)*(b*x^2+a)^(3/2)+1/4*(4*b*c*(2*A*d+B*c)+a*d*(B*d+2*C*c))*
x*(b*x^2+a)^(3/2)/a+1/5*C*d^2*(b*x^2+a)^(5/2)/b-c*(2*A*d+B*c)*(b*x^2+a)^(5
/2)/a/x+3/8*a*(4*b*c*(2*A*d+B*c)+a*d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2
+a)^(1/2))/b^(1/2)-1/2*a^(1/2)*(2*a*c*(2*B*d+C*c)+A*(2*a*d^2+3*b*c^2))*arc
tanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)^2\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^3}\,dx = \frac{\sqrt{a+bx^2}(24a^2Cd^2x^2+2b^2x^2(20A(3c^2+3cdx+d^2x^2)+$$

$$+3\sqrt{a}Abc^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$-2a^{3/2}\left(c^2C+2Bcd+Ad^2\right)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$-\frac{3a(4bc(Bc+2Ad)+ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(24*a^2*C*d^2*x^2 + 2*b^2*x^2*(20*A*(3*c^2 + 3*c*d*x + d^
2*x^2) + x*(5*B*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + 2*C*x*(10*c^2 + 15*c*d*x +
 6*d^2*x^2))) + a*b*(-20*A*(3*c^2 + 12*c*d*x - 8*d^2*x^2) + x*(2*C*x*(80*c
^2 + 75*c*d*x + 24*d^2*x^2) + B*(-120*c^2 + 320*c*d*x + 75*d^2*x^2)))))/(1
20*b*x^2) + 3*Sqrt[a]*A*b*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a
]] - 2*a^(3/2)*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a +
b*x^2])/Sqrt[a]] - (3*a*(4*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {2338, 25, 2338, 25, 2340, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right)}{x^3} \, dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2} \left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^2}dx}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2} \left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^2}dx}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2} \left(2a^2Cd^2x^2+2a(4bc(Bc+2Ad)+ad(2cC+Bd))x+a\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)\right)}{x}dx}{a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2} \left(2a^2Cd^2x^2+2a(4bc(Bc+2Ad)+ad(2cC+Bd))x+a\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)\right)}{x}dx}{a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\frac{\int \frac{5ab\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)+2(4bc(Bc+2Ad)+ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x}dx}{5b}+\frac{2a^2Cd^2\left(a+bx^2\right)^{5/2}}{5b}}{a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 27

$$\frac{\frac{a\int \frac{\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)+2(4bc(Bc+2Ad)+ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x}dx+\frac{2a^2Cd^2\left(a+bx^2\right)^{5/2}}{5b}}{a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 535

$$\frac{\frac{a\left(\frac{1}{4}a\int \frac{2\left(2\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)+3(4bc(Bc+2Ad)+ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}\left(a+bx^2\right)^{3/2}\left(2\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)+3x(ad(\right.\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 27

$$\frac{\frac{a\left(\frac{1}{2}a\int \frac{\left(2\left(2ac(cC+2Bd)+\frac{1}{2}A\left(6bc^2+4ad^2\right)\right)+3(4bc(Bc+2Ad)+ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}\left(a+bx^2\right)^{3/2}\left(2\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)+3x(ad\right.\right.}{a}}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 535

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{4\left(2ac(cC+2Bd)+A\left(3bc^2+2ad^2\right)\right)+3(4bc(Bc+2Ad)+ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)+3x(ad(Bd+2cC)\right.\right.\right.}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 538

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+3(ad(Bd+2cC)+4bc(2Ad+Bc))\int\frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+3bc^2\right)+2a\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 224

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+3(ad(Bd+2cC)+4bc(2Ad+Bc))\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 219

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+4bc(2Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+3bc\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 243

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(2\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+4bc(2Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+3\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 73

$$\frac{a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{4\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+4bc(2Ad+Bc))}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(4\left(A\left(2ad^2+3b\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

$\downarrow$ 221

$$\frac{\frac{2a^2Cd^2\left(a+bx^2\right)^{5/2}}{5b}+a\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad(Bd+2cC)+4bc(2Ad+Bc))}{\sqrt{b}}-\frac{4\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(A\left(2ad^2+3bc^2\right)+2ac(2Bd+cC)\right)}{\sqrt{a}}\right)+\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{2ax^2}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^3,x]
```

output
```
-1/2*(A*c^2*(a + b*x^2)^(5/2))/(a*x^2) + ((-2*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/x + ((2*a^2*C*d^2*(a + b*x^2)^(5/2))/(5*b) + a*(((2*(2*a*c*(c*C + 2
*B*d) + A*(3*b*c^2 + 2*a*d^2)) + 3*(4*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d
))*x)*(a + b*x^2)^(3/2))/6 + (a*(((4*(2*a*c*(c*C + 2*B*d) + A*(3*b*c^2 + 2
*a*d^2)) + 3*(4*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*x)*Sqrt[a + b*x^2])
/2 + (a*((3*(4*b*c*(B*c + 2*A*d) + a*d*(2*c*C + B*d))*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/Sqrt[b] - (4*(2*a*c*(c*C + 2*B*d) + A*(3*b*c^2 + 2*a*d^2
))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/2))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.10

method	result
default	$B\,d^{2} \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\,x +\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)+\left(A\,d^{2}+2 B c d +C\,c^{2}\right) \left(\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x\right.\right.$
risch	$-\frac{a c \sqrt{b\,x^{2}+a}\,\left(4 A d x +2 B c x +A c \right)}{2 x^{2}}+b^{2} d \left(B d +2 C c \right) \left(\frac{x^{3} \sqrt{b\,x^{2}+a}}{4 b}-\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2 b}-\frac{a \ln \left(\sqrt{b}\,x +\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}\right)}{4 b}\right)+b \left(A b\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
B*d^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d^2+2*B*c*d+C*c^2)*(1/3*(b*x^2+a)^(3/2)+a
*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+A*c^2*(-
1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-
a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+1/5*C*d^2*(b*x^2+a)^(5/2)
/b+c*(2*A*d+B*c)*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*
a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+2*
C*c*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 1.67 (sec) , antiderivative size = 1331, normalized size of antiderivative = 4.00

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="fricas
")
```

output
```
[1/240*(45*(4*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 4*A*a*b)*c*d)*sqrt(b)*x^2
*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 60*(4*B*a*b*c*d + 2*A*a
*b*d^2 + (2*C*a*b + 3*A*b^2)*c^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(24*C*b^2*d^2*x^6 + 30*(2*C*b^2*c*d + B*b^2*d^
2)*x^5 - 60*A*a*b*c^2 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d + (6*C*a*b + 5*A*b^2
)*d^2)*x^4 + 15*(4*B*b^2*c^2 + 5*B*a*b*d^2 + 2*(5*C*a*b + 4*A*b^2)*c*d)*x^
3 + 8*(40*B*a*b*c*d + 5*(4*C*a*b + 3*A*b^2)*c^2 + (3*C*a^2 + 20*A*a*b)*d^2
)*x^2 - 120*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(b*x^2), -1/120*
(45*(4*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 4*A*a*b)*c*d)*sqrt(-b)*x^2*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - 30*(4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b +
 3*A*b^2)*c^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) - (24*C*b^2*d^2*x^6 + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^5 - 60*A*a*b*c^2
 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d + (6*C*a*b + 5*A*b^2)*d^2)*x^4 + 15*(4*B*
b^2*c^2 + 5*B*a*b*d^2 + 2*(5*C*a*b + 4*A*b^2)*c*d)*x^3 + 8*(40*B*a*b*c*d +
 5*(4*C*a*b + 3*A*b^2)*c^2 + (3*C*a^2 + 20*A*a*b)*d^2)*x^2 - 120*(B*a*b*c^
2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(b*x^2), 1/240*(120*(4*B*a*b*c*d + 2*
A*a*b*d^2 + (2*C*a*b + 3*A*b^2)*c^2)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*s
qrt(-a)/a) + 45*(4*B*a*b*c^2 + B*a^2*d^2 + 2*(C*a^2 + 4*A*a*b)*c*d)*sqrt(b
)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*C*b^2*d^2*x^
6 + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^5 - 60*A*a*b*c^2 + 8*(5*C*b^2*c^2 + ...
```

Sympy [A] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 1251, normalized size of antiderivative = 3.76

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^3} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**3,x)
```

output
```
-2*A*a**(3/2)*c*d/(x*sqrt(1 + b*x**2/a)) - A*a**(3/2)*d**2*asinh(sqrt(a)/(
sqrt(b)*x)) - 3*A*sqrt(a)*b*c**2*asinh(sqrt(a)/(sqrt(b)*x))/2 - 2*A*sqrt(a
)*b*c*d*x/sqrt(1 + b*x**2/a) + A*a**2*d**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) - A*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(2*x) + A*a*sqrt(b)*c**2/(x*sqrt
(a/(b*x**2) + 1)) + 2*A*a*sqrt(b)*c*d*asinh(sqrt(b)*x/sqrt(a)) + A*a*sqrt(
b)*d**2*x/sqrt(a/(b*x**2) + 1) + A*b**(3/2)*c**2*x/sqrt(a/(b*x**2) + 1) +
2*A*b*c*d*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/
sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/
2, Ne(b, 0)), (sqrt(a)*x, True)) + A*b*d**2*Piecewise((a*sqrt(a + b*x**2)/
(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) - B*a*
*(3/2)*c**2/(x*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*c*d*asinh(sqrt(a)/(sqrt(
b)*x)) - B*sqrt(a)*b*c**2*x/sqrt(1 + b*x**2/a) + 2*B*a**2*c*d/(sqrt(b)*x*s
qrt(a/(b*x**2) + 1)) + B*a*sqrt(b)*c**2*asinh(sqrt(b)*x/sqrt(a)) + 2*B*a*s
qrt(b)*c*d*x/sqrt(a/(b*x**2) + 1) + B*a*d**2*Piecewise((a*Piecewise((log(2
*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x*
*2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*b*
c**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne
(b, 0)), (sqrt(a)*x, True)) + 2*B*b*c*d*Piecewise((a*sqrt(a + b*x**2)/(3*b
) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) + B*b*d...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^3}\,dx = -\frac{3}{2}A\sqrt{a}bc^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}Abc^2 + \frac{(bx^2+a)^{\frac{3}{2}}Abc^2}{2a} + \frac{(bx^2+a)^{\frac{5}{2}}Cd^2}{5b} + \frac{1}{4}\left(2Ccd+Bd^2\right)\left(bx^2+a\right)^{\frac{3}{2}}x + \frac{3}{8}\left(2Ccd+Bd^2\right)\sqrt{bx^2+a}ax + \frac{3}{2}\left(Bc^2+2Acd\right)\sqrt{bx^2+a}bx + \frac{3(2Ccd+Bd^2)a^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} + \frac{3}{2}(Bc^2+2Acd)a\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \left(Cc^2+2Bcd+Ad^2\right)a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(Cc^2+2Bcd+Ad^2\right)\left(bx^2+a\right)^{\frac{3}{2}} + \left(Cc^2+2Bcd+Ad^2\right)\sqrt{bx^2+a}a - \frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{2ax^2} - \frac{(Bc^2+2Acd)(bx^2+a)^{\frac{3}{2}}}{x}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="maxima
")
```

output
```
-3/2*A*sqrt(a)*b*c^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A
*b*c^2 + 1/2*(b*x^2 + a)^(3/2)*A*b*c^2/a + 1/5*(b*x^2 + a)^(5/2)*C*d^2/b +
 1/4*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*x + 3/8*(2*C*c*d + B*d^2)*sqrt(b*
x^2 + a)*a*x + 3/2*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b*x + 3/8*(2*C*c*d +
B*d^2)*a^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/2*(B*c^2 + 2*A*c*d)*a*sqrt(b
)*arcsinh(b*x/sqrt(a*b)) - (C*c^2 + 2*B*c*d + A*d^2)*a^(3/2)*arcsinh(a/(sq
rt(a*b)*abs(x))) + 1/3*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2) + (C*c^
2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a - 1/2*(b*x^2 + a)^(5/2)*A*c^2/(a*x^
2) - (B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)/x
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^3}\,dx = \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(4\,Cbd^2x+\frac{5\,(2\,Cb^4cd+Bb^4d^2)}{b^3}\right)x\right.\right.\right.$$
$$+\frac{(2\,Ca^2c^2+3\,Aabc^2+4\,Ba^2cd+2\,Aa^2d^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{3\,(4\,Babc^2+2\,Ca^2cd+8\,Aabcd+Ba^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Aabc^2+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba^2\sqrt{b}c^2+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Aa^2\sqrt{b}cd+\left(\sqrt{b}x\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*C*b*d^2*x + 5*(2*C*b^4*c*d + B*b^4*d^2)/b^
3)*x + 4*(5*C*b^4*c^2 + 10*B*b^4*c*d + 6*C*a*b^3*d^2 + 5*A*b^4*d^2)/b^3)*x
 + 15*(4*B*b^4*c^2 + 10*C*a*b^3*c*d + 8*A*b^4*c*d + 5*B*a*b^3*d^2)/b^3)*x
+ 8*(20*C*a*b^3*c^2 + 15*A*b^4*c^2 + 40*B*a*b^3*c*d + 3*C*a^2*b^2*d^2 + 20
*A*a*b^3*d^2)/b^3) + (2*C*a^2*c^2 + 3*A*a*b*c^2 + 4*B*a^2*c*d + 2*A*a^2*d^
2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*(4*B*a*b
*c^2 + 2*C*a^2*c*d + 8*A*a*b*c*d + B*a^2*d^2)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/sqrt(b) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b*c^2 + 2*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c^2 + 4*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a^2*sqrt(b)*c*d + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*c^2 - 2*B*
a^3*sqrt(b)*c^2 - 4*A*a^3*sqrt(b)*c*d)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
a)^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^3}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2\,\left(C\,x^2+B\,x+A\right)}{x^3}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^3,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 783, normalized size of antiderivative = 2.35

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3,x)
```

output
```
( - 60*sqrt(a + b*x**2)*a**2*b*c**2 - 240*sqrt(a + b*x**2)*a**2*b*c*d*x +
160*sqrt(a + b*x**2)*a**2*b*d**2*x**2 + 24*sqrt(a + b*x**2)*a**2*c*d**2*x*
*2 + 120*sqrt(a + b*x**2)*a*b**2*c**2*x**2 - 120*sqrt(a + b*x**2)*a*b**2*c
**2*x + 120*sqrt(a + b*x**2)*a*b**2*c*d*x**3 + 320*sqrt(a + b*x**2)*a*b**2
*c*d*x**2 + 40*sqrt(a + b*x**2)*a*b**2*d**2*x**4 + 75*sqrt(a + b*x**2)*a*b
**2*d**2*x**3 + 160*sqrt(a + b*x**2)*a*b*c**3*x**2 + 150*sqrt(a + b*x**2)*
a*b*c**2*d*x**3 + 48*sqrt(a + b*x**2)*a*b*c*d**2*x**4 + 60*sqrt(a + b*x**2
)*b**3*c**2*x**3 + 80*sqrt(a + b*x**2)*b**3*c*d*x**4 + 30*sqrt(a + b*x**2)
*b**3*d**2*x**5 + 40*sqrt(a + b*x**2)*b**2*c**3*x**4 + 60*sqrt(a + b*x**2)
*b**2*c**2*d*x**5 + 24*sqrt(a + b*x**2)*b**2*c*d**2*x**6 + 120*sqrt(a)*log
((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2*x**2 + 180*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*
x**2 + 240*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a
*b**2*c*d*x**2 + 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b*c**3*x**2 - 120*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqr
t(b)*x)/sqrt(a))*a**2*b*d**2*x**2 - 180*sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*x**2 - 240*sqrt(a)*log((sqrt(a + b
*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**2 - 120*sqrt(a)*log((
sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**3*x**2 + 360*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d*x**2 + 45*sqr...
```

3.68 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4} dx$

Optimal result

Integrand size = 32, antiderivative size = 426

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^4} dx = \frac{1}{2}(3bc(Bc+2Ad) + 2ad(2cC+Bd))\sqrt{a+bx^2} + \frac{(4Ab(2bc^2+3ad^2)+3a(aCd^2+4bc(cC+2Bd)))x\sqrt{a+bx^2}}{8a} + \frac{(3bc(Bc+2Ad)+2ad(2cC+Bd))(a+bx^2)^{3/2}}{6a} + \frac{(4Ab(2bc^2+3ad^2)+3a(aCd^2+4bc(cC+2Bd)))x(a+bx^2)^{3/2}}{12a^2} - \frac{Ac^2(a+bx^2)^{5/2}}{3ax^3} - \frac{c(Bc+2Ad)(a+bx^2)^{5/2}}{2ax^2} - \frac{(3ac(cC+2Bd)+A(2bc^2+3ad^2))(a+bx^2)^{5/2}}{3a^2x} + \frac{(4Ab(2bc^2+3ad^2)+3a(aCd^2+4bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - \frac{1}{2}\sqrt{a}(3bc(Bc+2Ad)+2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/2*(3*b*c*(2*A*d+B*c)+2*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)+1/8*(4*A*b*(3*a*
d^2+2*b*c^2)+3*a*(a*C*d^2+4*b*c*(2*B*d+C*c)))*x*(b*x^2+a)^(1/2)/a+1/6*(3*b
*c*(2*A*d+B*c)+2*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a+1/12*(4*A*b*(3*a*d^2+2
*b*c^2)+3*a*(a*C*d^2+4*b*c*(2*B*d+C*c)))*x*(b*x^2+a)^(3/2)/a^2-1/3*A*c^2*(
b*x^2+a)^(5/2)/a/x^3-1/2*c*(2*A*d+B*c)*(b*x^2+a)^(5/2)/a/x^2-1/3*(3*a*c*(2
*B*d+C*c)+A*(3*a*d^2+2*b*c^2))*(b*x^2+a)^(5/2)/a^2/x+1/8*(4*A*b*(3*a*d^2+2
*b*c^2)+3*a*(a*C*d^2+4*b*c*(2*B*d+C*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2)
)/b^(1/2)-1/2*a^(1/2)*(3*b*c*(2*A*d+B*c)+2*a*d*(B*d+2*C*c))*arctanh((b*x^2
+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^4} \, dx = \frac{1}{24} \left(\frac{\sqrt{a+bx^2}(-a(8A(c^2+3cdx+3d^2x^2)+x(Cx(24c^2-\right.$$

$$+ \frac{6(4Ab(2bc^2+3ad^2)+3a(aCd^2+4bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

$$-12\sqrt{a}(3bc(Bc+2Ad)+2ad(2cC+Bd))\log(x)$$

$$\left. +12\sqrt{a}(3bc(Bc+2Ad)+2ad(2cC+Bd))\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^4,x]
```

output
```
((Sqrt[a + b*x^2]*(-(a*(8*A*(c^2 + 3*c*d*x + 3*d^2*x^2) + x*(C*x*(24*c^2 -
 64*c*d*x - 15*d^2*x^2) + 4*B*(3*c^2 + 12*c*d*x - 8*d^2*x^2)))) + 2*b*x^2*
(A*(-16*c^2 + 24*c*d*x + 6*d^2*x^2) + x*(4*B*(3*c^2 + 3*c*d*x + d^2*x^2) +
 C*x*(6*c^2 + 8*c*d*x + 3*d^2*x^2)))))/x^3 + (6*(4*A*b*(2*b*c^2 + 3*a*d^2)
 + 3*a*(a*C*d^2 + 4*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sq
rt[a + b*x^2])])/Sqrt[b] - 12*Sqrt[a]*(3*b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C
+ B*d))*Log[x] + 12*Sqrt[a]*(3*b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d))*Lo
g[-Sqrt[a] + Sqrt[a + b*x^2]])/24
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2338, 25, 2338, 25, 2338, 25, 27, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx)^2(A+Bx+Cx^2)}{x^4}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{(bx^2+a)^{3/2}(3aCd^2x^3+3ad(2cC+Bd)x^2+(3ac(cC+2Bd)+A(2bc^2+3ad^2))x+3ac(Bc+2Ad))}{x^3}dx}{3a}-\frac{Ac^2(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(3aCd^2x^3+3ad(2cC+Bd)x^2+(3ac(cC+2Bd)+A(2bc^2+3ad^2))x+3ac(Bc+2Ad))}{x^3}dx}{3a}-\frac{Ac^2(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{(bx^2+a)^{3/2}(6a^2Cd^2x^2+3a(3bc(Bc+2Ad)+2ad(2cC+Bd))x+2a(3ac(cC+2Bd)+A(2bc^2+3ad^2)))}{x^2}dx}{2a}-\frac{3c(a+bx^2)^{5/2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{(bx^2+a)^{3/2}(6a^2Cd^2x^2+3a(3bc(Bc+2Ad)+2ad(2cC+Bd))x+2a(3ac(cC+2Bd)+A(2bc^2+3ad^2)))}{x^2}dx}{2a}-\frac{3c(a+bx^2)^{5/2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2(a+bx^2)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{\frac{-\frac{\int -\frac{a\left(3a(3bc(Bc+2Ad)+2ad(2cC+Bd))+2\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{5/2}\left(A\left(3ad^2+2bc^2\right)+3ac(2Bd+cC)\right)}{x}}{2a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a\left(3a(3bc(Bc+2Ad)+2ad(2cC+Bd))+2\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x}dx}{a}-\frac{2\left(a+bx^2\right)^{5/2}\left(A\left(3ad^2+2bc^2\right)+3ac(2Bd+cC)\right)}{x}}{2a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(3a(3bc(Bc+2Ad)+2ad(2cC+Bd))+2\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x}dx-\frac{2\left(a+bx^2\right)^{5/2}\left(A\left(3ad^2+2bc^2\right)+3ac(2Bd+cC)\right)}{x}}{2a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 535

$$\frac{\frac{1}{4}a\int \frac{6\left(2a(3bc(Bc+2Ad)+2ad(2cC+Bd))+\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx-\frac{\frac{2\left(a+bx^2\right)^{5/2}\left(A\left(3ad^2+2bc^2\right)+3ac(2Bd+cC)\right)}{x}+\frac{1}{2}}{2a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 27

$$\frac{\frac{3}{2}a\int \frac{\left(2a(3bc(Bc+2Ad)+2ad(2cC+Bd))+\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x}dx-\frac{\frac{2\left(a+bx^2\right)^{5/2}\left(A\left(3ad^2+2bc^2\right)+3ac(2Bd+cC)\right)}{x}+\frac{1}{2}(}{2a}}{3a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$\downarrow$ 535

$$\frac{3}{2}a\left(\frac{1}{2}a\int\frac{4a(3bc(Bc+2Ad)+2ad(2cC+Bd))+\left(4Ab\left(2bc^2+3ad^2\right)+3a\left(aCd^2+4bc(cC+2Bd)\right)\right)x}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab\left(3ad^2+2bc^2\right)+3a\left(aCd^2+4bc(2Bd+\right.\right.\right.\right.$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{538}$$

$$\frac{3}{2}a\left(\frac{1}{2}a\left(\left(4Ab(3ad^2+2bc^2)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx+4a(2ad(Bd+2cC)+3bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4Ab(3\right.\right.\right.$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{2}a\left(\frac{1}{2}a\left(\left(4Ab(3ad^2+2bc^2)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+4a(2ad(Bd+2cC)+3bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}\right.$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}a\left(\frac{1}{2}a\left(4a(2ad(Bd+2cC)+3bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(3ad^2+2bc^2\right)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\left(4\right.\right.\right.$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{3}{2}a\left(\frac{1}{2}a\left(2a(2ad(Bd+2cC)+3bc(2Ad+Bc))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(3ad^2+2bc^2\right)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}\left(x\right.\right.$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{3}{2}a\left(\frac{1}{2}a\left(\frac{4a(2ad(Bd+2cC)+3bc(2Ad+Bc))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(3ad^2+2bc^2\right)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(x(4\cdots}{\cdots}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3}{2}a\left(\frac{1}{2}a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(3ad^2+2bc^2\right)+3a\left(aCd^2+4bc(2Bd+cC)\right)\right)}{\sqrt{b}}-4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2ad(Bd+2cC)+3bc(2Ad+Bc))\right)+\frac{1}{2}\sqrt{a}\cdots}{\cdots}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{3ax^3}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^4,x]
```

output
```
-1/3*(A*c^2*(a + b*x^2)^(5/2))/(a*x^3) + ((-3*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(2*x^2) + (((2*a*(3*b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d)) + (4*A
*b*(2*b*c^2 + 3*a*d^2) + 3*a*(a*C*d^2 + 4*b*c*(c*C + 2*B*d)))*x)*(a + b*x^
2)^(3/2))/2 - (2*(3*a*c*(c*C + 2*B*d) + A*(2*b*c^2 + 3*a*d^2))*(a + b*x^2)
^(5/2))/x + (3*a*(((4*a*(3*b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d)) + (4*A
*b*(2*b*c^2 + 3*a*d^2) + 3*a*(a*C*d^2 + 4*b*c*(c*C + 2*B*d)))*x)*Sqrt[a +
b*x^2])/2 + (a*(((4*A*b*(2*b*c^2 + 3*a*d^2) + 3*a*(a*C*d^2 + 4*b*c*(c*C +
2*B*d)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - 4*Sqrt[a]*(3*b*c*
(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)
)/2)/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 402, normalized size of antiderivative = 0.94

method	result
default	$C\,d^2\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(\right.}{}\right.}{}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,(6Aa\,d^2x^2+8Ab\,c^2x^2+12Bacd\,x^2+6Ca\,c^2x^2+6Aacdx+3Ba\,c^2x+2A\,c^2a)}{6x^3}+b^2d(Bd+2Cc)\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
C*d^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))))+(A*d^2+2*B*c*d+C*c^2)*(-1/a/x*(b*x^2+a)^(5/2
)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+A*c^2*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a
*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))+c*(2*A*d+B*c)*(-1
/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a
^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+d*(B*d+2*C*c)*(1/3*(b*x^2+
a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
)
```

Fricas [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 1339, normalized size of antiderivative = 3.14

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^4} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="fricas
")
```

output
```
[1/48*(3*(24*B*a*b*c*d + 4*(3*C*a*b + 2*A*b^2)*c^2 + 3*(C*a^2 + 4*A*a*b)*d
^2)*sqrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 12*(3*B*
b^2*c^2 + 2*B*a*b*d^2 + 2*(2*C*a*b + 3*A*b^2)*c*d)*sqrt(a)*x^3*log(-(b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*C*b^2*d^2*x^6 + 8*(2*C*b^2
*c*d + B*b^2*d^2)*x^5 - 8*A*a*b*c^2 + 3*(4*C*b^2*c^2 + 8*B*b^2*c*d + (5*C*
a*b + 4*A*b^2)*d^2)*x^4 + 8*(3*B*b^2*c^2 + 4*B*a*b*d^2 + 2*(4*C*a*b + 3*A*
b^2)*c*d)*x^3 - 8*(6*B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b + 4*A*b^2)*c^2)*x^
2 - 12*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(b*x^3), -1/24*(3*(24
*B*a*b*c*d + 4*(3*C*a*b + 2*A*b^2)*c^2 + 3*(C*a^2 + 4*A*a*b)*d^2)*sqrt(-b)
*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*(3*B*b^2*c^2 + 2*B*a*b*d^2 + 2
*(2*C*a*b + 3*A*b^2)*c*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(a) + 2*a)/x^2) - (6*C*b^2*d^2*x^6 + 8*(2*C*b^2*c*d + B*b^2*d^2)*x^5 - 8*A
*a*b*c^2 + 3*(4*C*b^2*c^2 + 8*B*b^2*c*d + (5*C*a*b + 4*A*b^2)*d^2)*x^4 + 8
*(3*B*b^2*c^2 + 4*B*a*b*d^2 + 2*(4*C*a*b + 3*A*b^2)*c*d)*x^3 - 8*(6*B*a*b*
c*d + 3*A*a*b*d^2 + (3*C*a*b + 4*A*b^2)*c^2)*x^2 - 12*(B*a*b*c^2 + 2*A*a*b
*c*d)*x)*sqrt(b*x^2 + a))/(b*x^3), 1/48*(24*(3*B*b^2*c^2 + 2*B*a*b*d^2 + 2
*(2*C*a*b + 3*A*b^2)*c*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
+ 3*(24*B*a*b*c*d + 4*(3*C*a*b + 2*A*b^2)*c^2 + 3*(C*a^2 + 4*A*a*b)*d^2)*s
qrt(b)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d^
2*x^6 + 8*(2*C*b^2*c*d + B*b^2*d^2)*x^5 - 8*A*a*b*c^2 + 3*(4*C*b^2*c^2 ...
```

Sympy [A] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 1216, normalized size of antiderivative = 2.85

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^4} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**4,x)
```

output

```
-A*a**(3/2)*d**2/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*c**2/(x*sqrt(1 + b*x
**2/a)) - 3*A*sqrt(a)*b*c*d*asinh(sqrt(a)/(sqrt(b)*x)) - A*sqrt(a)*b*d**2*
x/sqrt(1 + b*x**2/a) - A*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(3*x**2) - A*
a*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/x + 2*A*a*sqrt(b)*c*d/(x*sqrt(a/(b*x**2
) + 1)) + A*a*sqrt(b)*d**2*asinh(sqrt(b)*x/sqrt(a)) - A*b**(3/2)*c**2*sqrt
(a/(b*x**2) + 1)/3 + A*b**(3/2)*c**2*asinh(sqrt(b)*x/sqrt(a)) + 2*A*b**(3/
2)*c*d*x/sqrt(a/(b*x**2) + 1) + A*b*d**2*Piecewise((a*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - A*b**2*c
**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - 2*B*a**(3/2)*c*d/(x*sqrt(1 + b*x**2/a
)) - B*a**(3/2)*d**2*asinh(sqrt(a)/(sqrt(b)*x)) - 3*B*sqrt(a)*b*c**2*asinh
(sqrt(a)/(sqrt(b)*x))/2 - 2*B*sqrt(a)*b*c*d*x/sqrt(1 + b*x**2/a) + B*a**2*
d**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) - B*a*sqrt(b)*c**2*sqrt(a/(b*x**2) +
 1)/(2*x) + B*a*sqrt(b)*c**2/(x*sqrt(a/(b*x**2) + 1)) + 2*B*a*sqrt(b)*c*d*
asinh(sqrt(b)*x/sqrt(a)) + B*a*sqrt(b)*d**2*x/sqrt(a/(b*x**2) + 1) + B*b**
(3/2)*c**2*x/sqrt(a/(b*x**2) + 1) + 2*B*b*c*d*Piecewise((a*Piecewise((log(
2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x
**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*b
*d**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b,
 0)), (sqrt(a)*x**2/2, True)) - C*a**(3/2)*c**2/(x*sqrt(1 + b*x**2/a)) ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^4} \, dx = \frac{\sqrt{bx^2+a}Ab^2c^2x}{a} + \frac{1}{4} \left(bx^2+a\right)^{\frac{3}{2}} Cd^2x + \frac{3}{8} \sqrt{bx^2+a}Cad^2x + Ab^{\frac{3}{2}}c^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + \frac{3\,Ca^2d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}} - \frac{2\left(bx^2+a\right)^{\frac{3}{2}}Abc^2}{3\,ax} + \frac{3}{2}\left(Cc^2+2\,Bcd+Ad^2\right)\sqrt{bx^2+a}bx + \frac{3}{2}\left(Cc^2+2\,Bcd+Ad^2\right)a\sqrt{b} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \left(2\,Ccd+Bd^2\right)a^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3}{2}\left(Bc^2+2\,Acd\right)\sqrt{a}b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(2\,Ccd+Bd^2\right)\left(bx^2+a\right)^{\frac{3}{2}} + \left(2\,Ccd+Bd^2\right)\sqrt{bx^2+a}a + \frac{3}{2}\left(Bc^2+2\,Acd\right)\sqrt{bx^2+a}b + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b}{2\,a} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ac^2}{3\,ax^3} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}}{x} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}}{2\,ax^2}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="maxima
")
```

output
```
sqrt(b*x^2 + a)*A*b^2*c^2*x/a + 1/4*(b*x^2 + a)^(3/2)*C*d^2*x + 3/8*sqrt(b
*x^2 + a)*C*a*d^2*x + A*b^(3/2)*c^2*arcsinh(b*x/sqrt(a*b)) + 3/8*C*a^2*d^2
*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2/3*(b*x^2 + a)^(3/2)*A*b*c^2/(a*x) + 3/
2*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*b*x + 3/2*(C*c^2 + 2*B*c*d + A
*d^2)*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (2*C*c*d + B*d^2)*a^(3/2)*arcsinh
(a/(sqrt(a*b)*abs(x))) - 3/2*(B*c^2 + 2*A*c*d)*sqrt(a)*b*arcsinh(a/(sqrt(a
*b)*abs(x))) + 1/3*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2) + (2*C*c*d + B*d^2)
*sqrt(b*x^2 + a)*a + 3/2*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b + 1/2*(B*c^2
+ 2*A*c*d)*(b*x^2 + a)^(3/2)*b/a - 1/3*(b*x^2 + a)^(5/2)*A*c^2/(a*x^3) - (
C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)/x - 1/2*(B*c^2 + 2*A*c*d)*(b*x^
2 + a)^(5/2)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 688, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(3\,Cbd^2x+\frac{4\left(2\,Cb^3cd+Bb^3d^2\right)}{b^2}\right)x+\frac{3}{}\right.\right.$$

$$+\frac{\left(3\,Babc^2+4\,Ca^2cd+6\,Aabcd+2\,Ba^2d^2\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

$$-\frac{\left(12\,Cabc^2+8\,Ab^2c^2+24\,Babcd+3\,Ca^2d^2+12\,Aabd^2\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Babc^2+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Aabcd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca^2\sqrt{b}c^2+12\left(\sqrt{b}\right.}{}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*C*b*d^2*x + 4*(2*C*b^3*c*d + B*b^3*d^2)/b^2)*x
 + 3*(4*C*b^3*c^2 + 8*B*b^3*c*d + 5*C*a*b^2*d^2 + 4*A*b^3*d^2)/b^2)*x + 8*
(3*B*b^3*c^2 + 8*C*a*b^2*c*d + 6*A*b^3*c*d + 4*B*a*b^2*d^2)/b^2) + (3*B*a*
b*c^2 + 4*C*a^2*c*d + 6*A*a*b*c*d + 2*B*a^2*d^2)*arctan(-(sqrt(b)*x - sqrt
(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/8*(12*C*a*b*c^2 + 8*A*b^2*c^2 + 24*B*a
*b*c*d + 3*C*a^2*d^2 + 12*A*a*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))
)/sqrt(b) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a*b*c^2 + 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^5*A*a*b*c*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^2
*sqrt(b)*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*b^(3/2)*c^2 + 12*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*sqrt(b)*c*d + 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*a^2*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^3*sqrt
(b)*c^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*b^(3/2)*c^2 - 24*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b)*c*d - 12*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*A*a^3*sqrt(b)*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^3*b*c^2 - 6
*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b*c*d + 6*C*a^4*sqrt(b)*c^2 + 8*A*a^3
*b^(3/2)*c^2 + 12*B*a^4*sqrt(b)*c*d + 6*A*a^4*sqrt(b)*d^2)/((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2\,\left(C\,x^2+B\,x+A\right)}{x^4}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^4,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^4, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^4} \, dx = \int \frac{(dx+c)^2 (b\,x^2+a)^{\frac{3}{2}} (C\,x^2+Bx+A)}{x^4} dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4,x)
```

3.69 $\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 367

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5} \, dx = \left(aCd^2+b\left(c^2C+2Bcd+Ad^2\right)\right)\sqrt{a+bx^2}$$
$$-\frac{aAc^2\sqrt{a+bx^2}}{4x^4}-\frac{\left(4ac(cC+2Bd)+A\left(5bc^2+4ad^2\right)\right)\sqrt{a+bx^2}}{8x^2}$$
$$+\frac{b(2bc(Bc+2Ad)+3ad(2cC+Bd))x\sqrt{a+bx^2}}{2a}+\frac{1}{3}Cd^2\left(a+bx^2\right)^{3/2}$$
$$-\frac{(2bc(Bc+2Ad)+3ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{3ax}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{3ax^3}$$
$$+\frac{1}{2}\sqrt{b}(2bc(Bc+2Ad)+3ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2\right.\right.}{8\sqrt{a}}$$

output
```
(a*C*d^2+b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(1/2)-1/4*a*A*c^2*(b*x^2+a)^(1
/2)/x^4-1/8*(4*a*c*(2*B*d+C*c)+A*(4*a*d^2+5*b*c^2))*(b*x^2+a)^(1/2)/x^2+1/
2*b*(2*b*c*(2*A*d+B*c)+3*a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/a+1/3*C*d^2*(b
*x^2+a)^(3/2)-1/3*(2*b*c*(2*A*d+B*c)+3*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a/
x-1/3*c*(2*A*d+B*c)*(b*x^2+a)^(5/2)/a/x^3+1/2*b^(1/2)*(2*b*c*(2*A*d+B*c)+3
*a*d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))-1/8*(3*A*b*(4*a*d^2+b
*c^2)+4*a*(2*a*C*d^2+3*b*c*(2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(1/2)
```

Mathematica [A] (verified)

Time = 2.07 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} dx = \frac{1}{24} \left(\frac{\sqrt{a+bx^2}(-2a(A(3c^2+8cdx+6d^2x^2)+2x(Cx(3c^2\right.$$

$$+\frac{6(3Ab^2c^2+8a^2Cd^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

$$-72\sqrt{a}b(c^2C+2Bcd+Ad^2)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

$$\left.-12\sqrt{b}(2bc(Bc+2Ad)+3ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^5,x]
```

output
```
((Sqrt[a + b*x^2]*(-2*a*(A*(3*c^2 + 8*c*d*x + 6*d^2*x^2) + 2*x*(C*x*(3*c^2
 + 12*c*d*x - 8*d^2*x^2) + 2*B*(c^2 + 3*c*d*x + 3*d^2*x^2))) + b*x^2*(A*(-
15*c^2 - 64*c*d*x + 24*d^2*x^2) + 4*x*(2*C*x*(3*c^2 + 3*c*d*x + d^2*x^2) +
 B*(-8*c^2 + 12*c*d*x + 3*d^2*x^2)))))/x^4 + (6*(3*A*b^2*c^2 + 8*a^2*C*d^2
)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - 72*Sqrt[a]*b*(
c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]]
 - 12*Sqrt[b]*(2*b*c*(B*c + 2*A*d) + 3*a*d*(2*c*C + B*d))*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/24
```

Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 25, 2338, 25, 27, 536, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^5}\,dx$$

$\Big\downarrow$ 2338

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(4aCd^2x^3+4ad(2cC+Bd)x^2+\left(4ac(cC+2Bd)+A\left(bc^2+4ad^2\right)\right)x+4ac(Bc+2Ad)\right)}{x^4}dx}{4a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(4aCd^2x^3+4ad(2cC+Bd)x^2+\left(4ac(cC+2Bd)+A\left(bc^2+4ad^2\right)\right)x+4ac(Bc+2Ad)\right)}{x^4}dx}{4a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(12a^2Cd^2x^2+4a(2bc(Bc+2Ad)+3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)+A\left(bc^2+4ad^2\right)\right)\right)}{x^3}dx}{3a}-\frac{4c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(12a^2Cd^2x^2+4a(2bc(Bc+2Ad)+3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)+A\left(bc^2+4ad^2\right)\right)\right)}{x^3}dx}{3a}-\frac{4c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 2338

$$\frac{\frac{-\frac{\int -\frac{a\left(8a(2bc(Bc+2Ad)+3ad(2cC+Bd))+3\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^2}dx}{2a}-\frac{3\left(a+bx^2\right)^{5/2}\left(A\left(4ad^2+bc^2\right)+4ac(2Bd+cC)\right)}{2x^2}}{3a}}{4a}\quad \frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a\left(8a(2bc(Bc+2Ad)+3ad(2cC+Bd))+3\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^2}dx}{2a}-\frac{3\left(a+bx^2\right)^{5/2}\left(A\left(4ad^2+bc^2\right)+4ac(2Bd+cC)\right)}{2x^2}}{3a}-}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{1}{2}\int \frac{\left(8a(2bc(Bc+2Ad)+3ad(2cC+Bd))+3\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^2}dx-\frac{3\left(a+bx^2\right)^{5/2}\left(A\left(4ad^2+bc^2\right)+4ac(2Bd+cC)\right)}{2x^2}}{3a}-}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 536

$$\frac{\frac{\frac{1}{2}\left(\int \frac{\left(3a\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)\right)+24ab(2bc(Bc+2Ad)+3ad(2cC+Bd))x\right)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}\left(8a(3ad(Bd+2cC)+2bc(2Ad+Bc))\right.}{}\right.}{3a}}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 535

$$\frac{\frac{1}{2}\left(\frac{1}{2}a\int \frac{6a\left(3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)\right)+4b(2bc(Bc+2Ad)+3ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{3/2}\left(8a(3ad(Bd+2cC)+2bc(2Ad+Bc))-x\left(3Ab\left(4\right.\right.\right.}{x}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}\left(3a^2\int \frac{3Ab\left(bc^2+4ad^2\right)+4a\left(2aCd^2+3bc(cC+2Bd)\right)+4b(2bc(Bc+2Ad)+3ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx+3a\sqrt{a+bx^2}(4bx(3ad(Bd+2cC)+2bc(2Ad+Bc))+3Ab(4a\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\Big\downarrow$ 538

$$\frac{\frac{1}{2}\left(3a^2\left(\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+4b(3ad(Bd+2cC)+2bc(2Ad+Bc))\int\frac{1}{\sqrt{bx^2+a}}dx\right)+3a\sqrt{a+bx^2}\left(4bx(3ad\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\big\downarrow$ 224

$$\frac{\frac{1}{2}\left(3a^2\left(\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+4b(3ad(Bd+2cC)+2bc(2Ad+Bc))\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+3a\sqrt{a+bx^2}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}\left(3a^2\left(\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx+4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad(Bd+2cC)+2bc(2Ad+Bc))\right)+3a\sqrt{a+bx}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\big\downarrow$ 243

$$\frac{\frac{1}{2}\left(3a^2\left(\frac{1}{2}\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad(Bd+2cC)+2bc(2Ad+Bc))\right)+3a\sqrt{a}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\big\downarrow$ 73

$$\frac{\frac{1}{2}\left(3a^2\left(\frac{\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad(Bd+2cC)+2bc(2Ad+Bc))\right)+3a\sqrt{a+bx}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

$\big\downarrow$ 221

$$\frac{\frac{1}{2}\left(3a^2\left(4\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad(Bd+2cC)+2bc(2Ad+Bc))-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(3Ab\left(4ad^2+bc^2\right)+4a\left(2aCd^2+3bc(2Bd+cC)\right)\right)}{\sqrt{a}}\right)+3a\sqrt{a}\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{4ax^4}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^5,x]
```

output
```
-1/4*(A*c^2*(a + b*x^2)^(5/2))/(a*x^4) + ((-4*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(3*x^3) + ((-3*(4*a*c*(c*C + 2*B*d) + A*(b*c^2 + 4*a*d^2))*(a + b*x
^2)^(5/2))/(2*x^2) + (3*a*(3*A*b*(b*c^2 + 4*a*d^2) + 4*a*(2*a*C*d^2 + 3*b*
c*(c*C + 2*B*d)) + 4*b*(2*b*c*(B*c + 2*A*d) + 3*a*d*(2*c*C + B*d))*x)*Sqrt
[a + b*x^2] - ((8*a*(2*b*c*(B*c + 2*A*d) + 3*a*d*(2*c*C + B*d)) - (3*A*b*(
b*c^2 + 4*a*d^2) + 4*a*(2*a*C*d^2 + 3*b*c*(c*C + 2*B*d)))*x)*(a + b*x^2)^(
3/2))/x + 3*a^2*(4*Sqrt[b]*(2*b*c*(B*c + 2*A*d) + 3*a*d*(2*c*C + B*d))*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - ((3*A*b*(b*c^2 + 4*a*d^2) + 4*a*(2*a*C
*d^2 + 3*b*c*(c*C + 2*B*d)))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2
)/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(64Abcd\,x^3+24Ba\,d^2x^3+32Bb\,c^2x^3+48Cacd\,x^3+12Aa\,d^2x^2+15Ab\,c^2x^2+24Bacd\,x^2+12Ca\,c^2x^2+16Aacdx+8Ba\,c^2x}{24x^4}$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)+A\,c^2\Bigg(-$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(64*A*b*c*d*x^3+24*B*a*d^2*x^3+32*B*b*c^2*x^3+48*C*a
*c*d*x^3+12*A*a*d^2*x^2+15*A*b*c^2*x^2+24*B*a*c*d*x^2+12*C*a*c^2*x^2+16*A*
a*c*d*x+8*B*a*c^2*x+6*A*a*c^2)/x^4+b^2*d*(B*d+2*C*c)*(1/2*x/b*(b*x^2+a)^(1
/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+(A*b*d^2+2*B*b*c*d+2*C*a*
d^2+C*b*c^2)*(b*x^2+a)^(1/2)-1/8*(12*A*a*b*d^2+3*A*b^2*c^2+24*B*a*b*c*d+8*
C*a^2*d^2+12*C*a*b*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*b^
(3/2)*c^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+2*A*b^(3/2)*c*d*ln(b^(1/2)*x+(b*x^
2+a)^(1/2))+2*B*b^(1/2)*d^2*a*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+C*b^2*d^2*(1/3
*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+4*C*b^(1/2)*c*d*a*ln(b^(
1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 1338, normalized size of antiderivative = 3.65

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="fricas
")
```

output
```
[1/48*(12*(2*B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + 2*A*a*b)*c*d)*sqrt(b)*
x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3*(24*B*a*b*c*d + 3*
(4*C*a*b + A*b^2)*c^2 + 4*(2*C*a^2 + 3*A*a*b)*d^2)*sqrt(a)*x^4*log(-(b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*C*a*b*d^2*x^6 + 12*(2*C*a*
b*c*d + B*a*b*d^2)*x^5 - 6*A*a^2*c^2 + 8*(3*C*a*b*c^2 + 6*B*a*b*c*d + (4*C
*a^2 + 3*A*a*b)*d^2)*x^4 - 8*(4*B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + 4*A
*a*b)*c*d)*x^3 - 3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 + 5*A*a*b)*c^2)*x
^2 - 8*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a*x^4), -1/48*(24*(2
*B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + 2*A*a*b)*c*d)*sqrt(-b)*x^4*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*(24*B*a*b*c*d + 3*(4*C*a*b + A*b^2)*c^2 +
4*(2*C*a^2 + 3*A*a*b)*d^2)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(8*C*a*b*d^2*x^6 + 12*(2*C*a*b*c*d + B*a*b*d^2)*x^5 -
 6*A*a^2*c^2 + 8*(3*C*a*b*c^2 + 6*B*a*b*c*d + (4*C*a^2 + 3*A*a*b)*d^2)*x^4
 - 8*(4*B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + 4*A*a*b)*c*d)*x^3 - 3*(8*B*
a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 + 5*A*a*b)*c^2)*x^2 - 8*(B*a^2*c^2 + 2*A*
a^2*c*d)*x)*sqrt(b*x^2 + a))/(a*x^4), 1/24*(3*(24*B*a*b*c*d + 3*(4*C*a*b +
 A*b^2)*c^2 + 4*(2*C*a^2 + 3*A*a*b)*d^2)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 +
a)*sqrt(-a)/a) + 6*(2*B*a*b*c^2 + 3*B*a^2*d^2 + 2*(3*C*a^2 + 2*A*a*b)*c*d)
*sqrt(b)*x^4*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (8*C*a*b*d^
2*x^6 + 12*(2*C*a*b*c*d + B*a*b*d^2)*x^5 - 6*A*a^2*c^2 + 8*(3*C*a*b*c^2...
```

Sympy [A] (verification not implemented)

Time = 12.18 (sec) , antiderivative size = 1151, normalized size of antiderivative = 3.14

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**5,x)
```

output
```
-2*A*sqrt(a)*b*c*d/(x*sqrt(1 + b*x**2/a)) - 3*A*sqrt(a)*b*d**2*asinh(sqrt(
a)/(sqrt(b)*x))/2 - A*a**2*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*
A*a*sqrt(b)*c**2/(8*x**3*sqrt(a/(b*x**2) + 1)) - 2*A*a*sqrt(b)*c*d*sqrt(a/
(b*x**2) + 1)/(3*x**2) - A*a*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(2*x) + A*a
*sqrt(b)*d**2/(x*sqrt(a/(b*x**2) + 1)) - A*b**(3/2)*c**2*sqrt(a/(b*x**2) +
 1)/(2*x) - A*b**(3/2)*c**2/(8*x*sqrt(a/(b*x**2) + 1)) - 2*A*b**(3/2)*c*d*
sqrt(a/(b*x**2) + 1)/3 + 2*A*b**(3/2)*c*d*asinh(sqrt(b)*x/sqrt(a)) + A*b**
(3/2)*d**2*x/sqrt(a/(b*x**2) + 1) - 3*A*b**2*c**2*asinh(sqrt(a)/(sqrt(b)*x
))/(8*sqrt(a)) - 2*A*b**2*c*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*a**(3/2)*
d**2/(x*sqrt(1 + b*x**2/a)) - B*sqrt(a)*b*c**2/(x*sqrt(1 + b*x**2/a)) - 3*
B*sqrt(a)*b*c*d*asinh(sqrt(a)/(sqrt(b)*x)) - B*sqrt(a)*b*d**2*x/sqrt(1 + b
*x**2/a) - B*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*sqrt(b)*c*
d*sqrt(a/(b*x**2) + 1)/x + 2*B*a*sqrt(b)*c*d/(x*sqrt(a/(b*x**2) + 1)) + B*
a*sqrt(b)*d**2*asinh(sqrt(b)*x/sqrt(a)) - B*b**(3/2)*c**2*sqrt(a/(b*x**2)
+ 1)/3 + B*b**(3/2)*c**2*asinh(sqrt(b)*x/sqrt(a)) + 2*B*b**(3/2)*c*d*x/sqr
t(a/(b*x**2) + 1) + B*b*d**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 +
x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - B*b**2*c**2*x/(sqrt(
a)*sqrt(1 + b*x**2/a)) - 2*C*a**(3/2)*c*d/(x*sqrt(1 + b*x**2/a)) - C*a**(3
/2)*d**2*asinh(sqrt(a)/(sqrt(b)*x)) - 3*C*sqrt(a)*b*c**2*asinh(sqrt(a)/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} \, dx = -\frac{3 Ab^2c^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8 \sqrt{a}} - Ca^{\frac{3}{2}}d^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{(bx^2+a)^{\frac{3}{2}} Ab^2c^2}{8 a^2} + \frac{3 \sqrt{bx^2+a}Ab^2c^2}{8 a} + \frac{1}{3} (bx^2+a)^{\frac{3}{2}} Cd^2 + \sqrt{bx^2+a}Cad^2 + \frac{3}{2} (2 Ccd+Bd^2) \sqrt{bx^2+a}bx + \frac{(Bc^2+2 Acd)\sqrt{bx^2+a}b^2x}{a} + \frac{3}{2} (2 Ccd+Bd^2) a\sqrt{b} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) + (Bc^2+2 Acd) b^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2} (Cc^2+2 Bcd+Ad^2) \sqrt{a}b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2} (Cc^2+2 Bcd+Ad^2) \sqrt{bx^2+a}b + \frac{(Cc^2+2 Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}} b}{2 a} - \frac{(bx^2+a)^{\frac{5}{2}} Abc^2}{8 a^2x^2} - \frac{(2 Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}}{x} - \frac{2 (Bc^2+2 Acd)(bx^2+a)^{\frac{3}{2}} b}{3 ax} - \frac{(bx^2+a)^{\frac{5}{2}} Ac^2}{4 ax^4} - \frac{(Cc^2+2 Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}}{2 ax^2} - \frac{(Bc^2+2 Acd)(bx^2+a)^{\frac{5}{2}}}{3 ax^3}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="maxima
")
```

output
```
-3/8*A*b^2*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - C*a^(3/2)*d^2*arcsi
nh(a/(sqrt(a*b)*abs(x))) + 1/8*(b*x^2 + a)^(3/2)*A*b^2*c^2/a^2 + 3/8*sqrt(
b*x^2 + a)*A*b^2*c^2/a + 1/3*(b*x^2 + a)^(3/2)*C*d^2 + sqrt(b*x^2 + a)*C*a
*d^2 + 3/2*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*b*x + (B*c^2 + 2*A*c*d)*sqrt(
b*x^2 + a)*b^2*x/a + 3/2*(2*C*c*d + B*d^2)*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)
) + (B*c^2 + 2*A*c*d)*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/2*(C*c^2 + 2*B*c*
d + A*d^2)*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*(C*c^2 + 2*B*c*d
+ A*d^2)*sqrt(b*x^2 + a)*b + 1/2*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/
2)*b/a - 1/8*(b*x^2 + a)^(5/2)*A*b*c^2/(a^2*x^2) - (2*C*c*d + B*d^2)*(b*x^
2 + a)^(3/2)/x - 2/3*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b/(a*x) - 1/4*(b*
x^2 + a)^(5/2)*A*c^2/(a*x^4) - 1/2*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(
5/2)/(a*x^2) - 1/3*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1102 vs. $2(325) = 650$.

Time = 0.23 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.00

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x, algorithm="giac")
```

output
```
-1/2*(2*B*b^(3/2)*c^2 + 6*C*a*sqrt(b)*c*d + 4*A*b^(3/2)*c*d + 3*B*a*sqrt(b
)*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/6*sqrt(b*x^2 + a)*((2*C*
b*d^2*x + 3*(2*C*b^2*c*d + B*b^2*d^2)/b)*x + 2*(3*C*b^2*c^2 + 6*B*b^2*c*d
+ 4*C*a*b*d^2 + 3*A*b^2*d^2)/b) + 1/4*(12*C*a*b*c^2 + 3*A*b^2*c^2 + 24*B*a
*b*c*d + 8*C*a^2*d^2 + 12*A*a*b*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))
/sqrt(-a))/sqrt(-a) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^2 +
 15*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*b^2*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2
 + a))^7*B*a*b*c*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b*d^2 + 48*(sq
rt(b)*x - sqrt(b*x^2 + a))^6*B*a*b^(3/2)*c^2 + 48*(sqrt(b)*x - sqrt(b*x^2
+ a))^6*C*a^2*sqrt(b)*c*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2)
*c*d + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*sqrt(b)*d^2 - 12*(sqrt(b)*
x - sqrt(b*x^2 + a))^5*C*a^2*b*c^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a
*b^2*c^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*c*d - 12*(sqrt(b)*x
- sqrt(b*x^2 + a))^5*A*a^2*b*d^2 - 96*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^
2*b^(3/2)*c^2 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*sqrt(b)*c*d - 19
2*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2)*c*d - 72*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*B*a^3*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^
3*b*c^2 + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c^2 - 24*(sqrt(b)*x
- sqrt(b*x^2 + a))^3*B*a^3*b*c*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^
3*b*d^2 + 80*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c^2 + 144*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} dx = \int \frac{(b x^2+a)^{3/2} (c+d x)^2 (C x^2+B x+A)}{x^5} dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^5,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 835, normalized size of antiderivative = 2.28

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^5} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5,x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**2*c**2 - 64*sqrt(a + b*x**2)*a**2*c*d*x - 48*sq
rt(a + b*x**2)*a**2*d**2*x**2 - 60*sqrt(a + b*x**2)*a*b*c**2*x**2 - 32*sqr
t(a + b*x**2)*a*b*c**2*x - 256*sqrt(a + b*x**2)*a*b*c*d*x**3 - 96*sqrt(a +
 b*x**2)*a*b*c*d*x**2 + 96*sqrt(a + b*x**2)*a*b*d**2*x**4 - 96*sqrt(a + b*
x**2)*a*b*d**2*x**3 - 48*sqrt(a + b*x**2)*a*c**3*x**2 - 192*sqrt(a + b*x**
2)*a*c**2*d*x**3 + 128*sqrt(a + b*x**2)*a*c*d**2*x**4 - 128*sqrt(a + b*x**
2)*b**2*c**2*x**3 + 192*sqrt(a + b*x**2)*b**2*c*d*x**4 + 48*sqrt(a + b*x**
2)*b**2*d**2*x**5 + 96*sqrt(a + b*x**2)*b*c**3*x**4 + 96*sqrt(a + b*x**2)*
b*c**2*d*x**5 + 32*sqrt(a + b*x**2)*b*c*d**2*x**6 + 144*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**4 + 96*sqrt(a)*log
((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*c*d**2*x**4 + 36*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 +
 288*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*
d*x**4 + 144*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))
*b*c**3*x**4 - 144*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sq
rt(a))*a*b*d**2*x**4 - 96*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*c*d**2*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 - 288*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**4 - 144*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*x**4 + 192*sqrt(b)*log((sq...
```

3.70 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 383

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx = \frac{3b(bc(Bc+2Ad)+4ad(2cC+Bd))\sqrt{a+bx^2}}{8a}$$
$$+\frac{b(3aCd^2+2b(c^2C+2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{2a}$$
$$-\frac{(bc(Bc+2Ad)+4ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{8ax^2}$$
$$-\frac{(3aCd^2+2b(c^2C+2Bcd+Ad^2))\left(a+bx^2\right)^{3/2}}{3ax}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$
$$-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{4ax^4}-\frac{\left(c^2C+2Bcd+Ad^2\right)\left(a+bx^2\right)^{5/2}}{3ax^3}$$
$$+\frac{1}{2}\sqrt{b}\left(3aCd^2+2b\left(c^2C+2Bcd+Ad^2\right)\right)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{3b(bc(Bc+2Ad)+4ad(2cC+Bd))\text{arctan}}{8\sqrt{a}}$$

output
```
3/8*b*(b*c*(2*A*d+B*c)+4*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a+1/2*b*(3*a*C*d
^2+2*b*(A*d^2+2*B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/a-1/8*(b*c*(2*A*d+B*c)+4*a
*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a/x^2-1/3*(3*a*C*d^2+2*b*(A*d^2+2*B*c*d+C*
c^2))*(b*x^2+a)^(3/2)/a/x-1/5*A*c^2*(b*x^2+a)^(5/2)/a/x^5-1/4*c*(2*A*d+B*c
)*(b*x^2+a)^(5/2)/a/x^4-1/3*(A*d^2+2*B*c*d+C*c^2)*(b*x^2+a)^(5/2)/a/x^3+1/
2*b^(1/2)*(3*a*C*d^2+2*b*(A*d^2+2*B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a
)^(1/2))-3/8*b*(b*c*(2*A*d+B*c)+4*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6}\,dx=\frac{1}{120}\left(-\frac{\sqrt{a+bx^2}(24Ab^2c^2x^4+abx^2(2A(24c^2+75cdx+8\right.$$

$$-\frac{90b(bc(Bc+2Ad)+4ad(2cC+Bd))\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

$$\left.-60\sqrt{b}\left(3aCd^2+2b\left(c^2C+2Bcd+Ad^2\right)\right)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^6,x]
```

output
```
(-((Sqrt[a + b*x^2]*(24*A*b^2*c^2*x^4 + a*b*x^2*(2*A*(24*c^2 + 75*c*d*x +
80*d^2*x^2) + 5*x*(B*(15*c^2 + 64*c*d*x - 24*d^2*x^2) - 4*C*x*(-8*c^2 + 12
*c*d*x + 3*d^2*x^2))) + 2*a^2*(2*A*(6*c^2 + 15*c*d*x + 10*d^2*x^2) + 5*x*(
4*C*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + B*(3*c^2 + 8*c*d*x + 6*d^2*x^2)))))/(a
*x^5)) - (90*b*(b*c*(B*c + 2*A*d) + 4*a*d*(2*c*C + B*d))*ArcTanh[(-(Sqrt[b
]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - 60*Sqrt[b]*(3*a*C*d^2 + 2*b*(c
^2*C + 2*B*c*d + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/120
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.95, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$, Rules used = {2338, 27, 2338, 25, 2338, 25, 27, 537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(c+dx)^2(A+Bx+Cx^2)}{x^6}dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{5(bx^2+a)^{3/2}(aCd^2x^3+ad(2cC+Bd)x^2+a(Cc^2+2Bdc+Ad^2)x+ac(Bc+2Ad))}{x^5}dx}{5a}-\frac{Ac^2(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(bx^2+a)^{3/2}(aCd^2x^3+ad(2cC+Bd)x^2+a(Cc^2+2Bdc+Ad^2)x+ac(Bc+2Ad))}{x^5}dx}{a}-\frac{Ac^2(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{(bx^2+a)^{3/2}\left(4Cd^2x^2a^2+4\left(Cc^2+2Bdc+Ad^2\right)a^2+(bc(Bc+2Ad)+4ad(2cC+Bd))xa\right)}{x^4}dx}{4a}-\frac{c(a+bx^2)^{5/2}(2Ad+Bc)}{4x^4}}{a}-\frac{Ac^2(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{(bx^2+a)^{3/2}\left(4Cd^2x^2a^2+4\left(Cc^2+2Bdc+Ad^2\right)a^2+(bc(Bc+2Ad)+4ad(2cC+Bd))xa\right)}{x^4}dx}{4a}-\frac{c(a+bx^2)^{5/2}(2Ad+Bc)}{4x^4}}{a}-\frac{Ac^2(a+bx^2)^{5/2}}{5ax^5}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{\frac{-\frac{\int -\frac{a^2\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+4\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)(bx^2+a)^{3/2}}{x^3}dx}{3a}-\frac{4a(a+bx^2)^{5/2}(Ad^2+2Bcd+c^2C)}{3x^3}}{4a}-\frac{c(a+bx^2)^{5/2}(2Ad+}{4x^4}}{a}-\frac{Ac^2(a+bx^2)^{5/2}}{5ax^5}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a^2\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+4\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^3}dx}{3a}-\frac{4a\left(a+bx^2\right)^{5/2}\left(Ad^2+2Bcd+c^2C\right)}{3x^3}}{4a}-\frac{c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{4x^4}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{3}a\int \frac{\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+4\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^3}dx-\frac{4a\left(a+bx^2\right)^{5/2}\left(Ad^2+2Bcd+c^2C\right)}{3x^3}}{4a}-\frac{c\left(a+bx^2\right)^{5/2}(2Ad+Bc}{4x^4}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 537

$$\frac{\frac{\frac{1}{3}a\left(-\frac{3}{2}b\int -\frac{\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+8\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}\left(8x\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+3(4ad(Bd\right.\right.}{2x^2}}{4a}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 25

$$\frac{\frac{\frac{1}{3}a\left(\frac{3}{2}b\int \frac{\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+8\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}\left(8x\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+3(4ad(Bd+2c\right.\right.}{2x^2}}{4a}}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 535

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(\frac{1}{2}a\int \frac{2\left(3(bc(Bc+2Ad)+4ad(2cC+Bd))+4\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+3(4ad(Bd+2c\right.\right.\right.}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 27

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\int\frac{3(bc(Bc+2Ad)+4ad(2cC+Bd))+4\left(3aCd^2+2b\left(Cc^2+2Bdc+Ad^2\right)\right)x}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)+3(4ad(Bd+2cC)+b\right.\right.\right.}{4a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 538

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(4\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx+3(4ad(Bd+2cC)+bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\left(Ad^2+2B\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 224

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(4\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+3(4ad(Bd+2cC)+bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 219

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(3(4ad(Bd+2cC)+bc(2Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\left(A\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 243

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(\frac{3}{2}(4ad(Bd+2cC)+bc(2Ad+Bc))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\downarrow$ 73

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(\frac{3(4ad(Bd+2cC)+bc(2Ad+Bc))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2b\left(A\right.\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

$\Big\downarrow$ 221

$$\frac{\frac{1}{3}a\left(\frac{3}{2}b\left(a\left(\frac{4\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2+2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}-\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4ad(Bd+2cC)+bc(2Ad+Bc))}{\sqrt{a}}\right)+\sqrt{a+bx^2}\left(4x\left(3aCd^2+2\right.\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{5ax^5}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^6,x]
```

output
```
-1/5*(A*c^2*(a + b*x^2)^(5/2))/(a*x^5) + (-1/4*(c*(B*c + 2*A*d)*(a + b*x^2
)^(5/2))/x^4 + ((-4*a*(c^2*C + 2*B*c*d + A*d^2)*(a + b*x^2)^(5/2))/(3*x^3)
 + (a*(-1/2*((3*(b*c*(B*c + 2*A*d) + 4*a*d*(2*c*C + B*d)) + 8*(3*a*C*d^2 +
 2*b*(c^2*C + 2*B*c*d + A*d^2))*x)*(a + b*x^2)^(3/2))/x^2 + (3*b*((3*(b*c*
(B*c + 2*A*d) + 4*a*d*(2*c*C + B*d)) + 4*(3*a*C*d^2 + 2*b*(c^2*C + 2*B*c*d
 + A*d^2))*x)*Sqrt[a + b*x^2] + a*((4*(3*a*C*d^2 + 2*b*(c^2*C + 2*B*c*d +
A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (3*(b*c*(B*c + 2*A
*d) + 4*a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])))/2)
)/3)/(4*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.08

method	result
default	$\left(A\,d^{2}+2Bcd+C\,c^{2}\right)\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{3a\,x^{3}}+\frac{2b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(160Ab\,d^{2}x^{4}a+24A\,b^{2}c^{2}x^{4}+320Bbcd\,x^{4}a+120C\,a^{2}d^{2}x^{4}+160Cb\,c^{2}x^{4}a+150Abcd\,x^{3}a+60B\,a^{2}d^{2}x^{3}+75Bb\,c^{2}x^{3}a+120\right.}{120x^{5}a}$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x,method=_RETURNVERBOSE)
```

output
```
(A*d^2+2*B*c*d+C*c^2)*(-1/3/a/x^3*(b*x^2+a)^(5/2)+2/3*b/a*(-1/a/x*(b*x^2+a
)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^
(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))))-1/5*A*c^2*(b*x^2+a)^(5/2)/a/x^5+C*
d^2*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x
^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))))+c*(2*A*d+B*c)*(
-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/
3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x)))))+d*(B*d+2*C*c)*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2
+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
)))
```

Fricas [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 1330, normalized size of antiderivative = 3.47

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^6} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="fricas
")
```

output
```
[1/240*(60*(2*C*a*b*c^2 + 4*B*a*b*c*d + (3*C*a^2 + 2*A*a*b)*d^2)*sqrt(b)*x
^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 45*(B*b^2*c^2 + 4*B*a
*b*d^2 + 2*(4*C*a*b + A*b^2)*c*d)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) + 2*(60*C*a*b*d^2*x^6 + 120*(2*C*a*b*c*d + B*a*b*d
^2)*x^5 - 24*A*a^2*c^2 - 8*(40*B*a*b*c*d + (20*C*a*b + 3*A*b^2)*c^2 + 5*(3
*C*a^2 + 4*A*a*b)*d^2)*x^4 - 15*(5*B*a*b*c^2 + 4*B*a^2*d^2 + 2*(4*C*a^2 +
5*A*a*b)*c*d)*x^3 - 8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 + 6*A*a*b)*c^
2)*x^2 - 30*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a*x^5), -1/240*
(120*(2*C*a*b*c^2 + 4*B*a*b*c*d + (3*C*a^2 + 2*A*a*b)*d^2)*sqrt(-b)*x^5*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 45*(B*b^2*c^2 + 4*B*a*b*d^2 + 2*(4*C*a*
b + A*b^2)*c*d)*sqrt(a)*x^5*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) - 2*(60*C*a*b*d^2*x^6 + 120*(2*C*a*b*c*d + B*a*b*d^2)*x^5 - 24*A*a^2
*c^2 - 8*(40*B*a*b*c*d + (20*C*a*b + 3*A*b^2)*c^2 + 5*(3*C*a^2 + 4*A*a*b)*
d^2)*x^4 - 15*(5*B*a*b*c^2 + 4*B*a^2*d^2 + 2*(4*C*a^2 + 5*A*a*b)*c*d)*x^3
- 8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 + 6*A*a*b)*c^2)*x^2 - 30*(B*a^2
*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a*x^5), 1/120*(45*(B*b^2*c^2 + 4*
B*a*b*d^2 + 2*(4*C*a*b + A*b^2)*c*d)*sqrt(-a)*x^5*arctan(sqrt(b*x^2 + a)*s
qrt(-a)/a) + 30*(2*C*a*b*c^2 + 4*B*a*b*c*d + (3*C*a^2 + 2*A*a*b)*d^2)*sqrt
(b)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (60*C*a*b*d^2*x^
6 + 120*(2*C*a*b*c*d + B*a*b*d^2)*x^5 - 24*A*a^2*c^2 - 8*(40*B*a*b*c*d ...
```

Sympy [A] (verification not implemented)

Time = 13.04 (sec) , antiderivative size = 1120, normalized size of antiderivative = 2.92

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^6} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**6,x)
```

output

```
-A*sqrt(a)*b*d**2/(x*sqrt(1 + b*x**2/a)) - A*a**2*c*d/(2*sqrt(b)*x**5*sqrt
(a/(b*x**2) + 1)) - A*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*A*a
*sqrt(b)*c*d/(4*x**3*sqrt(a/(b*x**2) + 1)) - A*a*sqrt(b)*d**2*sqrt(a/(b*x*
*2) + 1)/(3*x**2) - 2*A*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(5*x**2) - A*b*
*(3/2)*c*d*sqrt(a/(b*x**2) + 1)/x - A*b**(3/2)*c*d/(4*x*sqrt(a/(b*x**2) +
1)) - A*b**(3/2)*d**2*sqrt(a/(b*x**2) + 1)/3 + A*b**(3/2)*d**2*asinh(sqrt(
b)*x/sqrt(a)) - A*b**(5/2)*c**2*sqrt(a/(b*x**2) + 1)/(5*a) - 3*A*b**2*c*d*
asinh(sqrt(a)/(sqrt(b)*x))/(4*sqrt(a)) - A*b**2*d**2*x/(sqrt(a)*sqrt(1 + b
*x**2/a)) - 2*B*sqrt(a)*b*c*d/(x*sqrt(1 + b*x**2/a)) - 3*B*sqrt(a)*b*d**2*
asinh(sqrt(a)/(sqrt(b)*x))/2 - B*a**2*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2)
 + 1)) - 3*B*a*sqrt(b)*c**2/(8*x**3*sqrt(a/(b*x**2) + 1)) - 2*B*a*sqrt(b)*
c*d*sqrt(a/(b*x**2) + 1)/(3*x**2) - B*a*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/
(2*x) + B*a*sqrt(b)*d**2/(x*sqrt(a/(b*x**2) + 1)) - B*b**(3/2)*c**2*sqrt(a
/(b*x**2) + 1)/(2*x) - B*b**(3/2)*c**2/(8*x*sqrt(a/(b*x**2) + 1)) - 2*B*b*
*(3/2)*c*d*sqrt(a/(b*x**2) + 1)/3 + 2*B*b**(3/2)*c*d*asinh(sqrt(b)*x/sqrt(
a)) + B*b**(3/2)*d**2*x/sqrt(a/(b*x**2) + 1) - 3*B*b**2*c**2*asinh(sqrt(a)
/(sqrt(b)*x))/(8*sqrt(a)) - 2*B*b**2*c*d*x/(sqrt(a)*sqrt(1 + b*x**2/a)) -
C*a**(3/2)*d**2/(x*sqrt(1 + b*x**2/a)) - C*sqrt(a)*b*c**2/(x*sqrt(1 + b*x*
*2/a)) - 3*C*sqrt(a)*b*c*d*asinh(sqrt(a)/(sqrt(b)*x)) - C*sqrt(a)*b*d**2*x
/sqrt(1 + b*x**2/a) - C*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(3*x**2) - ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^6}\,dx = \frac{3}{2}\sqrt{bx^2+a}Cbd^2x + \frac{3}{2}Ca\sqrt{b}d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}Cd^2}{x} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b^2x}{a} + (Cc^2+2\,Bcd+Ad^2)b^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}\left(2\,Ccd+Bd^2\right)\sqrt{a}b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{3\,(Bc^2+2\,Acd)b^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\sqrt{a}} + \frac{3}{2}\left(2\,Ccd+Bd^2\right)\sqrt{bx^2+a}b + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b}{2\,a} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b^2}{8\,a^2} + \frac{3\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b^2}{8\,a} - \frac{2\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b}{3\,ax} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}}{2\,ax^2} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{5\,ax^5} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}}{3\,ax^3} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}}{4\,ax^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="maxima
")
```

output
```
3/2*sqrt(b*x^2 + a)*C*b*d^2*x + 3/2*C*a*sqrt(b)*d^2*arcsinh(b*x/sqrt(a*b))
 - (b*x^2 + a)^(3/2)*C*d^2/x + (C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*b
^2*x/a + (C*c^2 + 2*B*c*d + A*d^2)*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/2*(2
*C*c*d + B*d^2)*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) - 3/8*(B*c^2 + 2*A
*c*d)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 3/2*(2*C*c*d + B*d^2)*sq
rt(b*x^2 + a)*b + 1/2*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b/a + 1/8*(B*c^2
 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b^2/a^2 + 3/8*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 +
 a)*b^2/a - 2/3*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(3/2)*b/(a*x) - 1/2*
(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)/(a*x^2) - 1/8*(B*c^2 + 2*A*c*d)*(b*x^2
 + a)^(5/2)*b/(a^2*x^2) - 1/5*(b*x^2 + a)^(5/2)*A*c^2/(a*x^5) - 1/3*(C*c^2
 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)/(a*x^3) - 1/4*(B*c^2 + 2*A*c*d)*(b*x
^2 + a)^(5/2)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1236 vs. $2(339) = 678$.

Time = 0.24 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.23

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^6} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x, algorithm="giac")
```

output
```
-1/2*(2*C*b^(3/2)*c^2 + 4*B*b^(3/2)*c*d + 3*C*a*sqrt(b)*d^2 + 2*A*b^(3/2)*
d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(C*b*d^2*x + 4*C*b*c*d +
 2*B*b*d^2)*sqrt(b*x^2 + a) + 3/4*(B*b^2*c^2 + 8*C*a*b*c*d + 2*A*b^2*c*d +
 4*B*a*b*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 1
/60*(75*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*b^2*c^2 + 120*(sqrt(b)*x - sqrt(
b*x^2 + a))^9*C*a*b*c*d + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*b^2*c*d +
60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a*b*d^2 + 240*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*C*a*b^(3/2)*c^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2)*c
^2 + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a*b^(3/2)*c*d + 120*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*C*a^2*sqrt(b)*d^2 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))
^8*A*a*b^(3/2)*d^2 - 30*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2*c^2 - 240*
(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^2*b*c*d - 60*(sqrt(b)*x - sqrt(b*x^2 +
 a))^7*A*a*b^2*c*d - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a^2*b*d^2 - 720
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2)*c^2 - 1440*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*B*a^2*b^(3/2)*c*d - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^3*sqrt(b)*d^2 - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(3/2)*d^2 +
880*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*b^(3/2)*c^2 + 240*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*A*a^2*b^(5/2)*c^2 + 1760*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*B*a^3*b^(3/2)*c*d + 720*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b)*d^2
 + 880*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(3/2)*d^2 + 30*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^6} \, dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^6} \, dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^6,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^6, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^6}\,dx = \int \frac{(dx+c)^2 (b\,x^2+a)^{\frac{3}{2}} (C\,x^2+Bx+A)}{x^6}dx$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x)
```

output
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6,x)
```

3.71 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 330

$$\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7}\,dx = bCd^2\sqrt{a+bx^2} - \frac{aAc^2\sqrt{a+bx^2}}{6x^6} - \frac{(6ac(cC+2Bd)+A(7bc^2+6ad^2))\sqrt{a+bx^2}}{24x^4} - \frac{(Ab(bc^2+10ad^2)+2a(4aCd^2+5bc(cC+2Bd)))\sqrt{a+bx^2}}{16ax^2} - \frac{bd(2cC+Bd)\sqrt{a+bx^2}}{x} - \frac{d(2cC+Bd)(a+bx^2)^{3/2}}{3x^3} - \frac{c(Bc+2Ad)(a+bx^2)^{5/2}}{5ax^5} + b^{3/2}d(2cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) + \frac{b(Ab(bc^2-6ad^2)-6a(4aCd^2+bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
b*C*d^2*(b*x^2+a)^(1/2)-1/6*a*A*c^2*(b*x^2+a)^(1/2)/x^6-1/24*(6*a*c*(2*B*d
+C*c)+A*(6*a*d^2+7*b*c^2))*(b*x^2+a)^(1/2)/x^4-1/16*(A*b*(10*a*d^2+b*c^2)+
2*a*(4*a*C*d^2+5*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a/x^2-b*d*(B*d+2*C*c)*(
b*x^2+a)^(1/2)/x-1/3*d*(B*d+2*C*c)*(b*x^2+a)^(3/2)/x^3-1/5*c*(2*A*d+B*c)*(
b*x^2+a)^(5/2)/a/x^5+b^(3/2)*d*(B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/
2))+1/16*b*(A*b*(-6*a*d^2+b*c^2)-6*a*(4*a*C*d^2+b*c*(2*B*d+C*c)))*arctanh(
(b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 3.86 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7} dx =$$
$$-\frac{\sqrt{a+bx^2}(3b^2cx^4(5Ac+16Bcx+32Adx)+4a^2(A(10c^2+24cdx+15d^2x^2)+x(5Cx(3c^2+8cdx+6d^2x}{}$$
$$+\frac{b(-Ab^2c^2+24a^2Cd^2)\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$
$$-\frac{3b^2(c^2C+2Bcd+Ad^2)\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}}$$
$$-b^{3/2}d(2cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^7,x]
```

output
```
-1/240*(Sqrt[a + b*x^2]*(3*b^2*c*x^4*(5*A*c + 16*B*c*x + 32*A*d*x) + 4*a^2
*(A*(10*c^2 + 24*c*d*x + 15*d^2*x^2) + x*(5*C*x*(3*c^2 + 8*c*d*x + 6*d^2*x
^2) + 2*B*(6*c^2 + 15*c*d*x + 10*d^2*x^2))) + 2*a*b*x^2*(A*(35*c^2 + 96*c*
d*x + 75*d^2*x^2) + x*(5*C*x*(15*c^2 + 64*c*d*x - 24*d^2*x^2) + 2*B*(24*c^
2 + 75*c*d*x + 80*d^2*x^2)))))/(a*x^6) + (b*(-(A*b^2*c^2) + 24*a^2*C*d^2)*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(8*a^(3/2)) - (3*b^2*(c^2*
C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(4
*Sqrt[a]) - b^(3/2)*d*(2*c*C + B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2338, 25, 2338, 27, 2338, 25, 27, 537, 27, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^7}\,dx$$

2338

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(6aCd^2x^3+6ad(2cC+Bd)x^2+\left(6ac(cC+2Bd)-A\left(bc^2-6ad^2\right)\right)x+6ac(Bc+2Ad)\right)}{x^6}dx}{6a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

25

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(6aCd^2x^3+6ad(2cC+Bd)x^2+\left(6ac(cC+2Bd)-A\left(bc^2-6ad^2\right)\right)x+6ac(Bc+2Ad)\right)}{x^6}dx}{6a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

2338

$$\frac{-\frac{\int -\frac{5\left(bx^2+a\right)^{3/2}\left(6Cd^2x^2a^2+6d(2cC+Bd)xa^2+\left(6ac(cC+2Bd)-A\left(bc^2-6ad^2\right)\right)a\right)}{x^5}dx}{5a}-\frac{6c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{5x^5}}{6a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

27

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(6Cd^2x^2a^2+6d(2cC+Bd)xa^2+\left(6ac(cC+2Bd)-A\left(bc^2-6ad^2\right)\right)a\right)}{x^5}dx}{a}-\frac{6c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{5x^5}}{6a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

2338

$$\frac{\frac{-\frac{\int -\frac{a\left(24a^2d(2cC+Bd)-\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{\left(a+bx^2\right)^{5/2}\left(6ac(2Bd+cC)-A\left(bc^2-6ad^2\right)\right)}{4x^4}}{a}-\frac{6c\left(a+bx^2\right)^{5/2}(}{5x^5}}{6a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

25

$$\frac{\frac{\frac{\int \frac{a\left(24a^2d(2cC+Bd)-\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^4}dx}{4a}-\frac{\left(a+bx^2\right)^{5/2}\left(6ac(2Bd+cC)-A\left(bc^2-6ad^2\right)\right)}{4x^4}}{a}-\frac{6c\left(a+bx^2\right)^{5/2}(2A}{5x^5}}{6a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{4}\int \frac{\left(24a^2d(2cC+Bd)-\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^4}dx-\frac{\left(a+bx^2\right)^{5/2}\left(6ac(2Bd+cC)-A\left(bc^2-6ad^2\right)\right)}{4x^4}}{a}-\frac{6c\left(a+bx^2\right)^{5/2}(2A}{5x^5}}{6a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 537$$

$$\frac{\frac{\frac{1}{4}\left(-\frac{1}{2}b\int -\frac{3\left(16a^2d(2cC+Bd)-\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(16a^2d(Bd+2cC)-x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+\right.\right.\right.}{2x^3}\right.}{a}}{6a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{4}\left(\frac{3}{2}b\int \frac{\left(16a^2d(2cC+Bd)-\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(cC+2Bd)\right)\right)x\right)\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}\left(16a^2d(Bd+2cC)-x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2\right.\right.\right.}{2x^3}\right.}{a}}{6a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 536$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}b\left(\int \frac{16bd(2cC+Bd)xa^2+\left(6a\left(4aCd^2+bc(cC+2Bd)\right)-Ab\left(bc^2-6ad^2\right)\right)a}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}\right.\right.}{a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 538$$

$$\frac{1}{4}\Bigg(\frac{3}{2}b\Bigg(16a^2bd(Bd+2cC)\int\frac{1}{\sqrt{bx^2+a}}dx-a\left(Ab(bc^2-6ad^2)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right.\right.\right.}{x}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{4}\Bigg(\frac{3}{2}b\Bigg(16a^2bd(Bd+2cC)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-a\left(Ab(bc^2-6ad^2)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\Bigg(\frac{3}{2}b\Bigg(-a\left(Ab(bc^2-6ad^2)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+1$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}\Bigg(\frac{3}{2}b\Bigg(-\frac{1}{2}a\left(Ab(bc^2-6ad^2)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\right.}{x}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}\Bigg(\frac{3}{2}b\Bigg(-\frac{a\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}b\left(-\frac{\sqrt{a+bx^2}\left(16a^2d(Bd+2cC)+x\left(Ab\left(bc^2-6ad^2\right)-6a\left(4aCd^2+bc(2Bd+cC)\right)\right)\right)}{x}+16a^2\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)+\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{6ax^6}$$

input

```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^7,x]
```

output

```
-1/6*(A*c^2*(a + b*x^2)^(5/2))/(a*x^6) + ((-6*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(5*x^5) + (-1/4*((6*a*c*(c*C + 2*B*d) - A*(b*c^2 - 6*a*d^2))*(a + b
*x^2)^(5/2))/x^4 + (-1/2*((16*a^2*d*(2*c*C + B*d) - (A*b*(b*c^2 - 6*a*d^2)
 - 6*a*(4*a*C*d^2 + b*c*(c*C + 2*B*d)))*x)*(a + b*x^2)^(3/2))/x^3 + (3*b*(
-(((16*a^2*d*(2*c*C + B*d) + (A*b*(b*c^2 - 6*a*d^2) - 6*a*(4*a*C*d^2 + b*c
*(c*C + 2*B*d)))*x)*Sqrt[a + b*x^2])/x) + 16*a^2*Sqrt[b]*d*(2*c*C + B*d)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] + Sqrt[a]*(A*b*(b*c^2 - 6*a*d^2) - 6*a
*(4*a*C*d^2 + b*c*(c*C + 2*B*d)))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2)/4)
/a)/(6*a)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(96A\,b^2cd\,x^5+320Bab\,d^2x^5+48B\,b^2c^2x^5+640Cabcd\,x^5+150Ab\,d^2x^4a+15A\,b^2c^2x^4+300Bbcd\,x^4a+120C\,a^2d^2x^4+150$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(96*A*b^2*c*d*x^5+320*B*a*b*d^2*x^5+48*B*b^2*c^2*x^
5+640*C*a*b*c*d*x^5+150*A*a*b*d^2*x^4+15*A*b^2*c^2*x^4+300*B*a*b*c*d*x^4+1
20*C*a^2*d^2*x^4+150*C*a*b*c^2*x^4+192*A*a*b*c*d*x^3+80*B*a^2*d^2*x^3+96*B
*a*b*c^2*x^3+160*C*a^2*c*d*x^3+60*A*a^2*d^2*x^2+70*A*a*b*c^2*x^2+120*B*a^2
*c*d*x^2+60*C*a^2*c^2*x^2+96*A*a^2*c*d*x+48*B*a^2*c^2*x+40*A*a^2*c^2)/x^6/
a+1/16*b/a*(-(6*A*a*b*d^2-A*b^2*c^2+12*B*a*b*c*d+24*C*a^2*d^2+6*C*a*b*c^2)
/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+16*B*b^(1/2)*d^2*a*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))+16*C*d^2*a*(b*x^2+a)^(1/2)+32*C*b^(1/2)*c*d*a*ln(b^(
1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 1494, normalized size of antiderivative = 4.53

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="fricas
")
```

output
```
[1/480*(240*(2*C*a^2*b*c*d + B*a^2*b*d^2)*sqrt(b)*x^6*log(-2*b*x^2 - 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 15*(12*B*a*b^2*c*d + (6*C*a*b^2 - A*b^3)*c^2
 + 6*(4*C*a^2*b + A*a*b^2)*d^2)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(240*C*a^2*b*d^2*x^6 - 40*A*a^3*c^2 - 16*(3*B*a*
b^2*c^2 + 20*B*a^2*b*d^2 + 2*(20*C*a^2*b + 3*A*a*b^2)*c*d)*x^5 - 15*(20*B*
a^2*b*c*d + (10*C*a^2*b + A*a*b^2)*c^2 + 2*(4*C*a^3 + 5*A*a^2*b)*d^2)*x^4
- 16*(6*B*a^2*b*c^2 + 5*B*a^3*d^2 + 2*(5*C*a^3 + 6*A*a^2*b)*c*d)*x^3 - 10*
(12*B*a^3*c*d + 6*A*a^3*d^2 + (6*C*a^3 + 7*A*a^2*b)*c^2)*x^2 - 48*(B*a^3*c
^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^2*x^6), -1/480*(480*(2*C*a^2*b*c*
d + B*a^2*b*d^2)*sqrt(-b)*x^6*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 15*(12*
B*a*b^2*c*d + (6*C*a*b^2 - A*b^3)*c^2 + 6*(4*C*a^2*b + A*a*b^2)*d^2)*sqrt(
a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(240*C*a^2*
b*d^2*x^6 - 40*A*a^3*c^2 - 16*(3*B*a*b^2*c^2 + 20*B*a^2*b*d^2 + 2*(20*C*a^
2*b + 3*A*a*b^2)*c*d)*x^5 - 15*(20*B*a^2*b*c*d + (10*C*a^2*b + A*a*b^2)*c^
2 + 2*(4*C*a^3 + 5*A*a^2*b)*d^2)*x^4 - 16*(6*B*a^2*b*c^2 + 5*B*a^3*d^2 + 2
*(5*C*a^3 + 6*A*a^2*b)*c*d)*x^3 - 10*(12*B*a^3*c*d + 6*A*a^3*d^2 + (6*C*a^
3 + 7*A*a^2*b)*c^2)*x^2 - 48*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))
/(a^2*x^6), 1/240*(15*(12*B*a*b^2*c*d + (6*C*a*b^2 - A*b^3)*c^2 + 6*(4*C*a
^2*b + A*a*b^2)*d^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 120
*(2*C*a^2*b*c*d + B*a^2*b*d^2)*sqrt(b)*x^6*log(-2*b*x^2 - 2*sqrt(b*x^2 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(314) = 628$.

Time = 19.20 (sec) , antiderivative size = 1114, normalized size of antiderivative = 3.38

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7} dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**7,x)
```

output
```
-A*a**2*c**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - A*a**2*d**2/(4*sqrt(b
)*x**5*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)*c**2/(24*x**5*sqrt(a/(b*x**2
) + 1)) - 2*A*a*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*A*a*sqrt(b)*
d**2/(8*x**3*sqrt(a/(b*x**2) + 1)) - 17*A*b**(3/2)*c**2/(48*x**3*sqrt(a/(b
*x**2) + 1)) - 4*A*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(5*x**2) - A*b**(3/2)
*d**2*sqrt(a/(b*x**2) + 1)/(2*x) - A*b**(3/2)*d**2/(8*x*sqrt(a/(b*x**2) +
1)) - A*b**(5/2)*c**2/(16*a*x*sqrt(a/(b*x**2) + 1)) - 2*A*b**(5/2)*c*d*sqr
t(a/(b*x**2) + 1)/(5*a) - 3*A*b**2*d**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt
(a)) + A*b**3*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2)) - B*sqrt(a)*b*
d**2/(x*sqrt(1 + b*x**2/a)) - B*a**2*c*d/(2*sqrt(b)*x**5*sqrt(a/(b*x**2) +
 1)) - B*a*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*B*a*sqrt(b)*c*d/
(4*x**3*sqrt(a/(b*x**2) + 1)) - B*a*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(3*x
**2) - 2*B*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(5*x**2) - B*b**(3/2)*c*d*sq
rt(a/(b*x**2) + 1)/x - B*b**(3/2)*c*d/(4*x*sqrt(a/(b*x**2) + 1)) - B*b**(3
/2)*d**2*sqrt(a/(b*x**2) + 1)/3 + B*b**(3/2)*d**2*asinh(sqrt(b)*x/sqrt(a))
 - B*b**(5/2)*c**2*sqrt(a/(b*x**2) + 1)/(5*a) - 3*B*b**2*c*d*asinh(sqrt(a)
/(sqrt(b)*x))/(4*sqrt(a)) - B*b**2*d**2*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - 2
*C*sqrt(a)*b*c*d/(x*sqrt(1 + b*x**2/a)) - 3*C*sqrt(a)*b*d**2*asinh(sqrt(a)
/(sqrt(b)*x))/2 - C*a**2*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*C*
a*sqrt(b)*c**2/(8*x**3*sqrt(a/(b*x**2) + 1)) - 2*C*a*sqrt(b)*c*d*sqrt(a...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7}\, dx = \frac{Ab^3c^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{3}{2}\,C\sqrt{a}bd^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) - \frac{(bx^2+a)^{\frac{3}{2}}Ab^3c^2}{48\,a^3} - \frac{\sqrt{bx^2+a}Ab^3c^2}{16\,a^2} + \frac{3}{2}\sqrt{bx^2+a}Cbd^2 + \frac{(bx^2+a)^{\frac{3}{2}}Cbd^2}{2\,a} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}b^2x}{a} + (2\,Ccd+Bd^2)b^{\frac{3}{2}} \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3\,(Cc^2+2\,Bcd+Ad^2)b^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{3}{2}}b^2}{8\,a^2} + \frac{3\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b^2}{8\,a} + \frac{(bx^2+a)^{\frac{5}{2}}Ab^2c^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cd^2}{2\,ax^2} - \frac{2\,(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b}{3\,ax} + \frac{(bx^2+a)^{\frac{5}{2}}Abc^2}{24\,a^2x^4} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Bc^2}{5\,ax^5} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Acd}{5\,ax^5} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}}{3\,ax^3} - \frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{6\,ax^6} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}}{4\,ax^4}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="maxima
")
```

output
```
1/16*A*b^3*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/2*C*sqrt(a)*b*d^2
*arcsinh(a/(sqrt(a*b)*abs(x))) - 1/48*(b*x^2 + a)^(3/2)*A*b^3*c^2/a^3 - 1/
16*sqrt(b*x^2 + a)*A*b^3*c^2/a^2 + 3/2*sqrt(b*x^2 + a)*C*b*d^2 + 1/2*(b*x^
2 + a)^(3/2)*C*b*d^2/a + (2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*b^2*x/a + (2*C*
c*d + B*d^2)*b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 3/8*(C*c^2 + 2*B*c*d + A*d^2
)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(C*c^2 + 2*B*c*d + A*d^2
)*(b*x^2 + a)^(3/2)*b^2/a^2 + 3/8*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a
)*b^2/a + 1/48*(b*x^2 + a)^(5/2)*A*b^2*c^2/(a^3*x^2) - 1/2*(b*x^2 + a)^(5/
2)*C*d^2/(a*x^2) - 2/3*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b/(a*x) + 1/24*
(b*x^2 + a)^(5/2)*A*b*c^2/(a^2*x^4) - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2
 + a)^(5/2)*b/(a^2*x^2) - 1/5*(b*x^2 + a)^(5/2)*B*c^2/(a*x^5) - 2/5*(b*x^2
 + a)^(5/2)*A*c*d/(a*x^5) - 1/3*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)/(a*x^3
) - 1/6*(b*x^2 + a)^(5/2)*A*c^2/(a*x^6) - 1/4*(C*c^2 + 2*B*c*d + A*d^2)*(b
*x^2 + a)^(5/2)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1711 vs. $2(292) = 584$.

Time = 0.24 (sec) , antiderivative size = 1711, normalized size of antiderivative = 5.18

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*C*b*d^2 - (2*C*b^(3/2)*c*d + B*b^(3/2)*d^2)*log(abs(-sqrt(
b)*x + sqrt(b*x^2 + a))) + 1/8*(6*C*a*b^2*c^2 - A*b^3*c^2 + 12*B*a*b^2*c*d
 + 24*C*a^2*b*d^2 + 6*A*a*b^2*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a) + 1/120*(150*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^
2*c^2 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*b^3*c^2 + 300*(sqrt(b)*x - s
qrt(b*x^2 + a))^11*B*a*b^2*c*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^
2*b*d^2 + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a*b^2*d^2 + 240*(sqrt(b)*
x - sqrt(b*x^2 + a))^10*B*a*b^(5/2)*c^2 + 960*(sqrt(b)*x - sqrt(b*x^2 + a)
)^10*C*a^2*b^(3/2)*c*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(5/2)*
c*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(3/2)*d^2 - 210*(sqrt(b
)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2*c^2 + 235*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*A*a*b^3*c^2 - 420*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^2*c*d - 360*(
sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^3*b*d^2 - 210*(sqrt(b)*x - sqrt(b*x^2 +
 a))^9*A*a^2*b^2*d^2 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*b^(5/2)*c
^2 - 3840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2)*c*d - 480*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*A*a^2*b^(5/2)*c*d - 1920*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*B*a^3*b^(3/2)*d^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^3*b^2*c^
2 + 390*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c^2 + 120*(sqrt(b)*x - s
qrt(b*x^2 + a))^7*B*a^3*b^2*c*d + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^
4*b*d^2 + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^3*b^2*d^2 + 480*(sqrt(...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^7}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^7}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^7,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 876, normalized size of antiderivative = 2.65

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^7} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7,x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c**2 - 96*sqrt(a + b*x**2)*a**3*c*d*x - 60*sq
rt(a + b*x**2)*a**3*d**2*x**2 - 70*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 48*
sqrt(a + b*x**2)*a**2*b*c**2*x - 192*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 12
0*sqrt(a + b*x**2)*a**2*b*c*d*x**2 - 150*sqrt(a + b*x**2)*a**2*b*d**2*x**4
 - 80*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 60*sqrt(a + b*x**2)*a**2*c**3*x*
*2 - 160*sqrt(a + b*x**2)*a**2*c**2*d*x**3 - 120*sqrt(a + b*x**2)*a**2*c*d
**2*x**4 - 15*sqrt(a + b*x**2)*a*b**2*c**2*x**4 - 96*sqrt(a + b*x**2)*a*b*
*2*c**2*x**3 - 96*sqrt(a + b*x**2)*a*b**2*c*d*x**5 - 300*sqrt(a + b*x**2)*
a*b**2*c*d*x**4 - 320*sqrt(a + b*x**2)*a*b**2*d**2*x**5 - 150*sqrt(a + b*x
**2)*a*b*c**3*x**4 - 640*sqrt(a + b*x**2)*a*b*c**2*d*x**5 + 240*sqrt(a + b
*x**2)*a*b*c*d**2*x**6 - 48*sqrt(a + b*x**2)*b**3*c**2*x**5 + 90*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**6 + 36
0*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2
*x**6 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b
**3*c**2*x**6 + 180*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**3*c*d*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*b**2*c**3*x**6 - 90*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**6 - 360*sqrt(a)*log((sqrt(a + b*x**2
) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2*x**6 + 15*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**6 - 180*sqrt(a)...
```

3.72 $\int \frac{(c+dx)^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^8}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 323

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx=\frac{b(bc(Bc+2Ad)-6ad(2cC+Bd))\sqrt{a+bx^2}}{16ax^2}$$
$$-\frac{bCd^2\sqrt{a+bx^2}}{x}+\frac{(bc(Bc+2Ad)-6ad(2cC+Bd))\left(a+bx^2\right)^{3/2}}{24ax^4}$$
$$-\frac{Cd^2\left(a+bx^2\right)^{3/2}}{3x^3}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}-\frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{6ax^6}$$
$$-\frac{(7ac(cC+2Bd)-A(2bc^2-7ad^2))\left(a+bx^2\right)^{5/2}}{35a^2x^5}$$
$$+b^{3/2}Cd^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)+\frac{b^2(bc(Bc+2Ad)-6ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
1/16*b*(b*c*(2*A*d+B*c)-6*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a/x^2-b*C*d^2*(
b*x^2+a)^(1/2)/x+1/24*(b*c*(2*A*d+B*c)-6*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/
a/x^4-1/3*C*d^2*(b*x^2+a)^(3/2)/x^3-1/7*A*c^2*(b*x^2+a)^(5/2)/a/x^7-1/6*c*
(2*A*d+B*c)*(b*x^2+a)^(5/2)/a/x^6-1/35*(7*a*c*(2*B*d+C*c)-A*(-7*a*d^2+2*b*
c^2))*(b*x^2+a)^(5/2)/a^2/x^5+b^(3/2)*C*d^2*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))+1/16*b^2*(b*c*(2*A*d+B*c)-6*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/
a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx =$$

$$-\frac{\sqrt{a+bx^2}(-96Ab^3c^2x^6+3ab^2x^4(7cx(5Bc+16cCx+32Bdx)+2A(8c^2+35cdx+56d^2x^2))+4a^3(4A(1}{}$$

$$+\frac{b^2(bc(Bc+2Ad)-6ad(2cC+Bd))\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

$$-b^{3/2}Cd^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^8,x]
```

output
```
-1/1680*(Sqrt[a + b*x^2]*(-96*A*b^3*c^2*x^6 + 3*a*b^2*x^4*(7*c*x*(5*B*c +
16*c*C*x + 32*B*d*x) + 2*A*(8*c^2 + 35*c*d*x + 56*d^2*x^2)) + 4*a^3*(4*A*(
15*c^2 + 35*c*d*x + 21*d^2*x^2) + 7*x*(2*C*x*(6*c^2 + 15*c*d*x + 10*d^2*x^
2) + B*(10*c^2 + 24*c*d*x + 15*d^2*x^2))) + 2*a^2*b*x^2*(2*A*(96*c^2 + 245
*c*d*x + 168*d^2*x^2) + 7*x*(B*(35*c^2 + 96*c*d*x + 75*d^2*x^2) + 2*C*x*(2
4*c^2 + 75*c*d*x + 80*d^2*x^2)))))/(a^2*x^7) + (b^2*(b*c*(B*c + 2*A*d) - 6
*a*d*(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(8*
a^(3/2)) - b^(3/2)*C*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.97, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2338, 25, 2338, 25, 2338, 27, 537, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^8}\,dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(7aCd^2x^3+7ad(2cC+Bd)x^2+\left(7ac(cC+2Bd)-A\left(2bc^2-7ad^2\right)\right)x+7ac(Bc+2Ad)\right)}{x^7}dx}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(7aCd^2x^3+7ad(2cC+Bd)x^2+\left(7ac(cC+2Bd)-A\left(2bc^2-7ad^2\right)\right)x+7ac(Bc+2Ad)\right)}{x^7}dx}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(42a^2Cd^2x^2-7a(bc(Bc+2Ad)-6ad(2cC+Bd))x+6a\left(7ac(cC+2Bd)-A\left(2bc^2-7ad^2\right)\right)\right)}{x^6}dx}{6a}-\frac{7c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(42a^2Cd^2x^2-7a(bc(Bc+2Ad)-6ad(2cC+Bd))x+6a\left(7ac(cC+2Bd)-A\left(2bc^2-7ad^2\right)\right)\right)}{x^6}dx}{6a}-\frac{7c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{\frac{-\frac{\int \frac{35a^2\left(-6aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)\right)\left(bx^2+a\right)^{3/2}}{x^5}dx}{5a}-\frac{6\left(a+bx^2\right)^{5/2}\left(7ac(2Bd+cC)-A\left(2bc^2-7ad^2\right)\right)}{5x^5}}{6a}-\frac{7c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{-7a\int \frac{\left(-6aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)\right)\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{6\left(a+bx^2\right)^{5/2}\left(7ac(2Bd+cC)-A\left(2bc^2-7ad^2\right)\right)}{5x^5}}{6a}-\frac{7c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{6x^6}}{7a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{537}$$

$$\frac{-7a\left(-\frac{1}{4}b\int -\frac{3\left(-8aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)\right)\sqrt{bx^2+a}}{x^3}dx-\frac{\left(a+bx^2\right)^{3/2}\left(-6ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{4x^4}\right)-\frac{6\left(a+bx^2\right)^{5/2}\left(7ac(2Bd+\right.}{5x^5}}{6a}$$
$$7a$$
$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{27}$$

$$\frac{-7a\left(\frac{3}{4}b\int \frac{\left(-8aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)\right)\sqrt{bx^2+a}}{x^3}dx-\frac{\left(a+bx^2\right)^{3/2}\left(-6ad(Bd+2cC)-8aCd^2x+bc(2Ad+Bc)\right)}{4x^4}\right)-\frac{6\left(a+bx^2\right)^{5/2}\left(7ac(2Bd+cC)-\right.}{5x^5}}{6a}$$
$$7a$$
$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{537}$$

$$\frac{-7a\left(\frac{3}{4}b\left(-\frac{1}{2}b\int -\frac{-16aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}\left(-6ad(Bd+2cC)-8aC\right.}{4x^4}}{6a}$$
$$7a$$
$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{25}$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{-16aCxd^2-6a(2cC+Bd)d+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}\left(-6ad(Bd+2cC)-8aCd^2\right.}{4x^4}}{6a}$$
$$7a$$
$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{538}$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{1}{2}b\left((bc(2Ad+Bc)-6ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-16aCd^2\int \frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}\right)-\left(a+bx\right.}{6a}$$
$$7a$$
$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\Big\downarrow \text{224}$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{\frac{1}{2}b\left((bc(2Ad+Bc)-6ad(Bd+2cC))\int\frac{1}{x\sqrt{bx^2+a}}dx-16aCd^2\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{6a}\right)\right)}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\downarrow 219$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{\frac{1}{2}b\left((bc(2Ad+Bc)-6ad(Bd+2cC))\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{16aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{6a}\right)\right)}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\downarrow 243$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{\frac{1}{2}b\left(\frac{1}{2}(bc(2Ad+Bc)-6ad(Bd+2cC))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{16aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{6a}\right)\right)}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\downarrow 73$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{\frac{1}{2}b\left(\frac{(bc(2Ad+Bc)-6ad(Bd+2cC))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{16aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{6a}\right)\right)}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

$$\downarrow 221$$

$$\frac{-7a\left(\frac{3}{4}b\left(\frac{\frac{1}{2}b\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bc(2Ad+Bc)-6ad(Bd+2cC))}{\sqrt{a}}-\frac{16aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}\left(-6ad(Bd+2cC)-16aCd^2x+bc(2Ad+Bc)\right)}{2x^2}}{6a}\right)\right)}{7a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{7ax^7}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^8,x]
```

output
```
-1/7*(A*c^2*(a + b*x^2)^(5/2))/(a*x^7) + ((-7*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(6*x^6) + ((-6*(7*a*c*(c*C + 2*B*d) - A*(2*b*c^2 - 7*a*d^2))*(a + b
*x^2)^(5/2))/(5*x^5) - 7*a*(-1/4*((b*c*(B*c + 2*A*d) - 6*a*d*(2*c*C + B*d)
 - 8*a*C*d^2*x)*(a + b*x^2)^(3/2))/x^4 + (3*b*(-1/2*((b*c*(B*c + 2*A*d) -
6*a*d*(2*c*C + B*d) - 16*a*C*d^2*x)*Sqrt[a + b*x^2])/x^2 + (b*((-16*a*C*d^
2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((b*c*(B*c + 2*A*d) - 6*
a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4))/(6*
a))/(7*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(336 A a b^2 d^2 x^6-96 A b^3 c^2 x^6+672 B a b^2 c d x^6+2240 C a^2 b d^2 x^6+336 C a b^2 c^2 x^6+210 A a b^2 c d x^5+1050 B a^2 b d^2 x^5+105 B$
default	$-\frac{(A d^2+2Bcd+C c^2)(b x^2+a)^{\frac{5}{2}}}{5a x^5}+A c^2\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{7a x^7}+\frac{2b(b x^2+a)^{\frac{5}{2}}}{35a^2 x^5}\right)+C d^2\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{3a x^3}+\frac{2b\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{a x}\right.}{}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/1680*(b*x^2+a)^(1/2)*(336*A*a*b^2*d^2*x^6-96*A*b^3*c^2*x^6+672*B*a*b^2*
c*d*x^6+2240*C*a^2*b*d^2*x^6+336*C*a*b^2*c^2*x^6+210*A*a*b^2*c*d*x^5+1050*
B*a^2*b*d^2*x^5+105*B*a*b^2*c^2*x^5+2100*C*a^2*b*c*d*x^5+672*A*a^2*b*d^2*x
^4+48*A*a*b^2*c^2*x^4+1344*B*a^2*b*c*d*x^4+560*C*a^3*d^2*x^4+672*C*a^2*b*c
^2*x^4+980*A*a^2*b*c*d*x^3+420*B*a^3*d^2*x^3+490*B*a^2*b*c^2*x^3+840*C*a^3
*c*d*x^3+336*A*a^3*d^2*x^2+384*A*a^2*b*c^2*x^2+672*B*a^3*c*d*x^2+336*C*a^3
*c^2*x^2+560*A*a^3*c*d*x+280*B*a^3*c^2*x+240*A*a^3*c^2)/x^7/a^2-1/16/a*b^2
*(-(2*A*b*c*d-6*B*a*d^2+B*b*c^2-12*C*a*c*d)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)-16*a*C*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 1568, normalized size of antiderivative = 4.85

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^8} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="fricas
")
```

output
```
[1/3360*(1680*C*a^2*b^(3/2)*d^2*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) - 105*(B*b^3*c^2 - 6*B*a*b^2*d^2 - 2*(6*C*a*b^2 - A*b^3)*c*d)*sq
rt(a)*x^7*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*(42*
B*a*b^2*c*d + 3*(7*C*a*b^2 - 2*A*b^3)*c^2 + 7*(20*C*a^2*b + 3*A*a*b^2)*d^2
)*x^6 + 240*A*a^3*c^2 + 105*(B*a*b^2*c^2 + 10*B*a^2*b*d^2 + 2*(10*C*a^2*b
+ A*a*b^2)*c*d)*x^5 + 16*(84*B*a^2*b*c*d + 3*(14*C*a^2*b + A*a*b^2)*c^2 +
7*(5*C*a^3 + 6*A*a^2*b)*d^2)*x^4 + 70*(7*B*a^2*b*c^2 + 6*B*a^3*d^2 + 2*(6*
C*a^3 + 7*A*a^2*b)*c*d)*x^3 + 48*(14*B*a^3*c*d + 7*A*a^3*d^2 + (7*C*a^3 +
8*A*a^2*b)*c^2)*x^2 + 280*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a
^2*x^7), -1/3360*(3360*C*a^2*sqrt(-b)*b*d^2*x^7*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) + 105*(B*b^3*c^2 - 6*B*a*b^2*d^2 - 2*(6*C*a*b^2 - A*b^3)*c*d)*sqr
t(a)*x^7*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(42*B
*a*b^2*c*d + 3*(7*C*a*b^2 - 2*A*b^3)*c^2 + 7*(20*C*a^2*b + 3*A*a*b^2)*d^2)
*x^6 + 240*A*a^3*c^2 + 105*(B*a*b^2*c^2 + 10*B*a^2*b*d^2 + 2*(10*C*a^2*b +
 A*a*b^2)*c*d)*x^5 + 16*(84*B*a^2*b*c*d + 3*(14*C*a^2*b + A*a*b^2)*c^2 + 7
*(5*C*a^3 + 6*A*a^2*b)*d^2)*x^4 + 70*(7*B*a^2*b*c^2 + 6*B*a^3*d^2 + 2*(6*C
*a^3 + 7*A*a^2*b)*c*d)*x^3 + 48*(14*B*a^3*c*d + 7*A*a^3*d^2 + (7*C*a^3 + 8
*A*a^2*b)*c^2)*x^2 + 280*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^
2*x^7), 1/1680*(840*C*a^2*b^(3/2)*d^2*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 105*(B*b^3*c^2 - 6*B*a*b^2*d^2 - 2*(6*C*a*b^2 - A*b^3...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(303) = 606$.

Time = 22.11 (sec) , antiderivative size = 1435, normalized size of antiderivative = 4.44

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^8}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**8,x)
```

output
```
-15*A*a**6*b**(9/2)*c**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a*
*4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*c**2*x**2*sqrt(a
/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x*
*10) - 17*A*a**4*b**(13/2)*c**2*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x
**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 3*A*a**3*b**(15/2)*c**2*
x**6*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a
**3*b**6*x**10) - 12*A*a**2*b**(17/2)*c**2*x**8*sqrt(a/(b*x**2) + 1)/(105*
a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a**2*c*d/(3
*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 8*A*a*b**(19/2)*c**2*x**10*sqrt(a/(b
*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10
) - 11*A*a*sqrt(b)*c*d/(12*x**5*sqrt(a/(b*x**2) + 1)) - A*a*sqrt(b)*d**2*s
qrt(a/(b*x**2) + 1)/(5*x**4) - A*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(5*x**
4) - 17*A*b**(3/2)*c*d/(24*x**3*sqrt(a/(b*x**2) + 1)) - 2*A*b**(3/2)*d**2*
sqrt(a/(b*x**2) + 1)/(5*x**2) - A*b**(5/2)*c**2*sqrt(a/(b*x**2) + 1)/(15*a
*x**2) - A*b**(5/2)*c*d/(8*a*x*sqrt(a/(b*x**2) + 1)) - A*b**(5/2)*d**2*sqr
t(a/(b*x**2) + 1)/(5*a) + 2*A*b**(7/2)*c**2*sqrt(a/(b*x**2) + 1)/(15*a**2)
 + A*b**3*c*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(3/2)) - B*a**2*c**2/(6*sqr
t(b)*x**7*sqrt(a/(b*x**2) + 1)) - B*a**2*d**2/(4*sqrt(b)*x**5*sqrt(a/(b*x*
*2) + 1)) - 11*B*a*sqrt(b)*c**2/(24*x**5*sqrt(a/(b*x**2) + 1)) - 2*B*a*sqr
t(b)*c*d*sqrt(a/(b*x**2) + 1)/(5*x**4) - 3*B*a*sqrt(b)*d**2/(8*x**3*sqr...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^8}\,dx = \frac{\sqrt{bx^2+a}Cb^2d^2x}{a} + Cb^{\frac{3}{2}}d^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{2\,(bx^2+a)^{\frac{3}{2}}Cbd^2}{3\,ax} - \frac{3\,(2\,Ccd+Bd^2)b^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{(Bc^2+2\,Acd)b^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b^2}{8\,a^2} + \frac{3\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}b^2}{8\,a} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b^3}{48\,a^3} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}b^3}{16\,a^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cd^2}{3\,ax^3} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}b}{8\,a^2x^2} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b^2}{48\,a^3x^2} - \frac{(bx^2+a)^{\frac{5}{2}}Cc^2}{5\,ax^5} + \frac{2\,(bx^2+a)^{\frac{5}{2}}Abc^2}{35\,a^2x^5} - \frac{2\,(bx^2+a)^{\frac{5}{2}}Bcd}{5\,ax^5} - \frac{(bx^2+a)^{\frac{5}{2}}Ad^2}{5\,ax^5} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}}{4\,ax^4} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{7\,ax^7} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}}{6\,ax^6}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="maxima
")
```

output

```
sqrt(b*x^2 + a)*C*b^2*d^2*x/a + C*b^(3/2)*d^2*arcsinh(b*x/sqrt(a*b)) - 2/3
*(b*x^2 + a)^(3/2)*C*b*d^2/(a*x) - 3/8*(2*C*c*d + B*d^2)*b^2*arcsinh(a/(sq
rt(a*b)*abs(x)))/sqrt(a) + 1/16*(B*c^2 + 2*A*c*d)*b^3*arcsinh(a/(sqrt(a*b)
*abs(x)))/a^(3/2) + 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(3/2)*b^2/a^2 + 3/8*
(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*b^2/a - 1/48*(B*c^2 + 2*A*c*d)*(b*x^2 +
a)^(3/2)*b^3/a^3 - 1/16*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b^3/a^2 - 1/3*(b
*x^2 + a)^(5/2)*C*d^2/(a*x^3) - 1/8*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*b/
(a^2*x^2) + 1/48*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b^2/(a^3*x^2) - 1/5*(
b*x^2 + a)^(5/2)*C*c^2/(a*x^5) + 2/35*(b*x^2 + a)^(5/2)*A*b*c^2/(a^2*x^5)
- 2/5*(b*x^2 + a)^(5/2)*B*c*d/(a*x^5) - 1/5*(b*x^2 + a)^(5/2)*A*d^2/(a*x^5
) - 1/4*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)/(a*x^4) + 1/24*(B*c^2 + 2*A*c*
d)*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1/7*(b*x^2 + a)^(5/2)*A*c^2/(a*x^7) - 1
/6*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)/(a*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1784 vs. $2(283) = 566$.

Time = 0.18 (sec) , antiderivative size = 1784, normalized size of antiderivative = 5.52

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^8}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x, algorithm="giac")
```

output
```
-C*b^(3/2)*d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) - 1/8*(B*b^3*c^2 - 1
2*C*a*b^2*c*d + 2*A*b^3*c*d - 6*B*a*b^2*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x
^2 + a))/sqrt(-a))/(sqrt(-a)*a) + 1/840*(105*(sqrt(b)*x - sqrt(b*x^2 + a))
^13*B*b^3*c^2 + 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a*b^2*c*d + 210*(s
qrt(b)*x - sqrt(b*x^2 + a))^13*A*b^3*c*d + 1050*(sqrt(b)*x - sqrt(b*x^2 +
a))^13*B*a*b^2*d^2 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a*b^(5/2)*c^2
 + 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a*b^(5/2)*c*d + 3360*(sqrt(b)*x
 - sqrt(b*x^2 + a))^12*C*a^2*b^(3/2)*d^2 + 1680*(sqrt(b)*x - sqrt(b*x^2 +
a))^12*A*a*b^(5/2)*d^2 + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^3*c^2
 - 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b^2*c*d + 3080*(sqrt(b)*x -
 sqrt(b*x^2 + a))^11*A*a*b^3*c*d - 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B
*a^2*b^2*d^2 - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^2*b^(5/2)*c^2 + 3
360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2)*c^2 - 6720*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*B*a^2*b^(5/2)*c*d - 16800*(sqrt(b)*x - sqrt(b*x^2 + a))
^10*C*a^3*b^(3/2)*d^2 - 3360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^2*b^(5/2
)*d^2 + 1085*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^3*c^2 + 3780*(sqrt(b)
*x - sqrt(b*x^2 + a))^9*C*a^3*b^2*c*d + 2170*(sqrt(b)*x - sqrt(b*x^2 + a))
^9*A*a^2*b^3*c*d + 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a^3*b^2*d^2 + 50
40*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(5/2)*c^2 + 3360*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*A*a^2*b^(7/2)*c^2 + 10080*(sqrt(b)*x - sqrt(b*x^2 + a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^8}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^8}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^8,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 898, normalized size of antiderivative = 2.78

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^8} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^8,x)
```

output
```
( - 240*sqrt(a + b*x**2)*a**4*c**2 - 560*sqrt(a + b*x**2)*a**4*c*d*x - 336
*sqrt(a + b*x**2)*a**4*d**2*x**2 - 384*sqrt(a + b*x**2)*a**3*b*c**2*x**2 -
 280*sqrt(a + b*x**2)*a**3*b*c**2*x - 980*sqrt(a + b*x**2)*a**3*b*c*d*x**3
 - 672*sqrt(a + b*x**2)*a**3*b*c*d*x**2 - 672*sqrt(a + b*x**2)*a**3*b*d**2
*x**4 - 420*sqrt(a + b*x**2)*a**3*b*d**2*x**3 - 336*sqrt(a + b*x**2)*a**3*
c**3*x**2 - 840*sqrt(a + b*x**2)*a**3*c**2*d*x**3 - 560*sqrt(a + b*x**2)*a
**3*c*d**2*x**4 - 48*sqrt(a + b*x**2)*a**2*b**2*c**2*x**4 - 490*sqrt(a + b
*x**2)*a**2*b**2*c**2*x**3 - 210*sqrt(a + b*x**2)*a**2*b**2*c*d*x**5 - 134
4*sqrt(a + b*x**2)*a**2*b**2*c*d*x**4 - 336*sqrt(a + b*x**2)*a**2*b**2*d**
2*x**6 - 1050*sqrt(a + b*x**2)*a**2*b**2*d**2*x**5 - 672*sqrt(a + b*x**2)*
a**2*b*c**3*x**4 - 2100*sqrt(a + b*x**2)*a**2*b*c**2*d*x**5 - 2240*sqrt(a
+ b*x**2)*a**2*b*c*d**2*x**6 + 96*sqrt(a + b*x**2)*a*b**3*c**2*x**6 - 105*
sqrt(a + b*x**2)*a*b**3*c**2*x**5 - 672*sqrt(a + b*x**2)*a*b**3*c*d*x**6 -
 336*sqrt(a + b*x**2)*a*b**2*c**3*x**6 - 210*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c*d*x**7 + 630*sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*d**2*x**7 + 1260*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*d*x**7
- 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*c
**2*x**7 + 210*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a
))*a*b**3*c*d*x**7 - 630*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt...
```

3.73 $\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 338

$$\begin{aligned}\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx &= -\frac{aAc^2\sqrt{a+bx^2}}{8x^8} \\ &- \frac{(8ac(cC+2Bd)+A(9bc^2+8ad^2))\sqrt{a+bx^2}}{48x^6} \\ &- \frac{(Ab(3bc^2+56ad^2)+8a(6aCd^2+7bc(cC+2Bd)))\sqrt{a+bx^2}}{192ax^4} \\ &+ \frac{b(Ab(3bc^2-8ad^2)-8a(10aCd^2+bc(cC+2Bd)))\sqrt{a+bx^2}}{128a^2x^2} \\ &- \frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{7ax^7}+\frac{(2bc(Bc+2Ad)-7ad(2cC+Bd))\left(a+bx^2\right)^{5/2}}{35a^2x^5} \\ &- \frac{b^2(Ab(3bc^2-8ad^2)+8a(6aCd^2-bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}\end{aligned}$$

output
```
-1/8*a*A*c^2*(b*x^2+a)^(1/2)/x^8-1/48*(8*a*c*(2*B*d+C*c)+A*(8*a*d^2+9*b*c^
2))*(b*x^2+a)^(1/2)/x^6-1/192*(A*b*(56*a*d^2+3*b*c^2)+8*a*(6*a*C*d^2+7*b*c
*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a/x^4+1/128*b*(A*b*(-8*a*d^2+3*b*c^2)-8*a*(
10*a*C*d^2+b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^2/x^2-1/7*c*(2*A*d+B*c)*(b*
x^2+a)^(5/2)/a/x^7+1/35*(2*b*c*(2*A*d+B*c)-7*a*d*(B*d+2*C*c))*(b*x^2+a)^(5
/2)/a^2/x^5-1/128*b^2*(A*b*(-8*a*d^2+3*b*c^2)+8*a*(6*a*C*d^2-b*c*(2*B*d+C*
c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 4.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^9}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^2}\left(-3b^3cx^6(105Ac+256Bcx+512Adx)+16a^3\left(5A\left(21c^2+48cdx+28\right.\right.\right.}{}}{}$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^9,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(-3*b^3*c*x^6*(105*A*c + 256*B*c*x + 512*A*d*x
) + 16*a^3*(5*A*(21*c^2 + 48*c*d*x + 28*d^2*x^2) + 2*x*(7*C*x*(10*c^2 + 24
*c*d*x + 15*d^2*x^2) + 4*B*(15*c^2 + 35*c*d*x + 21*d^2*x^2))) + 6*a*b^2*x^
4*(A*(35*c^2 + 128*c*d*x + 140*d^2*x^2) + 4*x*(7*c*C*x*(5*c + 32*d*x) + 2*
B*(8*c^2 + 35*c*d*x + 56*d^2*x^2))) + 8*a^2*b*x^2*(A*(315*c^2 + 768*c*d*x
+ 490*d^2*x^2) + 2*x*(7*C*x*(35*c^2 + 96*c*d*x + 75*d^2*x^2) + 2*B*(96*c^2
 + 245*c*d*x + 168*d^2*x^2)))))/x^8) + 630*b^2*(A*b^2*c^2 + 16*a^2*C*d^2)*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + 1680*a*b^3*(c^2*C + 2*B*c
*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(13440*a^(5
/2))
```

Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 25, 2338, 25, 2338, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^9}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(8aCd^2x^3+8ad(2cC+Bd)x^2+\left(8ac(cC+2Bd)-A\left(3bc^2-8ad^2\right)\right)x+8ac(Bc+2Ad)\right)}{x^8}dx}{8a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(8aCd^2x^3+8ad(2cC+Bd)x^2+\left(8ac(cC+2Bd)-A\left(3bc^2-8ad^2\right)\right)x+8ac(Bc+2Ad)\right)}{x^8}dx}{8a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(56a^2Cd^2x^2-8a(2bc(Bc+2Ad)-7ad(2cC+Bd))x+7a\left(8ac(cC+2Bd)-A\left(3bc^2-8ad^2\right)\right)\right)}{x^7}dx}{7a}-\frac{8c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{7x^7}}{8a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(56a^2Cd^2x^2-8a(2bc(Bc+2Ad)-7ad(2cC+Bd))x+7a\left(8ac(cC+2Bd)-A\left(3bc^2-8ad^2\right)\right)\right)}{x^7}dx}{7a}-\frac{8c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{7x^7}}{8a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{-\frac{\int \frac{a\left(48a(2bc(Bc+2Ad)-7ad(2cC+Bd))-7\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{7\left(a+bx^2\right)^{5/2}\left(8ac(2Bd+cC)-A\left(3bc^2-8ad^2\right)\right)}{6x^6}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{1}{6}\int \frac{\left(48a(2bc(Bc+2Ad)-7ad(2cC+Bd))-7\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx-\frac{7\left(a+bx^2\right)^{5/2}\left(8ac(2Bd+cC)-A\left(3bc^2-8ad^2\right)\right)}{6x^6}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{1}{6}\left(7\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)\int\frac{\left(bx^2+a\right)^{3/2}}{x^5}dx+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7ad(Bd+2cC))}{5x^5}\right)-\frac{7\left(a+bx^2\right)^{5/2}\left(8ac(2Bd+cC)-\right.}{6x^6}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 243

$$\frac{\frac{\frac{1}{6}\left(\frac{7}{2}\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)\int\frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7ad(Bd+2cC))}{5x^5}\right)-\frac{7\left(a+bx^2\right)^{5/2}\left(8ac(2Bd+cC)\right.}{6x^6}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 51

$$\frac{\frac{\frac{1}{6}\left(\frac{7}{2}\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7ad(Bd+2cC))}{5x^5}\right)-\frac{7\left(a+bx^2\right)}{}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 51

$$\frac{\frac{\frac{1}{6}\left(\frac{7}{2}\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7ad(Bd+}{5x^5}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 73

$$\frac{\frac{\frac{1}{6}\left(\frac{7}{2}\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7ad(Bd+2}{5x^5}}{7a}}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{6}\left(\frac{7}{2}\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\left(Ab\left(3bc^2-8ad^2\right)+8a\left(6aCd^2-bc(2Bd+cC)\right)\right)+\frac{48\left(a+bx^2\right)^{5/2}(2bc(2Ad+Bc)-7a\cdots}{5x^5}\cdots}{7a}\cdots}{8a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{8ax^8}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^9,x]
```

output
```
-1/8*(A*c^2*(a + b*x^2)^(5/2))/(a*x^8) + ((-8*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(7*x^7) + ((-7*(8*a*c*(c*C + 2*B*d) - A*(3*b*c^2 - 8*a*d^2))*(a + b
*x^2)^(5/2))/(6*x^6) + ((48*(2*b*c*(B*c + 2*A*d) - 7*a*d*(2*c*C + B*d))*(a
 + b*x^2)^(5/2))/(5*x^5) + (7*(A*b*(3*b*c^2 - 8*a*d^2) + 8*a*(6*a*C*d^2 -
b*c*(c*C + 2*B*d)))*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/
x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)/6)/(7*a))/(8*
a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.30

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-1536 A\,b^3 c d\,x^7+2688 B a\,b^2 d^2 x^7-768 B\,b^3 c^2 x^7+5376 C a\,b^2 c d\,x^7+840 A a\,b^2 d^2 x^6-315 A\,b^3 c^2 x^6+1680 B a\,b^2 c d\,x^6+840}{}$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}}{}\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/13440*(b*x^2+a)^(1/2)*(-1536*A*b^3*c*d*x^7+2688*B*a*b^2*d^2*x^7-768*B*b
^3*c^2*x^7+5376*C*a*b^2*c*d*x^7+840*A*a*b^2*d^2*x^6-315*A*b^3*c^2*x^6+1680
*B*a*b^2*c*d*x^6+8400*C*a^2*b*d^2*x^6+840*C*a*b^2*c^2*x^6+768*A*a*b^2*c*d*
x^5+5376*B*a^2*b*d^2*x^5+384*B*a*b^2*c^2*x^5+10752*C*a^2*b*c*d*x^5+3920*A*
a^2*b*d^2*x^4+210*A*a*b^2*c^2*x^4+7840*B*a^2*b*c*d*x^4+3360*C*a^3*d^2*x^4+
3920*C*a^2*b*c^2*x^4+6144*A*a^2*b*c*d*x^3+2688*B*a^3*d^2*x^3+3072*B*a^2*b*
c^2*x^3+5376*C*a^3*c*d*x^3+2240*A*a^3*d^2*x^2+2520*A*a^2*b*c^2*x^2+4480*B*
a^3*c*d*x^2+2240*C*a^3*c^2*x^2+3840*A*a^3*c*d*x+1920*B*a^3*c^2*x+1680*A*a^
3*c^2)/x^8/a^2+1/128*(8*A*a*b*d^2-3*A*b^2*c^2+16*B*a*b*c*d-48*C*a^2*d^2+8*
C*a*b*c^2)*b^2/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="fricas
")
```

output
```
[1/26880*(105*(16*B*a*b^3*c*d + (8*C*a*b^3 - 3*A*b^4)*c^2 - 8*(6*C*a^2*b^2
 - A*a*b^3)*d^2)*sqrt(a)*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(384*(2*B*a*b^3*c^2 - 7*B*a^2*b^2*d^2 - 2*(7*C*a^2*b^2 - 2*A*a*
b^3)*c*d)*x^7 - 1680*A*a^4*c^2 - 105*(16*B*a^2*b^2*c*d + (8*C*a^2*b^2 - 3*
A*a*b^3)*c^2 + 8*(10*C*a^3*b + A*a^2*b^2)*d^2)*x^6 - 384*(B*a^2*b^2*c^2 +
14*B*a^3*b*d^2 + 2*(14*C*a^3*b + A*a^2*b^2)*c*d)*x^5 - 70*(112*B*a^3*b*c*d
 + (56*C*a^3*b + 3*A*a^2*b^2)*c^2 + 8*(6*C*a^4 + 7*A*a^3*b)*d^2)*x^4 - 384
*(8*B*a^3*b*c^2 + 7*B*a^4*d^2 + 2*(7*C*a^4 + 8*A*a^3*b)*c*d)*x^3 - 280*(16
*B*a^4*c*d + 8*A*a^4*d^2 + (8*C*a^4 + 9*A*a^3*b)*c^2)*x^2 - 1920*(B*a^4*c^
2 + 2*A*a^4*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^8), -1/13440*(105*(16*B*a*b^3*
c*d + (8*C*a*b^3 - 3*A*b^4)*c^2 - 8*(6*C*a^2*b^2 - A*a*b^3)*d^2)*sqrt(-a)*
x^8*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (384*(2*B*a*b^3*c^2 - 7*B*a^2*b^2
*d^2 - 2*(7*C*a^2*b^2 - 2*A*a*b^3)*c*d)*x^7 - 1680*A*a^4*c^2 - 105*(16*B*a
^2*b^2*c*d + (8*C*a^2*b^2 - 3*A*a*b^3)*c^2 + 8*(10*C*a^3*b + A*a^2*b^2)*d^
2)*x^6 - 384*(B*a^2*b^2*c^2 + 14*B*a^3*b*d^2 + 2*(14*C*a^3*b + A*a^2*b^2)*
c*d)*x^5 - 70*(112*B*a^3*b*c*d + (56*C*a^3*b + 3*A*a^2*b^2)*c^2 + 8*(6*C*a
^4 + 7*A*a^3*b)*d^2)*x^4 - 384*(8*B*a^3*b*c^2 + 7*B*a^4*d^2 + 2*(7*C*a^4 +
 8*A*a^3*b)*c*d)*x^3 - 280*(16*B*a^4*c*d + 8*A*a^4*d^2 + (8*C*a^4 + 9*A*a^
3*b)*c^2)*x^2 - 1920*(B*a^4*c^2 + 2*A*a^4*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^
8)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1889 vs. $2(326) = 652$.

Time = 47.89 (sec) , antiderivative size = 1889, normalized size of antiderivative = 5.59

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**9,x)
```

output
```
-30*A*a**6*b**(9/2)*c*d*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**
4*b**5*x**8 + 105*a**3*b**6*x**10) - 66*A*a**5*b**(11/2)*c*d*x**2*sqrt(a/(
b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**1
0) - 34*A*a**4*b**(13/2)*c*d*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6
 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 6*A*a**3*b**(15/2)*c*d*x**6
*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*
b**6*x**10) - 24*A*a**2*b**(17/2)*c*d*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*
b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - A*a**2*c**2/(8*sqr
t(b)*x**9*sqrt(a/(b*x**2) + 1)) - A*a**2*d**2/(6*sqrt(b)*x**7*sqrt(a/(b*x*
*2) + 1)) - 16*A*a*b**(19/2)*c*d*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4
*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 5*A*a*sqrt(b)*c**2/(16
*x**7*sqrt(a/(b*x**2) + 1)) - 11*A*a*sqrt(b)*d**2/(24*x**5*sqrt(a/(b*x**2)
 + 1)) - 13*A*b**(3/2)*c**2/(64*x**5*sqrt(a/(b*x**2) + 1)) - 2*A*b**(3/2)*
c*d*sqrt(a/(b*x**2) + 1)/(5*x**4) - 17*A*b**(3/2)*d**2/(48*x**3*sqrt(a/(b*
x**2) + 1)) + A*b**(5/2)*c**2/(128*a*x**3*sqrt(a/(b*x**2) + 1)) - 2*A*b**(
5/2)*c*d*sqrt(a/(b*x**2) + 1)/(15*a*x**2) - A*b**(5/2)*d**2/(16*a*x*sqrt(a
/(b*x**2) + 1)) + 3*A*b**(7/2)*c**2/(128*a**2*x*sqrt(a/(b*x**2) + 1)) + 4*
A*b**(7/2)*c*d*sqrt(a/(b*x**2) + 1)/(15*a**2) + A*b**3*d**2*asinh(sqrt(a)/
(sqrt(b)*x))/(16*a**(3/2)) - 3*A*b**4*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(128
*a**(5/2)) - 15*B*a**6*b**(9/2)*c**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="maxima
")
```

output
```
-3/128*A*b^4*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/8*C*b^2*d^2*arc
sinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/128*(b*x^2 + a)^(3/2)*A*b^4*c^2/a^4
 + 3/128*sqrt(b*x^2 + a)*A*b^4*c^2/a^3 + 1/8*(b*x^2 + a)^(3/2)*C*b^2*d^2/a
^2 + 3/8*sqrt(b*x^2 + a)*C*b^2*d^2/a + 1/16*(C*c^2 + 2*B*c*d + A*d^2)*b^3*
arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(C*c^2 + 2*B*c*d + A*d^2)*(b*
x^2 + a)^(3/2)*b^3/a^3 - 1/16*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*b^
3/a^2 - 1/128*(b*x^2 + a)^(5/2)*A*b^3*c^2/(a^4*x^2) - 1/8*(b*x^2 + a)^(5/2
)*C*b*d^2/(a^2*x^2) - 1/64*(b*x^2 + a)^(5/2)*A*b^2*c^2/(a^3*x^4) - 1/4*(b*
x^2 + a)^(5/2)*C*d^2/(a*x^4) + 1/48*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^
(5/2)*b^2/(a^3*x^2) - 2/5*(b*x^2 + a)^(5/2)*C*c*d/(a*x^5) - 1/5*(b*x^2 + a
)^(5/2)*B*d^2/(a*x^5) + 1/16*(b*x^2 + a)^(5/2)*A*b*c^2/(a^2*x^6) + 1/24*(C
*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*b/(a^2*x^4) + 2/35*(B*c^2 + 2*A*
c*d)*(b*x^2 + a)^(5/2)*b/(a^2*x^5) - 1/8*(b*x^2 + a)^(5/2)*A*c^2/(a*x^8) -
 1/6*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)/(a*x^6) - 1/7*(B*c^2 + 2*
A*c*d)*(b*x^2 + a)^(5/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2107 vs. $2(306) = 612$.

Time = 0.17 (sec) , antiderivative size = 2107, normalized size of antiderivative = 6.23

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^9} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x, algorithm="giac")
```

output
```
-1/64*(8*C*a*b^3*c^2 - 3*A*b^4*c^2 + 16*B*a*b^3*c*d - 48*C*a^2*b^2*d^2 + 8
*A*a*b^3*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
2) + 1/6720*(840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a*b^3*c^2 - 315*(sqrt(
b)*x - sqrt(b*x^2 + a))^15*A*b^4*c^2 + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^
15*B*a*b^3*c*d + 8400*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*b^2*d^2 + 840
*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a*b^3*d^2 + 26880*(sqrt(b)*x - sqrt(b*
x^2 + a))^14*C*a^2*b^(5/2)*c*d + 13440*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*
a^2*b^(5/2)*d^2 + 11480*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^2*b^3*c^2 + 2
415*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a*b^4*c^2 + 22960*(sqrt(b)*x - sqrt
(b*x^2 + a))^13*B*a^2*b^3*c*d - 28560*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a
^3*b^2*d^2 + 11480*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a^2*b^3*d^2 + 26880*
(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^2*b^(7/2)*c^2 - 80640*(sqrt(b)*x - sq
rt(b*x^2 + a))^12*C*a^3*b^(5/2)*c*d + 53760*(sqrt(b)*x - sqrt(b*x^2 + a))^
12*A*a^2*b^(7/2)*c*d - 40320*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(5/2
)*d^2 - 3640*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^3*b^3*c^2 + 34965*(sqrt(
b)*x - sqrt(b*x^2 + a))^11*A*a^2*b^4*c^2 - 7280*(sqrt(b)*x - sqrt(b*x^2 +
a))^11*B*a^3*b^3*c*d + 35280*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^4*b^2*d^
2 - 3640*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^3*b^3*d^2 + 134400*(sqrt(b)*
x - sqrt(b*x^2 + a))^10*C*a^4*b^(5/2)*c*d + 67200*(sqrt(b)*x - sqrt(b*x^2
+ a))^10*B*a^4*b^(5/2)*d^2 - 8680*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^9} \, dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^9} \, dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^9,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 994, normalized size of antiderivative = 2.94

$$\int \frac{(c+dx)^2 \left(a+bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x^9} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^9,x)
```

output
```
( - 1680*sqrt(a + b*x**2)*a**4*c**2 - 3840*sqrt(a + b*x**2)*a**4*c*d*x - 2
240*sqrt(a + b*x**2)*a**4*d**2*x**2 - 2520*sqrt(a + b*x**2)*a**3*b*c**2*x*
*2 - 1920*sqrt(a + b*x**2)*a**3*b*c**2*x - 6144*sqrt(a + b*x**2)*a**3*b*c*
d*x**3 - 4480*sqrt(a + b*x**2)*a**3*b*c*d*x**2 - 3920*sqrt(a + b*x**2)*a**
3*b*d**2*x**4 - 2688*sqrt(a + b*x**2)*a**3*b*d**2*x**3 - 2240*sqrt(a + b*x
**2)*a**3*c**3*x**2 - 5376*sqrt(a + b*x**2)*a**3*c**2*d*x**3 - 3360*sqrt(a
 + b*x**2)*a**3*c*d**2*x**4 - 210*sqrt(a + b*x**2)*a**2*b**2*c**2*x**4 - 3
072*sqrt(a + b*x**2)*a**2*b**2*c**2*x**3 - 768*sqrt(a + b*x**2)*a**2*b**2*
c*d*x**5 - 7840*sqrt(a + b*x**2)*a**2*b**2*c*d*x**4 - 840*sqrt(a + b*x**2)
*a**2*b**2*d**2*x**6 - 5376*sqrt(a + b*x**2)*a**2*b**2*d**2*x**5 - 3920*sq
rt(a + b*x**2)*a**2*b*c**3*x**4 - 10752*sqrt(a + b*x**2)*a**2*b*c**2*d*x**
5 - 8400*sqrt(a + b*x**2)*a**2*b*c*d**2*x**6 + 315*sqrt(a + b*x**2)*a*b**3
*c**2*x**6 - 384*sqrt(a + b*x**2)*a*b**3*c**2*x**5 + 1536*sqrt(a + b*x**2)
*a*b**3*c*d*x**7 - 1680*sqrt(a + b*x**2)*a*b**3*c*d*x**6 - 2688*sqrt(a + b
*x**2)*a*b**3*d**2*x**7 - 840*sqrt(a + b*x**2)*a*b**2*c**3*x**6 - 5376*sqr
t(a + b*x**2)*a*b**2*c**2*d*x**7 + 768*sqrt(a + b*x**2)*b**4*c**2*x**7 - 8
40*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*d*
*2*x**8 + 5040*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a
))*a*b**2*c*d**2*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**4*c**2*x**8 - 1680*sqrt(a)*log((sqrt(a + b*x**2) - s...
```

3.74 $\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 365

$$\begin{aligned}\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx &= \frac{b(3bc(Bc+2Ad)-8ad(2cC+Bd))\sqrt{a+bx^2}}{64ax^4} \\ &+ \frac{b^2(3bc(Bc+2Ad)-8ad(2cC+Bd))\sqrt{a+bx^2}}{128a^2x^2} \\ &+ \frac{(3bc(Bc+2Ad)-8ad(2cC+Bd))(a+bx^2)^{3/2}}{48ax^6} - \frac{Ac^2(a+bx^2)^{5/2}}{9ax^9} \\ &- \frac{c(Bc+2Ad)(a+bx^2)^{5/2}}{8ax^8} - \frac{(9ac(cC+2Bd)-A(4bc^2-9ad^2))(a+bx^2)^{5/2}}{63a^2x^7} \\ &- \frac{(2Ab(4bc^2-9ad^2)+9a(7aCd^2-2bc(cC+2Bd)))(a+bx^2)^{5/2}}{315a^3x^5} \\ &- \frac{b^3(3bc(Bc+2Ad)-8ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}\end{aligned}$$

output
```
1/64*b*(3*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a/x^4+1/128*b
^2*(3*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x^2+1/48*(3*b
*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/a/x^6-1/9*A*c^2*(b*x^2+a
)^(5/2)/a/x^9-1/8*c*(2*A*d+B*c)*(b*x^2+a)^(5/2)/a/x^8-1/63*(9*a*c*(2*B*d+C
*c)-A*(-9*a*d^2+4*b*c^2))*(b*x^2+a)^(5/2)/a^2/x^7-1/315*(2*A*b*(-9*a*d^2+4
*b*c^2)+9*a*(7*a*C*d^2-2*b*c*(2*B*d+C*c)))*(b*x^2+a)^(5/2)/a^3/x^5-1/128*b
^3*(3*b*c*(2*A*d+B*c)-8*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(5/2)
```

Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx =$$
$$-\frac{\sqrt{a+bx^2}(1024Ab^4c^2x^8 - ab^3x^6(9cx(105Bc+256cCx+512Bdx)+2A(256c^2+945cdx+1152d^2x^2))+}{}$$
$$+\frac{b^3(-3bc(Bc+2Ad)+8ad(2cC+Bd))\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{64a^{5/2}}$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/40320*(Sqrt[a + b*x^2]*(1024*A*b^4*c^2*x^8 - a*b^3*x^6*(9*c*x*(105*B*c
+ 256*c*C*x + 512*B*d*x) + 2*A*(256*c^2 + 945*c*d*x + 1152*d^2*x^2)) + 16*
a^4*(10*A*(28*c^2 + 63*c*d*x + 36*d^2*x^2) + 3*x*(8*C*x*(15*c^2 + 35*c*d*x
 + 21*d^2*x^2) + 5*B*(21*c^2 + 48*c*d*x + 28*d^2*x^2))) + 6*a^2*b^2*x^4*(2
*A*(32*c^2 + 105*c*d*x + 96*d^2*x^2) + 3*x*(8*C*x*(8*c^2 + 35*c*d*x + 56*d
^2*x^2) + B*(35*c^2 + 128*c*d*x + 140*d^2*x^2))) + 8*a^3*b*x^2*(2*A*(400*c
^2 + 945*c*d*x + 576*d^2*x^2) + 3*x*(4*C*x*(96*c^2 + 245*c*d*x + 168*d^2*x
^2) + B*(315*c^2 + 768*c*d*x + 490*d^2*x^2))))/(a^3*x^9) + (b^3*(-3*b*c*(
B*c + 2*A*d) + 8*a*d*(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2
])/Sqrt[a]])/(64*a^(5/2))
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.90, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2338, 25, 2338, 25, 2338, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(c+dx)^2\left(A+Bx+Cx^2\right)}{x^{10}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(9aCd^2x^3+9ad(2cC+Bd)x^2+\left(9ac(cC+2Bd)-A\left(4bc^2-9ad^2\right)\right)x+9ac(Bc+2Ad)\right)}{x^9}dx}{9a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(9aCd^2x^3+9ad(2cC+Bd)x^2+\left(9ac(cC+2Bd)-A\left(4bc^2-9ad^2\right)\right)x+9ac(Bc+2Ad)\right)}{x^9}dx}{9a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(72a^2Cd^2x^2-9a(3bc(Bc+2Ad)-8ad(2cC+Bd))x+8a\left(9ac(cC+2Bd)-A\left(4bc^2-9ad^2\right)\right)\right)}{x^8}dx}{8a}-\frac{9c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{8x^8}}{9a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(72a^2Cd^2x^2-9a(3bc(Bc+2Ad)-8ad(2cC+Bd))x+8a\left(9ac(cC+2Bd)-A\left(4bc^2-9ad^2\right)\right)\right)}{x^8}dx}{8a}-\frac{9c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{8x^8}}{9a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{\int \frac{a\left(63a(3bc(Bc+2Ad)-8ad(2cC+Bd))-8\left(2Ab\left(4bc^2-9ad^2\right)+9a\left(7aCd^2-2bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}-\frac{8\left(a+bx^2\right)^{5/2}\left(9ac(2Bd+cC)-A\left(4bc^2-9ad^2\right)\cdots\right.}{7x^7}}{8a}\cdots}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{1}{7}\int \frac{\left(63a(3bc(Bc+2Ad)-8ad(2cC+Bd))-8\left(2Ab\left(4bc^2-9ad^2\right)+9a\left(7aCd^2-2bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^7}dx-\frac{8\left(a+bx^2\right)^{5/2}\left(9ac(2Bd+cC)-A\left(4bc^2-9ad^2\cdots\right.\right.}{7x^7}}{8a}\cdots}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{\frac{1}{7}\left(\frac{\int \frac{3a\left(16\left(2Ab\left(4bc^2-9ad^2\right)+9a\left(7aCd^2-2bc(cC+2Bd)\right)\right)+21b(3bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}+\frac{21\left(a+bx^2\right)^{5/2}(3bc(2Ad+Bc)-8ad(Bd+2\cdots}{2x^6}\cdots\right.}{8a}\cdots}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{7}\left(\frac{1}{2}\int \frac{\left(16\left(2Ab\left(4bc^2-9ad^2\right)+9a\left(7aCd^2-2bc(cC+2Bd)\right)\right)+21b(3bc(Bc+2Ad)-8ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx+\frac{21\left(a+bx^2\right)^{5/2}(3bc(2Ad+Bc)-8ad(Bd+2\cdots}{2x^6}\cdots\right.}{8a}\cdots}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{1}{7}\left(\frac{1}{2}\left(21b(3bc(2Ad+Bc)-8ad(Bd+2cC))\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{16\left(a+bx^2\right)^{5/2}\left(2Ab\left(4bc^2-9ad^2\right)+9a\left(7aCd^2-2bc(2Bd+cC)\right)\right)}{5ax^5}\right)+\frac{21\left(a+bx^2\right)^{5/2}(3bc(2A\cdots}{2\cdots}\cdots\right.}{8a}\cdots}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{9ax^9}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{21}{2}b(3bc(2Ad+Bc)-8ad(Bd+2cC))\int\frac{(bx^2+a)^{3/2}}{x^6}dx^2-\frac{16(a+bx^2)^{5/2}(2Ab(4bc^2-9ad^2)+9a(7aCd^2-2bc(2Bd+cC)))}{5ax^5}\right)+\frac{21(a+bx^2)^{5/2}(3bc(2}{8a}}{9a}$$

$$\frac{Ac^2(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 51

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{21}{2}b(3bc(2Ad+Bc)-8ad(Bd+2cC))\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{16(a+bx^2)^{5/2}(2Ab(4bc^2-9ad^2)+9a(7aCd^2-2bc(2Bd+cC)))}{5ax^5}\right)+\frac{21}{}}{8a}}{9a}$$

$$\frac{Ac^2(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 51

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{21}{2}b(3bc(2Ad+Bc)-8ad(Bd+2cC))\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{16(a+bx^2)^{5/2}(2Ab(4bc^2-9ad^2)+9a(7aCd^2-2b}{5ax^5}}{8a}}{9a}$$

$$\frac{Ac^2(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 73

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{21}{2}b(3bc(2Ad+Bc)-8ad(Bd+2cC))\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{16(a+bx^2)^{5/2}(2Ab(4bc^2-9ad^2)+9a(7aCd^2-2bc}{5ax^5}}{8a}}{9a}$$

$$\frac{Ac^2(a+bx^2)^{5/2}}{9ax^9}$$

$\downarrow$ 221

$$\frac{\frac{1}{7}\left(\frac{\frac{1}{2}\left(\frac{21}{2}b\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)(3bc(2Ad+Bc)-8ad(Bd+2cC))-\frac{16(a+bx^2)^{5/2}(2Ab(4bc^2-9ad^2)+9a(7aC}{5ax^5}}{8a}}{9a}$$

$$\frac{Ac^2(a+bx^2)^{5/2}}{9ax^9}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^10,x]
```

output
```
-1/9*(A*c^2*(a + b*x^2)^(5/2))/(a*x^9) + ((-9*c*(B*c + 2*A*d)*(a + b*x^2)^
(5/2))/(8*x^8) + ((-8*(9*a*c*(c*C + 2*B*d) - A*(4*b*c^2 - 9*a*d^2))*(a + b
*x^2)^(5/2))/(7*x^7) + ((21*(3*b*c*(B*c + 2*A*d) - 8*a*d*(2*c*C + B*d))*(a
 + b*x^2)^(5/2))/(2*x^6) + ((-16*(2*A*b*(4*b*c^2 - 9*a*d^2) + 9*a*(7*a*C*d
^2 - 2*b*c*(c*C + 2*B*d)))*(a + b*x^2)^(5/2))/(5*a*x^5) + (21*b*(3*b*c*(B*
c + 2*A*d) - 8*a*d*(2*c*C + B*d))*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sq
rt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/2)
/2)/7)/(8*a))/(9*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.19

method	result
default	$(A\,d^{2}+2Bcd+C\,c^{2})\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b(b\,x^{2}+a)^{\frac{5}{2}}}{35a^{2}x^{5}}\right)+A\,c^{2}\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{9a\,x^{9}}-\frac{4b\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35a^{2}x^{5}}\right.}{9a}\right.$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-2304Aa\,b^{3}d^{2}x^{8}+1024A\,b^{4}c^{2}x^{8}-4608Ba\,b^{3}cd\,x^{8}+8064C\,a^{2}b^{2}d^{2}x^{8}-2304Ca\,b^{3}c^{2}x^{8}-1890Aa\,b^{3}cd\,x^{7}+2520B\,a^{2}b^{2}d}{}$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x,method=_RETURNVERBOSE)
```

output
```
(A*d^2+2*B*c*d+C*c^2)*(-1/7/a/x^7*(b*x^2+a)^(5/2)+2/35*b/a^2/x^5*(b*x^2+a)
^(5/2))+A*c^2*(-1/9/a/x^9*(b*x^2+a)^(5/2)-4/9*b/a*(-1/7/a/x^7*(b*x^2+a)^(5
/2)+2/35*b/a^2/x^5*(b*x^2+a)^(5/2)))-1/5*C*d^2/a/x^5*(b*x^2+a)^(5/2)+c*(2*
A*d+B*c)*(-1/8/a/x^8*(b*x^2+a)^(5/2)-3/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1
/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2
*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)))))))+d*(B*d+2*C*c)*(-1/6/a/x^6*(b*x^2+a)^(5/2)-1/6*b/a*(
-1/4/a/x^4*(b*x^2+a)^(5/2)+1/4*b/a*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/
3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x)))))
```

Fricas [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 944, normalized size of antiderivative = 2.59

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="frica
s")
```

output
```
[-1/80640*(315*(3*B*b^4*c^2 - 8*B*a*b^3*d^2 - 2*(8*C*a*b^3 - 3*A*b^4)*c*d)
*sqrt(a)*x^9*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(128*
(36*B*a*b^3*c*d + 2*(9*C*a*b^3 - 4*A*b^4)*c^2 - 9*(7*C*a^2*b^2 - 2*A*a*b^3
)*d^2)*x^8 + 315*(3*B*a*b^3*c^2 - 8*B*a^2*b^2*d^2 - 2*(8*C*a^2*b^2 - 3*A*a
*b^3)*c*d)*x^7 - 4480*A*a^4*c^2 - 128*(18*B*a^2*b^2*c*d + (9*C*a^2*b^2 - 4
*A*a*b^3)*c^2 + 9*(14*C*a^3*b + A*a^2*b^2)*d^2)*x^6 - 210*(3*B*a^2*b^2*c^2
 + 56*B*a^3*b*d^2 + 2*(56*C*a^3*b + 3*A*a^2*b^2)*c*d)*x^5 - 384*(48*B*a^3*
b*c*d + (24*C*a^3*b + A*a^2*b^2)*c^2 + 3*(7*C*a^4 + 8*A*a^3*b)*d^2)*x^4 -
840*(9*B*a^3*b*c^2 + 8*B*a^4*d^2 + 2*(8*C*a^4 + 9*A*a^3*b)*c*d)*x^3 - 640*
(18*B*a^4*c*d + 9*A*a^4*d^2 + (9*C*a^4 + 10*A*a^3*b)*c^2)*x^2 - 5040*(B*a^
4*c^2 + 2*A*a^4*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^9), 1/40320*(315*(3*B*b^4*
c^2 - 8*B*a*b^3*d^2 - 2*(8*C*a*b^3 - 3*A*b^4)*c*d)*sqrt(-a)*x^9*arctan(sqr
t(b*x^2 + a)*sqrt(-a)/a) + (128*(36*B*a*b^3*c*d + 2*(9*C*a*b^3 - 4*A*b^4)*
c^2 - 9*(7*C*a^2*b^2 - 2*A*a*b^3)*d^2)*x^8 + 315*(3*B*a*b^3*c^2 - 8*B*a^2*
b^2*d^2 - 2*(8*C*a^2*b^2 - 3*A*a*b^3)*c*d)*x^7 - 4480*A*a^4*c^2 - 128*(18*
B*a^2*b^2*c*d + (9*C*a^2*b^2 - 4*A*a*b^3)*c^2 + 9*(14*C*a^3*b + A*a^2*b^2)
*d^2)*x^6 - 210*(3*B*a^2*b^2*c^2 + 56*B*a^3*b*d^2 + 2*(56*C*a^3*b + 3*A*a^
2*b^2)*c*d)*x^5 - 384*(48*B*a^3*b*c*d + (24*C*a^3*b + A*a^2*b^2)*c^2 + 3*(
7*C*a^4 + 8*A*a^3*b)*d^2)*x^4 - 840*(9*B*a^3*b*c^2 + 8*B*a^4*d^2 + 2*(8*C*
a^4 + 9*A*a^3*b)*c*d)*x^3 - 640*(18*B*a^4*c*d + 9*A*a^4*d^2 + (9*C*a^4 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3181 vs. $2(350) = 700$.

Time = 64.34 (sec) , antiderivative size = 3181, normalized size of antiderivative = 8.72

$$\int \frac{(c+dx)^2\,(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^{10}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**10,x)
```

output

```
-35*A*a**8*b**(19/2)*c**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a
**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*A*a**
7*b**(21/2)*c**2*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*
b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*A*a**6*b*
*(23/2)*c**2*x**4*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**1
0*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 15*A*a**6*b**(9/2
)*d**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105
*a**3*b**6*x**10) - 40*A*a**5*b**(25/2)*c**2*x**6*sqrt(a/(b*x**2) + 1)/(31
5*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*
b**12*x**14) - 15*A*a**5*b**(11/2)*c**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**
4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 33*A*a**5*b**(11/2)*d
**2*x**2*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 1
05*a**3*b**6*x**10) + 5*A*a**4*b**(27/2)*c**2*x**8*sqrt(a/(b*x**2) + 1)/(3
15*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4
*b**12*x**14) - 33*A*a**4*b**(13/2)*c**2*x**2*sqrt(a/(b*x**2) + 1)/(105*a*
*5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*A*a**4*b**(1
3/2)*d**2*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x*
*8 + 105*a**3*b**6*x**10) + 30*A*a**3*b**(29/2)*c**2*x**10*sqrt(a/(b*x**2)
 + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
315*a**4*b**12*x**14) - 17*A*a**3*b**(15/2)*c**2*x**4*sqrt(a/(b*x**2) +...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.47

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx = \frac{(2\,Ccd+Bd^2)b^3 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{3\,(Bc^2+2\,Acd)b^4 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{3}{2}}b^3}{48\,a^3} - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}b^3}{16\,a^2} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{3}{2}}b^4}{128\,a^4} + \frac{3\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b^4}{128\,a^3} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}b^2}{48\,a^3x^2} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b^3}{128\,a^4x^2} - \frac{8\,(bx^2+a)^{\frac{5}{2}}Ab^2c^2}{315\,a^3x^5} - \frac{(bx^2+a)^{\frac{5}{2}}Cd^2}{5\,ax^5} + \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}b}{24\,a^2x^4} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b^2}{64\,a^3x^4} + \frac{4\,(bx^2+a)^{\frac{5}{2}}Abc^2}{63\,a^2x^7} + \frac{2\,(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}b}{35\,a^2x^5} - \frac{(2\,Ccd+Bd^2)(bx^2+a)^{\frac{5}{2}}}{6\,ax^6} + \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}b}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}Ac^2}{9\,ax^9} - \frac{(Cc^2+2\,Bcd+Ad^2)(bx^2+a)^{\frac{5}{2}}}{7\,ax^7} - \frac{(Bc^2+2\,Acd)(bx^2+a)^{\frac{5}{2}}}{8\,ax^8}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="maxim
a")
```

output
```
1/16*(2*C*c*d + B*d^2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/128*(
B*c^2 + 2*A*c*d)*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 1/48*(2*C*c*d
 + B*d^2)*(b*x^2 + a)^(3/2)*b^3/a^3 - 1/16*(2*C*c*d + B*d^2)*sqrt(b*x^2 +
a)*b^3/a^2 + 1/128*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(3/2)*b^4/a^4 + 3/128*(B*
c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b^4/a^3 + 1/48*(2*C*c*d + B*d^2)*(b*x^2 + a
)^(5/2)*b^2/(a^3*x^2) - 1/128*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b^3/(a^4
*x^2) - 8/315*(b*x^2 + a)^(5/2)*A*b^2*c^2/(a^3*x^5) - 1/5*(b*x^2 + a)^(5/2
)*C*d^2/(a*x^5) + 1/24*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1
/64*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 4/63*(b*x^2 + a)^(
5/2)*A*b*c^2/(a^2*x^7) + 2/35*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)*
b/(a^2*x^5) - 1/6*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2)/(a*x^6) + 1/16*(B*c^
2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b/(a^2*x^6) - 1/9*(b*x^2 + a)^(5/2)*A*c^2/(
a*x^9) - 1/7*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)^(5/2)/(a*x^7) - 1/8*(B*
c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)/(a*x^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. $2(329) = 658$.

Time = 0.18 (sec) , antiderivative size = 2195, normalized size of antiderivative = 6.01

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x, algorithm="giac"
)
```

output
```
1/64*(3*B*b^4*c^2 - 16*C*a*b^3*c*d + 6*A*b^4*c*d - 8*B*a*b^3*d^2)*arctan(-
(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/20160*(945*(sqr
t(b)*x - sqrt(b*x^2 + a))^17*B*b^4*c^2 - 5040*(sqrt(b)*x - sqrt(b*x^2 + a)
)^17*C*a*b^3*c*d + 1890*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*b^4*c*d - 2520*
(sqrt(b)*x - sqrt(b*x^2 + a))^17*B*a*b^3*d^2 - 40320*(sqrt(b)*x - sqrt(b*x
^2 + a))^16*C*a^2*b^(5/2)*d^2 - 8190*(sqrt(b)*x - sqrt(b*x^2 + a))^15*B*a*
b^4*c^2 - 63840*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^2*b^3*c*d - 16380*(sq
rt(b)*x - sqrt(b*x^2 + a))^15*A*a*b^4*c*d - 31920*(sqrt(b)*x - sqrt(b*x^2
+ a))^15*B*a^2*b^3*d^2 - 80640*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^2*b^(7
/2)*c^2 - 161280*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^2*b^(7/2)*c*d + 1612
80*(sqrt(b)*x - sqrt(b*x^2 + a))^14*C*a^3*b^(5/2)*d^2 - 80640*(sqrt(b)*x -
 sqrt(b*x^2 + a))^14*A*a^2*b^(7/2)*d^2 - 97650*(sqrt(b)*x - sqrt(b*x^2 + a
))^13*B*a^2*b^4*c^2 + 90720*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^3*b^3*c*d
 - 195300*(sqrt(b)*x - sqrt(b*x^2 + a))^13*A*a^2*b^4*c*d + 45360*(sqrt(b)*
x - sqrt(b*x^2 + a))^13*B*a^3*b^3*d^2 + 80640*(sqrt(b)*x - sqrt(b*x^2 + a)
)^12*C*a^3*b^(7/2)*c^2 - 215040*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(
9/2)*c^2 + 161280*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b^(7/2)*c*d - 322
560*(sqrt(b)*x - sqrt(b*x^2 + a))^12*C*a^4*b^(5/2)*d^2 + 80640*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*A*a^3*b^(7/2)*d^2 - 106470*(sqrt(b)*x - sqrt(b*x^2 +
 a))^11*B*a^3*b^4*c^2 + 30240*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^4*b^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^{10}} \, dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^10,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 23.69 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.92

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{10}} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^10,x)
```

output
```
( - 4480*sqrt(a + b*x**2)*a**5*c**2 - 10080*sqrt(a + b*x**2)*a**5*c*d*x -
5760*sqrt(a + b*x**2)*a**5*d**2*x**2 - 6400*sqrt(a + b*x**2)*a**4*b*c**2*x
**2 - 5040*sqrt(a + b*x**2)*a**4*b*c**2*x - 15120*sqrt(a + b*x**2)*a**4*b*
c*d*x**3 - 11520*sqrt(a + b*x**2)*a**4*b*c*d*x**2 - 9216*sqrt(a + b*x**2)*
a**4*b*d**2*x**4 - 6720*sqrt(a + b*x**2)*a**4*b*d**2*x**3 - 5760*sqrt(a +
b*x**2)*a**4*c**3*x**2 - 13440*sqrt(a + b*x**2)*a**4*c**2*d*x**3 - 8064*sq
rt(a + b*x**2)*a**4*c*d**2*x**4 - 384*sqrt(a + b*x**2)*a**3*b**2*c**2*x**4
 - 7560*sqrt(a + b*x**2)*a**3*b**2*c**2*x**3 - 1260*sqrt(a + b*x**2)*a**3*
b**2*c*d*x**5 - 18432*sqrt(a + b*x**2)*a**3*b**2*c*d*x**4 - 1152*sqrt(a +
b*x**2)*a**3*b**2*d**2*x**6 - 11760*sqrt(a + b*x**2)*a**3*b**2*d**2*x**5 -
 9216*sqrt(a + b*x**2)*a**3*b*c**3*x**4 - 23520*sqrt(a + b*x**2)*a**3*b*c*
*2*d*x**5 - 16128*sqrt(a + b*x**2)*a**3*b*c*d**2*x**6 + 512*sqrt(a + b*x**
2)*a**2*b**3*c**2*x**6 - 630*sqrt(a + b*x**2)*a**2*b**3*c**2*x**5 + 1890*s
qrt(a + b*x**2)*a**2*b**3*c*d*x**7 - 2304*sqrt(a + b*x**2)*a**2*b**3*c*d*x
**6 + 2304*sqrt(a + b*x**2)*a**2*b**3*d**2*x**8 - 2520*sqrt(a + b*x**2)*a*
*2*b**3*d**2*x**7 - 1152*sqrt(a + b*x**2)*a**2*b**2*c**3*x**6 - 5040*sqrt(
a + b*x**2)*a**2*b**2*c**2*d*x**7 - 8064*sqrt(a + b*x**2)*a**2*b**2*c*d**2
*x**8 - 1024*sqrt(a + b*x**2)*a*b**4*c**2*x**8 + 945*sqrt(a + b*x**2)*a*b*
*4*c**2*x**7 + 4608*sqrt(a + b*x**2)*a*b**4*c*d*x**8 + 2304*sqrt(a + b*x**
2)*a*b**3*c**3*x**8 + 1890*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq...
```

3.75 $\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{11}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 450

$$\begin{aligned}\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{11}}\,dx &= -\frac{aAc^2\sqrt{a+bx^2}}{10x^{10}} \\
&- \frac{(10ac(cC+2Bd)+A(11bc^2+10ad^2))\sqrt{a+bx^2}}{80x^8} \\
&- \frac{(3Ab(bc^2+30ad^2)+10a(8aCd^2+9bc(cC+2Bd)))\sqrt{a+bx^2}}{480ax^6} \\
&+ \frac{b(3Ab(bc^2-2ad^2)-2a(56aCd^2+3bc(cC+2Bd)))\sqrt{a+bx^2}}{384a^2x^4} \\
&- \frac{b^2(3Ab(bc^2-2ad^2)+2a(8aCd^2-3bc(cC+2Bd)))\sqrt{a+bx^2}}{256a^3x^2} \\
&- \frac{c(Bc+2Ad)\left(a+bx^2\right)^{5/2}}{9ax^9} \\
&+ \frac{(4bc(Bc+2Ad)-9ad(2cC+Bd))\left(a+bx^2\right)^{5/2}}{63a^2x^7} \\
&- \frac{2b(4bc(Bc+2Ad)-9ad(2cC+Bd))\left(a+bx^2\right)^{5/2}}{315a^3x^5} \\
&+ \frac{b^3(3Ab(bc^2-2ad^2)+2a(8aCd^2-3bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{7/2}}\end{aligned}$$

output
```
-1/10*a*A*c^2*(b*x^2+a)^(1/2)/x^10-1/80*(10*a*c*(2*B*d+C*c)+A*(10*a*d^2+11
*b*c^2))*(b*x^2+a)^(1/2)/x^8-1/480*(3*A*b*(30*a*d^2+b*c^2)+10*a*(8*a*C*d^2
+9*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a/x^6+1/384*b*(3*A*b*(-2*a*d^2+b*c^2)
-2*a*(56*a*C*d^2+3*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^2/x^4-1/256*b^2*(3*
A*b*(-2*a*d^2+b*c^2)+2*a*(8*a*C*d^2-3*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^
3/x^2-1/9*c*(2*A*d+B*c)*(b*x^2+a)^(5/2)/a/x^9+1/63*(4*b*c*(2*A*d+B*c)-9*a*
d*(B*d+2*C*c))*(b*x^2+a)^(5/2)/a^2/x^7-2/315*b*(4*b*c*(2*A*d+B*c)-9*a*d*(B
*d+2*C*c))*(b*x^2+a)^(5/2)/a^3/x^5+1/256*b^3*(3*A*b*(-2*a*d^2+b*c^2)+2*a*(
8*a*C*d^2-3*b*c*(2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 6.08 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^{11}}\,dx = -\frac{\sqrt{a}\sqrt{a+bx^2}\left(b^4cx^8(945Ac+2048Bcx+4096Adx)+32a^4\left(7A\left(36c^2+80cdx+45c\right.\right.\right.}{}$$

input
```
Integrate[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^11,x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(b^4*c*x^8*(945*A*c + 2048*B*c*x + 4096*A*d*x)
 + 32*a^4*(7*A*(36*c^2 + 80*c*d*x + 45*d^2*x^2) + 5*x*(3*C*x*(21*c^2 + 48*
c*d*x + 28*d^2*x^2) + 2*B*(28*c^2 + 63*c*d*x + 36*d^2*x^2))) + 12*a^2*b^2*
x^4*(A*(42*c^2 + 128*c*d*x + 105*d^2*x^2) + x*(2*B*(32*c^2 + 105*c*d*x + 9
6*d^2*x^2) + 3*C*x*(35*c^2 + 128*c*d*x + 140*d^2*x^2))) + 16*a^3*b*x^2*(A*
(693*c^2 + 1600*c*d*x + 945*d^2*x^2) + x*(3*C*x*(315*c^2 + 768*c*d*x + 490
*d^2*x^2) + 2*B*(400*c^2 + 945*c*d*x + 576*d^2*x^2))) - 2*a*b^3*x^6*(A*(31
5*c^2 + 1024*c*d*x + 945*d^2*x^2) + x*(9*c*C*x*(105*c + 512*d*x) + 2*B*(25
6*c^2 + 945*c*d*x + 1152*d^2*x^2)))))/x^10) - 630*b^3*(3*A*b^2*c^2 + 16*a^
2*C*d^2)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 3780*a*b^4*(c^2*
C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(8
0640*a^(7/2))
```

Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.87, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2338, 27, 2338, 25, 2338, 27, 539, 27, 539, 27, 534, 243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2} (c+dx)^2 \left(A+Bx+Cx^2\right)}{x^{11}} \, dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{5\left(bx^2+a\right)^{3/2}\left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^{10}}dx}{10a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)\right)}{x^{10}}dx}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(18a^2Cd^2x^2-2a(4bc(Bc+2Ad)-9ad(2cC+Bd))x+9a\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)\right)}{x^9}dx}{9a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{9x^9}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(18a^2Cd^2x^2-2a(4bc(Bc+2Ad)-9ad(2cC+Bd))x+9a\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)\right)}{x^9}dx}{9a}-\frac{2c\left(a+bx^2\right)^{5/2}(2Ad+Bc)}{9x^9}}{2a}-\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{\int \frac{a\left(16a(4bc(Bc+2Ad)-9ad(2cC+Bd))-9\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^8}dx}{8a}-\frac{9\left(a+bx^2\right)^{5/2}\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)}{8x^8}\cdots}{9a}\cdots}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{1}{8}\int \frac{\left(16a(4bc(Bc+2Ad)-9ad(2cC+Bd))-9\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^8}dx-\frac{9\left(a+bx^2\right)^{5/2}\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)}{8x^8}\cdots}{9a}\cdots}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 539

$$\frac{\frac{\frac{1}{8}\left(\frac{\int \frac{a\left(63\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)+32b(4bc(Bc+2Ad)-9ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x^7}dx}{7a}+\frac{16\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(Bd+2cC\cdots}{7x^7}\cdots\right.}{9a}\cdots}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{8}\left(\frac{1}{7}\int \frac{\left(63\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)+32b(4bc(Bc+2Ad)-9ad(2cC+Bd))x\right)\left(bx^2+a\right)^{3/2}}{x^7}dx+\frac{16\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(Bd+2c\cdots}{7x^7}\cdots\right.}{9a}\cdots}{2a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 539

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(-\frac{\int -\frac{3b\left(64a(4bc(Bc+2Ad)-9ad(2cC+Bd))-21\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx}{6a}-\frac{21\left(a+bx^2\right)^{5/2}\left(3Ab\left(bc^2-2ad^2\right)+\cdots\right.}{2ax^6}\cdots\right.\right.}{9a}\cdots$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\downarrow$ 27

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\int \frac{\left(64a(4bc(Bc+2Ad)-9ad(2cC+Bd))-21\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(cC+2Bd)\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{x^6}dx}{2a}-\frac{21\left(a+bx^2\right)^{5/2}\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8\right.\right.}{2ax^6}\right.\right.}{9a}$$

$$2a$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\big\downarrow$ 534

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-21\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)\int \frac{\left(bx^2+a\right)^{3/2}}{x^5}dx-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(Bd+2cC))}{5x^5}\right)}{2a}-\frac{21\left(a+bx^2\right)^{5/2}\left(3Ab\left(bc^2-\right.\right.}{}\right.\right.}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\big\downarrow$ 243

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-\frac{21}{2}\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(Bd+2cC))}{5x^5}\right)}{2a}-\frac{21\left(a+bx^2\right)^{5/2}\left(3Ab\left(bc\right.\right.}{}\right.\right.}{9a}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\big\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-\frac{21}{2}\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(Bd+2cC))}{5x^5}\right)}{2a}-\frac{21\left(a+\right.}{}\right.\right.}{9}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$\big\downarrow$ 51

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-\frac{21}{2}\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(B}{5x^5}}{2a}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-\frac{21}{2}\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)-9ad(B}{5x^5}}{2a}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{1}{8}\left(\frac{1}{7}\left(\frac{b\left(-\frac{21}{2}\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{2x^4}\right)\left(3Ab\left(bc^2-2ad^2\right)+2a\left(8aCd^2-3bc(2Bd+cC)\right)\right)-\frac{64\left(a+bx^2\right)^{5/2}(4bc(2Ad+Bc)}{5x^5}}{2a}\right.\right.}{}$$

$$\frac{Ac^2\left(a+bx^2\right)^{5/2}}{10ax^{10}}$$

input
```
Int[((c + d*x)^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/x^11,x]
```

output
```
-1/10*(A*c^2*(a + b*x^2)^(5/2))/(a*x^10) + ((-2*c*(B*c + 2*A*d)*(a + b*x^2
)^(5/2))/(9*x^9) + ((-9*(2*a*c*(c*C + 2*B*d) - A*(b*c^2 - 2*a*d^2))*(a + b
*x^2)^(5/2))/(8*x^8) + ((16*(4*b*c*(B*c + 2*A*d) - 9*a*d*(2*c*C + B*d))*(a
 + b*x^2)^(5/2))/(7*x^7) + ((-21*(3*A*b*(b*c^2 - 2*a*d^2) + 2*a*(8*a*C*d^2
 - 3*b*c*(c*C + 2*B*d)))*(a + b*x^2)^(5/2))/(2*a*x^6) + (b*((-64*(4*b*c*(B
*c + 2*A*d) - 9*a*d*(2*c*C + B*d))*(a + b*x^2)^(5/2))/(5*x^5) - (21*(3*A*b
*(b*c^2 - 2*a*d^2) + 2*a*(8*a*C*d^2 - 3*b*c*(c*C + 2*B*d)))*(-1/2*(a + b*x
^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/Sqrt[a]))/4))/2))/(2*a))/7)/8)/(9*a))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.27

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(4096 A\,b^4 c d\,x^9-4608 B a\,b^3 d^2 x^9+2048 B\,b^4 c^2 x^9-9216 C a\,b^3 c d\,x^9-1890 A a\,b^3 d^2 x^8+945 A\,b^4 c^2 x^8-3780 B a\,b^3 c d\,x^8+50\cdots\right.}{\cdots}$
default	$\left(A\,d^2+2 B c d+C\,c^2\right)\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{8 a\,x^8}-\frac{3 b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{6 a\,x^6}-\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{4 a\,x^4}+\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{2 a\,x^2}+\frac{3 b\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2\cdots}\right.\right.}{\cdots}\right)}{4 a}\right)}{6 a}\right)}{8 a}\cdots\right.$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/80640*(b*x^2+a)^(1/2)*(4096*A*b^4*c*d*x^9-4608*B*a*b^3*d^2*x^9+2048*B*b
^4*c^2*x^9-9216*C*a*b^3*c*d*x^9-1890*A*a*b^3*d^2*x^8+945*A*b^4*c^2*x^8-378
0*B*a*b^3*c*d*x^8+5040*C*a^2*b^2*d^2*x^8-1890*C*a*b^3*c^2*x^8-2048*A*a*b^3
*c*d*x^7+2304*B*a^2*b^2*d^2*x^7-1024*B*a*b^3*c^2*x^7+4608*C*a^2*b^2*c*d*x^
7+1260*A*a^2*b^2*d^2*x^6-630*A*a*b^3*c^2*x^6+2520*B*a^2*b^2*c*d*x^6+23520*
C*a^3*b*d^2*x^6+1260*C*a^2*b^2*c^2*x^6+1536*A*a^2*b^2*c*d*x^5+18432*B*a^3*
b*d^2*x^5+768*B*a^2*b^2*c^2*x^5+36864*C*a^3*b*c*d*x^5+15120*A*a^3*b*d^2*x^
4+504*A*a^2*b^2*c^2*x^4+30240*B*a^3*b*c*d*x^4+13440*C*a^4*d^2*x^4+15120*C*
a^3*b*c^2*x^4+25600*A*a^3*b*c*d*x^3+11520*B*a^4*d^2*x^3+12800*B*a^3*b*c^2*
x^3+23040*C*a^4*c*d*x^3+10080*A*a^4*d^2*x^2+11088*A*a^3*b*c^2*x^2+20160*B*
a^4*c*d*x^2+10080*C*a^4*c^2*x^2+17920*A*a^4*c*d*x+8960*B*a^4*c^2*x+8064*A*
a^4*c^2)/x^10/a^3-1/256*(6*A*a*b*d^2-3*A*b^2*c^2+12*B*a*b*c*d-16*C*a^2*d^2
+6*C*a*b*c^2)*b^3/a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 1097, normalized size of antiderivative = 2.44

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^11,x, algorithm="frica
s")
```

output
```
[1/161280*(315*(12*B*a*b^4*c*d + 3*(2*C*a*b^4 - A*b^5)*c^2 - 2*(8*C*a^2*b^
3 - 3*A*a*b^4)*d^2)*sqrt(a)*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(512*(4*B*a*b^4*c^2 - 9*B*a^2*b^3*d^2 - 2*(9*C*a^2*b^3 - 4*
A*a*b^4)*c*d)*x^9 - 315*(12*B*a^2*b^3*c*d + 3*(2*C*a^2*b^3 - A*a*b^4)*c^2
- 2*(8*C*a^3*b^2 - 3*A*a^2*b^3)*d^2)*x^8 + 8064*A*a^5*c^2 - 256*(4*B*a^2*b
^3*c^2 - 9*B*a^3*b^2*d^2 - 2*(9*C*a^3*b^2 - 4*A*a^2*b^3)*c*d)*x^7 + 210*(1
2*B*a^3*b^2*c*d + 3*(2*C*a^3*b^2 - A*a^2*b^3)*c^2 + 2*(56*C*a^4*b + 3*A*a^
3*b^2)*d^2)*x^6 + 768*(B*a^3*b^2*c^2 + 24*B*a^4*b*d^2 + 2*(24*C*a^4*b + A*
a^3*b^2)*c*d)*x^5 + 168*(180*B*a^4*b*c*d + 3*(30*C*a^4*b + A*a^3*b^2)*c^2
+ 10*(8*C*a^5 + 9*A*a^4*b)*d^2)*x^4 + 1280*(10*B*a^4*b*c^2 + 9*B*a^5*d^2 +
 2*(9*C*a^5 + 10*A*a^4*b)*c*d)*x^3 + 1008*(20*B*a^5*c*d + 10*A*a^5*d^2 + (
10*C*a^5 + 11*A*a^4*b)*c^2)*x^2 + 8960*(B*a^5*c^2 + 2*A*a^5*c*d)*x)*sqrt(b
*x^2 + a))/(a^4*x^10), 1/80640*(315*(12*B*a*b^4*c*d + 3*(2*C*a*b^4 - A*b^5
)*c^2 - 2*(8*C*a^2*b^3 - 3*A*a*b^4)*d^2)*sqrt(-a)*x^10*arctan(sqrt(b*x^2 +
 a)*sqrt(-a)/a) - (512*(4*B*a*b^4*c^2 - 9*B*a^2*b^3*d^2 - 2*(9*C*a^2*b^3 -
 4*A*a*b^4)*c*d)*x^9 - 315*(12*B*a^2*b^3*c*d + 3*(2*C*a^2*b^3 - A*a*b^4)*c
^2 - 2*(8*C*a^3*b^2 - 3*A*a^2*b^3)*d^2)*x^8 + 8064*A*a^5*c^2 - 256*(4*B*a^
2*b^3*c^2 - 9*B*a^3*b^2*d^2 - 2*(9*C*a^3*b^2 - 4*A*a^2*b^3)*c*d)*x^7 + 210
*(12*B*a^3*b^2*c*d + 3*(2*C*a^3*b^2 - A*a^2*b^3)*c^2 + 2*(56*C*a^4*b + 3*A
*a^3*b^2)*d^2)*x^6 + 768*(B*a^3*b^2*c^2 + 24*B*a^4*b*d^2 + 2*(24*C*a^4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**11,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 701, normalized size of antiderivative = 1.56

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^11,x, algorithm="maxim
a")
```

output
```
3/256*A*b^5*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/16*C*b^3*d^2*arc
sinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/256*(b*x^2 + a)^(3/2)*A*b^5*c^2/a^5
 - 3/256*sqrt(b*x^2 + a)*A*b^5*c^2/a^4 - 1/48*(b*x^2 + a)^(3/2)*C*b^3*d^2/
a^3 - 1/16*sqrt(b*x^2 + a)*C*b^3*d^2/a^2 - 3/128*(C*c^2 + 2*B*c*d + A*d^2)
*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(C*c^2 + 2*B*c*d + A*d^
2)*(b*x^2 + a)^(3/2)*b^4/a^4 + 3/128*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2
+ a)*b^4/a^3 + 1/256*(b*x^2 + a)^(5/2)*A*b^4*c^2/(a^5*x^2) + 1/48*(b*x^2 +
 a)^(5/2)*C*b^2*d^2/(a^3*x^2) + 1/128*(b*x^2 + a)^(5/2)*A*b^3*c^2/(a^4*x^4
) + 1/24*(b*x^2 + a)^(5/2)*C*b*d^2/(a^2*x^4) - 1/128*(C*c^2 + 2*B*c*d + A*
d^2)*(b*x^2 + a)^(5/2)*b^3/(a^4*x^2) - 1/32*(b*x^2 + a)^(5/2)*A*b^2*c^2/(a
^3*x^6) - 1/6*(b*x^2 + a)^(5/2)*C*d^2/(a*x^6) - 1/64*(C*c^2 + 2*B*c*d + A*
d^2)*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 2/35*(2*C*c*d + B*d^2)*(b*x^2 + a)^
(5/2)*b/(a^2*x^5) - 8/315*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b^2/(a^3*x^5
) + 1/16*(b*x^2 + a)^(5/2)*A*b*c^2/(a^2*x^8) + 1/16*(C*c^2 + 2*B*c*d + A*d
^2)*(b*x^2 + a)^(5/2)*b/(a^2*x^6) - 1/7*(2*C*c*d + B*d^2)*(b*x^2 + a)^(5/2
)/(a*x^7) + 4/63*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)*b/(a^2*x^7) - 1/10*(b
*x^2 + a)^(5/2)*A*c^2/(a*x^10) - 1/8*(C*c^2 + 2*B*c*d + A*d^2)*(b*x^2 + a)
^(5/2)/(a*x^8) - 1/9*(B*c^2 + 2*A*c*d)*(b*x^2 + a)^(5/2)/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2598 vs. 2(410) = 820.

Time = 0.24 (sec) , antiderivative size = 2598, normalized size of antiderivative = 5.77

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^11,x, algorithm="giac"
)
```

output
```
1/128*(6*C*a*b^4*c^2 - 3*A*b^5*c^2 + 12*B*a*b^4*c*d - 16*C*a^2*b^3*d^2 + 6
*A*a*b^4*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^
3) - 1/40320*(1890*(sqrt(b)*x - sqrt(b*x^2 + a))^19*C*a*b^4*c^2 - 945*(sqr
t(b)*x - sqrt(b*x^2 + a))^19*A*b^5*c^2 + 3780*(sqrt(b)*x - sqrt(b*x^2 + a)
)^19*B*a*b^4*c*d - 5040*(sqrt(b)*x - sqrt(b*x^2 + a))^19*C*a^2*b^3*d^2 + 1
890*(sqrt(b)*x - sqrt(b*x^2 + a))^19*A*a*b^4*d^2 - 18270*(sqrt(b)*x - sqrt
(b*x^2 + a))^17*C*a^2*b^4*c^2 + 9135*(sqrt(b)*x - sqrt(b*x^2 + a))^17*A*a*
b^5*c^2 - 36540*(sqrt(b)*x - sqrt(b*x^2 + a))^17*B*a^2*b^4*c*d - 58800*(sq
rt(b)*x - sqrt(b*x^2 + a))^17*C*a^3*b^3*d^2 - 18270*(sqrt(b)*x - sqrt(b*x^
2 + a))^17*A*a^2*b^4*d^2 - 322560*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a^3*b
^(7/2)*c*d - 161280*(sqrt(b)*x - sqrt(b*x^2 + a))^16*B*a^3*b^(7/2)*d^2 - 1
78920*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^3*b^4*c^2 - 39564*(sqrt(b)*x -
sqrt(b*x^2 + a))^15*A*a^2*b^5*c^2 - 357840*(sqrt(b)*x - sqrt(b*x^2 + a))^1
5*B*a^3*b^4*c*d + 154560*(sqrt(b)*x - sqrt(b*x^2 + a))^15*C*a^4*b^3*d^2 -
178920*(sqrt(b)*x - sqrt(b*x^2 + a))^15*A*a^3*b^4*d^2 - 430080*(sqrt(b)*x
- sqrt(b*x^2 + a))^14*B*a^3*b^(9/2)*c^2 + 645120*(sqrt(b)*x - sqrt(b*x^2 +
 a))^14*C*a^4*b^(7/2)*c*d - 860160*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a^3*
b^(9/2)*c*d + 322560*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a^4*b^(7/2)*d^2 -
17640*(sqrt(b)*x - sqrt(b*x^2 + a))^13*C*a^4*b^4*c^2 - 636300*(sqrt(b)*x -
 sqrt(b*x^2 + a))^13*A*a^3*b^5*c^2 - 35280*(sqrt(b)*x - sqrt(b*x^2 + a)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \int \frac{(b\,x^2+a)^{3/2}\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^{11}} \,dx$$

input
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^11,x)
```

output
```
int(((a + b*x^2)^(3/2)*(c + d*x)^2*(A + B*x + C*x^2))/x^11, x)
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.65

$$\int \frac{(c+dx)^2 (a+bx^2)^{3/2} (A+Bx+Cx^2)}{x^{11}} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^11,x)
```

output
```
( - 8064*sqrt(a + b*x**2)*a**5*c**2 - 17920*sqrt(a + b*x**2)*a**5*c*d*x -
10080*sqrt(a + b*x**2)*a**5*d**2*x**2 - 11088*sqrt(a + b*x**2)*a**4*b*c**2
*x**2 - 8960*sqrt(a + b*x**2)*a**4*b*c**2*x - 25600*sqrt(a + b*x**2)*a**4*
b*c*d*x**3 - 20160*sqrt(a + b*x**2)*a**4*b*c*d*x**2 - 15120*sqrt(a + b*x**
2)*a**4*b*d**2*x**4 - 11520*sqrt(a + b*x**2)*a**4*b*d**2*x**3 - 10080*sqrt
(a + b*x**2)*a**4*c**3*x**2 - 23040*sqrt(a + b*x**2)*a**4*c**2*d*x**3 - 13
440*sqrt(a + b*x**2)*a**4*c*d**2*x**4 - 504*sqrt(a + b*x**2)*a**3*b**2*c**
2*x**4 - 12800*sqrt(a + b*x**2)*a**3*b**2*c**2*x**3 - 1536*sqrt(a + b*x**2
)*a**3*b**2*c*d*x**5 - 30240*sqrt(a + b*x**2)*a**3*b**2*c*d*x**4 - 1260*sq
rt(a + b*x**2)*a**3*b**2*d**2*x**6 - 18432*sqrt(a + b*x**2)*a**3*b**2*d**2
*x**5 - 15120*sqrt(a + b*x**2)*a**3*b*c**3*x**4 - 36864*sqrt(a + b*x**2)*a
**3*b*c**2*d*x**5 - 23520*sqrt(a + b*x**2)*a**3*b*c*d**2*x**6 + 630*sqrt(a
 + b*x**2)*a**2*b**3*c**2*x**6 - 768*sqrt(a + b*x**2)*a**2*b**3*c**2*x**5
+ 2048*sqrt(a + b*x**2)*a**2*b**3*c*d*x**7 - 2520*sqrt(a + b*x**2)*a**2*b*
*3*c*d*x**6 + 1890*sqrt(a + b*x**2)*a**2*b**3*d**2*x**8 - 2304*sqrt(a + b*
x**2)*a**2*b**3*d**2*x**7 - 1260*sqrt(a + b*x**2)*a**2*b**2*c**3*x**6 - 46
08*sqrt(a + b*x**2)*a**2*b**2*c**2*d*x**7 - 5040*sqrt(a + b*x**2)*a**2*b**
2*c*d**2*x**8 - 945*sqrt(a + b*x**2)*a*b**4*c**2*x**8 + 1024*sqrt(a + b*x*
*2)*a*b**4*c**2*x**7 - 4096*sqrt(a + b*x**2)*a*b**4*c*d*x**9 + 3780*sqrt(a
 + b*x**2)*a*b**4*c*d*x**8 + 4608*sqrt(a + b*x**2)*a*b**4*d**2*x**9 + 1...
```

3.76 $\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 529

$$
\begin{aligned}
\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx &= \frac{c^2(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^7} \\
&- \frac{\left(2Abc(4bc^2+3ad^2)+\frac{(cC-Bd)(8b^2c^4+6abc^2d^2-a^2d^4)}{d^2}\right)x\sqrt{a+bx^2}}{16bd^4} \\
&+ \frac{c^2(c^2C-Bcd+Ad^2)(a+bx^2)^{3/2}}{3d^5} \\
&+ \frac{(ad^2(cC-Bd)-6bc(c^2C-Bcd+Ad^2))x(a+bx^2)^{3/2}}{24bd^4} \\
&+ \frac{\left(42Ab-12aC+\frac{bc(107cC-77Bd)}{d^2}\right)(a+bx^2)^{5/2}}{210b^2d} \\
&- \frac{(19cC-7Bd)(c+dx)(a+bx^2)^{5/2}}{42bd^3} + \frac{C(c+dx)^2(a+bx^2)^{5/2}}{7bd^3} \\
&+ \frac{(a^3d^6(cC-Bd)-16b^3c^5(c^2C-Bcd+Ad^2)-24ab^2c^3d^2(c^2C-Bcd+Ad^2)-6a^2bcd^4(c^2C-Bcd+A}{16b^{3/2}d^8} \\
&- \frac{c^2(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^8}
\end{aligned}
$$

output
```
c^2*(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^7-1/16*(2*A*b*c*(3
*a*d^2+4*b*c^2)+(-B*d+C*c)*(-a^2*d^4+6*a*b*c^2*d^2+8*b^2*c^4)/d^2)*x*(b*x^
2+a)^(1/2)/b/d^4+1/3*c^2*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(3/2)/d^5+1/24*(a*d
^2*(-B*d+C*c)-6*b*c*(A*d^2-B*c*d+C*c^2))*x*(b*x^2+a)^(3/2)/b/d^4+1/210*(42
*A*b-12*a*C+b*c*(-77*B*d+107*C*c)/d^2)*(b*x^2+a)^(5/2)/b^2/d-1/42*(-7*B*d+
19*C*c)*(d*x+c)*(b*x^2+a)^(5/2)/b/d^3+1/7*C*(d*x+c)^2*(b*x^2+a)^(5/2)/b/d^
3+1/16*(a^3*d^6*(-B*d+C*c)-16*b^3*c^5*(A*d^2-B*c*d+C*c^2)-24*a*b^2*c^3*d^2
*(A*d^2-B*c*d+C*c^2)-6*a^2*b*c*d^4*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(3/2)/d^8-c^2*(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*a
rctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^8
```

Mathematica [A] (verified)

Time = 3.91 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.99

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(-96a^3Cd^6+3a^2bd^4\left(112c^2C-7cd(16B+5Cx)+d^2\left(112A+35Bx+16Cx^2\right)\right)+2ab\cdots\right.}{\cdots}$$

input
```
Integrate[(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(-96*a^3*C*d^6 + 3*a^2*b*d^4*(112*c^2*C - 7*c*d*(16*B
+ 5*C*x) + d^2*(112*A + 35*B*x + 16*C*x^2)) + 2*a*b^2*d^2*(1120*c^4*C - 35
*c^3*d*(32*B + 15*C*x) - 7*c*d^3*x*(75*A + 48*B*x + 35*C*x^2) + 7*c^2*d^2*
(160*A + 75*B*x + 48*C*x^2) + d^4*x^2*(336*A + 245*B*x + 192*C*x^2)) + 4*b
^3*(420*c^6*C - 210*c^5*d*(2*B + C*x) + 70*c^4*d^2*(6*A + x*(3*B + 2*C*x))
 - 35*c^3*d^3*x*(6*A + x*(4*B + 3*C*x)) + 7*c^2*d^4*x^2*(20*A + 3*x*(5*B +
 4*C*x)) - 7*c*d^5*x^3*(15*A + 2*x*(6*B + 5*C*x)) + 2*d^6*x^4*(42*A + 5*x*
(7*B + 6*C*x)))))/b^2 - 3360*c^2*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d +
 A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d
^2]] + (105*(a^3*d^6*(-(c*C) + B*d) + 16*b^3*c^5*(c^2*C - B*c*d + A*d^2) +
 24*a*b^2*c^3*d^2*(c^2*C - B*c*d + A*d^2) + 6*a^2*b*c*d^4*(c^2*C - B*c*d +
 A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(1680*d^8)
```

Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {2185, 25, 2185, 25, 2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{c+dx}\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(bd^3(19cC-7Bd)x^3+d^2\left(17bCc^2-7Abd^2+2aCd^2\right)x^2+cCd\left(5bc^2+4ad^2\right)x+2ac^2Cd^2\right)}{c+dx}dx}{7bd^4}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(bd^3(19cC-7Bd)x^3+d^2\left(17bCc^2-7Abd^2+2aCd^2\right)x^2+cCd\left(5bc^2+4ad^2\right)x+2ac^2Cd^2\right)}{c+dx}dx}{7bd^4}$$

$$\Big\downarrow \text{2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(-b\left(12aCd^2-b\left(107Cc^2-77Bdc+42Ad^2\right)\right)x^2d^5+7abc(cC-Bd)d^5+b\left(5bc^2(13cC-7Bd)-ad^2(5cC+7Bd)\right)xd^4\right)}{c+dx}dx}{6bd^3}+\frac{1}{6}d\left(a+bx^2\right)^{5/2}(\ldots}{7bd^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(-b\left(12aCd^2-b\left(107Cc^2-77Bdc+42Ad^2\right)\right)x^2d^5+7abc(cC-Bd)d^5+b\left(5bc^2(13cC-7B\ldots\right.\right.}{c+dx}}{6bd^3}}{7bd^4}$$

$$\Big\downarrow \text{2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3} - \frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd) - \frac{\frac{\int \frac{35b^2d^6\left(acd(cC-Bd)+\left(ad^2(cC-Bd)-6bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{5bd^2} - \frac{1}{5}d^4\left(a+bx^2\right)^{5/}}{6bd^3}}{7bd^4}$$

$\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3} - \frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd) - \frac{7bd^4\int \frac{\left(acd(cC-Bd)+\left(ad^2(cC-Bd)-6bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx - \frac{1}{5}d^4\left(a+bx^2\right)^{5/2}}{6bd^3}}{7bd^4}$$

$\downarrow$ 682

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3} - \frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd) - 7bd^4\left(\frac{\int \frac{3b\left(acd\left(a(cC-Bd)d^2+2bc\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^2(cC-Bd)d^4-6abc\left(Cc^2-Bdc+Ad^2\right)d^2-\right.\right.}{c+dx}}{4bd^2}\right.}{}$$

$\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3} - \frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd) - 7bd^4\left(\frac{3\int \frac{\left(acd\left(a(cC-Bd)d^2+2bc\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^2(cC-Bd)d^4-6abc\left(Cc^2-Bdc+Ad^2\right)d^2-\right.\right.}{c+dx}}{4d^2}\right.}{}$$

$\downarrow$ 682

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3} - \frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd) - 7bd^4\left(3\left(\int \frac{b\left(acd\left(a^2(cC-Bd)d^4+10abc\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^3(cC-\right.\right.}{(c+}\right.\right.}{}$$

$\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-$$

$$7bd^4 \quad 3\left(\int \frac{acd\left(a^2(cC-Bd)d^4+10abc\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^3(cC-Bd\right.}{(c+dx}\right.$$

$$\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-$$

$\downarrow$ 719

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-$$

$$7bd^4 \quad 3\left(\frac{\frac{\left(a^3d^6(cC-Bd)-6a^2bcd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^3d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^5\left(Ad^2-\right.\right.}{d}}{2d^2}\right.$$

$$\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-$$

$\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-$$

$$7bd^4 \quad 3\left(\frac{\left(a^3d^6(cC-Bd)-6a^2bcd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^3d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^5\left(Ad^2-\right.\right.}{d}\right.$$

$$\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-$$

$\downarrow$ 219

$$\frac{\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-\frac{3\left(\frac{\frac{16bc^2\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3d^6\right.}{}}{}\right.}{7bd^4}}{}}{}$$

$\downarrow$ 488

$$\frac{\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-\frac{3\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3d^6(cC-Bd)-6a^2bcd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^3d^2\left(Ad^2-Bc\right.\right.}{\sqrt{b}d}}{}\right.}{7bd^4}}{}}{}$$

$\downarrow$ 219

$$\frac{\frac{C\left(a+bx^2\right)^{5/2}(c+dx)^2}{7bd^3}-\frac{\frac{1}{6}d\left(a+bx^2\right)^{5/2}(c+dx)(19cC-7Bd)-\frac{3\left(\frac{\frac{\sqrt{a+bx^2}\left(dx\left(a^2d^4(cC-Bd)-6abcd^2\left(Ad^2-Bcd+c^2C\right)-8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)+16bc^2\left(\right.\right.}{2d^2}}{}\right.}{7bd^4}}{}}{}$$

input
```
Int[(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(c + d*x)^2*(a + b*x^2)^(5/2))/(7*b*d^3) - ((d*(19*c*C - 7*B*d)*(c + d*
x)*(a + b*x^2)^(5/2))/6 - (-1/5*(d^4*(12*a*C*d^2 - b*(107*c^2*C - 77*B*c*d
 + 42*A*d^2))*(a + b*x^2)^(5/2)) + 7*b*d^4*(((8*b*c^2*(c^2*C - B*c*d + A*d
^2) + d*(a*d^2*(c*C - B*d) - 6*b*c*(c^2*C - B*c*d + A*d^2))*x)*(a + b*x^2)
^(3/2))/(4*d^2) + (3*(((16*b*c^2*(b*c^2 + a*d^2)*(c^2*C - B*c*d + A*d^2) +
 d*(a^2*d^4*(c*C - B*d) - 8*b^2*c^3*(c^2*C - B*c*d + A*d^2) - 6*a*b*c*d^2*
(c^2*C - B*c*d + A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) + (((a^3*d^6*(c*C - B
*d) - 16*b^3*c^5*(c^2*C - B*c*d + A*d^2) - 24*a*b^2*c^3*d^2*(c^2*C - B*c*d
 + A*d^2) - 6*a^2*b*c*d^4*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(Sqrt[b]*d) - (16*b*c^2*(b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*
d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d
)/(2*d^2)))/(4*d^2)))/(6*b*d^3))/(7*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.60

method	result
default	$\frac{c^2(A d^2-Bcd+C c^2)\left(\frac{\left(b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}\right)^{\frac{3}{2}}}{3}-\frac{bc\left(\frac{\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a d^2+b c^2}{d^2}}}{4b}+\frac{\left(\frac{4b\left(a d^2+b\right.}{d^2}\right.}{}\right.}{}\right.}{}$
risch	$\frac{(240C b^3 d^6 x^6+280B b^3 d^6 x^5-280C b^3 c d^5 x^5+336A b^3 d^6 x^4-336B b^3 c d^5 x^4+384C a b^2 d^6 x^4+336C b^3 c^2 d^4 x^4-420A b^3 c d^5 x^3+490B}{}$

input
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
c^2*(A*d^2-B*c*d+C*c^2)/d^5*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3
/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^
2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2
)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))-1/d^4*
(C*c^3*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))))-d^2*(B*d-C*c)*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*
a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2)))))-1/5*d*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(5/2)/b+A
*c*d^2*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2))))-B*c^2*d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x
*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))))-C*d^3*(1/7*
x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x^2(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(x**2*(a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 971, normalized size of antiderivative = 1.84

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*C*x^2/(b*d) - 1/2*sqrt(b*x^2 + a)*C*b*c^5*x/d^6 + 1/
2*sqrt(b*x^2 + a)*B*b*c^4*x/d^5 - 1/4*(b*x^2 + a)^(3/2)*C*c^3*x/d^4 - 3/8*
sqrt(b*x^2 + a)*C*a*c^3*x/d^4 - 1/2*sqrt(b*x^2 + a)*A*b*c^3*x/d^4 + 1/4*(b
*x^2 + a)^(3/2)*B*c^2*x/d^3 + 3/8*sqrt(b*x^2 + a)*B*a*c^2*x/d^3 - 1/4*(b*x
^2 + a)^(3/2)*A*c*x/d^2 - 3/8*sqrt(b*x^2 + a)*A*a*c*x/d^2 - 1/6*(b*x^2 + a
)^(5/2)*C*c*x/(b*d^2) + 1/24*(b*x^2 + a)^(3/2)*C*a*c*x/(b*d^2) + 1/16*sqrt
(b*x^2 + a)*C*a^2*c*x/(b*d^2) + 1/6*(b*x^2 + a)^(5/2)*B*x/(b*d) - 1/24*(b*
x^2 + a)^(3/2)*B*a*x/(b*d) - 1/16*sqrt(b*x^2 + a)*B*a^2*x/(b*d) - C*b^(3/2
)*c^7*arcsinh(b*x/sqrt(a*b))/d^8 + B*b^(3/2)*c^6*arcsinh(b*x/sqrt(a*b))/d^
7 - 3/2*C*a*sqrt(b)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 - A*b^(3/2)*c^5*arcsinh
(b*x/sqrt(a*b))/d^6 + 3/2*B*a*sqrt(b)*c^4*arcsinh(b*x/sqrt(a*b))/d^5 - 3/8
*C*a^2*c^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) - 3/2*A*a*sqrt(b)*c^3*arcs
inh(b*x/sqrt(a*b))/d^4 + 3/8*B*a^2*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3
) + 1/16*C*a^3*c*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) - 3/8*A*a^2*c*arcsin
h(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/(b^(3/2
)*d) + C*(a + b*c^2/d^2)^(3/2)*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c))
- a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 - B*(a + b*c^2/d^2)^(3/2)*c^3*arcsinh(
b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 + A*(a
+ b*c^2/d^2)^(3/2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(
a*b)*abs(d*x + c)))/d^3 + sqrt(b*x^2 + a)*C*b*c^6/d^7 - sqrt(b*x^2 + a)...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x^2\,(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{c+d\,x}\,dx$$

input
```
int((x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int((x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x^2(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{dx+c}dx$$

input
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

3.77 $\int \frac{x\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{c+dx}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 452

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = -\frac{c(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^6}$$
$$-\frac{\left(a^2Cd^3+8b^2c^2\left(Bc-\frac{c^2C}{d}-Ad\right)-6abd(c^2C-Bcd+Ad^2)\right)x\sqrt{a+bx^2}}{16bd^4}$$
$$-\frac{c(c^2C-Bcd+Ad^2)(a+bx^2)^{3/2}}{3d^4}-\frac{(aCd^2-6b(c^2C-Bcd+Ad^2))x(a+bx^2)^{3/2}}{24bd^3}$$
$$-\frac{(11cC-6Bd)(a+bx^2)^{5/2}}{30bd^2}+\frac{C(c+dx)(a+bx^2)^{5/2}}{6bd^2}$$
$$-\frac{(a^3Cd^6-16b^3c^4(c^2C-Bcd+Ad^2)-24ab^2c^2d^2(c^2C-Bcd+Ad^2)-6a^2bd^4(c^2C-Bcd+Ad^2))\,\text{arcta}}{16b^{3/2}d^7}$$
$$+\frac{c(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}$$

output

```
-c*(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^6-1/16*(a^2*C*d^3+8
*b^2*c^2*(B*c-c^2*C/d-A*d)-6*a*b*d*(A*d^2-B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/
b/d^4-1/3*c*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(3/2)/d^4-1/24*(a*C*d^2-6*b*(A*d
^2-B*c*d+C*c^2))*x*(b*x^2+a)^(3/2)/b/d^3-1/30*(-6*B*d+11*C*c)*(b*x^2+a)^(5
/2)/b/d^2+1/6*C*(d*x+c)*(b*x^2+a)^(5/2)/b/d^2-1/16*(a^3*C*d^6-16*b^3*c^4*(
A*d^2-B*c*d+C*c^2)-24*a*b^2*c^2*d^2*(A*d^2-B*c*d+C*c^2)-6*a^2*b*d^4*(A*d^2
-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^7+c*(a*d^2+b*c
^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*
x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 2.66 (sec) , antiderivative size = 426, normalized size of antiderivative = 0.94

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(3a^2d^4(-16cC+16Bd+5Cdx)-2abd^2\left(160c^3C-5c^2d(32B+15Cx)-d^3x\left(75A+48\right.\right.\right.}{}$$

input

```
Integrate[(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output

```
((d*Sqrt[a + b*x^2]*(3*a^2*d^4*(-16*c*C + 16*B*d + 5*C*d*x) - 2*a*b*d^2*(1
60*c^3*C - 5*c^2*d*(32*B + 15*C*x) - d^3*x*(75*A + 48*B*x + 35*C*x^2) + c*
d^2*(160*A + 75*B*x + 48*C*x^2)) - 4*b^2*(60*c^5*C - 30*c^4*d*(2*B + C*x)
+ 10*c^3*d^2*(6*A + x*(3*B + 2*C*x)) - 5*c^2*d^3*x*(6*A + x*(4*B + 3*C*x))
 + c*d^4*x^2*(20*A + 3*x*(5*B + 4*C*x)) - d^5*x^3*(15*A + 2*x*(6*B + 5*C*x
)))))/b + 480*c*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTan[(S
qrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (15*(-(a^3
*C*d^6) + 16*b^3*c^4*(c^2*C - B*c*d + A*d^2) + 24*a*b^2*c^2*d^2*(c^2*C - B
*c*d + A*d^2) + 6*a^2*b*d^4*(c^2*C - B*c*d + A*d^2))*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/b^(3/2))/(240*d^7)
```

Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.99, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.433$, Rules used = {2185, 25, 2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{c+dx}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int -\frac{\left(bx^2+a\right)^{3/2}\left(b(11cC-6Bd)x^2d^2+acCd^2+\left(5bCc^2-6Abd^2+aCd^2\right)xd\right)}{c+dx}dx}{6bd^3}+\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(b(11cC-6Bd)x^2d^2+acCd^2+\left(5bCc^2-6Abd^2+aCd^2\right)xd\right)}{c+dx}dx}{6bd^3}$$

$$\Big\downarrow \text{2185}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-\frac{\frac{\int \frac{5bd^3\left(acCd+\left(aCd^2-6b\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{5bd^2}+\frac{1}{5}d\left(a+bx^2\right)^{5/2}(11cC-6Bd)}{6bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-\frac{d\int \frac{\left(acCd+\left(aCd^2-6b\left(Cc^2-Bdc+Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx+\frac{1}{5}d\left(a+bx^2\right)^{5/2}(11cC-6Bd)}{6bd^3}$$

$$\Big\downarrow \text{682}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-\frac{d\left(\frac{\int \frac{b\left(3acd\left(aCd^2+2b\left(Cc^2-Bdc+Ad^2\right)\right)-\left(4abc^2Cd^2-\left(4bc^2+3ad^2\right)\left(aCd^2-6b\left(Cc^2-Bdc+Ad^2\right)\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(dx\left(aCd^2-6b\left(Ad^2-\right.\right.\right.}{}\right.}{6bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$
$$\frac{d\left(\frac{\int \frac{\left(3acd\left(aCd^2+2b\left(Cc^2-Bdc+Ad^2\right)\right)-\left(4abc^2Cd^2-\left(4bc^2+3ad^2\right)\left(aCd^2-6b\left(Cc^2-Bdc+Ad^2\right)\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(dx\left(aCd^2-6b\left(Ad^2-\right.\right.\right.}{4}\right.}{6bd^3}$$

$\downarrow$ 682

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$
$$d\left(\frac{\frac{\int \frac{3b\left(acd\left(a^2Cd^4+10ab\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^2\left(Cc^2-Bdc+Ad^2\right)\right)\right)+\left(a^3Cd^6-6a^2b\left(Cc^2-Bdc+Ad^2\right)d^4-24ab^2c^2\left(Cc^2-Bdc+Ad^2\right)d^2-16b^3c^4\left(Cc^2-\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}}{4d^2}\right.$$

$\downarrow$ 27

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$
$$d\left(\frac{\frac{3\int \frac{acd\left(a^2Cd^4+10ab\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^2\left(Cc^2-Bdc+Ad^2\right)\right)+\left(a^3Cd^6-6a^2b\left(Cc^2-Bdc+Ad^2\right)d^4-24ab^2c^2\left(Cc^2-Bdc+Ad^2\right)d^2-16b^3c^4\left(Cc^2-B\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}}{4d^2}\right.$$

$\downarrow$ 719

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$
$$d\left(\frac{\frac{3\left(\frac{\left(a^3Cd^6-6a^2bd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^2d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^4\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{16bc\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int \frac{}{(c+}}{d}\right.}{2d^2}}{4d^2}\right.$$

$\downarrow$ 224

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$

$$d\left(\frac{\frac{3\left(\frac{\left(a^3Cd^6-6a^2bd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^2d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^4\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}+\frac{16bc\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c\right.}{d}\right.}{2d^2}}{4d^2}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$

$$d\left(\frac{\frac{3\left(\frac{16bc\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3Cd^6-6a^2bd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^2d^2\left(Ad^2-Bcd+c^2C\right)-1\right.}{\sqrt{b}d}\right.}{2d^2}}{4d^2}\right.$$

$$\Big\downarrow \text{488}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$

$$d\left(\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3Cd^6-6a^2bd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^2d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^4\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{16bc\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c\right.}{}\right.}{2d^2}}{4d^2}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{C\left(a+bx^2\right)^{5/2}(c+dx)}{6bd^2}-$$

$$d\left(\frac{\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3Cd^6-6a^2bd^4\left(Ad^2-Bcd+c^2C\right)-24ab^2c^2d^2\left(Ad^2-Bcd+c^2C\right)-16b^3c^4\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{16bc\left(ad^2+bc^2\right)^{3/2}\left(Ad^2-Bcd\right.}{}\right.}{2d^2}}{4d}\right.$$

input
```
Int[(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(c + d*x)*(a + b*x^2)^(5/2))/(6*b*d^2) - ((d*(11*c*C - 6*B*d)*(a + b*x^
2)^(5/2))/5 + d*(((8*b*c*(c^2*C - B*c*d + A*d^2) + d*(a*C*d^2 - 6*b*(c^2*C
 - B*c*d + A*d^2))*x)*(a + b*x^2)^(3/2))/(4*d^2) + (((48*b*c*(b*c^2 + a*d^
2)*(c^2*C - B*c*d + A*d^2) - d*(4*a*b*c^2*C*d^2 - (4*b*c^2 + 3*a*d^2)*(a*C
*d^2 - 6*b*(c^2*C - B*c*d + A*d^2)))*x)*Sqrt[a + b*x^2])/(2*d^2) + (3*(((a
^3*C*d^6 - 16*b^3*c^4*(c^2*C - B*c*d + A*d^2) - 24*a*b^2*c^2*d^2*(c^2*C -
B*c*d + A*d^2) - 6*a^2*b*d^4*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (16*b*c*(b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c
*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/
d))/(2*d^2))/(4*d^2)))/(6*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{(-40C b^2 d^5 x^5 - 48B b^2 d^5 x^4 + 48C b^2 c d^4 x^4 - 60A b^2 d^5 x^3 + 60B b^2 c d^4 x^3 - 70C a b d^5 x^3 - 60C b^2 c^2 d^3 x^3 + 80A b^2 c d^4 x^2 - 96B a b d^5 x}{}$
default	$A d^2 \left(\frac{x (b x^2+a)^{\frac{3}{2}}}{4} + \frac{3a \left(\frac{x\sqrt{b x^2+a}}{2} + \frac{a \ln\left(\sqrt{b}\, x + \sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right) + C c^2 \left(\frac{x (b x^2+a)^{\frac{3}{2}}}{4} + \frac{3a \left(\frac{x\sqrt{b x^2+a}}{2} + \frac{a \ln\left(\sqrt{b}\, x + \sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right) + \frac{d(Bd-}{}$

input
```
int(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/240/b*(-40*C*b^2*d^5*x^5-48*B*b^2*d^5*x^4+48*C*b^2*c*d^4*x^4-60*A*b^2*d
^5*x^3+60*B*b^2*c*d^4*x^3-70*C*a*b*d^5*x^3-60*C*b^2*c^2*d^3*x^3+80*A*b^2*c
*d^4*x^2-96*B*a*b*d^5*x^2-80*B*b^2*c^2*d^3*x^2+96*C*a*b*c*d^4*x^2+80*C*b^2
*c^3*d^2*x^2-150*A*a*b*d^5*x-120*A*b^2*c^2*d^3*x+150*B*a*b*c*d^4*x+120*B*b
^2*c^3*d^2*x-15*C*a^2*d^5*x-150*C*a*b*c^2*d^3*x-120*C*b^2*c^4*d*x+320*A*a*
b*c*d^4+240*A*b^2*c^3*d^2-48*B*a^2*d^5-320*B*a*b*c^2*d^3-240*B*b^2*c^4*d+4
8*C*a^2*c*d^4+320*C*a*b*c^3*d^2+240*C*b^2*c^5)*(b*x^2+a)^(1/2)/d^6+1/16/d^
6/b*((6*A*a^2*b*d^6+24*A*a*b^2*c^2*d^4+16*A*b^3*c^4*d^2-6*B*a^2*b*c*d^5-24
*B*a*b^2*c^3*d^3-16*B*b^3*c^5*d-C*a^3*d^6+6*C*a^2*b*c^2*d^4+24*C*a*b^2*c^4
*d^2+16*C*b^3*c^6)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+16*c*(A*a^2*d^6
+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a
^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)*b/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((
2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{x(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate(x*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral(x*(a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.79

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*b*c^4*x/d^5 - 1/2*sqrt(b*x^2 + a)*B*b*c^3*x/d^4 + 1/
4*(b*x^2 + a)^(3/2)*C*c^2*x/d^3 + 3/8*sqrt(b*x^2 + a)*C*a*c^2*x/d^3 + 1/2*
sqrt(b*x^2 + a)*A*b*c^2*x/d^3 - 1/4*(b*x^2 + a)^(3/2)*B*c*x/d^2 - 3/8*sqrt
(b*x^2 + a)*B*a*c*x/d^2 + 1/4*(b*x^2 + a)^(3/2)*A*x/d + 3/8*sqrt(b*x^2 + a
)*A*a*x/d + 1/6*(b*x^2 + a)^(5/2)*C*x/(b*d) - 1/24*(b*x^2 + a)^(3/2)*C*a*x
/(b*d) - 1/16*sqrt(b*x^2 + a)*C*a^2*x/(b*d) + C*b^(3/2)*c^6*arcsinh(b*x/sq
rt(a*b))/d^7 - B*b^(3/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 + 3/2*C*a*sqrt(b)*
c^4*arcsinh(b*x/sqrt(a*b))/d^5 + A*b^(3/2)*c^4*arcsinh(b*x/sqrt(a*b))/d^5
- 3/2*B*a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 3/8*C*a^2*c^2*arcsinh(b
*x/sqrt(a*b))/(sqrt(b)*d^3) + 3/2*A*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d
^3 - 3/8*B*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 1/16*C*a^3*arcsinh
(b*x/sqrt(a*b))/(b^(3/2)*d) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d)
 - C*(a + b*c^2/d^2)^(3/2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*
d/(sqrt(a*b)*abs(d*x + c)))/d^4 + B*(a + b*c^2/d^2)^(3/2)*c^2*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - A*(a + b*
c^2/d^2)^(3/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*a
bs(d*x + c)))/d^2 - sqrt(b*x^2 + a)*C*b*c^5/d^6 + sqrt(b*x^2 + a)*B*b*c^4/
d^5 - 1/3*(b*x^2 + a)^(3/2)*C*c^3/d^4 - sqrt(b*x^2 + a)*C*a*c^3/d^4 - sqrt
(b*x^2 + a)*A*b*c^3/d^4 + 1/3*(b*x^2 + a)^(3/2)*B*c^2/d^3 + sqrt(b*x^2 + a
)*B*a*c^2/d^3 - 1/3*(b*x^2 + a)^(3/2)*A*c/d^2 - sqrt(b*x^2 + a)*A*a*c/d...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x\,(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{c+d\,x}\,dx$$

input
```
int((x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int((x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{x(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{dx+c}dx$$

input
```
int(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

3.78 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 349

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^5} - \frac{(3ad^2(cC-Bd)+4bc(c^2C-Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8d^4} + \frac{(c^2C-Bcd+Ad^2)(a+bx^2)^{3/2}}{3d^3} - \frac{(cC-Bd)x(a+bx^2)^{3/2}}{4d^2} + \frac{C(a+bx^2)^{5/2}}{5bd} - \frac{(3a^2d^4(cC-Bd)+8b^2c^3(c^2C-Bcd+Ad^2)+12abcd^2(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^6} - \frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6}$$

output
```
(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^5-1/8*(3*a*d^2*(-B*d+C
*c)+4*b*c*(A*d^2-B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/d^4+1/3*(A*d^2-B*c*d+C*c^
2)*(b*x^2+a)^(3/2)/d^3-1/4*(-B*d+C*c)*x*(b*x^2+a)^(3/2)/d^2+1/5*C*(b*x^2+a
)^(5/2)/b/d-1/8*(3*a^2*d^4*(-B*d+C*c)+8*b^2*c^3*(A*d^2-B*c*d+C*c^2)+12*a*b
*c*d^2*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^6
-(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2
)^(1/2)/(b*x^2+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 1.87 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \frac{d\sqrt{a+bx^2}\left(24a^2Cd^4+abd^2\left(160c^2C-5cd(32B+15Cx)+d^2\left(160A+75Bx+48Cx^2\right)\right)+2b^2\left(60c^4\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
((d*Sqrt[a + b*x^2]*(24*a^2*C*d^4 + a*b*d^2*(160*c^2*C - 5*c*d*(32*B + 15*
C*x) + d^2*(160*A + 75*B*x + 48*C*x^2)) + 2*b^2*(60*c^4*C - 30*c^3*d*(2*B
+ C*x) + 10*c^2*d^2*(6*A + x*(3*B + 2*C*x)) - 5*c*d^3*x*(6*A + x*(4*B + 3*
C*x)) + d^4*x^2*(20*A + 3*x*(5*B + 4*C*x)))))/b - 240*(-(b*c^2) - a*d^2)^(
3/2)*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2]
)/Sqrt[-(b*c^2) - a*d^2]] + (15*(3*a^2*d^4*(c*C - B*d) + 8*b^2*c^3*(c^2*C
- B*c*d + A*d^2) + 12*a*b*c*d^2*(c^2*C - B*c*d + A*d^2))*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b])/(120*d^6)
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{5bd(Ad-(cC-Bd)x)(bx^2+a)^{3/2}}{c+dx}dx}{5bd^2}+\frac{C(a+bx^2)^{5/2}}{5bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(Ad-(cC-Bd)x)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\big\downarrow$ 682

$$\frac{\frac{\int \frac{b\left(ad\left(Cc^2-Bdc+4Ad^2\right)-\left(4Abcd^2+(cC-Bd)\left(4bc^2+3ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(Ad^2-Bcd+c^2C\right)-3dx(cC-Bd)\right)}{12d^2}}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(ad\left(Cc^2-Bdc+4Ad^2\right)-\left(4Abcd^2+(cC-Bd)\left(4bc^2+3ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(Ad^2-Bcd+c^2C\right)-3dx(cC-Bd)\right)}{12d^2}}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\big\downarrow$ 682

$$\frac{\frac{\frac{\int \frac{b\left(ad\left(4b\left(Cc^2-Bdc+Ad^2\right)c^2+ad^2\left(5Cc^2-5Bdc+8Ad^2\right)\right)-\left(3a^2(cC-Bd)d^4+12abc\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+b\right.\right.}{}}{4d^2}}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{ad\left(4b\left(Cc^2-Bdc+Ad^2\right)c^2+ad^2\left(5Cc^2-5Bdc+8Ad^2\right)\right)-\left(3a^2(cC-Bd)d^4+12abc\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)\right.}{}}{4d^2}}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\big\downarrow$ 719

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4(cC-Bd)+12abcd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad\right.\right.}{}}{4d^2}}{d}+\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\downarrow$ 224

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4(cC-Bd)+12abcd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\sqrt{a+\cdots}}{4d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(cC-Bd)+12abcd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}}{2d^2}+\sqrt{a+\cdots}}{4d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\downarrow$ 488

$$\frac{\frac{\frac{-\frac{8\left(ad^2+bc^2\right)^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(cC-Bd)+12abcd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^3\left(Ad^2-Bcd\cdots\right.\right.}{\sqrt{b}d}}{2d^2}}{4d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

$\downarrow$ 219

$$\frac{\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(cC-Bd)+12abcd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{8\left(ad^2+bc^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-b\cdots}{\sqrt{a+bx^2}\sqrt{a\cdots}}\right.}{d}}{2d^2}}{4d^2}}{d}$$

$$\frac{C\left(a+bx^2\right)^{5/2}}{5bd}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x]
```

output
```
(C*(a + b*x^2)^(5/2))/(5*b*d) + (((4*(c^2*C - B*c*d + A*d^2) - 3*d*(c*C -
B*d)*x)*(a + b*x^2)^(3/2))/(12*d^2) + (((8*(b*c^2 + a*d^2)*(c^2*C - B*c*d
+ A*d^2) - d*(4*A*b*c*d^2 + (c*C - B*d)*(4*b*c^2 + 3*a*d^2))*x)*Sqrt[a + b
*x^2])/(2*d^2) + (-(((3*a^2*d^4*(c*C - B*d) + 8*b^2*c^3*(c^2*C - B*c*d + A
*d^2) + 12*a*b*c*d^2*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(Sqrt[b]*d)) - (8*(b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/
(4*d^2))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.60

method	result
risch	$\frac{(24C\,d^4b^2x^4+30B\,b^2d^4x^3-30C\,b^2c\,d^3x^3+40A\,b^2d^4x^2-40B\,b^2c\,d^3x^2+48Cab\,d^4x^2+40C\,b^2c^2d^2x^2-60A\,b^2c\,d^3x+75Bab\,d^4x+60B\cdots}{1\cdots}$
default	$\frac{Bd\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{Cd\left(b\,x^2+a\right)^{\frac{5}{2}}}{5b}-Cc\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\cdots}{d^2}$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/120/b*(24*C*b^2*d^4*x^4+30*B*b^2*d^4*x^3-30*C*b^2*c*d^3*x^3+40*A*b^2*d^4
*x^2-40*B*b^2*c*d^3*x^2+48*C*a*b*d^4*x^2+40*C*b^2*c^2*d^2*x^2-60*A*b^2*c*d
^3*x+75*B*a*b*d^4*x+60*B*b^2*c^2*d^2*x-75*C*a*b*c*d^3*x-60*C*b^2*c^3*d*x+1
60*A*a*b*d^4+120*A*b^2*c^2*d^2-160*B*a*b*c*d^3-120*B*b^2*c^3*d+24*C*a^2*d^
4+160*C*a*b*c^2*d^2+120*C*b^2*c^4)*(b*x^2+a)^(1/2)/d^5-1/8/d^5*((12*A*a*b*
c*d^4+8*A*b^2*c^3*d^2-3*B*a^2*d^5-12*B*a*b*c^2*d^3-8*B*b^2*c^4*d+3*C*a^2*c
*d^4+12*C*a*b*c^3*d^2+8*C*b^2*c^5)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
+8*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*
b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^2/((a*d^2+b*c^2)/d^2)
^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)
*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{c+dx}\,dx=\text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{c+dx}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{c+dx}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*sqrt(b*x^2 + a)*C*b*c^3*x/d^4 + 1/2*sqrt(b*x^2 + a)*B*b*c^2*x/d^3 - 1
/4*(b*x^2 + a)^(3/2)*C*c*x/d^2 - 3/8*sqrt(b*x^2 + a)*C*a*c*x/d^2 - 1/2*sqr
t(b*x^2 + a)*A*b*c*x/d^2 + 1/4*(b*x^2 + a)^(3/2)*B*x/d + 3/8*sqrt(b*x^2 +
a)*B*a*x/d - C*b^(3/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 + B*b^(3/2)*c^4*arcs
inh(b*x/sqrt(a*b))/d^5 - 3/2*C*a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 -
A*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4 + 3/2*B*a*sqrt(b)*c^2*arcsinh(b*x
/sqrt(a*b))/d^3 - 3/8*C*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 3/2*A
*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))
/(sqrt(b)*d) + C*(a + b*c^2/d^2)^(3/2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*
x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - B*(a + b*c^2/d^2)^(3/2)*c*ar
csinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 +
 A*(a + b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqr
t(a*b)*abs(d*x + c)))/d + sqrt(b*x^2 + a)*C*b*c^4/d^5 - sqrt(b*x^2 + a)*B*
b*c^3/d^4 + 1/3*(b*x^2 + a)^(3/2)*C*c^2/d^3 + sqrt(b*x^2 + a)*C*a*c^2/d^3
+ sqrt(b*x^2 + a)*A*b*c^2/d^3 - 1/3*(b*x^2 + a)^(3/2)*B*c/d^2 - sqrt(b*x^2
 + a)*B*a*c/d^2 + 1/3*(b*x^2 + a)^(3/2)*A/d + sqrt(b*x^2 + a)*A*a/d + 1/5*
(b*x^2 + a)^(5/2)*C/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{c+d\,x}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{c+dx}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{dx+c}dx$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c),x)
```

3.79 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 345

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx &= \\
&-\frac{(4ad^2(cC-Bd)+3bc(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3d^4} \\
&+\frac{(5aCd^2+4b(c^2C-Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8d^3} \\
&-\frac{b(cC-Bd)x^2\sqrt{a+bx^2}}{3d^2}+\frac{bCx^3\sqrt{a+bx^2}}{4d} \\
&+\frac{(3a^2Cd^4+8b^2c^2(c^2C-Bcd+Ad^2)+12abd^2(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^5} \\
&+\frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{cd^5}-\frac{a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}
\end{aligned}$$

output

```
-1/3*(4*a*d^2*(-B*d+C*c)+3*b*c*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/d^4+1/
8*(5*a*C*d^2+4*b*(A*d^2-B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/d^3-1/3*b*(-B*d+C*
c)*x^2*(b*x^2+a)^(1/2)/d^2+1/4*b*C*x^3*(b*x^2+a)^(1/2)/d+1/8*(3*a^2*C*d^4+
8*b^2*c^2*(A*d^2-B*c*d+C*c^2)+12*a*b*d^2*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^5+(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)
*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c/d^5-a^(3/2)*A
*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \frac{\sqrt{a+bx^2}(ad^2(-32cC+32Bd+15Cdx)-2b(12c^3C-6c^2d(2B+C}{24d^4}$$

$$+\frac{2(-bc^2-ad^2)^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{-bc^2-ad^2}\,x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{cd^5}$$

$$+\frac{(3a^2Cd^4+8b^2c^2(c^2C-Bcd+Ad^2)+12abd^2(c^2C-Bcd+Ad^2))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4\sqrt{b}d^5}$$

$$-\frac{a^{3/2}A\log(x)}{c}+\frac{a^{3/2}A\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c}$$

input

```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)),x]
```

output

```
(Sqrt[a + b*x^2]*(a*d^2*(-32*c*C + 32*B*d + 15*C*d*x) - 2*b*(12*c^3*C - 6*
c^2*d*(2*B + C*x) + 2*c*d^2*(6*A + 3*B*x + 2*C*x^2) - d^3*x*(6*A + 4*B*x +
 3*C*x^2))))/(24*d^4) + (2*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2
)*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a + b*x^2]
)])/(c*d^5) + ((3*a^2*C*d^4 + 8*b^2*c^2*(c^2*C - B*c*d + A*d^2) + 12*a*b*d
^2*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2
]])])/(4*Sqrt[b]*d^5) - (a^(3/2)*A*Log[x])/c + (a^(3/2)*A*Log[-Sqrt[a] + Sq
rt[a + b*x^2]])/c
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.31, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2351, 606, 243, 60, 73, 221, 682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{(bx^2+a)^{3/2}}{x(c+dx)}dx+\int \frac{(B+Cx)(bx^2+a)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{606}$$

$$A\left(\frac{a\int \frac{\sqrt{bx^2+a}}{x}dx}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{c}\right)+\int \frac{(B+Cx)(bx^2+a)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{243}$$

$$A\left(\frac{a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2}{2c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{c}\right)+\int \frac{(B+Cx)(bx^2+a)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{60}$$

$$A\left(\frac{a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)}{2c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{c}\right)+\int \frac{(B+Cx)(bx^2+a)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{73}$$

$$A\left(\frac{a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)}{2c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{c}\right)+$$
$$\int \frac{(B+Cx)(bx^2+a)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{221}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\int\frac{(ad-bcx)\sqrt{bx^2+a}}{c+dx}dx}{c}\right)+\int\frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{c+dx}dx$$

$$\Big\downarrow \text{682}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\int\frac{b\left(ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{c}\right)+\frac{\int-\frac{b\left(ad(cC-4Bd)-\left(3aCd^2+4bc(cC-Bd)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \text{25}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\int\frac{b\left(ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{c}\right)-\frac{\int\frac{b\left(ad(cC-4Bd)-\left(3aCd^2+4bc(cC-Bd)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \text{27}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\int\frac{ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{c}\right)-\frac{\int\frac{\left(ad(cC-4Bd)-\left(3aCd^2+4bc(cC-Bd)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}-\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \text{682}$$

$$-\frac{\frac{\int \frac{b\left(ad\left(4b(cC-Bd)c^2+ad^2(5cC-8Bd)\right)-\left(3a^2Cd^4+12abc(cC-Bd)d^2+8b^2c^3(cC-Bd)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(cC-Bd)-dx\left(3aCd^2+4bc(cC\right.\right.}{2d^2}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\int \frac{ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{c}\right)-$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{ad\left(4b(cC-Bd)c^2+ad^2(5cC-8Bd)\right)-\left(3a^2Cd^4+12abc(cC-Bd)d^2+8b^2c^3(cC-Bd)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(cC-Bd)-dx\left(3aCd^2+4bc(cC-B\right.\right.}{2d^2}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\int \frac{ad\left(bc^2+2ad^2\right)-bc\left(2bc^2+3ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)}{2d^2}}{c}\right)-$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 719$$

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Cd^4+12abcd^2(cC-Bd)+8b^2c^3(cC-Bd)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(cC-Bd)-dx\left(3aC\right.\right.}{2d^2}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+2bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+b\right.\right.}{2d^2}}{c}\right.$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2Cd^4+12abcd^2(cC-Bd)+8b^2c^3(cC-Bd)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(cC-Bd)\right.}{2d^2}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\left(3ad^2+2bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(\right.}{}}{c}\right.$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{8\left(ad^2+bc^2\right)^2(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4+12abcd^2(cC-Bd)+8b^2c^3(cC-Bd)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+bc^2\right)(cC-Bd)\right.}{2d^2}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx}}{}}{c}\right.$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 488$$

$$-\frac{\frac{-\frac{8\left(ad^2+bc^2\right)^2(cC-Bd)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4+12abcd^2(cC-Bd)+8b^2c^3(cC-Bd)\right)}{\sqrt{b}d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(ad^2+\right.\right.}{}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{-\frac{2\left(ad^2+bc^2\right)^2\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc\right.}{d}}{2d^2}}{c}\right.$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4+12abcd^2(cC-Bd)+8b^2c^3(cC-Bd)\right)}{\sqrt{b}d}-\frac{8\left(ad^2+bc^2\right)^{3/2}(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}(8(ad^2 \cdots}{\cdots}}{4d^2}$$

$$A\left(\frac{a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{2c}-\frac{\frac{-\frac{2\left(ad^2+bc^2\right)^{3/2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2 \cdots\right.}{d}}{2d^2}}{c}\cdots\right.$$

$$\frac{\left(a+bx^2\right)^{3/2}(4(cC-Bd)-3Cdx)}{12d^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)),x]
```

output
```
-1/12*((4*(c*C - B*d) - 3*C*d*x)*(a + b*x^2)^(3/2))/d^2 - (((8*(c*C - B*d)
*(b*c^2 + a*d^2) - d*(3*a*C*d^2 + 4*b*c*(c*C - B*d))*x)*Sqrt[a + b*x^2])/(
2*d^2) + (-(((3*a^2*C*d^4 + 8*b^2*c^3*(c*C - B*d) + 12*a*b*c*d^2*(c*C - B*
d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (8*(c*C - B*d)*(b
*c^2 + a*d^2)^(3/2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2])])/d)/(2*d^2))/(4*d^2) + A*(-((((2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a
 + b*x^2])/(2*d^2) + (-((Sqrt[b]*c*(2*b*c^2 + 3*a*d^2)*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/d) - (2*(b*c^2 + a*d^2)^(3/2)*ArcTanh[(a*d - b*c*x)/(Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2))/c) + (a*(2*Sqrt[a + b*x^2]
 - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(2*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(309) = 618$.

Time $= 0.19$ (sec) , antiderivative size $= 630$, normalized size of antiderivative $= 1.83$

method	result
default	$(A$ $\frac{C\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{d}+\frac{A\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{c}-$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
C/d*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b
^(1/2)*x+(b*x^2+a)^(1/2))))+A/c*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))-1/d^2*(A*d^2-B*c*d+C*c^2)/c*
(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b
*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+
1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^
(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2
)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*l
n((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/
d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{x\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(sqrt(b*x^2 + a)*b*c*x/d^2 + 2*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^4
+ 3*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^2 - 2*(a + b*c^2/d^2)^(3/2)*arcsi
nh(2*b*c*x/(sqrt(a*b)*abs(2*d*x + 2*c)) - 2*a*d/(sqrt(a*b)*abs(2*d*x + 2*c
)))/d - 2*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x)))/d - 2*sqrt(b*x^2 + a)*b*c^
2/d^3)*A*d/c - 1/6*(3*sqrt(b*x^2 + a)*b*c^2*x/d^3 + 6*b^(3/2)*c^4*arcsinh(
b*x/sqrt(a*b))/d^5 + 9*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^3 - 6*(a + b
*c^2/d^2)^(3/2)*c*arcsinh(2*b*c*x/(sqrt(a*b)*abs(2*d*x + 2*c)) - 2*a*d/(sq
rt(a*b)*abs(2*d*x + 2*c)))/d^2 - 6*sqrt(b*x^2 + a)*b*c^3/d^4 - 2*(b*x^2 +
a)^(3/2)*c/d^2 - 6*sqrt(b*x^2 + a)*a*c/d^2)*B*d/c + 1/24*(12*sqrt(b*x^2 +
a)*b*c^3*x/d^4 + 6*(b*x^2 + a)^(3/2)*c*x/d^2 + 9*sqrt(b*x^2 + a)*a*c*x/d^2
 + 24*b^(3/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^6 + 36*a*sqrt(b)*c^3*arcsinh(b*
x/sqrt(a*b))/d^4 + 9*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 24*(a +
b*c^2/d^2)^(3/2)*c^2*arcsinh(2*b*c*x/(sqrt(a*b)*abs(2*d*x + 2*c)) - 2*a*d/
(sqrt(a*b)*abs(2*d*x + 2*c)))/d^3 - 24*sqrt(b*x^2 + a)*b*c^4/d^5 - 8*(b*x^
2 + a)^(3/2)*c^2/d^3 - 24*sqrt(b*x^2 + a)*a*c^2/d^3)*C*d/c
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1157, normalized size of antiderivative = 3.35

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c),x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*b*d**4 + 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**2 - 48*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*
*2*c*d**3 + 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 - 48*sqrt(a*d**2 + b*c**2)*log( - sq
rt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d + 48*sqrt(
a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c
*x)*b**2*c**5 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**4 - 48*sqr
t(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2 + 48*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a*b**2*c*d**3 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c
**3*d**2 + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**3*d - 48*sqrt(a*d
**2 + b*c**2)*log(c + d*x)*b**2*c**5 - 48*sqrt(a + b*x**2)*a*b**2*c**2*d**
3 + 24*sqrt(a + b*x**2)*a*b**2*c*d**4*x + 64*sqrt(a + b*x**2)*a*b**2*c*d**
4 - 64*sqrt(a + b*x**2)*a*b*c**3*d**3 + 30*sqrt(a + b*x**2)*a*b*c**2*d**4*
x + 48*sqrt(a + b*x**2)*b**3*c**3*d**2 - 24*sqrt(a + b*x**2)*b**3*c**2*d**
3*x + 16*sqrt(a + b*x**2)*b**3*c*d**4*x**2 - 48*sqrt(a + b*x**2)*b**2*c**5
*d + 24*sqrt(a + b*x**2)*b**2*c**4*d**2*x - 16*sqrt(a + b*x**2)*b**2*c**3*
d**3*x**2 + 12*sqrt(a + b*x**2)*b**2*c**2*d**4*x**3 + 24*sqrt(a)*log(sqrt(
a + b*x**2) - sqrt(a))*a**2*b*d**5 - 24*sqrt(a)*log(sqrt(a + b*x**2) + ...
```

3.80 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \frac{(4aCd^2+3b(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3d^3} - \frac{aA\sqrt{a+bx^2}}{cx} - \frac{b(cC-Bd)x\sqrt{a+bx^2}}{2d^2} + \frac{bCx^2\sqrt{a+bx^2}}{3d} - \frac{\sqrt{b}(3ad^2(cC-Bd)+2bc(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^4} - \frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2d^4} - \frac{a^{3/2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}$$

output
```
1/3*(4*a*C*d^2+3*b*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/d^3-a*A*(b*x^2+a)^
(1/2)/c/x-1/2*b*(-B*d+C*c)*x*(b*x^2+a)^(1/2)/d^2+1/3*b*C*x^2*(b*x^2+a)^(1/
2)/d-1/2*b^(1/2)*(3*a*d^2*(-B*d+C*c)+2*b*c*(A*d^2-B*c*d+C*c^2))*arctanh(b^
(1/2)*x/(b*x^2+a)^(1/2))/d^4-(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arcta
nh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2/d^4-a^(3/2)*(-A*d
+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \frac{cd\sqrt{a+bx^2}(a(-6Ad^3+8cCd^2x)+bcx(6c^2C-3cd(2B+Cx)+d^2$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)),x]
```

output
```
(c*d*Sqrt[a + b*x^2]*(a*(-6*A*d^3 + 8*c*C*d^2*x) + b*c*x*(6*c^2*C - 3*c*d*
(2*B + C*x) + d^2*(6*A + 3*B*x + 2*C*x^2))) - 12*(-(b*c^2) - a*d^2)^(3/2)*
(c^2*C - B*c*d + A*d^2)*x*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/S
qrt[-(b*c^2) - a*d^2]] + 12*a^(3/2)*d^4*(-(B*c) + A*d)*x*ArcTanh[(-(Sqrt[b
]*x) + Sqrt[a + b*x^2])/Sqrt[a]] + 3*Sqrt[b]*c^2*(3*a*d^2*(c*C - B*d) + 2*
b*c*(c^2*C - B*c*d + A*d^2))*x*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(6*c^2
*d^4*x)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.33, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^2(c+dx)}+\frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x}+\frac{A(a+bx^2)^{3/2}}{cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{c^2}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2cd^4}-\frac{(ad^2+bc^2)^{3/2}(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2d^4}+\frac{3aA\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2c}+\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{3c^2d}+\frac{\sqrt{a+bx^2}(2(ad^2+bc^2)-bcdx)(Ad^2-Bcd+c^2C)}{2c^2d^3}+\frac{a\sqrt{a+bx^2}(Bc-Ad)}{c^2}+\frac{(a+bx^2)^{3/2}(Bc-Ad)}{3c^2}-\frac{A(a+bx^2)^{3/2}}{cx}+\frac{3Abx\sqrt{a+bx^2}}{2c}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)),x]
```

output
```
(a*(B*c - A*d)*Sqrt[a + b*x^2])/c^2 + (3*A*b*x*Sqrt[a + b*x^2])/(2*c) + ((
c^2*C - B*c*d + A*d^2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c
^2*d^3) + ((B*c - A*d)*(a + b*x^2)^(3/2))/(3*c^2) + ((c^2*C - B*c*d + A*d^
2)*(a + b*x^2)^(3/2))/(3*c^2*d) - (A*(a + b*x^2)^(3/2))/(c*x) + (3*a*A*Sqr
t[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c) - (Sqrt[b]*(2*b*c^2 + 3*a
*d^2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c*d
^4) - ((b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)
/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*d^4) - (a^(3/2)*(B*c - A*d)*
ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(262) = 524$.

Time = 0.25 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{aA\sqrt{b\,x^2+a}}{cx}+\frac{bc\left(b\,d^2(Bd-Cc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{d\left(Ab\,d^2-Bbcd+2aC\,d^2+Cb\,c^2\right)\sqrt{b\,x^2+a}}{b}+B\sqrt{b}\,c^2d\ln\left(\sqrt{b}\,x+\sqrt{b}\right.}{}$
default	$\frac{A\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{c}-\frac{(Ad-Bc)\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2}{}\right.\right.\right.}{c^2}$

input

```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
-a*A*(b*x^2+a)^(1/2)/c/x+1/c*(b*c/d^4*(b*d^2*(B*d-C*c)*(1/2*x/b*(b*x^2+a)^
(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+d*(A*b*d^2-B*b*c*d+2*C*
a*d^2+C*b*c^2)/b*(b*x^2+a)^(1/2)+B*b^(1/2)*c^2*d*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))+C*b*d^3*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+2*B*a*d
^3*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-C*b^(1/2)*c^3*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))-A*b^(1/2)*c*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))-2*C*a*c*d^2*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))-1/d^5*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c
^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d
^2+C*b^2*c^6)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*
(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2))/(x+c/d))+a^(3/2)*(A*d-B*c)/c*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**2/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**2*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^2\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^2\,(dx+c)}dx$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c),x)
```

3.81 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 287

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = -\frac{b(cC-Bd)\sqrt{a+bx^2}}{d^2} - \frac{aA\sqrt{a+bx^2}}{2cx^2} - \frac{a(Bc-Ad)\sqrt{a+bx^2}}{c^2x} + \frac{bCx\sqrt{a+bx^2}}{2d} + \frac{\sqrt{b}(3aCd^2+2b(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^3} + \frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3d^3} - \frac{\sqrt{a}\left(3Abc+2acC-2aBd+\frac{2aAd^2}{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c^2}$$

output
```
-b*(-B*d+C*c)*(b*x^2+a)^(1/2)/d^2-1/2*a*A*(b*x^2+a)^(1/2)/c/x^2-a*(-A*d+B*
c)*(b*x^2+a)^(1/2)/c^2/x+1/2*b*C*x*(b*x^2+a)^(1/2)/d+1/2*b^(1/2)*(3*a*C*d^
2+2*b*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^3+(a*d^2+b
*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(
b*x^2+a)^(1/2))/c^3/d^3-1/2*a^(1/2)*(3*A*b*c+2*C*a*c-2*B*a*d+2*a*A*d^2/c)*
arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \frac{1}{2}\left(\frac{\sqrt{a+bx^2}(ad^2(-Ac-2Bcx+2Adx)+bc^2x^2(-2cC+2Bd+C}{c^2d^2x^2}\right.$$

$$+\frac{4(-bc^2-ad^2)^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c^3d^3}$$

$$+\frac{6\sqrt{a}Ab\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}-\frac{4a^{3/2}(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^3}$$

$$\left.-\frac{\sqrt{b}(3aCd^2+2b(c^2C-Bcd+Ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^3}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)),x]
```

output
```
((Sqrt[a + b*x^2]*(a*d^2*(-(A*c) - 2*B*c*x + 2*A*d*x) + b*c^2*x^2*(-2*c*C
+ 2*B*d + C*d*x)))/(c^2*d^2*x^2) + (4*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*
c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2)
- a*d^2]])/(c^3*d^3) + (6*Sqrt[a]*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2]
)/Sqrt[a]])/c - (4*a^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) +
 Sqrt[a + b*x^2])/Sqrt[a]])/c^3 - (Sqrt[b]*(3*a*C*d^2 + 2*b*(c^2*C - B*c*d
 + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^3)/2
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.54, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x^2} + \frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^3x} - \frac{d(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^3(c+dx)} + \frac{A(a+\cdots}{c\cdots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{c^3} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2c^2d^3} + \frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-Ad)}{2c^2} + \frac{(ad^2+bc^2)^{3/2}(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3d^3} - \frac{3\sqrt{a}A b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c} - \frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x} + \frac{3bx\sqrt{a+bx^2}(Bc-Ad)}{2c^2} + \frac{a\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3} - \frac{\sqrt{a+bx^2}(2(ad^2+bc^2)-bcdx)(Ad^2-Bcd+c^2C)}{2c^3d^2} - \frac{A(a+bx^2)^{3/2}}{2cx^2} + \frac{3Ab\sqrt{a+bx^2}}{2c}$$

input

```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)),x]
```

output

```
(3*A*b*Sqrt[a + b*x^2])/(2*c) + (a*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2]
)/c^3 + (3*b*(B*c - A*d)*x*Sqrt[a + b*x^2])/(2*c^2) - ((c^2*C - B*c*d + A*
d^2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^3*d^2) - (A*(a +
b*x^2)^(3/2))/(2*c*x^2) - ((B*c - A*d)*(a + b*x^2)^(3/2))/(c^2*x) + (3*a*S
qrt[b]*(B*c - A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^2) + (Sqrt[b
]*(2*b*c^2 + 3*a*d^2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
 b*x^2]])/(2*c^2*d^3) + ((b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*Arc
Tanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^3*d^3) - (3*
Sqrt[a]*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c) - (a^(3/2)*(c^2*C - B*
c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{a\sqrt{b x^2+a}\,(-2Adx+2Bcx+Ac)}{2c^2x^2}+\frac{-\frac{\sqrt{a}\left(2Aa d^2+3bAc^2-2Bacd+2Cac^2\right)\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{c}+\frac{2\left(A a^2 d^6+2Aabc^2d^4+Ab^2c^4d^2-\right.}{}}{}$
default	$\frac{A\left(-\frac{\left(b x^2+a\right)^{\frac{5}{2}}}{2a x^2}+\frac{3b\left(\frac{\left(b x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)\right)\right)}{2a}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\left(b x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\right.}{}\right.}{}\right.}{c^2}$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(b*x^2+a)^(1/2)*(-2*A*d*x+2*B*c*x+A*c)/c^2/x^2+1/2/c^2*(-a^(1/2)*(2
*A*a*d^2+3*A*b*c^2-2*B*a*c*d+2*C*a*c^2)/c*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)+2/d^4*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c
^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/c/((a*d^2+b*c^
2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2
)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*
b*c^2/d^3*(d*(B*d-C*c)*(b*x^2+a)^(1/2)+A*b^(1/2)*d^2*ln(b^(1/2)*x+(b*x^2+a
)^(1/2))+C*b^(1/2)*c^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+C*b*d^2*(1/2*x/b*(b*x
^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+2*a*C*d^2*ln(b^(1
/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-B*b^(1/2)*c*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^3\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**3/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**3*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^3\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1074, normalized size of antiderivative = 3.74

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c),x)
```

output
```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*d**4*x**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**2*x**2 - 4*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b*c*d**3*x**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a*c**3*d**2*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(
 - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x**2
+ 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*b*c**5*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**4*
x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2*x**2 + 4*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(
c + d*x)*a*c**3*d**2*x**2 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**3
*d*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**5*x**2 - 2*sqrt(a + b*
x**2)*a**2*c**2*d**3 + 4*sqrt(a + b*x**2)*a**2*c*d**4*x - 4*sqrt(a + b*x**
2)*a*b*c**2*d**3*x + 4*sqrt(a + b*x**2)*b**2*c**3*d**2*x**2 - 4*sqrt(a + b
*x**2)*b*c**5*d*x**2 + 2*sqrt(a + b*x**2)*b*c**4*d**2*x**3 + 2*sqrt(a)*log
(sqrt(a + b*x**2) - sqrt(a))*a**2*d**5*x**2 + 3*sqrt(a)*log(sqrt(a + b*x**
2) - sqrt(a))*a*b*c**2*d**3*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a
))*a*b*c*d**4*x**2 + 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a*c**3*d**3
*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a**2*d**5*x**2 - 3*sq...
```

3.82 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 296

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \frac{bC\sqrt{a+bx^2}}{d} - \frac{aA\sqrt{a+bx^2}}{3cx^3} - \frac{a(Bc-Ad)\sqrt{a+bx^2}}{2c^2x^2} - \frac{\left(4Abc+3acC-3aBd+\frac{3aAd^2}{c}\right)\sqrt{a+bx^2}}{3c^2x} - \frac{b^{3/2}(cC-Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} - \frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4d^2} - \frac{\sqrt{a}(3bc^2(Bc-Ad)-2ad(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c^4}$$

output
```
b*C*(b*x^2+a)^(1/2)/d-1/3*a*A*(b*x^2+a)^(1/2)/c/x^3-1/2*a*(-A*d+B*c)*(b*x^
2+a)^(1/2)/c^2/x^2-1/3*(4*A*b*c+3*C*a*c-3*B*a*d+3*a*A*d^2/c)*(b*x^2+a)^(1/
2)/c^2/x-b^(3/2)*(-B*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^2-(a*d^2+
b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/
(b*x^2+a)^(1/2))/c^4/d^2-1/2*a^(1/2)*(3*b*c^2*(-A*d+B*c)-2*a*d*(A*d^2-B*c*
d+C*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^4
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = -\frac{\sqrt{a+bx^2}(2bc^2x^2(4Ad-3cCx)+ad(3cx(Bc+2cCx-2Bdx)+A(2c^2-3cdx+6d^2x^2)))}{6c^3dx^3} - \frac{2(-bc^2-ad^2)^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c^4d^2} + \frac{\sqrt{a}(3bc^2(-Bc+Ad)+2ad(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^4} + \frac{b^{3/2}(cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^2}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)),x]
```

output
```
-1/6*(Sqrt[a + b*x^2]*(2*b*c^2*x^2*(4*A*d - 3*c*C*x) + a*d*(3*c*x*(B*c + 2
*c*C*x - 2*B*d*x) + A*(2*c^2 - 3*c*d*x + 6*d^2*x^2))))/(c^3*d*x^3) - (2*(-
(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) -
 d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(c^4*d^2) + (Sqrt[a]*(3*b*c^2
*(-(B*c) + A*d) + 2*a*d*(c^2*C - B*c*d + A*d^2))*ArcTanh[(-(Sqrt[b]*x) + S
qrt[a + b*x^2])/Sqrt[a]])/c^4 + (b^(3/2)*(c*C - B*d)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/d^2
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.90, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^4(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2353}$$

$$\int \left(\frac{\left(a+bx^2\right)^{3/2}(Bc-Ad)}{c^2x^3}+\frac{d^2\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{c^4(c+dx)}-\frac{d\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{c^4x}+\frac{(a+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(Ad^2-Bcd+c^2C\right)}{c^4}+\frac{Ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{c}-\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{2c^2}-\frac{\left(ad^2+bc^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4d^2}+\frac{3a\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(Ad^2-Bcd+c^2C\right)}{2c^3}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2+2bc^2\right)\left(Ad^2-Bcd+c^2C\right)}{2c^3d^2}-\frac{\left(a+bx^2\right)^{3/2}(Bc-Ad)}{2c^2x^2}+\frac{3b\sqrt{a+bx^2}(Bc-Ad)}{2c^2}-\frac{ad\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c^4}+\frac{\sqrt{a+bx^2}\left(2\left(ad^2+bc^2\right)-bcdx\right)\left(Ad^2-Bcd+c^2C\right)}{2c^4d}-\frac{\left(a+bx^2\right)^{3/2}\left(Ad^2-Bcd+c^2C\right)}{c^3x}+\frac{3bx\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{2c^3}-\frac{Ab\sqrt{a+bx^2}}{cx}-\frac{A\left(a+bx^2\right)^{3/2}}{3cx^3}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)),x]
```

output
```
(3*b*(B*c - A*d)*Sqrt[a + b*x^2])/(2*c^2) - (a*d*(c^2*C - B*c*d + A*d^2)*S
qrt[a + b*x^2])/c^4 - (A*b*Sqrt[a + b*x^2])/(c*x) + (3*b*(c^2*C - B*c*d +
A*d^2)*x*Sqrt[a + b*x^2])/(2*c^3) + ((c^2*C - B*c*d + A*d^2)*(2*(b*c^2 + a
*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^4*d) - (A*(a + b*x^2)^(3/2))/(3*c*x
^3) - ((B*c - A*d)*(a + b*x^2)^(3/2))/(2*c^2*x^2) - ((c^2*C - B*c*d + A*d^
2)*(a + b*x^2)^(3/2))/(c^3*x) + (A*b^(3/2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/c + (3*a*Sqrt[b]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(2*c^3) - (Sqrt[b]*(2*b*c^2 + 3*a*d^2)*(c^2*C - B*c*d + A*d^2)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^3*d^2) - ((b*c^2 + a*d^2)^(3/2
)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[
a + b*x^2])])/(c^4*d^2) - (3*Sqrt[a]*b*(B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/(2*c^2) + (a^(3/2)*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.52

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(6Aa\,d^2x^2+8Ab\,c^2x^2-6Bacd\,x^2+6Ca\,c^2x^2-3Aacdx+3Ba\,c^2x+2A\,c^2a\right)}{6c^3x^3}+\frac{-\frac{2\left(A\,a^2d^6+2Aab\,c^2d^4+A\,b^2c^4d^2-B\,a^2c\,d^5 \cdots\right.}{\cdots}}{\cdots}$
default	$\frac{A\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{3a\,x^3}+\frac{2b\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)}{3a}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{2a\,x^2}+\cdots\right.}{\cdots}$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(6*A*a*d^2*x^2+8*A*b*c^2*x^2-6*B*a*c*d*x^2+6*C*a*c^2*
x^2-3*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/c^3/x^3+1/2/c^3*(-2/d^3*(A*a^2*d^6+
2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^
2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*
d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+a^(1/2)*(2*A*a*d^3+3*A*b*
c^2*d-2*B*a*c*d^2-3*B*b*c^3+2*C*a*c^2*d)/c*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x)+2*b^2*c^3/d^2*(B*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+C*d*(b*x^2
+a)^(1/2)/b-C*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^4\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**4/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**4*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^4\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^4\,(dx+c)}dx$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c),x)
```

3.83 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 346

$$\begin{aligned}\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx &= -\frac{aA\sqrt{a+bx^2}}{4cx^4}\\
&-\frac{a(Bc-Ad)\sqrt{a+bx^2}}{3c^2x^3}-\frac{\left(5Abc+4acC-4aBd+\frac{4aAd^2}{c}\right)\sqrt{a+bx^2}}{8c^2x^2}\\
&-\frac{(4bc^2(Bc-Ad)-3ad(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3c^4x}+\frac{b^{3/2}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}\\
&+\frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5d}\\
&-\frac{(4ac(cC-Bd)(3bc^2+2ad^2)+A(3b^2c^4+12abc^2d^2+8a^2d^4))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}c^5}\end{aligned}$$

output

```
-1/4*a*A*(b*x^2+a)^(1/2)/c/x^4-1/3*a*(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x^3-1/
8*(5*A*b*c+4*C*a*c-4*B*a*d+4*a*A*d^2/c)*(b*x^2+a)^(1/2)/c^2/x^2-1/3*(4*b*c
^2*(-A*d+B*c)-3*a*d*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/c^4/x+b^(3/2)*C*a
rctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/d+(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2
)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^5/d-1/8*(4*a
*c*(-B*d+C*c)*(2*a*d^2+3*b*c^2)+A*(8*a^2*d^4+12*a*b*c^2*d^2+3*b^2*c^4))*ar
ctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^5
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \frac{\sqrt{a+bx^2}(bc^2x^2(-15Ac-32Bcx+32Adx)-2a(A(3c^3-4c^2dx+ \cdots}{24c \cdots}$$
$$+\frac{2(-bc^2-ad^2)^{3/2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c^5d}$$
$$+\frac{2a^{3/2}Ad^4\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^5}$$
$$-\frac{(4a(cC-Bd)(3bc^2+2ad^2)+3Abc(bc^2+4ad^2))\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4\sqrt{a}c^4}$$
$$-\frac{b^{3/2}C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)),x]
```

output
```
(Sqrt[a + b*x^2]*(b*c^2*x^2*(-15*A*c - 32*B*c*x + 32*A*d*x) - 2*a*(A*(3*c^
3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3) + 2*c*x*(3*c*C*x*(c - 2*d*x) + B
*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))))/(24*c^4*x^4) + (2*(-(b*c^2) - a*d^2)^(3
/2)*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])
/Sqrt[-(b*c^2) - a*d^2]])/(c^5*d) + (2*a^(3/2)*A*d^4*ArcTanh[(Sqrt[b]*x -
Sqrt[a + b*x^2])/Sqrt[a]])/c^5 - ((4*a*(c*C - B*d)*(3*b*c^2 + 2*a*d^2) + 3
*A*b*c*(b*c^2 + 4*a*d^2))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]
])/(4*Sqrt[a]*c^4) - (b^(3/2)*C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx$$

$\Big\downarrow$ 2353

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x^4}+\frac{d^2(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^5x}-\frac{d^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^5(c+dx)}-\frac{d(a$$

$\Big\downarrow$ 2009

$$-\frac{a^{3/2}d^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{c^5}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-Ad)}{c^2}-\frac{3Ab^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}c}+\frac{(ad^2+bc^2)^{3/2}(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^5d}-\frac{3a\sqrt{b}d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{2c^4}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2c^4d}-\frac{3\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{2c^3}-\frac{b\sqrt{a+bx^2}(Bc-Ad)}{c^2x}-\frac{(a+bx^2)^{3/2}(Bc-Ad)}{3c^2x^3}+\frac{ad^2\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^5}-\frac{\sqrt{a+bx^2}(2(ad^2+bc^2)-bcdx)(Ad^2-Bcd+c^2C)}{2c^5}-\frac{3bdx\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{2c^4}+\frac{d(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^4x}+\frac{3b\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{2c^3}-\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{2c^3x^2}-\frac{3Ab\sqrt{a+bx^2}}{8cx^2}-\frac{A(a+bx^2)^{3/2}}{4cx^4}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)),x]
```

output
```
(3*b*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(2*c^3) + (a*d^2*(c^2*C - B*
c*d + A*d^2)*Sqrt[a + b*x^2])/c^5 - (3*A*b*Sqrt[a + b*x^2])/(8*c*x^2) - (b
*(B*c - A*d)*Sqrt[a + b*x^2])/(c^2*x) - (3*b*d*(c^2*C - B*c*d + A*d^2)*x*S
qrt[a + b*x^2])/(2*c^4) - ((c^2*C - B*c*d + A*d^2)*(2*(b*c^2 + a*d^2) - b*
c*d*x)*Sqrt[a + b*x^2])/(2*c^5) - (A*(a + b*x^2)^(3/2))/(4*c*x^4) - ((B*c
- A*d)*(a + b*x^2)^(3/2))/(3*c^2*x^3) - ((c^2*C - B*c*d + A*d^2)*(a + b*x^
2)^(3/2))/(2*c^3*x^2) + (d*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(c^4
*x) + (b^(3/2)*(B*c - A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^2 - (3*
a*Sqrt[b]*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/
(2*c^4) + (Sqrt[b]*(2*b*c^2 + 3*a*d^2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])/(2*c^4*d) + ((b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c
*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/
(c^5*d) - (3*A*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a]*c) - (3*Sq
rt[a]*b*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^3)
- (a^(3/2)*d^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c
^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-24 A a\,d^{3} x^{3}-32 A b\,c^{2} d\,x^{3}+24 B a c\,d^{2} x^{3}+32 B b\,c^{3} x^{3}-24 C a\,c^{2} d\,x^{3}+12 A a c\,d^{2} x^{2}+15 A b\,c^{3} x^{2}-12 B a\,c^{2} d\,x^{2}+12 C a\,c^{3} x^{2}\right.}{24 c^{4} x^{4}}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-24*A*a*d^3*x^3-32*A*b*c^2*d*x^3+24*B*a*c*d^2*x^3+3
2*B*b*c^3*x^3-24*C*a*c^2*d*x^3+12*A*a*c*d^2*x^2+15*A*b*c^3*x^2-12*B*a*c^2*
d*x^2+12*C*a*c^3*x^2-8*A*a*c^2*d*x+8*B*a*c^3*x+6*A*a*c^3)/c^4/x^4-1/8/c^4*
((8*A*a^2*d^4+12*A*a*b*c^2*d^2+3*A*b^2*c^4-8*B*a^2*c*d^3-12*B*a*b*c^3*d+8*
C*a^2*c^2*d^2+12*C*a*b*c^4)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)-8/d^2*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d
^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/c/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))-8*C*b^
(3/2)*c^4/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^5\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**5/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**5*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^5}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^5), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^5\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1183, normalized size of antiderivative = 3.42

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c),x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*d**4*x**4 + 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**2*x**4 - 48*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a*b*c*d**3*x**4 + 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a*c**3*d**2*x**4 - 48*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*
x**4 + 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*b*c**5*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**
2*d**4*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**2*x**4 + 4
8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x**4 - 48*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a*c**3*d**2*x**4 + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*b**2*c**3*d*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**5*x**4 - 1
2*sqrt(a + b*x**2)*a**2*c**4*d + 16*sqrt(a + b*x**2)*a**2*c**3*d**2*x - 24
*sqrt(a + b*x**2)*a**2*c**2*d**3*x**2 + 48*sqrt(a + b*x**2)*a**2*c*d**4*x*
*3 - 30*sqrt(a + b*x**2)*a*b*c**4*d*x**2 - 16*sqrt(a + b*x**2)*a*b*c**4*d*
x + 64*sqrt(a + b*x**2)*a*b*c**3*d**2*x**3 + 24*sqrt(a + b*x**2)*a*b*c**3*
d**2*x**2 - 48*sqrt(a + b*x**2)*a*b*c**2*d**3*x**3 - 24*sqrt(a + b*x**2)*a
*c**5*d*x**2 + 48*sqrt(a + b*x**2)*a*c**4*d**2*x**3 - 64*sqrt(a + b*x**2)*
b**2*c**4*d*x**3 + 24*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**2*d**5...
```

3.84 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 404

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6(c+dx)}\,dx = -\frac{aA\sqrt{a+bx^2}}{5cx^5} - \frac{a(Bc-Ad)\sqrt{a+bx^2}}{4c^2x^4} - \frac{\left(6Abc+5acC-5aBd+\frac{5aAd^2}{c}\right)\sqrt{a+bx^2}}{15c^2x^3} - \frac{(5bc^2(Bc-Ad)-4ad(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{8c^4x^2} - \frac{(5ac(cC-Bd)(4bc^2+3ad^2)+A(3b^2c^4+20abc^2d^2+15a^2d^4))\sqrt{a+bx^2}}{15ac^5x} - \frac{(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^6} - \frac{(3b^2c^4(Bc-Ad)-12abc^2d(c^2C-Bcd+Ad^2)-8a^2d^3(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}c^6}$$

output
```
-1/5*a*A*(b*x^2+a)^(1/2)/c/x^5-1/4*a*(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x^4-1/
15*(6*A*b*c+5*C*a*c-5*B*a*d+5*a*A*d^2/c)*(b*x^2+a)^(1/2)/c^2/x^3-1/8*(5*b*
c^2*(-A*d+B*c)-4*a*d*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/c^4/x^2-1/15*(5*
a*c*(-B*d+C*c)*(3*a*d^2+4*b*c^2)+A*(15*a^2*d^4+20*a*b*c^2*d^2+3*b^2*c^4))*
(b*x^2+a)^(1/2)/a/c^5/x-(a*d^2+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-
b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^6-1/8*(3*b^2*c^4*(-A*d+B
*c)-12*a*b*c^2*d*(A*d^2-B*c*d+C*c^2)-8*a^2*d^3*(A*d^2-B*c*d+C*c^2))*arctan
h((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^6
```

Mathematica [A] (verified)

Time = 2.74 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6(c+dx)}\,dx =$$

$$-\frac{\frac{c\sqrt{a+bx^2}\left(24Ab^2c^4x^4+abc^2x^2(5cx(15Bc+32cCx-32Bdx)+A(48c^2-75cdx+160d^2x^2))+2a^2(A(12c^4-15c^3dx+20c^2d^2x^2-30cd^3x^3+60d^4x^4)\cdots\right.}{ax^5}}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)),x]
```

output
```
-1/120*((c*Sqrt[a + b*x^2]*(24*A*b^2*c^4*x^4 + a*b*c^2*x^2*(5*c*x*(15*B*c
+ 32*c*C*x - 32*B*d*x) + A*(48*c^2 - 75*c*d*x + 160*d^2*x^2)) + 2*a^2*(A*(
12*c^4 - 15*c^3*d*x + 20*c^2*d^2*x^2 - 30*c*d^3*x^3 + 60*d^4*x^4) + 5*c*x*
(2*c*C*x*(2*c^2 - 3*c*d*x + 6*d^2*x^2) + B*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^
2 - 12*d^3*x^3)))))/(a*x^5) + 240*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d
+ A*d^2)*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a +
 b*x^2])] - (15*(3*b^2*c^4*(-(B*c) + A*d) + 12*a*b*c^2*d*(c^2*C - B*c*d +
A*d^2) + 8*a^2*d^3*(c^2*C - B*c*d + A*d^2))*Log[x])/Sqrt[a] + (15*(3*b^2*c
^4*(-(B*c) + A*d) + 12*a*b*c^2*d*(c^2*C - B*c*d + A*d^2) + 8*a^2*d^3*(c^2*
C - B*c*d + A*d^2))*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/Sqrt[a])/c^6
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.89, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6(c+dx)}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x^5}+\frac{d^4(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^6(c+dx)}-\frac{d^3(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^6x}+\frac{d^2(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}d^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{c^6}+\frac{b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{c^3}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{8\sqrt{a}c^2}-\frac{(ad^2+bc^2)^{3/2}(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^6}+\frac{3a\sqrt{b}d^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Ad^2-Bcd+c^2C)}{2c^5}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2c^5}+\frac{3\sqrt{a}bd\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{2c^4}-\frac{3b\sqrt{a+bx^2}(Bc-Ad)}{8c^2x^2}-\frac{(a+bx^2)^{3/2}(Bc-Ad)}{4c^2x^4}+\frac{d\sqrt{a+bx^2}(2(ad^2+bc^2)-bcdx)(Ad^2-Bcd+c^2C)}{2c^6}-\frac{ad^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^6}-\frac{d^2(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^5x}+\frac{3bd^2x\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{2c^5}+\frac{d(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{2c^4x^2}-\frac{3bd\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{2c^4}-\frac{b\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^3x}-\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{3c^3x^3}-\frac{A(a+bx^2)^{5/2}}{5acx^5}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)),x]
```

output
```
(-3*b*d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(2*c^4) - (a*d^3*(c^2*C -
 B*c*d + A*d^2)*Sqrt[a + b*x^2])/c^6 - (3*b*(B*c - A*d)*Sqrt[a + b*x^2])/(
8*c^2*x^2) - (b*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c^3*x) + (3*b*d^
2*(c^2*C - B*c*d + A*d^2)*x*Sqrt[a + b*x^2])/(2*c^5) + (d*(c^2*C - B*c*d +
 A*d^2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^6) - ((B*c - A
*d)*(a + b*x^2)^(3/2))/(4*c^2*x^4) - ((c^2*C - B*c*d + A*d^2)*(a + b*x^2)^
(3/2))/(3*c^3*x^3) + (d*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(2*c^4*
x^2) - (d^2*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(c^5*x) - (A*(a + b
*x^2)^(5/2))/(5*a*c*x^5) + (b^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/c^3 + (3*a*Sqrt[b]*d^2*(c^2*C - B*c*d + A*d^2)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^5) - (Sqrt[b]*(2*b*c^2 + 3*a*d^2)*
(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^5) - ((
b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b
*c^2 + a*d^2]*Sqrt[a + b*x^2])])/c^6 - (3*b^2*(B*c - A*d)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/(8*Sqrt[a]*c^2) + (3*Sqrt[a]*b*d*(c^2*C - B*c*d + A*d^2)
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^4) + (a^(3/2)*d^3*(c^2*C - B*c*d +
 A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(120A\,a^{2}d^{4}x^{4}+160Aab\,c^{2}d^{2}x^{4}+24A\,b^{2}c^{4}x^{4}-120B\,a^{2}c\,d^{3}x^{4}-160Bab\,c^{3}d\,x^{4}+120C\,a^{2}c^{2}d^{2}x^{4}+160Cab\,c^{4}x^{4}-60A\,a^{2}c}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(120*A*a^2*d^4*x^4+160*A*a*b*c^2*d^2*x^4+24*A*b^2*c
^4*x^4-120*B*a^2*c*d^3*x^4-160*B*a*b*c^3*d*x^4+120*C*a^2*c^2*d^2*x^4+160*C
*a*b*c^4*x^4-60*A*a^2*c*d^3*x^3-75*A*a*b*c^3*d*x^3+60*B*a^2*c^2*d^2*x^3+75
*B*a*b*c^4*x^3-60*C*a^2*c^3*d*x^3+40*A*a^2*c^2*d^2*x^2+48*A*a*b*c^4*x^2-40
*B*a^2*c^3*d*x^2+40*C*a^2*c^4*x^2-30*A*a^2*c^3*d*x+30*B*a^2*c^4*x+24*A*a^2
*c^4)/c^5/x^5/a+1/8/c^5*((8*A*a^2*d^5+12*A*a*b*c^2*d^3+3*A*b^2*c^4*d-8*B*a
^2*c*d^4-12*B*a*b*c^3*d^2-3*B*b^2*c^5+8*C*a^2*c^2*d^3+12*C*a*b*c^4*d)/c/a^
(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-8*(A*a^2*d^6+2*A*a*b*c^2*d^4+A
*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b
*c^4*d^2+C*b^2*c^6)/c/d/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-
2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 14.68 (sec) , antiderivative size = 1974, normalized size of antiderivative = 4.89

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^6(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x, algorithm="fricas")
```

output
```
[1/240*(120*(C*a*b*c^4 - B*a*b*c^3*d - B*a^2*c*d^3 + A*a^2*d^4 + (C*a^2 +
A*a*b)*c^2*d^2)*sqrt(b*c^2 + a*d^2)*x^5*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2
*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqr
t(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 15*(3*B*b^2*c^5 + 12*B*a*b*c^3*
d^2 + 8*B*a^2*c*d^4 - 8*A*a^2*d^5 - 3*(4*C*a*b + A*b^2)*c^4*d - 4*(2*C*a^2
 + 3*A*a*b)*c^2*d^3)*sqrt(a)*x^5*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - 2*(24*A*a^2*c^5 - 8*(20*B*a*b*c^4*d + 15*B*a^2*c^2*d^3 - 15*A
*a^2*c*d^4 - (20*C*a*b + 3*A*b^2)*c^5 - 5*(3*C*a^2 + 4*A*a*b)*c^3*d^2)*x^4
 + 15*(5*B*a*b*c^5 + 4*B*a^2*c^3*d^2 - 4*A*a^2*c^2*d^3 - (4*C*a^2 + 5*A*a*
b)*c^4*d)*x^3 - 8*(5*B*a^2*c^4*d - 5*A*a^2*c^3*d^2 - (5*C*a^2 + 6*A*a*b)*c
^5)*x^2 + 30*(B*a^2*c^5 - A*a^2*c^4*d)*x)*sqrt(b*x^2 + a))/(a*c^6*x^5), -1
/240*(240*(C*a*b*c^4 - B*a*b*c^3*d - B*a^2*c*d^3 + A*a^2*d^4 + (C*a^2 + A*
a*b)*c^2*d^2)*sqrt(-b*c^2 - a*d^2)*x^5*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x
- a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + 15
*(3*B*b^2*c^5 + 12*B*a*b*c^3*d^2 + 8*B*a^2*c*d^4 - 8*A*a^2*d^5 - 3*(4*C*a*
b + A*b^2)*c^4*d - 4*(2*C*a^2 + 3*A*a*b)*c^2*d^3)*sqrt(a)*x^5*log(-(b*x^2
+ 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(24*A*a^2*c^5 - 8*(20*B*a*b*c^
4*d + 15*B*a^2*c^2*d^3 - 15*A*a^2*c*d^4 - (20*C*a*b + 3*A*b^2)*c^5 - 5*(3*
C*a^2 + 4*A*a*b)*c^3*d^2)*x^4 + 15*(5*B*a*b*c^5 + 4*B*a^2*c^3*d^2 - 4*A*a^
2*c^2*d^3 - (4*C*a^2 + 5*A*a*b)*c^4*d)*x^3 - 8*(5*B*a^2*c^4*d - 5*A*a^2...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^6(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{x^6\,(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**6/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**6*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6(c+dx)}\,dx=\int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^6), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1819 vs. $2(371) = 742$.

Time = 0.18 (sec) , antiderivative size = 1819, normalized size of antiderivative = 4.50

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x, algorithm="giac")
```

output
```
2*(C*b^2*c^6 - B*b^2*c^5*d + 2*C*a*b*c^4*d^2 + A*b^2*c^4*d^2 - 2*B*a*b*c^3
*d^3 + C*a^2*c^2*d^4 + 2*A*a*b*c^2*d^4 - B*a^2*c*d^5 + A*a^2*d^6)*arctan(-
((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(
-b*c^2 - a*d^2)*c^6) + 1/4*(3*B*b^2*c^5 - 12*C*a*b*c^4*d - 3*A*b^2*c^4*d +
 12*B*a*b*c^3*d^2 - 8*C*a^2*c^2*d^3 - 12*A*a*b*c^2*d^3 + 8*B*a^2*c*d^4 - 8
*A*a^2*d^5)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*c^6)
 + 1/60*(75*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*b^2*c^4 - 60*(sqrt(b)*x - sq
rt(b*x^2 + a))^9*C*a*b*c^3*d - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*b^2*c^
3*d + 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*a*b*c^2*d^2 - 60*(sqrt(b)*x - s
qrt(b*x^2 + a))^9*A*a*b*c*d^3 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a*b^
(3/2)*c^4 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2)*c^4 - 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*B*a*b^(3/2)*c^3*d + 120*(sqrt(b)*x - sqrt(b*x^2
+ a))^8*C*a^2*sqrt(b)*c^2*d^2 + 240*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a*b^
(3/2)*c^2*d^2 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^2*sqrt(b)*c*d^3 +
120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*sqrt(b)*d^4 - 30*(sqrt(b)*x - sq
rt(b*x^2 + a))^7*B*a*b^2*c^4 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^2*b
*c^3*d + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b^2*c^3*d - 120*(sqrt(b)*x
 - sqrt(b*x^2 + a))^7*B*a^2*b*c^2*d^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^
7*A*a^2*b*c*d^3 - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2)*c^4 +
720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)*c^3*d - 480*(sqrt(b)*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^6(c+dx)}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^6\,(c+d\,x)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^6(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^6\,(dx+c)}dx$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c),x)
```

3.85 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 510

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx = -\frac{aA\sqrt{a+bx^2}}{6cx^6}$$
$$-\frac{a(Bc-Ad)\sqrt{a+bx^2}}{5c^2x^5}-\frac{\left(7Abc+6acC-6aBd+\frac{6aAd^2}{c}\right)\sqrt{a+bx^2}}{24c^2x^4}$$
$$-\frac{(6bc^2(Bc-Ad)-5ad(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{15c^4x^3}$$
$$-\frac{(2ac(cC-Bd)(5bc^2+4ad^2)+A(b^2c^4+10abc^2d^2+8a^2d^4))\sqrt{a+bx^2}}{16ac^5x^2}$$
$$-\frac{(3b^2c^4(Bc-Ad)-20abc^2d(c^2C-Bcd+Ad^2)-15a^2d^3(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{15ac^6x}$$
$$+\frac{d(bc^2+ad^2)^{3/2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^7}$$
$$-\frac{(2ac(cC-Bd)(3b^2c^4+12abc^2d^2+8a^2d^4)-A(b^3c^6-6ab^2c^4d^2-24a^2bc^2d^4-16a^3d^6))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right.}{16a^{3/2}c^7}$$

output
```
-1/6*a*A*(b*x^2+a)^(1/2)/c/x^6-1/5*a*(-A*d+B*c)*(b*x^2+a)^(1/2)/c^2/x^5-1/
24*(7*A*b*c+6*C*a*c-6*B*a*d+6*a*A*d^2/c)*(b*x^2+a)^(1/2)/c^2/x^4-1/15*(6*b
*c^2*(-A*d+B*c)-5*a*d*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/c^4/x^3-1/16*(2
*a*c*(-B*d+C*c)*(4*a*d^2+5*b*c^2)+A*(8*a^2*d^4+10*a*b*c^2*d^2+b^2*c^4))*(b
*x^2+a)^(1/2)/a/c^5/x^2-1/15*(3*b^2*c^4*(-A*d+B*c)-20*a*b*c^2*d*(A*d^2-B*c
*d+C*c^2)-15*a^2*d^3*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/a/c^6/x+d*(a*d^2
+b*c^2)^(3/2)*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)
/(b*x^2+a)^(1/2))/c^7-1/16*(2*a*c*(-B*d+C*c)*(8*a^2*d^4+12*a*b*c^2*d^2+3*b
^2*c^4)-A*(-16*a^3*d^6-24*a^2*b*c^2*d^4-6*a*b^2*c^4*d^2+b^3*c^6))*arctanh(
(b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^7
```

Mathematica [A] (verified)

Time = 5.46 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx = \frac{c\sqrt{a+bx^2}\left(-3b^2c^4x^4(5Ac+16Bcx-16Adx)-2abc^2x^2\left(A\left(35c^3-48c^2dx+75cd^2x^2-160d^3x^3\right)+\right.\right.}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^7*(c + d*x)),x]
```

output
```
((c*Sqrt[a + b*x^2]*(-3*b^2*c^4*x^4*(5*A*c + 16*B*c*x - 16*A*d*x) - 2*a*b*
c^2*x^2*(A*(35*c^3 - 48*c^2*d*x + 75*c*d^2*x^2 - 160*d^3*x^3) + c*x*(5*c*C
*x*(15*c - 32*d*x) + B*(48*c^2 - 75*c*d*x + 160*d^2*x^2))) - 4*a^2*(A*(10*
c^5 - 12*c^4*d*x + 15*c^3*d^2*x^2 - 20*c^2*d^3*x^3 + 30*c*d^4*x^4 - 60*d^5
*x^5) + c*x*(5*c*C*x*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3) + B*(1
2*c^4 - 15*c^3*d*x + 20*c^2*d^2*x^2 - 30*c*d^3*x^3 + 60*d^4*x^4)))))/(a*x^
6) + 480*d*(-(b*c^2) - a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b
]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + 480*a^(3/2)*A*d
^6*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (30*c*(A*b*c*(b^2*c^4
- 6*a*b*c^2*d^2 - 24*a^2*d^4) - 2*a*(c*C - B*d)*(3*b^2*c^4 + 12*a*b*c^2*d^
2 + 8*a^2*d^4))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
)/(240*c^7)
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.79, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-Ad)}{c^2x^6}-\frac{d^5(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^7(c+dx)}+\frac{d^4(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^7x}-\frac{d^3(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)d^4}{c^7}+\frac{a(Cc^2-Bdc+Ad^2)\sqrt{bx^2+a}d^4}{c^7}+\frac{(Cc^2-Bdc+Ad^2)(bx^2+a)^{3/2}d^3}{c^6x}-\frac{3a\sqrt{b}(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)d^3}{2c^6}-\frac{3b(Cc^2-Bdc+Ad^2)x\sqrt{bx^2+a}d^3}{2c^6}-\frac{(Cc^2-Bdc+Ad^2)(bx^2+a)^{3/2}d^2}{2c^5x^2}-\frac{3\sqrt{a}b(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)d^2}{2c^5}+\frac{3b(Cc^2-Bdc+Ad^2)\sqrt{bx^2+a}d^2}{2c^5}-\frac{(Cc^2-Bdc+Ad^2)(2(bc^2+ad^2)-bcdx)\sqrt{bx^2+a}d^2}{2c^7}+\frac{(Cc^2-Bdc+Ad^2)(bx^2+a)^{3/2}d}{3c^4x^3}+\frac{\sqrt{b}(2bc^2+3ad^2)(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)d}{2c^6}-\frac{b^{3/2}(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)d}{c^4}+\frac{(bc^2+ad^2)^{3/2}(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{bx^2+a}}\right)d}{c^7}+\frac{b(Cc^2-Bdc+Ad^2)\sqrt{bx^2+a}d}{c^4x}-\frac{(Bc-Ad)(bx^2+a)^{5/2}}{5ac^2x^5}-\frac{(Cc^2-Bdc+Ad^2)(bx^2+a)^{3/2}}{4c^3x^4}-\frac{A(bx^2+a)^{3/2}}{6cx^6}-\frac{3b^2(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{8\sqrt{a}c^3}+\frac{Ab^3\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{16a^{3/2}c}-\frac{3b(Cc^2-Bdc+Ad^2)\sqrt{bx^2+a}}{8c^3x^2}-\frac{Ab^2\sqrt{bx^2+a}}{16acx^2}-\frac{Ab\sqrt{bx^2+a}}{8cx^4}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^7*(c + d*x)),x]
```

output
```
(3*b*d^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(2*c^5) + (a*d^4*(c^2*C
- B*c*d + A*d^2)*Sqrt[a + b*x^2])/c^7 - (A*b*Sqrt[a + b*x^2])/(8*c*x^4) -
(A*b^2*Sqrt[a + b*x^2])/(16*a*c*x^2) - (3*b*(c^2*C - B*c*d + A*d^2)*Sqrt[a
 + b*x^2])/(8*c^3*x^2) + (b*d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c^
4*x) - (3*b*d^3*(c^2*C - B*c*d + A*d^2)*x*Sqrt[a + b*x^2])/(2*c^6) - (d^2*
(c^2*C - B*c*d + A*d^2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*
c^7) - (A*(a + b*x^2)^(3/2))/(6*c*x^6) - ((c^2*C - B*c*d + A*d^2)*(a + b*x
^2)^(3/2))/(4*c^3*x^4) + (d*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(3*
c^4*x^3) - (d^2*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(2*c^5*x^2) + (
d^3*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(c^6*x) - ((B*c - A*d)*(a +
 b*x^2)^(5/2))/(5*a*c^2*x^5) - (b^(3/2)*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/c^4 - (3*a*Sqrt[b]*d^3*(c^2*C - B*c*d + A*d^
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^6) + (Sqrt[b]*d*(2*b*c^2 + 3
*a*d^2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c
^6) + (d*(b*c^2 + a*d^2)^(3/2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*
x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])]])/c^7 + (A*b^3*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(16*a^(3/2)*c) - (3*b^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a]*c^3) - (3*Sqrt[a]*b*d^2*(c^2*C - B*c*d
+ A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^5) - (a^(3/2)*d^4*(c^2*C -
 B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^7
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-240A\,a^2d^5x^5-320Aab\,c^2d^3x^5-48A\,b^2c^4d\,x^5+240B\,a^2c\,d^4x^5+320Bab\,c^3d^2x^5+48B\,b^2c^5x^5-240C\,a^2c^2d^3x^5-320C$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(-240*A*a^2*d^5*x^5-320*A*a*b*c^2*d^3*x^5-48*A*b^2*
c^4*d*x^5+240*B*a^2*c*d^4*x^5+320*B*a*b*c^3*d^2*x^5+48*B*b^2*c^5*x^5-240*C
*a^2*c^2*d^3*x^5-320*C*a*b*c^4*d*x^5+120*A*a^2*c*d^4*x^4+150*A*a*b*c^3*d^2
*x^4+15*A*b^2*c^5*x^4-120*B*a^2*c^2*d^3*x^4-150*B*a*b*c^4*d*x^4+120*C*a^2*
c^3*d^2*x^4+150*C*a*b*c^5*x^4-80*A*a^2*c^2*d^3*x^3-96*A*a*b*c^4*d*x^3+80*B
*a^2*c^3*d^2*x^3+96*B*a*b*c^5*x^3-80*C*a^2*c^4*d*x^3+60*A*a^2*c^3*d^2*x^2+
70*A*a*b*c^5*x^2-60*B*a^2*c^4*d*x^2+60*C*a^2*c^5*x^2-48*A*a^2*c^4*d*x+48*B
*a^2*c^5*x+40*A*a^2*c^5)/c^6/x^6/a-1/16/c^6/a*((16*A*a^3*d^6+24*A*a^2*b*c^
2*d^4+6*A*a*b^2*c^4*d^2-A*b^3*c^6-16*B*a^3*c*d^5-24*B*a^2*b*c^3*d^3-6*B*a*
b^2*c^5*d+16*C*a^3*c^2*d^4+24*C*a^2*b*c^4*d^2+6*C*a*b^2*c^6)/c/a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-16*a*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c
^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d
^2+C*b^2*c^6)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*
(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*
c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 29.49 (sec) , antiderivative size = 2562, normalized size of antiderivative = 5.02

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x, algorithm="fricas")
```

output
```
[1/480*(240*(C*a^2*b*c^4*d - B*a^2*b*c^3*d^2 - B*a^3*c*d^4 + A*a^3*d^5 + (
C*a^3 + A*a^2*b)*c^2*d^3)*sqrt(b*c^2 + a*d^2)*x^6*log((2*a*b*c*d*x - a*b*c
^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x
- a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 15*(6*B*a*b^2*c^5*d +
 24*B*a^2*b*c^3*d^3 + 16*B*a^3*c*d^5 - 16*A*a^3*d^6 - (6*C*a*b^2 - A*b^3)*
c^6 - 6*(4*C*a^2*b + A*a*b^2)*c^4*d^2 - 8*(2*C*a^3 + 3*A*a^2*b)*c^2*d^4)*s
qrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(40*A*a
^3*c^6 + 16*(3*B*a*b^2*c^6 + 20*B*a^2*b*c^4*d^2 + 15*B*a^3*c^2*d^4 - 15*A*
a^3*c*d^5 - (20*C*a^2*b + 3*A*a*b^2)*c^5*d - 5*(3*C*a^3 + 4*A*a^2*b)*c^3*d
^3)*x^5 - 15*(10*B*a^2*b*c^5*d + 8*B*a^3*c^3*d^3 - 8*A*a^3*c^2*d^4 - (10*C
*a^2*b + A*a*b^2)*c^6 - 2*(4*C*a^3 + 5*A*a^2*b)*c^4*d^2)*x^4 + 16*(6*B*a^2
*b*c^6 + 5*B*a^3*c^4*d^2 - 5*A*a^3*c^3*d^3 - (5*C*a^3 + 6*A*a^2*b)*c^5*d)*
x^3 - 10*(6*B*a^3*c^5*d - 6*A*a^3*c^4*d^2 - (6*C*a^3 + 7*A*a^2*b)*c^6)*x^2
 + 48*(B*a^3*c^6 - A*a^3*c^5*d)*x)*sqrt(b*x^2 + a))/(a^2*c^7*x^6), 1/480*(
480*(C*a^2*b*c^4*d - B*a^2*b*c^3*d^2 - B*a^3*c*d^4 + A*a^3*d^5 + (C*a^3 +
A*a^2*b)*c^2*d^3)*sqrt(-b*c^2 - a*d^2)*x^6*arctan(sqrt(-b*c^2 - a*d^2)*(b*
c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2))
- 15*(6*B*a*b^2*c^5*d + 24*B*a^2*b*c^3*d^3 + 16*B*a^3*c*d^5 - 16*A*a^3*d^6
 - (6*C*a*b^2 - A*b^3)*c^6 - 6*(4*C*a^2*b + A*a*b^2)*c^4*d^2 - 8*(2*C*a^3
+ 3*A*a^2*b)*c^2*d^4)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**7/(d*x+c),x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**7*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx=\int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)x^7}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)*x^7), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3056 vs. $2(473) = 946$.

Time = 0.24 (sec) , antiderivative size = 3056, normalized size of antiderivative = 5.99

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x, algorithm="giac")
```

output
```
-2*(C*b^2*c^6*d - B*b^2*c^5*d^2 + 2*C*a*b*c^4*d^3 + A*b^2*c^4*d^3 - 2*B*a*
b*c^3*d^4 + C*a^2*c^2*d^5 + 2*A*a*b*c^2*d^5 - B*a^2*c*d^6 + A*a^2*d^7)*arc
tan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(
sqrt(-b*c^2 - a*d^2)*c^7) + 1/8*(6*C*a*b^2*c^6 - A*b^3*c^6 - 6*B*a*b^2*c^5
*d + 24*C*a^2*b*c^4*d^2 + 6*A*a*b^2*c^4*d^2 - 24*B*a^2*b*c^3*d^3 + 16*C*a^
3*c^2*d^4 + 24*A*a^2*b*c^2*d^4 - 16*B*a^3*c*d^5 + 16*A*a^3*d^6)*arctan(-(s
qrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^7) + 1/120*(150*(sqrt(
b)*x - sqrt(b*x^2 + a))^11*C*a*b^2*c^5 + 15*(sqrt(b)*x - sqrt(b*x^2 + a))^
11*A*b^3*c^5 - 150*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^2*c^4*d + 120*(s
qrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b*c^3*d^2 + 150*(sqrt(b)*x - sqrt(b*x
^2 + a))^11*A*a*b^2*c^3*d^2 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a^2*b
*c^2*d^3 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*a^2*b*c*d^4 + 240*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*B*a*b^(5/2)*c^5 - 480*(sqrt(b)*x - sqrt(b*x^2 +
 a))^10*C*a^2*b^(3/2)*c^4*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(
5/2)*c^4*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(3/2)*c^3*d^2 -
240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*C*a^3*sqrt(b)*c^2*d^3 - 480*(sqrt(b)*
x - sqrt(b*x^2 + a))^10*A*a^2*b^(3/2)*c^2*d^3 + 240*(sqrt(b)*x - sqrt(b*x^
2 + a))^10*B*a^3*sqrt(b)*c*d^4 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^
3*sqrt(b)*d^5 - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a^2*b^2*c^5 + 235*(s
qrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3*c^5 + 210*(sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{3/2}\,\left(C\,x^2+B\,x+A\right)}{x^7\,\left(c+d\,x\right)}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^7*(c + d*x)),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^7*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^7(c+dx)}\,dx = \int \frac{(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^7\,(dx+c)}dx$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x)
```

output
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^7/(d*x+c),x)
```

3.86 $\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 614

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx =$$

$$-\frac{(3a^2d^4(2cC-Bd)+20abcd^2(4c^2C-3Bcd+2Ad^2)+15b^2c^3(6c^2C-5Bcd+4Ad^2))\sqrt{a+bx^2}}{15bd^7}$$

$$+\frac{(a^2Cd^4+10abd^2(3c^2C-2Bcd+Ad^2)+8b^2c^2(5c^2C-4Bcd+3Ad^2))\,x\sqrt{a+bx^2}}{16bd^6}$$

$$-\frac{(6ad^2(2cC-Bd)+5bc(4c^2C-3Bcd+2Ad^2))\,x^2\sqrt{a+bx^2}}{15d^5}$$

$$+\frac{(7aCd^2+6b(3c^2C-2Bcd+Ad^2))\,x^3\sqrt{a+bx^2}}{24d^4}-\frac{b(2cC-Bd)x^4\sqrt{a+bx^2}}{5d^3}$$

$$+\frac{bCx^5\sqrt{a+bx^2}}{6d^2}-\frac{c^2(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^7(c+dx)}$$

$$-\frac{(a^3Cd^6-6a^2bd^4(3c^2C-2Bcd+Ad^2)-24ab^2c^2d^2(5c^2C-4Bcd+3Ad^2)-16b^3c^4(7c^2C-6Bcd+5A}{16b^{3/2}d^8}$$

$$+\frac{c\sqrt{bc^2+ad^2}(ad^2(4c^2C-3Bcd+2Ad^2)+bc^2(7c^2C-6Bcd+5Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^8}$$

output
```
-1/15*(3*a^2*d^4*(-B*d+2*C*c)+20*a*b*c*d^2*(2*A*d^2-3*B*c*d+4*C*c^2)+15*b^
2*c^3*(4*A*d^2-5*B*c*d+6*C*c^2))*(b*x^2+a)^(1/2)/b/d^7+1/16*(a^2*C*d^4+10*
a*b*d^2*(A*d^2-2*B*c*d+3*C*c^2)+8*b^2*c^2*(3*A*d^2-4*B*c*d+5*C*c^2))*x*(b*
x^2+a)^(1/2)/b/d^6-1/15*(6*a*d^2*(-B*d+2*C*c)+5*b*c*(2*A*d^2-3*B*c*d+4*C*c
^2))*x^2*(b*x^2+a)^(1/2)/d^5+1/24*(7*a*C*d^2+6*b*(A*d^2-2*B*c*d+3*C*c^2))*
x^3*(b*x^2+a)^(1/2)/d^4-1/5*b*(-B*d+2*C*c)*x^4*(b*x^2+a)^(1/2)/d^3+1/6*b*C
*x^5*(b*x^2+a)^(1/2)/d^2-c^2*(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(
1/2)/d^7/(d*x+c)-1/16*(a^3*C*d^6-6*a^2*b*d^4*(A*d^2-2*B*c*d+3*C*c^2)-24*a*
b^2*c^2*d^2*(3*A*d^2-4*B*c*d+5*C*c^2)-16*b^3*c^4*(5*A*d^2-6*B*c*d+7*C*c^2)
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^8+c*(a*d^2+b*c^2)^(1/2)*(a*
d^2*(2*A*d^2-3*B*c*d+4*C*c^2)+b*c^2*(5*A*d^2-6*B*c*d+7*C*c^2))*arctanh((-b
*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^8
```

Mathematica [A] (verified)

Time = 9.45 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.88

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \frac{d\sqrt{a+bx^2}\left(3a^2d^4(c+dx)(-32cC+16Bd+5Cdx)-4b^2\left(420c^6C-30c^5d(12B-7Cx)-d^6x^4(1\right.\right.}{}$$

input
```
Integrate[(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(3*a^2*d^4*(c + d*x)*(-32*c*C + 16*B*d + 5*C*d*x) - 4*
b^2*(420*c^6*C - 30*c^5*d*(12*B - 7*C*x) - d^6*x^4*(15*A + 2*x*(6*B + 5*C*
x)) + c*d^5*x^3*(25*A + 2*x*(9*B + 7*C*x)) - c^2*d^4*x^2*(50*A + 3*x*(10*B
 + 7*C*x)) + 5*c^3*d^3*x*(30*A + x*(12*B + 7*C*x)) + 10*c^4*d^2*(30*A - x*
(18*B + 7*C*x))) - 2*a*b*d^2*(760*c^4*C + c^3*(-600*B*d + 415*C*d*x) - d^4
*x^2*(75*A + x*(48*B + 35*C*x)) + c^2*d^2*(440*A - 3*x*(110*B + 43*C*x)) +
 c*d^3*x*(245*A + x*(102*B + 61*C*x)))))/(b*(c + d*x)) - 480*c*Sqrt[-(b*c^
2) - a*d^2]*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(7*c^2*C - 6*B*c*
d + 5*A*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqr
t[a + b*x^2])] + (30*(-(a^3*C*d^6) + 6*a^2*b*d^4*(3*c^2*C - 2*B*c*d + A*d^
2) + 24*a*b^2*c^2*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) + 16*b^3*c^4*(7*c^2*C
- 6*B*c*d + 5*A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b
^(3/2))/(240*d^8)
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.10, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.406, Rules used = {2182, 2185, 2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 2182}$$

$$-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(-C\left(\frac{bc^2}{d}+ad\right)x^3+\frac{(cC-Bd)\left(bc^2+ad^2\right)x^2}{d^2}-\frac{\left(5bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}\right)}{c+dx}dx}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(bd(17cC-6Bd)\left(bc^2+ad^2\right)x^2+\left(5bc^2+ad^2\right)\left(aCd^2-b\left(5Cc^2-6Bdc+6Ad^2\right)\right)x+acd\left(aCd^2+b\left(7Cc^2-6Bdc+6Ad^2\right)\right)\right)}{c+dx}dx}{6bd^3}-\frac{C\left(a+bx^2\right)^{5/2}(\cdots}{6\cdots}}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{\frac{\frac{\int \frac{5bd^2\left(acd\left(aCd^2+b\left(7Cc^2-6Bdc+6Ad^2\right)\right)+\left(a^2Cd^4-ab\left(17Cc^2-12Bdc+6Ad^2\right)d^2-6b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{5bd^2}+\frac{1}{5}\left(a+bx^2\right)^{5/2}\left(ad^2+\cdots\right.}{6bd^3}\cdots}{ad^2+bc^2}-\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{\int \frac{\left(acd\left(aCd^2+b\left(7Cc^2-6Bdc+6Ad^2\right)\right)+\left(a^2Cd^4-ab\left(17Cc^2-12Bdc+6Ad^2\right)d^2-6b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx+\frac{1}{5}\left(a+bx^2\right)^{5/2}\left(ad^2+bc^2\right.}{6bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 682

$$-\frac{\frac{\int \frac{3b\left(bc^2+ad^2\right)\left(acd\left(14bCc^2-12bBdc+10Abd^2+aCd^2\right)+\left(a^2Cd^4-6ab\left(3Cc^2-2Bdc+Ad^2\right)d^2-8b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(dx\right.}{}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$-\frac{\frac{3\left(ad^2+bc^2\right)\int \frac{\left(acd\left(14bCc^2-12bBdc+10Abd^2+aCd^2\right)+\left(a^2Cd^4-6ab\left(3Cc^2-2Bdc+Ad^2\right)d^2-8b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(dx\right.}{}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 682

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\int \frac{b\left(acd\left(a^2Cd^4+2ab\left(23Cc^2-18Bdc+13Ad^2\right)d^2+8b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)+\left(a^3Cd^6-6a^2b\left(3Cc^2-2Bdc+Ad^2\right)d^4-24ab^2c^2\left(5Cc^2-4Bdc\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}\right.}{}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\int \frac{acd\left(a^2Cd^4+2ab\left(23Cc^2-18Bdc+13Ad^2\right)d^2+8b^2c^2\left(7Cc^2-6Bdc+5Ad^2\right)\right)+\left(a^3Cd^6-6a^2b\left(3Cc^2-2Bdc+Ad^2\right)d^4-24ab^2c^2\left(5Cc^2-4Bdc+3\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\right.}{}}{}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 719

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\frac{\left(a^3Cd^6-6a^2bd^4\left(Ad^2-2Bcd+3c^2C\right)-24ab^2c^2d^2\left(3Ad^2-4Bcd+5c^2C\right)-16b^3c^4\left(5Ad^2-6Bcd+7c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{16bc\left(ad^2+bc^2\right)\left(ad^2\left(\cdots\right.\right.}{\cdots}}{2d^2}\right.}{\cdots}}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 224

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\frac{\left(a^3Cd^6-6a^2bd^4\left(Ad^2-2Bcd+3c^2C\right)-24ab^2c^2d^2\left(3Ad^2-4Bcd+5c^2C\right)-16b^3c^4\left(5Ad^2-6Bcd+7c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}+\frac{16bc\left(ad^2+b\cdots\right.}{\cdots}}{2d^2}\right.}{\cdots}}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 219

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\frac{16bc\left(ad^2+bc^2\right)\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(5Ad^2-6Bcd+7c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3Cd^6-6a^2bd^4\left(Ad^2-\cdots\right.\right.}{\cdots}}{2d^2}\right.}{\cdots}}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 488

$$-\frac{\frac{3\left(ad^2+bc^2\right)\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(a^3Cd^6-6a^2bd^4\left(Ad^2-2Bcd+3c^2C\right)-24ab^2c^2d^2\left(3Ad^2-4Bcd+5c^2C\right)-16b^3c^4\left(5Ad^2-6Bcd+7c^2C\right)\right)}{\sqrt{b}d}-\frac{16bc\left(ad^2+b\cdots\right.}{\cdots}}{2d^2}\right.}{\cdots}}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 219

$$-\frac{\frac{\left(a+bx^2\right)^{3/2}\left(dx\left(a^2Cd^4-abd^2\left(6Ad^2-12Bcd+17c^2C\right)-6b^2c^2\left(5Ad^2-6Bcd+7c^2C\right)\right)+8bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(5Ad^2-6Bcd+7c^2C\right)\right)\right)}{4d^2}+\frac{3\left(ad^2+\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{c^2\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
-((c^2*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(5/2))/(d^3*(b*c^2 + a*d^2)*(c
+ d*x))) - (-1/6*(C*(b*c^2 + a*d^2)*(c + d*x)*(a + b*x^2)^(5/2))/(b*d^3) +
 (((8*b*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(7*c^2*C - 6*B*c*d
+ 5*A*d^2)) + d*(a^2*C*d^4 - 6*b^2*c^2*(7*c^2*C - 6*B*c*d + 5*A*d^2) - a*b
*d^2*(17*c^2*C - 12*B*c*d + 6*A*d^2))*x)*(a + b*x^2)^(3/2))/(4*d^2) + ((17
*c*C - 6*B*d)*(b*c^2 + a*d^2)*(a + b*x^2)^(5/2))/5 + (3*(b*c^2 + a*d^2)*((
(16*b*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(7*c^2*C - 6*B*c*d +
5*A*d^2)) + d*(a^2*C*d^4 - 6*a*b*d^2*(3*c^2*C - 2*B*c*d + A*d^2) - 8*b^2*c
^2*(7*c^2*C - 6*B*c*d + 5*A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) + (((a^3*C*d
^6 - 6*a^2*b*d^4*(3*c^2*C - 2*B*c*d + A*d^2) - 24*a*b^2*c^2*d^2*(5*c^2*C -
 4*B*c*d + 3*A*d^2) - 16*b^3*c^4*(7*c^2*C - 6*B*c*d + 5*A*d^2))*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (16*b*c*Sqrt[b*c^2 + a*d^2]*(a*d
^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*c^2*(7*c^2*C - 6*B*c*d + 5*A*d^2))*Ar
cTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(
4*d^2))/(6*b*d^3))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 1014, normalized size of antiderivative = 1.65

method	result	size
risch	Expression too large to display	1014
default	Expression too large to display	1682

input
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/240/b*(-40*C*b^2*d^5*x^5-48*B*b^2*d^5*x^4+96*C*b^2*c*d^4*x^4-60*A*b^2*d
^5*x^3+120*B*b^2*c*d^4*x^3-70*C*a*b*d^5*x^3-180*C*b^2*c^2*d^3*x^3+160*A*b^
2*c*d^4*x^2-96*B*a*b*d^5*x^2-240*B*b^2*c^2*d^3*x^2+192*C*a*b*c*d^4*x^2+320
*C*b^2*c^3*d^2*x^2-150*A*a*b*d^5*x-360*A*b^2*c^2*d^3*x+300*B*a*b*c*d^4*x+4
80*B*b^2*c^3*d^2*x-15*C*a^2*d^5*x-450*C*a*b*c^2*d^3*x-600*C*b^2*c^4*d*x+64
0*A*a*b*c*d^4+960*A*b^2*c^3*d^2-48*B*a^2*d^5-960*B*a*b*c^2*d^3-1200*B*b^2*
c^4*d+96*C*a^2*c*d^4+1280*C*a*b*c^3*d^2+1440*C*b^2*c^5)*(b*x^2+a)^(1/2)/d^
7+1/16/d^7/b*((6*A*a^2*b*d^6+72*A*a*b^2*c^2*d^4+80*A*b^3*c^4*d^2-12*B*a^2*
b*c*d^5-96*B*a*b^2*c^3*d^3-96*B*b^3*c^5*d-C*a^3*d^6+18*C*a^2*b*c^2*d^4+120
*C*a*b^2*c^4*d^2+112*C*b^3*c^6)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+16
*b*c/d^2*(2*A*a^2*d^6+8*A*a*b*c^2*d^4+6*A*b^2*c^4*d^2-3*B*a^2*c*d^5-10*B*a
*b*c^3*d^3-7*B*b^2*c^5*d+4*C*a^2*c^2*d^4+12*C*a*b*c^4*d^2+8*C*b^2*c^6)/((a
*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b
*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x
+c/d))+16*b*c^2*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a
*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^3*(-1/(a
*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1
/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-
2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \int \frac{x^2(a+bx^2)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(x**2*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(x**2*(a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.75

$$\int \frac{x^2(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
-(b*x^2 + a)^(3/2)*C*c^4/(d^6*x + c*d^5) + (b*x^2 + a)^(3/2)*B*c^3/(d^5*x
+ c*d^4) - (b*x^2 + a)^(3/2)*A*c^2/(d^4*x + c*d^3) + 7/2*sqrt(b*x^2 + a)*C
*b*c^4*x/d^6 - 3*sqrt(b*x^2 + a)*B*b*c^3*x/d^5 + 3/4*(b*x^2 + a)^(3/2)*C*c
^2*x/d^4 + 9/8*sqrt(b*x^2 + a)*C*a*c^2*x/d^4 + 5/2*sqrt(b*x^2 + a)*A*b*c^2
*x/d^4 - 1/2*(b*x^2 + a)^(3/2)*B*c*x/d^3 - 3/4*sqrt(b*x^2 + a)*B*a*c*x/d^3
 + 1/4*(b*x^2 + a)^(3/2)*A*x/d^2 + 3/8*sqrt(b*x^2 + a)*A*a*x/d^2 + 1/6*(b*
x^2 + a)^(5/2)*C*x/(b*d^2) - 1/24*(b*x^2 + a)^(3/2)*C*a*x/(b*d^2) - 1/16*s
qrt(b*x^2 + a)*C*a^2*x/(b*d^2) + 7*C*b^(3/2)*c^6*arcsinh(b*x/sqrt(a*b))/d^
8 - 6*B*b^(3/2)*c^5*arcsinh(b*x/sqrt(a*b))/d^7 + 15/2*C*a*sqrt(b)*c^4*arcs
inh(b*x/sqrt(a*b))/d^6 + 5*A*b^(3/2)*c^4*arcsinh(b*x/sqrt(a*b))/d^6 - 6*B*
a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 + 9/8*C*a^2*c^2*arcsinh(b*x/sqrt(
a*b))/(sqrt(b)*d^4) + 9/2*A*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - 3/4
*B*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 1/16*C*a^3*arcsinh(b*x/sqr
t(a*b))/(b^(3/2)*d^2) + 3/8*A*a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 3
*C*sqrt(a + b*c^2/d^2)*b*c^5*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/d^7 + 3*B*sqrt(a + b*c^2/d^2)*b*c^4*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^6 - 4*C*(a +
b*c^2/d^2)^(3/2)*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*
b)*abs(d*x + c)))/d^5 - 3*A*sqrt(a + b*c^2/d^2)*b*c^3*arcsinh(b*c*x/(sqrt(
a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 3*B*(a + b*c^2...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^2\,(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{(c+d\,x)^2}\,dx$$

input
```
int((x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int((x^2*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{x^2(b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{(dx+c)^2}dx$$

input
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output
```
int(x^2*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

3.87 $\int \frac{x(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 496

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \frac{(3a^2Cd^4+20abd^2(3c^2C-2Bcd+Ad^2)+15b^2c^2(5c^2C-4Bcd+}{15bd^6}$$
$$-\frac{(5ad^2(2cC-Bd)+4bc(4c^2C-3Bcd+2Ad^2))\,x\sqrt{a+bx^2}}{8d^5}$$
$$+\frac{(6aCd^2+5b(3c^2C-2Bcd+Ad^2))\,x^2\sqrt{a+bx^2}}{15d^4}-\frac{b(2cC-Bd)x^3\sqrt{a+bx^2}}{4d^3}$$
$$+\frac{bCx^4\sqrt{a+bx^2}}{5d^2}+\frac{c(bc^2+ad^2)\left(c^2C-Bcd+Ad^2\right)\sqrt{a+bx^2}}{d^6(c+dx)}$$
$$-\frac{(3a^2d^4(2cC-Bd)+12abcd^2(4c^2C-3Bcd+2Ad^2)+8b^2c^3(6c^2C-5Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^7}$$
$$-\frac{\sqrt{bc^2+ad^2}(ad^2(3c^2C-2Bcd+Ad^2)+bc^2(6c^2C-5Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^7}$$

output
```
1/15*(3*a^2*C*d^4+20*a*b*d^2*(A*d^2-2*B*c*d+3*C*c^2)+15*b^2*c^2*(3*A*d^2-4
*B*c*d+5*C*c^2))*(b*x^2+a)^(1/2)/b/d^6-1/8*(5*a*d^2*(-B*d+2*C*c)+4*b*c*(2*
A*d^2-3*B*c*d+4*C*c^2))*x*(b*x^2+a)^(1/2)/d^5+1/15*(6*a*C*d^2+5*b*(A*d^2-2
*B*c*d+3*C*c^2))*x^2*(b*x^2+a)^(1/2)/d^4-1/4*b*(-B*d+2*C*c)*x^3*(b*x^2+a)^
(1/2)/d^3+1/5*b*C*x^4*(b*x^2+a)^(1/2)/d^2+c*(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c
^2)*(b*x^2+a)^(1/2)/d^6/(d*x+c)-1/8*(3*a^2*d^4*(-B*d+2*C*c)+12*a*b*c*d^2*(
2*A*d^2-3*B*c*d+4*C*c^2)+8*b^2*c^3*(4*A*d^2-5*B*c*d+6*C*c^2))*arctanh(b^(1
/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^7-(a*d^2+b*c^2)^(1/2)*(a*d^2*(A*d^2-2*B*c
*d+3*C*c^2)+b*c^2*(4*A*d^2-5*B*c*d+6*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b
*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^7
```

Mathematica [A] (verified)

Time = 3.05 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.88

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \frac{d\sqrt{a+bx^2}\left(24a^2Cd^4(c+dx)+abd^2\left(600c^3C-110c^2d(4B-3Cx)+cd^2\left(280A-245Bx-102Cx^2\right.\right.\right.}{}$$

input
```
Integrate[(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(24*a^2*C*d^4*(c + d*x) + a*b*d^2*(600*c^3*C - 110*c^2
*d*(4*B - 3*C*x) + c*d^2*(280*A - 245*B*x - 102*C*x^2) + d^3*x*(160*A + 75
*B*x + 48*C*x^2)) + 2*b^2*(360*c^5*C - 60*c^4*d*(5*B - 3*C*x) + 30*c^3*d^2
*(8*A - x*(5*B + 2*C*x)) + 10*c^2*d^3*x*(12*A + x*(5*B + 3*C*x)) + d^5*x^3
*(20*A + 3*x*(5*B + 4*C*x)) - c*d^4*x^2*(40*A + x*(25*B + 18*C*x)))))/(b*(
c + d*x)) + 240*Sqrt[-(b*c^2) - a*d^2]*(a*d^2*(3*c^2*C - 2*B*c*d + A*d^2)
+ b*c^2*(6*c^2*C - 5*B*c*d + 4*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[
a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] + (15*(-3*a^2*d^4*(-2*c*C + B*d) + 12*
a*b*c*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + 8*b^2*c^3*(6*c^2*C - 5*B*c*d + 4
*A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(120*d^7)
```

Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2182, 2185, 27, 682, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2182$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\int \frac{\left(bx^2+a\right)^{3/2}\left(-C\left(\frac{bc^2}{d}+ad\right)x^2+\left(4Abc+\frac{(cC-Bd)\left(5bc^2+ad^2\right)}{d^2}\right)x+a\left(-\frac{Cc^2}{d}+Bc-Ad\right)\right)}{c+dx}dx}{ad^2+bc^2}$$

$$\Big\downarrow \ 2185$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\frac{\int -\frac{5b\left(ad\left(Cc^2-Bdc+Ad^2\right)-\left(a(2cC-Bd)d^2+bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{5bd^2}-\frac{1}{5}C\left(a+bx^2\right)^{5/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \ 27$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\int \frac{\left(ad\left(Cc^2-Bdc+Ad^2\right)-\left(a(2cC-Bd)d^2+bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{d^2}-\frac{1}{5}C\left(a+bx^2\right)^{5/2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \ 682$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\frac{\int \frac{b\left(bc^2+ad^2\right)\left(ad\left(6Cc^2-5Bdc+4Ad^2\right)-\left(3a(2cC-Bd)d^2+4bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4bd^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-\right.\right.\right.}{}}{d^2}}{ad^2+bc^2}$$

$$\Big\downarrow 27$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\int\frac{\left(ad\left(6Cc^2-5Bdc+4Ad^2\right)-\left(3a(2cC-Bd)d^2+4bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^2}+\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-\cdots\right.\right.\right.}{d^2}}{ad^2+bc^2}$$

$$\Big\downarrow 682$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{\int\frac{b\left(ad\left(4b\left(6Cc^2-5Bdc+4Ad^2\right)c^2+ad^2\left(18Cc^2-13Bdc+8Ad^2\right)\right)-\left(3a^2(2cC-Bd)d^4+12abc\left(4Cc^2-3Bdc+2Ad^2\right)d^2+8b^2c^3\left(6Cc^2-5Bdc+4Ad^2\cdots\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^2}\cdots\right.}{4d^2}\cdots}{\cdots}$$

$$\Big\downarrow 27$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{\int\frac{ad\left(4b\left(6Cc^2-5Bdc+4Ad^2\right)c^2+ad^2\left(18Cc^2-13Bdc+8Ad^2\right)\right)-\left(3a^2(2cC-Bd)d^4+12abc\left(4Cc^2-3Bdc+2Ad^2\right)d^2+8b^2c^3\left(6Cc^2-5Bdc+4Ad^2\right)\cdots\right.}{(c+dx)\sqrt{bx^2+a}}}{2d^2}\cdots\right.}{4d^2}\cdots}{\cdots}$$

$$\Big\downarrow 719$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-5Bcd+6c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4(2cC-Bd)+12abcd^2\left(2Ad^2-3Bcd+4c^2C\right)+8b\cdots\right.}{d}}{2d^2}\cdots\right.}{\cdots}\cdots}{\cdots}$$

$$\Big\downarrow 224$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-5Bcd+6c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(3a^2d^4(2cC-Bd)+12abcd^2\left(2Ad^2-3Bcd+4c^2C\right)+8b\cdots}{d\cdots}}{2d^2}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\left(ad^2+bc^2\right)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-5Bcd+6c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(2cC-Bd)+12abcd^2\left(2\cdots\right.\right.}{\sqrt{b}c\cdots}}{2d^2}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 488$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\left(ad^2+bc^2\right)\left(\frac{-\frac{8\left(ad^2+bc^2\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^2\left(4Ad^2-5Bcd+6c^2C\right)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(2cC-\cdots\right.}{\cdots}}{2d^2}\cdots\right.}{\cdots}$$

$$\Big\downarrow \; 219$$

$$\frac{c\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\left(ad^2+bc^2\right)\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2d^4(2cC-Bd)+12abcd^2\left(2Ad^2-3Bcd+4c^2C\right)+8b^2c^3\left(4Ad^2-5Bcd+6c^2C\right)\right)}{\sqrt{b}d}-\frac{8\sqrt{ad^2+bc^2}\,\text{arctanh}\left(\frac{ad-\cdots}{\sqrt{a+bx^2}\sqrt{\cdots}}\right)\cdots}{\cdots}}{2d^2}\cdots\right.}{\cdots}$$

input

```
Int[(x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
(c*(c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(5/2))/(d^2*(b*c^2 + a*d^2)*(c + d*
x)) - (-1/5*(C*(a/b + c^2/d^2)*(a + b*x^2)^(5/2)) - (((4*(a*d^2*(3*c^2*C -
 2*B*c*d + A*d^2) + b*c^2*(6*c^2*C - 5*B*c*d + 4*A*d^2)) - 3*d*(a*d^2*(2*c
*C - B*d) + b*c*(6*c^2*C - 5*B*c*d + 4*A*d^2))*x)*(a + b*x^2)^(3/2))/(12*d
^2) + ((b*c^2 + a*d^2)*(((8*(a*d^2*(3*c^2*C - 2*B*c*d + A*d^2) + b*c^2*(6*
c^2*C - 5*B*c*d + 4*A*d^2)) - d*(3*a*d^2*(2*c*C - B*d) + 4*b*c*(6*c^2*C -
5*B*c*d + 4*A*d^2))*x)*Sqrt[a + b*x^2])/(2*d^2) + (-(((3*a^2*d^4*(2*c*C -
B*d) + 12*a*b*c*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + 8*b^2*c^3*(6*c^2*C - 5
*B*c*d + 4*A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (8
*Sqrt[b*c^2 + a*d^2]*(a*d^2*(3*c^2*C - 2*B*c*d + A*d^2) + b*c^2*(6*c^2*C -
 5*B*c*d + 4*A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b
*x^2])])/d)/(2*d^2)))/(4*d^2))/d^2)/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 870, normalized size of antiderivative = 1.75

method	result
risch	$\frac{(24C\,d^4b^2x^4+30B\,b^2d^4x^3-60C\,b^2c\,d^3x^3+40A\,b^2d^4x^2-80B\,b^2c\,d^3x^2+48Cab\,d^4x^2+120C\,b^2c^2d^2x^2-120A\,b^2c\,d^3x+75Bab\,d^4x+1}{}$
default	Expression too large to display

input
```
int(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/120/b*(24*C*b^2*d^4*x^4+30*B*b^2*d^4*x^3-60*C*b^2*c*d^3*x^3+40*A*b^2*d^4
*x^2-80*B*b^2*c*d^3*x^2+48*C*a*b*d^4*x^2+120*C*b^2*c^2*d^2*x^2-120*A*b^2*c
*d^3*x+75*B*a*b*d^4*x+180*B*b^2*c^2*d^2*x-150*C*a*b*c*d^3*x-240*C*b^2*c^3*
d*x+160*A*a*b*d^4+360*A*b^2*c^2*d^2-320*B*a*b*c*d^3-480*B*b^2*c^3*d+24*C*a
^2*d^4+480*C*a*b*c^2*d^2+600*C*b^2*c^4)*(b*x^2+a)^(1/2)/d^6-1/8/d^6*((24*A
*a*b*c*d^4+32*A*b^2*c^3*d^2-3*B*a^2*d^5-36*B*a*b*c^2*d^3-40*B*b^2*c^4*d+6*
C*a^2*c*d^4+48*C*a*b*c^3*d^2+48*C*b^2*c^5)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
/b^(1/2)+8/d^2*(A*a^2*d^6+6*A*a*b*c^2*d^4+5*A*b^2*c^4*d^2-2*B*a^2*c*d^5-8*
B*a*b*c^3*d^3-6*B*b^2*c^5*d+3*C*a^2*c^2*d^4+10*C*a*b*c^4*d^2+7*C*b^2*c^6)/
((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^
2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
/(x+c/d))+8*c*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b
*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^3*(-1/(a*d
^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*
b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*
d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \int \frac{x(a+bx^2)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(x*(b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral(x*(a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 890, normalized size of antiderivative = 1.79

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*x^2 + a)^(3/2)*C*c^3/(d^5*x + c*d^4) - (b*x^2 + a)^(3/2)*B*c^2/(d^4*x +
 c*d^3) + (b*x^2 + a)^(3/2)*A*c/(d^3*x + c*d^2) - 3*sqrt(b*x^2 + a)*C*b*c^
3*x/d^5 + 5/2*sqrt(b*x^2 + a)*B*b*c^2*x/d^4 - 1/2*(b*x^2 + a)^(3/2)*C*c*x/
d^3 - 3/4*sqrt(b*x^2 + a)*C*a*c*x/d^3 - 2*sqrt(b*x^2 + a)*A*b*c*x/d^3 + 1/
4*(b*x^2 + a)^(3/2)*B*x/d^2 + 3/8*sqrt(b*x^2 + a)*B*a*x/d^2 - 6*C*b^(3/2)*
c^5*arcsinh(b*x/sqrt(a*b))/d^7 + 5*B*b^(3/2)*c^4*arcsinh(b*x/sqrt(a*b))/d^
6 - 6*C*a*sqrt(b)*c^3*arcsinh(b*x/sqrt(a*b))/d^5 - 4*A*b^(3/2)*c^3*arcsinh
(b*x/sqrt(a*b))/d^5 + 9/2*B*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 - 3/4
*C*a^2*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 3*A*a*sqrt(b)*c*arcsinh(b*
x/sqrt(a*b))/d^3 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + 3*C*sq
rt(a + b*c^2/d^2)*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt
(a*b)*abs(d*x + c)))/d^6 - 3*B*sqrt(a + b*c^2/d^2)*b*c^3*arcsinh(b*c*x/(sq
rt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 3*C*(a + b*c^2
/d^2)^(3/2)*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*ab
s(d*x + c)))/d^4 + 3*A*sqrt(a + b*c^2/d^2)*b*c^2*arcsinh(b*c*x/(sqrt(a*b)*
abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^4 - 2*B*(a + b*c^2/d^2)^(3
/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)
))/d^3 + A*(a + b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
a*d/(sqrt(a*b)*abs(d*x + c)))/d^2 + 6*sqrt(b*x^2 + a)*C*b*c^4/d^6 - 5*sqrt
(b*x^2 + a)*B*b*c^3/d^5 + (b*x^2 + a)^(3/2)*C*c^2/d^4 + 3*sqrt(b*x^2 + ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \int \frac{x\,\left(b\,x^2+a\right)^{3/2}\,\left(C\,x^2+B\,x+A\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int((x*(a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2398, normalized size of antiderivative = 4.83

$$\int \frac{x(a+bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output

```
(240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a**2*b*c*d**4 + 240*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5*x + 960*sqrt(a*d**2 + b*c
**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3
*d**2 + 960*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a*b**2*c**2*d**3*x - 480*sqrt(a*d**2 + b*c**2)*log(sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3 - 480*sq
rt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c
*x)*a*b**2*c*d**4*x + 720*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(
a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**4*d**2 + 720*sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**3*x
- 1200*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*b**3*c**4*d - 1200*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d**2*x + 1440*sqrt(a*d**2
+ b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c
**6 + 1440*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*b**2*c**5*d*x - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*
*2*b*c*d**4 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**5*x - 960*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**2 - 960*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a*b**2*c**2*d**3*x + 480*sqrt(a*d**2 + b*c**2)*log(c ...
```

3.88 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 391

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = -\frac{(4ad^2(2cC-Bd)+3bc(4c^2C-3Bcd+2Ad^2))\sqrt{a+bx^2}}{3d^5} + \frac{(5aCd^2+4b(3c^2C-2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8d^4} - \frac{b(2cC-Bd)x^2\sqrt{a+bx^2}}{3d^3} + \frac{bCx^3\sqrt{a+bx^2}}{4d^2} - \frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^5(c+dx)} + \frac{(3a^2Cd^4+12abd^2(3c^2C-2Bcd+Ad^2)+8b^2c^2(5c^2C-4Bcd+3Ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}d^6} + \frac{\sqrt{bc^2+ad^2}(ad^2(2cC-Bd)+bc(5c^2C-4Bcd+3Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^6}$$

output
```
-1/3*(4*a*d^2*(-B*d+2*C*c)+3*b*c*(2*A*d^2-3*B*c*d+4*C*c^2))*(b*x^2+a)^(1/2
)/d^5+1/8*(5*a*C*d^2+4*b*(A*d^2-2*B*c*d+3*C*c^2))*x*(b*x^2+a)^(1/2)/d^4-1/
3*b*(-B*d+2*C*c)*x^2*(b*x^2+a)^(1/2)/d^3+1/4*b*C*x^3*(b*x^2+a)^(1/2)/d^2-(
a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^5/(d*x+c)+1/8*(3*a^2*C*
d^4+12*a*b*d^2*(A*d^2-2*B*c*d+3*C*c^2)+8*b^2*c^2*(3*A*d^2-4*B*c*d+5*C*c^2)
)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^6+(a*d^2+b*c^2)^(1/2)*(a*d^
2*(-B*d+2*C*c)+b*c*(3*A*d^2-4*B*c*d+5*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+
b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^6
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \frac{\frac{d\sqrt{a+bx^2}(-2b(60c^4C-6c^3d(8B-5Cx)+2c^2d^2(18A-12Bx-5Cx^2)-d^4x^2(6A+4Bx+3Cx^2)}{c+dx}}{}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
((d*Sqrt[a + b*x^2]*(-2*b*(60*c^4*C - 6*c^3*d*(8*B - 5*C*x) + 2*c^2*d^2*(1
8*A - 12*B*x - 5*C*x^2) - d^4*x^2*(6*A + 4*B*x + 3*C*x^2) + c*d^3*x*(18*A
+ 8*B*x + 5*C*x^2)) + a*d^2*(-88*c^2*C + 7*c*d*(8*B - 7*C*x) + d^2*(-24*A
+ 32*B*x + 15*C*x^2))))/(c + d*x) - 48*Sqrt[-(b*c^2) - a*d^2]*(a*d^2*(2*c*
C - B*d) + b*c*(5*c^2*C - 4*B*c*d + 3*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) -
d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]] - (3*(3*a^2*C*d^4 + 12*a*b*d^2*
(3*c^2*C - 2*B*c*d + A*d^2) + 8*b^2*c^2*(5*c^2*C - 4*B*c*d + 3*A*d^2))*Log
[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/(24*d^6)
```

Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$, Rules used = {2182, 25, 682, 25, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{(c+dx)^2}dx$$

$\downarrow$ 2182

$$-\frac{\int -\frac{\left(Abc-aCc+aBd+\left(aCd-b\left(-\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 25

$$\frac{\int \frac{\left(Abc-aCc+aBd+\left(aCd-b\left(-\frac{5Cc^2}{d}+4Bc-4Ad\right)\right)x\right)\left(bx^2+a\right)^{3/2}}{c+dx}dx}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 682

$$\frac{\frac{\int -\frac{b\left(bc^2+ad^2\right)\left(ad(5cC-4Bd)-\left(20bCc^2-16bBdc+12Abd^2+3aCd^2\right)x\right)\sqrt{bx^2+a}}{d(c+dx)}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2(2cC-Bd)+bc\left(3Ad^2-4Bcd+5c^2C\right)\right)-3dx(\ldots}{12d^3}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 25

$$\frac{-\frac{\int \frac{b\left(bc^2+ad^2\right)\left(ad(5cC-4Bd)-\left(20bCc^2-16bBdc+12Abd^2+3aCd^2\right)x\right)\sqrt{bx^2+a}}{d(c+dx)}dx}{4bd^2}-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2(2cC-Bd)+bc\left(3Ad^2-4Bcd+5c^2C\right)\right)-3dx(\ldots}{12d^3}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{\left(ad^2+bc^2\right)\int \frac{\left(ad(5cC-4Bd)-\left(20bCc^2-16bBdc+12Abd^2+3aCd^2\right)x\right)\sqrt{bx^2+a}}{c+dx}dx}{4d^3}-\frac{\left(a+bx^2\right)^{3/2}\left(4\left(ad^2(2cC-Bd)+bc\left(3Ad^2-4Bcd+5c^2C\right)\right)-3dx(\ldots}{12d^3}}{ad^2+bc^2}-\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 682

$$-\frac{(ad^2+bc^2)\left(\frac{\int \frac{b\left(ad\left(a(13cC-8Bd)d^2+4bc\left(5Cc^2-4Bdc+3Ad^2\right)\right)-\left(2abc(5cC-4Bd)d^2+\left(2bc^2+ad^2\right)\left(3aCd^2+4b\left(5Cc^2-4Bdc+3Ad^2\right)\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(\cdots\right.}{\cdots}\right.}{4d^3}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{(ad^2+bc^2)\left(\frac{\int \frac{ad\left(a(13cC-8Bd)d^2+4bc\left(5Cc^2-4Bdc+3Ad^2\right)\right)-\left(2abc(5cC-4Bd)d^2+\left(2bc^2+ad^2\right)\left(3aCd^2+4b\left(5Cc^2-4Bdc+3Ad^2\right)\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(8\left(a\cdots\right.\right.}{\cdots}\right.}{4d^3}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$-\frac{(ad^2+bc^2)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2(2cC-Bd)+bc\left(3Ad^2-4Bcd+5c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)\left(3aCd^2+4b\left(3Ad^2-4Bcd+5c^2C\right)\right)+2abcd^2(5cC-\cdots\right.}{d}}{2d^2}\cdots\right.}{4d^3}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$-\frac{(ad^2+bc^2)\left(\frac{\frac{8\left(ad^2+bc^2\right)\left(ad^2(2cC-Bd)+bc\left(3Ad^2-4Bcd+5c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(\left(ad^2+2bc^2\right)\left(3aCd^2+4b\left(3Ad^2-4Bcd+5c^2C\right)\right)+2abcd^2(5cC-\cdots\right.}{d}}{2d^2}\cdots\right.}{4d^3}$$

$$\frac{\left(a+bx^2\right)^{5/2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$-\frac{(ad^2+bc^2)\left(\frac{\frac{8(ad^2+bc^2)(ad^2(2cC-Bd)+bc(3Ad^2-4Bcd+5c^2C))\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)((ad^2+2bc^2)(3aCd^2+4b(3Ad^2-4Bcd+\cdots}{\sqrt{b}d}}{2d^2}\cdots\right.}{4d^3}\cdots$$

$$\frac{(a+bx^2)^{5/2}(Ad^2-Bcd+c^2C)}{d(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 488

$$-\frac{(ad^2+bc^2)\left(\frac{-\frac{8(ad^2+bc^2)(ad^2(2cC-Bd)+bc(3Ad^2-4Bcd+5c^2C))\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)((ad^2+2bc^2)(3aCd^2+4\cdots}{\sqrt{b}\cdots}}{2d^2}\cdots\right.}{4d^3}\cdots$$

$$\frac{(a+bx^2)^{5/2}(Ad^2-Bcd+c^2C)}{d(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 219

$$-\frac{(ad^2+bc^2)\left(\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)((ad^2+2bc^2)(3aCd^2+4b(3Ad^2-4Bcd+5c^2C))+2abcd^2(5cC-4Bd))}{\sqrt{b}d}-\frac{8\sqrt{ad^2+bc^2}\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\cdots\right.}{\cdots}}{2d^2}\cdots\right.}{4d^3}\cdots$$

$$\frac{(a+bx^2)^{5/2}(Ad^2-Bcd+c^2C)}{d(c+dx)(ad^2+bc^2)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x]
```

output
```
-(((c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(5/2))/(d*(b*c^2 + a*d^2)*(c + d*x)
)) + (-1/12*((4*(a*d^2*(2*c*C - B*d) + b*c*(5*c^2*C - 4*B*c*d + 3*A*d^2))
- 3*d*(a*C*d^2 + b*(5*c^2*C - 4*B*c*d + 4*A*d^2))*x)*(a + b*x^2)^(3/2))/d^
3 - ((b*c^2 + a*d^2)*(((8*(a*d^2*(2*c*C - B*d) + b*c*(5*c^2*C - 4*B*c*d +
3*A*d^2)) - d*(20*b*c^2*C - 16*b*B*c*d + 12*A*b*d^2 + 3*a*C*d^2)*x)*Sqrt[a
 + b*x^2])/(2*d^2) + (-(((2*a*b*c*d^2*(5*c*C - 4*B*d) + (2*b*c^2 + a*d^2)*
(3*a*C*d^2 + 4*b*(5*c^2*C - 4*B*c*d + 3*A*d^2)))*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(Sqrt[b]*d)) - (8*Sqrt[b*c^2 + a*d^2]*(a*d^2*(2*c*C - B*d) +
b*c*(5*c^2*C - 4*B*c*d + 3*A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d
^2]*Sqrt[a + b*x^2])])/d)/(2*d^2)))/(4*d^3))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(359) = 718$.

Time = 0.24 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{(-6C\,d^3b\,x^3-8Bb\,d^3x^2+16Cbc\,d^2x^2-12Ab\,d^3x+24Bbc\,d^2x-15Ca\,d^3x-36Cb\,c^2dx+48Abc\,d^2-32Ba\,d^3-72Bb\,c^2d+64Cac\,d^2+}{24d^5}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*C*b*d^3*x^3-8*B*b*d^3*x^2+16*C*b*c*d^2*x^2-12*A*b*d^3*x+24*B*b*c
*d^2*x-15*C*a*d^3*x-36*C*b*c^2*d*x+48*A*b*c*d^2-32*B*a*d^3-72*B*b*c^2*d+64
*C*a*c*d^2+96*C*b*c^3)*(b*x^2+a)^(1/2)/d^5+1/8/d^5*((12*A*a*b*d^4+24*A*b^2
*c^2*d^2-24*B*a*b*c*d^3-32*B*b^2*c^3*d+3*C*a^2*d^4+36*C*a*b*c^2*d^2+40*C*b
^2*c^4)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+8/d^2*(4*A*a*b*c*d^4+4*A*b
^2*c^3*d^2-B*a^2*d^5-6*B*a*b*c^2*d^3-5*B*b^2*c^4*d+2*C*a^2*c*d^4+8*C*a*b*c
^3*d^2+6*C*b^2*c^5)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*
c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2))/(x+c/d))+8*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B
*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2
*c^6)/d^3*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^
2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a
*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2
*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(b*x^2 + a)^(3/2)*C*c^2/(d^4*x + c*d^3) + (b*x^2 + a)^(3/2)*B*c/(d^3*x +
c*d^2) - (b*x^2 + a)^(3/2)*A/(d^2*x + c*d) + 5/2*sqrt(b*x^2 + a)*C*b*c^2*x
/d^4 - 2*sqrt(b*x^2 + a)*B*b*c*x/d^3 + 1/4*(b*x^2 + a)^(3/2)*C*x/d^2 + 3/8
*sqrt(b*x^2 + a)*C*a*x/d^2 + 3/2*sqrt(b*x^2 + a)*A*b*x/d^2 + 5*C*b^(3/2)*c
^4*arcsinh(b*x/sqrt(a*b))/d^6 - 4*B*b^(3/2)*c^3*arcsinh(b*x/sqrt(a*b))/d^5
 + 9/2*C*a*sqrt(b)*c^2*arcsinh(b*x/sqrt(a*b))/d^4 + 3*A*b^(3/2)*c^2*arcsin
h(b*x/sqrt(a*b))/d^4 - 3*B*a*sqrt(b)*c*arcsinh(b*x/sqrt(a*b))/d^3 + 3/8*C*
a^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqr
t(a*b))/d^2 - 3*C*sqrt(a + b*c^2/d^2)*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d
*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^5 + 3*B*sqrt(a + b*c^2/d^2)*b*c
^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/
d^4 - 2*C*(a + b*c^2/d^2)^(3/2)*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
 a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 - 3*A*sqrt(a + b*c^2/d^2)*b*c*arcsinh(b
*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/d^3 + B*(a +
 b*c^2/d^2)^(3/2)*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*
abs(d*x + c)))/d^2 - 5*sqrt(b*x^2 + a)*C*b*c^3/d^5 + 4*sqrt(b*x^2 + a)*B*b
*c^2/d^4 - 2/3*(b*x^2 + a)^(3/2)*C*c/d^3 - 2*sqrt(b*x^2 + a)*C*a*c/d^3 - 3
*sqrt(b*x^2 + a)*A*b*c/d^3 + 1/3*(b*x^2 + a)^(3/2)*B/d^2 + sqrt(b*x^2 + a)
*B*a/d^2
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2,x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1949, normalized size of antiderivative = 4.98

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^2,x)
```

output

```
(144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b**2*c**2*d**2 + 144*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x - 48*sqrt(a*
d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a*b**2*c*d**3 - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a*b**2*d**4*x + 96*sqrt(a*d**2 + b*c**2)*log
( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 +
96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*c**2*d**3*x - 192*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d - 192*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3
*c**2*d**2*x + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*b**2*c**5 + 240*sqrt(a*d**2 + b*c**2)*log( - s
qrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x - 144*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2 - 144*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a*b**2*c*d**3*x + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*
a*b**2*c*d**3 + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*d**4*x - 96*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2 - 96*sqrt(a*d**2 + b*c**2)
*log(c + d*x)*a*b*c**2*d**3*x + 192*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**
3*c**3*d + 192*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**2*d**2*x - 24...
```

3.89 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 350

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2} \, dx = \frac{(4aCd^2+3b(3c^2C-2Bcd+Ad^2))\sqrt{a+bx^2}}{3d^4} - \frac{b(2cC-Bd)x\sqrt{a+bx^2}}{2d^3} + \frac{bCx^2\sqrt{a+bx^2}}{3d^2} + \frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^4(c+dx)} - \frac{\sqrt{b}(3ad^2(2cC-Bd)+2bc(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^5} - \frac{\sqrt{bc^2+ad^2}(ad^2(c^2C-Ad^2)+bc^2(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2d^5} - \frac{a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}$$

output
```
1/3*(4*a*C*d^2+3*b*(A*d^2-2*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2)/d^4-1/2*b*(-B*
d+2*C*c)*x*(b*x^2+a)^(1/2)/d^3+1/3*b*C*x^2*(b*x^2+a)^(1/2)/d^2+(a*d^2+b*c^
2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^4/(d*x+c)-1/2*b^(1/2)*(3*a*d^2*
(-B*d+2*C*c)+2*b*c*(2*A*d^2-3*B*c*d+4*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/d^5-(a*d^2+b*c^2)^(1/2)*(a*d^2*(-A*d^2+C*c^2)+b*c^2*(2*A*d^2-3*B*c*
d+4*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2/
d^5-a^(3/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^2
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \frac{\sqrt{a+bx^2}(2ad^2(7c^2C-3Bcd+3Ad^2+4cCdx)+bc(24c^3C-6c^2d(\ldots}{6cd^4(c\ldots}$$
$$+\frac{2\sqrt{-bc^2-ad^2}(ad^2(c^2C-Ad^2)+bc^2(4c^2C-3Bcd+2Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c^2d^5}$$
$$+\frac{2a^{3/2}A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}$$
$$+\frac{\sqrt{b}(-3ad^2(-2cC+Bd)+2bc(4c^2C-3Bcd+2Ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2d^5}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2),x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*d^2*(7*c^2*C - 3*B*c*d + 3*A*d^2 + 4*c*C*d*x) + b*c*
(24*c^3*C - 6*c^2*d*(3*B - 2*C*x) + c*d^2*(12*A - 9*B*x - 4*C*x^2) + d^3*x
*(6*A + 3*B*x + 2*C*x^2))))/(6*c*d^4*(c + d*x)) + (2*Sqrt[-(b*c^2) - a*d^2
]*(a*d^2*(c^2*C - A*d^2) + b*c^2*(4*c^2*C - 3*B*c*d + 2*A*d^2))*ArcTan[(Sq
rt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(c^2*d^5) +
(2*a^(3/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/c^2 + (Sqrt[b
]*(-3*a*d^2*(-2*c*C + B*d) + 2*b*c*(4*c^2*C - 3*B*c*d + 2*A*d^2))*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(2*d^5)
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.50, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2351, 606, 606, 243, 73, 221, 681, 27, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{\left(bx^2+a\right)^{3/2}}{x(c+dx)^2}dx+\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{606}$$

$$A\left(\frac{a\int \frac{\sqrt{bx^2+a}}{x(c+dx)}dx}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{c}\right)+\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{606}$$

$$A\left(\frac{a\left(\frac{a\int \frac{1}{x\sqrt{bx^2+a}}dx}{c}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{c}\right)+$$
$$\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{243}$$

$$A\left(\frac{a\left(\frac{a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2c}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{c}\right)+$$
$$\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{73}$$

$$A\left(\frac{a\left(\frac{a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{bc}-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}\right)}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{c}\right)+$$
$$\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{221}$$

$$A\left(\frac{a\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{\int \frac{(ad-bcx)\sqrt{bx^2+a}}{(c+dx)^2}dx}{c}\right)+$$

$$\int \frac{(B+Cx)\left(bx^2+a\right)^{3/2}}{(c+dx)^2}dx$$

$$\Big\downarrow \text{681}$$

$$A\left(\frac{a\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{\int \frac{2b\left(acd-\left(2bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+2bc^2+bcdx\right)}{d^2(c+dx)}}{c}\right)-$$

$$\frac{\int -\frac{2(aCd-b(4cC-3Bd)x)\sqrt{bx^2+a}}{c+dx}dx}{2d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$A\left(\frac{a\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{b\int \frac{acd-\left(2bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+2bc^2+bcdx\right)}{d^2(c+dx)}}{c}\right)+$$

$$\frac{\int \frac{(aCd-b(4cC-3Bd)x)\sqrt{bx^2+a}}{c+dx}dx}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \text{682}$$

$$A\left(\frac{a\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{b\int \frac{acd-\left(2bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+2bc^2+bcdx\right)}{d^2(c+dx)}}{c}\right)+$$

$$\frac{\frac{\int \frac{b\left(ad\left(2aCd^2+bc(4cC-3Bd)\right)-b\left(2acCd^2+(4cC-3Bd)\left(2bc^2+ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^2}+\frac{\sqrt{a+bx^2}\left(2\left(aCd^2+bc(4cC-3Bd)\right)-bdx(4cC-3Bd)\right)}{2d^2}}{d^2}+$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \; 27$$

$$A\left(\frac{a\left(-\frac{\int \frac{ad-bcx}{(c+dx)\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{b\int \frac{acd-\left(2bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{\sqrt{a+bx^2}\left(ad^2+2bc^2+bcdx\right)}{d^2(c+dx)}}{c}\right)+\frac{\frac{\int \frac{ad\left(2aCd^2+bc(4cC-3Bd)\right)-b\left(2acCd^2+(4cC-3Bd)\left(2bc^2+ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aCd^2+bc(4cC-3Bd)\right)-bdx(4cC-3Bd)\right)}{2d^2}}{d^2}+\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \; 719$$

$$A\left(\frac{a\left(-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{b\left(\frac{2c\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+2bc\cdots\right.}{\cdots}\cdots\right.}{d^2}\cdots}{c\cdots}\right.$$
$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aCd^2+bc(4cC-3Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(\left(ad^2+2bc^2\right)(4cC-3Bd)+2acCd^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aCd^2+bc(4cC-3Bd)\right)-bdx\cdots\right.}{2d^2}\cdots}{d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$$\Big\downarrow \; 224$$

$$A\left(\frac{a\left(-\frac{\frac{\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{bc\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{-\frac{b\left(\frac{2c\left(ad^2+bc^2\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\cdots\right.}{d^2\cdots}}{\cdots}\right.$$
$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aCd^2+bc(4cC-3Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{b\left(\left(ad^2+2bc^2\right)(4cC-3Bd)+2acCd^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aCd^2+bc(4cC-3B\cdots\right.\right.}{2d^2}\cdots}{d^2}$$
$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$\downarrow$ 219

$$A\left(\frac{a\left(-\frac{\frac{\left(ad^2+bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{b\left(\frac{2c\left(ad^2+bc^2\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\ldots\right.}{\ldots}\right.$$

$$\frac{\frac{\frac{2\left(ad^2+bc^2\right)\left(aCd^2+bc(4cC-3Bd)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4cC-3Bd)+2acCd^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aCd^2+bc(4cC-3\ldots\right.\right.}{2d^2}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$\downarrow$ 488

$$A\left(\frac{a\left(-\frac{-\frac{\left(ad^2+bc^2\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{b\left(-\frac{2c\left(ad^2+bc^2\right)\int\ldots}{\ldots}\right.}{\ldots}\right.$$

$$\frac{\frac{-\frac{2\left(ad^2+bc^2\right)\left(aCd^2+bc(4cC-3Bd)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4cC-3Bd)+2acCd^2\right)}{d}}{2d^2}+\frac{\sqrt{a+bx^2}\left(2\left(aC\ldots\right.\right.}{\ldots}}{d^2}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

$\downarrow$ 219

$$A\left(\frac{a\left(-\frac{-\frac{\sqrt{ad^2+bc^2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d}-\frac{\sqrt{b}c\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d}}{c}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c}\right)}{c}-\frac{b\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}\cdots}{\sqrt{a+\cdots}}\right)}{\sqrt{b}\cdots}\right.}{\cdots}\right.$$

$$\frac{\frac{-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(\left(ad^2+2bc^2\right)(4cC-3Bd)+2acCd^2\right)}{d}-\frac{2\sqrt{ad^2+bc^2}\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(aCd^2+bc(4cC-3Bd)\right)}{d}}{2d^2}}{d^2}+\frac{\sqrt{a+bx^2}(2(aC\cdots}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{3/2}(-3Bd+4cC+Cdx)}{3d^2(c+dx)}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2),x]
```

output
```
((4*c*C - 3*B*d + C*d*x)*(a + b*x^2)^(3/2))/(3*d^2*(c + d*x)) + (((2*(a*C*
d^2 + b*c*(4*c*C - 3*B*d)) - b*d*(4*c*C - 3*B*d)*x)*Sqrt[a + b*x^2])/(2*d^
2) + (-((Sqrt[b]*(2*a*c*C*d^2 + (4*c*C - 3*B*d)*(2*b*c^2 + a*d^2))*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - (2*Sqrt[b*c^2 + a*d^2]*(a*C*d^2 + b*c*
(4*c*C - 3*B*d))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
]])/d)/(2*d^2))/d^2 + A*(-((-(((2*b*c^2 + a*d^2 + b*c*d*x)*Sqrt[a + b*x^2
])/(d^2*(c + d*x))) - (b*(-(((2*b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a
+ b*x^2]])/(Sqrt[b]*d)) - (2*c*Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*d - b*c*x)/(
Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d))/d^2)/c) + (a*(-((-((Sqrt[b]*c*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/d) - (Sqrt[b*c^2 + a*d^2]*ArcTanh[(a*
d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/d)/c) - (Sqrt[a]*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/c))/c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 606
```
Int[(((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :
> Simp[a/c   Int[(c + d*x)^(n + 1)*((a + b*x^2)^(p - 1)/x), x], x] - Simp[1
/c   Int[(c + d*x)^n*(a*d - b*c*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && GtQ[p, 0] && ILtQ[n, 0]
```

rule 681
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Simp[p/
(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Sim
p[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x]
, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -1] ||
EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2
*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1460 vs. $2(316) = 632$.

Time = 0.23 (sec) , antiderivative size = 1461, normalized size of antiderivative = 4.17

method	result	size
default	Expression too large to display	1461

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
A/c^2*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)))-(A*d^2-C*c^2)/c^2/d^2*(1/3*(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)-2*b*c/d)/b*(b*(x+c/d)^2
-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b*(a*d^2+b*c^2)/d^2-4*b^2
*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/
d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-(a*d^2+b*c^2)/d^2/((a*
d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*
c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+
c/d))))-(A*d^2-B*c*d+C*c^2)/d^3/c*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(5/2)-3*b*c*d/(a*d^2+b*c^2)*(1/3*(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(3/2)-b*c/d*(1/4*(2*b*(x+c/d)
-2*b*c/d)/b*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+1/8*(4*b
*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/b^(3/2)*ln((-b*c/d+b*(x+c/d))/b^(1/2)+(b
*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)))+(a*d^2+b*c^2)/d^2*((
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b^(1/2)*c/d*ln((-b*c/
d+b*(x+c/d))/b^(1/2)+(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
)-(a*d^2+b*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*
c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\,(A+Bx+Cx^2)}{x\,(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)^2x}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1681, normalized size of antiderivative = 4.80

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^2,x)
```

output
```
(12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*c*d**4 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x - 24*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*
d**2 - 24*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*a*b*c**2*d**3*x - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d**2 - 12*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a*c**3*d**3*x + 36*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d + 36*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d**2*x -
48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*b*c**6 - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**5*d*x - 12*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**2*c*d**4 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**5*x
+ 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2 + 24*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b*c**2*d**3*x + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a*c**4*d**2 + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**3*d**3*x - 36*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4*d - 36*sqrt(a*d**2 + b*c**2)*l
og(c + d*x)*b**2*c**3*d**2*x + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*...
```

3.90 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 359

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx &= -\frac{b(2cC-Bd)\sqrt{a+bx^2}}{d^3} \\
&- \frac{(bc^2(c^2C-Bcd+Ad^2)+ad^2(c^2C-Bcd+2Ad^2))\sqrt{a+bx^2}}{c^2d^4x} \\
&+ \frac{bCx\sqrt{a+bx^2}}{2d^2} + \frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^4x(c+dx)} \\
&+ \frac{\sqrt{b}(3aCd^2+2b(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2d^4} \\
&+ \frac{\sqrt{bc^2+ad^2}(ad^3(Bc-2Ad)+bc^2(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3d^4} \\
&- \frac{a^{3/2}(Bc-2Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^3}
\end{aligned}$$

output
```
-b*(-B*d+2*C*c)*(b*x^2+a)^(1/2)/d^3-(b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(2*A*
d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/c^2/d^4/x+1/2*b*C*x*(b*x^2+a)^(1/2)/d^2+
(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^4/x/(d*x+c)+1/2*b^(1
/2)*(3*a*C*d^2+2*b*(A*d^2-2*B*c*d+3*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/d^4+(a*d^2+b*c^2)^(1/2)*(a*d^3*(-2*A*d+B*c)+b*c^2*(A*d^2-2*B*c*d+3*C*
c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^3/d^4-a^
(3/2)*(-2*A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^3
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx =$$
$$-\frac{\sqrt{a+bx^2}(2ad^2(c(cC-Bd)x+Ad(c+2dx))+bc^2x(6c^2C+c(-4Bd+3Cdx)+d^2(2A-x(2B+Cx))}{2c^2d^3x(c+dx)}$$
$$-\frac{2\sqrt{-bc^2-ad^2}(ad^3(Bc-2Ad)+bc^2(3c^2C-2Bcd+Ad^2))\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{c^3d^4}$$
$$+\frac{\sqrt{b}(3aCd^2+2b(3c^2C-2Bcd+Ad^2))\,\mathrm{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{d^4}$$
$$-\frac{a^{3/2}(Bc-2Ad)\log(x)}{c^3}+\frac{a^{3/2}(Bc-2Ad)\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c^3}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x]
```

output
```
-1/2*(Sqrt[a + b*x^2]*(2*a*d^2*(c*(c*C - B*d)*x + A*d*(c + 2*d*x)) + b*c^2
*x*(6*c^2*C + c*(-4*B*d + 3*C*d*x) + d^2*(2*A - x*(2*B + C*x)))))/(c^2*d^3
*x*(c + d*x)) - (2*Sqrt[-(b*c^2) - a*d^2]*(a*d^3*(B*c - 2*A*d) + b*c^2*(3*
c^2*C - 2*B*c*d + A*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c +
d*x) - c*Sqrt[a + b*x^2])])/(c^3*d^4) + (Sqrt[b]*(3*a*C*d^2 + 2*b*(3*c^2*C
 - 2*B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/d^
4 - (a^(3/2)*(B*c - 2*A*d)*Log[x])/c^3 + (a^(3/2)*(B*c - 2*A*d)*Log[-Sqrt[
a] + Sqrt[a + b*x^2]])/c^3
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{c^3x}-\frac{d(a+bx^2)^{3/2}(Bc-2Ad)}{c^3(c+dx)}+\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^2(c+dx)^2}+\frac{A(a+bx^2)^{3/2}}{c^2x^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{c^3}+\frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2c^2d^4}+\frac{3b\sqrt{ad^2+bc^2}(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{cd^4}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(Bc-2Ad)}{2c^2d^3}+\frac{(ad^2+bc^2)^{3/2}(Bc-2Ad)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3d^3}+\frac{3aA\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2c^2}+\frac{a\sqrt{a+bx^2}(Bc-2Ad)}{c^3}-\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^2d(c+dx)}-\frac{3b\sqrt{a+bx^2}(2c-dx)(Ad^2-Bcd+c^2C)}{2c^2d^3}-\frac{\sqrt{a+bx^2}(Bc-2Ad)(2(ad^2+bc^2)-bcdx)}{2c^3d^2}-\frac{A(a+bx^2)^{3/2}}{c^2x}+\frac{3Abx\sqrt{a+bx^2}}{2c^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x]
```

output
```
(a*(B*c - 2*A*d)*Sqrt[a + b*x^2])/c^3 + (3*A*b*x*Sqrt[a + b*x^2])/(2*c^2)
- (3*b*(c^2*C - B*c*d + A*d^2)*(2*c - d*x)*Sqrt[a + b*x^2])/(2*c^2*d^3) -
((B*c - 2*A*d)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^3*d^2)
- (A*(a + b*x^2)^(3/2))/(c^2*x) - ((c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/
2))/(c^2*d*(c + d*x)) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(2*c^2) + (Sqrt[b]*(B*c - 2*A*d)*(2*b*c^2 + 3*a*d^2)*ArcTanh[(Sqrt[b]*x
)/Sqrt[a + b*x^2]])/(2*c^2*d^3) + (3*Sqrt[b]*(2*b*c^2 + a*d^2)*(c^2*C - B*
c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^2*d^4) + ((B*c - 2
*A*d)*(b*c^2 + a*d^2)^(3/2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqr
t[a + b*x^2])])/(c^3*d^3) + (3*b*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^
2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c*d^4) -
 (a^(3/2)*(B*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(329) = 658$.

Time = 0.33 (sec) , antiderivative size = 744, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{aA\sqrt{b\,x^2+a}}{c^2x}+\frac{\frac{\left(A\,a^2d^6+2Aab\,c^2d^4+A\,b^2c^4d^2-B\,a^2c\,d^5-2Bab\,c^3d^3-c^5B\,b^2d+C\,a^2c^2d^4+2Cab\,c^4d^2+c^6C\,b^2\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x\right.}{c}}}{\left(a\,d^2+b\,c^2\right)}\right.}{d^6}}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a*A/c^2*(b*x^2+a)^(1/2)/x+1/c^2*(1/d^6*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c
^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d
^2+C*b^2*c^6)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(
a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((
2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+b*c^2/d^4*(d*(B*d-2
*C*c)*(b*x^2+a)^(1/2)+A*b^(1/2)*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+C*b*d^2*
(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+3*C*
b^(1/2)*c^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+2*a*C*d^2*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))/b^(1/2)-2*B*b^(1/2)*c*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-1/d^5*(2*A*
a^2*d^6-2*A*b^2*c^4*d^2-B*a^2*c*d^5+2*B*a*b*c^3*d^3+3*B*b^2*c^5*d-4*C*a*b*
c^4*d^2-4*C*b^2*c^6)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2
*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+a^(3/2)/c*(2*A*d-B*c)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**2*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)^2x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)^2*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^2\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1680, normalized size of antiderivative = 4.68

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^2(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^2,x)
```

output

```
(8*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*c*d**4*x + 8*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x**2 - 4*sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2*x
- 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b*c**2*d**3*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**3*x - 4*sqrt(a*d**2 + b
*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**
4*x**2 + 8*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2
) - a*d + b*c*x)*b**2*c**4*d*x + 8*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d**2*x**2 - 12*sqrt(a*d
**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*
c**6*x - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*b*c**5*d*x**2 - 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**
2*c*d**4*x - 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**5*x**2 + 4*sqrt(
a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2*x + 4*sqrt(a*d**2 + b*c**2)*lo
g(c + d*x)*a*b*c**2*d**3*x**2 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c
**2*d**3*x + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**4*x**2 - 8*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4*d*x - 8*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*b**2*c**3*d**2*x**2 + 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b...
```

3.91 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 402

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx &= \frac{bC\sqrt{a+bx^2}}{d^2} \\
&- \frac{(2bc^2(c^2C-Bcd+Ad^2)+ad^2(2c^2C-2Bcd+3Ad^2))\sqrt{a+bx^2}}{2c^2d^4x^2} \\
&+ \frac{(bc^2(c^2C-Bcd+Ad^2)+ad^2(c^2C-2Bcd+3Ad^2))\sqrt{a+bx^2}}{c^3d^3x} \\
&+ \frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^4x^2(c+dx)} - \frac{b^{3/2}(2cC-Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^3} \\
&- \frac{\sqrt{bc^2+ad^2}(bc^3(2cC-Bd)-ad^2(c^2C-2Bcd+3Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4d^3} \\
&- \frac{\sqrt{a}(2ac(cC-2Bd)+3A(bc^2+2ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c^4}
\end{aligned}$$

output
```
b*C*(b*x^2+a)^(1/2)/d^2-1/2*(2*b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(3*A*d^2-2*
B*c*d+2*C*c^2))*(b*x^2+a)^(1/2)/c^2/d^4/x^2+(b*c^2*(A*d^2-B*c*d+C*c^2)+a*d
^2*(3*A*d^2-2*B*c*d+C*c^2))*(b*x^2+a)^(1/2)/c^3/d^3/x+(a*d^2+b*c^2)*(A*d^2
-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^4/x^2/(d*x+c)-b^(3/2)*(-B*d+2*C*c)*arcta
nh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^3-(a*d^2+b*c^2)^(1/2)*(b*c^3*(-B*d+2*C*c)-
a*d^2*(3*A*d^2-2*B*c*d+C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b
*x^2+a)^(1/2))/c^4/d^3-1/2*a^(1/2)*(2*a*c*(-2*B*d+C*c)+3*A*(2*a*d^2+b*c^2)
)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^4
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \frac{\sqrt{a+bx^2}(2bc^2x^2(2c^2C-Bcd+Ad^2+cCdx)+ad^2(A(-c^2+3cdx}{2c^3d^2x^2(c+dx)}$$
$$+\frac{2\sqrt{-bc^2-ad^2}(bc^3(2cC-Bd)-ad^2(c^2C-2Bcd+3Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c^4d^3}$$
$$+\frac{3\sqrt{a}Ab\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^2}-\frac{2a^{3/2}(c^2C-2Bcd+3Ad^2)\,\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{c^4}$$
$$+\frac{b^{3/2}(2cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{d^3}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x]
```

output
```
(Sqrt[a + b*x^2]*(2*b*c^2*x^2*(2*c^2*C - B*c*d + A*d^2 + c*C*d*x) + a*d^2*
(A*(-c^2 + 3*c*d*x + 6*d^2*x^2) + 2*c*x*(c*C*x - B*(c + 2*d*x)))))/(2*c^3*
d^2*x^2*(c + d*x)) + (2*Sqrt[-(b*c^2) - a*d^2]*(b*c^3*(2*c*C - B*d) - a*d^
2*(c^2*C - 2*B*c*d + 3*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^
2])/Sqrt[-(b*c^2) - a*d^2]])/(c^4*d^3) + (3*Sqrt[a]*A*b*ArcTanh[(Sqrt[b]*x
 - Sqrt[a + b*x^2])/Sqrt[a]])/c^2 - (2*a^(3/2)*(c^2*C - 2*B*c*d + 3*A*d^2)
*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/c^4 + (b^(3/2)*(2*c*C
- B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/d^3
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{c^3x^2}+\frac{(a+bx^2)^{3/2}(3Ad^2-2Bcd+c^2C)}{c^4x}-\frac{d(a+bx^2)^{3/2}(3Ad^2-2Bcd+c^2C)}{c^4(c+dx)}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3Ad^2-2Bcd+c^2C)}{c^4}+\frac{3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bc-2Ad)}{2c^3}-\frac{3b\sqrt{ad^2+bc^2}(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2d^3}+\frac{(ad^2+bc^2)^{3/2}(3Ad^2-2Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4d^3}-\frac{3\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+2bc^2)(Ad^2-Bcd+c^2C)}{2c^3d^3}+\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3ad^2+2bc^2)(3Ad^2-2Bcd+c^2C)}{2c^3d^3}-\frac{3\sqrt{a}Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c^2}-\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{c^3x}+\frac{3bx\sqrt{a+bx^2}(Bc-2Ad)}{2c^3}+\frac{a\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{c^4}-\frac{\sqrt{a+bx^2}(2(ad^2+bc^2)-bcdx)(3Ad^2-2Bcd+c^2C)}{2c^4d^2}+\frac{(a+bx^2)^{3/2}(Ad^2-Bcd+c^2C)}{c^3(c+dx)}+\frac{3b\sqrt{a+bx^2}(2c-dx)(Ad^2-Bcd+c^2C)}{2c^3d^2}-\frac{A(a+bx^2)^{3/2}}{2c^2x^2}+\frac{3Ab\sqrt{a+bx^2}}{2c^2}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x]
```

output
```
(3*A*b*Sqrt[a + b*x^2])/(2*c^2) + (a*(c^2*C - 2*B*c*d + 3*A*d^2)*Sqrt[a +
b*x^2])/c^4 + (3*b*(B*c - 2*A*d)*x*Sqrt[a + b*x^2])/(2*c^3) + (3*b*(c^2*C
- B*c*d + A*d^2)*(2*c - d*x)*Sqrt[a + b*x^2])/(2*c^3*d^2) - ((c^2*C - 2*B*
c*d + 3*A*d^2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^4*d^2)
- (A*(a + b*x^2)^(3/2))/(2*c^2*x^2) - ((B*c - 2*A*d)*(a + b*x^2)^(3/2))/(c
^3*x) + ((c^2*C - B*c*d + A*d^2)*(a + b*x^2)^(3/2))/(c^3*(c + d*x)) + (3*a
*Sqrt[b]*(B*c - 2*A*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^3) - (3*
Sqrt[b]*(2*b*c^2 + a*d^2)*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*c^3*d^3) + (Sqrt[b]*(2*b*c^2 + 3*a*d^2)*(c^2*C - 2*B*c*d
+ 3*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^3*d^3) - (3*b*Sqrt[b
*c^2 + a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 +
a*d^2]*Sqrt[a + b*x^2])])/(c^2*d^3) + ((b*c^2 + a*d^2)^(3/2)*(c^2*C - 2*B*
c*d + 3*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])
])/(c^4*d^3) - (3*Sqrt[a]*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^2) -
(a^(3/2)*(c^2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 690, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,(-4Adx+2Bcx+Ac)}{2c^3x^2}-\frac{\frac{2\left(A\,a^2d^6+2Aab\,c^2d^4+A\,b^2c^4d^2-B\,a^2c\,d^5-2Bab\,c^3d^3-c^5B\,b^2d+C\,a^2c^2d^4+2Cab\,c^4d^2+c^6C\,b^2\right)\left(\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(b*x^2+a)^(1/2)*(-4*A*d*x+2*B*c*x+A*c)/c^3/x^2-1/2/c^3*(2*(A*a^2*d^
6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*
a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^5*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/(
(a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2
+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/
(x+c/d)))-2/d^4*(3*A*a^2*d^6+2*A*a*b*c^2*d^4-A*b^2*c^4*d^2-2*B*a^2*c*d^5+2
*B*b^2*c^5*d+C*a^2*c^2*d^4-2*C*a*b*c^4*d^2-3*C*b^2*c^6)/c/((a*d^2+b*c^2)/d
^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1
/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+a^(1/2
)/c*(6*A*a*d^2+3*A*b*c^2-4*B*a*c*d+2*C*a*c^2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^
(1/2))/x)-2*b^2*c^3/d^3*(B*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+C*d*(b*
x^2+a)^(1/2)/b-2*C*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**3/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**3*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)^2x^3}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)^2*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:16.7888 interp horner, loop index 0
16.7889 interp resultant evaled at -3, 0% done22.132 interp dd 22.2036 int
erp build
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx=\int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^3\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.30

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^3(c+dx)^2}\,dx=\text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^2,x)
```

output

```
(12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*c*d**4*x**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x**3 - 8*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
b*c**2*d**3*x**2 - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*
d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**4*x**3 + 4*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d**2*x**
2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a*c**3*d**3*x**3 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x**2 + 4*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*b**2*c**3*d**2*x**3 - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**6*x**2 - 8*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**5*d*x**3 -
 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*c*d**4*x**2 - 12*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a**2*d**5*x**3 + 8*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a*b*c**2*d**3*x**2 + 8*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**4*x**
3 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**4*d**2*x**2 - 4*sqrt(a*d**2
+ b*c**2)*log(c + d*x)*a*c**3*d**3*x**3 - 4*sqrt(a*d**2 + b*c**2)*log(c +
d*x)*b**2*c**4*d*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**3*...
```

3.92 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 466

$$\begin{aligned}
\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = & -\frac{(3bc^2(c^2C-Bcd+Ad^2)+ad^2(3c^2C-3Bcd+4Ad^2))\sqrt{a+bx^2}}{3c^2d^4x^3} \\
& +\frac{(2bc^2(c^2C-Bcd+Ad^2)+ad^2(2c^2C-3Bcd+4Ad^2))\sqrt{a+bx^2}}{2c^3d^3x^2} \\
& -\frac{(3ad^2(2c^2C-3Bcd+4Ad^2)+bc^2(3c^2C-3Bcd+7Ad^2))\sqrt{a+bx^2}}{3c^4d^2x} \\
& +\frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^4x^3(c+dx)}+\frac{b^{3/2}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{d^2} \\
& -\frac{\sqrt{bc^2+ad^2}(ad^2(2c^2C-3Bcd+4Ad^2)-b(c^4C-Ac^2d^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5d^2} \\
& -\frac{\sqrt{a}(3bc^2(Bc-2Ad)-2ad(2c^2C-3Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2c^5}
\end{aligned}$$

output
```
-1/3*(3*b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(4*A*d^2-3*B*c*d+3*C*c^2))*(b*x^2+
a)^(1/2)/c^2/d^4/x^3+1/2*(2*b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(4*A*d^2-3*B*c
*d+2*C*c^2))*(b*x^2+a)^(1/2)/c^3/d^3/x^2-1/3*(3*a*d^2*(4*A*d^2-3*B*c*d+2*C
*c^2)+b*c^2*(7*A*d^2-3*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2)/c^4/d^2/x+(a*d^2+b*
c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^4/x^3/(d*x+c)+b^(3/2)*C*arcta
nh(b^(1/2)*x/(b*x^2+a)^(1/2))/d^2-(a*d^2+b*c^2)^(1/2)*(a*d^2*(4*A*d^2-3*B*
c*d+2*C*c^2)-b*(-A*c^2*d^2+C*c^4))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2
)/(b*x^2+a)^(1/2))/c^5/d^2-1/2*a^(1/2)*(3*b*c^2*(-2*A*d+B*c)-2*a*d*(4*A*d^
2-3*B*c*d+2*C*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/c^5
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx=\frac{1}{6}\left(-\frac{\sqrt{a+bx^2}(2bc^2x^2(3c(cC-Bd)x+Ad(4c+7dx))+ad(2A(c^3\right.$$
$$-\frac{12\sqrt{-bc^2-ad^2}(ad^2(-2c^2C+3Bcd-4Ad^2)+b(c^4C-Ac^2d^2))\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{c^5d^2}$$
$$+\frac{12b^{3/2}C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{d^2}$$
$$+\frac{3\sqrt{a}(-3bc^2(Bc-2Ad)+2ad(2c^2C-3Bcd+4Ad^2))\log(x)}{c^5}$$
$$\left.+\frac{3\sqrt{a}(3bc^2(Bc-2Ad)-2ad(2c^2C-3Bcd+4Ad^2))\log\left(-\sqrt{a}+\sqrt{a+bx^2}\right)}{c^5}\right)$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x]
```

output
```
(-((Sqrt[a + b*x^2]*(2*b*c^2*x^2*(3*c*(c*C - B*d)*x + A*d*(4*c + 7*d*x)) +
 a*d*(2*A*(c^3 - 2*c^2*d*x + 6*c*d^2*x^2 + 12*d^3*x^3) + 3*c*x*(2*c*C*x*(c
 + 2*d*x) + B*(c^2 - 3*c*d*x - 6*d^2*x^2)))))/(c^4*d*x^3*(c + d*x))) - (12
*Sqrt[-(b*c^2) - a*d^2]*(a*d^2*(-2*c^2*C + 3*B*c*d - 4*A*d^2) + b*(c^4*C -
 A*c^2*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt
[a + b*x^2])])/(c^5*d^2) + (12*b^(3/2)*C*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + S
qrt[a + b*x^2])])/d^2 + (3*Sqrt[a]*(-3*b*c^2*(B*c - 2*A*d) + 2*a*d*(2*c^2*
C - 3*B*c*d + 4*A*d^2))*Log[x])/c^5 + (3*Sqrt[a]*(3*b*c^2*(B*c - 2*A*d) -
2*a*d*(2*c^2*C - 3*B*c*d + 4*A*d^2))*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/c^5)
/6
```

Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 799, normalized size of antiderivative = 1.71, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{c^3x^3}+\frac{d^2(a+bx^2)^{3/2}(4Ad^2-3Bcd+2c^2C)}{c^5(c+dx)}-\frac{d(a+bx^2)^{3/2}(4Ad^2-3Bcd+2c^2C)}{c^5x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(2Cc^2-3Bdc+4Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)a^{3/2}}{c^5}+\frac{3\sqrt{b}(Cc^2-2Bdc+3Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)a}{2c^4}-\frac{d(2Cc^2-3Bdc+4Ad^2)\sqrt{bx^2+a}a}{c^5}-\frac{3b(Bc-2Ad)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)\sqrt{a}}{2c^3}-\frac{(Cc^2-2Bdc+3Ad^2)(bx^2+a)^{3/2}}{c^4x}-\frac{d(Cc^2-Bdc+Ad^2)(bx^2+a)^{3/2}}{c^4(c+dx)}-\frac{(Bc-2Ad)(bx^2+a)^{3/2}}{2c^3x^2}-\frac{A(bx^2+a)^{3/2}}{3c^2x^3}+\frac{3\sqrt{b}(2bc^2+ad^2)(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2c^4d^2}-\frac{\sqrt{b}(2bc^2+3ad^2)(2Cc^2-3Bdc+4Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{2c^4d^2}+\frac{Ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)}{c^2}+\frac{3b\sqrt{bc^2+ad^2}(Cc^2-Bdc+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{bx^2+a}}\right)}{c^3d^2}-\frac{(bc^2+ad^2)^{3/2}(2Cc^2-3Bdc+4Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{bx^2+a}}\right)}{c^5d^2}+\frac{3b(Bc-2Ad)\sqrt{bx^2+a}}{2c^3}+\frac{3b(Cc^2-2Bdc+3Ad^2)x\sqrt{bx^2+a}}{2c^4}-\frac{3b(Cc^2-Bdc+Ad^2)(2c-dx)\sqrt{bx^2+a}}{2c^4d}+\frac{(2Cc^2-3Bdc+4Ad^2)\left(2\left(bc^2+ad^2\right)-bcdx\right)\sqrt{bx^2+a}}{2c^5d}-\frac{Ab\sqrt{bx^2+a}}{c^2x}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x]
```

output
```
(3*b*(B*c - 2*A*d)*Sqrt[a + b*x^2])/(2*c^3) - (a*d*(2*c^2*C - 3*B*c*d + 4*
A*d^2)*Sqrt[a + b*x^2])/c^5 - (A*b*Sqrt[a + b*x^2])/(c^2*x) + (3*b*(c^2*C
- 2*B*c*d + 3*A*d^2)*x*Sqrt[a + b*x^2])/(2*c^4) - (3*b*(c^2*C - B*c*d + A*
d^2)*(2*c - d*x)*Sqrt[a + b*x^2])/(2*c^4*d) + ((2*c^2*C - 3*B*c*d + 4*A*d^
2)*(2*(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^5*d) - (A*(a + b*x^
2)^(3/2))/(3*c^2*x^3) - ((B*c - 2*A*d)*(a + b*x^2)^(3/2))/(2*c^3*x^2) - ((
c^2*C - 2*B*c*d + 3*A*d^2)*(a + b*x^2)^(3/2))/(c^4*x) - (d*(c^2*C - B*c*d
 + A*d^2)*(a + b*x^2)^(3/2))/(c^4*(c + d*x)) + (A*b^(3/2)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/c^2 + (3*Sqrt[b]*(2*b*c^2 + a*d^2)*(c^2*C - B*c*d + A
*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^4*d^2) + (3*a*Sqrt[b]*(c^
2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^4) - (
Sqrt[b]*(2*b*c^2 + 3*a*d^2)*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(2*c^4*d^2) + (3*b*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*
d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(
c^3*d^2) - ((b*c^2 + a*d^2)^(3/2)*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(a
*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^5*d^2) - (3*Sqrt[a]
*b*(B*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^3) + (a^(3/2)*d*(2
*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/c^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(18Aa\,d^2x^2+8Ab\,c^2x^2-12Bacd\,x^2+6Ca\,c^2x^2-6Aacdx+3Ba\,c^2x+2A\,c^2a)}{6c^4x^3}+\frac{2\left(A\,a^2d^6+2Aab\,c^2d^4+A\,b^2c^4d^2-B\,a^2c\,d\right.}{}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(18*A*a*d^2*x^2+8*A*b*c^2*x^2-12*B*a*c*d*x^2+6*C*a*c^
2*x^2-6*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/c^4/x^3+1/2/c^4*(2*(A*a^2*d^6+2*A
*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3*d^3-B*b^2*c^5*d+C*a^2*c
^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^4*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^
2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^
2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/
d)))-2/d^3*(4*A*a^2*d^6+4*A*a*b*c^2*d^4-3*B*a^2*c*d^5-2*B*a*b*c^3*d^3+B*b^
2*c^5*d+2*C*a^2*c^2*d^4-2*C*b^2*c^6)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*
d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*C*b^(3/2)*c^4/d^2*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))+a^(1/2)/c*(8*A*a*d^3+6*A*b*c^2*d-6*B*a*c*d^2-3*B*
b*c^3+4*C*a*c^2*d)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\,(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**4/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**4*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)^2x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)^2*x^4), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^4\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 11.50 (sec) , antiderivative size = 1779, normalized size of antiderivative = 3.82

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^4(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^2,x)
```

output

```
(48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*c*d**4*x**3 + 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**5*x**4 + 12*sqrt(a*d**2 + b*
c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d
**2*x**3 + 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c
**2) - a*d + b*c*x)*a*b*c**2*d**3*x**4 - 36*sqrt(a*d**2 + b*c**2)*log(sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**3*x**3 - 36*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a*b*c*d**4*x**4 + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d**2*x**3 + 24*sqrt(a*d**2 + b*c**
2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**3*d**3*x
**4 - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*b*c**6*x**3 - 12*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**5*d*x**4 - 48*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a**2*c*d**4*x**3 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*a**2*d**5*x**4 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2*x**3
 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**3*x**4 + 36*sqrt(a*d*
*2 + b*c**2)*log(c + d*x)*a*b*c**2*d**3*x**3 + 36*sqrt(a*d**2 + b*c**2)*lo
g(c + d*x)*a*b*c*d**4*x**4 - 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**4*
d**2*x**3 - 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**3*d**3*x**4 + 12...
```

3.93 $\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 527

$$
\begin{aligned}
&\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \\
&-\frac{(4bc^2(c^2C-Bcd+Ad^2)+ad^2(4c^2C-4Bcd+5Ad^2))\sqrt{a+bx^2}}{4c^2d^4x^4} \\
&+\frac{(3bc^2(c^2C-Bcd+Ad^2)+ad^2(3c^2C-4Bcd+5Ad^2))\sqrt{a+bx^2}}{3c^3d^3x^3} \\
&-\frac{(4ad^2(3c^2C-4Bcd+5Ad^2)+bc^2(8c^2C-8Bcd+13Ad^2))\sqrt{a+bx^2}}{8c^4d^2x^2} \\
&+\frac{(3ad^2(3c^2C-4Bcd+5Ad^2)+bc^2(3c^2C-7Bcd+11Ad^2))\sqrt{a+bx^2}}{3c^5dx} \\
&+\frac{(bc^2+ad^2)(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{cd^4x^4(c+dx)} \\
&-\frac{\sqrt{bc^2+ad^2}(bc^2(Bc-2Ad)-ad(3c^2C-4Bcd+5Ad^2))\,\mathrm{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^6} \\
&-\frac{(A(3b^2c^4+36abc^2d^2+40a^2d^4)+4ac(2ad^2(3cC-4Bd)+3bc^2(cC-2Bd)))\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}c^6}
\end{aligned}
$$

output
```
-1/4*(4*b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(5*A*d^2-4*B*c*d+4*C*c^2))*(b*x^2+
a)^(1/2)/c^2/d^4/x^4+1/3*(3*b*c^2*(A*d^2-B*c*d+C*c^2)+a*d^2*(5*A*d^2-4*B*c
*d+3*C*c^2))*(b*x^2+a)^(1/2)/c^3/d^3/x^3-1/8*(4*a*d^2*(5*A*d^2-4*B*c*d+3*C
*c^2)+b*c^2*(13*A*d^2-8*B*c*d+8*C*c^2))*(b*x^2+a)^(1/2)/c^4/d^2/x^2+1/3*(3
*a*d^2*(5*A*d^2-4*B*c*d+3*C*c^2)+b*c^2*(11*A*d^2-7*B*c*d+3*C*c^2))*(b*x^2+
a)^(1/2)/c^5/d/x+(a*d^2+b*c^2)*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/d^4/x
^4/(d*x+c)-(a*d^2+b*c^2)^(1/2)*(b*c^2*(-2*A*d+B*c)-a*d*(5*A*d^2-4*B*c*d+3*
C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^6-1/8*
(A*(40*a^2*d^4+36*a*b*c^2*d^2+3*b^2*c^4)+4*a*c*(2*a*d^2*(-4*B*d+3*C*c)+3*b
*c^2*(-2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^6
```

Mathematica [A] (verified)

Time = 4.02 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \frac{\frac{c\sqrt{a+bx^2}\left(bc^2x^2\left(8cx(-4Bc+3cCx-7Bdx)+A\left(-15c^2+49cdx+88d^2x^2\right)\right)-2a\left(A\left(3c^4-5c^3dx\cdots\right.\right.\right.}{x^4\cdots}}{\cdots}$$

input
```
Integrate[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x]
```

output
```
((c*Sqrt[a + b*x^2]*(b*c^2*x^2*(8*c*x*(-4*B*c + 3*c*C*x - 7*B*d*x) + A*(-1
5*c^2 + 49*c*d*x + 88*d^2*x^2)) - 2*a*(A*(3*c^4 - 5*c^3*d*x + 10*c^2*d^2*x
^2 - 30*c*d^3*x^3 - 60*d^4*x^4) + 2*c*x*(3*c*C*x*(c^2 - 3*c*d*x - 6*d^2*x^
2) + 2*B*(c^3 - 2*c^2*d*x + 6*c*d^2*x^2 + 12*d^3*x^3)))))/(x^4*(c + d*x))
+ 48*Sqrt[-(b*c^2) - a*d^2]*(b*c^2*(B*c - 2*A*d) + a*d*(-3*c^2*C + 4*B*c*d
 - 5*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2)
- a*d^2]] + 240*a^(3/2)*A*d^4*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a
]] - (6*c*(3*A*b*c*(b*c^2 + 12*a*d^2) + 4*a*(2*a*d^2*(3*c*C - 4*B*d) + 3*b
*c^2*(c*C - 2*B*d)))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sq
rt[a])/(24*c^6)
```

Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.76, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{(a+bx^2)^{3/2}(Bc-2Ad)}{c^3x^4}+\frac{d^2(a+bx^2)^{3/2}(5Ad^2-4Bcd+3c^2C)}{c^6x}-\frac{d^3(a+bx^2)^{3/2}(5Ad^2-4Bcd+3c^2C}{c^6(c+dx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3A\text{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)b^2}{8\sqrt{a}c^2}+\frac{(Bc-2Ad)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)b^{3/2}}{c^3}-\frac{3\sqrt{bc^2+ad^2}(Cc^2-Bdc+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{bx^2+a}}\right)b}{c^4d}-\frac{3\sqrt{a}(Cc^2-2Bdc+3Ad^2)\,\text{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)b}{2c^4}+\frac{3(Cc^2-2Bdc+3Ad^2)\sqrt{bx^2+a}b}{2c^4}-\frac{3d(2Cc^2-3Bdc+4Ad^2)\,x\sqrt{bx^2+a}b}{2c^5}+\frac{3(Cc^2-Bdc+Ad^2)\,(2c-dx)\sqrt{bx^2+a}b}{2c^5}-\frac{(Bc-2Ad)\sqrt{bx^2+a}b}{c^3x}-\frac{3A\sqrt{bx^2+a}b}{8c^2x^2}-\frac{3(2bc^2+ad^2)\,(Cc^2-Bdc+Ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)\sqrt{b}}{2c^5d}-\frac{3ad(2Cc^2-3Bdc+4Ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)\sqrt{b}}{2c^5}+\frac{(2bc^2+3ad^2)\,(3Cc^2-4Bdc+5Ad^2)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+a}}\right)\sqrt{b}}{2c^5d}+\frac{d(2Cc^2-3Bdc+4Ad^2)\,(bx^2+a)^{3/2}}{c^5x}+\frac{d^2(Cc^2-Bdc+Ad^2)\,(bx^2+a)^{3/2}}{c^5(c+dx)}-\frac{(Cc^2-2Bdc+3Ad^2)\,(bx^2+a)^{3/2}}{2c^4x^2}-\frac{(Bc-2Ad)\,(bx^2+a)^{3/2}}{3c^3x^3}-\frac{A(bx^2+a)^{3/2}}{4c^2x^4}+\frac{(bc^2+ad^2)^{3/2}\,(3Cc^2-4Bdc+5Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{bx^2+a}}\right)}{c^6d}-\frac{a^{3/2}d^2(3Cc^2-4Bdc+5Ad^2)\,\text{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)}{c^6}+\frac{ad^2(3Cc^2-4Bdc+5Ad^2)\sqrt{bx^2+a}}{c^6}-\frac{(3Cc^2-4Bdc+5Ad^2)\,(2(bc^2+ad^2)-bcdx)\sqrt{bx^2+a}}{2c^6}$$

input
```
Int[((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x]
```

output
```
(3*b*(c^2*C - 2*B*c*d + 3*A*d^2)*Sqrt[a + b*x^2])/(2*c^4) + (a*d^2*(3*c^2*
C - 4*B*c*d + 5*A*d^2)*Sqrt[a + b*x^2])/c^6 - (3*A*b*Sqrt[a + b*x^2])/(8*c
^2*x^2) - (b*(B*c - 2*A*d)*Sqrt[a + b*x^2])/(c^3*x) - (3*b*d*(2*c^2*C - 3*
B*c*d + 4*A*d^2)*x*Sqrt[a + b*x^2])/(2*c^5) + (3*b*(c^2*C - B*c*d + A*d^2)
*(2*c - d*x)*Sqrt[a + b*x^2])/(2*c^5) - ((3*c^2*C - 4*B*c*d + 5*A*d^2)*(2*
(b*c^2 + a*d^2) - b*c*d*x)*Sqrt[a + b*x^2])/(2*c^6) - (A*(a + b*x^2)^(3/2)
)/(4*c^2*x^4) - ((B*c - 2*A*d)*(a + b*x^2)^(3/2))/(3*c^3*x^3) - ((c^2*C -
2*B*c*d + 3*A*d^2)*(a + b*x^2)^(3/2))/(2*c^4*x^2) + (d*(2*c^2*C - 3*B*c*d
+ 4*A*d^2)*(a + b*x^2)^(3/2))/(c^5*x) + (d^2*(c^2*C - B*c*d + A*d^2)*(a +
b*x^2)^(3/2))/(c^5*(c + d*x)) + (b^(3/2)*(B*c - 2*A*d)*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/c^3 - (3*Sqrt[b]*(2*b*c^2 + a*d^2)*(c^2*C - B*c*d + A*d
^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^5*d) - (3*a*Sqrt[b]*d*(2*c^
2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*c^5) + (
Sqrt[b]*(2*b*c^2 + 3*a*d^2)*(3*c^2*C - 4*B*c*d + 5*A*d^2)*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(2*c^5*d) - (3*b*Sqrt[b*c^2 + a*d^2]*(c^2*C - B*c*d
+ A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^
4*d) + ((b*c^2 + a*d^2)^(3/2)*(3*c^2*C - 4*B*c*d + 5*A*d^2)*ArcTanh[(a*d -
 b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^6*d) - (3*A*b^2*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(8*Sqrt[a]*c^2) - (3*Sqrt[a]*b*(c^2*C - 2*B*c*d
 + 3*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*c^4) - (a^(3/2)*d^2*(3...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 765, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-96Aa\,d^3x^3-64Ab\,c^2d\,x^3+72Bac\,d^2x^3+32Bb\,c^3x^3-48Ca\,c^2d\,x^3+36Aac\,d^2x^2+15Ab\,c^3x^2-24Ba\,c^2d\,x^2+12Ca\,c^3x^2}{24c^5x^4}$
default	Expression too large to display

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-96*A*a*d^3*x^3-64*A*b*c^2*d*x^3+72*B*a*c*d^2*x^3+3
2*B*b*c^3*x^3-48*C*a*c^2*d*x^3+36*A*a*c*d^2*x^2+15*A*b*c^3*x^2-24*B*a*c^2*
d*x^2+12*C*a*c^3*x^2-16*A*a*c^2*d*x+8*B*a*c^3*x+6*A*a*c^3)/c^5/x^4-1/8/c^5
*(1/c*(40*A*a^2*d^4+36*A*a*b*c^2*d^2+3*A*b^2*c^4-32*B*a^2*c*d^3-24*B*a*b*c
^3*d+24*C*a^2*c^2*d^2+12*C*a*b*c^4)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x)+8*(A*a^2*d^6+2*A*a*b*c^2*d^4+A*b^2*c^4*d^2-B*a^2*c*d^5-2*B*a*b*c^3
*d^3-B*b^2*c^5*d+C*a^2*c^2*d^4+2*C*a*b*c^4*d^2+C*b^2*c^6)/d^3*(-1/(a*d^2+b
*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*
c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/
d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2))/(x+c/d)))-8*(5*A*a^2*d^6+6*A*a*b*c^2*d^4+A*b^2*c^4*d^2-
4*B*a^2*c*d^5-4*B*a*b*c^3*d^3+3*C*a^2*c^2*d^4+2*C*a*b*c^4*d^2-C*b^2*c^6)/c
/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*(
(a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 14.86 (sec) , antiderivative size = 2437, normalized size of antiderivative = 4.62

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="fricas
")
```

output
```
[-1/48*(24*((B*a*b*c^3*d + 4*B*a^2*c*d^3 - 5*A*a^2*d^4 - (3*C*a^2 + 2*A*a*
b)*c^2*d^2)*x^5 + (B*a*b*c^4 + 4*B*a^2*c^2*d^2 - 5*A*a^2*c*d^3 - (3*C*a^2
+ 2*A*a*b)*c^3*d)*x^4)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*
a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*
sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + 3*((24*B*a*b*c^3*d^2 + 32*B*
a^2*c*d^4 - 40*A*a^2*d^5 - 3*(4*C*a*b + A*b^2)*c^4*d - 12*(2*C*a^2 + 3*A*a
*b)*c^2*d^3)*x^5 + (24*B*a*b*c^4*d + 32*B*a^2*c^2*d^3 - 40*A*a^2*c*d^4 - 3
*(4*C*a*b + A*b^2)*c^5 - 12*(2*C*a^2 + 3*A*a*b)*c^3*d^2)*x^4)*sqrt(a)*log(
-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*A*a^2*c^5 - 8*(3*C*
a*b*c^5 - 7*B*a*b*c^4*d - 12*B*a^2*c^2*d^3 + 15*A*a^2*c*d^4 + (9*C*a^2 + 1
1*A*a*b)*c^3*d^2)*x^4 + (32*B*a*b*c^5 + 48*B*a^2*c^3*d^2 - 60*A*a^2*c^2*d^
3 - (36*C*a^2 + 49*A*a*b)*c^4*d)*x^3 - (16*B*a^2*c^4*d - 20*A*a^2*c^3*d^2
- 3*(4*C*a^2 + 5*A*a*b)*c^5)*x^2 + 2*(4*B*a^2*c^5 - 5*A*a^2*c^4*d)*x)*sqrt
(b*x^2 + a))/(a*c^6*d*x^5 + a*c^7*x^4), -1/48*(48*((B*a*b*c^3*d + 4*B*a^2*
c*d^3 - 5*A*a^2*d^4 - (3*C*a^2 + 2*A*a*b)*c^2*d^2)*x^5 + (B*a*b*c^4 + 4*B*
a^2*c^2*d^2 - 5*A*a^2*c*d^3 - (3*C*a^2 + 2*A*a*b)*c^3*d)*x^4)*sqrt(-b*c^2
- a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^
2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + 3*((24*B*a*b*c^3*d^2 + 32*B*a^2*
c*d^4 - 40*A*a^2*d^5 - 3*(4*C*a*b + A*b^2)*c^4*d - 12*(2*C*a^2 + 3*A*a*b)*
c^2*d^3)*x^5 + (24*B*a*b*c^4*d + 32*B*a^2*c^2*d^3 - 40*A*a^2*c*d^4 - 3*...
```

Sympy [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx$$

input
```
integrate((b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**5/(d*x+c)**2,x)
```

output
```
Integral((a + b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**5*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \int \frac{(Cx^2+Bx+A)(bx^2+a)^{\frac{3}{2}}}{(dx+c)^2x^5}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)*(b*x^2 + a)^(3/2)/((d*x + c)^2*x^5), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \int \frac{(b\,x^2+a)^{3/2}\,(C\,x^2+B\,x+A)}{x^5\,(c+d\,x)^2}\,dx$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2),x)
```

output
```
int(((a + b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1962, normalized size of antiderivative = 3.72

$$\int \frac{(a+bx^2)^{3/2}(A+Bx+Cx^2)}{x^5(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^2,x)
```

output
```
(240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a**2*c*d**3*x**4 + 240*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
 b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**4*x**5 + 96*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a*b*c**3*d*x**4 + 96*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*d**2*x**5 - 192*sqrt(a*d**2 + b*c*
*2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**2*
d**2*x**4 - 192*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2
+ b*c**2) - a*d + b*c*x)*a*b*c*d**3*x**5 + 144*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d*x**4 + 14
4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*
d + b*c*x)*a*c**3*d**2*x**5 - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*x**4 - 48*sqrt(a*d**2
+ b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**
2*c**3*d*x**5 - 240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*c*d**3*x**4 -
240*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**4*x**5 - 96*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b*c**3*d*x**4 - 96*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*a*b*c**2*d**2*x**5 + 192*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**2*d**
2*x**4 + 192*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c*d**3*x**5 - 144*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a*c**4*d*x**4 - 144*sqrt(a*d**2 + b*c**2...
```

3.94 $\int \frac{x^3(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = -\frac{a(Abc-acC-aBd)\sqrt{a+bx^2}}{b^3} + \frac{a(5aCd-6b(Bc+Ad))x\sqrt{a+bx^2}}{16b^3} - \frac{(5aCd-6b(Bc+Ad))x^3\sqrt{a+bx^2}}{24b^2} + \frac{Cdx^5\sqrt{a+bx^2}}{6b} + \frac{(Abc-2a(cC+Bd))\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{(cC+Bd)\left(a+bx^2\right)^{5/2}}{5b^3} - \frac{a^2(5aCd-6b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
-a*(A*b*c-B*a*d-C*a*c)*(b*x^2+a)^(1/2)/b^3+1/16*a*(5*a*C*d-6*b*(A*d+B*c))*
x*(b*x^2+a)^(1/2)/b^3-1/24*(5*a*C*d-6*b*(A*d+B*c))*x^3*(b*x^2+a)^(1/2)/b^2
+1/6*C*d*x^5*(b*x^2+a)^(1/2)/b+1/3*(A*b*c-2*a*(B*d+C*c))*(b*x^2+a)^(3/2)/b
^3+1/5*(B*d+C*c)*(b*x^2+a)^(5/2)/b^3-1/16*a^2*(5*a*C*d-6*b*(A*d+B*c))*arct
anh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(a^2(128cC+128Bd+75Cdx)-2ab(5A(16c+9dx)+x(45Bc+32cCx+32Bdx+25Cdx^2)}{240b^3}$$
$$+ \frac{a^2(5aCd-6b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{7/2}}$$

input
```
Integrate[(x^3*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(a^2*(128*c*C + 128*B*d + 75*C*d*x) - 2*a*b*(5*A*(16*c +
9*d*x) + x*(45*B*c + 32*c*C*x + 32*B*d*x + 25*C*d*x^2)) + 4*b^2*x^2*(5*A*(
4*c + 3*d*x) + x*(3*B*(5*c + 4*d*x) + 2*C*x*(6*c + 5*d*x)))))/(240*b^3) +
(a^2*(5*a*C*d - 6*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*
b^(7/2))
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2185, 25, 2185, 25, 2185, 27, 2185, 25, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int -\frac{(c+dx)\left(bd^4(19cC-6Bd)x^4+d^3\left(21bCc^2-6Abd^2+5aCd^2\right)x^3+3cCd^2\left(3bc^2+5ad^2\right)x^2+c^2Cd\left(bc^2+15ad^2\right)x+5ac^3Cd^2\right)}{\sqrt{bx^2+a}}dx}{6bd^5}+\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}$$

$\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\int \frac{(c+dx)\left(bd^4(19cC-6Bd)x^4+d^3\left(21bCc^2-6Abd^2+5aCd^2\right)x^3+3cCd^2\left(3bc^2+5ad^2\right)x^2+c^2Cd\left(bc^2+15ad^2\right)x+5ac^3Cd^2\right)}{\sqrt{bx^2+a}}dx}{6bd^5}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{\int -\frac{(c+dx)\left(-b\left(25aCd^2-2b\left(52Cc^2-33Bdc+15Ad^2\right)\right)x^3d^7+b\left(2b(44cC-21Bd)c^2+ad^2(cC-24Bd)\right)x^2d^6+3abc^2(17cC-8Bd)d^6+bc\left(2b(7cC-3Bd)c^2+ad^2(77cC\right.\right.}{\sqrt{bx^2+a}}}{5bd^4}}{6bd^5}$$

$\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{\int \frac{(c+dx)\left(-b\left(25aCd^2-2b\left(52Cc^2-33Bdc+15Ad^2\right)\right)x^3d^7+b\left(2b(44cC-21Bd)c^2+ad^2(cC-24Bd)\right)x^2d^6+\right.}{\sqrt{bx^2+a}}}{5bd^4}}{6bd^5}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{\int \frac{3(c+dx)\left(b^2\left(ad^2(43cC-32Bd)-2bc\left(28Cc^2-27Bdc+25Ad^2\right)\right)x^2d^9+abc\left(25aCd^2-b\left(36Cc^2-34Bdc+3\right.\right.\right.}{\sqrt{bx^2+a}}}{4bd^3}}{6}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\int \frac{(c+dx)\left(b^2\left(ad^2(43cC-32Bd)-2bc\left(28Cc^2-27Bdc+25Ad^2\right)\right)x^2d^9+abc\left(25aCd^2-b\left(36Cc^2-34Bdc+3\right.\right.\right.}{\sqrt{bx^2+a}}}{4bd^3}}{6}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{\int -\frac{b^2d^{10}(c+dx)\left(ad\left(ad^2(11cC-64Bd)-2bc\left(2Cc^2-3Bdc+5Ad^2\right)\right)-\left(75a^2Cd^4-2ab\left(11Cc^2-19Bdc-\right.\right.\right.}{\sqrt{bx^2+a}}}{3bd^2}\right.}{}}{}$$

$\Big\downarrow$ 25

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{1}{3}bd^8\sqrt{a+bx^2}(c+dx)^2\left(ad^2(43cC-32Bd)-2bc\left(25Ad^2-27Bcd+28c^2C\right)\right)-\frac{\int\frac{b^2d^{10}(c+dx)\left(ad\left(ad^2(11\cdots\right.\right.}{\cdots}}{\cdots}\right)}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 27

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{1}{3}bd^8\sqrt{a+bx^2}(c+dx)^2\left(ad^2(43cC-32Bd)-2bc\left(25Ad^2-27Bcd+28c^2C\right)\right)-\frac{1}{3}bd^8\int\frac{(c+dx)\left(ad\left(ad^2(11\cdots\right.\right.}{\cdots}\right)}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 676

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{1}{3}bd^8\sqrt{a+bx^2}(c+dx)^2\left(ad^2(43cC-32Bd)-2bc\left(25Ad^2-27Bcd+28c^2C\right)\right)-\frac{1}{3}bd^8\left(\frac{15a^2d^4(5aCd-6b(A\cdots}{\cdots}\right.\right)}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 224

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{1}{3}bd^8\sqrt{a+bx^2}(c+dx)^2\left(ad^2(43cC-32Bd)-2bc\left(25Ad^2-27Bcd+28c^2C\right)\right)-\frac{1}{3}bd^8\left(\frac{15a^2d^4(5aCd-6b(A\cdots}{\cdots}\right.\right)}{\cdots}}{\cdots}}{\cdots}$$

$\Big\downarrow$ 219

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^4}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(19cC-6Bd)-\frac{3\left(\frac{1}{3}bd^8\sqrt{a+bx^2}(c+dx)^2\left(ad^2(43cC-32Bd)-2bc\left(25Ad^2-27Bcd+28c^2C\right)\right)-\frac{1}{3}bd^8\left(\frac{15a^2d^4\text{arctanh}(\cdots}{\cdots}\right.\right)}{\cdots}}{\cdots}}{\cdots}$$

input
```
Int[(x^3*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^5*Sqrt[a + b*x^2])/(6*b*d^4) - ((d*(19*c*C - 6*B*d)*(c + d*x)
^4*Sqrt[a + b*x^2])/5 - ((d^5*(104*b*c^2*C - 66*b*B*c*d + 30*A*b*d^2 - 25*
a*C*d^2)*(c + d*x)^3*Sqrt[a + b*x^2])/4 + (3*((b*d^8*(a*d^2*(43*c*C - 32*B
*d) - 2*b*c*(28*c^2*C - 27*B*c*d + 25*A*d^2))*(c + d*x)^2*Sqrt[a + b*x^2])
/3 - (b*d^8*((-2*(32*a^2*d^4*(c*C + B*d) + 2*b^2*c^3*(2*c^2*C - 3*B*c*d +
5*A*d^2) - a*b*c*d^2*(9*c^2*C - 16*B*c*d + 40*A*d^2))*Sqrt[a + b*x^2])/b -
 (d*(75*a^2*C*d^4 + 4*b^2*c^2*(2*c^2*C - 3*B*c*d + 5*A*d^2) - 2*a*b*d^2*(1
1*c^2*C - 19*B*c*d + 45*A*d^2))*x*Sqrt[a + b*x^2])/(2*b) + (15*a^2*d^4*(5*
a*C*d - 6*b*(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
))/3))/(4*b*d^3))/(5*b*d^4))/(6*b*d^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(-40dC\,b^{2}x^{5}-48B\,b^{2}d\,x^{4}-48C\,b^{2}c\,x^{4}-60A\,b^{2}d\,x^{3}-60B\,b^{2}c\,x^{3}+50aCdb\,x^{3}-80A\,b^{2}c\,x^{2}+64Badb\,x^{2}+64Cacb\,x^{2}+90Axadb+90}{240b^{3}}$
default	$(Ad+Bc)\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+(Bd+Cc)\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}\right.}{5b}\right.$

input
```
int(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*C*b^2*d*x^5-48*B*b^2*d*x^4-48*C*b^2*c*x^4-60*A*b^2*d*x^3-60*B*
b^2*c*x^3+50*C*a*b*d*x^3-80*A*b^2*c*x^2+64*B*a*b*d*x^2+64*C*a*b*c*x^2+90*A
*a*b*d*x+90*B*a*b*c*x-75*C*a^2*d*x+160*A*a*b*c-128*B*a^2*d-128*C*a^2*c)*(b
*x^2+a)^(1/2)/b^3+1/16*a^2*(6*A*b*d+6*B*b*c-5*C*a*d)/b^(7/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.85

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{15\left(6\,Ba^2bc-\left(5\,Ca^3-6\,Aa^2b\right)d\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(40\,Cb^3dx^5+128\,Ba^2bd\right.}{}\right.$$
$$-\frac{15\left(6\,Ba^2bc-\left(5\,Ca^3-6\,Aa^2b\right)d\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(40\,Cb^3dx^5+128\,Ba^2bd+48\left(Cb^3c+Bb^3\right.\right.}{}$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(15*(6*B*a^2*b*c - (5*C*a^3 - 6*A*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*C*b^3*d*x^5 + 128*B*a^2*b*d + 48*
(C*b^3*c + B*b^3*d)*x^4 + 10*(6*B*b^3*c - (5*C*a*b^2 - 6*A*b^3)*d)*x^3 - 1
6*(4*B*a*b^2*d + (4*C*a*b^2 - 5*A*b^3)*c)*x^2 + 32*(4*C*a^2*b - 5*A*a*b^2)
*c - 15*(6*B*a*b^2*c - (5*C*a^2*b - 6*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^4,
 -1/240*(15*(6*B*a^2*b*c - (5*C*a^3 - 6*A*a^2*b)*d)*sqrt(-b)*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) - (40*C*b^3*d*x^5 + 128*B*a^2*b*d + 48*(C*b^3*c + B*
b^3*d)*x^4 + 10*(6*B*b^3*c - (5*C*a*b^2 - 6*A*b^3)*d)*x^3 - 16*(4*B*a*b^2*
d + (4*C*a*b^2 - 5*A*b^3)*c)*x^2 + 32*(4*C*a^2*b - 5*A*a*b^2)*c - 15*(6*B*
a*b^2*c - (5*C*a^2*b - 6*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{3a^2\left(Ad+Bc-\frac{5Cad}{6b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(\frac{Cdx^5}{6b}-\frac{3ax\left(Ad+Bc-\frac{5Cad}{6b}\right)}{8b^2}-\frac{2a\left(Ac-\frac{4a(B}{5}\right.}{3b^2}\right. \\ \frac{\frac{Acx^4}{4}+\frac{Cdx^7}{7}+\frac{x^6(Bd+Cc)}{6}+\frac{x^5(Ad+Bc)}{5}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**3*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(A*d + B*c - 5*C*a*d/(6*b))*Piecewise((log(2*sqrt(b)*sqr
t(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/
(8*b**2) + sqrt(a + b*x**2)*(C*d*x**5/(6*b) - 3*a*x*(A*d + B*c - 5*C*a*d/(
6*b))/(8*b**2) - 2*a*(A*c - 4*a*(B*d + C*c)/(5*b))/(3*b**2) + x**4*(B*d +
C*c)/(5*b) + x**3*(A*d + B*c - 5*C*a*d/(6*b))/(4*b) + x**2*(A*c - 4*a*(B*d
 + C*c)/(5*b))/(3*b)), Ne(b, 0)), ((A*c*x**4/4 + C*d*x**7/7 + x**6*(B*d +
C*c)/6 + x**5*(A*d + B*c)/5)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.11

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cdx^5}{6\,b} - \frac{5\,\sqrt{bx^2+a}Cadx^3}{24\,b^2} + \frac{\sqrt{bx^2+a}(Cc+Bd)x^4}{5\,b} + \frac{\sqrt{bx^2+a}Acx^2}{3\,b} + \frac{\sqrt{bx^2+a}(Bc+Ad)x^3}{4\,b} + \frac{5\,\sqrt{bx^2+a}Ca^2dx}{16\,b^3} - \frac{4\,\sqrt{bx^2+a}(Cc+Bd)ax^2}{15\,b^2} - \frac{5\,Ca^3d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} - \frac{2\,\sqrt{bx^2+a}Aac}{3\,b^2} - \frac{3\,\sqrt{bx^2+a}(Bc+Ad)ax}{8\,b^2} + \frac{3\,(Bc+Ad)a^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{8\,\sqrt{bx^2+a}(Cc+Bd)a^2}{15\,b^3}$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*C*d*x^5/b - 5/24*sqrt(b*x^2 + a)*C*a*d*x^3/b^2 + 1/5*s
qrt(b*x^2 + a)*(C*c + B*d)*x^4/b + 1/3*sqrt(b*x^2 + a)*A*c*x^2/b + 1/4*sqr
t(b*x^2 + a)*(B*c + A*d)*x^3/b + 5/16*sqrt(b*x^2 + a)*C*a^2*d*x/b^3 - 4/15
*sqrt(b*x^2 + a)*(C*c + B*d)*a*x^2/b^2 - 5/16*C*a^3*d*arcsinh(b*x/sqrt(a*b
))/b^(7/2) - 2/3*sqrt(b*x^2 + a)*A*a*c/b^2 - 3/8*sqrt(b*x^2 + a)*(B*c + A*
d)*a*x/b^2 + 3/8*(B*c + A*d)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/15*sqr
t(b*x^2 + a)*(C*c + B*d)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.95

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$

$$= \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\frac{5\,Cdx}{b}+\frac{6\,(Cb^5c+Bb^5d)}{b^6}\right)x+\frac{5\,(6\,Bb^5c-5\,Cab^4d+6\,Ab^5d)}{b^6}\right)x-\frac{8\,(4\,Cab^4\right.\right.\right.$$

$$-\frac{(6\,Ba^2bc-5\,Ca^3d+6\,Aa^2bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*C*d*x/b + 6*(C*b^5*c + B*b^5*d)/b^6)*x +
5*(6*B*b^5*c - 5*C*a*b^4*d + 6*A*b^5*d)/b^6)*x - 8*(4*C*a*b^4*c - 5*A*b^5*
c + 4*B*a*b^4*d)/b^6)*x - 15*(6*B*a*b^4*c - 5*C*a^2*b^3*d + 6*A*a*b^4*d)/b
^6)*x + 32*(4*C*a^2*b^3*c - 5*A*a*b^4*c + 4*B*a^2*b^3*d)/b^6) - 1/16*(6*B*
a^2*b*c - 5*C*a^3*d + 6*A*a^2*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^3\,(c+d\,x)\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.53

$$\int \frac{x^3(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{-160\sqrt{b\,x^2+a}\,a^2b^2c-90\sqrt{b\,x^2+a}\,a^2b^2dx+128\sqrt{b\,x^2+a}\,a^2b^2d+128\sqrt{b\,x^2+a}\,a^2b\,c^2+75\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 160*sqrt(a + b*x**2)*a**2*b**2*c - 90*sqrt(a + b*x**2)*a**2*b**2*d*x +
 128*sqrt(a + b*x**2)*a**2*b**2*d + 128*sqrt(a + b*x**2)*a**2*b*c**2 + 75*
sqrt(a + b*x**2)*a**2*b*c*d*x + 80*sqrt(a + b*x**2)*a*b**3*c*x**2 - 90*sqr
t(a + b*x**2)*a*b**3*c*x + 60*sqrt(a + b*x**2)*a*b**3*d*x**3 - 64*sqrt(a +
 b*x**2)*a*b**3*d*x**2 - 64*sqrt(a + b*x**2)*a*b**2*c**2*x**2 - 50*sqrt(a
+ b*x**2)*a*b**2*c*d*x**3 + 60*sqrt(a + b*x**2)*b**4*c*x**3 + 48*sqrt(a +
b*x**2)*b**4*d*x**4 + 48*sqrt(a + b*x**2)*b**3*c**2*x**4 + 40*sqrt(a + b*x
**2)*b**3*c*d*x**5 + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**3*b*d - 75*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c
*d + 90*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c)/(
240*b**4)
```

3.95 $\int \frac{x^2(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 192

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{a(aCd-b(Bc+Ad))\sqrt{a+bx^2}}{b^3} + \frac{(4Abc-3a(cC+Bd))x\sqrt{a+bx^2}}{8b^2} + \frac{(cC+Bd)x^3\sqrt{a+bx^2}}{4b} + \frac{(bBc+Abd-2aCd)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{Cd\left(a+bx^2\right)^{5/2}}{5b^3} - \frac{a(4Abc-3a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
a*(a*C*d-b*(A*d+B*c))*(b*x^2+a)^(1/2)/b^3+1/8*(4*A*b*c-3*a*(B*d+C*c))*x*(b
*x^2+a)^(1/2)/b^2+1/4*(B*d+C*c)*x^3*(b*x^2+a)^(1/2)/b+1/3*(A*b*d+B*b*c-2*C
*a*d)*(b*x^2+a)^(3/2)/b^3+1/5*C*d*(b*x^2+a)^(5/2)/b^3-1/8*a*(4*A*b*c-3*a*(
B*d+C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.81

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(64a^2Cd-ab(80Bc+80Ad+45cCx+45Bdx+32Cdx^2)+2b^2x(10A(3c+2dx)+x(5B(4c}{120b^3}$$

input
```
Integrate[(x^2*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(64*a^2*C*d - a*b*(80*B*c + 80*A*d + 45*c*C*x + 45*B*d*x
+ 32*C*d*x^2) + 2*b^2*x*(10*A*(3*c + 2*d*x) + x*(5*B*(4*c + 3*d*x) + 3*C*x
*(5*c + 4*d*x)))) + 30*a*Sqrt[b]*(-4*A*b*c + 3*a*(c*C + B*d))*ArcTanh[(Sqr
t[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(120*b^3)
```

Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.69, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2185, 25, 2185, 25, 2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int -\frac{(c+dx)\left(bd^3(11cC-5Bd)x^3+d^2\left(7bCc^2-5Abd^2+4aCd^2\right)x^2+cCd\left(bc^2+8ad^2\right)x+4ac^2Cd^2\right)}{\sqrt{bx^2+a}}dx}{5bd^4}+\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}$$
$$\Big\downarrow \; 25$$

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\int \frac{(c+dx)(bd^3(11cC-5Bd)x^3+d^2(7bCc^2-5Abd^2+4aCd^2)x^2+cCd(bc^2+8ad^2)x+4ac^2Cd^2)}{\sqrt{bx^2+a}}dx}{5bd^4}$$

$\Big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\frac{\frac{\int -\frac{(c+dx)\left(-b\left(16aCd^2-b\left(27Cc^2-25Bdc+20Ad^2\right)\right)x^2d^5+abc(17cC-15Bd)d^5+b\left(b(7cC-5Bd)c^2+ad^2(cC-15Bd)\right)xd^4\right)}{\sqrt{bx^2+a}}dx}{4bd^3}+\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3}{5bd^4}$$

$\Big\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd)-\frac{\int \frac{(c+dx)\left(-b\left(16aCd^2-b\left(27Cc^2-25Bdc+20Ad^2\right)\right)x^2d^5+abc(17cC-15Bd)d^5+b\left(b(7cC-5Bd)c^2+ad^2(c\right.}{\sqrt{bx^2+a}}}{4bd^3}}{5bd^4}$$

$\Big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd)-\frac{\frac{\int \frac{bd^6(c+dx)\left(ad\left(32aCd^2-b\left(3Cc^2-5Bdc+40Ad^2\right)\right)+b\left(ad^2(19cC-45Bd)-2bc\left(3Cc^2-5Bdc+10Ad^2\right)\right)x\right.}{\sqrt{bx^2+a}}}{3bd^2}}{4bd^3}}{5bd^4}$$

$\Big\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd)-\frac{\frac{1}{3}d^4\int \frac{(c+dx)\left(ad\left(32aCd^2-b\left(3Cc^2-5Bdc+40Ad^2\right)\right)+b\left(ad^2(19cC-45Bd)-2bc\left(3Cc^2-5Bdc+10Ad^2\right)\right)\right.}{\sqrt{bx^2+a}}}{4bd^3}}{5bd^4}$$

$\Big\downarrow$ 676

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd)-\frac{\frac{1}{3}d^4\left(-\frac{15}{2}ad^3(4Abc-3a(Bd+cC))\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(16a^2Cd^4+4abd^2\left(-5Ad^2-5Bcd+2c^2C\right.\right.}{b}\right.}{}}{}$$

$\Big\downarrow$ 224

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3} - \frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd) - \frac{\frac{1}{3}d^4\left(-\frac{15}{2}ad^3(4Abc-3a(Bd+cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} + \frac{2\sqrt{a+bx^2}\left(16a^2Cd^4+4abd^2\left(-5Ad^2-5\right.\right.}{}\right.}{}}{}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^3} - \frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(11cC-5Bd) - \frac{\frac{1}{3}d^4\left(\frac{2\sqrt{a+bx^2}\left(16a^2Cd^4+4abd^2\left(-5Ad^2-5Bcd+2c^2C\right)-b^2c^2\left(10Ad^2-5Bcd+3c^2C\right)\right)}{b} - \frac{15ad^3\text{arcta}}{}\right.}{}}{}$$

input
```
Int[(x^2*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^4*Sqrt[a + b*x^2])/(5*b*d^3) - ((d*(11*c*C - 5*B*d)*(c + d*x)
^3*Sqrt[a + b*x^2])/4 - (-1/3*(d^4*(16*a*C*d^2 - b*(27*c^2*C - 25*B*c*d +
20*A*d^2))*(c + d*x)^2*Sqrt[a + b*x^2]) + (d^4*((2*(16*a^2*C*d^4 + 4*a*b*d
^2*(2*c^2*C - 5*B*c*d - 5*A*d^2) - b^2*c^2*(3*c^2*C - 5*B*c*d + 10*A*d^2))
*Sqrt[a + b*x^2])/b + (d*(a*d^2*(19*c*C - 45*B*d) - 2*b*c*(3*c^2*C - 5*B*c
*d + 10*A*d^2))*x*Sqrt[a + b*x^2])/2 - (15*a*d^3*(4*A*b*c - 3*a*(c*C + B*d
))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/3)/(4*b*d^3))/(5*b*
d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(-24C\,b^2d\,x^4-30B\,b^2d\,x^3-30C\,b^2c\,x^3-40A\,b^2d\,x^2-40B\,b^2c\,x^2+32Cabd\,x^2-60A\,b^2cx+45Badxb+45Cacxb+80Aabd+80Babc}{120b^3}$
default	$(Ad+Bc)\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+(Bd+Cc)\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+$

input
```
int(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*C*b^2*d*x^4-30*B*b^2*d*x^3-30*C*b^2*c*x^3-40*A*b^2*d*x^2-40*B*
b^2*c*x^2+32*C*a*b*d*x^2-60*A*b^2*c*x+45*B*a*b*d*x+45*C*a*b*c*x+80*A*a*b*d
+80*B*a*b*c-64*C*a^2*d)*(b*x^2+a)^(1/2)/b^3-1/8*a/b^(5/2)*(4*A*b*c-3*B*a*d
-3*C*a*c)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.76

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{15\left(3\,Ba^2d+(3\,Ca^2-4\,Aab)c\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(24\,Cb^2dx^4-80\,Babc+30\right.}{}\right.$$

$$-\frac{15\left(3\,Ba^2d+(3\,Ca^2-4\,Aab)c\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(24\,Cb^2dx^4-80\,Babc+30\left(Cb^2c+Bb^2d\right)x^3\right.}{12}$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*(3*B*a^2*d + (3*C*a^2 - 4*A*a*b)*c)*sqrt(b)*log(-2*b*x^2 - 2*sq
rt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*C*b^2*d*x^4 - 80*B*a*b*c + 30*(C*b^2*
c + B*b^2*d)*x^3 + 8*(5*B*b^2*c - (4*C*a*b - 5*A*b^2)*d)*x^2 + 16*(4*C*a^2
 - 5*A*a*b)*d - 15*(3*B*a*b*d + (3*C*a*b - 4*A*b^2)*c)*x)*sqrt(b*x^2 + a))
/b^3, -1/120*(15*(3*B*a^2*d + (3*C*a^2 - 4*A*a*b)*c)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (24*C*b^2*d*x^4 - 80*B*a*b*c + 30*(C*b^2*c + B*b^
2*d)*x^3 + 8*(5*B*b^2*c - (4*C*a*b - 5*A*b^2)*d)*x^2 + 16*(4*C*a^2 - 5*A*a
*b)*d - 15*(3*B*a*b*d + (3*C*a*b - 4*A*b^2)*c)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{a\left(Ac-\frac{3a(Bd+Cc)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cdx^4}{5b}-\frac{2a\left(Ad+Bc-\frac{4Cad}{5b}\right)}{3b^2}+\frac{x^3(Bd+Cc)}{4b}\right. \\ \frac{\frac{Acx^3}{3}+\frac{Cdx^6}{6}+\frac{x^5(Bd+Cc)}{5}+\frac{x^4(Ad+Bc)}{4}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A*c - 3*a*(B*d + C*c)/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(
a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2
*b) + sqrt(a + b*x**2)*(C*d*x**4/(5*b) - 2*a*(A*d + B*c - 4*C*a*d/(5*b))/(
3*b**2) + x**3*(B*d + C*c)/(4*b) + x**2*(A*d + B*c - 4*C*a*d/(5*b))/(3*b)
+ x*(A*c - 3*a*(B*d + C*c)/(4*b))/(2*b)), Ne(b, 0)), ((A*c*x**3/3 + C*d*x*
*6/6 + x**5*(B*d + C*c)/5 + x**4*(A*d + B*c)/4)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.09

$$\int \frac{x^2(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cdx^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Cadx^2}{15\,b^2} + \frac{\sqrt{bx^2+a}(Cc+Bd)x^3}{4\,b} + \frac{\sqrt{bx^2+a}Acx}{2\,b} + \frac{\sqrt{bx^2+a}(Bc+Ad)x^2}{3\,b} - \frac{Aac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{8\,\sqrt{bx^2+a}Ca^2d}{15\,b^3} - \frac{3\,\sqrt{bx^2+a}(Cc+Bd)ax}{8\,b^2} + \frac{3\,(Cc+Bd)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{2\,\sqrt{bx^2+a}(Bc+Ad)a}{3\,b^2}$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*C*d*x^4/b - 4/15*sqrt(b*x^2 + a)*C*a*d*x^2/b^2 + 1/4*s
qrt(b*x^2 + a)*(C*c + B*d)*x^3/b + 1/2*sqrt(b*x^2 + a)*A*c*x/b + 1/3*sqrt(
b*x^2 + a)*(B*c + A*d)*x^2/b - 1/2*A*a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) +
8/15*sqrt(b*x^2 + a)*C*a^2*d/b^3 - 3/8*sqrt(b*x^2 + a)*(C*c + B*d)*a*x/b^2
 + 3/8*(C*c + B*d)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^2 + a
)*(B*c + A*d)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{1}{120}\,\sqrt{bx^2+a}\left(\left(2\left(3\left(\frac{4\,Cdx}{b} + \frac{5\,(Cb^4c+Bb^4d)}{b^5}\right)x + \frac{4\,(5\,Bb^4c-4\,Cab^3d+5\,Ab^4d)}{b^5}\right)x - \frac{15\,(3\,Cab^3 \cdots\right.\right. - \frac{(3\,Ca^2c-4\,Aabc+3\,Ba^2d)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*C*d*x/b + 5*(C*b^4*c + B*b^4*d)/b^5)*x + 4
*(5*B*b^4*c - 4*C*a*b^3*d + 5*A*b^4*d)/b^5)*x - 15*(3*C*a*b^3*c - 4*A*b^4*
c + 3*B*a*b^3*d)/b^5)*x - 16*(5*B*a*b^3*c - 4*C*a^2*b^2*d + 5*A*a*b^3*d)/b
^5) - 1/8*(3*C*a^2*c - 4*A*a*b*c + 3*B*a^2*d)*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^2\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.51

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{-80\sqrt{b\,x^2+a}\,a^2bd+64\sqrt{b\,x^2+a}\,a^2cd+60\sqrt{b\,x^2+a}\,a\,b^2cx-80\sqrt{b\,x^2+a}\,a\,b^2c+40\sqrt{b\,x^2+a}\,a\,b^2d\,x^2}{}$$

input
```
int(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output

```
( - 80*sqrt(a + b*x**2)*a**2*b*d + 64*sqrt(a + b*x**2)*a**2*c*d + 60*sqrt(
a + b*x**2)*a*b**2*c*x - 80*sqrt(a + b*x**2)*a*b**2*c + 40*sqrt(a + b*x**2
)*a*b**2*d*x**2 - 45*sqrt(a + b*x**2)*a*b**2*d*x - 45*sqrt(a + b*x**2)*a*b
*c**2*x - 32*sqrt(a + b*x**2)*a*b*c*d*x**2 + 40*sqrt(a + b*x**2)*b**3*c*x*
*2 + 30*sqrt(a + b*x**2)*b**3*d*x**3 + 30*sqrt(a + b*x**2)*b**2*c**2*x**3
+ 24*sqrt(a + b*x**2)*b**2*c*d*x**4 - 60*sqrt(b)*log((sqrt(a + b*x**2) + s
qrt(b)*x)/sqrt(a))*a**2*b*c + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a**2*b*d + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a
))*a**2*c**2)/(120*b**3)
```

3.96 $\int \frac{x(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 159

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{(Abc-a(cC+Bd))\sqrt{a+bx^2}}{b^2} - \frac{(3aCd-4b(Bc+Ad))x\sqrt{a+bx^2}}{8b^2} + \frac{Cdx^3\sqrt{a+bx^2}}{4b} + \frac{(cC+Bd)\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{a(3aCd-4b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
(A*b*c-a*(B*d+C*c))*(b*x^2+a)^(1/2)/b^2-1/8*(3*a*C*d-4*b*(A*d+B*c))*x*(b*x
^2+a)^(1/2)/b^2+1/4*C*d*x^3*(b*x^2+a)^(1/2)/b+1/3*(B*d+C*c)*(b*x^2+a)^(3/2
)/b^2+1/8*a*(3*a*C*d-4*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(
5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{\sqrt{a+bx^2}(24Abc-16acC-16aBd+12bBcx+12Abdx-9aCdx+8bcCx^2+8bBdx^2+6bCdx^3)}{24b^2}$$
$$-\frac{a(-4bBc-4Abd+3aCd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(x*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(24*A*b*c - 16*a*c*C - 16*a*B*d + 12*b*B*c*x + 12*A*b*d*x
 - 9*a*C*d*x + 8*b*c*C*x^2 + 8*b*B*d*x^2 + 6*b*C*d*x^3))/(24*b^2) - (a*(-4
*b*B*c - 4*A*b*d + 3*a*C*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2
))
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2185, 25, 2185, 25, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int -\frac{(c+dx)\left(b(5cC-4Bd)x^2d^2+3acCd^2+\left(bCc^2-4Abd^2+3aCd^2\right)xd\right)}{\sqrt{bx^2+a}}dx}{4bd^3}+\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}$$
$$\Big\downarrow \text{25}$$
$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\int \frac{(c+dx)\left(b(5cC-4Bd)x^2d^2+3acCd^2+\left(bCc^2-4Abd^2+3aCd^2\right)xd\right)}{\sqrt{bx^2+a}}dx}{4bd^3}$$

$\Big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{\int -\frac{bd^3(c+dx)\left(ad(cC-8Bd)-\left(9aCd^2-2b\left(Cc^2-2Bdc+6Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3bd^2}+\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)}{4bd^3}$$

$\Big\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)-\frac{\int \frac{bd^3(c+dx)\left(ad(cC-8Bd)-\left(9aCd^2-2b\left(Cc^2-2Bdc+6Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3bd^2}}{4bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)-\frac{1}{3}d\int \frac{(c+dx)\left(ad(cC-8Bd)-\left(9aCd^2-2b\left(Cc^2-2Bdc+6Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4bd^3}$$

$\Big\downarrow$ 676

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)-\frac{1}{3}d\left(\frac{3ad^2(3aCd-4b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{dx\sqrt{a+bx^2}\left(9aCd^2-2b\left(6Ad^2-2Bcd+c^2C\right)\right)}{2b}-\right.}{4bd^3}$$

$\Big\downarrow$ 224

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)-\frac{1}{3}d\left(\frac{3ad^2(3aCd-4b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{dx\sqrt{a+bx^2}\left(9aCd^2-2b\left(6Ad^2-2Bcd+\right.\right.}{2b}\right.}{4bd^3}$$

$\Big\downarrow$ 219

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^2}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(5cC-4Bd)-\frac{1}{3}d\left(\frac{3ad^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd-4b(Ad+Bc))}{2b^{3/2}}-\frac{dx\sqrt{a+bx^2}\left(9aCd^2-2b\left(6Ad^2-2Bcd+\right.\right.}{2b}\right.}{4bd^3}$$

input
```
Int[(x*(c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b*d^2) - ((d*(5*c*C - 4*B*d)*(c + d*x)^
2*Sqrt[a + b*x^2])/3 - (d*((-2*(4*a*d^2*(c*C + B*d) - b*c*(c^2*C - 2*B*c*d
 + 6*A*d^2))*Sqrt[a + b*x^2])/b - (d*(9*a*C*d^2 - 2*b*(c^2*C - 2*B*c*d + 6
*A*d^2))*x*Sqrt[a + b*x^2])/(2*b) + (3*a*d^2*(3*a*C*d - 4*b*(B*c + A*d))*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/3)/(4*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(6Cbd\,x^3+8Bbd\,x^2+8Ccb\,x^2+12xAbd+12Bbcx-9Cadx+24Abc-16Bad-16Cac)\sqrt{b\,x^2+a}}{24b^2}-\frac{a(4Abd+4Bbc-3aCd)\ln\left(\sqrt{b}\,x+\sqrt{\right.}}{8b^{\frac{5}{2}}}$
default	$(Ad+Bc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+(Bd+Cc)\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{Ac\sqrt{b\,x^2+a}}{b}+c$

input
```
int(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*b*d*x^3+8*B*b*d*x^2+8*C*b*c*x^2+12*A*b*d*x+12*B*b*c*x-9*C*a*d*x+
24*A*b*c-16*B*a*d-16*C*a*c)*(b*x^2+a)^(1/2)/b^2-1/8*a*(4*A*b*d+4*B*b*c-3*C
*a*d)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.79

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\left(4\,Babc-(3\,Ca^2-4\,Aab)d\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(6\,Cb^2dx^3-16\,Babd+8\,(C\right.}{48\,b^3}\right.$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(4*B*a*b*c - (3*C*a^2 - 4*A*a*b)*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d*x^3 - 16*B*a*b*d + 8*(C*b^2*c +
B*b^2*d)*x^2 - 8*(2*C*a*b - 3*A*b^2)*c + 3*(4*B*b^2*c - (3*C*a*b - 4*A*b^2
)*d)*x)*sqrt(b*x^2 + a))/b^3, 1/24*(3*(4*B*a*b*c - (3*C*a^2 - 4*A*a*b)*d)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*C*b^2*d*x^3 - 16*B*a*b*d
+ 8*(C*b^2*c + B*b^2*d)*x^2 - 8*(2*C*a*b - 3*A*b^2)*c + 3*(4*B*b^2*c - (3*
C*a*b - 4*A*b^2)*d)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.16

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{a\left(Ad+Bc-\frac{3Cad}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cdx^3}{4b}+\frac{x^2(Bd+Cc)}{3b}+\frac{x\left(Ad+Bc-\frac{3Cad}{4b}\right)}{2b}+\right. \\ \frac{\frac{Acx^2}{2}+\frac{Cdx^5}{5}+\frac{x^4(Bd+Cc)}{4}+\frac{x^3(Ad+Bc)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A*d + B*c - 3*C*a*d/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b
) + sqrt(a + b*x**2)*(C*d*x**3/(4*b) + x**2*(B*d + C*c)/(3*b) + x*(A*d + B
*c - 3*C*a*d/(4*b))/(2*b) + (A*c - 2*a*(B*d + C*c)/(3*b))/b), Ne(b, 0)), (
(A*c*x**2/2 + C*d*x**5/5 + x**4*(B*d + C*c)/4 + x**3*(A*d + B*c)/3)/sqrt(a
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.02

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cdx^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Cadx}{8\,b^2} + \frac{\sqrt{bx^2+a}(Cc+Bd)x^2}{3\,b} + \frac{3\,Ca^2d\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}Ac}{b} + \frac{\sqrt{bx^2+a}(Bc+Ad)x}{2\,b} - \frac{(Bc+Ad)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}(Cc+Bd)a}{3\,b^2}$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*C*d*x^3/b - 3/8*sqrt(b*x^2 + a)*C*a*d*x/b^2 + 1/3*sqrt
(b*x^2 + a)*(C*c + B*d)*x^2/b + 3/8*C*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2)
 + sqrt(b*x^2 + a)*A*c/b + 1/2*sqrt(b*x^2 + a)*(B*c + A*d)*x/b - 1/2*(B*c
+ A*d)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*(C*c + B*d)*
a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.92

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Cdx}{b}+\frac{4\,(Cb^3c+Bb^3d)}{b^4}\right)x+\frac{3\,(4\,Bb^3c-3\,Cab^2d+4\,Ab^3d)}{b^4}\right)x-\frac{8\,(2\,Cab^2c-3\,\ldots}{b\ldots}\right.$$
$$+\frac{(4\,Babc-3\,Ca^2d+4\,Aabd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*C*d*x/b + 4*(C*b^3*c + B*b^3*d)/b^4)*x + 3*(4*
B*b^3*c - 3*C*a*b^2*d + 4*A*b^3*d)/b^4)*x - 8*(2*C*a*b^2*c - 3*A*b^3*c + 2
*B*a*b^2*d)/b^4) + 1/8*(4*B*a*b*c - 3*C*a^2*d + 4*A*a*b*d)*log(abs(-sqrt(b
)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx=\int \frac{x\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.48

$$\int \frac{x(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{24\sqrt{b\,x^2+a}\,a\,b^2c+12\sqrt{b\,x^2+a}\,a\,b^2dx-16\sqrt{b\,x^2+a}\,a\,b^2d-16\sqrt{b\,x^2+a}\,ab\,c^2-9\sqrt{b\,x^2+a}\,abcdx+1}{}$$

input
```
int(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(24*sqrt(a + b*x**2)*a*b**2*c + 12*sqrt(a + b*x**2)*a*b**2*d*x - 16*sqrt(a
 + b*x**2)*a*b**2*d - 16*sqrt(a + b*x**2)*a*b*c**2 - 9*sqrt(a + b*x**2)*a*
b*c*d*x + 12*sqrt(a + b*x**2)*b**3*c*x + 8*sqrt(a + b*x**2)*b**3*d*x**2 +
8*sqrt(a + b*x**2)*b**2*c**2*x**2 + 6*sqrt(a + b*x**2)*b**2*c*d*x**3 - 12*
sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d + 9*sqrt(b)*l
og((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d - 12*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c)/(24*b**3)
```

3.97 $\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 120

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}} \, dx = \frac{(bBc+Abd-aCd)\sqrt{a+bx^2}}{b^2} + \frac{(cC+Bd)x\sqrt{a+bx^2}}{2b} + \frac{Cd\left(a+bx^2\right)^{3/2}}{3b^2} + \frac{(2Abc-acC-aBd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
(A*b*d+B*b*c-C*a*d)*(b*x^2+a)^(1/2)/b^2+1/2*(B*d+C*c)*x*(b*x^2+a)^(1/2)/b+
1/3*C*d*(b*x^2+a)^(3/2)/b^2+1/2*(2*A*b*c-B*a*d-C*a*c)*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}} \, dx = \frac{\sqrt{a+bx^2}(-4aCd+b(6Bc+6Ad+3cCx+3Bdx+2Cdx^2))+3\sqrt{b}(-2Abc+acC+aBd)\log\left(-\sqrt{b}x\right.}{6b^2}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-4*a*C*d + b*(6*B*c + 6*A*d + 3*c*C*x + 3*B*d*x + 2*C*d*
x^2)) + 3*Sqrt[b]*(-2*A*b*c + a*c*C + a*B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(6*b^2)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{d(c+dx)((3Ab-2aC)d-b(cC-3Bd)x)}{\sqrt{bx^2+a}}dx}{3bd^2}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)((3Ab-2aC)d-b(cC-3Bd)x)}{\sqrt{bx^2+a}}dx}{3bd}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd}$$

$$\Big\downarrow \text{676}$$

$$\frac{\frac{3}{2}d(-aBd-acC+2Abc)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{\sqrt{a+bx^2}\left(d^2(3Ab-2aC)-bc(cC-3Bd)\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(cC-3Bd)}{3bd}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3}{2}d(-aBd-acC+2Abc)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{\sqrt{a+bx^2}\left(d^2(3Ab-2aC)-bc(cC-3Bd)\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(cC-3Bd)}{3bd}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{3d\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(-aBd-acC+2Abc)}{2\sqrt{b}}+\frac{\sqrt{a+bx^2}\left(d^2(3Ab-2aC)-bc(cC-3Bd)\right)}{b}-\frac{1}{2}dx\sqrt{a+bx^2}(cC-3Bd)}{3bd}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b*d) + ((((3*A*b - 2*a*C)*d^2 - b*c*(c*
C - 3*B*d))*Sqrt[a + b*x^2])/b - (d*(c*C - 3*B*d)*x*Sqrt[a + b*x^2])/2 + (
3*d*(2*A*b*c - a*c*C - a*B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqr
t[b]))/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(2Cbd\,x^2+3Bbdx+3Ccxb+6Abd+6Bbc-4aCd)\sqrt{b\,x^2+a}}{6b^2}+\frac{(2Abc-Bad-Cac)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$
default	$\frac{Ac\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{(Ad+Bc)\sqrt{b\,x^2+a}}{b}+(Bd+Cc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+dC\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d*x^2+3*B*b*d*x+3*C*b*c*x+6*A*b*d+6*B*b*c-4*C*a*d)*(b*x^2+a)^(1
/2)/b^2+1/2/b^(3/2)*(2*A*b*c-B*a*d-C*a*c)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\,(Bad+(Ca-2\,Ab)c)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Cbdx^2+6\,Bbc-2\,(2\,Ca-3\,Ab}{12\,b^2}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(B*a*d + (C*a - 2*A*b)*c)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a
)*sqrt(b)*x - a) + 2*(2*C*b*d*x^2 + 6*B*b*c - 2*(2*C*a - 3*A*b)*d + 3*(C*b
*c + B*b*d)*x)*sqrt(b*x^2 + a))/b^2, 1/6*(3*(B*a*d + (C*a - 2*A*b)*c)*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*C*b*d*x^2 + 6*B*b*c - 2*(2*C*
a - 3*A*b)*d + 3*(C*b*c + B*b*d)*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Cdx^2}{3b}+\frac{x(Bd+Cc)}{2b}+\frac{Ad+Bc-\frac{2Cad}{3b}}{b}\right)+\left(Ac-\frac{a(Bd+Cc)}{2b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right) \\ \frac{Acx+\frac{Cdx^4}{4}+\frac{x^3(Bd+Cc)}{3}+\frac{x^2(Ad+Bc)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*d*x**2/(3*b) + x*(B*d + C*c)/(2*b) + (A*d +
 B*c - 2*C*a*d/(3*b))/b) + (A*c - a*(B*d + C*c)/(2*b))*Piecewise((log(2*sq
rt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2)
, True)), Ne(b, 0)), ((A*c*x + C*d*x**4/4 + x**3*(B*d + C*c)/3 + x**2*(A*d
 + B*c)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cdx^2}{3\,b} + \frac{Ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}Bc}{b} - \frac{2\sqrt{bx^2+a}Cad}{3\,b^2} + \frac{\sqrt{bx^2+a}Ad}{b} + \frac{\sqrt{bx^2+a}(Cc+Bd)x}{2\,b} - \frac{(Cc+Bd)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*C*d*x^2/b + A*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(
b*x^2 + a)*B*c/b - 2/3*sqrt(b*x^2 + a)*C*a*d/b^2 + sqrt(b*x^2 + a)*A*d/b +
 1/2*sqrt(b*x^2 + a)*(C*c + B*d)*x/b - 1/2*(C*c + B*d)*a*arcsinh(b*x/sqrt(
a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Cdx}{b} + \frac{3\left(Cb^2c+Bb^2d\right)}{b^3}\right)x + \frac{2\left(3\,Bb^2c-2\,Cabd+3\,Ab^2d\right)}{b^3}\right) + \frac{(Cac-2\,Abc+Bad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*C*d*x/b + 3*(C*b^2*c + B*b^2*d)/b^3)*x + 2*(3*B*b^
2*c - 2*C*a*b*d + 3*A*b^2*d)/b^3) + 1/2*(C*a*c - 2*A*b*c + B*a*d)*log(abs(
-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 18.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{3\,C\,d\,x^4+4\,B\,d\,x^3+6\,A\,d\,x^2}{12\,\sqrt{a}}+\frac{2\,C\,c\,x^3+3\,B\,c\,x^2+6\,A\,c\,x}{6\,\sqrt{a}} \\ \frac{A\,d\,\sqrt{b\,x^2+a}}{b}+\frac{B\,c\,\sqrt{b\,x^2+a}}{b}+\frac{A\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{C\,d\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}-\frac{B\,a\,d\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{C\,a\,c\,\ln\left(2\,\sqrt{\right.}}{2} \end{cases}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (6*A*d*x^2 + 4*B*d*x^3 + 3*C*d*x^4)/(12*a^(1/2)) + (6*A*
c*x + 3*B*c*x^2 + 2*C*c*x^3)/(6*a^(1/2)), b ~= 0, (A*d*(a + b*x^2)^(1/2))/
b + (B*c*(a + b*x^2)^(1/2))/b + (A*c*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b
^(1/2) - (C*d*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2) - (B*a*d*log(2*b^(1
/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) - (C*a*c*log(2*b^(1/2)*x + 2*(a
+ b*x^2)^(1/2)))/(2*b^(3/2)) + (B*d*x*(a + b*x^2)^(1/2))/(2*b) + (C*c*x*(a
 + b*x^2)^(1/2))/(2*b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{6\sqrt{b\,x^2+a}\,abd-4\sqrt{b\,x^2+a}\,acd+6\sqrt{b\,x^2+a}\,b^2c+3\sqrt{b\,x^2+a}\,b^2dx+3\sqrt{b\,x^2+a}\,b\,c^2x+2\sqrt{b\,x^2+a}\,b}{6b^2}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
(6*sqrt(a + b*x**2)*a*b*d - 4*sqrt(a + b*x**2)*a*c*d + 6*sqrt(a + b*x**2)*
b**2*c + 3*sqrt(a + b*x**2)*b**2*d*x + 3*sqrt(a + b*x**2)*b*c**2*x + 2*sqr
t(a + b*x**2)*b*c*d*x**2 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a*b*c - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d
 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c**2)/(6*b**2)
```

3.98 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 114

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \frac{(cC+Bd)\sqrt{a+bx^2}}{b}+\frac{Cdx\sqrt{a+bx^2}}{2b} - \frac{(aCd-2b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}} - \frac{A\text{carctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
(B*d+C*c)*(b*x^2+a)^(1/2)/b+1/2*C*d*x*(b*x^2+a)^(1/2)/b-1/2*(a*C*d-2*b*(A*
d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-A*c*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\sqrt{a+bx^2}}\,dx=\frac{1}{2}\left(\frac{(2cC+2Bd+Cdx)\sqrt{a+bx^2}}{b}+\frac{4Ac\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{(aCd-2b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}\right)$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x*Sqrt[a + b*x^2]),x]
```

output
```
(((2*c*C + 2*B*d + C*d*x)*Sqrt[a + b*x^2])/b + (4*A*c*ArcTanh[(Sqrt[b]*x -
 Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + ((a*C*d - 2*b*(B*c + A*d))*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/2
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2340, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{2b(cC+Bd)x^2-(aCd-2b(Bc+Ad))x+2Abc}{x\sqrt{bx^2+a}}dx}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{b(2Abc-(aCd-2b(Bc+Ad))x)}{x\sqrt{bx^2+a}}dx}{b}+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Abc-(aCd-2b(Bc+Ad))x}{x\sqrt{bx^2+a}}dx+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{538}$$

$$\frac{-(aCd-2b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx+2Abc\int \frac{1}{x\sqrt{bx^2+a}}dx+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{224}$$

$$\frac{-(aCd-2b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2Abc\int \frac{1}{x\sqrt{bx^2+a}}dx+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2Abc\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-2b(Ad+Bc))}{\sqrt{b}}+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{243}$$

$$\frac{Abc\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-2b(Ad+Bc))}{\sqrt{b}}+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{73}$$

$$\frac{2Ac\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-2b(Ad+Bc))}{\sqrt{b}}+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd-2b(Ad+Bc))}{\sqrt{b}}-\frac{2Abc\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+2\sqrt{a+bx^2}(Bd+cC)}{2b}+\frac{Cdx\sqrt{a+bx^2}}{2b}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x*Sqrt[a + b*x^2]),x]
```

output
```
(C*d*x*Sqrt[a + b*x^2])/(2*b) + (2*(c*C + B*d)*Sqrt[a + b*x^2] - ((a*C*d -
 2*b*(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (2*A*b*c
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.29

method	result
default	$\frac{Ad\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{Bc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{Ac\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+\frac{Bd\sqrt{b\,x^{2}+a}}{b}+\frac{Cc\sqrt{b\,x^{2}+a}}{b}+dC\Bigg(\frac{x\sqrt{}}{}$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+B*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)/b^(1/2)-A*c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+B*d/b*(b*x^2+a
)^(1/2)+C*c/b*(b*x^2+a)^(1/2)+d*C*(1/2*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 486, normalized size of antiderivative = 4.26

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{2\,A\sqrt{a}b^2c\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+(2\,Babc-(Ca^2-2\,Aab)d)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-\right.}{4\,ab^2}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*A*sqrt(a)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + (2*B*a*b*c - (C*a^2 - 2*A*a*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + 2*(C*a*b*d*x + 2*C*a*b*c + 2*B*a*b*d)*sqrt(b*x^2 + a
))/(a*b^2), 1/2*(A*sqrt(a)*b^2*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) - (2*B*a*b*c - (C*a^2 - 2*A*a*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) + (C*a*b*d*x + 2*C*a*b*c + 2*B*a*b*d)*sqrt(b*x^2 + a))/(a
*b^2), 1/4*(4*A*sqrt(-a)*b^2*c*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*B*a
*b*c - (C*a^2 - 2*A*a*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(C*a*b*d*x + 2*C*a*b*c + 2*B*a*b*d)*sqrt(b*x^2 + a))/(a*b^2)
, 1/2*(2*A*sqrt(-a)*b^2*c*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*B*a*b*c
- (C*a^2 - 2*A*a*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (C*a*
b*d*x + 2*C*a*b*c + 2*B*a*b*d)*sqrt(b*x^2 + a))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.22

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx$$
$$= Ad\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{Ac\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$
$$+ Bc\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$+ Bd\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) + Cc\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$+ Cd\left(\begin{cases}-\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0\\ \frac{x^3}{3\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
A*d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) &
 Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - A*c*as
inh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*c*Piecewise((log(2*sqrt(b)*sqrt(a + b
*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(
b, 0)), (x/sqrt(a), True)) + B*d*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)),
 (x**2/(2*sqrt(a)), True)) + C*c*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)),
 (x**2/(2*sqrt(a)), True)) + C*d*Piecewise((-a*Piecewise((log(2*sqrt(b)*sq
rt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))
/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cdx}{2\,b} + \frac{Bc\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Cad\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{Ad\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Ac\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}Cc}{b} + \frac{\sqrt{bx^2+a}Bd}{b}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*d*x/b + B*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*C*a
*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + A*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A
*c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*C*c/b + sqrt(b*
x^2 + a)*B*d/b
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{2\,C\,d\,x^3+3\,B\,d\,x^2+6\,A\,d\,x}{6\,\sqrt{a}}+\frac{2\,B\,c\,x+C\,c\,x^2+2\,A\,c\,\ln(x)}{2\,\sqrt{a}} \\ \frac{B\,d\,\sqrt{b\,x^2+a}}{b}-\frac{A\,c\,\ln\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}}{x}\right)}{\sqrt{a}}+\frac{C\,c\,\sqrt{b\,x^2+a}}{b}+\frac{A\,d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{B\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{C\,a\,d\,\ln\left(2\,\sqrt{b}\,x+2\right.}{2\,b^{3/2}} \\ \int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x\,\sqrt{b\,x^2+a}}\,dx \end{cases}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(1/2)),x)
```

output
```
piecewise(b == 0, (6*A*d*x + 3*B*d*x^2 + 2*C*d*x^3)/(6*a^(1/2)) + (2*B*c*x
 + C*c*x^2 + 2*A*c*log(x))/(2*a^(1/2)), 0 < b, - (A*c*log(((a + b*x^2)^(1/
2) + a^(1/2))/x))/a^(1/2) + (B*d*(a + b*x^2)^(1/2))/b + (C*c*(a + b*x^2)^(
1/2))/b + (A*d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) + (B*c*log(b^(1
/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (C*a*d*log(2*b^(1/2)*x + 2*(a + b*x^
2)^(1/2)))/(2*b^(3/2)) + (C*d*x*(a + b*x^2)^(1/2))/(2*b), ~0 <= b, int(((c
 + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(1/2)), x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\sqrt{a+bx^2}}\,dx$$
$$= \frac{2\sqrt{b\,x^2+a}\,b^2d+2\sqrt{b\,x^2+a}\,b\,c^2+\sqrt{b\,x^2+a}\,bcdx+2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2c-2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{\ }}{\sqrt{a}}\right.}{2b^2}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x)
```

output

```
(2*sqrt(a + b*x**2)*b**2*d + 2*sqrt(a + b*x**2)*b*c**2 + sqrt(a + b*x**2)*
b*c*d*x + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*
b**2*c - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b
**2*c + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d - sqrt
(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c*d + 2*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c)/(2*b**2)
```

3.99 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\sqrt{a+bx^2}}\,dx = \frac{Cd\sqrt{a+bx^2}}{b}-\frac{Ac\sqrt{a+bx^2}}{ax}+\frac{(cC+Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{(Bc+Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
C*d*(b*x^2+a)^(1/2)/b-A*c*(b*x^2+a)^(1/2)/a/x+(B*d+C*c)*arctanh(b^(1/2)*x/
(b*x^2+a)^(1/2))/b^(1/2)-(A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2
)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = \frac{(-Abc+aCdx)\sqrt{a+bx^2}}{abx} - \frac{2(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(-cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
((-(A*b*c) + a*C*d*x)*Sqrt[a + b*x^2])/(a*b*x) - (2*(B*c + A*d)*ArcTanh[(-
(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + ((-(c*C) - B*d)*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{aCdx^2+a(cC+Bd)x+a(Bc+Ad)}{x\sqrt{bx^2+a}}dx}{a} - \frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aCdx^2+a(cC+Bd)x+a(Bc+Ad)}{x\sqrt{bx^2+a}}dx}{a} - \frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 2340$$

$$\frac{\frac{\int \frac{ab(Bc+Ad+(cC+Bd)x)}{x\sqrt{bx^2+a}}dx}{b}+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 27$$

$$\frac{a\int \frac{Bc+Ad+(cC+Bd)x}{x\sqrt{bx^2+a}}dx+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 538$$

$$\frac{a\left((Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+(Bd+cC)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 224$$

$$\frac{a\left((Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+(Bd+cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left((Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)}{\sqrt{b}}\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 243$$

$$\frac{a\left(\frac{1}{2}(Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)}{\sqrt{b}}\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 73$$

$$\frac{a\left(\frac{(Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)}{\sqrt{b}}\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \; 221$$

$$\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad+Bc)}{\sqrt{a}}\right)+\frac{aCd\sqrt{a+bx^2}}{b}}{a}-\frac{Ac\sqrt{a+bx^2}}{ax}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*c*Sqrt[a + b*x^2])/(a*x)) + ((a*C*d*Sqrt[a + b*x^2])/b + a*(((c*C + B
*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((B*c + A*d)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

method	result	size
default	$\frac{Bd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{Cc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{(Ad+Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}-\frac{Ac\sqrt{b\,x^{2}+a}}{ax}+\frac{Cd\sqrt{b\,x^{2}+a}}{b}$	115
risch	$\frac{Bd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{Cc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{(Ad+Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}-\frac{Ac\sqrt{b\,x^{2}+a}}{ax}+\frac{Cd\sqrt{b\,x^{2}+a}}{b}$	115

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+C*c*ln(b^(1/2)*x+(b*x^2+a)^(1/2)
)/b^(1/2)-(A*d+B*c)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A*c*(b*x
^2+a)^(1/2)/a/x+C*d*(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 443, normalized size of antiderivative = 4.30

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(Cac+Bad)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(Bbc+Abd)\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+}{2\,abx}\right.$$
$$-\frac{2\,(Cac+Bad)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbc+Abd)\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(Cadx-Ab}{2\,abx}$$
$$-\frac{(Cac+Bad)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbc+Abd)\sqrt{-a}x\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(Cadx-Abc)\sqrt{bx^2}}{abx}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*c + B*a*d)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) + (B*b*c + A*b*d)*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a)
+ 2*a)/x^2) + 2*(C*a*d*x - A*b*c)*sqrt(b*x^2 + a))/(a*b*x), -1/2*(2*(C*a*c
 + B*a*d)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*c + A*b*d)*
sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(C*a*d*x
 - A*b*c)*sqrt(b*x^2 + a))/(a*b*x), 1/2*(2*(B*b*c + A*b*d)*sqrt(-a)*x*arct
an(sqrt(b*x^2 + a)*sqrt(-a)/a) + (C*a*c + B*a*d)*sqrt(b)*x*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a*d*x - A*b*c)*sqrt(b*x^2 + a))/(a
*b*x), -((C*a*c + B*a*d)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (
B*b*c + A*b*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (C*a*d*x -
A*b*c)*sqrt(b*x^2 + a))/(a*b*x)]
```

Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Ad\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + Bd\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{Bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + Cc\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + Cd\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a - A*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(
a) + B*d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - B
*c*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + C*c*Piecewise((log(2*sqrt(b)*sqrt(
a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2)
, Ne(b, 0)), (x/sqrt(a), True)) + C*d*Piecewise((sqrt(a + b*x**2)/b, Ne(b,
 0)), (x**2/(2*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\sqrt{a+bx^2}}\,dx=\frac{Cc\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{Bd\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{Bc\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}-\frac{Ad\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}Cd}{b}-\frac{\sqrt{bx^2+a}Ac}{ax}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
C*c*arcsinh(b*x/sqrt(a*b))/sqrt(b) + B*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) -
B*c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - A*d*arcsinh(a/(sqrt(a*b)*abs(x
)))/sqrt(a) + sqrt(b*x^2 + a)*C*d/b - sqrt(b*x^2 + a)*A*c/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\sqrt{a+bx^2}}\,dx=\frac{2\,A\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{\sqrt{bx^2+a}Cd}{b}+\frac{2\,(Bc+Ad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{(Cc+Bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*A*sqrt(b)*c/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + sqrt(b*x^2 + a)*C*d/
b + 2*(B*c + A*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a)
 - (C*c + B*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{C\,d\,\sqrt{b\,x^2+a}}{b} - \frac{B\,c\,\ln\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}}{x}\right)}{\sqrt{a}} - \frac{A\,d\,\ln\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}}{x}\right)}{\sqrt{a}} + \frac{B\,d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} + \frac{C\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{A\,c\,\sqrt{b\,x}}{a\,x} \\ \int \frac{(c+d\,x)\,(C\,x^2+B\,x+A)}{x^2\,\sqrt{b\,x^2+a}}\,dx \end{cases}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
piecewise(0 < b, - (A*d*log(((a + b*x^2)^(1/2) + a^(1/2))/x))/a^(1/2) - (B
*c*log(((a + b*x^2)^(1/2) + a^(1/2))/x))/a^(1/2) + (C*d*(a + b*x^2)^(1/2))
/b + (B*d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) + (C*c*log(b^(1/2)*x
 + (a + b*x^2)^(1/2)))/b^(1/2) - (A*c*(a + b*x^2)^(1/2))/(a*x), ~0 < b, in
t(((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(1/2)), x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.59

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-2\sqrt{b\,x^2+a}\,abc + 2\sqrt{b\,x^2+a}\,acdx + \sqrt{a}\log\left(\frac{-\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{b}\,\sqrt{b\,x^2+a}\,x-\sqrt{b}\,\sqrt{a}\,x+a+b\,x^2}{\sqrt{a}\,\sqrt{b\,x^2+a}+\sqrt{b}\,\sqrt{a}\,x}\right)abdx + \sqrt{a}\log\left(\frac{-\sqrt{}}{}\right.}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b*c + 2*sqrt(a + b*x**2)*a*c*d*x + sqrt(a)*log((
- sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*
x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b*d*x +
sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sq
rt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*
x))*b**2*c*x - sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*
x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt
(b)*sqrt(a)*x))*a*b*d*x - sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*
sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x
**2) + sqrt(b)*sqrt(a)*x))*b**2*c*x + 2*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a*b*d*x + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/s
qrt(a))*a*c**2*x - 2*sqrt(b)*a*b*c*x)/(2*a*b*x)
```

3.100 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 119

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = -\frac{Ac\sqrt{a+bx^2}}{2ax^2} - \frac{(Bc+Ad)\sqrt{a+bx^2}}{ax} + \frac{Cd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{(Abc-2a(cC+Bd))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2*A*c*(b*x^2+a)^(1/2)/a/x^2-(A*d+B*c)*(b*x^2+a)^(1/2)/a/x+C*d*arctanh(b
^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)+1/2*(A*b*c-2*a*(B*d+C*c))*arctanh((b*x^2
+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(2Bcx+A(c+2dx))}{2ax^2} - \frac{Abc\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2(cC+Bd)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{Cd\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(Sqrt[a + b*x^2]*(2*B*c*x + A*(c + 2*d*x)))/(a*x^2) - (A*b*c*ArcTanh[
(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (2*(c*C + B*d)*ArcTanh[(
-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (C*d*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2338, 25, 2338, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \textcolor{blue}{2338}$$

$$-\frac{\int -\frac{2aCdx^2-(Abc-2a(cC+Bd))x+2a(Bc+Ad)}{x^2\sqrt{bx^2+a}}dx}{2a} - \frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2aCdx^2-(Abc-2a(cC+Bd))x+2a(Bc+Ad)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int \frac{a(Abc-2a(cC+Bd)-2aCdx)}{x\sqrt{bx^2+a}}dx}{a}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$\frac{-\int \frac{Abc-2a(cC+Bd)-2aCdx}{x\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 538$$

$$\frac{-(Abc-2a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx+2aCd\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 224$$

$$\frac{-(Abc-2a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx+2aCd\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 219$$

$$\frac{-(Abc-2a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}+\frac{2aCd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 243$$

$$\frac{-\frac{1}{2}(Abc-2a(Bd+cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}+\frac{2aCd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 73$$

$$\frac{-\frac{(Abc-2a(Bd+cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}+\frac{2aCd\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Abc-2a(Bd+cC))}{\sqrt{a}}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}+\frac{2aCd\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*c*Sqrt[a + b*x^2])/(a*x^2) + ((-2*(B*c + A*d)*Sqrt[a + b*x^2])/x +
 (2*a*C*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] + ((A*b*c - 2*a*(c
*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2Adx+2Bcx+Ac)}{2a\,x^{2}}-\frac{-\frac{(Abc-2Bad-2Cac)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}-\frac{2aCd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}}{2a}$
default	$\frac{dC\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{(Ad+Bc)\sqrt{b\,x^{2}+a}}{ax}-\frac{(Bd+Cc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+Ac\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}}}{x}\right.}{2a^{\frac{3}{2}}}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(2*A*d*x+2*B*c*x+A*c)/a/x^2-1/2/a*(-(A*b*c-2*B*a*d-2*
C*a*c)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-2*a*C*d*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 527, normalized size of antiderivative = 4.43

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{2\,Ca^2\sqrt{b}dx^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(2\,Babd+(2\,Cab-Ab^2)c)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}}{x^2}\right.}{4\,a^2bx^2}\right.$$

$$-\frac{4\,Ca^2\sqrt{-b}dx^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Babd+(2\,Cab-Ab^2)c)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Aa}{4\,a^2bx^2}$$

$$-\frac{2\,Ca^2\sqrt{-b}dx^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Babd+(2\,Cab-Ab^2)c)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(Aabc+}{2\,a^2bx^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*C*a^2*sqrt(b)*d*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + (2*B*a*b*d + (2*C*a*b - A*b^2)*c)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(A*a*b*c + 2*(B*a*b*c + A*a*b*d)*x)*sqrt(b
*x^2 + a))/(a^2*b*x^2), -1/4*(4*C*a^2*sqrt(-b)*d*x^2*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (2*B*a*b*d + (2*C*a*b - A*b^2)*c)*sqrt(a)*x^2*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a*b*c + 2*(B*a*b*c + A*a*b*
d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^2), 1/2*(C*a^2*sqrt(b)*d*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (2*B*a*b*d + (2*C*a*b - A*b^2)*c)*sqr
t(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (A*a*b*c + 2*(B*a*b*c + A*a
*b*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^2), -1/2*(2*C*a^2*sqrt(-b)*d*x^2*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*a*b*d + (2*C*a*b - A*b^2)*c)*sqrt(-a)*
x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (A*a*b*c + 2*(B*a*b*c + A*a*b*d)*
x)*sqrt(b*x^2 + a))/(a^2*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{Abc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + Cd\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{Cc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x) - A*sqrt(b)*d*sqrt(a/(b*x**2) +
1)/a + A*b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - B*sqrt(b)*c*sqrt(a/
(b*x**2) + 1)/a - B*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + C*d*Piecewise((
log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*
log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - C*c*asinh(sqrt(a)/(sq
rt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \frac{Cd\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Cc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Abc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{Bd\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{\sqrt{bx^2+a}Bc}{ax} - \frac{\sqrt{bx^2+a}Ad}{ax} - \frac{\sqrt{bx^2+a}Ac}{2\,ax^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
C*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - C*c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqr
t(a) + 1/2*A*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - B*d*arcsinh(a/(sq
rt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*B*c/(a*x) - sqrt(b*x^2 + a)*A*d
/(a*x) - 1/2*sqrt(b*x^2 + a)*A*c/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(99) = 198$.

Time = 0.18 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.89

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = -\frac{Cd\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{(2\,Cac-Abc+2\,Bad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}d+\left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-C*d*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + (2*C*a*c - A*b*c + 2
*B*a*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((s
qrt(b)*x - sqrt(b*x^2 + a))^3*A*b*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*
a*sqrt(b)*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*d + (sqrt(b)*x
 - sqrt(b*x^2 + a))*A*a*b*c - 2*B*a^2*sqrt(b)*c - 2*A*a^2*sqrt(b)*d)/(((sq
rt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)\,(C\,x^2+B\,x+A)}{x^3\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.35

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,abc-2\sqrt{b\,x^2+a}\,abdx-2\sqrt{b\,x^2+a}\,b^2cx-\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2c\,x^2+2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2}\cdots}{}\right.}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*b*c - 2*sqrt(a + b*x**2)*a*b*d*x - 2*sqrt(a + b*x**
2)*b**2*c*x - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)
)*b**2*c*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*b**2*d*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*b*c**2*x**2 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*
x)/sqrt(a))*b**2*c*x**2 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*b**2*d*x**2 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
sqrt(b)*x)/sqrt(a))*b*c**2*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b
)*x)/sqrt(a))*a*c*d*x**2)/(2*a*b*x**2)
```

3.101 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 132

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{Ac\sqrt{a+bx^2}}{3ax^3} - \frac{(Bc+Ad)\sqrt{a+bx^2}}{2ax^2} + \frac{(2Abc-3a(cC+Bd))\sqrt{a+bx^2}}{3a^2x} - \frac{(2aCd-b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/3*A*c*(b*x^2+a)^(1/2)/a/x^3-1/2*(A*d+B*c)*(b*x^2+a)^(1/2)/a/x^2+1/3*(2*
A*b*c-3*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^2/x-1/2*(2*a*C*d-b*(A*d+B*c))*arcta
nh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx$$
$$= -\frac{\sqrt{a+bx^2}(-4Abcx^2+aA(2c+3dx)+3ax(2cCx+B(c+2dx)))}{6a^2x^3} + \frac{2Cd\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{b(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/6*(Sqrt[a + b*x^2]*(-4*A*b*c*x^2 + a*A*(2*c + 3*d*x) + 3*a*x*(2*c*C*x +
 B*(c + 2*d*x))))/(a^2*x^3) + (2*C*d*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])
/Sqrt[a]])/Sqrt[a] + (b*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2
])/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{3aCdx^2-(2Abc-3a(cC+Bd))x+3a(Bc+Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3aCdx^2-(2Abc-3a(cC+Bd))x+3a(Bc+Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{2338}$$

$$\frac{-\frac{\int \frac{a(2(2Abc-3a(cC+Bd))+3(bBc+Abd-2aCd)x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{-\frac{1}{2}\int \frac{2(2Abc-3a(cC+Bd))+3(bBc+Abd-2aCd)x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{534}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Abc-3a(Bd+cC))}{ax}-3(-2aCd+Abd+bBc)\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Abc-3a(Bd+cC))}{ax}-\frac{3}{2}(-2aCd+Abd+bBc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(2Abc-3a(Bd+cC))}{ax}-\frac{3(-2aCd+Abd+bBc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{1}{2}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(-2aCd+Abd+bBc)}{\sqrt{a}}+\frac{2\sqrt{a+bx^2}(2Abc-3a(Bd+cC))}{ax}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*c*Sqrt[a + b*x^2])/(a*x^3) + ((-3*(B*c + A*d)*Sqrt[a + b*x^2])/(2*
x^2) + ((2*(2*A*b*c - 3*a*(c*C + B*d))*Sqrt[a + b*x^2])/(a*x) + (3*(b*B*c
+ A*b*d - 2*a*C*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-4Abc\,x^{2}+6Bad\,x^{2}+6Cac\,x^{2}+3Aadx+3Bacx+2Aac\right)}{6a^{2}x^{3}}+\frac{\left(Abd+Bbc-2aCd\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$
default	$(Ad+Bc)\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{(Bd+Cc)\sqrt{b\,x^{2}+a}}{ax}+Ac\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{dC}{}$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(-4*A*b*c*x^2+6*B*a*d*x^2+6*C*a*c*x^2+3*A*a*d*x+3*B*a
*c*x+2*A*a*c)/a^2/x^3+1/2*(A*b*d+B*b*c-2*C*a*d)/a^(3/2)*ln((2*a+2*a^(1/2)*
(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx$$

$$=\left[\frac{3\,(Bbc-(2\,Ca-Ab)d)\sqrt{a}x^3\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Aac+2\,(3\,Bad+(3\,Ca-2\,Ab)c)x^2+3}{12\,a^2x^3}\right.$$

$$-\frac{3\,(Bbc-(2\,Ca-Ab)d)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(2\,Aac+2\,(3\,Bad+(3\,Ca-2\,Ab)c)x^2+3\,(B}{6\,a^2x^3}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(B*b*c - (2*C*a - A*b)*d)*sqrt(a)*x^3*log(-(b*x^2 + 2*sqrt(b*x^2
+ a)*sqrt(a) + 2*a)/x^2) - 2*(2*A*a*c + 2*(3*B*a*d + (3*C*a - 2*A*b)*c)*x^
2 + 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a))/(a^2*x^3), -1/6*(3*(B*b*c - (2*C
*a - A*b)*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*A*a*c +
2*(3*B*a*d + (3*C*a - 2*A*b)*c)*x^2 + 3*(B*a*c + A*a*d)*x)*sqrt(b*x^2 + a)
)/(a^2*x^3)]
```

Sympy [A] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{2Ab^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Abd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{B\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{Bbc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{C\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Cd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - A*sqrt(b)*d*sqrt(a/(b*x**2)
 + 1)/(2*a*x) + 2*A*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(3*a**2) + A*b*d*asinh
(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - B*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a
*x) - B*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + B*b*c*asinh(sqrt(a)/(sqrt(b)*x)
)/(2*a**(3/2)) - C*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a - C*d*asinh(sqrt(a)/(s
qrt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{Cd\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{(Bc+Ad)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}Cc}{ax} + \frac{2\sqrt{bx^2+a}Abc}{3\,a^2x} - \frac{\sqrt{bx^2+a}Bd}{ax} - \frac{\sqrt{bx^2+a}Ac}{3\,ax^3} - \frac{\sqrt{bx^2+a}(Bc+Ad)}{2\,ax^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-C*d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*(B*c + A*d)*b*arcsinh(a/(
sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*C*c/(a*x) + 2/3*sqrt(b*x^2 +
a)*A*b*c/(a^2*x) - sqrt(b*x^2 + a)*B*d/(a*x) - 1/3*sqrt(b*x^2 + a)*A*c/(a*
x^3) - 1/2*sqrt(b*x^2 + a)*(B*c + A*d)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(112) = 224.

Time = 0.15 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.64

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{(Bbc-2\,Cad+Abd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bbc+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Abd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}c+6\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(B*b*c - 2*C*a*d + A*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b*c + 3*(sqrt(b)*
x - sqrt(b*x^2 + a))^5*A*b*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a*sqrt(
b)*c + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*d - 12*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*C*a^2*sqrt(b)*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a
*b^(3/2)*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d - 3*(sqrt(
b)*x - sqrt(b*x^2 + a))*B*a^2*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*
b*d + 6*C*a^3*sqrt(b)*c - 4*A*a^2*b^(3/2)*c + 6*B*a^3*sqrt(b)*d)/(((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx=\int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^4\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.47

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx$$
$$=\frac{-2\sqrt{b\,x^2+a}\,a^2c-3\sqrt{b\,x^2+a}\,a^2dx+4\sqrt{b\,x^2+a}\,abc\,x^2-3\sqrt{b\,x^2+a}\,abcx-6\sqrt{b\,x^2+a}\,abd\,x^2-6\sqrt{b}}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*c - 3*sqrt(a + b*x**2)*a**2*d*x + 4*sqrt(a + b
*x**2)*a*b*c*x**2 - 3*sqrt(a + b*x**2)*a*b*c*x - 6*sqrt(a + b*x**2)*a*b*d*
x**2 - 6*sqrt(a + b*x**2)*a*c**2*x**2 - 3*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**3 + 6*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*c*d*x**3 - 3*sqrt(a)*log((sqrt(a + b*x*
*2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**3 + 3*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*x**3 - 6*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*c*d*x**3 + 3*sqrt(a)*log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**3 - 4*sqrt(b)*a*b*
c*x**3 + 2*sqrt(b)*a*b*d*x**3 + 2*sqrt(b)*a*c**2*x**3)/(6*a**2*x**3)
```

3.102 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 170

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{Ac\sqrt{a+bx^2}}{4ax^4}-\frac{(Bc+Ad)\sqrt{a+bx^2}}{3ax^3}+\frac{(3Abc-4a(cC+Bd))\sqrt{a+bx^2}}{8a^2x^2}-\frac{(3aCd-2b(Bc+Ad))\sqrt{a+bx^2}}{3a^2x}-\frac{b(3Abc-4a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/4*A*c*(b*x^2+a)^(1/2)/a/x^4-1/3*(A*d+B*c)*(b*x^2+a)^(1/2)/a/x^3+1/8*(3*
A*b*c-4*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^2/x^2-1/3*(3*a*C*d-2*b*(A*d+B*c))*(
b*x^2+a)^(1/2)/a^2/x-1/8*b*(3*A*b*c-4*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx$$
$$=\frac{\frac{\sqrt{a}\sqrt{a+bx^2}(bx^2(9Ac+16Bcx+16Adx)-2a(A(3c+4dx)+2x(3Cx(c+2dx)+B(2c+3dx))))}{x^4}+18Ab^2c\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+24a}{24a^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(b*x^2*(9*A*c + 16*B*c*x + 16*A*d*x) - 2*a*(A*(3
*c + 4*d*x) + 2*x*(3*C*x*(c + 2*d*x) + B*(2*c + 3*d*x)))))/x^4 + 18*A*b^2*
c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + 24*a*b*(c*C + B*d)*ArcT
anh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(24*a^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {2338, 25, 2338, 27, 539, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2338}$$
$$-\frac{\int -\frac{4aCdx^2-(3Abc-4a(cC+Bd))x+4a(Bc+Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{4aCdx^2-(3Abc-4a(cC+Bd))x+4a(Bc+Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$
$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(3(3Abc-4a(cC+Bd))-4(3aCd-2b(Bc+Ad))x)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 27

$$\frac{-\frac{1}{3}\int \frac{3(3Abc-4a(cC+Bd))-4(3aCd-2b(Bc+Ad))x}{x^3\sqrt{bx^2+a}}dx-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 539

$$\frac{\frac{1}{3}\left(\frac{\int \frac{8a(3aCd-2b(Bc+Ad))+3b(3Abc-4a(cC+Bd))x}{x^2\sqrt{bx^2+a}}dx}{2a}+\frac{3\sqrt{a+bx^2}(3Abc-4a(Bd+cC))}{2ax^2}\right)-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 534

$$\frac{\frac{1}{3}\left(\frac{3b(3Abc-4a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}(3aCd-2b(Ad+Bc))}{x}}{2a}+\frac{3\sqrt{a+bx^2}(3Abc-4a(Bd+cC))}{2ax^2}\right)-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 243

$$\frac{\frac{1}{3}\left(\frac{\frac{3}{2}b(3Abc-4a(Bd+cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8\sqrt{a+bx^2}(3aCd-2b(Ad+Bc))}{x}}{2a}+\frac{3\sqrt{a+bx^2}(3Abc-4a(Bd+cC))}{2ax^2}\right)-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 73

$$\frac{\frac{1}{3}\left(\frac{3(3Abc-4a(Bd+cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{8\sqrt{a+bx^2}(3aCd-2b(Ad+Bc))}{x}}{2a}+\frac{3\sqrt{a+bx^2}(3Abc-4a(Bd+cC))}{2ax^2}\right)-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 221

$$\frac{\frac{1}{3}\left(\frac{-\frac{3b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3Abc-4a(Bd+cC))}{\sqrt{a}}-\frac{8\sqrt{a+bx^2}(3aCd-2b(Ad+Bc))}{x}}{2a}+\frac{3\sqrt{a+bx^2}(3Abc-4a(Bd+cC))}{2ax^2}\right)-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
-1/4*(A*c*Sqrt[a + b*x^2])/(a*x^4) + ((-4*(B*c + A*d)*Sqrt[a + b*x^2])/(3*
x^3) + ((3*(3*A*b*c - 4*a*(c*C + B*d))*Sqrt[a + b*x^2])/(2*a*x^2) + ((-8*(
3*a*C*d - 2*b*(B*c + A*d))*Sqrt[a + b*x^2])/x - (3*b*(3*A*b*c - 4*a*(c*C +
 B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a))/3)/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-16Abd\,x^3-16Bbc\,x^3+24Cad\,x^3-9Abc\,x^2+12Bad\,x^2+12Cac\,x^2+8Aadx+8Bacx+6Aac)}{24a^2x^4}-\frac{(3Abc-4Bad-4Cac)}{8}$
default	$(Ad+Bc)\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)+(Bd+Cc)\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)+Ac\left(-\frac{\sqrt{b\,x}}{4a}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-16*A*b*d*x^3-16*B*b*c*x^3+24*C*a*d*x^3-9*A*b*c*x^2
+12*B*a*d*x^2+12*C*a*c*x^2+8*A*a*d*x+8*B*a*c*x+6*A*a*c)/a^2/x^4-1/8*(3*A*b
*c-4*B*a*d-4*C*a*c)*b/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\,(4\,Babd+(4\,Cab-3\,Ab^2)c)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(6\,Aa^2c-8\,(2\,Babc-(3\,Ca^2-2\,A\cdots}{48\,a^3x^4}\right.$$
$$-\frac{3\,(4\,Babd+(4\,Cab-3\,Ab^2)c)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(6\,Aa^2c-8\,(2\,Babc-(3\,Ca^2-2\,Aab)d\cdots}{24\,a^3x^4}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(4*B*a*b*d + (4*C*a*b - 3*A*b^2)*c)*sqrt(a)*x^4*log(-(b*x^2 + 2*s
qrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(6*A*a^2*c - 8*(2*B*a*b*c - (3*C*a^
2 - 2*A*a*b)*d)*x^3 + 3*(4*B*a^2*d + (4*C*a^2 - 3*A*a*b)*c)*x^2 + 8*(B*a^2
*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^3*x^4), -1/24*(3*(4*B*a*b*d + (4*C*a*
b - 3*A*b^2)*c)*sqrt(-a)*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (6*A*a^2
*c - 8*(2*B*a*b*c - (3*C*a^2 - 2*A*a*b)*d)*x^3 + 3*(4*B*a^2*d + (4*C*a^2 -
 3*A*a*b)*c)*x^2 + 8*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^3*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(155) = 310$.

Time = 5.76 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.01

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{Ac}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{A\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{3Ab^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Ab^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{3Ab^2c\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{B\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{B\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{2Bb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Bbd\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{C\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{C\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{Cbc\,\text{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-A*c/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c/(8*a*x**3*sqrt(a/
(b*x**2) + 1)) - A*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 3*A*b**(3/2
)*c/(8*a**2*x*sqrt(a/(b*x**2) + 1)) + 2*A*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/
(3*a**2) - 3*A*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*
c*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - B*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*
x) + 2*B*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/(3*a**2) + B*b*d*asinh(sqrt(a)/(s
qrt(b)*x))/(2*a**(3/2)) - C*sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x) - C*sqr
t(b)*d*sqrt(a/(b*x**2) + 1)/a + C*b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/
2))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{3\,Ab^2c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{(Cc+Bd)b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}Cd}{ax}+\frac{3\,\sqrt{bx^2+a}Abc}{8\,a^2x^2} + \frac{2\,\sqrt{bx^2+a}(Bc+Ad)b}{3\,a^2x}-\frac{\sqrt{bx^2+a}(Cc+Bd)}{2\,ax^2} - \frac{\sqrt{bx^2+a}Ac}{4\,ax^4}-\frac{\sqrt{bx^2+a}(Bc+Ad)}{3\,ax^3}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*A*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*(C*c + B*d)*b*arc
sinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*C*d/(a*x) + 3/8*sqrt(
b*x^2 + a)*A*b*c/(a^2*x^2) + 2/3*sqrt(b*x^2 + a)*(B*c + A*d)*b/(a^2*x) - 1
/2*sqrt(b*x^2 + a)*(C*c + B*d)/(a*x^2) - 1/4*sqrt(b*x^2 + a)*A*c/(a*x^4) -
 1/3*sqrt(b*x^2 + a)*(B*c + A*d)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(146) = 292.

Time = 0.17 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.75

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{\left(4\,Cabc-3\,Ab^2c+4\,Babd\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2} + \frac{12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Cabc-9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Ab^2c+12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^7Babd+24\left(\sqrt{b}x\right.}{}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*C*a*b*c - 3*A*b^2*c + 4*B*a*b*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
 a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C
*a*b*c - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*b^2*c + 12*(sqrt(b)*x - sqrt(
b*x^2 + a))^7*B*a*b*d + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*sqrt(b)*d
 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c + 33*(sqrt(b)*x - sqrt(b*x
^2 + a))^5*A*a*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*d + 48*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c - 72*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*C*a^3*sqrt(b)*d + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/
2)*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c + 33*(sqrt(b)*x - sqrt
(b*x^2 + a))^3*A*a^2*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b*d
- 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c + 72*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*C*a^4*sqrt(b)*d - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3
*b^(3/2)*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b*c - 9*(sqrt(b)*x - s
qrt(b*x^2 + a))*A*a^3*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b*d +
 16*B*a^4*b^(3/2)*c - 24*C*a^5*sqrt(b)*d + 16*A*a^4*b^(3/2)*d)/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^4*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx=\int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^5\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.23

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\sqrt{a+bx^2}}\,dx$$
$$=\frac{-6\sqrt{b\,x^2+a}\,a^2c-8\sqrt{b\,x^2+a}\,a^2dx+9\sqrt{b\,x^2+a}\,abc\,x^2-8\sqrt{b\,x^2+a}\,abcx+16\sqrt{b\,x^2+a}\,abd\,x^3-12\sqrt{}}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2*c - 8*sqrt(a + b*x**2)*a**2*d*x + 9*sqrt(a + b
*x**2)*a*b*c*x**2 - 8*sqrt(a + b*x**2)*a*b*c*x + 16*sqrt(a + b*x**2)*a*b*d
*x**3 - 12*sqrt(a + b*x**2)*a*b*d*x**2 - 12*sqrt(a + b*x**2)*a*c**2*x**2 -
 24*sqrt(a + b*x**2)*a*c*d*x**3 + 16*sqrt(a + b*x**2)*b**2*c*x**3 + 9*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*x**4 - 12
*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*d*x**4
 - 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**2
*x**4 - 9*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*
*2*c*x**4 + 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a
))*b**2*d*x**4 + 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*b*c**2*x**4 - 16*sqrt(b)*a*b*d*x**4 + 12*sqrt(b)*a*c*d*x**4 - 16*s
qrt(b)*b**2*c*x**4)/(24*a**2*x**4)
```

3.103 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 208

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = -\frac{Ac\sqrt{a+bx^2}}{5ax^5} - \frac{(Bc+Ad)\sqrt{a+bx^2}}{4ax^4} + \frac{(4Abc-5a(cC+Bd))\sqrt{a+bx^2}}{15a^2x^3} - \frac{(4aCd-3b(Bc+Ad))\sqrt{a+bx^2}}{8a^2x^2} - \frac{2b(4Abc-5a(cC+Bd))\sqrt{a+bx^2}}{15a^3x} + \frac{b(4aCd-3b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/5*A*c*(b*x^2+a)^(1/2)/a/x^5-1/4*(A*d+B*c)*(b*x^2+a)^(1/2)/a/x^4+1/15*(4
*A*b*c-5*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^2/x^3-1/8*(4*a*C*d-3*b*(A*d+B*c))*
(b*x^2+a)^(1/2)/a^2/x^2-2/15*b*(4*A*b*c-5*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^3
/x+1/8*b*(4*a*C*d-3*b*(A*d+B*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-64Ab^2cx^4+abx^2(A(32c+45dx)+5x(9Bc+16cCx+16Bdx))-2a^2(3A(4c+5dx)+5x(}{120a^3x^5}$$
$$-\frac{bCd\,\mathrm{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{3b^2(Bc+Ad)\mathrm{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-64*A*b^2*c*x^4 + a*b*x^2*(A*(32*c + 45*d*x) + 5*x*(9*B*
c + 16*c*C*x + 16*B*d*x)) - 2*a^2*(3*A*(4*c + 5*d*x) + 5*x*(3*B*c + 4*c*C*
x + 4*B*d*x + 6*C*d*x^2))))/(120*a^3*x^5) - (b*C*d*ArcTanh[(Sqrt[b]*x - Sq
rt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (3*b^2*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x
) + Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(5/2))
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2338, 25, 2338, 27, 539, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2338$$
$$-\frac{\int -\frac{5aCdx^2-(4Abc-5a(cC+Bd))x+5a(Bc+Ad)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5aCdx^2-(4Abc-5a(cC+Bd))x+5a(Bc+Ad)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 2338

$$\frac{-\frac{\int \frac{a(4(4Abc-5a(cC+Bd))-5(4aCd-3b(Bc+Ad))x)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 27

$$\frac{-\frac{1}{4}\int \frac{4(4Abc-5a(cC+Bd))-5(4aCd-3b(Bc+Ad))x}{x^4\sqrt{bx^2+a}}dx-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 539

$$\frac{\frac{1}{4}\left(\frac{\int \frac{15a(4aCd-3b(Bc+Ad))+8b(4Abc-5a(cC+Bd))x}{x^3\sqrt{bx^2+a}}dx}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{3ax^3}\right)-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 539

$$\frac{\frac{1}{4}\left(\frac{-\frac{\int -\frac{ab(16(4Abc-5a(cC+Bd))-15(4aCd-3b(Bc+Ad))x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{3ax^3}\right)-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{\frac{\int \frac{ab(16(4Abc-5a(cC+Bd))-15(4aCd-3b(Bc+Ad))x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{3ax^3}\right)-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 27

$$\frac{\frac{1}{4}\left(\frac{\frac{1}{2}b\int \frac{16(4Abc-5a(cC+Bd))-15(4aCd-3b(Bc+Ad))x}{x^2\sqrt{bx^2+a}}dx-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{3ax^3}\right)-\frac{5\sqrt{a+bx^2}(Ad+Bc)}{4x^4}}{5a}-\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 534$$

$$\frac{\frac{1}{4}\left(\frac{\frac{1}{2}b\left(-15(4aCd-3b(Ad+Bc))\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{16\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{ax}\right)-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+cC}{3ax^3}\right.}{5a}$$

$$\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{4}\left(\frac{\frac{1}{2}b\left(-\frac{15}{2}(4aCd-3b(Ad+Bc))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{16\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{ax}\right)-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+}{3ax^3}\right.}{5a}$$

$$\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{1}{4}\left(\frac{\frac{1}{2}b\left(-\frac{15(4aCd-3b(Ad+Bc))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{16\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{ax}\right)-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+c}{3ax^3}\right.}{5a}$$

$$\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{1}{4}\left(\frac{\frac{1}{2}b\left(\frac{15\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4aCd-3b(Ad+Bc))}{\sqrt{a}}-\frac{16\sqrt{a+bx^2}(4Abc-5a(Bd+cC))}{ax}\right)-\frac{15\sqrt{a+bx^2}(4aCd-3b(Ad+Bc))}{2x^2}}{3a}+\frac{4\sqrt{a+bx^2}(4Abc-5a(Bd+}{3ax^3}\right.}{5a}$$

$$\frac{Ac\sqrt{a+bx^2}}{5ax^5}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/5*(A*c*Sqrt[a + b*x^2])/(a*x^5) + ((-5*(B*c + A*d)*Sqrt[a + b*x^2])/(4*
x^4) + ((4*(4*A*b*c - 5*a*(c*C + B*d))*Sqrt[a + b*x^2])/(3*a*x^3) + ((-15*
(4*a*C*d - 3*b*(B*c + A*d))*Sqrt[a + b*x^2])/(2*x^2) + (b*((-16*(4*A*b*c -
 5*a*(c*C + B*d))*Sqrt[a + b*x^2])/(a*x) + (15*(4*a*C*d - 3*b*(B*c + A*d))
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2)/(3*a))/4)/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.83

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(64A\,b^{2}c\,x^{4}-80Babd\,x^{4}-80Cabc\,x^{4}-45Abd\,x^{3}a-45Bbc\,x^{3}a+60C\,a^{2}d\,x^{3}-32Aabc\,x^{2}+40B\,a^{2}d\,x^{2}+40C\,a^{2}c\,x^{2}+30A}{120a^{3}x^{5}}$
default	$(Ad+Bc)\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+(Bd+Cc)\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(64*A*b^2*c*x^4-80*B*a*b*d*x^4-80*C*a*b*c*x^4-45*A*
a*b*d*x^3-45*B*a*b*c*x^3+60*C*a^2*d*x^3-32*A*a*b*c*x^2+40*B*a^2*d*x^2+40*C
*a^2*c*x^2+30*A*a^2*d*x+30*B*a^2*c*x+24*A*a^2*c)/a^3/x^5-1/8*(3*A*b*d+3*B*
b*c-4*C*a*d)/a^(5/2)*b*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{15\,(3\,Bb^2c-(4\,Cab-3\,Ab^2)d)\sqrt{a}x^5\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(16\,(5\,Babd+(5\,Cab-4\,Ab^2)c)x^4}{}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*(3*B*b^2*c - (4*C*a*b - 3*A*b^2)*d)*sqrt(a)*x^5*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(16*(5*B*a*b*d + (5*C*a*b - 4*A*b
^2)*c)*x^4 - 24*A*a^2*c + 15*(3*B*a*b*c - (4*C*a^2 - 3*A*a*b)*d)*x^3 - 8*(
5*B*a^2*d + (5*C*a^2 - 4*A*a*b)*c)*x^2 - 30*(B*a^2*c + A*a^2*d)*x)*sqrt(b*
x^2 + a))/(a^3*x^5), 1/120*(15*(3*B*b^2*c - (4*C*a*b - 3*A*b^2)*d)*sqrt(-a
)*x^5*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (16*(5*B*a*b*d + (5*C*a*b - 4*A
*b^2)*c)*x^4 - 24*A*a^2*c + 15*(3*B*a*b*c - (4*C*a^2 - 3*A*a*b)*d)*x^3 - 8
*(5*B*a^2*d + (5*C*a^2 - 4*A*a*b)*c)*x^2 - 30*(B*a^2*c + A*a^2*d)*x)*sqrt(
b*x^2 + a))/(a^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(196) = 392$.

Time = 7.20 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.34

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3Aa^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2Aa^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3Aa^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12Aab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8Ab^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{Ad}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}d}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3Ab^{\frac{3}{2}}d}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3Ab^2d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{Bc}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{B\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{3Bb^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Bb^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2} - \frac{3Bb^2c\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{C\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{C\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{2Cb^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Cbd\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**
5*x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*
b**(13/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x*
*6 + 15*a**3*b**6*x**8) - 12*A*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*c
*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**
3*b**6*x**8) - A*d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*d/(8*
a*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x**2) +
1)) - 3*A*b**2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - B*c/(4*sqrt(b)*
x**5*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*c/(8*a*x**3*sqrt(a/(b*x**2) + 1)) -
 B*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 3*B*b**(3/2)*c/(8*a**2*x*sq
rt(a/(b*x**2) + 1)) + 2*B*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(3*a**2) - 3*B*b
**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - C*sqrt(b)*c*sqrt(a/(b*x**2
) + 1)/(3*a*x**2) - C*sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x) + 2*C*b**(3/2
)*c*sqrt(a/(b*x**2) + 1)/(3*a**2) + C*b*d*asinh(sqrt(a)/(sqrt(b)*x))/(2*a*
*(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx = \frac{Cbd\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,(Bc+Ad)b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} - \frac{8\,\sqrt{bx^2+a}Ab^2c}{15\,a^3x} - \frac{\sqrt{bx^2+a}Cd}{2\,ax^2} + \frac{2\,\sqrt{bx^2+a}(Cc+Bd)b}{3\,a^2x} + \frac{4\,\sqrt{bx^2+a}Abc}{15\,a^2x^3} + \frac{3\,\sqrt{bx^2+a}(Bc+Ad)b}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}(Cc+Bd)}{3\,ax^3} - \frac{\sqrt{bx^2+a}Ac}{5\,ax^5} - \frac{\sqrt{bx^2+a}(Bc+Ad)}{4\,ax^4}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*C*b*d*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*(B*c + A*d)*b^2*arcs
inh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 8/15*sqrt(b*x^2 + a)*A*b^2*c/(a^3*x) -
 1/2*sqrt(b*x^2 + a)*C*d/(a*x^2) + 2/3*sqrt(b*x^2 + a)*(C*c + B*d)*b/(a^2*
x) + 4/15*sqrt(b*x^2 + a)*A*b*c/(a^2*x^3) + 3/8*sqrt(b*x^2 + a)*(B*c + A*d
)*b/(a^2*x^2) - 1/3*sqrt(b*x^2 + a)*(C*c + B*d)/(a*x^3) - 1/5*sqrt(b*x^2 +
 a)*A*c/(a*x^5) - 1/4*sqrt(b*x^2 + a)*(B*c + A*d)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(180) = 360$.

Time = 0.15 (sec) , antiderivative size = 672, normalized size of antiderivative = 3.23

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx = \frac{\left(3\,Bb^2c-4\,Cabd+3\,Ab^2d\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2}$$
$$-\frac{45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9Bb^2c-60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9Cabd+45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^9Ab^2d-210\left(\sqrt{\right.}\right.}{}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*B*b^2*c - 4*C*a*b*d + 3*A*b^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/60*(45*(sqrt(b)*x - sqrt(b*x^2 + a))^9*B*
b^2*c - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a*b*d + 45*(sqrt(b)*x - sqrt(
b*x^2 + a))^9*A*b^2*d - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2*c + 12
0*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^2*b*d - 210*(sqrt(b)*x - sqrt(b*x^2
+ a))^7*A*a*b^2*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*b^(3/2)*c -
240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*b^(3/2)*d + 560*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*C*a^3*b^(3/2)*c - 640*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^
2*b^(5/2)*c + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)*d + 210*(s
qrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b^2*c - 120*(sqrt(b)*x - sqrt(b*x^2 +
a))^3*C*a^4*b*d + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b^2*d - 400*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/2)*c + 320*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*A*a^3*b^(5/2)*c - 400*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3
/2)*d - 45*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b^2*c + 60*(sqrt(b)*x - sqr
t(b*x^2 + a))*C*a^5*b*d - 45*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^4*b^2*d + 8
0*C*a^5*b^(3/2)*c - 64*A*a^4*b^(5/2)*c + 80*B*a^5*b^(3/2)*d)/(((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^5*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx=\int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^6\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^6*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.18

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^6\sqrt{a+bx^2}}\,dx$$
$$=\frac{-24\sqrt{b\,x^2+a}\,a^3c-30\sqrt{b\,x^2+a}\,a^3dx+32\sqrt{b\,x^2+a}\,a^2bc\,x^2-30\sqrt{b\,x^2+a}\,a^2bcx+45\sqrt{b\,x^2+a}\,a^2bd\,x^{\cdots}}{\cdots}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3*c - 30*sqrt(a + b*x**2)*a**3*d*x + 32*sqrt(a
+ b*x**2)*a**2*b*c*x**2 - 30*sqrt(a + b*x**2)*a**2*b*c*x + 45*sqrt(a + b*x
**2)*a**2*b*d*x**3 - 40*sqrt(a + b*x**2)*a**2*b*d*x**2 - 40*sqrt(a + b*x**
2)*a**2*c**2*x**2 - 60*sqrt(a + b*x**2)*a**2*c*d*x**3 - 64*sqrt(a + b*x**2
)*a*b**2*c*x**4 + 45*sqrt(a + b*x**2)*a*b**2*c*x**3 + 80*sqrt(a + b*x**2)*
a*b**2*d*x**4 + 80*sqrt(a + b*x**2)*a*b*c**2*x**4 + 45*sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5 - 60*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d*x**5 + 45*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**5 - 45*s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5
 + 60*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*
d*x**5 - 45*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*
b**3*c*x**5 + 64*sqrt(b)*a*b**2*c*x**5 - 80*sqrt(b)*a*b**2*d*x**5 - 80*sqr
t(b)*a*b*c**2*x**5)/(120*a**3*x**5)
```

3.104 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 248

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = -\frac{Ac\sqrt{a+bx^2}}{6ax^6}-\frac{(Bc+Ad)\sqrt{a+bx^2}}{5ax^5} + \frac{(5Abc-6a(cC+Bd))\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5aCd-4b(Bc+Ad))\sqrt{a+bx^2}}{15a^2x^3} - \frac{b(5Abc-6a(cC+Bd))\sqrt{a+bx^2}}{16a^3x^2} + \frac{2b(5aCd-4b(Bc+Ad))\sqrt{a+bx^2}}{15a^3x} + \frac{b^2(5Abc-6a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

output
```
-1/6*A*c*(b*x^2+a)^(1/2)/a/x^6-1/5*(A*d+B*c)*(b*x^2+a)^(1/2)/a/x^5+1/24*(5
*A*b*c-6*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^2/x^4-1/15*(5*a*C*d-4*b*(A*d+B*c))
*(b*x^2+a)^(1/2)/a^2/x^3-1/16*b*(5*A*b*c-6*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^
3/x^2+2/15*b*(5*a*C*d-4*b*(A*d+B*c))*(b*x^2+a)^(1/2)/a^3/x+1/16*b^2*(5*A*b
*c-6*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.65 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\frac{\sqrt{a}\sqrt{a+bx^2}\left(b^2x^4(75Ac+128Bcx+128Adx)-2abx^2\left(A(25c+32dx)+x\left(32Bc+45cCx+45Bdx+80Cdx^2\right)\right)+a^2(8A(5c+6dx)+4x(5Cx(3c+4dx}{x^6}}{240a^{7/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(b^2*x^4*(75*A*c + 128*B*c*x + 128*A*d*x) - 2*
a*b*x^2*(A*(25*c + 32*d*x) + x*(32*B*c + 45*c*C*x + 45*B*d*x + 80*C*d*x^2)
) + a^2*(8*A*(5*c + 6*d*x) + 4*x*(5*C*x*(3*c + 4*d*x) + 3*B*(4*c + 5*d*x))
)))/x^6) - 150*A*b^3*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 18
0*a*b^2*(c*C + B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(24
0*a^(7/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2338, 25, 2338, 27, 539, 27, 539, 25, 27, 539, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2338$$
$$-\frac{\int -\frac{6aCdx^2-(5Abc-6a(cC+Bd))x+6a(Bc+Ad)}{x^6\sqrt{bx^2+a}}dx}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{6aCdx^2-(5Abc-6a(cC+Bd))x+6a(Bc+Ad)}{x^6\sqrt{bx^2+a}}dx}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int \frac{a(5(5Abc-6a(cC+Bd))-6(5aCd-4b(Bc+Ad))x)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{6\sqrt{a+bx^2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{5}\int \frac{5(5Abc-6a(cC+Bd))-6(5aCd-4b(Bc+Ad))x}{x^5\sqrt{bx^2+a}}dx-\frac{6\sqrt{a+bx^2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{5}\left(\frac{\int \frac{3(8a(5aCd-4b(Bc+Ad))+5b(5Abc-6a(cC+Bd))x)}{x^4\sqrt{bx^2+a}}dx}{4a}+\frac{5\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{4ax^4}\right)-\frac{6\sqrt{a+bx^2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\left(\frac{3\int \frac{8a(5aCd-4b(Bc+Ad))+5b(5Abc-6a(cC+Bd))x}{x^4\sqrt{bx^2+a}}dx}{4a}+\frac{5\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{4ax^4}\right)-\frac{6\sqrt{a+bx^2}(Ad+Bc)}{5x^5}}{6a}-\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{539}$$

$$\frac{\frac{1}{5}\left(\frac{3\left(-\frac{\int -\frac{ab(15(5Abc-6a(cC+Bd))-16(5aCd-4b(Bc+Ad))x)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{3x^3}\right)}{4a}+\frac{5\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{4ax^4}\right)-\frac{6\sqrt{a+b}}{}}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{\int \frac{ab(15(5Abc-6a(cC+Bd))-16(5aCd-4b(Bc+Ad))x)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{3x^3}\right)}{4a}+\frac{5\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{4ax^4}\right)-\frac{6\sqrt{a+bx^2}(}{5x^5}}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\downarrow 27$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\int \frac{15(5Abc-6a(cC+Bd))-16(5aCd-4b(Bc+Ad))x}{x^3\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{3x^3}\right)}{4a}+\frac{5\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{4ax^4}\right)-\frac{6\sqrt{a+bx^2}(A}{5x^5}}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\downarrow 539$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\left(-\frac{\int \frac{32a(5aCd-4b(Bc+Ad))+15b(5Abc-6a(cC+Bd))x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{15\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{2ax^2}\right)-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{3x^3}\right)}{4a}+\frac{5\sqrt{a+bx^2}(}{}\right.}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\downarrow 534$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\left(-\frac{15b(5Abc-6a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{32\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{x}}{2a}-\frac{15\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{2ax^2}\right)-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{3x^3}\right)}{4a}\right.}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\downarrow 243$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\left(-\frac{\frac{15}{2}b(5Abc-6a(Bd+cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{32\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{x}}{2a}-\frac{15\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{2ax^2}\right)-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc)}{3x^3}\right.}{4a}\right.}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$$\downarrow 73$$

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\left(-\frac{15(5Abc-6a(Bd+cC))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{32\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{x}}{2a}-\frac{15\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{2ax^2}\right)-\frac{8\sqrt{a+bx^2}(5aCd-4b(Ad+Bc)}{3x^3}\cdots}{4a}\cdots\right.}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

$\big\downarrow$ 221

$$\frac{\frac{1}{5}\left(\frac{3\left(\frac{1}{3}b\left(-\frac{-\frac{15b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(5Abc-6a(Bd+cC))}{\sqrt{a}}-\frac{32\sqrt{a+bx^2}(5aCd-4b(Ad+Bc))}{x}}{2a}-\frac{15\sqrt{a+bx^2}(5Abc-6a(Bd+cC))}{2ax^2}\right)-\frac{8\sqrt{a+bx^2}(5aCd-4b(\cdots}{3x^3}\cdots}{4a}\cdots\right.}{6a}$$

$$\frac{Ac\sqrt{a+bx^2}}{6ax^6}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
-1/6*(A*c*Sqrt[a + b*x^2])/(a*x^6) + ((-6*(B*c + A*d)*Sqrt[a + b*x^2])/(5*
x^5) + ((5*(5*A*b*c - 6*a*(c*C + B*d))*Sqrt[a + b*x^2])/(4*a*x^4) + (3*((-
8*(5*a*C*d - 4*b*(B*c + A*d))*Sqrt[a + b*x^2])/(3*x^3) + (b*((-15*(5*A*b*c
 - 6*a*(c*C + B*d))*Sqrt[a + b*x^2])/(2*a*x^2) - ((-32*(5*a*C*d - 4*b*(B*c
 + A*d))*Sqrt[a + b*x^2])/x - (15*b*(5*A*b*c - 6*a*(c*C + B*d))*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)))/3))/(4*a))/5)/(6*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(128A\,b^2d\,x^5+128B\,b^2c\,x^5-160Cabd\,x^5+75A\,b^2c\,x^4-90Babd\,x^4-90Cabc\,x^4-64Abd\,x^3a-64Bbc\,x^3a+80C\,a^2d\,x^3-5}{240a^3x^6}$
default	$(Ad+Bc)\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)+(Bd+Cc)\left(-\frac{\sqrt{b\,x^2+a}}{4a\,x^4}-\frac{3b\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+}{}\right.}{}\right.}{4a}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(128*A*b^2*d*x^5+128*B*b^2*c*x^5-160*C*a*b*d*x^5+75
*A*b^2*c*x^4-90*B*a*b*d*x^4-90*C*a*b*c*x^4-64*A*a*b*d*x^3-64*B*a*b*c*x^3+8
0*C*a^2*d*x^3-50*A*a*b*c*x^2+60*B*a^2*d*x^2+60*C*a^2*c*x^2+48*A*a^2*d*x+48
*B*a^2*c*x+40*A*a^2*c)/a^3/x^6+1/16*(5*A*b*c-6*B*a*d-6*C*a*c)*b^2/a^(7/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{15\,(6\,Bab^2d+(6\,Cab^2-5\,Ab^3)c)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(32\,(4\,Bab^2c-(5\,Ca^2b-4\,Aab^2)}{}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(15*(6*B*a*b^2*d + (6*C*a*b^2 - 5*A*b^3)*c)*sqrt(a)*x^6*log(-(b*x^2
 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(32*(4*B*a*b^2*c - (5*C*a^2*b
 - 4*A*a*b^2)*d)*x^5 + 40*A*a^3*c - 15*(6*B*a^2*b*d + (6*C*a^2*b - 5*A*a*b
^2)*c)*x^4 - 16*(4*B*a^2*b*c - (5*C*a^3 - 4*A*a^2*b)*d)*x^3 + 10*(6*B*a^3*
d + (6*C*a^3 - 5*A*a^2*b)*c)*x^2 + 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 +
a))/(a^4*x^6), 1/240*(15*(6*B*a*b^2*d + (6*C*a*b^2 - 5*A*b^3)*c)*sqrt(-a)*
x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (32*(4*B*a*b^2*c - (5*C*a^2*b - 4
*A*a*b^2)*d)*x^5 + 40*A*a^3*c - 15*(6*B*a^2*b*d + (6*C*a^2*b - 5*A*a*b^2)*
c)*x^4 - 16*(4*B*a^2*b*c - (5*C*a^3 - 4*A*a^2*b)*d)*x^3 + 10*(6*B*a^3*d +
(6*C*a^3 - 5*A*a^2*b)*c)*x^2 + 48*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/
(a^4*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(235) = 470$.

Time = 13.23 (sec) , antiderivative size = 1044, normalized size of antiderivative = 4.21

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**7/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**
5*x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A*a**2*
b**(13/2)*d*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x*
*6 + 15*a**3*b**6*x**8) - 12*A*a*b**(15/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A*b**(17/2)*d
*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**
3*b**6*x**8) - A*c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c/(24
*a*x**5*sqrt(a/(b*x**2) + 1)) - 5*A*b**(3/2)*c/(48*a**2*x**3*sqrt(a/(b*x**
2) + 1)) - 5*A*b**(5/2)*c/(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 5*A*b**3*c*as
inh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2)) - 3*B*a**4*b**(9/2)*c*sqrt(a/(b*x**
2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 2*B*
a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b*
*5*x**6 + 15*a**3*b**6*x**8) - 3*B*a**2*b**(13/2)*c*x**4*sqrt(a/(b*x**2) +
 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*B*a*b
**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**
6 + 15*a**3*b**6*x**8) - 8*B*b**(17/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(15*a**
5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - B*d/(4*sqrt(b)*x**5
*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*d/(8*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*B
*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*B*b**2*d*asinh(sqrt(a)/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx &= \frac{5\,Ab^3c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} \\ &\quad - \frac{3\,(Cc+Bd)b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} \\ &\quad + \frac{2\,\sqrt{bx^2+a}Cbd}{3\,a^2x} - \frac{5\,\sqrt{bx^2+a}Ab^2c}{16\,a^3x^2} \\ &\quad - \frac{8\,\sqrt{bx^2+a}(Bc+Ad)b^2}{15\,a^3x} - \frac{\sqrt{bx^2+a}Cd}{3\,ax^3} \\ &\quad + \frac{3\,\sqrt{bx^2+a}(Cc+Bd)b}{8\,a^2x^2} + \frac{5\,\sqrt{bx^2+a}Abc}{24\,a^2x^4} \\ &\quad + \frac{4\,\sqrt{bx^2+a}(Bc+Ad)b}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}(Cc+Bd)}{4\,ax^4} \\ &\quad - \frac{\sqrt{bx^2+a}Ac}{6\,ax^6} - \frac{\sqrt{bx^2+a}(Bc+Ad)}{5\,ax^5}\end{aligned}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
5/16*A*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*(C*c + B*d)*b^2*a
rcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 2/3*sqrt(b*x^2 + a)*C*b*d/(a^2*x) -
 5/16*sqrt(b*x^2 + a)*A*b^2*c/(a^3*x^2) - 8/15*sqrt(b*x^2 + a)*(B*c + A*d)
*b^2/(a^3*x) - 1/3*sqrt(b*x^2 + a)*C*d/(a*x^3) + 3/8*sqrt(b*x^2 + a)*(C*c
+ B*d)*b/(a^2*x^2) + 5/24*sqrt(b*x^2 + a)*A*b*c/(a^2*x^4) + 4/15*sqrt(b*x^
2 + a)*(B*c + A*d)*b/(a^2*x^3) - 1/4*sqrt(b*x^2 + a)*(C*c + B*d)/(a*x^4) -
 1/6*sqrt(b*x^2 + a)*A*c/(a*x^6) - 1/5*sqrt(b*x^2 + a)*(B*c + A*d)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(216) = 432$.

Time = 0.17 (sec) , antiderivative size = 918, normalized size of antiderivative = 3.70

$$\int \frac{(c + dx)(A + Bx + Cx^2)}{x^7\sqrt{a + bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(6*C*a*b^2*c - 5*A*b^3*c + 6*B*a*b^2*d)*arctan(-(sqrt(b)*x - sqrt(b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/120*(90*(sqrt(b)*x - sqrt(b*x^2 + a))
^11*C*a*b^2*c - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^11*A*b^3*c + 90*(sqrt(b)*
x - sqrt(b*x^2 + a))^11*B*a*b^2*d - 510*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*
a^2*b^2*c + 425*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3*c - 510*(sqrt(b)*x
 - sqrt(b*x^2 + a))^9*B*a^2*b^2*d - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*
a^3*b^(3/2)*d + 420*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^3*b^2*c - 990*(sqr
t(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c + 420*(sqrt(b)*x - sqrt(b*x^2 + a)
)^7*B*a^3*b^2*d - 1280*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2)*c + 1
600*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^4*b^(3/2)*d - 1280*(sqrt(b)*x - sq
rt(b*x^2 + a))^6*A*a^3*b^(5/2)*d + 420*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a
^4*b^2*c - 990*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^3*b^3*c + 420*(sqrt(b)*
x - sqrt(b*x^2 + a))^5*B*a^4*b^2*d + 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^4*
B*a^4*b^(5/2)*c - 1920*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^5*b^(3/2)*d + 1
920*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^4*b^(5/2)*d - 510*(sqrt(b)*x - sqr
t(b*x^2 + a))^3*C*a^5*b^2*c + 425*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^4*b^
3*c - 510*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^5*b^2*d - 768*(sqrt(b)*x - s
qrt(b*x^2 + a))^2*B*a^5*b^(5/2)*c + 960*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*
a^6*b^(3/2)*d - 768*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^(5/2)*d + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))*C*a^6*b^2*c - 75*(sqrt(b)*x - sqrt(b*x^2 +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)\,(C\,x^2+B\,x+A)}{x^7\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^7*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^7*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.04

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx$$
$$= \frac{-40\sqrt{b\,x^2+a}\,a^3c-48\sqrt{b\,x^2+a}\,a^3dx+50\sqrt{b\,x^2+a}\,a^2bc\,x^2-48\sqrt{b\,x^2+a}\,a^2bcx+64\sqrt{b\,x^2+a}\,a^2bd\,x^3}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c - 48*sqrt(a + b*x**2)*a**3*d*x + 50*sqrt(a
+ b*x**2)*a**2*b*c*x**2 - 48*sqrt(a + b*x**2)*a**2*b*c*x + 64*sqrt(a + b*x
**2)*a**2*b*d*x**3 - 60*sqrt(a + b*x**2)*a**2*b*d*x**2 - 60*sqrt(a + b*x**
2)*a**2*c**2*x**2 - 80*sqrt(a + b*x**2)*a**2*c*d*x**3 - 75*sqrt(a + b*x**2
)*a*b**2*c*x**4 + 64*sqrt(a + b*x**2)*a*b**2*c*x**3 - 128*sqrt(a + b*x**2)
*a*b**2*d*x**5 + 90*sqrt(a + b*x**2)*a*b**2*d*x**4 + 90*sqrt(a + b*x**2)*a
*b*c**2*x**4 + 160*sqrt(a + b*x**2)*a*b*c*d*x**5 - 128*sqrt(a + b*x**2)*b*
*3*c*x**5 - 75*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a
))*b**3*c*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**3*d*x**6 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**2*c**2*x**6 + 75*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**3*c*x**6 - 90*sqrt(a)*log((sqrt(a + b*x**2) + sqrt
(a) + sqrt(b)*x)/sqrt(a))*b**3*d*x**6 - 90*sqrt(a)*log((sqrt(a + b*x**2) +
 sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**6 + 128*sqrt(b)*a*b**2*d*x**6
- 160*sqrt(b)*a*b*c*d*x**6 + 128*sqrt(b)*b**3*c*x**6)/(240*a**3*x**6)
```

3.105 $\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 310

$$\begin{aligned}
\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx
&= -\frac{a(bc(Bc+2Ad)-ad(2cC+Bd))\sqrt{a+bx^2}}{b^3} \\
&\quad+\frac{(8Ab^2c^2+a(5aCd^2-6b(c^2C+2Bcd+Ad^2)))\,x\sqrt{a+bx^2}}{16b^3} \\
&\quad-\frac{(5aCd^2-6b(c^2C+2Bcd+Ad^2))\,x^3\sqrt{a+bx^2}}{24b^2}+\frac{Cd^2x^5\sqrt{a+bx^2}}{6b} \\
&\quad+\frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\,(a+bx^2)^{3/2}}{3b^3}+\frac{d(2cC+Bd)\,(a+bx^2)^{5/2}}{5b^3} \\
&\quad-\frac{a(8Ab^2c^2+a(5aCd^2-6b(c^2C+2Bcd+Ad^2)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}
\end{aligned}$$

output
```
-a*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/b^3+1/16*(8*A*b^2*c^2
+a*(5*a*C*d^2-6*b*(A*d^2+2*B*c*d+C*c^2)))*x*(b*x^2+a)^(1/2)/b^3-1/24*(5*a*
C*d^2-6*b*(A*d^2+2*B*c*d+C*c^2))*x^3*(b*x^2+a)^(1/2)/b^2+1/6*C*d^2*x^5*(b*
x^2+a)^(1/2)/b+1/3*(b*c*(2*A*d+B*c)-2*a*d*(B*d+2*C*c))*(b*x^2+a)^(3/2)/b^3
+1/5*d*(B*d+2*C*c)*(b*x^2+a)^(5/2)/b^3-1/16*a*(8*A*b^2*c^2+a*(5*a*C*d^2-6*
b*(A*d^2+2*B*c*d+C*c^2)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.75 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(a^2d(256cC+128Bd+75Cdx)-2ab(5Ad(32c+9dx)+Cx(45c^2+64cdx+25d^2x^2)+B(80$$
$$-\frac{a(2Ab(4bc^2-3ad^2)+a(5aCd^2-6bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

input
```
Integrate[(x^2*(c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(a^2*d*(256*c*C + 128*B*d + 75*C*d*x) - 2*a*b*(5*A*d*(32*
c + 9*d*x) + C*x*(45*c^2 + 64*c*d*x + 25*d^2*x^2) + B*(80*c^2 + 90*c*d*x +
 32*d^2*x^2)) + 4*b^2*x*(5*A*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + x*(2*B*(10*c^
2 + 15*c*d*x + 6*d^2*x^2) + C*x*(15*c^2 + 24*c*d*x + 10*d^2*x^2)))))/(240*
b^3) - (a*(2*A*b*(4*b*c^2 - 3*a*d^2) + a*(5*a*C*d^2 - 6*b*c*(c*C + 2*B*d))
)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2185, 25, 2185, 25, 2185, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2185}$$
$$\frac{\int -\frac{(c+dx)^2\left(bd^3(13cC-6Bd)x^3+d^2\left(8bCc^2-6Abd^2+5aCd^2\right)x^2+cCd\left(bc^2+10ad^2\right)x+5ac^2Cd^2\right)}{\sqrt{bx^2+a}}dx}{6bd^4}+\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}$$

$\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\int \frac{(c+dx)^2\left(bd^3(13cC-6Bd)x^3+d^2\left(8bCc^2-6Abd^2+5aCd^2\right)x^2+cCd\left(bc^2+10ad^2\right)x+5ac^2Cd^2\right)}{\sqrt{bx^2+a}}dx}{6bd^4}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\frac{\int -\frac{(c+dx)^2\left(-b\left(25aCd^2-b\left(38Cc^2-36Bdc+30Ad^2\right)\right)x^2d^5+3abc(9cC-8Bd)d^5+2b\left(b(4cC-3Bd)c^2+ad^2(cC-12Bd)\right)xd^4\right)}{\sqrt{bx^2+a}}dx}{5bd^3}+\frac{1}{5}d\sqrt{a+bx^2}(c+dx)}{6bd^4}$$

$\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd)-\frac{\int \frac{(c+dx)^2\left(-b\left(25aCd^2-b\left(38Cc^2-36Bdc+30Ad^2\right)\right)x^2d^5+3abc(9cC-8Bd)d^5+2b\left(b(4cC-3Bd)c^2+ad^2(\right.\right.}{\sqrt{bx^2+a}}}{5bd^3}}{6bd^4}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd)-\frac{\frac{\int \frac{3bd^6(c+dx)^2\left(ad\left(25aCd^2-2b\left(Cc^2-2Bdc+15Ad^2\right)\right)+b\left(ad^2(11cC-32Bd)-2bc\left(Cc^2-2Bdc+5Ad^2\right)\right)x\right.}{\sqrt{bx^2+a}}}{4bd^2}}{5bd^3}}{6bd^4}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd)-\frac{\frac{3}{4}d^4\int \frac{(c+dx)^2\left(ad\left(25aCd^2-2b\left(Cc^2-2Bdc+15Ad^2\right)\right)+b\left(ad^2(11cC-32Bd)-2bc\left(Cc^2-2Bdc+5Ad^2\right)\right)x\right.}{\sqrt{bx^2+a}}}{5bd^3}}{6bd^4}$$

$\downarrow$ 687

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3}-\frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd)-\frac{\frac{3}{4}d^4\left(\frac{\int \frac{b(c+dx)\left(ad\left(ad^2(53cC+64Bd)-2bc\left(Cc^2-2Bdc+35Ad^2\right)\right)+\left(75a^2Cd^4+2ab\left(8Cc^2-26Bdc-45A\right.\right.\right.}{\sqrt{bx^2+a}}}{3b}\right.}{}}{}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3} - \frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd) - \frac{\frac{3}{4}d^4\left(\frac{1}{3}\int \frac{(c+dx)\left(ad\left(ad^2(53cC+64Bd)-2bc\left(Cc^2-2Bdc+35Ad^2\right)\right)+\left(75a^2Cd^4+2ab\left(8Cc^2-26Bdc-45\right.\right.\right.}{\sqrt{bx^2+a}}}{}}{}$$

$$\Big\downarrow \; 676$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3} - \frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd) - \frac{\frac{3}{4}d^4\left(\frac{1}{3}\left(-\frac{15ad^3\left(2Ab\left(4bc^2-3ad^2\right)+a\left(5aCd^2-6bc(2Bd+cC)\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{dx\sqrt{a+bx^2}\left(75a^2Cd^4\right.}{}\right.\right.}{}}{}$$

$$\Big\downarrow \; 224$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3} - \frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd) - \frac{\frac{3}{4}d^4\left(\frac{1}{3}\left(-\frac{15ad^3\left(2Ab\left(4bc^2-3ad^2\right)+a\left(5aCd^2-6bc(2Bd+cC)\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{dx\sqrt{a+bx^2}}{}\right.\right.}{}}{}$$

$$\Big\downarrow \; 219$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^5}{6bd^3} - \frac{\frac{1}{5}d\sqrt{a+bx^2}(c+dx)^4(13cC-6Bd) - \frac{\frac{3}{4}d^4\left(\frac{1}{3}\left(\frac{dx\sqrt{a+bx^2}\left(75a^2Cd^4+2abd^2\left(-45Ad^2-26Bcd+8c^2C\right)-4b^2c^2\left(5Ad^2-2Bcd+c^2C\right)\right)}{2b}+\frac{2\sqrt{a+}}{}\right.\right.}{}}{}$$

input
```
Int[(x^2*(c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^5*Sqrt[a + b*x^2])/(6*b*d^3) - ((d*(13*c*C - 6*B*d)*(c + d*x)
^4*Sqrt[a + b*x^2])/5 - (-1/4*(d^4*(25*a*C*d^2 - b*(38*c^2*C - 36*B*c*d +
30*A*d^2))*(c + d*x)^3*Sqrt[a + b*x^2]) + (3*d^4*(((a*d^2*(11*c*C - 32*B*d
) - 2*b*c*(c^2*C - 2*B*c*d + 5*A*d^2))*(c + d*x)^2*Sqrt[a + b*x^2])/3 + ((
2*(32*a^2*d^4*(2*c*C + B*d) + a*b*c*d^2*(7*c^2*C - 24*B*c*d - 80*A*d^2) -
2*b^2*c^3*(c^2*C - 2*B*c*d + 5*A*d^2))*Sqrt[a + b*x^2])/b + (d*(75*a^2*C*d
^4 + 2*a*b*d^2*(8*c^2*C - 26*B*c*d - 45*A*d^2) - 4*b^2*c^2*(c^2*C - 2*B*c*
d + 5*A*d^2))*x*Sqrt[a + b*x^2])/(2*b) - (15*a*d^3*(2*A*b*(4*b*c^2 - 3*a*d
^2) + a*(5*a*C*d^2 - 6*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(2*b^(3/2)))/3))/4)/(5*b*d^3))/(6*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{(-40C\,b^2d^2x^5-48B\,b^2d^2x^4-96C\,b^2cd\,x^4-60A\,b^2d^2x^3-120B\,b^2cd\,x^3+50Cab\,d^2x^3-60C\,b^2c^2x^3-160A\,b^2cd\,x^2+64Bab\,d^2x^2-}{}$
default	$c(2Ad+Bc)\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+d(Bd+2Cc)\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+($

input
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*C*b^2*d^2*x^5-48*B*b^2*d^2*x^4-96*C*b^2*c*d*x^4-60*A*b^2*d^2*x
^3-120*B*b^2*c*d*x^3+50*C*a*b*d^2*x^3-60*C*b^2*c^2*x^3-160*A*b^2*c*d*x^2+6
4*B*a*b*d^2*x^2-80*B*b^2*c^2*x^2+128*C*a*b*c*d*x^2+90*A*a*b*d^2*x-120*A*b^
2*c^2*x+180*B*a*b*c*d*x-75*C*a^2*d^2*x+90*C*a*b*c^2*x+320*A*a*b*c*d-128*B*
a^2*d^2+160*B*a*b*c^2-256*C*a^2*c*d)*(b*x^2+a)^(1/2)/b^3+1/16*a*(6*A*a*b*d
^2-8*A*b^2*c^2+12*B*a*b*c*d-5*C*a^2*d^2+6*C*a*b*c^2)/b^(7/2)*ln(b^(1/2)*x+
(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.97

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{15\left(12\,Ba^2bcd+2\left(3\,Ca^2b-4\,Aab^2\right)c^2-\left(5\,Ca^3-6\,Aa^2b\right)d^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+\right.$$

$$-\frac{15\left(12\,Ba^2bcd+2\left(3\,Ca^2b-4\,Aab^2\right)c^2-\left(5\,Ca^3-6\,Aa^2b\right)d^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\left(40\,Cb^3d^2x^5-\right.}{}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/480*(15*(12*B*a^2*b*c*d + 2*(3*C*a^2*b - 4*A*a*b^2)*c^2 - (5*C*a^3 - 6*
A*a^2*b)*d^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(40*C*b^3*d^2*x^5 - 160*B*a*b^2*c^2 + 128*B*a^2*b*d^2 + 48*(2*C*b^3*c*d +
B*b^3*d^2)*x^4 + 10*(6*C*b^3*c^2 + 12*B*b^3*c*d - (5*C*a*b^2 - 6*A*b^3)*d^
2)*x^3 + 64*(4*C*a^2*b - 5*A*a*b^2)*c*d + 16*(5*B*b^3*c^2 - 4*B*a*b^2*d^2
- 2*(4*C*a*b^2 - 5*A*b^3)*c*d)*x^2 - 15*(12*B*a*b^2*c*d + 2*(3*C*a*b^2 - 4
*A*b^3)*c^2 - (5*C*a^2*b - 6*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^4, -1/240
*(15*(12*B*a^2*b*c*d + 2*(3*C*a^2*b - 4*A*a*b^2)*c^2 - (5*C*a^3 - 6*A*a^2*
b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*C*b^3*d^2*x^5 -
160*B*a*b^2*c^2 + 128*B*a^2*b*d^2 + 48*(2*C*b^3*c*d + B*b^3*d^2)*x^4 + 10*
(6*C*b^3*c^2 + 12*B*b^3*c*d - (5*C*a*b^2 - 6*A*b^3)*d^2)*x^3 + 64*(4*C*a^2
*b - 5*A*a*b^2)*c*d + 16*(5*B*b^3*c^2 - 4*B*a*b^2*d^2 - 2*(4*C*a*b^2 - 5*A
*b^3)*c*d)*x^2 - 15*(12*B*a*b^2*c*d + 2*(3*C*a*b^2 - 4*A*b^3)*c^2 - (5*C*a
^2*b - 6*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.18

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{a\left(Ac^2-\frac{3a\left(Ad^2+2Bcd-\frac{5Cad^2}{6b}+Cc^2\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{Cd^2x^5}{6b}-\frac{2a\left(2Acd+Bc^2-\right.}{3b}\right. \\ \frac{\frac{Ac^2x^3}{3}+\frac{Cd^2x^7}{7}+\frac{x^6\left(Bd^2+2Ccd\right)}{6}+\frac{x^5\left(Ad^2+2Bcd+Cc^2\right)}{5}+\frac{x^4\cdot\left(2Acd+Bc^2\right)}{4}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**2*(d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(A*c**2 - 3*a*(A*d**2 + 2*B*c*d - 5*C*a*d**2/(6*b) + C*c**2)
/(4*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*d**2*x**5/
(6*b) - 2*a*(2*A*c*d + B*c**2 - 4*a*(B*d**2 + 2*C*c*d)/(5*b))/(3*b**2) + x
**4*(B*d**2 + 2*C*c*d)/(5*b) + x**3*(A*d**2 + 2*B*c*d - 5*C*a*d**2/(6*b) +
 C*c**2)/(4*b) + x**2*(2*A*c*d + B*c**2 - 4*a*(B*d**2 + 2*C*c*d)/(5*b))/(3
*b) + x*(A*c**2 - 3*a*(A*d**2 + 2*B*c*d - 5*C*a*d**2/(6*b) + C*c**2)/(4*b)
)/(2*b)), Ne(b, 0)), ((A*c**2*x**3/3 + C*d**2*x**7/7 + x**6*(B*d**2 + 2*C*
c*d)/6 + x**5*(A*d**2 + 2*B*c*d + C*c**2)/5 + x**4*(2*A*c*d + B*c**2)/4)/s
qrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.17

$$\int \frac{x^2(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cd^2x^5}{6\,b} - \frac{5\,\sqrt{bx^2+a}Cad^2x^3}{24\,b^2} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}x^4}{5\,b} + \frac{\sqrt{bx^2+a}Ac^2x}{2\,b} + \frac{5\,\sqrt{bx^2+a}Ca^2d^2x}{16\,b^3} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}x^3}{4\,b} - \frac{Aac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{5\,Ca^3d^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} - \frac{4\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}ax^2}{15\,b^2} + \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}x^2}{3\,b} - \frac{3\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{3\,(Cc^2+2\,Bcd+Ad^2)a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{8\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}a^2}{15\,b^3} - \frac{2\,(Bc^2+2\,Acd)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
1/6*sqrt(b*x^2 + a)*C*d^2*x^5/b - 5/24*sqrt(b*x^2 + a)*C*a*d^2*x^3/b^2 + 1
/5*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*x^4/b + 1/2*sqrt(b*x^2 + a)*A*c^2*x/b
 + 5/16*sqrt(b*x^2 + a)*C*a^2*d^2*x/b^3 + 1/4*(C*c^2 + 2*B*c*d + A*d^2)*sq
rt(b*x^2 + a)*x^3/b - 1/2*A*a*c^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 5/16*C*
a^3*d^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 4/15*(2*C*c*d + B*d^2)*sqrt(b*x^2
 + a)*a*x^2/b^2 + 1/3*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*x^2/b - 3/8*(C*c^2
 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*(C*c^2 + 2*B*c*d + A*d^2
)*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 8/15*(2*C*c*d + B*d^2)*sqrt(b*x^2 +
 a)*a^2/b^3 - 2/3*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\left(\frac{5\,Cd^2x}{b}+\frac{6\,(2\,Cb^5cd+Bb^5d^2)}{b^6}\right)x+\frac{5\,(6\,Cb^5c^2+12\,Bb^5cd-5\,Cab^4d^2+6\,Ab}{b^6}\right.\right.\right.\right.$$
$$-\frac{(6\,Ca^2bc^2-8\,Aab^2c^2+12\,Ba^2bcd-5\,Ca^3d^2+6\,Aa^2bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/240*sqrt(b*x^2 + a)*((2*((4*(5*C*d^2*x/b + 6*(2*C*b^5*c*d + B*b^5*d^2)/b
^6)*x + 5*(6*C*b^5*c^2 + 12*B*b^5*c*d - 5*C*a*b^4*d^2 + 6*A*b^5*d^2)/b^6)*
x + 8*(5*B*b^5*c^2 - 8*C*a*b^4*c*d + 10*A*b^5*c*d - 4*B*a*b^4*d^2)/b^6)*x
- 15*(6*C*a*b^4*c^2 - 8*A*b^5*c^2 + 12*B*a*b^4*c*d - 5*C*a^2*b^3*d^2 + 6*A
*a*b^4*d^2)/b^6)*x - 32*(5*B*a*b^4*c^2 - 8*C*a^2*b^3*c*d + 10*A*a*b^4*c*d
- 4*B*a^2*b^3*d^2)/b^6) - 1/16*(6*C*a^2*b*c^2 - 8*A*a*b^2*c^2 + 12*B*a^2*b
*c*d - 5*C*a^3*d^2 + 6*A*a^2*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))
/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^2\,{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^2 (dx+c)^2\left(C\,x^2+Bx+A\right)}{\sqrt{b\,x^2+a}}dx$$

input
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

3.106 $\int \frac{x(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 248

$$\int \frac{x(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\sqrt{a+bx^2}}{b^3} + \frac{(4bc(Bc+2Ad)-3ad(2cC+Bd))x\sqrt{a+bx^2}}{8b^2} + \frac{d(2cC+Bd)x^3\sqrt{a+bx^2}}{4b} - \frac{(2aCd^2-b(c^2C+2Bcd+Ad^2))(a+bx^2)^{3/2}}{3b^3} + \frac{Cd^2(a+bx^2)^{5/2}}{5b^3} - \frac{a(4bc(Bc+2Ad)-3ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/b^3+1/8*(
4*b*c*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*x*(b*x^2+a)^(1/2)/b^2+1/4*d*(B*d+2*C*
c)*x^3*(b*x^2+a)^(1/2)/b-1/3*(2*a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)
^(3/2)/b^3+1/5*C*d^2*(b*x^2+a)^(5/2)/b^3-1/8*a*(4*b*c*(2*A*d+B*c)-3*a*d*(B
*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.82

$$\int \frac{x(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(64a^2Cd^2 - ab(80c^2C + 10cd(16B+9Cx) + d^2x(45B+32Cx)) + 40Ab(-2ad^2 + b(3c^2 + 3cd}{}$$

input
```
Integrate[(x*(c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(64*a^2*C*d^2 - a*b*(80*c^2*C + 10*c*d*(16*B + 9*C*x) + d
^2*x*(45*B + 32*C*x)) + 40*A*b*(-2*a*d^2 + b*(3*c^2 + 3*c*d*x + d^2*x^2))
+ 2*b^2*x*(5*B*(6*c^2 + 8*c*d*x + 3*d^2*x^2) + 2*C*x*(10*c^2 + 15*c*d*x +
6*d^2*x^2))) - 15*a*Sqrt[b]*(-4*b*c*(B*c + 2*A*d) + 3*a*d*(2*c*C + B*d))*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^3)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2185, 25, 2185, 25, 27, 687, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int -\frac{(c+dx)^2\left(b(6cC-5Bd)x^2d^2+4acCd^2+\left(bCc^2-5Abd^2+4aCd^2\right)xd\right)}{\sqrt{bx^2+a}}dx}{5bd^3} + \frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}$$
$$\Big\downarrow \; 25$$
$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2} - \frac{\int \frac{(c+dx)^2\left(b(6cC-5Bd)x^2d^2+4acCd^2+\left(bCc^2-5Abd^2+4aCd^2\right)xd\right)}{\sqrt{bx^2+a}}dx}{5bd^3}$$

$\Big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{\int -\frac{bd^3(c+dx)^2\left(ad(2cC-15Bd)-\left(16aCd^2-b\left(2Cc^2-5Bdc+20Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4bd^2}+\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)}{5bd^3}$$

$\Big\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{\int \frac{bd^3(c+dx)^2\left(ad(2cC-15Bd)-\left(16aCd^2-b\left(2Cc^2-5Bdc+20Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{4bd^2}}{5bd^3}$$

$\Big\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{1}{4}d\int \frac{(c+dx)^2\left(ad(2cC-15Bd)-\left(16aCd^2-b\left(2Cc^2-5Bdc+20Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{5bd^3}$$

$\Big\downarrow$ 687

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{1}{4}d\left(\frac{\int \frac{(c+dx)\left(ad\left(32aCd^2+b\left(2Cc^2-35Bdc-40Ad^2\right)\right)-b\left(ad^2(26cC+45Bd)-2bc\left(2Cc^2-5Bdc+20Ad^2\right)\right.\right.}{\sqrt{bx^2+a}}}{3b}\right.}{5bd^3}$$

$\Big\downarrow$ 676

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{1}{4}d\left(\frac{-\frac{15}{2}ad^2(4bc(2Ad+Bc)-3ad(Bd+2cC))\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}\left(16a^2Cd^4-4abd^2\left(5Ad^2+10\right.\right.}{}}{}\right.}{}$$

$\Big\downarrow$ 224

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{1}{4}d\left(\frac{-\frac{15}{2}ad^2(4bc(2Ad+Bc)-3ad(Bd+2cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}\left(16a^2Cd^4-4abd^2\right.}{}}{}\right.}{}$$

$\Big\downarrow$ 219

$$\frac{C\sqrt{a+bx^2}(c+dx)^4}{5bd^2}-$$

$$\frac{\frac{1}{4}d\sqrt{a+bx^2}(c+dx)^3(6cC-5Bd)-\frac{1}{4}d\left(\frac{\frac{2\sqrt{a+bx^2}\left(16a^2Cd^4-4abd^2\left(5Ad^2+10Bcd+3c^2C\right)+b^2c^2\left(20Ad^2-5Bcd+2c^2C\right)\right)}{b}-\frac{15ad^2\text{arctan}}{}}{}\right.}{}$$

input
```
Int[(x*(c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^4*Sqrt[a + b*x^2])/(5*b*d^2) - ((d*(6*c*C - 5*B*d)*(c + d*x)^
3*Sqrt[a + b*x^2])/4 - (d*(-1/3*((16*a*C*d^2 - b*(2*c^2*C - 5*B*c*d + 20*A
*d^2))*(c + d*x)^2*Sqrt[a + b*x^2])/b + ((2*(16*a^2*C*d^4 - 4*a*b*d^2*(3*c
^2*C + 10*B*c*d + 5*A*d^2) + b^2*c^2*(2*c^2*C - 5*B*c*d + 20*A*d^2))*Sqrt[
a + b*x^2])/b - (d*(a*d^2*(26*c*C + 45*B*d) - 2*b*c*(2*c^2*C - 5*B*c*d + 2
0*A*d^2))*x*Sqrt[a + b*x^2])/2 - (15*a*d^2*(4*b*c*(B*c + 2*A*d) - 3*a*d*(2
*c*C + B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/(3*b)))/4)
/(5*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\left(-24C\,b^2d^2x^4-30B\,b^2d^2x^3-60C\,b^2cd\,x^3-40A\,b^2d^2x^2-80B\,b^2cd\,x^2+32Cab\,d^2x^2-40C\,b^2c^2x^2-120A\,b^2cdx+45Bb\,d^2xa-60B}{120b^3}$
default	$c(2Ad+Bc)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+d(Bd+2Cc)\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{}\right.}{2b^{\frac{3}{2}}}\right.}{4b}\right.$

input
```
int(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(-24*C*b^2*d^2*x^4-30*B*b^2*d^2*x^3-60*C*b^2*c*d*x^3-40*A*b^2*d^2*x
^2-80*B*b^2*c*d*x^2+32*C*a*b*d^2*x^2-40*C*b^2*c^2*x^2-120*A*b^2*c*d*x+45*B
*a*b*d^2*x-60*B*b^2*c^2*x+90*C*a*b*c*d*x+80*A*a*b*d^2-120*A*b^2*c^2+160*B*
a*b*c*d-64*C*a^2*d^2+80*C*a*b*c^2)*(b*x^2+a)^(1/2)/b^3-1/8*a/b^(5/2)*(8*A*
b*c*d-3*B*a*d^2+4*B*b*c^2-6*C*a*c*d)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.87

$$\int \frac{x(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \Bigg[-\frac{15\,(4\,Babc^2-3\,Ba^2d^2-2\,(3\,Ca^2-4\,Aab)cd)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(24\,Cb^2d^2x}{}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/240*(15*(4*B*a*b*c^2 - 3*B*a^2*d^2 - 2*(3*C*a^2 - 4*A*a*b)*c*d)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(24*C*b^2*d^2*x^4 -
160*B*a*b*c*d + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^3 - 40*(2*C*a*b - 3*A*b^2)*
c^2 + 16*(4*C*a^2 - 5*A*a*b)*d^2 + 8*(5*C*b^2*c^2 + 10*B*b^2*c*d - (4*C*a*
b - 5*A*b^2)*d^2)*x^2 + 15*(4*B*b^2*c^2 - 3*B*a*b*d^2 - 2*(3*C*a*b - 4*A*b
^2)*c*d)*x)*sqrt(b*x^2 + a))/b^3, 1/120*(15*(4*B*a*b*c^2 - 3*B*a^2*d^2 - 2
*(3*C*a^2 - 4*A*a*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2
4*C*b^2*d^2*x^4 - 160*B*a*b*c*d + 30*(2*C*b^2*c*d + B*b^2*d^2)*x^3 - 40*(2
*C*a*b - 3*A*b^2)*c^2 + 16*(4*C*a^2 - 5*A*a*b)*d^2 + 8*(5*C*b^2*c^2 + 10*B
*b^2*c*d - (4*C*a*b - 5*A*b^2)*d^2)*x^2 + 15*(4*B*b^2*c^2 - 3*B*a*b*d^2 -
2*(3*C*a*b - 4*A*b^2)*c*d)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.26

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$

$$= \begin{cases} -\frac{a\left(2Acd+Bc^2-\frac{3a\left(Bd^2+2Ccd\right)}{4b}\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Cd^2x^4}{5b} + \frac{x^3\left(Bd^2+2Ccd\right)}{4b} + \frac{x^2\left(\right.}{} \right. \\ \frac{\frac{Ac^2x^2}{2}+\frac{Cd^2x^6}{6}+\frac{x^5\left(Bd^2+2Ccd\right)}{5}+\frac{x^4\left(Ad^2+2Bcd+Cc^2\right)}{4}+\frac{x^3\cdot\left(2Acd+Bc^2\right)}{3}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(2*A*c*d + B*c**2 - 3*a*(B*d**2 + 2*C*c*d)/(4*b))*Piecewise(
(log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqr
t(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(C*d**2*x**4/(5*b) + x**3*(B*d*
*2 + 2*C*c*d)/(4*b) + x**2*(A*d**2 + 2*B*c*d - 4*C*a*d**2/(5*b) + C*c**2)/
(3*b) + x*(2*A*c*d + B*c**2 - 3*a*(B*d**2 + 2*C*c*d)/(4*b))/(2*b) + (A*c**
2 - 2*a*(A*d**2 + 2*B*c*d - 4*C*a*d**2/(5*b) + C*c**2)/(3*b))/b), Ne(b, 0)
), ((A*c**2*x**2/2 + C*d**2*x**6/6 + x**5*(B*d**2 + 2*C*c*d)/5 + x**4*(A*d
**2 + 2*B*c*d + C*c**2)/4 + x**3*(2*A*c*d + B*c**2)/3)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.13

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cd^2x^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}Cad^2x^2}{15\,b^2} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}x^3}{4\,b} + \frac{\sqrt{bx^2+a}Ac^2}{b} + \frac{8\,\sqrt{bx^2+a}Ca^2d^2}{15\,b^3} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}x^2}{3\,b} - \frac{3\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}ax}{8\,b^2} + \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}x}{2\,b} + \frac{3\,(2\,Ccd+Bd^2)a^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{(Bc^2+2\,Acd)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*C*d^2*x^4/b - 4/15*sqrt(b*x^2 + a)*C*a*d^2*x^2/b^2 + 1
/4*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*x^3/b + sqrt(b*x^2 + a)*A*c^2/b + 8/1
5*sqrt(b*x^2 + a)*C*a^2*d^2/b^3 + 1/3*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2
 + a)*x^2/b - 3/8*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a*x/b^2 + 1/2*(B*c^2 +
 2*A*c*d)*sqrt(b*x^2 + a)*x/b + 3/8*(2*C*c*d + B*d^2)*a^2*arcsinh(b*x/sqrt
(a*b))/b^(5/2) - 1/2*(B*c^2 + 2*A*c*d)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) -
2/3*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\left(\frac{4\,Cd^2x}{b}+\frac{5\,(2\,Cb^4cd+Bb^4d^2)}{b^5}\right)x+\frac{4\,(5\,Cb^4c^2+10\,Bb^4cd-4\,Cab^3d^2+5\,Ab^4d}{b^5}\right.\right.\right.$$
$$+\frac{(4\,Babc^2-6\,Ca^2cd+8\,Aabcd-3\,Ba^2d^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/120*sqrt(b*x^2 + a)*((2*(3*(4*C*d^2*x/b + 5*(2*C*b^4*c*d + B*b^4*d^2)/b^
5)*x + 4*(5*C*b^4*c^2 + 10*B*b^4*c*d - 4*C*a*b^3*d^2 + 5*A*b^4*d^2)/b^5)*x
 + 15*(4*B*b^4*c^2 - 6*C*a*b^3*c*d + 8*A*b^4*c*d - 3*B*a*b^3*d^2)/b^5)*x -
 8*(10*C*a*b^3*c^2 - 15*A*b^4*c^2 + 20*B*a*b^3*c*d - 8*C*a^2*b^2*d^2 + 10*
A*a*b^3*d^2)/b^5) + 1/8*(4*B*a*b*c^2 - 6*C*a^2*c*d + 8*A*a*b*c*d - 3*B*a^2
*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{x\,{(c+d\,x)}^2\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int((x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.66

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{-80\sqrt{b\,x^2+a}\,a^2b\,d^2+64\sqrt{b\,x^2+a}\,a^2c\,d^2+120\sqrt{b\,x^2+a}\,a\,b^2c^2+120\sqrt{b\,x^2+a}\,a\,b^2cdx-160\sqrt{b\,x^2+a}}{}$$

input
```
int(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
( - 80*sqrt(a + b*x**2)*a**2*b*d**2 + 64*sqrt(a + b*x**2)*a**2*c*d**2 + 12
0*sqrt(a + b*x**2)*a*b**2*c**2 + 120*sqrt(a + b*x**2)*a*b**2*c*d*x - 160*s
qrt(a + b*x**2)*a*b**2*c*d + 40*sqrt(a + b*x**2)*a*b**2*d**2*x**2 - 45*sqr
t(a + b*x**2)*a*b**2*d**2*x - 80*sqrt(a + b*x**2)*a*b*c**3 - 90*sqrt(a + b
*x**2)*a*b*c**2*d*x - 32*sqrt(a + b*x**2)*a*b*c*d**2*x**2 + 60*sqrt(a + b*
x**2)*b**3*c**2*x + 80*sqrt(a + b*x**2)*b**3*c*d*x**2 + 30*sqrt(a + b*x**2
)*b**3*d**2*x**3 + 40*sqrt(a + b*x**2)*b**2*c**3*x**2 + 60*sqrt(a + b*x**2
)*b**2*c**2*d*x**3 + 24*sqrt(a + b*x**2)*b**2*c*d**2*x**4 - 120*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d + 45*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2 + 90*sqrt(b)*log((sqrt(a +
 b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c**2*d - 60*sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2)/(120*b**3)
```

3.107 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 206

$$\begin{aligned}
\int \frac{(c+dx)^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx &= \frac{(bc(Bc+2Ad)-ad(2cC+Bd))\sqrt{a+bx^2}}{b^2} \\
&\quad - \frac{(3aCd^2-4b(c^2C+2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8b^2} \\
&\quad + \frac{Cd^2x^3\sqrt{a+bx^2}}{4b} + \frac{d(2cC+Bd)\,(a+bx^2)^{3/2}}{3b^2} \\
&\quad + \frac{(4Ab(2bc^2-ad^2)+a(3aCd^2-4bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}
\end{aligned}$$

output
```
(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/b^2-1/8*(3*a*C*d^2-4*b*(
A*d^2+2*B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/b^2+1/4*C*d^2*x^3*(b*x^2+a)^(1/2)/
b+1/3*d*(B*d+2*C*c)*(b*x^2+a)^(3/2)/b^2+1/8*(4*A*b*(-a*d^2+2*b*c^2)+a*(3*a
*C*d^2-4*b*c*(2*B*d+C*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}} \, dx$$
$$= \frac{\sqrt{a+bx^2}(-ad(32cC+16Bd+9Cdx)+2b(6Ad(4c+dx)+4B(3c^2+3cdx+d^2x^2)+Cx(6c^2+8cdx -}{24b^2}$$
$$- \frac{(4Ab(2bc^2-ad^2)+a(3aCd^2-4bc(cC+2Bd)))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-(a*d*(32*c*C + 16*B*d + 9*C*d*x)) + 2*b*(6*A*d*(4*c + d
*x) + 4*B*(3*c^2 + 3*c*d*x + d^2*x^2) + C*x*(6*c^2 + 8*c*d*x + 3*d^2*x^2))
))/(24*b^2) - ((4*A*b*(2*b*c^2 - a*d^2) + a*(3*a*C*d^2 - 4*b*c*(c*C + 2*B*
d)))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2185, 27, 687, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}} \, dx$$
$$\Big\downarrow \; 2185$$
$$\frac{\int \frac{d(c+dx)^2((4Ab-3aC)d-b(cC-4Bd)x)}{\sqrt{bx^2+a}}dx}{4bd^2} + \frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$
$$\Big\downarrow \; 27$$
$$\frac{\int \frac{(c+dx)^2((4Ab-3aC)d-b(cC-4Bd)x)}{\sqrt{bx^2+a}}dx}{4bd} + \frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

$$\Big\downarrow \ 687$$

$$\frac{\frac{\int \frac{b(c+dx)\left(d(12Abc-7aCc-8aBd)+\left(3(4Ab-3aC)d^2-2bc(cC-4Bd)\right)x\right)}{\sqrt{bx^2+a}}dx}{3b}-\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(cC-4Bd)}{4bd}+\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{3}\int \frac{(c+dx)(d(12Abc-7aCc-8aBd)+(3(4Ab-3aC)d^2-2bc(cC-4Bd))x)}{\sqrt{bx^2+a}}dx-\frac{1}{3}\sqrt{a+bx^2}(c+dx)^2(cC-4Bd)}{4bd}+\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

$$\Big\downarrow \ 676$$

$$\frac{\frac{1}{3}\left(\frac{3d(4Ab(2bc^2-ad^2)+a(3aCd^2-4bc(2Bd+cC)))\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2\sqrt{a+bx^2}(4ad^2(Bd+2cC)+bc(-12Ad^2-4Bcd+c^2C))}{b}+\frac{dx\sqrt{a+bx^2}(3d}{}\right.}{4bd}$$
$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{1}{3}\left(\frac{3d(4Ab(2bc^2-ad^2)+a(3aCd^2-4bc(2Bd+cC)))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2\sqrt{a+bx^2}(4ad^2(Bd+2cC)+bc(-12Ad^2-4Bcd+c^2C))}{b}+\frac{dx\sqrt{a}}{}\right.}{4bd}$$
$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{1}{3}\left(\frac{3d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(4Ab(2bc^2-ad^2)+a(3aCd^2-4bc(2Bd+cC)))}{2b^{3/2}}-\frac{2\sqrt{a+bx^2}(4ad^2(Bd+2cC)+bc(-12Ad^2-4Bcd+c^2C))}{b}+\frac{dx\sqrt{a}}{}\right.}{4bd}$$
$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(C*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b*d) + (-1/3*((c*C - 4*B*d)*(c + d*x)^2
*Sqrt[a + b*x^2]) + ((-2*(4*a*d^2*(2*c*C + B*d) + b*c*(c^2*C - 4*B*c*d - 1
2*A*d^2))*Sqrt[a + b*x^2])/b + (d*(3*(4*A*b - 3*a*C)*d^2 - 2*b*c*(c*C - 4*
B*d))*x*Sqrt[a + b*x^2])/(2*b) + (3*d*(4*A*b*(2*b*c^2 - a*d^2) + a*(3*a*C*
d^2 - 4*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/
2)))/3)/(4*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.81

method	result
risch	$\frac{\left(6Cb\,d^{2}x^{3}+8Bb\,d^{2}x^{2}+16Cbcd\,x^{2}+12A\,d^{2}xb+24Bcdxb-9Ca\,d^{2}x+12xb\,c^{2}C+48Abcd-16aB\,d^{2}+24bB\,c^{2}-32Cacd\right)\sqrt{b\,x^{2}+a}}{24b^{2}}-$
default	$\frac{A\,c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{c(2Ad+Bc)\sqrt{b\,x^{2}+a}}{b}+d(Bd+2Cc)\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+(A\,d^{2}+2Bcd$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*b*d^2*x^3+8*B*b*d^2*x^2+16*C*b*c*d*x^2+12*A*b*d^2*x+24*B*b*c*d*x
-9*C*a*d^2*x+12*C*b*c^2*x+48*A*b*c*d-16*B*a*d^2+24*B*b*c^2-32*C*a*c*d)*(b*
x^2+a)^(1/2)/b^2-1/8*(4*A*a*b*d^2-8*A*b^2*c^2+8*B*a*b*c*d-3*C*a^2*d^2+4*C*
a*b*c^2)/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.86

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3 (8 Babcd + 4 (Cab - 2 Ab^2)c^2 - (3 Ca^2 - 4 Aab)d^2)\sqrt{b}\log\left(-2 bx^2 + 2 \sqrt{bx^2+a}\sqrt{b}x - a\right) + 2 (6 Cb}{}\right.$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*B*a*b*c*d + 4*(C*a*b - 2*A*b^2)*c^2 - (3*C*a^2 - 4*A*a*b)*d^2)
*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*C*b^2*d^2*
x^3 + 24*B*b^2*c^2 - 16*B*a*b*d^2 - 16*(2*C*a*b - 3*A*b^2)*c*d + 8*(2*C*b^
2*c*d + B*b^2*d^2)*x^2 + 3*(4*C*b^2*c^2 + 8*B*b^2*c*d - (3*C*a*b - 4*A*b^2
)*d^2)*x)*sqrt(b*x^2 + a))/b^3, 1/24*(3*(8*B*a*b*c*d + 4*(C*a*b - 2*A*b^2)
*c^2 - (3*C*a^2 - 4*A*a*b)*d^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (6*C*b^2*d^2*x^3 + 24*B*b^2*c^2 - 16*B*a*b*d^2 - 16*(2*C*a*b - 3*A*b^2
)*c*d + 8*(2*C*b^2*c*d + B*b^2*d^2)*x^2 + 3*(4*C*b^2*c^2 + 8*B*b^2*c*d - (
3*C*a*b - 4*A*b^2)*d^2)*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{Cd^2x^3}{4b} + \frac{x^2(Bd^2+2Ccd)}{3b} + \frac{x\left(Ad^2+2Bcd-\frac{3Cad^2}{4b}+Cc^2\right)}{2b} + \frac{2Acd+Bc^2-\frac{2a\left(Bd^2+2Ccd\right)}{3b}}{b}\right) + \left(Ac^2 - \frac{a\left(Ad^2+2B\right.}{}\right. \\ \frac{Ac^2x+\frac{Cd^2x^5}{5}+\frac{x^4\left(Bd^2+2Ccd\right)}{4}+\frac{x^3\left(Ad^2+2Bcd+Cc^2\right)}{3}+\frac{x^2\cdot\left(2Acd+Bc^2\right)}{2}}{\sqrt{a}} \end{cases}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(C*d**2*x**3/(4*b) + x**2*(B*d**2 + 2*C*c*d)/(
3*b) + x*(A*d**2 + 2*B*c*d - 3*C*a*d**2/(4*b) + C*c**2)/(2*b) + (2*A*c*d +
 B*c**2 - 2*a*(B*d**2 + 2*C*c*d)/(3*b))/b) + (A*c**2 - a*(A*d**2 + 2*B*c*d
 - 3*C*a*d**2/(4*b) + C*c**2)/(2*b))*Piecewise((log(2*sqrt(b)*sqrt(a + b*x
**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)
), ((A*c**2*x + C*d**2*x**5/5 + x**4*(B*d**2 + 2*C*c*d)/4 + x**3*(A*d**2 +
 2*B*c*d + C*c**2)/3 + x**2*(2*A*c*d + B*c**2)/2)/sqrt(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cd^2x^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}Cad^2x}{8\,b^2} + \frac{Ac^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{3\,Ca^2d^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}Bc^2}{b} + \frac{2\,\sqrt{bx^2+a}Acd}{b} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}x^2}{3\,b} + \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}x}{2\,b} - \frac{(Cc^2+2\,Bcd+Ad^2)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*C*d^2*x^3/b - 3/8*sqrt(b*x^2 + a)*C*a*d^2*x/b^2 + A*c^
2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 3/8*C*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^
(5/2) + sqrt(b*x^2 + a)*B*c^2/b + 2*sqrt(b*x^2 + a)*A*c*d/b + 1/3*(2*C*c*d
 + B*d^2)*sqrt(b*x^2 + a)*x^2/b + 1/2*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2
 + a)*x/b - 1/2*(C*c^2 + 2*B*c*d + A*d^2)*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
 - 2/3*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Cd^2x}{b}+\frac{4\,(2\,Cb^3cd+Bb^3d^2)}{b^4}\right)x+\frac{3\,(4\,Cb^3c^2+8\,Bb^3cd-3\,Cab^2d^2+4\,Ab^3d^2)}{b^4}\right)x\right.$$
$$+\frac{(4\,Cabc^2-8\,Ab^2c^2+8\,Babcd-3\,Ca^2d^2+4\,Aabd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*C*d^2*x/b + 4*(2*C*b^3*c*d + B*b^3*d^2)/b^4)*x
 + 3*(4*C*b^3*c^2 + 8*B*b^3*c*d - 3*C*a*b^2*d^2 + 4*A*b^3*d^2)/b^4)*x + 8*
(3*B*b^3*c^2 - 4*C*a*b^2*c*d + 6*A*b^3*c*d - 2*B*a*b^2*d^2)/b^4) + 1/8*(4*
C*a*b*c^2 - 8*A*b^2*c^2 + 8*B*a*b*c*d - 3*C*a^2*d^2 + 4*A*a*b*d^2)*log(abs
(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(dx+c)^2\,(C\,x^2+Bx+A)}{\sqrt{b\,x^2+a}}dx$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

output
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(1/2),x)
```

3.108 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 175

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx &= -\frac{(aCd^2-b(c^2C+2Bcd+Ad^2))\sqrt{a+bx^2}}{b^2}\\&\quad+\frac{d(2cC+Bd)x\sqrt{a+bx^2}}{2b}+\frac{Cd^2(a+bx^2)^{3/2}}{3b^2}\\&\quad+\frac{(2bc(Bc+2Ad)-ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{Ac^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

output
```
-(a*C*d^2-b*(A*d^2+2*B*c*d+C*c^2))*(b*x^2+a)^(1/2)/b^2+1/2*d*(B*d+2*C*c)*x
*(b*x^2+a)^(1/2)/b+1/3*C*d^2*(b*x^2+a)^(3/2)/b^2+1/2*(2*b*c*(2*A*d+B*c)-a*
d*(B*d+2*C*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-A*c^2*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-4aCd^2+b(6c^2C+6cd(2B+Cx)+d^2(6A+3Bx+2Cx^2)))}{6b^2} + \frac{2Ac^2\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{(2bc(Bc+2Ad)-ad(2cC+Bd))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-4*a*C*d^2 + b*(6*c^2*C + 6*c*d*(2*B + C*x) + d^2*(6*A +
 3*B*x + 2*C*x^2))))/(6*b^2) + (2*A*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^
2])/Sqrt[a]])/Sqrt[a] - ((2*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2340, 2340, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2340}$$

$$\frac{\int \frac{3bd(2cC+Bd)x^3-(2aCd^2-3b(Cc^2+2Bdc+Ad^2))x^2+3bc(Bc+2Ad)x+3Abc^2}{x\sqrt{bx^2+a}}dx}{3b}+\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{6Ab^2c^2-2b\left(2aCd^2-3b\left(Cc^2+2Bdc+Ad^2\right)\right)x^2+3b(2bc(Bc+2Ad)-ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}(Bd+2cC)}{3b}+\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \textbf{2340}$$

$$\frac{\frac{\frac{\int \frac{3b^2\left(2Abc^2+(2bc(Bc+2Ad)-ad(2cC+Bd))x\right)}{x\sqrt{bx^2+a}}dx}{b}-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}(Bd+2cC)}{3b}+\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{3b\int \frac{2Abc^2+(2bc(Bc+2Ad)-ad(2cC+Bd))x}{x\sqrt{bx^2+a}}dx-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}(Bd+2cC)}{3b}+\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \textbf{538}$$

$$\frac{\frac{3b\left((2bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{\sqrt{bx^2+a}}dx+2Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}(Bd\ -}{3b}$$
$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \textbf{224}$$

$$\frac{\frac{3b\left((2bc(2Ad+Bc)-ad(Bd+2cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+b}}{3b}$$
$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{3b\left(2Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}(}{3b}$$

$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$\downarrow$ 243

$$\frac{\frac{3b\left(Abc^2\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+bx^2}}{3b}$$

$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$\downarrow$ 73

$$\frac{\frac{3b\left(2Ac^2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+b}}{3b}$$

$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

$\downarrow$ 221

$$\frac{\frac{3b\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc(2Ad+Bc)-ad(Bd+2cC))}{\sqrt{b}}-\frac{2Abc^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-2\sqrt{a+bx^2}\left(2aCd^2-3b\left(Ad^2+2Bcd+c^2C\right)\right)}{2b}+\frac{3}{2}dx\sqrt{a+}}{3b}$$

$$\frac{Cd^2x^2\sqrt{a+bx^2}}{3b}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x*Sqrt[a + b*x^2]),x]
```

output
```
(C*d^2*x^2*Sqrt[a + b*x^2])/(3*b) + ((3*d*(2*c*C + B*d)*x*Sqrt[a + b*x^2])
/2 + (-2*(2*a*C*d^2 - 3*b*(c^2*C + 2*B*c*d + A*d^2))*Sqrt[a + b*x^2] + 3*b
*(((2*b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/Sqrt[b] - (2*A*b*c^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/
(2*b))/(3*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.47

method	result
default	$\frac{c^{2} B \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\,c^{2} \ln\left(\frac{2 a+2 \sqrt{a}\, \sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+\frac{A\,d^{2} \sqrt{b\,x^{2}+a}}{b}+d^{2} B \left(\frac{x \sqrt{b\,x^{2}+a}}{2 b}-\frac{a \ln\left(\sqrt{b}\, x+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}\right)+$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
c^2*B*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A*c^2/a^(1/2)*ln((2*a+2*a^(1/2
)*(b*x^2+a)^(1/2))/x)+A*d^2/b*(b*x^2+a)^(1/2)+d^2*B*(1/2*x/b*(b*x^2+a)^(1/
2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+C*c^2/b*(b*x^2+a)^(1/2)+C*
d^2*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+2*c*d*A*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*B*c*d/b*(b*x^2+a)^(1/2)+2*c*d*C*(1/2*x/b*(
b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 717, normalized size of antiderivative = 4.10

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(6*A*sqrt(a)*b^2*c^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) - 3*(2*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2 - 2*A*a*b)*c*d)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*C*a*b*d^2*x^2 + 6*C*a*b*
c^2 + 12*B*a*b*c*d - 2*(2*C*a^2 - 3*A*a*b)*d^2 + 3*(2*C*a*b*c*d + B*a*b*d^
2)*x)*sqrt(b*x^2 + a))/(a*b^2), 1/6*(3*A*sqrt(a)*b^2*c^2*log(-(b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 3*(2*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a^2
 - 2*A*a*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*C*a*b*d^
2*x^2 + 6*C*a*b*c^2 + 12*B*a*b*c*d - 2*(2*C*a^2 - 3*A*a*b)*d^2 + 3*(2*C*a*
b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^2), 1/12*(12*A*sqrt(-a)*b^2*c^
2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 3*(2*B*a*b*c^2 - B*a^2*d^2 - 2*(C*a
^2 - 2*A*a*b)*c*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(2*C*a*b*d^2*x^2 + 6*C*a*b*c^2 + 12*B*a*b*c*d - 2*(2*C*a^2 - 3*A*a*b)
*d^2 + 3*(2*C*a*b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^2), 1/6*(6*A*s
qrt(-a)*b^2*c^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 3*(2*B*a*b*c^2 - B*a^
2*d^2 - 2*(C*a^2 - 2*A*a*b)*c*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (2*C*a*b*d^2*x^2 + 6*C*a*b*c^2 + 12*B*a*b*c*d - 2*(2*C*a^2 - 3*A*a*b)
*d^2 + 3*(2*C*a*b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 5.28 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
2*A*c*d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + A*
d**2*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) -
 A*c**2*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*c**2*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt
(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + 2*B*c*d*Piecewise((sqrt(a + b*x*
*2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) + B*d**2*Piecewise((-a*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3
*sqrt(a)), True)) + C*c**2*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2
/(2*sqrt(a)), True)) + 2*C*c*d*Piecewise((-a*Piecewise((log(2*sqrt(b)*sqrt
(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(
2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)), (x**3/(3*sqrt(a)), True)) + C*
d**2*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*
b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cd^2x^2}{3\,b} + \frac{Bc^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{2\,Acd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Ac^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}Cc^2}{b} + \frac{2\sqrt{bx^2+a}Bcd}{b} - \frac{2\sqrt{bx^2+a}Cad^2}{3\,b^2} + \frac{\sqrt{bx^2+a}Ad^2}{b} + \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}x}{2\,b} - \frac{(2\,Ccd+Bd^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*C*d^2*x^2/b + B*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 2
*A*c*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*c^2*arcsinh(a/(sqrt(a*b)*abs(x))
)/sqrt(a) + sqrt(b*x^2 + a)*C*c^2/b + 2*sqrt(b*x^2 + a)*B*c*d/b - 2/3*sqrt
(b*x^2 + a)*C*a*d^2/b^2 + sqrt(b*x^2 + a)*A*d^2/b + 1/2*(2*C*c*d + B*d^2)*
sqrt(b*x^2 + a)*x/b - 1/2*(2*C*c*d + B*d^2)*a*arcsinh(b*x/sqrt(a*b))/b^(3/
2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx = \int \frac{{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x\sqrt{a+bx^2}}\,dx$$
$$= \frac{6\sqrt{b\,x^2+a}\,ab\,d^2-4\sqrt{b\,x^2+a}\,ac\,d^2+12\sqrt{b\,x^2+a}\,b^2cd+3\sqrt{b\,x^2+a}\,b^2d^2x+6\sqrt{b\,x^2+a}\,b\,c^3+6\sqrt{b\,x^2}}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(1/2),x)
```

output
```
(6*sqrt(a + b*x**2)*a*b*d**2 - 4*sqrt(a + b*x**2)*a*c*d**2 + 12*sqrt(a + b
*x**2)*b**2*c*d + 3*sqrt(a + b*x**2)*b**2*d**2*x + 6*sqrt(a + b*x**2)*b*c*
*3 + 6*sqrt(a + b*x**2)*b*c**2*d*x + 2*sqrt(a + b*x**2)*b*c*d**2*x**2 + 6*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2 -
6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2
+ 12*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d - 3*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*d**2 - 6*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*c**2*d + 6*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*b**2*c**2)/(6*b**2)
```

3.109 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 159

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = \frac{d(2cC+Bd)\sqrt{a+bx^2}}{b} - \frac{Ac^2\sqrt{a+bx^2}}{ax} + \frac{Cd^2x\sqrt{a+bx^2}}{2b} - \frac{(aCd^2-2b(c^2C+2Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}} - \frac{c(Bc+2Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
d*(B*d+2*C*c)*(b*x^2+a)^(1/2)/b-A*c^2*(b*x^2+a)^(1/2)/a/x+1/2*C*d^2*x*(b*x
^2+a)^(1/2)/b-1/2*(a*C*d^2-2*b*(A*d^2+2*B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(3/2)-c*(2*A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1
/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = \frac{1}{2}\left(\frac{\sqrt{a+bx^2}(-2Abc^2+adx(4cC+2Bd+Cdx))}{abx} - \frac{4c(Bc+2Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{(aCd^2-2b(c^2C+2Bcd+Ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}\right)$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a + b*x^2]*(-2*A*b*c^2 + a*d*x*(4*c*C + 2*B*d + C*d*x)))/(a*b*x) -
(4*c*(B*c + 2*A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt
[a] + ((a*C*d^2 - 2*b*(c^2*C + 2*B*c*d + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a
 + b*x^2]])/b^(3/2))/2
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2338, 25, 2340, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{aCd^2x^3+ad(2cC+Bd)x^2+a(Cc^2+2Bdc+Ad^2)x+ac(Bc+2Ad)}{x\sqrt{bx^2+a}}dx}{a} - \frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{aCd^2x^3+ad(2cC+Bd)x^2+a(Cc^2+2Bdc+Ad^2)x+ac(Bc+2Ad)}{x\sqrt{bx^2+a}}dx}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\int \frac{2abd(2cC+Bd)x^2-a\left(aCd^2-2b\left(Cc^2+2Bdc+Ad^2\right)\right)x+2abc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\frac{\int \frac{ab\left(2bc(Bc+2Ad)-\left(aCd^2-2b\left(Cc^2+2Bdc+Ad^2\right)\right)x\right)}{x\sqrt{bx^2+a}}dx}{b}+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{a\int \frac{2bc(Bc+2Ad)-\left(aCd^2-2b\left(Cc^2+2Bdc+Ad^2\right)\right)x}{x\sqrt{bx^2+a}}dx+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{a\left(2bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{a\left(2bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{a\left(2bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{a\left(bc(2Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{a\left(2c(2Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{a\left(-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(aCd^2-2b\left(Ad^2+2Bcd+c^2C\right)\right)}{\sqrt{b}}-\frac{2bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2Ad+Bc)}{\sqrt{a}}\right)+2ad\sqrt{a+bx^2}(Bd+2cC)}{2b}+\frac{aCd^2x\sqrt{a+bx^2}}{2b}}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*c^2*Sqrt[a + b*x^2])/(a*x)) + ((a*C*d^2*x*Sqrt[a + b*x^2])/(2*b) + (2
*a*d*(2*c*C + B*d)*Sqrt[a + b*x^2] + a*(-(((a*C*d^2 - 2*b*(c^2*C + 2*B*c*d
 + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]) - (2*b*c*(B*c +
2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/(2*b))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.31

method	result
default	$\frac{A\,d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{C\,c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\,c^{2}\sqrt{b\,x^{2}+a}}{a x}+\frac{d^{2}B\sqrt{b\,x^{2}+a}}{b}+C\,d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\right.}{2b}\right.$
risch	$\frac{A\,d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{C\,c^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\,c^{2}\sqrt{b\,x^{2}+a}}{a x}+\frac{d^{2}B\sqrt{b\,x^{2}+a}}{b}+C\,d^{2}\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\right.}{2b}\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+C*c^2*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))/b^(1/2)-A*c^2*(b*x^2+a)^(1/2)/a/x+d^2*B/b*(b*x^2+a)^(1/2)+C*d^2*(1/2
*x/b*(b*x^2+a)^(1/2)-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-c*(2*A*d
+B*c)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+2*B*c*d*ln(b^(1/2)*x+(
b*x^2+a)^(1/2))/b^(1/2)+2*c*d*C/b*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 675, normalized size of antiderivative = 4.25

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{(2\,Cabc^2+4\,Babcd-(Ca^2-2\,Aab)d^2)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bb^2c^2+2\,Ab^2cd}{4\,ab^2x}\right.$$
$$-\frac{(2\,Cabc^2+4\,Babcd-(Ca^2-2\,Aab)d^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bb^2c^2+2\,Ab^2cd)\sqrt{a}x\log\left(-\frac{bx^2-2}{}\right.}{2\,ab^2x}$$
$$-\frac{(2\,Cabc^2+4\,Babcd-(Ca^2-2\,Aab)d^2)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\,(Bb^2c^2+2\,Ab^2cd)\sqrt{-a}x\arctan\left(\right.}{2\,ab^2x}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*((2*C*a*b*c^2 + 4*B*a*b*c*d - (C*a^2 - 2*A*a*b)*d^2)*sqrt(b)*x*log(-2
*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b^2*c^2 + 2*A*b^2*c*d)*sq
rt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(C*a*b*d^2
*x^2 - 2*A*b^2*c^2 + 2*(2*C*a*b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^
2*x), -1/2*((2*C*a*b*c^2 + 4*B*a*b*c*d - (C*a^2 - 2*A*a*b)*d^2)*sqrt(-b)*x
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b^2*c^2 + 2*A*b^2*c*d)*sqrt(a)*x*
log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (C*a*b*d^2*x^2 - 2*A
*b^2*c^2 + 2*(2*C*a*b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^2*x), 1/4*
(4*(B*b^2*c^2 + 2*A*b^2*c*d)*sqrt(-a)*x*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
 + (2*C*a*b*c^2 + 4*B*a*b*c*d - (C*a^2 - 2*A*a*b)*d^2)*sqrt(b)*x*log(-2*b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a*b*d^2*x^2 - 2*A*b^2*c^2 +
2*(2*C*a*b*c*d + B*a*b*d^2)*x)*sqrt(b*x^2 + a))/(a*b^2*x), -1/2*((2*C*a*b*
c^2 + 4*B*a*b*c*d - (C*a^2 - 2*A*a*b)*d^2)*sqrt(-b)*x*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - 2*(B*b^2*c^2 + 2*A*b^2*c*d)*sqrt(-a)*x*arctan(sqrt(b*x^2
+ a)*sqrt(-a)/a) - (C*a*b*d^2*x^2 - 2*A*b^2*c^2 + 2*(2*C*a*b*c*d + B*a*b*d
^2)*x)*sqrt(b*x^2 + a))/(a*b^2*x)]
```

Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.33

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$$

$$= Ad^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right) - \frac{A\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{a}$$

$$- \frac{2Acd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + 2Bcd\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

$$+ Bd^2\left(\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}\right) - \frac{Bc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

$$+ Cc^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right) + 2Ccd\left(\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

$$+ Cd^2\left(\begin{cases} -\frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
A*d**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - A*s
qrt(b)*c**2*sqrt(a/(b*x**2) + 1)/a - 2*A*c*d*asinh(sqrt(a)/(sqrt(b)*x))/sq
rt(a) + 2*B*c*d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), Tru
e)) + B*d**2*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)),
True)) - B*c**2*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + C*c**2*Piecewise((log
(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log
(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + 2*C*c*d*Piecewise((sqrt(
a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) + C*d**2*Piecewise((-a
*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x
*log(x)/sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b, 0)),
(x**3/(3*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cd^2x}{2\,b} + \frac{Cc^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{2\,Bcd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Cad^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{Ad^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Bc^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{2\,Acd\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{2\,\sqrt{bx^2+a}Ccd}{b} + \frac{\sqrt{bx^2+a}Bd^2}{b} - \frac{\sqrt{bx^2+a}Ac^2}{ax}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
1/2*sqrt(b*x^2 + a)*C*d^2*x/b + C*c^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 2*B
*c*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*C*a*d^2*arcsinh(b*x/sqrt(a*b))/b
^(3/2) + A*d^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - B*c^2*arcsinh(a/(sqrt(a*b)
*abs(x)))/sqrt(a) - 2*A*c*d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 2*sqrt
(b*x^2 + a)*C*c*d/b + sqrt(b*x^2 + a)*B*d^2/b - sqrt(b*x^2 + a)*A*c^2/(a*x
)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx \\
&= \frac{2\,A\sqrt{b}c^2}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{2}\left(\frac{Cd^2x}{b}+\frac{2\,(2\,Cbcd+Bbd^2)}{b^2}\right)\sqrt{bx^2+a} \\
&\quad+\frac{2\,(Bc^2+2\,Acd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} \\
&\quad-\frac{(2\,Cbc^2+4\,Bbcd-Cad^2+2\,Abd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}
\end{aligned}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*A*sqrt(b)*c^2/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/2*(C*d^2*x/b + 2
*(2*C*b*c*d + B*b*d^2)/b^2)*sqrt(b*x^2 + a) + 2*(B*c^2 + 2*A*c*d)*arctan(-
(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*(2*C*b*c^2 + 4*B*b*
c*d - C*a*d^2 + 2*A*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.11

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-2\sqrt{b\,x^2+a}\,a\,b^2c^2+2\sqrt{b\,x^2+a}\,a\,b^2d^2x+4\sqrt{b\,x^2+a}\,ab\,c^2dx+\sqrt{b\,x^2+a}\,abc\,d^2x^2+2\sqrt{a}\log\left(\frac{-\sqrt{a}\,\sqrt{b\,x^2}}{}\right.}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(1/2),x)
```

output

```
( - 2*sqrt(a + b*x**2)*a*b**2*c**2 + 2*sqrt(a + b*x**2)*a*b**2*d**2*x + 4*
sqrt(a + b*x**2)*a*b*c**2*d*x + sqrt(a + b*x**2)*a*b*c*d**2*x**2 + 2*sqrt(
a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)
*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a
*b**2*c*d*x + sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a +
b*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sq
rt(b)*sqrt(a)*x))*b**3*c**2*x - 2*sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) +
sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt
(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b**2*c*d*x - sqrt(a)*log((sqrt(a)*sqr
t(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**
2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*b**3*c**2*x + 2*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2*x - sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d**2*x + 4*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x + 2*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**3*x - 2*sqrt(b)*a*b**2*c**2*x)/(2*a*b
**2*x)
```

3.110 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 162

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx &= \frac{Cd^2\sqrt{a+bx^2}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}-\frac{c(Bc+2Ad)\sqrt{a+bx^2}}{ax} \\ &\quad+\frac{d(2cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} \\ &\quad-\frac{(2ac(cC+2Bd)-A(bc^2-2ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

output
```
C*d^2*(b*x^2+a)^(1/2)/b-1/2*A*c^2*(b*x^2+a)^(1/2)/a/x^2-c*(2*A*d+B*c)*(b*x
^2+a)^(1/2)/a/x+d*(B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)-1
/2*(2*a*c*(2*B*d+C*c)-A*(-2*a*d^2+b*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))
/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-Abc(c+4dx)+2x(-bBc^2+aCd^2x))}{2abx^2} - \frac{Abc^2\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2(c^2C+2Bcd+Ad^2)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{d(2cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-(A*b*c*(c + 4*d*x)) + 2*x*(-(b*B*c^2) + a*C*d^2*x)))/(2
*a*b*x^2) - (A*b*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/
2) - (2*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])
/Sqrt[a]])/Sqrt[a] - (d*(2*c*C + B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/Sqrt[b]
```

Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 25, 2338, 25, 2340, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)}{x^2\sqrt{bx^2+a}}dx}{2a} - \frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2aCd^2x^3+2ad(2cC+Bd)x^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)x+2ac(Bc+2Ad)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{2Cd^2x^2a^2+2d(2cC+Bd)xa^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)a}{x\sqrt{bx^2+a}}dx}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{2Cd^2x^2a^2+2d(2cC+Bd)xa^2+\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)\right)a}{x\sqrt{bx^2+a}}dx}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 2340$$

$$\frac{\frac{\frac{\int \frac{ab\left(2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)+2ad(2cC+Bd)x\right)}{x\sqrt{bx^2+a}}dx}{b}+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{a\int \frac{2ac(cC+2Bd)-A\left(bc^2-2ad^2\right)+2ad(2cC+Bd)x}{x\sqrt{bx^2+a}}dx+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 538$$

$$\frac{\frac{a\left(\left(2ac(2Bd+cC)-A(bc^2-2ad^2)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+2ad(Bd+2cC)\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{a\left(\left(2ac(2Bd+cC)-A(bc^2-2ad^2)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+2ad(Bd+2cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{a\left(\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{2ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}\right)+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{a\left(\frac{1}{2}\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{2ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}\right)+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{a\left(\frac{\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{2ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}\right)+\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{2a^2Cd^2\sqrt{a+bx^2}}{b}+a\left(\frac{2ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(2ac(2Bd+cC)-A\left(bc^2-2ad^2\right)\right)}{\sqrt{a}}\right)}{a}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*c^2*Sqrt[a + b*x^2])/(a*x^2) + ((-2*c*(B*c + 2*A*d)*Sqrt[a + b*x^2
])/x + ((2*a^2*C*d^2*Sqrt[a + b*x^2])/b + a*((2*a*d*(2*c*C + B*d)*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - ((2*a*c*(c*C + 2*B*d) - A*(b*c^2 -
 2*a*d^2))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{c\sqrt{b\,x^2+a}\,(4Adx+2Bcx+Ac)}{2a\,x^2}+\frac{-\frac{(2Aa\,d^2-bA\,c^2+4Bacd+2Ca\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+\frac{2aB\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{4Cacd\ln(\sqrt{}}{}}{2a}$
default	$\frac{d^2B\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{(A\,d^2+2Bcd+C\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+A\,c^2\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)+$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^2+a)^(1/2)*(4*A*d*x+2*B*c*x+A*c)/a/x^2+1/2/a*(-(2*A*a*d^2-A*b*
c^2+4*B*a*c*d+2*C*a*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+2*a
*B*d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+4*C*a*c*d*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))/b^(1/2)+2*a*C*d^2/b*(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 695, normalized size of antiderivative = 4.29

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{2\,(2\,Ca^2cd+Ba^2d^2)\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(4\,Babcd+2\,Aabd^2+(2\,Cab-Ab^2)c}{4\,a^2bx^2}\right.$$
$$-\frac{4\,(2\,Ca^2cd+Ba^2d^2)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(4\,Babcd+2\,Aabd^2+(2\,Cab-Ab^2)c^2)\sqrt{a}x^2\log\left(-\frac{b}{}\right.}{4\,a^2bx^2}$$
$$-\frac{2\,(2\,Ca^2cd+Ba^2d^2)\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(4\,Babcd+2\,Aabd^2+(2\,Cab-Ab^2)c^2)\sqrt{-a}x^2\,\mathrm{arctan}}{2\,a^2bx^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/4*(2*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + (4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b - A*b^2)*c^2)*
sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*C*a
^2*d^2*x^2 - A*a*b*c^2 - 2*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(
a^2*b*x^2), -1/4*(4*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(-b)*x^2*arctan(sqrt(-b)
*x/sqrt(b*x^2 + a)) - (4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b - A*b^2)*c^2)*
sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*C*a
^2*d^2*x^2 - A*a*b*c^2 - 2*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(
a^2*b*x^2), 1/2*((4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b - A*b^2)*c^2)*sqrt(
-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*C*a^2*c*d + B*a^2*d^2)*sqr
t(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (2*C*a^2*d^2*x^
2 - A*a*b*c^2 - 2*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^2
), -1/2*(2*(2*C*a^2*c*d + B*a^2*d^2)*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) - (4*B*a*b*c*d + 2*A*a*b*d^2 + (2*C*a*b - A*b^2)*c^2)*sqrt(-a)*
x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*C*a^2*d^2*x^2 - A*a*b*c^2 - 2*
(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^2)]
```

Sympy [A] (verification not implemented)

Time = 4.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.93

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{2A\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Ad^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{Abc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} + Bd^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{B\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{2Bcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + 2Ccd\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) + Cd^2\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for } b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{Cc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(2*a*x) - 2*A*sqrt(b)*c*d*sqrt(a/(b*x
**2) + 1)/a - A*d**2*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + A*b*c**2*asinh(s
qrt(a)/(sqrt(b)*x))/(2*a**(3/2)) + B*d**2*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2),
Ne(b, 0)), (x/sqrt(a), True)) - B*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/a - 2*
B*c*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + 2*C*c*d*Piecewise((log(2*sqrt(b
)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(
b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + C*d**2*Piecewise((sqrt(a + b*x**2
)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) - C*c**2*asinh(sqrt(a)/(sqrt(b)*
x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \frac{2\,Ccd\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{Bd^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{Cc^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{Abc^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{2\,Bcd\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{Ad^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}Cd^2}{b} - \frac{\sqrt{bx^2+a}Bc^2}{ax} - \frac{2\,\sqrt{bx^2+a}Acd}{ax} - \frac{\sqrt{bx^2+a}Ac^2}{2\,ax^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
2*C*c*d*arcsinh(b*x/sqrt(a*b))/sqrt(b) + B*d^2*arcsinh(b*x/sqrt(a*b))/sqrt
(b) - C*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*A*b*c^2*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(3/2) - 2*B*c*d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a
) - A*d^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*C*d^2/b
- sqrt(b*x^2 + a)*B*c^2/(a*x) - 2*sqrt(b*x^2 + a)*A*c*d/(a*x) - 1/2*sqrt(b
*x^2 + a)*A*c^2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{bx^2+a}Cd^2}{b} - \frac{(2\,Ccd+Bd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$
$$+ \frac{(2\,Cac^2-Abc^2+4\,Bacd+2\,Aad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$
$$+ \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Abc^2 + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b}c^2 + 4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Aa\sqrt{b}cd + \left(\sqrt{b}x\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2 a}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*C*d^2/b - (2*C*c*d + B*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/sqrt(b) + (2*C*a*c^2 - A*b*c^2 + 4*B*a*c*d + 2*A*a*d^2)*arctan(-(
sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(b)*x - sqrt(b
*x^2 + a))^3*A*b*c^2 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*c^2 +
 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*c*d + (sqrt(b)*x - sqrt(b*x
^2 + a))*A*a*b*c^2 - 2*B*a^2*sqrt(b)*c^2 - 4*A*a^2*sqrt(b)*c*d)/(((sqrt(b)
*x - sqrt(b*x^2 + a))^2 - a)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^3\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.53

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\sqrt{b\,x^2+a}\,ab\,c^2-4\sqrt{b\,x^2+a}\,abcdx+2\sqrt{b\,x^2+a}\,ac\,d^2x^2-2\sqrt{b\,x^2+a}\,b^2c^2x+2\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{}}{\sqrt{a}}\right.}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*b*c**2 - 4*sqrt(a + b*x**2)*a*b*c*d*x + 2*sqrt(a +
b*x**2)*a*c*d**2*x**2 - 2*sqrt(a + b*x**2)*b**2*c**2*x + 2*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**2 - sqrt(a)*log
((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**2 + 4*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**2 +
2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*x**
2 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d*
*2*x**2 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*
*2*c**2*x**2 - 4*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt
(a))*b**2*c*d*x**2 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x
)/sqrt(a))*b*c**3*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a*b*d**2*x**2 + 4*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*a*c**2*d*x**2)/(2*a*b*x**2)
```

3.111 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 186

$$\begin{aligned}\int \frac{(c+dx)^2\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}-\frac{c(Bc+2Ad)\sqrt{a+bx^2}}{2ax^2}\\ &\quad -\frac{(3ac(cC+2Bd)-A(2bc^2-3ad^2))\sqrt{a+bx^2}}{3a^2x}+\frac{Cd^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\\ &\quad +\frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}\end{aligned}$$

output
```
-1/3*A*c^2*(b*x^2+a)^(1/2)/a/x^3-1/2*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a/x^2-1
/3*(3*a*c*(2*B*d+C*c)-A*(-3*a*d^2+2*b*c^2))*(b*x^2+a)^(1/2)/a^2/x+C*d^2*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)+1/2*(b*c*(2*A*d+B*c)-2*a*d*(B*d+2
*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(4Abc^2x^2-2aA(c^2+3cdx+3d^2x^2)-3acx(2cCx+B(c+4dx)))}{6a^2x^3} + \frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{Cd^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(4*A*b*c^2*x^2 - 2*a*A*(c^2 + 3*c*d*x + 3*d^2*x^2) - 3*a*
c*x*(2*c*C*x + B*(c + 4*d*x))))/(6*a^2*x^3) + ((b*c*(B*c + 2*A*d) - 2*a*d*
(2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
- (C*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 25, 2338, 25, 2338, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2\right)}{x^4\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 2338}$$

$$-\frac{\int -\frac{3aCd^2x^3+3ad(2cC+Bd)x^2+\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)x+3ac(Bc+2Ad)}{x^3\sqrt{bx^2+a}}dx}{3a} - \frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3aCd^2x^3+3ad(2cC+Bd)x^2+\left(3ac(cC+2Bd)-A(2bc^2-3ad^2)\right)x+3ac(Bc+2Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{6a^2Cd^2x^2-3a(bc(Bc+2Ad)-2ad(2cC+Bd))x+2a\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{6a^2Cd^2x^2-3a(bc(Bc+2Ad)-2ad(2cC+Bd))x+2a\left(3ac(cC+2Bd)-A\left(2bc^2-3ad^2\right)\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 2338

$$\frac{\frac{-\frac{\int \frac{3a^2\left(-2aCxd^2-2a(2cC+Bd)d+bc(Bc+2Ad)\right)}{x\sqrt{bx^2+a}}dx}{a}-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{-3a\int \frac{-2aCxd^2-2a(2cC+Bd)d+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 538

$$\frac{\frac{-3a\left((bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-2aCd^2\int \frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 224

$$\frac{\frac{-3a\left((bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-2aCd^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2A}{2x^2}}{3a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\downarrow$ 219

$$\frac{\frac{-3a\left((bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2A}{2x^2}}{3a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\downarrow$ 243

$$\frac{\frac{-3a\left(\frac{1}{2}(bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}}{2}}{3a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\downarrow$ 73

$$\frac{\frac{-3a\left(\frac{(bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx^2}(2A}{2x^2}}{3a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

$\downarrow$ 221

$$\frac{\frac{-3a\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bc(2Ad+Bc)-2ad(Bd+2cC))}{\sqrt{a}}-\frac{2aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(2bc^2-3ad^2\right)\right)}{x}}{2a}-\frac{3c\sqrt{a+bx}}{}}{3a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}$$

input

```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*c^2*Sqrt[a + b*x^2])/(a*x^3) + ((-3*c*(B*c + 2*A*d)*Sqrt[a + b*x^2
])/(2*x^2) + ((-2*(3*a*c*(c*C + 2*B*d) - A*(2*b*c^2 - 3*a*d^2))*Sqrt[a + b
*x^2])/x - 3*a*((-2*a*C*d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
- ((b*c*(B*c + 2*A*d) - 2*a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/Sqrt[a]))/(2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(6Aa\,d^{2}x^{2}-4Ab\,c^{2}x^{2}+12Bacd\,x^{2}+6Ca\,c^{2}x^{2}+6Aacdx+3Ba\,c^{2}x+2A\,c^{2}a\right)}{6a^{2}x^{3}}-\frac{-\frac{\left(2Abcd-2aB\,d^{2}+bB\,c^{2}-4Cacd\right)\ln\left(\frac{2a+}{}\right.}{\sqrt{a}}}{}$
default	$\frac{C\,d^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{\left(A\,d^{2}+2Bcd+C\,c^{2}\right)\sqrt{b\,x^{2}+a}}{ax}+A\,c^{2}\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)+c(2Ad+Bc)\left(-\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(6*A*a*d^2*x^2-4*A*b*c^2*x^2+12*B*a*c*d*x^2+6*C*a*c^2
*x^2+6*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/a^2/x^3-1/2/a*(-(2*A*b*c*d-2*B*a*d
^2+B*b*c^2-4*C*a*c*d)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-2*a*C*
d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 762, normalized size of antiderivative = 4.10

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/12*(6*C*a^2*sqrt(b)*d^2*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) - 3*(B*b^2*c^2 - 2*B*a*b*d^2 - 2*(2*C*a*b - A*b^2)*c*d)*sqrt(a)*x^3*l
og(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*A*a*b*c^2 + 2*(6
*B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b - 2*A*b^2)*c^2)*x^2 + 3*(B*a*b*c^2 + 2
*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^3), -1/12*(12*C*a^2*sqrt(-b)*d^2*
x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + 3*(B*b^2*c^2 - 2*B*a*b*d^2 - 2*(2
*C*a*b - A*b^2)*c*d)*sqrt(a)*x^3*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
 2*a)/x^2) + 2*(2*A*a*b*c^2 + 2*(6*B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b - 2*
A*b^2)*c^2)*x^2 + 3*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x
^3), 1/6*(3*C*a^2*sqrt(b)*d^2*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) - 3*(B*b^2*c^2 - 2*B*a*b*d^2 - 2*(2*C*a*b - A*b^2)*c*d)*sqrt(-a)*x
^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (2*A*a*b*c^2 + 2*(6*B*a*b*c*d + 3*
A*a*b*d^2 + (3*C*a*b - 2*A*b^2)*c^2)*x^2 + 3*(B*a*b*c^2 + 2*A*a*b*c*d)*x)*
sqrt(b*x^2 + a))/(a^2*b*x^3), -1/6*(6*C*a^2*sqrt(-b)*d^2*x^3*arctan(sqrt(-
b)*x/sqrt(b*x^2 + a)) + 3*(B*b^2*c^2 - 2*B*a*b*d^2 - 2*(2*C*a*b - A*b^2)*c
*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*A*a*b*c^2 + 2*(6*
B*a*b*c*d + 3*A*a*b*d^2 + (3*C*a*b - 2*A*b^2)*c^2)*x^2 + 3*(B*a*b*c^2 + 2*
A*a*b*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b*x^3)]
```

Sympy [A] (verification not implemented)

Time = 4.80 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{A\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{ax} - \frac{A\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2Ab^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Abcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{a^{\frac{3}{2}}} - \frac{B\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{2B\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Bd^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{Bbc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} + Cd^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{C\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{2Ccd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - A*sqrt(b)*c*d*sqrt(a/(b*
x**2) + 1)/(a*x) - A*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/a + 2*A*b**(3/2)*c*
*2*sqrt(a/(b*x**2) + 1)/(3*a**2) + A*b*c*d*asinh(sqrt(a)/(sqrt(b)*x))/a**(
3/2) - B*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(2*a*x) - 2*B*sqrt(b)*c*d*sqrt(
a/(b*x**2) + 1)/a - B*d**2*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*b*c**2*a
sinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) + C*d**2*Piecewise((log(2*sqrt(b)*s
qrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x
**2), Ne(b, 0)), (x/sqrt(a), True)) - C*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/
a - 2*C*c*d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = \frac{Cd^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{2\,Ccd \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{Bd^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{\sqrt{bx^2+a}Cc^2}{ax} + \frac{2\,\sqrt{bx^2+a}Abc^2}{3\,a^2x} - \frac{2\,\sqrt{bx^2+a}Bcd}{ax} - \frac{\sqrt{bx^2+a}Ad^2}{ax} + \frac{(Bc^2+2\,Acd)b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}Ac^2}{3\,ax^3} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}}{2\,ax^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
C*d^2*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 2*C*c*d*arcsinh(a/(sqrt(a*b)*abs(x)
))/sqrt(a) - B*d^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)
*C*c^2/(a*x) + 2/3*sqrt(b*x^2 + a)*A*b*c^2/(a^2*x) - 2*sqrt(b*x^2 + a)*B*c
*d/(a*x) - sqrt(b*x^2 + a)*A*d^2/(a*x) + 1/2*(B*c^2 + 2*A*c*d)*b*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/3*sqrt(b*x^2 + a)*A*c^2/(a*x^3) - 1/2*(B*
c^2 + 2*A*c*d)*sqrt(b*x^2 + a)/(a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(160) = 320$.

Time = 0.20 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.57

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = -\frac{Cd^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} - \frac{(Bbc^2-4\,Cacd+2\,Abcd-2\,Bad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Bbc^2+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Abcd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 Ca\sqrt{b}c^2+12\left(\sqrt{b}x\right.}{}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-C*d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) - (B*b*c^2 - 4*C*a*c
*d + 2*A*b*c*d - 2*B*a*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a)
)/(sqrt(-a)*a) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*b*c^2 + 6*(sqrt(
b)*x - sqrt(b*x^2 + a))^5*A*b*c*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a*
sqrt(b)*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*c*d + 6*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*A*a*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*C*a^2*sqrt(b)*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*b^(3/2)*c
^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c*d - 12*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*A*a^2*sqrt(b)*d^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B
*a^2*b*c^2 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*c*d + 6*C*a^3*sqrt(b)
*c^2 - 4*A*a^2*b^(3/2)*c^2 + 12*B*a^3*sqrt(b)*c*d + 6*A*a^3*sqrt(b)*d^2)/(
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^4\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.74

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4\sqrt{a+bx^2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2b\,c^2-6\sqrt{b\,x^2+a}\,a^2bcdx-6\sqrt{b\,x^2+a}\,a^2b\,d^2x^2+4\sqrt{b\,x^2+a}\,a\,b^2c^2x^2-3\sqrt{b\,x^2+a}\,a\,b^2c}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*b*c**2 - 6*sqrt(a + b*x**2)*a**2*b*c*d*x - 6*s
qrt(a + b*x**2)*a**2*b*d**2*x**2 + 4*sqrt(a + b*x**2)*a*b**2*c**2*x**2 - 3
*sqrt(a + b*x**2)*a*b**2*c**2*x - 12*sqrt(a + b*x**2)*a*b**2*c*d*x**2 - 6*
sqrt(a + b*x**2)*a*b*c**3*x**2 - 6*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**3 + 6*sqrt(a)*log((sqrt(a + b*x**2) -
 sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**3 + 12*sqrt(a)*log((sqrt(a +
 b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x**3 - 3*sqrt(a)*log((
sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**3 + 6*sqrt(a
)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**3 -
6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**
2*x**3 - 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*
a*b*c**2*d*x**3 + 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**3*c**2*x**3 + 6*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**2*c*d**2*x**3 + 2*sqrt(b)*a**2*b*d**2*x**3 - 4*sqrt(b)*a*b**2*c**2
*x**3 + 4*sqrt(b)*a*b**2*c*d*x**3 + 2*sqrt(b)*a*b*c**3*x**3)/(6*a**2*b*x**
3)
```

3.112 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 219

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx &= -\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}-\frac{c(Bc+2Ad)\sqrt{a+bx^2}}{3ax^3}\\ &\quad-\frac{(4ac(cC+2Bd)-A(3bc^2-4ad^2))\sqrt{a+bx^2}}{8a^2x^2}\\ &\quad+\frac{(2bc(Bc+2Ad)-3ad(2cC+Bd))\sqrt{a+bx^2}}{3a^2x}\\ &\quad-\frac{(Ab(3bc^2-4ad^2)+4a(2aCd^2-bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}\end{aligned}$$

output

```
-1/4*A*c^2*(b*x^2+a)^(1/2)/a/x^4-1/3*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a/x^3-1
/8*(4*a*c*(2*B*d+C*c)-A*(-4*a*d^2+3*b*c^2))*(b*x^2+a)^(1/2)/a^2/x^2+1/3*(2
*b*c*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x-1/8*(A*b*(-4*a*d
^2+3*b*c^2)+4*a*(2*a*C*d^2-b*c*(2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(bcx^2(9Ac+16Bcx+32Adx)-2a(6cCx^2(c+4dx)+4Bx(c^2+3cdx+3d^2x^2)+A(3c^2+8cdx+6d^2x^2)))}{x^4}+6(3Ab^2c^2+8a^2Cd^2)}{24a^{5/2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(b*c*x^2*(9*A*c + 16*B*c*x + 32*A*d*x) - 2*a*(6*
c*C*x^2*(c + 4*d*x) + 4*B*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + A*(3*c^2 + 8*c*d
*x + 6*d^2*x^2))))/x^4 + 6*(3*A*b^2*c^2 + 8*a^2*C*d^2)*ArcTanh[(Sqrt[b]*x
- Sqrt[a + b*x^2])/Sqrt[a]] + 24*a*b*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(-(
Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(24*a^(5/2))
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2338, 25, 2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2338$$
$$-\frac{\int -\frac{4aCd^2x^3+4ad(2cC+Bd)x^2+(4ac(cC+2Bd)-A(3bc^2-4ad^2))x+4ac(Bc+2Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$
$$\Big\downarrow \; 25$$
$$\frac{\int \frac{4aCd^2x^3+4ad(2cC+Bd)x^2+(4ac(cC+2Bd)-A(3bc^2-4ad^2))x+4ac(Bc+2Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{12a^2Cd^2x^2-4a(2bc(Bc+2Ad)-3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)-A\left(3bc^2-4ad^2\right)\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4c\sqrt{a+bx^2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{12a^2Cd^2x^2-4a(2bc(Bc+2Ad)-3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)-A\left(3bc^2-4ad^2\right)\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4c\sqrt{a+bx^2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{-\frac{\int \frac{a\left(8a(2bc(Bc+2Ad)-3ad(2cC+Bd))-3\left(8a^2Cd^2-4abc(cC+2Bd)+Ab\left(3bc^2-4ad^2\right)\right)x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4ad^2\right)\right)}{2x^2}}{3a}-\frac{4c\sqrt{a+bx^2}(2\cdots}{3x^3}}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{1}{2}\int \frac{8a(2bc(Bc+2Ad)-3ad(2cC+Bd))-3\left(8a^2Cd^2-4abc(cC+2Bd)+Ab\left(3bc^2-4ad^2\right)\right)x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4ad^2\right)\right)}{2x^2}}{3a}-\frac{4c\sqrt{a+bx^2}(2A\cdots}{3x^3}}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{1}{2}\left(3\left(8a^2Cd^2+Ab\left(3bc^2-4ad^2\right)-4abc(2Bd+cC)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{8\sqrt{a+bx^2}(2bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4ad^2\cdots\right.\right.}{2x^2}}{3a}\cdots}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3}{2}\left(8a^2Cd^2+Ab\left(3bc^2-4ad^2\right)-4abc(2Bd+cC)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{8\sqrt{a+bx^2}(2bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4a\right.\right.}{2x^2}}{3a}}{4a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 73

$$\frac{\frac{\frac{1}{2}\left(\frac{3\left(8a^2Cd^2+Ab\left(3bc^2-4ad^2\right)-4abc(2Bd+cC)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{8\sqrt{a+bx^2}(2bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4ad^2\right.\right.}{2x^2}}{3a}}{4a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{2}\left(\frac{8\sqrt{a+bx^2}(2bc(2Ad+Bc)-3ad(Bd+2cC))}{x}-\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2Cd^2+Ab\left(3bc^2-4ad^2\right)-4abc(2Bd+cC)\right)}{\sqrt{a}}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(3bc^2-4\right.\right.}{2x^2}}{3a}}{4a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}$$

input

```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^5*Sqrt[a + b*x^2]),x]
```

output

```
-1/4*(A*c^2*Sqrt[a + b*x^2])/(a*x^4) + ((-4*c*(B*c + 2*A*d)*Sqrt[a + b*x^2
])/(3*x^3) + ((-3*(4*a*c*(c*C + 2*B*d) - A*(3*b*c^2 - 4*a*d^2))*Sqrt[a + b
*x^2])/(2*x^2) + ((8*(2*b*c*(B*c + 2*A*d) - 3*a*d*(2*c*C + B*d))*Sqrt[a +
b*x^2])/x - (3*(8*a^2*C*d^2 - 4*a*b*c*(c*C + 2*B*d) + A*b*(3*b*c^2 - 4*a*d
^2))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-32Abcd\,x^3+24Ba\,d^2x^3-16Bb\,c^2x^3+48Cacd\,x^3+12Aa\,d^2x^2-9Ab\,c^2x^2+24Bacd\,x^2+12Ca\,c^2x^2+16Aacdx+8Ba\,c^2}{24a^2x^4}$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)+A\,c^2\left(-\frac{\sqrt{b\,x^2+a}}{4a\,x^4}-\frac{3b\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(2a+\right.}{}\right.}{4a}\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-32*A*b*c*d*x^3+24*B*a*d^2*x^3-16*B*b*c^2*x^3+48*C*
a*c*d*x^3+12*A*a*d^2*x^2-9*A*b*c^2*x^2+24*B*a*c*d*x^2+12*C*a*c^2*x^2+16*A*
a*c*d*x+8*B*a*c^2*x+6*A*a*c^2)/a^2/x^4+1/8*(4*A*a*b*d^2-3*A*b^2*c^2+8*B*a*
b*c*d-8*C*a^2*d^2+4*C*a*b*c^2)/a^(5/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/
x)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.84

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$$

$$=\left[\frac{3\,(8\,Babcd+(4\,Cab-3\,Ab^2)c^2-4\,(2\,Ca^2-Aab)d^2)\sqrt{a}x^4\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(6\,Aa^2c^2-8}{}\right.$$

$$-\frac{3\,(8\,Babcd+(4\,Cab-3\,Ab^2)c^2-4\,(2\,Ca^2-Aab)d^2)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(6\,Aa^2c^2-8\,(2\,B}{}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/48*(3*(8*B*a*b*c*d + (4*C*a*b - 3*A*b^2)*c^2 - 4*(2*C*a^2 - A*a*b)*d^2)
*sqrt(a)*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(6*A*
a^2*c^2 - 8*(2*B*a*b*c^2 - 3*B*a^2*d^2 - 2*(3*C*a^2 - 2*A*a*b)*c*d)*x^3 +
3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 - 3*A*a*b)*c^2)*x^2 + 8*(B*a^2*c^2
 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^4), -1/24*(3*(8*B*a*b*c*d + (4*
C*a*b - 3*A*b^2)*c^2 - 4*(2*C*a^2 - A*a*b)*d^2)*sqrt(-a)*x^4*arctan(sqrt(b
*x^2 + a)*sqrt(-a)/a) + (6*A*a^2*c^2 - 8*(2*B*a*b*c^2 - 3*B*a^2*d^2 - 2*(3
*C*a^2 - 2*A*a*b)*c*d)*x^3 + 3*(8*B*a^2*c*d + 4*A*a^2*d^2 + (4*C*a^2 - 3*A
*a*b)*c^2)*x^2 + 8*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^4)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(202) = 404$.

Time = 7.50 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.13

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{Ac^2}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{A\sqrt{b}c^2}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{2A\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{A\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{3Ab^{\frac{3}{2}}c^2}{8a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{4Ab^{\frac{3}{2}}cd\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Abd^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3Ab^2c^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} - \frac{B\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{B\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{ax} - \frac{B\sqrt{b}d^2\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2Bb^{\frac{3}{2}}c^2\sqrt{\frac{a}{bx^2}+1}}{3a^2} + \frac{Bbcd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{a^{\frac{3}{2}}} - \frac{C\sqrt{b}c^2\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{2C\sqrt{b}cd\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{Cd^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{Cbc^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-A*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c**2/(8*a*x**3*s
qrt(a/(b*x**2) + 1)) - 2*A*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - A
*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(2*a*x) + 3*A*b**(3/2)*c**2/(8*a**2*x*s
qrt(a/(b*x**2) + 1)) + 4*A*b**(3/2)*c*d*sqrt(a/(b*x**2) + 1)/(3*a**2) + A*
b*d**2*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*A*b**2*c**2*asinh(sqrt(
a)/(sqrt(b)*x))/(8*a**(5/2)) - B*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(3*a*x*
*2) - B*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(a*x) - B*sqrt(b)*d**2*sqrt(a/(b*
x**2) + 1)/a + 2*B*b**(3/2)*c**2*sqrt(a/(b*x**2) + 1)/(3*a**2) + B*b*c*d*a
sinh(sqrt(a)/(sqrt(b)*x))/a**(3/2) - C*sqrt(b)*c**2*sqrt(a/(b*x**2) + 1)/(
2*a*x) - 2*C*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/a - C*d**2*asinh(sqrt(a)/(sq
rt(b)*x))/sqrt(a) + C*b*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx = -\frac{3\,Ab^2c^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} - \frac{Cd^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{2\,\sqrt{bx^2+a}Ccd}{ax} - \frac{\sqrt{bx^2+a}Bd^2}{ax} + \frac{(Cc^2+2\,Bcd+Ad^2)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} + \frac{3\,\sqrt{bx^2+a}Abc^2}{8\,a^2x^2} + \frac{2\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b}{3\,a^2x} - \frac{\sqrt{bx^2+a}Ac^2}{4\,ax^4} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}}{2\,ax^2} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}}{3\,ax^3}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
-3/8*A*b^2*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - C*d^2*arcsinh(a/(sq
rt(a*b)*abs(x)))/sqrt(a) - 2*sqrt(b*x^2 + a)*C*c*d/(a*x) - sqrt(b*x^2 + a)
*B*d^2/(a*x) + 1/2*(C*c^2 + 2*B*c*d + A*d^2)*b*arcsinh(a/(sqrt(a*b)*abs(x)
))/a^(3/2) + 3/8*sqrt(b*x^2 + a)*A*b*c^2/(a^2*x^2) + 2/3*(B*c^2 + 2*A*c*d)
*sqrt(b*x^2 + a)*b/(a^2*x) - 1/4*sqrt(b*x^2 + a)*A*c^2/(a*x^4) - 1/2*(C*c^
2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)/(a*x^2) - 1/3*(B*c^2 + 2*A*c*d)*sqrt(
b*x^2 + a)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(195) = 390$.

Time = 0.17 (sec) , antiderivative size = 909, normalized size of antiderivative = 4.15

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(4*C*a*b*c^2 - 3*A*b^2*c^2 + 8*B*a*b*c*d - 8*C*a^2*d^2 + 4*A*a*b*d^2)
*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/12*(12
*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^2 - 9*(sqrt(b)*x - sqrt(b*x^2 + a
))^7*A*b^2*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b*c*d + 12*(sqrt(b
)*x - sqrt(b*x^2 + a))^7*A*a*b*d^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^2*sqrt(b)*c*d + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*sqrt(b)*d^2 - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c^2 + 33*(sqrt(b)*x - sqrt(b*x^2
 + a))^5*A*a*b^2*c^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*c*d - 12
*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^2*b*d^2 + 48*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^2*b^(3/2)*c^2 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*sqrt
(b)*c*d + 96*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2)*c*d - 72*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*C*a^3*b*c^2 + 33*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c^2 - 24
*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b*c*d - 12*(sqrt(b)*x - sqrt(b*x^2
+ a))^3*A*a^3*b*d^2 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c^2
 + 144*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*sqrt(b)*c*d - 128*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*A*a^3*b^(3/2)*c*d + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*B*a^4*sqrt(b)*d^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b*c^2 - 9*(sq
rt(b)*x - sqrt(b*x^2 + a))*A*a^3*b^2*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a)
)*B*a^4*b*c*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^4*b*d^2 + 16*B*a^4...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^5\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 588, normalized size of antiderivative = 2.68

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \frac{16\sqrt{b\,x^2+a}\,b^2c^2x^3+32\sqrt{b\,x^2+a}\,abcd\,x^3-12\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab\,d^2x^4+24\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}-}{\sqrt{a}}\right.}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**2*c**2 - 16*sqrt(a + b*x**2)*a**2*c*d*x - 12*sqr
t(a + b*x**2)*a**2*d**2*x**2 + 9*sqrt(a + b*x**2)*a*b*c**2*x**2 - 8*sqrt(a
 + b*x**2)*a*b*c**2*x + 32*sqrt(a + b*x**2)*a*b*c*d*x**3 - 24*sqrt(a + b*x
**2)*a*b*c*d*x**2 - 24*sqrt(a + b*x**2)*a*b*d**2*x**3 - 12*sqrt(a + b*x**2
)*a*c**3*x**2 - 48*sqrt(a + b*x**2)*a*c**2*d*x**3 + 16*sqrt(a + b*x**2)*b*
*2*c**2*x**3 - 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr
t(a))*a*b*d**2*x**4 + 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*a*c*d**2*x**4 + 9*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*b**2*c**2*x**4 - 24*sqrt(a)*log((sqrt(a + b*x**2) - sqr
t(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**4 - 12*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*x**4 + 12*sqrt(a)*log((sqrt(a + b
*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*d**2*x**4 - 24*sqrt(a)*log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*c*d**2*x**4 - 9*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 + 24*sq
rt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c*d*x**4
 + 12*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*c**3*
x**4 - 32*sqrt(b)*a*b*c*d*x**4 + 12*sqrt(b)*a*b*d**2*x**4 + 24*sqrt(b)*a*c
**2*d*x**4 - 16*sqrt(b)*b**2*c**2*x**4)/(24*a**2*x**4)
```

3.113 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 267

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx &= -\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}-\frac{c(Bc+2Ad)\sqrt{a+bx^2}}{4ax^4}\\ &\quad-\frac{(5ac(cC+2Bd)-A(4bc^2-5ad^2))\sqrt{a+bx^2}}{15a^2x^3}\\ &\quad+\frac{(3bc(Bc+2Ad)-4ad(2cC+Bd))\sqrt{a+bx^2}}{8a^2x^2}\\ &\quad-\frac{(2Ab(4bc^2-5ad^2)+5a(3aCd^2-2bc(cC+2Bd)))\sqrt{a+bx^2}}{15a^3x}\\ &\quad-\frac{b(3bc(Bc+2Ad)-4ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}\end{aligned}$$

output
```
-1/5*A*c^2*(b*x^2+a)^(1/2)/a/x^5-1/4*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a/x^4-1
/15*(5*a*c*(2*B*d+C*c)-A*(-5*a*d^2+4*b*c^2))*(b*x^2+a)^(1/2)/a^2/x^3+1/8*(
3*b*c*(2*A*d+B*c)-4*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x^2-1/15*(2*A*b*(
-5*a*d^2+4*b*c^2)+5*a*(3*a*C*d^2-2*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^3/x
-1/8*b*(3*b*c*(2*A*d+B*c)-4*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = \frac{\frac{\sqrt{a+bx^2}\left(-64Ab^2c^2x^4+abx^2\left(5cx(9Bc+16cCx+32Bdx)+A\left(32c^2+90cdx+80d^2x^2\right)\right)-2a^2\left(2A\left(6c^2+15cdx+10d^2x^2\right)+5x\left(4Cx\left(c^2+3cdx+3d^2x\right.\right.\right.\right.}{x^5}}{120a^3}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a + b*x^2]*(-64*A*b^2*c^2*x^4 + a*b*x^2*(5*c*x*(9*B*c + 16*c*C*x +
32*B*d*x) + A*(32*c^2 + 90*c*d*x + 80*d^2*x^2)) - 2*a^2*(2*A*(6*c^2 + 15*c
*d*x + 10*d^2*x^2) + 5*x*(4*C*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + B*(3*c^2 + 8
*c*d*x + 6*d^2*x^2)))))/x^5 + 30*Sqrt[a]*b*(-3*b*c*(B*c + 2*A*d) + 4*a*d*(
2*c*C + B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(120*a^3)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2338, 25, 2338, 25, 2338, 27, 539, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 2338$$

$$-\frac{\int -\frac{5aCd^2x^3+5ad(2cC+Bd)x^2+\left(5ac(cC+2Bd)-A\left(4bc^2-5ad^2\right)\right)x+5ac(Bc+2Ad)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{5aCd^2x^3+5ad(2cC+Bd)x^2+\left(5ac(cC+2Bd)-A\left(4bc^2-5ad^2\right)\right)x+5ac(Bc+2Ad)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 2338$$

$$-\frac{-\frac{\int -\frac{20a^2Cd^2x^2-5a(3bc(Bc+2Ad)-4ad(2cC+Bd))x+4a\left(5ac(cC+2Bd)-A\left(4bc^2-5ad^2\right)\right)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{5c\sqrt{a+bx^2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{20a^2Cd^2x^2-5a(3bc(Bc+2Ad)-4ad(2cC+Bd))x+4a\left(5ac(cC+2Bd)-A\left(4bc^2-5ad^2\right)\right)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{5c\sqrt{a+bx^2}(2Ad+Bc)}{4x^4}}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{-\frac{\int \frac{a\left(15a(3bc(Bc+2Ad)-4ad(2cC+Bd))-4\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(cC+2Bd)\right)\right)x\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4\sqrt{a+bx^2}\left(5ac(2Bd+cC)-A\left(4bc^2-5ad^2\right)\right)}{3x^3}}{4a}-5c\sqrt{a+bx\cdots}}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{1}{3}\int \frac{15a(3bc(Bc+2Ad)-4ad(2cC+Bd))-4\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(cC+2Bd)\right)\right)x}{x^3\sqrt{bx^2+a}}dx-\frac{4\sqrt{a+bx^2}\left(5ac(2Bd+cC)-A\left(4bc^2-5ad^2\right)\right)}{3x^3}}{4a}-\frac{5c\sqrt{a+bx^2}\cdots}{4\cdots}}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 539$$

$$\frac{\frac{\frac{1}{3}\left(\frac{\int \frac{a\left(8\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(cC+2Bd)\right)\right)+15b(3bc(Bc+2Ad)-4ad(2cC+Bd))x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc)-4ad(Bd+2cC))}{2x^2}\right)-\frac{4\sqrt{a+bx^2}\cdots}{\cdots}}{4a}\cdots}{5a}-\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}\left(\frac{1}{2}\int \frac{8\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(cC+2Bd)\right)\right)+15b(3bc(Bc+2Ad)-4ad(2cC+Bd))x}{x^2\sqrt{bx^2+a}}dx+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc)-4ad(Bd+2cC))}{2x^2}\right)-\frac{4\sqrt{a+bx^2}(\cdots}{\cdots}}{4a}}{5a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 534

$$\frac{\frac{\frac{1}{3}\left(\frac{1}{2}\left(15b(3bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(2Bd+cC)\right)\right)}{ax}\right)+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc)-4ad\cdots}{2x^2}\right.}{4a}}{5a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 243

$$\frac{\frac{\frac{1}{3}\left(\frac{1}{2}\left(\frac{15}{2}b(3bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{8\sqrt{a+bx^2}\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(2Bd+cC)\right)\right)}{ax}\right)+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc)-4\cdots}{2x^2}\right.}{4a}}{5a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 73

$$\frac{\frac{\frac{1}{3}\left(\frac{1}{2}\left(15(3bc(2Ad+Bc)-4ad(Bd+2cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{8\sqrt{a+bx^2}\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(2Bd+cC)\right)\right)}{ax}\right)+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc)-\cdots}{2x^2}\right.}{4a}}{5a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

$\downarrow$ 221

$$\frac{\frac{\frac{1}{3}\left(\frac{1}{2}\left(-\frac{15b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3bc(2Ad+Bc)-4ad(Bd+2cC))}{\sqrt{a}}-\frac{8\sqrt{a+bx^2}\left(2Ab\left(4bc^2-5ad^2\right)+5a\left(3aCd^2-2bc(2Bd+cC)\right)\right)}{ax}\right)+\frac{15\sqrt{a+bx^2}(3bc(2Ad+Bc\cdots}{2x^2}\right.}{4a}}{5a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{5ax^5}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/5*(A*c^2*Sqrt[a + b*x^2])/(a*x^5) + ((-5*c*(B*c + 2*A*d)*Sqrt[a + b*x^2
])/(4*x^4) + ((-4*(5*a*c*(c*C + 2*B*d) - A*(4*b*c^2 - 5*a*d^2))*Sqrt[a + b
*x^2])/(3*x^3) + ((15*(3*b*c*(B*c + 2*A*d) - 4*a*d*(2*c*C + B*d))*Sqrt[a +
 b*x^2])/(2*x^2) + ((-8*(2*A*b*(4*b*c^2 - 5*a*d^2) + 5*a*(3*a*C*d^2 - 2*b*
c*(c*C + 2*B*d)))*Sqrt[a + b*x^2])/(a*x) - (15*b*(3*b*c*(B*c + 2*A*d) - 4*
a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/3)/(4*a))
/(5*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(-80 A b d^2 x^4 a+64 A b^2 c^2 x^4-160 B b c d x^4 a+120 C a^2 d^2 x^4-80 C b c^2 x^4 a-90 A b c d x^3 a+60 B a^2 d^2 x^3-45 B b c^2 x^3 a+120 C}{120 a^3 x^5}$
default	$(A d^2+2Bcd+C c^2)\left(-\frac{\sqrt{b x^2+a}}{3 a x^3}+\frac{2b\sqrt{b x^2+a}}{3a^2 x}\right)+A c^2\left(-\frac{\sqrt{b x^2+a}}{5 a x^5}-\frac{4b\left(-\frac{\sqrt{b x^2+a}}{3 a x^3}+\frac{2b\sqrt{b x^2+a}}{3a^2 x}\right)}{5a}\right)-C c$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(b*x^2+a)^(1/2)*(-80*A*a*b*d^2*x^4+64*A*b^2*c^2*x^4-160*B*a*b*c*d*x
^4+120*C*a^2*d^2*x^4-80*C*a*b*c^2*x^4-90*A*a*b*c*d*x^3+60*B*a^2*d^2*x^3-45
*B*a*b*c^2*x^3+120*C*a^2*c*d*x^3+40*A*a^2*d^2*x^2-32*A*a*b*c^2*x^2+80*B*a^
2*c*d*x^2+40*C*a^2*c^2*x^2+60*A*a^2*c*d*x+30*B*a^2*c^2*x+24*A*a^2*c^2)/a^3
/x^5-1/8*(6*A*b*c*d-4*B*a*d^2+3*B*b*c^2-8*C*a*c*d)/a^(5/2)*b*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{15\,(3\,Bb^2c^2-4\,Babd^2-2\,(4\,Cab-3\,Ab^2)cd)\sqrt{a}x^5\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(24\,Aa^2c^2-8\,(20\,B\right.$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[-1/240*(15*(3*B*b^2*c^2 - 4*B*a*b*d^2 - 2*(4*C*a*b - 3*A*b^2)*c*d)*sqrt(a
)*x^5*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(24*A*a^2*c^
2 - 8*(20*B*a*b*c*d + 2*(5*C*a*b - 4*A*b^2)*c^2 - 5*(3*C*a^2 - 2*A*a*b)*d^
2)*x^4 - 15*(3*B*a*b*c^2 - 4*B*a^2*d^2 - 2*(4*C*a^2 - 3*A*a*b)*c*d)*x^3 +
8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 - 4*A*a*b)*c^2)*x^2 + 30*(B*a^2*c
^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^5), 1/120*(15*(3*B*b^2*c^2 -
4*B*a*b*d^2 - 2*(4*C*a*b - 3*A*b^2)*c*d)*sqrt(-a)*x^5*arctan(sqrt(b*x^2 +
a)*sqrt(-a)/a) - (24*A*a^2*c^2 - 8*(20*B*a*b*c*d + 2*(5*C*a*b - 4*A*b^2)*c
^2 - 5*(3*C*a^2 - 2*A*a*b)*d^2)*x^4 - 15*(3*B*a*b*c^2 - 4*B*a^2*d^2 - 2*(4
*C*a^2 - 3*A*a*b)*c*d)*x^3 + 8*(10*B*a^2*c*d + 5*A*a^2*d^2 + (5*C*a^2 - 4*
A*a*b)*c^2)*x^2 + 30*(B*a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^3*x^
5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(255) = 510$.

Time = 8.62 (sec) , antiderivative size = 860, normalized size of antiderivative = 3.22

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*A*a**4*b**(9/2)*c**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*
b**5*x**6 + 15*a**3*b**6*x**8) - 2*A*a**3*b**(11/2)*c**2*x**2*sqrt(a/(b*x*
*2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*A
*a**2*b**(13/2)*c**2*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**
4*b**5*x**6 + 15*a**3*b**6*x**8) - 12*A*a*b**(15/2)*c**2*x**6*sqrt(a/(b*x*
*2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*A
*b**(17/2)*c**2*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**
5*x**6 + 15*a**3*b**6*x**8) - A*c*d/(2*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1))
+ A*sqrt(b)*c*d/(4*a*x**3*sqrt(a/(b*x**2) + 1)) - A*sqrt(b)*d**2*sqrt(a/(b
*x**2) + 1)/(3*a*x**2) + 3*A*b**(3/2)*c*d/(4*a**2*x*sqrt(a/(b*x**2) + 1))
+ 2*A*b**(3/2)*d**2*sqrt(a/(b*x**2) + 1)/(3*a**2) - 3*A*b**2*c*d*asinh(sqr
t(a)/(sqrt(b)*x))/(4*a**(5/2)) - B*c**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) +
1)) + B*sqrt(b)*c**2/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - 2*B*sqrt(b)*c*d*sqr
t(a/(b*x**2) + 1)/(3*a*x**2) - B*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/(2*a*x)
 + 3*B*b**(3/2)*c**2/(8*a**2*x*sqrt(a/(b*x**2) + 1)) + 4*B*b**(3/2)*c*d*sq
rt(a/(b*x**2) + 1)/(3*a**2) + B*b*d**2*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3
/2)) - 3*B*b**2*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) - C*sqrt(b)*c
**2*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - C*sqrt(b)*c*d*sqrt(a/(b*x**2) + 1)/(
a*x) - C*sqrt(b)*d**2*sqrt(a/(b*x**2) + 1)/a + 2*C*b**(3/2)*c**2*sqrt(a/(b
*x**2) + 1)/(3*a**2) + C*b*c*d*asinh(sqrt(a)/(sqrt(b)*x))/a**(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = -\frac{8\,\sqrt{bx^2+a}Ab^2c^2}{15\,a^3x} - \frac{\sqrt{bx^2+a}Cd^2}{ax} + \frac{(2\,Ccd+Bd^2)b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,(Bc^2+2\,Acd)b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{4\,\sqrt{bx^2+a}Abc^2}{15\,a^2x^3} + \frac{2\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b}{3\,a^2x} - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}}{2\,ax^2} + \frac{3\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}Ac^2}{5\,ax^5} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}}{3\,ax^3} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}}{4\,ax^4}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
-8/15*sqrt(b*x^2 + a)*A*b^2*c^2/(a^3*x) - sqrt(b*x^2 + a)*C*d^2/(a*x) + 1/
2*(2*C*c*d + B*d^2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*(B*c^2 +
 2*A*c*d)*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 4/15*sqrt(b*x^2 + a)
*A*b*c^2/(a^2*x^3) + 2/3*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)*b/(a^2*
x) - 1/2*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)/(a*x^2) + 3/8*(B*c^2 + 2*A*c*d)
*sqrt(b*x^2 + a)*b/(a^2*x^2) - 1/5*sqrt(b*x^2 + a)*A*c^2/(a*x^5) - 1/3*(C*
c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)/(a*x^3) - 1/4*(B*c^2 + 2*A*c*d)*sqr
t(b*x^2 + a)/(a*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(239) = 478$.

Time = 0.18 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.01

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/4*(3*B*b^2*c^2 - 8*C*a*b*c*d + 6*A*b^2*c*d - 4*B*a*b*d^2)*arctan(-(sqrt(
b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 1/60*(45*(sqrt(b)*x - s
qrt(b*x^2 + a))^9*B*b^2*c^2 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a*b*c*
d + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*b^2*c*d - 60*(sqrt(b)*x - sqrt(b*
x^2 + a))^9*B*a*b*d^2 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b)*
d^2 - 210*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b^2*c^2 + 240*(sqrt(b)*x - s
qrt(b*x^2 + a))^7*C*a^2*b*c*d - 420*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b^
2*c*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a^2*b*d^2 - 240*(sqrt(b)*x -
 sqrt(b*x^2 + a))^6*C*a^2*b^(3/2)*c^2 - 480*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*B*a^2*b^(3/2)*c*d + 480*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b)*d^
2 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^2*b^(3/2)*d^2 + 560*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*C*a^3*b^(3/2)*c^2 - 640*(sqrt(b)*x - sqrt(b*x^2 + a)
)^4*A*a^2*b^(5/2)*c^2 + 1120*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*b^(3/2)
*c*d - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b)*d^2 + 560*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*A*a^3*b^(3/2)*d^2 + 210*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*B*a^3*b^2*c^2 - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^4*b*c*d + 4
20*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b^2*c*d - 120*(sqrt(b)*x - sqrt(b
*x^2 + a))^3*B*a^4*b*d^2 - 400*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*b^(3/
2)*c^2 + 320*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(5/2)*c^2 - 800*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2)*c*d + 480*(sqrt(b)*x - sqrt(b*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^6\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^6*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.48

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^6\sqrt{a+bx^2}}\,dx$$
$$= \frac{-120\sqrt{b\,x^2+a}\,a^2c\,d^2x^4 - 64\sqrt{b\,x^2+a}\,a\,b^2c^2x^4 + 80\sqrt{b\,x^2+a}\,ab\,c^3x^4 + 45\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3c}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^6/(b*x^2+a)^(1/2),x)
```

output
```
( - 24*sqrt(a + b*x**2)*a**3*c**2 - 60*sqrt(a + b*x**2)*a**3*c*d*x - 40*sq
rt(a + b*x**2)*a**3*d**2*x**2 + 32*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 30*
sqrt(a + b*x**2)*a**2*b*c**2*x + 90*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 80*
sqrt(a + b*x**2)*a**2*b*c*d*x**2 + 80*sqrt(a + b*x**2)*a**2*b*d**2*x**4 -
60*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 40*sqrt(a + b*x**2)*a**2*c**3*x**2
- 120*sqrt(a + b*x**2)*a**2*c**2*d*x**3 - 120*sqrt(a + b*x**2)*a**2*c*d**2
*x**4 - 64*sqrt(a + b*x**2)*a*b**2*c**2*x**4 + 45*sqrt(a + b*x**2)*a*b**2*
c**2*x**3 + 160*sqrt(a + b*x**2)*a*b**2*c*d*x**4 + 80*sqrt(a + b*x**2)*a*b
*c**3*x**4 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**2*c*d*x**5 - 60*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*a*b**2*d**2*x**5 - 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x**5 + 45*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**5 - 90*sqrt(a)*log((sqrt(a +
 b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**5 + 60*sqrt(a)*log(
(sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**5 + 120*s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*d*x*
*5 - 45*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3
*c**2*x**5 - 80*sqrt(b)*a**2*b*d**2*x**5 + 72*sqrt(b)*a**2*c*d**2*x**5 + 6
4*sqrt(b)*a*b**2*c**2*x**5 - 160*sqrt(b)*a*b**2*c*d*x**5 - 80*sqrt(b)*a*b*
c**3*x**5)/(120*a**3*x**5)
```

3.114 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 328

$$\begin{aligned}\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx &= -\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}-\frac{c(Bc+2Ad)\sqrt{a+bx^2}}{5ax^5}\\
&-\frac{(6ac(cC+2Bd)-A(5bc^2-6ad^2))\sqrt{a+bx^2}}{24a^2x^4}\\
&+\frac{(4bc(Bc+2Ad)-5ad(2cC+Bd))\sqrt{a+bx^2}}{15a^2x^3}\\
&-\frac{(Ab(5bc^2-6ad^2)+2a(4aCd^2-3bc(cC+2Bd)))\sqrt{a+bx^2}}{16a^3x^2}\\
&-\frac{2b(4bc(Bc+2Ad)-5ad(2cC+Bd))\sqrt{a+bx^2}}{15a^3x}\\
&+\frac{b(Ab(5bc^2-6ad^2)+2a(4aCd^2-3bc(cC+2Bd)))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}\end{aligned}$$

output
```
-1/6*A*c^2*(b*x^2+a)^(1/2)/a/x^6-1/5*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a/x^5-1
/24*(6*a*c*(2*B*d+C*c)-A*(-6*a*d^2+5*b*c^2))*(b*x^2+a)^(1/2)/a^2/x^4+1/15*
(4*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^2/x^3-1/16*(A*b*(-
6*a*d^2+5*b*c^2)+2*a*(4*a*C*d^2-3*b*c*(2*B*d+C*c)))*(b*x^2+a)^(1/2)/a^3/x^
2-2/15*b*(4*b*c*(2*A*d+B*c)-5*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^3/x+1/16*
b*(A*b*(-6*a*d^2+5*b*c^2)+2*a*(4*a*C*d^2-3*b*c*(2*B*d+C*c)))*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \frac{-\frac{\sqrt{a}\sqrt{a+bx^2}\left(b^2cx^4(75Ac+128Bcx+256Adx)+4a^2\left(A\left(10c^2+24cdx+15d^2x^2\right)+x\left(5Cx\left(3c^2+8cdx+6d^2x^2\right)+2B\left(6c^2+15cdx+10d^2x^2\right)\right)\right)-2ab\ldots}{x^6}}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(-((Sqrt[a]*Sqrt[a + b*x^2]*(b^2*c*x^4*(75*A*c + 128*B*c*x + 256*A*d*x) +
4*a^2*(A*(10*c^2 + 24*c*d*x + 15*d^2*x^2) + x*(5*C*x*(3*c^2 + 8*c*d*x + 6*
d^2*x^2) + 2*B*(6*c^2 + 15*c*d*x + 10*d^2*x^2))) - 2*a*b*x^2*(A*(25*c^2 +
64*c*d*x + 45*d^2*x^2) + x*(5*c*C*x*(9*c + 32*d*x) + B*(32*c^2 + 90*c*d*x
+ 80*d^2*x^2)))))/x^6) - 30*b*(5*A*b^2*c^2 + 8*a^2*C*d^2)*ArcTanh[(Sqrt[b]
*x - Sqrt[a + b*x^2])/Sqrt[a]] - 180*a*b^2*(c^2*C + 2*B*c*d + A*d^2)*ArcTa
nh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(240*a^(7/2))
```

Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2338, 25, 2338, 25, 2338, 27, 539, 27, 539, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\left(A+Bx+Cx^2\right)}{x^7\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 2338$$

$$-\frac{\int -\frac{6aCd^2x^3+6ad(2cC+Bd)x^2+\left(6ac(cC+2Bd)-A\left(5bc^2-6ad^2\right)\right)x+6ac(Bc+2Ad)}{x^6\sqrt{bx^2+a}}dx}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{6aCd^2x^3+6ad(2cC+Bd)x^2+\left(6ac(cC+2Bd)-A\left(5bc^2-6ad^2\right)\right)x+6ac(Bc+2Ad)}{x^6\sqrt{bx^2+a}}dx}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 2338$$

$$\frac{-\frac{\int -\frac{30a^2Cd^2x^2-6a(4bc(Bc+2Ad)-5ad(2cC+Bd))x+5a\left(6ac(cC+2Bd)-A\left(5bc^2-6ad^2\right)\right)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{6c\sqrt{a+bx^2}(2Ad+Bc)}{5x^5}}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{30a^2Cd^2x^2-6a(4bc(Bc+2Ad)-5ad(2cC+Bd))x+5a\left(6ac(cC+2Bd)-A\left(5bc^2-6ad^2\right)\right)}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{6c\sqrt{a+bx^2}(2Ad+Bc)}{5x^5}}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 2338$$

$$\frac{\frac{-\frac{\int \frac{3a\left(8a(4bc(Bc+2Ad)-5ad(2cC+Bd))-5\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)x\right)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{5\sqrt{a+bx^2}\left(6ac(2Bd+cC)-A\left(5bc^2-6ad^2\right)\right)}{4x^4}}{5a}-\frac{6c\sqrt{a+bx^2}(2}{5x^5}}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{-\frac{3}{4}\int \frac{8a(4bc(Bc+2Ad)-5ad(2cC+Bd))-5\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)x}{x^4\sqrt{bx^2+a}}dx-\frac{5\sqrt{a+bx^2}\left(6ac(2Bd+cC)-A\left(5bc^2-6ad^2\right)\right)}{4x^4}}{5a}-\frac{6c\sqrt{a+bx^2}(2A}{5x^5}}{6a}-\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$$\Big\downarrow \ 539$$

$$\frac{-\frac{3}{4}\left(-\frac{\int \frac{a\left(15\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)+16b(4bc(Bc+2Ad)-5ad(2cC+Bd))x\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{8\sqrt{a+bx^2}(4bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\sqrt{a+bx^2}\cdots}{\cdots}}{5a} \cdots$$

$$6a$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 27

$$\frac{-\frac{3}{4}\left(-\frac{1}{3}\int \frac{15\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)+16b(4bc(Bc+2Ad)-5ad(2cC+Bd))x}{x^3\sqrt{bx^2+a}}dx-\frac{8\sqrt{a+bx^2}(4bc(2Ad+Bc)-5ad(Bd+2cC))}{3x^3}\right)-\frac{5\sqrt{a+bx^2}\left(\cdots\right.}{\cdots}}{5a} \cdots$$

$$6a$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 539

$$\frac{-\frac{3}{4}\left(\frac{1}{3}\left(\frac{\int -\frac{b\left(32a(4bc(Bc+2Ad)-5ad(2cC+Bd))-15\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}+\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+\cdots\right.}{2ax^2}\cdots\right.\right.}{5a} \cdots$$

$$6a$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 25

$$\frac{-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{\int \frac{b\left(32a(4bc(Bc+2Ad)-5ad(2cC+Bd))-15\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)x\right)}{x^2\sqrt{bx^2+a}}\cdots}{2a}\cdots\right.\right.}{5a} \cdots$$

$$6a$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 27

$$\frac{-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{b\int \frac{32a(4bc(Bc+2Ad)-5ad(2cC+Bd))-15\left(8a^2Cd^2-6abc(cC+2Bd)+Ab\left(5bc^2-6ad^2\right)\right)x}{x^2\sqrt{bx^2+a}}d\cdots}{2a}\cdots\right.\right.}{5a} \cdots$$

$$6a$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 534

$$-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{b\left(-15\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)\int\frac{1}{x\sqrt{bx^2+a}}dx-\frac{32\sqrt{a+bx^2}(4bc(2Ad+\ldots}{\ldots}\right.}{2a}\right.\right.\cdots}{5a}\cdots}{6a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 243

$$-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{b\left(-\frac{15}{2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{32\sqrt{a+bx^2}(4bc(2\ldots}{\ldots}\right.}{2a}\right.\right.\cdots}{5a}\cdots}{6a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 73

$$-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{b\left(-\frac{15\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{32\sqrt{a+bx^2}(4bc(\ldots}{\ldots}\right.}{2a}\right.\right.\cdots}{5a}\cdots}{6a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

$\downarrow$ 221

$$-\frac{3}{4}\left(\frac{1}{3}\left(\frac{15\sqrt{a+bx^2}\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{2ax^2}-\frac{b\left(\frac{15\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(8a^2Cd^2+Ab\left(5bc^2-6ad^2\right)-6abc(2Bd+cC)\right)}{\sqrt{a}}-\frac{32\sqrt{a+bx^2}(4b\ldots}{\ldots}\right.}{2a}\right.\right.\cdots}{5a}\cdots}{6a}$$

$$\frac{Ac^2\sqrt{a+bx^2}}{6ax^6}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
-1/6*(A*c^2*Sqrt[a + b*x^2])/(a*x^6) + ((-6*c*(B*c + 2*A*d)*Sqrt[a + b*x^2
])/(5*x^5) + ((-5*(6*a*c*(c*C + 2*B*d) - A*(5*b*c^2 - 6*a*d^2))*Sqrt[a + b
*x^2])/(4*x^4) - (3*((-8*(4*b*c*(B*c + 2*A*d) - 5*a*d*(2*c*C + B*d))*Sqrt[
a + b*x^2])/(3*x^3) + ((15*(8*a^2*C*d^2 - 6*a*b*c*(c*C + 2*B*d) + A*b*(5*b
*c^2 - 6*a*d^2))*Sqrt[a + b*x^2])/(2*a*x^2) - (b*((-32*(4*b*c*(B*c + 2*A*d
) - 5*a*d*(2*c*C + B*d))*Sqrt[a + b*x^2])/x + (15*(8*a^2*C*d^2 - 6*a*b*c*(
c*C + 2*B*d) + A*b*(5*b*c^2 - 6*a*d^2))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
Sqrt[a]))/(2*a))/3))/4)/(5*a))/(6*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{\sqrt{b x^2+a}\,(256 A b^2 c d x^5-160 B a b d^2 x^5+128 B b^2 c^2 x^5-320 C a b c d x^5-90 A b d^2 x^4 a+75 A b^2 c^2 x^4-180 B b c d x^4 a+120 C a^2 d^2 x^4-90 \ldots}{\ldots}$
default	$(A d^2+2Bcd+C c^2)\left(-\frac{\sqrt{b x^2+a}}{4a x^4}-\frac{3b\left(-\frac{\sqrt{b x^2+a}}{2a x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+A c^2\left(-\frac{\sqrt{b x^2+a}}{6a x^6}-\frac{5b\left(- \ldots\right.}{\ldots}\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(b*x^2+a)^(1/2)*(256*A*b^2*c*d*x^5-160*B*a*b*d^2*x^5+128*B*b^2*c^2*
x^5-320*C*a*b*c*d*x^5-90*A*a*b*d^2*x^4+75*A*b^2*c^2*x^4-180*B*a*b*c*d*x^4+
120*C*a^2*d^2*x^4-90*C*a*b*c^2*x^4-128*A*a*b*c*d*x^3+80*B*a^2*d^2*x^3-64*B
*a*b*c^2*x^3+160*C*a^2*c*d*x^3+60*A*a^2*d^2*x^2-50*A*a*b*c^2*x^2+120*B*a^2
*c*d*x^2+60*C*a^2*c^2*x^2+96*A*a^2*c*d*x+48*B*a^2*c^2*x+40*A*a^2*c^2)/a^3/
x^6-1/16*(6*A*a*b*d^2-5*A*b^2*c^2+12*B*a*b*c*d-8*C*a^2*d^2+6*C*a*b*c^2)*b/
a^(7/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{15\,(12\,Bab^2cd+(6\,Cab^2-5\,Ab^3)c^2-2\,(4\,Ca^2b-3\,Aab^2)d^2)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(40\,A\right.$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/480*(15*(12*B*a*b^2*c*d + (6*C*a*b^2 - 5*A*b^3)*c^2 - 2*(4*C*a^2*b - 3*
A*a*b^2)*d^2)*sqrt(a)*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(40*A*a^3*c^2 + 32*(4*B*a*b^2*c^2 - 5*B*a^2*b*d^2 - 2*(5*C*a^2*b -
 4*A*a*b^2)*c*d)*x^5 - 15*(12*B*a^2*b*c*d + (6*C*a^2*b - 5*A*a*b^2)*c^2 -
2*(4*C*a^3 - 3*A*a^2*b)*d^2)*x^4 - 16*(4*B*a^2*b*c^2 - 5*B*a^3*d^2 - 2*(5*
C*a^3 - 4*A*a^2*b)*c*d)*x^3 + 10*(12*B*a^3*c*d + 6*A*a^3*d^2 + (6*C*a^3 -
5*A*a^2*b)*c^2)*x^2 + 48*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^
4*x^6), 1/240*(15*(12*B*a*b^2*c*d + (6*C*a*b^2 - 5*A*b^3)*c^2 - 2*(4*C*a^2
*b - 3*A*a*b^2)*d^2)*sqrt(-a)*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (40
*A*a^3*c^2 + 32*(4*B*a*b^2*c^2 - 5*B*a^2*b*d^2 - 2*(5*C*a^2*b - 4*A*a*b^2)
*c*d)*x^5 - 15*(12*B*a^2*b*c*d + (6*C*a^2*b - 5*A*a*b^2)*c^2 - 2*(4*C*a^3
- 3*A*a^2*b)*d^2)*x^4 - 16*(4*B*a^2*b*c^2 - 5*B*a^3*d^2 - 2*(5*C*a^3 - 4*A
*a^2*b)*c*d)*x^3 + 10*(12*B*a^3*c*d + 6*A*a^3*d^2 + (6*C*a^3 - 5*A*a^2*b)*
c^2)*x^2 + 48*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(318) = 636$.

Time = 16.75 (sec) , antiderivative size = 1318, normalized size of antiderivative = 4.02

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**7/(b*x**2+a)**(1/2),x)
```

output
```
-6*A*a**4*b**(9/2)*c*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b
**5*x**6 + 15*a**3*b**6*x**8) - 4*A*a**3*b**(11/2)*c*d*x**2*sqrt(a/(b*x**2
) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 6*A*a
**2*b**(13/2)*c*d*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b
**5*x**6 + 15*a**3*b**6*x**8) - 24*A*a*b**(15/2)*c*d*x**6*sqrt(a/(b*x**2)
+ 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 16*A*b*
*(17/2)*c*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x*
*6 + 15*a**3*b**6*x**8) - A*c**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - A
*d**2/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + A*sqrt(b)*c**2/(24*a*x**5*sq
rt(a/(b*x**2) + 1)) + A*sqrt(b)*d**2/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - 5*A
*b**(3/2)*c**2/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*A*b**(3/2)*d**2/(8*
a**2*x*sqrt(a/(b*x**2) + 1)) - 5*A*b**(5/2)*c**2/(16*a**3*x*sqrt(a/(b*x**2
) + 1)) - 3*A*b**2*d**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*A*b**3
*c**2*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2)) - 3*B*a**4*b**(9/2)*c**2*sq
rt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) - 2*B*a**3*b**(11/2)*c**2*x**2*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*B*a**2*b**(13/2)*c**2*x**4*
sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - 12*B*a*b**(15/2)*c**2*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*B*b**(17/2)*c**2*x**8*sq...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \frac{5\,Ab^3c^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} + \frac{Cbd^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{3\,(Cc^2+2\,Bcd+Ad^2)b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} - \frac{5\,\sqrt{bx^2+a}Ab^2c^2}{16\,a^3x^2} - \frac{\sqrt{bx^2+a}Cd^2}{2\,ax^2} + \frac{2\,(2\,Ccd+Bd^2)\sqrt{bx^2+a}b}{3\,a^2x} - \frac{8\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b^2}{15\,a^3x} + \frac{5\,\sqrt{bx^2+a}Abc^2}{24\,a^2x^4} + \frac{3\,(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}b}{8\,a^2x^2} - \frac{(2\,Ccd+Bd^2)\sqrt{bx^2+a}}{3\,ax^3} + \frac{4\,(Bc^2+2\,Acd)\sqrt{bx^2+a}b}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}Ac^2}{6\,ax^6} - \frac{(Cc^2+2\,Bcd+Ad^2)\sqrt{bx^2+a}}{4\,ax^4} - \frac{(Bc^2+2\,Acd)\sqrt{bx^2+a}}{5\,ax^5}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
5/16*A*b^3*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) + 1/2*C*b*d^2*arcsinh
(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 3/8*(C*c^2 + 2*B*c*d + A*d^2)*b^2*arcsinh
(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 5/16*sqrt(b*x^2 + a)*A*b^2*c^2/(a^3*x^2)
- 1/2*sqrt(b*x^2 + a)*C*d^2/(a*x^2) + 2/3*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a
)*b/(a^2*x) - 8/15*(B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b^2/(a^3*x) + 5/24*sq
rt(b*x^2 + a)*A*b*c^2/(a^2*x^4) + 3/8*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2
 + a)*b/(a^2*x^2) - 1/3*(2*C*c*d + B*d^2)*sqrt(b*x^2 + a)/(a*x^3) + 4/15*(
B*c^2 + 2*A*c*d)*sqrt(b*x^2 + a)*b/(a^2*x^3) - 1/6*sqrt(b*x^2 + a)*A*c^2/(
a*x^6) - 1/4*(C*c^2 + 2*B*c*d + A*d^2)*sqrt(b*x^2 + a)/(a*x^4) - 1/5*(B*c^
2 + 2*A*c*d)*sqrt(b*x^2 + a)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1480 vs. $2(296) = 592$.

Time = 0.16 (sec) , antiderivative size = 1480, normalized size of antiderivative = 4.51

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(6*C*a*b^2*c^2 - 5*A*b^3*c^2 + 12*B*a*b^2*c*d - 8*C*a^2*b*d^2 + 6*A*a*
b^2*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) -
1/120*(90*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a*b^2*c^2 - 75*(sqrt(b)*x - s
qrt(b*x^2 + a))^11*A*b^3*c^2 + 180*(sqrt(b)*x - sqrt(b*x^2 + a))^11*B*a*b^
2*c*d - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^11*C*a^2*b*d^2 + 90*(sqrt(b)*x -
 sqrt(b*x^2 + a))^11*A*a*b^2*d^2 - 510*(sqrt(b)*x - sqrt(b*x^2 + a))^9*C*a
^2*b^2*c^2 + 425*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a*b^3*c^2 - 1020*(sqrt(
b)*x - sqrt(b*x^2 + a))^9*B*a^2*b^2*c*d + 360*(sqrt(b)*x - sqrt(b*x^2 + a)
)^9*C*a^3*b*d^2 - 510*(sqrt(b)*x - sqrt(b*x^2 + a))^9*A*a^2*b^2*d^2 - 960*
(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^3*b^(3/2)*c*d - 480*(sqrt(b)*x - sqrt(
b*x^2 + a))^8*B*a^3*b^(3/2)*d^2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a^
3*b^2*c^2 - 990*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^2*b^3*c^2 + 840*(sqrt(
b)*x - sqrt(b*x^2 + a))^7*B*a^3*b^2*c*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a)
)^7*C*a^4*b*d^2 + 420*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a^3*b^2*d^2 - 1280
*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^3*b^(5/2)*c^2 + 3200*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*C*a^4*b^(3/2)*c*d - 2560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A
*a^3*b^(5/2)*c*d + 1600*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(3/2)*d^2
+ 420*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^4*b^2*c^2 - 990*(sqrt(b)*x - sqr
t(b*x^2 + a))^5*A*a^3*b^3*c^2 + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^4*
b^2*c*d - 240*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^5*b*d^2 + 420*(sqrt(b...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^7\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^7*(a + b*x^2)^(1/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^7*(a + b*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.41

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^7\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^7/(b*x^2+a)^(1/2),x)
```

output
```
( - 40*sqrt(a + b*x**2)*a**3*c**2 - 96*sqrt(a + b*x**2)*a**3*c*d*x - 60*sq
rt(a + b*x**2)*a**3*d**2*x**2 + 50*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 48*
sqrt(a + b*x**2)*a**2*b*c**2*x + 128*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 12
0*sqrt(a + b*x**2)*a**2*b*c*d*x**2 + 90*sqrt(a + b*x**2)*a**2*b*d**2*x**4
- 80*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 60*sqrt(a + b*x**2)*a**2*c**3*x**
2 - 160*sqrt(a + b*x**2)*a**2*c**2*d*x**3 - 120*sqrt(a + b*x**2)*a**2*c*d*
*2*x**4 - 75*sqrt(a + b*x**2)*a*b**2*c**2*x**4 + 64*sqrt(a + b*x**2)*a*b**
2*c**2*x**3 - 256*sqrt(a + b*x**2)*a*b**2*c*d*x**5 + 180*sqrt(a + b*x**2)*
a*b**2*c*d*x**4 + 160*sqrt(a + b*x**2)*a*b**2*d**2*x**5 + 90*sqrt(a + b*x*
*2)*a*b*c**3*x**4 + 320*sqrt(a + b*x**2)*a*b*c**2*d*x**5 - 128*sqrt(a + b*
x**2)*b**3*c**2*x**5 + 90*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*b**2*d**2*x**6 - 120*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2*x**6 - 75*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**6 + 180*sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*d*x**6 + 90*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**3*x**6 - 90*sqrt
(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**6
 + 120*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c
*d**2*x**6 + 75*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*b**3*c**2*x**6 - 180*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt...
```

3.115 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 326

$$\begin{aligned}\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx &= \frac{(2ad^2(cC-Bd)-3bc(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3b^2d^4}\\ &\quad-\frac{(3aCd^2-4b(c^2C-Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8b^2d^3}\\ &\quad-\frac{(cC-Bd)x^2\sqrt{a+bx^2}}{3bd^2}+\frac{Cx^3\sqrt{a+bx^2}}{4bd}\\ &\quad+\frac{(3a^2Cd^4+8b^2c^2(c^2C-Bcd+Ad^2)-4abd^2(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}d^5}\\ &\quad+\frac{c^3(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
1/3*(2*a*d^2*(-B*d+C*c)-3*b*c*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/b^2/d^4
-1/8*(3*a*C*d^2-4*b*(A*d^2-B*c*d+C*c^2))*x*(b*x^2+a)^(1/2)/b^2/d^3-1/3*(-B
*d+C*c)*x^2*(b*x^2+a)^(1/2)/b/d^2+1/4*C*x^3*(b*x^2+a)^(1/2)/b/d+1/8*(3*a^2
*C*d^4+8*b^2*c^2*(A*d^2-B*c*d+C*c^2)-4*a*b*d^2*(A*d^2-B*c*d+C*c^2))*arctan
h(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)/d^5+c^3*(A*d^2-B*c*d+C*c^2)*arctanh((
-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.84

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx$$

$$=\frac{\frac{d\sqrt{a+bx^2}\left(ad^2(16cC-16Bd-9Cdx)-2b\left(12c^3C-6c^2d(2B+Cx)+2cd^2\left(6A+3Bx+2Cx^2\right)-d^3x\left(6A+4Bx+3Cx^2\right)\right)\right)}{b^2}+\frac{48c^3\left(c^2C-Bcd+Ad^2\right)}{\sqrt{\ }}}{24d^5}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(16*c*C - 16*B*d - 9*C*d*x) - 2*b*(12*c^3*C - 6
*c^2*d*(2*B + C*x) + 2*c*d^2*(6*A + 3*B*x + 2*C*x^2) - d^3*x*(6*A + 4*B*x
+ 3*C*x^2))))/b^2 + (48*c^3*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d
*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] -
 (3*(3*a^2*C*d^4 + 8*b^2*c^2*(c^2*C - B*c*d + A*d^2) - 4*a*b*d^2*(c^2*C -
B*c*d + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2))/(24*d^5)
```

Rubi [A] (verified)

Time = 2.73 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2185, 25, 2185, 25, 2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \ 2185$$

$$\frac{\int -\frac{bd^4(13cC-4Bd)x^4+d^3\left(15bCc^2-4Abd^2+3aCd^2\right)x^3+cCd^2\left(7bc^2+9ad^2\right)x^2+c^2Cd\left(bc^2+9ad^2\right)x+3ac^3Cd^2}{(c+dx)\sqrt{bx^2+a}}dx}{4bd^5}+\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}$$

$$\Big\downarrow \ 25$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\int \frac{bd^4(13cC-4Bd)x^4+d^3\left(15bCc^2-4Abd^2+3aCd^2\right)x^3+cCd^2\left(7bc^2+9ad^2\right)x^2+c^2Cd\left(bc^2+9ad^2\right)x+3ac^3Cd^2}{(c+dx)\sqrt{bx^2+a}}dx}{4bd^5}$$

$\big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{\int -\frac{-b\left(9aCd^2-2b\left(23Cc^2-14Bdc+6Ad^2\right)\right)x^3d^7+b\left(4bc^2(11cC-5Bd)-ad^2(cC+8Bd)\right)x^2d^6+abc^2(17cC-8Bd)d^6+bc\left(2b(5cC-2Bd)c^2+ad^2(25cC-16Bd)\right)xd^5 \cdots}{(c+dx)\sqrt{bx^2+a}}}{3bd^4}\cdots}{4bd^5}$$

$\big\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{\int \frac{-b\left(9aCd^2-2b\left(23Cc^2-14Bdc+6Ad^2\right)\right)x^3d^7+b\left(4bc^2(11cC-5Bd)-ad^2(cC+8Bd)\right)x^2d^6+abc^2(17cC- \cdots}{(c+dx)\sqrt{bx^2+a}}}{3bd^4}}{4bd^5}$$

$\big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{\int \frac{b^2\left(ad^2(25cC-16Bd)-2bc\left(25Cc^2-22Bdc+18Ad^2\right)\right)x^2d^9+3abc\left(3aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)d^9+b\left(\cdots\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^3}}{4bd^5}$$

$\big\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{\frac{\int \frac{3b^2d^{10}\left(acd\left(3aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)+\left(3a^2Cd^4-4ab\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^2\left(Cc^2-Bdc+A\cdots\right.\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{bd^2}}{2bd^3}}{4bd^5}$$

$\big\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\int \frac{acd\left(3aCd^2-4b\left(Cc^2-Bdc+Ad^2\right)\right)+\left(3a^2Cd^4-4ab\left(Cc^2-Bdc+Ad^2\right)d^2+8b^2c^2\left(Cc^2-Bdc+Ad^2\right)\cdots\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^3}}{4bd^5}$$

$\big\downarrow$ 719

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\left(\frac{\left(3a^2Cd^4-4abd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^2\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{8b^2c^3\left(Ad^2-Bcd+\right.}{}\right.}{2bd^3}}{4b}}{}$$

$\downarrow$ 224

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\left(\frac{\left(3a^2Cd^4-4abd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^2\left(Ad^2-Bcd+c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{8b^2c^3\left(A\right.}{}\right.}{2bd}}{}$$

$\downarrow$ 219

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4-4abd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^2\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{8b^2c^3\left(A\right.}{}\right.}{2bd}}{}$$

$\downarrow$ 488

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\left(\frac{8b^2c^3\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4-4\right.}{}\right.}{}}{}$$

$\downarrow$ 219

$$\frac{C\sqrt{a+bx^2}(c+dx)^3}{4bd^4}-\frac{\frac{1}{3}d\sqrt{a+bx^2}(c+dx)^2(13cC-4Bd)-\frac{3bd^8\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3a^2Cd^4-4abd^2\left(Ad^2-Bcd+c^2C\right)+8b^2c^2\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}+\frac{8b^2c^3\left(A\right.}{}\right.}{}}{}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(C*(c + d*x)^3*Sqrt[a + b*x^2])/(4*b*d^4) - ((d*(13*c*C - 4*B*d)*(c + d*x)
^2*Sqrt[a + b*x^2])/3 - ((d^5*(46*b*c^2*C - 28*b*B*c*d + 12*A*b*d^2 - 9*a*
C*d^2)*(c + d*x)*Sqrt[a + b*x^2])/2 + (b*d^8*(a*d^2*(25*c*C - 16*B*d) - 2*
b*c*(25*c^2*C - 22*B*c*d + 18*A*d^2))*Sqrt[a + b*x^2] + 3*b*d^8*(((3*a^2*C
*d^4 + 8*b^2*c^2*(c^2*C - B*c*d + A*d^2) - 4*a*b*d^2*(c^2*C - B*c*d + A*d^
2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + (8*b^2*c^3*(c^2*C
- B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2
])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*b*d^3))/(3*b*d^4))/(4*b*d^5)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.14

method	result
risch	$-\frac{\left(-6C\,d^3 b\,x^3-8Bb\,d^3x^2+8Cbc\,d^2x^2-12Ab\,d^3x+12Bbc\,d^2x+9Ca\,d^3x-12Cb\,c^2dx+24Abc\,d^2+16Ba\,d^3-24Bb\,c^2d-16Cac\,d^2+2\right.}{24b^2d^4}$
default	$\frac{\frac{C\,c^4\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+d^3(Bd-Cc)\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+d^2\left(A\,d^2-Bcd+C\,c^2\right)\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)-\frac{cd\left(A\right.}{}}{}$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*C*b*d^3*x^3-8*B*b*d^3*x^2+8*C*b*c*d^2*x^2-12*A*b*d^3*x+12*B*b*c*
d^2*x+9*C*a*d^3*x-12*C*b*c^2*d*x+24*A*b*c*d^2+16*B*a*d^3-24*B*b*c^2*d-16*C
*a*c*d^2+24*C*b*c^3)*(b*x^2+a)^(1/2)/b^2/d^4-1/8/b^2/d^4*((4*A*a*b*d^4-8*A
*b^2*c^2*d^2-4*B*a*b*c*d^3+8*B*b^2*c^3*d-3*C*a^2*d^4+4*C*a*b*c^2*d^2-8*C*b
^2*c^4)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-8*b^2*c^3*(A*d^2-B*c*d+C*c
^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+
2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2
)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.71

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cx^3}{4\,bd} - \frac{\sqrt{bx^2+a}Ccx^2}{3\,bd^2} + \frac{\sqrt{bx^2+a}Bx^2}{3\,bd} + \frac{\sqrt{bx^2+a}Cc^2x}{2\,bd^3} - \frac{\sqrt{bx^2+a}Bcx}{2\,bd^2} - \frac{3\sqrt{bx^2+a}Cax}{8\,b^2d} + \frac{\sqrt{bx^2+a}Ax}{2\,bd} + \frac{Cc^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^5} - \frac{Bc^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^4} - \frac{Cac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d^3} + \frac{Ac^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} + \frac{Bac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d^2} + \frac{3\,Ca^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}d} - \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d} - \frac{Cc^5\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^6} + \frac{Bc^4\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^5} - \frac{Ac^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} - \frac{\sqrt{bx^2+a}Cc^3}{bd^4} + \frac{\sqrt{bx^2+a}Bc^2}{bd^3} + \frac{2\sqrt{bx^2+a}Cac}{3\,b^2d^2} - \frac{\sqrt{bx^2+a}Ac}{bd^2} - \frac{2\sqrt{bx^2+a}Ba}{3\,b^2d}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*C*x^3/(b*d) - 1/3*sqrt(b*x^2 + a)*C*c*x^2/(b*d^2) + 1/
3*sqrt(b*x^2 + a)*B*x^2/(b*d) + 1/2*sqrt(b*x^2 + a)*C*c^2*x/(b*d^3) - 1/2*
sqrt(b*x^2 + a)*B*c*x/(b*d^2) - 3/8*sqrt(b*x^2 + a)*C*a*x/(b^2*d) + 1/2*sq
rt(b*x^2 + a)*A*x/(b*d) + C*c^4*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^5) - B*c
^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) - 1/2*C*a*c^2*arcsinh(b*x/sqrt(a*b
))/(b^(3/2)*d^3) + A*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) + 1/2*B*a*c*
arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) + 3/8*C*a^2*arcsinh(b*x/sqrt(a*b))/(b
^(5/2)*d) - 1/2*A*a*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) - C*c^5*arcsinh(b*c
*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^
2/d^2)*d^6) + B*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b
)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5) - A*c^3*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^4)
- sqrt(b*x^2 + a)*C*c^3/(b*d^4) + sqrt(b*x^2 + a)*B*c^2/(b*d^3) + 2/3*sqrt
(b*x^2 + a)*C*a*c/(b^2*d^2) - sqrt(b*x^2 + a)*A*c/(b*d^2) - 2/3*sqrt(b*x^2
 + a)*B*a/(b^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{x^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)\,\sqrt{b\,x^2+a}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

3.116 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 245

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx &= -\frac{(2aCd^2-3b(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3b^2d^3}-\frac{(cC-Bd)x\sqrt{a+bx^2}}{2bd^2}\\ &\quad+\frac{Cx^2\sqrt{a+bx^2}}{3bd}+\frac{(ad^2(cC-Bd)-2bc(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}d^4}\\ &\quad-\frac{c^2(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4\sqrt{bc^2+ad^2}}\end{aligned}$$

output
```
-1/3*(2*a*C*d^2-3*b*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/b^2/d^3-1/2*(-B*d
+C*c)*x*(b*x^2+a)^(1/2)/b/d^2+1/3*C*x^2*(b*x^2+a)^(1/2)/b/d+1/2*(a*d^2*(-B
*d+C*c)-2*b*c*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3
/2)/d^4-c^2*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(
b*x^2+a)^(1/2))/d^4/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx$$
$$=\frac{\frac{d\sqrt{a+bx^2}(-4aCd^2+b(6c^2C-3cd(2B+Cx)+d^2(6A+3Bx+2Cx^2)))}{b^2}-\frac{12c^2(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{\sqrt{-bc^2-ad^2}}-\frac{6(ad^2(-cC}{}}{6d^4}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(-4*a*C*d^2 + b*(6*c^2*C - 3*c*d*(2*B + C*x) + d^2*(6*
A + 3*B*x + 2*C*x^2))))/b^2 - (12*c^2*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt
[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a + b*x^2])])/Sqrt[-(b*c
^2) - a*d^2] - (6*(a*d^2*(-(c*C) + B*d) + 2*b*c*(c^2*C - B*c*d + A*d^2))*A
rcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2))/(6*d^4)
```

Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2185, 25, 2185, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -\frac{bd^3(7cC-3Bd)x^3+d^2(5bCc^2-3Abd^2+2aCd^2)x^2+cCd(bc^2+4ad^2)x+2ac^2Cd^2}{(c+dx)\sqrt{bx^2+a}}dx}{3bd^4}+\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\int \frac{bd^3(7cC-3Bd)x^3+d^2(5bCc^2-3Abd^2+2aCd^2)x^2+cCd(bc^2+4ad^2)x+2ac^2Cd^2}{(c+dx)\sqrt{bx^2+a}}dx}{3bd^4}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{\int -\frac{-b\left(4aCd^2-b\left(11Cc^2-9Bdc+6Ad^2\right)\right)x^2d^5+3abc(cC-Bd)d^5+b\left(bc^2(5cC-3Bd)-ad^2(cC+3Bd)\right)xd^4}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}+\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd}{3bd^4}$$

$\downarrow$ 25

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{\int \frac{-b\left(4aCd^2-b\left(11Cc^2-9Bdc+6Ad^2\right)\right)x^2d^5+3abc(cC-Bd)d^5+b\left(bc^2(5cC-3Bd)-ad^2(cC+3Bd)\right)xd^4}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}}{3bd^4}$$

$\downarrow$ 2185

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{\frac{\int \frac{3b^2d^6\left(acd(cC-Bd)+\left(ad^2(cC-Bd)-2bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}-d^4\sqrt{a+bx^2}\left(4aCd^2-b\left(6Ad^2-9Bc\right.\right.}{2bd^3}}{3bd^4}$$

$\downarrow$ 27

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\int \frac{acd(cC-Bd)+\left(ad^2(cC-Bd)-2bc\left(Cc^2-Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx-d^4\sqrt{a+bx^2}\left(4aCd^2-b\left(6Ad^2-9Bcd\right.\right.}{2bd^3}}{3bd^4}$$

$\downarrow$ 719

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\left(\frac{2bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\left(ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right.}{2bd^3}}{3bd^4}$$

$\downarrow$ 224

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\left(\frac{2bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\left(ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d}{d}\right.}{2bd^3}}{3bd^4}$$

$$\Big\downarrow \ 219$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\left(\frac{2bc^2\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(cC-Bd)-2bc\left(Ad^2-Bc\right.\right.}{\sqrt{b}d}\right.}{2bd^3}}{3bd^4}$$

$$\Big\downarrow \ 488$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2bc^2\left(Ad^2-Bcd+c^2C\right)\int\frac{}{bc^2+ad^2}}{d}\right.}{2bd^3}}{3bd^4}$$

$$\Big\downarrow \ 219$$

$$\frac{C\sqrt{a+bx^2}(c+dx)^2}{3bd^3}-\frac{\frac{1}{2}d\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3bd^4\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2bc^2\left(Ad^2-Bcd+c^2C\right)\text{arctanh}(}{d\sqrt{ad^2+bc^2}}\right.}{2bd^3}}{3bd^4}$$

input

```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output

```
(C*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b*d^3) - ((d*(7*c*C - 3*B*d)*(c + d*x)*
Sqrt[a + b*x^2])/2 - (-(d^4*(4*a*C*d^2 - b*(11*c^2*C - 9*B*c*d + 6*A*d^2))
*Sqrt[a + b*x^2]) + 3*b*d^4*(((a*d^2*(c*C - B*d) - 2*b*c*(c^2*C - B*c*d +
A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (2*b*c^2*(c^2*
C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x
^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2*b*d^3))/(3*b*d^4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.18

method	result
risch	$\frac{\left(2C\,d^{2}b\,x^{2}+3Bb\,d^{2}x-3Cbcdx+6Ab\,d^{2}-6Bbcd-4aC\,d^{2}+6Cb\,c^{2}\right)\sqrt{b\,x^{2}+a}}{6b^{2}d^{3}}-\frac{\frac{\left(2Abc\,d^{2}+Ba\,d^{3}-2Bb\,c^{2}d-Cac\,d^{2}+2Cb\,c^{3}\right)\ln\left(\sqrt{b}\,x+\sqrt{\right.}}{d\sqrt{b}}}{}$
default	$-\frac{c^{2}\left(A\,d^{2}-Bcd+C\,c^{2}\right)\ln\left(\frac{\frac{2a\,d^{2}+2b\,c^{2}}{d^{2}}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}{x+\frac{c}{d}}\right)}{d^{5}\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}-\frac{\frac{C\,c^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}}{}$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x-3*C*b*c*d*x+6*A*b*d^2-6*B*b*c*d-4*C*a*d^2+6
*C*b*c^2)*(b*x^2+a)^(1/2)/b^2/d^3-1/2/b/d^3*((2*A*b*c*d^2+B*a*d^3-2*B*b*c^
2*d-C*a*c*d^2+2*C*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*b*c^2*(
A*d^2-B*c*d+C*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2
*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = {} & \frac{\sqrt{bx^2+a}Cx^2}{3\,bd} - \frac{\sqrt{bx^2+a}Ccx}{2\,bd^2} + \frac{\sqrt{bx^2+a}Bx}{2\,bd} \\ & - \frac{Cc^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^4} + \frac{Bc^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} \\ & + \frac{Cac\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d^2} - \frac{Ac\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} \\ & - \frac{Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d} + \frac{Cc^4\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^5} \\ & - \frac{Bc^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} \\ & + \frac{Ac^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} + \frac{\sqrt{bx^2+a}Cc^2}{bd^3} \\ & - \frac{\sqrt{bx^2+a}Bc}{bd^2} - \frac{2\,\sqrt{bx^2+a}Ca}{3\,b^2d} + \frac{\sqrt{bx^2+a}A}{bd}\end{aligned}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*C*x^2/(b*d) - 1/2*sqrt(b*x^2 + a)*C*c*x/(b*d^2) + 1/2*
sqrt(b*x^2 + a)*B*x/(b*d) - C*c^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) + B
*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) + 1/2*C*a*c*arcsinh(b*x/sqrt(a*b
))/(b^(3/2)*d^2) - A*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 1/2*B*a*arcs
inh(b*x/sqrt(a*b))/(b^(3/2)*d) + C*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x +
c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5) - B*c^3*arcs
inh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a
 + b*c^2/d^2)*d^4) + A*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(s
qrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) + sqrt(b*x^2 + a)*C*c^2/
(b*d^3) - sqrt(b*x^2 + a)*B*c/(b*d^2) - 2/3*sqrt(b*x^2 + a)*C*a/(b^2*d) +
sqrt(b*x^2 + a)*A/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{x^2\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2(A + Bx + Cx^2)}{(c + dx)\sqrt{a + bx^2}}\,dx = \int \frac{x^2(C\,x^2 + Bx + A)}{(dx + c)\,\sqrt{b\,x^2 + a}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

3.117 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 181

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}} \, dx = -\frac{(cC-Bd)\sqrt{a+bx^2}}{bd^2} + \frac{Cx\sqrt{a+bx^2}}{2bd} - \frac{(aCd^2-2b(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}d^3} + \frac{c(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3\sqrt{bc^2+ad^2}}$$

output
```
-(-B*d+C*c)*(b*x^2+a)^(1/2)/b/d^2+1/2*C*x*(b*x^2+a)^(1/2)/b/d-1/2*(a*C*d^2
-2*b*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^3+c
*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1
/2))/d^3/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{d(-2cC+2Bd+Cdx)\sqrt{a+bx^2}}{b}+\frac{4c(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+\frac{(aCd^2-2b(c^2C-Bcd+Ad^2))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}}{2d^3}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((d*(-2*c*C + 2*B*d + C*d*x)*Sqrt[a + b*x^2])/b + (4*c*(c^2*C - B*c*d + A*
d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]
])/Sqrt[-(b*c^2) - a*d^2] + ((a*C*d^2 - 2*b*(c^2*C - B*c*d + A*d^2))*Log[-
(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(2*d^3)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2185, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\int -\frac{b(3cC-2Bd)x^2d^2+acCd^2+(bCc^2-2Abd^2+aCd^2)xd}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}+\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{\int \frac{b(3cC-2Bd)x^2d^2+acCd^2+(bCc^2-2Abd^2+aCd^2)xd}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}$$

$$\Big\downarrow \text{2185}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{\frac{\int \frac{bd^3\left(acCd+\left(aCd^2-2b\left(Cc^2-Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+d\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\int \frac{acCd+(aCd^2-2b(Cc^2-Bdc+Ad^2))x}{(c+dx)\sqrt{bx^2+a}}dx+d\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}$$

$$\Big\downarrow \text{719}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\left(\frac{(aCd^2-2b(Ad^2-Bcd+c^2C))\int \frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{2bc(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}\right)+d\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}$$

$$\Big\downarrow \text{224}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\left(\frac{2bc(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{(aCd^2-2b(Ad^2-Bcd+c^2C))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)+d\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\left(\frac{2bc(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd^2-2b(Ad^2-Bcd+c^2C))}{\sqrt{b}d}\right)+d\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}$$

$$\Big\downarrow \text{488}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd^2-2b(Ad^2-Bcd+c^2C))}{\sqrt{b}d}-\frac{2bc(Ad^2-Bcd+c^2C)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}\right)+d\sqrt{a+bx^2}(3cC-}{2bd^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\sqrt{a+bx^2}(c+dx)}{2bd^2}-\frac{d\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(aCd^2-2b(Ad^2-Bcd+c^2C))}{\sqrt{b}d}-\frac{2bc(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}\right)+d\sqrt{a+bx^2}(3cC-}{2bd^3}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(C*(c + d*x)*Sqrt[a + b*x^2])/(2*b*d^2) - (d*(3*c*C - 2*B*d)*Sqrt[a + b*x^
2] + d*(((a*C*d^2 - 2*b*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(Sqrt[b]*d) - (2*b*c*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b
*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2])))/(2
*b*d^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.30

method	result
risch	$\frac{(Cxd+2Bd-2Cc)\sqrt{b\,x^2+a}}{2b\,d^2}+\frac{\frac{\left(2Ab\,d^2-2Bbcd-aC\,d^2+2Cb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{2bc\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}\cdots}{\cdots}\right)}{d^2\sqrt{a\cdots}}}{2b\,d^2}$
default	$\frac{\frac{A\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{C\,c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{d(Bd-Cc)\sqrt{b\,x^2+a}}{b}+C\,d^2\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)-\frac{Bcd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2\cdots}\right)}{\sqrt{b}}\cdots}{d^3}$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d-2*C*c)*(b*x^2+a)^(1/2)/b/d^2+1/2/b/d^2*((2*A*b*d^2-2*B*b*
c*d-C*a*d^2+2*C*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*b*c*(A*d^
2-B*c*d+C*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c
/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2
+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.65

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cx}{2\,bd} + \frac{Cc^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} - \frac{Bc\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} - \frac{Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}d} + \frac{A\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d} - \frac{Cc^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} + \frac{Bc^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} - \frac{Ac\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2} - \frac{\sqrt{bx^2+a}Cc}{bd^2} + \frac{\sqrt{bx^2+a}B}{bd}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*C*x/(b*d) + C*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3)
 - B*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - 1/2*C*a*arcsinh(b*x/sqrt(a*b
))/(b^(3/2)*d) + A*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d) - C*c^3*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2
/d^2)*d^4) + B*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)
*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) - A*c*arcsinh(b*c*x/(sqrt(a*b)*a
bs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) - s
qrt(b*x^2 + a)*C*c/(b*d^2) + sqrt(b*x^2 + a)*B/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3558, normalized size of antiderivative = 19.66

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output

```
(2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqr
t(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*
sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**2*c**2*d**2 - 2*sqrt(b)
*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 +
 b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2))*b**3*c**3*d + 2*sqrt(b)*sqrt(2*sqrt(b)
*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((
sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c -
 a*d**2 - 2*b*c**2))*b**2*c**5 + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c
- a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b
)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b**2*c*d**4 + 2*sqrt(
2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x*
*2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b
*c**2))*a*b**3*c**3*d**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d*
*2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b**3*c**2*d**3 + 2*sqrt(2*sqrt
(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d
+ sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)
)*a*b**2*c**4*d**2 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2
*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*...
```

3.118 $\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 130

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{C\sqrt{a+bx^2}}{bd} - \frac{(cC-Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d^2} - \frac{(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2\sqrt{bc^2+ad^2}}$$

output
```
C*(b*x^2+a)^(1/2)/b/d-(-B*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2
)/d^2-(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+
a)^(1/2))/d^2/(a*d^2+b*c^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\frac{Cd\sqrt{a+bx^2}}{b} - \frac{2(c^2C-Bcd+Ad^2)\,\text{arctan}\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}} + \frac{(cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{d^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((C*d*Sqrt[a + b*x^2])/b - (2*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c +
 d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2]
 + ((c*C - B*d)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{2185}$$

$$\frac{\int \frac{bd(Ad-(cC-Bd)x)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ad-(cC-Bd)x}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{(cC-Bd)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{(cC-Bd)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{488}$$

$$\frac{-\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}}{d}+\frac{C\sqrt{a+bx^2}}{bd}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(C*Sqrt[a + b*x^2])/(b*d) + (-(((c*C - B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/(Sqrt[b]*d)) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.52

method	result
risch	$\frac{C\sqrt{b\,x^2+a}}{bd}+\frac{\frac{(Bd-Cc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,c}{}}}{x+\frac{c}{d}}\right.}{d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}}{d}$
default	$\frac{\frac{Bd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{Cd\sqrt{b\,x^2+a}}{b}-\frac{Cc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}}{d^2}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\right.}}{x+\frac{c}{d}}\right.}{d^3\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}$

input
```
int((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C*(b*x^2+a)^(1/2)/b/d+1/d*((B*d-C*c)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/
2)-(A*d^2-B*c*d+C*c^2)/d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d
^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d
)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 120.08 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.78

$$\int \frac{A + Bx + Cx^2}{(c + dx)\sqrt{a + bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((C*b*c^3 - B*b*c^2*d + C*a*c*d^2 - B*a*d^3)*sqrt(b)*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (C*b*c^2 - B*b*c*d + A*b*d^2)*sqrt(b*c^
2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*
x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*
d*x + c^2)) - 2*(C*b*c^2*d + C*a*d^3)*sqrt(b*x^2 + a))/(b^2*c^2*d^2 + a*b*
d^4), 1/2*(2*(C*b*c^3 - B*b*c^2*d + C*a*c*d^2 - B*a*d^3)*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) + (C*b*c^2 - B*b*c*d + A*b*d^2)*sqrt(b*c^2 + a*
d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 -
2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x +
c^2)) + 2*(C*b*c^2*d + C*a*d^3)*sqrt(b*x^2 + a))/(b^2*c^2*d^2 + a*b*d^4),
-1/2*(2*(C*b*c^2 - B*b*c*d + A*b*d^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*
c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 +
 a*b*d^2)*x^2)) + (C*b*c^3 - B*b*c^2*d + C*a*c*d^2 - B*a*d^3)*sqrt(b)*log(
-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(C*b*c^2*d + C*a*d^3)*sqrt
(b*x^2 + a))/(b^2*c^2*d^2 + a*b*d^4), -((C*b*c^2 - B*b*c*d + A*b*d^2)*sqrt
(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)
/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (C*b*c^3 - B*b*c^2*d + C
*a*c*d^2 - B*a*d^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (C*b*c^2
*d + C*a*d^3)*sqrt(b*x^2 + a))/(b^2*c^2*d^2 + a*b*d^4)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.68

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{Cc\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d} + \frac{Cc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} - \frac{Bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2} + \frac{A\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|} - \frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d} + \frac{\sqrt{bx^2+a}C}{bd}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-C*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + B*arcsinh(b*x/sqrt(a*b))/(sqrt
(b)*d) + C*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs
(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^3) - B*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d
*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) + A*arc
sinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(
a + b*c^2/d^2)*d) + sqrt(b*x^2 + a)*C/(b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2)/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 3317, normalized size of antiderivative = 25.52

$$\int \frac{A+Bx+Cx^2}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(b)*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*
sqrt(a*d**2 + b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(
b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c*d**2 + 2*sqrt(b)*sq
rt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*
c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 +
 b*c**2)*c - a*d**2 - 2*b*c**2))*b**2*c**2*d - 2*sqrt(b)*sqrt(2*sqrt(b)*sq
rt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*sqrt(a*d**2 + b*c**2)*atan((sqr
t(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*
d**2 - 2*b*c**2))*b*c**4 - 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d*
*2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt
(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a**2*b*d**4 - 2*sqrt(2*sqrt(b)*s
qrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqr
t(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b
**2*c**2*d**2 + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c*
*2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b
*c**2)*c - a*d**2 - 2*b*c**2))*a*b**2*c*d**3 - 2*sqrt(2*sqrt(b)*sqrt(a*d**
2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sqrt(a + b*x**2)*d + sqrt(b)*d*x)
/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2))*a*b*c**3*d**
2 + 2*sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c - a*d**2 - 2*b*c**2)*atan((sq
rt(a + b*x**2)*d + sqrt(b)*d*x)/sqrt(2*sqrt(b)*sqrt(a*d**2 + b*c**2)*c ...
```

3.119 $\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 134

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx = \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d} + \frac{(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{cd\sqrt{bc^2+ad^2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c}$$

output
```
C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d+(A*d^2-B*c*d+C*c^2)*arctanh
((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c/d/(a*d^2+b*c^2)^(1/2)
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c
```

Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c} - \frac{\frac{2\sqrt{-bc^2-ad^2}(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{bc^3+acd^2}+\frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{d}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(Sqrt[a]*c) - ((2*Sqr
t[-(b*c^2) - a*d^2]*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*
Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(b*c^3 + a*c*d^2) + (C*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2351, 617, 719, 224, 219, 488, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{1}{x(c+dx)\sqrt{bx^2+a}}dx+\int \frac{B+Cx}{(c+dx)\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \text{617}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx+\int \frac{B+Cx}{(c+dx)\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \text{719}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx-\frac{(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx}{d}$$

$$\Big\downarrow \text{224}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx-\frac{(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}$$

$$\Big\downarrow \text{219}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx-\frac{(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{488}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx+\frac{(cC-Bd)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{219}$$

$$A\int \left(\frac{1}{cx\sqrt{bx^2+a}}-\frac{d}{c(c+dx)\sqrt{bx^2+a}}\right)dx+\frac{(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

$$\Big\downarrow \text{2009}$$

$$A\left(\frac{d\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c}\right)+\frac{(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\sqrt{ad^2+bc^2}}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) + ((c*C - B*d)*ArcTan
h[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*
d^2]) + A*((d*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])]
)/(c*Sqrt[b*c^2 + a*d^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(Sqrt[a]*c))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.51

method	result
default	$\frac{C\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\,\sqrt{b\left(x+\frac{c}{d}\right)^2-}}{x+\frac{c}{d}}\right.}{d^2c\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-A/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x)+1/d^2*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln
((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/
d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{x(c + dx)\sqrt{a + bx^2}}\,dx = \int \frac{A + Bx + Cx^2}{x\sqrt{a + bx^2}\,(c + dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{x(c + dx)\sqrt{a + bx^2}}\,dx = \int \frac{Cx^2 + Bx + A}{\sqrt{bx^2 + a}(dx + c)x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{x(c + dx)\sqrt{a + bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.25

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\sqrt{a+bx^2}}\,dx$$
$$=\frac{2\sqrt{a\,d^2+b\,c^2}\,\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c^2}-ad+bcx\right)ab\,d^2-2\sqrt{a\,d^2+b\,c^2}\,\log\left(-\sqrt{b\,x^2+a}\,\sqrt{a\,d^2+b\,c\right.}$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*d**2 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c*d + 2*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b*c**3 - 2*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a*b*d**2 + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x
)*b**2*c*d - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b*c**3 + sqrt(a)*log(sqr
t(a + b*x**2) - sqrt(a))*a*b*d**3 + sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a)
)*b**2*c**2*d - sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a*b*d**3 - sqrt(a)
*log(sqrt(a + b*x**2) + sqrt(a))*b**2*c**2*d - sqrt(b)*log(sqrt(a + b*x**2
) - sqrt(b)*x)*a*c**2*d**2 - sqrt(b)*log(sqrt(a + b*x**2) - sqrt(b)*x)*b*c
**4 + sqrt(b)*log(sqrt(a + b*x**2) + sqrt(b)*x)*a*c**2*d**2 + sqrt(b)*log(
sqrt(a + b*x**2) + sqrt(b)*x)*b*c**4)/(2*b*c*d*(a*d**2 + b*c**2))
```

3.120 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 133

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{acx} - \frac{(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2\sqrt{bc^2+ad^2}} - \frac{(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

output
```
-A*(b*x^2+a)^(1/2)/a/c/x-(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b
*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2/(a*d^2+b*c^2)^(1/2)-(-A*d+B*c)*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^2
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = \frac{-\frac{Ac\sqrt{a+bx^2}}{ax}-\frac{2(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+\frac{2(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-((A*c*Sqrt[a + b*x^2])/(a*x)) - (2*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[
b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/Sqrt[-(b*c^2) -
 a*d^2] + (2*(B*c - A*d)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/S
qrt[a])/c^2
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \; 2353$$

$$\int \left(\frac{Ad^2-Bcd+c^2C}{c^2\sqrt{a+bx^2}(c+dx)}+\frac{Bc-Ad}{c^2x\sqrt{a+bx^2}}+\frac{A}{cx^2\sqrt{a+bx^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{\sqrt{a}c^2}-\frac{A\sqrt{a+bx^2}}{acx}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*c*x)) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b
*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*Sqrt[b*c^2 + a*d^2]) -
((B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.55

method	result
default	$-\frac{A\sqrt{b x^2+a}}{a c x}+\frac{(A d-B c)\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{c^2\sqrt{a}}-\frac{(A d^2-B c d+C c^2)\ln\left(\frac{\frac{2a d^2+2b c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a d^2+b c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2b}{}}}{x+\frac{c}{d}}\right.}{c^2 d\sqrt{\frac{a d^2+b c^2}{d^2}}}$
risch	$-\frac{A\sqrt{b x^2+a}}{a c x}-\frac{-\frac{(A d-B c)\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{c\sqrt{a}}+\frac{\left(A d^2-B c d+C c^2\right)\ln\left(\frac{\frac{2a d^2+2b c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a d^2+b c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\right.}{d}}}{x+\frac{c}{d}}\right.}{c d\sqrt{\frac{a d^2+b c^2}{d^2}}}}{c}$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^2+a)^(1/2)/a/c/x+(A*d-B*c)/c^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x)-(A*d^2-B*c*d+C*c^2)/c^2/d/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
```

Fricas [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 910, normalized size of antiderivative = 6.84

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(b*c^2 + a*d^2)*x*log((2*a*b*c*d*x
 - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)
*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c^3 - A*
b*c^2*d + B*a*c*d^2 - A*a*d^3)*sqrt(a)*x*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(A*b*c^3 + A*a*c*d^2)*sqrt(b*x^2 + a))/((a*b*c^4 +
a^2*c^2*d^2)*x), -1/2*(2*(C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(-b*c^2 - a*d^2
)*x*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a
^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (B*b*c^3 - A*b*c^2*d + B*a*c*d^2 - A*
a*d^3)*sqrt(a)*x*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(
A*b*c^3 + A*a*c*d^2)*sqrt(b*x^2 + a))/((a*b*c^4 + a^2*c^2*d^2)*x), 1/2*(2*
(B*b*c^3 - A*b*c^2*d + B*a*c*d^2 - A*a*d^3)*sqrt(-a)*x*arctan(sqrt(b*x^2 +
 a)*sqrt(-a)/a) + (C*a*c^2 - B*a*c*d + A*a*d^2)*sqrt(b*c^2 + a*d^2)*x*log(
(2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*
c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2
*(A*b*c^3 + A*a*c*d^2)*sqrt(b*x^2 + a))/((a*b*c^4 + a^2*c^2*d^2)*x), -((C*
a*c^2 - B*a*c*d + A*a*d^2)*sqrt(-b*c^2 - a*d^2)*x*arctan(sqrt(-b*c^2 - a*d
^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)
*x^2)) - (B*b*c^3 - A*b*c^2*d + B*a*c*d^2 - A*a*d^3)*sqrt(-a)*x*arctan(sqr
t(b*x^2 + a)*sqrt(-a)/a) + (A*b*c^3 + A*a*c*d^2)*sqrt(b*x^2 + a))/((a*b*c^
4 + a^2*c^2*d^2)*x)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^2\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)c} + \frac{2\,(Cc^2-Bcd+Ad^2)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}c^2} + \frac{2\,(Bc-Ad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}c^2}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*c) + 2*(C*c^2 - B*c*d +
 A*d^2)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2
- a*d^2))/(sqrt(-b*c^2 - a*d^2)*c^2) + 2*(B*c - A*d)*arctan(-(sqrt(b)*x -
sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\,(dx+c)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(1/2),x)
```

3.121 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 187

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{2acx^2}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{ac^2x}+\frac{d(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3\sqrt{bc^2+ad^2}}-\frac{(2ac(cC-Bd)-A(bc^2-2ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c^3}$$

output
```
-1/2*A*(b*x^2+a)^(1/2)/a/c/x^2-(-A*d+B*c)*(b*x^2+a)^(1/2)/a/c^2/x+d*(A*d^2
-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^
3/(a*d^2+b*c^2)^(1/2)-1/2*(2*a*c*(-B*d+C*c)-A*(-2*a*d^2+b*c^2))*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^3
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\frac{c\sqrt{a+bx^2}(2Bcx+A(c-2dx))}{ax^2}+\frac{4d(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+\frac{4Ad^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2c(Abc-2acC}{}}{2c^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-((c*Sqrt[a + b*x^2]*(2*B*c*x + A*(c - 2*d*x)))/(a*x^2)) + (4*d*(c^2*C -
B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2
) - a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + (4*A*d^2*ArcTanh[(Sqrt[b]*x - Sqrt[a
 + b*x^2])/Sqrt[a]])/Sqrt[a] + (2*c*(A*b*c - 2*a*c*C + 2*a*B*d)*ArcTanh[(-
(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/(2*c^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{Bc-Ad}{c^2x^2\sqrt{a+bx^2}}+\frac{Ad^2-Bcd+c^2C}{c^3x\sqrt{a+bx^2}}-\frac{d(Ad^2-Bcd+c^2C)}{c^3\sqrt{a+bx^2}(c+dx)}+\frac{A}{cx^3\sqrt{a+bx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c}+\frac{d(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3\sqrt{ad^2+bc^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{\sqrt{a}c^3}-\frac{\sqrt{a+bx^2}(Bc-Ad)}{ac^2x}-\frac{A\sqrt{a+bx^2}}{2acx^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(a*c*x^2) - ((B*c - A*d)*Sqrt[a + b*x^2])/(a*c^2*
x) + (d*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]
*Sqrt[a + b*x^2])])/(c^3*Sqrt[b*c^2 + a*d^2]) + (A*b*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/(2*a^(3/2)*c) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/(Sqrt[a]*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-2Adx+2Bcx+Ac)}{2a\,c^2x^2}-\frac{\frac{\left(2Aa\,d^2-bA\,c^2-2Bacd+2Ca\,c^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}-\frac{2a\left(A\,d^2-Bcd+C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2}{}}{}\right.}{}}{2c^2a}$
default	$\frac{A\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{c}+\frac{(Ad-Bc)\sqrt{b\,x^2+a}}{c^2ax}-\frac{(A\,d^2-Bcd+C\,c^2)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c^3\sqrt{a}}+\frac{(A\,d^2-Bcd+C}{}$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-2*A*d*x+2*B*c*x+A*c)/a/c^2/x^2-1/2/c^2/a*((2*A*a*d^
2-A*b*c^2-2*B*a*c*d+2*C*a*c^2)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2)
)/x)-2*a*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^
2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 3.98 (sec) , antiderivative size = 1301, normalized size of antiderivative = 6.96

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(b*c^2 + a*d^2)*x^2*lo
g((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(
b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) -
 (2*B*a*b*c^3*d + 2*B*a^2*c*d^3 - 2*A*a^2*d^4 - (2*C*a*b - A*b^2)*c^4 - (2
*C*a^2 + A*a*b)*c^2*d^2)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - 2*(A*a*b*c^4 + A*a^2*c^2*d^2 + 2*(B*a*b*c^4 - A*a*b*c^3*d
 + B*a^2*c^2*d^2 - A*a^2*c*d^3)*x)*sqrt(b*x^2 + a))/((a^2*b*c^5 + a^3*c^3*
d^2)*x^2), 1/4*(4*(C*a^2*c^2*d - B*a^2*c*d^2 + A*a^2*d^3)*sqrt(-b*c^2 - a*
d^2)*x^2*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^
2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (2*B*a*b*c^3*d + 2*B*a^2*c*d^3 -
 2*A*a^2*d^4 - (2*C*a*b - A*b^2)*c^4 - (2*C*a^2 + A*a*b)*c^2*d^2)*sqrt(a)*
x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(A*a*b*c^4 + A
*a^2*c^2*d^2 + 2*(B*a*b*c^4 - A*a*b*c^3*d + B*a^2*c^2*d^2 - A*a^2*c*d^3)*x
)*sqrt(b*x^2 + a))/((a^2*b*c^5 + a^3*c^3*d^2)*x^2), -1/2*((2*B*a*b*c^3*d +
 2*B*a^2*c*d^3 - 2*A*a^2*d^4 - (2*C*a*b - A*b^2)*c^4 - (2*C*a^2 + A*a*b)*c
^2*d^2)*sqrt(-a)*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (C*a^2*c^2*d - B
*a^2*c*d^2 + A*a^2*d^3)*sqrt(b*c^2 + a*d^2)*x^2*log((2*a*b*c*d*x - a*b*c^2
 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x -
a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + (A*a*b*c^4 + A*a^2*c^2*
d^2 + 2*(B*a*b*c^4 - A*a*b*c^3*d + B*a^2*c^2*d^2 - A*a^2*c*d^3)*x)*sqrt...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x^3\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.60

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{2\,(Cc^2d-Bcd^2+Ad^3)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,c^3} + \frac{(2\,Cac^2-Abc^2-2\,Bacd+2\,Aad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ac^3} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}d+\left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2ac^2}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-2*(C*c^2*d - B*c*d^2 + A*d^3)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d +
sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*c^3) + (2*C*a*c^2 -
 A*b*c^2 - 2*B*a*c*d + 2*A*a*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a*c^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b*c + 2*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*c - 2*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*a*sqrt(b)*d + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b*c - 2*B*a^2*sqrt(b
)*c + 2*A*a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.99

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*d**3*x**2 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*d**2*x**2 + 4*sqrt(a*d**2 +
b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**
3*d*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*d**3*x**2 + 4*sqrt(a*
d**2 + b*c**2)*log(c + d*x)*a*b*c*d**2*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(
c + d*x)*a*c**3*d*x**2 - 2*sqrt(a + b*x**2)*a**2*c**2*d**2 + 4*sqrt(a + b*
x**2)*a**2*c*d**3*x - 2*sqrt(a + b*x**2)*a*b*c**4 + 4*sqrt(a + b*x**2)*a*b
*c**3*d*x - 4*sqrt(a + b*x**2)*a*b*c**2*d**2*x - 4*sqrt(a + b*x**2)*b**2*c
**4*x + 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**2*d**4*x**2 + sqrt(a)
*log(sqrt(a + b*x**2) - sqrt(a))*a*b*c**2*d**2*x**2 - 2*sqrt(a)*log(sqrt(a
 + b*x**2) - sqrt(a))*a*b*c*d**3*x**2 + 2*sqrt(a)*log(sqrt(a + b*x**2) - s
qrt(a))*a*c**3*d**2*x**2 - sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*b**2*c*
*4*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*b**2*c**3*d*x**2 + 2*s
qrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*b*c**5*x**2 - 2*sqrt(a)*log(sqrt(a
+ b*x**2) + sqrt(a))*a**2*d**4*x**2 - sqrt(a)*log(sqrt(a + b*x**2) + sqrt(
a))*a*b*c**2*d**2*x**2 + 2*sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a*b*c*d
**3*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2) + sqrt(a))*a*c**3*d**2*x**2 + sq
rt(a)*log(sqrt(a + b*x**2) + sqrt(a))*b**2*c**4*x**2 + 2*sqrt(a)*log(sqrt(
a + b*x**2) + sqrt(a))*b**2*c**3*d*x**2 - 2*sqrt(a)*log(sqrt(a + b*x**2...
```

3.122 $\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 251

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{3acx^3}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{2ac^2x^2}-\frac{(3ac(cC-Bd)-A(2bc^2-3ad^2))\sqrt{a+bx^2}}{3a^2c^3x}\\ &\quad-\frac{d^2(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4\sqrt{bc^2+ad^2}}\\ &\quad+\frac{(bc^2(Bc-Ad)+2ad(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c^4}\end{aligned}$$

output

```
-1/3*A*(b*x^2+a)^(1/2)/a/c/x^3-1/2*(-A*d+B*c)*(b*x^2+a)^(1/2)/a/c^2/x^2-1/
3*(3*a*c*(-B*d+C*c)-A*(-3*a*d^2+2*b*c^2))*(b*x^2+a)^(1/2)/a^2/c^3/x-d^2*(A
*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2)
)/c^4/(a*d^2+b*c^2)^(1/2)+1/2*(b*c^2*(-A*d+B*c)+2*a*d*(A*d^2-B*c*d+C*c^2))
*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^4
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx =$$

$$-\frac{\frac{c\sqrt{a+bx^2}\left(-4Abc^2x^2+aA\left(2c^2-3cdx+6d^2x^2\right)+3acx(2cCx+B(c-2dx))\right)}{a^2x^3}+\frac{12d^2\left(c^2C-Bcd+Ad^2\right)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}}+\frac{12A}{}}{6c^4}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^4*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
-1/6*((c*Sqrt[a + b*x^2]*(-4*A*b*c^2*x^2 + a*A*(2*c^2 - 3*c*d*x + 6*d^2*x^
2) + 3*a*c*x*(2*c*C*x + B*(c - 2*d*x))))/(a^2*x^3) + (12*d^2*(c^2*C - B*c*
d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) -
a*d^2]])/Sqrt[-(b*c^2) - a*d^2] + (12*A*d^3*ArcTanh[(Sqrt[b]*x - Sqrt[a +
b*x^2])/Sqrt[a]])/Sqrt[a] - (6*c*(b*c*(B*c - A*d) + 2*a*d*(c*C - B*d))*Arc
Tanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/c^4
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^4\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \; 2353$$

$$\int \left(\frac{Bc-Ad}{c^2x^3\sqrt{a+bx^2}}+\frac{d^2\left(Ad^2-Bcd+c^2C\right)}{c^4\sqrt{a+bx^2}(c+dx)}-\frac{d\left(Ad^2-Bcd+c^2C\right)}{c^4x\sqrt{a+bx^2}}+\frac{Ad^2-Bcd+c^2C}{c^3x^2\sqrt{a+bx^2}}+\frac{A}{cx^4\sqrt{a+bx^2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{2a^{3/2}c^2}+\frac{2Ab\sqrt{a+bx^2}}{3a^2cx}-\frac{d^2(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4\sqrt{ad^2+bc^2}}+\frac{d\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{\sqrt{a}c^4}-\frac{\sqrt{a+bx^2}(Bc-Ad)}{2ac^2x^2}-\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{ac^3x}-\frac{A\sqrt{a+bx^2}}{3acx^3}$$

input

```
Int[(A + B*x + C*x^2)/(x^4*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output

```
-1/3*(A*Sqrt[a + b*x^2])/(a*c*x^3) - ((B*c - A*d)*Sqrt[a + b*x^2])/(2*a*c^
2*x^2) + (2*A*b*Sqrt[a + b*x^2])/(3*a^2*c*x) - ((c^2*C - B*c*d + A*d^2)*Sq
rt[a + b*x^2])/(a*c^3*x) - (d^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c
*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^4*Sqrt[b*c^2 + a*d^2]) + (b
*(B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*c^2) + (d*(c^2*C
 - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353

```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(6Aa\,d^2x^2-4Ab\,c^2x^2-6Bacd\,x^2+6Ca\,c^2x^2-3Aacdx+3Ba\,c^2x+2A\,c^2a)}{6a^2c^3x^3}+\frac{\frac{(2Aa\,d^3-Ab\,c^2d-2Bac\,d^2+Bb\,c^3+2Ca\,c^2d)}{c\sqrt{a}}}{}$
default	$\frac{A\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{c^2}-\frac{(A\,d^2-Bcd+C\,c^2)\sqrt{b\,x^2+a}}{c^3ax}-\frac{(A\,d^2-Bc}{}$

input
```
int((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(6*A*a*d^2*x^2-4*A*b*c^2*x^2-6*B*a*c*d*x^2+6*C*a*c^2*
x^2-3*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/a^2/c^3/x^3+1/2/c^3/a*((2*A*a*d^3-A
*b*c^2*d-2*B*a*c*d^2+B*b*c^3+2*C*a*c^2*d)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)-2*d*a*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((
2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)
^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 1653, normalized size of antiderivative = 6.59

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(6*(C*a^2*c^2*d^2 - B*a^2*c*d^3 + A*a^2*d^4)*sqrt(b*c^2 + a*d^2)*x^3
*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sq
rt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)
) + 3*(B*b^2*c^5 - B*a*b*c^3*d^2 - 2*B*a^2*c*d^4 + 2*A*a^2*d^5 + (2*C*a*b
- A*b^2)*c^4*d + (2*C*a^2 + A*a*b)*c^2*d^3)*sqrt(a)*x^3*log(-(b*x^2 + 2*sq
rt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*A*a*b*c^5 + 2*A*a^2*c^3*d^2 - 2*(
3*B*a*b*c^4*d + 3*B*a^2*c^2*d^3 - 3*A*a^2*c*d^4 - (3*C*a*b - 2*A*b^2)*c^5
- (3*C*a^2 + A*a*b)*c^3*d^2)*x^2 + 3*(B*a*b*c^5 - A*a*b*c^4*d + B*a^2*c^3*
d^2 - A*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/((a^2*b*c^6 + a^3*c^4*d^2)*x^3),
-1/12*(12*(C*a^2*c^2*d^2 - B*a^2*c*d^3 + A*a^2*d^4)*sqrt(-b*c^2 - a*d^2)*x
^3*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^
2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - 3*(B*b^2*c^5 - B*a*b*c^3*d^2 - 2*B*a^2
*c*d^4 + 2*A*a^2*d^5 + (2*C*a*b - A*b^2)*c^4*d + (2*C*a^2 + A*a*b)*c^2*d^3
)*sqrt(a)*x^3*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*A
*a*b*c^5 + 2*A*a^2*c^3*d^2 - 2*(3*B*a*b*c^4*d + 3*B*a^2*c^2*d^3 - 3*A*a^2*
c*d^4 - (3*C*a*b - 2*A*b^2)*c^5 - (3*C*a^2 + A*a*b)*c^3*d^2)*x^2 + 3*(B*a*
b*c^5 - A*a*b*c^4*d + B*a^2*c^3*d^2 - A*a^2*c^2*d^3)*x)*sqrt(b*x^2 + a))/(
(a^2*b*c^6 + a^3*c^4*d^2)*x^3), -1/6*(3*(B*b^2*c^5 - B*a*b*c^3*d^2 - 2*B*a
^2*c*d^4 + 2*A*a^2*d^5 + (2*C*a*b - A*b^2)*c^4*d + (2*C*a^2 + A*a*b)*c^2*d
^3)*sqrt(-a)*x^3*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - 3*(C*a^2*c^2*d^2 ...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x^4\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**4/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**4*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)x^4}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*x^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(226) = 452.

Time = 0.14 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2\,(Cc^2d^2 - Bcd^3 + Ad^4)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\sqrt{-bc^2-ad^2}\,c^4} - \frac{(Bbc^3 + 2\,Cac^2d - Abc^2d - 2\,Bacd^2 + 2\,Aad^3)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,ac^4} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Bbc^2 - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5 Abcd + 6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 Ca\sqrt{b}c^2 - 6\left(\sqrt{b}x\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*(C*c^2*d^2 - B*c*d^3 + A*d^4)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d +
 sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*c^4) - (B*b*c^3 +
2*C*a*c^2*d - A*b*c^2*d - 2*B*a*c*d^2 + 2*A*a*d^3)*arctan(-(sqrt(b)*x - sq
rt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^4) + 1/3*(3*(sqrt(b)*x - sqrt(b*x^2
 + a))^5*B*b*c^2 - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b*c*d + 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^4*C*a*sqrt(b)*c^2 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*B*a*sqrt(b)*c*d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*sqrt(b)*d^2 - 12*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b)*c^2 + 12*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*A*a*b^(3/2)*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sq
rt(b)*c*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*sqrt(b)*d^2 - 3*(sqrt
(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c^2 + 3*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a
^2*b*c*d + 6*C*a^3*sqrt(b)*c^2 - 4*A*a^2*b^(3/2)*c^2 - 6*B*a^3*sqrt(b)*c*d
 + 6*A*a^3*sqrt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^4\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)} \,dx$$

input
```
int((A + B*x + C*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^4\,(dx+c)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^4/(d*x+c)/(b*x^2+a)^(1/2),x)
```

3.123 $\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 330

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx
&= -\frac{A\sqrt{a+bx^2}}{4acx^4}-\frac{(Bc-Ad)\sqrt{a+bx^2}}{3ac^2x^3}-\frac{(4ac(cC-Bd)-A(3bc^2-4ad^2))\sqrt{a+bx^2}}{8a^2c^3x^2} \\
&\quad+\frac{(2bc^2(Bc-Ad)+3ad(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3a^2c^4x} \\
&\quad+\frac{d^3(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5\sqrt{bc^2+ad^2}} \\
&\quad+\frac{(4ac(cC-Bd)(bc^2-2ad^2)-A(3b^2c^4-4abc^2d^2+8a^2d^4))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}c^5}
\end{aligned}$$

output
```
-1/4*A*(b*x^2+a)^(1/2)/a/c/x^4-1/3*(-A*d+B*c)*(b*x^2+a)^(1/2)/a/c^2/x^3-1/
8*(4*a*c*(-B*d+C*c)-A*(-4*a*d^2+3*b*c^2))*(b*x^2+a)^(1/2)/a^2/c^3/x^2+1/3*
(2*b*c^2*(-A*d+B*c)+3*a*d*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/a^2/c^4/x+d
^3*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^
(1/2))/c^5/(a*d^2+b*c^2)^(1/2)+1/8*(4*a*c*(-B*d+C*c)*(-2*a*d^2+b*c^2)-A*(8
*a^2*d^4-4*a*b*c^2*d^2+3*b^2*c^4))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2
)/c^5
```

Mathematica [A] (verified)

Time = 2.72 (sec) , antiderivative size = 323, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{c\sqrt{a+bx^2}(bc^2x^2(9Ac+16Bcx-16Adx)-2a(A(3c^3-4c^2dx+6cd^2x^2-12d^3x^3)+2cx(3cCx(c-2dx)+B(2c^2-3cdx+6d^2x^2))))}{a^2x^4}+\frac{48d^3(c^2C-B}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^5*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((c*Sqrt[a + b*x^2]*(b*c^2*x^2*(9*A*c + 16*B*c*x - 16*A*d*x) - 2*a*(A*(3*c
^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3) + 2*c*x*(3*c*C*x*(c - 2*d*x) +
B*(2*c^2 - 3*c*d*x + 6*d^2*x^2)))))/(a^2*x^4) + (48*d^3*(c^2*C - B*c*d + A
*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2
]])/Sqrt[-(b*c^2) - a*d^2] + (48*A*d^4*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2
])/Sqrt[a]])/Sqrt[a] - (6*c*(A*b*c*(3*b*c^2 - 4*a*d^2) + 4*a*(c*C - B*d)*(
-(b*c^2) + 2*a*d^2))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^
(5/2))/(24*c^5)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.21, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^5\sqrt{a+bx^2}(c+dx)}\,dx$$
$$\Big\downarrow \text{2353}$$
$$\int \left(\frac{Bc-Ad}{c^2x^4\sqrt{a+bx^2}}+\frac{d^2(Ad^2-Bcd+c^2C)}{c^5x\sqrt{a+bx^2}}-\frac{d^3(Ad^2-Bcd+c^2C)}{c^5\sqrt{a+bx^2}(c+dx)}-\frac{d(Ad^2-Bcd+c^2C)}{c^4x^2\sqrt{a+bx^2}}+\frac{Ad^2-Bcd+}{c^3x^3\sqrt{a+bx}}\right.$$

$$\downarrow \text{2009}$$

$$-\frac{3Ab^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}c}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{2a^{3/2}c^3}+\frac{2b\sqrt{a+bx^2}(Bc-Ad)}{3a^2c^2x}+\frac{3Ab\sqrt{a+bx^2}}{8a^2cx^2}-\frac{d^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{\sqrt{a}c^5}+\frac{d^3(Ad^2-Bcd+c^2C)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^5\sqrt{ad^2+bc^2}}-\frac{\sqrt{a+bx^2}(Bc-Ad)}{3ac^2x^3}+\frac{d\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{ac^4x}-\frac{\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{2ac^3x^2}-\frac{A\sqrt{a+bx^2}}{4acx^4}$$

input
```
Int[(A + B*x + C*x^2)/(x^5*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
-1/4*(A*Sqrt[a + b*x^2])/(a*c*x^4) - ((B*c - A*d)*Sqrt[a + b*x^2])/(3*a*c^
2*x^3) + (3*A*b*Sqrt[a + b*x^2])/(8*a^2*c*x^2) - ((c^2*C - B*c*d + A*d^2)*
Sqrt[a + b*x^2])/(2*a*c^3*x^2) + (2*b*(B*c - A*d)*Sqrt[a + b*x^2])/(3*a^2*
c^2*x) + (d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(a*c^4*x) + (d^3*(c^2
*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2])])/(c^5*Sqrt[b*c^2 + a*d^2]) - (3*A*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/(8*a^(5/2)*c) + (b*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sq
rt[a]])/(2*a^(3/2)*c^3) - (d^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*
x^2]/Sqrt[a]])/(Sqrt[a]*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.21

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-24Aa\,d^{3}x^{3}+16Ab\,c^{2}d\,x^{3}+24Bac\,d^{2}x^{3}-16Bb\,c^{3}x^{3}-24Ca\,c^{2}d\,x^{3}+12Aac\,d^{2}x^{2}-9Ab\,c^{3}x^{2}-12Ba\,c^{2}d\,x^{2}+12Ca\,c^{3}x^{2}\right.}{24a^{2}c^{4}x^{4}}$
default	$\frac{A\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{c^{2}}+\frac{(A\,d^{2}-Bcd+C\,c^{2})\left(-\sqrt{\right.}}{}$

input
```
int((C*x^2+B*x+A)/x^5/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-24*A*a*d^3*x^3+16*A*b*c^2*d*x^3+24*B*a*c*d^2*x^3-1
6*B*b*c^3*x^3-24*C*a*c^2*d*x^3+12*A*a*c*d^2*x^2-9*A*b*c^3*x^2-12*B*a*c^2*d
*x^2+12*C*a*c^3*x^2-8*A*a*c^2*d*x+8*B*a*c^3*x+6*A*a*c^3)/a^2/c^4/x^4-1/8/c
^4/a^2*((8*A*a^2*d^4-4*A*a*b*c^2*d^2+3*A*b^2*c^4-8*B*a^2*c*d^3+4*B*a*b*c^3
*d+8*C*a^2*c^2*d^2-4*C*a*b*c^4)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)-8*a^2*d^2*(A*d^2-B*c*d+C*c^2)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d
^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [A] (verification not implemented)

Time = 19.91 (sec) , antiderivative size = 2207, normalized size of antiderivative = 6.69

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^5/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(24*(C*a^3*c^2*d^3 - B*a^3*c*d^4 + A*a^3*d^5)*sqrt(b*c^2 + a*d^2)*x^
4*log((2*a*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*s
qrt(b*c^2 + a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2
)) + 3*(4*B*a*b^2*c^5*d - 4*B*a^2*b*c^3*d^3 - 8*B*a^3*c*d^5 + 8*A*a^3*d^6
- (4*C*a*b^2 - 3*A*b^3)*c^6 + (4*C*a^2*b - A*a*b^2)*c^4*d^2 + 4*(2*C*a^3 +
 A*a^2*b)*c^2*d^4)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(6*A*a^2*b*c^6 + 6*A*a^3*c^4*d^2 - 8*(2*B*a*b^2*c^6 - B*a^2*b
*c^4*d^2 - 3*B*a^3*c^2*d^4 + 3*A*a^3*c*d^5 + (3*C*a^2*b - 2*A*a*b^2)*c^5*d
 + (3*C*a^3 + A*a^2*b)*c^3*d^3)*x^3 - 3*(4*B*a^2*b*c^5*d + 4*B*a^3*c^3*d^3
 - 4*A*a^3*c^2*d^4 - (4*C*a^2*b - 3*A*a*b^2)*c^6 - (4*C*a^3 + A*a^2*b)*c^4
*d^2)*x^2 + 8*(B*a^2*b*c^6 - A*a^2*b*c^5*d + B*a^3*c^4*d^2 - A*a^3*c^3*d^3
)*x)*sqrt(b*x^2 + a))/((a^3*b*c^7 + a^4*c^5*d^2)*x^4), 1/48*(48*(C*a^3*c^2
*d^3 - B*a^3*c*d^4 + A*a^3*d^5)*sqrt(-b*c^2 - a*d^2)*x^4*arctan(sqrt(-b*c^
2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a
*b*d^2)*x^2)) + 3*(4*B*a*b^2*c^5*d - 4*B*a^2*b*c^3*d^3 - 8*B*a^3*c*d^5 + 8
*A*a^3*d^6 - (4*C*a*b^2 - 3*A*b^3)*c^6 + (4*C*a^2*b - A*a*b^2)*c^4*d^2 + 4
*(2*C*a^3 + A*a^2*b)*c^2*d^4)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) - 2*(6*A*a^2*b*c^6 + 6*A*a^3*c^4*d^2 - 8*(2*B*a*b^2*c^
6 - B*a^2*b*c^4*d^2 - 3*B*a^3*c^2*d^4 + 3*A*a^3*c*d^5 + (3*C*a^2*b - 2*A*a
*b^2)*c^5*d + (3*C*a^3 + A*a^2*b)*c^3*d^3)*x^3 - 3*(4*B*a^2*b*c^5*d + 4...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x^5\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**5/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**5*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)x^5}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^5/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*x^5), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(301) = 602$.

Time = 0.21 (sec) , antiderivative size = 1161, normalized size of antiderivative = 3.52

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^5/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-2*(C*c^2*d^3 - B*c*d^4 + A*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^2 + a))*d
+ sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/(sqrt(-b*c^2 - a*d^2)*c^5) - 1/4*(4*C*a
*b*c^4 - 3*A*b^2*c^4 - 4*B*a*b*c^3*d - 8*C*a^2*c^2*d^2 + 4*A*a*b*c^2*d^2 +
 8*B*a^2*c*d^3 - 8*A*a^2*d^4)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-
a))/(sqrt(-a)*a^2*c^5) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^
3 - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*b^2*c^3 - 12*(sqrt(b)*x - sqrt(b*x
^2 + a))^7*B*a*b*c^2*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b*c*d^2 -
24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^2*sqrt(b)*c^2*d + 24*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*B*a^2*sqrt(b)*c*d^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*A*a^2*sqrt(b)*d^3 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c^3 + 33*(
sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c^3 + 12*(sqrt(b)*x - sqrt(b*x^2 +
a))^5*B*a^2*b*c^2*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a^2*b*c*d^2 + 4
8*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c^3 + 72*(sqrt(b)*x - sqrt
(b*x^2 + a))^4*C*a^3*sqrt(b)*c^2*d - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*
a^2*b^(3/2)*c^2*d - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^3*sqrt(b)*c*d^2
 + 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^3*sqrt(b)*d^3 - 12*(sqrt(b)*x -
sqrt(b*x^2 + a))^3*C*a^3*b*c^3 + 33*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*
b^2*c^3 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b*c^2*d - 12*(sqrt(b)*x
 - sqrt(b*x^2 + a))^3*A*a^3*b*c*d^2 - 64*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B
*a^3*b^(3/2)*c^3 - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^4*sqrt(b)*c^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^5\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^5*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^5*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 1280, normalized size of antiderivative = 3.88

$$\int \frac{A+Bx+Cx^2}{x^5(c+dx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^5/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**3*d**5*x**4 - 48*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**4*x**4 + 48*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a**2*c**3*d**3*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*d**5*x**
4 + 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4*x**4 - 48*sqrt(a*d
**2 + b*c**2)*log(c + d*x)*a**2*c**3*d**3*x**4 - 12*sqrt(a + b*x**2)*a**3*
c**4*d**2 + 16*sqrt(a + b*x**2)*a**3*c**3*d**3*x - 24*sqrt(a + b*x**2)*a**
3*c**2*d**4*x**2 + 48*sqrt(a + b*x**2)*a**3*c*d**5*x**3 - 12*sqrt(a + b*x*
*2)*a**2*b*c**6 + 16*sqrt(a + b*x**2)*a**2*b*c**5*d*x - 6*sqrt(a + b*x**2)
*a**2*b*c**4*d**2*x**2 - 16*sqrt(a + b*x**2)*a**2*b*c**4*d**2*x + 16*sqrt(
a + b*x**2)*a**2*b*c**3*d**3*x**3 + 24*sqrt(a + b*x**2)*a**2*b*c**3*d**3*x
**2 - 48*sqrt(a + b*x**2)*a**2*b*c**2*d**4*x**3 - 24*sqrt(a + b*x**2)*a**2
*c**5*d**2*x**2 + 48*sqrt(a + b*x**2)*a**2*c**4*d**3*x**3 + 18*sqrt(a + b*
x**2)*a*b**2*c**6*x**2 - 16*sqrt(a + b*x**2)*a*b**2*c**6*x - 32*sqrt(a + b
*x**2)*a*b**2*c**5*d*x**3 + 24*sqrt(a + b*x**2)*a*b**2*c**5*d*x**2 - 16*sq
rt(a + b*x**2)*a*b**2*c**4*d**2*x**3 - 24*sqrt(a + b*x**2)*a*b*c**7*x**2 +
 48*sqrt(a + b*x**2)*a*b*c**6*d*x**3 + 32*sqrt(a + b*x**2)*b**3*c**6*x**3
+ 24*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**3*d**6*x**4 + 12*sqrt(a)*l
og(sqrt(a + b*x**2) - sqrt(a))*a**2*b*c**2*d**4*x**4 - 24*sqrt(a)*log(s...
```

3.124 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 335

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{(2aCd^2-3b(3c^2C-2Bcd+Ad^2))\sqrt{a+bx^2}}{3b^2d^4} - \frac{(2cC-Bd)x\sqrt{a+bx^2}}{2bd^3} + \frac{Cx^2\sqrt{a+bx^2}}{3bd^2} + \frac{c^3(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^4(bc^2+ad^2)(c+dx)} + \frac{(ad^2(2cC-Bd)-2bc(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}d^5} - \frac{c^2(bc^2(4c^2C-3Bcd+2Ad^2)+ad^2(5c^2C-4Bcd+3Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^5(bc^2+ad^2)^{3/2}}$$

output
```
-1/3*(2*a*C*d^2-3*b*(A*d^2-2*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2)/b^2/d^4-1/2*(
-B*d+2*C*c)*x*(b*x^2+a)^(1/2)/b/d^3+1/3*C*x^2*(b*x^2+a)^(1/2)/b/d^2+c^3*(A
*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^4/(a*d^2+b*c^2)/(d*x+c)+1/2*(a*d^2*(-B
*d+2*C*c)-2*b*c*(2*A*d^2-3*B*c*d+4*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/
2))/b^(3/2)/d^5-c^2*(b*c^2*(2*A*d^2-3*B*c*d+4*C*c^2)+a*d^2*(3*A*d^2-4*B*c*
d+5*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^5/
(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.04

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$

$$= \frac{\frac{d\sqrt{a+bx^2}\left(-4a^2Cd^4(c+dx)+abd^2(c+dx)\left(14c^2C-6cd(2B+Cx)+d^2\left(6A+3Bx+2Cx^2\right)\right)+b^2c^2\left(24c^3C-6c^2d(3B-2Cx)+cd^2\left(12A-9Bx-4Cx^2\right.\right.\right.}{b^2(bc^2+ad^2)(c+dx)}}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(-4*a^2*C*d^4*(c + d*x) + a*b*d^2*(c + d*x)*(14*c^2*C
- 6*c*d*(2*B + C*x) + d^2*(6*A + 3*B*x + 2*C*x^2)) + b^2*c^2*(24*c^3*C - 6
*c^2*d*(3*B - 2*C*x) + c*d^2*(12*A - 9*B*x - 4*C*x^2) + d^3*x*(6*A + 3*B*x
 + 2*C*x^2))))/(b^2*(b*c^2 + a*d^2)*(c + d*x)) + (12*c^2*(b*c^2*(4*c^2*C -
 3*B*c*d + 2*A*d^2) + a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2))*ArcTan[(Sqrt[b]
*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2
)^(3/2) + (3*(a*d^2*(-2*c*C + B*d) + 2*b*c*(4*c^2*C - 3*B*c*d + 2*A*d^2))*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/(6*d^5)
```

Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2182, 25, 2185, 25, 2185, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2182$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}-\frac{\int -\frac{C\left(\frac{bc^2}{d}+ad\right)x^4-\frac{(cC-Bd)\left(bc^2+ad^2\right)x^3}{d^2}+\frac{\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}-\frac{c\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}$$

$\downarrow$ 25

$$\frac{\int \frac{C\left(\frac{bc^2}{d}+ad\right)x^4-\frac{(cC-Bd)\left(bc^2+ad^2\right)x^3}{d^2}+\frac{\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}-\frac{c\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 2185

$$\frac{\int -\frac{\frac{bd^2(10cC-3Bd)\left(bc^2+ad^2\right)x^3+d\left(bc^2+ad^2\right)\left(2aCd^2+b\left(2Cc^2+3Bdc-3Ad^2\right)\right)x^2+c\left(bc^2+ad^2\right)\left(4aCd^2+b\left(4Cc^2-3Bdc+3Ad^2\right)\right)x+ac^2d\left(2aCd^2-b\left(Cc^2-3\cdots\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{3bd^4}}{ad^2+bc^2}\cdots$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}{3bd^4}-\frac{\int \frac{bd^2(10cC-3Bd)\left(bc^2+ad^2\right)x^3+d\left(bc^2+ad^2\right)\left(2aCd^2+b\left(2Cc^2+3Bdc-3Ad^2\right)\right)x^2+c\left(bc^2+ad^2\right)\left(4aCd^2+b\left(4Cc^2-3Bdc+\cdots\right.\right.}{(c+dx)\sqrt{bx^2+a}}}{3bd^4}}{ad^2+bc^2}\cdots$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 2185

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2\left(ad^2+bc^2\right)}{3bd^4}-\frac{\frac{\int -\frac{-b\left(bc^2+ad^2\right)\left(4aCd^2-b\left(26Cc^2-15Bdc+6Ad^2\right)\right)x^2d^4+3abc\left(a(2cC-Bd)d^2+bc\left(4Cc^2-3Bdc+2Ad^2\right)\right)d^4+b\left(bc^2+ad\cdots\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^3}}{3bd^4}}{ad^2+bc^2}\cdots$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$\downarrow$ 25

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-\frac{\int \frac{-b(bc^2+ad^2)(4aCd^2-b(26Cc^2-15Bdc+6Ad^2))x^2d^4+3abc(a(2cC-B}{}}{}}{3bd^4}}{ad^2+bc^2}$$

$$\frac{c^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 2185

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-\frac{\int \frac{3b^2d^5(acd(a(2cC-Bd)d^2+bc(4Cc^2-3Bdc+2Ad^2))+(bc^2+ad^2)(ad^2(}{(c+dx)\sqrt{bx^2+a}}}{bd^2}}{3bd^4}}{ad^2+bc^2}$$

$$\frac{c^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 27

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-3bd^3\int \frac{acd(a(2cC-Bd)d^2+bc(4Cc^2-3Bdc+2Ad^2))+(bc^2+ad^2)(ad^2(2c}{(c+dx)\sqrt{bx^2+a}}}{3bd^4}}{ad^2+bc^2}$$

$$\frac{c^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 719

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-3bd^3\left(\frac{2bc^2(ad^2(3Ad^2-4Bcd+5c^2C)+bc^2(2Ad^2-3Bcd+4c^2C))\int \frac{}{(c+d}}{d}\right.}{}}{ad^2+bc^2}$$

$$\frac{c^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$\downarrow$ 224

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-3bd^3\left(\frac{2bc^2(ad^2(3Ad^2-4Bcd+5c^2C)+bc^2(2Ad^2-3Bcd+4c^2C))\int \frac{}{(c+d}}{d}\right.}{}}{ad^2+}$$

$$\frac{c^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^4(c+dx)(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-\frac{3bd^3\left(\frac{2bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^2\left(2Ad^2-3Bcd+4c^2C\right)\right)\int\frac{}{(c+d\ldots}}{d}\ldots\right)}{\ldots}}{\ldots}}{ad^2+\ldots}$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-\frac{3bd^3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)\left(ad^2(2cC-Bd)-2bc\left(2Ad^2-3B\ldots\right.\right.}{\sqrt{b}d}\ldots\right)}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{C\sqrt{a+bx^2}(c+dx)^2(ad^2+bc^2)}{3bd^4}-\frac{\frac{1}{2}\sqrt{a+bx^2}(c+dx)(ad^2+bc^2)(10cC-3Bd)-\frac{3bd^3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)\left(ad^2(2cC-Bd)-2bc\left(2Ad^2-3B\ldots\right.\right.}{\sqrt{b}d}\ldots\right)}{\ldots}}{\ldots}}{\ldots}$$

$$\frac{c^3\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4(c+dx)\left(ad^2+bc^2\right)}$$

input

```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output

```
(c^3*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d^4*(b*c^2 + a*d^2)*(c + d*
x)) + ((C*(b*c^2 + a*d^2)*(c + d*x)^2*Sqrt[a + b*x^2])/(3*b*d^4) - (((10*c
*C - 3*B*d)*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a + b*x^2])/2 - (-(d^3*(b*c^2 +
 a*d^2)*(4*a*C*d^2 - b*(26*c^2*C - 15*B*c*d + 6*A*d^2))*Sqrt[a + b*x^2]) +
 3*b*d^3*(((b*c^2 + a*d^2)*(a*d^2*(2*c*C - B*d) - 2*b*c*(4*c^2*C - 3*B*c*d
 + 2*A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (2*b*c^2*
(b*c^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2)
)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*
c^2 + a*d^2])))/(2*b*d^3))/(3*b*d^4))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.57

method	result
risch	$\frac{\left(2C\,d^{2}b\,x^{2}+3Bb\,d^{2}x-6Cbcdx+6Ab\,d^{2}-12Bbcd-4aC\,d^{2}+18Cb\,c^{2}\right)\sqrt{b\,x^{2}+a}}{6b^{2}d^{4}}-\frac{\frac{\left(4Abc\,d^{2}+Ba\,d^{3}-6Bb\,c^{2}d-2Cac\,d^{2}+8Cb\,c^{3}\right)\ln\left(\sqrt{b}\,x\right.}{d\sqrt{b}}}{}$
default	$-\frac{c^{2}\left(3A\,d^{2}-4Bcd+5C\,c^{2}\right)\ln\left(\frac{\frac{2a\,d^{2}+2b\,c^{2}}{d^{2}}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}{x+\frac{c}{d}}\right)}{d^{6}\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}-\frac{-d^{2}\left(Bd-2Cc\right)\left(\frac{x\sqrt{}}{}\right.}{}$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x-6*C*b*c*d*x+6*A*b*d^2-12*B*b*c*d-4*C*a*d^2+
18*C*b*c^2)*(b*x^2+a)^(1/2)/b^2/d^4-1/2/b/d^4*((4*A*b*c*d^2+B*a*d^3-6*B*b*
c^2*d-2*C*a*c*d^2+8*C*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*b*c
^2/d^2*(3*A*d^2-4*B*c*d+5*C*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*
c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*c^3*b*(A*d^2-B*c*d+C*c^2)/d^3
*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^
2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(308) = 616$.

Time = 0.13 (sec) , antiderivative size = 719, normalized size of antiderivative = 2.15

$$\int \frac{x^3(A + Bx + Cx^2)}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
sqrt(b*x^2 + a)*C*c^5/(b*c^2*d^5*x + a*d^7*x + b*c^3*d^4 + a*c*d^6) - sqrt
(b*x^2 + a)*B*c^4/(b*c^2*d^4*x + a*d^6*x + b*c^3*d^3 + a*c*d^5) + sqrt(b*x
^2 + a)*A*c^3/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) + 1/3*sqrt(b*x
^2 + a)*C*x^2/(b*d^2) - sqrt(b*x^2 + a)*C*c*x/(b*d^3) + 1/2*sqrt(b*x^2 + a
)*B*x/(b*d^2) - 4*C*c^3*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^5) + 3*B*c^2*arc
sinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) + C*a*c*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*
d^3) - 2*A*c*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 1/2*B*a*arcsinh(b*x/sq
rt(a*b))/(b^(3/2)*d^2) - C*b*c^6*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^8) + B*b*c^5*arcsin
h(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c
^2/d^2)^(3/2)*d^7) + 5*C*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^6) - A*b*c^4*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2
)^(3/2)*d^6) - 4*B*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(
a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^5) + 3*A*c^2*arcsinh(b*c*x/(sqr
t(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*
d^4) + 3*sqrt(b*x^2 + a)*C*c^2/(b*d^4) - 2*sqrt(b*x^2 + a)*B*c/(b*d^3) - 2
/3*sqrt(b*x^2 + a)*C*a/(b^2*d^2) + sqrt(b*x^2 + a)*A/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3(A + Bx + Cx^2)}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\int \frac{x^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2937, normalized size of antiderivative = 8.77

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**2*b**2*c**3*d**4 + 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**5*x + 24*sqrt(a
*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*
a*b**3*c**5*d**2 + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**3*x - 48*sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4*d**3
 - 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b**3*c**3*d**4*x + 60*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x
**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**6*d**2 + 60*sqrt(a*d**
2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*
*2*c**5*d**3*x - 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2
 + b*c**2) - a*d + b*c*x)*b**4*c**6*d - 36*sqrt(a*d**2 + b*c**2)*log(sqrt(
a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d**2*x + 48*sqr
t(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*b**3*c**8 + 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*b**3*c**7*d*x - 36*sqrt(a*d**2 + b*c**2)*log(c + d
*x)*a**2*b**2*c**3*d**4 - 36*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*
c**2*d**5*x - 24*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**3*c**5*d**2 - 24*
sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**3*c**4*d**3*x + 48*sqrt(a*d**2 ...
```

3.125 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 268

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx &= -\frac{(2cC-Bd)\sqrt{a+bx^2}}{bd^3}+\frac{Cx\sqrt{a+bx^2}}{2bd^2}-\frac{c^2(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^3(bc^2+ad^2)(c+dx)} \\ &\quad-\frac{(aCd^2-2b(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}d^4} \\ &\quad+\frac{c(bc^2(3c^2C-2Bcd+Ad^2)+ad^2(4c^2C-3Bcd+2Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4(bc^2+ad^2)^{3/2}}\end{aligned}$$

output
```
-(-B*d+2*C*c)*(b*x^2+a)^(1/2)/b/d^3+1/2*C*x*(b*x^2+a)^(1/2)/b/d^2-c^2*(A*d
^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^3/(a*d^2+b*c^2)/(d*x+c)-1/2*(a*C*d^2-2*b
*(A*d^2-2*B*c*d+3*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^4+c
*(b*c^2*(A*d^2-2*B*c*d+3*C*c^2)+a*d^2*(2*A*d^2-3*B*c*d+4*C*c^2))*arctanh((
-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^4/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}\left(ad^2(c+dx)(-4cC+2Bd+Cdx)+bc^2\left(-6c^2C+cd(4B-3Cx)+d^2\left(-2A+2Bx+Cx^2\right)\right)\right)}{b(bc^2+ad^2)(c+dx)} - \frac{4c\left(bc^2\left(3c^2C-2Bcd+Ad^2\right)+ad^2\left(4c^2C-3\right.\right.}{(-bc^2-}}{2d^4}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*d^2*(c + d*x)*(-4*c*C + 2*B*d + C*d*x) + b*c^2*(-6*
c^2*C + c*d*(4*B - 3*C*x) + d^2*(-2*A + 2*B*x + C*x^2))))/(b*(b*c^2 + a*d^
2)*(c + d*x)) - (4*c*(b*c^2*(3*c^2*C - 2*B*c*d + A*d^2) + a*d^2*(4*c^2*C -
 3*B*c*d + 2*A*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x)
- c*Sqrt[a + b*x^2])])/(-(b*c^2) - a*d^2)^(3/2) + (2*(-(a*C*d^2) + 2*b*(3*
c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])
])/b^(3/2))/(2*d^4)
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2182, 2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}(c+dx)^2}\,dx$$
$$\Big\downarrow \; 2182$$
$$-\frac{\int \frac{-C\left(\frac{bc^2}{d}+ad\right)x^3+\frac{(cC-Bd)\left(bc^2+ad^2\right)x^2}{d^2}-\frac{\left(bc^2+ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{\int \frac{bd(5cC-2Bd)\left(bc^2+ad^2\right)x^2+\left(bc^2+ad^2\right)\left(aCd^2-b\left(Cc^2-2Bdc+2Ad^2\right)\right)x+acd\left(aCd^2+b\left(3Cc^2-2Bdc+2Ad^2\right)\right)}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}-\frac{C\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}{2bd^3}-}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{\frac{\int \frac{bd^2\left(acd\left(aCd^2+b\left(3Cc^2-2Bdc+2Ad^2\right)\right)+\left(bc^2+ad^2\right)\left(aCd^2-2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+\sqrt{a+bx^2}\left(ad^2+bc^2\right)(5cC-2Bd)}{2bd^3}-\frac{C\sqrt{a+bx^2}(c+dx)(}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{acd\left(aCd^2+b\left(3Cc^2-2Bdc+2Ad^2\right)\right)+\left(bc^2+ad^2\right)\left(aCd^2-2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}\left(ad^2+bc^2\right)(5cC-2Bd)}{2bd^3}-\frac{C\sqrt{a+bx^2}(c+dx)(ad^2+}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{719}$$

$$-\frac{\frac{\frac{\left(ad^2+bc^2\right)\left(aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}+\frac{2bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\sqrt{a+bx^2}(ad^2+}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{\frac{\left(ad^2+bc^2\right)\left(aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}+\frac{2bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\sqrt{a+b}}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{2bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}+\sqrt{a+b}}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 488

$$-\frac{\frac{-\frac{2bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(aCd^2-2b\left(Ad^2-2Bcd+\right.\right.}{\sqrt{b}d}}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

$\big\downarrow$ 219

$$-\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}-\frac{2bc\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2\right.\right.}{d\sqrt{ad^2+bc^2}}}{2bd^3}}{ad^2+bc^2}$$

$$\frac{c^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-((c^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d^3*(b*c^2 + a*d^2)*(c +
d*x))) - (-1/2*(C*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a + b*x^2])/(b*d^3) + ((5
*c*C - 2*B*d)*(b*c^2 + a*d^2)*Sqrt[a + b*x^2] + ((b*c^2 + a*d^2)*(a*C*d^2
- 2*b*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(
Sqrt[b]*d) - (2*b*c*(b*c^2*(3*c^2*C - 2*B*c*d + A*d^2) + a*d^2*(4*c^2*C -
3*B*c*d + 2*A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*
x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/(2*b*d^3))/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.76

method	result
risch	$\frac{(Cxd+2Bd-4Cc)\sqrt{b\,x^2+a}}{2b\,d^3}+\frac{\frac{\left(2Ab\,d^2-4Bbcd-aC\,d^2+6Cb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{2bc\left(2A\,d^2-3Bcd+4C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\right.}{d}}{}\right.}{d^2\sqrt{}}}{}$
default	$\frac{\frac{A\,d^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{d(Bd-2Cc)\sqrt{b\,x^2+a}}{b}+C\,d^2\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{3C\,c^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{2Bcd\ln\left(\sqrt{b}\,x+\sqrt{b}\right.}{\sqrt{b}}}{d^4}$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d-4*C*c)*(b*x^2+a)^(1/2)/b/d^3+1/2/b/d^3*((2*A*b*d^2-4*B*b*
c*d-C*a*d^2+6*C*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+2*b*c/d^2*(
2*A*d^2-3*B*c*d+4*C*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2
-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+2*b*c^2*(A*d^2-B*c*d+C*c^2)/d^3*(-1/(a*
d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/
2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2
*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(247) = 494$.

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.28

$$\int \frac{x^2(A + Bx + Cx^2)}{(c + dx)^2\sqrt{a + bx^2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
-sqrt(b*x^2 + a)*C*c^4/(b*c^2*d^4*x + a*d^6*x + b*c^3*d^3 + a*c*d^5) + sqr
t(b*x^2 + a)*B*c^3/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) - sqrt(b*
x^2 + a)*A*c^2/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) + 1/2*sqrt(b*x^
2 + a)*C*x/(b*d^2) + 3*C*c^2*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^4) - 2*B*c*
arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^3) - 1/2*C*a*arcsinh(b*x/sqrt(a*b))/(b^(
3/2)*d^2) + A*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) + C*b*c^5*arcsinh(b*c*x
/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)
^(3/2)*d^7) - B*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a
*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^6) - 4*C*c^3*arcsinh(b*c*x/(sq
rt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)
*d^5) + A*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*ab
s(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) + 3*B*c^2*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d^2)*d^4)
- 2*A*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x +
c)))/(sqrt(a + b*c^2/d^2)*d^3) - 2*sqrt(b*x^2 + a)*C*c/(b*d^3) + sqrt(b*x^
2 + a)*B/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^2(A + Bx + Cx^2)}{(c + dx)^2\sqrt{a + bx^2}}\, dx = \text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\int \frac{x^2\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}\,(c+d\,x)^2}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 2568, normalized size of antiderivative = 9.58

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\text{Too large to display}$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*b**2*c**2*d**4 + 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**5*x + 4*sqrt(a
*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*
x)*a*b**3*c**4*d**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d**3*x - 12*sqrt(a*d**2 + b*c
**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c
**3*d**3 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**4*x + 16*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**5*d**2 +
 16*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b**2*c**4*d**3*x - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4*c**5*d - 8*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**4
*c**4*d**2*x + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*b**3*c**7 + 12*sqrt(a*d**2 + b*c**2)*log( - sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**6*d*x - 8*sqrt(
a*d**2 + b*c**2)*log(c + d*x)*a**2*b**2*c**2*d**4 - 8*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a**2*b**2*c*d**5*x - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a
*b**3*c**4*d**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**3*c**3*d**3...
```

3.126 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 204

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{C\sqrt{a+bx^2}}{bd^2}+\frac{c(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^2(bc^2+ad^2)(c+dx)}-\frac{(2cC-Bd)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d^3}-\frac{(bc^3(2cC-Bd)+ad^2(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3(bc^2+ad^2)^{3/2}}$$

output
```
C*(b*x^2+a)^(1/2)/b/d^2+c*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^2/(a*d^2+b
*c^2)/(d*x+c)-(-B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^3-
(b*c^3*(-B*d+2*C*c)+a*d^2*(A*d^2-2*B*c*d+3*C*c^2))*arctanh((-b*c*x+a*d)/(a
*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.05

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{d\sqrt{a+bx^2}(aCd^2(c+dx)+bc(2c^2C-Bcd+Ad^2+cCdx))}{b(bc^2+ad^2)(c+dx)} + \frac{2(bc^3(2cC-Bd)+ad^2(3c^2C-2Bcd+Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}} + \frac{(2cC}{}}{d^3}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((d*Sqrt[a + b*x^2]*(a*C*d^2*(c + d*x) + b*c*(2*c^2*C - B*c*d + A*d^2 + c*
C*d*x)))/(b*(b*c^2 + a*d^2)*(c + d*x)) + (2*(b*c^3*(2*c*C - B*d) + a*d^2*(
3*c^2*C - 2*B*c*d + A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])
/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + ((2*c*C - B*d)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d^3
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2182, 2185, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\sqrt{a+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)} - \frac{\int \frac{-C\left(\frac{bc^2}{d}+ad\right)x^2+\frac{(cC-Bd)\left(bc^2+ad^2\right)x}{d^2}+a\left(-\frac{Cc^2}{d}+Bc-Ad\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{\int -\frac{b\left(ad\left(Cc^2-Bdc+Ad^2\right)-(2cC-Bd)\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}--\frac{\int \frac{b\left(ad\left(Cc^2-Bdc+Ad^2\right)-(2cC-Bd)\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}--\frac{\int \frac{ad\left(Cc^2-Bdc+Ad^2\right)-(2cC-Bd)\left(bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 719$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}--\frac{\frac{\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^3(2cC-Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+bc^2\right)(2cC-Bd)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}--\frac{\frac{\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^3(2cC-Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\left(ad^2+bc^2\right)(2cC-Bd)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}--\frac{\frac{\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^3(2cC-Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\frac{c^2}{d^2}\right)}{ad^2+bc^2}$$

$$\Big\downarrow \; 488$$

$$\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{-\frac{\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^3(2cC-Bd)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\right.}{ad^2+bc^2}$$

$$\Big\downarrow \; 219$$

$$\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2(c+dx)\left(ad^2+bc^2\right)}-\frac{-\frac{-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(Ad^2-2Bcd+3c^2C\right)+bc^3(2cC-Bd)\right)}{d\sqrt{ad^2+bc^2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}}{d^2}-C\sqrt{a+bx^2}\left(\frac{a}{b}+\right.}{ad^2+bc^2}$$

input

```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output

```
(c*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)*(c + d*x)
) - (-(C*(a/b + c^2/d^2)*Sqrt[a + b*x^2]) - (-(((2*c*C - B*d)*(b*c^2 + a*d
^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - ((b*c^3*(2*c*C -
B*d) + a*d^2*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^
2 + a*d^2]*Sqrt[a + b*x^2])])/(d*Sqrt[b*c^2 + a*d^2]))/d^2)/(b*c^2 + a*d^2
)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(188) = 376$.

Time = 0.24 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.11

method	result
risch	$\frac{C\sqrt{b\,x^2+a}}{b\,d^2} + \frac{\frac{(Bd-2Cc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}} - \frac{\left(A\,d^2-2Bcd+3C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+}}{x+\frac{c}{d}}\right.}{d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}}{}$
default	$\frac{\frac{Bd\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+\frac{Cd\sqrt{b\,x^2+a}}{b}-\frac{2Cc\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}}{d^3} - \frac{\left(A\,d^2-2Bcd+3C\,c^2\right)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b}}{x+\frac{c}{d}}\right.}{d^4\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C*(b*x^2+a)^(1/2)/b/d^2+1/d^2*((B*d-2*C*c)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
/b^(1/2)-1/d^2*(A*d^2-2*B*c*d+3*C*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*
d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*
b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))-c*(A*d^2-B*c*d+C*c^2)/d^3
*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/
d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^
2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(189) = 378$.

Time = 0.09 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.56

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Cc^3}{bc^2d^3x+ad^5x+bc^3d^2+acd^4} - \frac{\sqrt{bx^2+a}Bc^2}{bc^2d^2x+ad^4x+bc^3d+acd^3} + \frac{\sqrt{bx^2+a}Ac}{bc^2dx+ad^3x+bc^3+acd^2} - \frac{2\,Cc\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^3} + \frac{B\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2} - \frac{Cbc^4\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^6} + \frac{Bbc^3\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5} + \frac{3\,Cc^2\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^4} - \frac{Abc^2\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} - \frac{2\,Bc\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3} + \frac{A\,\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2} + \frac{\sqrt{bx^2+a}C}{bd^2}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*C*c^3/(b*c^2*d^3*x + a*d^5*x + b*c^3*d^2 + a*c*d^4) - sqrt
(b*x^2 + a)*B*c^2/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) + sqrt(b*x^2
 + a)*A*c/(b*c^2*d*x + a*d^3*x + b*c^3 + a*c*d^2) - 2*C*c*arcsinh(b*x/sqrt
(a*b))/(sqrt(b)*d^3) + B*arcsinh(b*x/sqrt(a*b))/(sqrt(b)*d^2) - C*b*c^4*ar
csinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a +
 b*c^2/d^2)^(3/2)*d^6) + B*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) -
a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) + 3*C*c^2*arcsin
h(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a +
 b*c^2/d^2)*d^4) - A*b*c^2*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(s
qrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^4) - 2*B*c*arcsinh(b*c*x/
(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a + b*c^2/d
^2)*d^3) + A*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/(sqrt(a + b*c^2/d^2)*d^2) + sqrt(b*x^2 + a)*C/(b*d^2)
```

Giac [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\int \frac{x\,(C\,x^2+B\,x+A)}{\sqrt{b\,x^2+a}\,{(c+d\,x)}^2}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1717, normalized size of antiderivative = 8.42

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**2*b*c*d**4 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**5*x - 4*sqrt(a*d**2 + b*c**2)*l
og(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d**3
- 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b**2*c*d**4*x + 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**4*d**2 + 6*sqrt(a*d**2 + b*c**2
)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**3*
x - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*b**3*c**4*d - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*b**3*c**3*d**2*x + 4*sqrt(a*d**2 + b*c**
2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**6 + 4
*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*b**2*c**5*d*x - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**4
- 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*d**5*x + 4*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a*b**2*c**2*d**3 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*
a*b**2*c*d**4*x - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**4*d**2 - 6*s
qrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**3*x + 2*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*b**3*c**4*d + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**3*c**
3*d**2*x - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**6 - 4*sqrt(a*d*...
```

3.127 $\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 168

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d(bc^2+ad^2)(c+dx)}+\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d^2}+\frac{(ad^2(2cC-Bd)+b(c^3C-Acd^2))\arctan\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2(bc^2+ad^2)^{3/2}}$$

output
```
-(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d/(a*d^2+b*c^2)/(d*x+c)+C*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)/d^2+(a*d^2*(-B*d+2*C*c)+b*(-A*c*d^2+C*c^3)
)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^2/(a*d^2+b*c
^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{\frac{d(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{(bc^2+ad^2)(c+dx)}+\frac{2(ad^2(2cC-Bd)+b(c^3C-Acd^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}+\frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}}{d^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-(((d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x))
 + (2*(a*d^2*(2*c*C - B*d) + b*(c^3*C - A*c*d^2))*ArcTan[(Sqrt[b]*(c + d*x
) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) +
 (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/d^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {2182, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\int -\frac{Abc-aCc+aBd+C\left(\frac{bc^2}{d}+ad\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Abc-aCc+aBd+C\left(\frac{bc^2}{d}+ad\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 719$$

$$\frac{\left(aBd-2acC+Abc-\frac{bc^3C}{d^2}\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx+\frac{C(ad^2+bc^2)\int \frac{1}{\sqrt{bx^2+a}}dx}{d^2}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 224$$

$$\frac{\left(aBd-2acC+Abc-\frac{bc^3C}{d^2}\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx+\frac{C(ad^2+bc^2)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d^2}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(aBd-2acC+Abc-\frac{bc^3C}{d^2}\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)}{\sqrt{b}d^2}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)}{\sqrt{b}d^2}-\left(aBd-2acC+Abc-\frac{bc^3C}{d^2}\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)}{\sqrt{b}d^2}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(aBd-2acC+Abc-\frac{bc^3C}{d^2}\right)}{\sqrt{ad^2+bc^2}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-(((c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d*(b*c^2 + a*d^2)*(c + d*x)))
 + ((C*(b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d^2)
 - ((A*b*c - 2*a*c*C - (b*c^3*C)/d^2 + a*B*d)*ArcTanh[(a*d - b*c*x)/(Sqrt[
b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/Sqrt[b*c^2 + a*d^2])/(b*c^2 + a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(154) = 308$.

Time = 0.19 (sec), antiderivative size = 390, normalized size of antiderivative = 2.32

method	result
default	$\frac{C \ln\left(\sqrt{b}\, x+\sqrt{b\, x^{2}+a}\right)}{d^{2} \sqrt{b}}-\frac{(B d -2 C c) \ln\left(\frac{\frac{2 a \,d^{2}+2 b \,c^{2}}{d^{2}}-\frac{2 b c \left(x +\frac{c}{d}\right)}{d}+2 \sqrt{\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}\, \sqrt{b \left(x +\frac{c}{d}\right)^{2}-\frac{2 b c \left(x +\frac{c}{d}\right)}{d}+\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}}{x +\frac{c}{d}}\right)}{d^{3} \sqrt{\frac{a \,d^{2}+b \,c^{2}}{d^{2}}}}+$ $(A d^{2}-B$

input
```
int((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C/d^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-1/d^3*(B*d-2*C*c)/((a*d^2+b*c^
2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2
)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))+1/
d^4*(A*d^2-B*c*d+C*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d
*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^
(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(155) = 310$.

Time = 0.08 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.49

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx = &-\frac{\sqrt{bx^2+a}Cc^2}{bc^2d^2x+ad^4x+bc^3d+acd^3}+\frac{\sqrt{bx^2+a}Bc}{bc^2dx+ad^3x+bc^3+acd^2}\\ &-\frac{\sqrt{bx^2+a}A}{bc^2x+ad^2x+\frac{bc^3}{d}+acd}+\frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}d^2}\\ &+\frac{Cbc^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5}\\ &-\frac{Bbc^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4}\\ &-\frac{2\,Cc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^3}\\ &+\frac{Abc\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}\\ &+\frac{B\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\sqrt{a+\frac{bc^2}{d^2}}d^2}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)*C*c^2/(b*c^2*d^2*x + a*d^4*x + b*c^3*d + a*c*d^3) + sqrt(
b*x^2 + a)*B*c/(b*c^2*d*x + a*d^3*x + b*c^3 + a*c*d^2) - sqrt(b*x^2 + a)*A
/(b*c^2*x + a*d^2*x + b*c^3/d + a*c*d) + C*arcsinh(b*x/sqrt(a*b))/(sqrt(b)
*d^2) + C*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*ab
s(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) - B*b*c^2*arcsinh(b*c*x/(sqrt(a*b
)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^4
) - 2*C*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x
+ c)))/(sqrt(a + b*c^2/d^2)*d^3) + A*b*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
+ c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^3) + B*arcs
inh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/(sqrt(a
 + b*c^2/d^2)*d^2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(c + dx)^2\sqrt{a + bx^2}}\,dx = \int \frac{C\,x^2 + B\,x + A}{\sqrt{b\,x^2 + a}\,{(c + d\,x)}^2}\,dx$$

input
```
int((A + B*x + C*x^2)/((a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/((a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1148, normalized size of antiderivative = 6.83

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a*b**2*c**2*d**2 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x - 2*sqrt(a*d*
*2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*
a*b**2*c*d**3 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**
2 + b*c**2) - a*d + b*c*x)*a*b**2*d**4*x + 4*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d**2 + 4*sq
rt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a*b*c**2*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*s
qrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**5 + 2*sqrt(a*d**2 + b*c**2)*lo
g( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**4*d*x +
 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**2*d**2 + 2*sqrt(a*d**2 + b
*c**2)*log(c + d*x)*a*b**2*c*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*a*b**2*c*d**3 + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*d**4*x - 4*sq
rt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d**2 - 4*sqrt(a*d**2 + b*c**2)*l
og(c + d*x)*a*b*c**2*d**3*x - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c*
*5 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4*d*x - 2*sqrt(a + b*x**
2)*a**2*b*d**5 - 2*sqrt(a + b*x**2)*a*b**2*c**2*d**3 + 2*sqrt(a + b*x**2)*
a*b**2*c*d**4 - 2*sqrt(a + b*x**2)*a*b*c**3*d**3 + 2*sqrt(a + b*x**2)*b**3
*c**3*d**2 - 2*sqrt(a + b*x**2)*b**2*c**5*d - sqrt(b)*log(sqrt(a + b*x*...
```

3.128 $\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 167

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{\left(c^2C-Bcd+Ad^2\right)\sqrt{a+bx^2}}{c\left(bc^2+ad^2\right)(c+dx)} - \frac{\left(bc^2(Bc-2Ad)+ad(c^2C-Ad^2)\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2\left(bc^2+ad^2\right)^{3/2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

output
```
(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c/(a*d^2+b*c^2)/(d*x+c)-(b*c^2*(-2*A*d
+B*c)+a*d*(-A*d^2+C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+
a)^(1/2))/c^2/(a*d^2+b*c^2)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/
2)/c^2
```

Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \frac{\frac{c(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{(bc^2+ad^2)(c+dx)}+\frac{2(bc^2(Bc-2Ad)+ad(c^2C-Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}+\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((c*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) +
 (2*(b*c^2*(B*c - 2*A*d) + a*d*(c^2*C - A*d^2))*ArcTan[(Sqrt[b]*(c + d*x)
- d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + (
2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a])/c^2
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2351, 617, 679, 488, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{a+bx^2}(c+dx)^2}dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{1}{x(c+dx)^2\sqrt{bx^2+a}}dx+\int \frac{B+Cx}{(c+dx)^2\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \text{617}$$

$$A\int \left(-\frac{d}{c^2(c+dx)\sqrt{bx^2+a}}-\frac{d}{c(c+dx)^2\sqrt{bx^2+a}}+\frac{1}{c^2x\sqrt{bx^2+a}}\right)dx+\int \frac{B+Cx}{(c+dx)^2\sqrt{bx^2+a}}dx$$

$\Big\downarrow$ 679

$$A\int \left(-\frac{d}{c^2(c+dx)\sqrt{bx^2+a}}-\frac{d}{c(c+dx)^2\sqrt{bx^2+a}}+\frac{1}{c^2x\sqrt{bx^2+a}}\right)dx+\frac{(aCd+bBc)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}(cC-Bd)}{(c+dx)\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 488

$$A\int \left(-\frac{d}{c^2(c+dx)\sqrt{bx^2+a}}-\frac{d}{c(c+dx)^2\sqrt{bx^2+a}}+\frac{1}{c^2x\sqrt{bx^2+a}}\right)dx-\frac{(aCd+bBc)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}+\frac{\sqrt{a+bx^2}(cC-Bd)}{(c+dx)\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 219

$$A\int \left(-\frac{d}{c^2(c+dx)\sqrt{bx^2+a}}-\frac{d}{c(c+dx)^2\sqrt{bx^2+a}}+\frac{1}{c^2x\sqrt{bx^2+a}}\right)dx-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)(aCd+bBc)}{\left(ad^2+bc^2\right)^{3/2}}+\frac{\sqrt{a+bx^2}(cC-Bd)}{(c+dx)\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2009

$$A\left(\frac{d\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\sqrt{ad^2+bc^2}}+\frac{bd\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{d^2\sqrt{a+bx^2}}{c(c+dx)\left(ad^2+bc^2\right)}\right)-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)(aCd+bBc)}{\left(ad^2+bc^2\right)^{3/2}}+\frac{\sqrt{a+bx^2}(cC-Bd)}{(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((c*C - B*d)*Sqrt[a + b*x^2])/((b*c^2 + a*d^2)*(c + d*x)) - ((b*B*c + a*C*
d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 +
a*d^2)^(3/2) + A*((d^2*Sqrt[a + b*x^2])/(c*(b*c^2 + a*d^2)*(c + d*x)) + (b
*d*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 +
a*d^2)^(3/2) + (d*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])])/(c^2*Sqrt[b*c^2 + a*d^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(Sqrt[a
]*c^2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(153) = 306$.

Time = 0.22 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.44

method	result
default	$-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c^2\sqrt{a}}+\frac{(A\,d^2-C\,c^2)\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\,\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{x+\frac{c}{d}}\right)}{c^2d^2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}-\frac{(A}{}$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A/c^2/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)+(A*d^2-C*c^2)/c^2/d^2
/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d
^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)
)/(x+c/d))-(A*d^2-B*c*d+C*c^2)/d^3/c*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2
+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2
)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(154) = 308$.

Time = 7.93 (sec) , antiderivative size = 1585, normalized size of antiderivative = 9.49

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((B*a*b*c^4 - A*a^2*c*d^3 + (C*a^2 - 2*A*a*b)*c^3*d + (B*a*b*c^3*d -
 A*a^2*d^4 + (C*a^2 - 2*A*a*b)*c^2*d^2)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*
c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a
*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - (A*b^2*c
^5 + 2*A*a*b*c^3*d^2 + A*a^2*c*d^4 + (A*b^2*c^4*d + 2*A*a*b*c^2*d^3 + A*a^
2*d^5)*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
(C*a*b*c^5 - B*a*b*c^4*d - B*a^2*c^2*d^3 + A*a^2*c*d^4 + (C*a^2 + A*a*b)*c
^3*d^2)*sqrt(b*x^2 + a))/(a*b^2*c^7 + 2*a^2*b*c^5*d^2 + a^3*c^3*d^4 + (a*b
^2*c^6*d + 2*a^2*b*c^4*d^3 + a^3*c^2*d^5)*x), -1/2*(2*(B*a*b*c^4 - A*a^2*c
*d^3 + (C*a^2 - 2*A*a*b)*c^3*d + (B*a*b*c^3*d - A*a^2*d^4 + (C*a^2 - 2*A*a
*b)*c^2*d^2)*x)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x -
a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) - (A*b
^2*c^5 + 2*A*a*b*c^3*d^2 + A*a^2*c*d^4 + (A*b^2*c^4*d + 2*A*a*b*c^2*d^3 +
A*a^2*d^5)*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(C*a*b*c^5 - B*a*b*c^4*d - B*a^2*c^2*d^3 + A*a^2*c*d^4 + (C*a^2 + A*a*
b)*c^3*d^2)*sqrt(b*x^2 + a))/(a*b^2*c^7 + 2*a^2*b*c^5*d^2 + a^3*c^3*d^4 +
(a*b^2*c^6*d + 2*a^2*b*c^4*d^3 + a^3*c^2*d^5)*x), 1/2*(2*(A*b^2*c^5 + 2*A*
a*b*c^3*d^2 + A*a^2*c*d^4 + (A*b^2*c^4*d + 2*A*a*b*c^2*d^3 + A*a^2*d^5)*x)
*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (B*a*b*c^4 - A*a^2*c*d^3 +
(C*a^2 - 2*A*a*b)*c^3*d + (B*a*b*c^3*d - A*a^2*d^4 + (C*a^2 - 2*A*a*b)*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)^2x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x\,\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1093, normalized size of antiderivative = 6.54

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**2*c*d**3 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*d**4*x + 4*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**3*d +
 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a*b*c**2*d**2*x - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x*
*2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**4*d - 2*sqrt(a*d**2 + b*c**2
)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*c**3*d**2
*x - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
 - a*d + b*c*x)*b**2*c**4 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2
)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*b**2*c**3*d*x - 2*sqrt(a*d**2 + b*c
**2)*log(c + d*x)*a**2*c*d**3 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*
d**4*x - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b*c**3*d - 4*sqrt(a*d**2 +
 b*c**2)*log(c + d*x)*a*b*c**2*d**2*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*
x)*a*c**4*d + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*c**3*d**2*x + 2*sqrt(
a*d**2 + b*c**2)*log(c + d*x)*b**2*c**4 + 2*sqrt(a*d**2 + b*c**2)*log(c +
d*x)*b**2*c**3*d*x + 2*sqrt(a + b*x**2)*a**2*c*d**4 + 2*sqrt(a + b*x**2)*a
*b*c**3*d**2 - 2*sqrt(a + b*x**2)*a*b*c**2*d**3 + 2*sqrt(a + b*x**2)*a*c**
4*d**2 - 2*sqrt(a + b*x**2)*b**2*c**4*d + 2*sqrt(a + b*x**2)*b*c**6 + sqrt
(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**2*c*d**4 + sqrt(a)*log(sqrt(a + ...
```

3.129 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 206

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ac^2x}-\frac{d(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{c^2(bc^2+ad^2)(c+dx)}\\ &\quad+\frac{(ad^3(Bc-2Ad)-bc^2(c^2C-2Bcd+3Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3(bc^2+ad^2)^{3/2}}\\ &\quad-\frac{(Bc-2Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}c^3}\end{aligned}$$

output

```
-A*(b*x^2+a)^(1/2)/a/c^2/x-d*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c^2/(a*d^
2+b*c^2)/(d*x+c)+(a*d^3*(-2*A*d+B*c)-b*c^2*(3*A*d^2-2*B*c*d+C*c^2))*arctan
h((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^3/(a*d^2+b*c^2)^(3/2
)-(-2*A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)/c^3
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\frac{c\sqrt{a+bx^2}\left(acd(cC-Bd)x+Abc^2(c+dx)+aAd^2(c+2dx)\right)}{a(bc^2+ad^2)x(c+dx)}+\frac{2\left(ad^3(-Bc+2Ad)+bc^2\left(c^2C-2Bcd+3Ad^2\right)\right)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}+}{c^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
(-((c*Sqrt[a + b*x^2]*(a*c*d*(c*C - B*d)*x + A*b*c^2*(c + d*x) + a*A*d^2*(
c + 2*d*x)))/(a*(b*c^2 + a*d^2)*x*(c + d*x))) + (2*(a*d^3*(-(B*c) + 2*A*d)
 + b*c^2*(c^2*C - 2*B*c*d + 3*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a
 + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + (2*(B*c - 2
*A*d)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a])/c^3
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{a+bx^2}(c+dx)^2}\,dx$$
$$\Big\downarrow \text{2353}$$
$$\int \left(\frac{Bc-2Ad}{c^3x\sqrt{a+bx^2}}-\frac{d(Bc-2Ad)}{c^3\sqrt{a+bx^2}(c+dx)}+\frac{Ad^2-Bcd+c^2C}{c^2\sqrt{a+bx^2}(c+dx)^2}+\frac{A}{c^2x^2\sqrt{a+bx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{\sqrt{a}c^3}-\frac{b(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c\,(ad^2+bc^2)^{3/2}}+\frac{d(Bc-2Ad)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3\sqrt{ad^2+bc^2}}-\frac{d\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^2(c+dx)\,(ad^2+bc^2)}-\frac{A\sqrt{a+bx^2}}{ac^2x}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*c^2*x)) - (d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x
^2])/(c^2*(b*c^2 + a*d^2)*(c + d*x)) + (d*(B*c - 2*A*d)*ArcTanh[(a*d - b*c
*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^3*Sqrt[b*c^2 + a*d^2]) - (b
*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a
 + b*x^2])])/(c*(b*c^2 + a*d^2)^(3/2)) - ((B*c - 2*A*d)*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(Sqrt[a]*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(190) = 380$.

Time = 0.31 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.09

method	result
default	$-\frac{A\sqrt{b\,x^{2}+a}}{a\,c^{2}x}+\frac{(2Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{c^{3}\sqrt{a}}+\frac{(A\,d^{2}-Bcd+C\,c^{2})\left(-\frac{d^{2}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}{\left(a\,d^{2}+b\,c^{2}\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{2a\,d^{2}+}{d^{2}}\right.}{}\right.}{c^{2}d^{2}}$ $\frac{\left(-A\,d^{2}+Bcd-C\,c^{2}\right)\left(-\frac{d^{2}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}{\left(a\,d^{2}+b\,c^{2}\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{\frac{2a\,d^{2}+2b\,c^{2}}{d^{2}}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}\,\sqrt{b\left(x+\right.}}{x+\frac{c}{d}}\right.}{\left(a\,d^{2}+b\,c^{2}\right)\sqrt{\frac{a\,d^{2}+b\,c^{2}}{d^{2}}}}\right.}{d^{2}}$
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{a\,c^{2}x}-$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^2+a)^(1/2)/a/c^2/x+(2*A*d-B*c)/c^3/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^
2+a)^(1/2))/x)+(A*d^2-B*c*d+C*c^2)/c^2/d^2*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/(
(a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2
+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/
(x+c/d)))-(2*A*d-B*c)/c^3/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^
2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(191) = 382$.

Time = 11.54 (sec) , antiderivative size = 2210, normalized size of antiderivative = 10.73

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
[1/2*(sqrt(b*c^2 + a*d^2)*((C*a*b*c^4*d - 2*B*a*b*c^3*d^2 + 3*A*a*b*c^2*d^
3 - B*a^2*c*d^4 + 2*A*a^2*d^5)*x^2 + (C*a*b*c^5 - 2*B*a*b*c^4*d + 3*A*a*b*
c^3*d^2 - B*a^2*c^2*d^3 + 2*A*a^2*c*d^4)*x)*log((2*a*b*c*d*x - a*b*c^2 - 2
*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)
*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - ((B*b^2*c^5*d - 2*A*b^2*c^4
*d^2 + 2*B*a*b*c^3*d^3 - 4*A*a*b*c^2*d^4 + B*a^2*c*d^5 - 2*A*a^2*d^6)*x^2
+ (B*b^2*c^6 - 2*A*b^2*c^5*d + 2*B*a*b*c^4*d^2 - 4*A*a*b*c^3*d^3 + B*a^2*c
^2*d^4 - 2*A*a^2*c*d^5)*x)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) - 2*(A*b^2*c^6 + 2*A*a*b*c^4*d^2 + A*a^2*c^2*d^4 - (B*a*b*c^4
*d^2 + B*a^2*c^2*d^4 - 2*A*a^2*c*d^5 - (C*a*b + A*b^2)*c^5*d - (C*a^2 + 3*
A*a*b)*c^3*d^3)*x)*sqrt(b*x^2 + a))/((a*b^2*c^7*d + 2*a^2*b*c^5*d^3 + a^3*
c^3*d^5)*x^2 + (a*b^2*c^8 + 2*a^2*b*c^6*d^2 + a^3*c^4*d^4)*x), -1/2*(2*sqr
t(-b*c^2 - a*d^2)*((C*a*b*c^4*d - 2*B*a*b*c^3*d^2 + 3*A*a*b*c^2*d^3 - B*a^
2*c*d^4 + 2*A*a^2*d^5)*x^2 + (C*a*b*c^5 - 2*B*a*b*c^4*d + 3*A*a*b*c^3*d^2
- B*a^2*c^2*d^3 + 2*A*a^2*c*d^4)*x)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a
*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + ((B*b
^2*c^5*d - 2*A*b^2*c^4*d^2 + 2*B*a*b*c^3*d^3 - 4*A*a*b*c^2*d^4 + B*a^2*c*d
^5 - 2*A*a^2*d^6)*x^2 + (B*b^2*c^6 - 2*A*b^2*c^5*d + 2*B*a*b*c^4*d^2 - 4*A
*a*b*c^3*d^3 + B*a^2*c^2*d^4 - 2*A*a^2*c*d^5)*x)*sqrt(a)*log(-(b*x^2 + 2*s
qrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*b^2*c^6 + 2*A*a*b*c^4*d^2 + A...
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2\sqrt{a + bx^2}}\,dx = \int \frac{A + Bx + Cx^2}{x^2\sqrt{a + bx^2}\,(c + dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2\sqrt{a + bx^2}}\,dx = \int \frac{Cx^2 + Bx + A}{\sqrt{bx^2 + a}(dx + c)^2x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)^2*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2\sqrt{a + bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 1767, normalized size of antiderivative = 8.58

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output

```
(4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
+ b*c*x)*a**3*c*d**4*x + 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*d**5*x**2 + 6*sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**2
*x + 6*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**2*b*c**2*d**3*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**2*d**3*x - 2*sqrt(a
*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*
a**2*b*c*d**4*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d*x - 4*sqrt(a*d**2 + b*c**2)*log
(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d**2*x*
*2 + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a*b*c**6*x + 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**5*d*x**2 - 4*sqrt(a*d**2 + b*c**2
)*log(c + d*x)*a**3*c*d**4*x - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*d
**5*x**2 - 6*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**3*d**2*x - 6*sqr
t(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**2*d**3*x**2 + 2*sqrt(a*d**2 + b*
c**2)*log(c + d*x)*a**2*b*c**2*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log(c + d*
x)*a**2*b*c*d**4*x**2 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**4*d
*x + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c**3*d**2*x**2 - 2*sqr...
```

3.130 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 270

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{a+bx^2}}{2ac^2x^2}-\frac{(Bc-2Ad)\sqrt{a+bx^2}}{ac^3x}+\frac{d^2(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{c^3(bc^2+ad^2)(c+dx)} + \frac{d(ad^2(c^2C-2Bcd+3Ad^2)+bc^2(2c^2C-3Bcd+4Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4(bc^2+ad^2)^{3/2}} + \frac{(Abc^2-2ac^2C+4aBcd-6aAd^2)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c^4}$$

output
```
-1/2*A*(b*x^2+a)^(1/2)/a/c^2/x^2-(-2*A*d+B*c)*(b*x^2+a)^(1/2)/a/c^3/x+d^2*
(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c^3/(a*d^2+b*c^2)/(d*x+c)+d*(a*d^2*(3*
A*d^2-2*B*c*d+C*c^2)+b*c^2*(4*A*d^2-3*B*c*d+2*C*c^2))*arctanh((-b*c*x+a*d)
/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^4/(a*d^2+b*c^2)^(3/2)+1/2*(-6*A*a*
d^2+A*b*c^2+4*B*a*c*d-2*C*a*c^2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/
c^4
```

Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx$$

$$= \frac{\frac{c\sqrt{a+bx^2}(-2cx(bBc^2(c+dx)+ad^2(Bc-cCx+2Bdx))+A(bc^2(-c^2+3cdx+4d^2x^2)+ad^2(-c^2+3cdx+6d^2x^2)))}{a(bc^2+ad^2)x^2(c+dx)} - \frac{4d(ad^2(c^2C-2Bcd+3Ad^2)}{}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
((c*Sqrt[a + b*x^2]*(-2*c*x*(b*B*c^2*(c + d*x) + a*d^2*(B*c - c*C*x + 2*B*
d*x)) + A*(b*c^2*(-c^2 + 3*c*d*x + 4*d^2*x^2) + a*d^2*(-c^2 + 3*c*d*x + 6*
d^2*x^2))))/(a*(b*c^2 + a*d^2)*x^2*(c + d*x)) - (4*d*(a*d^2*(c^2*C - 2*B*c
*d + 3*A*d^2) + b*c^2*(2*c^2*C - 3*B*c*d + 4*A*d^2))*ArcTan[(Sqrt[b]*(c +
d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2
) + (12*A*d^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + (2
*c*(A*b*c - 2*a*c*C + 4*a*B*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sq
rt[a]])/a^(3/2))/(2*c^4)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{a+bx^2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{Bc-2Ad}{c^3x^2\sqrt{a+bx^2}}+\frac{3Ad^2-2Bcd+c^2C}{c^4x\sqrt{a+bx^2}}-\frac{d(3Ad^2-2Bcd+c^2C)}{c^4\sqrt{a+bx^2}(c+dx)}-\frac{d(Ad^2-Bcd+c^2C)}{c^3\sqrt{a+bx^2}(c+dx)^2}+\frac{A}{c^2x^3\sqrt{a+bx^2}}\right)$$

$$\downarrow \text{2009}$$

$$\frac{A b \operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c^2}+\frac{bd\left(Ad^2-Bcd+c^2C\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\left(ad^2+bc^2\right)^{3/2}}+\frac{d\left(3Ad^2-2Bcd+c^2C\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4\sqrt{ad^2+bc^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(3Ad^2-2Bcd+c^2C\right)}{\sqrt{a}c^4}-\frac{\sqrt{a+bx^2}(Bc-2Ad)}{ac^3x}+\frac{d^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c^3(c+dx)\left(ad^2+bc^2\right)}-\frac{A\sqrt{a+bx^2}}{2ac^2x^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(a*c^2*x^2) - ((B*c - 2*A*d)*Sqrt[a + b*x^2])/(a*
c^3*x) + (d^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c^3*(b*c^2 + a*d^2
)*(c + d*x)) + (b*d*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*
c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*(b*c^2 + a*d^2)^(3/2)) + (d*(c^2*C -
2*B*c*d + 3*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x
^2])])/(c^4*Sqrt[b*c^2 + a*d^2]) + (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2*a^(3/2)*c^2) - ((c^2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqr
t[a]])/(Sqrt[a]*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.77

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-4Adx+2Bcx+Ac)}{2a\,c^{3}x^{2}}-\frac{\frac{\left(6Aa\,d^{2}-bA\,c^{2}-4Bacd+2Ca\,c^{2}\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{c\sqrt{a}}+\frac{2a\left(A\,d^{2}-Bcd+C\,c^{2}\right)\left(-\frac{d^{2}\sqrt{b\left(x+\frac{c}{d}\right)^{2}-}}{\left(a\,d^{2}+\right.}\right.}{}}{}$
default	$\frac{A\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{c^{2}}+\frac{(2Ad-Bc)\sqrt{b\,x^{2}+a}}{c^{3}ax}-\frac{\left(3A\,d^{2}-2Bcd+C\,c^{2}\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{c^{4}\sqrt{a}}+\frac{\left(3A\,d^{2}-2B\right.}{}$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-4*A*d*x+2*B*c*x+A*c)/a/c^3/x^2-1/2/c^3/a*(1/c*(6*A*
a*d^2-A*b*c^2-4*B*a*c*d+2*C*a*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x)+2*a/d*(A*d^2-B*c*d+C*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c
^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^
2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-
2*a/c*(3*A*d^2-2*B*c*d+C*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2
)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. 2(250) = 500.

Time = 31.18 (sec) , antiderivative size = 3165, normalized size of antiderivative = 11.72

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^3\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)^2x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)^2*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 2620, normalized size of antiderivative = 9.70

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output

```
(12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
a*d + b*c*x)*a**3*c*d**5*x**2 + 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b
*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*d**6*x**3 + 16*sqrt(a*d**
2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a
**2*b*c**3*d**3*x**2 + 16*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**2*d**4*x**3 - 8*sqrt(a*d**2 +
 b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2
*b*c**2*d**4*x**2 - 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**5*x**3 + 4*sqrt(a*d**2 + b*c**2
)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*c**4*d
**3*x**2 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*c**3*d**4*x**3 - 12*sqrt(a*d**2 + b*c**2)*log(
- sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**4*d**2*x
**2 - 12*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a*b**2*c**3*d**3*x**3 + 8*sqrt(a*d**2 + b*c**2)*log( - s
qrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c**6*d*x**2 + 8*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a*b*c**5*d**2*x**3 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*c*
d**5*x**2 - 12*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*d**6*x**3 - 16*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c**3*d**3*x**2 - 16*sqrt(a*d**2 +...
```

3.131 $\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 338

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{3ac^2x^3}-\frac{(Bc-2Ad)\sqrt{a+bx^2}}{2ac^3x^2}\\ &+\frac{(2Abc^2-3ac^2C+6aBcd-9aAd^2)\sqrt{a+bx^2}}{3a^2c^4x}-\frac{d^3(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{c^4(bc^2+ad^2)(c+dx)}\\ &-\frac{d^2(ad^2(2c^2C-3Bcd+4Ad^2)+bc^2(3c^2C-4Bcd+5Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^5(bc^2+ad^2)^{3/2}}\\ &+\frac{(bc^2(Bc-2Ad)+2ad(2c^2C-3Bcd+4Ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}c^5}\end{aligned}$$

output
```
-1/3*A*(b*x^2+a)^(1/2)/a/c^2/x^3-1/2*(-2*A*d+B*c)*(b*x^2+a)^(1/2)/a/c^3/x^
2+1/3*(-9*A*a*d^2+2*A*b*c^2+6*B*a*c*d-3*C*a*c^2)*(b*x^2+a)^(1/2)/a^2/c^4/x
-d^3*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/c^4/(a*d^2+b*c^2)/(d*x+c)-d^2*(a*
d^2*(4*A*d^2-3*B*c*d+2*C*c^2)+b*c^2*(5*A*d^2-4*B*c*d+3*C*c^2))*arctanh((-b
*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^5/(a*d^2+b*c^2)^(3/2)+1/2
*(b*c^2*(-2*A*d+B*c)+2*a*d*(4*A*d^2-3*B*c*d+2*C*c^2))*arctanh((b*x^2+a)^(1
/2)/a^(1/2))/a^(3/2)/c^5
```

Mathematica [A] (verified)

Time = 3.42 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\frac{c\sqrt{a+bx^2}\left(-4Ab^2c^4x^2(c+dx)+abc^2(c+dx)\left(3cx(Bc+2cCx-4Bdx)+2A\left(c^2-3cdx+7d^2x^2\right)\right)+a^2d^2\left(2A\left(c^3-2c^2dx+6cd^2x^2+12d^3x^3\right)+3cx(2\right.\right.}{a^2(bc^2+ad^2)x^3(c+dx)}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^4*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
(-((c*Sqrt[a + b*x^2]*(-4*A*b^2*c^4*x^2*(c + d*x) + a*b*c^2*(c + d*x)*(3*c
*x*(B*c + 2*c*C*x - 4*B*d*x) + 2*A*(c^2 - 3*c*d*x + 7*d^2*x^2)) + a^2*d^2*
(2*A*(c^3 - 2*c^2*d*x + 6*c*d^2*x^2 + 12*d^3*x^3) + 3*c*x*(2*c*C*x*(c + 2*
d*x) + B*(c^2 - 3*c*d*x - 6*d^2*x^2)))))/(a^2*(b*c^2 + a*d^2)*x^3*(c + d*x
))) + (12*d^2*(a*d^2*(2*c^2*C - 3*B*c*d + 4*A*d^2) + b*c^2*(3*c^2*C - 4*B*
c*d + 5*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^
2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) - (48*A*d^3*ArcTanh[(Sqrt[b]*x - Sq
rt[a + b*x^2])/Sqrt[a]])/Sqrt[a] + (6*c*(b*c*(B*c - 2*A*d) + 2*a*d*(2*c*C
- 3*B*d))*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/(6*c
^5)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^4\sqrt{a+bx^2}(c+dx)^2}\,dx$$
$$\Big\downarrow \ 2353$$

$$\int \left(\frac{Bc-2Ad}{c^3x^3\sqrt{a+bx^2}}+\frac{d^2(4Ad^2-3Bcd+2c^2C)}{c^5\sqrt{a+bx^2}(c+dx)}-\frac{d(4Ad^2-3Bcd+2c^2C)}{c^5x\sqrt{a+bx^2}}+\frac{d^2(Ad^2-Bcd+c^2C)}{c^4\sqrt{a+bx^2}(c+dx)^2}+\frac{3Ad^2-}{c^4x^2}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{2a^{3/2}c^3}+\frac{2Ab\sqrt{a+bx^2}}{3a^2c^2x}-\frac{d^2(4Ad^2-3Bcd+2c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^5\sqrt{ad^2+bc^2}}+\frac{d\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(4Ad^2-3Bcd+2c^2C)}{\sqrt{a}c^5}-\frac{bd^2(Ad^2-Bcd+c^2C)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3(ad^2+bc^2)^{3/2}}-\frac{\sqrt{a+bx^2}(Bc-2Ad)}{2ac^3x^2}-\frac{\sqrt{a+bx^2}(3Ad^2-2Bcd+c^2C)}{ac^4x}-\frac{d^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{c^4(c+dx)(ad^2+bc^2)}-\frac{A\sqrt{a+bx^2}}{3ac^2x^3}$$

input
```
Int[(A + B*x + C*x^2)/(x^4*(c + d*x)^2*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(A*Sqrt[a + b*x^2])/(a*c^2*x^3) - ((B*c - 2*A*d)*Sqrt[a + b*x^2])/(2*
a*c^3*x^2) + (2*A*b*Sqrt[a + b*x^2])/(3*a^2*c^2*x) - ((c^2*C - 2*B*c*d + 3
*A*d^2)*Sqrt[a + b*x^2])/(a*c^4*x) - (d^3*(c^2*C - B*c*d + A*d^2)*Sqrt[a +
 b*x^2])/(c^4*(b*c^2 + a*d^2)*(c + d*x)) - (b*d^2*(c^2*C - B*c*d + A*d^2)*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^3*(b*c^2
+ a*d^2)^(3/2)) - (d^2*(2*c^2*C - 3*B*c*d + 4*A*d^2)*ArcTanh[(a*d - b*c*x)
/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^5*Sqrt[b*c^2 + a*d^2]) + (b*(B
*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*c^3) + (d*(2*c^2*
C - 3*B*c*d + 4*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(Sqrt[a]*c^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(18 A a\,d^{2} x^{2}-4 A b\,c^{2} x^{2}-12 B a c d\,x^{2}+6 C a\,c^{2} x^{2}-6 A a c d x+3 B a\,c^{2} x+2 A\,c^{2} a\right)}{6 a^{2} c^{4} x^{3}}+\frac{\frac{\left(8 A a\,d^{3}-2 A b\,c^{2} d-6 B a c\,d^{2}+B b\,c^{3}+4 C a\,c \cdots\right.}{c\sqrt{a}}}{\cdots}$
default	$\frac{A\left(-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b\sqrt{b\,x^{2}+a}}{3 a^{2} x}\right)}{c^{2}}-\frac{\left(2 A d-B c\right)\left(-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{c^{3}}-\frac{\left(3 A\,d^{2}-2 B c d+C\,c^{2}\right)\sqrt{b\,x^{2}+a}}{c^{4} a x}-\frac{d\left(4 A \cdots\right.}{\cdots}$

input
```
int((C*x^2+B*x+A)/x^4/(d*x+c)^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(18*A*a*d^2*x^2-4*A*b*c^2*x^2-12*B*a*c*d*x^2+6*C*a*c^
2*x^2-6*A*a*c*d*x+3*B*a*c^2*x+2*A*a*c^2)/a^2/c^4/x^3+1/2/c^4/a*(1/c*(8*A*a
*d^3-2*A*b*c^2*d-6*B*a*c*d^2+B*b*c^3+4*C*a*c^2*d)/a^(1/2)*ln((2*a+2*a^(1/2
)*(b*x^2+a)^(1/2))/x)+2*a*(A*d^2-B*c*d+C*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d
)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2
)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*
d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2
))/(x+c/d)))-2*a*d*(4*A*d^2-3*B*c*d+2*C*c^2)/c/((a*d^2+b*c^2)/d^2)^(1/2)*l
n((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c
/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(311) = 622$.

Time = 33.92 (sec) , antiderivative size = 3853, normalized size of antiderivative = 11.40

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^4\sqrt{a+bx^2}\,(c+dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**4/(d*x+c)**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**4*sqrt(a + b*x**2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}(dx+c)^2x^4}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)^2*x^4), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^4/(d*x+c)^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^4\,\sqrt{b\,x^2+a}\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x^4*(a + b*x^2)^(1/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 3070, normalized size of antiderivative = 9.08

$$\int \frac{A+Bx+Cx^2}{x^4(c+dx)^2\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^4/(d*x+c)^2/(b*x^2+a)^(1/2),x)
```

output
```
(48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a**4*c*d**6*x**3 + 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*d**7*x**4 + 60*sqrt(a*d**2 + b*
c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c**
3*d**4*x**3 + 60*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**3*b*c**2*d**5*x**4 - 36*sqrt(a*d**2 + b*c**2)*lo
g(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c**2*d**5*x
**3 - 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**3*b*c*d**6*x**4 + 24*sqrt(a*d**2 + b*c**2)*log(sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*c**4*d**4*x**3 + 24*sqrt
(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x
)*a**3*c**3*d**5*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**4*d**3*x**3 - 48*sqrt(a*d**2
 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*
b**2*c**3*d**4*x**4 + 36*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**6*d**2*x**3 + 36*sqrt(a*d**2 + b*
c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**
5*d**3*x**4 - 48*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**4*c*d**6*x**3 - 48*
sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**4*d**7*x**4 - 60*sqrt(a*d**2 + b*c**
2)*log(c + d*x)*a**3*b*c**3*d**4*x**3 - 60*sqrt(a*d**2 + b*c**2)*log(c ...
```

3.132 $\int \frac{x^3(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 221

$$
\begin{aligned}
\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx &= \frac{a(Abc-acC-aBd)}{b^3\sqrt{a+bx^2}} \\
&+ \frac{a(bBc+Abd-aCd)x}{b^3\sqrt{a+bx^2}} + \frac{(Abc-2a(cC+Bd))\sqrt{a+bx^2}}{b^3} \\
&- \frac{(7aCd-4b(Bc+Ad))x\sqrt{a+bx^2}}{8b^3} + \frac{Cdx^3\sqrt{a+bx^2}}{4b^2} \\
&+ \frac{(cC+Bd)\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{3a(5aCd-4b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}
\end{aligned}
$$

output

```
a*(A*b*c-B*a*d-C*a*c)/b^3/(b*x^2+a)^(1/2)+a*(A*b*d+B*b*c-C*a*d)*x/b^3/(b*x
^2+a)^(1/2)+(A*b*c-2*a*(B*d+C*c))*(b*x^2+a)^(1/2)/b^3-1/8*(7*a*C*d-4*b*(A*
d+B*c))*x*(b*x^2+a)^(1/2)/b^3+1/4*C*d*x^3*(b*x^2+a)^(1/2)/b^2+1/3*(B*d+C*c
)*(b*x^2+a)^(3/2)/b^3+3/8*a*(5*a*C*d-4*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.77

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx = \frac{-a^2(64cC+64Bd+45Cdx)+ab(4Bx(9c-8dx)+12A(4c+3dx)}{24}$$
$$-\frac{3a(5aCd-4b(Bc+Ad))\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{7/2}}$$

input
```
Integrate[(x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-(a^2*(64*c*C + 64*B*d + 45*C*d*x)) + a*b*(4*B*x*(9*c - 8*d*x) + 12*A*(4*
c + 3*d*x) - C*x^2*(32*c + 15*d*x)) + 2*b^2*x^2*(6*A*(2*c + d*x) + x*(C*x*
(4*c + 3*d*x) + B*(6*c + 4*d*x))))/(24*b^3*Sqrt[a + b*x^2]) - (3*a*(5*a*C*
d - 4*b*(B*c + A*d))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2176, 2346, 2346, 2346, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\int \frac{-aCdx^4-a(cC+Bd)x^3-a\left(Bc+\left(A-\frac{aC}{b}\right)d\right)x^2-\frac{a(Abc-a(cC+2Bd))x}{b}+\frac{a^2(bBc+Abd-aCd)}{b^2}}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{2346}$$

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\int\frac{-4ab(cC+Bd)x^3+a(7aCd-4b(Bc+Ad))x^2-4a(Abc-a(cC+2Bd))x+\frac{4a^2(bBc+Abd-aCd)}{b}}{\sqrt{bx^2+a}}dx}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 2346

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\int\frac{12(bBc+Abd-aCd)a^2+3b(7aCd-4b(Bc+Ad))x^2a-4b(3Abc-5aCc-8aBd)xa}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 2346

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{\int-\frac{ab(9a(5aCd-4b(Bc+Ad))+8b(3Abc-5aCc-8aBd)x)}{\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 25

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))-\frac{\int\frac{ab(9a(5aCd-4b(Bc+Ad))+8b(3Abc-5aCc-8aBd)x)}{\sqrt{bx^2+a}}dx}{2b}}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 27

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))-\frac{1}{2}a\int\frac{9a(5aCd-4b(Bc+Ad))+8b(3Abc-5aCc-8aBd)x}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 455

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))-\frac{1}{2}a\left(9a(5aCd-4b(Ad+Bc))\int\frac{1}{\sqrt{bx^2+a}}dx+8\sqrt{a+bx^2}(-8aBd-5acC+3Abc)\right)}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aCdx^3\sqrt{a+bx^2}}{4b}}{ab}$$

$\Big\downarrow$ 224

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))-\frac{1}{2}a\left(9a(5aCd-4b(Ad+Bc))\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8\sqrt{a+bx^2}(-8aBd-5acC+3Abc)\right)}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aC\cdots}{}}{ab}$$

$\Big\downarrow$ 219

$$\frac{a(c+dx)(-aC+Ab+bBx)}{b^3\sqrt{a+bx^2}}-\frac{\frac{\frac{\frac{3}{2}ax\sqrt{a+bx^2}(7aCd-4b(Ad+Bc))-\frac{1}{2}a\left(\frac{9a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(5aCd-4b(Ad+Bc))}{\sqrt{b}}+8\sqrt{a+bx^2}(-8aBd-5acC+3Abc)\right)}{3b}-\frac{4}{3}ax^2\sqrt{a+bx^2}(Bd+cC)}{4b}-\frac{aC\cdots}{}}{ab}$$

input
```
Int[(x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(a*(A*b - a*C + b*B*x)*(c + d*x))/(b^3*Sqrt[a + b*x^2]) - (-1/4*(a*C*d*x^3
*Sqrt[a + b*x^2])/b + ((-4*a*(c*C + B*d)*x^2*Sqrt[a + b*x^2])/3 + ((3*a*(7
*a*C*d - 4*b*(B*c + A*d))*x*Sqrt[a + b*x^2])/2 - (a*(8*(3*A*b*c - 5*a*c*C
- 8*a*B*d)*Sqrt[a + b*x^2] + (9*a*(5*a*C*d - 4*b*(B*c + A*d))*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/(3*b))/(4*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.94

method	result
risch	$\frac{\left(6Cbd\,x^{3}+8Bbd\,x^{2}+8Ccb\,x^{2}+12xAbd+12Bbcx-21Cadx+24Abc-40Bad-40Cac\right)\sqrt{b\,x^{2}+a}}{24b^{3}}-\frac{a\left(3b\left(4Abd+4Bbc-5aCd\right)\left(-\frac{}{b\sqrt{}}\right.\right.}{}$
default	$\left(Ad+Bc\right)\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+\left(Bd+Cc\right)\left(\frac{x^{4}}{3b\sqrt{b\,x^{2}+a}}-\frac{4a\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\right.}{3b}\right.$

input
```
int(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*b*d*x^3+8*B*b*d*x^2+8*C*b*c*x^2+12*A*b*d*x+12*B*b*c*x-21*C*a*d*x
+24*A*b*c-40*B*a*d-40*C*a*c)*(b*x^2+a)^(1/2)/b^3-1/8*a/b^3*(3*b*(4*A*b*d+4
*B*b*c-5*C*a*d)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/
2)))-8*(A*b*c-B*a*d-C*a*c)/(b*x^2+a)^(1/2)-7*C*a*d*x/(b*x^2+a)^(1/2)+4*A*b
*d*x/(b*x^2+a)^(1/2)+4*B*b*c*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.34

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{9\left(4\,Ba^2bc+\left(4\,Bab^2c-\left(5\,Ca^2b-4\,Aab^2\right)d\right)x^2-\left(5\,Ca^3-4\,Aa^2b\right)\right.}{}\right.$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(9*(4*B*a^2*b*c + (4*B*a*b^2*c - (5*C*a^2*b - 4*A*a*b^2)*d)*x^2 - (5
*C*a^3 - 4*A*a^2*b)*d)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(6*C*b^3*d*x^5 - 64*B*a^2*b*d + 8*(C*b^3*c + B*b^3*d)*x^4 + 3*(4*
B*b^3*c - (5*C*a*b^2 - 4*A*b^3)*d)*x^3 - 8*(4*B*a*b^2*d + (4*C*a*b^2 - 3*A
*b^3)*c)*x^2 - 16*(4*C*a^2*b - 3*A*a*b^2)*c + 9*(4*B*a*b^2*c - (5*C*a^2*b
- 4*A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/(b^5*x^2 + a*b^4), 1/24*(9*(4*B*a^2*b*
c + (4*B*a*b^2*c - (5*C*a^2*b - 4*A*a*b^2)*d)*x^2 - (5*C*a^3 - 4*A*a^2*b)*
d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*C*b^3*d*x^5 - 64*B*a^2
*b*d + 8*(C*b^3*c + B*b^3*d)*x^4 + 3*(4*B*b^3*c - (5*C*a*b^2 - 4*A*b^3)*d)
*x^3 - 8*(4*B*a*b^2*d + (4*C*a*b^2 - 3*A*b^3)*c)*x^2 - 16*(4*C*a^2*b - 3*A
*a*b^2)*c + 9*(4*B*a*b^2*c - (5*C*a^2*b - 4*A*a*b^2)*d)*x)*sqrt(b*x^2 + a)
)/(b^5*x^2 + a*b^4)]
```

Sympy [A] (verification not implemented)

Time = 12.03 (sec) , antiderivative size = 449, normalized size of antiderivative = 2.03

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = Ac\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Ad\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Bc\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Bd\left(\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Cc\left(\begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Cd\left(-\frac{15a^{\frac{3}{2}}x}{8b^3\sqrt{1+\frac{bx^2}{a}}} - \frac{5\sqrt{a}x^3}{8b^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}} + \frac{x^5}{4\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**3*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*c*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(3/2)), True)) + A*d*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*
x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*s
qrt(1 + b*x**2/a))) + B*c*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*a
sinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a
))) + B*d*Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*
sqrt(a + b*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3
/2)), True)) + C*c*Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2
/(3*b**2*sqrt(a + b*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6
/(6*a**(3/2)), True)) + C*d*(-15*a**(3/2)*x/(8*b**3*sqrt(1 + b*x**2/a)) -
5*sqrt(a)*x**3/(8*b**2*sqrt(1 + b*x**2/a)) + 15*a**2*asinh(sqrt(b)*x/sqrt(
a))/(8*b**(7/2)) + x**5/(4*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.16

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Cdx^5}{4\sqrt{bx^2+a}b} - \frac{5\,Cadx^3}{8\sqrt{bx^2+a}b^2} + \frac{(Cc+Bd)x^4}{3\sqrt{bx^2+a}b} + \frac{Acx^2}{\sqrt{bx^2+a}b} + \frac{(Bc+Ad)x^3}{2\sqrt{bx^2+a}b} - \frac{15\,Ca^2dx}{8\sqrt{bx^2+a}b^3} - \frac{4\,(Cc+Bd)ax^2}{3\sqrt{bx^2+a}b^2} + \frac{15\,Ca^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} + \frac{2\,Aac}{\sqrt{bx^2+a}b^2} + \frac{3\,(Bc+Ad)ax}{2\sqrt{bx^2+a}b^2} - \frac{3\,(Bc+Ad)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{8\,(Cc+Bd)a^2}{3\sqrt{bx^2+a}b^3}$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*C*d*x^5/(sqrt(b*x^2 + a)*b) - 5/8*C*a*d*x^3/(sqrt(b*x^2 + a)*b^2) + 1/
3*(C*c + B*d)*x^4/(sqrt(b*x^2 + a)*b) + A*c*x^2/(sqrt(b*x^2 + a)*b) + 1/2*
(B*c + A*d)*x^3/(sqrt(b*x^2 + a)*b) - 15/8*C*a^2*d*x/(sqrt(b*x^2 + a)*b^3)
 - 4/3*(C*c + B*d)*a*x^2/(sqrt(b*x^2 + a)*b^2) + 15/8*C*a^2*d*arcsinh(b*x/
sqrt(a*b))/b^(7/2) + 2*A*a*c/(sqrt(b*x^2 + a)*b^2) + 3/2*(B*c + A*d)*a*x/(
sqrt(b*x^2 + a)*b^2) - 3/2*(B*c + A*d)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) -
8/3*(C*c + B*d)*a^2/(sqrt(b*x^2 + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{3\,Cdx}{b} + \frac{4\,(Cb^5c+Bb^5d)}{b^6}\right)x + \frac{3\,(4\,Bb^5c-5\,Cab^4d+4\,Ab^5d)}{b^6}\right)x - \frac{8\,(4\,Cab^4c-}{}\right.\right.}{24\,\sqrt{}} + \frac{3\,(4\,Babc-5\,Ca^2d+4\,Aabd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

input
```
integrate(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*((((2*(3*C*d*x/b + 4*(C*b^5*c + B*b^5*d)/b^6)*x + 3*(4*B*b^5*c - 5*C*
a*b^4*d + 4*A*b^5*d)/b^6)*x - 8*(4*C*a*b^4*c - 3*A*b^5*c + 4*B*a*b^4*d)/b^
6)*x + 9*(4*B*a*b^4*c - 5*C*a^2*b^3*d + 4*A*a*b^4*d)/b^6)*x - 16*(4*C*a^2*
b^3*c - 3*A*a*b^4*c + 4*B*a^2*b^3*d)/b^6)/sqrt(b*x^2 + a) + 3/8*(4*B*a*b*c
 - 5*C*a^2*d + 4*A*a*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^3\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^3*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.40

$$\int \frac{x^3(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{27\sqrt{b}\,a^3bd-30\sqrt{b}\,a^3cd+27\sqrt{b}\,a^2b^2c+27\sqrt{b}\,a^2b^2d\,x^2+27\sqrt{b}\,a\,b^3c\,x}{}$$

input
```
int(x^3*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**2*b**2*c + 36*sqrt(a + b*x**2)*a**2*b**2*d*x - 64*
sqrt(a + b*x**2)*a**2*b**2*d - 64*sqrt(a + b*x**2)*a**2*b*c**2 - 45*sqrt(a
 + b*x**2)*a**2*b*c*d*x + 24*sqrt(a + b*x**2)*a*b**3*c*x**2 + 36*sqrt(a +
b*x**2)*a*b**3*c*x + 12*sqrt(a + b*x**2)*a*b**3*d*x**3 - 32*sqrt(a + b*x**
2)*a*b**3*d*x**2 - 32*sqrt(a + b*x**2)*a*b**2*c**2*x**2 - 15*sqrt(a + b*x*
*2)*a*b**2*c*d*x**3 + 12*sqrt(a + b*x**2)*b**4*c*x**3 + 8*sqrt(a + b*x**2)
*b**4*d*x**4 + 8*sqrt(a + b*x**2)*b**3*c**2*x**4 + 6*sqrt(a + b*x**2)*b**3
*c*d*x**5 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*
d + 45*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*c*d - 36*s
qrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c - 36*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d*x**2 + 45*sqrt(b
)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d*x**2 - 36*sqrt(b)
*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*c*x**2 + 27*sqrt(b)*a*
*3*b*d - 30*sqrt(b)*a**3*c*d + 27*sqrt(b)*a**2*b**2*c + 27*sqrt(b)*a**2*b*
*2*d*x**2 - 30*sqrt(b)*a**2*b*c*d*x**2 + 27*sqrt(b)*a*b**3*c*x**2)/(24*b**
4*(a + b*x**2))
```

3.133 $\int \frac{x^2(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 184

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{a(aCd-b(Bc+Ad))}{b^3\sqrt{a+bx^2}} - \frac{(Abc-acC-aBd)x}{b^2\sqrt{a+bx^2}} + \frac{(bBc+Abd-2aCd)\sqrt{a+bx^2}}{b^3} + \frac{(cC+Bd)x\sqrt{a+bx^2}}{2b^2} + \frac{Cd\left(a+bx^2\right)^{3/2}}{3b^3} + \frac{(2Abc-3a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-a*(a*C*d-b*(A*d+B*c))/b^3/(b*x^2+a)^(1/2)-(A*b*c-B*a*d-C*a*c)*x/b^2/(b*x^
2+a)^(1/2)+(A*b*d+B*b*c-2*C*a*d)*(b*x^2+a)^(1/2)/b^3+1/2*(B*d+C*c)*x*(b*x^
2+a)^(1/2)/b^2+1/3*C*d*(b*x^2+a)^(3/2)/b^3+1/2*(2*A*b*c-3*a*(B*d+C*c))*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.79

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx = \frac{-16a^2Cd+ab(12Bc+12Ad+9cCx+9Bdx-8Cdx^2)+b^2x(-6A(}{6b^3\sqrt{a+bx^2}}$$
$$+\frac{(2Abc-3a(cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

input
```
Integrate[(x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-16*a^2*C*d + a*b*(12*B*c + 12*A*d + 9*c*C*x + 9*B*d*x - 8*C*d*x^2) + b^2
*x*(-6*A*(c - d*x) + x*(6*B*c + 3*c*C*x + 3*B*d*x + 2*C*d*x^2)))/(6*b^3*Sq
rt[a + b*x^2]) + ((2*A*b*c - 3*a*(c*C + B*d))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a
] + Sqrt[a + b*x^2])])/b^(5/2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2176, 25, 2346, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}-\frac{\int -\frac{aCdx^3+a(cC+Bd)x^2+a\left(Bc+2\left(A-\frac{aC}{b}\right)d\right)x+\frac{a(Abc-aCc-aBd)}{b}}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aCdx^3+a(cC+Bd)x^2+a\left(Bc+2\left(A-\frac{aC}{b}\right)d\right)x+\frac{a(Abc-aCc-aBd)}{b}}{\sqrt{bx^2+a}}dx}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\int \frac{3ab(cC+Bd)x^2+a(3bBc+6Abd-8aCd)x+3a(Abc-aCc-aBd)}{\sqrt{bx^2+a}}dx}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2346}$$

$$\frac{\frac{\frac{\int \frac{ab(3(2Abc-3a(cC+Bd))+2(3bBc+6Abd-8aCd)x)}{\sqrt{bx^2+a}}dx}{2b}+\frac{3}{2}ax\sqrt{a+bx^2}(Bd+cC)}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{2}a\int \frac{3(2Abc-3a(cC+Bd))+2(3bBc+6Abd-8aCd)x}{\sqrt{bx^2+a}}dx+\frac{3}{2}ax\sqrt{a+bx^2}(Bd+cC)}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{\frac{\frac{1}{2}a\left(3(2Abc-3a(Bd+cC))\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}(-8aCd+6Abd+3bBc)}{b}\right)+\frac{3}{2}ax\sqrt{a+bx^2}(Bd+cC)}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{1}{2}a\left(3(2Abc-3a(Bd+cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{2\sqrt{a+bx^2}(-8aCd+6Abd+3bBc)}{b}\right)+\frac{3}{2}ax\sqrt{a+bx^2}(Bd+cC)}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{1}{2}a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2Abc-3a(Bd+cC))}{\sqrt{b}}+\frac{2\sqrt{a+bx^2}(-8aCd+6Abd+3bBc)}{b}\right)+\frac{3}{2}ax\sqrt{a+bx^2}(Bd+cC)}{3b}+\frac{aCdx^2\sqrt{a+bx^2}}{3b}}{ab}+\frac{(c+dx)\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((a*B - b*(A - (a*C)/b)*x)*(c + d*x))/(b^2*Sqrt[a + b*x^2]) + ((a*C*d*x^2*
Sqrt[a + b*x^2])/(3*b) + ((3*a*(c*C + B*d)*x*Sqrt[a + b*x^2])/2 + (a*((2*(
3*b*B*c + 6*A*b*d - 8*a*C*d)*Sqrt[a + b*x^2])/b + (3*(2*A*b*c - 3*a*(c*C +
 B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/(3*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.93

method	result
risch	$\frac{\left(2Cbd\,x^{2}+3Bbdx+3Ccxb+6Abd+6Bbc-10aCd\right)\sqrt{b\,x^{2}+a}}{6b^{3}}+\frac{b(2Abc-3Bad-3Cac)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{2a(Abd\cdots}{b\sqrt{\cdots}}}{2b^{2}}$
default	$(Ad+Bc)\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+(Bd+Cc)\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+\cdots$

input
```
int(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d*x^2+3*B*b*d*x+3*C*b*c*x+6*A*b*d+6*B*b*c-10*C*a*d)*(b*x^2+a)^(
1/2)/b^3+1/2/b^2*(b*(2*A*b*c-3*B*a*d-3*C*a*c)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3
/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+2*a*(A*b*d+B*b*c-C*a*d)/b/(b*x^2+a)^(1/
2)-B*a*d*x/(b*x^2+a)^(1/2)-C*a*c*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 411, normalized size of antiderivative = 2.23

$$\int \frac{x^2(c+dx)\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{3\,(3\,Ba^2d+(3\,Babd+(3\,Cab-2\,Ab^2)c)x^2+(3\,Ca^2-2\,Aab)c)\sqrt{b}}{}\right.$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(3*B*a^2*d + (3*B*a*b*d + (3*C*a*b - 2*A*b^2)*c)*x^2 + (3*C*a^2 -
 2*A*a*b)*c)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*C*b^2*d*x^4 + 12*B*a*b*c + 3*(C*b^2*c + B*b^2*d)*x^3 + 2*(3*B*b^2*c - (4
*C*a*b - 3*A*b^2)*d)*x^2 - 4*(4*C*a^2 - 3*A*a*b)*d + 3*(3*B*a*b*d + (3*C*a
*b - 2*A*b^2)*c)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/6*(3*(3*B*a^2*d
+ (3*B*a*b*d + (3*C*a*b - 2*A*b^2)*c)*x^2 + (3*C*a^2 - 2*A*a*b)*c)*sqrt(-b
)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*C*b^2*d*x^4 + 12*B*a*b*c + 3*(C*
b^2*c + B*b^2*d)*x^3 + 2*(3*B*b^2*c - (4*C*a*b - 3*A*b^2)*d)*x^2 - 4*(4*C*
a^2 - 3*A*a*b)*d + 3*(3*B*a*b*d + (3*C*a*b - 2*A*b^2)*c)*x)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.95

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx = Ac\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Ad\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + Bc\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + Bd\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cc\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cd\left(\begin{cases}-\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
 + A*d*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)),
 Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + B*c*Piecewise((2*a/(b**2*sqrt(a +
 b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True
)) + B*d*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sq
rt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*c*(3*sqrt
(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/
2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*d*Piecewise((-8*a**2/(3*b
**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b*x**2)) + x**4/(3*b*sqr
t(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Cdx^4}{3\sqrt{bx^2+a}b} - \frac{4\,Cadx^2}{3\sqrt{bx^2+a}b^2} + \frac{(Cc+Bd)x^3}{2\sqrt{bx^2+a}b} - \frac{Acx}{\sqrt{bx^2+a}b} + \frac{(Bc+Ad)x^2}{\sqrt{bx^2+a}b} + \frac{Ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{8\,Ca^2d}{3\sqrt{bx^2+a}b^3} + \frac{3\,(Cc+Bd)ax}{2\sqrt{bx^2+a}b^2} - \frac{3\,(Cc+Bd)a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{2\,(Bc+Ad)a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*C*d*x^4/(sqrt(b*x^2 + a)*b) - 4/3*C*a*d*x^2/(sqrt(b*x^2 + a)*b^2) + 1/
2*(C*c + B*d)*x^3/(sqrt(b*x^2 + a)*b) - A*c*x/(sqrt(b*x^2 + a)*b) + (B*c +
 A*d)*x^2/(sqrt(b*x^2 + a)*b) + A*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 8/3*C
*a^2*d/(sqrt(b*x^2 + a)*b^3) + 3/2*(C*c + B*d)*a*x/(sqrt(b*x^2 + a)*b^2) -
 3/2*(C*c + B*d)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*(B*c + A*d)*a/(sqrt(
b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{\left(\left(\left(\frac{2\,Cdx}{b} + \frac{3\,(Cb^4c+Bb^4d)}{b^5}\right)x + \frac{2\,(3\,Bb^4c-4\,Cab^3d+3\,Ab^4d)}{b^5}\right)x + \frac{3\,(3\,Cab^3c-2\,Ab}{b^5}\right.}{6\sqrt{bx^2+a}} + \frac{(3\,Cac-2\,Abc+3\,Bad)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*C*d*x/b + 3*(C*b^4*c + B*b^4*d)/b^5)*x + 2*(3*B*b^4*c - 4*C*a*b^
3*d + 3*A*b^4*d)/b^5)*x + 3*(3*C*a*b^3*c - 2*A*b^4*c + 3*B*a*b^3*d)/b^5)*x
 + 4*(3*B*a*b^3*c - 4*C*a^2*b^2*d + 3*A*a*b^3*d)/b^5)/sqrt(b*x^2 + a) + 1/
2*(3*C*a*c - 2*A*b*c + 3*B*a*d)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^2\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^2*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.48

$$\int \frac{x^2(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{48\sqrt{b\,x^2+a}\,a^2bd-64\sqrt{b\,x^2+a}\,a^2cd-24\sqrt{b\,x^2+a}\,a\,b^2cx+48\sqrt{b\,x^2}}{}$$

input
```
int(x^2*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**2*b*d - 64*sqrt(a + b*x**2)*a**2*c*d - 24*sqrt(a +
 b*x**2)*a*b**2*c*x + 48*sqrt(a + b*x**2)*a*b**2*c + 24*sqrt(a + b*x**2)*a
*b**2*d*x**2 + 36*sqrt(a + b*x**2)*a*b**2*d*x + 36*sqrt(a + b*x**2)*a*b*c*
*2*x - 32*sqrt(a + b*x**2)*a*b*c*d*x**2 + 24*sqrt(a + b*x**2)*b**3*c*x**2
+ 12*sqrt(a + b*x**2)*b**3*d*x**3 + 12*sqrt(a + b*x**2)*b**2*c**2*x**3 + 8
*sqrt(a + b*x**2)*b**2*c*d*x**4 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(
b)*x)/sqrt(a))*a**2*b*c - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**2*b*d - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**2*c**2 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c
*x**2 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x*
*2 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c**2*x**2
- 24*sqrt(b)*a**2*b*c + 27*sqrt(b)*a**2*b*d + 27*sqrt(b)*a**2*c**2 - 24*sq
rt(b)*a*b**2*c*x**2 + 27*sqrt(b)*a*b**2*d*x**2 + 27*sqrt(b)*a*b*c**2*x**2)
/(24*b**3*(a + b*x**2))
```

3.134 $\int \frac{x(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 150

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{Abc-a(cC+Bd)}{b^2\sqrt{a+bx^2}}-\frac{(bBc+Abd-aCd)x}{b^2\sqrt{a+bx^2}}+\frac{(cC+Bd)\sqrt{a+bx^2}}{b^2}+\frac{Cdx\sqrt{a+bx^2}}{2b^2}-\frac{(3aCd-2b(Bc+Ad))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output

```
-(A*b*c-a*(B*d+C*c))/b^2/(b*x^2+a)^(1/2)-(A*b*d+B*b*c-C*a*d)*x/b^2/(b*x^2+
a)^(1/2)+(B*d+C*c)*(b*x^2+a)^(1/2)/b^2+1/2*C*d*x*(b*x^2+a)^(1/2)/b^2-1/2*(
3*a*C*d-2*b*(A*d+B*c))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{\frac{\sqrt{b}(-2Ab(c+dx)+a(4cC+4Bd+3Cdx)+bx(-2Bc+2cCx+2Bdx+Cdx^2))}{\sqrt{a+bx^2}}+(3aCd-2b(}{2b^{5/2}}$$

input

```
Integrate[(x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*(-2*A*b*(c + d*x) + a*(4*c*C + 4*B*d + 3*C*d*x) + b*x*(-2*B*c +
2*c*C*x + 2*B*d*x + C*d*x^2)))/Sqrt[a + b*x^2] + (3*a*C*d - 2*b*(B*c + A*d
))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2176, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int \frac{-aCdx^2-a(cC+2Bd)x+\frac{a(aCd-b(Bc+Ad))}{b}}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2346}$$

$$-\frac{\frac{\int \frac{a(3aCd-2b(Bc+Ad)-2b(cC+2Bd)x)}{\sqrt{bx^2+a}}dx}{2b}-\frac{aCdx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{a\int \frac{3aCd-2b(Bc+Ad)-2b(cC+2Bd)x}{\sqrt{bx^2+a}}dx}{2b}-\frac{aCdx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$-\frac{\frac{a\left((3aCd-2b(Ad+Bc))\int \frac{1}{\sqrt{bx^2+a}}dx-2\sqrt{a+bx^2}(2Bd+cC)\right)}{2b}-\frac{aCdx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-\frac{a\left((3aCd-2b(Ad+Bc))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2\sqrt{a+bx^2}(2Bd+cC)\right)}{2b}-\frac{aCdx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{-\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3aCd-2b(Ad+Bc))}{\sqrt{b}}-2\sqrt{a+bx^2}(2Bd+cC)\right)}{2b}-\frac{aCdx\sqrt{a+bx^2}}{2b}}{ab}-\frac{(c+dx)\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

input
```
Int[(x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
-(((A - (a*C)/b + B*x)*(c + d*x))/(b*Sqrt[a + b*x^2])) - (-1/2*(a*C*d*x*Sq
rt[a + b*x^2])/b + (a*(-2*(c*C + 2*B*d)*Sqrt[a + b*x^2] + ((3*a*C*d - 2*b*
(B*c + A*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/(2*b))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.87

method	result
risch	$\frac{(Cxd+2Bd+2Cc)\sqrt{b\,x^2+a}}{2b^2}+\frac{b(2Abd+2Bbc-3aCd)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{2(Abc-Bad-Cac)}{\sqrt{b\,x^2+a}}-\frac{Cadx}{\sqrt{b\,x^2+a}}}{2b^2}$
default	$(Ad+Bc)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+(Bd+Cc)\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)-\frac{Ac}{b\sqrt{b\,x^2+a}}+dC$

input
```
int(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d+2*C*c)*(b*x^2+a)^(1/2)/b^2+1/2/b^2*(b*(2*A*b*d+2*B*b*c-3*
C*a*d)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-2*(A
*b*c-B*a*d-C*a*c)/(b*x^2+a)^(1/2)-C*a*d*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.40

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(2\,Babc+(2\,Bb^2c-(3\,Cab-2\,Ab^2)d)x^2-(3\,Ca^2-2\,Aab)d)\sqrt{b}\log\right.$$

$$-\frac{(2\,Babc+(2\,Bb^2c-(3\,Cab-2\,Ab^2)d)x^2-(3\,Ca^2-2\,Aab)d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Cb^2dx^3+4\,Bab}{2\,(b^4x^2+ab^3)}$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((2*B*a*b*c + (2*B*b^2*c - (3*C*a*b - 2*A*b^2)*d)*x^2 - (3*C*a^2 - 2*
A*a*b)*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*b
^2*d*x^3 + 4*B*a*b*d + 2*(C*b^2*c + B*b^2*d)*x^2 + 2*(2*C*a*b - A*b^2)*c -
 (2*B*b^2*c - (3*C*a*b - 2*A*b^2)*d)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)
, -1/2*((2*B*a*b*c + (2*B*b^2*c - (3*C*a*b - 2*A*b^2)*d)*x^2 - (3*C*a^2 -
2*A*a*b)*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (C*b^2*d*x^3 + 4
*B*a*b*d + 2*(C*b^2*c + B*b^2*d)*x^2 + 2*(2*C*a*b - A*b^2)*c - (2*B*b^2*c
- (3*C*a*b - 2*A*b^2)*d)*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 7.60 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.87

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{(a+bx^2)^{3/2}}\,dx = Ac\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Ad\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Bc\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Bd\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Cc\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + Cd\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x*(d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*c*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), Tru
e)) + A*d*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**
2/a))) + B*c*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*
x**2/a))) + B*d*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a +
b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/2)), True)) + C*c*Piecewise((2*a/(b**2
*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(3/
2)), True)) + C*d*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqr
t(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.05

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{Cdx^3}{2\sqrt{bx^2+a}b}+\frac{3\,Cadx}{2\sqrt{bx^2+a}b^2}+\frac{(Cc+Bd)x^2}{\sqrt{bx^2+a}b}-\frac{3\,Cad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}-\frac{Ac}{\sqrt{bx^2+a}b}-\frac{(Bc+Ad)x}{\sqrt{bx^2+a}b}+\frac{(Bc+Ad)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\,(Cc+Bd)a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*C*d*x^3/(sqrt(b*x^2 + a)*b) + 3/2*C*a*d*x/(sqrt(b*x^2 + a)*b^2) + (C*c
 + B*d)*x^2/(sqrt(b*x^2 + a)*b) - 3/2*C*a*d*arcsinh(b*x/sqrt(a*b))/b^(5/2)
 - A*c/(sqrt(b*x^2 + a)*b) - (B*c + A*d)*x/(sqrt(b*x^2 + a)*b) + (B*c + A*
d)*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*(C*c + B*d)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.93

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{\left(\left(\frac{Cdx}{b}+\frac{2\,(Cb^3c+Bb^3d)}{b^4}\right)x-\frac{2\,Bb^3c-3\,Cab^2d+2\,Ab^3d}{b^4}\right)x+\frac{2\,(2\,Cab^2c-Ab^3c+2\,Bab^2}{b^4}}{2\sqrt{bx^2+a}}-\frac{(2\,Bbc-3\,Cad+2\,Abd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((C*d*x/b + 2*(C*b^3*c + B*b^3*d)/b^4)*x - (2*B*b^3*c - 3*C*a*b^2*d +
 2*A*b^3*d)/b^4)*x + 2*(2*C*a*b^2*c - A*b^3*c + 2*B*a*b^2*d)/b^4)/sqrt(b*x
^2 + a) - 1/2*(2*B*b*c - 3*C*a*d + 2*A*b*d)*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x\,\left(c+d\,x\right)\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int((x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x*(c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.63

$$\int \frac{x(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\frac{-2\sqrt{b\,x^2+a}\,a\,b^2c-2\sqrt{b\,x^2+a}\,a\,b^2dx+4\sqrt{b\,x^2+a}\,a\,b^2d+4\sqrt{b\,x^2+}}{}$$

input
```
int(x*(d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a*b**2*c - 2*sqrt(a + b*x**2)*a*b**2*d*x + 4*sqrt(a
 + b*x**2)*a*b**2*d + 4*sqrt(a + b*x**2)*a*b*c**2 + 3*sqrt(a + b*x**2)*a*b
*c*d*x - 2*sqrt(a + b*x**2)*b**3*c*x + 2*sqrt(a + b*x**2)*b**3*d*x**2 + 2*
sqrt(a + b*x**2)*b**2*c**2*x**2 + sqrt(a + b*x**2)*b**2*c*d*x**3 + 2*sqrt(
b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d - 3*sqrt(b)*log((s
qrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*c*d + 2*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*c + 2*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*a*b**2*d*x**2 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a*b*c*d*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*b**3*c*x**2 - 2*sqrt(b)*a**2*b*d + 2*sqrt(b)*a**2*c*d - 2*sqrt(
b)*a*b**2*c - 2*sqrt(b)*a*b**2*d*x**2 + 2*sqrt(b)*a*b*c*d*x**2 - 2*sqrt(b)
*b**3*c*x**2)/(2*b**3*(a + b*x**2))
```

3.135 $\int \frac{(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 114

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{bBc+Abd-aCd}{b^2\sqrt{a+bx^2}} + \frac{(Abc-a(cC+Bd))x}{ab\sqrt{a+bx^2}} + \frac{Cd\sqrt{a+bx^2}}{b^2} + \frac{(cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-(A*b*d+B*b*c-C*a*d)/b^2/(b*x^2+a)^(1/2)+(A*b*c-a*(B*d+C*c))*x/a/b/(b*x^2+
a)^(1/2)+C*d*(b*x^2+a)^(1/2)/b^2+(B*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/
2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{-abBc-aAbd+2a^2Cd+Ab^2cx-abcCx-abBdx+abCdx^2}{ab^2\sqrt{a+bx^2}} + \frac{(-cC-Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*b*B*c) - a*A*b*d + 2*a^2*C*d + A*b^2*c*x - a*b*c*C*x - a*b*B*d*x + a*
b*C*d*x^2)/(a*b^2*Sqrt[a + b*x^2]) + ((-(c*C) - B*d)*Log[-(Sqrt[b]*x) + Sq
rt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2176, 25, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2176}$$

$$-\frac{\int -\frac{a(cC+Bd)-(Ab-2aC)dx}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(cC+Bd)-(Ab-2aC)dx}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{a(Bd+cC)\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{d\sqrt{a+bx^2}(Ab-2aC)}{b}}{ab}-\frac{(c+dx)(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{a(Bd+cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{d\sqrt{a+bx^2}(Ab-2aC)}{b}}{ab}-\frac{(c+dx)(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+cC)}{\sqrt{b}}-\frac{d\sqrt{a+bx^2}(Ab-2aC)}{b}}{ab}-\frac{(c+dx)(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*B - (A*b - a*C)*x)*(c + d*x))/(a*b*Sqrt[a + b*x^2])) + (-(((A*b - 2*
a*C)*d*Sqrt[a + b*x^2])/b) + (a*(c*C + B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]])/Sqrt[b])/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

method	result
risch	$\frac{Cd\sqrt{b\,x^{2}+a}}{b^{2}}+\frac{b(Bd+Cc)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{Abd+Bbc-aCd}{b\sqrt{b\,x^{2}+a}}+\frac{Abcx}{a\sqrt{b\,x^{2}+a}}}{b}$
default	$\frac{Acx}{a\sqrt{b\,x^{2}+a}}-\frac{Ad+Bc}{b\sqrt{b\,x^{2}+a}}+(Bd+Cc)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+dC\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)$

input
```
int((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C*d*(b*x^2+a)^(1/2)/b^2+1/b*(b*(B*d+C*c)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*l
n(b^(1/2)*x+(b*x^2+a)^(1/2)))-(A*b*d+B*b*c-C*a*d)/b/(b*x^2+a)^(1/2)+A*b*c*
x/a/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.42

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(Ca^2c+Ba^2d+(Cabc+Babd)x^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x\right.}{2}\right.$$

$$-\frac{(Ca^2c+Ba^2d+(Cabc+Babd)x^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Cabdx^2-Babc+(2\,Ca^2-Aab)d-(Bab}{ab^3x^2+a^2b^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a^2*c + B*a^2*d + (C*a*b*c + B*a*b*d)*x^2)*sqrt(b)*log(-2*b*x^2 -
 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a*b*d*x^2 - B*a*b*c + (2*C*a^2 -
A*a*b)*d - (B*a*b*d + (C*a*b - A*b^2)*c)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 +
a^2*b^2), -((C*a^2*c + B*a^2*d + (C*a*b*c + B*a*b*d)*x^2)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) - (C*a*b*d*x^2 - B*a*b*c + (2*C*a^2 - A*a*b)*d
 - (B*a*b*d + (C*a*b - A*b^2)*c)*x)*sqrt(b*x^2 + a))/(a*b^3*x^2 + a^2*b^2)
]
```

Sympy [A] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = Ad\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + \frac{Acx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + Bc\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right) + Bd\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cc\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + Cd\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*d*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), Tru
e)) + A*c*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*c*Piecewise((-1/(b*sqrt(a +
b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + B*d*(asinh(sqrt(b)*x/sqr
t(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*c*(asinh(sqrt(b)*x/
sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) + C*d*Piecewise((2*a
/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*
a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{Cdx^2}{\sqrt{bx^2+a}b} + \frac{Acx}{\sqrt{bx^2+a}a} - \frac{Bc}{\sqrt{bx^2+a}b} + \frac{2\,Cad}{\sqrt{bx^2+a}b^2} - \frac{Ad}{\sqrt{bx^2+a}b} - \frac{(Cc+Bd)x}{\sqrt{bx^2+a}b} + \frac{(Cc+Bd)\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*d*x^2/(sqrt(b*x^2 + a)*b) + A*c*x/(sqrt(b*x^2 + a)*a) - B*c/(sqrt(b*x^2
+ a)*b) + 2*C*a*d/(sqrt(b*x^2 + a)*b^2) - A*d/(sqrt(b*x^2 + a)*b) - (C*c +
 B*d)*x/(sqrt(b*x^2 + a)*b) + (C*c + B*d)*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Cdx}{b} - \frac{Cab^2c-Ab^3c+Bab^2d}{ab^3}\right)x - \frac{Bab^2c-2\,Ca^2bd+Aab^2d}{ab^3}}{\sqrt{bx^2+a}} - \frac{(Cc+Bd)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((C*d*x/b - (C*a*b^2*c - A*b^3*c + B*a*b^2*d)/(a*b^3))*x - (B*a*b^2*c - 2*
C*a^2*b*d + A*a*b^2*d)/(a*b^3))/sqrt(b*x^2 + a) - (C*c + B*d)*log(abs(-sqr
t(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 17.96 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{B\,d\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{B\,c}{b\,\sqrt{b\,x^2+a}} - \frac{A\,d}{b\,\sqrt{b\,x^2+a}} + \frac{C\,c\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} + \frac{C\,d\,(b\,x^2+2\,a)}{b^2\,\sqrt{b\,x^2+a}} + \frac{A\,c\,x}{a\,\sqrt{b\,x^2+a}} - \frac{B\,d\,x}{b\,\sqrt{b\,x^2+a}} - \frac{C\,c\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
(B*d*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (B*c)/(b*(a + b*x^2)^(1
/2)) - (A*d)/(b*(a + b*x^2)^(1/2)) + (C*c*log(b^(1/2)*x + (a + b*x^2)^(1/2
)))/b^(3/2) + (C*d*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2)) + (A*c*x)/(a*(a
+ b*x^2)^(1/2)) - (B*d*x)/(b*(a + b*x^2)^(1/2)) - (C*c*x)/(b*(a + b*x^2)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.41

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,abd+2\sqrt{b\,x^2+a}\,acd+\sqrt{b\,x^2+a}\,b^2cx-\sqrt{b\,x^2+a}\,b^2c-\sqrt{}}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a*b*d + 2*sqrt(a + b*x**2)*a*c*d + sqrt(a + b*x**2)*b
**2*c*x - sqrt(a + b*x**2)*b**2*c - sqrt(a + b*x**2)*b**2*d*x - sqrt(a + b
*x**2)*b*c**2*x + sqrt(a + b*x**2)*b*c*d*x**2 + sqrt(b)*log((sqrt(a + b*x*
*2) + sqrt(b)*x)/sqrt(a))*a*b*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a*c**2 + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b
**2*d*x**2 + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*c**2*x*
*2 + sqrt(b)*a*b*c - sqrt(b)*a*b*d - sqrt(b)*a*c**2 + sqrt(b)*b**2*c*x**2
- sqrt(b)*b**2*d*x**2 - sqrt(b)*b*c**2*x**2)/(b**2*(a + b*x**2))
```

3.136 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx=\frac{Abc-a(cC+Bd)}{ab\sqrt{a+bx^2}}+\frac{(bBc+Abd-aCd)x}{ab\sqrt{a+bx^2}}+\frac{Cd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{Ac\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(A*b*c-a*(B*d+C*c))/a/b/(b*x^2+a)^(1/2)+(A*b*d+B*b*c-C*a*d)*x/a/b/(b*x^2+a
)^(1/2)+C*d*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-A*c*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx=\frac{bBcx+Ab(c+dx)-a(cC+Bd+Cdx)}{ab\sqrt{a+bx^2}}+\frac{2Ac\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{Cd\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(b*B*c*x + A*b*(c + d*x) - a*(c*C + B*d + C*d*x))/(a*b*Sqrt[a + b*x^2]) +
(2*A*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (C*d*Log[
-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2336, 25, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}-\frac{\int -\frac{Abc+aCdx}{bx\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Abc+aCdx}{bx\sqrt{bx^2+a}}dx}{a}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Abc+aCdx}{x\sqrt{bx^2+a}}dx}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{538}$$

$$\frac{Abc\int \frac{1}{x\sqrt{bx^2+a}}dx+aCd\int \frac{1}{\sqrt{bx^2+a}}dx}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{Abc\int \frac{1}{x\sqrt{bx^2+a}}dx+aCd\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{Abc\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{\frac{1}{2}Abc\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{Ac\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{aCd\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{Abc\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{ab}+\frac{x(-aCd+Abd+bBc)-aBd-acC+Abc}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b*c - a*c*C - a*B*d + (b*B*c + A*b*d - a*C*d)*x)/(a*b*Sqrt[a + b*x^2])
+ ((a*C*d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*b*c*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
default	$\frac{Adx}{a\sqrt{b\,x^2+a}}+\frac{Bcx}{a\sqrt{b\,x^2+a}}+Ac\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{Bd}{b\sqrt{b\,x^2+a}}-\frac{Cc}{b\sqrt{b\,x^2+a}}+dC\left(-\frac{x}{b\sqrt{b\,x^2+a}}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*d*x/a/(b*x^2+a)^(1/2)+B*c*x/a/(b*x^2+a)^(1/2)+A*c*(1/a/(b*x^2+a)^(1/2)-1
/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-B*d/b/(b*x^2+a)^(1/2)-C*c/
b/(b*x^2+a)^(1/2)+d*C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+
a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 706, normalized size of antiderivative = 5.83

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x\,(a+bx^2)^{3/2}}\,dx = \left[\frac{(Ca^2bdx^2+Ca^3d)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+(Ab^3cx^2\cdots}{\cdots}\right.$$

$$-\frac{2\,(Ca^2bdx^2+Ca^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Ab^3cx^2+Aab^2c)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Ba^2bd\cdots}{2\,(a^2b^3x^2+a^3b^2)}$$

$$-\frac{(Ca^2bdx^2+Ca^3d)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Ab^3cx^2+Aab^2c)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(Ba^2bd+(Ca\cdots}{a^2b^3x^2+a^3b^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a^2*b*d*x^2 + C*a^3*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a) + (A*b^3*c*x^2 + A*a*b^2*c)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*a^2*b*d + (C*a^2*b - A*a*b^2)*c - (B*a*
b^2*c - (C*a^2*b - A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2)
, -1/2*(2*(C*a^2*b*d*x^2 + C*a^3*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) - (A*b^3*c*x^2 + A*a*b^2*c)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) + 2*(B*a^2*b*d + (C*a^2*b - A*a*b^2)*c - (B*a*b^2*c -
(C*a^2*b - A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(2
*(A*b^3*c*x^2 + A*a*b^2*c)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (
C*a^2*b*d*x^2 + C*a^3*d)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) - 2*(B*a^2*b*d + (C*a^2*b - A*a*b^2)*c - (B*a*b^2*c - (C*a^2*b - A*
a*b^2)*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), -((C*a^2*b*d*x^2 +
C*a^3*d)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (A*b^3*c*x^2 + A*a*
b^2*c)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (B*a^2*b*d + (C*a^2*b
 - A*a*b^2)*c - (B*a*b^2*c - (C*a^2*b - A*a*b^2)*d)*x)*sqrt(b*x^2 + a))/(a
^2*b^3*x^2 + a^3*b^2)]
```

Sympy [A] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.74

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\left(a+bx^2\right)^{3/2}}\,dx = Ac\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Adx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+Bd\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Bcx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+Cc\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+Cd\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log
(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a
) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**
(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*
a**(9/2) + 2*a**(7/2)*b*x**2)) + A*d*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*d
*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*c*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + C*c*Piecewise((-1/(b*sqrt(a + b*x
**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*d*(asinh(sqrt(b)*x/sqrt(a
))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bcx}{\sqrt{bx^2+a}a}+\frac{Adx}{\sqrt{bx^2+a}a}-\frac{Cdx}{\sqrt{bx^2+a}b}+\frac{Cd\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{Ac\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{Ac}{\sqrt{bx^2+a}a}-\frac{Cc}{\sqrt{bx^2+a}b}-\frac{Bd}{\sqrt{bx^2+a}b}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*c*x/(sqrt(b*x^2 + a)*a) + A*d*x/(sqrt(b*x^2 + a)*a) - C*d*x/(sqrt(b*x^2
+ a)*b) + C*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A*c*arcsinh(a/(sqrt(a*b)*ab
s(x)))/a^(3/2) + A*c/(sqrt(b*x^2 + a)*a) - C*c/(sqrt(b*x^2 + a)*b) - B*d/(
sqrt(b*x^2 + a)*b)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x\left(a+bx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)\,(C\,x^2+B\,x+A)}{x\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,a\,b^2c+\sqrt{b\,x^2+a}\,a\,b^2dx-\sqrt{b\,x^2+a}\,a\,b^2d-\sqrt{b\,x^2+a}\,ab\,c^2}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*a*b**2*c + sqrt(a + b*x**2)*a*b**2*d*x - sqrt(a + b*x**2
)*a*b**2*d - sqrt(a + b*x**2)*a*b*c**2 - sqrt(a + b*x**2)*a*b*c*d*x + sqrt
(a + b*x**2)*b**3*c*x + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*
x)/sqrt(a))*a*b**2*c + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*b**3*c*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)
*x)/sqrt(a))*a*b**2*c - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*
x)/sqrt(a))*b**3*c*x**2 + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**2*c*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b*c*d
*x**2 + sqrt(b)*a**2*b*d - sqrt(b)*a**2*c*d + sqrt(b)*a*b**2*c + sqrt(b)*a
*b**2*d*x**2 - sqrt(b)*a*b*c*d*x**2 + sqrt(b)*b**3*c*x**2)/(a*b**2*(a + b*
x**2))
```

3.137 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 120

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = -\frac{aCd-b(Bc+Ad)}{ab\sqrt{a+bx^2}} - \frac{(Abc-acC-aBd)x}{a^2\sqrt{a+bx^2}} - \frac{Ac\sqrt{a+bx^2}}{a^2x} - \frac{(Bc+Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-(a*C*d-b*(A*d+B*c))/a/b/(b*x^2+a)^(1/2)-(A*b*c-B*a*d-C*a*c)*x/a^2/(b*x^2+
a)^(1/2)-A*c*(b*x^2+a)^(1/2)/a^2/x-(A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \frac{\frac{-a^2Cdx-2Ab^2cx^2+ab(A(-c+dx)+x(Bc+cCx+Bdx))}{bx\sqrt{a+bx^2}} - \sqrt{a}(Bc+Ad)\log(x)+\sqrt{a}(}{a^2}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
((-(a^2*C*d*x) - 2*A*b^2*c*x^2 + a*b*(A*(-c + d*x) + x*(B*c + c*C*x + B*d*
x)))/(b*x*Sqrt[a + b*x^2]) - Sqrt[a]*(B*c + A*d)*Log[x] + Sqrt[a]*(B*c + A
*d)*Log[-Sqrt[a] + Sqrt[a + b*x^2]])/a^2
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2336, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{Ac+(Bc+Ad)x}{x^2\sqrt{bx^2+a}}dx}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ac+(Bc+Ad)x}{x^2\sqrt{bx^2+a}}dx}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{(Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{Ac\sqrt{a+bx^2}}{ax}}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}(Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{Ac\sqrt{a+bx^2}}{ax}}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{(Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{Ac\sqrt{a+bx^2}}{ax}}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad+Bc)}{\sqrt{a}}-\frac{Ac\sqrt{a+bx^2}}{ax}}{a}-\frac{bx\left(\frac{Abc}{a}-Bd-cC\right)+aCd-b(Ad+Bc)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*C*d - b*(B*c + A*d) + b*((A*b*c)/a - c*C - B*d)*x)/(a*b*Sqrt[a + b*x^
2])) + (-((A*c*Sqrt[a + b*x^2])/(a*x)) - ((B*c + A*d)*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/Sqrt[a])/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.14

method	result
default	$\frac{Bdx}{a\sqrt{b\,x^2+a}}+\frac{Ccx}{a\sqrt{b\,x^2+a}}+(Ad+Bc)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+Ac\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)$
risch	$-\frac{Ac\sqrt{b\,x^2+a}}{a^2x}-\frac{\frac{Abcx}{a\sqrt{b\,x^2+a}}-\frac{Bdx}{\sqrt{b\,x^2+a}}-\frac{Ccx}{\sqrt{b\,x^2+a}}-a(Ad+Bc)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+\frac{aCd}{b\sqrt{b\,x^2+a}}}{a}$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*d*x/a/(b*x^2+a)^(1/2)+C*c*x/a/(b*x^2+a)^(1/2)+(A*d+B*c)*(1/a/(b*x^2+a)^(
1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+A*c*(-1/a/x/(b*x^2+a
)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))-d*C/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.43

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{((Bb^2c+Ab^2d)x^3+(Babc+Aabd)x)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-}{2\,(a^2}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((B*b^2*c + A*b^2*d)*x^3 + (B*a*b*c + A*a*b*d)*x)*sqrt(a)*log(-(b*x^
2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(A*a*b*c - (B*a*b*d + (C*a*b
 - 2*A*b^2)*c)*x^2 - (B*a*b*c - (C*a^2 - A*a*b)*d)*x)*sqrt(b*x^2 + a))/(a^
2*b^2*x^3 + a^3*b*x), (((B*b^2*c + A*b^2*d)*x^3 + (B*a*b*c + A*a*b*d)*x)*s
qrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (A*a*b*c - (B*a*b*d + (C*a*b
- 2*A*b^2)*c)*x^2 - (B*a*b*c - (C*a^2 - A*a*b)*d)*x)*sqrt(b*x^2 + a))/(a^2
*b^2*x^3 + a^3*b*x)]
```

Sympy [A] (verification not implemented)

Time = 10.84 (sec) , antiderivative size = 500, normalized size of antiderivative = 4.17

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\left(a+bx^2\right)^{3/2}}\,dx = Ac\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$
$$+Ad\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right.$$
$$\left.+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$
$$+Bc\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right.$$
$$\left.+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$
$$+\frac{Bdx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+Cd\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Ccx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*
x**2) + 1))) + A*d*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x
**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sq
rt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b
*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x
**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)) + B*c*(2*a**3*sqrt(1 + b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2
*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**
(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)
 - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x*
*2)) + B*d*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + C*d*Piecewise((-1/(b*sqrt(a +
 b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + C*c*x/(a**(3/2)*sqrt(1
+ b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{Ccx}{\sqrt{bx^2+a}a} - \frac{2\,Abcx}{\sqrt{bx^2+a}a^2} + \frac{Bdx}{\sqrt{bx^2+a}a} - \frac{Cd}{\sqrt{bx^2+a}b} - \frac{(Bc+Ad)\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{Bc+Ad}{\sqrt{bx^2+a}a} - \frac{Ac}{\sqrt{bx^2+a}ax}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*c*x/(sqrt(b*x^2 + a)*a) - 2*A*b*c*x/(sqrt(b*x^2 + a)*a^2) + B*d*x/(sqrt(
b*x^2 + a)*a) - C*d/(sqrt(b*x^2 + a)*b) - (B*c + A*d)*arcsinh(a/(sqrt(a*b)
*abs(x)))/a^(3/2) + (B*c + A*d)/(sqrt(b*x^2 + a)*a) - A*c/(sqrt(b*x^2 + a)
*a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \frac{2\,A\sqrt{b}c}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a} + \frac{\frac{(Ca^2bc-Aab^2c+Ba^2bd)x}{a^3b}+\frac{Ba^2bc-Ca^3d+Aa^2bd}{a^3b}}{\sqrt{bx^2+a}} + \frac{2\,(Bc+Ad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
2*A*sqrt(b)*c/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + ((C*a^2*b*c - A*
a*b^2*c + B*a^2*b*d)*x/(a^3*b) + (B*a^2*b*c - C*a^3*d + A*a^2*b*d)/(a^3*b)
)/sqrt(b*x^2 + a) + 2*(B*c + A*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sq
rt(-a))/(sqrt(-a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)\,(C\,x^2+B\,x+A)}{x^2\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 454, normalized size of antiderivative = 3.78

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^2\,(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2bc+\sqrt{b\,x^2+a}\,a^2bdx-\sqrt{b\,x^2+a}\,a^2cdx-2\sqrt{b\,x^2+a}\,a\,b$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b*c + sqrt(a + b*x**2)*a**2*b*d*x - sqrt(a + b*x
**2)*a**2*c*d*x - 2*sqrt(a + b*x**2)*a*b**2*c*x**2 + sqrt(a + b*x**2)*a*b*
*2*c*x + sqrt(a + b*x**2)*a*b**2*d*x**2 + sqrt(a + b*x**2)*a*b*c**2*x**2 +
 sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x
+ sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x
 + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*
x**3 + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*
c*x**3 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**
2*b*d*x - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
b**2*c*x - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a
*b**2*d*x**3 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a
))*b**3*c*x**3 - 2*sqrt(b)*a**2*b*c*x + sqrt(b)*a**2*b*d*x + sqrt(b)*a**2*
c**2*x - 2*sqrt(b)*a*b**2*c*x**3 + sqrt(b)*a*b**2*d*x**3 + sqrt(b)*a*b*c**
2*x**3)/(a**2*b*x*(a + b*x**2))
```

3.138 $\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 158

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{3Abc-2a(cC+Bd)}{2a^2\sqrt{a+bx^2}} - \frac{Ac}{2ax^2\sqrt{a+bx^2}} + \frac{(aCd-b(Bc+Ad))x}{a^2\sqrt{a+bx^2}} - \frac{(Bc+Ad)\sqrt{a+bx^2}}{a^2x} + \frac{(3Abc-2a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/2*(3*A*b*c-2*a*(B*d+C*c))/a^2/(b*x^2+a)^(1/2)-1/2*A*c/a/x^2/(b*x^2+a)^(
1/2)+(a*C*d-b*(A*d+B*c))*x/a^2/(b*x^2+a)^(1/2)-(A*d+B*c)*(b*x^2+a)^(1/2)/a
^2/x+1/2*(3*A*b*c-2*a*(B*d+C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{-\frac{\sqrt{a}(aA(c+2dx)+bx^2(3Ac+4Bcx+4Adx)-2ax(B(-c+dx)+Cx(c+dx)))}{x^2\sqrt{a+bx^2}} - 6Abc\text{arctanh}}{2a^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-((Sqrt[a]*(a*A*(c + 2*d*x) + b*x^2*(3*A*c + 4*B*c*x + 4*A*d*x) - 2*a*x*(
B*(-c + d*x) + C*x*(c + d*x))))/(x^2*Sqrt[a + b*x^2])) - 6*A*b*c*ArcTanh[(
Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 4*a*(c*C + B*d)*ArcTanh[(-(Sqrt[b]
*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2336, 25, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\left(\left(\frac{Abc}{a}-Cc-Bd\right)x^2\right)+(Bc+Ad)x+Ac}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\left(\left(\frac{Abc}{a}-Cc-Bd\right)x^2\right)+(Bc+Ad)x+Ac}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{2a(Bc+Ad)-(3Abc-2a(cC+Bd))x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2a(Bc+Ad)-(3Abc-2a(cC+Bd))x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{-(3Abc-2a(Bd+cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{-\frac{1}{2}(3Abc-2a(Bd+cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{-\frac{(3Abc-2a(Bd+cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3Abc-2a(Bd+cC))}{\sqrt{a}}-\frac{2\sqrt{a+bx^2}(Ad+Bc)}{x}}{2a}-\frac{Ac\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{-x(aCd-b(Ad+Bc))-a(Bd+cC)+Abc}{a^2\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((A*b*c - a*(c*C + B*d) - (a*C*d - b*(B*c + A*d))*x)/(a^2*Sqrt[a + b*x^2]
)) + (-1/2*(A*c*Sqrt[a + b*x^2])/(a*x^2) + ((-2*(B*c + A*d)*Sqrt[a + b*x^2
])/x + ((3*A*b*c - 2*a*(c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt
[a])/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(2Adx+2Bcx+Ac)}{2a^2x^2}-\frac{a(3Abc-2Bad-2Cac)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{Abc}{\sqrt{b\,x^2+a}}-\frac{2Cadx}{\sqrt{b\,x^2+a}}+\frac{2Abdx}{\sqrt{b\,x^2+a}}+}{2a^2}$
default	$\frac{dCx}{a\sqrt{b\,x^2+a}}+(Ad+Bc)\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)+(Bd+Cc)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(2*A*d*x+2*B*c*x+A*c)/a^2/x^2-1/2/a^2*(a*(3*A*b*c-2*B
*a*d-2*C*a*c)*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(
1/2))/x))-A*b*c/(b*x^2+a)^(1/2)-2*C*a*d*x/(b*x^2+a)^(1/2)+2*A*b*d*x/(b*x^2
+a)^(1/2)+2*B*b*c*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.40

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^3\,(a+bx^2)^{3/2}}\,dx = \left[\frac{((2\,Babd+(2\,Cab-3\,Ab^2)c)x^4+(2\,Ba^2d+(2\,Ca^2-3\,Aab)c)x^2)\sqrt{a}}{}\right.$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((2*B*a*b*d + (2*C*a*b - 3*A*b^2)*c)*x^4 + (2*B*a^2*d + (2*C*a^2 - 3
*A*a*b)*c)*x^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) - 2*(A*a^2*c + 2*(2*B*a*b*c - (C*a^2 - 2*A*a*b)*d)*x^3 - (2*B*a^2*d + (2
*C*a^2 - 3*A*a*b)*c)*x^2 + 2*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^3*
b*x^4 + a^4*x^2), 1/2*(((2*B*a*b*d + (2*C*a*b - 3*A*b^2)*c)*x^4 + (2*B*a^2
*d + (2*C*a^2 - 3*A*a*b)*c)*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/
a) - (A*a^2*c + 2*(2*B*a*b*c - (C*a^2 - 2*A*a*b)*d)*x^3 - (2*B*a^2*d + (2*
C*a^2 - 3*A*a*b)*c)*x^2 + 2*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^3*b
*x^4 + a^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(139) = 278$.

Time = 9.22 (sec) , antiderivative size = 578, normalized size of antiderivative = 3.66

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = Ac\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)+Ad\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+Bc\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+Bd\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+Cc\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Cdx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt
(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + A*d*(-1
/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) +
 1))) + B*c*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sq
rt(a/(b*x**2) + 1))) + B*d*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(
7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**
3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x*
*2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt
(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)) + C*c*(2*a**3*sqrt(1
 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(
9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)
*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7
/2)*b*x**2)) + C*d*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{Cdx}{\sqrt{bx^2+a}a} + \frac{3\,Abc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,Abc}{2\,\sqrt{bx^2+a}a^2} - \frac{2\,(Bc+Ad)bx}{\sqrt{bx^2+a}a^2} - \frac{(Cc+Bd)\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{Cc+Bd}{\sqrt{bx^2+a}a} - \frac{Ac}{2\,\sqrt{bx^2+a}ax^2} - \frac{Bc+Ad}{\sqrt{bx^2+a}ax}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*d*x/(sqrt(b*x^2 + a)*a) + 3/2*A*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2
) - 3/2*A*b*c/(sqrt(b*x^2 + a)*a^2) - 2*(B*c + A*d)*b*x/(sqrt(b*x^2 + a)*a
^2) - (C*c + B*d)*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + (C*c + B*d)/(sqr
t(b*x^2 + a)*a) - 1/2*A*c/(sqrt(b*x^2 + a)*a*x^2) - (B*c + A*d)/(sqrt(b*x^
2 + a)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.68

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{(Ba^2bc-Ca^3d+Aa^2bd)x}{a^4}-\frac{Ca^3c-Aa^2bc+Ba^3d}{a^4}}{\sqrt{bx^2+a}}$$
$$+\frac{(2\,Cac-3\,Abc+2\,Bad)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}d+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((B*a^2*b*c - C*a^3*d + A*a^2*b*d)*x/a^4 - (C*a^3*c - A*a^2*b*c + B*a^3*d
)/a^4)/sqrt(b*x^2 + a) + (2*C*a*c - 3*A*b*c + 2*B*a*d)*arctan(-(sqrt(b)*x
- sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a
))^3*A*b*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*c + 2*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*d + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b
*c - 2*B*a^2*sqrt(b)*c - 2*A*a^2*sqrt(b)*d)/(((sqrt(b)*x - sqrt(b*x^2 + a)
)^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^3\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 650, normalized size of antiderivative = 4.11

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{2\sqrt{b\,x^2+a}\,abcd\,x^3-6\sqrt{b}\,abcd\,x^4-3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3c\,x^4+}{}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b*c - 2*sqrt(a + b*x**2)*a**2*b*d*x - 3*sqrt(a +
 b*x**2)*a*b**2*c*x**2 - 2*sqrt(a + b*x**2)*a*b**2*c*x - 4*sqrt(a + b*x**2
)*a*b**2*d*x**3 + 2*sqrt(a + b*x**2)*a*b**2*d*x**2 + 2*sqrt(a + b*x**2)*a*
b*c**2*x**2 + 2*sqrt(a + b*x**2)*a*b*c*d*x**3 - 4*sqrt(a + b*x**2)*b**3*c*
x**3 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
**2*c*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a
))*a*b**2*d*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b*c**2*x**2 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*b**3*c*x**4 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*b**3*d*x**4 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) +
sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**2 - 2*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**2 - 2*sqrt(a)*log((sqrt(a
+ b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*x**2 + 3*sqrt(a)*log((s
qrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**4 - 2*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*d*x**4 - 2*sqrt(a
)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**4 + 4
*sqrt(b)*a**2*b*d*x**2 - 6*sqrt(b)*a**2*c*d*x**2 + 4*sqrt(b)*a*b**2*c*x**2
 + 4*sqrt(b)*a*b**2*d*x**4 - 6*sqrt(b)*a*b*c*d*x**4 + 4*sqrt(b)*b**3*c*x**
4)/(2*a**2*b*x**2*(a + b*x**2))
```

3.139 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 198

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx = \frac{2aCd-3b(Bc+Ad)}{2a^2\sqrt{a+bx^2}} - \frac{Bc+Ad}{2ax^2\sqrt{a+bx^2}} + \frac{b(Abc-a(cC+Bd))x}{a^3\sqrt{a+bx^2}} - \frac{Ac\sqrt{a+bx^2}}{3a^2x^3} + \frac{(5Abc-3a(cC+Bd))\sqrt{a+bx^2}}{3a^3x} - \frac{(2aCd-3b(Bc+Ad))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
1/2*(2*a*C*d-3*b*(A*d+B*c))/a^2/(b*x^2+a)^(1/2)-1/2*(A*d+B*c)/a/x^2/(b*x^2
+a)^(1/2)+b*(A*b*c-a*(B*d+C*c))*x/a^3/(b*x^2+a)^(1/2)-1/3*A*c*(b*x^2+a)^(1
/2)/a^2/x^3+1/3*(5*A*b*c-3*a*(B*d+C*c))*(b*x^2+a)^(1/2)/a^3/x-1/2*(2*a*C*d
-3*b*(A*d+B*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\left(a+bx^2\right)^{3/2}}\,dx = \frac{16Ab^2cx^4-abx^2(A(-8c+9dx)+3x(3Bc+4cCx+4Bdx))-a^2(A(2}{6a^3x^3\sqrt{a+bx^2}} + \frac{2Cd\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{3b(Bc+Ad)\text{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(16*A*b^2*c*x^4 - a*b*x^2*(A*(-8*c + 9*d*x) + 3*x*(3*B*c + 4*c*C*x + 4*B*d
*x)) - a^2*(A*(2*c + 3*d*x) + 3*x*(2*C*x*(c - d*x) + B*(c + 2*d*x))))/(6*a
^3*x^3*Sqrt[a + b*x^2]) + (2*C*d*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqr
t[a]])/a^(3/2) + (3*b*(B*c + A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])
/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2336, 25, 2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{\int -\frac{\frac{(aCd-b(Bc+Ad))x^3}{a}-\left(\frac{Abc}{a}-Cc-Bd\right)x^2+(Bc+Ad)x+Ac}{x^4\sqrt{bx^2+a}}dx}{a} - \frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{(aCd-b(Bc+Ad))x^3}{a}-\left(\frac{Abc}{a}-Cc-Bd\right)x^2+(Bc+Ad)x+Ac}{x^4\sqrt{bx^2+a}}dx}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{-3(bBc+Abd-aCd)x^2-(5Abc-3a(cC+Bd))x+3a(Bc+Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-3(bBc+Abd-aCd)x^2-(5Abc-3a(cC+Bd))x+3a(Bc+Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 2338

$$\frac{\frac{-\frac{\int \frac{a(2(5Abc-3a(cC+Bd))-3(2aCd-3b(Bc+Ad))x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{-\frac{1}{2}\int \frac{2(5Abc-3a(cC+Bd))-3(2aCd-3b(Bc+Ad))x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{1}{2}\left(3(2aCd-3b(Ad+Bc))\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{2\sqrt{a+bx^2}(5Abc-3a(Bd+cC))}{ax}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{1}{2}\left(\frac{3}{2}(2aCd-3b(Ad+Bc))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{2\sqrt{a+bx^2}(5Abc-3a(Bd+cC))}{ax}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3(2aCd-3b(Ad+Bc))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{2\sqrt{a+bx^2}(5Abc-3a(Bd+cC))}{ax}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(5Abc-3a(Bd+cC))}{ax}-\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2aCd-3b(Ad+Bc))}{\sqrt{a}}\right)-\frac{3\sqrt{a+bx^2}(Ad+Bc)}{2x^2}}{3a}-\frac{Ac\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(-aCd+Abd+bBc)-bx(Abc-a(Bd+cC))}{a^3\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b*B*c + A*b*d - a*C*d) - b*(A*b*c - a*(c*C + B*d))*x)/(a^3*Sqrt[a +
b*x^2])) + (-1/3*(A*c*Sqrt[a + b*x^2])/(a*x^3) + ((-3*(B*c + A*d)*Sqrt[a +
 b*x^2])/(2*x^2) + ((2*(5*A*b*c - 3*a*(c*C + B*d))*Sqrt[a + b*x^2])/(a*x)
- (3*(2*a*C*d - 3*b*(B*c + A*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
)/2)/(3*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-10Abc\,x^{2}+6Bad\,x^{2}+6Cac\,x^{2}+3Aadx+3Bacx+2Aac\right)}{6a^{3}x^{3}}-\frac{a(3Abd+3Bbc-2aCd)\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2 \cdots}$
default	$(Ad+Bc)\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+(Bd+Cc)\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2b \cdots}{a^{2}\sqrt{b \cdots}}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(-10*A*b*c*x^2+6*B*a*d*x^2+6*C*a*c*x^2+3*A*a*d*x+3*B*
a*c*x+2*A*a*c)/a^3/x^3-1/2/a^2*(a*(3*A*b*d+3*B*b*c-2*C*a*d)*(1/a/(b*x^2+a)
^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-A*b*d/(b*x^2+a)^(1
/2)-B*b*c/(b*x^2+a)^(1/2)-2*A*b^2*c*x/a/(b*x^2+a)^(1/2)+2*B*d/(b*x^2+a)^(1
/2)*b*x+2*C*c/(b*x^2+a)^(1/2)*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 440, normalized size of antiderivative = 2.22

$$\int \frac{(c+dx)\left(A+Bx+Cx^{2}\right)}{x^{4}\left(a+bx^{2}\right)^{3/2}}\,dx=\left[\frac{3\left((3\,Bb^{2}c-(2\,Cab-3\,Ab^{2})d)x^{5}+(3\,Babc-(2\,Ca^{2}-3\,Aab)d)x^{3}\right)\sqrt{\cdots}}{\cdots}\right.$$

$$-\frac{3\left((3\,Bb^{2}c-(2\,Cab-3\,Ab^{2})d)x^{5}+(3\,Babc-(2\,Ca^{2}-3\,Aab)d)x^{3}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^{2}+a}\sqrt{-a}}{a}\right)+(4\,(3\,B\cdots}{\cdots}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((3*B*b^2*c - (2*C*a*b - 3*A*b^2)*d)*x^5 + (3*B*a*b*c - (2*C*a^2
- 3*A*a*b)*d)*x^3)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) - 2*(4*(3*B*a*b*d + (3*C*a*b - 4*A*b^2)*c)*x^4 + 2*A*a^2*c + 3*(3*B*a
*b*c - (2*C*a^2 - 3*A*a*b)*d)*x^3 + 2*(3*B*a^2*d + (3*C*a^2 - 4*A*a*b)*c)*
x^2 + 3*(B*a^2*c + A*a^2*d)*x)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x^3), -1/
6*(3*((3*B*b^2*c - (2*C*a*b - 3*A*b^2)*d)*x^5 + (3*B*a*b*c - (2*C*a^2 - 3*
A*a*b)*d)*x^3)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (4*(3*B*a*b*d
 + (3*C*a*b - 4*A*b^2)*c)*x^4 + 2*A*a^2*c + 3*(3*B*a*b*c - (2*C*a^2 - 3*A*
a*b)*d)*x^3 + 2*(3*B*a^2*d + (3*C*a^2 - 4*A*a*b)*c)*x^2 + 3*(B*a^2*c + A*a
^2*d)*x)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(178) = 356$.

Time = 21.73 (sec) , antiderivative size = 683, normalized size of antiderivative = 3.45

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx = Ac\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + Ad\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + Bc\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + Bd\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + Cc\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + Cd\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**4/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x
**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a*
*5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*
sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x*
*6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**
5*x**4 + 3*a**3*b**6*x**6)) + A*d*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) +
1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(
b)*x))/(2*a**(5/2))) + B*c*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3
*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/
(2*a**(5/2))) + B*d*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/
(a**2*sqrt(a/(b*x**2) + 1))) + C*c*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1
)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1))) + C*d*(2*a**3*sqrt(1 + b*x**2/
a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a
**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7
/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) -
 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2
))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx = \frac{8\,Ab^2cx}{3\,\sqrt{bx^2+a}a^3} - \frac{Cd\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{Cd}{\sqrt{bx^2+a}a} - \frac{2\,(Cc+Bd)bx}{\sqrt{bx^2+a}a^2} + \frac{3\,(Bc+Ad)b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,(Bc+Ad)b}{2\,\sqrt{bx^2+a}a^2} + \frac{4\,Abc}{3\,\sqrt{bx^2+a}a^2x} - \frac{Cc+Bd}{\sqrt{bx^2+a}ax} - \frac{Ac}{3\,\sqrt{bx^2+a}ax^3} - \frac{Bc+Ad}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
8/3*A*b^2*c*x/(sqrt(b*x^2 + a)*a^3) - C*d*arcsinh(a/(sqrt(a*b)*abs(x)))/a^
(3/2) + C*d/(sqrt(b*x^2 + a)*a) - 2*(C*c + B*d)*b*x/(sqrt(b*x^2 + a)*a^2)
+ 3/2*(B*c + A*d)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*(B*c + A*d
)*b/(sqrt(b*x^2 + a)*a^2) + 4/3*A*b*c/(sqrt(b*x^2 + a)*a^2*x) - (C*c + B*d
)/(sqrt(b*x^2 + a)*a*x) - 1/3*A*c/(sqrt(b*x^2 + a)*a*x^3) - 1/2*(B*c + A*d
)/(sqrt(b*x^2 + a)*a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(172) = 344$.

Time = 0.13 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.24

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{\left(Ca^3bc-Aa^2b^2c+Ba^3bd\right)x}{a^5}+\frac{Ba^3bc-Ca^4d+Aa^3bd}{a^5}}{\sqrt{bx^2+a}}$$
$$-\frac{(3\,Bbc-2\,Cad+3\,Abd)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$
$$+\frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bbc+3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Abd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}c-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right.}{}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((C*a^3*b*c - A*a^2*b^2*c + B*a^3*b*d)*x/a^5 + (B*a^3*b*c - C*a^4*d + A*a
^3*b*d)/a^5)/sqrt(b*x^2 + a) - (3*B*b*c - 2*C*a*d + 3*A*b*d)*arctan(-(sqrt
(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*(sqrt(b)*x - sq
rt(b*x^2 + a))^5*B*b*c + 3*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b*d + 6*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*C*a*sqrt(b)*c - 6*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*A*b^(3/2)*c + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*d - 12*(sqr
t(b)*x - sqrt(b*x^2 + a))^2*C*a^2*sqrt(b)*c + 24*(sqrt(b)*x - sqrt(b*x^2 +
 a))^2*A*a*b^(3/2)*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*d
- 3*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^2*b*c - 3*(sqrt(b)*x - sqrt(b*x^2 +
a))*A*a^2*b*d + 6*C*a^3*sqrt(b)*c - 10*A*a^2*b^(3/2)*c + 6*B*a^3*sqrt(b)*d
)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^4\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 703, normalized size of antiderivative = 3.55

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^4\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**3*c - 3*sqrt(a + b*x**2)*a**3*d*x + 8*sqrt(a + b
*x**2)*a**2*b*c*x**2 - 3*sqrt(a + b*x**2)*a**2*b*c*x - 9*sqrt(a + b*x**2)*
a**2*b*d*x**3 - 6*sqrt(a + b*x**2)*a**2*b*d*x**2 - 6*sqrt(a + b*x**2)*a**2
*c**2*x**2 + 6*sqrt(a + b*x**2)*a**2*c*d*x**3 + 16*sqrt(a + b*x**2)*a*b**2
*c*x**4 - 9*sqrt(a + b*x**2)*a*b**2*c*x**3 - 12*sqrt(a + b*x**2)*a*b**2*d*
x**4 - 12*sqrt(a + b*x**2)*a*b*c**2*x**4 - 9*sqrt(a)*log((sqrt(a + b*x**2)
 - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d*x**3 + 6*sqrt(a)*log((sqrt(a + b
*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*c*d*x**3 - 9*sqrt(a)*log((sqrt
(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**3 - 9*sqrt(a)*log
((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**5 + 6*sqrt(
a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d*x**5 - 9*
sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**5
+ 9*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d
*x**3 - 6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*
*2*c*d*x**3 + 9*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**2*c*x**3 + 9*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*a*b**2*d*x**5 - 6*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt
(b)*x)/sqrt(a))*a*b*c*d*x**5 + 9*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*b**3*c*x**5 - 16*sqrt(b)*a**2*b*c*x**3 + 12*sqrt(b)*a
**2*b*d*x**3 + 12*sqrt(b)*a**2*c**2*x**3 - 16*sqrt(b)*a*b**2*c*x**5 + 1...
```

3.140 $\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 236

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \frac{3b(5Abc-4a(cC+Bd))}{8a^3\sqrt{a+bx^2}}-\frac{Ac}{4ax^4\sqrt{a+bx^2}}+\frac{5Abc-4a(cC+Bd)}{8a^2x^2\sqrt{a+bx^2}}+\frac{b(bBc+Abd-aCd)x}{a^3\sqrt{a+bx^2}}-\frac{(Bc+Ad)\sqrt{a+bx^2}}{3a^2x^3}-\frac{(3aCd-5b(Bc+Ad))\sqrt{a+bx^2}}{3a^3x}-\frac{3b(5Abc-4a(cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
3/8*b*(5*A*b*c-4*a*(B*d+C*c))/a^3/(b*x^2+a)^(1/2)-1/4*A*c/a/x^4/(b*x^2+a)^
(1/2)+1/8*(5*A*b*c-4*a*(B*d+C*c))/a^2/x^2/(b*x^2+a)^(1/2)+b*(A*b*d+B*b*c-C
*a*d)*x/a^3/(b*x^2+a)^(1/2)-1/3*(A*d+B*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/3*(3*a
*C*d-5*b*(A*d+B*c))*(b*x^2+a)^(1/2)/a^3/x-3/8*b*(5*A*b*c-4*a*(B*d+C*c))*ar
ctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx = \frac{15Ab^2c\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{7/2}}$$
$$+\frac{b^2x^4(45Ac+64Bcx+64Adx)+abx^2(A(15c+32dx)-4x(-8Bc+9cCx+9Bdx+12Cdx^2))-2a^2(A}{24a^3x^4\sqrt{}}$$

input
```
Integrate[((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(15*A*b^2*c*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(4*a^(7/2)) +
(b^2*x^4*(45*A*c + 64*B*c*x + 64*A*d*x) + a*b*x^2*(A*(15*c + 32*d*x) - 4*x
*(-8*B*c + 9*c*C*x + 9*B*d*x + 12*C*d*x^2)) - 2*a^2*(A*(3*c + 4*d*x) + 2*x
*(3*C*x*(c + 2*d*x) + B*(2*c + 3*d*x))) + 72*Sqrt[a]*b*(c*C + B*d)*x^4*Sqr
t[a + b*x^2]*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(24*a^3*x^
4*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules used = {2336, 25, 2338, 25, 2338, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{b(Abc-a(cC+Bd))x^4}{a^2}+\frac{(aCd-b(Bc+Ad))x^3}{a}-\left(\frac{Abc}{a}-Cc-Bd\right)x^2+(Bc+Ad)x+Ac}{x^5\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{b(Abc-a(cC+Bd))x^4}{a^2}+\frac{(aCd-b(Bc+Ad))x^3}{a}-\left(\frac{Abc}{a}-Cc-Bd\right)x^2+(Bc+Ad)x+Ac}{x^5\sqrt{bx^2+a}}dx}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{4b\left(\frac{Abc}{a}-Cc-Bd\right)x^3-4(bBc+Abd-aCd)x^2-(7Abc-4a(cC+Bd))x+4a(Bc+Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{4b\left(\frac{Abc}{a}-Cc-Bd\right)x^3-4(bBc+Abd-aCd)x^2-(7Abc-4a(cC+Bd))x+4a(Bc+Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{\int \frac{-12b(Abc-a(cC+Bd))x^2-4a(3aCd-5b(Bc+Ad))x+3a(7Abc-4a(cC+Bd))}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{-\frac{-\frac{\int \frac{a(8a(3aCd-5b(Bc+Ad))+9b(5Abc-4a(cC+Bd))x)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{-\frac{-\frac{1}{2}\int \frac{8a(3aCd-5b(Bc+Ad))+9b(5Abc-4a(cC+Bd))x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 534

$$\frac{-\frac{-\frac{\frac{1}{2}\left(\frac{8\sqrt{a+bx^2}(3aCd-5b(Ad+Bc))}{x}-9b(5Abc-4a(Bd+cC))\int\frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{-\frac{-\frac{\frac{1}{2}\left(\frac{8\sqrt{a+bx^2}(3aCd-5b(Ad+Bc))}{x}-\frac{9}{2}b(5Abc-4a(Bd+cC))\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2\right)-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{-\frac{-\frac{\frac{1}{2}\left(\frac{8\sqrt{a+bx^2}(3aCd-5b(Ad+Bc))}{x}-9(5Abc-4a(Bd+cC))\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}\right)-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}+\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}$$

$\Big\downarrow$ 221

$$\frac{b(x(-aCd+Abd+bBc)-aBd-acC+Abc)}{a^3\sqrt{a+bx^2}}+\frac{-\frac{-\frac{\frac{1}{2}\left(\frac{9b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(5Abc-4a(Bd+cC))}{\sqrt{a}}+\frac{8\sqrt{a+bx^2}(3aCd-5b(Ad+Bc))}{x}\right)-\frac{3\sqrt{a+bx^2}(7Abc-4a(Bd+cC))}{2x^2}}{3a}-\frac{4\sqrt{a+bx^2}(Ad+Bc)}{3x^3}}{4a}-\frac{Ac\sqrt{a+bx^2}}{4ax^4}}{a}$$

input
```
Int[((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(b*(A*b*c - a*c*C - a*B*d + (b*B*c + A*b*d - a*C*d)*x))/(a^3*Sqrt[a + b*x^
2]) + (-1/4*(A*c*Sqrt[a + b*x^2])/(a*x^4) + ((-4*(B*c + A*d)*Sqrt[a + b*x^
2])/(3*x^3) - ((-3*(7*A*b*c - 4*a*(c*C + B*d))*Sqrt[a + b*x^2])/(2*x^2) +
((8*(3*a*C*d - 5*b*(B*c + A*d))*Sqrt[a + b*x^2])/x + (9*b*(5*A*b*c - 4*a*(
c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a))/(4*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-40 A b d\,x^{3}-40 B b c\,x^{3}+24 C a d\,x^{3}-21 A b c\,x^{2}+12 B a d\,x^{2}+12 C a c\,x^{2}+8 A a d x+8 B a c x+6 A a c\right)}{24 a^{3} x^{4}}+\frac{b\left(-\frac{7 A b c-4 B a d-4 C}{\sqrt{b\,x^{2}+a}} \ldots\right.}{\ldots}$
default	$\left(A d+B c\right)\left(-\frac{1}{3 a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4 b\left(-\frac{1}{a x\sqrt{b\,x^{2}+a}}-\frac{2 b x}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3 a}\right)+\left(B d+C c\right)\left(-\frac{1}{2 a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3 b\left(\frac{1}{a\sqrt{b\,x^{2} \ldots}} \ldots\right.}{\ldots}\right.$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-40*A*b*d*x^3-40*B*b*c*x^3+24*C*a*d*x^3-21*A*b*c*x^
2+12*B*a*d*x^2+12*C*a*c*x^2+8*A*a*d*x+8*B*a*c*x+6*A*a*c)/a^3/x^4+1/8/a^3*b
*(-(7*A*b*c-4*B*a*d-4*C*a*c)/(b*x^2+a)^(1/2)-8*C*a*d*x/(b*x^2+a)^(1/2)+3*a
*(5*A*b*c-4*B*a*d-4*C*a*c)*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2
)*(b*x^2+a)^(1/2))/x))+8*A*b*d*x/(b*x^2+a)^(1/2)+8*B*b*c*x/(b*x^2+a)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.28

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{9\left((4\,Bab^2d+(4\,Cab^2-5\,Ab^3)c)x^6+(4\,Ba^2bd+(4\,Ca^2b-5\,Aab^2)c)\right.}{}\right.$$

$$-\frac{9\left((4\,Bab^2d+(4\,Cab^2-5\,Ab^3)c)x^6+(4\,Ba^2bd+(4\,Ca^2b-5\,Aab^2)c)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-(1}{}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/48*(9*((4*B*a*b^2*d + (4*C*a*b^2 - 5*A*b^3)*c)*x^6 + (4*B*a^2*b*d + (4*
C*a^2*b - 5*A*a*b^2)*c)*x^4)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) + 2*(16*(4*B*a*b^2*c - (3*C*a^2*b - 4*A*a*b^2)*d)*x^5 - 6*A
*a^3*c - 9*(4*B*a^2*b*d + (4*C*a^2*b - 5*A*a*b^2)*c)*x^4 + 8*(4*B*a^2*b*c
- (3*C*a^3 - 4*A*a^2*b)*d)*x^3 - 3*(4*B*a^3*d + (4*C*a^3 - 5*A*a^2*b)*c)*x
^2 - 8*(B*a^3*c + A*a^3*d)*x)*sqrt(b*x^2 + a))/(a^4*b*x^6 + a^5*x^4), -1/2
4*(9*((4*B*a*b^2*d + (4*C*a*b^2 - 5*A*b^3)*c)*x^6 + (4*B*a^2*b*d + (4*C*a^
2*b - 5*A*a*b^2)*c)*x^4)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (16
*(4*B*a*b^2*c - (3*C*a^2*b - 4*A*a*b^2)*d)*x^5 - 6*A*a^3*c - 9*(4*B*a^2*b*
d + (4*C*a^2*b - 5*A*a*b^2)*c)*x^4 + 8*(4*B*a^2*b*c - (3*C*a^3 - 4*A*a^2*b
)*d)*x^3 - 3*(4*B*a^3*d + (4*C*a^3 - 5*A*a^2*b)*c)*x^2 - 8*(B*a^3*c + A*a^
3*d)*x)*sqrt(b*x^2 + a))/(a^4*b*x^6 + a^5*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(226) = 452$.

Time = 14.49 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.34

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\left(a+bx^2\right)^{3/2}}\,dx = Ac\left(-\frac{1}{4a\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{5\sqrt{b}}{8a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{15b^{\frac{3}{2}}}{8a^3x\sqrt{\frac{a}{bx^2}+1}} - \frac{15b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{7}{2}}}\right) + Ad\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + Bc\left(-\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}\right) + Bd\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + Cc\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right) + Cd\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

input
```
integrate((d*x+c)*(C*x**2+B*x+A)/x**5/(b*x**2+a)**(3/2),x)
```

output
```
A*c*(-1/(4*a*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + 5*sqrt(b)/(8*a**2*x**3*s
qrt(a/(b*x**2) + 1)) + 15*b**(3/2)/(8*a**3*x*sqrt(a/(b*x**2) + 1)) - 15*b*
*2*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(7/2))) + A*d*(-a**3*b**(9/2)*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 3*
a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x
**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**5
*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 8*b**(15/2)*x**6*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6))
+ B*c*(-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5
*x**4 + 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*
a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**
4*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*
x**6) + 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b
**5*x**4 + 3*a**3*b**6*x**6)) + B*d*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2)
+ 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqr
t(b)*x))/(2*a**(5/2))) + C*c*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) -
 3*sqrt(b)/(2*a**2*x*sqrt(a/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x)
)/(2*a**(5/2))) + C*d*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b
)/(a**2*sqrt(a/(b*x**2) + 1)))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx = -\frac{2\,Cbdx}{\sqrt{bx^2+a}a^2} - \frac{15\,Ab^2c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} + \frac{15\,Ab^2c}{8\,\sqrt{bx^2+a}a^3} + \frac{8\,(Bc+Ad)b^2x}{3\,\sqrt{bx^2+a}a^3} + \frac{3\,(Cc+Bd)b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,(Cc+Bd)b}{2\,\sqrt{bx^2+a}a^2} - \frac{Cd}{\sqrt{bx^2+a}ax} + \frac{5\,Abc}{8\,\sqrt{bx^2+a}a^2x^2} + \frac{4\,(Bc+Ad)b}{3\,\sqrt{bx^2+a}a^2x} - \frac{Cc+Bd}{2\,\sqrt{bx^2+a}ax^2} - \frac{Ac}{4\,\sqrt{bx^2+a}ax^4} - \frac{Bc+Ad}{3\,\sqrt{bx^2+a}ax^3}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*C*b*d*x/(sqrt(b*x^2 + a)*a^2) - 15/8*A*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x
)))/a^(7/2) + 15/8*A*b^2*c/(sqrt(b*x^2 + a)*a^3) + 8/3*(B*c + A*d)*b^2*x/(
sqrt(b*x^2 + a)*a^3) + 3/2*(C*c + B*d)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(
5/2) - 3/2*(C*c + B*d)*b/(sqrt(b*x^2 + a)*a^2) - C*d/(sqrt(b*x^2 + a)*a*x)
 + 5/8*A*b*c/(sqrt(b*x^2 + a)*a^2*x^2) + 4/3*(B*c + A*d)*b/(sqrt(b*x^2 + a
)*a^2*x) - 1/2*(C*c + B*d)/(sqrt(b*x^2 + a)*a*x^2) - 1/4*A*c/(sqrt(b*x^2 +
 a)*a*x^4) - 1/3*(B*c + A*d)/(sqrt(b*x^2 + a)*a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(206) = 412$.

Time = 0.14 (sec) , antiderivative size = 764, normalized size of antiderivative = 3.24

$$\int \frac{(c+dx)\left(A+Bx+Cx^2\right)}{x^5\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((B*a^3*b^2*c - C*a^4*b*d + A*a^3*b^2*d)*x/a^6 - (C*a^4*b*c - A*a^3*b^2*c
+ B*a^4*b*d)/a^6)/sqrt(b*x^2 + a) - 3/4*(4*C*a*b*c - 5*A*b^2*c + 4*B*a*b*d
)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) + 1/12*(1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c - 21*(sqrt(b)*x - sqrt(b*x^2 + a
))^7*A*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*B*a*b*d - 24*(sqrt(b)*x
- sqrt(b*x^2 + a))^6*B*a*b^(3/2)*c + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^2*sqrt(b)*d - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2)*d - 12*(sqr
t(b)*x - sqrt(b*x^2 + a))^5*C*a^2*b*c + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^5
*A*a*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*d + 120*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*B*a^2*b^(3/2)*c - 72*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*C*a^3*sqrt(b)*d + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2)*d - 1
2*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c + 45*(sqrt(b)*x - sqrt(b*x^2 +
 a))^3*A*a^2*b^2*c - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*B*a^3*b*d - 136*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c + 72*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*C*a^4*sqrt(b)*d - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(3/
2)*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*C*a^4*b*c - 21*(sqrt(b)*x - sqrt(b
*x^2 + a))*A*a^3*b^2*c + 12*(sqrt(b)*x - sqrt(b*x^2 + a))*B*a^4*b*d + 40*B
*a^4*b^(3/2)*c - 24*C*a^5*sqrt(b)*d + 40*A*a^4*b^(3/2)*d)/(((sqrt(b)*x - s
qrt(b*x^2 + a))^2 - a)^4*a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^5\,(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)\,\left(C\,x^2+B\,x+A\right)}{x^5\,\left(b\,x^2+a\right)^{3/2}}\,dx$$

input
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 754, normalized size of antiderivative = 3.19

$$\int \frac{(c+dx)\,(A+Bx+Cx^2)}{x^5\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x)
```

output

```
( - 6*sqrt(a + b*x**2)*a**3*c - 8*sqrt(a + b*x**2)*a**3*d*x + 15*sqrt(a +
b*x**2)*a**2*b*c*x**2 - 8*sqrt(a + b*x**2)*a**2*b*c*x + 32*sqrt(a + b*x**2
)*a**2*b*d*x**3 - 12*sqrt(a + b*x**2)*a**2*b*d*x**2 - 12*sqrt(a + b*x**2)*
a**2*c**2*x**2 - 24*sqrt(a + b*x**2)*a**2*c*d*x**3 + 45*sqrt(a + b*x**2)*a
*b**2*c*x**4 + 32*sqrt(a + b*x**2)*a*b**2*c*x**3 + 64*sqrt(a + b*x**2)*a*b
**2*d*x**5 - 36*sqrt(a + b*x**2)*a*b**2*d*x**4 - 36*sqrt(a + b*x**2)*a*b*c
**2*x**4 - 48*sqrt(a + b*x**2)*a*b*c*d*x**5 + 64*sqrt(a + b*x**2)*b**3*c*x
**5 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
**2*c*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**2*d*x**4 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x
)/sqrt(a))*a*b*c**2*x**4 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**3*c*x**6 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**3*d*x**6 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sq
rt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**2*x**6 - 45*sqrt(a)*log((sqrt(a + b*x*
*2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*x**4 + 36*sqrt(a)*log((sqrt(a
 + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d*x**4 + 36*sqrt(a)*log(
(sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**2*x**4 - 45*sqrt(
a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*x**6 + 36*
sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*d*x**6
+ 36*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2...
```

3.141 $\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 301

$$\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{a(bc(Bc+2Ad)-ad(2cC+Bd))}{b^3\sqrt{a+bx^2}} - \frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\,x}{b^3\sqrt{a+bx^2}} + \frac{(bc(Bc+2Ad)-2ad(2cC+Bd))\sqrt{a+bx^2}}{b^3} - \frac{(7aCd^2-4b(c^2C+2Bcd+Ad^2))\,x\sqrt{a+bx^2}}{8b^3} + \frac{Cd^2x^3\sqrt{a+bx^2}}{4b^2} + \frac{d(2cC+Bd)\,(a+bx^2)^{3/2}}{3b^3} + \frac{(4Ab(2bc^2-3ad^2)+3a(5aCd^2-4bc(cC+2Bd)))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

output
```
a*(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))/b^3/(b*x^2+a)^(1/2)-(A*b*(-a*d^2+b*c^2
)+a*(a*C*d^2-b*c*(2*B*d+C*c)))*x/b^3/(b*x^2+a)^(1/2)+(b*c*(2*A*d+B*c)-2*a*
d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/b^3-1/8*(7*a*C*d^2-4*b*(A*d^2+2*B*c*d+C*c^2
))*x*(b*x^2+a)^(1/2)/b^3+1/4*C*d^2*x^3*(b*x^2+a)^(1/2)/b^2+1/3*d*(B*d+2*C*
c)*(b*x^2+a)^(3/2)/b^3+1/8*(4*A*b*(-3*a*d^2+2*b*c^2)+3*a*(5*a*C*d^2-4*b*c*
(2*B*d+C*c)))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b}(-a^2d(128cC+64Bd+45Cdx)+ab(12Ad(8c+3dx)+Cx(36c^2-64cdx-15d^2x^2)+8B(6c^2+\ldots}{\sqrt{\ldots}}$$

input
```
Integrate[(x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*(-(a^2*d*(128*c*C + 64*B*d + 45*C*d*x)) + a*b*(12*A*d*(8*c + 3*d
*x) + C*x*(36*c^2 - 64*c*d*x - 15*d^2*x^2) + 8*B*(6*c^2 + 9*c*d*x - 4*d^2*
x^2)) + 2*b^2*x*(6*A*(-2*c^2 + 4*c*d*x + d^2*x^2) + x*(4*B*(3*c^2 + 3*c*d*
x + d^2*x^2) + C*x*(6*c^2 + 8*c*d*x + 3*d^2*x^2)))))/Sqrt[a + b*x^2] + 72*
a*b*(c^2*C + 2*B*c*d + A*d^2)*ArcTanh[(Sqrt[b]*x)/(Sqrt[a] - Sqrt[a + b*x^
2])] + 6*(8*A*b^2*c^2 + 15*a^2*C*d^2)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt
[a + b*x^2])])/(24*b^(7/2))
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2176, 25, 2185, 2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{2176}$$

$$\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}-\frac{\int -\frac{(c+dx)\left(aCdx^3+a(cC+Bd)x^2+a\left(Bc+3\left(A-\frac{aC}{b}\right)d\right)x+\frac{a(Abc-aCc-2aBd)}{b}\right)}{\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{(c+dx)\left(aCdx^3+a(cC+Bd)x^2+a\left(Bc+3\left(A-\frac{aC}{b}\right)d\right)x+\frac{a(Abc-a(cC+2Bd))}{b}\right)}{\sqrt{bx^2+a}}dx}{ab}+\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}}{}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\int \frac{(c+dx)\left(-ab(cC-4Bd)x^2d^3+a(4Abc-7aCc-8aBd)d^3-a\left(15aCd^2+b\left(Cc^2-4Bdc-12Ad^2\right)\right)xd^2\right)}{\sqrt{bx^2+a}}dx}{4bd^3}+\frac{aC\sqrt{a+bx^2}(c+dx)^3}{4bd}}{ab}+\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{\frac{\int \frac{abd^4(c+dx)\left(d(12Abc-19aCc-32aBd)-\left(45aCd^2+2b\left(Cc^2-4Bdc-18Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx}{3bd^2}-\frac{1}{3}ad^2\sqrt{a+bx^2}(c+dx)^2(cC-4Bd)}{4bd^3}+\frac{aC\sqrt{a+bx^2}(c+dx)^3}{4bd}}{ab}+\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{3}ad^2\int \frac{(c+dx)\left(d(12Abc-19aCc-32aBd)-\left(45aCd^2+2b\left(Cc^2-4Bdc-18Ad^2\right)\right)x\right)}{\sqrt{bx^2+a}}dx-\frac{1}{3}ad^2\sqrt{a+bx^2}(c+dx)^2(cC-4Bd)}{4bd^3}+\frac{aC\sqrt{a+bx^2}(c+dx)^3}{4bd}}{ab}+\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{\frac{1}{3}ad^2\left(\frac{3d\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(5aCd^2-4bc(2Bd+cC)\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2\sqrt{a+bx^2}\left(16ad^2(Bd+2cC)+bc\left(-24Ad^2-4Bcd+c^2C\right)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(45aCd^2+b\left(\cdots\right.\right.}{2\cdots}\right.}{4bd^3}}{ab}$$

$$\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{1}{3}ad^2\left(\frac{3d\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(5aCd^2-4bc(2Bd+cC)\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2\sqrt{a+bx^2}\left(16ad^2(Bd+2cC)+bc\left(-24Ad^2-4Bcd+c^2C\right)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(45c\right.}{}\right.}{4bd^3}}{ab}$$

$$\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow 219$$

$$\frac{\frac{\frac{1}{3}ad^2\left(\frac{3d\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(4Ab\left(2bc^2-3ad^2\right)+3a\left(5aCd^2-4bc(2Bd+cC)\right)\right)}{2b^{3/2}}-\frac{2\sqrt{a+bx^2}\left(16ad^2(Bd+2cC)+bc\left(-24Ad^2-4Bcd+c^2C\right)\right)}{b}-\frac{dx\sqrt{a+bx^2}\left(45c\right.}{}\right.}{4bd^3}}{ab}$$

$$\frac{(c+dx)^2\left(aB-bx\left(A-\frac{aC}{b}\right)\right)}{b^2\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((a*B - b*(A - (a*C)/b)*x)*(c + d*x)^2)/(b^2*Sqrt[a + b*x^2]) + ((a*C*(c +
 d*x)^3*Sqrt[a + b*x^2])/(4*b*d) + (-1/3*(a*d^2*(c*C - 4*B*d)*(c + d*x)^2*
Sqrt[a + b*x^2]) + (a*d^2*((-2*(16*a*d^2*(2*c*C + B*d) + b*c*(c^2*C - 4*B*
c*d - 24*A*d^2))*Sqrt[a + b*x^2])/b - (d*(45*a*C*d^2 + b*(2*c^2*C - 8*B*c*
d - 36*A*d^2))*x*Sqrt[a + b*x^2])/(2*b) + (3*d*(4*A*b*(2*b*c^2 - 3*a*d^2)
+ 3*a*(5*a*C*d^2 - 4*b*c*(c*C + 2*B*d)))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/(2*b^(3/2))))/3)/(4*b*d^3))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.99

method	result
risch	$\frac{(6Cb\,d^2x^3+8Bb\,d^2x^2+16Cbcd\,x^2+12A\,d^2xb+24Bcdxb-21Ca\,d^2x+12xb\,c^2C+48Abcd-40aB\,d^2+24bB\,c^2-80Cacd)\sqrt{b\,x^2+a}}{24b^3} -$
default	$c(2Ad+Bc)\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+d(Bd+2Cc)\left(\frac{x^4}{3b\sqrt{b\,x^2+a}}-\frac{4a\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)}{3b}\right)+(A$

input
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*C*b*d^2*x^3+8*B*b*d^2*x^2+16*C*b*c*d*x^2+12*A*b*d^2*x+24*B*b*c*d*x
-21*C*a*d^2*x+12*C*b*c^2*x+48*A*b*c*d-40*B*a*d^2+24*B*b*c^2-80*C*a*c*d)*(b
*x^2+a)^(1/2)/b^3-1/8/b^3*(-8*a*(2*A*b*c*d-B*a*d^2+B*b*c^2-2*C*a*c*d)/(b*x
^2+a)^(1/2)+b*(12*A*a*b*d^2-8*A*b^2*c^2+24*B*a*b*c*d-15*C*a^2*d^2+12*C*a*b
*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-7*C*a
^2*d^2*x/(b*x^2+a)^(1/2)+4*A*a*b*d^2*x/(b*x^2+a)^(1/2)+4*C*a*b*c^2*x/(b*x^
2+a)^(1/2)+8*B*a*b*c*d*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.46

$$\int \frac{x^2(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/48*(3*(24*B*a^2*b*c*d + 4*(3*C*a^2*b - 2*A*a*b^2)*c^2 - 3*(5*C*a^3 - 4*
A*a^2*b)*d^2 + (24*B*a*b^2*c*d + 4*(3*C*a*b^2 - 2*A*b^3)*c^2 - 3*(5*C*a^2*
b - 4*A*a*b^2)*d^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*
x - a) + 2*(6*C*b^3*d^2*x^5 + 48*B*a*b^2*c^2 - 64*B*a^2*b*d^2 + 8*(2*C*b^3
*c*d + B*b^3*d^2)*x^4 + 3*(4*C*b^3*c^2 + 8*B*b^3*c*d - (5*C*a*b^2 - 4*A*b^
3)*d^2)*x^3 - 32*(4*C*a^2*b - 3*A*a*b^2)*c*d + 8*(3*B*b^3*c^2 - 4*B*a*b^2*
d^2 - 2*(4*C*a*b^2 - 3*A*b^3)*c*d)*x^2 + 3*(24*B*a*b^2*c*d + 4*(3*C*a*b^2
- 2*A*b^3)*c^2 - 3*(5*C*a^2*b - 4*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/(b^5*x
^2 + a*b^4), 1/24*(3*(24*B*a^2*b*c*d + 4*(3*C*a^2*b - 2*A*a*b^2)*c^2 - 3*(
5*C*a^3 - 4*A*a^2*b)*d^2 + (24*B*a*b^2*c*d + 4*(3*C*a*b^2 - 2*A*b^3)*c^2 -
 3*(5*C*a^2*b - 4*A*a*b^2)*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + (6*C*b^3*d^2*x^5 + 48*B*a*b^2*c^2 - 64*B*a^2*b*d^2 + 8*(2*C*b^3*c
*d + B*b^3*d^2)*x^4 + 3*(4*C*b^3*c^2 + 8*B*b^3*c*d - (5*C*a*b^2 - 4*A*b^3)
*d^2)*x^3 - 32*(4*C*a^2*b - 3*A*a*b^2)*c*d + 8*(3*B*b^3*c^2 - 4*B*a*b^2*d^
2 - 2*(4*C*a*b^2 - 3*A*b^3)*c*d)*x^2 + 3*(24*B*a*b^2*c*d + 4*(3*C*a*b^2 -
2*A*b^3)*c^2 - 3*(5*C*a^2*b - 4*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/(b^5*x^2
 + a*b^4)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^2(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2*(c + d*x)**2*(A + B*x + C*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.19

$$\int \frac{x^2(c+dx)^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx = \frac{Cd^2x^5}{4\sqrt{bx^2+a}b} - \frac{5\,Cad^2x^3}{8\sqrt{bx^2+a}b^2} + \frac{(2\,Ccd+Bd^2)x^4}{3\sqrt{bx^2+a}b} - \frac{Ac^2x}{\sqrt{bx^2+a}b} - \frac{15\,Ca^2d^2x}{8\sqrt{bx^2+a}b^3} + \frac{(Cc^2+2\,Bcd+Ad^2)x^3}{2\sqrt{bx^2+a}b} + \frac{Ac^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{15\,Ca^2d^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{7}{2}}} - \frac{4\,(2\,Ccd+Bd^2)ax^2}{3\sqrt{bx^2+a}b^2} + \frac{(Bc^2+2\,Acd)x^2}{\sqrt{bx^2+a}b} + \frac{3\,(Cc^2+2\,Bcd+Ad^2)ax}{2\sqrt{bx^2+a}b^2} - \frac{3\,(Cc^2+2\,Bcd+Ad^2)a\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{8\,(2\,Ccd+Bd^2)a^2}{3\sqrt{bx^2+a}b^3} + \frac{2\,(Bc^2+2\,Acd)a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
1/4*C*d^2*x^5/(sqrt(b*x^2 + a)*b) - 5/8*C*a*d^2*x^3/(sqrt(b*x^2 + a)*b^2)
+ 1/3*(2*C*c*d + B*d^2)*x^4/(sqrt(b*x^2 + a)*b) - A*c^2*x/(sqrt(b*x^2 + a)
*b) - 15/8*C*a^2*d^2*x/(sqrt(b*x^2 + a)*b^3) + 1/2*(C*c^2 + 2*B*c*d + A*d^
2)*x^3/(sqrt(b*x^2 + a)*b) + A*c^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 15/8*C
*a^2*d^2*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 4/3*(2*C*c*d + B*d^2)*a*x^2/(sqr
t(b*x^2 + a)*b^2) + (B*c^2 + 2*A*c*d)*x^2/(sqrt(b*x^2 + a)*b) + 3/2*(C*c^2
 + 2*B*c*d + A*d^2)*a*x/(sqrt(b*x^2 + a)*b^2) - 3/2*(C*c^2 + 2*B*c*d + A*d
^2)*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 8/3*(2*C*c*d + B*d^2)*a^2/(sqrt(b*x
^2 + a)*b^3) + 2*(B*c^2 + 2*A*c*d)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(2\left(\frac{3Cd^2x}{b}+\frac{4(2Cb^5cd+Bb^5d^2)}{b^6}\right)x+\frac{3(4Cb^5c^2+8Bb^5cd-5Cab^4d^2+4Ab^5d^2)}{b^6}\right)\cdots}{\cdots}$$
$$+\frac{(12Cabc^2-8Ab^2c^2+24Babcd-15Ca^2d^2+12Aabd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8b^{\frac{7}{2}}}$$

input
```
integrate(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/24*((((2*(3*C*d^2*x/b + 4*(2*C*b^5*c*d + B*b^5*d^2)/b^6)*x + 3*(4*C*b^5*
c^2 + 8*B*b^5*c*d - 5*C*a*b^4*d^2 + 4*A*b^5*d^2)/b^6)*x + 8*(3*B*b^5*c^2 -
 8*C*a*b^4*c*d + 6*A*b^5*c*d - 4*B*a*b^4*d^2)/b^6)*x + 3*(12*C*a*b^4*c^2 -
 8*A*b^5*c^2 + 24*B*a*b^4*c*d - 15*C*a^2*b^3*d^2 + 12*A*a*b^4*d^2)/b^6)*x
+ 16*(3*B*a*b^4*c^2 - 8*C*a^2*b^3*c*d + 6*A*a*b^4*c*d - 4*B*a^2*b^3*d^2)/b
^6)/sqrt(b*x^2 + a) + 1/8*(12*C*a*b*c^2 - 8*A*b^2*c^2 + 24*B*a*b*c*d - 15*
C*a^2*d^2 + 12*A*a*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^2\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^2*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^2(dx+c)^2\,(C\,x^2+Bx+A)}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

3.142 $\int \frac{x(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 241

$$\begin{aligned}\int \frac{x(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = & -\frac{Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd))}{b^3\sqrt{a+bx^2}} \\ & -\frac{(bc(Bc+2Ad)-ad(2cC+Bd))x}{b^2\sqrt{a+bx^2}} \\ & -\frac{(2aCd^2-b(c^2C+2Bcd+Ad^2))\sqrt{a+bx^2}}{b^3}+\frac{d(2cC+Bd)x\sqrt{a+bx^2}}{2b^2} \\ & +\frac{Cd^2(a+bx^2)^{3/2}}{3b^3}+\frac{(2bc(Bc+2Ad)-3ad(2cC+Bd))\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

output
```
-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))/b^3/(b*x^2+a)^(1/2)-(b*c
*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x/b^2/(b*x^2+a)^(1/2)-(2*a*C*d^2-b*(A*d^2+2*
B*c*d+C*c^2))*(b*x^2+a)^(1/2)/b^3+1/2*d*(B*d+2*C*c)*x*(b*x^2+a)^(1/2)/b^2+
1/3*C*d^2*(b*x^2+a)^(3/2)/b^3+1/2*(2*b*c*(2*A*d+B*c)-3*a*d*(B*d+2*C*c))*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.98

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}} dx = \frac{-6Ab^2c^2+12abc^2C+24abBcd+12aAbd^2-16a^2Cd^2-6b^2Bc^2x-}{}$$
$$+\frac{(-2bBc^2-4Abcd+6acCd+3aBd^2)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(-6*A*b^2*c^2 + 12*a*b*c^2*C + 24*a*b*B*c*d + 12*a*A*b*d^2 - 16*a^2*C*d^2
- 6*b^2*B*c^2*x - 12*A*b^2*c*d*x + 18*a*b*c*C*d*x + 9*a*b*B*d^2*x + 6*b^2*
c^2*C*x^2 + 12*b^2*B*c*d*x^2 + 6*A*b^2*d^2*x^2 - 8*a*b*C*d^2*x^2 + 6*b^2*c
*C*d*x^3 + 3*b^2*B*d^2*x^3 + 2*b^2*C*d^2*x^4)/(6*b^3*Sqrt[a + b*x^2]) + ((
-2*b*B*c^2 - 4*A*b*c*d + 6*a*c*C*d + 3*a*B*d^2)*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2176, 25, 2185, 27, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{ 2176}$$

$$-\frac{\int -\frac{(c+dx)\left(aCdx^2+a(cC+3Bd)x+a\left(Bc+2\left(A-\frac{aC}{b}\right)d\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(c+dx)\left(aCdx^2+a(cC+3Bd)x+a\left(Bc+2\left(A-\frac{aC}{b}\right)d\right)\right)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 2185

$$\frac{\frac{\int \frac{ad^2(c+dx)(3bBc+6Abd-8aCd+b(2cC+9Bd)x)}{\sqrt{bx^2+a}}dx}{3bd^2}+\frac{aC\sqrt{a+bx^2}(c+dx)^2}{3b}}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{a\int \frac{(c+dx)(3bBc+6Abd-8aCd+b(2cC+9Bd)x)}{\sqrt{bx^2+a}}dx}{3b}+\frac{aC\sqrt{a+bx^2}(c+dx)^2}{3b}}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 676

$$\frac{\frac{a\left(\frac{3}{2}(2bc(2Ad+Bc)-3ad(Bd+2cC))\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}\left(4aCd^2-b\left(3Ad^2+6Bcd+c^2C\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(9Bd+2cC)\right)}{3b}+\frac{aC\sqrt{a+bx^2}(c+dx)^2}{3b}}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{a\left(\frac{3}{2}(2bc(2Ad+Bc)-3ad(Bd+2cC))\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{2\sqrt{a+bx^2}\left(4aCd^2-b\left(3Ad^2+6Bcd+c^2C\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(9Bd+2cC)\right)}{3b}+\frac{aC\sqrt{a+bx^2}}{3b}}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{a\left(\frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bc(2Ad+Bc)-3ad(Bd+2cC))}{2\sqrt{b}}-\frac{2\sqrt{a+bx^2}\left(4aCd^2-b\left(3Ad^2+6Bcd+c^2C\right)\right)}{b}+\frac{1}{2}dx\sqrt{a+bx^2}(9Bd+2cC)\right)}{3b}+\frac{aC\sqrt{a+bx^2}(c+}{3b}}{ab}-\frac{(c+dx)^2\left(-\frac{aC}{b}+A+Bx\right)}{b\sqrt{a+bx^2}}$$

input
```
Int[(x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
-(((A - (a*C)/b + B*x)*(c + d*x)^2)/(b*Sqrt[a + b*x^2])) + ((a*C*(c + d*x)
^2*Sqrt[a + b*x^2])/(3*b) + (a*((-2*(4*a*C*d^2 - b*(c^2*C + 6*B*c*d + 3*A*
d^2))*Sqrt[a + b*x^2])/b + (d*(2*c*C + 9*B*d)*x*Sqrt[a + b*x^2])/2 + (3*(2
*b*c*(B*c + 2*A*d) - 3*a*d*(2*c*C + B*d))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*Sqrt[b])))/(3*b))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] &&  !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

method	result
risch	$\frac{\left(2C\,d^{2}b\,x^{2}+3Bb\,d^{2}x+6Cbcdx+6Ab\,d^{2}+12Bbcd-10aC\,d^{2}+6Cb\,c^{2}\right)\sqrt{b\,x^{2}+a}}{6b^{3}}+\frac{b\left(4Abcd-3aB\,d^{2}+2bB\,c^{2}-6Cacd\right)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}\right.}{}$
default	$c(2Ad+Bc)\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+d(Bd+2Cc)\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\right.}{b^{\frac{3}{2}}}\right.}{2b}\right.$

input
```
int(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x+6*C*b*c*d*x+6*A*b*d^2+12*B*b*c*d-10*C*a*d^2
+6*C*b*c^2)*(b*x^2+a)^(1/2)/b^3+1/2/b^2*(b*(4*A*b*c*d-3*B*a*d^2+2*B*b*c^2-
6*C*a*c*d)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-
(-2*A*a*b*d^2+2*A*b^2*c^2-4*B*a*b*c*d+2*C*a^2*d^2-2*C*a*b*c^2)/b/(b*x^2+a)
^(1/2)-B*a*d^2*x/(b*x^2+a)^(1/2)-2*C*a*c*d*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.34

$$\int \frac{x(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \Bigg[-\frac{3\,(2\,Babc^2-3\,Ba^2d^2-2\,(3\,Ca^2-2\,Aab)cd+(2\,Bb^2c^2-3\,Babd^2}{}$$

$$-\frac{3\,(2\,Babc^2-3\,Ba^2d^2-2\,(3\,Ca^2-2\,Aab)cd+(2\,Bb^2c^2-3\,Babd^2-2\,(3\,Cab-2\,Ab^2)cd)x^2)\sqrt{-b}\arctan}{}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*B*a*b*c^2 - 3*B*a^2*d^2 - 2*(3*C*a^2 - 2*A*a*b)*c*d + (2*B*b^
2*c^2 - 3*B*a*b*d^2 - 2*(3*C*a*b - 2*A*b^2)*c*d)*x^2)*sqrt(b)*log(-2*b*x^2
 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(2*C*b^2*d^2*x^4 + 24*B*a*b*c*d +
3*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 6*(2*C*a*b - A*b^2)*c^2 - 4*(4*C*a^2 - 3
*A*a*b)*d^2 + 2*(3*C*b^2*c^2 + 6*B*b^2*c*d - (4*C*a*b - 3*A*b^2)*d^2)*x^2
- 3*(2*B*b^2*c^2 - 3*B*a*b*d^2 - 2*(3*C*a*b - 2*A*b^2)*c*d)*x)*sqrt(b*x^2
+ a))/(b^4*x^2 + a*b^3), -1/6*(3*(2*B*a*b*c^2 - 3*B*a^2*d^2 - 2*(3*C*a^2 -
 2*A*a*b)*c*d + (2*B*b^2*c^2 - 3*B*a*b*d^2 - 2*(3*C*a*b - 2*A*b^2)*c*d)*x^
2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*C*b^2*d^2*x^4 + 24*B*a
*b*c*d + 3*(2*C*b^2*c*d + B*b^2*d^2)*x^3 + 6*(2*C*a*b - A*b^2)*c^2 - 4*(4*
C*a^2 - 3*A*a*b)*d^2 + 2*(3*C*b^2*c^2 + 6*B*b^2*c*d - (4*C*a*b - 3*A*b^2)*
d^2)*x^2 - 3*(2*B*b^2*c^2 - 3*B*a*b*d^2 - 2*(3*C*a*b - 2*A*b^2)*c*d)*x)*sq
rt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [F]

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(c + d*x)**2*(A + B*x + C*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\begin{aligned}\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx &= \frac{Cd^2x^4}{3\sqrt{bx^2+a}b} - \frac{4\,Cad^2x^2}{3\sqrt{bx^2+a}b^2} \\ &+ \frac{(2\,Ccd+Bd^2)x^3}{2\sqrt{bx^2+a}b} - \frac{Ac^2}{\sqrt{bx^2+a}b} - \frac{8\,Ca^2d^2}{3\sqrt{bx^2+a}b^3} + \frac{(Cc^2+2\,Bcd+Ad^2)x^2}{\sqrt{bx^2+a}b} \\ &+ \frac{3\,(2\,Ccd+Bd^2)ax}{2\sqrt{bx^2+a}b^2} - \frac{(Bc^2+2\,Acd)x}{\sqrt{bx^2+a}b} - \frac{3\,(2\,Ccd+Bd^2)a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} \\ &+ \frac{(Bc^2+2\,Acd)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,(Cc^2+2\,Bcd+Ad^2)a}{\sqrt{bx^2+a}b^2}\end{aligned}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*C*d^2*x^4/(sqrt(b*x^2 + a)*b) - 4/3*C*a*d^2*x^2/(sqrt(b*x^2 + a)*b^2)
+ 1/2*(2*C*c*d + B*d^2)*x^3/(sqrt(b*x^2 + a)*b) - A*c^2/(sqrt(b*x^2 + a)*b
) - 8/3*C*a^2*d^2/(sqrt(b*x^2 + a)*b^3) + (C*c^2 + 2*B*c*d + A*d^2)*x^2/(s
qrt(b*x^2 + a)*b) + 3/2*(2*C*c*d + B*d^2)*a*x/(sqrt(b*x^2 + a)*b^2) - (B*c
^2 + 2*A*c*d)*x/(sqrt(b*x^2 + a)*b) - 3/2*(2*C*c*d + B*d^2)*a*arcsinh(b*x/
sqrt(a*b))/b^(5/2) + (B*c^2 + 2*A*c*d)*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*
(C*c^2 + 2*B*c*d + A*d^2)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.01

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\left(\frac{2\,Cd^2x}{b}+\frac{3\,(2\,Cb^4cd+Bb^4d^2)}{b^5}\right)x+\frac{2\,(3\,Cb^4c^2+6\,Bb^4cd-4\,Cab^3d^2+3\,Ab^4d^2)}{b^5}\right)x-\right.}{}$$

$$-\frac{(2\,Bbc^2-6\,Cacd+4\,Abcd-3\,Bad^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/6*((((2*C*d^2*x/b + 3*(2*C*b^4*c*d + B*b^4*d^2)/b^5)*x + 2*(3*C*b^4*c^2
+ 6*B*b^4*c*d - 4*C*a*b^3*d^2 + 3*A*b^4*d^2)/b^5)*x - 3*(2*B*b^4*c^2 - 6*C
*a*b^3*c*d + 4*A*b^4*c*d - 3*B*a*b^3*d^2)/b^5)*x + 2*(6*C*a*b^3*c^2 - 3*A*
b^4*c^2 + 12*B*a*b^3*c*d - 8*C*a^2*b^2*d^2 + 6*A*a*b^3*d^2)/b^5)/sqrt(b*x^
2 + a) - 1/2*(2*B*b*c^2 - 6*C*a*c*d + 4*A*b*c*d - 3*B*a*d^2)*log(abs(-sqrt
(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x\,(c+d\,x)^2\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int((x*(c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 645, normalized size of antiderivative = 2.68

$$\int \frac{x(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{27\sqrt{b}\,a^2b\,d^2+54\sqrt{b}\,a^2c^2d-24\sqrt{b}\,a\,b^2c^2-24\sqrt{b}\,b^3c^2x^2-36\sqrt{b}\log\Big(}{}$$

input
```
int(x*(d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
(48*sqrt(a + b*x**2)*a**2*b*d**2 - 64*sqrt(a + b*x**2)*a**2*c*d**2 - 24*sq
rt(a + b*x**2)*a*b**2*c**2 - 48*sqrt(a + b*x**2)*a*b**2*c*d*x + 96*sqrt(a
+ b*x**2)*a*b**2*c*d + 24*sqrt(a + b*x**2)*a*b**2*d**2*x**2 + 36*sqrt(a +
b*x**2)*a*b**2*d**2*x + 48*sqrt(a + b*x**2)*a*b*c**3 + 72*sqrt(a + b*x**2)
*a*b*c**2*d*x - 32*sqrt(a + b*x**2)*a*b*c*d**2*x**2 - 24*sqrt(a + b*x**2)*
b**3*c**2*x + 48*sqrt(a + b*x**2)*b**3*c*d*x**2 + 12*sqrt(a + b*x**2)*b**3
*d**2*x**3 + 24*sqrt(a + b*x**2)*b**2*c**3*x**2 + 24*sqrt(a + b*x**2)*b**2
*c**2*d*x**3 + 8*sqrt(a + b*x**2)*b**2*c*d**2*x**4 + 48*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*c*d - 36*sqrt(b)*log((sqrt(a + b*
x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2 - 72*sqrt(b)*log((sqrt(a + b*x**2)
 + sqrt(b)*x)/sqrt(a))*a**2*c**2*d + 24*sqrt(b)*log((sqrt(a + b*x**2) + sq
rt(b)*x)/sqrt(a))*a*b**2*c**2 + 48*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*a*b**2*c*d*x**2 - 36*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a*b**2*d**2*x**2 - 72*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a*b*c**2*d*x**2 + 24*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*b**3*c**2*x**2 - 48*sqrt(b)*a**2*b*c*d + 27*sqrt(b)*a**2*b*d**2
 + 54*sqrt(b)*a**2*c**2*d - 24*sqrt(b)*a*b**2*c**2 - 48*sqrt(b)*a*b**2*c*d
*x**2 + 27*sqrt(b)*a*b**2*d**2*x**2 + 54*sqrt(b)*a*b*c**2*d*x**2 - 24*sqrt
(b)*b**3*c**2*x**2)/(24*b**3*(a + b*x**2))
```

3.143 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 199

$$
\begin{aligned}
\int \frac{(c+dx)^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx &= -\frac{bc(Bc+2Ad)-ad(2cC+Bd)}{b^2\sqrt{a+bx^2}} \\
&+ \frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\,x}{ab^2\sqrt{a+bx^2}} + \frac{d(2cC+Bd)\sqrt{a+bx^2}}{b^2} \\
&+ \frac{Cd^2x\sqrt{a+bx^2}}{2b^2} - \frac{(3aCd^2-2b(c^2C+2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}
\end{aligned}
$$

output

```
-(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))/b^2/(b*x^2+a)^(1/2)+(A*b*(-a*d^2+b*c^2)
+a*(a*C*d^2-b*c*(2*B*d+C*c)))*x/a/b^2/(b*x^2+a)^(1/2)+d*(B*d+2*C*c)*(b*x^2
+a)^(1/2)/b^2+1/2*C*d^2*x*(b*x^2+a)^(1/2)/b^2-1/2*(3*a*C*d^2-2*b*(A*d^2+2*
B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}} \, dx = \frac{\frac{\sqrt{b}\left(2Ab^2c^2x+a^2d(8cC+4Bd+3Cdx)+ab\left(-2Ad(2c+dx)-2B\left(c^2+2cdx-d^2x^2\right)+Cx\left(-2c^2+4cdx\right.\right.\right.}{a\sqrt{a+bx^2}}}{2b^{5}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
((Sqrt[b]*(2*A*b^2*c^2*x + a^2*d*(8*c*C + 4*B*d + 3*C*d*x) + a*b*(-2*A*d*(
2*c + d*x) - 2*B*(c^2 + 2*c*d*x - d^2*x^2) + C*x*(-2*c^2 + 4*c*d*x + d^2*x
^2))))/(a*Sqrt[a + b*x^2]) + (3*a*C*d^2 - 2*b*(c^2*C + 2*B*c*d + A*d^2))*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2176, 25, 676, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \; 2176$$

$$-\frac{\int -\frac{(c+dx)(a(cC+2Bd)-(2Ab-3aC)dx)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(c+dx)(a(cC+2Bd)-(2Ab-3aC)dx)}{\sqrt{bx^2+a}}dx}{ab}-\frac{(c+dx)^2(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 676$$

$$\frac{\frac{a\left(d^2(2Ab-3aC)+2bc(2Bd+cC)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}-\frac{2d\sqrt{a+bx^2}(-aBd-2acC+Abc)}{b}-\frac{d^2x\sqrt{a+bx^2}(2Ab-3aC)}{2b}}{ab}-\frac{(c+dx)^2(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a\left(d^2(2Ab-3aC)+2bc(2Bd+cC)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}-\frac{2d\sqrt{a+bx^2}(-aBd-2acC+Abc)}{b}-\frac{d^2x\sqrt{a+bx^2}(2Ab-3aC)}{2b}}{ab}-\frac{(c+dx)^2(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(d^2(2Ab-3aC)+2bc(2Bd+cC)\right)}{2b^{3/2}}-\frac{2d\sqrt{a+bx^2}(-aBd-2acC+Abc)}{b}-\frac{d^2x\sqrt{a+bx^2}(2Ab-3aC)}{2b}}{ab}-\frac{(c+dx)^2(aB-x(Ab-aC))}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
-(((a*B - (A*b - a*C)*x)*(c + d*x)^2)/(a*b*Sqrt[a + b*x^2])) + ((-2*d*(A*b
*c - 2*a*c*C - a*B*d)*Sqrt[a + b*x^2])/b - ((2*A*b - 3*a*C)*d^2*x*Sqrt[a +
 b*x^2])/(2*b) + (a*((2*A*b - 3*a*C)*d^2 + 2*b*c*(c*C + 2*B*d))*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2)))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] && !LeQ[p, -1]
```

rule 2176
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(d + e*x)^m*(a + b*x^2)^(p + 1)*((a*S - b*R*x)/(2*a*b*(p
 + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(p
 + 1)*ExpandToSum[2*a*b*(p + 1)*(d + e*x)*Qx - a*e*S*m + b*d*R*(2*p + 3) +
b*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && !(IGtQ[m, 0] && R
ationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.89

method	result
risch	$\frac{d(Cxd+2Bd+4Cc)\sqrt{b\,x^2+a}}{2b^2}+\frac{b(2Ab\,d^2+4Bbcd-3aC\,d^2+2Cb\,c^2)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{2\left(2Abcd-aB\,d^2+bB\,c^2-2\right.}{\sqrt{b\,x^2+a}}}{2b^2}$
default	$\frac{A\,c^2x}{a\sqrt{b\,x^2+a}}-\frac{c(2Ad+Bc)}{b\sqrt{b\,x^2+a}}+d(Bd+2Cc)\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+(A\,d^2+2Bcd+C\,c^2)\left(-\frac{x}{b\sqrt{b\,x^2+a}}\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(C*d*x+2*B*d+4*C*c)*(b*x^2+a)^(1/2)/b^2+1/2/b^2*(b*(2*A*b*d^2+4*B*b*
c*d-3*C*a*d^2+2*C*b*c^2)*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x
^2+a)^(1/2)))-2*(2*A*b*c*d-B*a*d^2+B*b*c^2-2*C*a*c*d)/(b*x^2+a)^(1/2)+2*A*
b^2*c^2*x/a/(b*x^2+a)^(1/2)-a*C*d^2/(b*x^2+a)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 538, normalized size of antiderivative = 2.70

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \Bigg[\frac{(2\,Ca^2bc^2+4\,Ba^2bcd-(3\,Ca^3-2\,Aa^2b)d^2+(2\,Cab^2c^2+4\,Bab^2cd$$

$$-\frac{(2\,Ca^2bc^2+4\,Ba^2bcd-(3\,Ca^3-2\,Aa^2b)d^2+(2\,Cab^2c^2+4\,Bab^2cd-(3\,Ca^2b-2\,Aab^2)d^2)x^2)\sqrt{-b}\,\mathrm{arct}}{}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((2*C*a^2*b*c^2 + 4*B*a^2*b*c*d - (3*C*a^3 - 2*A*a^2*b)*d^2 + (2*C*a*
b^2*c^2 + 4*B*a*b^2*c*d - (3*C*a^2*b - 2*A*a*b^2)*d^2)*x^2)*sqrt(b)*log(-2
*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a*b^2*d^2*x^3 - 2*B*a*b^2
*c^2 + 4*B*a^2*b*d^2 + 4*(2*C*a^2*b - A*a*b^2)*c*d + 2*(2*C*a*b^2*c*d + B*
a*b^2*d^2)*x^2 - (4*B*a*b^2*c*d + 2*(C*a*b^2 - A*b^3)*c^2 - (3*C*a^2*b - 2
*A*a*b^2)*d^2)*x)*sqrt(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), -1/2*((2*C*a^2*b
*c^2 + 4*B*a^2*b*c*d - (3*C*a^3 - 2*A*a^2*b)*d^2 + (2*C*a*b^2*c^2 + 4*B*a*
b^2*c*d - (3*C*a^2*b - 2*A*a*b^2)*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (C*a*b^2*d^2*x^3 - 2*B*a*b^2*c^2 + 4*B*a^2*b*d^2 + 4*(2*C*
a^2*b - A*a*b^2)*c*d + 2*(2*C*a*b^2*c*d + B*a*b^2*d^2)*x^2 - (4*B*a*b^2*c*
d + 2*(C*a*b^2 - A*b^3)*c^2 - (3*C*a^2*b - 2*A*a*b^2)*d^2)*x)*sqrt(b*x^2 +
 a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\, dx = \int \frac{(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(a + b*x**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.14

$$\begin{aligned}\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\, dx &= \frac{Cd^2x^3}{2\sqrt{bx^2+a}b} + \frac{Ac^2x}{\sqrt{bx^2+a}a} \\ &+ \frac{3\,Cad^2x}{2\sqrt{bx^2+a}b^2} - \frac{3\,Cad^2 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} - \frac{Bc^2}{\sqrt{bx^2+a}b} \\ &- \frac{2\,Acd}{\sqrt{bx^2+a}b} + \frac{(2\,Ccd+Bd^2)x^2}{\sqrt{bx^2+a}b} - \frac{(Cc^2+2\,Bcd+Ad^2)x}{\sqrt{bx^2+a}b} \\ &+ \frac{(Cc^2+2\,Bcd+Ad^2)\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,(2\,Ccd+Bd^2)a}{\sqrt{bx^2+a}b^2}\end{aligned}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*C*d^2*x^3/(sqrt(b*x^2 + a)*b) + A*c^2*x/(sqrt(b*x^2 + a)*a) + 3/2*C*a*
d^2*x/(sqrt(b*x^2 + a)*b^2) - 3/2*C*a*d^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) -
 B*c^2/(sqrt(b*x^2 + a)*b) - 2*A*c*d/(sqrt(b*x^2 + a)*b) + (2*C*c*d + B*d^
2)*x^2/(sqrt(b*x^2 + a)*b) - (C*c^2 + 2*B*c*d + A*d^2)*x/(sqrt(b*x^2 + a)*
b) + (C*c^2 + 2*B*c*d + A*d^2)*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 2*(2*C*c*d
 + B*d^2)*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{Cd^2x}{b}+\frac{2\,(2\,Cab^3cd+Bab^3d^2)}{ab^4}\right)x-\frac{2\,Cab^3c^2-2\,Ab^4c^2+4\,Bab^3cd-3\,Ca^2b^2d^2+2\,Aab^3d^{\cdots}}{ab^4}\cdots}{2\,\sqrt{bx^2+a}}$$
$$-\frac{(2\,Cbc^2+4\,Bbcd-3\,Cad^2+2\,Abd^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((C*d^2*x/b + 2*(2*C*a*b^3*c*d + B*a*b^3*d^2)/(a*b^4))*x - (2*C*a*b^3
*c^2 - 2*A*b^4*c^2 + 4*B*a*b^3*c*d - 3*C*a^2*b^2*d^2 + 2*A*a*b^3*d^2)/(a*b
^4))*x - 2*(B*a*b^3*c^2 - 4*C*a^2*b^2*c*d + 2*A*a*b^3*c*d - 2*B*a^2*b^2*d^
2)/(a*b^4))/sqrt(b*x^2 + a) - 1/2*(2*C*b*c^2 + 4*B*b*c*d - 3*C*a*d^2 + 2*A
*b*d^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{{(c+d\,x)}^2\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{(a+bx^2)^{3/2}}\,dx = \int \frac{(dx+c)^2 (C\,x^2+Bx+A)}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

output
```
int((d*x+c)^2*(C*x^2+B*x+A)/(b*x^2+a)^(3/2),x)
```

3.144 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 181

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x(a+bx^2)^{3/2}}\,dx = \frac{Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd))}{ab^2\sqrt{a+bx^2}} + \frac{(bc(Bc+2Ad)-ad(2cC+Bd))x}{ab\sqrt{a+bx^2}} + \frac{Cd^2\sqrt{a+bx^2}}{b^2} + \frac{d(2cC+Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{Ac^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output

```
(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))/a/b^2/(b*x^2+a)^(1/2)+(b*
c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x/a/b/(b*x^2+a)^(1/2)+C*d^2*(b*x^2+a)^(1/2)
/b^2+d*(B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-A*c^2*arctan
h((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{3/2}} dx = \frac{2a^2Cd^2 + b^2Bc^2x - ab(c^2C + d^2x(B-Cx) + 2cd(B+Cx)) + Ab(-a}{ab^2\sqrt{a+bx^2}}$$
$$+ \frac{2Ac^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{d(2cC+Bd)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(2*a^2*C*d^2 + b^2*B*c^2*x - a*b*(c^2*C + d^2*x*(B - C*x) + 2*c*d*(B + C*x
)) + A*b*(-(a*d^2) + b*c*(c + 2*d*x)))/(a*b^2*Sqrt[a + b*x^2]) + (2*A*c^2*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2) - (d*(2*c*C + B*d)
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2336, 25, 2340, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{a^2Cd^2}{b} + x(bc(2Ad+Bc) - ad(Bd+2cC)) - aAd^2 - 2aBcd - ac^2C + Abc^2}{ab\sqrt{a+bx^2}} - \frac{\int -\frac{Ac^2 + \frac{aCd^2x^2}{b} + \frac{ad(2cC+Bd)x}{b}}{x\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{Ac^2+\frac{aCd^2x^2}{b}+\frac{ad(2cC+Bd)x}{b}}{x\sqrt{bx^2+a}}dx}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}}{}$$

$$\Big\downarrow \text{2340}$$

$$\frac{\frac{\int \frac{Abc^2+ad(2cC+Bd)x}{x\sqrt{bx^2+a}}dx}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{538}$$

$$\frac{\frac{Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+ad(Bd+2cC)\int \frac{1}{\sqrt{bx^2+a}}dx}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+ad(Bd+2cC)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{Abc^2\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{1}{2}Abc^2\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{ad\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{Ac^2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{ad\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}+\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{a^2Cd^2}{b}+x(bc(2Ad+Bc)-ad(Bd+2cC))-aAd^2-2aBcd-ac^2C+Abc^2}{ab\sqrt{a+bx^2}}+\frac{\frac{\frac{ad\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(Bd+2cC)}{\sqrt{b}}-\frac{Abc^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{b}+\frac{aCd^2\sqrt{a+bx^2}}{b^2}}{a}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b*c^2 - a*c^2*C - 2*a*B*c*d - a*A*d^2 + (a^2*C*d^2)/b + (b*c*(B*c + 2*A
*d) - a*d*(2*c*C + B*d))*x)/(a*b*Sqrt[a + b*x^2]) + ((a*C*d^2*Sqrt[a + b*x
^2])/b^2 + ((a*d*(2*c*C + B*d)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[
b] - (A*b*c^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/b)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.42

method	result
default	$\frac{c^{2} B x}{a \sqrt{b \,x^{2}+a}}+A \,c^{2} \left(\frac{1}{a \sqrt{b \,x^{2}+a}}-\frac{\ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)-\frac{A \,d^{2}}{b \sqrt{b \,x^{2}+a}}+d^{2} B \left(-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
c^2*B*x/a/(b*x^2+a)^(1/2)+A*c^2*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x))-A*d^2/b/(b*x^2+a)^(1/2)+d^2*B*(-x/b/(b*x^2+a)^
(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))-C*c^2/b/(b*x^2+a)^(1/2)+C*d
^2*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+2*c*d*A*x/a/(b*x^2+a)^(
1/2)-2*B*c*d/b/(b*x^2+a)^(1/2)+2*c*d*C*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(
b^(1/2)*x+(b*x^2+a)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1003, normalized size of antiderivative = 5.54

$$\int \frac{(c+dx)^2 \left(A+Bx+Cx^2\right)}{x \left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((2*C*a^3*c*d + B*a^3*d^2 + (2*C*a^2*b*c*d + B*a^2*b*d^2)*x^2)*sqrt(b
)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + (A*b^3*c^2*x^2 + A*a*b
^2*c^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(C
*a^2*b*d^2*x^2 - 2*B*a^2*b*c*d - (C*a^2*b - A*a*b^2)*c^2 + (2*C*a^3 - A*a^
2*b)*d^2 + (B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a^2*b - A*a*b^2)*c*d)*x)*sqrt
(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), -1/2*(2*(2*C*a^3*c*d + B*a^3*d^2 + (
2*C*a^2*b*c*d + B*a^2*b*d^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) - (A*b^3*c^2*x^2 + A*a*b^2*c^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) - 2*(C*a^2*b*d^2*x^2 - 2*B*a^2*b*c*d - (C*a^2*b - A*
a*b^2)*c^2 + (2*C*a^3 - A*a^2*b)*d^2 + (B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a
^2*b - A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(2*(
A*b^3*c^2*x^2 + A*a*b^2*c^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) +
 (2*C*a^3*c*d + B*a^3*d^2 + (2*C*a^2*b*c*d + B*a^2*b*d^2)*x^2)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(C*a^2*b*d^2*x^2 - 2*B*a^
2*b*c*d - (C*a^2*b - A*a*b^2)*c^2 + (2*C*a^3 - A*a^2*b)*d^2 + (B*a*b^2*c^2
 - B*a^2*b*d^2 - 2*(C*a^2*b - A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x
^2 + a^3*b^2), -((2*C*a^3*c*d + B*a^3*d^2 + (2*C*a^2*b*c*d + B*a^2*b*d^2)*
x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (A*b^3*c^2*x^2 + A*a*b^
2*c^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (C*a^2*b*d^2*x^2 - 2*
B*a^2*b*c*d - (C*a^2*b - A*a*b^2)*c^2 + (2*C*a^3 - A*a^2*b)*d^2 + (B*a*...
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(x*(a + b*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{3/2}}\,dx = \frac{Cd^2x^2}{\sqrt{bx^2+a}b} + \frac{Bc^2x}{\sqrt{bx^2+a}a} + \frac{2\,Acdx}{\sqrt{bx^2+a}a} - \frac{Ac^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{Ac^2}{\sqrt{bx^2+a}a} - \frac{Cc^2}{\sqrt{bx^2+a}b} - \frac{2\,Bcd}{\sqrt{bx^2+a}b} + \frac{2\,Cad^2}{\sqrt{bx^2+a}b^2} - \frac{Ad^2}{\sqrt{bx^2+a}b} - \frac{(2\,Ccd+Bd^2)x}{\sqrt{bx^2+a}b} + \frac{(2\,Ccd+Bd^2) \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*d^2*x^2/(sqrt(b*x^2 + a)*b) + B*c^2*x/(sqrt(b*x^2 + a)*a) + 2*A*c*d*x/(s
qrt(b*x^2 + a)*a) - A*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A*c^2/(s
qrt(b*x^2 + a)*a) - C*c^2/(sqrt(b*x^2 + a)*b) - 2*B*c*d/(sqrt(b*x^2 + a)*b
) + 2*C*a*d^2/(sqrt(b*x^2 + a)*b^2) - A*d^2/(sqrt(b*x^2 + a)*b) - (2*C*c*d
 + B*d^2)*x/(sqrt(b*x^2 + a)*b) + (2*C*c*d + B*d^2)*arcsinh(b*x/sqrt(a*b))
/b^(3/2)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x (a+bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x\,(a+bx^2)^{3/2}}\,dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.96

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x\,(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b}\,a^2b\,d^2-2\sqrt{b}\,a^2c^2d+\sqrt{b}\,a\,b^2c^2+\sqrt{b}\,b^3c^2x^2+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right.}{}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b*d**2 + 2*sqrt(a + b*x**2)*a**2*c*d**2 + sqrt(a
 + b*x**2)*a*b**2*c**2 + 2*sqrt(a + b*x**2)*a*b**2*c*d*x - 2*sqrt(a + b*x*
*2)*a*b**2*c*d - sqrt(a + b*x**2)*a*b**2*d**2*x - sqrt(a + b*x**2)*a*b*c**
3 - 2*sqrt(a + b*x**2)*a*b*c**2*d*x + sqrt(a + b*x**2)*a*b*c*d**2*x**2 + s
qrt(a + b*x**2)*b**3*c**2*x + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sq
rt(b)*x)/sqrt(a))*a*b**2*c**2 + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
sqrt(b)*x)/sqrt(a))*b**3*c**2*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(
a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqr
t(a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**2 + sqrt(b)*log((sqrt(a + b*x**2)
+ sqrt(b)*x)/sqrt(a))*a**2*b*d**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt
(b)*x)/sqrt(a))*a**2*c**2*d + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/s
qrt(a))*a*b**2*d**2*x**2 + 2*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a*b*c**2*d*x**2 + 2*sqrt(b)*a**2*b*c*d - sqrt(b)*a**2*b*d**2 - 2*sq
rt(b)*a**2*c**2*d + sqrt(b)*a*b**2*c**2 + 2*sqrt(b)*a*b**2*c*d*x**2 - sqrt
(b)*a*b**2*d**2*x**2 - 2*sqrt(b)*a*b*c**2*d*x**2 + sqrt(b)*b**3*c**2*x**2)
/(a*b**2*(a + b*x**2))
```

3.145 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 186

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \frac{bc(Bc+2Ad)-ad(2cC+Bd)}{ab\sqrt{a+bx^2}} - \frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\,x}{a^2b\sqrt{a+bx^2}} - \frac{Ac^2\sqrt{a+bx^2}}{a^2x} + \frac{Cd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{c(Bc+2Ad)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
(b*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))/a/b/(b*x^2+a)^(1/2)-(A*b*(-a*d^2+b*c^2)+
a*(a*C*d^2-b*c*(2*B*d+C*c)))*x/a^2/b/(b*x^2+a)^(1/2)-A*c^2*(b*x^2+a)^(1/2)
/a^2/x+C*d^2*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)-c*(2*A*d+B*c)*arct
anh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2 (a+bx^2)^{3/2}}\,dx = \frac{-2Ab^2c^2x^2 - a^2dx(2cC+d(B+Cx)) + ab(cx(Bc+cCx+2Bdx)+}{a^2bx\sqrt{a+bx^2}}$$
$$-\frac{2c(Bc+2Ad)\operatorname{arctanh}\left(\frac{-\sqrt{b}x+\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{Cd^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*A*b^2*c^2*x^2 - a^2*d*x*(2*c*C + d*(B + C*x)) + a*b*(c*x*(B*c + c*C*x
+ 2*B*d*x) + A*(-c^2 + 2*c*d*x + d^2*x^2)))/(a^2*b*x*Sqrt[a + b*x^2]) - (2
*c*(B*c + 2*A*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2
) - (C*d^2*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2336, 25, 2338, 25, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2 (a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2336$$

$$\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}} - \frac{\int -\frac{Ac^2+(Bc+2Ad)xc+\frac{aCd^2x^2}{b}}{x^2\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{Ac^2+(Bc+2Ad)xc+\frac{aCd^2x^2}{b}}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}}{}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{a\left(aCxd^2+bc(Bc+2Ad)\right)}{bx\sqrt{bx^2+a}}dx}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{a\left(aCxd^2+bc(Bc+2Ad)\right)}{bx\sqrt{bx^2+a}}dx}{a}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{aCxd^2+bc(Bc+2Ad)}{x\sqrt{bx^2+a}}dx}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 538$$

$$\frac{\frac{bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+aCd^2\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+aCd^2\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{bc(2Ad+Bc)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{\frac{1}{2}bc(2Ad+Bc)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{c(2Ad+Bc)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}+\frac{aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{aCd^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{bc\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(2Ad+Bc)}{\sqrt{a}}}{b}-\frac{Ac^2\sqrt{a+bx^2}}{ax}}{a}+\frac{-bx\left(\frac{Abc^2}{a}+\frac{aCd^2}{b}-Ad^2-2Bcd-c^2C\right)-ad(Bd+2cC)+bc(2Ad+Bc)}{ab\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d) - b*((A*b*c^2)/a - c^2*C - 2*B*c*d
- A*d^2 + (a*C*d^2)/b)*x)/(a*b*Sqrt[a + b*x^2]) + (-((A*c^2*Sqrt[a + b*x^2
])/(a*x)) + ((a*C*d^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (b*c
*(B*c + 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/b)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.20

method	result
default	$\frac{A d^2 x}{a\sqrt{b x^2+a}}+\frac{C c^2 x}{a\sqrt{b x^2+a}}+A c^2\left(-\frac{1}{a x\sqrt{b x^2+a}}-\frac{2 b x}{a^2\sqrt{b x^2+a}}\right)-\frac{d^2 B}{b\sqrt{b x^2+a}}+C d^2\left(-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\, x+\sqrt{b x^2 \cdots\right.}{b^{\frac{3}{2}}}\cdots\right.$
risch	$-\frac{A c^2\sqrt{b x^2+a}}{a^2 x}+\frac{\frac{A d^2 x}{\sqrt{b x^2+a}}+\frac{C c^2 x}{\sqrt{b x^2+a}}-\frac{a B d^2}{b\sqrt{b x^2+a}}+a C d^2\left(-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}\right)+a c(2 A d+B c)\left(\frac{1}{a\sqrt{b x^2+a}}-\frac{\ln\left(2a \cdots\right.}{\cdots}\cdots\right.}{a}$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*d^2*x/a/(b*x^2+a)^(1/2)+C*c^2*x/a/(b*x^2+a)^(1/2)+A*c^2*(-1/a/x/(b*x^2+a
)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))-d^2*B/b/(b*x^2+a)^(1/2)+C*d^2*(-x/b/(b*
x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+c*(2*A*d+B*c)*(1/a/(
b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+2*B*c*d*x/
a/(b*x^2+a)^(1/2)-2*c*d*C/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 1034, normalized size of antiderivative = 5.56

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^2\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/2*((C*a^2*b*d^2*x^3 + C*a^3*d^2*x)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2
+ a)*sqrt(b)*x - a) + ((B*b^3*c^2 + 2*A*b^3*c*d)*x^3 + (B*a*b^2*c^2 + 2*A*
a*b^2*c*d)*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(A*a*b^2*c^2 - (2*B*a*b^2*c*d + (C*a*b^2 - 2*A*b^3)*c^2 - (C*a^2*b - A
*a*b^2)*d^2)*x^2 - (B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a^2*b - A*a*b^2)*c*d)
*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^3 + a^3*b^2*x), -1/2*(2*(C*a^2*b*d^2*x^3 +
 C*a^3*d^2*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - ((B*b^3*c^2 +
2*A*b^3*c*d)*x^3 + (B*a*b^2*c^2 + 2*A*a*b^2*c*d)*x)*sqrt(a)*log(-(b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(A*a*b^2*c^2 - (2*B*a*b^2*c*d +
(C*a*b^2 - 2*A*b^3)*c^2 - (C*a^2*b - A*a*b^2)*d^2)*x^2 - (B*a*b^2*c^2 - B*
a^2*b*d^2 - 2*(C*a^2*b - A*a*b^2)*c*d)*x)*sqrt(b*x^2 + a))/(a^2*b^3*x^3 +
a^3*b^2*x), 1/2*(2*((B*b^3*c^2 + 2*A*b^3*c*d)*x^3 + (B*a*b^2*c^2 + 2*A*a*b
^2*c*d)*x)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (C*a^2*b*d^2*x^3
+ C*a^3*d^2*x)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2
*(A*a*b^2*c^2 - (2*B*a*b^2*c*d + (C*a*b^2 - 2*A*b^3)*c^2 - (C*a^2*b - A*a*
b^2)*d^2)*x^2 - (B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a^2*b - A*a*b^2)*c*d)*x)
*sqrt(b*x^2 + a))/(a^2*b^3*x^3 + a^3*b^2*x), -((C*a^2*b*d^2*x^3 + C*a^3*d^
2*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - ((B*b^3*c^2 + 2*A*b^3*c
*d)*x^3 + (B*a*b^2*c^2 + 2*A*a*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt(b*x^2 + a)
*sqrt(-a)/a) + (A*a*b^2*c^2 - (2*B*a*b^2*c*d + (C*a*b^2 - 2*A*b^3)*c^2 ...
```

Sympy [F]

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(x**2*(a + b*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^2(a+bx^2)^{3/2}}\,dx = \frac{Cc^2x}{\sqrt{bx^2+a}a} - \frac{2\,Abc^2x}{\sqrt{bx^2+a}a^2} + \frac{2\,Bcdx}{\sqrt{bx^2+a}a} + \frac{Ad^2x}{\sqrt{bx^2+a}a} - \frac{Cd^2x}{\sqrt{bx^2+a}b} + \frac{Cd^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{2\,Ccd}{\sqrt{bx^2+a}b} - \frac{Bd^2}{\sqrt{bx^2+a}b} - \frac{Ac^2}{\sqrt{bx^2+a}ax} - \frac{(Bc^2+2\,Acd)\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{Bc^2+2\,Acd}{\sqrt{bx^2+a}a}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
C*c^2*x/(sqrt(b*x^2 + a)*a) - 2*A*b*c^2*x/(sqrt(b*x^2 + a)*a^2) + 2*B*c*d*
x/(sqrt(b*x^2 + a)*a) + A*d^2*x/(sqrt(b*x^2 + a)*a) - C*d^2*x/(sqrt(b*x^2
+ a)*b) + C*d^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2*C*c*d/(sqrt(b*x^2 + a)*
b) - B*d^2/(sqrt(b*x^2 + a)*b) - A*c^2/(sqrt(b*x^2 + a)*a*x) - (B*c^2 + 2*
A*c*d)*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + (B*c^2 + 2*A*c*d)/(sqrt(b*x
^2 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2 (a+bx^2)^{3/2}} \, dx =$$
$$-\frac{Cd^2 \log \left(\left| -\sqrt{b}x + \sqrt{bx^2+a} \right| \right)}{b^{\frac{3}{2}}} + \frac{2 A\sqrt{b}c^2}{\left(\left(\sqrt{b}x - \sqrt{bx^2+a} \right)^2 - a \right) a}$$
$$+ \frac{\frac{(Ca^2b^2c^2 - Aab^3c^2 + 2 Ba^2b^2cd - Ca^3bd^2 + Aa^2b^2d^2)x}{a^3b^2} + \frac{Ba^2b^2c^2 - 2 Ca^3bcd + 2 Aa^2b^2cd - Ba^3bd^2}{a^3b^2}}{\sqrt{bx^2+a}}$$
$$+ \frac{2 (Bc^2 + 2 Acd) \arctan \left(-\frac{\sqrt{b}x - \sqrt{bx^2+a}}{\sqrt{-a}} \right)}{\sqrt{-a}a}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-C*d^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 2*A*sqrt(b)*c^2/((
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a) + ((C*a^2*b^2*c^2 - A*a*b^3*c^2 +
2*B*a^2*b^2*c*d - C*a^3*b*d^2 + A*a^2*b^2*d^2)*x/(a^3*b^2) + (B*a^2*b^2*c^
2 - 2*C*a^3*b*c*d + 2*A*a^2*b^2*c*d - B*a^3*b*d^2)/(a^3*b^2))/sqrt(b*x^2 +
 a) + 2*(B*c^2 + 2*A*c*d)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/
(sqrt(-a)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2 (a+bx^2)^{3/2}} \, dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^2\,(b\,x^2+a)^{3/2}} \, dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^2*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 990, normalized size of antiderivative = 5.32

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^2 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^2/(b*x^2+a)^(3/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2*b**2*c**2 + 4*sqrt(a + b*x**2)*a**2*b**2*c*d*x
 + 2*sqrt(a + b*x**2)*a**2*b**2*d**2*x**2 - 2*sqrt(a + b*x**2)*a**2*b**2*d
**2*x - 4*sqrt(a + b*x**2)*a**2*b*c**2*d*x - 2*sqrt(a + b*x**2)*a**2*b*c*d
**2*x**2 - 4*sqrt(a + b*x**2)*a*b**3*c**2*x**2 + 2*sqrt(a + b*x**2)*a*b**3
*c**2*x + 4*sqrt(a + b*x**2)*a*b**3*c*d*x**2 + 2*sqrt(a + b*x**2)*a*b**2*c
**3*x**2 + 2*sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b
*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqr
t(b)*sqrt(a)*x))*a**2*b**2*c*d*x + sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2
) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*
sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b**3*c**2*x + 2*sqrt(a)*log(( - s
qrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x - sqrt(b)*sqrt(a)*x +
 a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b**3*c*d*x*
*3 + sqrt(a)*log(( - sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x
 - sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqr
t(a)*x))*b**4*c**2*x**3 - 2*sqrt(a)*log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b
)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/(sqrt(a)*sqrt(a + b
*x**2) + sqrt(b)*sqrt(a)*x))*a**2*b**2*c*d*x - sqrt(a)*log((sqrt(a)*sqrt(a
 + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sqrt(a)*x + a + b*x**2)/
(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x))*a*b**3*c**2*x - 2*sqrt(a)*
log((sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a + b*x**2)*x + sqrt(b)*sq...
```

3.146 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 207

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^3(a+bx^2)^{3/2}}\,dx = -\frac{Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd))}{a^2b\sqrt{a+bx^2}} - \frac{(bc(Bc+2Ad)-ad(2cC+Bd))x}{a^2\sqrt{a+bx^2}} - \frac{Ac^2\sqrt{a+bx^2}}{2a^2x^2} - \frac{c(Bc+2Ad)\sqrt{a+bx^2}}{a^2x} - \frac{(2ac(cC+2Bd)-A(3bc^2-2ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))/a^2/b/(b*x^2+a)^(1/2)-(b
*c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x/a^2/(b*x^2+a)^(1/2)-1/2*A*c^2*(b*x^2+a)^
(1/2)/a^2/x^2-c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a^2/x-1/2*(2*a*c*(2*B*d+C*c)-A
*(-2*a*d^2+3*b*c^2))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^3\,(a+bx^2)^{3/2}}\,dx = -\frac{3Abc^2\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$
$$-\frac{2a^2Cd^2x^2+b^2cx^2(3Ac+4Bcx+8Adx)+ab(A(c^2+4cdx-2d^2x^2)-2x(cCx(c+2dx)+B(-c^2+2c}{2a^2bx^2\sqrt{a+bx^2}}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-3*A*b*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2) - (2*a
^2*C*d^2*x^2 + b^2*c*x^2*(3*A*c + 4*B*c*x + 8*A*d*x) + a*b*(A*(c^2 + 4*c*d
*x - 2*d^2*x^2) - 2*x*(c*C*x*(c + 2*d*x) + B*(-c^2 + 2*c*d*x + d^2*x^2)))
+ 4*Sqrt[a]*b*(c^2*C + 2*B*c*d + A*d^2)*x^2*Sqrt[a + b*x^2]*ArcTanh[(-(Sqr
t[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(2*a^2*b*x^2*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2336, 25, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^3\,(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{\int -\frac{Ac^2+(Bc+2Ad)xc-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{Ac^2+(Bc+2Ad)xc-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 2338

$$\frac{-\frac{\int -\frac{2ac(Bc+2Ad)+\left(2ac(cC+2Bd)-A\left(3bc^2-2ad^2\right)\right)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{2ac(Bc+2Ad)+\left(2ac(cC+2Bd)-A\left(3bc^2-2ad^2\right)\right)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\left(2ac(2Bd+cC)-A\left(3bc^2-2ad^2\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{1}{2}\left(2ac(2Bd+cC)-A\left(3bc^2-2ad^2\right)\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 73

$$\frac{\frac{\frac{\left(2ac(2Bd+cC)-A\left(3bc^2-2ad^2\right)\right)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(2ac(2Bd+cC)-A\left(3bc^2-2ad^2\right)\right)}{\sqrt{a}}-\frac{2c\sqrt{a+bx^2}(2Ad+Bc)}{x}}{2a}-\frac{Ac^2\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{a\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Ad^2+2Bcd+c^2C\right)\right)+bx(bc(2Ad+Bc)-ad(Bd+2cC))}{a^2b\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*((A*b^2*c^2)/a + a*C*d^2 - b*(c^2*C + 2*B*c*d + A*d^2)) + b*(b*c*(B*c
 + 2*A*d) - a*d*(2*c*C + B*d))*x)/(a^2*b*Sqrt[a + b*x^2])) + (-1/2*(A*c^2*
Sqrt[a + b*x^2])/(a*x^2) + ((-2*c*(B*c + 2*A*d)*Sqrt[a + b*x^2])/x - ((2*a
*c*(c*C + 2*B*d) - A*(3*b*c^2 - 2*a*d^2))*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/Sqrt[a])/(2*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{c\sqrt{b\,x^{2}+a}\,(4Adx+2Bcx+Ac)}{2a^{2}x^{2}}-\frac{-\frac{A b\,c^{2}}{\sqrt{b\,x^{2}+a}}-\frac{2Ba\,d^{2}x}{\sqrt{b\,x^{2}+a}}-a(2Aa\,d^{2}-3bA\,c^{2}+4Bacd+2Ca\,c^{2})\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right.}{2a^{2}}$
default	$\frac{d^{2}Bx}{a\sqrt{b\,x^{2}+a}}+(A\,d^{2}+2Bcd+C\,c^{2})\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+A\,c^{2}\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{}{a\sqrt{}}\right.}{}\right.$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(b*x^2+a)^(1/2)*(4*A*d*x+2*B*c*x+A*c)/a^2/x^2-1/2/a^2*(-A*b*c^2/(b*
x^2+a)^(1/2)-2*B*a*d^2*x/(b*x^2+a)^(1/2)-a*(2*A*a*d^2-3*A*b*c^2+4*B*a*c*d+
2*C*a*c^2)*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x))+2*B*b*c^2*x/(b*x^2+a)^(1/2)+2*a^2*C*d^2/b/(b*x^2+a)^(1/2)-4*C*a*c*d
*x/(b*x^2+a)^(1/2)+4*A*b*c*d*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.69

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^3\,(a+bx^2)^{3/2}}\,dx = \left[\frac{((4\,Bab^2cd+2\,Aab^2d^2+(2\,Cab^2-3\,Ab^3)c^2)x^4+(4\,Ba^2bcd+2\,Aa^2b}{}\right.$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/4*(((4*B*a*b^2*c*d + 2*A*a*b^2*d^2 + (2*C*a*b^2 - 3*A*b^3)*c^2)*x^4 + (
4*B*a^2*b*c*d + 2*A*a^2*b*d^2 + (2*C*a^2*b - 3*A*a*b^2)*c^2)*x^2)*sqrt(a)*
log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(A*a^2*b*c^2 + 2*(
2*B*a*b^2*c^2 - B*a^2*b*d^2 - 2*(C*a^2*b - 2*A*a*b^2)*c*d)*x^3 - (4*B*a^2*
b*c*d + (2*C*a^2*b - 3*A*a*b^2)*c^2 - 2*(C*a^3 - A*a^2*b)*d^2)*x^2 + 2*(B*
a^2*b*c^2 + 2*A*a^2*b*c*d)*x)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + a^4*b*x^2),
1/2*(((4*B*a*b^2*c*d + 2*A*a*b^2*d^2 + (2*C*a*b^2 - 3*A*b^3)*c^2)*x^4 + (4
*B*a^2*b*c*d + 2*A*a^2*b*d^2 + (2*C*a^2*b - 3*A*a*b^2)*c^2)*x^2)*sqrt(-a)*
arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (A*a^2*b*c^2 + 2*(2*B*a*b^2*c^2 - B*a
^2*b*d^2 - 2*(C*a^2*b - 2*A*a*b^2)*c*d)*x^3 - (4*B*a^2*b*c*d + (2*C*a^2*b
- 3*A*a*b^2)*c^2 - 2*(C*a^3 - A*a^2*b)*d^2)*x^2 + 2*(B*a^2*b*c^2 + 2*A*a^2
*b*c*d)*x)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + a^4*b*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{3/2}} \, dx = \int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{\frac{3}{2}}} \, dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(x**3*(a + b*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{3/2}} \, dx &= \frac{2\,Ccdx}{\sqrt{bx^2+a}a} + \frac{Bd^2x}{\sqrt{bx^2+a}a} \\ &+ \frac{3\,Abc^2 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,Abc^2}{2\,\sqrt{bx^2+a}a^2} - \frac{Cd^2}{\sqrt{bx^2+a}b} \\ &- \frac{2\,(Bc^2+2\,Acd)bx}{\sqrt{bx^2+a}a^2} - \frac{(Cc^2+2\,Bcd+Ad^2)\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} \\ &+ \frac{Cc^2+2\,Bcd+Ad^2}{\sqrt{bx^2+a}a} - \frac{Ac^2}{2\,\sqrt{bx^2+a}ax^2} - \frac{Bc^2+2\,Acd}{\sqrt{bx^2+a}ax}\end{aligned}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
2*C*c*d*x/(sqrt(b*x^2 + a)*a) + B*d^2*x/(sqrt(b*x^2 + a)*a) + 3/2*A*b*c^2*
arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*A*b*c^2/(sqrt(b*x^2 + a)*a^2)
- C*d^2/(sqrt(b*x^2 + a)*b) - 2*(B*c^2 + 2*A*c*d)*b*x/(sqrt(b*x^2 + a)*a^2
) - (C*c^2 + 2*B*c*d + A*d^2)*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + (C*c
^2 + 2*B*c*d + A*d^2)/(sqrt(b*x^2 + a)*a) - 1/2*A*c^2/(sqrt(b*x^2 + a)*a*x
^2) - (B*c^2 + 2*A*c*d)/(sqrt(b*x^2 + a)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{3/2}}\, dx =$$
$$-\frac{\frac{(Ba^2b^2c^2-2\,Ca^3bcd+2\,Aa^2b^2cd-Ba^3bd^2)x}{a^4b}-\frac{Ca^3bc^2-Aa^2b^2c^2+2\,Ba^3bcd-Ca^4d^2+Aa^3bd^2}{a^4b}}{\sqrt{bx^2+a}}$$
$$+\frac{(2\,Cac^2-3\,Abc^2+4\,Bacd+2\,Aad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc^2+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c^2+4\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}cd+\left(\sqrt{b}x-\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((B*a^2*b^2*c^2 - 2*C*a^3*b*c*d + 2*A*a^2*b^2*c*d - B*a^3*b*d^2)*x/(a^4*b
) - (C*a^3*b*c^2 - A*a^2*b^2*c^2 + 2*B*a^3*b*c*d - C*a^4*d^2 + A*a^3*b*d^2
)/(a^4*b))/sqrt(b*x^2 + a) + (2*C*a*c^2 - 3*A*b*c^2 + 4*B*a*c*d + 2*A*a*d^
2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt
(b)*x - sqrt(b*x^2 + a))^3*A*b*c^2 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a
*sqrt(b)*c^2 + 4*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*c*d + (sqrt(b
)*x - sqrt(b*x^2 + a))*A*a*b*c^2 - 2*B*a^2*sqrt(b)*c^2 - 4*A*a^2*sqrt(b)*c
*d)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{3/2}}\, dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^3\,(b\,x^2+a)^{3/2}}\, dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^3*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 917, normalized size of antiderivative = 4.43

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^3 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^3/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2*b*c**2 - 4*sqrt(a + b*x**2)*a**2*b*c*d*x + 2*sqr
t(a + b*x**2)*a**2*b*d**2*x**2 - 2*sqrt(a + b*x**2)*a**2*c*d**2*x**2 - 3*s
qrt(a + b*x**2)*a*b**2*c**2*x**2 - 2*sqrt(a + b*x**2)*a*b**2*c**2*x - 8*sq
rt(a + b*x**2)*a*b**2*c*d*x**3 + 4*sqrt(a + b*x**2)*a*b**2*c*d*x**2 + 2*sq
rt(a + b*x**2)*a*b**2*d**2*x**3 + 2*sqrt(a + b*x**2)*a*b*c**3*x**2 + 4*sqr
t(a + b*x**2)*a*b*c**2*d*x**3 - 4*sqrt(a + b*x**2)*b**3*c**2*x**3 + 2*sqrt
(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2*x**2
 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*
c**2*x**2 + 4*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)
)*a*b**2*c*d*x**2 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*a*b**2*d**2*x**4 + 2*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b*c**3*x**2 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**3*c**2*x**4 + 4*sqrt(a)*log((sqrt(a + b*x**2)
- sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*c*d*x**4 + 2*sqrt(a)*log((sqrt(a + b*
x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*c**3*x**4 - 2*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*d**2*x**2 + 3*sqrt(a)*
log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*x**2 - 4
*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*
x**2 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b
**2*d**2*x**4 - 2*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/...
```

3.147 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 254

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^4(a+bx^2)^{3/2}}\,dx = -\frac{3bc(Bc+2Ad)-2ad(2cC+Bd)}{2a^2\sqrt{a+bx^2}} - \frac{c(Bc+2Ad)}{2ax^2\sqrt{a+bx^2}} + \frac{(Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd)))\,x}{a^3\sqrt{a+bx^2}} - \frac{Ac^2\sqrt{a+bx^2}}{3a^2x^3} - \frac{(3ac(cC+2Bd)-A(5bc^2-3ad^2))\sqrt{a+bx^2}}{3a^3x} + \frac{(3bc(Bc+2Ad)-2ad(2cC+Bd))\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/2*(3*b*c*(2*A*d+B*c)-2*a*d*(B*d+2*C*c))/a^2/(b*x^2+a)^(1/2)-1/2*c*(2*A*
d+B*c)/a/x^2/(b*x^2+a)^(1/2)+(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c
)))*x/a^3/(b*x^2+a)^(1/2)-1/3*A*c^2*(b*x^2+a)^(1/2)/a^2/x^3-1/3*(3*a*c*(2*
B*d+C*c)-A*(-3*a*d^2+5*b*c^2))*(b*x^2+a)^(1/2)/a^3/x+1/2*(3*b*c*(2*A*d+B*c
)-2*a*d*(B*d+2*C*c))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}} \, dx = \frac{16Ab^2c^2x^4 - abx^2(3cx(3Bc+4cCx+8Bdx)+A(-8c^2+18cdx+12d}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(16*A*b^2*c^2*x^4 - a*b*x^2*(3*c*x*(3*B*c + 4*c*C*x + 8*B*d*x) + A*(-8*c^2
 + 18*c*d*x + 12*d^2*x^2)) - a^2*(2*A*(c^2 + 3*c*d*x + 3*d^2*x^2) + 3*x*(B
*(c^2 + 4*c*d*x - 2*d^2*x^2) + 2*C*x*(c^2 - 2*c*d*x - d^2*x^2))) - 6*Sqrt[
a]*(-3*b*c*(B*c + 2*A*d) + 2*a*d*(2*c*C + B*d))*x^3*Sqrt[a + b*x^2]*ArcTan
h[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(6*a^3*x^3*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2336, 25, 2338, 25, 2338, 25, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{\int -\frac{-\frac{(bc(Bc+2Ad)-ad(2cC+Bd))x^3}{a}-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2+c(Bc+2Ad)x+Ac^2}{x^4\sqrt{bx^2+a}}dx}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{(bc(Bc+2Ad)-ad(2cC+Bd))x^3}{a}-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2+c(Bc+2Ad)x+Ac^2}{x^4\sqrt{bx^2+a}}dx}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{-\frac{\int -\frac{-3(bc(Bc+2Ad)-ad(2cC+Bd))x^2+\left(3ac(cC+2Bd)-A\left(5bc^2-3ad^2\right)\right)x+3ac(Bc+2Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{-3(bc(Bc+2Ad)-ad(2cC+Bd))x^2+\left(3ac(cC+2Bd)-A\left(5bc^2-3ad^2\right)\right)x+3ac(Bc+2Ad)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 2338}$$

$$\frac{\frac{-\frac{\int -\frac{a\left(2\left(3ac(cC+2Bd)-A\left(5bc^2-3ad^2\right)\right)-3(3bc(Bc+2Ad)-2ad(2cC+Bd))x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{a\left(2\left(3ac(cC+2Bd)-\frac{1}{2}A\left(10bc^2-6ad^2\right)\right)-3(3bc(Bc+2Ad)-2ad(2cC+Bd))x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{1}{2}\int \frac{2\left(3ac(cC+2Bd)-\frac{1}{2}A\left(10bc^2-6ad^2\right)\right)-3(3bc(Bc+2Ad)-2ad(2cC+Bd))x}{x^2\sqrt{bx^2+a}}dx-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\frac{\frac{1}{2}\left(-3(3bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(5bc^2-3ad^2\right)\right)}{ax}\right)-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{243}$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{3}{2}(3bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(5bc^2-3ad^2\right)\right)}{ax}\right)-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{73}$$

$$\frac{\frac{\frac{1}{2}\left(-\frac{3(3bc(2Ad+Bc)-2ad(Bd+2cC))\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(5bc^2-3ad^2\right)\right)}{ax}\right)-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{\frac{1}{2}\left(\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(3bc(2Ad+Bc)-2ad(Bd+2cC))}{\sqrt{a}}-\frac{2\sqrt{a+bx^2}\left(3ac(2Bd+cC)-A\left(5bc^2-3ad^2\right)\right)}{ax}\right)-\frac{3c\sqrt{a+bx^2}(2Ad+Bc)}{2x^2}}{3a}-\frac{Ac^2\sqrt{a+bx^2}}{3ax^3}}{a}-\frac{a(bc(2Ad+Bc)-ad(Bd+2cC))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)\right)}{a^3\sqrt{a+bx^2}}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b*c*(B*c + 2*A*d) - a*d*(2*c*C + B*d)) - (A*b*(b*c^2 - a*d^2) + a*(a
*C*d^2 - b*c*(c*C + 2*B*d)))*x)/(a^3*Sqrt[a + b*x^2])) + (-1/3*(A*c^2*Sqrt
[a + b*x^2])/(a*x^3) + ((-3*c*(B*c + 2*A*d)*Sqrt[a + b*x^2])/(2*x^2) + ((-
2*(3*a*c*(c*C + 2*B*d) - A*(5*b*c^2 - 3*a*d^2))*Sqrt[a + b*x^2])/(a*x) + (
3*(3*b*c*(B*c + 2*A*d) - 2*a*d*(2*c*C + B*d))*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a])/2)/(3*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.03

method	result
default	$\frac{C\,d^2x}{a\sqrt{b\,x^2+a}}+\left(A\,d^2+2Bcd+C\,c^2\right)\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)+A\,c^2\left(-\frac{1}{3a\,x^3\sqrt{b\,x^2+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^2+a}}\right.}{3a}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(6Aa\,d^2x^2-10Ab\,c^2x^2+12Bacd\,x^2+6Ca\,c^2x^2+6Aacdx+3Ba\,c^2x+2A\,c^2a\right)}{6a^3x^3}-\frac{a\left(6Abcd-2aB\,d^2+3bB\,c^2-4Cacd\right)\left(\right.}{}$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C*d^2*x/a/(b*x^2+a)^(1/2)+(A*d^2+2*B*c*d+C*c^2)*(-1/a/x/(b*x^2+a)^(1/2)-2*
b/a^2*x/(b*x^2+a)^(1/2))+A*c^2*(-1/3/a/x^3/(b*x^2+a)^(1/2)-4/3*b/a*(-1/a/x
/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2)))+c*(2*A*d+B*c)*(-1/2/a/x^2/(b*
x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x)))+d*(B*d+2*C*c)*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a
^(1/2)*(b*x^2+a)^(1/2))/x))
```

Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}}\,dx = \left[-\frac{3\left((3\,Bb^2c^2-2\,Babd^2-2\,(2\,Cab-3\,Ab^2)cd)x^5+(3\,Babc^2-2\,Ba\right.}{}\right.$$

$$-\frac{3\left((3\,Bb^2c^2-2\,Babd^2-2\,(2\,Cab-3\,Ab^2)cd)x^5+(3\,Babc^2-2\,Ba^2d^2-2\,(2\,Ca^2-3\,Aab)cd)x^3\right)\sqrt{-a}\,\mathrm{ar}}{}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[-1/12*(3*((3*B*b^2*c^2 - 2*B*a*b*d^2 - 2*(2*C*a*b - 3*A*b^2)*c*d)*x^5 + (
3*B*a*b*c^2 - 2*B*a^2*d^2 - 2*(2*C*a^2 - 3*A*a*b)*c*d)*x^3)*sqrt(a)*log(-(
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*A*a^2*c^2 + 2*(12*B*a
*b*c*d + 2*(3*C*a*b - 4*A*b^2)*c^2 - 3*(C*a^2 - 2*A*a*b)*d^2)*x^4 + 3*(3*B
*a*b*c^2 - 2*B*a^2*d^2 - 2*(2*C*a^2 - 3*A*a*b)*c*d)*x^3 + 2*(6*B*a^2*c*d +
 3*A*a^2*d^2 + (3*C*a^2 - 4*A*a*b)*c^2)*x^2 + 3*(B*a^2*c^2 + 2*A*a^2*c*d)*
x)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x^3), -1/6*(3*((3*B*b^2*c^2 - 2*B*a*b
*d^2 - 2*(2*C*a*b - 3*A*b^2)*c*d)*x^5 + (3*B*a*b*c^2 - 2*B*a^2*d^2 - 2*(2*
C*a^2 - 3*A*a*b)*c*d)*x^3)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (
2*A*a^2*c^2 + 2*(12*B*a*b*c*d + 2*(3*C*a*b - 4*A*b^2)*c^2 - 3*(C*a^2 - 2*A
*a*b)*d^2)*x^4 + 3*(3*B*a*b*c^2 - 2*B*a^2*d^2 - 2*(2*C*a^2 - 3*A*a*b)*c*d)
*x^3 + 2*(6*B*a^2*c*d + 3*A*a^2*d^2 + (3*C*a^2 - 4*A*a*b)*c^2)*x^2 + 3*(B*
a^2*c^2 + 2*A*a^2*c*d)*x)*sqrt(b*x^2 + a))/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^4\,(a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**4/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(x**4*(a + b*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}}\,dx = \frac{8\,Ab^2c^2x}{3\,\sqrt{bx^2+a}a^3} + \frac{Cd^2x}{\sqrt{bx^2+a}a} + \frac{4\,Abc^2}{3\,\sqrt{bx^2+a}a^2x} - \frac{2\,(Cc^2+2\,Bcd+Ad^2)bx}{\sqrt{bx^2+a}a^2} - \frac{(2\,Ccd+Bd^2)\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{3\,(Bc^2+2\,Acd)b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} + \frac{2\,Ccd+Bd^2}{\sqrt{bx^2+a}a} - \frac{3\,(Bc^2+2\,Acd)b}{2\,\sqrt{bx^2+a}a^2} - \frac{Ac^2}{3\,\sqrt{bx^2+a}ax^3} - \frac{Cc^2+2\,Bcd+Ad^2}{\sqrt{bx^2+a}ax} - \frac{Bc^2+2\,Acd}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
8/3*A*b^2*c^2*x/(sqrt(b*x^2 + a)*a^3) + C*d^2*x/(sqrt(b*x^2 + a)*a) + 4/3*
A*b*c^2/(sqrt(b*x^2 + a)*a^2*x) - 2*(C*c^2 + 2*B*c*d + A*d^2)*b*x/(sqrt(b*
x^2 + a)*a^2) - (2*C*c*d + B*d^2)*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
3/2*(B*c^2 + 2*A*c*d)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + (2*C*c*d +
 B*d^2)/(sqrt(b*x^2 + a)*a) - 3/2*(B*c^2 + 2*A*c*d)*b/(sqrt(b*x^2 + a)*a^2
) - 1/3*A*c^2/(sqrt(b*x^2 + a)*a*x^3) - (C*c^2 + 2*B*c*d + A*d^2)/(sqrt(b*
x^2 + a)*a*x) - 1/2*(B*c^2 + 2*A*c*d)/(sqrt(b*x^2 + a)*a*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(228) = 456$.

Time = 0.14 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.31

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^4 (a+bx^2)^{3/2}}\,dx = -\frac{\frac{(Ca^3bc^2-Aa^2b^2c^2+2\,Ba^3bcd-Ca^4d^2+Aa^3bd^2)x}{a^5}+\frac{Ba^3bc^2-2\,Ca^4cd+2\,Aa^3bcd-Ba^4d^2}{a^5}}{\sqrt{bx^2+a}} - \frac{(3\,Bbc^2-4\,Cacd+6\,Abcd-2\,Bad^2)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Bbc^2+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^5Abcd+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ca\sqrt{b}c^2-6\left(\sqrt{b}x-\right.}{}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((C*a^3*b*c^2 - A*a^2*b^2*c^2 + 2*B*a^3*b*c*d - C*a^4*d^2 + A*a^3*b*d^2)*
x/a^5 + (B*a^3*b*c^2 - 2*C*a^4*c*d + 2*A*a^3*b*c*d - B*a^4*d^2)/a^5)/sqrt(
b*x^2 + a) - (3*B*b*c^2 - 4*C*a*c*d + 6*A*b*c*d - 2*B*a*d^2)*arctan(-(sqrt
(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*(sqrt(b)*x - sq
rt(b*x^2 + a))^5*B*b*c^2 + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*b*c*d + 6*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a*sqrt(b)*c^2 - 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*A*b^(3/2)*c^2 + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a*sqrt(b)*c*
d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a*sqrt(b)*d^2 - 12*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*C*a^2*sqrt(b)*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*
a*b^(3/2)*c^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b)*c*d - 12*
(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^2*sqrt(b)*d^2 - 3*(sqrt(b)*x - sqrt(b*
x^2 + a))*B*a^2*b*c^2 - 6*(sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b*c*d + 6*C*
a^3*sqrt(b)*c^2 - 10*A*a^2*b^(3/2)*c^2 + 12*B*a^3*sqrt(b)*c*d + 6*A*a^3*sq
rt(b)*d^2)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^4\,(a+bx^2)^{3/2}}\,dx=\int \frac{{\left(c+d\,x\right)}^2\,\left(C\,x^2+B\,x+A\right)}{x^4\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^4*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.22

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^4\,(a+bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^4/(b*x^2+a)^(3/2),x)
```

output

```
( - 2*sqrt(a + b*x**2)*a**3*b*c**2 - 6*sqrt(a + b*x**2)*a**3*b*c*d*x - 6*s
qrt(a + b*x**2)*a**3*b*d**2*x**2 + 8*sqrt(a + b*x**2)*a**2*b**2*c**2*x**2
- 3*sqrt(a + b*x**2)*a**2*b**2*c**2*x - 18*sqrt(a + b*x**2)*a**2*b**2*c*d*
x**3 - 12*sqrt(a + b*x**2)*a**2*b**2*c*d*x**2 - 12*sqrt(a + b*x**2)*a**2*b
**2*d**2*x**4 + 6*sqrt(a + b*x**2)*a**2*b**2*d**2*x**3 - 6*sqrt(a + b*x**2
)*a**2*b*c**3*x**2 + 12*sqrt(a + b*x**2)*a**2*b*c**2*d*x**3 + 6*sqrt(a + b
*x**2)*a**2*b*c*d**2*x**4 + 16*sqrt(a + b*x**2)*a*b**3*c**2*x**4 - 9*sqrt(
a + b*x**2)*a*b**3*c**2*x**3 - 24*sqrt(a + b*x**2)*a*b**3*c*d*x**4 - 12*sq
rt(a + b*x**2)*a*b**2*c**3*x**4 - 18*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*d*x**3 + 6*sqrt(a)*log((sqrt(a + b*x*
*2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d**2*x**3 + 12*sqrt(a)*log((
sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*c**2*d*x**3 - 9*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**3*c**2*x*
*3 - 18*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*
*3*c*d*x**5 + 6*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a*b**3*d**2*x**5 + 12*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*b**2*c**2*d*x**5 - 9*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**4*c**2*x**5 + 18*sqrt(a)*log((sqrt(a + b*x**2)
 + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*c*d*x**3 - 6*sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*d**2*x**3 - 12*sq...
```

3.148 $\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 313

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx = \frac{Ab(bc^2-ad^2)+a(aCd^2-bc(cC+2Bd))}{a^3\sqrt{a+bx^2}} + \frac{b(bc(Bc+2Ad)-ad(2cC+Bd))x}{a^3\sqrt{a+bx^2}} - \frac{Ac^2\sqrt{a+bx^2}}{4a^2x^4} - \frac{c(Bc+2Ad)\sqrt{a+bx^2}}{3a^2x^3} - \frac{(4ac(cC+2Bd)-A(7bc^2-4ad^2))\sqrt{a+bx^2}}{8a^3x^2} + \frac{(5bc(Bc+2Ad)-3ad(2cC+Bd))\sqrt{a+bx^2}}{3a^3x} - \frac{(3Ab(5bc^2-4ad^2)+4a(2aCd^2-3bc(cC+2Bd)))\,\mathrm{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(2*B*d+C*c)))/a^3/(b*x^2+a)^(1/2)+b*(b*
c*(2*A*d+B*c)-a*d*(B*d+2*C*c))*x/a^3/(b*x^2+a)^(1/2)-1/4*A*c^2*(b*x^2+a)^(
1/2)/a^2/x^4-1/3*c*(2*A*d+B*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/8*(4*a*c*(2*B*d+C
*c)-A*(-4*a*d^2+7*b*c^2))*(b*x^2+a)^(1/2)/a^3/x^2+1/3*(5*b*c*(2*A*d+B*c)-3
*a*d*(B*d+2*C*c))*(b*x^2+a)^(1/2)/a^3/x-1/8*(3*A*b*(-4*a*d^2+5*b*c^2)+4*a*
(2*a*C*d^2-3*b*c*(2*B*d+C*c)))*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}} dx = \frac{(15Ab^2c^2+8a^2Cd^2) \operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{4a^{7/2}}$$
$$+ \frac{b^2cx^4(45Ac+64Bcx+128Adx) - 2a^2(6Cx^2(c^2+4cdx-2d^2x^2)+4Bx(c^2+3cdx+3d^2x^2)+A(3c^2+}{}$$

input
```
Integrate[((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
((15*A*b^2*c^2 + 8*a^2*C*d^2)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a
]])/(4*a^(7/2)) + (b^2*c*x^4*(45*A*c + 64*B*c*x + 128*A*d*x) - 2*a^2*(6*C*
x^2*(c^2 + 4*c*d*x - 2*d^2*x^2) + 4*B*x*(c^2 + 3*c*d*x + 3*d^2*x^2) + A*(3
*c^2 + 8*c*d*x + 6*d^2*x^2)) - a*b*x^2*(A*(-15*c^2 - 64*c*d*x + 36*d^2*x^2
) + 4*x*(3*c*C*x*(3*c + 8*d*x) + B*(-8*c^2 + 18*c*d*x + 12*d^2*x^2))) + 72
*Sqrt[a]*b*(c^2*C + 2*B*c*d + A*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(-(Sqrt[b
]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/(24*a^3*x^4*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2336, 25, 2338, 25, 2338, 25, 2338, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}} dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab(bc^2-ad^2)+a(aCd^2-bc(2Bd+cC))}{a^3\sqrt{a+bx^2}}-\frac{\int -\frac{\frac{\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)x^4}{a^2}-\frac{(bc(Bc+2Ad)-ad(2cC+Bd))x^3}{a}-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2+c(Bc+2Ad)x+Ac^2}{x^5\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)x^4}{a^2}-\frac{(bc(Bc+2Ad)-ad(2cC+Bd))x^3}{a}-\left(\frac{Abc^2}{a}-Cc^2-2Bdc-Ad^2\right)x^2+c(Bc+2Ad)x+Ac^2}{x^5\sqrt{bx^2+a}}dx}{a}+\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab(bc^2-ad^2)+a(aCd^2-bc(2Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\int -\frac{4\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Cc^2+2Bdc+Ad^2\right)\right)x^3-4(bc(Bc+2Ad)-ad(2cC+Bd))x^2+\left(4ac(cC+2Bd)-A\left(7bc^2-4ad^2\right)\right)x+4ac(Bc+2Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}}{a} \cdots$$
$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab(bc^2-ad^2)+a(aCd^2-bc(2Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{4\left(\frac{Ab^2c^2}{a}+aCd^2-b\left(Cc^2+2Bdc+Ad^2\right)\right)x^3-4(bc(Bc+2Ad)-ad(2cC+Bd))x^2+\left(4ac(cC+2Bd)-A\left(7bc^2-4ad^2\right)\right)x+4ac(Bc+2Ad)}{x^4\sqrt{bx^2+a}}dx}{4a}-\frac{Ac^2\sqrt{a+bx^2}}{4ax^4}}{a}+$$
$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab(bc^2-ad^2)+a(aCd^2-bc(2Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{-\frac{\frac{\int -\frac{12\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)x^2-4a(5bc(Bc+2Ad)-3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)-A\left(7bc^2-4ad^2\right)\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4c\sqrt{a+bx^2}(2Ad+Bc)}{3x^3}}{4a}- \cdots}{a}$$
$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab(bc^2-ad^2)+a(aCd^2-bc(2Bd+cC))}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\int \frac{12\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC+2Bd)\right)\right)x^2-4a(5bc(Bc+2Ad)-3ad(2cC+Bd))x+3a\left(4ac(cC+2Bd)-A\left(7bc^2-4ad^2\right)\right)}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{4c\sqrt{a+bx^2}(2Ad+Bc)}{3x^3}}{4a}-\frac{Ac^2}{\ldots}}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 2338$$

$$\frac{\frac{\frac{-\frac{\int \frac{a\left(8a(5bc(Bc+2Ad)-3ad(2cC+Bd))-3\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(cC+2Bd)\right)\right)x\right)}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2-4ad^2\right)\right)}{2x^2}}{3a}-\frac{4c\sqrt{a+bx^2}(2\ldots}{3x^3}}{4a}\ldots}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{-\frac{1}{2}\int \frac{8a(5bc(Bc+2Ad)-3ad(2cC+Bd))-3\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(cC+2Bd)\right)\right)x}{x^2\sqrt{bx^2+a}}dx-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2-4ad^2\right)\right)}{2x^2}}{3a}-\frac{4c\sqrt{a+bx^2}(2Ad\ldots}{3x^3}}{4a}\ldots}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{\frac{1}{2}\left(3\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(2Bd+cC)\right)\right)\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{8\sqrt{a+bx^2}(5bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2-4ad^2\right)\right)}{2x^2}\ldots}{3a}\ldots}{4a}\ldots}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{\frac{1}{2}\left(\frac{3}{2}\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(2Bd+cC)\right)\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{8\sqrt{a+bx^2}(5bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2-4ad^2\ldots\right.\right.}{2x^2}\ldots}{3a}\ldots}{4a}\ldots}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\frac{1}{2}\left(\frac{3\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(2Bd+cC)\right)\right)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{8\sqrt{a+bx^2}(5bc(2Ad+Bc)-3ad(Bd+2cC))}{x}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2-4a\right.\right.}{2x^2}}{3a}}{4a}}{a}$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{bx(bc(2Ad+Bc)-ad(Bd+2cC))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(2Bd+cC)\right)}{a^3\sqrt{a+bx^2}}+$$

$$\frac{\frac{\frac{\frac{1}{2}\left(\frac{8\sqrt{a+bx^2}(5bc(2Ad+Bc)-3ad(Bd+2cC))}{x}-\frac{3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(3Ab\left(5bc^2-4ad^2\right)+4a\left(2aCd^2-3bc(2Bd+cC)\right)\right)}{\sqrt{a}}\right)-\frac{3\sqrt{a+bx^2}\left(4ac(2Bd+cC)-A\left(7bc^2\right.\right.}{2x^2}}{3a}}{4a}}{a}$$

input
```
Int[((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b*(b*c^2 - a*d^2) + a*(a*C*d^2 - b*c*(c*C + 2*B*d)) + b*(b*c*(B*c + 2*A
*d) - a*d*(2*c*C + B*d))*x)/(a^3*Sqrt[a + b*x^2]) + (-1/4*(A*c^2*Sqrt[a +
b*x^2])/(a*x^4) + ((-4*c*(B*c + 2*A*d)*Sqrt[a + b*x^2])/(3*x^3) + ((-3*(4*
a*c*(c*C + 2*B*d) - A*(7*b*c^2 - 4*a*d^2))*Sqrt[a + b*x^2])/(2*x^2) + ((8*
(5*b*c*(B*c + 2*A*d) - 3*a*d*(2*c*C + B*d))*Sqrt[a + b*x^2])/x - (3*(3*A*b
*(5*b*c^2 - 4*a*d^2) + 4*a*(2*a*C*d^2 - 3*b*c*(c*C + 2*B*d)))*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2)/(3*a))/(4*a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-80Abcd\,x^3+24Ba\,d^2x^3-40Bb\,c^2x^3+48Cacd\,x^3+12Aa\,d^2x^2-21Ab\,c^2x^2+24Bacd\,x^2+12Ca\,c^2x^2+16Aacdx+8Ba\,c$ ... $}{24a^3x^4}$
default	$(A\,d^2+2Bcd+C\,c^2)\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+A\,c^2\left(-\frac{1}{4a\,x^4\sqrt{b\,x^2+a}}\right.$ ...

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(b*x^2+a)^(1/2)*(-80*A*b*c*d*x^3+24*B*a*d^2*x^3-40*B*b*c^2*x^3+48*C*
a*c*d*x^3+12*A*a*d^2*x^2-21*A*b*c^2*x^2+24*B*a*c*d*x^2+12*C*a*c^2*x^2+16*A
*a*c*d*x+8*B*a*c^2*x+6*A*a*c^2)/a^3/x^4-1/8/a^3*(-b*(4*A*a*d^2-7*A*b*c^2+8
*B*a*c*d+4*C*a*c^2)/(b*x^2+a)^(1/2)+a*(12*A*a*b*d^2-15*A*b^2*c^2+24*B*a*b*
c*d-8*C*a^2*d^2+12*C*a*b*c^2)*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x))-8*B*b^2*c^2*x/(b*x^2+a)^(1/2)+8*B*a*b*d^2*x/(b*x
^2+a)^(1/2)-16*A*b^2*c*d*x/(b*x^2+a)^(1/2)+16*C*a*b*c*d*x/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 764, normalized size of antiderivative = 2.44

$$\int \frac{(c+dx)^2\,(A+Bx+Cx^2)}{x^5\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
[1/48*(3*((24*B*a*b^2*c*d + 3*(4*C*a*b^2 - 5*A*b^3)*c^2 - 4*(2*C*a^2*b - 3
*A*a*b^2)*d^2)*x^6 + (24*B*a^2*b*c*d + 3*(4*C*a^2*b - 5*A*a*b^2)*c^2 - 4*(
2*C*a^3 - 3*A*a^2*b)*d^2)*x^4)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(6*A*a^3*c^2 - 16*(4*B*a*b^2*c^2 - 3*B*a^2*b*d^2 - 2*
(3*C*a^2*b - 4*A*a*b^2)*c*d)*x^5 + 3*(24*B*a^2*b*c*d + 3*(4*C*a^2*b - 5*A*
a*b^2)*c^2 - 4*(2*C*a^3 - 3*A*a^2*b)*d^2)*x^4 - 8*(4*B*a^2*b*c^2 - 3*B*a^3
*d^2 - 2*(3*C*a^3 - 4*A*a^2*b)*c*d)*x^3 + 3*(8*B*a^3*c*d + 4*A*a^3*d^2 + (
4*C*a^3 - 5*A*a^2*b)*c^2)*x^2 + 8*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*sqrt(b*x^2
+ a))/(a^4*b*x^6 + a^5*x^4), -1/24*(3*((24*B*a*b^2*c*d + 3*(4*C*a*b^2 - 5*
A*b^3)*c^2 - 4*(2*C*a^2*b - 3*A*a*b^2)*d^2)*x^6 + (24*B*a^2*b*c*d + 3*(4*C
*a^2*b - 5*A*a*b^2)*c^2 - 4*(2*C*a^3 - 3*A*a^2*b)*d^2)*x^4)*sqrt(-a)*arcta
n(sqrt(b*x^2 + a)*sqrt(-a)/a) + (6*A*a^3*c^2 - 16*(4*B*a*b^2*c^2 - 3*B*a^2
*b*d^2 - 2*(3*C*a^2*b - 4*A*a*b^2)*c*d)*x^5 + 3*(24*B*a^2*b*c*d + 3*(4*C*a
^2*b - 5*A*a*b^2)*c^2 - 4*(2*C*a^3 - 3*A*a^2*b)*d^2)*x^4 - 8*(4*B*a^2*b*c^
2 - 3*B*a^3*d^2 - 2*(3*C*a^3 - 4*A*a^2*b)*c*d)*x^3 + 3*(8*B*a^3*c*d + 4*A*
a^3*d^2 + (4*C*a^3 - 5*A*a^2*b)*c^2)*x^2 + 8*(B*a^3*c^2 + 2*A*a^3*c*d)*x)*
sqrt(b*x^2 + a))/(a^4*b*x^6 + a^5*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}}\,dx = \int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2*(C*x**2+B*x+A)/x**5/(b*x**2+a)**(3/2),x)
```

output
```
Integral((c + d*x)**2*(A + B*x + C*x**2)/(x**5*(a + b*x**2)**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2(A+Bx+Cx^2)}{x^5(a+bx^2)^{3/2}}\,dx = -\frac{15\,Ab^2c^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{7}{2}}} - \frac{Cd^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{15\,Ab^2c^2}{8\sqrt{bx^2+a}a^3} + \frac{Cd^2}{\sqrt{bx^2+a}a} - \frac{2\,(2\,Ccd+Bd^2)bx}{\sqrt{bx^2+a}a^2} + \frac{8\,(Bc^2+2\,Acd)b^2x}{3\sqrt{bx^2+a}a^3} + \frac{3\,(Cc^2+2\,Bcd+Ad^2)b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}} - \frac{3\,(Cc^2+2\,Bcd+Ad^2)b}{2\sqrt{bx^2+a}a^2} + \frac{5\,Abc^2}{8\sqrt{bx^2+a}a^2x^2} - \frac{2\,Ccd+Bd^2}{\sqrt{bx^2+a}ax} + \frac{4\,(Bc^2+2\,Acd)b}{3\sqrt{bx^2+a}a^2x} - \frac{Ac^2}{4\sqrt{bx^2+a}ax^4} - \frac{Cc^2+2\,Bcd+Ad^2}{2\sqrt{bx^2+a}ax^2} - \frac{Bc^2+2\,Acd}{3\sqrt{bx^2+a}ax^3}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
-15/8*A*b^2*c^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - C*d^2*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(3/2) + 15/8*A*b^2*c^2/(sqrt(b*x^2 + a)*a^3) + C*d^2/(
sqrt(b*x^2 + a)*a) - 2*(2*C*c*d + B*d^2)*b*x/(sqrt(b*x^2 + a)*a^2) + 8/3*(
B*c^2 + 2*A*c*d)*b^2*x/(sqrt(b*x^2 + a)*a^3) + 3/2*(C*c^2 + 2*B*c*d + A*d^
2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*(C*c^2 + 2*B*c*d + A*d^2)
*b/(sqrt(b*x^2 + a)*a^2) + 5/8*A*b*c^2/(sqrt(b*x^2 + a)*a^2*x^2) - (2*C*c*
d + B*d^2)/(sqrt(b*x^2 + a)*a*x) + 4/3*(B*c^2 + 2*A*c*d)*b/(sqrt(b*x^2 + a
)*a^2*x) - 1/4*A*c^2/(sqrt(b*x^2 + a)*a*x^4) - 1/2*(C*c^2 + 2*B*c*d + A*d^
2)/(sqrt(b*x^2 + a)*a*x^2) - 1/3*(B*c^2 + 2*A*c*d)/(sqrt(b*x^2 + a)*a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(285) = 570$.

Time = 0.14 (sec) , antiderivative size = 1078, normalized size of antiderivative = 3.44

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((B*a^3*b^2*c^2 - 2*C*a^4*b*c*d + 2*A*a^3*b^2*c*d - B*a^4*b*d^2)*x/a^6 - (
C*a^4*b*c^2 - A*a^3*b^2*c^2 + 2*B*a^4*b*c*d - C*a^5*d^2 + A*a^4*b*d^2)/a^6
)/sqrt(b*x^2 + a) - 1/4*(12*C*a*b*c^2 - 15*A*b^2*c^2 + 24*B*a*b*c*d - 8*C*
a^2*d^2 + 12*A*a*b*d^2)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(s
qrt(-a)*a^3) + 1/12*(12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*C*a*b*c^2 - 21*(sq
rt(b)*x - sqrt(b*x^2 + a))^7*A*b^2*c^2 + 24*(sqrt(b)*x - sqrt(b*x^2 + a))^
7*B*a*b*c*d + 12*(sqrt(b)*x - sqrt(b*x^2 + a))^7*A*a*b*d^2 - 24*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*B*a*b^(3/2)*c^2 + 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6
*C*a^2*sqrt(b)*c*d - 48*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(3/2)*c*d +
24*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^2*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqr
t(b*x^2 + a))^5*C*a^2*b*c^2 + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^5*A*a*b^2*c
^2 - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^5*B*a^2*b*c*d - 12*(sqrt(b)*x - sqrt
(b*x^2 + a))^5*A*a^2*b*d^2 + 120*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^2*b^(
3/2)*c^2 - 144*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^3*sqrt(b)*c*d + 240*(sq
rt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(3/2)*c*d - 72*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*B*a^3*sqrt(b)*d^2 - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*C*a^3*b*c
^2 + 45*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^2*b^2*c^2 - 24*(sqrt(b)*x - sq
rt(b*x^2 + a))^3*B*a^3*b*c*d - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a^3*b*
d^2 - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*b^(3/2)*c^2 + 144*(sqrt(b)
*x - sqrt(b*x^2 + a))^2*C*a^4*sqrt(b)*c*d - 272*(sqrt(b)*x - sqrt(b*x^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}}\, dx = \int \frac{(c+d\,x)^2\,(C\,x^2+B\,x+A)}{x^5\,(b\,x^2+a)^{3/2}}\, dx$$

input
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)),x)
```

output
```
int(((c + d*x)^2*(A + B*x + C*x^2))/(x^5*(a + b*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1210, normalized size of antiderivative = 3.87

$$\int \frac{(c+dx)^2 (A+Bx+Cx^2)}{x^5 (a+bx^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(C*x^2+B*x+A)/x^5/(b*x^2+a)^(3/2),x)
```

output
```
( - 6*sqrt(a + b*x**2)*a**3*c**2 - 16*sqrt(a + b*x**2)*a**3*c*d*x - 12*sqr
t(a + b*x**2)*a**3*d**2*x**2 + 15*sqrt(a + b*x**2)*a**2*b*c**2*x**2 - 8*sq
rt(a + b*x**2)*a**2*b*c**2*x + 64*sqrt(a + b*x**2)*a**2*b*c*d*x**3 - 24*sq
rt(a + b*x**2)*a**2*b*c*d*x**2 - 36*sqrt(a + b*x**2)*a**2*b*d**2*x**4 - 24
*sqrt(a + b*x**2)*a**2*b*d**2*x**3 - 12*sqrt(a + b*x**2)*a**2*c**3*x**2 -
48*sqrt(a + b*x**2)*a**2*c**2*d*x**3 + 24*sqrt(a + b*x**2)*a**2*c*d**2*x**
4 + 45*sqrt(a + b*x**2)*a*b**2*c**2*x**4 + 32*sqrt(a + b*x**2)*a*b**2*c**2
*x**3 + 128*sqrt(a + b*x**2)*a*b**2*c*d*x**5 - 72*sqrt(a + b*x**2)*a*b**2*
c*d*x**4 - 48*sqrt(a + b*x**2)*a*b**2*d**2*x**5 - 36*sqrt(a + b*x**2)*a*b*
c**3*x**4 - 96*sqrt(a + b*x**2)*a*b*c**2*d*x**5 + 64*sqrt(a + b*x**2)*b**3
*c**2*x**5 - 36*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**2*b*d**2*x**4 + 24*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a**2*c*d**2*x**4 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*a*b**2*c**2*x**4 - 72*sqrt(a)*log((sqrt(a + b*x**2
) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*c*d*x**4 - 36*sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*d**2*x**6 - 36*sqrt(a)*l
og((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c**3*x**4 + 24*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*c*d**2*x**
6 + 45*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*
c**2*x**6 - 72*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqr...
```

3.149 $\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 311

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx = -\frac{a(a(bBc-Abd+aCd)-b(Abc-acC+aBd)x)}{b^3(bc^2+ad^2)\sqrt{a+bx^2}} - \frac{(5aCd^2-3b(c^2C-Bcd+Ad^2))\sqrt{a+bx^2}}{3b^3d^3} - \frac{(cC-Bd)x\sqrt{a+bx^2}}{2b^2d^2} + \frac{Cx^2\sqrt{a+bx^2}}{3b^2d} + \frac{(3ad^2(cC-Bd)-2bc(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}d^4} - \frac{c^4(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4(bc^2+ad^2)^{3/2}}$$

output
```
-a*(a*(-A*b*d+B*b*c+C*a*d)-b*(A*b*c+B*a*d-C*a*c)*x)/b^3/(a*d^2+b*c^2)/(b*x
^2+a)^(1/2)-1/3*(5*a*C*d^2-3*b*(A*d^2-B*c*d+C*c^2))*(b*x^2+a)^(1/2)/b^3/d^
3-1/2*(-B*d+C*c)*x*(b*x^2+a)^(1/2)/b^2/d^2+1/3*C*x^2*(b*x^2+a)^(1/2)/b^2/d
+1/2*(3*a*d^2*(-B*d+C*c)-2*b*c*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x
^2+a)^(1/2))/b^(5/2)/d^4-c^4*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d
^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^4/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.23

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \frac{d\left(-16a^3Cd^4+b^3c^2x^2\left(6c^2C-3cd(2B+Cx)+d^2\left(6A+3Bx+2Cx^2\right)\right)+ab^2\left(6c^4C-3c^3d(2B+Cx)+c^2d^2\left(6A\right.\right.\right.}{}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
((d*(-16*a^3*C*d^4 + b^3*c^2*x^2*(6*c^2*C - 3*c*d*(2*B + C*x) + d^2*(6*A +
 3*B*x + 2*C*x^2)) + a*b^2*(6*c^4*C - 3*c^3*d*(2*B + C*x) + c^2*d^2*(6*A +
 3*B*x - 2*C*x^2) - 3*c*d^3*x*(-2*A + 2*B*x + C*x^2) + d^4*x^2*(6*A + 3*B*
x + 2*C*x^2)) - a^2*b*d^2*(4*c^2*C + 3*c*d*(4*B + 3*C*x) + d^2*(-12*A - 9*
B*x + 8*C*x^2))))/(b^3*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + (12*c^4*(c^2*C -
 B*c*d + A*d^2)*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*S
qrt[a + b*x^2])])/(-(b*c^2) - a*d^2)^(3/2) - (6*(3*a*d^2*(-(c*C) + B*d) +
2*b*c*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*
x^2])])/b^(5/2))/(6*d^4)
```

Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2178, 2185, 2185, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}(c+dx)}\,dx$$

$$\Big\downarrow \; 2178$$

$$-\frac{\int \frac{-aCx^4-aBx^3-\frac{a(Ab-aC)x^2}{b}+\frac{a^2Bx}{b}+\frac{a^2c(Abc-aCc+aBd)}{b\left(bc^2+ad^2\right)}}{(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{\int \frac{abd^3(7cC-3Bd)x^3+ad^2\left(5bCc^2-3Abd^2+5aCd^2\right)x^2+ad\left(bCc^3+ad^2(4cC+3Bd)\right)x+\frac{a^2cd^2\left(2bCc^3+3Abd^2c-aCd^2c+3aBd^3\right)}{bc^2+ad^2}}{(c+dx)\sqrt{bx^2+a}}dx}{3bd^4}-\frac{aC\sqrt{a+bx^2}(c+dx)^2}{3bd^3}}{ab}-\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{\frac{\int -\frac{-ab\left(10aCd^2-b\left(11Cc^2-9Bdc+6Ad^2\right)\right)x^2d^5+\frac{3a^2bc\left(3a(cC-Bd)d^2+bc\left(Cc^2-Bdc-2Ad^2\right)\right)d^5}{bc^2+ad^2}+ab\left(bc^2(5cC-3Bd)-ad^2(cC+9Bd)\right)xd^4}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}+\frac{1}{2}ad\sqrt{a+b}}{3bd^4}}{ab}-\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{\int \frac{-ab\left(10aCd^2-b\left(11Cc^2-9Bdc+6Ad^2\right)\right)x^2d^5+\frac{3a^2bc\left(3a(cC-Bd)d^2+bc\left(Cc^2-Bdc-2Ad^2\right)\right)d^5}{bc^2+ad^2}+ab\left(bc^2(5cC-3Bd)-\right.}{(c+dx)\sqrt{bx^2+a}}}{2bd^3}}{3bd^4}}{ab}-\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{\frac{\int \frac{3ab^2d^6\left(\frac{acd\left(3a(cC-Bd)d^2+bc\left(Cc^2-Bdc-2Ad^2\right)\right)}{bc^2+ad^2}+\left(3ad^2(cC-Bd)-2bc\left(Cc^2-Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}-ad^4\sqrt{a+bx^2}\left(\right.}{2bd^3}}{3bd^4}}{ab}-\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\int \frac{\frac{acd\left(3a(cC-Bd)d^2+bc\left(Cc^2-Bdc-2Ad^2\right)\right)}{bc^2+ad^2}+\left(3ad^2(cC-Bd)-2bc\left(Cc^2-Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx-ad^4\sqrt{a+bx^2}\left(10aC\right.}{2bd^3}}{3bd^4}}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$\Big\downarrow$ 719

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\left(\frac{2b^2c^4\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(3ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)-ad^4\sqrt{}}{2bd^3}}{3bd^4}}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$\Big\downarrow$ 224

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\left(\frac{2b^2c^4\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(3ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right.}{2bd^3}}{3bd^4}}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\left(\frac{2b^2c^4\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}\right.}{2bd^3}}{3bd^4}}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$\Big\downarrow$ 488

$$-\frac{\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^4\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bc}{bx^2+}}}{d\left(ad^2+bc^2\right)}\right.}{2bd^3}}{3bd^4}}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$\downarrow$ 219

$$-\frac{\frac{1}{2}ad\sqrt{a+bx^2}(c+dx)(7cC-3Bd)-\frac{3abd^4\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3ad^2(cC-Bd)-2bc\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^4\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{a\cdots}{\sqrt{a+bx^2}\cdots}\right)}{d\left(ad^2+bc^2\right)^{3/2}}\right.}{2bd^3}}{3bd^4} \cdots \frac{}{ab}$$

$$\frac{a(a(aCd-Abd+bBc)-bx(aBd-acC+Abc))}{b^3\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

input
```
Int[(x^4*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(a*(b*B*c - A*b*d + a*C*d) - b*(A*b*c - a*c*C + a*B*d)*x))/(b^3*(b*c^
2 + a*d^2)*Sqrt[a + b*x^2])) - (-1/3*(a*C*(c + d*x)^2*Sqrt[a + b*x^2])/(b*
d^3) + ((a*d*(7*c*C - 3*B*d)*(c + d*x)*Sqrt[a + b*x^2])/2 - (-(a*d^4*(10*a
*C*d^2 - b*(11*c^2*C - 9*B*c*d + 6*A*d^2))*Sqrt[a + b*x^2]) + 3*a*b*d^4*((
(3*a*d^2*(c*C - B*d) - 2*b*c*(c^2*C - B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/
Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (2*b^2*c^4*(c^2*C - B*c*d + A*d^2)*ArcTanh
[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d*(b*c^2 + a*d^2)^
(3/2))))/(2*b*d^3))/(3*b*d^4))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.63

method	result
risch	$\frac{(2Cd^2bx^2+3Bbd^2x-3Cbcdx+6Abd^2-6Bbcd-10aCd^2+6Cbc^2)\sqrt{bx^2+a}}{6b^3d^3}-\frac{\frac{(2Abcd^2+3Bad^3-2Bbc^2d-3Cacd^2+2Cbc^3)\ln(\sqrt{b}x\cdots}{d\sqrt{b}}\cdots}{\cdots}$
default	$\frac{c^4(Ad^2-Bcd+Cc^2)\left(\frac{d^2}{(ad^2+bc^2)\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\frac{ad^2+bc^2}{d^2}}}+\frac{2bcd\left(2b(x+\frac{c}{d})-\frac{2bc}{d}\right)}{(ad^2+bc^2)\left(\frac{4b(ad^2+bc^2)}{d^2}-\frac{4b^2c^2}{d^2}\right)\sqrt{b(x+\frac{c}{d})^2-\frac{2bc(x+\frac{c}{d})}{d}+\cdots}}\cdots\right.}{d^7}$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*C*b*d^2*x^2+3*B*b*d^2*x-3*C*b*c*d*x+6*A*b*d^2-6*B*b*c*d-10*C*a*d^2+
6*C*b*c^2)*(b*x^2+a)^(1/2)/b^3/d^3-1/2/d^3/b^2*((2*A*b*c*d^2+3*B*a*d^3-2*B
*b*c^2*d-3*C*a*c*d^2+2*C*b*c^3)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+d^
3*a*(A*b-B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)-b*c)/b/(x+(-a*b)^(1/2)/b)*(b*
(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-d^3*a*(A*b+B
*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)+b*c)/b/(x-(-a*b)^(1/2)/b)*(b*(x-(-a*b)^
(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)-2*b^3/d^2*c^4*(A*d^2-B
*c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b)^(1/2)-b*c)/((a*d^2+b*c^2)/d^2)^
(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*
(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(284) = 568$.

Time $= 0.35$ (sec) , antiderivative size $= 983$, normalized size of antiderivative $= 3.16$

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*b*c^7*x/(sqrt(b*x^2 + a)*a*b*c^2*d^6 + sqrt(b*x^2 + a)*a^2*d^8) - B*b*c^
6*x/(sqrt(b*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7) + A*b*c^5*x/(s
qrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) + C*c^6/(sqrt(b*x^2
+ a)*b*c^2*d^5 + sqrt(b*x^2 + a)*a*d^7) - B*c^5/(sqrt(b*x^2 + a)*b*c^2*d^4
 + sqrt(b*x^2 + a)*a*d^6) + A*c^4/(sqrt(b*x^2 + a)*b*c^2*d^3 + sqrt(b*x^2
+ a)*a*d^5) + 1/3*C*x^4/(sqrt(b*x^2 + a)*b*d) - 1/2*C*c*x^3/(sqrt(b*x^2 +
a)*b*d^2) + 1/2*B*x^3/(sqrt(b*x^2 + a)*b*d) + C*c^2*x^2/(sqrt(b*x^2 + a)*b
*d^3) - B*c*x^2/(sqrt(b*x^2 + a)*b*d^2) - 4/3*C*a*x^2/(sqrt(b*x^2 + a)*b^2
*d) + A*x^2/(sqrt(b*x^2 + a)*b*d) - C*c^5*x/(sqrt(b*x^2 + a)*a*d^6) + B*c^
4*x/(sqrt(b*x^2 + a)*a*d^5) - A*c^3*x/(sqrt(b*x^2 + a)*a*d^4) + C*c^3*x/(s
qrt(b*x^2 + a)*b*d^4) - B*c^2*x/(sqrt(b*x^2 + a)*b*d^3) - 3/2*C*a*c*x/(sqr
t(b*x^2 + a)*b^2*d^2) + A*c*x/(sqrt(b*x^2 + a)*b*d^2) + 3/2*B*a*x/(sqrt(b*
x^2 + a)*b^2*d) - C*c^3*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^4) + B*c^2*arcsi
nh(b*x/sqrt(a*b))/(b^(3/2)*d^3) + 3/2*C*a*c*arcsinh(b*x/sqrt(a*b))/(b^(5/2
)*d^2) - A*c*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) - 3/2*B*a*arcsinh(b*x/sq
rt(a*b))/(b^(5/2)*d) + C*c^6*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/
(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^7) - B*c^5*arcsinh(b*c*
x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2
)^(3/2)*d^6) + A*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*
b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) - C*c^4/(sqrt(b*x^2 + a)*...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^4\left(C\,x^2+B\,x+A\right)}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^4(C\,x^2+Bx+A)}{(dx+c)\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.150 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 243

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx = \frac{a(Abc-acC+aBd+(bBc-Abd+aCd)x)}{b^2(bc^2+ad^2)\sqrt{a+bx^2}} - \frac{(cC-Bd)\sqrt{a+bx^2}}{b^2d^2} + \frac{Cx\sqrt{a+bx^2}}{2b^2d} - \frac{(3aCd^2-2b(c^2C-Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}d^3} + \frac{c^3(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3(bc^2+ad^2)^{3/2}}$$

output
```
a*(A*b*c-C*a*c+B*a*d+(-A*b*d+B*b*c+C*a*d)*x)/b^2/(a*d^2+b*c^2)/(b*x^2+a)^(
1/2)-(-B*d+C*c)*(b*x^2+a)^(1/2)/b^2/d^2+1/2*C*x*(b*x^2+a)^(1/2)/b^2/d-1/2*
(3*a*C*d^2-2*b*(A*d^2-B*c*d+C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(
5/2)/d^3+c^3*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/
(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.13

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \frac{\frac{d(b^2c^2x^2(-2cC+2Bd+Cdx)+a^2d^2(-4cC+4Bd+3Cdx)+ab(-2c^3C+c^2d(2B+Cx)+2cd^2(A+x(B-Cx)}{b^2(bc^2+ad^2)\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
((d*(b^2*c^2*x^2*(-2*c*C + 2*B*d + C*d*x) + a^2*d^2*(-4*c*C + 4*B*d + 3*C*
d*x) + a*b*(-2*c^3*C + c^2*d*(2*B + C*x) + 2*c*d^2*(A + x*(B - C*x)) + d^3
*x*(-2*A + x*(2*B + C*x)))))/(b^2*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (4*c^
3*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/S
qrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + ((3*a*C*d^2 - 2*b*(c^2*
C - B*c*d + A*d^2))*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2))/(2*d^3)
```

Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2178, 2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{3/2}\,(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2178}$$

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\int \frac{-aCx^3-aBx^2-\frac{a(Ab-aC)x}{b}+\frac{a^2c(bBc-Abd+aCd)}{b\left(bc^2+ad^2\right)}}{(c+dx)\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{ 2185}$$

$$\frac{\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{\int \frac{ab(3cC-2Bd)x^2d^2+\frac{a^2c\left(3aCd^2+b\left(Cc^2+2Bdc-2Ad^2\right)\right)d^2}{bc^2+ad^2}+a\left(bCc^2-2Abd^2+3aCd^2\right)xd}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+dx)}{2bd^2}}{ab}}{}$$

$\Big\downarrow$ 2185

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{\frac{\int \frac{abd^3\left(\frac{acd\left(3aCd^2+b\left(Cc^2+2Bdc-2Ad^2\right)\right)}{bc^2+ad^2}+\left(3aCd^2-2b\left(Cc^2-Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+dx)}{2bd^2}}{ab}$$

$\Big\downarrow$ 27

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\int \frac{\frac{acd\left(3aCd^2+b\left(Cc^2+2Bdc-2Ad^2\right)\right)}{bc^2+ad^2}+\left(3aCd^2-2b\left(Cc^2-Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+dx)}{2bd^2}}{ab}$$

$\Big\downarrow$ 719

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\left(\frac{2b^2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(3aCd^2-2b\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+dx)}{2bd^2}}{ab}$$

$\Big\downarrow$ 224

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\left(\frac{2b^2c^3\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(3aCd^2-2b\left(Ad^2-Bcd+c^2C\right)\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+dx)}{2bd^2}}{ab}$$

$\Big\downarrow$ 219

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\left(\frac{2b^2c^3\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2-2b\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}\right)+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC\sqrt{a+bx^2}(c+}{2bd^2}}{ab}$$

$$\Big\downarrow \text{488}$$

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2-2b\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^3\left(Ad^2-Bcd+c^2C\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d\left(ad^2+bc^2\right)}\right)+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC}{}}{ab}$$

$$\Big\downarrow \text{219}$$

$$\frac{a(x(aCd-Abd+bBc)+aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}-\frac{\frac{ad\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(3aCd^2-2b\left(Ad^2-Bcd+c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^3\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\left(ad^2+bc^2\right)^{3/2}}\right)+ad\sqrt{a+bx^2}(3cC-2Bd)}{2bd^3}-\frac{aC}{}}{ab}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(a*(A*b*c - a*c*C + a*B*d + (b*B*c - A*b*d + a*C*d)*x))/(b^2*(b*c^2 + a*d^
2)*Sqrt[a + b*x^2]) - (-1/2*(a*C*(c + d*x)*Sqrt[a + b*x^2])/(b*d^2) + (a*d
*(3*c*C - 2*B*d)*Sqrt[a + b*x^2] + a*d*(((3*a*C*d^2 - 2*b*(c^2*C - B*c*d +
 A*d^2))*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) - (2*b^2*c^3*(c
^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a +
b*x^2])])/(d*(b*c^2 + a*d^2)^(3/2))))/(2*b*d^3))/(a*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(221) = 442$.

Time = 0.31 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.94

method	result
risch	$\frac{(Cxd+2Bd-2Cc)\sqrt{b\,x^2+a}}{2b^2d^2}+\frac{\frac{\left(2Ab\,d^2-2Bbcd-3aC\,d^2+2Cb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}-\frac{d^2\left(A\sqrt{-ab}\,b+Bab-C\sqrt{-ab}\,a\right)\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{}}}{(d\sqrt{-ab}-bc)b\left(x+\frac{\sqrt{-ab}}{b}\right)}}{}$
default	$\frac{\frac{C\,c^4x}{a\sqrt{b\,x^2+a}}+d^3(Bd-Cc)\left(\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}\right)+d^2\left(A\,d^2-Bcd+C\,c^2\right)\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{cd\left(A\,d^2-Bcd+C\,c^2\right)}{b\sqrt{b\,x^2+a}}}{d^5}$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d-2*C*c)*(b*x^2+a)^(1/2)/b^2/d^2+1/2/b^2/d^2*((2*A*b*d^2-2*
B*b*c*d-3*C*a*d^2+2*C*b*c^2)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)-d^2*(
A*(-a*b)^(1/2)*b+B*a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b)^(1/2)-b*c)/b/(x+(-a*b)^
(1/2)/b)*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-
d^2*(A*(-a*b)^(1/2)*b-B*a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b)^(1/2)+b*c)/b/(x-(-
a*b)^(1/2)/b)*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(
1/2)-2*b^3/d^2*c^3*(A*d^2-B*c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b)^(1/2
)-b*c)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2
*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2))/(x+c/d))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^3(A+Bx+Cx^2)}{\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(225) = 450$.

Time = 0.23 (sec) , antiderivative size = 804, normalized size of antiderivative = 3.31

$$\int \frac{x^3(A + Bx + Cx^2)}{(c + dx)\,(a + bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-C*b*c^6*x/(sqrt(b*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7) + B*b*c
^5*x/(sqrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) - A*b*c^4*x/(
sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) - C*c^5/(sqrt(b*x^2
 + a)*b*c^2*d^4 + sqrt(b*x^2 + a)*a*d^6) + B*c^4/(sqrt(b*x^2 + a)*b*c^2*d^
3 + sqrt(b*x^2 + a)*a*d^5) - A*c^3/(sqrt(b*x^2 + a)*b*c^2*d^2 + sqrt(b*x^2
 + a)*a*d^4) + 1/2*C*x^3/(sqrt(b*x^2 + a)*b*d) - C*c*x^2/(sqrt(b*x^2 + a)*
b*d^2) + B*x^2/(sqrt(b*x^2 + a)*b*d) + C*c^4*x/(sqrt(b*x^2 + a)*a*d^5) - B
*c^3*x/(sqrt(b*x^2 + a)*a*d^4) + A*c^2*x/(sqrt(b*x^2 + a)*a*d^3) - C*c^2*x
/(sqrt(b*x^2 + a)*b*d^3) + B*c*x/(sqrt(b*x^2 + a)*b*d^2) + 3/2*C*a*x/(sqrt
(b*x^2 + a)*b^2*d) - A*x/(sqrt(b*x^2 + a)*b*d) + C*c^2*arcsinh(b*x/sqrt(a*
b))/(b^(3/2)*d^3) - B*c*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d^2) - 3/2*C*a*arc
sinh(b*x/sqrt(a*b))/(b^(5/2)*d) + A*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) - C
*c^5*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c))
)/((a + b*c^2/d^2)^(3/2)*d^6) + B*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c
)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^5) - A*c^3*arc
sinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a +
b*c^2/d^2)^(3/2)*d^4) + C*c^3/(sqrt(b*x^2 + a)*b*d^4) - B*c^2/(sqrt(b*x^2
+ a)*b*d^3) - 2*C*a*c/(sqrt(b*x^2 + a)*b^2*d^2) + A*c/(sqrt(b*x^2 + a)*b*d
^2) + 2*B*a/(sqrt(b*x^2 + a)*b^2*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(A + Bx + Cx^2)}{(c + dx)\,(a + bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^3\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.151 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 196

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \frac{a(bBc-Abd+aCd)-b(Abc-acC+aBd)x}{b^2\,(bc^2+ad^2)\,\sqrt{a+bx^2}} + \frac{C\sqrt{a+bx^2}}{b^2d} - \frac{(cC-Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}d^2} - \frac{c^2(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2\,(bc^2+ad^2)^{3/2}}$$

output
```
(a*(-A*b*d+B*b*c+C*a*d)-b*(A*b*c+B*a*d-C*a*c)*x)/b^2/(a*d^2+b*c^2)/(b*x^2+
a)^(1/2)+C*(b*x^2+a)^(1/2)/b^2/d-(-B*d+C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1
/2))/b^(3/2)/d^2-c^2*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2
)^(1/2)/(b*x^2+a)^(1/2))/d^2/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)(a+bx^2)^{3/2}}\,dx = \frac{\frac{d(2a^2Cd^2+b^2cx(-Ad+cCx)+ab(c^2C+cd(B+Cx)-d^2(A+x(B-Cx))))}{b^2(bc^2+ad^2)\sqrt{a+bx^2}} + \frac{2c^2(c^2C-Bcd+Ad^2)\,\text{arcta}}{(-bc^2-a}}{d^2}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
((d*(2*a^2*C*d^2 + b^2*c*x*(-(A*d) + c*C*x) + a*b*(c^2*C + c*d*(B + C*x) -
 d^2*(A + x*(B - C*x)))))/(b^2*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + (2*c^2*(
c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x
) - c*Sqrt[a + b*x^2])])/(-(b*c^2) - a*d^2)^(3/2) + (2*(-(c*C) + B*d)*ArcT
anh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2))/d^2
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2178, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}(c+dx)}\,dx$$

$$\Big\downarrow \ 2178$$

$$\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)} - \frac{\int -\frac{aCx^2+aBx+\frac{ac(Abc-aCc+aBd)}{bc^2+ad^2}}{(c+dx)\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{aCx^2+aBx+\frac{ac(Abc-aCc+aBd)}{bc^2+ad^2}}{(c+dx)\sqrt{bx^2+a}}dx}{ab} + \frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{\int \frac{abd\left(\frac{cd(Abc-aCc+aBd)}{bc^2+ad^2}-(cC-Bd)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{\frac{cd(Abc-aCc+aBd)}{bc^2+ad^2}-(cC-Bd)x}{(c+dx)\sqrt{bx^2+a}}dx}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{a\left(\frac{bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}-\frac{(cC-Bd)\int \frac{1}{\sqrt{bx^2+a}}dx}{d}\right)}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{a\left(\frac{bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}-\frac{(cC-Bd)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\left(\frac{bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}\right)}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{488}$$

$$\frac{\frac{a\left(-\frac{bc^2\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d\left(ad^2+bc^2\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}\right)}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\left(-\frac{bc^2\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\left(ad^2+bc^2\right)^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(cC-Bd)}{\sqrt{b}d}\right)}{d}+\frac{aC\sqrt{a+bx^2}}{bd}}{ab}+\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{b^2\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(a*(b*B*c - A*b*d + a*C*d) - b*(A*b*c - a*c*C + a*B*d)*x)/(b^2*(b*c^2 + a*
d^2)*Sqrt[a + b*x^2]) + ((a*C*Sqrt[a + b*x^2])/(b*d) + (a*(-(((c*C - B*d)*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d)) - (b*c^2*(c^2*C - B*c*d
 + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d
*(b*c^2 + a*d^2)^(3/2))))/d)/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(180) = 360$.

Time = 0.29 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.13

method	result
risch	$\frac{C\sqrt{b\,x^2+a}}{b^2 d}+\frac{\frac{(Bd-Cc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{d(Ab-B\sqrt{-ab}-aC)\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^2-2\sqrt{-ab}\left(x+\frac{\sqrt{-ab}}{b}\right)}}{2(d\sqrt{-ab}-bc)b\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{d(Ab+B\sqrt{-ab}-aC)\sqrt{b\left(x-\frac{\sqrt{-a}}{b}\right.}}{2(d\sqrt{-ab}+bc)b\left(\right.}}{}$
default	$\frac{c^2\left(A\,d^2-Bcd+C\,c^2\right)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{2bcd\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\,c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+}}\right.}{d^5}$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C*(b*x^2+a)^(1/2)/b^2/d+1/d/b*((B*d-C*c)/d*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b
^(1/2)+1/2*d*(A*b-B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)-b*c)/b/(x+(-a*b)^(1/
2)/b)*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2
*d*(A*b+B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)+b*c)/b/(x-(-a*b)^(1/2)/b)*(b*(
x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+b^2/d^2*c^2*(
A*d^2-B*c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b)^(1/2)-b*c)/((a*d^2+b*c^2
)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)
^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{x^2(A+Bx+Cx^2)}{\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(181) = 362$.

Time = 0.17 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.33

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx&=\frac{Cbc^5x}{\sqrt{bx^2+a}abc^2d^4+\sqrt{bx^2+a}a^2d^6}\\&-\frac{Bbc^4x}{\sqrt{bx^2+a}abc^2d^3+\sqrt{bx^2+a}a^2d^5}+\frac{Abc^3x}{\sqrt{bx^2+a}abc^2d^2+\sqrt{bx^2+a}a^2d^4}\\&+\frac{Cc^4}{\sqrt{bx^2+a}bc^2d^3+\sqrt{bx^2+a}ad^5}-\frac{Bc^3}{\sqrt{bx^2+a}bc^2d^2+\sqrt{bx^2+a}ad^4}\\&+\frac{Ac^2}{\sqrt{bx^2+a}bc^2d+\sqrt{bx^2+a}ad^3}+\frac{Cx^2}{\sqrt{bx^2+a}bd}-\frac{Cc^3x}{\sqrt{bx^2+a}ad^4}\\&+\frac{Bc^2x}{\sqrt{bx^2+a}ad^3}-\frac{Acx}{\sqrt{bx^2+a}ad^2}+\frac{Ccx}{\sqrt{bx^2+a}bd^2}-\frac{Bx}{\sqrt{bx^2+a}bd}\\&-\frac{Cc\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}d^2}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}d}+\frac{Cc^4\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^5}\\&-\frac{Bc^3\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4}+\frac{Ac^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}\\&-\frac{Cc^2}{\sqrt{bx^2+a}bd^3}+\frac{Bc}{\sqrt{bx^2+a}bd^2}+\frac{2\,Ca}{\sqrt{bx^2+a}b^2d}-\frac{A}{\sqrt{bx^2+a}bd}\end{aligned}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output

```
C*b*c^5*x/(sqrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) - B*b*c^
4*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) + A*b*c^3*x/(s
qrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) + C*c^4/(sqrt(b*x^2
+ a)*b*c^2*d^3 + sqrt(b*x^2 + a)*a*d^5) - B*c^3/(sqrt(b*x^2 + a)*b*c^2*d^2
 + sqrt(b*x^2 + a)*a*d^4) + A*c^2/(sqrt(b*x^2 + a)*b*c^2*d + sqrt(b*x^2 +
a)*a*d^3) + C*x^2/(sqrt(b*x^2 + a)*b*d) - C*c^3*x/(sqrt(b*x^2 + a)*a*d^4)
+ B*c^2*x/(sqrt(b*x^2 + a)*a*d^3) - A*c*x/(sqrt(b*x^2 + a)*a*d^2) + C*c*x/
(sqrt(b*x^2 + a)*b*d^2) - B*x/(sqrt(b*x^2 + a)*b*d) - C*c*arcsinh(b*x/sqrt
(a*b))/(b^(3/2)*d^2) + B*arcsinh(b*x/sqrt(a*b))/(b^(3/2)*d) + C*c^4*arcsin
h(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c
^2/d^2)^(3/2)*d^5) - B*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(s
qrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^4) + A*c^2*arcsinh(b*c*x/
(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^
(3/2)*d^3) - C*c^2/(sqrt(b*x^2 + a)*b*d^3) + B*c/(sqrt(b*x^2 + a)*b*d^2) +
 2*C*a/(sqrt(b*x^2 + a)*b^2*d) - A/(sqrt(b*x^2 + a)*b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output

```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx=\int \frac{x^2\,(C\,x^2+B\,x+A)}{{(b\,x^2+a)}^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx=\int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.152 $\int \frac{x\left(A+Bx+Cx^2\right)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 162

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=-\frac{Abc-acC+aBd+(bBc-Abd+aCd)x}{b\left(bc^2+ad^2\right)\sqrt{a+bx^2}}+\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}d}+\frac{c(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d\left(bc^2+ad^2\right)^{3/2}}$$

output
```
-(A*b*c-C*a*c+B*a*d+(-A*b*d+B*b*c+C*a*d)*x)/b/(a*d^2+b*c^2)/(b*x^2+a)^(1/2
)+C*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d+c*(A*d^2-B*c*d+C*c^2)*arc
tanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d/(a*d^2+b*c^2)^(3/
2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{-bBcx+Ab(-c+dx)+a(cC-Bd-Cdx)}{b\left(bc^2+ad^2\right)\sqrt{a+bx^2}}-\frac{2c(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{d\left(-bc^2-ad^2\right)^{3/2}}-\frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}d}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(-(b*B*c*x) + A*b*(-c + d*x) + a*(c*C - B*d - C*d*x))/(b*(b*c^2 + a*d^2)*S
qrt[a + b*x^2]) - (2*c*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) -
 d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(d*(-(b*c^2) - a*d^2)^(3/2))
- (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2178, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{a\left(\frac{c(bBc-Abd+aCd)}{bc^2+ad^2}+Cx\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a\left(\frac{c(bBc-Abd+aCd)}{bc^2+ad^2}+Cx\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\frac{c(bBc-Abd+aCd)}{bc^2+ad^2}+Cx}{(c+dx)\sqrt{bx^2+a}}dx}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \text{719}$$

$$\frac{\frac{C\int \frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{bc(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d(ad^2+bc^2)}}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{bc\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}-\frac{bc\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{bc\left(Ad^2-Bcd+c^2C\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d\left(ad^2+bc^2\right)}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{bc\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{d\left(ad^2+bc^2\right)^{3/2}}+\frac{C\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}d}}{b}-\frac{x(aCd-Abd+bBc)+aBd-acC+Abc}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-((A*b*c - a*c*C + a*B*d + (b*B*c - A*b*d + a*C*d)*x)/(b*(b*c^2 + a*d^2)*S
qrt[a + b*x^2])) + ((C*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*d) +
 (b*c*(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*S
qrt[a + b*x^2])])/(d*(b*c^2 + a*d^2)^(3/2)))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(148) = 296$.

Time = 0.21 (sec) , antiderivative size = 457, normalized size of antiderivative = 2.82

method	result
default	$\frac{\frac{A d^2 x}{a\sqrt{b x^2+a}}+\frac{C c^2 x}{a\sqrt{b x^2+a}}-\frac{d(Bd-Cc)}{b\sqrt{b x^2+a}}+C d^2\left(-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{Bcdx}{a\sqrt{b x^2+a}}}{d^3} - \frac{c(A d^2-Bcd+C c^2)\left(\frac{}{(a d^2+b c^2)\sqrt{b(}}\right.}{}$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(A*d^2*x/a/(b*x^2+a)^(1/2)+C*c^2*x/a/(b*x^2+a)^(1/2)-d*(B*d-C*c)/b/(
b*x^2+a)^(1/2)+C*d^2*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a
)^(1/2)))-B*c*d*x/a/(b*x^2+a)^(1/2))-c*(A*d^2-B*c*d+C*c^2)/d^4*(1/(a*d^2+b
*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a
*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b
*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((
a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+
b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(
x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. 2(150) = 300.

Time = 0.12 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.36

$$\begin{aligned}\int \frac{x(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx =& -\frac{Cbc^4x}{\sqrt{bx^2+a}abc^2d^3+\sqrt{bx^2+a}a^2d^5} \\ &+ \frac{Bbc^3x}{\sqrt{bx^2+a}abc^2d^2+\sqrt{bx^2+a}a^2d^4} - \frac{Abc^2x}{\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3} \\ &- \frac{Cc^3}{\sqrt{bx^2+a}bc^2d^2+\sqrt{bx^2+a}ad^4} + \frac{Bc^2}{\sqrt{bx^2+a}bc^2d+\sqrt{bx^2+a}ad^3} \\ &- \frac{Ac}{\sqrt{bx^2+a}bc^2+\sqrt{bx^2+a}ad^2} + \frac{Cc^2x}{\sqrt{bx^2+a}ad^3} - \frac{Bcx}{\sqrt{bx^2+a}ad^2} \\ &+ \frac{Ax}{\sqrt{bx^2+a}ad} - \frac{Cx}{\sqrt{bx^2+a}bd} + \frac{C\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}d} \\ &- \frac{Cc^3\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^4} + \frac{Bc^2\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3} \\ &- \frac{Ac\operatorname{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^2} + \frac{Cc}{\sqrt{bx^2+a}bd^2} - \frac{B}{\sqrt{bx^2+a}bd}\end{aligned}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-C*b*c^4*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) + B*b*c
^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - A*b*c^2*x/(
sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - C*c^3/(sqrt(b*x^2 +
 a)*b*c^2*d^2 + sqrt(b*x^2 + a)*a*d^4) + B*c^2/(sqrt(b*x^2 + a)*b*c^2*d +
sqrt(b*x^2 + a)*a*d^3) - A*c/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 + a)*a*d^
2) + C*c^2*x/(sqrt(b*x^2 + a)*a*d^3) - B*c*x/(sqrt(b*x^2 + a)*a*d^2) + A*x
/(sqrt(b*x^2 + a)*a*d) - C*x/(sqrt(b*x^2 + a)*b*d) + C*arcsinh(b*x/sqrt(a*
b))/(b^(3/2)*d) - C*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt
(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^4) + B*c^2*arcsinh(b*c*x/(sq
rt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/
2)*d^3) - A*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(
d*x + c)))/((a + b*c^2/d^2)^(3/2)*d^2) + C*c/(sqrt(b*x^2 + a)*b*d^2) - B/(
sqrt(b*x^2 + a)*b*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)\,(a+bx^2)^{3/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x(A + Bx + Cx^2)}{(c + dx)\,(a + bx^2)^{3/2}}\,dx = \int \frac{x(C\,x^2 + Bx + A)}{(dx + c)\,(b\,x^2 + a)^{\frac{3}{2}}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.153 $\int \frac{A+Bx+Cx^2}{(c+dx)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{A+Bx+Cx^2}{(c+dx)(a+bx^2)^{3/2}}\,dx = -\frac{a(bBc-Abd+aCd)-b(Abc-acC+aBd)x}{ab(bc^2+ad^2)\sqrt{a+bx^2}} - \frac{(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{3/2}}$$

output
```
-(a*(-A*b*d+B*b*c+C*a*d)-b*(A*b*c+B*a*d-C*a*c)*x)/a/b/(a*d^2+b*c^2)/(b*x^2
+a)^(1/2)-(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*
x^2+a)^(1/2))/(a*d^2+b*c^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2}{(c+dx)(a+bx^2)^{3/2}}\,dx = \frac{-a^2Cd+Ab^2cx+ab(-Bc+Ad-cCx+Bdx)}{ab(bc^2+ad^2)\sqrt{a+bx^2}} + \frac{2(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{(-bc^2-ad^2)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a^2*C*d) + A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x))/(a*b*(b*c^2
+ a*d^2)*Sqrt[a + b*x^2]) + (2*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c
+ d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3
/2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2178, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{ab(Cc^2-Bdc+Ad^2)}{(bc^2+ad^2)(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{ab\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ab(Cc^2-Bdc+Ad^2)}{(bc^2+ad^2)(c+dx)\sqrt{bx^2+a}}dx}{ab}-\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{ab\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{ab\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{488}$$

$$-\frac{(Ad^2-Bcd+c^2C)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{ab\sqrt{a+bx^2}(ad^2+bc^2)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{(ad^2+bc^2)^{3/2}}-\frac{a(aCd-Abd+bBc)-bx(aBd-acC+Abc)}{ab\sqrt{a+bx^2}\,(ad^2+bc^2)}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b*B*c - A*b*d + a*C*d) - b*(A*b*c - a*c*C + a*B*d)*x)/(a*b*(b*c^2 +
a*d^2)*Sqrt[a + b*x^2])) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/
(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(130) = 260$.

Time = 0.19 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.80

method	result
default	$\frac{\frac{Bdx}{a\sqrt{bx^2+a}}-\frac{dC}{b\sqrt{bx^2+a}}-\frac{Ccx}{a\sqrt{bx^2+a}}}{d^2}+\frac{(Ad^2-Bcd+Cc^2)\left(\frac{d^2}{(ad^2+bc^2)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{ad^2+bc^2}{d^2}}}+\frac{\cdots}{(ad^2+bc^2)\left(\frac{4b\left(ad^2+bc^2\right)}{d^2}\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(B*d*x/a/(b*x^2+a)^(1/2)-d*C/b/(b*x^2+a)^(1/2)-C*c*x/a/(b*x^2+a)^(1/
2))+(A*d^2-B*c*d+C*c^2)/d^3*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(
4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(131) = 262$.

Time = 0.86 (sec) , antiderivative size = 721, normalized size of antiderivative = 5.22

$$\int \frac{A + Bx + Cx^2}{(c + dx)(a + bx^2)^{3/2}} \, dx = \left[\frac{(Ca^2bc^2 - Ba^2bcd + Aa^2bd^2 + (Cab^2c^2 - Bab^2cd + Aab^2d^2)x^2)\sqrt{bc^2 + ad^2}}{} \right.$$

$$- \frac{(Ca^2bc^2 - Ba^2bcd + Aa^2bd^2 + (Cab^2c^2 - Bab^2cd + Aab^2d^2)x^2)\sqrt{-bc^2 - ad^2}\arctan\left(\frac{\sqrt{-bc^2-ad^2}(bcx-ad)\sqrt{b}}{abc^2+a^2d^2+(b^2c^2+abd}\right.}{a^2b^3c^4 + 2a}$$

input

```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output

```
[1/2*((C*a^2*b*c^2 - B*a^2*b*c*d + A*a^2*b*d^2 + (C*a*b^2*c^2 - B*a*b^2*c*
d + A*a*b^2*d^2)*x^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2*a
^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)*s
qrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(B*a*b^2*c^3 + B*a^2*b*c*d^
2 + (C*a^2*b - A*a*b^2)*c^2*d + (C*a^3 - A*a^2*b)*d^3 - (B*a*b^2*c^2*d + B
*a^2*b*d^3 - (C*a*b^2 - A*b^3)*c^3 - (C*a^2*b - A*a*b^2)*c*d^2)*x)*sqrt(b*
x^2 + a))/(a^2*b^3*c^4 + 2*a^3*b^2*c^2*d^2 + a^4*b*d^4 + (a*b^4*c^4 + 2*a^
2*b^3*c^2*d^2 + a^3*b^2*d^4)*x^2), -((C*a^2*b*c^2 - B*a^2*b*c*d + A*a^2*b*
d^2 + (C*a*b^2*c^2 - B*a*b^2*c*d + A*a*b^2*d^2)*x^2)*sqrt(-b*c^2 - a*d^2)*
arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a*b*c^2 + a^2*d
^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (B*a*b^2*c^3 + B*a^2*b*c*d^2 + (C*a^2*b -
 A*a*b^2)*c^2*d + (C*a^3 - A*a^2*b)*d^3 - (B*a*b^2*c^2*d + B*a^2*b*d^3 - (
C*a*b^2 - A*b^3)*c^3 - (C*a^2*b - A*a*b^2)*c*d^2)*x)*sqrt(b*x^2 + a))/(a^2
*b^3*c^4 + 2*a^3*b^2*c^2*d^2 + a^4*b*d^4 + (a*b^4*c^4 + 2*a^2*b^3*c^2*d^2
+ a^3*b^2*d^4)*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(131) = 262.

Time = 0.10 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.28

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx &= \frac{Cbc^3x}{\sqrt{bx^2+a}abc^2d^2+\sqrt{bx^2+a}a^2d^4} \\ &- \frac{Bbc^2x}{\sqrt{bx^2+a}abc^2d+\sqrt{bx^2+a}a^2d^3}+\frac{Abcx}{\sqrt{bx^2+a}abc^2+\sqrt{bx^2+a}a^2d^2} \\ &+ \frac{Cc^2}{\sqrt{bx^2+a}bc^2d+\sqrt{bx^2+a}ad^3}-\frac{Bc}{\sqrt{bx^2+a}bc^2+\sqrt{bx^2+a}ad^2} \\ &+ \frac{A}{\frac{\sqrt{bx^2+a}bc^2}{d}+\sqrt{bx^2+a}ad}-\frac{Ccx}{\sqrt{bx^2+a}ad^2}+\frac{Bx}{\sqrt{bx^2+a}ad} \\ &+ \frac{Cc^2\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^3}-\frac{Bc\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d^2} \\ &+ \frac{A\,\mathrm{arsinh}\left(\frac{bcx}{\sqrt{ab}|dx+c|}-\frac{ad}{\sqrt{ab}|dx+c|}\right)}{\left(a+\frac{bc^2}{d^2}\right)^{\frac{3}{2}}d}-\frac{C}{\sqrt{bx^2+a}bd}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
C*b*c^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - B*b*c^
2*x/(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) + A*b*c*x/(sqrt(
b*x^2 + a)*a*b*c^2 + sqrt(b*x^2 + a)*a^2*d^2) + C*c^2/(sqrt(b*x^2 + a)*b*c
^2*d + sqrt(b*x^2 + a)*a*d^3) - B*c/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 +
a)*a*d^2) + A/(sqrt(b*x^2 + a)*b*c^2/d + sqrt(b*x^2 + a)*a*d) - C*c*x/(sqr
t(b*x^2 + a)*a*d^2) + B*x/(sqrt(b*x^2 + a)*a*d) + C*c^2*arcsinh(b*c*x/(sqr
t(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2
)*d^3) - B*c*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d
*x + c)))/((a + b*c^2/d^2)^(3/2)*d^2) + A*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x
 + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(3/2)*d) - C/(sqrt
(b*x^2 + a)*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(131) = 262$.

Time = 0.14 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.10

$$\int \frac{A + Bx + Cx^2}{(c + dx)(a + bx^2)^{3/2}} \, dx = -\frac{\frac{(Cab^2c^3 - Ab^3c^3 - Bab^2c^2d + Ca^2bcd^2 - Aab^2cd^2 - Ba^2bd^3)x}{ab^3c^4 + 2a^2b^2c^2d^2 + a^3bd^4} + \frac{Bab^2c^3 + Ca^2bc^2d - Aab^2c^2d + Ba^2bcd^2 + Ca^3d^3 - Aa^2bd^3}{ab^3c^4 + 2a^2b^2c^2d^2 + a^3bd^4}}{\sqrt{bx^2 + a}} - \frac{2(Cc^2 - Bcd + Ad^2)\arctan\left(\frac{\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)d + \sqrt{b}c}{\sqrt{-bc^2 - ad^2}}\right)}{(bc^2 + ad^2)\sqrt{-bc^2 - ad^2}}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((C*a*b^2*c^3 - A*b^3*c^3 - B*a*b^2*c^2*d + C*a^2*b*c*d^2 - A*a*b^2*c*d^2
 - B*a^2*b*d^3)*x/(a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4) + (B*a*b^2*c
^3 + C*a^2*b*c^2*d - A*a*b^2*c^2*d + B*a^2*b*c*d^2 + C*a^3*d^3 - A*a^2*b*d
^3)/(a*b^3*c^4 + 2*a^2*b^2*c^2*d^2 + a^3*b*d^4))/sqrt(b*x^2 + a) - 2*(C*c^
2 - B*c*d + A*d^2)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d + sqrt(b)*c)/sq
rt(-b*c^2 - a*d^2))/((b*c^2 + a*d^2)*sqrt(-b*c^2 - a*d^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/((a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/((a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{(dx+c)\left(b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.154 $\int \frac{A+Bx+Cx^2}{x(c+dx)(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 161

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \frac{Abc-acC+aBd+(bBc-Abd+aCd)x}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}} + \frac{d(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c\left(bc^2+ad^2\right)^{3/2}} - \frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c}$$

output

```
(A*b*c-C*a*c+B*a*d+(-A*b*d+B*b*c+C*a*d)*x)/a/(a*d^2+b*c^2)/(b*x^2+a)^(1/2)
+d*(A*d^2-B*c*d+C*c^2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^
(1/2))/c/(a*d^2+b*c^2)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \frac{bBcx+Ab(c-dx)+a(-cC+Bd+Cdx)}{a\left(bc^2+ad^2\right)\sqrt{a+bx^2}} - \frac{2d(c^2C-Bcd+Ad^2)\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{c\left(-bc^2-ad^2\right)^{3/2}} + \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(b*B*c*x + A*b*(c - d*x) + a*(-(c*C) + B*d + C*d*x))/(a*(b*c^2 + a*d^2)*Sq
rt[a + b*x^2]) - (2*d*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) -
d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(c*(-(b*c^2) - a*d^2)^(3/2)) +
 (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(a^(3/2)*c)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.64, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2351, 617, 686, 27, 488, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{3/2}(c+dx)}dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x(c+dx)\left(bx^2+a\right)^{3/2}}dx+\int \frac{B+Cx}{(c+dx)\left(bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 617}$$

$$A\int \left(\frac{1}{cx\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)\left(bx^2+a\right)^{3/2}}\right)dx+\int \frac{B+Cx}{(c+dx)\left(bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$A\int \left(\frac{1}{cx\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)\left(bx^2+a\right)^{3/2}}\right)dx-\frac{\int \frac{abd(cC-Bd)}{(c+dx)\sqrt{bx^2+a}}dx}{ab\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$A\int \left(\frac{1}{cx\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)\left(bx^2+a\right)^{3/2}}\right)dx-\frac{d(cC-Bd)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{488}$$

$$A\int \left(\frac{1}{cx\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)\left(bx^2+a\right)^{3/2}}\right)dx+\frac{d(cC-Bd)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{219}$$

$$A\int \left(\frac{1}{cx\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)\left(bx^2+a\right)^{3/2}}\right)dx+\frac{d(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$A\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c}+\frac{d^3\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c\left(ad^2+bc^2\right)^{3/2}}-\frac{d(ad+bcx)}{ac\sqrt{a+bx^2}\left(ad^2+bc^2\right)}+\frac{1}{ac\sqrt{a+bx^2}}\right)+\frac{d(cC-Bd)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)}$$

input

```
Int[(A + B*x + C*x^2)/(x*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output

```
-((a*(c*C - B*d) - (b*B*c + a*C*d)*x)/(a*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]))
 + (d*(c*C - B*d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])])/(b*c^2 + a*d^2)^(3/2) + A*(1/(a*c*Sqrt[a + b*x^2]) - (d*(a*d + b*c*x
))/(a*c*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + (d^3*ArcTanh[(a*d - b*c*x)/(Sqr
t[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c*(b*c^2 + a*d^2)^(3/2)) - ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]/(a^(3/2)*c))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(147) = 294$.

Time = 0.20 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.49

method	result
default	$\frac{Cx}{da\sqrt{b\,x^2+a}}+\frac{A\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{c}-\frac{\left(A\,d^2-Bcd+C\,c^2\right)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\cdots\right.}{\cdots}$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C/d*x/a/(b*x^2+a)^(1/2)+A/c*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x))-1/d^2*(A*d^2-B*c*d+C*c^2)/c*(1/(a*d^2+b*c^2)*d^2/(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)
*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2
)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)
^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. $2(151) = 302$.

Time = 5.01 (sec) , antiderivative size = 1847, normalized size of antiderivative = 11.47

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a^3*c^2*d - B*a^3*c*d^2 + A*a^3*d^3 + (C*a^2*b*c^2*d - B*a^2*b*c*
d^2 + A*a^2*b*d^3)*x^2)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b*c^2 - 2
*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 + 2*sqrt(b*c^2 + a*d^2)*(b*c*x - a*d)
*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) + (A*a*b^2*c^4 + 2*A*a^2*b*c^
2*d^2 + A*a^3*d^4 + (A*b^3*c^4 + 2*A*a*b^2*c^2*d^2 + A*a^2*b*d^4)*x^2)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*a^2*b*c^3*
d + B*a^3*c*d^3 - (C*a^2*b - A*a*b^2)*c^4 - (C*a^3 - A*a^2*b)*c^2*d^2 + (B
*a*b^2*c^4 + B*a^2*b*c^2*d^2 + (C*a^2*b - A*a*b^2)*c^3*d + (C*a^3 - A*a^2*
b)*c*d^3)*x)*sqrt(b*x^2 + a))/(a^3*b^2*c^5 + 2*a^4*b*c^3*d^2 + a^5*c*d^4 +
 (a^2*b^3*c^5 + 2*a^3*b^2*c^3*d^2 + a^4*b*c*d^4)*x^2), 1/2*(2*(C*a^3*c^2*d
 - B*a^3*c*d^2 + A*a^3*d^3 + (C*a^2*b*c^2*d - B*a^2*b*c*d^2 + A*a^2*b*d^3)
*x^2)*sqrt(-b*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(
b*x^2 + a)/(a*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + (A*a*b^2*c^4 +
 2*A*a^2*b*c^2*d^2 + A*a^3*d^4 + (A*b^3*c^4 + 2*A*a*b^2*c^2*d^2 + A*a^2*b*
d^4)*x^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(B*a^2*b*c^3*d + B*a^3*c*d^3 - (C*a^2*b - A*a*b^2)*c^4 - (C*a^3 - A*a^2*b)
*c^2*d^2 + (B*a*b^2*c^4 + B*a^2*b*c^2*d^2 + (C*a^2*b - A*a*b^2)*c^3*d + (C
*a^3 - A*a^2*b)*c*d^3)*x)*sqrt(b*x^2 + a))/(a^3*b^2*c^5 + 2*a^4*b*c^3*d^2
+ a^5*c*d^4 + (a^2*b^3*c^5 + 2*a^3*b^2*c^3*d^2 + a^4*b*c*d^4)*x^2), 1/2*(2
*(A*a*b^2*c^4 + 2*A*a^2*b*c^2*d^2 + A*a^3*d^4 + (A*b^3*c^4 + 2*A*a*b^2*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*(a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{3}{2}}(dx+c)x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,{\left(b\,x^2+a\right)}^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1103, normalized size of antiderivative = 6.85

$$\int \frac{A+Bx+Cx^2}{x(c+dx)(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**3*d**3 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sq
rt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**2 + 2*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*d**3*
x**2 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a**2*c**3*d - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*
x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**2*x**2 + 2*sqrt(a*d
**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)
*a*b*c**3*d*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*d**3 + 2*sqrt
(a*d**2 + b*c**2)*log(c + d*x)*a**2*b*c*d**2 - 2*sqrt(a*d**2 + b*c**2)*log
(c + d*x)*a**2*b*d**3*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**2*c**
3*d + 2*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a*b**2*c*d**2*x**2 - 2*sqrt(a*d
**2 + b*c**2)*log(c + d*x)*a*b*c**3*d*x**2 + 2*sqrt(a + b*x**2)*a**2*b*c**
2*d**2 - 2*sqrt(a + b*x**2)*a**2*b*c*d**3*x + 2*sqrt(a + b*x**2)*a**2*b*c*
d**3 - 2*sqrt(a + b*x**2)*a**2*c**3*d**2 + 2*sqrt(a + b*x**2)*a**2*c**2*d*
*3*x + 2*sqrt(a + b*x**2)*a*b**2*c**4 - 2*sqrt(a + b*x**2)*a*b**2*c**3*d*x
 + 2*sqrt(a + b*x**2)*a*b**2*c**3*d + 2*sqrt(a + b*x**2)*a*b**2*c**2*d**2*
x - 2*sqrt(a + b*x**2)*a*b*c**5 + 2*sqrt(a + b*x**2)*a*b*c**4*d*x + 2*sqrt
(a + b*x**2)*b**3*c**4*x + sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**3*d*
*4 + 2*sqrt(a)*log(sqrt(a + b*x**2) - sqrt(a))*a**2*b*c**2*d**2 + sqrt(...
```

3.155 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 215

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{acx\sqrt{a+bx^2}} + \frac{ac(bBc-Abd+aCd)+b(ac(cC-Bd)-A(2bc^2+ad^2))\,x}{a^2c\left(bc^2+ad^2\right)\sqrt{a+bx^2}} - \frac{d^2(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2\left(bc^2+ad^2\right)^{3/2}} - \frac{(Bc-Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

output
```
-A/a/c/x/(b*x^2+a)^(1/2)+(a*c*(-A*b*d+B*b*c+C*a*d)+b*(a*c*(-B*d+C*c)-A*(a*
d^2+2*b*c^2))*x)/a^2/c/(a*d^2+b*c^2)/(b*x^2+a)^(1/2)-d^2*(A*d^2-B*c*d+C*c^
2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2/(a*d^2+b*
c^2)^(3/2)-(-A*d+B*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^2
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \frac{-\frac{c\left(2Ab^2c^2x^2+a^2d(Ad-cCx)+ab\left(-cx(Bc+cCx-Bdx)+A\left(c^2+cdx+d^2x^2\right)\right)\right)}{a^2\left(bc^2+ad^2\right)x\sqrt{a+bx^2}}+\frac{2d^2\left(c^2C-Bcd+A\right.}{}}{c^2}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(-((c*(2*A*b^2*c^2*x^2 + a^2*d*(A*d - c*C*x) + a*b*(-(c*x*(B*c + c*C*x - B
*d*x)) + A*(c^2 + c*d*x + d^2*x^2))))/(a^2*(b*c^2 + a*d^2)*x*Sqrt[a + b*x^
2])) + (2*d^2*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a
 + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) - (2*(B*c - A
*d)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/c^2
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{3/2}(c+dx)}\,dx$$

$$\Big\downarrow \; 2353$$

$$\int \left(\frac{Ad^2-Bcd+c^2C}{c^2\left(a+bx^2\right)^{3/2}(c+dx)}+\frac{Bc-Ad}{c^2x\left(a+bx^2\right)^{3/2}}+\frac{A}{cx^2\left(a+bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-Ad)}{a^{3/2}c^2}-\frac{2Abx}{a^2c\sqrt{a+bx^2}}-\frac{d^2\left(Ad^2-Bcd+c^2C\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\left(ad^2+bc^2\right)^{3/2}}+\frac{(ad+bcx)\left(Ad^2-Bcd+c^2C\right)}{ac^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)}+\frac{Bc-Ad}{ac^2\sqrt{a+bx^2}}-\frac{A}{acx\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(B*c - A*d)/(a*c^2*Sqrt[a + b*x^2]) - A/(a*c*x*Sqrt[a + b*x^2]) - (2*A*b*x
)/(a^2*c*Sqrt[a + b*x^2]) + ((c^2*C - B*c*d + A*d^2)*(a*d + b*c*x))/(a*c^2
*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (d^2*(c^2*C - B*c*d + A*d^2)*ArcTanh[(
a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*(b*c^2 + a*d^2)^
(3/2)) - ((B*c - A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a^(3/2)*c^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(199) = 398$.

Time = 0.27 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.00

method	result
default	$\frac{A\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{c}-\frac{(Ad-Bc)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{c^2}+\frac{(A\,d^2-Bcd+C\,c^2)\left(\frac{\cdots}{(a\,d^2+b\,c^2)\sqrt{b\left(x+\frac{c}{d}\right)^2\cdots}}\cdots\right.}{\cdots}$
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^2cx}-\frac{-\frac{(Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}-\frac{ba\left(A\,d^2-Bcd+C\,c^2\right)d\ln\left(\frac{\frac{2a\,d^2+2b\,c^2}{d^2}-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+2\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\cdots}{\cdots}}}{x+\frac{c}{d}}\right)}{(d\sqrt{-ab}+bc)(d\sqrt{-ab}-bc)c\sqrt{\frac{a\,d^2+b\,c^2}{d^2}}}\cdots}{\cdots}$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A/c*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))-(A*d-B*c)/c^2*(1/a/
(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+(A*d^2-B*
c*d+C*c^2)/c^2/d*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+
b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^
2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(200) = 400$.

Time = 8.52 (sec) , antiderivative size = 2398, normalized size of antiderivative = 11.15

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*(((C*a^2*b*c^2*d^2 - B*a^2*b*c*d^3 + A*a^2*b*d^4)*x^3 + (C*a^3*c^2*d^
2 - B*a^3*c*d^3 + A*a^3*d^4)*x)*sqrt(b*c^2 + a*d^2)*log((2*a*b*c*d*x - a*b
*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2 + a*d^2)*(b*c*
x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - ((B*b^3*c^5 - A*b^3
*c^4*d + 2*B*a*b^2*c^3*d^2 - 2*A*a*b^2*c^2*d^3 + B*a^2*b*c*d^4 - A*a^2*b*d
^5)*x^3 + (B*a*b^2*c^5 - A*a*b^2*c^4*d + 2*B*a^2*b*c^3*d^2 - 2*A*a^2*b*c^2
*d^3 + B*a^3*c*d^4 - A*a^3*d^5)*x)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(a) + 2*a)/x^2) - 2*(A*a*b^2*c^5 + 2*A*a^2*b*c^3*d^2 + A*a^3*c*d^4 +
(B*a*b^2*c^4*d + B*a^2*b*c^2*d^3 + A*a^2*b*c*d^4 - (C*a*b^2 - 2*A*b^3)*c^5
 - (C*a^2*b - 3*A*a*b^2)*c^3*d^2)*x^2 - (B*a*b^2*c^5 + B*a^2*b*c^3*d^2 + (
C*a^2*b - A*a*b^2)*c^4*d + (C*a^3 - A*a^2*b)*c^2*d^3)*x)*sqrt(b*x^2 + a))/
((a^2*b^3*c^6 + 2*a^3*b^2*c^4*d^2 + a^4*b*c^2*d^4)*x^3 + (a^3*b^2*c^6 + 2*
a^4*b*c^4*d^2 + a^5*c^2*d^4)*x), -1/2*(2*((C*a^2*b*c^2*d^2 - B*a^2*b*c*d^3
 + A*a^2*b*d^4)*x^3 + (C*a^3*c^2*d^2 - B*a^3*c*d^3 + A*a^3*d^4)*x)*sqrt(-b
*c^2 - a*d^2)*arctan(sqrt(-b*c^2 - a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a)/(a
*b*c^2 + a^2*d^2 + (b^2*c^2 + a*b*d^2)*x^2)) + ((B*b^3*c^5 - A*b^3*c^4*d +
 2*B*a*b^2*c^3*d^2 - 2*A*a*b^2*c^2*d^3 + B*a^2*b*c*d^4 - A*a^2*b*d^5)*x^3
+ (B*a*b^2*c^5 - A*a*b^2*c^4*d + 2*B*a^2*b*c^3*d^2 - 2*A*a^2*b*c^2*d^3 + B
*a^3*c*d^4 - A*a^3*d^5)*x)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
 + 2*a)/x^2) + 2*(A*a*b^2*c^5 + 2*A*a^2*b*c^3*d^2 + A*a^3*c*d^4 + (B*a*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*(a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{3}{2}}(dx+c)x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.81

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\frac{\frac{(Ca^2b^2c^3-Aab^3c^3-Ba^2b^2c^2d+Ca^3bcd^2-Aa^2b^2cd^2-Ba^3bd^3)x}{a^3b^2c^4+2\,a^4bc^2d^2+a^5d^4}+\frac{Ba^2b^2c^3+Ca^3bc^2d-Aa^2b^2c^2d+}{a^3b^2c^4+2\,a^4bc^2}}{\sqrt{bx^2+a}}$$
$$-\frac{2\left(Cc^2d^2-Bcd^3+Ad^4\right)\arctan\left(\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{(bc^4+ac^2d^2)\sqrt{-bc^2-ad^2}}$$
$$+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)ac}+\frac{2\left(Bc-Ad\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ac^2}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((C*a^2*b^2*c^3 - A*a*b^3*c^3 - B*a^2*b^2*c^2*d + C*a^3*b*c*d^2 - A*a^2*b^
2*c*d^2 - B*a^3*b*d^3)*x/(a^3*b^2*c^4 + 2*a^4*b*c^2*d^2 + a^5*d^4) + (B*a^
2*b^2*c^3 + C*a^3*b*c^2*d - A*a^2*b^2*c^2*d + B*a^3*b*c*d^2 + C*a^4*d^3 -
A*a^3*b*d^3)/(a^3*b^2*c^4 + 2*a^4*b*c^2*d^2 + a^5*d^4))/sqrt(b*x^2 + a) -
2*(C*c^2*d^2 - B*c*d^3 + A*d^4)*arctan(((sqrt(b)*x - sqrt(b*x^2 + a))*d +
sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b*c^4 + a*c^2*d^2)*sqrt(-b*c^2 - a*d^2)
) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a*c) + 2*(B*c - A*d
)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\,(dx+c)\,\left(b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)/(b*x^2+a)^(3/2),x)
```

3.156 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 293

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx = -\frac{A}{2acx^2\sqrt{a+bx^2}}-\frac{Bc-Ad}{ac^2x\sqrt{a+bx^2}} + \frac{b\left(c(2ac(cC-Bd)-A(3bc^2+ad^2))-2\left(2bc^2(Bc-Ad)+ad(c^2C+Bcd-Ad^2)\right)x\right)}{2a^2c^2\left(bc^2+ad^2\right)\sqrt{a+bx^2}} + \frac{d^3(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3\left(bc^2+ad^2\right)^{3/2}} - \frac{\left(2ac(cC-Bd)-A(3bc^2-2ad^2)\right)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}c^3}$$

output
```
-1/2*A/a/c/x^2/(b*x^2+a)^(1/2)-(-A*d+B*c)/a/c^2/x/(b*x^2+a)^(1/2)+1/2*b*(c
*(2*a*c*(-B*d+C*c)-A*(a*d^2+3*b*c^2))-2*(2*b*c^2*(-A*d+B*c)+a*d*(-A*d^2+B*
c*d+C*c^2))*x)/a^2/c^2/(a*d^2+b*c^2)/(b*x^2+a)^(1/2)+d^3*(A*d^2-B*c*d+C*c^
2)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^3/(a*d^2+b*
c^2)^(3/2)-1/2*(2*a*c*(-B*d+C*c)-A*(-2*a*d^2+3*b*c^2))*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(5/2)/c^3
```

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{c\left(b^2c^2x^2(3Ac+4Bcx-4Adx)+a^2d^2(2Bcx+A(c-2dx))+ab\left(A\left(c^3-2c^2dx+cd^2x^2-2d^3x^3\right)+2cx\left(cCx(-c+dx)+B\left(c^2+cdx+d^2x^2\right)\right)\right)\right)}{a^2(bc^2+ad^2)x^2\sqrt{a+bx^2}}+\frac{4d^3(c}{}}{2c^3}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
-1/2*((c*(b^2*c^2*x^2*(3*A*c + 4*B*c*x - 4*A*d*x) + a^2*d^2*(2*B*c*x + A*(
c - 2*d*x)) + a*b*(A*(c^3 - 2*c^2*d*x + c*d^2*x^2 - 2*d^3*x^3) + 2*c*x*(c*
C*x*(-c + d*x) + B*(c^2 + c*d*x + d^2*x^2)))))/(a^2*(b*c^2 + a*d^2)*x^2*Sq
rt[a + b*x^2]) + (4*d^3*(c^2*C - B*c*d + A*d^2)*ArcTan[(Sqrt[b]*(c + d*x)
- d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(3/2) + (
6*A*b*c^2*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2) + (4*(c^
2*C - B*c*d + A*d^2)*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^
(3/2))/c^3
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{3/2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{Bc-Ad}{c^2x^2\left(a+bx^2\right)^{3/2}}+\frac{Ad^2-Bcd+c^2C}{c^3x\left(a+bx^2\right)^{3/2}}-\frac{d\left(Ad^2-Bcd+c^2C\right)}{c^3\left(a+bx^2\right)^{3/2}(c+dx)}+\frac{A}{cx^3\left(a+bx^2\right)^{3/2}}\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Ad^2-Bcd+c^2C)}{a^{3/2}c^3}+\frac{3Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}c}-\frac{2bx(Bc-Ad)}{a^2c^2\sqrt{a+bx^2}}-\frac{3Ab}{2a^2c\sqrt{a+bx^2}}+\frac{d^3(Ad^2-Bcd+c^2C)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3(ad^2+bc^2)^{3/2}}-\frac{Bc-Ad}{ac^2x\sqrt{a+bx^2}}-\frac{d(ad+bcx)(Ad^2-Bcd+c^2C)}{ac^3\sqrt{a+bx^2}(ad^2+bc^2)}+\frac{Ad^2-Bcd+c^2C}{ac^3\sqrt{a+bx^2}}-\frac{A}{2acx^2\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)*(a + b*x^2)^(3/2)),x]
```

output
```
(-3*A*b)/(2*a^2*c*Sqrt[a + b*x^2]) + (c^2*C - B*c*d + A*d^2)/(a*c^3*Sqrt[a
 + b*x^2]) - A/(2*a*c*x^2*Sqrt[a + b*x^2]) - (B*c - A*d)/(a*c^2*x*Sqrt[a +
 b*x^2]) - (2*b*(B*c - A*d)*x)/(a^2*c^2*Sqrt[a + b*x^2]) - (d*(c^2*C - B*c
*d + A*d^2)*(a*d + b*c*x))/(a*c^3*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + (d^3*
(c^2*C - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a
+ b*x^2])])/(c^3*(b*c^2 + a*d^2)^(3/2)) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/(2*a^(5/2)*c) - ((c^2*C - B*c*d + A*d^2)*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(a^(3/2)*c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-2Adx+2Bcx+Ac)}{2a^2c^2x^2}-\frac{\frac{\left(2Aa\,d^2-3bA\,c^2-2Bacd+2Ca\,c^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}+\frac{b\,c^2\,(Ab-B\sqrt{-ab}-aC)\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^2 \cdots}}{(d\sqrt{-ab}-bc)\sqrt{-ab}\left(x+\cdots\right.}}{\cdots}$
default	$\frac{A\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)}{c}-\frac{(Ad-Bc)\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{c^2}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\cdots}{\cdots}$

input

```
int((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/2*(b*x^2+a)^(1/2)*(-2*A*d*x+2*B*c*x+A*c)/a^2/c^2/x^2-1/2/c^2/a^2*((2*A*
a*d^2-3*A*b*c^2-2*B*a*c*d+2*C*a*c^2)/c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)
^(1/2))/x)+b*c^2*(A*b-B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)-b*c)/(-a*b)^(1/2
)/(x+(-a*b)^(1/2)/b)*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2
)/b))^(1/2)+b*c^2*(A*b+B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)+b*c)/(-a*b)^(1/
2)/(x-(-a*b)^(1/2)/b)*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/
2)/b))^(1/2)+2*b*a^2*(A*d^2-B*c*d+C*c^2)*d^2/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b
)^(1/2)-b*c)/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(
x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c
^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. 2(270) = 540.

Time = 21.32 (sec) , antiderivative size = 3317, normalized size of antiderivative = 11.32

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{\frac{3}{2}}(c+dx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*(a + b*x**2)**(3/2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{3}{2}}(dx+c)x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.83

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{\left(Ba^2b^3c^3+Ca^3b^2c^2d-Aa^2b^3c^2d+Ba^3b^2cd^2+Ca^4bd^3-Aa^3b^2d^3\right)x}{a^4b^2c^4+2\,a^5bc^2d^2+a^6d^4}-\frac{Ca^3b^2c^3-Aa^2b^3c^3-Ba^3b^2c^2d+Ca^4bcd^2-Aa^3b^2cd^2-Ba^4bd^3}{a^4b^2c^4+2\,a^5bc^2d^2+a^6d^4}}{\sqrt{bx^2+a}}$$

$$-\frac{2\left(Cc^2d^3-Bcd^4+Ad^5\right)\arctan\left(-\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)d+\sqrt{b}c}{\sqrt{-bc^2-ad^2}}\right)}{\left(bc^5+ac^3d^2\right)\sqrt{-bc^2-ad^2}}$$

$$+\frac{\left(2\,Cac^2-3\,Abc^2-2\,Bacd+2\,Aad^2\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2c^3}$$

$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Abc+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}c-2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Aa\sqrt{b}d+\left(\sqrt{b}x-\sqrt{b}\right.}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2c^2}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-((B*a^2*b^3*c^3 + C*a^3*b^2*c^2*d - A*a^2*b^3*c^2*d + B*a^3*b^2*c*d^2 + C
*a^4*b*d^3 - A*a^3*b^2*d^3)*x/(a^4*b^2*c^4 + 2*a^5*b*c^2*d^2 + a^6*d^4) -
(C*a^3*b^2*c^3 - A*a^2*b^3*c^3 - B*a^3*b^2*c^2*d + C*a^4*b*c*d^2 - A*a^3*b
^2*c*d^2 - B*a^4*b*d^3)/(a^4*b^2*c^4 + 2*a^5*b*c^2*d^2 + a^6*d^4))/sqrt(b*
x^2 + a) - 2*(C*c^2*d^3 - B*c*d^4 + A*d^5)*arctan(-((sqrt(b)*x - sqrt(b*x^
2 + a))*d + sqrt(b)*c)/sqrt(-b*c^2 - a*d^2))/((b*c^5 + a*c^3*d^2)*sqrt(-b*
c^2 - a*d^2)) + (2*C*a*c^2 - 3*A*b*c^2 - 2*B*a*c*d + 2*A*a*d^2)*arctan(-(s
qrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*c^3) + ((sqrt(b)*x - s
qrt(b*x^2 + a))^3*A*b*c + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b)*c
- 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a*sqrt(b)*d + (sqrt(b)*x - sqrt(b*x^
2 + a))*A*a*b*c - 2*B*a^2*sqrt(b)*c + 2*A*a^2*sqrt(b)*d)/(((sqrt(b)*x - sq
rt(b*x^2 + a))^2 - a)^2*a^2*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2277, normalized size of antiderivative = 7.77

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)/(b*x^2+a)^(3/2),x)
```

output

```
(4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**4*d**5*x**2 - 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c*d**4*x**2 + 4*sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
*3*b*d**5*x**4 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d*
*2 + b*c**2) - a*d + b*c*x)*a**3*c**3*d**3*x**2 - 4*sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c*d
**4*x**4 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*b*c**3*d**3*x**4 - 4*sqrt(a*d**2 + b*c**2)*log(
c + d*x)*a**4*d**5*x**2 + 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b*c*d*
*4*x**2 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)*a**3*b*d**5*x**4 - 4*sqrt(a
*d**2 + b*c**2)*log(c + d*x)*a**3*c**3*d**3*x**2 + 4*sqrt(a*d**2 + b*c**2)
*log(c + d*x)*a**2*b**2*c*d**4*x**4 - 4*sqrt(a*d**2 + b*c**2)*log(c + d*x)
*a**2*b*c**3*d**3*x**4 - 2*sqrt(a + b*x**2)*a**4*c**2*d**4 + 4*sqrt(a + b*
x**2)*a**4*c*d**5*x - 4*sqrt(a + b*x**2)*a**3*b*c**4*d**2 + 8*sqrt(a + b*x
**2)*a**3*b*c**3*d**3*x - 2*sqrt(a + b*x**2)*a**3*b*c**2*d**4*x**2 - 4*sqr
t(a + b*x**2)*a**3*b*c**2*d**4*x + 4*sqrt(a + b*x**2)*a**3*b*c*d**5*x**3 -
 2*sqrt(a + b*x**2)*a**2*b**2*c**6 + 4*sqrt(a + b*x**2)*a**2*b**2*c**5*d*x
 - 8*sqrt(a + b*x**2)*a**2*b**2*c**4*d**2*x**2 - 8*sqrt(a + b*x**2)*a**2*b
**2*c**4*d**2*x + 12*sqrt(a + b*x**2)*a**2*b**2*c**3*d**3*x**3 - 4*sqrt...
```

3.157 $\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 368

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = -\frac{a(a(bc(Bc-2Ad)+ad(2cC-Bd))-(Ab(bc^2-ad^2)+a(aCd^2-bc(cC-2Bd)))x)}{b^2(bc^2+ad^2)^2\sqrt{a+bx^2}} - \frac{(2cC-Bd)\sqrt{a+bx^2}}{b^2d^3} + \frac{Cx\sqrt{a+bx^2}}{2b^2d^2} - \frac{c^4(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^3(bc^2+ad^2)^2(c+dx)} - \frac{(3aCd^2-2b(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}d^4} + \frac{c^3(bc^2(3c^2C-2Bcd+Ad^2)+ad^2(6c^2C-5Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^4(bc^2+ad^2)^{5/2}}$$

output

```
-a*(a*(b*c*(-2*A*d+B*c)+a*d*(-B*d+2*C*c))-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b
*c*(-2*B*d+C*c)))*x)/b^2/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)-(-B*d+2*C*c)*(b*x
^2+a)^(1/2)/b^2/d^3+1/2*C*x*(b*x^2+a)^(1/2)/b^2/d^2-c^4*(A*d^2-B*c*d+C*c^2
)*(b*x^2+a)^(1/2)/d^3/(a*d^2+b*c^2)^2/(d*x+c)-1/2*(3*a*C*d^2-2*b*(A*d^2-2*
B*c*d+3*C*c^2))*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)/d^4+c^3*(b*c^2*
(A*d^2-2*B*c*d+3*C*c^2)+a*d^2*(4*A*d^2-5*B*c*d+6*C*c^2))*arctanh((-b*c*x+a
*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^4/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 3.85 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.21

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \frac{d\left(a^3d^4(c+dx)(-8cC+4Bd+3Cdx)+b^3c^4x^2\left(-6c^2C+cd(4B-3Cx)+d^2(-2A+x(2B+Cx))\right)-a^2bd^2(c+\cdots\right.}{\cdots}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
((d*(a^3*d^4*(c + d*x)*(-8*c*C + 4*B*d + 3*C*d*x) + b^3*c^4*x^2*(-6*c^2*C
+ c*d*(4*B - 3*C*x) + d^2*(-2*A + x*(2*B + C*x))) - a^2*b*d^2*(c + d*x)*(8
*c^3*C - 2*B*c^2*d - 4*c*d^2*(A + x*(B - C*x)) - d^3*x*(-2*A + x*(2*B + C*
x))) + a*b^2*c^2*(-6*c^4*C + c^3*d*(4*B - 3*C*x) + c^2*d^2*(-2*A + x*(2*B
- 7*C*x)) + 2*c*d^3*x*(A + x*(2*B - 3*C*x)) + 2*d^4*x^2*(A + x*(2*B + C*x)
))))/(b^2*(b*c^2 + a*d^2)^2*(c + d*x)*Sqrt[a + b*x^2]) + (4*c^3*(b*c^2*(3*
c^2*C - 2*B*c*d + A*d^2) + a*d^2*(6*c^2*C - 5*B*c*d + 4*A*d^2))*ArcTan[(Sq
rt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a + b*x^2])])/(-(b*c^2
) - a*d^2)^(5/2) + (2*(-3*a*C*d^2 + 2*b*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTa
nh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(5/2))/(2*d^4)
```

Rubi [A] (verified)

Time = 3.95 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2178, 2182, 25, 2185, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \; 2178$$

$$-\frac{\int \frac{-aCx^4-aBx^3-\frac{a(Ab-aC)x^2}{b}+\frac{a^2c\left(b^2Bc^3+2a^2Cd^3+abd^2(3Bc-2Ad)\right)x}{b\left(bc^2+ad^2\right)^2}+\frac{a^2c^2\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\left(bc^2+ad^2\right)^2}}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 2182

$$-\frac{\frac{abc^4\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^3(c+dx)(ad^2+bc^2)^2}-\frac{\int -\frac{-aC\left(\frac{bc^2}{d}+ad\right)x^3+\frac{a(cC-Bd)\left(bc^2+ad^2\right)x^2}{d^2}+\frac{a\left(bc^2+ad^2\right)\left(aCd^2-b\left(Cc^2-Bdc+Ad^2\right)\right)x}{bd^3}+\frac{a^2c\left(a^2Cd^4-ab\left(Cc^2\right.\right.}{}}{(c+dx)\sqrt{bx^2+a}}}{ad^2+bc^2}}{ab}-\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{-aC\left(\frac{bc^2}{d}+ad\right)x^3+\frac{a(cC-Bd)\left(bc^2+ad^2\right)x^2}{d^2}+\frac{a\left(bc^2+ad^2\right)\left(aCd^2-b\left(Cc^2-Bdc+Ad^2\right)\right)x}{bd^3}+\frac{a^2c\left(a^2Cd^4-ab\left(Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(Cc^2-Bdc+2Ad^2\right.\right.}{bd^2\left(bc^2+ad^2\right)}}{(c+dx)\sqrt{bx^2+a}}}{ad^2+bc^2}}{ab}-\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{\frac{\int \frac{\frac{cd\left(3a^2Cd^4+2ab(2Bc-Ad)d^3+b^2c^2\left(3Cc^2-2Bdc+4Ad^2\right)\right)a^2}{bc^2+ad^2}+bd(5cC-2Bd)\left(bc^2+ad^2\right)x^2a+\left(bc^2+ad^2\right)\left(3aCd^2-b\left(Cc^2-2Bdc+2Ad^2\right)\right)xa}{(c+dx)\sqrt{bx^2+a}}dx}{2bd^3}-\frac{aC\sqrt{a+}}{}}{ad^2+bc^2}}{ab}-\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{\frac{\frac{\int \frac{abd^2\left(\frac{acd\left(3a^2Cd^4+2ab(2Bc-Ad)d^3+b^2c^2\left(3Cc^2-2Bdc+4Ad^2\right)\right)}{bc^2+ad^2}+\left(bc^2+ad^2\right)\left(3aCd^2-2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}+a\sqrt{a+bx^2}\left(ad^2+bc^2\right)(5cC-2Bd}{2bd^3}}{ad^2+bc^2}}{ab}-\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{a\int \frac{\frac{acd\left(3a^2Cd^4+2ab(2Bc-Ad)d^3+b^2c^2\left(3Cc^2-2Bdc+4Ad^2\right)\right)}{bc^2+ad^2}+\left(bc^2+ad^2\right)\left(3aCd^2-2b\left(3Cc^2-2Bdc+Ad^2\right)\right)x}{(c+dx)\sqrt{bx^2+a}}dx+a\sqrt{a+bx^2}\left(ad^2+bc^2\right)(5cC-2Bd)}{2bd^3}-\frac{aC\sqrt{}}{}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 719$$

$$-\frac{\frac{\frac{a\left(\frac{2b^2c^3\left(ad^2\left(4Ad^2-5Bcd+6c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(ad^2+bc^2\right)\left(3aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}\right)+a\sqrt{a+b}}{2bd^3}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{\frac{a\left(\frac{2b^2c^3\left(ad^2\left(4Ad^2-5Bcd+6c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\left(ad^2+bc^2\right)\left(3aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}\right)}{2bd^3}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{a\left(\frac{2b^2c^3\left(ad^2\left(4Ad^2-5Bcd+6c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)\left(3aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}\right)}{2bd^3}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 488$$

$$-\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)\left(3aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^3\left(ad^2\left(4Ad^2-5Bcd+6c^2C\right)+bc^2\left(Ad^2-2Bcd+3c^2C\right)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)}{bx^2+a}}}{d(ad^2+bc^2)}}{2bd^3}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\downarrow$ 219

$$-\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(ad^2+bc^2)\left(3aCd^2-2b\left(Ad^2-2Bcd+3c^2C\right)\right)}{\sqrt{b}d}-\frac{2b^2c^3\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(4Ad^2-5Bcd+6c^2C\right)+bc^2\left(Ad^2-2B\right.\right.}{d(ad^2+bc^2)^{3/2}}}{2bd^3}}{ad^2+bc^2}}{ab}$$

$$\frac{a\left(a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

input

```
Int[(x^4*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output

```
-((a*(a*(b*c*(B*c - 2*A*d) + a*d*(2*c*C - B*d)) - (A*b*(b*c^2 - a*d^2) + a
*(a*C*d^2 - b*c*(c*C - 2*B*d)))*x))/(b^2*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2]
)) - ((a*b*c^4*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d^3*(b*c^2 + a*d^
2)^2*(c + d*x)) + (-1/2*(a*C*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a + b*x^2])/(b
*d^3) + (a*(5*c*C - 2*B*d)*(b*c^2 + a*d^2)*Sqrt[a + b*x^2] + a*(((b*c^2 +
a*d^2)*(3*a*C*d^2 - 2*b*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(Sqrt[b]*d) - (2*b^2*c^3*(b*c^2*(3*c^2*C - 2*B*c*d + A*d^
2) + a*d^2*(6*c^2*C - 5*B*c*d + 4*A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^
2 + a*d^2]*Sqrt[a + b*x^2])])/(d*(b*c^2 + a*d^2)^(3/2))))/(2*b*d^3))/(b*c^
2 + a*d^2))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(344) = 688$.

Time = 0.45 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.04

method	result
risch	$\frac{(Cxd+2Bd-4Cc)\sqrt{b\,x^2+a}}{2b^2d^3}+\frac{\frac{\left(2Ab\,d^2-4Bbcd-3aC\,d^2+6Cb\,c^2\right)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{d^3a\left(Ab+B\sqrt{-ab}-aC\right)\sqrt{b\left(x-\frac{\sqrt{-ab}}{b}\right)^2+2\sqrt{-ab}\left(x \cdots\right.}}{\left(d\sqrt{-ab}+bc\right)^2\left(x-\frac{\sqrt{-ab}}{b}\right)}\cdots}{\cdots}$
default	Expression too large to display

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(C*d*x+2*B*d-4*C*c)*(b*x^2+a)^(1/2)/b^2/d^3+1/2/b^2/d^3*(1/d*(2*A*b*d^
2-4*B*b*c*d-3*C*a*d^2+6*C*b*c^2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)+d^3
*a*(A*b+B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)+b*c)^2/(x-(-a*b)^(1/2)/b)*(b*(
x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+d^3*a*(A*b-B*
(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)-b*c)^2/(x+(-a*b)^(1/2)/b)*(b*(x+(-a*b)^(
1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-2*b^3/d^3*c^4*(A*d^2-B*
c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b)^(1/2)-b*c)*(-1/(a*d^2+b*c^2)*d^2
/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^
2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)
+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^
2)^(1/2))/(x+c/d)))+2*b^4/d^2*c^3*(4*A*a*d^4+2*A*b*c^2*d^2-5*B*a*c*d^3-3*B
*b*c^3*d+6*C*a*c^2*d^2+4*C*b*c^4)/(d*(-a*b)^(1/2)+b*c)^2/(d*(-a*b)^(1/2)-b
*c)^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*
((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)^2}\,dx$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. 2(345) = 690.

Time = 0.32 (sec) , antiderivative size = 1610, normalized size of antiderivative = 4.38

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2 (a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
3*C*b^2*c^8*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^6 + 2*sqrt(b*x^2 + a)*a^2*b*c^2
*d^8 + sqrt(b*x^2 + a)*a^3*d^10) - 3*B*b^2*c^7*x/(sqrt(b*x^2 + a)*a*b^2*c^
4*d^5 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^7 + sqrt(b*x^2 + a)*a^3*d^9) + 3*A*b
^2*c^6*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^6
+ sqrt(b*x^2 + a)*a^3*d^8) + 3*C*b*c^7/(sqrt(b*x^2 + a)*b^2*c^4*d^5 + 2*sq
rt(b*x^2 + a)*a*b*c^2*d^7 + sqrt(b*x^2 + a)*a^2*d^9) - 8*C*b*c^6*x/(sqrt(b
*x^2 + a)*a*b*c^2*d^6 + sqrt(b*x^2 + a)*a^2*d^8) - 3*B*b*c^6/(sqrt(b*x^2 +
 a)*b^2*c^4*d^4 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^6 + sqrt(b*x^2 + a)*a^2*d^8)
 + 7*B*b*c^5*x/(sqrt(b*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7) + 3
*A*b*c^5/(sqrt(b*x^2 + a)*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^5 + sq
rt(b*x^2 + a)*a^2*d^7) - C*c^6/(sqrt(b*x^2 + a)*b*c^2*d^6*x + sqrt(b*x^2 +
 a)*a*d^8*x + sqrt(b*x^2 + a)*b*c^3*d^5 + sqrt(b*x^2 + a)*a*c*d^7) - 6*A*b
*c^4*x/(sqrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) + B*c^5/(sq
rt(b*x^2 + a)*b*c^2*d^5*x + sqrt(b*x^2 + a)*a*d^7*x + sqrt(b*x^2 + a)*b*c^
3*d^4 + sqrt(b*x^2 + a)*a*c*d^6) - 6*C*c^5/(sqrt(b*x^2 + a)*b*c^2*d^5 + sq
rt(b*x^2 + a)*a*d^7) - A*c^4/(sqrt(b*x^2 + a)*b*c^2*d^4*x + sqrt(b*x^2 + a
)*a*d^6*x + sqrt(b*x^2 + a)*b*c^3*d^3 + sqrt(b*x^2 + a)*a*c*d^5) + 5*B*c^4
/(sqrt(b*x^2 + a)*b*c^2*d^4 + sqrt(b*x^2 + a)*a*d^6) - 4*A*c^3/(sqrt(b*x^2
 + a)*b*c^2*d^3 + sqrt(b*x^2 + a)*a*d^5) + 1/2*C*x^3/(sqrt(b*x^2 + a)*b*d^
2) - 2*C*c*x^2/(sqrt(b*x^2 + a)*b*d^3) + B*x^2/(sqrt(b*x^2 + a)*b*d^2) ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^4\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^4(C\,x^2+Bx+A)}{(dx+c)^2\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

3.158 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 303

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{a(Ab(bc^2-ad^2)+a(aCd^2-bc(cC-2Bd))+b(bc(Bc-2Ad)+ad(2cC-}{b^2(bc^2+ad^2)^2\sqrt{a+bx^2}}$$

$$+\frac{C\sqrt{a+bx^2}}{b^2d^2}+\frac{c^3(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d^2(bc^2+ad^2)^2(c+dx)}-\frac{(2cC-Bd)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}d^3}$$

$$-\frac{c^2(bc^3(2cC-Bd)+ad^2(5c^2C-4Bcd+3Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^3(bc^2+ad^2)^{5/2}}$$

output
```
a*(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(-2*B*d+C*c))+b*(b*c*(-2*A*d+B*c)+a*d
*(-B*d+2*C*c))*x)/b^2/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)+C*(b*x^2+a)^(1/2)/b^
2/d^2+c^3*(A*d^2-B*c*d+C*c^2)*(b*x^2+a)^(1/2)/d^2/(a*d^2+b*c^2)^2/(d*x+c)-
(-B*d+2*C*c)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)/d^3-c^2*(b*c^3*(-B
*d+2*C*c)+a*d^2*(3*A*d^2-4*B*c*d+5*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c
^2)^(1/2)/(b*x^2+a)^(1/2))/d^3/(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 11.88 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.79

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{\frac{aCd^3}{b^2\sqrt{a+bx^2}} - \frac{d(3c^2C-2Bcd+Ad^2)}{b\sqrt{a+bx^2}} - \frac{c(4c^2C-3Bcd+2Ad^2)x}{a\sqrt{a+bx^2}} + \frac{c^2(5c^2C-4Bcd+3Ad^2)(ad+bcx)}{a(bc^2+ad^2)\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
((a*C*d^3)/(b^2*Sqrt[a + b*x^2]) - (d*(3*c^2*C - 2*B*c*d + A*d^2))/(b*Sqrt
[a + b*x^2]) - (c*(4*c^2*C - 3*B*c*d + 2*A*d^2)*x)/(a*Sqrt[a + b*x^2]) + (
c^2*(5*c^2*C - 4*B*c*d + 3*A*d^2)*(a*d + b*c*x))/(a*(b*c^2 + a*d^2)*Sqrt[a
 + b*x^2]) - (c^3*(c^2*C - B*c*d + A*d^2)*(a*d + b*c*x))/(a*(b*c^2 + a*d^2
)*(c + d*x)*Sqrt[a + b*x^2]) + (C*d^3*Sqrt[a + b*x^2])/b^2 + (Sqrt[a]*d^2*
(-2*c*C + B*d)*Sqrt[1 + (b*x^2)/a]*(-(Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a
]) + (a + b*x^2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(b^(3/2)*(a + b*x^2)^(3/2)
) - (c^2*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2) + (c^3*d*(c^2*C - B*c
*d + A*d^2)*(Sqrt[b*c^2 + a*d^2]*(-(b*c^2) + 2*a*d^2)*Sqrt[a + b*x^2] + 3*
a*b*c*d*(c + d*x)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])]))/(a*(b*c^2 + a*d^2)^(5/2)*(c + d*x)))/d^5
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2178, 2182, 25, 2185, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{3/2}(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2178$$

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\int \frac{-aCx^3-aBx^2-\frac{ac\left(Abc\left(bc^2+3ad^2\right)-a\left(bCc^3+ad^2(3cC-2Bd)\right)\right)x}{\left(bc^2+ad^2\right)^2}+\frac{a^2c^2(bc(Bc-2Ad)+ad(2cC-Bd))}{\left(bc^2+ad^2\right)^2}}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab}$$

$\Big\downarrow$ 2182

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{-\frac{\int -\frac{\frac{c\left(ad^2(2cC-Bd)-bc\left(Cc^2-2Bdc+3Ad^2\right)\right)a^2}{d\left(bc^2+ad^2\right)}-C\left(\frac{bc^2}{d}+ad\right)x^2a+\frac{(cC-Bd)\left(bc^2+ad^2\right)xa}{d^2}}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{abc^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^2(c+dx)(ad^2+bc^2)^2}}{ab}$$

$\Big\downarrow$ 25

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\int \frac{\frac{c\left(ad^2(2cC-Bd)-bc\left(Cc^2-2Bdc+3Ad^2\right)\right)a^2}{d\left(bc^2+ad^2\right)}-C\left(\frac{bc^2}{d}+ad\right)x^2a+\frac{(cC-Bd)\left(bc^2+ad^2\right)xa}{d^2}}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{abc^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^2(c+dx)(ad^2+bc^2)^2}}{ab}$$

$\Big\downarrow$ 2185

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{\int \frac{ab\left(\frac{acd\left(ad^2(2cC-Bd)-bc\left(Cc^2-2Bdc+3Ad^2\right)\right)}{bc^2+ad^2}+(2cC-Bd)\left(bc^2+ad^2\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{bd^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\right)}{bd^2}}{ad^2+bc^2}-\frac{abc^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^2(c+dx)(ad^2+bc^2)^2}}{ab}$$

$\Big\downarrow$ 27

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\int \frac{\frac{acd\left(ad^2(2cC-Bd)-bc\left(Cc^2-2Bdc+3Ad^2\right)\right)}{bc^2+ad^2}+(2cC-Bd)\left(bc^2+ad^2\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\right)}{bd^2}}{ad^2+bc^2}-\frac{abc^3\sqrt{a+bx^2}(Ad^2-Bcd+c^2C)}{d^2(c+dx)(ad^2+bc^2)^2}}{ab}$$

$\Big\downarrow$ 719

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\left(\frac{\left(ad^2+bc^2\right)(2cC-Bd)\int\frac{1}{\sqrt{bx^2+a}}dx}{d}-\frac{bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^3(2cC-Bd)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}\right)}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\right)}{bd^2}}{ad^2+bc^2}-\frac{abc^3\sqrt{a+bx^2}(\cdots}{d^2(c+dx)(\cdots}}{ab}$$

$\downarrow$ 224

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\left(\frac{\left(ad^2+bc^2\right)(2cC-Bd)\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d}-\frac{bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^3(2cC-Bd)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}\right)}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\right)}{bd^2}}{ad^2+bc^2}-\frac{abc^3\sqrt{\cdots}}{d\cdots}}{ab}$$

$\downarrow$ 219

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}-\frac{bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^3(2cC-Bd)\right)\int\frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d\left(ad^2+bc^2\right)}\right)}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\right)}{bd^2}}{ad^2+bc^2}-\frac{abc^3\sqrt{\cdots}}{d\cdots}}{ab}$$

$\downarrow$ 488

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\left(\frac{bc^2\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^3(2cC-Bd)\right)\int\frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d\left(ad^2+bc^2\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}\right)}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\cdots\right.}{bd^2}}{ad^2+bc^2}}{ab}$$

$\downarrow$ 219

$$\frac{a\left(bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b^2\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\frac{\frac{a\left(\frac{bc^2\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(3Ad^2-4Bcd+5c^2C\right)+bc^3(2cC-Bd)\right)}{d\left(ad^2+bc^2\right)^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)(2cC-Bd)}{\sqrt{b}d}\right)}{d^2}-\frac{aC\sqrt{a+bx^2}\left(ad^2+bc^2\cdots\right.}{bd^2}}{ad^2+bc^2}}{ab}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(a*(A*b*(b*c^2 - a*d^2) + a*(a*C*d^2 - b*c*(c*C - 2*B*d)) + b*(b*c*(B*c -
2*A*d) + a*d*(2*c*C - B*d))*x))/(b^2*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2]) -
(-((a*b*c^3*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d^2*(b*c^2 + a*d^2)^
2*(c + d*x))) + (-((a*C*(b*c^2 + a*d^2)*Sqrt[a + b*x^2])/(b*d^2)) + (a*(((
2*c*C - B*d)*(b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b
]*d) + (b*c^2*(b*c^3*(2*c*C - B*d) + a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2))*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2]])])/(d*(b*c^2 +
a*d^2)^(3/2))))/d^2)/(b*c^2 + a*d^2))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(285) = 570$.

Time = 0.35 (sec) , antiderivative size = 737, normalized size of antiderivative = 2.43

method	result
risch	$\frac{C\sqrt{b\,x^2+a}}{b^2d^2}+\frac{\frac{(Bd-2Cc)\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{d\sqrt{b}}+\frac{b^2c^3\left(A\,d^2-Bcd+C\,c^2\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{2a\,d^2+2b\,c^2}{d^2}-\ldots\right)}{\ldots}\right)}{d^3\left(d\sqrt{-ab}+bc\right)\left(d\sqrt{-ab}-bc\ldots\right)}}{\ldots}$
default	Expression too large to display

input

```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
C*(b*x^2+a)^(1/2)/b^2/d^2+1/b/d^2*((B*d-2*C*c)/d*ln(b^(1/2)*x+(b*x^2+a)^(1
/2))/b^(1/2)+b^2/d^3*c^3*(A*d^2-B*c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/(d*(-a*b
)^(1/2)-b*c)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a
*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2
*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^
2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-b^3/d^2*c^2*(3*A*a*d
^4+A*b*c^2*d^2-4*B*a*c*d^3-2*B*b*c^3*d+5*C*a*c^2*d^2+3*C*b*c^4)/(d*(-a*b)^
(1/2)+b*c)^2/(d*(-a*b)^(1/2)-b*c)^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2
+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d))-1/2*d^2*(A*(-a*b)^(1/2)*b-B*
a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b)^(1/2)+b*c)^2/b/(x-(-a*b)^(1/2)/b)*(b*(x-(-
a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d^2*(A*(-a*b)
^(1/2)*b+B*a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b)^(1/2)-b*c)^2/b/(x+(-a*b)^(1/2)/
b)*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}(c+dx)^2}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(286) = 572$.

Time = 0.23 (sec) , antiderivative size = 1459, normalized size of antiderivative = 4.82

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
-3*C*b^2*c^7*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^5 + 2*sqrt(b*x^2 + a)*a^2*b*c^
2*d^7 + sqrt(b*x^2 + a)*a^3*d^9) + 3*B*b^2*c^6*x/(sqrt(b*x^2 + a)*a*b^2*c^
4*d^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^6 + sqrt(b*x^2 + a)*a^3*d^8) - 3*A*b
^2*c^5*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^5
+ sqrt(b*x^2 + a)*a^3*d^7) - 3*C*b*c^6/(sqrt(b*x^2 + a)*b^2*c^4*d^4 + 2*sq
rt(b*x^2 + a)*a*b*c^2*d^6 + sqrt(b*x^2 + a)*a^2*d^8) + 7*C*b*c^5*x/(sqrt(b
*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7) + 3*B*b*c^5/(sqrt(b*x^2 +
 a)*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7)
 - 6*B*b*c^4*x/(sqrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) - 3
*A*b*c^4/(sqrt(b*x^2 + a)*b^2*c^4*d^2 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^4 + sq
rt(b*x^2 + a)*a^2*d^6) + C*c^5/(sqrt(b*x^2 + a)*b*c^2*d^5*x + sqrt(b*x^2 +
 a)*a*d^7*x + sqrt(b*x^2 + a)*b*c^3*d^4 + sqrt(b*x^2 + a)*a*c*d^6) + 5*A*b
*c^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) - B*c^4/(sq
rt(b*x^2 + a)*b*c^2*d^4*x + sqrt(b*x^2 + a)*a*d^6*x + sqrt(b*x^2 + a)*b*c^
3*d^3 + sqrt(b*x^2 + a)*a*c*d^5) + 5*C*c^4/(sqrt(b*x^2 + a)*b*c^2*d^4 + sq
rt(b*x^2 + a)*a*d^6) + A*c^3/(sqrt(b*x^2 + a)*b*c^2*d^3*x + sqrt(b*x^2 + a
)*a*d^5*x + sqrt(b*x^2 + a)*b*c^3*d^2 + sqrt(b*x^2 + a)*a*c*d^4) - 4*B*c^3
/(sqrt(b*x^2 + a)*b*c^2*d^3 + sqrt(b*x^2 + a)*a*d^5) + 3*A*c^2/(sqrt(b*x^2
 + a)*b*c^2*d^2 + sqrt(b*x^2 + a)*a*d^4) + C*x^2/(sqrt(b*x^2 + a)*b*d^2) -
 4*C*c^3*x/(sqrt(b*x^2 + a)*a*d^5) + 3*B*c^2*x/(sqrt(b*x^2 + a)*a*d^4) ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{x^3\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)^2\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

3.159 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 278

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{a(bc(Bc-2Ad)+ad(2cC-Bd))-(Ab(bc^2-ad^2)+a(aCd^2-bc(cC-2}{b(bc^2+ad^2)^2\sqrt{a+bx^2}}$$
$$-\frac{c^2(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{d(bc^2+ad^2)^2(c+dx)}+\frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}d^2}$$
$$+\frac{c(ad^2(4c^2C-3Bcd+2Ad^2)+b(c^4C-Ac^2d^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{d^2(bc^2+ad^2)^{5/2}}$$

output
```
(a*(b*c*(-2*A*d+B*c)+a*d*(-B*d+2*C*c))-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*
(-2*B*d+C*c)))*x)/b/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)-c^2*(A*d^2-B*c*d+C*c^2
)*(b*x^2+a)^(1/2)/d/(a*d^2+b*c^2)^2/(d*x+c)+C*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(3/2)/d^2+c*(a*d^2*(2*A*d^2-3*B*c*d+4*C*c^2)+b*(-A*c^2*d^2+C*c^4)
)*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/d^2/(a*d^2+b*c
^2)^(5/2)
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.09

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \frac{-\frac{d\left(a^2d^2(c+dx)(-2cC+d(B+Cx))+b^2c^2x(c(cC-Bd)x+Ad(c+2dx))+ab\left(c^4C-Ad^4x^2+cd^3x(A+2Bx\right.\right.}{b(bc^2+ad^2)^2(c+dx)\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-((d*(a^2*d^2*(c + d*x)*(-2*c*C + d*(B + C*x)) + b^2*c^2*x*(c*(c*C - B*d)
*x + A*d*(c + 2*d*x)) + a*b*(c^4*C - A*d^4*x^2 + c*d^3*x*(A + 2*B*x) - c^3
*d*(2*B + C*x) + c^2*d^2*(3*A + x*(B - C*x)))))/(b*(b*c^2 + a*d^2)^2*(c +
d*x)*Sqrt[a + b*x^2])) + (2*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*(c^
4*C - A*c^2*d^2))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c
*Sqrt[a + b*x^2])])/(-(b*c^2) - a*d^2)^(5/2) + (2*C*ArcTanh[(Sqrt[b]*x)/(-
Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2))/d^2
```

Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2178, 25, 2182, 25, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{3/2}(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2178}$$

$$\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}-\frac{\int -\frac{\frac{a\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)c^2}{\left(bc^2+ad^2\right)^2}+\frac{a\left(b^2Bc^3+2a^2Cd^3+abd^2(3Bc-2Ad)\right)xc}{\left(bc^2+ad^2\right)^2}+aCx^2}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{a\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)c^2}{\left(bc^2+ad^2\right)^2}+\frac{a\left(b^2Bc^3+2a^2Cd^3+abd^2(3Bc-2Ad)\right)xc}{\left(bc^2+ad^2\right)^2}+aCx^2}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{-\frac{\int -\frac{a\left(\frac{c\left(Ab\left(bc^2-2ad^2\right)+a\left(aCd^2-bc(2cC-3Bd)\right)\right)}{bc^2+ad^2}+C\left(\frac{bc^2}{d}+ad\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{a\left(\frac{c\left(Ab\left(bc^2-2ad^2\right)+a\left(aCd^2-bc(2cC-3Bd)\right)\right)}{bc^2+ad^2}+C\left(\frac{bc^2}{d}+ad\right)x\right)}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{a\int \frac{\frac{c\left(Ab\left(bc^2-2ad^2\right)+a\left(aCd^2-bc(2cC-3Bd)\right)\right)}{bc^2+ad^2}+C\left(\frac{bc^2}{d}+ad\right)x}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 719}$$

$$\frac{\frac{a\left(\frac{C\left(ad^2+bc^2\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{d^2}-\frac{bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+b\left(c^4C-Ac^2d^2\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{a\left(\frac{C\left(ad^2+bc^2\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{d^2}-\frac{bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+b\left(c^4C-Ac^2d^2\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{a\left(\frac{C\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d^2}-\frac{bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+b\left(c^4C-Ac^2d^2\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{d^2\left(ad^2+bc^2\right)}\right)}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 488$$

$$\frac{\frac{a\left(\frac{bc\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+b\left(c^4C-Ac^2d^2\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{d^2\left(ad^2+bc^2\right)}+\frac{C\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d^2}\right)}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{a\left(\frac{bc\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2\left(2Ad^2-3Bcd+4c^2C\right)+b\left(c^4C-Ac^2d^2\right)\right)}{d^2\left(ad^2+bc^2\right)^{3/2}}+\frac{C\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(ad^2+bc^2\right)}{\sqrt{b}d^2}\right)}{ad^2+bc^2}-\frac{abc^2\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{d(c+dx)\left(ad^2+bc^2\right)^2}}{ab}+\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(a*(b*c*(B*c - 2*A*d) + a*d*(2*c*C - B*d)) - (A*b*(b*c^2 - a*d^2) + a*(a*C
*d^2 - b*c*(c*C - 2*B*d)))*x)/(b*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2]) + (-((
a*b*c^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(d*(b*c^2 + a*d^2)^2*(c +
 d*x))) + (a*((C*(b*c^2 + a*d^2)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sq
rt[b]*d^2) + (b*c*(a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2) + b*(c^4*C - A*c^2*
d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(d^2*(
b*c^2 + a*d^2)^(3/2))))/(b*c^2 + a*d^2))/(a*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(262) = 524$.

Time = 0.28 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.57

method	result
default	$\frac{\frac{A\,d^{2} x}{a\sqrt{b\,x^{2}+a}}-\frac{d(Bd-2Cc)}{b\sqrt{b\,x^{2}+a}}+C\,d^{2}\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)+\frac{3C\,c^{2} x}{a\sqrt{b\,x^{2}+a}}-\frac{2Bcdx}{a\sqrt{b\,x^{2}+a}}}{d^{4}}+\frac{c^{2}\left(A\,d^{2}-Bcd+C\,c^{2}\right)\left(-\frac{\ldots}{\left(a\,d^{2}+b\,c^{2}\right)\left(\ldots\right.}\ldots\right.}{\ldots}$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^4*(A*d^2*x/a/(b*x^2+a)^(1/2)-d*(B*d-2*C*c)/b/(b*x^2+a)^(1/2)+C*d^2*(-x
/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2)))+3*C*c^2*x/a/(b
*x^2+a)^(1/2)-2*B*c*d*x/a/(b*x^2+a)^(1/2))+c^2*(A*d^2-B*c*d+C*c^2)/d^6*(-1
/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)
^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+
c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/
(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d
^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b
*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2)*d^2*(2*
b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))-c/d^5*(2*A*d^2-3*B*c*d+4*C*c^2)*(1/(
a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b
*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/
d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)
*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*(
(a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(
1/2))/(x+c/d)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)^2}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(263) = 526$.

Time = 0.21 (sec) , antiderivative size = 1347, normalized size of antiderivative = 4.85

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
3*C*b^2*c^6*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2
*d^6 + sqrt(b*x^2 + a)*a^3*d^8) - 3*B*b^2*c^5*x/(sqrt(b*x^2 + a)*a*b^2*c^4
*d^3 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^5 + sqrt(b*x^2 + a)*a^3*d^7) + 3*A*b^
2*c^4*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^2 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^4 +
 sqrt(b*x^2 + a)*a^3*d^6) + 3*C*b*c^5/(sqrt(b*x^2 + a)*b^2*c^4*d^3 + 2*sqr
t(b*x^2 + a)*a*b*c^2*d^5 + sqrt(b*x^2 + a)*a^2*d^7) - 6*C*b*c^4*x/(sqrt(b*
x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) - 3*B*b*c^4/(sqrt(b*x^2 +
a)*b^2*c^4*d^2 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6)
+ 5*B*b*c^3*x/(sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) + 3*
A*b*c^3/(sqrt(b*x^2 + a)*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(
b*x^2 + a)*a^2*d^5) - C*c^4/(sqrt(b*x^2 + a)*b*c^2*d^4*x + sqrt(b*x^2 + a)
*a*d^6*x + sqrt(b*x^2 + a)*b*c^3*d^3 + sqrt(b*x^2 + a)*a*c*d^5) - 4*A*b*c^
2*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) + B*c^3/(sqrt(
b*x^2 + a)*b*c^2*d^3*x + sqrt(b*x^2 + a)*a*d^5*x + sqrt(b*x^2 + a)*b*c^3*d
^2 + sqrt(b*x^2 + a)*a*c*d^4) - 4*C*c^3/(sqrt(b*x^2 + a)*b*c^2*d^3 + sqrt(
b*x^2 + a)*a*d^5) - A*c^2/(sqrt(b*x^2 + a)*b*c^2*d^2*x + sqrt(b*x^2 + a)*a
*d^4*x + sqrt(b*x^2 + a)*b*c^3*d + sqrt(b*x^2 + a)*a*c*d^3) + 3*B*c^2/(sqr
t(b*x^2 + a)*b*c^2*d^2 + sqrt(b*x^2 + a)*a*d^4) - 2*A*c/(sqrt(b*x^2 + a)*b
*c^2*d + sqrt(b*x^2 + a)*a*d^3) + 3*C*c^2*x/(sqrt(b*x^2 + a)*a*d^4) - 2*B*
c*x/(sqrt(b*x^2 + a)*a*d^3) + A*x/(sqrt(b*x^2 + a)*a*d^2) - C*x/(sqrt(b...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Recursive assumption sageVARa>=((-s
ageVARb*sageVARc^2*sageVARd^2*t_nostep^2-2*sageVARb*sageVARc*sageVARd*t_no
step-sage
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^2\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)^2\,(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

3.160 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 231

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = -\frac{Ab(bc^2-ad^2)+a(aCd^2-bc(cC-2Bd))+b(bc(Bc-2Ad)+ad(2cC-Bd))x}{b(bc^2+ad^2)^2\sqrt{a+bx^2}} + \frac{c(c^2C-Bcd+Ad^2)\sqrt{a+bx^2}}{(bc^2+ad^2)^2(c+dx)} - \frac{(bc^2(Bc-2Ad)+ad(3c^2C-2Bcd+Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{(bc^2+ad^2)^{5/2}}$$

output
```
-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c*(-2*B*d+C*c))+b*(b*c*(-2*A*d+B*c)+a*d*
(-B*d+2*C*c))*x)/b/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)+c*(A*d^2-B*c*d+C*c^2)*(
b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)-(b*c^2*(-2*A*d+B*c)+a*d*(A*d^2-2*B*
c*d+3*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/(a
*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\frac{-a^2Cd^2(c+dx)+ab(2c^3C+Bd^3x^2-c^2d(3B+Cx)-cd^2x(B+2Cx))+}{b\left(bc^2+ad^2\right)^2(c-}$$

$$-\frac{2(bc^2(Bc-2Ad)+ad(3c^2C-2Bcd+Ad^2))\arctan\left(\frac{\sqrt{-bc^2-ad^2}x}{\sqrt{a}(c+dx)-c\sqrt{a+bx^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a^2*C*d^2*(c + d*x)) + a*b*(2*c^3*C + B*d^3*x^2 - c^2*d*(3*B + C*x) - c
*d^2*x*(B + 2*C*x)) + b^2*c^2*x*(c*C*x - B*(c + 2*d*x)) + A*b*(a*d^2*(2*c
+ d*x) + b*c*(-c^2 + c*d*x + 3*d^2*x^2)))/(b*(b*c^2 + a*d^2)^2*(c + d*x)*S
qrt[a + b*x^2]) - (2*(b*c^2*(B*c - 2*A*d) + a*d*(3*c^2*C - 2*B*c*d + A*d^2
))*ArcTan[(Sqrt[-(b*c^2) - a*d^2]*x)/(Sqrt[a]*(c + d*x) - c*Sqrt[a + b*x^2
])])/(-(b*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2178, 25, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{2178}$$

$$-\frac{\int -\frac{ab\left(\left(bc(Bc-2Ad)+ad(2cC-Bd)\right)c^2+\left(a\left(3Cc^2-2Bdc+Ad^2\right)d^2+b\left(c^4C-Ac^2d^2\right)\right)x\right)}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ab\left((bc(Bc-2Ad)+ad(2cC-Bd))c^2+\left(a\left(3Cc^2-2Bdc+Ad^2\right)d^2+b\left(c^4C-Ac^2d^2\right)\right)x\right)}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{(bc(Bc-2Ad)+ad(2cC-Bd))c^2+\left(a\left(3Cc^2-2Bdc+Ad^2\right)d^2+b\left(c^4C-Ac^2d^2\right)\right)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{\left(ad^2+bc^2\right)^2}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 679

$$\frac{\left(ad\left(Ad^2-2Bcd+3c^2C\right)+bc^2(Bc-2Ad)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx+\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}}{\left(ad^2+bc^2\right)^2}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 488

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}-\left(ad\left(Ad^2-2Bcd+3c^2C\right)+bc^2(Bc-2Ad)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{\left(ad^2+bc^2\right)^2}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 219

$$\frac{\frac{c\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad\left(Ad^2-2Bcd+3c^2C\right)+bc^2(Bc-2Ad)\right)}{\sqrt{ad^2+bc^2}}}{\left(ad^2+bc^2\right)^2}-\frac{bx(ad(2cC-Bd)+bc(Bc-2Ad))+Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)}{b\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((A*b*(b*c^2 - a*d^2) + a*(a*C*d^2 - b*c*(c*C - 2*B*d)) + b*(b*c*(B*c - 2
*A*d) + a*d*(2*c*C - B*d))*x)/(b*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2])) + ((c
*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c + d*x) - ((b*c^2*(B*c - 2*A*d
) + a*d*(3*c^2*C - 2*B*c*d + A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a
*d^2]*Sqrt[a + b*x^2])])/Sqrt[b*c^2 + a*d^2])/(b*c^2 + a*d^2)^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 918 vs. $2(221) = 442$.

Time = 0.24 (sec) , antiderivative size = 919, normalized size of antiderivative = 3.98

method	result
default	$\frac{\frac{Bdx}{a\sqrt{b\,x^2+a}}-\frac{dC}{b\sqrt{b\,x^2+a}}-\frac{2Ccx}{a\sqrt{b\,x^2+a}}}{d^3}+\frac{(A\,d^2-2Bcd+3C\,c^2)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\right.}{d^2}\right.}\right.}{}$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d*x/a/(b*x^2+a)^(1/2)-d*C/b/(b*x^2+a)^(1/2)-2*C*c*x/a/(b*x^2+a)^(
1/2))+1/d^4*(A*d^2-2*B*c*d+3*C*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*
b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln(
(2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-c*(A*d^2-B*c*d+C*c
^2)/d^5*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+
b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-
2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d
)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)
*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c
^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(222) = 444$.

Time = 1.56 (sec) , antiderivative size = 1571, normalized size of antiderivative = 6.80

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*a*b^2*c^4 - 2*B*a^2*b*c^2*d^2 + A*a^2*b*c*d^3 + (3*C*a^2*b - 2*A*
a*b^2)*c^3*d + (B*b^3*c^3*d - 2*B*a*b^2*c*d^3 + A*a*b^2*d^4 + (3*C*a*b^2 -
 2*A*b^3)*c^2*d^2)*x^3 + (B*b^3*c^4 - 2*B*a*b^2*c^2*d^2 + A*a*b^2*c*d^3 +
(3*C*a*b^2 - 2*A*b^3)*c^3*d)*x^2 + (B*a*b^2*c^3*d - 2*B*a^2*b*c*d^3 + A*a^
2*b*d^4 + (3*C*a^2*b - 2*A*a*b^2)*c^2*d^2)*x)*sqrt(b*c^2 + a*d^2)*log((2*a
*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2
+ a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(3*
B*a*b^2*c^4*d + 3*B*a^2*b*c^2*d^3 - (2*C*a*b^2 - A*b^3)*c^5 - (C*a^2*b + A
*a*b^2)*c^3*d^2 + (C*a^3 - 2*A*a^2*b)*c*d^4 - (C*b^3*c^5 - 2*B*b^3*c^4*d -
 B*a*b^2*c^2*d^3 + B*a^2*b*d^5 - (C*a*b^2 - 3*A*b^3)*c^3*d^2 - (2*C*a^2*b
- 3*A*a*b^2)*c*d^4)*x^2 + (B*b^3*c^5 + 2*B*a*b^2*c^3*d^2 + B*a^2*b*c*d^4 +
 (C*a*b^2 - A*b^3)*c^4*d + 2*(C*a^2*b - A*a*b^2)*c^2*d^3 + (C*a^3 - A*a^2*
b)*d^5)*x)*sqrt(b*x^2 + a))/(a*b^4*c^7 + 3*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^3
*d^4 + a^4*b*c*d^6 + (b^5*c^6*d + 3*a*b^4*c^4*d^3 + 3*a^2*b^3*c^2*d^5 + a^
3*b^2*d^7)*x^3 + (b^5*c^7 + 3*a*b^4*c^5*d^2 + 3*a^2*b^3*c^3*d^4 + a^3*b^2*
c*d^6)*x^2 + (a*b^4*c^6*d + 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^2*d^5 + a^4*b*
d^7)*x), -((B*a*b^2*c^4 - 2*B*a^2*b*c^2*d^2 + A*a^2*b*c*d^3 + (3*C*a^2*b -
 2*A*a*b^2)*c^3*d + (B*b^3*c^3*d - 2*B*a*b^2*c*d^3 + A*a*b^2*d^4 + (3*C*a*
b^2 - 2*A*b^3)*c^2*d^2)*x^3 + (B*b^3*c^4 - 2*B*a*b^2*c^2*d^2 + A*a*b^2*c*d
^3 + (3*C*a*b^2 - 2*A*b^3)*c^3*d)*x^2 + (B*a*b^2*c^3*d - 2*B*a^2*b*c*d^...
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)^2}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(222) = 444$.

Time = 0.17 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.38

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-3*C*b^2*c^5*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^3 + 2*sqrt(b*x^2 + a)*a^2*b*c^
2*d^5 + sqrt(b*x^2 + a)*a^3*d^7) + 3*B*b^2*c^4*x/(sqrt(b*x^2 + a)*a*b^2*c^
4*d^2 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^4 + sqrt(b*x^2 + a)*a^3*d^6) - 3*A*b
^2*c^3*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 +
sqrt(b*x^2 + a)*a^3*d^5) - 3*C*b*c^4/(sqrt(b*x^2 + a)*b^2*c^4*d^2 + 2*sqrt
(b*x^2 + a)*a*b*c^2*d^4 + sqrt(b*x^2 + a)*a^2*d^6) + 5*C*b*c^3*x/(sqrt(b*x
^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) + 3*B*b*c^3/(sqrt(b*x^2 + a
)*b^2*c^4*d + 2*sqrt(b*x^2 + a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) - 4
*B*b*c^2*x/(sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - 3*A*b
*c^2/(sqrt(b*x^2 + a)*b^2*c^4 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2
 + a)*a^2*d^4) + C*c^3/(sqrt(b*x^2 + a)*b*c^2*d^3*x + sqrt(b*x^2 + a)*a*d^
5*x + sqrt(b*x^2 + a)*b*c^3*d^2 + sqrt(b*x^2 + a)*a*c*d^4) + 3*A*b*c*x/(sq
rt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) - B*c^2/(sqrt(b*x^2 + a
)*b*c^2*d^2*x + sqrt(b*x^2 + a)*a*d^4*x + sqrt(b*x^2 + a)*b*c^3*d + sqrt(b
*x^2 + a)*a*c*d^3) + 3*C*c^2/(sqrt(b*x^2 + a)*b*c^2*d^2 + sqrt(b*x^2 + a)*
a*d^4) + A*c/(sqrt(b*x^2 + a)*b*c^2*d*x + sqrt(b*x^2 + a)*a*d^3*x + sqrt(b
*x^2 + a)*b*c^3 + sqrt(b*x^2 + a)*a*c*d^2) - 2*B*c/(sqrt(b*x^2 + a)*b*c^2*
d + sqrt(b*x^2 + a)*a*d^3) + A/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 + a)*a*
d^2) - 2*C*c*x/(sqrt(b*x^2 + a)*a*d^3) + B*x/(sqrt(b*x^2 + a)*a*d^2) - 3*C
*b*c^4*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x ...
```

Giac [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{3/2}\,(c+d\,x)^2}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 35.35 (sec) , antiderivative size = 2422, normalized size of antiderivative = 10.48

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^2(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a**3*b*c*d**3 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d
**2 + b*c**2) - a*d + b*c*x)*a**3*b*d**4*x - 2*sqrt(a*d**2 + b*c**2)*log(s
qrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d - 2*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a**2*b**2*c**2*d**2*x - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**2 + sqrt(a*d**2 +
b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**
2*c*d**3*x**2 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 +
 b*c**2) - a*d + b*c*x)*a**2*b**2*c*d**3*x + sqrt(a*d**2 + b*c**2)*log(sqr
t(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*d**4*x**3 + 3
*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
b*c*x)*a**2*b*c**4*d + 3*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a
*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**3*d**2*x + sqrt(a*d**2 + b*c**2)*
log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**4 - 2*
sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b
*c*x)*a*b**3*c**3*d*x**2 + sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt
(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**3*d*x - 2*sqrt(a*d**2 + b*c**2)
*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**3*c**2*d**
2*x**3 - 2*sqrt(a*d**2 + b*c**2)*log(sqrt(a + b*x**2)*sqrt(a*d**2 + b*c...
```

3.161 $\int \frac{A+Bx+Cx^2}{(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 236

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = -\frac{a(bc(Bc-2Ad)+ad(2cC-Bd))-\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)x}{a\left(bc^2+ad^2\right)^2\sqrt{a+bx^2}} - \frac{d\left(c^2C-Bcd+Ad^2\right)\sqrt{a+bx^2}}{\left(bc^2+ad^2\right)^2(c+dx)} + \frac{\left(ad^2(2cC-Bd)-bc\left(c^2C-2Bcd+3Ad^2\right)\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{\left(bc^2+ad^2\right)^{5/2}}$$

output
```
-(a*(b*c*(-2*A*d+B*c)+a*d*(-B*d+2*C*c))-(A*b*(-a*d^2+b*c^2)+a*(a*C*d^2-b*c
*(-2*B*d+C*c)))*x)/a/(a*d^2+b*c^2)^2/(b*x^2+a)^(1/2)-d*(A*d^2-B*c*d+C*c^2)
*(b*x^2+a)^(1/2)/(a*d^2+b*c^2)^2/(d*x+c)+(a*d^2*(-B*d+2*C*c)-b*c*(3*A*d^2-
2*B*c*d+C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/
(a*d^2+b*c^2)^(5/2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{Ab^2c^2x(c+dx)+ab(-c^2Cx(c+2dx)+Ad(2c^2+cdx-2d^2x^2)+Bc(-c^2}{a\left(bc^2+ad^2\right)^2(c+} - \frac{2(ad^2(-2cC+Bd)+bc(c^2C-2Bcd+3Ad^2))\arctan\left(\frac{\sqrt{b}(c+dx)-d\sqrt{a+bx^2}}{\sqrt{-bc^2-ad^2}}\right)}{\left(-bc^2-ad^2\right)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(A*b^2*c^2*x*(c + d*x) + a*b*(-(c^2*C*x*(c + 2*d*x)) + A*d*(2*c^2 + c*d*x
- 2*d^2*x^2) + B*c*(-c^2 + c*d*x + 3*d^2*x^2)) + a^2*d*(-3*c^2*C + c*d*(2*
B - C*x) + d^2*(-A + x*(B + C*x))))/(a*(b*c^2 + a*d^2)^2*(c + d*x)*Sqrt[a
+ b*x^2]) - (2*(a*d^2*(-2*c*C + B*d) + b*c*(c^2*C - 2*B*c*d + 3*A*d^2))*Ar
cTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b
*c^2) - a*d^2)^(5/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2178, 27, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{ 2178}$$

$$-\frac{\int \frac{ab\left(-b\left(Cc^2-2Bdc+3Ad^2\right)c^2+ad^2\left(c^2C-Ad^2\right)+d^2(bc(Bc-2Ad)+ad(2cC-Bd))x\right)}{\left(bc^2+ad^2\right)^2(c+dx)^2\sqrt{bx^2+a}}dx}{ab}-\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{-b(Cc^2-2Bdc+3Ad^2)c^2+ad^2(c^2C-Ad^2)+d^2(bc(Bc-2Ad)+ad(2cC-Bd))x}{(c+dx)^2\sqrt{bx^2+a}}dx}{(ad^2+bc^2)^2}-\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 679

$$-\frac{\left(ad^2(2cC-Bd)-bc\left(3Ad^2-2Bcd+c^2C\right)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx+\frac{d\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}}{(ad^2+bc^2)^2}-\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 488

$$-\frac{\frac{d\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}-\left(ad^2(2cC-Bd)-bc\left(3Ad^2-2Bcd+c^2C\right)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{(ad^2+bc^2)^2}-\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

$\Big\downarrow$ 219

$$-\frac{\frac{d\sqrt{a+bx^2}\left(Ad^2-Bcd+c^2C\right)}{c+dx}-\frac{\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(ad^2(2cC-Bd)-bc\left(3Ad^2-2Bcd+c^2C\right)\right)}{\sqrt{ad^2+bc^2}}}{(ad^2+bc^2)^2}-\frac{a(ad(2cC-Bd)+bc(Bc-2Ad))-x\left(Ab\left(bc^2-ad^2\right)+a\left(aCd^2-bc(cC-2Bd)\right)\right)}{a\sqrt{a+bx^2}\left(ad^2+bc^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(b*c*(B*c - 2*A*d) + a*d*(2*c*C - B*d)) - (A*b*(b*c^2 - a*d^2) + a*(a
*C*d^2 - b*c*(c*C - 2*B*d)))*x)/(a*(b*c^2 + a*d^2)^2*Sqrt[a + b*x^2])) - (
(d*(c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(c + d*x) - ((a*d^2*(2*c*C - B
*d) - b*c*(c^2*C - 2*B*c*d + 3*A*d^2))*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 +
 a*d^2]*Sqrt[a + b*x^2])])/Sqrt[b*c^2 + a*d^2])/(b*c^2 + a*d^2)^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(226) = 452$.

Time = 0.22 (sec) , antiderivative size = 872, normalized size of antiderivative = 3.69

method	result
default	$\frac{Cx}{d^2 a\sqrt{b\,x^2+a}}+\frac{(Bd-2Cc)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{2bcd\left(2b\left(x+\frac{c}{d}\right)-\frac{2bc}{d}\right)}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\,c^2\right)}{d^2}-\frac{4b^2c^2}{d^2}\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x\right.}{}}}\right.}{d^3}$

input
```
int((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
C/d^2*x/a/(b*x^2+a)^(1/2)+1/d^3*(B*d-2*C*c)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d
)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x
+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(
1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(
b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/d^4*(A*d
^2-B*c*d+C*c^2)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)
+(a*d^2+b*c^2)/d^2)^(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x
+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*
b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^
2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/
2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a
*d^2+b*c^2)*d^2*(2*b*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2
)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(227) = 454$.

Time = 1.63 (sec) , antiderivative size = 1583, normalized size of antiderivative = 6.71

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((C*a^2*b*c^4 - 2*B*a^2*b*c^3*d + B*a^3*c*d^3 - (2*C*a^3 - 3*A*a^2*b)
*c^2*d^2 + (C*a*b^2*c^3*d - 2*B*a*b^2*c^2*d^2 + B*a^2*b*d^4 - (2*C*a^2*b -
 3*A*a*b^2)*c*d^3)*x^3 + (C*a*b^2*c^4 - 2*B*a*b^2*c^3*d + B*a^2*b*c*d^3 -
(2*C*a^2*b - 3*A*a*b^2)*c^2*d^2)*x^2 + (C*a^2*b*c^3*d - 2*B*a^2*b*c^2*d^2
+ B*a^3*d^4 - (2*C*a^3 - 3*A*a^2*b)*c*d^3)*x)*sqrt(b*c^2 + a*d^2)*log((2*a
*b*c*d*x - a*b*c^2 - 2*a^2*d^2 - (2*b^2*c^2 + a*b*d^2)*x^2 - 2*sqrt(b*c^2
+ a*d^2)*(b*c*x - a*d)*sqrt(b*x^2 + a))/(d^2*x^2 + 2*c*d*x + c^2)) - 2*(B*
a*b^2*c^5 - B*a^2*b*c^3*d^2 - 2*B*a^3*c*d^4 + A*a^3*d^5 + (3*C*a^2*b - 2*A
*a*b^2)*c^4*d + (3*C*a^3 - A*a^2*b)*c^2*d^3 - (3*B*a*b^2*c^3*d^2 + 3*B*a^2
*b*c*d^4 - (2*C*a*b^2 - A*b^3)*c^4*d - (C*a^2*b + A*a*b^2)*c^2*d^3 + (C*a^
3 - 2*A*a^2*b)*d^5)*x^2 - (B*a*b^2*c^4*d + 2*B*a^2*b*c^2*d^3 + B*a^3*d^5 -
 (C*a*b^2 - A*b^3)*c^5 - 2*(C*a^2*b - A*a*b^2)*c^3*d^2 - (C*a^3 - A*a^2*b)
*c*d^4)*x)*sqrt(b*x^2 + a))/(a^2*b^3*c^7 + 3*a^3*b^2*c^5*d^2 + 3*a^4*b*c^3
*d^4 + a^5*c*d^6 + (a*b^4*c^6*d + 3*a^2*b^3*c^4*d^3 + 3*a^3*b^2*c^2*d^5 +
a^4*b*d^7)*x^3 + (a*b^4*c^7 + 3*a^2*b^3*c^5*d^2 + 3*a^3*b^2*c^3*d^4 + a^4*
b*c*d^6)*x^2 + (a^2*b^3*c^6*d + 3*a^3*b^2*c^4*d^3 + 3*a^4*b*c^2*d^5 + a^5*
d^7)*x), -((C*a^2*b*c^4 - 2*B*a^2*b*c^3*d + B*a^3*c*d^3 - (2*C*a^3 - 3*A*a
^2*b)*c^2*d^2 + (C*a*b^2*c^3*d - 2*B*a*b^2*c^2*d^2 + B*a^2*b*d^4 - (2*C*a^
2*b - 3*A*a*b^2)*c*d^3)*x^3 + (C*a*b^2*c^4 - 2*B*a*b^2*c^3*d + B*a^2*b*c*d
^3 - (2*C*a^2*b - 3*A*a*b^2)*c^2*d^2)*x^2 + (C*a^2*b*c^3*d - 2*B*a^2*b*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{(a+bx^2)^{\frac{3}{2}}\,(c+dx)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(227) = 454$.

Time = 0.14 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.60

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
3*C*b^2*c^4*x/(sqrt(b*x^2 + a)*a*b^2*c^4*d^2 + 2*sqrt(b*x^2 + a)*a^2*b*c^2
*d^4 + sqrt(b*x^2 + a)*a^3*d^6) - 3*B*b^2*c^3*x/(sqrt(b*x^2 + a)*a*b^2*c^4
*d + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^3 + sqrt(b*x^2 + a)*a^3*d^5) + 3*A*b^2*
c^2*x/(sqrt(b*x^2 + a)*a*b^2*c^4 + 2*sqrt(b*x^2 + a)*a^2*b*c^2*d^2 + sqrt(
b*x^2 + a)*a^3*d^4) + 3*C*b*c^3/(sqrt(b*x^2 + a)*b^2*c^4*d + 2*sqrt(b*x^2
+ a)*a*b*c^2*d^3 + sqrt(b*x^2 + a)*a^2*d^5) - 4*C*b*c^2*x/(sqrt(b*x^2 + a)
*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) - 3*B*b*c^2/(sqrt(b*x^2 + a)*b^2*c
^4 + 2*sqrt(b*x^2 + a)*a*b*c^2*d^2 + sqrt(b*x^2 + a)*a^2*d^4) + 3*B*b*c*x/
(sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3) + 3*A*b*c/(sqrt(b*x^
2 + a)*b^2*c^4/d + 2*sqrt(b*x^2 + a)*a*b*c^2*d + sqrt(b*x^2 + a)*a^2*d^3)
- C*c^2/(sqrt(b*x^2 + a)*b*c^2*d^2*x + sqrt(b*x^2 + a)*a*d^4*x + sqrt(b*x^
2 + a)*b*c^3*d + sqrt(b*x^2 + a)*a*c*d^3) - 2*A*b*x/(sqrt(b*x^2 + a)*a*b*c
^2 + sqrt(b*x^2 + a)*a^2*d^2) + B*c/(sqrt(b*x^2 + a)*b*c^2*d*x + sqrt(b*x^
2 + a)*a*d^3*x + sqrt(b*x^2 + a)*b*c^3 + sqrt(b*x^2 + a)*a*c*d^2) - 2*C*c/
(sqrt(b*x^2 + a)*b*c^2*d + sqrt(b*x^2 + a)*a*d^3) - A/(sqrt(b*x^2 + a)*b*c
^2*x + sqrt(b*x^2 + a)*a*d^2*x + sqrt(b*x^2 + a)*b*c^3/d + sqrt(b*x^2 + a)
*a*c*d) + B/(sqrt(b*x^2 + a)*b*c^2 + sqrt(b*x^2 + a)*a*d^2) + C*x/(sqrt(b*
x^2 + a)*a*d^2) + 3*C*b*c^3*arcsinh(b*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(
sqrt(a*b)*abs(d*x + c)))/((a + b*c^2/d^2)^(5/2)*d^5) - 3*B*b*c^2*arcsinh(b
*c*x/(sqrt(a*b)*abs(d*x + c)) - a*d/(sqrt(a*b)*abs(d*x + c)))/((a + b*c...
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(c + dx)^2 (a + bx^2)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2)/((a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/((a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 15.84 (sec) , antiderivative size = 2371, normalized size of antiderivative = 10.05

$$\int \frac{A+Bx+Cx^2}{(c+dx)^2\,(a+bx^2)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
( - 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2)
- a*d + b*c*x)*a**2*b*c**2*d**2 - 3*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c*d**3*x - sqrt(a*d**2
 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*
*2*b*c*d**3 - sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 +
b*c**2) - a*d + b*c*x)*a**2*b*d**4*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*c**3*d**2 + 2*sqrt(
a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c
*x)*a**2*c**2*d**3*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqr
t(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**3*d - 3*sqrt(a*d**2 + b*c**2)*
log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c**2*d
**2*x**2 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a*b**2*c**2*d**2*x - 3*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*c*d**3*x**3 -
 sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d
 + b*c*x)*a*b**2*c*d**3*x**2 - sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**
2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b**2*d**4*x**3 - sqrt(a*d**2 + b
*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a*b*c*
*5 - sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) -
 a*d + b*c*x)*a*b*c**4*d*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*...
```

3.162 $\int \frac{A+Bx+Cx^2}{x(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 268

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2(a+bx^2)^{3/2}}\,dx=\frac{Abc-acC+aBd+(bBc-Abd+aCd)x}{a(bc^2+ad^2)(c+dx)\sqrt{a+bx^2}}+\frac{d(bc^2(Bc-2Ad)+ad(3c^2C-2Bcd+Ad^2))\sqrt{a+bx^2}}{ac(bc^2+ad^2)^2(c+dx)}-\frac{d(ad^2(c^2C-Ad^2)-bc^2(2c^2C-3Bcd+4Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^2(bc^2+ad^2)^{5/2}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c^2}$$

output

```
(A*b*c-C*a*c+B*a*d+(-A*b*d+B*b*c+C*a*d)*x)/a/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+
a)^(1/2)+d*(b*c^2*(-2*A*d+B*c)+a*d*(A*d^2-2*B*c*d+3*C*c^2))*(b*x^2+a)^(1/2
)/a/c/(a*d^2+b*c^2)^2/(d*x+c)-d*(a*d^2*(-A*d^2+C*c^2)-b*c^2*(4*A*d^2-3*B*c
*d+2*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^2
/(a*d^2+b*c^2)^(5/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/c^2
```

Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{\frac{c\left(a^2Ad^4+b^2Bc^3x(c+dx)+a^2cd^2(2cC-Bd+Cdx)+Ab^2c^2\left(c^2-cdx-2d^2x^2\right)-aAbd^2\left(c^2+cdx-d^2x^2\right)+\right.}{a\left(bc^2+ad^2\right)^2(c+dx)\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
((c*(a^2*A*d^4 + b^2*B*c^3*x*(c + d*x) + a^2*c*d^2*(2*c*C - B*d + C*d*x) +
 A*b^2*c^2*(c^2 - c*d*x - 2*d^2*x^2) - a*A*b*d^2*(c^2 + c*d*x - d^2*x^2) +
 a*b*c*(-(c^3*C) - 2*B*d^3*x^2 + c^2*d*(2*B + C*x) + c*d^2*x*(B + 3*C*x)))
)/(a*(b*c^2 + a*d^2)^2*(c + d*x)*Sqrt[a + b*x^2]) + (2*d*(b*c^2*(2*c^2*C -
 3*B*c*d + 4*A*d^2) + a*(-(c^2*C*d^2) + A*d^4))*ArcTan[(Sqrt[b]*(c + d*x)
- d*Sqrt[a + b*x^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2) + (
2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/c^2
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2351, 617, 686, 27, 679, 488, 219, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x(c+dx)^2\left(bx^2+a\right)^{3/2}}dx+\int \frac{B+Cx}{(c+dx)^2\left(bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 617}$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx+\int \frac{B+Cx}{(c+dx)^2\left(bx^2+a\right)^{3/2}}dx$$

$$\Big\downarrow \ 686$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx-\frac{\int \frac{bd(2a(cC-Bd)-(bBc+aCd)x)}{(c+dx)^2\sqrt{bx^2+a}}dx}{ab\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 27$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx-\frac{d\int \frac{2a(cC-Bd)-(bBc+aCd)x}{(c+dx)^2\sqrt{bx^2+a}}dx}{a\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 679$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx-\frac{d\left(-\frac{a\left(aCd^2-bc(2cC-3Bd)\right)\int \frac{1}{(c+dx)\sqrt{bx^2+a}}dx}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(ad(3cC-2Bd)+bBc^2\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 488$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx-\frac{d\left(\frac{a\left(aCd^2-bc(2cC-3Bd)\right)\int \frac{1}{bc^2+ad^2-\frac{(ad-bcx)^2}{bx^2+a}}d\frac{ad-bcx}{\sqrt{bx^2+a}}}{ad^2+bc^2}-\frac{\sqrt{a+bx^2}\left(ad(3cC-2Bd)+bBc^2\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \ 219$$

$$A\int \left(-\frac{d}{c^2(c+dx)\left(bx^2+a\right)^{3/2}}-\frac{d}{c(c+dx)^2\left(bx^2+a\right)^{3/2}}+\frac{1}{c^2x\left(bx^2+a\right)^{3/2}}\right)dx-\frac{d\left(\frac{a\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(aCd^2-bc(2cC-3Bd)\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\left(ad(3cC-2Bd)+bBc^2\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c^2}+\frac{d^3\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\left(ad^2+bc^2\right)^{3/2}}+\frac{3bd^3\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{\left(ad^2+bc^2\right)^{5/2}}-\frac{d^2\sqrt{a+bx^2}\left(bc^2-2\right.}{ac(c+dx)\left(ad^2+b\right.}\right.$$

$$-\frac{d\left(\frac{a\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)\left(aCd^2-bc(2cC-3Bd)\right)}{\left(ad^2+bc^2\right)^{3/2}}-\frac{\sqrt{a+bx^2}\left(ad(3cC-2Bd)+bBc^2\right)}{(c+dx)\left(ad^2+bc^2\right)}\right)}{a\left(ad^2+bc^2\right)}-\frac{a(cC-Bd)-x(aCd+bBc)}{a\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-((a*(c*C - B*d) - (b*B*c + a*C*d)*x)/(a*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a
+ b*x^2])) - (d*(-(((b*B*c^2 + a*d*(3*c*C - 2*B*d))*Sqrt[a + b*x^2])/((b*c
^2 + a*d^2)*(c + d*x))) + (a*(a*C*d^2 - b*c*(2*c*C - 3*B*d))*ArcTanh[(a*d
- b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(3/2)))/(
a*(b*c^2 + a*d^2)) + A*(1/(a*c^2*Sqrt[a + b*x^2]) - (d*(a*d + b*c*x))/(a*c
^2*(b*c^2 + a*d^2)*Sqrt[a + b*x^2]) - (d*(a*d + b*c*x))/(a*c*(b*c^2 + a*d^
2)*(c + d*x)*Sqrt[a + b*x^2]) - (d^2*(b*c^2 - 2*a*d^2)*Sqrt[a + b*x^2])/(a
*c*(b*c^2 + a*d^2)^2*(c + d*x)) + (3*b*d^3*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c
^2 + a*d^2]*Sqrt[a + b*x^2])])/(b*c^2 + a*d^2)^(5/2) + (d^3*ArcTanh[(a*d -
 b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^2*(b*c^2 + a*d^2)^(3/2)
) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(a^(3/2)*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 617
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p, x^m*(c + d*x)^n, x], x] /; FreeQ[{
a, b, c, d, p}, x] && ILtQ[n, 0] && IntegerQ[m] && IntegerQ[2*p]
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(252) = 504$.

Time = 0.26 (sec) , antiderivative size = 913, normalized size of antiderivative = 3.41

method	result
default	$\frac{A\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{c^2}-\frac{\left(A\,d^2-C\,c^2\right)\left(\frac{d^2}{\left(a\,d^2+b\,c^2\right)\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}+\frac{}{\left(a\,d^2+b\,c^2\right)\left(\frac{4b\left(a\,d^2+b\,c\right.}{d^2}\right.}\right.}{}$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A/c^2*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
)-(A*d^2-C*c^2)/c^2/d^2*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+
(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(4*b*
(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2
)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*
c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*
(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-(A*d^2-B*c*d+C*c^2)/d^3/c*(-1/
(a*d^2+b*c^2)*d^2/(x+c/d)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2)+3*b*c*d/(a*d^2+b*c^2)*(1/(a*d^2+b*c^2)*d^2/(b*(x+c/d)^2-2*b*c/d*(x+c
/d)+(a*d^2+b*c^2)/d^2)^(1/2)+2*b*c*d/(a*d^2+b*c^2)*(2*b*(x+c/d)-2*b*c/d)/(
4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b
*c^2)/d^2)^(1/2)-1/(a*d^2+b*c^2)*d^2/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^
2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*
c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-4*b/(a*d^2+b*c^2)*d^2*(2*b
*(x+c/d)-2*b*c/d)/(4*b*(a*d^2+b*c^2)/d^2-4*b^2*c^2/d^2)/(b*(x+c/d)^2-2*b*c
/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(255) = 510$.

Time = 34.62 (sec) , antiderivative size = 4367, normalized size of antiderivative = 16.29

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{\frac{3}{2}}\left(c+dx\right)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*(a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(bx^2+a)^{\frac{3}{2}}(dx+c)^2 x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)^2*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2 (a+bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 3375, normalized size of antiderivative = 12.59

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a
*d + b*c*x)*a**4*c*d**5 + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*
sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**4*d**6*x + 8*sqrt(a*d**2 + b*c**2)
*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c**3*
d**3 + 8*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a**3*b*c**2*d**4*x + 2*sqrt(a*d**2 + b*c**2)*log( - sqrt
(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*b*c*d**5*x**2 + 2*s
qrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d +
 b*c*x)*a**3*b*d**6*x**3 - 2*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)
*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*c**4*d**3 - 2*sqrt(a*d**2 + b*c
**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**3*c**
3*d**4*x - 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b
*c**2) - a*d + b*c*x)*a**2*b**2*c**4*d**2 + 8*sqrt(a*d**2 + b*c**2)*log( -
 sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**3*d**3
*x**2 - 6*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c*
*2) - a*d + b*c*x)*a**2*b**2*c**3*d**3*x + 8*sqrt(a*d**2 + b*c**2)*log( -
sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b**2*c**2*d**4*
x**3 + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a + b*x**2)*sqrt(a*d**2 + b*c**
2) - a*d + b*c*x)*a**2*b*c**6*d + 4*sqrt(a*d**2 + b*c**2)*log( - sqrt(a +
b*x**2)*sqrt(a*d**2 + b*c**2) - a*d + b*c*x)*a**2*b*c**5*d**2*x - 2*sqr...
```

3.163 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 351

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{ac^2x\sqrt{a+bx^2}}\\
&+\frac{a(bBc^3+ac^2Cd+aAd^3)+bc(ac(cC-Bd)-A(2bc^2+ad^2))\,x}{a^2c^2\left(bc^2+ad^2\right)(c+dx)\sqrt{a+bx^2}}\\
&-\frac{d(A(2b^2c^4+abc^2d^2+2a^2d^4)-ac(bc^2(cC-2Bd)-ad^2(2cC-Bd)))\sqrt{a+bx^2}}{a^2c^2\left(bc^2+ad^2\right)^2(c+dx)}\\
&+\frac{d^2(ad^3(Bc-2Ad)-bc^2(3c^2C-4Bcd+5Ad^2))\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^3\left(bc^2+ad^2\right)^{5/2}}\\
&-\frac{(Bc-2Ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}c^3}\end{aligned}$$

output

```
-A/a/c^2/x/(b*x^2+a)^(1/2)+(a*(A*a*d^3+B*b*c^3+C*a*c^2*d)+b*c*(a*c*(-B*d+C
*c)-A*(a*d^2+2*b*c^2))*x)/a^2/c^2/(a*d^2+b*c^2)/(d*x+c)/(b*x^2+a)^(1/2)-d*
(A*(2*a^2*d^4+a*b*c^2*d^2+2*b^2*c^4)-a*c*(b*c^2*(-2*B*d+C*c)-a*d^2*(-B*d+2
*C*c)))*(b*x^2+a)^(1/2)/a^2/c^2/(a*d^2+b*c^2)^2/(d*x+c)+d^2*(a*d^3*(-2*A*d
+B*c)-b*c^2*(5*A*d^2-4*B*c*d+3*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^
(1/2)/(b*x^2+a)^(1/2))/c^3/(a*d^2+b*c^2)^(5/2)-(-2*A*d+B*c)*arctanh((b*x^2
+a)^(1/2)/a^(1/2))/a^(3/2)/c^3
```

Mathematica [A] (verified)

Time = 3.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{c\left(2Ab^3c^4x^2(c+dx)+a^3d^3(c(cC-Bd)x+Ad(c+2dx))+ab^2c^2(c+dx)\left(A(c+dx)^2-cx(Bc+cCx-2Bdx)\right)+a^2bd\left(Ad\left(2c^3+2c^2dx+cd^2x^2+2d^3x^3\right)+\right.\right.}{a^2\left(bc^2+ad^2\right)^2x(c+dx)\sqrt{a+bx^2}}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-(((c*(2*A*b^3*c^4*x^2*(c + d*x) + a^3*d^3*(c*(c*C - B*d)*x + A*d*(c + 2*d
*x)) + a*b^2*c^2*(c + d*x)*(A*(c + d*x)^2 - c*x*(B*c + c*C*x - 2*B*d*x)) +
 a^2*b*d*(A*d*(2*c^3 + 2*c^2*d*x + c*d^2*x^2 + 2*d^3*x^3) + c*x*(-2*c^3*C
- B*d^3*x^2 + c^2*d*(B - C*x) + c*d^2*x*(B + 2*C*x)))))/(a^2*(b*c^2 + a*d^
2)^2*x*(c + d*x)*Sqrt[a + b*x^2]) + (2*d^2*(a*d^3*(-(B*c) + 2*A*d) + b*c^2
*(3*c^2*C - 4*B*c*d + 5*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x
^2])/Sqrt[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2) + (2*(B*c - 2*A*d)*
ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/c^3)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{ 2353}$$

$$\int \left(\frac{Bc-2Ad}{c^3x\left(a+bx^2\right)^{3/2}}-\frac{d(Bc-2Ad)}{c^3\left(a+bx^2\right)^{3/2}(c+dx)}+\frac{Ad^2-Bcd+c^2C}{c^2\left(a+bx^2\right)^{3/2}(c+dx)^2}+\frac{A}{c^2x^2\left(a+bx^2\right)^{3/2}}\right)dx$$

$$\downarrow 2009$$

$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(Bc-2Ad)}{a^{3/2}c^3}-\frac{2Abx}{a^2c^2\sqrt{a+bx^2}}-\frac{3bd^2\left(Ad^2-Bcd+c^2C\right)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c\left(ad^2+bc^2\right)^{5/2}}+\frac{d^3(Bc-2Ad)\operatorname{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^3\left(ad^2+bc^2\right)^{3/2}}+\frac{Bc-2Ad}{ac^3\sqrt{a+bx^2}}+\frac{d\sqrt{a+bx^2}\left(bc^2-2ad^2\right)\left(Ad^2-Bcd+c^2C\right)}{ac^2(c+dx)\left(ad^2+bc^2\right)^2}+\frac{(ad+bcx)\left(Ad^2-Bcd+c^2C\right)}{ac^2\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}-\frac{d(Bc-2Ad)(ad+bcx)}{ac^3\sqrt{a+bx^2}\left(ad^2+bc^2\right)}-\frac{A}{ac^2x\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(B*c - 2*A*d)/(a*c^3*Sqrt[a + b*x^2]) - A/(a*c^2*x*Sqrt[a + b*x^2]) - (2*A
*b*x)/(a^2*c^2*Sqrt[a + b*x^2]) - (d*(B*c - 2*A*d)*(a*d + b*c*x))/(a*c^3*(
b*c^2 + a*d^2)*Sqrt[a + b*x^2]) + ((c^2*C - B*c*d + A*d^2)*(a*d + b*c*x))/
(a*c^2*(b*c^2 + a*d^2)*(c + d*x)*Sqrt[a + b*x^2]) + (d*(b*c^2 - 2*a*d^2)*(
c^2*C - B*c*d + A*d^2)*Sqrt[a + b*x^2])/(a*c^2*(b*c^2 + a*d^2)^2*(c + d*x)
) + (d^3*(B*c - 2*A*d)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a +
 b*x^2])])/(c^3*(b*c^2 + a*d^2)^(3/2)) - (3*b*d^2*(c^2*C - B*c*d + A*d^2)*
ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c*(b*c^2 +
a*d^2)^(5/2)) - ((B*c - 2*A*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a^(3/2)*
c^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(333) = 666.

Time = 0.33 (sec) , antiderivative size = 747, normalized size of antiderivative = 2.13

method	result
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^2c^2x}-\frac{-\frac{(2Ad-Bc)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}+\frac{ba\left(A\,d^2-Bcd+C\,c^2\right)\left(-\frac{d^2\sqrt{b\left(x+\frac{c}{d}\right)^2-\frac{2bc\left(x+\frac{c}{d}\right)}{d}+\frac{a\,d^2+b\,c^2}{d^2}}}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}-\frac{bcd\ln\left(\frac{2a\,d^2+2b\cdots}{d^2}\cdots\right)}{\cdots}\right)}{(d\sqrt{-ab}+bc)(d\sqrt{-\cdots}}}{\cdots}$
default	$\frac{A\left(-\frac{1}{ax\sqrt{b\,x^2+a}}-\frac{2bx}{a^2\sqrt{b\,x^2+a}}\right)}{c^2}-\frac{(2Ad-Bc)\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{c^3}+\frac{\left(A\,d^2-Bcd+C\,c^2\right)\left(-\frac{\cdots}{\left(a\,d^2+b\,c^2\right)\left(x+\frac{c}{d}\right)}\cdots\right)}{\cdots}$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*A/c^2*(b*x^2+a)^(1/2)/x-1/c^2/a*(-1/c*(2*A*d-B*c)/a^(1/2)*ln((2*a+2
*a^(1/2)*(b*x^2+a)^(1/2))/x)+b*a*(A*d^2-B*c*d+C*c^2)/(d*(-a*b)^(1/2)+b*c)/
(d*(-a*b)^(1/2)-b*c)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/
2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))+1/2*b*c^2*(A
*(-a*b)^(1/2)*b-B*a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b)^(1/2)+b*c)^2/a/(-a*b)^(1
/2)/(x-(-a*b)^(1/2)/b)*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1
/2)/b))^(1/2)+1/2*b*c^2*(A*(-a*b)^(1/2)*b+B*a*b-C*(-a*b)^(1/2)*a)/(d*(-a*b
)^(1/2)-b*c)^2/a/(-a*b)^(1/2)/(x+(-a*b)^(1/2)/b)*(b*(x+(-a*b)^(1/2)/b)^2-2
*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+b^2*a*d*(2*A*a*d^4+4*A*b*c^2*d^2-B
*a*c*d^3-3*B*b*c^3*d+2*C*b*c^4)/(d*(-a*b)^(1/2)+b*c)^2/(d*(-a*b)^(1/2)-b*c
)^2/c/((a*d^2+b*c^2)/d^2)^(1/2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*
((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^
(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1409 vs. $2(333) = 666$.

Time = 55.65 (sec) , antiderivative size = 5706, normalized size of antiderivative = 16.26

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2 (a + bx^2)^{3/2}} dx = \int \frac{A + Bx + Cx^2}{x^2 (a + bx^2)^{\frac{3}{2}} (c + dx)^2} dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*(a + b*x**2)**(3/2)*(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2 (a + bx^2)^{3/2}} dx = \int \frac{Cx^2 + Bx + A}{(bx^2 + a)^{\frac{3}{2}} (dx + c)^2 x^2} dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)^2*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x^2(c + dx)^2 (a + bx^2)^{3/2}} dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\,\left(dx+c\right)^2\,\left(b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

3.164 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 456

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx &= -\frac{A}{2ac^2x^2\sqrt{a+bx^2}}-\frac{Bc-2Ad}{ac^3x\sqrt{a+bx^2}}\\
&+\frac{2ac(bc^3C+aBd^3)-A(3b^2c^4+5abc^2d^2+4a^2d^4)-bc(bc^2(4Bc-5Ad)+ad(2c^2C+2Bcd-3Ad^2))x}{2a^2c^3\left(bc^2+ad^2\right)(c+dx)\sqrt{a+bx^2}}\\
&-\frac{d(2b^2c^4(Bc-2Ad)+abc^2d(2c^2C+Bcd-4Ad^2)-a^2d^3(c^2C-2Bcd+3Ad^2))\sqrt{a+bx^2}}{a^2c^3\left(bc^2+ad^2\right)^2(c+dx)}\\
&+\frac{d^3(ad^2(c^2C-2Bcd+3Ad^2)+bc^2(4c^2C-5Bcd+6Ad^2))\,\text{arctanh}\left(\frac{ad-bcx}{\sqrt{bc^2+ad^2}\sqrt{a+bx^2}}\right)}{c^4\left(bc^2+ad^2\right)^{5/2}}\\
&-\frac{(2ac(cC-2Bd)-A(3bc^2-6ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}c^4}\end{aligned}$$

output
```
-1/2*A/a/c^2/x^2/(b*x^2+a)^(1/2)-(-2*A*d+B*c)/a/c^3/x/(b*x^2+a)^(1/2)+1/2*
(2*a*c*(B*a*d^3+C*b*c^3)-A*(4*a^2*d^4+5*a*b*c^2*d^2+3*b^2*c^4)-b*c*(b*c^2*
(-5*A*d+4*B*c)+a*d*(-3*A*d^2+2*B*c*d+2*C*c^2))*x)/a^2/c^3/(a*d^2+b*c^2)/(d
*x+c)/(b*x^2+a)^(1/2)-d*(2*b^2*c^4*(-2*A*d+B*c)+a*b*c^2*d*(-4*A*d^2+B*c*d+
2*C*c^2)-a^2*d^3*(3*A*d^2-2*B*c*d+C*c^2))*(b*x^2+a)^(1/2)/a^2/c^3/(a*d^2+b
*c^2)^2/(d*x+c)+d^3*(a*d^2*(3*A*d^2-2*B*c*d+C*c^2)+b*c^2*(6*A*d^2-5*B*c*d+
4*C*c^2))*arctanh((-b*c*x+a*d)/(a*d^2+b*c^2)^(1/2)/(b*x^2+a)^(1/2))/c^4/(a
*d^2+b*c^2)^(5/2)-1/2*(2*a*c*(-2*B*d+C*c)-A*(-6*a*d^2+3*b*c^2))*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(5/2)/c^4
```

Mathematica [A] (verified)

Time = 4.31 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=$$

$$-\frac{\frac{c(b^3c^4x^2(c+dx)(3Ac+4Bcx-8Adx)+a^3d^4(2cx(Bc-cCx+2Bdx)+A(c^2-3cdx-6d^2x^2))+ab^2c^2(c+dx)(A(c^3-4c^2dx-8d^3x^3)+2cx(-cCx(c-}{a^2(bc^2+ad^2)^2x^2(c+d}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
-1/2*((c*(b^3*c^4*x^2*(c + d*x)*(3*A*c + 4*B*c*x - 8*A*d*x) + a^3*d^4*(2*c
*x*(B*c - c*C*x + 2*B*d*x) + A*(c^2 - 3*c*d*x - 6*d^2*x^2)) + a*b^2*c^2*(c
 + d*x)*(A*(c^3 - 4*c^2*d*x - 8*d^3*x^3) + 2*c*x*(-(c*C*x*(c - 2*d*x)) + B
*(c + d*x)^2)) + a^2*b*d^2*(A*(2*c^4 - 6*c^3*d*x - 7*c^2*d^2*x^2 - 3*c*d^3
*x^3 - 6*d^4*x^4) + 2*c*x*(c*C*x*(c^2 + c*d*x - d^2*x^2) + B*(2*c^3 + 2*c^
2*d*x + c*d^2*x^2 + 2*d^3*x^3)))))/(a^2*(b*c^2 + a*d^2)^2*x^2*(c + d*x)*Sq
rt[a + b*x^2]) - (4*d^3*(a*d^2*(c^2*C - 2*B*c*d + 3*A*d^2) + b*c^2*(4*c^2*
C - 5*B*c*d + 6*A*d^2))*ArcTan[(Sqrt[b]*(c + d*x) - d*Sqrt[a + b*x^2])/Sqr
t[-(b*c^2) - a*d^2]])/(-(b*c^2) - a*d^2)^(5/2) + (6*A*b*c^2*ArcTanh[(Sqrt[
b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2) + (4*(c^2*C - 2*B*c*d + 3*A*d^2)
*ArcTanh[(-(Sqrt[b]*x) + Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2))/c^4
```

Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 565, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2353, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{3/2}(c+dx)^2}dx$$

$$\Big\downarrow \text{2353}$$

$$\int \left(\frac{Bc-2Ad}{c^3x^2\left(a+bx^2\right)^{3/2}}+\frac{3Ad^2-2Bcd+c^2C}{c^4x\left(a+bx^2\right)^{3/2}}-\frac{d\left(3Ad^2-2Bcd+c^2C\right)}{c^4\left(a+bx^2\right)^{3/2}(c+dx)}-\frac{d\left(Ad^2-Bcd+c^2C\right)}{c^3\left(a+bx^2\right)^{3/2}(c+dx)^2}+\frac{A}{c^2x^3\left(a+\cdots\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(3Ad^2-2Bcd+c^2C\right)}{a^{3/2}c^4}+\frac{3Ab\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}c^2}-\frac{2bx(Bc-2Ad)}{a^2c^3\sqrt{a+bx^2}}-\frac{3Ab}{2a^2c^2\sqrt{a+bx^2}}+\frac{3bd^3\left(Ad^2-Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^2\left(ad^2+bc^2\right)^{5/2}}+\frac{d^3\left(3Ad^2-2Bcd+c^2C\right)\text{arctanh}\left(\frac{ad-bcx}{\sqrt{a+bx^2}\sqrt{ad^2+bc^2}}\right)}{c^4\left(ad^2+bc^2\right)^{3/2}}-\frac{Bc-2Ad}{ac^3x\sqrt{a+bx^2}}-\frac{d(ad+bcx)\left(3Ad^2-2Bcd+c^2C\right)}{ac^4\sqrt{a+bx^2}\left(ad^2+bc^2\right)}+\frac{3Ad^2-2Bcd+c^2C}{ac^4\sqrt{a+bx^2}}-\frac{d^2\sqrt{a+bx^2}\left(bc^2-2ad^2\right)\left(Ad^2-Bcd+c^2C\right)}{ac^3(c+dx)\left(ad^2+bc^2\right)^2}-\frac{d(ad+bcx)\left(Ad^2-Bcd+c^2C\right)}{ac^3\sqrt{a+bx^2}(c+dx)\left(ad^2+bc^2\right)}-\frac{A}{2ac^2x^2\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-3*A*b)/(2*a^2*c^2*Sqrt[a + b*x^2]) + (c^2*C - 2*B*c*d + 3*A*d^2)/(a*c^4*
Sqrt[a + b*x^2]) - A/(2*a*c^2*x^2*Sqrt[a + b*x^2]) - (B*c - 2*A*d)/(a*c^3*
x*Sqrt[a + b*x^2]) - (2*b*(B*c - 2*A*d)*x)/(a^2*c^3*Sqrt[a + b*x^2]) - (d*
(c^2*C - 2*B*c*d + 3*A*d^2)*(a*d + b*c*x))/(a*c^4*(b*c^2 + a*d^2)*Sqrt[a +
 b*x^2]) - (d*(c^2*C - B*c*d + A*d^2)*(a*d + b*c*x))/(a*c^3*(b*c^2 + a*d^2
)*(c + d*x)*Sqrt[a + b*x^2]) - (d^2*(b*c^2 - 2*a*d^2)*(c^2*C - B*c*d + A*d
^2)*Sqrt[a + b*x^2])/(a*c^3*(b*c^2 + a*d^2)^2*(c + d*x)) + (3*b*d^3*(c^2*C
 - B*c*d + A*d^2)*ArcTanh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^
2])])/(c^2*(b*c^2 + a*d^2)^(5/2)) + (d^3*(c^2*C - 2*B*c*d + 3*A*d^2)*ArcTa
nh[(a*d - b*c*x)/(Sqrt[b*c^2 + a*d^2]*Sqrt[a + b*x^2])])/(c^4*(b*c^2 + a*d
^2)^(3/2)) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2)*c^2) - ((
c^2*C - 2*B*c*d + 3*A*d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a^(3/2)*c^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2353
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_), x_Symbol] :> Int[ExpandIntegrand[Px*(e*x)^m*(c + d*x)^n*(a + b*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && PolyQ[Px, x] && (Integer
Q[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(-4Adx+2Bcx+Ac)}{2a^2c^3x^2}-\frac{\frac{\left(6Aa\,d^2-3bA\,c^2-4Bacd+2Ca\,c^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{c\sqrt{a}}-\frac{b^2c^3(Ab-B\sqrt{-ab}-aC)\sqrt{b\left(x+\frac{\sqrt{-ab}}{b}\right)^2}}{(d\sqrt{-ab}-bc)^2\sqrt{-ab}\left(x+\right.}}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-4*A*d*x+2*B*c*x+A*c)/a^2/c^3/x^2-1/2/c^3/a^2*(1/c*(
6*A*a*d^2-3*A*b*c^2-4*B*a*c*d+2*C*a*c^2)/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)-b^2*c^3*(A*b-B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)-b*c)^2/(-a*b
)^(1/2)/(x+(-a*b)^(1/2)/b)*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b
)^(1/2)/b))^(1/2)+b^2*c^3*(A*b+B*(-a*b)^(1/2)-a*C)/(d*(-a*b)^(1/2)+b*c)^2/
(-a*b)^(1/2)/(x-(-a*b)^(1/2)/b)*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-
(-a*b)^(1/2)/b))^(1/2)-2*b*a^2*(A*d^2-B*c*d+C*c^2)*d/(d*(-a*b)^(1/2)+b*c)/
(d*(-a*b)^(1/2)-b*c)*(-1/(a*d^2+b*c^2)*d^2/(x+c/d)*(b*(x+c/d)^2-2*b*c/d*(x
+c/d)+(a*d^2+b*c^2)/d^2)^(1/2)-b*c*d/(a*d^2+b*c^2)/((a*d^2+b*c^2)/d^2)^(1/
2)*ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*
(x+c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))-2*b^2*d^2*a^
2*(3*A*a*d^4+5*A*b*c^2*d^2-2*B*a*c*d^3-4*B*b*c^3*d+C*a*c^2*d^2+3*C*b*c^4)/
(d*(-a*b)^(1/2)+b*c)^2/(d*(-a*b)^(1/2)-b*c)^2/c/((a*d^2+b*c^2)/d^2)^(1/2)*
ln((2*(a*d^2+b*c^2)/d^2-2*b*c/d*(x+c/d)+2*((a*d^2+b*c^2)/d^2)^(1/2)*(b*(x+
c/d)^2-2*b*c/d*(x+c/d)+(a*d^2+b*c^2)/d^2)^(1/2))/(x+c/d)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1895 vs. 2(428) = 856.

Time = 145.91 (sec) , antiderivative size = 7645, normalized size of antiderivative = 16.77

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**2/(b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(bx^2+a\right)^{\frac{3}{2}}(dx+c)^2x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
integrate((C*x^2 + B*x + A)/((b*x^2 + a)^(3/2)*(d*x + c)^2*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\left(b\,x^2+a\right)^{3/2}\,\left(c+d\,x\right)^2}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(3/2)*(c + d*x)^2),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(3/2)*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^2\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\,\left(dx+c\right)^2\left(b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^2/(b*x^2+a)^(3/2),x)
```

3.165 $\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 498

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2cx^2} - \frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4c^2x}$$
$$+ \frac{\sqrt{a}\sqrt{b}(4Bc-3Ad)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+ \frac{\sqrt{a}\sqrt{b}(4Bc+Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$+ \frac{(4Abc^2+4aBcd-3aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^2-1/4*(-3*A*d+4*B*c)*(d*x+c)^(1/
2)*(-b*x^2+a)^(1/2)/c^2/x+1/4*a^(1/2)*b^(1/2)*(-3*A*d+4*B*c)*(d*x+c)^(1/2)
*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^
(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/4*a^(1/2)*b^(1/2)*(A*d+4*B*c)*((d*x+c)/(
c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+1/4*(-3*A*a*d^2+4*A*b*c^2+4*B*a*c*d)*((d*
x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b
^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.33 (sec) , antiderivative size = 1192, normalized size of antiderivative = 2.39

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((A + B*x)*Sqrt[a - b*x^2])/(x^3*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(2*A*c + 4*B*c*x - 3*A*d*x))/(c^2*x^2)) + (
4*b*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*A*b*c^3*d*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]] - 4*a*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 3*a*A*c*d^3*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*b*B*c^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(
c + d*x) + 6*A*b*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 4*b*B*c^
2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 3*A*b*c*d*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(4*B*c - 3
*A*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]
*d)] + I*(4*B*c - 3*A*d)*(2*b*c^2 - Sqrt[a]*Sqrt[b]*c*d - a*d^2)*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + (4*I)*A*
b*c^2*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c -
 Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqr
t[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + (4*I)*a*B*c*d^2*Sqrt[(d*(Sq
rt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x
))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*A...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1057 vs. $2(498) = 996$.

Time = 4.64 (sec) , antiderivative size = 1057, normalized size of antiderivative = 2.12, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.633$, Rules used = {2355, 27, 628, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}(A+Bx)}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A-\frac{Bc}{d}\right)\int \frac{\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}dx+\int \frac{B\sqrt{c+dx}\sqrt{a-bx^2}}{dx^3}dx$$

$$\Big\downarrow 27$$

$$\left(A-\frac{Bc}{d}\right)\int \frac{\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}dx+\frac{B\int \frac{\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx}{d}$$

$$\Big\downarrow 628$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\int -\frac{-\frac{bdx^2}{c}+2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\frac{B\left(\frac{1}{4}\int \frac{-3bdx^2-2bcx+ad}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$$\Big\downarrow 25$$

$$\left(A-\frac{Bc}{d}\right)\left(-\frac{1}{4}\int \frac{-\frac{bdx^2}{c}+2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\frac{B\left(\frac{1}{4}\int \frac{-3bdx^2-2bcx+ad}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$$\Big\downarrow 2352$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{\int -\frac{-\frac{3abd^2x^2}{c}-2abdx+a\left(4bc-\frac{3ad^2}{c}\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}+\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\frac{B\left(\frac{1}{4}\left(-\frac{\int \frac{abd^2x^2+6abcdx+a\left(4bc^2+ad^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{d\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$$\Big\downarrow 25$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\int \frac{-\frac{3abd^2x^2}{c}-2abdx+a\left(4bc-\frac{3ad^2}{c}\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\frac{B\left(\frac{1}{4}\left(-\frac{\int \frac{abd^2x^2+6abcdx+a\left(4bc^2+ad^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{d\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$$\Big\downarrow 2351$$

$$\frac{\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{-\frac{3abxd^2}{c}-2abd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}\right)-\frac{\sqrt{a-bx^2}\sqrt{}}{2cx^2}\right.}{}$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{a(ad^2+4bc^2)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{abxd^2+6abcd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{d\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$\downarrow$ 600

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{3abd\int\frac{\sqrt{c+}}{\sqrt{a-b}}}{c}}{2ac}\right.\right.$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{a(ad^2+4bc^2)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+5abcd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{2ac}-\frac{d\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2x^2}\right)}{d}$$

$\downarrow$ 509

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{3abd\sqrt{1-\frac{bx^2}{a}}}{c\sqrt{}}}{2ac}\right.\right.$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{a(ad^2+4bc^2)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+5abcd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{abd\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{2ac}-\frac{d\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+d}}{2x^2}\right.}{d}$$

$\downarrow$ 508

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\frac{6a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{c\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{}{x\sqrt{c+dx}\cdots}}{2ac}\cdots\right.\right.$$

$$B\left(\frac{1}{4}\left(-\frac{-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+a\left(ad^2+4bc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+5abcd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{d\sqrt{a-bx\cdots}}{c\cdots}\cdots\right.\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 327

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{6a^{3/2}\sqrt{b}d\sqrt{1\cdots}}{\cdots}}{2ac}\cdots\right.\right.$$

$$B\left(\frac{1}{4}\left(-\frac{a\left(ad^2+4bc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+5abcd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}-\frac{d\sqrt{a-bx\cdots}}{c\cdots}\cdots\right.\right.$$

$$\overline{\qquad\qquad d\qquad\qquad}$$

$\downarrow$ 512

$$\frac{\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{abd\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{6a^{3/2}\sqrt{\cdots}}{\cdots}}{2ac}\cdots\right.\right.}{}$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{a\left(ad^2+4bc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{5abcd\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}-\cdots\right.\right.}{d}$$

$\downarrow$ 511

$$\frac{\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\cdots}{\cdots}}{\cdots}\cdots\right.\right.}{}$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\cdots\right.}\cdots}\cdots}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}\cdots\right.\right.}{d}$$

$\downarrow$ 321

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\text{ar}\right.}{\sqrt{a-bx^2}\sqrt{c+dx}}}{2}\right.\right.$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{a\left(ad^2+4bc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{10a^{3/2}\sqrt{b}cd\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E}{\sqrt{a-bx}}}{2ac}\right.\right.}{d}$$

$\downarrow$ 633

$$\frac{B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}}{\sqrt{c}}}}{\sqrt{2}}\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\right.\right.}{d}$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}}{}}{}\right.\right.$$

$\downarrow$ 632

$$B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\right.\right.$$

$$d$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}}{}}{}\right.\right.$$

$\downarrow$ 186

$$B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\right.\right.$$

$$d$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}}{}}{}\right.\right.$$

$\downarrow$ 413

$$\frac{B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\right.\right.}{d}$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}}{}}{}\right.\right.$$

$$\Big\downarrow \text{412}$$

$$\frac{B\left(\frac{1}{4}\left(-\frac{\sqrt{c+dx}\sqrt{a-bx^2}d}{cx}-\frac{-\frac{2\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{10\sqrt{b}cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}}{\sqrt{}}}}{\sqrt{2}}\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\right.\right.}{d}$$

$$\left(A-\frac{Bc}{d}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x}-\frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}}{}}{}\right.\right.$$

input
```
Int[((A + B*x)*Sqrt[a - b*x^2])/(x^3*Sqrt[c + d*x]),x]
```

output
```
(B*(-1/2*(Sqrt[c + d*x]*Sqrt[a - b*x^2])/x^2 + (-((d*Sqrt[c + d*x]*Sqrt[a
- b*x^2])/(c*x)) - ((-2*a^(3/2)*Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]
*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[a - b*x^2]) - (10*a^(3/2)*Sqrt[b]*c*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[c + d*x]*Sqrt[a
- b*x^2]) - (2*a*(4*b*c^2 + a*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d
*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[
Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]
*d)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqr
t[b]*x)/Sqrt[a]))/Sqrt[b]]))/(2*a*c))/4))/d + (A - (B*c)/d)*(-1/2*(Sqrt[c
+ d*x]*Sqrt[a - b*x^2])/(c*x^2) + ((3*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c^
2*x) - ((6*a^(3/2)*Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(c*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]
) - (2*a^(3/2)*Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]
*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2
*a*(4*b*c - (3*a*d^2)/c)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(409) = 818$.

Time = 5.92 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.78

method	result
elliptic	$\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{A\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{2c\,x^2}+\frac{(3Ad-4Bc)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{4c^2x}+\frac{2\left(-Bb+\frac{Abd}{4c}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\sqrt{\cdots}}{-\frac{c}{d}-\cdots}}}{\sqrt{-bd\,x^3\cdots}}\cdots\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/2/c*A*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x^2+1/4*(3*A*d-4*B*c)/c^2*(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)/x+2*(-B*b+1/4*A*b*d/c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/4*(3*A*d-4*B*c)*b*d/c^2*(c/d-
1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2)
)/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)
))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Ellipt
icE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(
1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4*(
3*A*a*d^2-4*A*b*c^2-4*B*a*c*d)/c^3*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a
*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-bx^2+a}(Bx+A)}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x, algorithm="fricas"
)
```

output
```
integral(sqrt(-b*x^2 + a)*(B*x + A)*sqrt(d*x + c)/(d*x^4 + c*x^3), x)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx$$

input
```
integrate((B*x+A)*(-b*x**2+a)**(1/2)/x**3/(d*x+c)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(a - b*x**2)/(x**3*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{-bx^2+a}(Bx+A)}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x, algorithm="maxima"
)
```

output
```
integrate(sqrt(-b*x^2 + a)*(B*x + A)/(sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{-bx^2+a}(Bx+A)}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-b*x^2 + a)*(B*x + A)/(sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,(A+B\,x)}{x^3\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x))/(x^3*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x))/(x^3*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}\,dx = \int \frac{(Bx+A)\sqrt{-b\,x^2+a}}{x^3\sqrt{dx+c}}dx$$

input
```
int((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x)
```

output
```
int((B*x+A)*(-b*x^2+a)^(1/2)/x^3/(d*x+c)^(1/2),x)
```

3.166 $\int \frac{A+Bx}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 459

$$\begin{aligned}\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx &= \frac{\sqrt{c+dx}(Abc-aBd+b(Bc-Ad)x)}{a\,(bc^2-ad^2)\,\sqrt{a-bx^2}}\\
&+\frac{\sqrt{b}(Bc-Ad)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\,(bc^2-ad^2)\,\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}\\
&-\frac{B\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}\\
&-\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a\sqrt{c+dx}\sqrt{a-bx^2}}\end{aligned}$$

output
```
(d*x+c)^(1/2)*(A*b*c-B*a*d+b*(-A*d+B*c)*x)/a/(-a*d^2+b*c^2)/(-b*x^2+a)^(1/
2)+b^(1/2)*(-A*d+B*c)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-
b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))
^(1/2))/a^(1/2)/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x
^2+a)^(1/2)-B*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*E
llipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A
*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/a/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.66 (sec) , antiderivative size = 1049, normalized size of antiderivative = 2.29

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(-(b*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]) + A*b*c^3*d*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]] + a*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - a*A*c*d^3*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]] + 2*b*B*c^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c +
d*x) - 2*A*b*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - b*B*c^2*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + A*b*c*d*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(c + d*x)^2 - c*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)*(-(A*b*c)
 + a*B*d - b*B*c*x + A*b*d*x) + I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(B*c -
 A*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b
] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]
*d)] - I*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*Sqrt[b]*c - Sqrt[a]*B*c + 2*Sqrt[a]*
A*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
 - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*
d)] + (2*I)*A*b*c^2*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((S
qrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c
)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - (2*I)*a*A*d^3
*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d
*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqr...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 2351$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{B}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 27$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\int \frac{1}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \ 496$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\left(\frac{\int -\frac{d(ad+bcx)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\left(-\frac{d\int \frac{ad+bcx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right)$$

$$\Big\downarrow \text{600}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$B\left(-\frac{d\left(\frac{bc\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right)$$

$$\Big\downarrow \text{509}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$B\left(-\frac{d\left(\frac{bc\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right)$$

$$\Big\downarrow \text{508}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\left(-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right)$$

$\downarrow$ 327

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\left(-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-bcx)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}\right.$$

$\downarrow$ 512

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+B\left(-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}-\frac{\sqrt{c+dx}(ad-}{a\sqrt{a-bx^2}\left(bc^2\right.}\right.$$

$$\Big\downarrow \; 511$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$

$$B\left(-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a\left(bc^2-ad^2\right)}\right.$$

$$\Big\downarrow \; 321$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$

$$B\left(-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a\left(bc^2-ad^2\right)}\right.$$

$$\Big\downarrow \; 638$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$

$$B\left(-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a\left(bc^2-ad^2\right)}\right.$$

input
```
Int[(A + B*x)/(x*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(384) = 768$.

Time = 6.07 (sec) , antiderivative size = 958, normalized size of antiderivative = 2.09

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(\frac{(Ad-Bc)x}{2\left(a\,d^2-b\,c^2\right)a}-\frac{Abc-Bad}{2\left(a\,d^2-b\,c^2\right)ab}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(\frac{B}{a}+\frac{d(Abc-Bad)}{2\left(a\,d^2-b\,c^2\right)a}-\frac{bc(Ad-Bc)}{\left(a\,d^2-b\,c^2\right)a}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\ \sqrt{\ \ }}{\sqrt{-bd\,x^3-bc}}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(A*d-B*c)/(a*d^2-b*c^2)/a*x-1/2*(A*b*c-B*a*d)/(a*d^2-b*c^2)/a/b)/((x
^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(B/a+1/2*d*(A*b*c-B*a*d)/(a*d^2-b*c^2)/a-b*c
*(A*d-B*c)/(a*d^2-b*c^2)/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-b*d/a*(A*d-B*c)/(a*d^2-b*c^2)*(c/d-1/b*(a*
b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-2/a*A*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/c*d*EllipticPi(((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+cx}}\,dx$$

input
```
integrate((B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(-b*x^2 + a)*(B*x + A)*sqrt(d*x + c)/(b^2*d*x^6 + b^2*c*x^5 -
 2*a*b*d*x^4 - 2*a*b*c*x^3 + a^2*d*x^2 + a^2*c*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{A+Bx}{x\,(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate((B*x+A)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x)/(x*(a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+cx}}\,dx$$

input
```
integrate((B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+cx}}\,dx$$

input
```
integrate((B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{A+B\,x}{x\,(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{b^2d\,x^6+b^2c\,x^5-2abd\,x^4-2abc\,x^3+a^2d\,x^2+a^2cx}dx\right)a+\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{b^2d\,x^5+b^2c\,x^4-2abd\,x^3-2abc\,x^2+a^2dx+a^2c}dx\right)b$$

input
```
int((B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a**2*c*x + a**2*d*x**2 - 2*a*b*c*x**
3 - 2*a*b*d*x**4 + b**2*c*x**5 + b**2*d*x**6),x)*a + int((sqrt(c + d*x)*sq
rt(a - b*x**2))/(a**2*c + a**2*d*x - 2*a*b*c*x**2 - 2*a*b*d*x**3 + b**2*c*
x**4 + b**2*d*x**5),x)*b
```

3.167 $\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 831

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = -\frac{2(3a^2d^4(432cC-325Bd)+64b^2c^3(120c^2C-130Bcd+143Ad^2)+2abcd^2(1312c^2C-1326Bcd+12}{45045b^2d^6}$$

$$+\frac{2(539a^2Cd^4+16b^2c^2(120c^2C-130Bcd+143Ad^2)+abd^2(956c^2C-988Bcd+1001Ad^2))\,x\sqrt{c+dx}}{15015b^2d^5}$$

$$+\frac{2(ad^2(367cC-195Bd)+bc(1083c^2C-923Bcd+715Ad^2))\sqrt{c+dx}(a-bx^2)^{3/2}}{3003b^2d^4}$$

$$-\frac{2(77aCd^2+b(633c^2C-364Bcd+143Ad^2))(c+dx)^{3/2}(a-bx^2)^{3/2}}{1287b^2d^4}$$

$$+\frac{2(45cC-13Bd)(c+dx)^{5/2}(a-bx^2)^{3/2}}{143bd^4}-\frac{2C(c+dx)^{7/2}(a-bx^2)^{3/2}}{13bd^4}$$

$$-\frac{4\sqrt{a}(1617a^3Cd^6-64b^3c^4(120c^2C-130Bcd+143Ad^2)+3a^2bd^4(524c^2C-663Bcd+1001Ad^2)+2a}{45045b^{5/2}d^7\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}}$$

$$-\frac{4\sqrt{a}(bc^2-ad^2)(3a^2d^4(432cC-325Bd)+64b^2c^3(120c^2C-130Bcd+143Ad^2)+2abcd^2(1312c^2C-}{45045b^{5/2}d^7\sqrt{c+dx}\sqrt{a-}}$$

output
```
-2/45045*(3*a^2*d^4*(-325*B*d+432*C*c)+64*b^2*c^3*(143*A*d^2-130*B*c*d+120
*C*c^2)+2*a*b*c*d^2*(1287*A*d^2-1326*B*c*d+1312*C*c^2))*(d*x+c)^(1/2)*(-b*
x^2+a)^(1/2)/b^2/d^6+2/15015*(539*a^2*C*d^4+16*b^2*c^2*(143*A*d^2-130*B*c*
d+120*C*c^2)+a*b*d^2*(1001*A*d^2-988*B*c*d+956*C*c^2))*x*(d*x+c)^(1/2)*(-b
*x^2+a)^(1/2)/b^2/d^5+2/3003*(a*d^2*(-195*B*d+367*C*c)+b*c*(715*A*d^2-923*
B*c*d+1083*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b^2/d^4-2/1287*(77*a*C*d
^2+b*(143*A*d^2-364*B*c*d+633*C*c^2))*(d*x+c)^(3/2)*(-b*x^2+a)^(3/2)/b^2/d
^4+2/143*(-13*B*d+45*C*c)*(d*x+c)^(5/2)*(-b*x^2+a)^(3/2)/b/d^4-2/13*C*(d*x
+c)^(7/2)*(-b*x^2+a)^(3/2)/b/d^4-4/45045*a^(1/2)*(1617*a^3*C*d^6-64*b^3*c^
4*(143*A*d^2-130*B*c*d+120*C*c^2)+3*a^2*b*d^4*(1001*A*d^2-663*B*c*d+524*C*
c^2)+2*a*b^2*c^2*d^2*(2145*A*d^2-1794*B*c*d+1568*C*c^2))*(d*x+c)^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^7/((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-4/45045*a^(1/2)*(-a*d^2+b*c^2)*(3*a^2*
d^4*(-325*B*d+432*C*c)+64*b^2*c^3*(143*A*d^2-130*B*c*d+120*C*c^2)+2*a*b*c*
d^2*(1287*A*d^2-1326*B*c*d+1312*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1
/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^7/(d*x+c)^(1/2
)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.63 (sec) , antiderivative size = 982, normalized size of antiderivative = 1.18

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-(b*(c + d*x)*(2*a^2*d^4*(-757*c*C + 975*B*d + 539*C*d
*x) - 2*a*b*d^2*(1088*c^3*C - 2*c^2*d*(637*B + 333*C*x) - d^3*x*(1001*A +
585*B*x + 385*C*x^2) + c*d^2*(1573*A + 793*B*x + 485*C*x^2)) + b^2*(7680*c
^5*C - 640*c^4*d*(13*B + 9*C*x) + 32*c^3*d^2*(286*A + 15*x*(13*B + 10*C*x)
) - 35*d^5*x^3*(143*A + 9*x*(13*B + 11*C*x)) - 8*c^2*d^3*x*(858*A + 25*x*(
26*B + 21*C*x)) + 10*c*d^4*x^2*(572*A + 7*x*(65*B + 54*C*x))))) - (2*(d^2*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(1617*a^3*C*d^6 - 64*b^3*c^4*(120*c^2*C - 1
30*B*c*d + 143*A*d^2) + 3*a^2*b*d^4*(524*c^2*C - 663*B*c*d + 1001*A*d^2) +
 2*a*b^2*c^2*d^2*(1568*c^2*C - 1794*B*c*d + 2145*A*d^2))*(a - b*x^2) + I*S
qrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(1617*a^3*C*d^6 - 64*b^3*c^4*(120*c^2*C - 1
30*B*c*d + 143*A*d^2) + 3*a^2*b*d^4*(524*c^2*C - 663*B*c*d + 1001*A*d^2) +
 2*a*b^2*c^2*d^2*(1568*c^2*C - 1794*B*c*d + 2145*A*d^2))*Sqrt[(d*(Sqrt[a]/
Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
 + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c +
d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b
]*d*(Sqrt[b]*c - Sqrt[a]*d)*(1617*a^(5/2)*C*d^5 + 3*a^2*Sqrt[b]*d^4*(432*c
*C - 325*B*d) + 64*b^(5/2)*c^3*(120*c^2*C - 130*B*c*d + 143*A*d^2) + 48*Sq
rt[a]*b^2*c^2*d*(120*c^2*C - 130*B*c*d + 143*A*d^2) + 3*a^(3/2)*b*d^3*(956
*c^2*C - 988*B*c*d + 1001*A*d^2) + 2*a*b^(3/2)*c*d^2*(1312*c^2*C - 1326*B*
c*d + 1287*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((S...
```

Rubi [A] (verified)

Time = 4.07 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2185, 27, 2185, 27, 2185, 27, 2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{\sqrt{a-bx^2}\left(-bd^4(45cC-13Bd)x^4-d^3\left(57bCc^2-13Abd^2-7aCd^2\right)x^3-cCd^2\left(31bc^2-21ad^2\right)x^2-3c^2Cd\left(2bc^2-7ad^2\right)x+7ac^3Cd^2\right)}{2\sqrt{c+dx}}dx}{13bd^5}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{a-bx^2}\left(-bd^4(45cC-13Bd)x^4-d^3\left(57bCc^2-13Abd^2-7aCd^2\right)x^3-cCd^2\left(31bc^2-21ad^2\right)x^2-3c^2Cd\left(2bc^2-7ad^2\right)x+7ac^3Cd^2\right)}{\sqrt{c+dx}}dx}{13bd^5}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{2\int \frac{\sqrt{a-bx^2}\left(-b\left(77aCd^2+b\left(633Cc^2-364Bdc+143Ad^2\right)\right)x^3d^7-b\left(b(694cC-299Bd)c^2+ad^2(6\right.\right.}{2\sqrt{}}}{11}}{13bd^5}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\int \frac{\sqrt{a-bx^2}\left(-b\left(77aCd^2+b\left(633Cc^2-364Bdc+143Ad^2\right)\right)x^3d^7-b\left(b(694cC-299Bd)c^2+ad^2(6c\right.\right.}{\sqrt{c+}}}{11b}}{13bd^5}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{2\int \frac{3\sqrt{a-bx^2}\left(-\right.}{}}{}}{}}{}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\int\frac{\sqrt{a-bx^2}\left(-b^2\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{2}{7}bd^8\left(a-bx^2\right)^{3/\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\int\frac{\left(ad\left(a(1\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 682

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(-\frac{4\int-\frac{b\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}}{\cdots}}{\cdots}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(\frac{2\int\frac{ad\left(3\cdots\right.}{\cdots}}{\cdots}\cdots\right.}{\cdots}}{\cdots}}{\cdots}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 600

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(2\left(\frac{\left(bc^2\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 509

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(2\left(\frac{\left(bc^2\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\Big\downarrow$ 508

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(2\left(\frac{\left(bc^2\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\downarrow$ 327

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(\frac{2\left(\frac{\left(bc^2-\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\downarrow$ 512

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}(45cC-13Bd)-\frac{\frac{2}{9}d^5\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}\left(77aCd^2+b\left(143Ad^2-364Bcd+633c^2C\right)\right)-\frac{\frac{1}{7}bd^8\left(\frac{2\left(\frac{\sqrt{1-\cdots}}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{7/2}}{13bd^4}$$

$\downarrow$ 511

$\frac{2}{7}b\Big(a(367cC-19$

$$\frac{\frac{2}{11}d(45cC-13Bd)(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}-\frac{\frac{2}{9}d^5\left(77aCd^2+b\left(633Cc^2-364Bdc+143Ad^2\right)\right)(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}-}{}}{}$$

$$\frac{2C(c+dx)^{7/2}\left(a-bx^2\right)^{3/2}}{13bd^4}$$

$\big\downarrow$ 321

$\frac{2}{7}b\Big(a(367cC-19$

$$\frac{\frac{2}{11}d(45cC-13Bd)(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}-\frac{\frac{2}{9}d^5\left(77aCd^2+b\left(633Cc^2-364Bdc+143Ad^2\right)\right)(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}-}{}}{}$$

$$\frac{2C(c+dx)^{7/2}\left(a-bx^2\right)^{3/2}}{13bd^4}$$

input
```
Int[(x^3*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*(c + d*x)^(7/2)*(a - b*x^2)^(3/2))/(13*b*d^4) + ((2*d*(45*c*C - 13*B
*d)*(c + d*x)^(5/2)*(a - b*x^2)^(3/2))/11 - ((2*d^5*(77*a*C*d^2 + b*(633*c
^2*C - 364*B*c*d + 143*A*d^2))*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/9 - ((2*
b*d^8*(a*d^2*(367*c*C - 195*B*d) + b*c*(1083*c^2*C - 923*B*c*d + 715*A*d^2
))*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/7 + (b*d^8*((-2*Sqrt[c + d*x]*(3*a^2*d
^4*(432*c*C - 325*B*d) + 64*b^2*c^3*(120*c^2*C - 130*B*c*d + 143*A*d^2) +
2*a*b*c*d^2*(1312*c^2*C - 1326*B*c*d + 1287*A*d^2) - 3*d*(539*a^2*C*d^4 +
16*b^2*c^2*(120*c^2*C - 130*B*c*d + 143*A*d^2) + a*b*d^2*(956*c^2*C - 988*
B*c*d + 1001*A*d^2))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((-2*Sqrt[a]*(1617*
a^3*C*d^6 - 64*b^3*c^4*(120*c^2*C - 130*B*c*d + 143*A*d^2) + 3*a^2*b*d^4*(
524*c^2*C - 663*B*c*d + 1001*A*d^2) + 2*a*b^2*c^2*d^2*(1568*c^2*C - 1794*B
*c*d + 2145*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqr
t[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sq
rt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
 - (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a^2*d^4*(432*c*C - 325*B*d) + 64*b^2*c^3*
(120*c^2*C - 130*B*c*d + 143*A*d^2) + 2*a*b*c*d^2*(1312*c^2*C - 1326*B*c*d
 + 1287*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 -
 (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)
/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(
15*d^2)))/7)/(3*b*d^3))/(11*b*d^4))/(13*b*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2090 vs. $2(741) = 1482$.

Time = 9.38 (sec) , antiderivative size = 2091, normalized size of antiderivative = 2.52

method	result	size
elliptic	Expression too large to display	2091
risch	Expression too large to display	2093
default	Expression too large to display	5767

input
```
int(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/13*C/d*x^5*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/11*(-B*b+12/13*C/d*b*c)/b/d*x^4*(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/
d*c)/b/d*x^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(B*a-10/13*C/d*a*c+9/1
1*(-B*b+12/13*C/d*b*c)/b*a-8/9*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d
*c)/d*c)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(A*a+8/11*(-B*b+12
/13*C/d*b*c)/b/d*a*c+7/9*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d*c)/b*
a-6/7*(B*a-10/13*C/d*a*c+9/11*(-B*b+12/13*C/d*b*c)/b*a-8/9*(-A*b+2/13*a*C-
10/11*(-B*b+12/13*C/d*b*c)/d*c)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*
c)^(1/2)-2/3*(2/3*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d*c)/b/d*a*c+5
/7*(B*a-10/13*C/d*a*c+9/11*(-B*b+12/13*C/d*b*c)/b*a-8/9*(-A*b+2/13*a*C-10/
11*(-B*b+12/13*C/d*b*c)/d*c)/d*c)/b*a-4/5*(A*a+8/11*(-B*b+12/13*C/d*b*c)/b
/d*a*c+7/9*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d*c)/b*a-6/7*(B*a-10/
13*C/d*a*c+9/11*(-B*b+12/13*C/d*b*c)/b*a-8/9*(-A*b+2/13*a*C-10/11*(-B*b+12
/13*C/d*b*c)/d*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*
(2/5*(A*a+8/11*(-B*b+12/13*C/d*b*c)/b/d*a*c+7/9*(-A*b+2/13*a*C-10/11*(-B*b
+12/13*C/d*b*c)/d*c)/b*a-6/7*(B*a-10/13*C/d*a*c+9/11*(-B*b+12/13*C/d*b*c)/
b*a-8/9*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d*c)/d*c)/d*c)/b/d*a*c+1
/3*(2/3*(-A*b+2/13*a*C-10/11*(-B*b+12/13*C/d*b*c)/d*c)/b/d*a*c+5/7*(B*a-10
/13*C/d*a*c+9/11*(-B*b+12/13*C/d*b*c)/b*a-8/9*(-A*b+2/13*a*C-10/11*(-B*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 739, normalized size of antiderivative = 0.89

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-2/135135*(2*(7680*C*b^3*c^7 - 8320*B*b^3*c^6*d + 9828*B*a*b^2*c^4*d^3 + 1
053*B*a^2*b*c^2*d^5 + 2925*B*a^3*d^7 - 64*(139*C*a*b^2 - 143*A*b^3)*c^5*d^
2 - 6*(140*C*a^2*b + 1859*A*a*b^2)*c^3*d^4 - 6*(109*C*a^3 + 286*A*a^2*b)*c
*d^6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*
(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(7680*C*b^3*c^6*d - 83
20*B*b^3*c^5*d^2 + 3588*B*a*b^2*c^3*d^4 + 1989*B*a^2*b*c*d^6 - 64*(49*C*a*
b^2 - 143*A*b^3)*c^4*d^3 - 6*(262*C*a^2*b + 715*A*a*b^2)*c^2*d^5 - 231*(7*
C*a^3 + 13*A*a^2*b)*d^7)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)
) - 3*(3465*C*b^3*d^7*x^5 - 7680*C*b^3*c^5*d^2 + 8320*B*b^3*c^4*d^3 - 2548
*B*a*b^2*c^2*d^5 - 1950*B*a^2*b*d^7 + 64*(34*C*a*b^2 - 143*A*b^3)*c^3*d^4
+ 2*(757*C*a^2*b + 1573*A*a*b^2)*c*d^6 - 315*(12*C*b^3*c*d^6 - 13*B*b^3*d^
7)*x^4 + 35*(120*C*b^3*c^2*d^5 - 130*B*b^3*c*d^6 - 11*(2*C*a*b^2 - 13*A*b^
3)*d^7)*x^3 - 10*(480*C*b^3*c^3*d^4 - 520*B*b^3*c^2*d^5 + 117*B*a*b^2*d^7
- (97*C*a*b^2 - 572*A*b^3)*c*d^6)*x^2 + 2*(2880*C*b^3*c^4*d^3 - 3120*B*b^3
*c^3*d^4 + 793*B*a*b^2*c*d^6 - 6*(111*C*a*b^2 - 572*A*b^3)*c^2*d^5 - 77*(7
*C*a^2*b + 13*A*a*b^2)*d^7)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d^8)
```

Sympy [F]

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(x**3*(-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a - b*x**2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}x^3}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x^3/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}x^3}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x^3/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x^3\,\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int((x^3*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int((x^3*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x^3\sqrt{-b\,x^2+a}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int(x^3*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.168 $\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 700

$$\begin{aligned}\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx &= \frac{2(75a^2Cd^4+3abd^2(68c^2C-66Bcd+55Ad^2)+8b^2c^2(80c^2C-88Bcd+99Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{3465b^2d^5} \\ &- \frac{2(ad^2(76cC-77Bd)+2bc(80c^2C-88Bcd+99Ad^2))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{1155bd^4} \\ &- \frac{2(15aCd^2+b(71c^2C-55Bcd+33Ad^2))\sqrt{c+dx}(a-bx^2)^{3/2}}{231b^2d^3} \\ &+ \frac{2(28cC-11Bd)(c+dx)^{3/2}(a-bx^2)^{3/2}}{99bd^3} - \frac{2C(c+dx)^{5/2}(a-bx^2)^{3/2}}{11bd^3} \\ &+ \frac{4\sqrt{a}(3a^2d^4(51cC-77Bd)-8b^2c^3(80c^2C-88Bcd+99Ad^2)+3abcd^2(92c^2C-110Bcd+143Ad^2))}{3465b^{3/2}d^6\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\ &+ \frac{4\sqrt{a}(bc^2-ad^2)(75a^2Cd^4+3abd^2(68c^2C-66Bcd+55Ad^2)+8b^2c^2(80c^2C-88Bcd+99Ad^2))\sqrt{\frac{c}{c+}}}{3465b^{5/2}d^6\sqrt{c+dx}\sqrt{a-bx^2}}\end{aligned}$$

output
```
2/3465*(75*a^2*C*d^4+3*a*b*d^2*(55*A*d^2-66*B*c*d+68*C*c^2)+8*b^2*c^2*(99*
A*d^2-88*B*c*d+80*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^5-2/1155*(a
*d^2*(-77*B*d+76*C*c)+2*b*c*(99*A*d^2-88*B*c*d+80*C*c^2))*x*(d*x+c)^(1/2)*
(-b*x^2+a)^(1/2)/b/d^4-2/231*(15*a*C*d^2+b*(33*A*d^2-55*B*c*d+71*C*c^2))*(
d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b^2/d^3+2/99*(-11*B*d+28*C*c)*(d*x+c)^(3/2)*
(-b*x^2+a)^(3/2)/b/d^3-2/11*C*(d*x+c)^(5/2)*(-b*x^2+a)^(3/2)/b/d^3+4/3465*
a^(1/2)*(3*a^2*d^4*(-77*B*d+51*C*c)-8*b^2*c^3*(99*A*d^2-88*B*c*d+80*C*c^2)
+3*a*b*c*d^2*(143*A*d^2-110*B*c*d+92*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^
(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d
/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^6/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))
^(1/2)/(-b*x^2+a)^(1/2)+4/3465*a^(1/2)*(-a*d^2+b*c^2)*(75*a^2*C*d^4+3*a*b*
d^2*(55*A*d^2-66*B*c*d+68*C*c^2)+8*b^2*c^2*(99*A*d^2-88*B*c*d+80*C*c^2))*(
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1
-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/b^(5/2)/d^6/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.97 (sec) , antiderivative size = 830, normalized size of antiderivative = 1.19

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left((c+dx)\left(-150a^2Cd^4-2abd^2(98c^2C-cd(121B+61Cx)+d^2(165A+77Bx+45Cx^2))+\right.\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(-150*a^2*C*d^4 - 2*a*b*d^2*(98*c^2*C - c*d*
(121*B + 61*C*x) + d^2*(165*A + 77*B*x + 45*C*x^2)) + b^2*(640*c^4*C - 32*
c^3*d*(22*B + 15*C*x) + 8*c^2*d^2*(99*A + 66*B*x + 50*C*x^2) + 5*d^4*x^2*(
99*A + 7*x*(11*B + 9*C*x)) - 2*c*d^3*x*(297*A + 5*x*(44*B + 35*C*x)))) - (
2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*d^4*(-51*c*C + 77*B*d) + 8*b^
2*c^3*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*a*b*c*d^2*(92*c^2*C - 110*B*c*d
 + 143*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*
(-51*c*C + 77*B*d) + 8*b^2*c^3*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*a*b*c*
d^2*(92*c^2*C - 110*B*c*d + 143*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
+ d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
 + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*
d)*(75*a^2*C*d^4 + 3*a^(3/2)*Sqrt[b]*d^3*(76*c*C - 77*B*d) + 3*a*b*d^2*(68
*c^2*C - 66*B*c*d + 55*A*d^2) + 8*b^2*c^2*(80*c^2*C - 88*B*c*d + 99*A*d^2)
 + 6*Sqrt[a]*b^(3/2)*c*d*(80*c^2*C - 88*B*c*d + 99*A*d^2))*Sqrt[(d*(Sqrt[a
]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*
(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
+ d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(3465*b^2*d^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.75 (sec) , antiderivative size = 694, normalized size of antiderivative = 0.99, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2185, 27, 2185, 27, 2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{\sqrt{a-bx^2}\left(-bd^3(28cC-11Bd)x^3-d^2\left(23bCc^2-11Abd^2-5aCd^2\right)x^2-2cCd\left(3bc^2-5ad^2\right)x+5ac^2Cd^2\right)}{2\sqrt{c+dx}}dx}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(-bd^3(28cC-11Bd)x^3-d^2\left(23bCc^2-11Abd^2-5aCd^2\right)x^2-2cCd\left(3bc^2-5ad^2\right)x+5ac^2Cd^2\right)}{\sqrt{c+dx}}dx}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{2\int \frac{3\sqrt{a-bx^2}\left(-b\left(15aCd^2+b\left(71Cc^2-55Bdc+33Ad^2\right)\right)x^2d^5+abc(13cC-11Bd)d^5-b\left(2b(19cC-\right.\right.}{2\sqrt{c+dx}}}{9bd^3}}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\int \frac{\sqrt{a-bx^2}\left(-b\left(15aCd^2+b\left(71Cc^2-55Bdc+33Ad^2\right)\right)x^2d^5+abc(13cC-11Bd)d^5-b\left(2b(19cC-11\right.\right.}{\sqrt{c+dx}}}{3bd^3}}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{2\int \frac{bd^6\left(ad\left(15aCd^2-b\left(2\right.\right.\right.}{}}{}}{3bd^3}}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\int \frac{\left(ad\left(15aCd^2-b\left(2\right.\right.\right.}{}}{3bd^3}}{11bd^4}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 682

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(\cdots}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\int\frac{ad\left(75a^2Cd^4-\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 600

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 509

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\downarrow$ 508

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx}{a}}}{}\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\big\downarrow$ 327

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx}{a}}}{}\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\big\downarrow$ 512

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx}{a}}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\Big\downarrow$ 511

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(\cdots\right.}{\cdots}\right)}{\cdots}\right)}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

$\Big\downarrow$ 321

$$\frac{\frac{2}{9}d\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}(28cC-11Bd)-\frac{\frac{2}{7}d^4\left(a-bx^2\right)^{3/2}\sqrt{c+dx}\left(15aCd^2+b\left(33Ad^2-55Bcd+71c^2C\right)\right)-\frac{1}{7}d^4\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}{11bd^3}$$

input

```
Int[(x^2*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output

```
(-2*C*(c + d*x)^(5/2)*(a - b*x^2)^(3/2))/(11*b*d^3) + ((2*d*(28*c*C - 11*B
*d)*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/9 - ((2*d^4*(15*a*C*d^2 + b*(71*c^2
*C - 55*B*c*d + 33*A*d^2))*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/7 - (d^4*((2*S
qrt[c + d*x]*(75*a^2*C*d^4 + 3*a*b*d^2*(68*c^2*C - 66*B*c*d + 55*A*d^2) +
8*b^2*c^2*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*b*d*(a*d^2*(76*c*C - 77*B*d
) + 2*b*c*(80*c^2*C - 88*B*c*d + 99*A*d^2))*x)*Sqrt[a - b*x^2])/(15*d^2) +
 (2*((2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(51*c*C - 77*B*d) - 8*b^2*c^3*(80*c^2*C
 - 88*B*c*d + 99*A*d^2) + 3*a*b*c*d^2*(92*c^2*C - 110*B*c*d + 143*A*d^2))*
Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sq
rt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d
*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2
)*(75*a^2*C*d^4 + 3*a*b*d^2*(68*c^2*C - 66*B*c*d + 55*A*d^2) + 8*b^2*c^2*(
80*c^2*C - 88*B*c*d + 99*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]
]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqr
t[a - b*x^2])))/(15*d^2)))/7)/(3*b*d^3))/(11*b*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. 2(616) = 1232.

Time = 7.00 (sec) , antiderivative size = 1405, normalized size of antiderivative = 2.01

method	result	size
elliptic	Expression too large to display	1405
risch	Expression too large to display	1782
default	Expression too large to display	4856

input
```
int(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/11*C/d*x^4*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(-B*b+10/11*C/d*b*c)/b/d*x^3*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*c
)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(B*a-8/11*C/d*a*c+7/9*(-B
*b+10/11*C/d*b*c)/b*a-6/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*c)/d*c
)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(A*a+2/3*(-B*b+10/11*C/d*b*
c)/b/d*a*c+5/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*c)/b*a-4/5*(B*a-8
/11*C/d*a*c+7/9*(-B*b+10/11*C/d*b*c)/b*a-6/7*(-A*b+2/11*a*C-8/9*(-B*b+10/1
1*C/d*b*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(2/5*(B
*a-8/11*C/d*a*c+7/9*(-B*b+10/11*C/d*b*c)/b*a-6/7*(-A*b+2/11*a*C-8/9*(-B*b+
10/11*C/d*b*c)/d*c)/d*c)/b/d*a*c+1/3*(A*a+2/3*(-B*b+10/11*C/d*b*c)/b/d*a*c
+5/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*c)/b*a-4/5*(B*a-8/11*C/d*a*
c+7/9*(-B*b+10/11*C/d*b*c)/b*a-6/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)
/d*c)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1
/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elli
pticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2))+2*(4/7*(-A*b+2/11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*
c)/b/d*a*c+3/5*(B*a-8/11*C/d*a*c+7/9*(-B*b+10/11*C/d*b*c)/b*a-6/7*(-A*b+2/
11*a*C-8/9*(-B*b+10/11*C/d*b*c)/d*c)/d*c)/b*a-2/3*(A*a+2/3*(-B*b+10/11*...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 609, normalized size of antiderivative = 0.87

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(2\,(640\,Cb^3c^6-704\,Bb^3c^5d+858\,Bab^2c^3d^3+132\,Ba^2bcd^5-36\,(21\,Cab^2-22\,Ab^3)c^4d^2-3\,(27\,Ca^2b\right.}{}$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
2/10395*(2*(640*C*b^3*c^6 - 704*B*b^3*c^5*d + 858*B*a*b^2*c^3*d^3 + 132*B*
a^2*b*c*d^5 - 36*(21*C*a*b^2 - 22*A*b^3)*c^4*d^2 - 3*(27*C*a^2*b + 341*A*a
*b^2)*c^2*d^4 - 45*(5*C*a^3 + 11*A*a^2*b)*d^6)*sqrt(-b*d)*weierstrassPInve
rse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*
(3*d*x + c)/d) + 6*(640*C*b^3*c^5*d - 704*B*b^3*c^4*d^2 + 330*B*a*b^2*c^2*
d^4 + 231*B*a^2*b*d^6 - 12*(23*C*a*b^2 - 66*A*b^3)*c^3*d^3 - 3*(51*C*a^2*b
 + 143*A*a*b^2)*c*d^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 +
 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))
+ 3*(315*C*b^3*d^6*x^4 + 640*C*b^3*c^4*d^2 - 704*B*b^3*c^3*d^3 + 242*B*a*b
^2*c*d^5 - 4*(49*C*a*b^2 - 198*A*b^3)*c^2*d^4 - 30*(5*C*a^2*b + 11*A*a*b^2
)*d^6 - 35*(10*C*b^3*c*d^5 - 11*B*b^3*d^6)*x^3 + 5*(80*C*b^3*c^2*d^4 - 88*
B*b^3*c*d^5 - 9*(2*C*a*b^2 - 11*A*b^3)*d^6)*x^2 - 2*(240*C*b^3*c^3*d^3 - 2
64*B*b^3*c^2*d^4 + 77*B*a*b^2*d^6 - (61*C*a*b^2 - 297*A*b^3)*c*d^5)*x)*sqr
t(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d^7)
```

Sympy [F]

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2*(-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a - b*x**2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}x^2}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x^2/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}x^2}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x^2/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x^2\,\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int((x^2*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x^2\sqrt{-b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{\sqrt{dx+c}}dx$$

input
```
int(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int(x^2*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.169 $\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 577

$$\begin{aligned}
&\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx \\
&= -\frac{2(3ad^2(6cC-5Bd)+4bc(16c^2C-18Bcd+21Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{315bd^4} \\
&\quad+\frac{2(7aCd^2+b(16c^2C-18Bcd+21Ad^2))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{105bd^3} \\
&\quad+\frac{2(5cC-3Bd)\sqrt{c+dx}(a-bx^2)^{3/2}}{21bd^2}-\frac{2C(c+dx)^{3/2}\,(a-bx^2)^{3/2}}{9bd^2} \\
&\quad-\frac{4\sqrt{a}(21a^2Cd^4-4b^2c^2(16c^2C-18Bcd+21Ad^2)+3abd^2(10c^2C-13Bcd+21Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}}{315b^{3/2}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad-\frac{4\sqrt{a}(bc^2-ad^2)\,(3ad^2(6cC-5Bd)+4bc(16c^2C-18Bcd+21Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\Big(\mathrm{arcsi}}{315b^{3/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-2/315*(3*a*d^2*(-5*B*d+6*C*c)+4*b*c*(21*A*d^2-18*B*c*d+16*C*c^2))*(d*x+c)
^(1/2)*(-b*x^2+a)^(1/2)/b/d^4+2/105*(7*a*C*d^2+b*(21*A*d^2-18*B*c*d+16*C*c
^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^3+2/21*(-3*B*d+5*C*c)*(d*x+c)^(1
/2)*(-b*x^2+a)^(3/2)/b/d^2-2/9*C*(d*x+c)^(3/2)*(-b*x^2+a)^(3/2)/b/d^2-4/31
5*a^(1/2)*(21*a^2*C*d^4-4*b^2*c^2*(21*A*d^2-18*B*c*d+16*C*c^2)+3*a*b*d^2*(
21*A*d^2-13*B*c*d+10*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(3/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a
)^(1/2)-4/315*a^(1/2)*(-a*d^2+b*c^2)*(3*a*d^2*(-5*B*d+6*C*c)+4*b*c*(21*A*d
^2-18*B*c*d+16*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a
)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)
*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.71 (sec) , antiderivative size = 726, normalized size of antiderivative = 1.26

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(-b(c+dx)\left(2ad^2(-11cC+15Bd+7Cdx)+b(64c^3C-24c^2d(3B+2Cx)-d^3x(63A+5\right.\right.}{}$$

input
```
Integrate[(x*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-(b*(c + d*x)*(2*a*d^2*(-11*c*C + 15*B*d + 7*C*d*x) +
b*(64*c^3*C - 24*c^2*d*(3*B + 2*C*x) - d^3*x*(63*A + 5*x*(9*B + 7*C*x)) +
2*c*d^2*(42*A + x*(27*B + 20*C*x))))) - (2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]*(21*a^2*C*d^4 - 4*b^2*c^2*(16*c^2*C - 18*B*c*d + 21*A*d^2) + 3*a*b*d^
2*(10*c^2*C - 13*B*c*d + 21*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - S
qrt[a]*d)*(21*a^2*C*d^4 - 4*b^2*c^2*(16*c^2*C - 18*B*c*d + 21*A*d^2) + 3*a
*b*d^2*(10*c^2*C - 13*B*c*d + 21*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
 + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*El
lipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*
c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c -
 Sqrt[a]*d)*(21*a^(3/2)*C*d^3 + 3*a*Sqrt[b]*d^2*(6*c*C - 5*B*d) + 4*b^(3/2
)*c*(16*c^2*C - 18*B*c*d + 21*A*d^2) + 3*Sqrt[a]*b*d*(16*c^2*C - 18*B*c*d
+ 21*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)
/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c -
Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(315*b^2*
d^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.03 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2185, 27, 2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \textbf{2185}$$

$$-\frac{2\int -\frac{3\sqrt{a-bx^2}\left(-b(5cC-3Bd)x^2d^2+acCd^2-\left(2bCc^2-3Abd^2-aCd^2\right)xd\right)}{2\sqrt{c+dx}}dx}{9bd^3}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\sqrt{a-bx^2}\left(-b(5cC-3Bd)x^2d^2+acCd^2-\left(2bCc^2-3Abd^2-aCd^2\right)xd\right)}{\sqrt{c+dx}}dx}{3bd^3}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \textbf{2185}$$

$$\frac{\frac{2}{7}d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(5cC-3Bd)-\frac{2\int -\frac{bd^3\left(ad(2cC+3Bd)+\left(7aCd^2+b\left(16Cc^2-18Bdc+21Ad^2\right)\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7bd^2}}{3bd^3}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\int \frac{(ad(2cC+3Bd)+(7aCd^2+b(16Cc^2-18Bdc+21Ad^2))x)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx+\frac{2}{7}d\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(5cC-3Bd)}{3bd^3}-\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{1}{7}d\left(-\frac{4\int -\frac{b\left(ad\left(3ad^2(cC+5Bd)-bc\left(16Cc^2-18Bdc+21Ad^2\right)\right)+\left(5abcd^2(2cC+3Bd)-\left(4bc^2-3ad^2\right)\left(7aCd^2+b\left(16Cc^2-18Bdc+21Ad^2\right)\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}-\frac{2\sqrt{a-}\cdots}{}\right.}{}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{7}d\left(\frac{2\int \frac{ad\left(3ad^2(cC+5Bd)-bc\left(16Cc^2-18Bdc+21Ad^2\right)\right)+\left(5abcd^2(2cC+3Bd)-\left(4bc^2-3ad^2\right)\left(7aCd^2+b\left(16Cc^2-18Bdc+21Ad^2\right)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}-\frac{2\sqrt{a-bx^2}\sqrt{c+}\cdots}{}\right.}{}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{1}{7}d\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\left(5abcd^2(3Bd+2cC)-\left(4bc^2-3ad^2\right)\left(7aCd^2+b\left(21Ad^2-18Bcd+1\cdots\right.\right.\right.}{d}\right.}{15d^2}\right.}{}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$$\Big\downarrow \; 509$$

$$\frac{1}{7}d\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}\left(5abcd^2(3Bd+2cC)-\left(4bc^2-3ad^2\right)\left(7aCd^2+b\left(21Ad^2-\cdots\right.\right.\right.}{d\sqrt{a-bx^2}\cdots}\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$\downarrow$ 508

$$\frac{1}{7}d\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5abcd^2(3Bd+2cC)-\left(4bc^2-3ad^2\right)\left(7aCd^2+\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\cdots}}\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$\downarrow$ 327

$$\frac{1}{7}d\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5abcd^2(3Bd+2cC)-\left(4bc^2-3ad^2\right)\left(7aCd^2+\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\cdots}\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$\downarrow$ 512

$$\frac{1}{7}d\left(\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5abcd^2(3Bd+2cC)-\left(4bc^2-3ad^2\right)\cdots\right.}{\sqrt{b}\cdots}\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$\downarrow$ 511

$$\frac{1}{7}d\left(\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1\cdots}}{\cdots}\right)}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

$\Big\downarrow$ 321

$$\frac{1}{7}d\left(\frac{2\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(6cC-5Bd)+4bc\left(21Ad^2-18Bcd+16c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\cdots}}{\cdots}\right)}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}{9bd^2}$$

input
```
Int[(x*Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*(c + d*x)^(3/2)*(a - b*x^2)^(3/2))/(9*b*d^2) + ((2*d*(5*c*C - 3*B*d)
*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/7 + (d*((-2*Sqrt[c + d*x]*(3*a*d^2*(6*c*
C - 5*B*d) + 4*b*c*(16*c^2*C - 18*B*c*d + 21*A*d^2) - 3*d*(7*a*C*d^2 + b*(
16*c^2*C - 18*B*c*d + 21*A*d^2))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((-2*Sq
rt[a]*(5*a*b*c*d^2*(2*c*C + 3*B*d) - (4*b*c^2 - 3*a*d^2)*(7*a*C*d^2 + b*(1
6*c^2*C - 18*B*c*d + 21*A*d^2)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellipti
cE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[
a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqr
t[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*d^2*(6*c*C - 5*B*d) + 4*b*
c*(16*c^2*C - 18*B*c*d + 21*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt
[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*
Sqrt[a - b*x^2])))/(15*d^2)))/7)/(3*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(499) = 998$.

Time = 5.29 (sec) , antiderivative size = 1018, normalized size of antiderivative = 1.76

method	result	size
elliptic	Expression too large to display	1018
risch	Expression too large to display	1326
default	Expression too large to display	4057

input
```
int(x*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/9*C/d*x^3*(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(-B*b+8/9*C/d*b*c)/b/d*x^2*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-A*b+2/9*a*C-6/7*(-B*b+8/9*C/d*b*c)/d*c)/b/d*
x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(B*a-2/3*C/d*a*c+5/7*(-B*b+8/9*C/
d*b*c)/b*a-4/5*(-A*b+2/9*a*C-6/7*(-B*b+8/9*C/d*b*c)/d*c)/d*c)/b/d*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(2/5*(-A*b+2/9*a*C-6/7*(-B*b+8/9*C/d*b*c)/d*c
)/b/d*a*c+1/3*(B*a-2/3*C/d*a*c+5/7*(-B*b+8/9*C/d*b*c)/b*a-4/5*(-A*b+2/9*a*
C-6/7*(-B*b+8/9*C/d*b*c)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/
d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*
d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(A*a+4/7*(-B*b+8/9*C/d*b*c)/
b/d*a*c+3/5*(-A*b+2/9*a*C-6/7*(-B*b+8/9*C/d*b*c)/d*c)/b*a-2/3*(B*a-2/3*C/d
*a*c+5/7*(-B*b+8/9*C/d*b*c)/b*a-4/5*(-A*b+2/9*a*C-6/7*(-B*b+8/9*C/d*b*c)/d
*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*
((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-
c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*
(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 468, normalized size of antiderivative = 0.81

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx =$$
$$-\frac{2\left(2\left(64\,Cb^2c^5-72\,Bb^2c^4d+93\,Babc^2d^3+45\,Ba^2d^5-6\left(13\,Cab-14\,Ab^2\right)c^3d^2-6\left(2\,Ca^2+21\,Aab\right)\right.\right.}{}$$

input
```
integrate(x*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
-2/945*(2*(64*C*b^2*c^5 - 72*B*b^2*c^4*d + 93*B*a*b*c^2*d^3 + 45*B*a^2*d^5
 - 6*(13*C*a*b - 14*A*b^2)*c^3*d^2 - 6*(2*C*a^2 + 21*A*a*b)*c*d^4)*sqrt(-b
*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*
c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 6*(64*C*b^2*c^4*d - 72*B*b^2*c^3*d^2
+ 39*B*a*b*c*d^4 - 6*(5*C*a*b - 14*A*b^2)*c^2*d^3 - 21*(C*a^2 + 3*A*a*b)*d
^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
-8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(35*C*b^2*d^5*x
^3 - 64*C*b^2*c^3*d^2 + 72*B*b^2*c^2*d^3 - 30*B*a*b*d^5 + 2*(11*C*a*b - 42
*A*b^2)*c*d^4 - 5*(8*C*b^2*c*d^4 - 9*B*b^2*d^5)*x^2 + (48*C*b^2*c^2*d^3 -
54*B*b^2*c*d^4 - 7*(2*C*a*b - 9*A*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x +
 c))/(b^2*d^6)
```

Sympy [F]

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(x*(-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(a - b*x**2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}x}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+ax}}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*x/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{x\,\sqrt{a-b\,x^2}\,(C\,x^2+B\,x+A)}{\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int((x*(a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\text{Too large to display}$$

input
```
int(x*(-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output

```
( - 126*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*b*d**3 - 42*sqrt(c + d*x)*sqrt
(a - b*x**2)*a**2*c*d**3 + 126*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d**
2*x + 18*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d**2 - 16*sqrt(c + d*x)*s
qrt(a - b*x**2)*a*b*c**3*d - 28*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*d*
*2*x - 108*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c**2*d*x + 90*sqrt(c + d*x)
*sqrt(a - b*x**2)*b**3*c*d**2*x**2 + 96*sqrt(c + d*x)*sqrt(a - b*x**2)*b**
2*c**4*x - 80*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*d*x**2 + 70*sqrt(c
+ d*x)*sqrt(a - b*x**2)*b**2*c**2*d**2*x**3 - 189*int((sqrt(c + d*x)*sqrt(
a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b**2*d**4 -
63*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d
*x**3),x)*a**2*b*c*d**4 + 252*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a
*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c**2*d**2 + 117*int((sqrt(c +
d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*
c*d**3 - 90*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x
**2 - b*d*x**3),x)*a*b**2*c**3*d**2 - 216*int((sqrt(c + d*x)*sqrt(a - b*x*
*2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**4*c**3*d + 192*int((sq
rt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*
b**3*c**5 + 63*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**
2 - b*d*x**3),x)*a**3*b*d**4 + 21*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*
c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*c*d**4 - 126*int((sqrt(c + d*x...
```

3.170 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 468

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \frac{2\left(35Ab+5aC+\frac{4bc(6cC-7Bd)}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{105bd} - \frac{2(6cC-7Bd)x\sqrt{c+dx}\sqrt{a-bx^2}}{35d^2} - \frac{2C\sqrt{c+dx}(a-bx^2)^{3/2}}{7bd}$$

$$+ \frac{4\sqrt{a}(ad^2(13cC-21Bd)-bc(24c^2C-28Bcd+35Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{c}}{\sqrt{b}c+}\right.}{105\sqrt{b}d^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+ \frac{4\sqrt{a}(bc^2-ad^2)(5aCd^2+b(24c^2C-28Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{105b^{3/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/105*(35*A*b+5*a*C+4*b*c*(-7*B*d+6*C*c)/d^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/
2)/b/d-2/35*(-7*B*d+6*C*c)*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^2-2/7*C*(d*x
+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d+4/105*a^(1/2)*(a*d^2*(-21*B*d+13*C*c)-b*c*(
35*A*d^2-28*B*c*d+24*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(1/2)/d^4/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a
)^(1/2)+4/105*a^(1/2)*(-a*d^2+b*c^2)*(5*a*C*d^2+b*(35*A*d^2-28*B*c*d+24*C*
c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF
(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(
1/2)*d))^(1/2))/b^(3/2)/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.06 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left((c+dx)\left(-10aCd^2+b(24c^2C-2cd(14B+9Cx)+d^2(35A+3x(7B+5Cx)))\right)-\frac{2\left(d^2\sqrt{-\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*((c + d*x)*(-10*a*C*d^2 + b*(24*c^2*C - 2*c*d*(14*B + 9
*C*x) + d^2*(35*A + 3*x*(7*B + 5*C*x)))) - (2*(d^2*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]*(a*d^2*(-13*c*C + 21*B*d) + b*c*(24*c^2*C - 28*B*c*d + 35*A*d^2))*
(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^2*(-13*c*C + 21*B*d)
+ b*c*(24*c^2*C - 28*B*c*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
+ d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
 + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*
d)*(5*a*C*d^2 + 3*Sqrt[a]*Sqrt[b]*d*(6*c*C - 7*B*d) + b*(24*c^2*C - 28*B*c
*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]
*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c
 - Sqrt[a]*d)]))/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(105*b
*d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 457, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2185, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{2\int -\frac{d((7Ab+aC)d-b(6cC-7Bd)x)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{7bd^2}-\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{((7Ab+aC)d-b(6cC-7Bd)x)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{7bd}-\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$$\Big\downarrow \text{ 682}$$

$$\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5d^2(aC+7Ab)-3bdx(6cC-7Bd)+4bc(6cC-7Bd)\right)}{15d^2}-\frac{4\int -\frac{b\left(ad\left(5(7Ab+aC)d^2+bc(6cC-7Bd)\right)+b\left(5c(7Ab+aC)d^2+(6cC-7Bd)\left(4b\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{15bd^2}}{7bd}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\downarrow$ 27

$$\frac{\frac{2\int \frac{ad\left(5(7Ab+aC)d^2+bc(6cC-7Bd)\right)+b\left(5c(7Ab+aC)d^2+(6cC-7Bd)\left(4bc^2-3ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(5d^2(aC+7Ab)-3bdx(6cC-7Bd)+4bc(6\right.}{15d^2}}{7bd}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\downarrow$ 600

$$\frac{\frac{2\left(\frac{b\left(5cd^2(aC+7Ab)+\left(4bc^2-3ad^2\right)(6cC-7Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}+\frac{2\sqrt{a-bx^2}\sqrt{c+\ldots}}{}}{7bd}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\downarrow$ 509

$$\frac{\frac{2\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(5cd^2(aC+7Ab)+\left(4bc^2-3ad^2\right)(6cC-7Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}+\frac{2\sqrt{a-\ldots}}{}}{7bd}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\downarrow$ 508

$$\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5cd^2(aC+7Ab)+\left(4bc^2-3ad^2\right)(6cC-7Bd)\right)\int \frac{\sqrt{1-\frac{d\left(1-\cdots\right.}{\frac{\sqrt{b}}{\sqrt{\cdots}}}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}\cdots\right.}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right.}{15d^2}}{7bd}\cdots$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\Big\downarrow$ 327

$$\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5cd^2(aC+7Ab)+\left(4bc^2-3ad^2\right)(6cC-7Bd)\right)E\left(\arcsin\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right.}{15d^2}}{7bd}\cdots$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\Big\downarrow$ 512

$$\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5cd^2(aC+7Ab)+\left(4bc^2-3ad^2\right)(6cC-7Bd)\right)E\left(\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right.}{15d^2}}{7bd}\cdots$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\Big\downarrow$ 511

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5cd\right.}{}\right.}{15d^2}}{7b}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

$\Big\downarrow$ 321

$$\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5aCd^2+b\left(35Ad^2-28Bcd+24c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(5cd^2(aC+7\right.}{}\right.}{15d^2}}{7bd}$$

$$\frac{2C\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}{7bd}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*Sqrt[c + d*x]*(a - b*x^2)^(3/2))/(7*b*d) + ((2*Sqrt[c + d*x]*(5*(7*A
*b + a*C)*d^2 + 4*b*c*(6*c*C - 7*B*d) - 3*b*d*(6*c*C - 7*B*d)*x)*Sqrt[a -
b*x^2])/(15*d^2) + (2*((-2*Sqrt[a]*Sqrt[b]*(5*c*(7*A*b + a*C)*d^2 + (6*c*C
 - 7*B*d)*(4*b*c^2 - 3*a*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE
[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
 + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^
2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(5*a*C*d^2 + b*(24*c^2*C - 28*B*c*d + 35*
A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2))
/(7*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(396) = 792$.

Time = 3.24 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.71

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2C\,x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7d}-\frac{2\left(-Bb+\frac{6Cbc}{7d}\right)x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5bd}-\frac{2\left(-Ab+\frac{2aC}{7}-\frac{4\left(-Bb+\frac{6Cbc}{7d}\right)c}{5d}\right)\sqrt{-b}}{3bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/7*C/d*x^2*(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-B*b+6/7*C/d*b*c)/b/d*x*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)-2/3*(-A*b+2/7*a*C-4/5*(-B*b+6/7*C/d*b*c)/d*c)/b/d*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*a+2/5*(-B*b+6/7*C/d*b*c)/b/d*a*c+1/3
*(-A*b+2/7*a*C-4/5*(-B*b+6/7*C/d*b*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+
c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*
c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-
c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B*a-4/7*C/d*a*c+3/5
*(-B*b+6/7*C/d*b*c)/b*a-2/3*(-A*b+2/7*a*C-4/5*(-B*b+6/7*C/d*b*c)/d*c)/d*c)
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)
^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*
EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$
$$= \frac{2\left(2\left(24\,Cb^2c^4 - 28\,Bb^2c^3d + 42\,Babcd^3 - (31\,Cab - 35\,Ab^2)c^2d^2 - 15\,(Ca^2 + 7\,Aab)d^4\right)\sqrt{-bd}\text{weierstra}\right.}{}$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
2/315*(2*(24*C*b^2*c^4 - 28*B*b^2*c^3*d + 42*B*a*b*c*d^3 - (31*C*a*b - 35*
A*b^2)*c^2*d^2 - 15*(C*a^2 + 7*A*a*b)*d^4)*sqrt(-b*d)*weierstrassPInverse(
4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d
*x + c)/d) + 6*(24*C*b^2*c^3*d - 28*B*b^2*c^2*d^2 + 21*B*a*b*d^4 - (13*C*a
*b - 35*A*b^2)*c*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*
d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 +
3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) +
 3*(15*C*b^2*d^4*x^2 + 24*C*b^2*c^2*d^2 - 28*B*b^2*c*d^3 - 5*(2*C*a*b - 7*
A*b^2)*d^4 - 3*(6*C*b^2*c*d^3 - 7*B*b^2*d^4)*x)*sqrt(-b*x^2 + a)*sqrt(d*x
+ c))/(b^2*d^5)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,(C\,x^2+B\,x+A)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.171 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 519

$$\begin{aligned}
&\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx \\
&= -\frac{2(4cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15d^2}+\frac{2Cx\sqrt{c+dx}\sqrt{a-bx^2}}{5d} \\
&\quad -\frac{2\sqrt{a}(6aCd^2-b(8c^2C-10Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15\sqrt{b}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad +\frac{2\sqrt{a}(2ad^2(4cC-5Bd)-bc(8c^2C-10Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{15\sqrt{b}d^3\sqrt{c+dx}\sqrt{a-bx^2}} \\
&\quad -\frac{2aA\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-2/15*(-5*B*d+4*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^2+2/5*C*x*(d*x+c)^(1
/2)*(-b*x^2+a)^(1/2)/d-2/15*a^(1/2)*(6*a*C*d^2-b*(15*A*d^2-10*B*c*d+8*C*c^
2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^
3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2/15*a^(1/2)*(2*a
*d^2*(-5*B*d+4*C*c)-b*c*(15*A*d^2-10*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)*d
/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*a*A*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2
)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(d*x+c)^(1/2)/(-b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.43 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx$$

$$= \frac{\sqrt{a-bx^2}\left(\frac{2(c+dx)(-4cC+5Bd+3Cdx)}{d^2} - \frac{2i\left(icd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(-6aCd^2+b\left(8c^2C-10Bcd+15Ad^2\right)\right)\left(-a+bx^2\right)+c\left(bc-\sqrt{a}\sqrt{b}d\right)\left(-6aC\right.\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a - b*x^2]*((2*(c + d*x)*(-4*c*C + 5*B*d + 3*C*d*x))/d^2 - ((2*I)*(I
*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-6*a*C*d^2 + b*(8*c^2*C - 10*B*c*d
+ 15*A*d^2))*(-a + b*x^2) + c*(b*c - Sqrt[a]*Sqrt[b]*d)*(-6*a*C*d^2 + b*(8
*c^2*C - 10*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*S
qrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*(EllipticE[I
*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[
a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
) + b*(b*c^2 - a*d^2)*(8*c^2*C - 10*B*c*d + 15*A*d^2)*Sqrt[(d*(Sqrt[a]/Sqr
t[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + 15*a*A*b*d^4*Sqrt[(
d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c
+ d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I
*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[
a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(
-a + b*x^2))))/(15*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.53, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {2351, 626, 25, 27, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{x\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{\sqrt{a-bx^2}}{x\sqrt{c+dx}}dx+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{626}$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int -\frac{bx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 25$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\int \frac{bx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 27$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\int \frac{x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 600$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 509$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 508$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)\right)+\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\downarrow 327$$

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$\Big\downarrow$ 512

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(-\frac{c\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$\Big\downarrow$ 511

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1}}{}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$\Big\downarrow$ 321

$$A\left(a\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{}}{}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 633$$

$$A\left(\frac{a\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{}}{}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 632$$

$$A\left(\frac{a\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 186$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsi\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+d}}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \; 413$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{Ell}}{\sqrt{b}d\sqrt{a}}\right.\right.$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow 412$$

$$\int \frac{(B+Cx)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx+$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\Big\downarrow 682$$

$$-\frac{4\int \frac{b\left(ad(cC-5Bd)-\left(3aCd^2-bc(4cC-5Bd)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15bd^2}+$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$$\Big\downarrow 27$$

$$-\frac{2\int \frac{ad(cC-5Bd)-\left(3aCd^2-bc(4cC-5Bd)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}+$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$$\Big\downarrow 600$$

$$-\frac{2\left(-\frac{(bc^2-ad^2)(4cC-5Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{(3aCd^2-bc(4cC-5Bd))\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}+A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$$\Big\downarrow \text{509}$$

$$-\frac{2\left(-\frac{(bc^2-ad^2)(4cC-5Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}(3aCd^2-bc(4cC-5Bd))\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{15d^2}+A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$$\Big\downarrow \text{508}$$

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3aCd^2-bc(4cC-5Bd))\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{(bc^2-ad^2)(4cC-5Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}+A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$\Big\downarrow$ 327

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3aCd^2-bc(4cC-5Bd))E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{(bc^2-ad^2)(4cC-5Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}+$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$\Big\downarrow$ 512

$$-\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3aCd^2-bc(4cC-5Bd))E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)(4cC-5Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{15d^2}$$

$$A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

$\Big\downarrow$ 511

$$-\frac{2\sqrt{c+dx}\sqrt{a-bx^2}(4cC-3dxC-5Bd)}{15d^2}+$$

$$\frac{A\left(-b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\right.\right.}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a}}\right.\right.}{}$$

$$2\left(\frac{2\sqrt{a}(3aCd^2-bc(4cC-5Bd))\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{a}(4cC-5Bd)(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{}{\sqrt{1-}}}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx}}\right.$$

$$\overline{\qquad\qquad 15d^2 \qquad\qquad}$$

$\big\downarrow$ 321

$$\frac{A\left(-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-b\left(\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{}}\right.\right.}{}$$

$$2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)(4cC-5Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(3aCd^2-bc(4cC-5Bd))}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}}\right.$$

$$\overline{\qquad\qquad 15d^2 \qquad\qquad}$$

$$\frac{2\sqrt{a-bx^2}\sqrt{c+dx}(-5Bd+4cC-3Cdx)}{15d^2}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x*Sqrt[c + d*x]),x]
```

output
```
(-2*Sqrt[c + d*x]*(4*c*C - 5*B*d - 3*C*d*x)*Sqrt[a - b*x^2])/(15*d^2) - (2
*((2*Sqrt[a]*(3*a*C*d^2 - b*c*(4*c*C - 5*B*d))*Sqrt[c + d*x]*Sqrt[1 - (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sq
rt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(4*c*C - 5*B*d)*(b*c^2 - a*d^2)*S
qrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2) + A*(-(b*
((-2*Sqrt[a]*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (
Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*S
qrt[a]*c*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2
)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt
[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))) - (2*a*S
qrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]
*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqr
t[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 626
```
Int[(((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2])/(x_), x_Symbol] :>
 Simp[a*c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int
[(1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[((-a)*c^(n + 1/2) + a*(c +
 d*x)^(n + 1/2) + b*x^2*(c + d*x)^(n + 1/2))/x, x], x] /; FreeQ[{a, b, c, d
}, x] && IGtQ[n + 3/2, 0]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(432) = 864$.

Time = 1.25 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.74

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2Cx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d}-\frac{2\left(-Bb+\frac{4Cbc}{5d}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(Ba-\frac{2Cac}{5d}+\frac{\left(-Bb+\frac{4Cbc}{5d}\right)a}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{a}}{b}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/5*C/d*x*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-B*b+4/5*C/d*b*c)/b/d*(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)+2*(B*a-2/5*C/d*a*c+1/3*(-B*b+4/5*C/d*b*c)/b*a)*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-A
*b+2/5*a*C-2/3*(-B*b+4/5*C/d*b*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*
b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-2*A*a*(c/d-1/b*(a*b)^(1/2))*((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)/c*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-
(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/(x*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{x\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{x\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x)
```

3.172 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 505

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3d} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{cx}$$
$$- \frac{\sqrt{a}\sqrt{b}(4c^2C-6Bcd-3Ad^2)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3cd^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$- \frac{\sqrt{a}(4aCd^2-b(4c^2C-6Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$- \frac{a(2Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/
x-1/3*a^(1/2)*b^(1/2)*(-3*A*d^2-6*B*c*d+4*C*c^2)*(d*x+c)^(1/2)*((-b*x^2+a)
/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c/d^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(
1/2)/(-b*x^2+a)^(1/2)-1/3*a^(1/2)*(4*a*C*d^2-b*(3*A*d^2-6*B*c*d+4*C*c^2))*
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/b^(1/2)/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-a*(-A*d+2*B*c)*((d*x+
c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(
1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^
(1/2))/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.08 (sec) , antiderivative size = 994, normalized size of antiderivative = 1.97

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^2*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a - b*x^2]*((c*(-3*A*d + 2*c*C*x)*(c + d*x))/x + (-3*A*b*c^3 - 4*a*c
^3*C + (4*b*c^5*C)/d^2 - (6*b*B*c^4)/d + 6*a*B*c^2*d + 3*a*A*c*d^2 + 6*A*b
*c^2*(c + d*x) - (8*b*c^4*C*(c + d*x))/d^2 + (12*b*B*c^3*(c + d*x))/d - 3*
A*b*c*(c + d*x)^2 + (4*b*c^3*C*(c + d*x)^2)/d^2 - (6*b*B*c^2*(c + d*x)^2)/
d + (I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(-4*c^2*C + 6*B*c*d + 3*A*d^2)*Sq
rt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)
/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]) - (I*(Sqrt[b]*c - Sqrt[a]*d)*(6*A*Sqrt[
b]*c*d + Sqrt[a]*(4*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] +
 x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(
3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (S
qrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]) + ((6*I)*a*B*c*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-
(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[
b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sq
rt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]] - ((3*I)*a*A*d^2*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1365 vs. $2(505) = 1010$.

Time = 7.87 (sec) , antiderivative size = 1365, normalized size of antiderivative = 2.70, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {2355, 628, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\sqrt{a-bx^2}}{x^2\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 628

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\int -\frac{\frac{bdx^2}{c}+2bx+\frac{ad}{c}}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{1}{2}\int \frac{\frac{bdx^2}{c}+2bx+\frac{ad}{c}}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 2351

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\int \frac{\frac{dxb}{c}+2b}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{cx}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 600

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-b\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ad\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{b\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{c}\right)-\frac{\sqrt{a-bx^2}\sqrt{c}}{cx}\right)\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 509

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-b\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ad\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{c\sqrt{a-bx^2}}\right)-\sqrt{}\right)\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 508

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-b\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ad\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int\frac{\sqrt{1\ldots}}{\sqrt{\frac{1}{2}\ldots}}}{c\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(\ldots}{\sqrt{ad\ldots}}}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 327

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-b\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ad\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{ar}\ldots\right.}{c\sqrt{a-bx^2}\sqrt{\ldots}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 512

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{b\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\ldots}{c\sqrt{a-b\ldots}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$\downarrow$ 511

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d}{\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\downarrow 321$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\downarrow 633$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{c\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\downarrow 632$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(-\frac{ad\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{c\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(a\right.}{\sqrt{a-bx^2}\sqrt{c+}}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\Big\downarrow 186$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(\frac{2ad\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c}{\sqrt{a}d}}}{}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\Big\downarrow 413$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(\frac{2ad\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{}}{}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx$$

$$\Big\downarrow 412$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^2}dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-}}{}\right.\right.$$

$$\Big\downarrow 7293$$

$$\int\left(\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{dx}+\frac{(Bd-cC)\sqrt{c+dx}\sqrt{a-bx^2}}{d^2x^2}\right)dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-}}{}\right.\right.$$

$$\Big\downarrow 2009$$

$$\frac{2\sqrt{a}\sqrt{b}c\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{3d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}\left(bc^2+2ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{3\sqrt{b}d^2\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{2ac\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)C}{d\sqrt{c+dx}\sqrt{a-bx^2}} +$$

$$\frac{2\sqrt{c+dx}\sqrt{a-bx^2}C}{3d} - \frac{3\sqrt{a}\sqrt{b}(cC-Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{\sqrt{a}\sqrt{b}c(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d^2\sqrt{c+dx}\sqrt{a-bx^2}} + \frac{a(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{d\sqrt{c+dx}\sqrt{a-bx^2}} +$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{2}\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-}}{}\right.\right.$$

$$\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{d^2x}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^2*Sqrt[c + d*x]),x]
```

output
```
(2*C*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*d) + ((c*C - B*d)*Sqrt[c + d*x]*Sqr
t[a - b*x^2])/(d^2*x) + (2*Sqrt[a]*Sqrt[b]*c*C*Sqrt[c + d*x]*Sqrt[1 - (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sq
rt[b]*c)/Sqrt[a] + d)])/(3*d^2*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[
a]*d)]*Sqrt[a - b*x^2]) - (3*Sqrt[a]*Sqrt[b]*(c*C - B*d)*Sqrt[c + d*x]*Sqr
t[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d^2*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (Sqrt[a]*Sqrt[b]*c*(c*C - B*d)*Sqrt[(Sqr
t[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[Arc
Sin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d
)])/(d^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*C*(b*c^2 + 2*a*d^2)*S
qrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellip
ticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(3*Sqrt[b]*d^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*a*c*C*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticP
i[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]
*c + Sqrt[a]*d)])/(d*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (a*(c*C - B*d)*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticP
i[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]
*c + Sqrt[a]*d)])/(d*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (A + (c*(c*C - B*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(420) = 840$.

Time = 3.98 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.72

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(-\frac{A\sqrt{-bd x^3-bc x^2+adx+ac}}{xc}+\frac{2C\sqrt{-bd x^3-bc x^2+adx+ac}}{3d}+\frac{2\left(-Ab+\frac{2aC}{3}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\cdots}}{\sqrt{-bd x^3-bc x^2+\cdots}}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-A/x/c*(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)+2/3*C/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-
A*b+2/3*a*C)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c
/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((
x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b
)^(1/2)))^(1/2))+2*(-B*b-1/2*A*b*d/c+2/3*C/d*b*c)*(c/d-1/b*(a*b)^(1/2))*((
x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(
1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1
/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d
+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+a*(A*d-2*B*c)/c^2*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**2/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/(x**2*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{x^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{x^2\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x)
```

3.173 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 532

$$\begin{aligned}
&\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx \\
&= -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2cx^2} - \frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4c^2x} \\
&+ \frac{\sqrt{a}\sqrt{b}(8c^2C+4Bcd-3Ad^2)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c^2d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&- \frac{\sqrt{a}\sqrt{b}(8c^2C-4Bcd-Ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4cd\sqrt{c+dx}\sqrt{a-bx^2}} \\
&- \frac{(4ac(2cC-Bd)-A(4bc^2-3ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c^2\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^2-1/4*(-3*A*d+4*B*c)*(d*x+c)^(1/
2)*(-b*x^2+a)^(1/2)/c^2/x+1/4*a^(1/2)*b^(1/2)*(-3*A*d^2+4*B*c*d+8*C*c^2)*(
d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2
)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/d/((d*x+c)/
(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/4*a^(1/2)*b^(1/2)*(-A*d^2-
4*B*c*d+8*C*c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/c/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(4*a*c*(
-B*d+2*C*c)-A*(-3*a*d^2+4*b*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((
-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^
(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/(d*x+c)^(1/2)/(-b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.76 (sec) , antiderivative size = 1530, normalized size of antiderivative = 2.88

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^3*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(2*A*c + 4*B*c*x - 3*A*d*x))/(c^2*x^2)) + (
8*b*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 4*b*B*c^4*d*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]] - 3*A*b*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*a*c^3*C*d^2
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 4*a*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]] + 3*a*A*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 16*b*c^4*C*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 8*b*B*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(c + d*x) + 6*A*b*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 8*b*
c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 4*b*B*c^2*d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 3*A*b*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(c + d*x)^2 + I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(-8*c^2*C - 4*B*c*d +
 3*A*d^2)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sq
rt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqr
t[a]*d)] + I*d*(Sqrt[b]*c - Sqrt[a]*d)*(2*Sqrt[b]*c*(4*B*c - 3*A*d) + Sqrt
[a]*(-8*c^2*C + 4*B*c*d - 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*
x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipti
cF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + S
qrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + (4*I)*A*b*c^2*d^2*Sqrt[(d*(Sqrt[a]/Sq
rt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
 d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[S...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1470 vs. $2(532) = 1064$.

Time = 9.43 (sec) , antiderivative size = 1470, normalized size of antiderivative = 2.76, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {2355, 628, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{x^3\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{2355}$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\sqrt{a-bx^2}}{x^3\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 628$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\int -\frac{-\frac{bdx^2}{c}+2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{1}{4}\int \frac{-\frac{bdx^2}{c}+2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 2352$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{\int -\frac{-\frac{3abd^2x^2}{c}-2abdx+a\left(4bc-\frac{3ad^2}{c}\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}+\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\int \frac{-\frac{3abd^2x^2}{c}-2abdx+a\left(4bc-\frac{3ad^2}{c}\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{2cx^2}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 2351$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{-\frac{3abxd^2}{c}-2abd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}\right)-\sqrt{a\ldots}\right.$$
$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow \; 600$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{3a}{\ldots}}{2ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow 509$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\ldots}{2ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow 508$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\frac{6a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{c\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+a\left(4bc-\frac{3ad^2}{c}\right)\ldots}{2ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$$\Big\downarrow 327$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+-\cdots}{2ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\Big\downarrow$ 512

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{abd\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}\cdots}{2ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\Big\downarrow$ 511

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\cdots}{\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\Big\downarrow$ 321

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{a\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\mathrm{E}}{\sqrt{a-}}}{}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\downarrow$ 633

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx}{\sqrt{a}d+\sqrt{b}}}}{\sqrt{}}}{}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\downarrow$ 632

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{3ad^2}{c}\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{}}{}}{}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\downarrow$ 186

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{3ad^2}{c}\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}}{\ldots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\downarrow$ 413

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{3ad^2}{c}\right)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}\ldots}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{\ldots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx$$

$\downarrow$ 412

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^3}dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x}-\frac{-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\ldots}{\ldots}\right.\right.$$

$\downarrow$ 7293

$$\int \left(\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{dx^2} + \frac{(Bd-cC)\sqrt{c+dx}\sqrt{a-bx^2}}{d^2x^3} \right) dx +$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{a-bx^2}\sqrt{c+dx}}{c^2x} - \frac{-\frac{2a^{3/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}} + \right.\right.\right.$$

↓ 2009

$$\frac{3\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{\sqrt{a}\sqrt{b}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{d\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{a\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)C}{\sqrt{c+dx}\sqrt{a-bx^2}} - $$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{dx} - \frac{\sqrt{a}\sqrt{b}(cC-Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{4cd\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{5\sqrt{a}\sqrt{b}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{4d\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{(cC-Bd)\left(4bc^2+ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4cd^2\sqrt{c+dx}\sqrt{a-bx^2}} + $$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{4}\left(\frac{3d\sqrt{c+dx}\sqrt{a-bx^2}}{c^2x} - \frac{\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+c}{\sqrt{b}c+\sqrt{}}}}{}}{}\right.\right.$$

$$\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{4cdx} + \frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{2d^2x^2}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^3*Sqrt[c + d*x]),x]
```

output
```
((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(2*d^2*x^2) - (C*Sqrt[c + d*x]
*Sqrt[a - b*x^2])/(d*x) + ((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(4*c
*d*x) + (3*Sqrt[a]*Sqrt[b]*C*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]
) - (Sqrt[a]*Sqrt[b]*(c*C - B*d)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellipti
cE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[
a] + d)])/(4*c*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a
- b*x^2]) - (Sqrt[a]*Sqrt[b]*c*C*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]
]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[c + d*x]*Sqrt[a - b*
x^2]) - (5*Sqrt[a]*Sqrt[b]*(c*C - B*d)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(4*d*Sqrt[c + d*x]*Sqr
t[a - b*x^2]) - (a*C*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqr
t[1 - (b*x^2)/a]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[c + d*x]*Sqrt[a - b*x^2]
) - ((c*C - B*d)*(4*b*c^2 + a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(4*c*d^2*Sqrt...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 849, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{\sqrt{-b\,x^{2}+a}\,\sqrt{dx+c}\,(-3Adx+4Bcx+2Ac)}{4c^{2}x^{2}}+\frac{\left(\left(3A\,d^{2}-4Bcd-8C\,c^{2}\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right.\right.}{\sqrt{-}}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(dx+c)}\left(-\frac{A\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{2c\,x^{2}}+\frac{(3Ad-4Bc)\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{4c^{2}x}+\frac{2\left(-Bb+\frac{Abd}{4c}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x-\sqrt{}}{-\frac{c}{d}-}}}{\sqrt{-bd\,x^{3}}}\right.}{}$
default	Expression too large to display

input

```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/4*(-b*x^2+a)^(1/2)*(d*x+c)^(1/2)*(-3*A*d*x+4*B*c*x+2*A*c)/c^2/x^2+1/8/c
^2*((3*A*d^2-4*B*c*d-8*C*c^2)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(
a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(
1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a
*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)
,(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)
*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b
)^(1/2)))^(1/2)))-(3*A*a*d^2-4*A*b*c^2-4*B*a*c*d+8*C*a*c^2)*2^(1/2)*((x+1/
b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*
(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*EllipticPi(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),2,(
-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-8*B*c^2*(a*b)^(1/2)*2^(1/2)
*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/
2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))+2*A*c*d*(a*b)^(1/2)*2^(
1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))
^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))*((-b*x^2+a)*(d*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**3/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/(x**3*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-b\,x^2}\,(C\,x^2+B\,x+A)}{x^3\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{\sqrt{-b\,x^2+a}\,(C\,x^2+Bx+A)}{x^3\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x)
```

3.174 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 617

$$
\begin{aligned}
&\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx \\
&= -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{3cx^3} - \frac{(6Bc-5Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{12c^2x^2} \\
&\quad - \frac{(6ac(4cC-3Bd)-A(8bc^2-15ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{24ac^3x} \\
&\quad - \frac{\sqrt{b}(8Abc^2-24ac^2C+18aBcd-15aAd^2)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{24\sqrt{a}c^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad + \frac{\sqrt{b}(6ac(4cC+Bd)+A(8bc^2-5ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{24\sqrt{a}c^2\sqrt{c+dx}\sqrt{a-bx^2}} \\
&\quad + \frac{(4bc^2(2Bc-Ad)+ad(8c^2C-6Bcd+5Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{\sqrt{b}c+}\right.}{8c^3\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output

```
-1/3*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^3-1/12*(-5*A*d+6*B*c)*(d*x+c)^(1
/2)*(-b*x^2+a)^(1/2)/c^2/x^2-1/24*(6*a*c*(-3*B*d+4*C*c)-A*(-15*a*d^2+8*b*c
^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c^3/x-1/24*b^(1/2)*(-15*A*a*d^2+8*A*
b*c^2+18*B*a*c*d-24*C*a*c^2)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/a^(1/2)/c^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a
)^(1/2)+1/24*b^(1/2)*(6*a*c*(B*d+4*C*c)+A*(-5*a*d^2+8*b*c^2))*((d*x+c)/(c+
a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^
(1/2)/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+1/8*(4*b*c^2*(-A*d+2*B*c)+a*d*(5*
A*d^2-6*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.83 (sec) , antiderivative size = 1895, normalized size of antiderivative = 3.07

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input

```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^4*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[a - b*x^2]*(-((c*(c + d*x)*(-8*A*b*c^2*x^2 + 6*a*c*x*(2*B*c + 4*c*C*
x - 3*B*d*x) + a*A*(8*c^2 - 10*c*d*x + 15*d^2*x^2)))/x^3) + (8*A*b^2*c^5*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 24*a*b*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
] + 18*a*b*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 23*a*A*b*c^3*d^2*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]] + 24*a^2*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
] - 18*a^2*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 15*a^2*A*c*d^4*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]] - 16*A*b^2*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c
 + d*x) + 48*a*b*c^4*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 36*a*b*B
*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 30*a*A*b*c^2*d^2*Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 8*A*b^2*c^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]*(c + d*x)^2 - 24*a*b*c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2
 + 18*a*b*B*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 15*a*A*b*c*
d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c -
Sqrt[a]*d)*(6*a*c*(-4*c*C + 3*B*d) + A*(8*b*c^2 - 15*a*d^2))*Sqrt[(d*(Sqrt
[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))
]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*(S
qrt[b]*c - Sqrt[a]*d)*(8*A*b*c^2*d + 6*Sqrt[a]*Sqrt[b]*c*(8*c^2*C - 6*B*c*
d + 5*A*d^2) + 3*a*d*(8*c^2*C - 6*B*c*d + 5*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[
b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1676 vs. $2(617) = 1234$.

Time = 11.60 (sec) , antiderivative size = 1676, normalized size of antiderivative = 2.72, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {2355, 628, 25, 2352, 25, 2352, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{x^4\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\sqrt{a-bx^2}}{x^4\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{628}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\int -\frac{-\frac{3bdx^2}{c}+2bx+\frac{5ad}{c}}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{3cx^3}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{1}{6}\int \frac{-\frac{3bdx^2}{c}+2bx+\frac{5ad}{c}}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{3cx^3}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{2352}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{\int -\frac{\frac{5abd^2x^2}{c}-2abdx+a\left(8bc-\frac{15ad^2}{c}\right)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}+\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{3cx^3}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{\int \frac{\frac{5abd^2x^2}{c}-2abdx+a\left(8bc-\frac{15ad^2}{c}\right)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{3cx^3}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{2352}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{\int \frac{3d\left(4bc-\frac{5ad^2}{c}\right)a^2-10bd^2xa^2+bd\left(8bc-\frac{15ad^2}{c}\right)x^2a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}\left(8b-\right.}{x}}{4ac}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{2351}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{abd\left(8bc-\frac{15ad^2}{c}\right)x-10a^2bd^2}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\cdots}{4ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{600}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-ab\left(8bc^2-5ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\cdots}{2ac}}{4ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{509}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-ab\left(8bc^2-5ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\cdots}{2ac}}{4ac}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{508}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(8bc-\frac{15ad^2}{c}\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+3a^2d\left(4\cdots\right.}{2ac}\cdots\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{327}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-ab\left(8bc^2-5ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\cdots}{2ac}}{\cdots}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{512}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ab\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\cdots}}\cdots}{\sqrt{a-bx^2}}}{2a\cdots}}{\cdots}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{511}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\cdots}{\sqrt{a-bx^2}\sqrt{c+dx}}}{\cdots}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$$\Big\downarrow \text{321}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{3a^2d\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\sqrt{\frac{\sqrt{b}(c+\cdots}{\sqrt{a}d+\cdots}}\cdots}{\sqrt{a-\cdots}}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$\downarrow$ 633

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{\frac{3a^2d\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$\downarrow$ 632

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{\frac{3a^2d\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{5ad^2}{c}\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$\downarrow$ 186

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{-\frac{6a^2d\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{5ad^2}{c}\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\cdots}{\sqrt{a-bx^2}}}{\cdots}}{\cdots}\right)\cdots\right)$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$\downarrow$ 413

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{-\frac{6a^2d\sqrt{1-\frac{bx^2}{a}}\left(4bc-\frac{5ad^2}{c}\right)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}\cdots}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{\cdots}}{\cdots}\right)\cdots\right)$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx$$

$\downarrow$ 412

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^4}dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2}-\frac{-\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\cdots}{\sqrt{a-bx^2}\sqrt{c+dx}}}{\cdots}\right)\cdots\right)$$

$\downarrow$ 7293

$$\int \left(\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{dx^3} + \frac{(Bd-cC)\sqrt{c+dx}\sqrt{a-bx^2}}{d^2x^4}\right)dx +$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{a-bx^2}\sqrt{c+dx}}{2c^2x^2} - \frac{-\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\left(8bc^2-5ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}}{}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{4c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+$$

$$\frac{5\sqrt{a}\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{4\sqrt{c+dx}\sqrt{a-bx^2}}+$$

$$\frac{\left(\frac{4bc}{d}+\frac{ad}{c}\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)C}{4\sqrt{c+dx}\sqrt{a-bx^2}}-$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{4cx}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{2dx^2}+$$

$$\frac{\sqrt{b}(cC-Bd)\left(8bc^2+3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{24\sqrt{a}c^2d^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-$$

$$\frac{\sqrt{b}(cC-Bd)\left(8bc^2+ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{24\sqrt{a}cd^2\sqrt{c+dx}\sqrt{a-bx^2}}-$$

$$\frac{(cC-Bd)\left(4b-\frac{ad^2}{c^2}\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{8d\sqrt{c+dx}\sqrt{a-bx^2}}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{6}\left(\frac{5d\sqrt{c+dx}\sqrt{a-bx^2}}{2c^2x^2}-\frac{-\frac{\sqrt{c+dx}\sqrt{a-bx^2}\left(8b-\frac{15ad^2}{c^2}\right)}{x}-\frac{-\frac{6d\left(4bc-\frac{5ad^2}{c}\right)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}{\sqrt{a-bx^2}\sqrt{c+\frac{\sqrt{}}{\sqrt{}}}}}{}}{}\right.\right.$$

$$\frac{(cC-Bd)\left(\frac{3d^2}{c^2}+\frac{8b}{a}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{24d^2x}+\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{12cdx^2}+$$

$$\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{3d^2x^3}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^4*Sqrt[c + d*x]),x]
```

output
```
((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*d^2*x^3) - (C*Sqrt[c + d*x]
*Sqrt[a - b*x^2])/(2*d*x^2) + ((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/
(12*c*d*x^2) - (C*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(4*c*x) - ((c*C - B*d)*((
8*b)/a + (3*d^2)/c^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(24*d^2*x) + (Sqrt[a]
*Sqrt[b]*C*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sq
rt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(4*c*Sqrt[(S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (Sqrt[b]*(c*
C - B*d)*(8*b*c^2 + 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(24*Sqrt[a]*c^2*d^2*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)
]*Sqrt[a - b*x^2]) + (5*Sqrt[a]*Sqrt[b]*C*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b
]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x
)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(4*Sqrt[c + d*x]*Sq
rt[a - b*x^2]) - (Sqrt[b]*(c*C - B*d)*(8*b*c^2 + a*d^2)*Sqrt[(Sqrt[b]*(c +
 d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[
1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(24*S
qrt[a]*c*d^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (C*((4*b*c)/d + (a*d)/c)*Sqr
t[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipti
cPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[
b]*c + Sqrt[a]*d)])/(4*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((c*C - B*d)*(4...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 6.30 (sec) , antiderivative size = 999, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(-\frac{A\sqrt{-bd x^3-bc x^2+adx+ac}}{3c x^3}+\frac{(5Ad-6Bc)\sqrt{-bd x^3-bc x^2+adx+ac}}{12c^2x^2}-\frac{\left(15Aa d^2-8bA c^2-18Bacd+24Ca c^2\right)\sqrt{-bd x^3}}{24c^3ax}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/3*A/c/x^3*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/12*(5*A*d-6*B*c)/c^2*(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)/x^2-1/24/c^3/a*(15*A*a*d^2-8*A*b*c^2-18*B*a*c*d+24*C*a*c^
2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(-C*b-1/24*b*d*(5*A*d-6*B*c)/c^2
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))-1/24*b*d*(15*A*a*d^2-8*A*b*c^2-18*B*a*c*d+24*C*a*c^2)/a/c^3*(c/d-1/b
*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^
(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE
(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+1/8*(5*A
*a*d^3-4*A*b*c^2*d-6*B*a*c*d^2+8*B*b*c^3+8*C*a*c^2*d)/c^4*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b...
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(d*x^5 + c*x^4),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**4/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/(x**4*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^4), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{x^4\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{x^4\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x)
```

3.175 $\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 738

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{4cx^4}-\frac{(8Bc-7Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{24c^2x^3}$$
$$-\frac{(8ac(6cC-5Bd)-A(12bc^2-35ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{96ac^3x^2}$$
$$+\frac{(4bc^2(16Bc-11Ad)+3ad(48c^2C-40Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{192ac^4x}$$
$$-\frac{\sqrt{b}(4bc^2(16Bc-11Ad)+3ad(48c^2C-40Bcd+35Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{}}{\sqrt{bc-}}\right.}{192\sqrt{a}c^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{\sqrt{b}(4bc^2(16Bc-5Ad)+ad(48c^2C-40Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{192\sqrt{a}c^3\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$+\frac{(A(16b^2c^4+24abc^2d^2-35a^2d^4)-8ac(ad^2(6cC-5Bd)-4bc^2(2cC-Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{Elliptic}}{64ac^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/4*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^4-1/24*(-7*A*d+8*B*c)*(d*x+c)^(1
/2)*(-b*x^2+a)^(1/2)/c^2/x^3-1/96*(8*a*c*(-5*B*d+6*C*c)-A*(-35*a*d^2+12*b*
c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c^3/x^2+1/192*(4*b*c^2*(-11*A*d+16*
B*c)+3*a*d*(35*A*d^2-40*B*c*d+48*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/
c^4/x-1/192*b^(1/2)*(4*b*c^2*(-11*A*d+16*B*c)+3*a*d*(35*A*d^2-40*B*c*d+48*
C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2
)/c^4/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/192*b^(1/2)
*(4*b*c^2*(-5*A*d+16*B*c)+a*d*(35*A*d^2-40*B*c*d+48*C*c^2))*((d*x+c)/(c+a^
(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1
/2)/c^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+1/64*(A*(-35*a^2*d^4+24*a*b*c^2*d^2
+16*b^2*c^4)-8*a*c*(a*d^2*(-5*B*d+6*C*c)-4*b*c^2*(-B*d+2*C*c)))*((d*x+c)/(
c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/a/c^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.74 (sec) , antiderivative size = 2679, normalized size of antiderivative = 3.63

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^5*Sqrt[c + d*x]),x]
```

output
```
(-1/4*A/(c*x^4) + (-8*B*c + 7*A*d)/(24*c^2*x^3) + (12*A*b*c^2 - 48*a*c^2*C
 + 40*a*B*c*d - 35*a*A*d^2)/(96*a*c^3*x^2) + (64*b*B*c^3 - 44*A*b*c^2*d +
144*a*c^2*C*d - 120*a*B*c*d^2 + 105*a*A*d^3)/(192*a*c^4*x))*Sqrt[c + d*x]*
Sqrt[a - b*x^2] + (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-
64*b^2*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 44*A*b^2*c^3*d*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]] - 144*a*b*c^3*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 120*a
*b*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 105*a*A*b*c*d^3*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]] - (64*b^2*B*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*
x)^2 + (44*A*b^2*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (144*
a*b*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (184*a*b*B*c^4*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (149*a*A*b*c^3*d^3*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (144*a^2*c^3*C*d^3*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]])/(c + d*x)^2 - (120*a^2*B*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]])/(c + d*x)^2 + (105*a^2*A*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c
+ d*x)^2 + (128*b^2*B*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (88*
A*b^2*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (288*a*b*c^4*C*d*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (240*a*b*B*c^3*d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]])/(c + d*x) + (210*a*A*b*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]])/(c + d*x) + (I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(4*b*c^2*(16*B*c
 - 11*A*d) + 3*a*d*(48*c^2*C - 40*B*c*d + 35*A*d^2))*Sqrt[1 - c/(c + d*...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1907 vs. $2(738) = 1476$.

Time = 14.33 (sec) , antiderivative size = 1907, normalized size of antiderivative = 2.58, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.629$, Rules used = {2355, 628, 25, 2352, 25, 2352, 2352, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-bx^2}\left(A+Bx+Cx^2\right)}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\sqrt{a-bx^2}}{x^5\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 628$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\int -\frac{-\frac{5bdx^2}{c}+2bx+\frac{7ad}{c}}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{4cx^4}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{1}{8}\int \frac{-\frac{5bdx^2}{c}+2bx+\frac{7ad}{c}}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{4cx^4}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 2352$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{\int -\frac{\frac{21abd^2x^2}{c}-2abdx+a\left(12bc-\frac{35ad^2}{c}\right)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}+\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{4cx^4}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{\int \frac{\frac{21abd^2x^2}{c}-2abdx+a\left(12bc-\frac{35ad^2}{c}\right)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}\right)-\frac{\sqrt{a-bx^2}\sqrt{c+dx}}{4cx^4}\right)+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 2352$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{\int \frac{d\left(44bc-\frac{105ad^2}{c}\right)a^2-bd\left(12bc-\frac{35ad^2}{c}\right)x^2a-2b\left(12bc^2+7ad^2\right)xa}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{\sqrt{a-bx^2}}{}}{6ac}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$$\Big\downarrow \; 2352$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{-\frac{\int \frac{bd^2\left(44bc-\frac{105ad^2}{c}\right)x^2a^2+3\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)a^2+2bd\left(12bc^2-35ad\right.}{x\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4ac}}{6a}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 2351

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{-70a^3bd^3+a^2b\left(44bc\right.}{\sqrt{c+}}}{2ac}}{4ac}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 600

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-5a^2bd\left(4bc^2-7ad^2\right)\int \frac{}{\sqrt{}}}{2ac}}{}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 509

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-5a^2bd\left(4bc^2-7ad^2\right)\int \frac{}{\sqrt{}}}{2ac}}{}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 508

$$-\frac{2a^{5/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(44bc-\frac{105ad^2}{c}\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+3a\ldots$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\ldots\right)\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\Big\downarrow$ 327

$$3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-5a^2bd\left(4bc^2-7ad^2\right)\int \ldots$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\ldots\right)\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\Big\downarrow$ 512

$$3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{5a^2bd\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-\ldots\right.}{\sqrt{\ldots}}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\ldots\right)\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 511

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{10a^{5/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-7ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)\cdots}}}{\sqrt{a-bx^2}\sqrt{c+dx}}}{\cdots}\right)\right)$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 321

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{3a^2\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{10a^{5/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\left(4b\cdots\right.}{\cdots}}{\cdots}}{\cdots}\right)\right)$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 633

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{-\frac{\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{10a^{5/2}\sqrt{b}d\sqrt{\cdots}}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 632

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{-\frac{\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\frac{10\cdots}{\cdots}}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 186

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{-\frac{-\frac{6a^2\sqrt{1-\frac{bx^2}{a}}\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\cdots\right)}{\sqrt{\cdots}}}}\cdots}{\sqrt{a-bx^2}}\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\int\frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 413

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{-\frac{6a^2\sqrt{1-\frac{bx^2}{a}}\left(-\frac{35a^2d^4}{c}+24abcd^2+16b^2c^3\right)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \cdots}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+\cdots}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{\cdots}\right.\right.$$

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx$$

$\downarrow$ 412

$$\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{x^5}dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{\frac{10a^{5/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-7ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\cdots\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 7293

$$\int \left(\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{dx^4}+\frac{(Bd-cC)\sqrt{c+dx}\sqrt{a-bx^2}}{d^2x^5}\right)dx+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{a-bx^2}\sqrt{c+dx}}{3c^2x^3}-\frac{-\frac{10a^{5/2}\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}\left(4bc^2-7ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\ldots}{\ldots}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}\ldots}{\ldots}\right.\right.$$

↓ 2009

$$-\frac{\sqrt{b}\left(8bc^2+3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{24\sqrt{a}c^2d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+$$

$$\frac{\sqrt{b}\left(8bc^2+ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)C}{24\sqrt{a}cd\sqrt{c+dx}\sqrt{a-bx^2}}+$$

$$\frac{\left(4b-\frac{ad^2}{c^2}\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)C}{8\sqrt{c+dx}\sqrt{a-bx^2}}+$$

$$\frac{\left(\frac{3d^2}{c^2}+\frac{8b}{a}\right)\sqrt{c+dx}\sqrt{a-bx^2}C}{24dx}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{12cx^2}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}C}{3dx^3}+$$

$$\frac{5\sqrt{b}(cC-Bd)\left(4bc^2-3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{192\sqrt{a}c^3d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-$$

$$\frac{\sqrt{b}(cC-Bd)\left(44bc^2-5ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{192\sqrt{a}c^2d\sqrt{c+dx}\sqrt{a-bx^2}}-$$

$$\frac{(cC-Bd)\left(16b^2c^4-8abd^2c^2+5a^2d^4\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{64ac^3d^2\sqrt{c+dx}\sqrt{a-bx^2}}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{1}{8}\left(\frac{7d\sqrt{c+dx}\sqrt{a-bx^2}}{3c^2x^3}-\frac{-\frac{\sqrt{c+dx}\sqrt{a-bx^2}\left(12b-\frac{35ad^2}{c^2}\right)}{2x^2}-\frac{-\frac{ad\sqrt{c+dx}\sqrt{a-bx^2}\left(44bc^2-105ad^2\right)}{c^2x}-\frac{2\sqrt{b}c\cdots}{\cdots}}{\cdots}}{\cdots}\right.\right.$$

$$\frac{5(cC-Bd)\left(4bc^2-3ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}{192ac^3dx}-\frac{(cC-Bd)\left(\frac{5d^2}{c^2}+\frac{12b}{a}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{96d^2x^2}+$$

$$\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{24cdx^3}+\frac{(cC-Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{4d^2x^4}$$

input
```
Int[(Sqrt[a - b*x^2]*(A + B*x + C*x^2))/(x^5*Sqrt[c + d*x]),x]
```

output
```
((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(4*d^2*x^4) - (C*Sqrt[c + d*x]
*Sqrt[a - b*x^2])/(3*d*x^3) + ((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/
(24*c*d*x^3) - (C*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(12*c*x^2) - ((c*C - B*d)
*((12*b)/a + (5*d^2)/c^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(96*d^2*x^2) - (5
*(c*C - B*d)*(4*b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(192*a*c^3
*d*x) + (C*((8*b)/a + (3*d^2)/c^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(24*d*x)
 + (5*Sqrt[b]*(c*C - B*d)*(4*b*c^2 - 3*a*d^2)*Sqrt[c + d*x]*Sqrt[1 - (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqr
t[b]*c)/Sqrt[a] + d)])/(192*Sqrt[a]*c^3*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b
]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (Sqrt[b]*C*(8*b*c^2 + 3*a*d^2)*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(24*Sqrt[a]*c^2*d*Sqrt[(Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (Sqrt[b]*(c*C -
B*d)*(44*b*c^2 - 5*a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)
]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[
2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(192*Sqrt[a]*c^2*d*Sqrt[c + d*x]*Sq
rt[a - b*x^2]) + (Sqrt[b]*C*(8*b*c^2 + a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(24*Sqrt[a]*c*d
*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (C*(4*b - (a*d^2)/c^2)*Sqrt[(Sqrt[b]*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 628
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*Sqrt[(a_) + (b_.)*(x_)^2], x
_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(
e*(m + 1))), x] - Simp[1/(2*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*
Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2)*d*(2
*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x^3 - 2
*a*(m + 1)*(c + d*x)^(n + 1/2) - 2*b*(m + 1)*x^2*(c + d*x)^(n + 1/2))/x, x]
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] &&
IntegerQ[2*m]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 7.96 (sec) , antiderivative size = 1132, normalized size of antiderivative = 1.53

method	result	size
elliptic	Expression too large to display	1132
risch	Expression too large to display	1362
default	Expression too large to display	5800

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/4*A/c/x^4*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/24*(7*A*d-8*B*c)/c^2*(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)/x^3-1/96/a/c^3*(35*A*a*d^2-12*A*b*c^2-40*B*a*c*d+48*C*a*c
^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x^2+1/192*(105*A*a*d^3-44*A*b*c^2*d
-120*B*a*c*d^2+64*B*b*c^3+144*C*a*c^2*d)/a/c^4*(-b*d*x^3-b*c*x^2+a*d*x+a*c
)^(1/2)/x+1/96*b*d*(35*A*a*d^2-12*A*b*c^2-40*B*a*c*d+48*C*a*c^2)/a/c^3*(c/
d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/
2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))
+1/192*(105*A*a*d^3-44*A*b*c^2*d-120*B*a*c*d^2+64*B*b*c^3+144*C*a*c^2*d)*d
/a*b/c^4*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1
/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)
^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)))-1/64*(35*A*a^2*d^4-24*A*a*b*c^2*d^2-16*A*b^2*c^4-40*B*a^2*c*d^3+3
2*B*a*b*c^3*d+48*C*a^2*c^2*d^2-64*C*a*b*c^4)/c^5/a*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*...
```

Fricas [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}x^5}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(d*x^6 + c*x^5),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{a-bx^2}\,(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{\sqrt{a-bx^2}\,(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(1/2)*(C*x**2+B*x+A)/x**5/(d*x+c)**(1/2),x)
```

output
```
Integral(sqrt(a - b*x**2)*(A + B*x + C*x**2)/(x**5*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a-bx^2}\,(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}\,x^5}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^5), x)
```

Giac [F]

$$\int \frac{\sqrt{a-bx^2}\,(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{-bx^2+a}}{\sqrt{dx+c}\,x^5}\,dx$$

input
```
integrate((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)/(sqrt(d*x + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx=\int \frac{\sqrt{a-b\,x^2}\,\left(C\,x^2+B\,x+A\right)}{x^5\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(1/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{a-bx^2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx=\int \frac{\sqrt{-b\,x^2+a}\,\left(C\,x^2+Bx+A\right)}{x^5\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x)
```

3.176 $\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 990

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

output
```
4/45045*(195*a^3*C*d^6+9*a^2*b*d^4*(65*A*d^2-71*B*c*d+68*C*c^2)-64*b^3*c^4
*(65*A*d^2-60*B*c*d+56*C*c^2)+6*a*b^2*c^2*d^2*(637*A*d^2-544*B*c*d+480*C*c
^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^7-4/15015*(3*a^2*d^4*(-77*B*d+71
*C*c)-16*b^2*c^3*(65*A*d^2-60*B*c*d+56*C*c^2)+a*b*c*d^2*(793*A*d^2-666*B*c
*d+580*C*c^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^6+2/9009*(39*a^2*C*d^4
+3*a*b*d^2*(39*A*d^2-58*B*c*d+68*C*c^2)+16*b^2*c^2*(65*A*d^2-60*B*c*d+56*C
*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b^2/d^5-2/1287*(3*a*d^2*(-11*B*d+12*
C*c)+2*b*c*(65*A*d^2-60*B*c*d+56*C*c^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/
b/d^4-2/429*(13*a*C*d^2+b*(39*A*d^2-69*B*c*d+93*C*c^2))*(d*x+c)^(1/2)*(-b*
x^2+a)^(5/2)/b^2/d^3+2/39*(-3*B*d+8*C*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(5/2)/b/
d^3-2/15*C*(d*x+c)^(5/2)*(-b*x^2+a)^(5/2)/b/d^3+8/45045*a^(1/2)*(3*a^3*d^6
*(-231*B*d+148*C*c)+64*b^3*c^5*(65*A*d^2-60*B*c*d+56*C*c^2)+3*a^2*b*c*d^4*
(598*A*d^2-453*B*c*d+376*C*c^2)-6*a*b^2*c^3*d^2*(1157*A*d^2-1024*B*c*d+928
*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/
2)/d^8/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+8/45045*a^(1
/2)*(-a*d^2+b*c^2)*(195*a^3*C*d^6+9*a^2*b*d^4*(65*A*d^2-71*B*c*d+68*C*c^2)
-64*b^3*c^4*(65*A*d^2-60*B*c*d+56*C*c^2)+6*a*b^2*c^2*d^2*(637*A*d^2-544*B*
c*d+480*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)
*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.82 (sec) , antiderivative size = 1335, normalized size of antiderivative = 1.35

$$\int \frac{x^2(a-bx^2)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-8*(1792*b^3*c^6*C - 1920*b^3*B*c^5*d + 20
80*A*b^3*c^4*d^2 - 2560*a*b^2*c^4*C*d^2 + 2832*a*b^2*B*c^3*d^3 - 3211*a*A*
b^2*c^2*d^4 + 321*a^2*b*c^2*C*d^4 - 408*a^2*b*B*c*d^5 + 585*a^2*A*b*d^6 +
195*a^3*C*d^6))/(45045*b^2*d^7) - (8*(-1344*b^2*c^5*C + 1440*b^2*B*c^4*d -
 1560*A*b^2*c^3*d^2 + 1850*a*b*c^3*C*d^2 - 2049*a*b*B*c^2*d^3 + 2327*a*A*b
*c*d^4 - 174*a^2*c*C*d^4 + 231*a^2*B*d^5)*x)/(45045*b*d^6) + (2*(-4480*b^2
*c^4*C + 4800*b^2*B*c^3*d - 5200*A*b^2*c^2*d^2 + 6036*a*b*c^2*C*d^2 - 6690
*a*b*B*c*d^3 + 7605*a*A*b*d^4 - 468*a^2*C*d^4)*x^2)/(45045*b*d^5) + (2*(56
0*b*c^3*C - 600*b*B*c^2*d + 650*A*b*c*d^2 - 744*a*c*C*d^2 + 825*a*B*d^3)*x
^3)/(6435*d^4) - (2*(168*b*c^2*C - 180*b*B*c*d + 195*A*b*d^2 - 221*a*C*d^2
)*x^4)/(2145*d^3) - (2*b*(-14*c*C + 15*B*d)*x^5)/(195*d^2) - (2*b*C*x^6)/(
15*d)) + (8*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(3*a^3*d^6*(148*c*C - 231*B*d) + 64*b^3*c^5*(56*c^2*C
 - 60*B*c*d + 65*A*d^2) + 3*a^2*b*c*d^4*(376*c^2*C - 453*B*c*d + 598*A*d^2
) - 6*a*b^2*c^3*d^2*(928*c^2*C - 1024*B*c*d + 1157*A*d^2))*(-((a*d^2)/(c +
 d*x)^2) + b*(-1 + c/(c + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3
*a^3*d^6*(148*c*C - 231*B*d) + 64*b^3*c^5*(56*c^2*C - 60*B*c*d + 65*A*d^2)
 + 3*a^2*b*c*d^4*(376*c^2*C - 453*B*c*d + 598*A*d^2) - 6*a*b^2*c^3*d^2*(92
8*c^2*C - 1024*B*c*d + 1157*A*d^2))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sq
rt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x)...
```

Rubi [A] (verified)

Time = 3.62 (sec) , antiderivative size = 962, normalized size of antiderivative = 0.97, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2185, 27, 2185, 27, 2185, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{5\left(a-bx^2\right)^{3/2}\left(-bd^3(8cC-3Bd)x^3-d^2\left(7bCc^2-3Abd^2-aCd^2\right)x^2-2cCd\left(bc^2-ad^2\right)x+ac^2Cd^2\right)}{2\sqrt{c+dx}}dx}{15bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 27

$$\frac{\int \frac{\left(a-bx^2\right)^{3/2}\left(-bd^3(8cC-3Bd)x^3-d^2\left(7bCc^2-3Abd^2-aCd^2\right)x^2-2cCd\left(bc^2-ad^2\right)x+ac^2Cd^2\right)}{\sqrt{c+dx}}dx}{3bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{2\int \frac{\left(a-bx^2\right)^{3/2}\left(-b\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)x^2d^5+abc(11cC-9Bd)d^5-b\left(6b(9cC-\right.\right.}{2\sqrt{c+dx}}}{13bd^3}}{3bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\int \frac{\left(a-bx^2\right)^{3/2}\left(-b\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)x^2d^5+abc(11cC-9Bd)d^5-b\left(6b(9cC-5\right.\right.}{\sqrt{c+dx}}}{13bd^3}}{3bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{2\int \frac{bd^6\left(ad\left(13aCd^2-b\left(28\right.\right.\right.}{}}{}}{13b}}{3bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\int \frac{\left(ad\left(13aCd^2-b\left(2\right.\right.\right.}{}}{13}}{3bd^4}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 682

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\left(\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+}}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\big\downarrow$ 27

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\left(2\int\frac{\left(ad\left(39a^2Cd^4\right.\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\big\downarrow$ 682

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\left(\frac{2\left(\frac{2\sqrt{a-bx^2}\sqrt{c+}}{}\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\big\downarrow$ 27

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\left(\frac{2\left(2\int\frac{ad\left(195a^3C\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\big\downarrow$ 600

$$\frac{\frac{2}{13}d\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}(8cC-3Bd)-\frac{\frac{2}{11}d^4\left(a-bx^2\right)^{5/2}\sqrt{c+dx}\left(13aCd^2+b\left(39Ad^2-69Bcd+93c^2C\right)\right)-\frac{1}{11}d^4\left(\frac{2\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right.}{}\right.}{}\right.}{}\right.}{}}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{5/2}}{15bd^3}$$

$\downarrow$ 509

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

$\downarrow$ 508

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

$\downarrow$ 327

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

$\downarrow$ 512

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

$\downarrow$ 511

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

$\downarrow$ 321

$$\frac{2}{11}d^4\left(13aCd^2+b\left(93Cc^2-69Bdc+39Ad^2\right)\right)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}-\frac{1}{11}d^4\left(\frac{2\sqrt{c+dx}\left(39a^2Cd^4\right.}{}\right.$$

$$\frac{\frac{2}{13}d(8cC-3Bd)(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}-\frac{}{}}{}$$

$$\frac{2C(c+dx)^{5/2}\left(a-bx^2\right)^{5/2}}{15bd^3}$$

input
```
Int[(x^2*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*(c + d*x)^(5/2)*(a - b*x^2)^(5/2))/(15*b*d^3) + ((2*d*(8*c*C - 3*B*d
)*(c + d*x)^(3/2)*(a - b*x^2)^(5/2))/13 - ((2*d^4*(13*a*C*d^2 + b*(93*c^2*
C - 69*B*c*d + 39*A*d^2))*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/11 - (d^4*((2*S
qrt[c + d*x]*(39*a^2*C*d^4 + 3*a*b*d^2*(68*c^2*C - 58*B*c*d + 39*A*d^2) +
16*b^2*c^2*(56*c^2*C - 60*B*c*d + 65*A*d^2) - 7*b*d*(3*a*d^2*(12*c*C - 11*
B*d) + 2*b*c*(56*c^2*C - 60*B*c*d + 65*A*d^2))*x)*(a - b*x^2)^(3/2))/(21*d
^2) + (2*((2*Sqrt[c + d*x]*(195*a^3*C*d^6 + 9*a^2*b*d^4*(68*c^2*C - 71*B*c
*d + 65*A*d^2) - 64*b^3*c^4*(56*c^2*C - 60*B*c*d + 65*A*d^2) + 6*a*b^2*c^2
*d^2*(480*c^2*C - 544*B*c*d + 637*A*d^2) - 3*b*d*(3*a^2*d^4*(71*c*C - 77*B
*d) - 16*b^2*c^3*(56*c^2*C - 60*B*c*d + 65*A*d^2) + a*b*c*d^2*(580*c^2*C -
 666*B*c*d + 793*A*d^2))*x)*Sqrt[a - b*x^2])/(15*d^2) + (2*((2*Sqrt[a]*Sqr
t[b]*(3*a^3*d^6*(148*c*C - 231*B*d) + 64*b^3*c^5*(56*c^2*C - 60*B*c*d + 65
*A*d^2) + 3*a^2*b*c*d^4*(376*c^2*C - 453*B*c*d + 598*A*d^2) - 6*a*b^2*c^3*
d^2*(928*c^2*C - 1024*B*c*d + 1157*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b
]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*S
qrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(195*a^3*C*d^6 + 9*a^2*b*d^4*
(68*c^2*C - 71*B*c*d + 65*A*d^2) - 64*b^3*c^4*(56*c^2*C - 60*B*c*d + 65*A*
d^2) + 6*a*b^2*c^2*d^2*(480*c^2*C - 544*B*c*d + 637*A*d^2))*Sqrt[(Sqrt[b]*
(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSi...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2609 vs. $2(894) = 1788$.

Time = 9.05 (sec) , antiderivative size = 2610, normalized size of antiderivative = 2.64

method	result	size
risch	Expression too large to display	2610
elliptic	Expression too large to display	3813
default	Expression too large to display	6620

input
```
int(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-2/45045/b^2*(3003*C*b^3*d^6*x^6+3465*B*b^3*d^6*x^5-3234*C*b^3*c*d^5*x^5+4
095*A*b^3*d^6*x^4-3780*B*b^3*c*d^5*x^4-4641*C*a*b^2*d^6*x^4+3528*C*b^3*c^2
*d^4*x^4-4550*A*b^3*c*d^5*x^3-5775*B*a*b^2*d^6*x^3+4200*B*b^3*c^2*d^4*x^3+
5208*C*a*b^2*c*d^5*x^3-3920*C*b^3*c^3*d^3*x^3-7605*A*a*b^2*d^6*x^2+5200*A*
b^3*c^2*d^4*x^2+6690*B*a*b^2*c*d^5*x^2-4800*B*b^3*c^3*d^3*x^2+468*C*a^2*b*
d^6*x^2-6036*C*a*b^2*c^2*d^4*x^2+4480*C*b^3*c^4*d^2*x^2+9308*A*a*b^2*c*d^5
*x-6240*A*b^3*c^3*d^3*x+924*B*a^2*b*d^6*x-8196*B*a*b^2*c^2*d^4*x+5760*B*b^
3*c^4*d^2*x-696*C*a^2*b*c*d^5*x+7400*C*a*b^2*c^3*d^3*x-5376*C*b^3*c^5*d*x+
2340*A*a^2*b*d^6-12844*A*a*b^2*c^2*d^4+8320*A*b^3*c^4*d^2-1632*B*a^2*b*c*d
^5+11328*B*a*b^2*c^3*d^3-7680*B*b^3*c^5*d+780*C*a^3*d^6+1284*C*a^2*b*c^2*d
^4-10240*C*a*b^2*c^4*d^2+7168*C*b^3*c^6)/d^7*(-b*x^2+a)^(1/2)*(d*x+c)^(1/2
)+4/45045/b^2/d^7*(-(1794*A*a^2*b*c*d^6-6942*A*a*b^2*c^3*d^4+4160*A*b^3*c^
5*d^2-693*B*a^3*d^7-1359*B*a^2*b*c^2*d^5+6144*B*a*b^2*c^4*d^3-3840*B*b^3*c
^6*d+444*C*a^3*c*d^6+1128*C*a^2*b*c^3*d^4-5568*C*a*b^2*c^5*d^2+3584*C*b^3*
c^7)*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*EllipticE(1/2*
2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1
/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(
a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))+195*...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 916, normalized size of antiderivative = 0.93

$$\int \frac{x^2(a-bx^2)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
-2/135135*(4*(3584*C*b^4*c^8 - 3840*B*b^4*c^7*d + 9024*B*a*b^3*c^5*d^3 - 5
157*B*a^2*b^2*c^3*d^5 - 531*B*a^3*b*c*d^7 - 64*(129*C*a*b^3 - 65*A*b^4)*c^
6*d^2 + 6*(758*C*a^2*b^2 - 1677*A*a*b^3)*c^4*d^4 + 3*(121*C*a^3*b + 2041*A
*a^2*b^2)*c^2*d^6 + 585*(C*a^4 + 3*A*a^3*b)*d^8)*sqrt(-b*d)*weierstrassPIn
verse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/
3*(3*d*x + c)/d) + 12*(3584*C*b^4*c^7*d - 3840*B*b^4*c^6*d^2 + 6144*B*a*b^
3*c^4*d^4 - 1359*B*a^2*b^2*c^2*d^6 - 693*B*a^3*b*d^8 - 64*(87*C*a*b^3 - 65
*A*b^4)*c^5*d^3 + 6*(188*C*a^2*b^2 - 1157*A*a*b^3)*c^3*d^5 + 6*(74*C*a^3*b
 + 299*A*a^2*b^2)*c*d^7)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)
) + 3*(3003*C*b^4*d^8*x^6 + 7168*C*b^4*c^6*d^2 - 7680*B*b^4*c^5*d^3 + 1132
8*B*a*b^3*c^3*d^5 - 1632*B*a^2*b^2*c*d^7 - 640*(16*C*a*b^3 - 13*A*b^4)*c^4
*d^4 + 4*(321*C*a^2*b^2 - 3211*A*a*b^3)*c^2*d^6 + 780*(C*a^3*b + 3*A*a^2*b
^2)*d^8 - 231*(14*C*b^4*c*d^7 - 15*B*b^4*d^8)*x^5 + 21*(168*C*b^4*c^2*d^6
- 180*B*b^4*c*d^7 - 13*(17*C*a*b^3 - 15*A*b^4)*d^8)*x^4 - 7*(560*C*b^4*c^3
*d^5 - 600*B*b^4*c^2*d^6 + 825*B*a*b^3*d^8 - 2*(372*C*a*b^3 - 325*A*b^4)*c
*d^7)*x^3 + (4480*C*b^4*c^4*d^4 - 4800*B*b^4*c^3*d^5 + 6690*B*a*b^3*c*d^7
- 4*(1509*C*a*b^3 - 1300*A*b^4)*c^2*d^6 + 117*(4*C*a^2*b^2 - 65*A*a*b^3)*d
^8)*x^2 - 4*(1344*C*b^4*c^5*d^3 - 1440*B*b^4*c^4*d^4 + 2049*B*a*b^3*c^2...
```

Sympy [F]

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x^2(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2*(-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(x**2*(a - b*x**2)**(3/2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*x^2/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}x^2}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*x^2/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x^2\,(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int((x^2*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x^2(-b\,x^2+a)^{\frac{3}{2}}(C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int(x^2*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.177 $\int \frac{x(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 831

$$\int \frac{x(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx =$$

$$-\frac{4(9a^2d^4(71cC-65Bd)-32b^2c^3(120c^2C-130Bcd+143Ad^2)+3abcd^2(1088c^2C-1274Bcd+1573Ad}{45045bd^6}$$

$$+\frac{4(231a^2Cd^4-8b^2c^2(120c^2C-130Bcd+143Ad^2)+abd^2(666c^2C-793Bcd+1001Ad^2))\,x\sqrt{c+dx}\sqrt{a}}{15015bd^5}$$

$$-\frac{2(3ad^2(58cC-39Bd)+8bc(120c^2C-130Bcd+143Ad^2))\sqrt{c+dx}(a-bx^2)^{3/2}}{9009bd^4}$$

$$+\frac{2\left(143Ab+33aC+\frac{10bc(12cC-13Bd)}{d^2}\right)x\sqrt{c+dx}(a-bx^2)^{3/2}}{1287bd}$$

$$+\frac{2(23cC-13Bd)\sqrt{c+dx}(a-bx^2)^{5/2}}{143bd^2}-\frac{2C(c+dx)^{3/2}(a-bx^2)^{5/2}}{13bd^2}$$

$$-\frac{8\sqrt{a}(693a^3Cd^6+32b^3c^4(120c^2C-130Bcd+143Ad^2)+3a^2bd^4(453c^2C-598Bcd+1001Ad^2)-3ab^2c}{45045b^{3/2}d^7\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a}}$$

$$-\frac{8\sqrt{a}(bc^2-ad^2)(9a^2d^4(71cC-65Bd)-32b^2c^3(120c^2C-130Bcd+143Ad^2)+3abcd^2(1088c^2C-1274}{45045b^{3/2}d^7\sqrt{c+dx}\sqrt{a-bx}}$$

output
```
-4/45045*(9*a^2*d^4*(-65*B*d+71*C*c)-32*b^2*c^3*(143*A*d^2-130*B*c*d+120*C
*c^2)+3*a*b*c*d^2*(1573*A*d^2-1274*B*c*d+1088*C*c^2))*(d*x+c)^(1/2)*(-b*x^
2+a)^(1/2)/b/d^6+4/15015*(231*a^2*C*d^4-8*b^2*c^2*(143*A*d^2-130*B*c*d+120
*C*c^2)+a*b*d^2*(1001*A*d^2-793*B*c*d+666*C*c^2))*x*(d*x+c)^(1/2)*(-b*x^2+
a)^(1/2)/b/d^5-2/9009*(3*a*d^2*(-39*B*d+58*C*c)+8*b*c*(143*A*d^2-130*B*c*d
+120*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d^4+2/1287*(143*A*b+33*a*C+1
0*b*c*(-13*B*d+12*C*c)/d^2)*x*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/b/d+2/143*(-1
3*B*d+23*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(5/2)/b/d^2-2/13*C*(d*x+c)^(3/2)*(-
b*x^2+a)^(5/2)/b/d^2-8/45045*a^(1/2)*(693*a^3*C*d^6+32*b^3*c^4*(143*A*d^2-
130*B*c*d+120*C*c^2)+3*a^2*b*d^4*(1001*A*d^2-598*B*c*d+453*C*c^2)-3*a*b^2*
c^2*d^2*(2717*A*d^2-2314*B*c*d+2048*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(
1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/
(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^7/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^
(1/2)/(-b*x^2+a)^(1/2)-8/45045*a^(1/2)*(-a*d^2+b*c^2)*(9*a^2*d^4*(-65*B*d+
71*C*c)-32*b^2*c^3*(143*A*d^2-130*B*c*d+120*C*c^2)+3*a*b*c*d^2*(1573*A*d^2
-1274*B*c*d+1088*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)
/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^7/(d*x+c)^(1/2)/(-b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.54 (sec) , antiderivative size = 980, normalized size of antiderivative = 1.18

$$\int \frac{x(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(b*(c + d*x)*(-12*a^2*d^4*(-136*c*C + 195*B*d + 77*C*d*
x) + a*b*d^2*(-11328*c^3*C + 4*c^2*d*(3211*B + 2049*C*x) - 2*c*d^2*(7579*A
 + 4654*B*x + 3345*C*x^2) + d^3*x*(11011*A + 7605*B*x + 5775*C*x^2)) + b^2
*(7680*c^5*C - 640*c^4*d*(13*B + 9*C*x) + 32*c^3*d^2*(286*A + 15*x*(13*B +
 10*C*x)) - 35*d^5*x^3*(143*A + 9*x*(13*B + 11*C*x)) - 8*c^2*d^3*x*(858*A
+ 25*x*(26*B + 21*C*x)) + 10*c*d^4*x^2*(572*A + 7*x*(65*B + 54*C*x)))) - (
4*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(693*a^3*C*d^6 + 32*b^3*c^4*(120*c^2
*C - 130*B*c*d + 143*A*d^2) + 3*a^2*b*d^4*(453*c^2*C - 598*B*c*d + 1001*A*
d^2) - 3*a*b^2*c^2*d^2*(2048*c^2*C - 2314*B*c*d + 2717*A*d^2))*(a - b*x^2)
 + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(693*a^3*C*d^6 + 32*b^3*c^4*(120*c^2*
C - 130*B*c*d + 143*A*d^2) + 3*a^2*b*d^4*(453*c^2*C - 598*B*c*d + 1001*A*d
^2) - 3*a*b^2*c^2*d^2*(2048*c^2*C - 2314*B*c*d + 2717*A*d^2))*Sqrt[(d*(Sqr
t[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x)
)]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt
[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*S
qrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(693*a^(5/2)*C*d^5 + 9*a^2*Sqrt[b]*d^4*(7
1*c*C - 65*B*d) - 32*b^(5/2)*c^3*(120*c^2*C - 130*B*c*d + 143*A*d^2) - 24*
Sqrt[a]*b^2*c^2*d*(120*c^2*C - 130*B*c*d + 143*A*d^2) + 3*a^(3/2)*b*d^3*(6
66*c^2*C - 793*B*c*d + 1001*A*d^2) + 3*a*b^(3/2)*c*d^2*(1088*c^2*C - 1274*
B*c*d + 1573*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-((...
```

Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 795, normalized size of antiderivative = 0.96, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {2185, 27, 2185, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{\left(a-bx^2\right)^{3/2}\left(-b(23cC-13Bd)x^2d^2+3acCd^2-\left(10bCc^2-13Abd^2-3aCd^2\right)xd\right)}{2\sqrt{c+dx}}dx}{13bd^3}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(a-bx^2)^{3/2}\left(-b(23cC-13Bd)x^2d^2+3acCd^2-\left(10bCc^2-13Abd^2-3aCd^2\right)xd\right)}{\sqrt{c+dx}}dx}{13bd^3}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{2}{11}d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}(23cC-13Bd)-\frac{2\int -\frac{bd^3\left(ad(10cC+13Bd)+\left(33aCd^2+b\left(120Cc^2-130Bdc+143Ad^2\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{2\sqrt{c+dx}}dx}{11bd^2}}{13bd^3}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{11}d\int \frac{\left(ad(10cC+13Bd)+\left(33aCd^2+b\left(120Cc^2-130Bdc+143Ad^2\right)\right)x\right)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx+\frac{2}{11}d\left(a-bx^2\right)^{5/2}\sqrt{c+dx}(23cC-13Bd)}{13bd^3}-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{\frac{1}{11}d\left(-\frac{4\int -\frac{b\left(ad\left(3ad^2(19cC+39Bd)-bc\left(120Cc^2-130Bdc+143Ad^2\right)\right)+\left(9abcd^2(10cC+13Bd)-\left(8bc^2-7ad^2\right)\left(33aCd^2+b\left(120Cc^2-130Bdc+143Ad^2\right)\right)\right)x\right)}{2\sqrt{c+dx}}}{21bd^2}\right.}{}$$

$$-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{11}d\left(\frac{2\int \frac{\left(ad\left(3ad^2(19cC+39Bd)-bc\left(120Cc^2-130Bdc+143Ad^2\right)\right)+\left(9abcd^2(10cC+13Bd)-\left(8bc^2-7ad^2\right)\left(33aCd^2+b\left(120Cc^2-130Bdc+143Ad^2\right)\right)\right)x\right)\sqrt{a-b}}{\sqrt{c+dx}}}{21d^2}\right.}{}$$

$$-\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$$\Big\downarrow \; 682$$

$$\frac{1}{11}d\left(\frac{2\left(-\frac{4\int -\frac{b\left(ad\left(9a^2(6cC+65Bd)d^4-3abc\left(422Cc^2-481Bdc+572Ad^2\right)d^2+8b^2c^3\left(120Cc^2-130Bdc+143Ad^2\right)\right)+\left(693a^3Cd^6+3a^2b\left(453Cc^2-598Bdc+10\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{15bd^2}\right.}{}\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 27

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\int \frac{ad\left(9a^2(6cC+65Bd)d^4-3abc\left(422Cc^2-481Bdc+572Ad^2\right)d^2+8b^2c^3\left(120Cc^2-130Bdc+143Ad^2\right)\right)+\left(693a^3Cd^6+3a^2b\left(453Cc^2-598Bdc+1001Ad^2\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\right.}{}\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 600

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(9a^2d^4(71cC-65Bd)+3abcd^2\left(1573Ad^2-1274Bcd+1088c^2C\right)-32b^2c^3\left(143Ad^2-130Bcd+120c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\left(693a^3C\right.}{}\right.}{15d^2}\right.}{}\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 509

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(9a^2d^4(71cC-65Bd)+3abcd^2\left(1573Ad^2-1274Bcd+1088c^2C\right)-32b^2c^3\left(143Ad^2-130Bcd+120c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}}{\cdots}}{15\cdots}\right)}{\cdots}\right)}{\cdots}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 508

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(9a^2d^4(71cC-65Bd)+3abcd^2\left(1573Ad^2-1274Bcd+1088c^2C\right)-32b^2c^3\left(143Ad^2-130Bcd+120c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-}}{}\right.}{}\right.}{}\right.}{}$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\downarrow$ 327

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\left(bc^2-ad^2\right)\left(9a^2d^4(71cC-65Bd)+3abcd^2\left(1573Ad^2-1274Bcd+1088c^2C\right)-32b^2c^3\left(143Ad^2-130Bcd+120c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\cdots}}{\cdots}}{\cdots}}{\cdots}\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\big\downarrow$ 512

$$\frac{1}{11}d\left(\frac{2\left(\frac{2\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(9a^2d^4(71cC-65Bd)+3abcd^2\left(1573Ad^2-1274Bcd+1088c^2C\right)-32b^2c^3\left(143Ad^2-130Bcd+120c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\cdots}{\cdots}}{\cdots}\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}(c+dx)^{3/2}}{13bd^2}$$

$\Big\downarrow$ 511

$$\frac{\frac{2}{11}d(23cC-13Bd)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+\frac{1}{11}d\left(\frac{2\left(\frac{2\left(-\frac{2\sqrt{a}\left(693a^3Cd^6+3a^2b\left(453Cc^2-598Bdc+1001Ad^2\right)d^4-3ab^2c^2\left(2048Cc^2-2314B\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2C(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13bd^2}$$

$\Big\downarrow$ 321

$$\frac{\frac{2}{11}d(23cC-13Bd)\sqrt{c+dx}\left(a-bx^2\right)^{5/2}+\frac{1}{11}d\left(\frac{2\left(\frac{2\left(-\frac{2\sqrt{a}\left(693a^3Cd^6+3a^2b\left(453Cc^2-598Bdc+1001Ad^2\right)d^4-3ab^2c^2\left(2048Cc^2-2314Bc\cdots\right.\right.}{\sqrt{\cdots}}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{2C(c+dx)^{3/2}\left(a-bx^2\right)^{5/2}}{13bd^2}$$

input
```
Int[(x*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*(c + d*x)^(3/2)*(a - b*x^2)^(5/2))/(13*b*d^2) + ((2*d*(23*c*C - 13*B
*d)*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/11 + (d*((-2*Sqrt[c + d*x]*(3*a*d^2*(
58*c*C - 39*B*d) + 8*b*c*(120*c^2*C - 130*B*c*d + 143*A*d^2) - 7*d*(33*a*C
*d^2 + b*(120*c^2*C - 130*B*c*d + 143*A*d^2))*x)*(a - b*x^2)^(3/2))/(63*d^
2) + (2*((-2*Sqrt[c + d*x]*(9*a^2*d^4*(71*c*C - 65*B*d) - 32*b^2*c^3*(120*
c^2*C - 130*B*c*d + 143*A*d^2) + 3*a*b*c*d^2*(1088*c^2*C - 1274*B*c*d + 15
73*A*d^2) - 3*d*(9*a*b*c*d^2*(10*c*C + 13*B*d) - (8*b*c^2 - 7*a*d^2)*(33*a
*C*d^2 + b*(120*c^2*C - 130*B*c*d + 143*A*d^2)))*x)*Sqrt[a - b*x^2])/(15*d
^2) + (2*((-2*Sqrt[a]*(693*a^3*C*d^6 + 32*b^3*c^4*(120*c^2*C - 130*B*c*d +
 143*A*d^2) + 3*a^2*b*d^4*(453*c^2*C - 598*B*c*d + 1001*A*d^2) - 3*a*b^2*c
^2*d^2*(2048*c^2*C - 2314*B*c*d + 2717*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sq
rt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(9*a^2*d^4*(71*c*
C - 65*B*d) - 32*b^2*c^3*(120*c^2*C - 130*B*c*d + 143*A*d^2) + 3*a*b*c*d^2
*(1088*c^2*C - 1274*B*c*d + 1573*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]
*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)
/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c +
d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(21*d^2)))/11)/(13*b*d^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2093 vs. $2(741) = 1482$.

Time = 6.65 (sec) , antiderivative size = 2094, normalized size of antiderivative = 2.52

method	result	size
risch	Expression too large to display	2094
elliptic	Expression too large to display	2346
default	Expression too large to display	5767

input
```
int(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/45045/b*(3465*C*b^2*d^5*x^5+4095*B*b^2*d^5*x^4-3780*C*b^2*c*d^4*x^4+500
5*A*b^2*d^5*x^3-4550*B*b^2*c*d^4*x^3-5775*C*a*b*d^5*x^3+4200*C*b^2*c^2*d^3
*x^3-5720*A*b^2*c*d^4*x^2-7605*B*a*b*d^5*x^2+5200*B*b^2*c^2*d^3*x^2+6690*C
*a*b*c*d^4*x^2-4800*C*b^2*c^3*d^2*x^2-11011*A*a*b*d^5*x+6864*A*b^2*c^2*d^3
*x+9308*B*a*b*c*d^4*x-6240*B*b^2*c^3*d^2*x+924*C*a^2*d^5*x-8196*C*a*b*c^2*
d^3*x+5760*C*b^2*c^4*d*x+15158*A*a*b*c*d^4-9152*A*b^2*c^3*d^2+2340*B*a^2*d
^5-12844*B*a*b*c^2*d^3+8320*B*b^2*c^4*d-1632*C*a^2*c*d^4+11328*C*a*b*c^3*d
^2-7680*C*b^2*c^5)/d^6*(-b*x^2+a)^(1/2)*(d*x+c)^(1/2)-4/45045/b/d^6*(-(300
3*A*a^2*b*d^6-8151*A*a*b^2*c^2*d^4+4576*A*b^3*c^4*d^2-1794*B*a^2*b*c*d^5+6
942*B*a*b^2*c^3*d^3-4160*B*b^3*c^5*d+693*C*a^3*d^6+1359*C*a^2*b*c^2*d^4-61
44*C*a*b^2*c^4*d^2+3840*C*b^3*c^6)/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)^(1/
2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-1/b*
(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((c/d
-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2)
)^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-c/d*EllipticF(1/2*
2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1
/b*(a*b)^(1/2)))^(1/2)))-585*B*a^3*d^6/b*(a*b)^(1/2)*2^(1/2)*((x+1/b*(a*b)
^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(-2*(x-
1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*E
llipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 738, normalized size of antiderivative = 0.89

$$\int \frac{x(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
2/135135*(4*(3840*C*b^3*c^7 - 4160*B*b^3*c^6*d + 10062*B*a*b^2*c^4*d^3 - 6
123*B*a^2*b*c^2*d^5 - 1755*B*a^3*d^7 - 32*(282*C*a*b^2 - 143*A*b^3)*c^5*d^
2 + 27*(191*C*a^2*b - 429*A*a*b^2)*c^3*d^4 + 3*(177*C*a^3 + 2717*A*a^2*b)*
c*d^6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 12*(3840*C*b^3*c^6*d -
4160*B*b^3*c^5*d^2 + 6942*B*a*b^2*c^3*d^4 - 1794*B*a^2*b*c*d^6 - 32*(192*C
*a*b^2 - 143*A*b^3)*c^4*d^3 + 3*(453*C*a^2*b - 2717*A*a*b^2)*c^2*d^5 + 231
*(3*C*a^3 + 13*A*a^2*b)*d^7)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d
^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b
*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c
)/d)) - 3*(3465*C*b^3*d^7*x^5 - 7680*C*b^3*c^5*d^2 + 8320*B*b^3*c^4*d^3 -
12844*B*a*b^2*c^2*d^5 + 2340*B*a^2*b*d^7 + 64*(177*C*a*b^2 - 143*A*b^3)*c^
3*d^4 - 2*(816*C*a^2*b - 7579*A*a*b^2)*c*d^6 - 315*(12*C*b^3*c*d^6 - 13*B*
b^3*d^7)*x^4 + 35*(120*C*b^3*c^2*d^5 - 130*B*b^3*c*d^6 - 11*(15*C*a*b^2 -
13*A*b^3)*d^7)*x^3 - 5*(960*C*b^3*c^3*d^4 - 1040*B*b^3*c^2*d^5 + 1521*B*a*
b^2*d^7 - 2*(669*C*a*b^2 - 572*A*b^3)*c*d^6)*x^2 + (5760*C*b^3*c^4*d^3 - 6
240*B*b^3*c^3*d^4 + 9308*B*a*b^2*c*d^6 - 12*(683*C*a*b^2 - 572*A*b^3)*c^2*
d^5 + 77*(12*C*a^2*b - 143*A*a*b^2)*d^7)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c)
)/(b^2*d^8)
```

Sympy [F]

$$\int \frac{x(a-bx^2)^{3/2}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}\,dx=\int \frac{x(a-bx^2)^{\frac{3}{2}}\left(A+Bx+Cx^2\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(x*(-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral(x*(a - b*x**2)**(3/2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{x(a-bx^2)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}x}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*x/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{x(a-bx^2)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}x}{\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)*x/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a-bx^2)^{3/2}\,(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x\,(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int((x*(a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{x(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{x(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int(x*(-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.178 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 692

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \frac{4(45a^2Cd^4-4b^2c^2(80c^2C-88Bcd+99Ad^2)+3abd^2(98c^2C-121 \cdots}{3465bd^5}$$
$$-\frac{4(ad^2(61cC-77Bd)-bc(80c^2C-88Bcd+99Ad^2))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{1155d^4}$$
$$+\frac{2\left(99Ab+9aC+\frac{8bc(10cC-11Bd)}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{693bd}$$
$$-\frac{2(10cC-11Bd)x\sqrt{c+dx}(a-bx^2)^{3/2}}{99d^2}-\frac{2C\sqrt{c+dx}(a-bx^2)^{5/2}}{11bd}$$
$$+\frac{8\sqrt{a}(3a^2d^4(46cC-77Bd)+4b^2c^3(80c^2C-88Bcd+99Ad^2)-3abcd^2(178c^2C-209Bcd+264Ad^2))\sqrt{\cdots}}{3465\sqrt{b}d^6\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{8\sqrt{a}(bc^2-ad^2)(45a^2Cd^4-4b^2c^2(80c^2C-88Bcd+99Ad^2)+3abd^2(98c^2C-121Bcd+165Ad^2))\sqrt{\frac{c+\cdots}{c+\cdots}}}{3465b^{3/2}d^6\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
4/3465*(45*a^2*C*d^4-4*b^2*c^2*(99*A*d^2-88*B*c*d+80*C*c^2)+3*a*b*d^2*(165
*A*d^2-121*B*c*d+98*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^5-4/1155*(a
*d^2*(-77*B*d+61*C*c)-b*c*(99*A*d^2-88*B*c*d+80*C*c^2))*x*(d*x+c)^(1/2)*(-
b*x^2+a)^(1/2)/d^4+2/693*(99*A*b+9*a*C+8*b*c*(-11*B*d+10*C*c)/d^2)*(d*x+c)
^(1/2)*(-b*x^2+a)^(3/2)/b/d-2/99*(-11*B*d+10*C*c)*x*(d*x+c)^(1/2)*(-b*x^2+
a)^(3/2)/d^2-2/11*C*(d*x+c)^(1/2)*(-b*x^2+a)^(5/2)/b/d+8/3465*a^(1/2)*(3*a
^2*d^4*(-77*B*d+46*C*c)+4*b^2*c^3*(99*A*d^2-88*B*c*d+80*C*c^2)-3*a*b*c*d^2
*(264*A*d^2-209*B*c*d+178*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/b^(1/2)/d^6/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*
x^2+a)^(1/2)+8/3465*a^(1/2)*(-a*d^2+b*c^2)*(45*a^2*C*d^4-4*b^2*c^2*(99*A*d
^2-88*B*c*d+80*C*c^2)+3*a*b*d^2*(165*A*d^2-121*B*c*d+98*C*c^2))*((d*x+c)/(
c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
b^(3/2)/d^6/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.36 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.20

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \frac{2\sqrt{a-bx^2}\left(-((c+dx)(180a^2Cd^4-abd^2(988c^2C-2cd(583B+3\right.}{}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(2*Sqrt[a - b*x^2]*(-((c + d*x)*(180*a^2*C*d^4 - a*b*d^2*(988*c^2*C - 2*c*
d*(583*B + 358*C*x) + d^2*(1485*A + 847*B*x + 585*C*x^2)) + b^2*(640*c^4*C
 - 32*c^3*d*(22*B + 15*C*x) + 8*c^2*d^2*(99*A + 66*B*x + 50*C*x^2) + 5*d^4
*x^2*(99*A + 7*x*(11*B + 9*C*x)) - 2*c*d^3*x*(297*A + 5*x*(44*B + 35*C*x))
))) + (4*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*d^4*(46*c*C - 77*B*d)
+ 4*b^2*c^3*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*a*b*c*d^2*(178*c^2*C - 20
9*B*c*d + 264*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a
^2*d^4*(46*c*C - 77*B*d) + 4*b^2*c^3*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*
a*b*c*d^2*(178*c^2*C - 209*B*c*d + 264*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] +
x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3
/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sq
rt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*d*(Sqrt[b]*c - S
qrt[a]*d)*(45*a^2*C*d^4 + 3*a^(3/2)*Sqrt[b]*d^3*(61*c*C - 77*B*d) - 4*b^2*
c^2*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*Sqrt[a]*b^(3/2)*c*d*(80*c^2*C - 8
8*B*c*d + 99*A*d^2) + 3*a*b*d^2*(98*c^2*C - 121*B*c*d + 165*A*d^2))*Sqrt[(
d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c
+ d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d^2*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(3465*b*d^5*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 662, normalized size of antiderivative = 0.96, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2185, 27, 682, 27, 682, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{\sqrt{c+dx}} \, dx$$

$$\Big\downarrow \ 2185$$

$$-\frac{2 \int -\frac{d((11Ab+aC)d-b(10cC-11Bd)x)\left(a-bx^2\right)^{3/2}}{2\sqrt{c+dx}}dx}{11bd^2}-\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{((11Ab+aC)d-b(10cC-11Bd)x)(a-bx^2)^{3/2}}{\sqrt{c+dx}}dx}{11bd}-\frac{2C(a-bx^2)^{5/2}\sqrt{c+dx}}{11bd}$$

$\big\downarrow$ 682

$$\frac{\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}(9aCd^2+b(99Ad^2-88Bcd+80c^2C)-7bdx(10cC-11Bd))}{63d^2}-\frac{4\int -\frac{b\left(ad\left(9aCd^2+b\left(10Cc^2-11Bdc+99Ad^2\right)\right)-b\left(ad^2(61cC-77Bc\right.\right.}{2\sqrt{c+dx}}}{21bd^2}}{11bd}$$

$$\frac{2C(a-bx^2)^{5/2}\sqrt{c+dx}}{11bd}$$

$\big\downarrow$ 27

$$\frac{\frac{2\int \frac{\left(ad\left(9aCd^2+b\left(10Cc^2-11Bdc+99Ad^2\right)\right)-b\left(ad^2(61cC-77Bd)-bc\left(80Cc^2-88Bdc+99Ad^2\right)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}+\frac{2(a-bx^2)^{3/2}\sqrt{c+dx}(9aCd^2+b(99Ad^2-}{63}}{11bd}$$

$$\frac{2C(a-bx^2)^{5/2}\sqrt{c+dx}}{11bd}$$

$\big\downarrow$ 682

$$2\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(45a^2Cd^4-3bdx\left(ad^2(61cC-77Bd)-bc\left(99Ad^2-88Bcd+80c^2C\right)\right)+3abd^2\left(165Ad^2-121Bcd+98c^2C\right)-4b^2c^2\left(99Ad^2-88Bcd+80c^2C\right)\right)}{15d^2}-\frac{4}{}\right.$$

$$\frac{2C(a-bx^2)^{5/2}\sqrt{c+dx}}{11bd}$$

$\big\downarrow$ 27

$$2\left(\frac{2\int \frac{ad\left(45a^2Cd^4+3ab\left(37Cc^2-44Bdc+165Ad^2\right)d^2-b^2c^2\left(80Cc^2-88Bdc+99Ad^2\right)\right)-b\left(3a^2(46cC-77Bd)d^4-3abc\left(178Cc^2-209Bdc+264Ad^2\right)d^2+4b^2c^3\left(80C\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{15d^2}\right.$$

$$\frac{2C(a-bx^2)^{5/2}\sqrt{c+dx}}{11bd}$$

$\big\downarrow$ 600

$$2\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(45a^2Cd^4+3abd^2\left(165Ad^2-121Bcd+98c^2C\right)-4b^2c^2\left(99Ad^2-88Bcd+80c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(3a^2d^4(46cC-77Bd)-3abcd^2\left(264A\cdots\right.\right.}{\cdots}\cdots\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \text{509}$$

$$2\left(\frac{2\left(-\frac{\left(bc^2-ad^2\right)\left(45a^2Cd^4+3abd^2\left(165Ad^2-121Bcd+98c^2C\right)-4b^2c^2\left(99Ad^2-88Bcd+80c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(46cC-77Bd)-3ab\cdots\right.}{\cdots}\cdots\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \text{508}$$

$$2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(46cC-77Bd)-3abcd^2\left(264Ad^2-209Bcd+178c^2C\right)+4b^2c^3\left(99Ad^2-88Bcd+80c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(\cdots\right.}{\cdots}\cdots\right.}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \; 327$$

$$2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(46cC-77Bd)-3abcd^2\left(264Ad^2-209Bcd+178c^2C\right)+4b^2c^3\left(99Ad^2-88Bcd+80c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\cdots\right)}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \; 512$$

$$2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(46cC-77Bd)-3abcd^2\left(264Ad^2-209Bcd+178c^2C\right)+4b^2c^3\left(99Ad^2-88Bcd+80c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\cdots\right)}{15d^2}\cdots\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$$\Big\downarrow \; 511$$

$$2\left(2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(45a^2Cd^4+3abd^2\left(165Ad^2-121Bcd+98c^2C\right)-4b^2c^2\left(99Ad^2-88Bcd+80c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)\cdots}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots\right.\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

$\downarrow$ 321

$$2\left(2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(45a^2Cd^4+3abd^2\left(165Ad^2-121Bcd+98c^2C\right)-4b^2c^2\left(99Ad^2-88Bcd+80c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots\right.\right.$$

$$\frac{2C\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}{11bd}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/Sqrt[c + d*x],x]
```

output
```
(-2*C*Sqrt[c + d*x]*(a - b*x^2)^(5/2))/(11*b*d) + ((2*Sqrt[c + d*x]*(9*a*C
*d^2 + b*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 7*b*d*(10*c*C - 11*B*d)*x)*(a
- b*x^2)^(3/2))/(63*d^2) + (2*((2*Sqrt[c + d*x]*(45*a^2*C*d^4 - 4*b^2*c^2*
(80*c^2*C - 88*B*c*d + 99*A*d^2) + 3*a*b*d^2*(98*c^2*C - 121*B*c*d + 165*A
*d^2) - 3*b*d*(a*d^2*(61*c*C - 77*B*d) - b*c*(80*c^2*C - 88*B*c*d + 99*A*d
^2))*x)*Sqrt[a - b*x^2]])/(15*d^2) + (2*((2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(46*
c*C - 77*B*d) + 4*b^2*c^3*(80*c^2*C - 88*B*c*d + 99*A*d^2) - 3*a*b*c*d^2*(
178*c^2*C - 209*B*c*d + 264*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Elli
pticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sq
rt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a -
 b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(45*a^2*C*d^4 - 4*b^2*c^2*(80*c^2*C
- 88*B*c*d + 99*A*d^2) + 3*a*b*d^2*(98*c^2*C - 121*B*c*d + 165*A*d^2))*Sqr
t[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipti
cF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[
a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(15*d^2)))/(21*d^2))
/(11*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1539 vs. $2(608) = 1216$.

Time = 6.66 (sec) , antiderivative size = 1540, normalized size of antiderivative = 2.23

method	result	size
elliptic	Expression too large to display	1540
risch	Expression too large to display	1781
default	Expression too large to display	4856

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/11*C*b/d*x^4
*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(B*b^2-10/11*C*b^2/d*c)/b/d*x^3*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(A*b^2-13/11*C*b*a-8/9*(B*b^2-10/11*C
*b^2/d*c)/d*c)/b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-2*B*a*b+8/
11*C*b/d*a*c+7/9*(B*b^2-10/11*C*b^2/d*c)/b*a-6/7*(A*b^2-13/11*C*b*a-8/9*(B
*b^2-10/11*C*b^2/d*c)/d*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2
/3*(-2*A*a*b+a^2*C+2/3*(B*b^2-10/11*C*b^2/d*c)/b/d*a*c+5/7*(A*b^2-13/11*C*
b*a-8/9*(B*b^2-10/11*C*b^2/d*c)/d*c)/b*a-4/5*(-2*B*a*b+8/11*C*b/d*a*c+7/9*
(B*b^2-10/11*C*b^2/d*c)/b*a-6/7*(A*b^2-13/11*C*b*a-8/9*(B*b^2-10/11*C*b^2/
d*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A*a^2+2/5*(-
2*B*a*b+8/11*C*b/d*a*c+7/9*(B*b^2-10/11*C*b^2/d*c)/b*a-6/7*(A*b^2-13/11*C*
b*a-8/9*(B*b^2-10/11*C*b^2/d*c)/d*c)/d*c)/b/d*a*c+1/3*(-2*A*a*b+a^2*C+2/3*
(B*b^2-10/11*C*b^2/d*c)/b/d*a*c+5/7*(A*b^2-13/11*C*b*a-8/9*(B*b^2-10/11*C*
b^2/d*c)/d*c)/b*a-4/5*(-2*B*a*b+8/11*C*b/d*a*c+7/9*(B*b^2-10/11*C*b^2/d*c)
/b*a-6/7*(A*b^2-13/11*C*b*a-8/9*(B*b^2-10/11*C*b^2/d*c)/d*c)/d*c)/d*c)/b*a
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+2*(B*a^2+4/7*(A*b^2-13/11*C*b*a-8/9*(B*b^2-10/11*C*b^2/d*c)/d*c)/...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 608, normalized size of antiderivative = 0.88

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx =$$

$$-\frac{2\left(4\left(320\,Cb^3c^6-352\,Bb^3c^5d+891\,Bab^2c^3d^3-627\,Ba^2bcd^5-18\,(43\,Cab^2-22\,Ab^3)c^4d^2+3\,(157\,Ca^2b\right.\right.}{}$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="frica
s")
```

output
```
-2/10395*(4*(320*C*b^3*c^6 - 352*B*b^3*c^5*d + 891*B*a*b^2*c^3*d^3 - 627*B
*a^2*b*c*d^5 - 18*(43*C*a*b^2 - 22*A*b^3)*c^4*d^2 + 3*(157*C*a^2*b - 363*A
*a*b^2)*c^2*d^4 + 135*(C*a^3 + 11*A*a^2*b)*d^6)*sqrt(-b*d)*weierstrassPInv
erse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3
*(3*d*x + c)/d) + 12*(320*C*b^3*c^5*d - 352*B*b^3*c^4*d^2 + 627*B*a*b^2*c^
2*d^4 - 231*B*a^2*b*d^6 - 6*(89*C*a*b^2 - 66*A*b^3)*c^3*d^3 + 6*(23*C*a^2*
b - 132*A*a*b^2)*c*d^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2
+ 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))
 + 3*(315*C*b^3*d^6*x^4 + 640*C*b^3*c^4*d^2 - 704*B*b^3*c^3*d^3 + 1166*B*a
*b^2*c*d^5 - 4*(247*C*a*b^2 - 198*A*b^3)*c^2*d^4 + 45*(4*C*a^2*b - 33*A*a*
b^2)*d^6 - 35*(10*C*b^3*c*d^5 - 11*B*b^3*d^6)*x^3 + 5*(80*C*b^3*c^2*d^4 -
88*B*b^3*c*d^5 - 9*(13*C*a*b^2 - 11*A*b^3)*d^6)*x^2 - (480*C*b^3*c^3*d^3 -
 528*B*b^3*c^2*d^4 + 847*B*a*b^2*d^6 - 2*(358*C*a*b^2 - 297*A*b^3)*c*d^5)*
x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*d^7)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/sqrt(c + d*x), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{\sqrt{c+dx}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/(d*x+c)^(1/2),x)
```

3.179 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 709

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx =$$

$$-\frac{4(3ad^2(11cC-15Bd)-2bc(16c^2C-18Bcd+21Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{315d^4}$$
$$+\frac{2(14aCd^2-b(16c^2C-18Bcd+21Ad^2))\,x\sqrt{c+dx}\sqrt{a-bx^2}}{105d^3}$$
$$-\frac{2(8cC-9Bd)\sqrt{c+dx}(a-bx^2)^{3/2}}{63d^2}+\frac{2Cx\sqrt{c+dx}(a-bx^2)^{3/2}}{9d}$$
$$-\frac{2\sqrt{a}(84a^2Cd^4+8b^2c^2(16c^2C-18Bcd+21Ad^2)-3abd^2(76c^2C-96Bcd+147Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E}{315\sqrt{b}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(12a^2d^4(11cC-15Bd)+8b^2c^3(16c^2C-18Bcd+21Ad^2)-abcd^2(260c^2C-324Bcd+483Ad^2))\sqrt{}}{315\sqrt{b}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{2a^2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-4/315*(3*a*d^2*(-15*B*d+11*C*c)-2*b*c*(21*A*d^2-18*B*c*d+16*C*c^2))*(d*x+
c)^(1/2)*(-b*x^2+a)^(1/2)/d^4+2/105*(14*a*C*d^2-b*(21*A*d^2-18*B*c*d+16*C*
c^2))*x*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^3-2/63*(-9*B*d+8*C*c)*(d*x+c)^(1/
2)*(-b*x^2+a)^(3/2)/d^2+2/9*C*x*(d*x+c)^(1/2)*(-b*x^2+a)^(3/2)/d-2/315*a^(
1/2)*(84*a^2*C*d^4+8*b^2*c^2*(21*A*d^2-18*B*c*d+16*C*c^2)-3*a*b*d^2*(147*A
*d^2-96*B*c*d+76*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*
(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*
d))^(1/2))/b^(1/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1
/2)+2/315*a^(1/2)*(12*a^2*d^4*(-15*B*d+11*C*c)+8*b^2*c^3*(21*A*d^2-18*B*c*
d+16*C*c^2)-a*b*c*d^2*(483*A*d^2-324*B*c*d+260*C*c^2))*((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d
^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*a^2*A*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^
(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1
/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(d*x+c)^(1/2)/(-b*x
^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.67 (sec) , antiderivative size = 1669, normalized size of antiderivative = 2.35

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*Sqrt[c + d*x]),x]
```

output
```
(2*Sqrt[c + d*x]*Sqrt[a - b*x^2]*(a*d^2*(-106*c*C + 135*B*d + 77*C*d*x) +
b*(64*c^3*C - 24*c^2*d*(3*B + 2*C*x) - d^3*x*(63*A + 5*x*(9*B + 7*C*x)) +
2*c*d^2*(42*A + x*(27*B + 20*C*x)))))/(315*d^4) + (2*(128*b^3*c^7*C*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]] - 144*b^3*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
+ 168*A*b^3*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 356*a*b^2*c^5*C*d^2*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 432*a*b^2*B*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]] - 609*a*A*b^2*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 312*a^2*b*
c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 288*a^2*b*B*c^2*d^5*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]] + 441*a^2*A*b*c*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8
4*a^3*c*C*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 256*b^3*c^6*C*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*(c + d*x) + 288*b^3*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(c + d*x) - 336*A*b^3*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)
+ 456*a*b^2*c^4*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 576*a*b^2
*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 882*a*A*b^2*c^2*d^4*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 168*a^2*b*c^2*C*d^4*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 128*b^3*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(c + d*x)^2 - 144*b^3*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^
2 + 168*A*b^3*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 228*a*b
^2*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 288*a*b^2*B*c^2*
d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 441*a*A*b^2*c*d^4*Sqrt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2} \left(A+Bx+Cx^2\right)}{x\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 2351$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx+\int \frac{(B+Cx)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 638$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx+\int \frac{(B+Cx)\left(a-bx^2\right)^{3/2}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 682$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{4\int \frac{b\left(ad(cC-9Bd)-\left(7aCd^2-bc(8cC-9Bd)\right)x\right)\sqrt{a-bx^2}}{2\sqrt{c+dx}}dx}{21bd^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 27

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\int \frac{\left(ad(cC-9Bd)-\left(7aCd^2-bc(8cC-9Bd)\right)x\right)\sqrt{a-bx^2}}{\sqrt{c+dx}}dx}{21d^2}-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 682

$$-\frac{2\left(\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-3dx\left(7aCd^2-bc(8cC-9Bd)\right)+3ad^2(11cC-15Bd)-4bc^2(8cC-9Bd)\right)}{15d^2}-\frac{4\int -\frac{b\left(ad\left(3ad^2(4cC-15Bd)-bc^2(8cC-9Bd)\right)-\right.}{2\sqrt{}}}{}\right.}{21d^2}$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 27

$$-\frac{2\left(\frac{2\int \frac{ad\left(3ad^2(4cC-15Bd)-bc^2(8cC-9Bd)\right)-\left(21a^2Cd^4-3abc(19cC-24Bd)d^2+4b^2c^3(8cC-9Bd)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{15d^2}+\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(-3dx\left(7aCd^2-bc(8cC\right.\right.}{}\right.}{21d^2}$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 600

$$-\frac{2\left(\frac{2\left(-\frac{\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(11cC-15Bd)-4bc^2(8cC-9Bd)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{15d^2}-\right.}{21d^2}$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 509

$$-\frac{2\left(\frac{2\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(11cC-15Bd)-4bc^2(8cC-9Bd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-b}}}{d}\right.}{15d^2}\right.}{21d^2}$$

$$A\int\frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 508

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(11cC-15Bd)-4bc^2(8\right.}{d}\right.}{15d^2}\right.}{21d^2}$$

$$A\int\frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 327

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(3ad^2(11cC-15Bd)-4bc^2\right.}{d}\right.}{15d^2}\right.}{21d^2}$$

$$A\int\frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 512

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(11cC-15I\right.}{d}\right.}{15d^2}\right.}{21d}$$

$$A\int\frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$\downarrow$ 511

$$-\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(11cC-15Bd)-4bc^2(8cC-9Bd)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+d}}{}\right)}{15d^2}}{}$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

$$\Big\downarrow \text{321}$$

$$A\int \frac{\left(a-bx^2\right)^{3/2}}{x\sqrt{c+dx}}dx-$$

$$\frac{2\left(\frac{2\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(21a^2Cd^4-3abcd^2(19cC-24Bd)+4b^2c^3(8cC-9Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3\right.}{}\right)}{15d^2}}{}$$

$$\frac{2\left(a-bx^2\right)^{3/2}\sqrt{c+dx}(-9Bd+8cC-7Cdx)}{63d^2}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(610) = 1220$.

Time = 3.92 (sec) , antiderivative size = 1318, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1318
default	Expression too large to display	4424

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/9*C*b/d*x^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(B*b^2-8/9*C*b^2/d*c)/b/d*x^2*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(A*b^2-11/9*C*b*a-6/7*(B*b^2-8/9*C*b^2/d
*c)/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-2*B*a*b+2/3*C*b/d*
a*c+5/7*(B*b^2-8/9*C*b^2/d*c)/b*a-4/5*(A*b^2-11/9*C*b*a-6/7*(B*b^2-8/9*C*b
^2/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(B*a^2+2/5*(A*b
^2-11/9*C*b*a-6/7*(B*b^2-8/9*C*b^2/d*c)/d*c)/b/d*a*c+1/3*(-2*B*a*b+2/3*C*b
/d*a*c+5/7*(B*b^2-8/9*C*b^2/d*c)/b*a-4/5*(A*b^2-11/9*C*b*a-6/7*(B*b^2-8/9*
C*b^2/d*c)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*
b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-2*A*a*b+a^2*C+4/7*(B*b^2-8/9*C*b^2/d*c)/
b/d*a*c+3/5*(A*b^2-11/9*C*b*a-6/7*(B*b^2-8/9*C*b^2/d*c)/d*c)/b*a-2/3*(-2*B
*a*b+2/3*C*b/d*a*c+5/7*(B*b^2-8/9*C*b^2/d*c)/b*a-4/5*(A*b^2-11/9*C*b*a-6/7
*(B*b^2-8/9*C*b^2/d*c)/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+cx}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+cx}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx=\int \frac{{\left(a-b\,x^2\right)}^{3/2}\,\left(C\,x^2+B\,x+A\right)}{x\,\sqrt{c+d\,x}} \,d x$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x\sqrt{c+dx}}\,dx=\int \frac{\left(-b\,x^{2}+a\right)^{\frac{3}{2}} \left(C\,x^{2}+Bx+A\right)}{x\sqrt{dx+c}}d x$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x/(d*x+c)^(1/2),x)
```

3.180 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 667

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \frac{2(45aCd^2-b(57c^2C-49Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{105d^3}$$
$$-\frac{aA\sqrt{c+dx}\sqrt{a-bx^2}}{cx}+\frac{2b(16cC-7Bd)(c+dx)^{3/2}\sqrt{a-bx^2}}{35d^3}$$
$$-\frac{2bC(c+dx)^{5/2}\sqrt{a-bx^2}}{7d^3}$$
$$-\frac{\sqrt{a}\sqrt{b}(3ad^2(64c^2C-98Bcd-35Ad^2)-4bc^2(24c^2C-28Bcd+35Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\right.\right.}{105cd^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{a}(120a^2Cd^4+4b^2c^2(24c^2C-28Bcd+35Ad^2)-abd^2(216c^2C-322Bcd+245Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}}{105\sqrt{b}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{a^2(2Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/105*(45*a*C*d^2-b*(35*A*d^2-49*B*c*d+57*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)
^(1/2)/d^3-a*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x+2/35*b*(-7*B*d+16*C*c)*(
d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^3-2/7*b*C*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/d
^3-1/105*a^(1/2)*b^(1/2)*(3*a*d^2*(-35*A*d^2-98*B*c*d+64*C*c^2)-4*b*c^2*(3
5*A*d^2-28*B*c*d+24*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/c/d^4/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)
-1/105*a^(1/2)*(120*a^2*C*d^4+4*b^2*c^2*(35*A*d^2-28*B*c*d+24*C*c^2)-a*b*d
^2*(245*A*d^2-322*B*c*d+216*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*
((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(
1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(d*x+c)^(1/2)/(-
b*x^2+a)^(1/2)-a^2*(-A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b
*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1
/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.91 (sec) , antiderivative size = 1658, normalized size of antiderivative = 2.49

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*Sqrt[c + d*x]),x]
```

output
```
-1/105*(Sqrt[c + d*x]*Sqrt[a - b*x^2]*(15*a*d^2*(7*A*d - 6*c*C*x) + 2*b*c*
x*(24*c^2*C - 2*c*d*(14*B + 9*C*x) + d^2*(35*A + 3*x*(7*B + 5*C*x)))))/(c*
d^3*x) - (96*b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 112*b^2*B*c^6*d*Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]] + 140*A*b^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]] - 288*a*b*c^5*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 406*a*b*B*c^4*d
^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 35*a*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]] + 192*a^2*c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 294*a^2*B*c
^2*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 105*a^2*A*c*d^6*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]] - 192*b^2*c^6*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 22
4*b^2*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 280*A*b^2*c^4*d^2
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 384*a*b*c^4*C*d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*(c + d*x) - 588*a*b*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(c + d*x) - 210*a*A*b*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*
x) + 96*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 112*b^2*B*c
^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 140*A*b^2*c^3*d^2*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 192*a*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]*(c + d*x)^2 + 294*a*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(c + d*x)^2 + 105*a*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^
2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(4*b*c^2*(24*c^2*C - 28*B*c*d + 35
*A*d^2) + 3*a*d^2*(-64*c^2*C + 98*B*c*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/S...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^2\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^2\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx}+\frac{(Bd-cC)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^2}\right)dx$$

$$\Big\downarrow \ 7293$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx}-\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^2}\right)dx$$

$$\Big\downarrow \ 7239$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)\left(a-bx^2\right)^{3/2}}{d^2x^2}dx$$

$$\Big\downarrow \ 27$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^2}dx}{d^2}$$

$$\Big\downarrow \ 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^2}dx}{d^2}$$

$$\Big\downarrow \ 7293$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^2}-\frac{Cd\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x}\right)dx}{d^2}$$

$$\Big\downarrow \ 7296$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx+\frac{2\int -\frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^2x^2}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \ 25$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-\frac{2\int \frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^2x^2}d\sqrt{c+dx}}{d^3}$$

$$\Big\downarrow \ 2011$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-\frac{2\int \frac{(c+dx)\left(a-bx^2\right)^{3/2}(2cC-(c+dx)C-Bd)}{d^2x^2}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^2x^2}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-$$

$$2\int\left(-\frac{b^2C(c+dx)^4}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{b^2(Bd-4cC)(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(2aCd^2-bc(5cC-2Bd)\right)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{b(Bd-2cC)}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b}{}}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^2\sqrt{c+dx}}dx-$$

$$2\left(\frac{bC\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}(c+dx)^{5/2}}{7d^2}-\frac{b(4cC-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}(c+dx)^{3/2}}{5d^2}+\frac{12bcC\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b}{}}}{35d^2}\right.$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 8.48 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1119
risch	Expression too large to display	3045
default	Expression too large to display	4283

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-A*a/x/c*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*C*b/d*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1
/2)-2/5*(B*b^2-6/7*C*b^2/d*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3
*(A*b^2-9/7*C*b*a-4/5*(B*b^2-6/7*C*b^2/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)+2*(-2*A*a*b+a^2*C+2/5*(B*b^2-6/7*C*b^2/d*c)/b/d*a*c+1/3*(A*b
^2-9/7*C*b*a-4/5*(B*b^2-6/7*C*b^2/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((
-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-2*B*a*b-1/2*A*a*b
*d/c+4/7*C*b/d*a*c+3/5*(B*b^2-6/7*C*b^2/d*c)/b*a-2/3*(A*b^2-9/7*C*b*a-4/5*
(B*b^2-6/7*C*b^2/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1
/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)
^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2
)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+a^2*(A*d-2*B*c)/c^2*(c/d-1/b*(a*b)^(1/2))*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d...
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(C*b*x^4 + B*b*x^3 - B*a*x - (C*a - A*b)*x^2 - A*a)*sqrt(-b*x^2
+ a)*sqrt(d*x + c)/(d*x^3 + c*x^2), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**2/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**2*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+cx^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+cx^2}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{x^2\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^2*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^2\sqrt{c+dx}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^2\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^2/(d*x+c)^(1/2),x)
```

3.181 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 664

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \frac{2b(7cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15d^2} - \frac{aA\sqrt{c+dx}\sqrt{a-bx^2}}{2cx^2} - \frac{a(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4c^2x} - \frac{2bC(c+dx)^{3/2}\sqrt{a-bx^2}}{5d^2}$$

$$+ \frac{\sqrt{a}\sqrt{b}(3ad^2(56c^2C+20Bcd-15Ad^2)-8bc^2(8c^2C-10Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-}}{}\right.\right.}{60c^2d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$- \frac{\sqrt{a}\sqrt{b}(ad^2(184c^2C-140Bcd-15Ad^2)-8bc^2(8c^2C-10Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\text{ar}\right.}{60cd^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$- \frac{a(4ac(2cC-Bd)-3A(4bc^2-ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4c^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/15*b*(-5*B*d+7*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d^2-1/2*a*A*(d*x+c)^(
1/2)*(-b*x^2+a)^(1/2)/c/x^2-1/4*a*(-3*A*d+4*B*c)*(d*x+c)^(1/2)*(-b*x^2+a)^
(1/2)/c^2/x-2/5*b*C*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/d^2+1/60*a^(1/2)*b^(1/2
)*(3*a*d^2*(-15*A*d^2+20*B*c*d+56*C*c^2)-8*b*c^2*(15*A*d^2-10*B*c*d+8*C*c^
2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/d^3/((
d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/60*a^(1/2)*b^(1/2)*
(a*d^2*(-15*A*d^2-140*B*c*d+184*C*c^2)-8*b*c^2*(15*A*d^2-10*B*c*d+8*C*c^2)
)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)
*d))^(1/2))/c/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*a*(4*a*c*(-B*d+2*C*c)
-3*A*(-a*d^2+4*b*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a
)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1
/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.32 (sec) , antiderivative size = 1990, normalized size of antiderivative = 3.00

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*Sqrt[c + d*x]),x]
```

output
```
(Sqrt[c + d*x]*Sqrt[a - b*x^2]*(15*a*d^2*(-2*A*c - 4*B*c*x + 3*A*d*x) + 8*
b*c^2*x^2*(4*c*C - 5*B*d - 3*C*d*x)))/(60*c^2*d^2*x^2) + (64*b^2*c^7*C*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]] - 80*b^2*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
] + 120*A*b^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 232*a*b*c^5*C*d^2*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 20*a*b*B*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]] - 75*a*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 168*a^2*c^3*C*d^
4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 60*a^2*B*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]] - 45*a^2*A*c*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 128*b^2*c^6*C*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 160*b^2*B*c^5*d*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]]*(c + d*x) - 240*A*b^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*(c + d*x) + 336*a*b*c^4*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) +
 120*a*b*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 90*a*A*b*c^2
*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 64*b^2*c^5*C*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 80*b^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]*(c + d*x)^2 + 120*A*b^2*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*
x)^2 - 168*a*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 60*a
*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 45*a*A*b*c*d^4*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[
a]*d)*(3*a*d^2*(-56*c^2*C - 20*B*c*d + 15*A*d^2) + 8*b*c^2*(8*c^2*C - 10*B
*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sq...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^3\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^3\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^2}+\frac{(Bd-cC)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^2}-\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)\left(a-bx^2\right)^{3/2}}{d^2x^3}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^3}dx}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^3}dx}{d^2}$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^3}-\frac{Cd\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^2}\right)dx}{d^2}$$

$$\Big\downarrow \text{7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+\frac{2\int -\frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^3x^3}d\sqrt{c+dx}}{d^2}$$

$$\Big\downarrow \text{2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+2\int -\frac{(c+dx)\left(a-bx^2\right)^{3/2}(2cC-(c+dx)C-Bd)}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\ (}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{a}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\ (}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{a}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\cdots}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+\cdots}{\cdots}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$

$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\cdots}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+\cdots}{\cdots}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$

$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\cdots}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+\cdots}{\cdots}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$

$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\left(\right.}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$

$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\left(\right.}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$

$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)\left(\right.}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^3x^3}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{b^2C(c+dx)^3}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b^2(Bd-3cC)(c+dx)^2}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\left(bc(2cC-Bd)-2aCd^2\right)}{d^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+}{}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^3\sqrt{c+dx}}dx$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 5.76 (sec) , antiderivative size = 1047, normalized size of antiderivative = 1.58

method	result	size
elliptic	Expression too large to display	1047
risch	Expression too large to display	1817
default	Expression too large to display	4804

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/2*A*a/x^2/c*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4*a*(3*A*d-4*B*c)/c^2*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)/x-2/5*C*b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(
B*b^2-4/5*C*b^2/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-2*B*a*b+1/
4*A*a*b*d/c+2/5*C*b/d*a*c+1/3*(B*b^2-4/5*C*b^2/d*c)/b*a)*(c/d-1/b*(a*b)^(1
/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b
*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(A*b^2-7/5*
C*b*a+1/8*b*d*a*(3*A*d-4*B*c)/c^2-2/3*(B*b^2-4/5*C*b^2/d*c)/d*c)*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4*a*(3*
A*a*d^2-12*A*b*c^2-4*B*a*c*d+8*C*a*c^2)/c^3*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-...
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(C*b*x^4 + B*b*x^3 - B*a*x - (C*a - A*b)*x^2 - A*a)*sqrt(-b*x^2
+ a)*sqrt(d*x + c)/(d*x^4 + c*x^3), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**3/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**3*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx=\int \frac{{\left(a-b\,x^2\right)}^{3/2}\,\left(C\,x^2+B\,x+A\right)}{x^3\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^3*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^3\sqrt{c+dx}}\,dx=\int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^3\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^3/(d*x+c)^(1/2),x)
```

3.182 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 693

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = -\frac{2bC\sqrt{c+dx}\sqrt{a-bx^2}}{3d}$$
$$-\frac{aA\sqrt{c+dx}\sqrt{a-bx^2}}{3cx^3}-\frac{a(6Bc-5Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{12c^2x^2}$$
$$-\frac{(6ac(4cC-3Bd)-A(32bc^2-15ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{24c^3x}$$
$$+\frac{\sqrt{a}\sqrt{b}(16bc^2(2c^2C-3Bcd-2Ad^2)+3ad^2(8c^2C-6Bcd+5Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{24c^3d^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{\sqrt{a}\sqrt{b}(ad^2(56c^2C+6Bcd-5Ad^2)-16bc^2(2c^2C-3Bcd+Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{24c^2d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$+\frac{a(12bc^2(2Bc-Ad)+ad(8c^2C-6Bcd+5Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{\sqrt{bc+}}\right.}{8c^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*b*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/d-1/3*a*A*(d*x+c)^(1/2)*(-b*x^2+a)
^(1/2)/c/x^3-1/12*a*(-5*A*d+6*B*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c^2/x^2-
1/24*(6*a*c*(-3*B*d+4*C*c)-A*(-15*a*d^2+32*b*c^2))*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)/c^3/x+1/24*a^(1/2)*b^(1/2)*(16*b*c^2*(-2*A*d^2-3*B*c*d+2*C*c^2)+3*
a*d^2*(5*A*d^2-6*B*c*d+8*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipt
icE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+
a^(1/2)*d))^(1/2))/c^3/d^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a
)^(1/2)+1/24*a^(1/2)*b^(1/2)*(a*d^2*(-5*A*d^2+6*B*c*d+56*C*c^2)-16*b*c^2*(
A*d^2-3*B*c*d+2*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2
)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+1
/8*a*(12*b*c^2*(-A*d+2*B*c)+a*d*(5*A*d^2-6*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(
1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^
3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.90 (sec) , antiderivative size = 2192, normalized size of antiderivative = 3.16

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*Sqrt[c + d*x]),x]
```

output

```
-1/24*(Sqrt[c + d*x]*Sqrt[a - b*x^2]*(16*b*c^2*x^2*(-2*A*d + c*C*x) + a*d*
(6*c*x*(2*B*c + 4*c*C*x - 3*B*d*x) + A*(8*c^2 - 10*c*d*x + 15*d^2*x^2))))/
(c^3*d*x^3) - (32*b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 48*b^2*B*c^6*
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 32*A*b^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]] - 8*a*b*c^5*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 30*a*b*B*c^4*d
^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 47*a*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]] - 24*a^2*c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 18*a^2*B*c^2
*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 15*a^2*A*c*d^6*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]] - 64*b^2*c^6*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 96*b^2
*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 64*A*b^2*c^4*d^2*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 48*a*b*c^4*C*d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]*(c + d*x) + 36*a*b*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c
 + d*x) - 30*a*A*b*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 32*b
^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 48*b^2*B*c^4*d*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 32*A*b^2*c^3*d^2*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]]*(c + d*x)^2 + 24*a*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*(c + d*x)^2 - 18*a*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^
2 + 15*a*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + I*Sqrt[b]*
c*(Sqrt[b]*c - Sqrt[a]*d)*(16*b*c^2*(-2*c^2*C + 3*B*c*d + 2*A*d^2) - 3*a*d
^2*(8*c^2*C - 6*B*c*d + 5*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^4\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^4\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}(a-bx^2)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^3}+\frac{(Bd-cC)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^4}\right)dx$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^3}-\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^4}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)\left(a-bx^2\right)^{3/2}}{d^2x^4}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^4}dx}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^4}dx}{d^2}$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^4}-\frac{Cd\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^3}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx+\frac{2\int -\frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^4x^4}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-\frac{2\int \frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^4x^4}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \frac{(c+dx)\left(a-bx^2\right)^{3/2}(2cC-(c+dx)C-Bd)}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2091

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int\left(-\frac{Ba^2}{d^2x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^4x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2bB}{d^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^4\sqrt{c+dx}}dx-$$
$$2d\int\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^4x^4}d\sqrt{c+dx}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 6.19 (sec) , antiderivative size = 1052, normalized size of antiderivative = 1.52

method	result	size
elliptic	Expression too large to display	1052
risch	Expression too large to display	1717
default	Expression too large to display	5353

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/3*A*a/x^3/c*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/12*a*(5*A*d-6*B*c)/c^2*(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)/x^2-1/24/c^3*(15*A*a*d^2-32*A*b*c^2-18*B*a*c*d+24*C*a
*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x-2/3*C*b/d*(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)+2*(A*b^2-5/3*C*b*a-1/24*b*d*a*(5*A*d-6*B*c)/c^2)*(c/d-1/b*(a
*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/
2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B*b^
2-1/48*b*d*(15*A*a*d^2-32*A*b*c^2-18*B*a*c*d+24*C*a*c^2)/c^3-2/3*C*b^2/d*c
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))
*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))
)+1/8*a*(5*A*a*d^3-12*A*b*c^2*d-6*B*a*c*d^2+24*B*b*c^3+8*C*a*c^2*d)/c^4*(c
/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1
/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1
/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c...
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx=\int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(C*b*x^4 + B*b*x^3 - B*a*x - (C*a - A*b)*x^2 - A*a)*sqrt(-b*x^2
+ a)*sqrt(d*x + c)/(d*x^5 + c*x^4), x)
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**4/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**4*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^4), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{x^4\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^4*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^4\sqrt{c+dx}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^4\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^4/(d*x+c)^(1/2),x)
```

3.183 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 757

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = -\frac{aA\sqrt{c+dx}\sqrt{a-bx^2}}{4cx^4} - \frac{a(8Bc-7Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{24c^2x^3} + \frac{(60Abc^2-48ac^2C+40aBcd-35aAd^2)\sqrt{c+dx}\sqrt{a-bx^2}}{96c^3x^2} + \frac{(4bc^2(64Bc-47Ad)+3ad(48c^2C-40Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{192c^4x} - \frac{\sqrt{a}\sqrt{b}(4bc^2(96c^2C+64Bcd-47Ad^2)+3ad^2(48c^2C-40Bcd+35Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{}}{}\right.\right.}{192c^4d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{\sqrt{a}\sqrt{b}(4bc^2(96c^2C-32Bcd-17Ad^2)+ad^2(48c^2C-40Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\operatorname{arcs}\right.}{192c^3d\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{(A(48b^2c^4-72abc^2d^2+35a^2d^4)+8ac(ad^2(6cC-5Bd)-12bc^2(2cC-Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticP}}{64c^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/4*a*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^4-1/24*a*(-7*A*d+8*B*c)*(d*x+c
)^(1/2)*(-b*x^2+a)^(1/2)/c^2/x^3+1/96*(-35*A*a*d^2+60*A*b*c^2+40*B*a*c*d-4
8*C*a*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c^3/x^2+1/192*(4*b*c^2*(-47*A*d+
64*B*c)+3*a*d*(35*A*d^2-40*B*c*d+48*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)
/c^4/x-1/192*a^(1/2)*b^(1/2)*(4*b*c^2*(-47*A*d^2+64*B*c*d+96*C*c^2)+3*a*d^
2*(35*A*d^2-40*B*c*d+48*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipti
cE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a
^(1/2)*d))^(1/2))/c^4/d/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(
1/2)+1/192*a^(1/2)*b^(1/2)*(4*b*c^2*(-17*A*d^2-32*B*c*d+96*C*c^2)+a*d^2*(3
5*A*d^2-40*B*c*d+48*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^
(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^3/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
-1/64*(A*(35*a^2*d^4-72*a*b*c^2*d^2+48*b^2*c^4)+8*a*c*(a*d^2*(-5*B*d+6*C*c
)-12*b*c^2*(-B*d+2*C*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a
)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a
^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.41 (sec) , antiderivative size = 2808, normalized size of antiderivative = 3.71

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*Sqrt[c + d*x]),x]
```

output
```
(-1/4*(a*A)/(c*x^4) - (a*(8*B*c - 7*A*d))/(24*c^2*x^3) + (60*A*b*c^2 - 48*
a*c^2*C + 40*a*B*c*d - 35*a*A*d^2)/(96*c^3*x^2) + (256*b*B*c^3 - 188*A*b*c
^2*d + 144*a*c^2*C*d - 120*a*B*c*d^2 + 105*a*A*d^3)/(192*c^4*x))*Sqrt[c +
d*x]*Sqrt[a - b*x^2] + (Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]
*(-384*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 256*b^2*B*c^4*d*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]] + 188*A*b^2*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
- 144*a*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 120*a*b*B*c^2*d^3*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]] - 105*a*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]] - (384*b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (256*b^2
*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (188*A*b^2*c^5*d^2*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (240*a*b*c^5*C*d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (376*a*b*B*c^4*d^3*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]])/(c + d*x)^2 - (293*a*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]])/(c + d*x)^2 + (144*a^2*c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c +
 d*x)^2 - (120*a^2*B*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 +
 (105*a^2*A*c*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (768*b^2*c
^6*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (512*b^2*B*c^5*d*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (376*A*b^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]])/(c + d*x) + (288*a*b*c^4*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
)/(c + d*x) - (240*a*b*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^5\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^4}+\frac{(Bd-cC)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^5}\right)dx$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^4}-\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^5}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)\left(a-bx^2\right)^{3/2}}{d^2x^5}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^5}dx}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^5}dx}{d^2}$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^5}-\frac{Cd\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^4}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+2\int -\frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+2d^2\int -\frac{(c+dx)\left(a-bx^2\right)^{3/2}(2cC-(c+dx)C-Bd)}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+$$
$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+$$
$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+$$
$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int\left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx+$$

$$2d^2\int-\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$2d^2\int\left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx+$$

$$2d^2\int-\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$2d^2\int\left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx+$$

$$2d^2\int-\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+$$

$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx+$$

$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx+$$
$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx+$$
$$2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^5x^5}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$2d^2\int \left(\frac{Ba^2}{d^3x^4\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{c(cC-Bd)a^2}{d^5x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bB}{d^3x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+d}{d^2}}}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{(a-bx^2)^{3/2}}{x^5\sqrt{c+dx}}dx$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 1135, normalized size of antiderivative = 1.50

method	result	size
elliptic	Expression too large to display	1135
risch	Expression too large to display	1532
default	Expression too large to display	6267

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/4*A*a/c/x^4*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/24*a*(7*A*d-8*B*c)/c^2*(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)/x^3-1/96/c^3*(35*A*a*d^2-60*A*b*c^2-40*B*a*c*d+48*C*a
*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x^2+1/192*(105*A*a*d^3-188*A*b*c^
2*d-120*B*a*c*d^2+256*B*b*c^3+144*C*a*c^2*d)/c^4*(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)/x+2*(B*b^2+1/192*b*d*(35*A*a*d^2-60*A*b*c^2-40*B*a*c*d+48*C*a*c^
2)/c^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/
b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))+2*(C*b^2+1/384*b*d*(105*A*a*d^3-188*A*b*c^2*d-120*B*a*c*d^2+25
6*B*b*c^3+144*C*a*c^2*d)/c^4)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b
)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b
*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(
1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/64*(35*A*a^2*d^4-72*A*a*b*c^2*d^2+48*A*b^2
*c^4-40*B*a^2*c*d^3+96*B*a*b*c^3*d+48*C*a^2*c^2*d^2-192*C*a*b*c^4)/c^5*(c/
d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**5/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**5*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^5}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^5), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^5}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx=\int \frac{(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{x^5\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^5*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^5\sqrt{c+dx}}\,dx=\int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^5\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^5/(d*x+c)^(1/2),x)
```

3.184 $\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 893

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx =$$

$$-\frac{aA\sqrt{c+dx}\sqrt{a-bx^2}}{5cx^5}-\frac{a(10Bc-9Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{40c^2x^4}$$

$$-\frac{(10ac(8cC-7Bd)-A(96bc^2-63ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{240c^3x^3}$$

$$+\frac{(12bc^2(50Bc-41Ad)+5ad(80c^2C-70Bcd+63Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{960c^4x^2}$$

$$-\frac{(3A(128b^2c^4-516abc^2d^2+315a^2d^4)-10ac(4bc^2(64cC-47Bd)-15ad^2(8cC-7Bd)))\sqrt{c+dx}\sqrt{a-}}{1920ac^5x}$$

$$+\frac{\sqrt{b}(3A(128b^2c^4-516abc^2d^2+315a^2d^4)-10ac(4bc^2(64cC-47Bd)-15ad^2(8cC-7Bd)))\sqrt{c+dx}\sqrt{\frac{a}{}}}{1920\sqrt{a}c^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{b}(A(384b^2c^4-564abc^2d^2+315a^2d^4)+10ac(5ad^2(8cC-7Bd)+4bc^2(32cC+17Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-b}{a}}}{1920\sqrt{a}c^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(48b^2c^4(2Bc-Ad)+24abc^2d(8c^2C-6Bcd+5Ad^2)-a^2d^3(80c^2C-70Bcd+63Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}}{128c^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/5*a*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c/x^5-1/40*a*(-9*A*d+10*B*c)*(d*x+
c)^(1/2)*(-b*x^2+a)^(1/2)/c^2/x^4-1/240*(10*a*c*(-7*B*d+8*C*c)-A*(-63*a*d^
2+96*b*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/c^3/x^3+1/960*(12*b*c^2*(-41*A
*d+50*B*c)+5*a*d*(63*A*d^2-70*B*c*d+80*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a)^(1
/2)/c^4/x^2-1/1920*(3*A*(315*a^2*d^4-516*a*b*c^2*d^2+128*b^2*c^4)-10*a*c*(
4*b*c^2*(-47*B*d+64*C*c)-15*a*d^2*(-7*B*d+8*C*c)))*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)/a/c^5/x+1/1920*b^(1/2)*(3*A*(315*a^2*d^4-516*a*b*c^2*d^2+128*b^2*c
^4)-10*a*c*(4*b*c^2*(-47*B*d+64*C*c)-15*a*d^2*(-7*B*d+8*C*c)))*(d*x+c)^(1/
2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c^5/((d*x+c)/(c+a
^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/1920*b^(1/2)*(A*(315*a^2*d^4-5
64*a*b*c^2*d^2+384*b^2*c^4)+10*a*c*(5*a*d^2*(-7*B*d+8*C*c)+4*b*c^2*(17*B*d
+32*C*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/a^(1/2)/c^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/128*(48*
b^2*c^4*(-A*d+2*B*c)+24*a*b*c^2*d*(5*A*d^2-6*B*c*d+8*C*c^2)-a^2*d^3*(63*A*
d^2-70*B*c*d+80*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.78 (sec) , antiderivative size = 3014, normalized size of antiderivative = 3.38

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*Sqrt[c + d*x]),x]
```

output

```
(Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-384*A*b^2*c^4*x^4 + 4*a*b*c^2*x^2*(10*c*x
*(30*B*c + 64*c*C*x - 47*B*d*x) + 3*A*(64*c^2 - 82*c*d*x + 129*d^2*x^2)) -
 a^2*(A*(384*c^4 - 432*c^3*d*x + 504*c^2*d^2*x^2 - 630*c*d^3*x^3 + 945*d^4
*x^4) + 10*c*x*(8*c*C*x*(8*c^2 - 10*c*d*x + 15*d^2*x^2) + B*(48*c^3 - 56*c
^2*d*x + 70*c*d^2*x^2 - 105*d^3*x^3)))))/(1920*a*c^5*x^5) - (384*A*b^3*c^7
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 2560*a*b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]] + 1880*a*b^2*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 1932*a*A*b^2
*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 3760*a^2*b*c^5*C*d^2*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]] - 2930*a^2*b*B*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
+ 2493*a^2*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 1200*a^3*c^3*C*d^4
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 1050*a^3*B*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]] - 945*a^3*A*c*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 768*A*b^3*c^6
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 5120*a*b^2*c^6*C*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*(c + d*x) - 3760*a*b^2*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(c + d*x) + 3096*a*A*b^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c +
 d*x) - 2400*a^2*b*c^4*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 21
00*a^2*b*B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 1890*a^2*A*b
*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 384*A*b^3*c^5*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 2560*a*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(c + d*x)^2 + 1880*a*b^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/2}\left(A+Bx+Cx^2\right)}{x^6\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^5}+\frac{(Bd-cC)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^6}\right)dx$$

$\Big\downarrow$ 7293

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\int \left(\frac{C\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{dx^5}-\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{d^2x^6}\right)dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)\left(a-bx^2\right)^{3/2}}{d^2x^6}dx$$

$\Big\downarrow$ 27

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^6}dx}{d^2}$$

$\Big\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)\left(a-bx^2\right)^{3/2}}{x^6}dx}{d^2}$$

$\Big\downarrow$ 7293

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^6}-\frac{Cd\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}{x^5}\right)dx}{d^2}$$

$\Big\downarrow$ 7296

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx+2d\int -\frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^6x^6}d\sqrt{c+dx}$$

$\Big\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d\int \frac{(c+dx)\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)(2cC-(c+dx)C-Bd)}{d^6x^6}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \frac{(c+dx)\left(a-bx^2\right)^{3/2}(2cC-(c+dx)C-Bd)}{d^6x^6}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c-}{d}}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c-}{d}}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c}{}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c}{}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c}{}}}\right.$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c-}{d}}}\right.$$

$\big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\big\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c-}{d}}}\right.$$

$\big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\big\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c-}{d}}}\right.$$

$\big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

$\big\downarrow$ 2248

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \left(-\frac{Ba^2}{d^4x^5\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC-Bd)a^2}{d^6x^6\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{2b}{d^4x^3\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c}{d}}}\right.$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{\left(a-bx^2\right)^{3/2}}{x^6\sqrt{c+dx}}dx-$$
$$2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}{d^6x^6}d\sqrt{c+dx}$$

input
```
Int[((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*Sqrt[c + d*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 10.72 (sec) , antiderivative size = 1288, normalized size of antiderivative = 1.44

method	result	size
elliptic	Expression too large to display	1288
risch	Expression too large to display	1865
default	Expression too large to display	7291

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/5*A*a/c/x^5*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/40*a*(9*A*d-10*B*c)/c^2*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)/x^4-1/240/c^3*(63*A*a*d^2-96*A*b*c^2-70*B*a*c*d+80*C
*a*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x^3+1/960*(315*A*a*d^3-492*A*b*
c^2*d-350*B*a*c*d^2+600*B*b*c^3+400*C*a*c^2*d)/c^4*(-b*d*x^3-b*c*x^2+a*d*x
+a*c)^(1/2)/x^2-1/1920/c^5/a*(945*A*a^2*d^4-1548*A*a*b*c^2*d^2+384*A*b^2*c
^4-1050*B*a^2*c*d^3+1880*B*a*b*c^3*d+1200*C*a^2*c^2*d^2-2560*C*a*b*c^4)*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(C*b^2-1/1920*b*d*(315*A*a*d^3-492*A*
b*c^2*d-350*B*a*c*d^2+600*B*b*c^3+400*C*a*c^2*d)/c^4)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-1/1920*(945*A*a^
2*d^4-1548*A*a*b*c^2*d^2+384*A*b^2*c^4-1050*B*a^2*c*d^3+1880*B*a*b*c^3*d+1
200*C*a^2*c^2*d^2-2560*C*a*b*c^4)*b*d/a/c^5*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/
b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx = \int \frac{(a-bx^2)^{\frac{3}{2}}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx$$

input
```
integrate((-b*x**2+a)**(3/2)*(C*x**2+B*x+A)/x**6/(d*x+c)**(1/2),x)
```

output
```
Integral((a - b*x**2)**(3/2)*(A + B*x + C*x**2)/(x**6*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^6), x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx = \int \frac{(Cx^2+Bx+A)(-bx^2+a)^{\frac{3}{2}}}{\sqrt{dx+c}x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*(-b*x^2 + a)^(3/2)/(sqrt(d*x + c)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx=\int \frac{(a-b\,x^2)^{3/2}\,(C\,x^2+B\,x+A)}{x^6\,\sqrt{c+d\,x}}\,dx$$

input
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)^(1/2)),x)
```

output
```
int(((a - b*x^2)^(3/2)*(A + B*x + C*x^2))/(x^6*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/2}(A+Bx+Cx^2)}{x^6\sqrt{c+dx}}\,dx=\int \frac{(-b\,x^2+a)^{\frac{3}{2}}\,(C\,x^2+Bx+A)}{x^6\sqrt{dx+c}}dx$$

input
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x)
```

output
```
int((-b*x^2+a)^(3/2)*(C*x^2+B*x+A)/x^6/(d*x+c)^(1/2),x)
```

3.185 $\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 594

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2(3ad^2(37cC-25Bd)+bc(187c^2C-171Bcd+147Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{315b^2d^4} - \frac{2(49aCd^2+b(233c^2C-144Bcd+63Ad^2))(c+dx)^{3/2}\sqrt{a-bx^2}}{315b^2d^4} + \frac{2(29cC-9Bd)(c+dx)^{5/2}\sqrt{a-bx^2}}{63bd^4} - \frac{2C(c+dx)^{7/2}\sqrt{a-bx^2}}{9bd^4} - \frac{2\sqrt{a}(147a^2Cd^4+8b^2c^2(16c^2C-18Bcd+21Ad^2)+3abd^2(36c^2C-44Bcd+63Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx}{a}}}{315b^{5/2}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(3a^2d^4(37cC-25Bd)+8b^2c^3(16c^2C-18Bcd+21Ad^2)+abcd^2(76c^2C-96Bcd+147Ad^2))\sqrt{}}{315b^{5/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/315*(3*a*d^2*(-25*B*d+37*C*c)+b*c*(147*A*d^2-171*B*c*d+187*C*c^2))*(d*x+
c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^4-2/315*(49*a*C*d^2+b*(63*A*d^2-144*B*c*d+
233*C*c^2))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d^4+2/63*(-9*B*d+29*C*c)*(d
*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^4-2/9*C*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/b/
d^4-2/315*a^(1/2)*(147*a^2*C*d^4+8*b^2*c^2*(21*A*d^2-18*B*c*d+16*C*c^2)+3*
a*b*d^2*(63*A*d^2-44*B*c*d+36*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*E
llipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^5/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/
(-b*x^2+a)^(1/2)+2/315*a^(1/2)*(3*a^2*d^4*(-25*B*d+37*C*c)+8*b^2*c^3*(21*A
*d^2-18*B*c*d+16*C*c^2)+a*b*c*d^2*(147*A*d^2-96*B*c*d+76*C*c^2))*((d*x+c)/
(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))
/b^(5/2)/d^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.57 (sec) , antiderivative size = 738, normalized size of antiderivative = 1.24

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(-147a^2Cd^4-8b^2c^2(16c^2C-18Bcd+21Ad^2)-3abd^2(36c^2C-44Bcd+63Ad^2)+b(c+\cdots\right.}{\cdots}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-147*a^2*C*d^4 - 8*b^2*c^2*(16*c^2*C - 18*B*c*d + 21*A
*d^2) - 3*a*b*d^2*(36*c^2*C - 44*B*c*d + 63*A*d^2) + b*(c + d*x)*(a*d^2*(6
2*c*C - 75*B*d - 49*C*d*x) + b*(64*c^3*C - 24*c^2*d*(3*B + 2*C*x) - d^3*x*
(63*A + 5*x*(9*B + 7*C*x)) + 2*c*d^2*(42*A + x*(27*B + 20*C*x)))) + (I*Sqr
t[b]*(Sqrt[b]*c - Sqrt[a]*d)*(147*a^2*C*d^4 + 8*b^2*c^2*(16*c^2*C - 18*B*c
*d + 21*A*d^2) + 3*a*b*d^2*(36*c^2*C - 44*B*c*d + 63*A*d^2))*Sqrt[(d*(Sqrt
[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))
]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) + (I*Sqrt[a]*Sqrt[b]*(147*a^2*C*d^4 -
 3*a^(3/2)*Sqrt[b]*d^3*(12*c*C + 25*B*d) + 8*b^2*c^2*(16*c^2*C - 18*B*c*d
+ 21*A*d^2) - 2*Sqrt[a]*b^(3/2)*c*d*(16*c^2*C - 18*B*c*d + 21*A*d^2) + 3*a
*b*d^2*(36*c^2*C - 44*B*c*d + 63*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
 + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*El
lipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*
c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(-a + b*x^2))))/(315*b^3*d^4*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 3.67 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2185, 27, 2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2185}$$

$$-\frac{2\int -\frac{-bd^4(29cC-9Bd)x^4-d^3\left(33bCc^2-9Abd^2-7aCd^2\right)x^3-3cCd^2\left(5bc^2-7ad^2\right)x^2-c^2Cd\left(2bc^2-21ad^2\right)x+7ac^3Cd^2}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{9bd^5}-\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-bd^4(29cC-9Bd)x^4-d^3(33bCc^2-9Abd^2-7aCd^2)x^3-3cCd^2(5bc^2-7ad^2)x^2-c^2Cd(2bc^2-21ad^2)x+7ac^3Cd^2}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{9bd^5} - \frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd) - \frac{2\int \frac{-b\left(49aCd^2+b\left(233Cc^2-144Bdc+63Ad^2\right)\right)x^3d^7-b\left(b(214cC-99Bd)c^2+ad^2(2cC+45Bd)\right)x^2d^6+3}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{9bd^5} - \frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd) - \frac{\int \frac{-b\left(49aCd^2+b\left(233Cc^2-144Bdc+63Ad^2\right)\right)x^3d^7-b\left(b(214cC-99Bd)c^2+ad^2(2cC+45Bd)\right)x^2d^6+3a}{\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{9bd^5} - \frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd) - \frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right) - \frac{2\int \frac{3\left(-b^2\left(3a(37cC-25Bd)d^2\right.\right.}{}}{}}{}}{} - \frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 27

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd) - \frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right) - \frac{3\int \frac{-b^2\left(3a(37cC-25Bd)d^2+\right.}{}}{}}{}}{} - \frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 2185

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{2}{3}bd^8\sqrt{a-bx^2}\sqrt{c+dx}\left(3ad^2\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\big\downarrow$ 27

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\int\frac{ad\left(3a(12cC+25B\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\big\downarrow$ 600

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(\frac{\left(147a^2Cd^4+3abd^2\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\big\downarrow$ 509

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(147a^2C\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\big\downarrow$ 508

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c\cdots}}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 327

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(-\frac{\left(3a^2d^4(37cC-25\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 512

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4\cdots\right.}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 511

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\cdots}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 321

$$\frac{\frac{2}{7}d\sqrt{a-bx^2}(c+dx)^{5/2}(29cC-9Bd)-\frac{\frac{2}{5}d^5\sqrt{a-bx^2}(c+dx)^{3/2}\left(49aCd^2+b\left(63Ad^2-144Bcd+233c^2C\right)\right)-\frac{3\left(\frac{1}{3}bd^8\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\cdots}{\cdots}\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*C*(c + d*x)^(7/2)*Sqrt[a - b*x^2])/(9*b*d^4) + ((2*d*(29*c*C - 9*B*d)*
(c + d*x)^(5/2)*Sqrt[a - b*x^2])/7 - ((2*d^5*(49*a*C*d^2 + b*(233*c^2*C -
144*B*c*d + 63*A*d^2))*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 - (3*((2*b*d^8*(
3*a*d^2*(37*c*C - 25*B*d) + b*c*(187*c^2*C - 171*B*c*d + 147*A*d^2))*Sqrt[
c + d*x]*Sqrt[a - b*x^2])/3 + (b*d^8*((-2*Sqrt[a]*(147*a^2*C*d^4 + 8*b^2*c
^2*(16*c^2*C - 18*B*c*d + 21*A*d^2) + 3*a*b*d^2*(36*c^2*C - 44*B*c*d + 63*
A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[
b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt
[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a
]*(3*a^2*d^4*(37*c*C - 25*B*d) + 8*b^2*c^3*(16*c^2*C - 18*B*c*d + 21*A*d^2
) + a*b*c*d^2*(76*c^2*C - 96*B*c*d + 147*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/
(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sq
rt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*S
qrt[c + d*x]*Sqrt[a - b*x^2])))/3))/(5*b*d^3))/(7*b*d^4))/(9*b*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && !(EqQ[d, 0] &&
True) && !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 6.40 (sec) , antiderivative size = 974, normalized size of antiderivative = 1.64

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2C\,x^3\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{9bd}-\frac{2\left(B-\frac{8cC}{9d}\right)x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7bd}-\frac{2\left(A+\frac{7aC}{9b}-\frac{6\left(B-\frac{8cC}{9d}\right)c}{7d}\right)x\sqrt{-bd\,x^3-}}{5bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/9*C/b/d*x^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(B-8/9*c*C/d)/b/d*x^2*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)-2/5*(A+7/9*a*C/b-6/7*(B-8/9*c*C/d)/d*c)/b/d*x*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(2/3*C/b/d*a*c+5/7*(B-8/9*c*C/d)/b*a-4/5*
(A+7/9*a*C/b-6/7*(B-8/9*c*C/d)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)+2*(2/5*(A+7/9*a*C/b-6/7*(B-8/9*c*C/d)/d*c)/b/d*a*c+1/3*(2/3*C/b/d*a*
c+5/7*(B-8/9*c*C/d)/b*a-4/5*(A+7/9*a*C/b-6/7*(B-8/9*c*C/d)/d*c)/d*c)/b*a)*
(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^
(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1
/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2))+2*(4/7*(B-8/9*c*C/d)/b/d*a*c+3/5*(A+7/9*a*C/b-6/7*(B-8/9*c*C/d)/d*c)/b
*a-2/3*(2/3*C/b/d*a*c+5/7*(B-8/9*c*C/d)/b*a-4/5*(A+7/9*a*C/b-6/7*(B-8/9*c*
C/d)/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/
2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/
d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2\left((128\,Cb^2c^5-144\,Bb^2c^4d-24\,Babc^2d^3-225\,Ba^2d^5+12\,(Cab+14\,Ab^2)c^3d^2+3\,(13\,Ca^2+21\,Aab)c\right.}{}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
2/945*((128*C*b^2*c^5 - 144*B*b^2*c^4*d - 24*B*a*b*c^2*d^3 - 225*B*a^2*d^5
 + 12*(C*a*b + 14*A*b^2)*c^3*d^2 + 3*(13*C*a^2 + 21*A*a*b)*c*d^4)*sqrt(-b*
d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(128*C*b^2*c^4*d - 144*B*b^2*c^3*d^2
 - 132*B*a*b*c*d^4 + 12*(9*C*a*b + 14*A*b^2)*c^2*d^3 + 21*(7*C*a^2 + 9*A*a
*b)*d^5)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(
b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d
^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(35*C*b^2*
d^5*x^3 - 64*C*b^2*c^3*d^2 + 72*B*b^2*c^2*d^3 + 75*B*a*b*d^5 - 2*(31*C*a*b
 + 42*A*b^2)*c*d^4 - 5*(8*C*b^2*c*d^4 - 9*B*b^2*d^5)*x^2 + (48*C*b^2*c^2*d
^3 - 54*B*b^2*c*d^4 + 7*(7*C*a*b + 9*A*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(
d*x + c))/(b^3*d^6)
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{x^3\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 378*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*b*d**3 - 294*sqrt(c + d*x)*sqr
t(a - b*x**2)*a**2*c*d**3 - 252*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d*
*2*x - 36*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*c*d**2 + 32*sqrt(c + d*x)*
sqrt(a - b*x**2)*a*b*c**3*d - 196*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*
d**2*x + 216*sqrt(c + d*x)*sqrt(a - b*x**2)*b**3*c**2*d*x - 180*sqrt(c + d
*x)*sqrt(a - b*x**2)*b**3*c*d**2*x**2 - 192*sqrt(c + d*x)*sqrt(a - b*x**2)
*b**2*c**4*x + 160*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*d*x**2 - 140*s
qrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d**2*x**3 - 567*int((sqrt(c + d*x)
*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b**2*d
**4 - 441*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**
2 - b*d*x**3),x)*a**2*b*c*d**4 - 504*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x
**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c**2*d**2 + 396*int((sq
rt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*
a*b**3*c*d**3 - 324*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x
 - b*c*x**2 - b*d*x**3),x)*a*b**2*c**3*d**2 + 432*int((sqrt(c + d*x)*sqrt(
a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**4*c**3*d - 384
*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x
**3),x)*b**3*c**5 + 189*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x
- b*c*x**2 - b*d*x**3),x)*a**3*b*d**4 + 147*int((sqrt(c + d*x)*sqrt(a - b*
x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*c*d**4 + 252*int((sq...
```

3.186 $\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 493

$$\begin{aligned}
&\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx \\
&= -\frac{2(25aCd^2+b(57c^2C-49Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{105b^2d^3} \\
&\quad+\frac{2(16cC-7Bd)(c+dx)^{3/2}\sqrt{a-bx^2}}{35bd^3}-\frac{2C(c+dx)^{5/2}\sqrt{a-bx^2}}{7bd^3} \\
&\quad+\frac{2\sqrt{a}(ad^2(44cC-63Bd)+2bc(24c^2C-28Bcd+35Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{\ldots}}{\sqrt{b}c\ldots}\right.}{105b^{3/2}d^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad-\frac{2\sqrt{a}(25a^2Cd^4+abd^2(32c^2C-49Bcd+35Ad^2)+2b^2c^2(24c^2C-28Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{E}\ldots}{105b^{5/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-2/105*(25*a*C*d^2+b*(35*A*d^2-49*B*c*d+57*C*c^2))*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)/b^2/d^3+2/35*(-7*B*d+16*C*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b/d^3-
2/7*C*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^3+2/105*a^(1/2)*(a*d^2*(-63*B*d+4
4*C*c)+2*b*c*(35*A*d^2-28*B*c*d+24*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1
/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^4/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(
1/2)/(-b*x^2+a)^(1/2)-2/105*a^(1/2)*(25*a^2*C*d^4+a*b*d^2*(35*A*d^2-49*B*c
*d+32*C*c^2)+2*b^2*c^2*(35*A*d^2-28*B*c*d+24*C*c^2))*((d*x+c)/(c+a^(1/2)*d
/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^4
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.95 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.25

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(ad^2(44cC-63Bd)+2bc(24c^2C-28Bcd+35Ad^2)-(c+dx)\left(25aCd^2+b(24c^2C-2cd(\right.\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(a*d^2*(44*c*C - 63*B*d) + 2*b*c*(24*c^2*C - 28*B*c*d +
 35*A*d^2) - (c + d*x)*(25*a*C*d^2 + b*(24*c^2*C - 2*c*d*(14*B + 9*C*x) +
d^2*(35*A + 3*x*(7*B + 5*C*x)))) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d
^2*(44*c*C - 63*B*d) + 2*b*c*(24*c^2*C - 28*B*c*d + 35*A*d^2))*Sqrt[(d*(Sq
rt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x
))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqr
t[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[a]*(25*a^(3/2)*C*d^3 + a*
Sqrt[b]*d^2*(44*c*C - 63*B*d) + 2*b^(3/2)*c*(24*c^2*C - 28*B*c*d + 35*A*d^
2) + Sqrt[a]*b*d*(-12*c^2*C + 14*B*c*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[
b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*
x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]]
, (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]*(-a + b*x^2))))/(105*b^2*d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{-bd^3(16cC-7Bd)x^3-d^2(11bCc^2-7Abd^2-5aCd^2)x^2-2cCd(bc^2-5ad^2)x+5ac^2Cd^2}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{7bd^4}-\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-bd^3(16cC-7Bd)x^3-d^2(11bCc^2-7Abd^2-5aCd^2)x^2-2cCd(bc^2-5ad^2)x+5ac^2Cd^2}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{7bd^4}-\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2\int\frac{-b\left(25aCd^2+b\left(57Cc^2-49Bdc+35Ad^2\right)\right)x^2d^5+abc(23cC-21Bd)d^5-b\left(2b(11cC-7Bd)c^2+ad^2(2cC\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4}}{}\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\int\frac{-b\left(25aCd^2+b\left(57Cc^2-49Bdc+35Ad^2\right)\right)x^2d^5+abc(23cC-21Bd)d^5-b\left(2b(11cC-7Bd)c^2+ad^2(2cC\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4}\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{2\int\frac{bd^6\left(ad\left(25aCd^2-b\left(12Cc^2-14B\right.\right.\right.}{}}{5bd^3}}{7bd^4}\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 27

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\int\frac{ad\left(25aCd^2-b\left(12Cc^2-14Bc\right.\right.}{}}{5bd^3}}{7bd^4}\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 600

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{\left(25a^2Cd^4+abd^2\left(35Ad^2-49\right.\right.}{}\right.}{7bd^4}}{}\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 509

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{\left(25a^2Cd^4+abd^2\left(35Ad^2-49\right.\right.}{\cdots}\right.}{7bd^4}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\big\downarrow$ 508

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{\left(25a^2Cd^4+abd^2\left(35Ad^2-49\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\big\downarrow$ 327

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{\left(25a^2Cd^4+abd^2\left(35Ad^2-49\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\big\downarrow$ 512

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(25a^2Cd^4+abd^2\left(3\right.\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\big\downarrow$ 511

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2\right.}{}\right.}{}}{}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{2}{5}d\sqrt{a-bx^2}(c+dx)^{3/2}(16cC-7Bd)-\frac{\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}\left(25aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)-\frac{1}{3}d^4\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2\right.}{}\right.}{}}{}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*C*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b*d^3) + ((2*d*(16*c*C - 7*B*d)*
(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 - ((2*d^4*(25*a*C*d^2 + b*(57*c^2*C - 4
9*B*c*d + 35*A*d^2))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 - (d^4*((2*Sqrt[a]*S
qrt[b]*(a*d^2*(44*c*C - 63*B*d) + 2*b*c*(24*c^2*C - 28*B*c*d + 35*A*d^2))*
Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sq
rt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d
*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(25*a^2*C*d^4
 + a*b*d^2*(32*c^2*C - 49*B*c*d + 35*A*d^2) + 2*b^2*c^2*(24*c^2*C - 28*B*c*
d + 35*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 -
(b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/
((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3)
/(5*b*d^3))/(7*b*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 6.03 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2C\,x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7bd}-\frac{2\left(B-\frac{6cC}{7d}\right)x\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5bd}-\frac{2\left(A+\frac{5aC}{7b}-\frac{4\left(B-\frac{6cC}{7d}\right)c}{5d}\right)\sqrt{-bd\,x^3-bc}}{3bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/7*C/b/d*x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(B-6/7*c*C/d)/b/d*x*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)-2/3*(A+5/7*a*C/b-4/5*(B-6/7*c*C/d)/d*c)/b/d*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)+2*(2/5*(B-6/7*c*C/d)/b/d*a*c+1/3*(A+5/7*a*C/b-4/5
*(B-6/7*c*C/d)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(4/7*C/b/d*a*c+3/5*(B-6/7*c*C/d)/b*a-2/3*(A
+5/7*a*C/b-4/5*(B-6/7*c*C/d)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*
b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 0.77

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\left((48\,Cb^2c^4-56\,Bb^2c^3d-21\,Babcd^3+2\,(4\,Cab+35\,Ab^2)c^2d^2+15\,(5\,Ca^2+7\,Aab)d^4)\sqrt{-bd}\text{weiers}\right.}{}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2/315*((48*C*b^2*c^4 - 56*B*b^2*c^3*d - 21*B*a*b*c*d^3 + 2*(4*C*a*b + 35*
A*b^2)*c^2*d^2 + 15*(5*C*a^2 + 7*A*a*b)*d^4)*sqrt(-b*d)*weierstrassPInvers
e(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3
*d*x + c)/d) + 3*(48*C*b^2*c^3*d - 56*B*b^2*c^2*d^2 - 63*B*a*b*d^4 + 2*(22
*C*a*b + 35*A*b^2)*c*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)
/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^
2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d
)) + 3*(15*C*b^2*d^4*x^2 + 24*C*b^2*c^2*d^2 - 28*B*b^2*c*d^3 + 5*(5*C*a*b
+ 7*A*b^2)*d^4 - 3*(6*C*b^2*c*d^3 - 7*B*b^2*d^4)*x)*sqrt(-b*x^2 + a)*sqrt(
d*x + c))/(b^3*d^5)
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x^2\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x^2(C\,x^2+Bx+A)}{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.187 $\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 398

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2(7cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15bd^2} - \frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5bd^2}$$
$$-\frac{2\sqrt{a}\left(15Ab+9aC+\frac{2bc(4cC-5Bd)}{d^2}\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15b^{3/2}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(ad^2(7cC-5Bd)+bc(8c^2C-10Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{15b^{3/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/15*(-5*B*d+7*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^2-2/5*C*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/b/d^2-2/15*a^(1/2)*(15*A*b+9*a*C+2*b*c*(-5*B*d+4*C*c)/
d^2)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d
/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2/15*a^(1/2)*(a*d^
2*(-5*B*d+7*C*c)+b*c*(15*A*d^2-10*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^3/(d
*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.61 (sec) , antiderivative size = 536, normalized size of antiderivative = 1.35

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-9aCd^2-b(8c^2C-10Bcd+15Ad^2)+b(c+dx)(4cC-5Bd-3Cdx)+\frac{i\sqrt{b}\left(\sqrt{b}c-\sqrt{a}d\right)(9aC}{}\right.}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-9*a*C*d^2 - b*(8*c^2*C - 10*B*c*d + 15*A*d^2) + b*(c
+ d*x)*(4*c*C - 5*B*d - 3*C*d*x) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(9*a
*C*d^2 + b*(8*c^2*C - 10*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))
/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)
*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[
b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]*(-a + b*x^2)) + (I*Sqrt[a]*Sqrt[b]*(9*a*C*d^2 - Sqrt[a]*Sqrt[b]*d*(2
*c*C + 5*B*d) + b*(8*c^2*C - 10*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b
] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x
)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]],
 (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(-a + b*x^2))))/(15*b^2*d^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{-b(7cC-5Bd)x^2d^2+3acCd^2-(2bCc^2-5Abd^2-3aCd^2)xd}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-b(7cC-5Bd)x^2d^2+3acCd^2-(2bCc^2-5Abd^2-3aCd^2)xd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{2185}$$

$$\frac{\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(7cC-5Bd)-\frac{2\int -\frac{bd^3\left(ad(2cC+5Bd)+\left(9aCd^2+b\left(8Cc^2-10Bdc+15Ad^2\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{3}d\int \frac{ad(2cC+5Bd)+(9aCd^2+b(8Cc^2-10Bdc+15Ad^2))x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(7cC-5Bd)}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{1}{3}d\left(\frac{(9aCd^2+b(15Ad^2-10Bcd+8c^2C))\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{(ad^2(7cC-5Bd)+bc(15Ad^2-10Bcd+8c^2C))\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)+\frac{2}{3}d\sqrt{a-b}}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{1}{3}d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(9aCd^2+b\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)+\frac{2}{3}c}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$\Big\downarrow$ 508

$$\frac{\frac{1}{3}d\left(-\frac{\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9aCd^2+b\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{\sqrt{1-\frac{d(}{}}}{\sqrt{\frac{1}{2}\left(\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$\Big\downarrow$ 327

$$\frac{\frac{1}{3}d\left(-\frac{\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9aCd^2+b\left(15Ad^2-10Bcd+8c^2C\right)\right)E\left(\arcsin\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$\Big\downarrow$ 512

$$\frac{\frac{1}{3}d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9aCd^2+b\left(15Ad^2-10Bcd+8c^2C\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$\Big\downarrow$ 511

$$\frac{\frac{1}{3}d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c}\cdots}{\cdots}\right)}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$\Big\downarrow$ 321

$$\frac{\frac{1}{3}d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(7cC-5Bd)+bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\cdots}{\cdots}\right)}{5bd^3}$$

$$\frac{2C\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

input

```
Int[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output

```
(-2*C*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b*d^2) + ((2*d*(7*c*C - 5*B*d)*S
qrt[c + d*x]*Sqrt[a - b*x^2])/3 + (d*((-2*Sqrt[a]*(9*a*C*d^2 + b*(8*c^2*C
- 10*B*c*d + 15*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])
/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x
^2]) + (2*Sqrt[a]*(a*d^2*(7*c*C - 5*B*d) + b*c*(8*c^2*C - 10*B*c*d + 15*A*
d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a
]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]
*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3)/(5*b*d^3
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(dx+c)}\left(-\frac{2Cx\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{5bd}-\frac{2\left(B-\frac{4cC}{5d}\right)\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{3bd}+\frac{2\left(\frac{2Cac}{5bd}+\frac{\left(B-\frac{4cC}{5d}\right)a}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+}{\frac{c}{d}-}}}{}\right.}{}$
risch	$-\frac{2(3Cxd+5Bd-4Cc)\sqrt{-b\,x^{2}+a}\,\sqrt{dx+c}}{15b\,d^{2}}+\frac{\left(\left(15Ab\,d^{2}-10Bbcd+9aC\,d^{2}+8Cb\,c^{2}\right)\sqrt{ab}\,\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\right.}{}$
default	Expression too large to display

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/5*C/b/d*x*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(B-4/5*c*C/d)/b/d*(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)+2*(2/5*C/b/d*a*c+1/3*(B-4/5*c*C/d)/b*a)*(c/d-1/b*(a*b)^(1/
2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(A+3/5*a*C/b
-2/3*(B-4/5*c*C/d)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)
^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*(
(-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-
c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*Ellipt
icF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.71

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2\left(\left(8\,Cbc^3-10\,Bbc^2d-15\,Bad^3+3\,(Ca+5\,Ab)cd^2\right)\sqrt{-bd}\,\text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2},-\frac{8\,(bc^3-9\,ac\ldots}{27\,bd^3}\right.\right.}{}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
2/45*((8*C*b*c^3 - 10*B*b*c^2*d - 15*B*a*d^3 + 3*(C*a + 5*A*b)*c*d^2)*sqrt
(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9
*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(8*C*b*c^2*d - 10*B*b*c*d^2 + 3*
(3*C*a + 5*A*b)*d^3)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d
^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3
*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) -
3*(3*C*b*d^3*x - 4*C*b*c*d^2 + 5*B*b*d^3)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/
(b^2*d^4)
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{-10\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,abd-6\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,acd-4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b\,c^2x-15\left(\int \frac{\sqrt{dx+c}\,\sqrt{}}{-bd\,x^3-b}\right.}{}$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 10*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*d - 6*sqrt(c + d*x)*sqrt(a - b*x
**2)*a*c*d - 4*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c**2*x - 15*int((sqrt(c +
d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*
d**2 - 9*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2
 - b*d*x**3),x)*a*b*c*d**2 + 10*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/
(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c*d - 8*int((sqrt(c + d*x)*sqr
t(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c**3 + 5*i
nt((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)
*a**2*b*d**2 + 3*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x
**2 - b*d*x**3),x)*a**2*c*d**2 + 4*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a
*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**3)/(10*b**2*c*d)
```

3.188 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 333

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3bd} + \frac{2\sqrt{a}(2cC-3Bd)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}(aCd^2+b(2c^2C-3Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{3/2}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d+2/3*a^(1/2)*(-3*B*d+2*C*c)*(d*x+
c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^
(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^2/((d*x+c
)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/3*a^(1/2)*(a*C*d^2+b*(3*
A*d^2-3*B*c*d+2*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2
)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.95 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(2cC-3Bd-C(c+dx)+\frac{i\sqrt{b}\left(\sqrt{b}c-\sqrt{a}d\right)(-2cC+3Bd)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\text{arcsinh}\left(\sqrt{\cdots}\right.\right.}{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(-a+bx^2)}\cdots\right.}{3b\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(2*c*C - 3*B*d - C*(c + d*x) + (I*Sqrt[b]*(Sqrt[b]*c -
Sqrt[a]*d)*(-2*c*C + 3*B*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt
[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d
)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^
2)) - (I*(3*A*b*d + a*C*d + Sqrt[a]*Sqrt[b]*(2*c*C - 3*B*d))*Sqrt[(d*(Sqrt
[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))
]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*b*d*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{d((3Ab+aC)d-b(2cC-3Bd)x)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(3Ab+aC)d-b(2cC-3Bd)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b(2cC-3Bd)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}(2cC-3Bd)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cC-3Bd)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cC-3Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cC-3Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cC-3Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\dots\right)}{\dots}}\dots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cC-3Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2+b\left(3Ad^2-3Bcd+2c^2C\right)\right)\text{EllipticF}\left(\dots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{3bd}-\frac{2C\sqrt{a-bx^2}\sqrt{c+dx}}{3bd}$$

input

```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output

```
(-2*C*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b*d) + ((2*Sqrt[a]*Sqrt[b]*(2*c*C
 - 3*B*d)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(a*C*
d^2 + b*(2*c^2*C - 3*B*c*d + 3*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*
x]*Sqrt[a - b*x^2]))/(3*b*d)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 587 vs. $2(273) = 546$.

Time = 3.50 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.77

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2C\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(A+\frac{aC}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\right.}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
risch	$-\frac{2C\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{3bd}+\frac{\left(\frac{(3Bd-2Cc)\sqrt{ab}\,\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\left(\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\mathrm{EllipticE}\left(\frac{\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{2},\right.\right.}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3*C/b/d*(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(A+1/3*a*C/b)*(c/d-1/b*(a*b)^(1/2))*((x+c
/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c
/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B-2/3*c*C/d)*(c/d-1/
b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))
^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Elliptic
E(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx =$$

$$-\frac{2\left(3\sqrt{-bx^2+a}\sqrt{dx+c}Cbd^2+(2\,Cbc^2-3\,Bbcd+3\,(Ca+3\,Ab)d^2)\sqrt{-bd}\text{weierstrassPInverse}\left(\frac{4\,(bc^2}{3}\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
-2/9*(3*sqrt(-b*x^2 + a)*sqrt(d*x + c)*C*b*d^2 + (2*C*b*c^2 - 3*B*b*c*d +
3*(C*a + 3*A*b)*d^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*C*b*
c*d - 3*B*b*d^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
 -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d
^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)))/(b^2*
d^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.189 $\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 389

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{a}C\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(cC-Bd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*a^(1/2)*C*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x
/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b
^(1/2)/d/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2*a^(1/2)*
(-B*d+C*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/b^(1/2)/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)
/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1
/2))/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.66 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= \frac{2\left(bc^3C\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}-acCd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}-2bc^2C\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)+bcC\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)^2-i\sqrt{\right.}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(2*(b*c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - a*c*C*d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]] - 2*b*c^2*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + b*c*C*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*C*(Sqrt[b]*c - Sqr
t[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)] - I*Sqrt[b]*d*(Sqrt[a]*c*C + Sqrt[b]*(-(B*c) + A*d))*Sqrt[(d*(Sqrt[
a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]
*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*A*b*d^2*Sqr
t[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/
(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d)
, I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sq
rt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{B+Cx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{600}$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{C\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{509}$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{C\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 508$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \ 327$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \ 512$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \ 511$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 321

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 633

$$\frac{A\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 632

$$\frac{A\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 186

$$-\frac{2A\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 413

$$-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

input
```
Int[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*Sqrt[a]*C*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(c*C - B*d)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])
 - (2*A*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))
/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a
]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[a - b*x^2]*Sqrt
[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(318) = 636$.

Time = 2.99 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.73

method	result
default	$-\frac{2\left(A\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\frac{-\sqrt{ab}\,d+bc}{bc},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)bc\,d^{2}-A\sqrt{ab}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\frac{-\sqrt{ab}\,d+bc}{bc},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)d^{3}-\right.}{}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(dx+c)}\left(\frac{2B\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}+\frac{2C\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\ldots}}}{}\right.}{}$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-2*(A*EllipticPi((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-(a*b)^(1/2)*d+b*
c)/b/c,(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*b*c*d^2-A*(a*b)^(
1/2)*EllipticPi((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-(a*b)^(1/2)*d+b*c
)/b/c,(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*d^3-B*EllipticF((-
(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+
b*c))^(1/2))*b*c^2*d+B*(a*b)^(1/2)*EllipticF((-(d*x+c)*b/((a*b)^(1/2)*d-b*
c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*c*d^2+C*Ellipt
icF((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1
/2)*d+b*c))^(1/2))*a*c*d^2-C*(a*b)^(1/2)*EllipticF((-(d*x+c)*b/((a*b)^(1/2
)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*c^2*d-C*
EllipticE((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a
*b)^(1/2)*d+b*c))^(1/2))*a*c*d^2+C*EllipticE((-(d*x+c)*b/((a*b)^(1/2)*d-b*
c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*b*c^3)*((b*x+(
a*b)^(1/2))*d/((a*b)^(1/2)*d-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/((a*b)^(1/2
)*d+b*c))^(1/2)*(-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2)*(-b*x^2+a)^(1/2)*(d
*x+c)^(1/2)/b/d^2/c/(-b*d*x^3-b*c*x^2+a*d*x+a*c)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+cx}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}\,x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.190 $\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 429

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{acx}+\frac{A\sqrt{b}\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}c\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(Ab+2aC)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(2Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x+A*b^(1/2)*(d*x+c)^(1/2)*((-b*x^2+a
)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1
/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c/((d*x+c)/(c+a^(1/2)*d/b^(1/2
)))^(1/2)/(-b*x^2+a)^(1/2)-(A*b+2*C*a)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/
2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(d*x+c)^(
1/2)/(-b*x^2+a)^(1/2)-(-A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*(
(-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2
^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c/(d*x+c)^(1/2)/(-b*x^2+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.76 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.03

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{Ac(c+dx)}{x}+\frac{Abc^3\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}-aAcd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}-2Abc^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)+Abc\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)^2-iA\sqrt{b}c\left(\sqrt{b}c-\sqrt{a}d\right)}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*(-((A*c*(c + d*x))/x) + (A*b*c^3*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]] - a*A*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 2*A*b*c^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*(c + d*x) + A*b*c*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d
*x)^2 - I*A*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x
))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/
2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqr
t[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*(A*Sqrt[b]*c*d +
Sqrt[a]*(2*c^2*C - 2*B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d
*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipt
icF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - (2*I)*a*B*c*d*Sqrt[(d*(Sqrt[a]/Sqrt[
b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*
x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c
 - Sqrt[a]*d)] + I*a*A*d^2*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt
[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*c^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2352}$$

$$-\frac{\int -\frac{-Abdx^2+2acCx+a(2Bc-Ad)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-Abdx^2+2acCx+a(2Bc-Ad)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{2351}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{2acC-Abdx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{600}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+c(2aC+Ab)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-Ab\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{509}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+c(2aC+Ab)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{Ab\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{508}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+c(2aC+Ab)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}A\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{327}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+c(2aC+Ab)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}A\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{512}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}A\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2}{\frac{\sqrt{b}}{\sqrt{a}}}\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 511

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}A\sqrt{b}\sqrt{}}{}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 321

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}A\sqrt{b}\sqrt{1-\frac{b}{}}}{}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 633

$$\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}A\sqrt{b}\sqrt{1-}}{}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 632

$$\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\cdots}{\cdots}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\big\downarrow$ 186

$$\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{2}}\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\big\downarrow$ 413

$$\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\big\downarrow$ 412

$$\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}(2aC+Ab)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
-((A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x)) + ((2*Sqrt[a]*A*Sqrt[b]*Sqrt[
c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]
]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(S
qrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*c*(A*b + 2*a*C)*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
 + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*a*(2*B*c - A*d)*Sqrt[
1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c +
 Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]
*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]]))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.50

method	result
risch	$-\frac{A\sqrt{dx+c}\,\sqrt{-b\,x^{2}+a}}{acx}-\frac{\left(-\frac{a(Ad-2Bc)\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{2},2,\sqrt{-\frac{2\sqrt{ab}}{b\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right.}}\right.}{\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}\right.}{}$
elliptic	$\frac{\sqrt{(-b\,x^{2}+a)(dx+c)}\left(-\frac{A\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}{acx}+\frac{2C\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\,\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\sqrt{}}{-\frac{c}{d}-\sqrt{}}}\right.}{\sqrt{-bd\,x^{3}-bc\,x^{2}+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x-1/2/c/a*(-a*(A*d-2*B*c)*2^(1/2)*((
x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*EllipticPi(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)
,2,(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))+A*d*(a*b)^(1/2)*2^(1/2)
*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*((c/d-1/b*(a*b)^(1/2))*EllipticE(1/2*2^(1/2)*((x+1/b*(a*b)^(1
/2))*b/(a*b)^(1/2))^(1/2),(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2))-
c/d*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),(-2/b*
(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))-2*C*a*c/b*(a*b)^(1/2)*2^(1/2)*(
(x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+
a*c)^(1/2)*EllipticF(1/2*2^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)
,(-2/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)))^(1/2)))*((-b*x^2+a)*(d*x+c))^(1/
2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b*d*x^5 + b*c*
x^4 - a*d*x^3 - a*c*x^2), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.191 $\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 515

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}-\frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4ac^2x}$$
$$+\frac{\sqrt{b}(4Bc-3Ad)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}c^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(4Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}c\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(4Abc^2+8ac^2C-4aBcd+3aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4ac^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x^2-1/4*(-3*A*d+4*B*c)*(d*x+c)^(
1/2)*(-b*x^2+a)^(1/2)/a/c^2/x+1/4*b^(1/2)*(-3*A*d+4*B*c)*(d*x+c)^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c^2/((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/4*b^(1/2)*(-A*d+4*B*c)*((d*x+c)/(c+a
^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a
^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(
1/2)/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(3*A*a*d^2+4*A*b*c^2-4*B*a*c*d+8
*C*a*c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/a/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.69 (sec) , antiderivative size = 1392, normalized size of antiderivative = 2.70

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(2*A*c + 4*B*c*x - 3*A*d*x))/(a*c^2*x^2)) +
 (4*b*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*A*b*c^3*d*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]] - 4*a*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 3*a*A*c*d^3
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*b*B*c^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(c + d*x) + 6*A*b*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 4*b*B*
c^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 3*A*b*c*d*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(4*B*c -
 3*A*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt
[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[
a]*d)] + I*d*(A*(2*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d + 3*a*d^2) - 4*(Sqrt[a]*S
qrt[b]*B*c^2 + a*c*(-2*c*C + B*d)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*
x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipti
cF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + S
qrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - (4*I)*A*b*c^2*d*Sqrt[(d*(Sqrt[a]/Sqrt
[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d
*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)] - (8*I)*a*c^2*C*d*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)
]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipt...
```

Rubi [A] (verified)

Time = 2.83 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2352, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2352}$$

$$-\frac{\int -\frac{Abdx^2+2c(Ab+2aC)x+a(4Bc-3Ad)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Abdx^2+2c(Ab+2aC)x+a(4Bc-3Ad)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 2352

$$\frac{-\frac{\int -\frac{-abd(4Bc-3Ad)x^2+2aAbcdx+a\left(4Abc^2+8aCc^2-4aBdc+3aAd^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{-abd(4Bc-3Ad)x^2+2aAbcdx+a\left(4Abc^2+8aCc^2-4aBdc+3aAd^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 2351

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{2aAbcd-abd(4Bc-3Ad)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 600

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abc(4Bc-Ad)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-ab(4Bc-3Ad)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4B}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 509

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abc(4Bc-Ad)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ab\sqrt{1-\frac{bx^2}{a}}(4Bc-3Ad)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{2ac}-\sqrt{a-bx^2}\sqrt{c}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 508

$$\frac{\frac{\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(4Bc-3Ad)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abc(4Bc-Ad)\int \frac{}{\sqrt{c+\cdots}}\cdots}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+abc(4Bc-Ad)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(4Bc-3Ad)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}\cdots}{\sqrt{\cdots}}}}{\sqrt{2}}\cdots\right.\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{abc\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(4Bc-3Ad)E\left(\arcsin\left(\cdots\right.\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{c+dx}}+a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx\cdots}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2a^{3/2}\sqrt{b}\cdots}{\cdots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 633

$$\frac{\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2a^{3}\cdots}{\cdots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 632

$$\frac{\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}\cdots}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 186

$$\frac{\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\cdots\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 413

$$\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+d}{\sqrt{a}d+\sqrt{}}}}{\sqrt{a-}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\Big\downarrow$ 412

$$\frac{-\frac{2a^{3/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}(4Bc-Ad)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(4Bc-3Ad)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x^2) + (-(((4*B*c - 3*A*d)*Sqr
t[c + d*x]*Sqrt[a - b*x^2])/(c*x)) + ((2*a^(3/2)*Sqrt[b]*(4*B*c - 3*A*d)*S
qrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqr
t[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x)
)/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*a^(3/2)*Sqrt[b]*c*(4*B*c
- A*d)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/
a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b
]*c)/Sqrt[a] + d)])/(Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*a*(4*A*b*c^2 + 8*
a*c^2*C - 4*a*B*c*d + 3*a*A*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(
1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sq
rt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d
)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[
b]*x)/Sqrt[a]))/Sqrt[b]]))/(2*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(426) = 852$.

Time = 5.22 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(-bx^2+a)(dx+c)}\left(-\frac{A\sqrt{-bdx^3-bcx^2+adx+ac}}{2cax^2}+\frac{(3Ad-4Bc)\sqrt{-bdx^3-bcx^2+adx+ac}}{4ac^2x}+\frac{bdA\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\cdots}{-\frac{c}{d}\cdots}}\cdots}{2ac\sqrt{-bdx^3-b\cdots}}\cdots\right.}{\cdots}$
risch	$-\frac{\sqrt{-bx^2+a}\,\sqrt{dx+c}\,(-3Adx+4Bcx+2Ac)}{4ac^2x^2}+\frac{\left(-\frac{\left(3Aad^2+4bAc^2-4Bacd+8Cac^2\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\mathrm{El}\cdots}{\sqrt{-bdx^3-bcx^2+adx+ac}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/2*A/c/a/x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4*(3*A*d-4*B*c)/a/c^2*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)/x+1/2*b*d/a/c*A*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+
a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/4*b*d*(3*A*d-4*B*c)/a/c^2*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*Ellipti
cE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4/c^
3/a*(3*A*a*d^2+4*A*b*c^2-4*B*a*c*d+8*C*a*c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(
-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^
(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b*d*x^6 + b*c*
x^5 - a*d*x^4 - a*c*x^3), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.192 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 597

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = -\frac{2c^3(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{d^4(bc^2-ad^2)\sqrt{c+dx}}$$

$$-\frac{2(25aCd^2+b(141c^2C-84Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{105b^2d^4}$$

$$+\frac{2(23cC-7Bd)(c+dx)^{3/2}\sqrt{a-bx^2}}{35bd^4}-\frac{2C(c+dx)^{5/2}\sqrt{a-bx^2}}{7bd^4}$$

$$-\frac{2\sqrt{a}(a^2d^4(107cC-63Bd)-8b^2c^3(48c^2C-42Bcd+35Ad^2)+abcd^2(172c^2C-168Bcd+175Ad^2))\sqrt{c}}{105b^{3/2}d^5(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{2\sqrt{a}(25a^2Cd^4+abd^2(116c^2C-84Bcd+35Ad^2)+8b^2c^2(48c^2C-42Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{El}}{105b^{5/2}d^5\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*c^3*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(1/
2)-2/105*(25*a*C*d^2+b*(35*A*d^2-84*B*c*d+141*C*c^2))*(d*x+c)^(1/2)*(-b*x^
2+a)^(1/2)/b^2/d^4+2/35*(-7*B*d+23*C*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b/d
^4-2/7*C*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^4-2/105*a^(1/2)*(a^2*d^4*(-63*
B*d+107*C*c)-8*b^2*c^3*(35*A*d^2-42*B*c*d+48*C*c^2)+a*b*c*d^2*(175*A*d^2-1
68*B*c*d+172*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b
^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^
(1/2))/b^(3/2)/d^5/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-
b*x^2+a)^(1/2)-2/105*a^(1/2)*(25*a^2*C*d^4+a*b*d^2*(35*A*d^2-84*B*c*d+116*
C*c^2)+8*b^2*c^2*(35*A*d^2-42*B*c*d+48*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/
2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^5/(d*x+
c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.54 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.32

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{2(c+dx)\left(-\frac{25aCd^2}{b^2}-\frac{105c^3\left(c^2C-Bcd+Ad^2\right)}{\left(bc^2-ad^2\right)(c+dx)}+\frac{-87c^2C+c(63Bd+39Cdx)-d^2(35A+3x(7B+5C}{b}\right.}{d^4}\right.}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*((2*(c + d*x)*((-25*a*C*d^2)/b^2 - (105*c^3*(c^2*C - B*c*
d + A*d^2))/((b*c^2 - a*d^2)*(c + d*x)) + (-87*c^2*C + c*(63*B*d + 39*C*d*
x) - d^2*(35*A + 3*x*(7*B + 5*C*x)))/b))/d^4 + (2*(d^2*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]*(a^2*d^4*(-107*c*C + 63*B*d) + a*b*c*d^2*(-172*c^2*C + 168*B*c
*d - 175*A*d^2) + 8*b^2*c^3*(48*c^2*C - 42*B*c*d + 35*A*d^2))*(a - b*x^2)
+ I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a^2*d^4*(-107*c*C + 63*B*d) + a*b*c*d
^2*(-172*c^2*C + 168*B*c*d - 175*A*d^2) + 8*b^2*c^3*(48*c^2*C - 42*B*c*d +
 35*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/
Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - S
qrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(25*a^2*C*d^4 + 3*a^(3/2)
*Sqrt[b]*d^3*(44*c*C - 21*B*d) + a*b*d^2*(116*c^2*C - 84*B*c*d + 35*A*d^2)
 + 8*b^2*c^2*(48*c^2*C - 42*B*c*d + 35*A*d^2) + 6*Sqrt[a]*b^(3/2)*c*d*(48*
c^2*C - 42*B*c*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b^2*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-(
b*c^2) + a*d^2)*(-a + b*x^2))))/(105*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 3.90 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2182, 27, 2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+Bx+Cx^2\right)}{\sqrt{a-bx^2}(c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{2 \int -\frac{-C\left(\frac{bc^2}{d}-ad\right)x^4+\frac{(cC-Bd)\left(bc^2-ad^2\right)x^3}{d^2}-\frac{\left(bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}+\frac{c\left(2bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{-C\left(\frac{bc^2}{d}-ad\right)x^4+\frac{(cC-Bd)\left(bc^2-ad^2\right)x^3}{d^2}-\frac{\left(bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}+\frac{c\left(2bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{2C\sqrt{a-bx^2}(c+dx)^{5/2}\left(bc^2-ad^2\right)}{7bd^4}-\frac{2\int -\frac{bd^2(23cC-7Bd)\left(bc^2-ad^2\right)x^3-d\left(bc^2-ad^2\right)\left(5aCd^2-b\left(4Cc^2+7Bdc-7Ad^2\right)\right)x^2+c\left(10a^2Cd^4-ab\left(19Cc^2-7B\cdots\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\int \frac{bd^2(23cC-7Bd)\left(bc^2-ad^2\right)x^3-d\left(bc^2-ad^2\right)\left(5aCd^2-b\left(4Cc^2+7Bdc-7Ad^2\right)\right)x^2+c\left(10a^2Cd^4-ab\left(19Cc^2-7Bdc+7Ad^2\right)d^2+2b^2c^2\left(8Cc^2-7Bdc+7Ad^2\right)\right)x\cdots}{\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}\cdots}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{-\frac{2\int \frac{b\left(bc^2-ad^2\right)\left(25aCd^2+b\left(141Cc^2-84Bdc+35Ad^2\right)\right)x^2d^4+abc\left(ad^2(44cC-21Bd)-bc\left(79Cc^2-56Bdc+35Ad^2\right)\right)d^4+b\left(a^2(19cC-21Bd)d^4-5abc\left(4c^2C-\cdots\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}\cdots}{7bd^4}\cdots}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{-\frac{\int \frac{b\left(bc^2-ad^2\right)\left(25aCd^2+b\left(141Cc^2-84Bdc+35Ad^2\right)\right)x^2d^4+abc\left(ad^2(44cC-21Bd)-bc\left(79Cc^2-56Bdc+35Ad^2\right)\right)d^4+b\left(a^2(19cC-21Bd)d^4-5abc\left(4c^2C-7A\cdots\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}\cdots}{7bd^4}\cdots}{bc^2-ad^2}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$-\frac{-\frac{-\frac{2\int \frac{bd^5\left(ad\left(25a^2Cd^4-ab\left(16Cc^2+21Bdc-35Ad^2\right)d^2+2b^2c^2\left(48Cc^2-42Bdc+35Ad^2\right)\right)-b\left(a^2(107cC-63Bd)d^4+abc\left(172Cc^2-168Bdc+175Ad^2\right)d^2-8b^2c\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}}{5bd^3}}{7b}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{-\frac{1}{3}d^3\int \frac{ad\left(25a^2Cd^4-ab\left(16Cc^2+21Bdc-35Ad^2\right)d^2+2b^2c^2\left(48Cc^2-42Bdc+35Ad^2\right)\right)-b\left(a^2(107cC-63Bd)d^4+abc\left(172Cc^2-168Bdc+175Ad^2\right)d^2-8b^2c^3\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$-\frac{-\frac{-\frac{1}{3}d^3\left(-\frac{\left(bc^2-ad^2\right)\left(25a^2Cd^4+abd^2\left(35Ad^2-84Bcd+116c^2C\right)+8b^2c^2\left(35Ad^2-42Bcd+48c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(a^2d^4(107cC-63Bd)+abcd^2\left(17\right.\right.}{}\right.}{5bd^3}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$-\frac{-\frac{-\frac{1}{3}d^3\left(-\frac{\left(bc^2-ad^2\right)\left(25a^2Cd^4+abd^2\left(35Ad^2-84Bcd+116c^2C\right)+8b^2c^2\left(35Ad^2-42Bcd+48c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(a^2d^4(107cC-63Bd)+\right.}{}\right.}{5}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$-\frac{-\frac{-\frac{1}{3}d^3\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(107cC-63Bd)+abcd^2\left(175Ad^2-168Bcd+172c^2C\right)-8b^2c^3\left(35Ad^2-42Bcd+48c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\downarrow \text{327}$$

$$-\frac{-\frac{-\frac{1}{3}d^3\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(107cC-63Bd)+abcd^2\left(175Ad^2-168Bcd+172c^2C\right)-8b^2c^3\left(35Ad^2-42Bcd+48c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\downarrow \text{512}$$

$$-\frac{-\frac{-\frac{1}{3}d^3\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(107cC-63Bd)+abcd^2\left(175Ad^2-168Bcd+172c^2C\right)-8b^2c^3\left(35Ad^2-42Bcd+48c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\downarrow \text{511}$$

$$-\frac{1}{3}d^3\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(25a^2Cd^4+abd^2\left(35Ad^2-84Bcd+116c^2C\right)+8b^2c^2\left(35Ad^2-42Bcd+48c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{1}{3}d^3\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(25a^2Cd^4+abd^2\left(35Ad^2-84Bcd+116c^2C\right)+8b^2c^2\left(35Ad^2-42Bcd+48c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*c^3*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(d^4*(b*c^2 - a*d^2)*Sqrt
[c + d*x]) - ((2*C*(b*c^2 - a*d^2)*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b*d
^4) + ((-2*(23*c*C - 7*B*d)*(b*c^2 - a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2
])/5 - ((-2*d^3*(b*c^2 - a*d^2)*(25*a*C*d^2 + b*(141*c^2*C - 84*B*c*d + 35
*A*d^2))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 - (d^3*((2*Sqrt[a]*Sqrt[b]*(a^2*
d^4*(107*c*C - 63*B*d) - 8*b^2*c^3*(48*c^2*C - 42*B*c*d + 35*A*d^2) + a*b*
c*d^2*(172*c^2*C - 168*B*c*d + 175*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/
a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b
]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*S
qrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(25*a^2*C*d^4 + a*b*d^2*(116*
c^2*C - 84*B*c*d + 35*A*d^2) + 8*b^2*c^2*(48*c^2*C - 42*B*c*d + 35*A*d^2))
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/S
qrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3)/(5*b*d^3))/(7
*b*d^4))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1100 vs. $2(521) = 1042$.

Time = 9.52 (sec) , antiderivative size = 1101, normalized size of antiderivative = 1.84

method	result	size
elliptic	Expression too large to display	1101
risch	Expression too large to display	1141
default	Expression too large to display	4795

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output

```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*(-b*d*x^2+a*d
)/(a*d^2-b*c^2)/d^5*c^3*(A*d^2-B*c*d+C*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)
-2/7*C/d^2/b*x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(1/d^2*(B*d-C*c)-6
/7*C/d^2*c)/b/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(1/d^3*(A*d^2-B*c
*d+C*c^2)+5/7*C/d/b*a-4/5*(1/d^2*(B*d-C*c)-6/7*C/d^2*c)/d*c)/b/d*(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)+2*(c^2*(A*d^2-B*c*d+C*c^2)/d^5+b/d^5*c^4*(A*d^2-
B*c*d+C*c^2)/(a*d^2-b*c^2)+2/5*(1/d^2*(B*d-C*c)-6/7*C/d^2*c)/b/d*a*c+1/3*(
1/d^3*(A*d^2-B*c*d+C*c^2)+5/7*C/d/b*a-4/5*(1/d^2*(B*d-C*c)-6/7*C/d^2*c)/d*
c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/
b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))+2*(-c/d^4*(A*d^2-B*c*d+C*c^2)+b/d^4*c^3*(A*d^2-B*c*d+C*c^2)/(a
*d^2-b*c^2)+4/7*C/d^2/b*a*c+3/5*(1/d^2*(B*d-C*c)-6/7*C/d^2*c)/b*a-2/3*(1/d
^3*(A*d^2-B*c*d+C*c^2)+5/7*C/d/b*a-4/5*(1/d^2*(B*d-C*c)-6/7*C/d^2*c)/d*c)/
d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(
a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/
2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2)
)/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 878, normalized size of antiderivative = 1.47

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input

```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2/315*((384*C*b^3*c^7 - 336*B*b^3*c^6*d + 420*B*a*b^2*c^4*d^3 + 126*B*a^2
*b*c^2*d^5 - 20*(23*C*a*b^2 - 14*A*b^3)*c^5*d^2 - (59*C*a^2*b + 385*A*a*b^
2)*c^3*d^4 - 15*(5*C*a^3 + 7*A*a^2*b)*c*d^6 + (384*C*b^3*c^6*d - 336*B*b^3
*c^5*d^2 + 420*B*a*b^2*c^3*d^4 + 126*B*a^2*b*c*d^6 - 20*(23*C*a*b^2 - 14*A
*b^3)*c^4*d^3 - (59*C*a^2*b + 385*A*a*b^2)*c^2*d^5 - 15*(5*C*a^3 + 7*A*a^2
*b)*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
-8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(384*C*b^3*c^6*d
 - 336*B*b^3*c^5*d^2 + 168*B*a*b^2*c^3*d^4 + 63*B*a^2*b*c*d^6 - 4*(43*C*a*
b^2 - 70*A*b^3)*c^4*d^3 - (107*C*a^2*b + 175*A*a*b^2)*c^2*d^5 + (384*C*b^3
*c^5*d^2 - 336*B*b^3*c^4*d^3 + 168*B*a*b^2*c^2*d^5 + 63*B*a^2*b*d^7 - 4*(4
3*C*a*b^2 - 70*A*b^3)*c^3*d^4 - (107*C*a^2*b + 175*A*a*b^2)*c*d^6)*x)*sqrt
(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(192*C*b^3*c^5*d^2 - 16
8*B*b^3*c^4*d^3 + 63*B*a*b^2*c^2*d^5 - 2*(31*C*a*b^2 - 70*A*b^3)*c^3*d^4 -
 5*(5*C*a^2*b + 7*A*a*b^2)*c*d^6 + 15*(C*b^3*c^2*d^5 - C*a*b^2*d^7)*x^3 -
3*(8*C*b^3*c^3*d^4 - 7*B*b^3*c^2*d^5 - 8*C*a*b^2*c*d^6 + 7*B*a*b^2*d^7)*x^
2 + (48*C*b^3*c^4*d^3 - 42*B*b^3*c^3*d^4 + 42*B*a*b^2*c*d^6 - (23*C*a*b^2
- 35*A*b^3)*c^2*d^5 - 5*(5*C*a^2*b + 7*A*a*b^2)*d^7)*x)*sqrt(-b*x^2 + a)*s
qrt(d*x + c))/(b^4*c^3*d^6 - a*b^3*c*d^8 + (b^4*c^2*d^7 - a*b^3*d^9)*x)
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^3\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.193 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 507

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2c^2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{d^3(bc^2-ad^2)\sqrt{c+dx}}$$
$$+\frac{2(12cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15bd^3}-\frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5bd^3}$$
$$+\frac{2\sqrt{a}(9a^2Cd^4+abd^2(24c^2C-25Bcd+15Ad^2)-2b^2c^2(24c^2C-20Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\text{a}\right.}{15b^{3/2}d^4(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(ad^2(12cC-5Bd)+2bc(24c^2C-20Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{15b^{3/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*c^2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2
)+2/15*(-5*B*d+12*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^3-2/5*C*(d*x+c)^
(3/2)*(-b*x^2+a)^(1/2)/b/d^3+2/15*a^(1/2)*(9*a^2*C*d^4+a*b*d^2*(15*A*d^2-2
5*B*c*d+24*C*c^2)-2*b^2*c^2*(15*A*d^2-20*B*c*d+24*C*c^2))*(d*x+c)^(1/2)*((
-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/
2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^4/(-a*d^2+b*c^2)/((d
*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2/15*a^(1/2)*(a*d^2*(-
5*B*d+12*C*c)+2*b*c*(15*A*d^2-20*B*c*d+24*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^4/(d
*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.24 (sec) , antiderivative size = 728, normalized size of antiderivative = 1.44

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(9a^2Cd^4+abd^2(24c^2C-25Bcd+15Ad^2)-2b^2c^2(24c^2C-\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(9*a^2*C*d^4 + a*b*d^2*(24*c^2*C -
25*B*c*d + 15*A*d^2) - 2*b^2*c^2*(24*c^2*C - 20*B*c*d + 15*A*d^2))*(a - b*
x^2) + b*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2)*(a*d^2*(c + d*x)*(
-9*c*C + 5*B*d + 3*C*d*x) + b*c^2*(24*c^2*C + c*(-20*B*d + 6*C*d*x) + d^2*
(15*A - 5*B*x - 3*C*x^2))) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(9*a^2*C*d^
4 + a*b*d^2*(24*c^2*C - 25*B*c*d + 15*A*d^2) - 2*b^2*c^2*(24*c^2*C - 20*B*
c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a
]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(9*a^(3/2)*C
*d^3 + a*Sqrt[b]*d^2*(12*c*C - 5*B*d) + 3*Sqrt[a]*b*d*(12*c^2*C - 10*B*c*d
 + 5*A*d^2) + 2*b^(3/2)*c*(24*c^2*C - 20*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[
a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]
*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(15*b^2*d^5*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2
])
```

Rubi [A] (verified)

Time = 2.60 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2182, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \ 2182$$

$$\frac{2\int \frac{C\left(\frac{bc^2}{d}-ad\right)x^3-\frac{(cC-Bd)\left(bc^2-ad^2\right)x^2}{d^2}+\frac{\left(2bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{C\left(\frac{bc^2}{d}-ad\right)x^3-\frac{(cC-Bd)\left(bc^2-ad^2\right)x^2}{d^2}+\frac{\left(2bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 2185

$$\frac{-\frac{2\int \frac{bd(12cC-5Bd)\left(bc^2-ad^2\right)x^2+\left(3a^2Cd^4-5ab(Bc-Ad)d^3-2b^2c^2\left(4Cc^2-5Bdc+5Ad^2\right)\right)x+acd\left(3aCd^2-b\left(8Cc^2-5Bdc+5Ad^2\right)\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+d}{5b}}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{bd(12cC-5Bd)\left(bc^2-ad^2\right)x^2+\left(3a^2Cd^4-5ab(Bc-Ad)d^3-2b^2c^2\left(4Cc^2-5Bdc+5Ad^2\right)\right)x+acd\left(3aCd^2-b\left(8Cc^2-5Bdc+5Ad^2\right)\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2C\sqrt{a-bx^2}(c+dx}{5bd}}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 2185

$$\frac{-\frac{-\frac{2\int \frac{bd^2\left(ad\left(a(3cC-5Bd)d^2+bc\left(12Cc^2-10Bdc+15Ad^2\right)\right)-\left(9a^2Cd^4+ab\left(24Cc^2-25Bdc+15Ad^2\right)d^2-2b^2c^2\left(24Cc^2-20Bdc+15Ad^2\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2}{3}\sqrt{a-bx^2}\sqrt{}}{5bd^3}}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{-\frac{-\frac{1}{3}\int \frac{ad\left(a(3cC-5Bd)d^2+bc\left(12Cc^2-10Bdc+15Ad^2\right)\right)-\left(9a^2Cd^4+ab\left(24Cc^2-25Bdc+15Ad^2\right)d^2-2b^2c^2\left(24Cc^2-20Bdc+15Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}\sqrt{a-bx^2}\sqrt{c+d}}{5bd^3}}{bc^2-ad^2}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 600

$$-\frac{\frac{\frac{1}{3}\left(\frac{\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2C\right)-2b^2c^2\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}+\frac{\left(bc^2-ad^2\right)\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+24c^2C\right)\right)\cdots}{d}\right.}{5bd^3}\cdots}{bc^2-ad^2}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2C\right)-2b^2c^2\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\left(bc^2-ad^2\right)\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+\cdots\right.\right.}{d}\right.}{5bd^3}\cdots}{bc^2-ad^2}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2C\right)-2b^2c^2\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{\cdots}}{\sqrt{\cdots}}}}\right.}{5bd^3}\cdots}{bc^2-a\cdots}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2C\right)-2b^2c^2\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\cdots}{\cdots}}}\right.}{5bd^3}\cdots}{bc^2-a\cdots}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{512}$$

$$-\frac{\frac{\frac{1}{3}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2\cdots\right.\right.}{\sqrt{b}d\sqrt{\cdots}}\right.}{5bd^3}}{bc^2\cdots}-\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 511}$$

$$-\frac{\frac{1}{3}\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(12cC-5Bd)+2bc\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{b\cdots}{}}}{\cdots}\right.}{\cdots}-\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{\frac{1}{3}\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2Cd^4+abd^2\left(15Ad^2-25Bcd+24c^2C\right)-2b^2c^2\left(15Ad^2-20Bcd+24c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2\cdots\right.}{\cdots}\right.}{5\cdots}-\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input

```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*c^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(d^3*(b*c^2 - a*d^2)*Sqrt[
c + d*x]) + ((-2*C*(b*c^2 - a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b*d
^3) - ((-2*(12*c*C - 5*B*d)*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])
/3 + ((-2*Sqrt[a]*(9*a^2*C*d^4 + a*b*d^2*(24*c^2*C - 25*B*c*d + 15*A*d^2)
- 2*b^2*c^2*(24*c^2*C - 20*B*c*d + 15*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^
2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqr
t[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(a*d^2*(12*c*C - 5
*B*d) + 2*b*c*(24*c^2*C - 20*B*c*d + 15*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sq
rt[c + d*x]*Sqrt[a - b*x^2]))/3)/(5*b*d^3))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. $2(437) = 874$.

Time = 7.65 (sec) , antiderivative size = 888, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2\left(-bd\,x^2+da\right)c^2\left(A\,d^2-Bcd+C\,c^2\right)}{\left(a\,d^2-b\,c^2\right)d^4\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}-\frac{2Cx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5d^2b}-\frac{2\left(\frac{Bd-Cc}{d^2}-\frac{4Cc}{5d^2}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x^2+a*
d)/(a*d^2-b*c^2)/d^4*c^2*(A*d^2-B*c*d+C*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2
)-2/5*C/d^2/b*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(1/d^2*(B*d-C*c)-4/
5*C/d^2*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-c/d^4*(A*d^2-B*c*d+C
*c^2)-b/d^4*c^3*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)+2/5*C/d^2/b*a*c+1/3*(1/d
^2*(B*d-C*c)-4/5*C/d^2*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/d^3*(A*d^2-B*c*d+C*c^2)-b/d^3*c^2*
(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)+3/5*C/d/b*a-2/3*(1/d^2*(B*d-C*c)-4/5*C/d
^2*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*
b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 662, normalized size of antiderivative = 1.31

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(\left(48\,Cb^2c^6-40\,Bb^2c^5d+55\,Babc^3d^3+15\,Ba^2cd^5-30\,(2\,Cab-Ab^2)c^4d^2\right.\right.}{}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
2/45*((48*C*b^2*c^6 - 40*B*b^2*c^5*d + 55*B*a*b*c^3*d^3 + 15*B*a^2*c*d^5 -
 30*(2*C*a*b - A*b^2)*c^4*d^2 - 6*(3*C*a^2 + 10*A*a*b)*c^2*d^4 + (48*C*b^2
*c^5*d - 40*B*b^2*c^4*d^2 + 55*B*a*b*c^2*d^4 + 15*B*a^2*d^6 - 30*(2*C*a*b
- A*b^2)*c^3*d^3 - 6*(3*C*a^2 + 10*A*a*b)*c*d^5)*x)*sqrt(-b*d)*weierstrass
PInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
 1/3*(3*d*x + c)/d) + 3*(48*C*b^2*c^5*d - 40*B*b^2*c^4*d^2 + 25*B*a*b*c^2*
d^4 - 6*(4*C*a*b - 5*A*b^2)*c^3*d^3 - 3*(3*C*a^2 + 5*A*a*b)*c*d^5 + (48*C*
b^2*c^4*d^2 - 40*B*b^2*c^3*d^3 + 25*B*a*b*c*d^5 - 6*(4*C*a*b - 5*A*b^2)*c^
2*d^4 - 3*(3*C*a^2 + 5*A*a*b)*d^6)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^
2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInver
se(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(
3*d*x + c)/d)) + 3*(24*C*b^2*c^4*d^2 - 20*B*b^2*c^3*d^3 + 5*B*a*b*c*d^5 -
3*(3*C*a*b - 5*A*b^2)*c^2*d^4 - 3*(C*b^2*c^2*d^4 - C*a*b*d^6)*x^2 + (6*C*b
^2*c^3*d^3 - 5*B*b^2*c^2*d^4 - 6*C*a*b*c*d^5 + 5*B*a*b*d^6)*x)*sqrt(-b*x^2
 + a)*sqrt(d*x + c))/(b^3*c^3*d^5 - a*b^2*c*d^7 + (b^3*c^2*d^6 - a*b^2*d^8
)*x)
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{x^2\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.194 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 432

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = -\frac{2c(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{d^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3bd^2}$$
$$-\frac{2\sqrt{a}(ad^2(5cC-3Bd)-bc(8c^2C-6Bcd+3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^3(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{2\sqrt{a}(aCd^2+b(8c^2C-6Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{3/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*c*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)
-2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^2-2/3*a^(1/2)*(a*d^2*(-3*B*d+5*C
*c)-b*c*(3*A*d^2-6*B*c*d+8*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/3*a^(1/2)*(a*C*d^2+b*(3*A*d^2-6*B*c*d+8*C*c
^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(3/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.97 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.41

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx =$$

$$-\frac{2\left(-d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(ad^2(-5cC+3Bd)+bc(8c^2C-6Bcd+3Ad^2))\left(a-bx^2\right)+d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(a-bx^2\right)\right.}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*(-(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a*d^2*(-5*c*C + 3*B*d) + b*c*(8
*c^2*C - 6*B*c*d + 3*A*d^2))*(a - b*x^2)) + d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]*(a - b*x^2)*(-(a*C*d^2*(c + d*x)) + b*c*(4*c^2*C - 3*B*c*d + 3*A*d^2
+ c*C*d*x)) - I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^2*(-5*c*C + 3*B*d) +
b*c*(8*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x
)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Elliptic
E[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sq
rt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(a
*C*d^2 - 3*Sqrt[a]*Sqrt[b]*d*(-2*c*C + B*d) + b*(8*c^2*C - 6*B*c*d + 3*A*d
^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
 - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*
d)]))/(3*b*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)*Sqrt[c + d*x
]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2182, 27, 25, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 2182$$

$$\frac{2\int -\frac{-C\left(\frac{bc^2}{d}-ad\right)x^2+\left(Abc+\frac{(cC-Bd)\left(2bc^2-ad^2\right)}{d^2}\right)x+\frac{a\left(Cc^2-Bdc+Ad^2\right)}{d}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -\frac{C\left(\frac{bc^2}{d}-ad\right)x^2-\left(Abc+\frac{(cC-Bd)\left(2bc^2-ad^2\right)}{d^2}\right)x+a\left(-\frac{Cc^2}{d}+Bc-Ad\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{C\left(\frac{bc^2}{d}-ad\right)x^2-\left(Abc+\frac{(cC-Bd)\left(2bc^2-ad^2\right)}{d^2}\right)x+a\left(-\frac{Cc^2}{d}+Bc-Ad\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2185$$

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{2\int \frac{ad\left(aCd^2+b\left(2Cc^2-3Bdc+3Ad^2\right)\right)-b\left(ad^2(5cC-3Bd)-bc\left(8Cc^2-6Bdc+3Ad^2\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\int \frac{ad\left(aCd^2+b\left(2Cc^2-3Bdc+3Ad^2\right)\right)-b\left(ad^2(5cC-3Bd)-bc\left(8Cc^2-6Bdc+3Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 600$$

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{-\frac{\left(bc^2-ad^2\right)\left(aCd^2+b\left(3Ad^2-6Bcd+8c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(ad^2(5cC-3Bd)-bc\left(3Ad^2-6Bcd+8c^2C \cdots\right)\right)}{d}}{3bd^2}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 509$$

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{-\frac{\left(bc^2-ad^2\right)\left(aCd^2+b\left(3Ad^2-6Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(5cC-3Bd)-bc\left(3Ad^2-6B\right.\right.}{d\sqrt{a-bx^2}}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 508

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(5cC-3Bd)-bc\left(3Ad^2-6Bcd+8c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}-\frac{\left(bc^2-ad\right.}{}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 327

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(5cC-3Bd)-bc\left(3Ad^2-6Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}-\frac{\left(bc^2-a\right.}{}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 512

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(5cC-3Bd)-bc\left(3Ad^2-6Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}-\frac{\sqrt{1-\frac{bx}{}}}{}}{3bd^2}}{bc^2-ad^2}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 511

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2+b\left(3Ad^2-6Bcd+8c^2C\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\cdots}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{3\cdots}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{2}{3}C\sqrt{a-bx^2}\sqrt{c+dx}\left(\frac{a}{b}-\frac{c^2}{d^2}\right)-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2+b\left(3Ad^2-6Bcd+8c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\cdots}{3bd^2}}{bc^2-ad^2}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*c*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(d^2*(b*c^2 - a*d^2)*Sqrt[c
 + d*x]) + ((2*C*(a/b - c^2/d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 - ((2*Sq
rt[a]*Sqrt[b]*(a*d^2*(5*c*C - 3*B*d) - b*c*(8*c^2*C - 6*B*c*d + 3*A*d^2))*
Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sq
rt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d
*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2
)*(a*C*d^2 + b*(8*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt
[c + d*x]*Sqrt[a - b*x^2]))/(3*b*d^2))/(b*c^2 - a*d^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(368) = 736.

Time = 5.96 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.77

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2\left(-bd\,x^2+da\right)c\left(A\,d^2-Bcd+C\,c^2\right)}{\left(a\,d^2-b\,c^2\right)d^3\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}-\frac{2C\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3b\,d^2}+\frac{2\left(\frac{A\,d^2-Bcd+C\,c^2}{d^3}+\frac{b\,c^2\left(A\,d^2-Bcd+C\,c^2\right)}{d^3\left(a\,d^2-b\,c^2\right)}+\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input

```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output

```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*(-b*d*x^2+a*d
)/(a*d^2-b*c^2)/d^3*c*(A*d^2-B*c*d+C*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2
/3*C/b/d^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(1/d^3*(A*d^2-B*c*d+C*c^2)
+b/d^3*c^2*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)+1/3*C/d/b*a)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/d^2*(B*
d-C*c)+b/d^2*c*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)-2/3*C/d^2*c)*(c/d-1/b*(a*
b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.24

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left((8\,Cb^2c^5-6\,Bb^2c^4d+12\,Babc^2d^3-(11\,Cab-3\,Ab^2)c^3d^2-3\,(Ca^2+3\,Aab)cd^4+(8\,Cb^2c^4d-6\,Bb^2\right.}{}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
-2/9*((8*C*b^2*c^5 - 6*B*b^2*c^4*d + 12*B*a*b*c^2*d^3 - (11*C*a*b - 3*A*b^
2)*c^3*d^2 - 3*(C*a^2 + 3*A*a*b)*c*d^4 + (8*C*b^2*c^4*d - 6*B*b^2*c^3*d^2
+ 12*B*a*b*c*d^4 - (11*C*a*b - 3*A*b^2)*c^2*d^3 - 3*(C*a^2 + 3*A*a*b)*d^5)
*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(8*C*b^2*c^4*d - 6*B*b^2
*c^3*d^2 + 3*B*a*b*c*d^4 - (5*C*a*b - 3*A*b^2)*c^2*d^3 + (8*C*b^2*c^3*d^2
- 6*B*b^2*c^2*d^3 + 3*B*a*b*d^5 - (5*C*a*b - 3*A*b^2)*c*d^4)*x)*sqrt(-b*d)
*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 -
 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(4*C*b^2*c^3*d^2 - 3*B*b^2*c^
2*d^3 - (C*a*b - 3*A*b^2)*c*d^4 + (C*b^2*c^2*d^3 - C*a*b*d^5)*x)*sqrt(-b*x
^2 + a)*sqrt(d*x + c))/(b^3*c^3*d^4 - a*b^2*c*d^6 + (b^3*c^2*d^5 - a*b^2*d
^7)*x)
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{x(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.195 $\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 367

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{d\,(bc^2-ad^2)\sqrt{c+dx}}$$
$$+\frac{2\sqrt{a}(aCd^2-b(2c^2C-Bcd+Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d^2\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(2cC-Bd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)+2*a^
(1/2)*(a*C*d^2-b*(A*d^2-B*c*d+2*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)
*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(
1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^2/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d
/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2*a^(1/2)*(-B*d+2*C*c)*((d*x+c)/(c+a^(1/
2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)
/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.69 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.45

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-c^2C+Bcd-Ad^2-\frac{-d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(-aCd^2+b\left(2c^2C-Bcd+Ad^2\right)\right)\left(-a+bx^2\right.}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-(c^2*C) + B*c*d - A*d^2 - (-(d^2*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(-(a*C*d^2) + b*(2*c^2*C - B*c*d + A*d^2))*(-a + b*x^2)) + I*Sq
rt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(-(a*C*d^2) + b*(2*c^2*C - B*c*d + A*d^2))*S
qrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x
)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] -
I*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*d - a*C*d + Sqrt[a]*Sqrt[b]*(-2*c
*C + B*d))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/S
qrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sq
rt[a]*d)])/(b*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/((-(b*c^2
*d) + a*d^3)*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2182, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2182}$$

$$\frac{2\int \frac{Abc+aCc-aBd-\left(aCd+b\left(-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{Abc+aCc-aBd-\left(aCd+b\left(-\frac{2Cc^2}{d}+Bc-Ad\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 508}$$

$$\frac{-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{-\frac{(bc^2-ad^2)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{512}$$

$$\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d^2\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{511}$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)(2cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(}{\sqrt{a}}}}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{321}$$

$$\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)(2cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-aC+Ab+\frac{bc(2cC-Bd)}{d^2}\right)E\Big(}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx}{\sqrt{a}d+\sqrt{b}}}}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{d\sqrt{c+dx}\,(bc^2-ad^2)}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(d*(b*c^2 - a*d^2)*Sqrt[c + d*
x]) + ((-2*Sqrt[a]*(A*b - a*C + (b*c*(2*c*C - B*d))/d^2)*Sqrt[c + d*x]*Sqr
t[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[
b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(2*c*C - B*d)*(b*c^2 - a*
d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]
*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*
c)/Sqrt[a] + d)])/(Sqrt[b]*d^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*
d^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(313) = 626$.

Time = 3.66 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2\left(-bd\,x^2+da\right)\left(A\,d^2-Bcd+C\,c^2\right)}{\left(a\,d^2-b\,c^2\right)d^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}+\frac{2\left(\frac{Bd-Cc}{d^2}-\frac{bc\left(A\,d^2-Bcd+C\,c^2\right)}{d^2\left(a\,d^2-b\,c^2\right)}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{}}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x^2+a*
d)/(a*d^2-b*c^2)/d^2*(A*d^2-B*c*d+C*c^2)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*
(1/d^2*(B*d-C*c)-b/d^2*c*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)
^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(C/d-b/d
*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+
a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^
(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2\left(\left(2\,Cbc^4-Bbc^3d+3\,Bacd^3-2\,(2\,Ca+Ab)c^2d^2+\left(2\,Cbc^3d-Bbc^2d^2+3\right.\right.\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
2/3*((2*C*b*c^4 - B*b*c^3*d + 3*B*a*c*d^3 - 2*(2*C*a + A*b)*c^2*d^2 + (2*C
*b*c^3*d - B*b*c^2*d^2 + 3*B*a*d^4 - 2*(2*C*a + A*b)*c*d^3)*x)*sqrt(-b*d)*
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*C*b*c^3*d - B*b*c^2*d^2 - (C*a - A*b
)*c*d^3 + (2*C*b*c^2*d^2 - B*b*c*d^3 - (C*a - A*b)*d^4)*x)*sqrt(-b*d)*weie
rstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^
3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*
c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(C*b*c^2*d^2 - B*b*c*d^3 + A*b*d^4
)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^2*c^3*d^3 - a*b*c*d^5 + (b^2*c^2*d^4
- a*b*d^6)*x)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{(c + dx)^{3/2}\sqrt{a - bx^2}}\, dx = \int \frac{A + Bx + Cx^2}{\sqrt{a - bx^2}\,(c + dx)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{(c + dx)^{3/2}\sqrt{a - bx^2}}\, dx = \int \frac{Cx^2 + Bx + A}{\sqrt{-bx^2 + a}(dx + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{a-b\,x^2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(dx+c)^{\frac{3}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.196 $\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 473

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{c\,(bc^2-ad^2)\sqrt{c+dx}} + \frac{2\sqrt{a}\sqrt{b}(c^2C-Bcd+Ad^2)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{cd\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}C\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c/(-a*d^2+b*c^2)/(d*x+c)^(1/2)+2*a
^(1/2)*b^(1/2)*(A*d^2-B*c*d+C*c^2)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/c/d/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1
/2)/(-b*x^2+a)^(1/2)-2*a^(1/2)*C*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d/(d*x+c)^(1/2)/(-b*x^2
+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*E
llipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.35 (sec) , antiderivative size = 435, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2i\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}\,(c+dx)\left(\sqrt{b}c(c^2C-Bcd+Ad^2)\,E\left(i\,\text{arcsinh}\left(\frac{\sqrt{\cdots}}{\cdots}\right)\right)\cdots\right)}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
((2*I)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[
b] - d*x)/(c + d*x))]*(c + d*x)*(Sqrt[b]*c*(c^2*C - B*c*d + A*d^2)*Ellipti
cE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + S
qrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + d*((Sqrt[b]*c*(B*c - 2*A*d) + Sqrt[a]
*(c^2*C - A*d^2))*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + A*d*(Sqrt[b]
*c + Sqrt[a]*d)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sq
rt[b]*c - Sqrt[a]*d)])))/(c^2*d^2*(Sqrt[b]*c + Sqrt[a]*d)*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 2.43 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.49, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.657, Rules used = {2351, 635, 25, 27, 498, 27, 509, 508, 327, 633, 632, 186, 413, 412, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \; 2351$$

$$A\int \frac{1}{x(c+dx)^{3/2}\sqrt{a-bx^2}}dx+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 635$$

$$A\left(\int -\frac{d}{c(c+dx)^{3/2}\sqrt{a-bx^2}}dx+\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 25$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\int \frac{d}{c(c+dx)^{3/2}\sqrt{a-bx^2}}dx\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 27$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\int \frac{1}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 498$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2b\int -\frac{\sqrt{c+dx}}{2\sqrt{a-bx^2}}dx}{bc^2-ad^2}\right)}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \; 27$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\left(\frac{b\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{509}$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{508}$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}\right)+\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{327}$$

$$A\left(\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}\right)+$$

$$\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{633}$$

$$A\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{c\sqrt{a-bx^2}}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}\right)+$$

$$\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{632}$$

$$A\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{c\sqrt{a-bx^2}}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}\right.$$

$$\int \frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{186}$$

$$A\left(-\frac{2\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c\sqrt{a-bx^2}}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{arcsin}\right.}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{}}\right.}{c}\right.$$

$$\int\frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{413}$$

$$A\left(-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}}{\sqrt{}}\right.}{}\right.$$

$$\int\frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{412}$$

$$\int\frac{B+Cx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx+$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\Big\downarrow \text{688}$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\int\frac{bBc-aCd-b(cC-Bd)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\downarrow$ 27

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{\int\frac{bBc-aCd-b(cC-Bd)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\downarrow$ 600

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{\frac{C(bc^2-ad^2)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b(cC-Bd)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{C(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{bc^2-ad^2}+A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(cC-Bd)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{C(bc^2-ad^2)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}+A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{C\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(cC-Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 512

$$\frac{\frac{C\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(cC-Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}+$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$\Big\downarrow$ 511

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(cC-Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 321}$$

$$A\left(-\frac{d\left(\frac{2d\sqrt{a-bx^2}}{\sqrt{c+dx}(bc^2-ad^2)}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{c}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{c\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{}}{}}}\right.$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(cC-Bd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{}{}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{\sqrt{c+dx}\,(bc^2-ad^2)}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*(c*C - B*d)*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((2*Sqr
t[a]*Sqrt[b]*(c*C - B*d)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) -
(2*Sqrt[a]*C*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]
*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sq
rt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a
- b*x^2]))/(b*c^2 - a*d^2) + A*(-((d*((2*d*Sqrt[a - b*x^2])/((b*c^2 - a*d^
2)*Sqrt[c + d*x]) - (2*Sqrt[a]*Sqrt[b]*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*E
llipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)
/Sqrt[a] + d)])/((b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[a - b*x^2])))/c) - (2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*
d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin
[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a
]*d)])/(c*Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (
Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(398) = 796$.

Time = 3.94 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.93

method	result
elliptic	$\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2\left(-bd\,x^2+da\right)\left(A\,d^2-Bcd+C\,c^2\right)}{\left(a\,d^2-b\,c^2\right)dc\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}+\frac{2\left(\frac{C}{d}+\frac{b\left(A\,d^2-Bcd+C\,c^2\right)}{d\left(a\,d^2-b\,c^2\right)}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{a}}{b}}}}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*(-b*d*x^2+a*d
)/(a*d^2-b*c^2)/d*(A*d^2-B*c*d+C*c^2)/c/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(
C/d+b/d*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*b*(A*d^2-B*c*d+C*c^2)/(a*
d^2-b*c^2)/c*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*(
(x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c
/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(
a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2)))-2*A/c^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1
/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*El
lipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-(C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b*d^2*x^5 + 2*
b*c*d*x^4 - 2*a*c*d*x^2 - a*c^2*x + (b*c^2 - a*d^2)*x^3), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x\,\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{x\,(dx+c)^{\frac{3}{2}}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.197 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 524

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{2d(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{ac^2x}$$
$$-\frac{\sqrt{b}(2ac(cC-Bd)-A(bc^2-3ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}c^2\left(bc^2-ad^2\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{A\sqrt{b}\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}c\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(2Bc-3Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2*d*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)-
A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c^2/x-b^(1/2)*(2*a*c*(-B*d+C*c)-A*(-3*a
*d^2+b*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x
/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a
^(1/2)/c^2/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)
^(1/2)-A*b^(1/2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(-3*A*
d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipt
icPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)
*c+a^(1/2)*d))^(1/2))/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.43 (sec) , antiderivative size = 1699, normalized size of antiderivative = 3.24

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
-((Sqrt[a - b*x^2]*((c*(2*a*c*d*(-(c*C) + B*d)*x + A*b*c^2*(c + d*x) - a*A
*d^2*(c + 3*d*x)))/x + (A*b^2*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 2*a*b*c
^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 2*a*b*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]] - 4*a*A*b*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 2*a^2*c^3*C*d^
2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 2*a^2*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]] + 3*a^2*A*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 2*A*b^2*c^4*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 4*a*b*c^4*C*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]*(c + d*x) - 4*a*b*B*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)
+ 6*a*A*b*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + A*b^2*c^3*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 2*a*b*c^3*C*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(c + d*x)^2 + 2*a*b*B*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c +
 d*x)^2 - 3*a*A*b*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqr
t[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(2*a*c*(-(c*C) + B*d) + A*(b*c^2 - 3*a*d^2)
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
 - I*Sqrt[a]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*c^2*d + a*d^2*(-2*B*c + 3*A*d) +
 2*Sqrt[a]*Sqrt[b]*c*(c^2*C - 2*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b]
 + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)
^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1131 vs. $2(524) = 1048$.

Time = 5.57 (sec) , antiderivative size = 1131, normalized size of antiderivative = 2.16, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {2355, 637, 2009, 2352, 2351, 27, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\sqrt{a-bx^2}}-\frac{d}{c^2x\sqrt{c+dx}\sqrt{a-bx^2}}+\frac{1}{cx^2\sqrt{c+dx}\sqrt{a-bx^2}}\right)dx+\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{2009}$$

$$\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a}\sqrt{b}d^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\right.}{\sqrt{a}c^2\sqrt{a-}}\right.$$

$$\Big\downarrow \text{2352}$$

$$-\frac{\int\frac{b\left(B-\frac{cC}{d}\right)x^2+a\left(B-\frac{3cC}{d}\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a}\sqrt{b}d^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\right.}{\sqrt{a}c^2\sqrt{a-}}\right.$$

$$\frac{\sqrt{a-bx^2}\sqrt{c+dx}(cC-Bd)}{acd^2x}$$

$$\Big\downarrow \text{2351}$$

$$-\frac{a\left(B-\frac{3cC}{d}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{b\left(B-\frac{cC}{d}\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a}\sqrt{b}d^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\right.}{\sqrt{a}c^2\sqrt{a-}}\right.$$

$$\frac{\sqrt{a-bx^2}\sqrt{c+dx}(cC-Bd)}{acd^2x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\left(B-\frac{3cC}{d}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\int \frac{x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a}\sqrt{b}d^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\right.}{\sqrt{a}c^2\sqrt{a-}}\right.$$

$$\frac{\sqrt{a-bx^2}\sqrt{c+dx}(cC-Bd)}{acd^2x}$$

$\downarrow$ 600

$$-\frac{a\left(B-\frac{3cC}{d}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\left(\frac{\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ac}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a}\sqrt{b}d^2\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{c^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\right.}{\sqrt{a}c^2\sqrt{a-}}\right.$$

$$\frac{\sqrt{a-bx^2}\sqrt{c+dx}(cC-Bd)}{acd^2x}$$

$\downarrow$ 509

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.$$

$$\frac{a\left(B-\frac{3cC}{d}\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2ac}$$

$\downarrow$ 508

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)+a\left(B-\frac{3cC}{d}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\left(-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)}{2ac}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)+b\left(B-\frac{cC}{d}\right)\left(-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)+a\left(B-\frac{3cC}{d}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}$$

$$\Big\downarrow \; 512$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\cdots\right)+a\left(B-\frac{3cC}{d}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\left(-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{c\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1\cdots}}\cdots}{d\sqrt{a-bx^2}}\cdots\right)}{2ac}$$

$$\Big\downarrow \; 511$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\cdots\right)+a\left(B-\frac{3cC}{d}\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c\cdots}}{\cdots}\cdots\right)}{2ac}$$

$$\Big\downarrow \; 321$$

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

$\downarrow$ 633

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

$\downarrow$ 632

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

$$\Big\downarrow \textcolor{blue}{186}$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.}{}$$

$$\frac{b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

$$\Big\downarrow \textcolor{blue}{413}$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.}{}$$

$$\frac{b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

$$\Big\downarrow \textcolor{blue}{412}$$

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{acd^2x}+$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\right.$$

$$\frac{b\left(B-\frac{cC}{d}\right)\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{2ac}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*d^2*x) + (A + (c*(c*C - B
*d))/d^2)*((2*d^3*Sqrt[a - b*x^2])/(c^2*(b*c^2 - a*d^2)*Sqrt[c + d*x]) - (
Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c^2*x) + (Sqrt[b]*Sqrt[c + d*x]*Sqrt[1 -
 (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)
/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[a]*c^2*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b
]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*Sqrt[b]*d^2*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(c^2*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c
 + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (Sqrt[b]*Sqrt[(Sqrt[b
]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])
/(Sqrt[a]*c*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (3*d*Sqrt[(Sqrt[b]*(c + d*x))
/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticPi[2, ArcSin[Sqrt[1
- (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(
c^2*Sqrt[c + d*x]*Sqrt[a - b*x^2])) - (b*(B - (c*C)/d)*((-2*Sqrt[a]*Sqrt[c
 + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c +
 d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*Sqrt[(Sqrt
[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcS
in[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(445) = 890$.

Time = 6.61 (sec) , antiderivative size = 960, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2\left(-bd\,x^2+da\right)\left(A\,d^2-Bcd+C\,c^2\right)}{\left(a\,d^2-b\,c^2\right)c^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}-\frac{A\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{c^2ax}-\frac{2b\left(A\,d^2-Bcd+C\,c^2\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{}}{\left(a\,d^2-b\,c^2\right.}\right.}{}$
risch	$-\frac{A\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{a\,c^2x}-\frac{\left(-\frac{(3Ad-2Bc)a\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\mathrm{EllipticPi}\left(\frac{\sqrt{2}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}}{2},2,\sqrt{-\frac{2\sqrt{ab}}{b\left(\frac{c}{d}-\frac{\sqrt{a}}{}\right.}}\right.}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x^2+a*
d)/(a*d^2-b*c^2)*(A*d^2-B*c*d+C*c^2)/c^2/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-A/
c^2/a/x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2*b*(A*d^2-B*c*d+C*c^2)/(a*d^2-
b*c^2)/c*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1
/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2))+2*(-b*d*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)/c^2-1/2*A*b*d/a/c^2
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))
*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))
)+(3*A*d-2*B*c)/c^3*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/
2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*Ell
ipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,
((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \frac{-2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}+\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x}{-b\,d^2x^4-2bcd\,x^3+a\,d^2x^2-b\,c^2x^2+2acdx+a\,c^2}dx\right)bcdx}{}$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(c + d*x)*sqrt(a - b*x**2) + int((sqrt(c + d*x)*sqrt(a - b*x**2)
*x)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**
2*x**4),x)*b*c*d*x + int((sqrt(c + d*x)*sqrt(a - b*x**2)*x)/(a*c**2 + 2*a*
c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*b*d**2*
x**2 - 3*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2*x + 2*a*c*d*x**2 + a
*d**2*x**3 - b*c**2*x**3 - 2*b*c*d*x**4 - b*d**2*x**5),x)*a*c*d*x - 3*int(
(sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2*x + 2*a*c*d*x**2 + a*d**2*x**3 -
b*c**2*x**3 - 2*b*c*d*x**4 - b*d**2*x**5),x)*a*d**2*x**2 + 2*int((sqrt(c +
 d*x)*sqrt(a - b*x**2))/(a*c**2*x + 2*a*c*d*x**2 + a*d**2*x**3 - b*c**2*x*
*3 - 2*b*c*d*x**4 - b*d**2*x**5),x)*b*c**2*x + 2*int((sqrt(c + d*x)*sqrt(a
 - b*x**2))/(a*c**2*x + 2*a*c*d*x**2 + a*d**2*x**3 - b*c**2*x**3 - 2*b*c*d
*x**4 - b*d**2*x**5),x)*b*c*d*x**2 + 2*int((sqrt(c + d*x)*sqrt(a - b*x**2)
)/(a*c**2 + 2*a*c*d*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*
x**4),x)*c**3*x + 2*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**2 + 2*a*c*d
*x + a*d**2*x**2 - b*c**2*x**2 - 2*b*c*d*x**3 - b*d**2*x**4),x)*c**2*d*x**
2)/(2*c*x*(c + d*x))
```

3.198 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 615

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = -\frac{2d^2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{c^3(bc^2-ad^2)\sqrt{c+dx}} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2ac^2x^2} - \frac{(4Bc-7Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4ac^3x} + \frac{\sqrt{b}(bc^2(4Bc-7Ad)+ad(8c^2C-12Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}c^3(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{\sqrt{b}(4Bc-5Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}c^2\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{(4Abc^2+8ac^2C-12aBcd+15aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4ac^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*d^2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c^3/(-a*d^2+b*c^2)/(d*x+c)^(1/
2)-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c^2/x^2-1/4*(-7*A*d+4*B*c)*(d*x+
c)^(1/2)*(-b*x^2+a)^(1/2)/a/c^3/x+1/4*b^(1/2)*(b*c^2*(-7*A*d+4*B*c)+a*d*(1
5*A*d^2-12*B*c*d+8*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/
2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2
)*d))^(1/2))/a^(1/2)/c^3/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1
/2)/(-b*x^2+a)^(1/2)-1/4*b^(1/2)*(-5*A*d+4*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c^2/(d*x
+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(15*A*a*d^2+4*A*b*c^2-12*B*a*c*d+8*C*a*c^2)
*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/a/c^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.46 (sec) , antiderivative size = 2647, normalized size of antiderivative = 4.30

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-1/2*A/(a*c^2*x^2) + (-4*B*c + 7*A*d)/(4*a*
c^3*x) - (2*d^2*(c^2*C - B*c*d + A*d^2))/(c^3*(b*c^2 - a*d^2)*(c + d*x)))
- (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-4*b^2*B*c^4*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]] + 7*A*b^2*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
- 8*a*b*c^3*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 12*a*b*B*c^2*d^2*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]] - 15*a*A*b*c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - (
4*b^2*B*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (7*A*b^2*c^5*d*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (8*a*b*c^5*C*d*Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]])/(c + d*x)^2 + (16*a*b*B*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]])/(c + d*x)^2 - (22*a*A*b*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c
 + d*x)^2 + (8*a^2*c^3*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 -
 (12*a^2*B*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (15*a^2*A
*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (8*b^2*B*c^5*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (14*A*b^2*c^4*d*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]])/(c + d*x) + (16*a*b*c^4*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c +
 d*x) - (24*a*b*B*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (30*
a*A*b*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (I*Sqrt[b]*c*(Sq
rt[b]*c - Sqrt[a]*d)*(b*c^2*(4*B*c - 7*A*d) + a*d*(8*c^2*C - 12*B*c*d + 15
*A*d^2))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 -
c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1401 vs. $2(615) = 1230$.

Time = 8.58 (sec) , antiderivative size = 1401, normalized size of antiderivative = 2.28, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.543$, Rules used = {2355, 637, 2009, 2352, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \; 2355$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\sqrt{a-bx^2}}+\frac{d^2}{c^3x\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{d}{c^2x^2\sqrt{c+dx}\sqrt{a-bx^2}}+\frac{}{cx^3\sqrt{c+}}\right.$$
$$\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$\downarrow$ 2009

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.$$
$$\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$\downarrow$ 2352

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.$$
$$\frac{\int\frac{-b\left(B-\frac{cC}{d}\right)x^2+\frac{2bc(cC-Bd)x}{d^2}+a\left(3B-\frac{7cC}{d}\right)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}$$

$\downarrow$ 2352

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.$$
$$\frac{-\frac{\sqrt{c+dx}\sqrt{a-bx^2}\left(3B-\frac{7cC}{d}\right)}{cx}-\frac{\int-\frac{ab(7cC-3Bd)x^2-2abc\left(B-\frac{cC}{d}\right)x+a\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}}{4ac}$$

$\downarrow$ 25

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}\frac{\int\frac{ab(7cC-3Bd)x^2-2abc\left(B-\frac{cC}{d}\right)x+a\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\left(3B-\frac{7cC}{d}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{cx}}{4ac}$$

$\downarrow$ 2351

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}\frac{a\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{\frac{2abCc^2}{d}-2abBc+ab(7cC-3Bd)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\left(3B-\frac{7cC}{d}\right)\sqrt{c+dx}\sqrt{a-bx^2}}{cx}}{4ac}$$

$\downarrow$ 600

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}\frac{-\frac{abc(5cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+a\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ab(7cC-3Bd)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{2ac}-\frac{\left(3B-\frac{7cC}{d}\right)\sqrt{c+dx}\sqrt{a}}{cx}}{4ac}$$

$\downarrow$ 509

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{}$$

$$\frac{-\frac{abc(5cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+a\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ab(7cC-3Bd)\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{2ac}-\frac{\left(3B-\frac{7cC}{d}\right)\ldots}{\ldots}$$

$$4ac$$

$\downarrow$ 508

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{bc(5cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dxa}{d}+\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{1}{x\sqrt{c+dx}\ldots}}{2ac}$$

$$4ac$$

$\downarrow$ 327

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.}{}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{bc(5cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dxa}{d}+\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{\ldots}{x\sqrt{c+dx}\ldots}}{2ac}$$

$$4ac$$

$\downarrow$ 512

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \cdots\right) \quad -\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\left(\frac{4b(cC-Bd)c^2}{d^2}+a(7cC-3Bd)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dxa-\frac{bc(5cC-Bd)\sqrt{1-\frac{bx^2}{a}}\cdots}{d\sqrt{c}\cdots}}{2ac}}{4ac}$$

$\downarrow$ 511

$$\frac{\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} \cdots\right) \quad -\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\sqrt{1-\cdots}\cdots}{d\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4ac}$$

$\downarrow$ 321

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}$$

$$4ac$$

$\downarrow$ 633

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}$$

$$4ac$$

$\downarrow$ 632

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}$$

$$4ac$$

$\downarrow$ 186

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\cdots\right)\;\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}\cdots}{d\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}}{4ac}$$

$\downarrow$ 413

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\cdots\right)\;\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}\cdots}{d\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}}{4ac}$$

$\downarrow$ 412

$$\frac{\sqrt{c+dx}\sqrt{a-bx^2}(cC-Bd)}{2acd^2x^2}+\frac{\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \cdots\right.}{\cdots}$$

$$\frac{-\frac{2\sqrt{b}(7cC-3Bd)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}c(5cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{d\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}$$

$$\cdots\ \overline{\qquad 4ac \qquad}$$

input

```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^(3/2)*Sqrt[a - b*x^2]),x]
```

output

```
((c*C - B*d)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(2*a*c*d^2*x^2) + (A + (c*(c*C
 - B*d))/d^2)*((-2*d^4*Sqrt[a - b*x^2])/(c^3*(b*c^2 - a*d^2)*Sqrt[c + d*x]
) - (Sqrt[c + d*x]*Sqrt[a - b*x^2])/(2*a*c^2*x^2) + (7*d*Sqrt[c + d*x]*Sqr
t[a - b*x^2])/(4*a*c^3*x) - (7*Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*
c)/Sqrt[a] + d)])/(4*Sqrt[a]*c^3*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*Sqrt[b]*d^3*Sqrt[c + d*x]*Sqrt[1 -
(b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/
((Sqrt[b]*c)/Sqrt[a] + d)])/(c^3*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/
(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (5*Sqrt[b]*d*Sqrt[(Sqrt[b]*(c
+ d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt
[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(4*S
qrt[a]*c^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (3*d^2*Sqrt[(Sqrt[b]*(c + d*x)
)/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticPi[2, ArcSin[Sqrt[1
 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/
(c^3*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((4*b*c^2 + 3*a*d^2)*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticPi[2, ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[
a]*d)])/(4*a*c^3*Sqrt[c + d*x]*Sqrt[a - b*x^2])) - (-(((3*B - (7*c*C)/d)*S
qrt[c + d*x]*Sqrt[a - b*x^2])/(c*x)) + ((-2*a^(3/2)*Sqrt[b]*(7*c*C - 3*...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1050 vs. $2(522) = 1044$.

Time = 7.41 (sec) , antiderivative size = 1051, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	1051
risch	Expression too large to display	1420
default	Expression too large to display	5536

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*(-b*d*x^2+a*d
)/(a*d^2-b*c^2)*d*(A*d^2-B*c*d+C*c^2)/c^3/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-1
/2*A/c^2/a/x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4*(7*A*d-4*B*c)/a/c^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(b*d*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c
^2)/c^2+1/4*A*b*d/a/c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b
)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*
EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(d^2*b*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)/c^
3+1/8*(7*A*d-4*B*c)*d/a*b/c^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)
*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(
-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4*(15*A*a*d^2+4*A*b*c^2-12*B*a*c*d+8*C*a*
c^2)/a/c^4*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi((
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}\,(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(a - b*x**2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\,(dx+c)^{\frac{3}{2}}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(1/2),x)
```

3.199 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 681

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = -\frac{2c^3(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3d^4(bc^2-ad^2)(c+dx)^{3/2}}$$

$$-\frac{2c^2(3ad^2(5c^2C-4Bcd+3Ad^2)-bc^2(11c^2C-8Bcd+5Ad^2))\sqrt{a-bx^2}}{3d^4(bc^2-ad^2)^2\sqrt{c+dx}}$$

$$+\frac{2(17cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15bd^4}-\frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5bd^4}$$

$$-\frac{2\sqrt{a}(9a^3Cd^6+5a^2bd^4(11c^2C-8Bcd+3Ad^2)+8b^3c^4(16c^2C-10Bcd+5Ad^2)-ab^2c^2d^2(212c^2C-14}{15b^{3/2}d^5(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx}}$$

$$-\frac{2\sqrt{a}(a^2d^4(17cC-5Bd)-8b^2c^3(16c^2C-10Bcd+5Ad^2)+abcd^2(116c^2C-80Bcd+45Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}}{15b^{3/2}d^5(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*c^3*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(
3/2)-2/3*c^2*(3*a*d^2*(3*A*d^2-4*B*c*d+5*C*c^2)-b*c^2*(5*A*d^2-8*B*c*d+11*
C*c^2))*(-b*x^2+a)^(1/2)/d^4/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)+2/15*(-5*B*d+1
7*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^4-2/5*C*(d*x+c)^(3/2)*(-b*x^2+a)
^(1/2)/b/d^4-2/15*a^(1/2)*(9*a^3*C*d^6+5*a^2*b*d^4*(3*A*d^2-8*B*c*d+11*C*c
^2)+8*b^3*c^4*(5*A*d^2-10*B*c*d+16*C*c^2)-a*b^2*c^2*d^2*(75*A*d^2-140*B*c*
d+212*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
b^(3/2)/d^5/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2
+a)^(1/2)-2/15*a^(1/2)*(a^2*d^4*(-5*B*d+17*C*c)-8*b^2*c^3*(5*A*d^2-10*B*c*
d+16*C*c^2)+a*b*c*d^2*(45*A*d^2-80*B*c*d+116*C*c^2))*((d*x+c)/(c+a^(1/2)*d
/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^5
/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.74 (sec) , antiderivative size = 921, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{2(c+dx)\left(\frac{14cC-5Bd}{b}-\frac{3Cdx}{b}+\frac{5c^3\left(c^2C-Bcd+Ad^2\right)}{\left(-bc^2+ad^2\right)(c+dx)^2}+\frac{5c^2\left(-3ad^2\left(5c^2C-4Bcd+3Ad^2\right)+bc^2\left(1\cdots\right.\right.}{\left(bc^2-ad^2\right)^2(c+dx)\cdots}\right)}{d^4}\cdots\right)}{\cdots}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*((2*(c + d*x)*((14*c*C - 5*B*d)/b - (3*C*d*x)/b + (5*c^3*
(c^2*C - B*c*d + A*d^2))/((-(b*c^2) + a*d^2)*(c + d*x)^2) + (5*c^2*(-3*a*d
^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) + b*c^2*(11*c^2*C - 8*B*c*d + 5*A*d^2)))/
((b*c^2 - a*d^2)^2*(c + d*x))))/d^4 + (2*(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(9*a^3*C*d^6 + a*b^2*c^2*d^2*(-212*c^2*C + 140*B*c*d - 75*A*d^2) + 5*a^
2*b*d^4*(11*c^2*C - 8*B*c*d + 3*A*d^2) + 8*b^3*c^4*(16*c^2*C - 10*B*c*d +
5*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(9*a^3*C*d^6 + a
*b^2*c^2*d^2*(-212*c^2*C + 140*B*c*d - 75*A*d^2) + 5*a^2*b*d^4*(11*c^2*C -
 8*B*c*d + 3*A*d^2) + 8*b^3*c^4*(16*c^2*C - 10*B*c*d + 5*A*d^2))*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a
]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(9*a^(5/2)*C*d^5 + a^2*Sqrt[b]*d^4*(17
*c*C - 5*B*d) + 3*a^(3/2)*b*d^3*(24*c^2*C - 15*B*c*d + 5*A*d^2) - 8*b^(5/2
)*c^3*(16*c^2*C - 10*B*c*d + 5*A*d^2) - 6*Sqrt[a]*b^2*c^2*d*(16*c^2*C - 10
*B*c*d + 5*A*d^2) + a*b^(3/2)*c*d^2*(116*c^2*C - 80*B*c*d + 45*A*d^2))*Sqr
t[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/
(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b
^2*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)^2*(-a + b*x^2)))...
```

Rubi [A] (verified)

Time = 4.14 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2182, 27, 2182, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a-bx^2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int -\frac{-3C\left(\frac{bc^2}{d}-ad\right)x^4+\frac{3(cC-Bd)\left(bc^2-ad^2\right)x^3}{d^2}-\frac{3\left(bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}+\frac{c\left(2bc^2-3ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{3ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-3C\left(\frac{bc^2}{d}-ad\right)x^4+\frac{3(cC-Bd)\left(bc^2-ad^2\right)x^3}{d^2}-\frac{3\left(bc^2-ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x^2}{d^3}+\frac{c\left(2bc^2-3ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^4}+\frac{3ac^2\left(Cc^2-Bdc+Ad^2\right)}{d^3}}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{2182}$$

$$-\frac{\frac{2\int -\frac{\frac{3C\left(bc^2-ad^2\right)^2x^3}{d^2}-\frac{3(2cC-Bd)\left(bc^2-ad^2\right)^2x^2}{d^3}+\frac{\left(2b^2\left(10Cc^2-7Bdc+4Ad^2\right)c^4-3abd^2\left(11Cc^2-8Bdc+5Ad^2\right)c^2+3a^2d^4\left(3Cc^2-2Bdc+Ad^2\right)\right)x}{d^4}+\frac{ac(b}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\int \frac{3C\left(bc^2-ad^2\right)^2x^3}{d^2}-\frac{3(2cC-Bd)\left(bc^2-ad^2\right)^2x^2}{d^3}+\frac{\left(2b^2\left(10Cc^2-7Bdc+4A\right.\right.}{}}{}}{3\left(bc^2-ad^2\right)}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{2\int -\frac{-3b(17cC-5Bd)\left(bc^2-ad^2\right)^2x^2+\frac{\left(9a^3Cd^6+3a^2b\left(7Cc^2-10Bdc+5A\right.\right.}{}}{}}{}}{}-\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\int \frac{-3b(17cC-5Bd)\left(bc^2-ad^2\right)^2x^2+\frac{\left(9a^3Cd^6+3a^2b\left(7Cc^2-10Bdc+5Ad^2\right)d^{\cdots}\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 2185

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{2\int \frac{3bd\left(ad\left(a^2(8cC-5Bd)d^4+\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 27

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{\int \frac{ad\left(a^2(8cC-5Bd)d^4+abc\left(44\cdots\right.\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 600

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{-\frac{\left(bc^2-ad^2\right)\left(a^2d^4(17cC-5B\cdots\right.}{\cdots}}{\cdots}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 509

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{-\frac{\left(bc^2-ad^2\right)\left(a^2d^4(17cC-5Bc\right.}{}}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^3Cd^6\right.}{}}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 327

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^3Cd^6\right.}{}}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^3Cd^6\right.}{}}{}}{}}{}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 511

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}(bc^2-ad^2)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$-\frac{\frac{2c^2\sqrt{a-bx^2}\left(3ad^2\left(3Ad^2-4Bcd+5c^2C\right)-bc^2\left(5Ad^2-8Bcd+11c^2C\right)\right)}{d^4\sqrt{c+dx}(bc^2-ad^2)}-\frac{\frac{2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(17cC-5Bd)}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2c^3\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^4(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*c^3*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(3*d^4*(b*c^2 - a*d^2)*(c
 + d*x)^(3/2)) - ((2*c^2*(3*a*d^2*(5*c^2*C - 4*B*c*d + 3*A*d^2) - b*c^2*(1
1*c^2*C - 8*B*c*d + 5*A*d^2))*Sqrt[a - b*x^2])/(d^4*(b*c^2 - a*d^2)*Sqrt[c
 + d*x]) - ((-6*C*(b*c^2 - a*d^2)^2*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b*
d^4) + ((2*(17*c*C - 5*B*d)*(b*c^2 - a*d^2)^2*Sqrt[c + d*x]*Sqrt[a - b*x^2
])/d - ((2*Sqrt[a]*(9*a^3*C*d^6 + 5*a^2*b*d^4*(11*c^2*C - 8*B*c*d + 3*A*d^
2) + 8*b^3*c^4*(16*c^2*C - 10*B*c*d + 5*A*d^2) - a*b^2*c^2*d^2*(212*c^2*C
- 140*B*c*d + 75*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSi
n[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)]
)/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*
x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(a^2*d^4*(17*c*C - 5*B*d) - 8*b^2*c^3*(
16*c^2*C - 10*B*c*d + 5*A*d^2) + a*b*c*d^2*(116*c^2*C - 80*B*c*d + 45*A*d^
2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*
EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c
)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/d)/(5*b*d^3))/
(b*c^2 - a*d^2))/(3*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 11.75 (sec) , antiderivative size = 1106, normalized size of antiderivative = 1.62

method	result	size
elliptic	Expression too large to display	1106
risch	Expression too large to display	1651
default	Expression too large to display	10668

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/(a*d^2-b*c^
2)/d^6*c^3*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^
2-2/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d^5*c^2*(9*A*a*d^4-5*A*b*c^2*d^2-12*B
*a*c*d^3+8*B*b*c^3*d+15*C*a*c^2*d^2-11*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(
1/2)-2/5*C/d^3/b*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(1/d^3*(B*d-2*C*
c)-4/5*C/d^3*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-c*(2*A*d^2-3*B*
c*d+4*C*c^2)/d^5-1/3*b/d^5*c^3*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)-1/3*b/d^5
*c^3*(9*A*a*d^4-5*A*b*c^2*d^2-12*B*a*c*d^3+8*B*b*c^3*d+15*C*a*c^2*d^2-11*C
*b*c^4)/(a*d^2-b*c^2)^2+2/5*C/d^3/b*a*c+1/3*(1/d^3*(B*d-2*C*c)-4/5*C/d^3*c
)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b
*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2
)))^(1/2))+2*(1/d^4*(A*d^2-2*B*c*d+3*C*c^2)-1/3*b/d^4*c^2*(9*A*a*d^4-5*A*b
*c^2*d^2-12*B*a*c*d^3+8*B*b*c^3*d+15*C*a*c^2*d^2-11*C*b*c^4)/(a*d^2-b*c^2)
^2+3/5*C/d^2/b*a-2/3*(1/d^3*(B*d-2*C*c)-4/5*C/d^3*c)/d*c)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(606) = 1212$.

Time = 0.15 (sec) , antiderivative size = 1336, normalized size of antiderivative = 1.96

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
2/45*((128*C*b^3*c^9 - 80*B*b^3*c^8*d + 200*B*a*b^2*c^6*d^3 - 145*B*a^2*b*
c^4*d^5 - 15*B*a^3*c^2*d^7 - 4*(77*C*a*b^2 - 10*A*b^3)*c^7*d^2 + (187*C*a^
2*b - 105*A*a*b^2)*c^5*d^4 + 3*(11*C*a^3 + 35*A*a^2*b)*c^3*d^6 + (128*C*b^
3*c^7*d^2 - 80*B*b^3*c^6*d^3 + 200*B*a*b^2*c^4*d^5 - 145*B*a^2*b*c^2*d^7 -
 15*B*a^3*d^9 - 4*(77*C*a*b^2 - 10*A*b^3)*c^5*d^4 + (187*C*a^2*b - 105*A*a
*b^2)*c^3*d^6 + 3*(11*C*a^3 + 35*A*a^2*b)*c*d^8)*x^2 + 2*(128*C*b^3*c^8*d
- 80*B*b^3*c^7*d^2 + 200*B*a*b^2*c^5*d^4 - 145*B*a^2*b*c^3*d^6 - 15*B*a^3*
c*d^8 - 4*(77*C*a*b^2 - 10*A*b^3)*c^6*d^3 + (187*C*a^2*b - 105*A*a*b^2)*c^
4*d^5 + 3*(11*C*a^3 + 35*A*a^2*b)*c^2*d^7)*x)*sqrt(-b*d)*weierstrassPInver
se(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(
3*d*x + c)/d) + 3*(128*C*b^3*c^8*d - 80*B*b^3*c^7*d^2 + 140*B*a*b^2*c^5*d^
4 - 40*B*a^2*b*c^3*d^6 - 4*(53*C*a*b^2 - 10*A*b^3)*c^6*d^3 + 5*(11*C*a^2*b
 - 15*A*a*b^2)*c^4*d^5 + 3*(3*C*a^3 + 5*A*a^2*b)*c^2*d^7 + (128*C*b^3*c^6*
d^3 - 80*B*b^3*c^5*d^4 + 140*B*a*b^2*c^3*d^6 - 40*B*a^2*b*c*d^8 - 4*(53*C*
a*b^2 - 10*A*b^3)*c^4*d^5 + 5*(11*C*a^2*b - 15*A*a*b^2)*c^2*d^7 + 3*(3*C*a
^3 + 5*A*a^2*b)*d^9)*x^2 + 2*(128*C*b^3*c^7*d^2 - 80*B*b^3*c^6*d^3 + 140*B
*a*b^2*c^4*d^5 - 40*B*a^2*b*c^2*d^7 - 4*(53*C*a*b^2 - 10*A*b^3)*c^5*d^4 +
5*(11*C*a^2*b - 15*A*a*b^2)*c^3*d^6 + 3*(3*C*a^3 + 5*A*a^2*b)*c*d^8)*x)*sq
rt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a
*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/...
```

Sympy [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^3(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^3\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,{(c+d\,x)}^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.200 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 603

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\frac{2c^2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3d^3(bc^2-ad^2)(c+dx)^{3/2}}$$
$$-\frac{2c(bc^2(8c^2C-5Bcd+2Ad^2)-3ad^2(4c^2C-3Bcd+2Ad^2))\sqrt{a-bx^2}}{3d^3(bc^2-ad^2)^2\sqrt{c+dx}}$$
$$-\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3bd^3}$$
$$+\frac{2\sqrt{a}(a^2d^4(8cC-3Bd)+2b^2c^3(8c^2C-4Bcd+Ad^2)-abcd^2(28c^2C-15Bcd+6Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}}{3\sqrt{b}d^4(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(a^2Cd^4-2b^2c^2(8c^2C-4Bcd+Ad^2)+abd^2(16c^2C-9Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\mathrm{ar}\right.}{3b^{3/2}d^4(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*c^2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(3
/2)-2/3*c*(b*c^2*(2*A*d^2-5*B*c*d+8*C*c^2)-3*a*d^2*(2*A*d^2-3*B*c*d+4*C*c^
2))*(-b*x^2+a)^(1/2)/d^3/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/3*C*(d*x+c)^(1/2
)*(-b*x^2+a)^(1/2)/b/d^3+2/3*a^(1/2)*(a^2*d^4*(-3*B*d+8*C*c)+2*b^2*c^3*(A*
d^2-4*B*c*d+8*C*c^2)-a*b*c*d^2*(6*A*d^2-15*B*c*d+28*C*c^2))*(d*x+c)^(1/2)*
((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(
1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^4/(-a*d^2+b*c^2)^2
/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2/3*a^(1/2)*(a^2*C
*d^4-2*b^2*c^2*(A*d^2-4*B*c*d+8*C*c^2)+a*b*d^2*(3*A*d^2-9*B*c*d+16*C*c^2))
*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*
(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*
d))^(1/2))/b^(3/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.81 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.32

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(a^2d^4(8cC-3Bd)+abcd^2(-28c^2C+15Bcd-6Ad^2)+2b^2c^3(8c\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output

```
(2*Sqrt[a - b*x^2]*(a^2*d^4*(8*c*C - 3*B*d) + a*b*c*d^2*(-28*c^2*C + 15*B*
c*d - 6*A*d^2) + 2*b^2*c^3*(8*c^2*C - 4*B*c*d + A*d^2) - b*c*(b*c^2*(8*c^2
*C - 5*B*c*d + 2*A*d^2) - 3*a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2)) + (b*c^2*
(b*c^2 - a*d^2)*(c^2*C - B*c*d + A*d^2))/(c + d*x) - C*(b*c^2 - a*d^2)^2*(
c + d*x) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a^2*d^4*(8*c*C - 3*B*d) + a
*b*c*d^2*(-28*c^2*C + 15*B*c*d - 6*A*d^2) + 2*b^2*c^3*(8*c^2*C - 4*B*c*d +
 A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqr
t[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt
[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[a]*(S
qrt[b]*c - Sqrt[a]*d)*(-(a^2*C*d^4) + 3*a^(3/2)*Sqrt[b]*d^3*(-3*c*C + B*d)
 + a*b*d^2*(-16*c^2*C + 9*B*c*d - 3*A*d^2) + 2*b^2*c^2*(8*c^2*C - 4*B*c*d
+ A*d^2) + 3*Sqrt[a]*b^(3/2)*c*d*(4*c^2*C - 2*B*c*d + A*d^2))*Sqrt[(d*(Sqr
t[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x)
)]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt
[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*b*d^3*(b*c^2 - a*d^2)^2*Sqrt[c +
d*x])
```

Rubi [A] (verified)

Time = 3.11 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2182, 27, 2182, 27, 25, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+B x+C x^2\right)}{\sqrt{a-b x^2} (c+d x)^{5/2}} \, dx$$

$$\Big\downarrow \textbf{2182}$$

$$\frac{2 \int \frac{3 C \left(\frac{b c^2}{d}-a d\right) x^3-\frac{3 (c C-B d) \left(b c^2-a d^2\right) x^2}{d^2}+\frac{\left(2 b c^2-3 a d^2\right) \left(C c^2-B d c+A d^2\right) x}{d^3}+\frac{3 a c \left(C c^2-B d c+A d^2\right)}{d^2}}{2 (c+d x)^{3/2} \sqrt{a-b x^2}}dx}{3 \left(b c^2-a d^2\right)}+\frac{2 c^2 \sqrt{a-b x^2} \left(A d^2-B c d+c^2 C\right)}{3 d^3 (c+d x)^{3/2} \left(b c^2-a d^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{3C\left(\frac{bc^2}{d}-ad\right)x^3-\frac{3(cC-Bd)\left(bc^2-ad^2\right)x^2}{d^2}+\frac{\left(2bc^2-3ad^2\right)\left(Cc^2-Bdc+Ad^2\right)x}{d^3}+\frac{3ac\left(Cc^2-Bdc+Ad^2\right)}{d^2}}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}+\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2182

$$\frac{\frac{2\int -\frac{-\frac{3C\left(bc^2-ad^2\right)^2x^2}{d^2}+\frac{\left(3a^2(2cC-Bd)d^4-3abc\left(8Cc^2-5Bdc+2Ad^2\right)d^2+2b^2c^3\left(7Cc^2-4Bdc+Ad^2\right)\right)x}{d^3}+\frac{a\left(bc^2\left(5Cc^2-2Bdc-Ad^2\right)-3ad^2\left(3Cc^2-2Bdc+A\right.\right.}{d^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{\int -\frac{\frac{3C\left(bc^2-ad^2\right)^2x^2}{d^2}-\frac{\left(3a^2(2cC-Bd)d^4-3abc\left(8Cc^2-5Bdc+2Ad^2\right)d^2+2b^2c^3\left(7Cc^2-4Bdc+Ad^2\right)\right)x}{d^3}+a\left(-\frac{b(5cC-2Bd)c^3}{d^2}+a(9cC-6Bd)c+A\left(bc^2+3ad^2\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\frac{3C\left(bc^2-ad^2\right)^2x^2}{d^2}-\frac{\left(3a^2(2cC-Bd)d^4-3abc\left(8Cc^2-5Bdc+2Ad^2\right)d^2+2b^2c^3\left(7Cc^2-4Bdc+Ad^2\right)\right)x}{d^3}+a\left(-\frac{b(5cC-2Bd)c^3}{d^2}+a(9cC-6Bd)c+A\left(bc^2+3ad^2\right)\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}d}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$\frac{-\frac{\frac{2\int -\frac{3\left(ad\left(a^2Cd^4+ab\left(7Cc^2-6Bdc+3Ad^2\right)d^2-b^2c^2\left(4Cc^2-2Bdc-Ad^2\right)\right)-b\left(a^2(8cC-3Bd)d^4-abc\left(28Cc^2-15Bdc+6Ad^2\right)d^2+2b^2c^3\left(8Cc^2-4Bdc+Ad^2\right)\right)x\right.}{2d\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{ad\left(a^2Cd^4+ab\left(7Cc^2-6Bdc+3Ad^2\right)d^2-b^2c^2\left(4Cc^2-2Bdc-Ad^2\right)\right)-b\left(a^2(8cC-3Bd)d^4-abc\left(28Cc^2-15Bdc+6Ad^2\right)d^2+2b^2c^3\left(8Cc^2-4Bdc+Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bd^3}-\frac{2C}{}}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{-\frac{\left(bc^2-ad^2\right)\left(a^2Cd^4+abd^2\left(3Ad^2-9Bcd+16c^2C\right)-2b^2c^2\left(Ad^2-4Bcd+8c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(a^2d^4(8cC-3Bd)-abcd^2\left(6Ad^2-15Bcd+28c^2C\right)+2b^2}{d}}{bd^3}}{bc^2-ad^2}}{3\left(bc^2-a\right.}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{-\frac{\left(bc^2-ad^2\right)\left(a^2Cd^4+abd^2\left(3Ad^2-9Bcd+16c^2C\right)-2b^2c^2\left(Ad^2-4Bcd+8c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(a^2d^4(8cC-3Bd)-abcd^2\left(6Ad^2-15Bcd+28c\right.\right.}{d\sqrt{a-bx}}}{bd^3}}{bc^2-ad^2}}{3\left(bc^2\right.}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(8cC-3Bd)-abcd^2\left(6Ad^2-15Bcd+28c^2C\right)+2b^2c^3\left(Ad^2-4Bcd+8c^2C\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(a^2Cd^4+\right.}{}}{bd^3}}{bc^2-ad^2}$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(8cC-3Bd)-abcd^2\left(6Ad^2-15Bcd+28c^2C\right)+2b^2c^3\left(Ad^2-4Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(a^2Cd^4\cdots\right.}{\cdots}}{bd^3}}{bc^2-ad^2}\cdots$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 512

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(8cC-3Bd)-abcd^2\left(6Ad^2-15Bcd+28c^2C\right)+2b^2c^3\left(Ad^2-4Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\cdots}{\cdots}}{bd^3}}{bc^2-ad^2}\cdots$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 511

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(a^2Cd^4+abd^2\left(3Ad^2-9Bcd+16c^2C\right)-2b^2c^2\left(Ad^2-4Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{\cdots}}{\cdots}}{bd^3}}{\cdots}\cdots$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(a^2Cd^4+abd^2\left(3Ad^2-9Bcd+16c^2C\right)-2b^2c^2\left(Ad^2-4Bcd+8c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1\cdots}}{\cdots}}{bd^3}}{bc^2\cdots}\cdots$$

$$\frac{2c^2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*c^2*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(3*d^3*(b*c^2 - a*d^2)*(c
+ d*x)^(3/2)) + ((-2*c*(b*c^2*(8*c^2*C - 5*B*c*d + 2*A*d^2) - 3*a*d^2*(4*c
^2*C - 3*B*c*d + 2*A*d^2))*Sqrt[a - b*x^2])/(d^3*(b*c^2 - a*d^2)*Sqrt[c +
d*x]) + ((-2*C*(b*c^2 - a*d^2)^2*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(b*d^3) +
((2*Sqrt[a]*Sqrt[b]*(a^2*d^4*(8*c*C - 3*B*d) + 2*b^2*c^3*(8*c^2*C - 4*B*c*
d + A*d^2) - a*b*c*d^2*(28*c^2*C - 15*B*c*d + 6*A*d^2))*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(a^2*C*d^4 - 2*b
^2*c^2*(8*c^2*C - 4*B*c*d + A*d^2) + a*b*d^2*(16*c^2*C - 9*B*c*d + 3*A*d^2
))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)
/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*d^3))/(b*c^2
 - a*d^2))/(3*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 10.18 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.63

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2c^2\left(A\,d^2-Bcd+C\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3\left(a\,d^2-b\,c^2\right)d^5\left(x+\frac{c}{d}\right)^2}+\frac{2\left(-bd\,x^2+da\right)c\left(6Aa\,d^4-2Ab\,c^2d^2-9Bac\,d^3+5c^3Bbd+12Ca\,c^2d^2\right.}{3\left(a\,d^2-b\,c^2\right)^2d^4\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/(a*d^2-b*c
^2)/d^5*c^2*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)
^2+2/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d^4*c*(6*A*a*d^4-2*A*b*c^2*d^2-9*B*a
*c*d^3+5*B*b*c^3*d+12*C*a*c^2*d^2-8*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1/2
)-2/3*C/b/d^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(1/d^4*(A*d^2-2*B*c*d+3
*C*c^2)+1/3*b/d^4*c^2*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)+1/3*b/d^4*c^2*(6*A
*a*d^4-2*A*b*c^2*d^2-9*B*a*c*d^3+5*B*b*c^3*d+12*C*a*c^2*d^2-8*C*b*c^4)/(a*
d^2-b*c^2)^2+1/3*C/d^2/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/d^3*(B*d-2*C*c)+1/3*b/d^3*c*(6*A*a*d^
4-2*A*b*c^2*d^2-9*B*a*c*d^3+5*B*b*c^3*d+12*C*a*c^2*d^2-8*C*b*c^4)/(a*d^2-b
*c^2)^2-2/3*C/d^3*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1
/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c
/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d
+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF
(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(535) = 1070$.

Time = 0.13 (sec) , antiderivative size = 1125, normalized size of antiderivative = 1.87

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
-2/9*((16*C*b^3*c^8 - 8*B*b^3*c^7*d + 21*B*a*b^2*c^5*d^3 - 21*B*a^2*b*c^3*
d^5 - 2*(20*C*a*b^2 - A*b^3)*c^6*d^2 + (29*C*a^2*b - 3*A*a*b^2)*c^4*d^4 +
3*(C*a^3 + 3*A*a^2*b)*c^2*d^6 + (16*C*b^3*c^6*d^2 - 8*B*b^3*c^5*d^3 + 21*B
*a*b^2*c^3*d^5 - 21*B*a^2*b*c*d^7 - 2*(20*C*a*b^2 - A*b^3)*c^4*d^4 + (29*C
*a^2*b - 3*A*a*b^2)*c^2*d^6 + 3*(C*a^3 + 3*A*a^2*b)*d^8)*x^2 + 2*(16*C*b^3
*c^7*d - 8*B*b^3*c^6*d^2 + 21*B*a*b^2*c^4*d^4 - 21*B*a^2*b*c^2*d^6 - 2*(20
*C*a*b^2 - A*b^3)*c^5*d^3 + (29*C*a^2*b - 3*A*a*b^2)*c^3*d^5 + 3*(C*a^3 +
3*A*a^2*b)*c*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/
(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(16*C*b
^3*c^7*d - 8*B*b^3*c^6*d^2 + 15*B*a*b^2*c^4*d^4 - 3*B*a^2*b*c^2*d^6 - 2*(1
4*C*a*b^2 - A*b^3)*c^5*d^3 + 2*(4*C*a^2*b - 3*A*a*b^2)*c^3*d^5 + (16*C*b^3
*c^5*d^3 - 8*B*b^3*c^4*d^4 + 15*B*a*b^2*c^2*d^6 - 3*B*a^2*b*d^8 - 2*(14*C*
a*b^2 - A*b^3)*c^3*d^5 + 2*(4*C*a^2*b - 3*A*a*b^2)*c*d^7)*x^2 + 2*(16*C*b^
3*c^6*d^2 - 8*B*b^3*c^5*d^3 + 15*B*a*b^2*c^3*d^5 - 3*B*a^2*b*c*d^7 - 2*(14
*C*a*b^2 - A*b^3)*c^4*d^4 + 2*(4*C*a^2*b - 3*A*a*b^2)*c^2*d^6)*x)*sqrt(-b*
d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2
)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(8*C*b^3*c^6*d^2 - 4*B*b^3*
c^5*d^3 + 8*B*a*b^2*c^3*d^5 - (13*C*a*b^2 - A*b^3)*c^4*d^4 + (C*a^2*b - 5*
A*a*b^2)*c^2*d^6 + (C*b^3*c^4*d^4 - 2*C*a*b^2*c^2*d^6 + C*a^2*b*d^8)*x^...
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^2\,\left(C\,x^2+B\,x+A\right)}{\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.201 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 538

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = -\frac{2c(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3d^2(bc^2-ad^2)(c+dx)^{3/2}} + \frac{2(bc^2(5c^2C-2Bcd-Ad^2)-3ad^2(3c^2C-2Bcd+Ad^2))\sqrt{a-bx^2}}{3d^2(bc^2-ad^2)^2\sqrt{c+dx}} - \frac{2\sqrt{a}(3a^2Cd^4+b^2c^2(8c^2C-2Bcd-Ad^2)-3abd^2(5c^2C-2Bcd+Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\sqrt{\right.\right.}{3\sqrt{b}d^3(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}(3ad^2(3cC-Bd)-bc(8c^2C-2Bcd-Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{\right.}{3\sqrt{b}d^3(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*c*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x+c)^(3/
2)+2/3*(b*c^2*(-A*d^2-2*B*c*d+5*C*c^2)-3*a*d^2*(A*d^2-2*B*c*d+3*C*c^2))*(-
b*x^2+a)^(1/2)/d^2/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/3*a^(1/2)*(3*a^2*C*d^4
+b^2*c^2*(-A*d^2-2*B*c*d+8*C*c^2)-3*a*b*d^2*(A*d^2-2*B*c*d+5*C*c^2))*(d*x+
c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^
(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^3/(-a*d^2
+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/3*a^(1/
2)*(3*a*d^2*(-B*d+3*C*c)-b*c*(-A*d^2-2*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d
^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.03 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.33

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(5bc^4C-2bBc^3d-Abc^2d^2-9ac^2Cd^2+6aBcd^3-3aAd^4-\frac{3a^2C}{b}\right.}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(5*b*c^4*C - 2*b*B*c^3*d - A*b*c^2*d^2 - 9*a*c^2*C*d^2
+ 6*a*B*c*d^3 - 3*a*A*d^4 - (3*a^2*C*d^4)/b + 3*a*d^2*(5*c^2*C - 2*B*c*d +
 A*d^2) + b*c^2*(-8*c^2*C + 2*B*c*d + A*d^2) - (c*(b*c^2 - a*d^2)*(c^2*C -
 B*c*d + A*d^2))/(c + d*x) - (I*(Sqrt[b]*c - Sqrt[a]*d)*(-3*a^2*C*d^4 + 3*
a*b*d^2*(5*c^2*C - 2*B*c*d + A*d^2) + b^2*c^2*(-8*c^2*C + 2*B*c*d + A*d^2)
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
)/(Sqrt[b]*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[a]*(
Sqrt[b]*c - Sqrt[a]*d)*(3*a^(3/2)*C*d^3 - 3*a*Sqrt[b]*d^2*(-3*c*C + B*d) -
 3*Sqrt[a]*b*d*(2*c^2*C - B*c*d + A*d^2) + b^(3/2)*c*(-8*c^2*C + 2*B*c*d +
 A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqr
t[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt
[a]*d)])/(Sqrt[b]*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*(b*c
^2*d - a*d^3)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2182, 27, 25, 2182, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int -\frac{-3C\left(\frac{bc^2}{d}-ad\right)x^2-\left(Abc-\frac{(cC-Bd)\left(2bc^2-3ad^2\right)}{d^2}\right)x+\frac{3a\left(Cc^2-Bdc+Ad^2\right)}{d}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{3C\left(\frac{bc^2}{d}-ad\right)x^2+\left(Abc-\frac{(cC-Bd)\left(2bc^2-3ad^2\right)}{d^2}\right)x+3a\left(-\frac{Cc^2}{d}+Bc-Ad\right)}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 25

$$\frac{\int \frac{3C\left(\frac{bc^2}{d}-ad\right)x^2+\left(Abc-\frac{(cC-Bd)\left(2bc^2-3ad^2\right)}{d^2}\right)x+3a\left(-\frac{Cc^2}{d}+Bc-Ad\right)}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 2182

$$\frac{\frac{2\int -\frac{ad\left(3ad^2(2cC-Bd)-bc\left(2Cc^2+Bdc-4Ad^2\right)\right)-\left(3a^2Cd^4-3ab\left(5Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(8Cc^2-2Bdc-Ad^2\right)\right)x}{2d^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd\right.\right.}{d^2\sqrt{c+}}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 27

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\int\frac{ad\left(3ad^2(2cC-Bd)-bc\left(2Cc^2+Bdc-4Ad^2\right)\right)-\left(3a^2Cd^4-3ab\left(5Cc^2-2Bdc+Ad^2\right)\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{d^2\left(bc^2-ad^2\right)}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 600

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{-\frac{\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{d^2\left(bc^2-\right.}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 509

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2C\right)\right)\int \cdots}{d\sqrt{a-bx^2}}\cdots}{d^2(\cdots}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}}{\cdots}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 327

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}}{\cdots}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}}{\cdots}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 511

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}(bc^2-ad^2)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3ad^2(3cC-Bd)-bc\left(-Ad^2-2Bcd+8c^2C\right)\right)\int}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{}}{3}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$\frac{\frac{2\sqrt{a-bx^2}\left(bc^2\left(-Ad^2-2Bcd+5c^2C\right)-3ad^2\left(Ad^2-2Bcd+3c^2C\right)\right)}{d^2\sqrt{c+dx}(bc^2-ad^2)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-Ad^2-2Bcd+8c^2\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{}}{3\left(bc\right.}$$

$$\frac{2c\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d^2(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*c*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(3*d^2*(b*c^2 - a*d^2)*(c +
 d*x)^(3/2)) + ((2*(b*c^2*(5*c^2*C - 2*B*c*d - A*d^2) - 3*a*d^2*(3*c^2*C -
 2*B*c*d + A*d^2))*Sqrt[a - b*x^2])/(d^2*(b*c^2 - a*d^2)*Sqrt[c + d*x]) -
((2*Sqrt[a]*(3*a^2*C*d^4 + b^2*c^2*(8*c^2*C - 2*B*c*d - A*d^2) - 3*a*b*d^2
*(5*c^2*C - 2*B*c*d + A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[
ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
+ d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a
 - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*d^2*(3*c*C - B*d) - b*c*(8*c^
2*C - 2*B*c*d - A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*
Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x
^2]))/(d^2*(b*c^2 - a*d^2)))/(3*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.38 (sec) , antiderivative size = 904, normalized size of antiderivative = 1.68

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(\frac{2c\left(A d^2-Bcd+C c^2\right)\sqrt{-bd x^3-bc x^2+adx+ac}}{3\left(a d^2-b c^2\right)d^4\left(x+\frac{c}{d}\right)^2}-\frac{2\left(-bd x^2+da\right)\left(3Aa d^4+Ab c^2 d^2-6Bac d^3+2c^3 Bbd+9Ca c^2 d^2-5c^4 C\right.}{3\left(a d^2-b c^2\right)^2 d^3\sqrt{\left(x+\frac{c}{d}\right)\left(-bd x^2+da\right)}}\right.}{}$
default	Expression too large to display

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/(a*d^2-b*c^
2)/d^4*c*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2-
2/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d^3*(3*A*a*d^4+A*b*c^2*d^2-6*B*a*c*d^3+
2*B*b*c^3*d+9*C*a*c^2*d^2-5*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(1/d
^3*(B*d-2*C*c)-1/3*b/d^3*c*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)-1/3*b/d^3*c*(
3*A*a*d^4+A*b*c^2*d^2-6*B*a*c*d^3+2*B*b*c^3*d+9*C*a*c^2*d^2-5*C*b*c^4)/(a*
d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*
((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-
c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF((
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2))+2*(C/d^2-1/3*b/d^2*(3*A*a*d^4+A*b*c^2*d^2-6*B*a*c*d^3+2*
B*b*c^3*d+9*C*a*c^2*d^2-5*C*b*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c
/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 892, normalized size of antiderivative = 1.66

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
2/9*((8*C*b^2*c^7 - 2*B*b^2*c^6*d + 3*B*a*b*c^4*d^3 - 9*B*a^2*c^2*d^5 - (2
1*C*a*b + A*b^2)*c^5*d^2 + 3*(7*C*a^2 + 3*A*a*b)*c^3*d^4 + (8*C*b^2*c^5*d^
2 - 2*B*b^2*c^4*d^3 + 3*B*a*b*c^2*d^5 - 9*B*a^2*d^7 - (21*C*a*b + A*b^2)*c
^3*d^4 + 3*(7*C*a^2 + 3*A*a*b)*c*d^6)*x^2 + 2*(8*C*b^2*c^6*d - 2*B*b^2*c^5
*d^2 + 3*B*a*b*c^3*d^4 - 9*B*a^2*c*d^6 - (21*C*a*b + A*b^2)*c^4*d^3 + 3*(7
*C*a^2 + 3*A*a*b)*c^2*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 +
3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) +
3*(8*C*b^2*c^6*d - 2*B*b^2*c^5*d^2 + 6*B*a*b*c^3*d^4 - (15*C*a*b + A*b^2)*
c^4*d^3 + 3*(C*a^2 - A*a*b)*c^2*d^5 + (8*C*b^2*c^4*d^3 - 2*B*b^2*c^3*d^4 +
 6*B*a*b*c*d^6 - (15*C*a*b + A*b^2)*c^2*d^5 + 3*(C*a^2 - A*a*b)*d^7)*x^2 +
 2*(8*C*b^2*c^5*d^2 - 2*B*b^2*c^4*d^3 + 6*B*a*b*c^2*d^5 - (15*C*a*b + A*b^
2)*c^3*d^4 + 3*(C*a^2 - A*a*b)*c*d^6)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b
*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPIn
verse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/
3*(3*d*x + c)/d)) + 3*(4*C*b^2*c^5*d^2 - B*b^2*c^4*d^3 + 5*B*a*b*c^2*d^5 -
 2*A*a*b*c*d^6 - 2*(4*C*a*b + A*b^2)*c^3*d^4 + (5*C*b^2*c^4*d^3 - 2*B*b^2*
c^3*d^4 + 6*B*a*b*c*d^6 - 3*A*a*b*d^7 - (9*C*a*b + A*b^2)*c^2*d^5)*x)*sqrt
(-b*x^2 + a)*sqrt(d*x + c))/(b^3*c^6*d^4 - 2*a*b^2*c^4*d^6 + a^2*b*c^2*d^8
 + (b^3*c^4*d^6 - 2*a*b^2*c^2*d^8 + a^2*b*d^10)*x^2 + 2*(b^3*c^5*d^5 - 2*a
*b^2*c^3*d^7 + a^2*b*c*d^9)*x)
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{\sqrt{a-b\,x^2}\,{(c+d\,x)}^{5/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{x(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.202 $\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 495

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3d\left(bc^2-ad^2\right)(c+dx)^{3/2}}$$
$$+\frac{2(3ad^2(2cC-Bd)-bc(2c^2C+Bcd-4Ad^2))\sqrt{a-bx^2}}{3d\left(bc^2-ad^2\right)^2\sqrt{c+dx}}$$
$$-\frac{2\sqrt{a}\sqrt{b}(3ad^2(2cC-Bd)-bc(2c^2C+Bcd-4Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3d^2\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(3aCd^2-b(2c^2C+Bcd-Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
2/3*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/d/(-a*d^2+b*c^2)/(d*x+c)^(3/2)+2/
3*(3*a*d^2*(-B*d+2*C*c)-b*c*(-4*A*d^2+B*c*d+2*C*c^2))*(-b*x^2+a)^(1/2)/d/(
-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-2/3*a^(1/2)*b^(1/2)*(3*a*d^2*(-B*d+2*C*c)-b*
c*(-4*A*d^2+B*c*d+2*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/d^2/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(
-b*x^2+a)^(1/2)+2/3*a^(1/2)*(3*a*C*d^2-b*(-A*d^2+B*c*d+2*C*c^2))*((d*x+c)/
(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))
/b^(1/2)/d^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.12 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.28

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \frac{2\sqrt{a-bx^2}\left(-2bc^3C-bBc^2d+4Abcd^2+6acCd^2-3aBd^3+3ad^2(-2cC+\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-2*b*c^3*C - b*B*c^2*d + 4*A*b*c*d^2 + 6*a*c*C*d^2 - 3
*a*B*d^3 + 3*a*d^2*(-2*c*C + B*d) + b*c*(2*c^2*C + B*c*d - 4*A*d^2) + ((b*
c^2 - a*d^2)*(c^2*C - B*c*d + A*d^2))/(c + d*x) - (I*Sqrt[b]*(Sqrt[b]*c -
Sqrt[a]*d)*(3*a*d^2*(-2*c*C + B*d) + b*c*(2*c^2*C + B*c*d - 4*A*d^2))*Sqrt
[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(
c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(Sqrt[b]*c - Sqrt[a]*d)
*(3*A*b^(3/2)*c*d - 3*a^(3/2)*C*d^2 + 3*a*Sqrt[b]*d*(c*C - B*d) + Sqrt[a]*
b*(2*c^2*C + B*c*d - A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqr
t[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*
d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2
))))/(3*d*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2182, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2182}$$

$$\frac{2\int \frac{3(Abc+aCc-aBd)-\left(3aCd-b\left(\frac{2Cc^2}{d}+Bc-Ad\right)\right)x}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(Ad^2-Bcd+c^2C)}{3d(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(Abc+aCc-aBd)-\left(3aCd-b\left(\frac{2Cc^2}{d}+Bc-Ad\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}(Ad^2-Bcd+c^2C)}{3d(c+dx)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{688}$$

$$\frac{\frac{2\int \frac{d\left(Ab\left(3bc^2+ad^2\right)+a\left(3aCd^2+bc(cC-4Bd)\right)\right)+b\left(3ad^2(2cC-Bd)-bc\left(2Cc^2+Bdc-4Ad^2\right)\right)x}{2d\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{a-bx^2}\left(3ad^2(2cC-Bd)-bc(-4Ad^2+Bcd+2c^2C\right.}{d\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{d\left(Ab\left(3bc^2+ad^2\right)+a\left(3aCd^2+bc(cC-4Bd)\right)\right)+b\left(3ad^2(2cC-Bd)-bc\left(2Cc^2+Bdc-4Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}\left(3ad^2(2cC-Bd)-bc(-4Ad^2+Bcd+2c^2C)\right.}{d\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 600

$$\frac{\frac{\frac{b\left(3ad^2(2cC-Bd)-bc\left(-4Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{d(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}\left(3ad^2(2cC-B\right.}{d\sqrt{c+d}}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 509

$$\frac{\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3ad^2(2cC-Bd)-bc\left(-4Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{d(bc^2-ad^2)}+\frac{2\sqrt{a-bx^2}\left(3ad^2\right.}{}}{3\left(bc^2-ad^2\right)}$$
$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$\frac{\frac{-\frac{\left(bc^2-ad^2\right)\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2(2cC-Bd)-bc\left(-4Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{d(bc^2-ad^2)}\cdots}{3\left(bc^2-ad^2\right)}\cdots$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{-\frac{\left(bc^2-ad^2\right)\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2(2cC-Bd)-bc\left(-4Ad^2+Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\cdots\right)\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{d(bc^2-ad^2)}\cdots}{3\left(bc^2-ad^2\right)}\cdots$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2(2cC-Bd)-bc\left(-4Ad^2+Bcd+2c^2C\right)\right)E\left(\arcsin\left(\cdots\right)\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{d(bc^2-ad^2)}\cdots}{3\left(bc^2-ad^2\right)}\cdots$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2(2cC-\cdots\right.}{\cdots}}{d(bc^2-ad^2)}\cdots}{3\left(bc^2-ad^2\right)}\cdots$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 321

$$\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3ad^2(2cC-Bd)-bc\cdots\right.}{d\sqrt{a\cdots}}}{d(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}$$

$$\frac{2\sqrt{a-bx^2}\left(Ad^2-Bcd+c^2C\right)}{3d(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(c^2*C - B*c*d + A*d^2)*Sqrt[a - b*x^2])/(3*d*(b*c^2 - a*d^2)*(c + d*x)
^(3/2)) + ((2*(3*a*d^2*(2*c*C - B*d) - b*c*(2*c^2*C + B*c*d - 4*A*d^2))*Sq
rt[a - b*x^2])/(d*(b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((-2*Sqrt[a]*Sqrt[b]*(3
*a*d^2*(2*c*C - B*d) - b*c*(2*c^2*C + B*c*d - 4*A*d^2))*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*C*d^2 - b*(
2*c^2*C + B*c*d - A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)
]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[
2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b
*x^2]))/(d*(b*c^2 - a*d^2)))/(3*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 5.37 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2\left(A\,d^2-Bcd+C\,c^2\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3\left(a\,d^2-b\,c^2\right)d^3\left(x+\frac{c}{d}\right)^2}+\frac{2\left(-bd\,x^2+da\right)\left(4Abc\,d^2-3Ba\,d^3-Bb\,c^2d+6Cac\,d^2-2Cb\,c^3\right)}{3\left(a\,d^2-b\,c^2\right)^2d^2\sqrt{\left(x+\frac{c}{d}\right)\left(-bd\,x^2+da\right)}}+\frac{2\left(\frac{C}{d^2}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/(a*d^2-b*c
^2)/d^3*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2
/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d^2*(4*A*b*c*d^2-3*B*a*d^3-B*b*c^2*d+6*C
*a*c*d^2-2*C*b*c^3)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(C/d^2+1/3*b/d^2*(A*d
^2-B*c*d+C*c^2)/(a*d^2-b*c^2)+1/3*b/d^2*c*(4*A*b*c*d^2-3*B*a*d^3-B*b*c^2*d
+6*C*a*c*d^2-2*C*b*c^3)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2/3*b/d*(4*A*b*c*d^2-3*B*a*d^
3-B*b*c^2*d+6*C*a*c*d^2-2*C*b*c^3)/(a*d^2-b*c^2)^2*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/
d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
-2/9*((2*C*b^2*c^6 + B*b^2*c^5*d - 9*B*a*b*c^3*d^3 - (3*C*a*b - 5*A*b^2)*c
^4*d^2 + 3*(3*C*a^2 + A*a*b)*c^2*d^4 + (2*C*b^2*c^4*d^2 + B*b^2*c^3*d^3 -
9*B*a*b*c*d^5 - (3*C*a*b - 5*A*b^2)*c^2*d^4 + 3*(3*C*a^2 + A*a*b)*d^6)*x^2
 + 2*(2*C*b^2*c^5*d + B*b^2*c^4*d^2 - 9*B*a*b*c^2*d^4 - (3*C*a*b - 5*A*b^2
)*c^3*d^3 + 3*(3*C*a^2 + A*a*b)*c*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4
/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*
x + c)/d) + 3*(2*C*b^2*c^5*d + B*b^2*c^4*d^2 + 3*B*a*b*c^2*d^4 - 2*(3*C*a*
b + 2*A*b^2)*c^3*d^3 + (2*C*b^2*c^3*d^3 + B*b^2*c^2*d^4 + 3*B*a*b*d^6 - 2*
(3*C*a*b + 2*A*b^2)*c*d^5)*x^2 + 2*(2*C*b^2*c^4*d^2 + B*b^2*c^3*d^3 + 3*B*
a*b*c*d^5 - 2*(3*C*a*b + 2*A*b^2)*c^2*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4
/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstra
ssPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3
), 1/3*(3*d*x + c)/d)) + 3*(C*b^2*c^4*d^2 + 2*B*b^2*c^3*d^3 + 2*B*a*b*c*d^
5 + A*a*b*d^6 - 5*(C*a*b + A*b^2)*c^2*d^4 + (2*C*b^2*c^3*d^3 + B*b^2*c^2*d
^4 + 3*B*a*b*d^6 - 2*(3*C*a*b + 2*A*b^2)*c*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d
*x + c))/(b^3*c^6*d^3 - 2*a*b^2*c^4*d^5 + a^2*b*c^2*d^7 + (b^3*c^4*d^5 - 2
*a*b^2*c^2*d^7 + a^2*b*d^9)*x^2 + 2*(b^3*c^5*d^4 - 2*a*b^2*c^3*d^6 + a^2*b
*c*d^8)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{a-b\,x^2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(dx+c)^{\frac{5}{2}}\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.203 $\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 619

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = -\frac{2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3c(bc^2-ad^2)(c+dx)^{3/2}} - \frac{2(3ad^2(c^2C-Ad^2)+bc^2(c^2C-4Bcd+7Ad^2))\sqrt{a-bx^2}}{3c^2(bc^2-ad^2)^2\sqrt{c+dx}} + \frac{2\sqrt{a}\sqrt{b}(3ad^2(c^2C-Ad^2)+bc^2(c^2C-4Bcd+7Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3c^2d(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}\sqrt{b}(c^2C-Bcd+Ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3cd(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c/(-a*d^2+b*c^2)/(d*x+c)^(3/2)-2
/3*(3*a*d^2*(-A*d^2+C*c^2)+b*c^2*(7*A*d^2-4*B*c*d+C*c^2))*(-b*x^2+a)^(1/2)
/c^2/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)+2/3*a^(1/2)*b^(1/2)*(3*a*d^2*(-A*d^2+C
*c^2)+b*c^2*(7*A*d^2-4*B*c*d+C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*El
lipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/c^2/d/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2
)))^(1/2)/(-b*x^2+a)^(1/2)-2/3*a^(1/2)*b^(1/2)*(A*d^2-B*c*d+C*c^2)*((d*x+c
)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/c/d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/
2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c^2/
(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.24 (sec) , antiderivative size = 1728, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-(c*(c*(b*c^2 - a*d^2)*(c^2*C - B*c*d + A*d^2) + (3*a*
d^2*(c^2*C - A*d^2) + b*c^2*(c^2*C - 4*B*c*d + 7*A*d^2))*(c + d*x))) - ((c
 + d*x)*(b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 4*b^2*B*c^6*d*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]] + 7*A*b^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] +
2*a*b*c^5*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 4*a*b*B*c^4*d^3*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]] - 10*a*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] -
3*a^2*c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 3*a^2*A*c*d^6*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]] - 2*b^2*c^6*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)
 + 8*b^2*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 14*A*b^2*c^4*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 6*a*b*c^4*C*d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*(c + d*x) + 6*a*A*b*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]*(c + d*x) + b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 4*b
^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 7*A*b^2*c^3*d^2*Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 3*a*b*c^3*C*d^2*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]*(c + d*x)^2 - 3*a*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(3*a*d^2*(c^2*C - A*d^2
) + b*c^2*(c^2*C - 4*B*c*d + 7*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
 d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Elli
pticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
+ Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*d*(Sqrt[b]*c - Sqrt[a]*d)*(3*...
```

Rubi [A] (verified)

Time = 3.10 (sec) , antiderivative size = 1037, normalized size of antiderivative = 1.68, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.514$, Rules used = {2351, 635, 633, 632, 186, 413, 412, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{635}$$

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\frac{\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c^2}\right)+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$\big\downarrow$ 633

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{c^2\sqrt{a-bx^2}}\right)+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$\big\downarrow$ 632

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\frac{\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{c^2\sqrt{a-bx^2}}\right)+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$\big\downarrow$ 186

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx-\frac{2\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c^2\sqrt{a-bx^2}}\right)+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$\big\downarrow$ 413

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\right)+\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$\big\downarrow$ 412

$$A\left(\int \frac{-\frac{xd^2}{c^2}-\frac{2d}{c}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\right)+$$

$$\int \frac{B+Cx}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{ 688}$$

$$A\left(\frac{2\int -\frac{d\left(3c\left(2b-\frac{ad^2}{c^2}\right)-bdx\right)}{2c(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{}{3}\right.$$

$$\frac{2\int \frac{3(bBc-aCd)+b(cC-Bd)x}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$A\left(-\frac{d\int \frac{3\left(2bc-\frac{ad^2}{c}\right)-bdx}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3c\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{}{3c(}\right.$$

$$\frac{\int \frac{3(bBc-aCd)+b(cC-Bd)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 688}$$

$$A\left(-\frac{d\left(\frac{2\int \frac{b\left(2c\left(3bc^2-ad^2\right)+d\left(7bc^2-3ad^2\right)x\right)}{2c\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}(7bc^2-3ad^2)}{c\sqrt{c+dx}(bc^2-ad^2)}\right)}{3c\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\ \text{EllipticPi}\left(2,\cdots\right.}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}\cdots}}\cdots\right.$$

$$\frac{\frac{2\int \frac{b\left(3bBc^2-ad(4cC-Bd)-\left(3aCd^2+bc(cC-4Bd)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}(3aCd^2+bc(cC-4Bd))}{\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$A\left(-\frac{d\left(\frac{b\int \frac{2c\left(3bc^2-ad^2\right)+d\left(7bc^2-3ad^2\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{c(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}(7bc^2-3ad^2)}{c\sqrt{c+dx}(bc^2-ad^2)}\right)}{3c\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\ \text{EllipticPi}\left(2,\text{arc}\cdots\right.}{c^2\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\cdots}}\cdots\right.$$

$$\frac{\frac{b\int \frac{3bBc^2-ad(4cC-Bd)-\left(3aCd^2+bc(cC-4Bd)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}(3aCd^2+bc(cC-4Bd))}{\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$A\left(-\frac{d\left(\frac{b\left(\left(7bc^2-3ad^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-c\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{c(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}(7bc^2-3ad^2)}{c\sqrt{c+dx}(bc^2-ad^2)}\right)}{3c\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\cdots\right)}{\sqrt{a}d+\cdots}}\cdots}{c^2\sqrt{a-\cdots}}\cdots\right.$$

$$\frac{\frac{b\left(\frac{\left(bc^2-ad^2\right)(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\left(3aCd^2+bc(cC-4Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}(3aCd^2+bc(cC-4Bd))}{\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$A\left(-\frac{d\left(\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(7bc^2-3ad^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{c(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c\sqrt{c+dx}(bc^2-ad^2)}\right)}{3c\left(bc^2-ad^2\right)}-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\cdots}}{\cdots}\right.$$

$$\frac{\frac{b\left(\frac{\left(bc^2-ad^2\right)(cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(3aCd^2+bc(cC-4Bd)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}(3aCd^2+bc(cC-4Bd))}{\sqrt{c+dx}(bc^2-ad^2)}}{3\left(bc^2-ad^2\right)}-$$

$$\frac{2\sqrt{a-bx^2}(cC-Bd)}{3(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{b\left(\frac{(cC-Bd)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\left(3aCd^2+bc(cC-4Bd)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)}{bc^2-ad^2}-\frac{2\left(3aCd^2+bc(cC-4Bd)\right)}{\left(bc^2-ad^2\right)\sqrt{c+d}}$$
$$\overline{\qquad\qquad 3\left(bc^2-ad^2\right)\qquad\qquad}$$

$$A\left(-\frac{2\sqrt{a-bx^2}d^2}{3c\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{\left(\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{b\left(-c\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\left(7bc^2-3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}{\sqrt{b}\sqrt{\frac{\sqrt{b}(c+}{\sqrt{b}c+}}}\right.}{c\left(bc^2-ad^2\right)}\right.}{3c\left(bc^2-ad^2\right)}\right.$$

↓ 327

$$-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{\frac{b\left(\frac{2\sqrt{a}\left(3aCd^2+bc(cC-4Bd)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{(cC-Bd)\left(bc^2-ad^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\left(3aCd^2+bc(cC-4Bd\right.}{(bc^2-ad^2)\sqrt{c+}}}{3\left(bc^2-ad^2\right)}$$

$$A\left(-\frac{2\sqrt{a-bx^2}d^2}{3c\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{\left(\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c(bc^2-ad^2)\sqrt{c+dx}}+\frac{b\left(-\frac{2\sqrt{a}\left(7bc^2-3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\right.}{c(bc^2-ad^2)}\right.}{3c\left(bc^2-ad^2\right)}\right.$$

↓ 512

$$-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{\frac{b\left(\frac{2\sqrt{a}\left(3aCd^2+bc(cC-4Bd)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{(cC-Bd)\left(bc^2-ad^2\right)\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{bc^2-ad^2}-\frac{2\left(3aCd^2+bc(\ldots\right.}{(bc^2-\ldots}}{3\left(bc^2-ad^2\right)}$$

$$A\left(-\frac{2\sqrt{a-bx^2}d^2}{3c\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{\left(\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c(bc^2-ad^2)\sqrt{c+dx}}+\frac{b\left(-\frac{2\sqrt{a}\left(7bc^2-3ad^2\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\ldots\right)}{c(bc^2-ad^2)\ldots}\right)}{3c\left(bc^2-ad^2\right)}\ldots\right.$$

↓ 511

$$-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+$$

$$b\left(\frac{2\sqrt{a}\left(3aCd^2+bc(cC-4Bd)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{a}(cC-Bd)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\cdots\right.}}}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}\right.$$

$$\frac{\cdots}{bc^2-ad^2}$$

$$\frac{\cdots}{3\left(bc^2-ad^2\right)}$$

$$A\left(-\frac{2\sqrt{a-bx^2}d^2}{3c\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{\left(\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{b\left(\frac{2\sqrt{a}c\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\cdots\right.}}}{\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}\cdots\right.}{\cdots}\cdots\right.}{3c\left(bc^2-ad\cdots\right.}\right.$$

↓ 321

$$-\frac{2\sqrt{a-bx^2}(cC-Bd)}{3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{b\left(\frac{2\sqrt{a}\left(3aCd^2+bc(cC-4Bd)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{a}(cC-Bd)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}\right.}{\frac{bc^2-ad^2}{3\left(bc^2-ad^2\right)}}$$

$$A\left(-\frac{2\sqrt{a-bx^2}d^2}{3c\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{\left(\frac{2d\sqrt{a-bx^2}\left(7bc^2-3ad^2\right)}{c\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{b\left(\frac{2\sqrt{a}c\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}\right.}{c(b}\right.}{3c\left(bc^2-ad^2\right)}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*(c*C - B*d)*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) + ((-
2*(3*a*C*d^2 + b*c*(c*C - 4*B*d))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c
 + d*x]) + (b*((2*Sqrt[a]*(3*a*C*d^2 + b*c*(c*C - 4*B*d))*Sqrt[c + d*x]*Sq
rt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
 (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(c*C - B*d)*(b*c^2 - a
*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a
]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]
*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b*c^2 - a*
d^2))/(3*(b*c^2 - a*d^2)) + A*((-2*d^2*Sqrt[a - b*x^2])/(3*c*(b*c^2 - a*d^
2)*(c + d*x)^(3/2)) - (d*((2*d*(7*b*c^2 - 3*a*d^2)*Sqrt[a - b*x^2])/(c*(b*
c^2 - a*d^2)*Sqrt[c + d*x]) + (b*((-2*Sqrt[a]*(7*b*c^2 - 3*a*d^2)*Sqrt[c +
 d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/S
qrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[(Sqrt[b]*(c + d*x
))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*c*(b*c^2 - a*d^2
)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*El
lipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/
Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(c*(b*c^2 - a*d^2
))))/(3*c*(b*c^2 - a*d^2)) - (2*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1
 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 635
```
Int[((c_) + (d_.)*(x_))^(n_)/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :>
 Simp[c^(n + 1/2)   Int[1/(x*Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] + Int[(
(c + d*x)^n/Sqrt[a + b*x^2])*ExpandToSum[(1 - c^(n + 1/2)*(c + d*x)^(-n - 1
/2))/x, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n + 1/2, 0]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1101 vs. 2(532) = 1064.

Time = 5.64 (sec) , antiderivative size = 1102, normalized size of antiderivative = 1.78

method	result	size
elliptic	Expression too large to display	1102
default	Expression too large to display	8465

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/(a*d^2-b*c^
2)/d^2*(A*d^2-B*c*d+C*c^2)/c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+
2/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2/d*(3*A*a*d^4-7*A*b*c^2*d^2+4*B*b*c^3*d-
3*C*a*c^2*d^2-C*b*c^4)/c^2/((x+c/d)*(-b*d*x^2+a*d))^(1/2)+2*(-1/3*b/d*(A*d
^2-B*c*d+C*c^2)/(a*d^2-b*c^2)/c+1/3*b/c/d*(3*A*a*d^4-7*A*b*c^2*d^2+4*B*b*c
^3*d-3*C*a*c^2*d^2-C*b*c^4)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d
+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2/3*b*(3*A*a*d^4-7*A*b*c^
2*d^2+4*B*b*c^3*d-3*C*a*c^2*d^2-C*b*c^4)/(a*d^2-b*c^2)^2/c^2*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)
/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+
c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-2*A/c^3*(c/d-
1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2)
)/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)
))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x}{-b\,d^3x^5-3bc\,d^2x^4+a\,d^3x^3-3b\,c^2d\,x^3+3ac\,d^2x^2-b\,c^3x^2+3a\,c^2dx+ \cdots}\right.$$

$$+\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,d^3x^6-3bc\,d^2x^5+a\,d^3x^4-3b\,c^2d\,x^4+3ac\,d^2x^3-b\,c^3x^3+3a\,c^2d\,x^2+a\,c^3x}dx\right)a$$

$$+\left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-b\,d^3x^5-3bc\,d^2x^4+a\,d^3x^3-3b\,c^2d\,x^3+3ac\,d^2x^2-b\,c^3x^2+3a\,c^2dx+a\,c^3}dx\right)b$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output

```
int((sqrt(c + d*x)*sqrt(a - b*x**2)*x)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2
*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*
d**3*x**5),x)*c + int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c**3*x + 3*a*c**
2*d*x**2 + 3*a*c*d**2*x**3 + a*d**3*x**4 - b*c**3*x**3 - 3*b*c**2*d*x**4 -
 3*b*c*d**2*x**5 - b*d**3*x**6),x)*a + int((sqrt(c + d*x)*sqrt(a - b*x**2)
)/(a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 - b*c**3*x**2 - 3
*b*c**2*d*x**3 - 3*b*c*d**2*x**4 - b*d**3*x**5),x)*b
```

3.204 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 681

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx &= \frac{2d(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3c^2(bc^2-ad^2)(c+dx)^{3/2}} \\
&+ \frac{2d(3ad^3(Bc-2Ad)+bc^2(4c^2C-7Bcd+10Ad^2))\sqrt{a-bx^2}}{3c^3(bc^2-ad^2)^2\sqrt{c+dx}} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{ac^3x} \\
&+ \frac{\sqrt{b}(A(3b^2c^4-26abc^2d^2+15a^2d^4)-2ac(3aBd^3+bc^2(4cC-7Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{3\sqrt{a}c^3(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&+ \frac{\sqrt{b}(2ac(cC-Bd)-A(3bc^2-5ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}c^2(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}} \\
&- \frac{(2Bc-5Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c^3\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output
```
2/3*d*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c^2/(-a*d^2+b*c^2)/(d*x+c)^(3/2
)+2/3*d*(3*a*d^3*(-2*A*d+B*c)+b*c^2*(10*A*d^2-7*B*c*d+4*C*c^2))*(-b*x^2+a)
^(1/2)/c^3/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)
/a/c^3/x+1/3*b^(1/2)*(A*(15*a^2*d^4-26*a*b*c^2*d^2+3*b^2*c^4)-2*a*c*(3*B*a
*d^3+b*c^2*(-7*B*d+4*C*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/a^(1/2)/c^3/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))
^(1/2)/(-b*x^2+a)^(1/2)+1/3*b^(1/2)*(2*a*c*(-B*d+C*c)-A*(-5*a*d^2+3*b*c^2)
)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)
*d))^(1/2))/a^(1/2)/c^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(-5*
A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/c^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.27 (sec) , antiderivative size = 2682, normalized size of antiderivative = 3.94

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-(A/(a*c^3*x)) + (2*d*(c^2*C - B*c*d + A*d^
2))/(3*c^2*(b*c^2 - a*d^2)*(c + d*x)^2) + (2*d*(4*b*c^4*C - 7*b*B*c^3*d +
10*A*b*c^2*d^2 + 3*a*B*c*d^3 - 6*a*A*d^4))/(3*c^3*(b*c^2 - a*d^2)^2*(c + d
*x))) - (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-3*A*b^3*c^
5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 8*a*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]] - 14*a*b^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 26*a*A*b^2*c^3*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 6*a^2*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]] - 15*a^2*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - (3*A*b^3*c^
7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (8*a*b^2*c^7*C*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (14*a*b^2*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]])/(c + d*x)^2 + (29*a*A*b^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
])/(c + d*x)^2 - (8*a^2*b*c^5*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d
*x)^2 + (20*a^2*b*B*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 -
(41*a^2*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (6*a^3*B
*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (15*a^3*A*c*d^6*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (6*A*b^3*c^6*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]])/(c + d*x) - (16*a*b^2*c^6*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/
(c + d*x) + (28*a*b^2*B*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) -
(52*a*A*b^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (12*a^2*b*
B*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (30*a^2*A*b*c^2*d...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2117 vs. $2(681) = 1362$.

Time = 9.90 (sec) , antiderivative size = 2117, normalized size of antiderivative = 3.11, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.686$, Rules used = {2355, 637, 2009, 2355, 27, 636, 25, 27, 637, 2009, 2351, 27, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\sqrt{a-bx^2}}+\frac{d^2}{c^2(c+dx)^{5/2}\sqrt{a-bx^2}}-\frac{2d}{c^3x\sqrt{c+dx}\sqrt{a-bx^2}}+\frac{}{c^2x^2\sqrt{c+}}\right.$$

$$\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{2009}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\int\frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\int\frac{C}{d^2x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\int\frac{1}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}$$

$$\Big\downarrow \text{636}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\left(\frac{\int-\frac{d\left(bx^2+a\right)}{cx\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\left(-\frac{\int\frac{d\left(bx^2+a\right)}{cx\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\downarrow$ 27

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\left(-\frac{d\int\frac{bx^2+a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{\ }}{}\right.$$

$$\frac{C\left(-\frac{d\int\frac{bx^2+a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}-$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\sqrt{a-bx^2}}-\frac{d}{c^2x\sqrt{c+dx}\sqrt{a-bx^2}}+\frac{1}{cx^2\sqrt{c+dx}\sqrt{a-bx^2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{\ }}{}\right.$$

$$\frac{(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{\ }}\right.}{}$$

$$\frac{C\left(-\frac{d\int\frac{bx^2+a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$$\Big\downarrow \text{2351}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{bx}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$$\Big\downarrow \; 27$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\int\frac{x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$$\Big\downarrow \; 600$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.}{}$$

$$\frac{C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(\frac{\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\Big\downarrow$ 509

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.}{}$$

$$\frac{C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\Big\downarrow$ 508

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)$$

$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$$\Big\downarrow \; 327$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(b\left(-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{c\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)+a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{acx}\right)}{d^2}$$

$\downarrow$ 512

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{c\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)\right)}{2ac}-\frac{\sqrt{c+dx}\sqrt{a-bx}}{acx}\right.}{d^2}$$

$$\Big\downarrow \text{511}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\operatorname{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$C\left(-\frac{d\left(a\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-}}\right.\right.}{2ac}\right.$$

$$\overline{\qquad\qquad d^2\qquad\qquad}$$

$\downarrow$ 321

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)+a\int\frac{}{x\sqrt{c+dx}}\right.}{2ac}\right.}{d^2}$$

$\downarrow$ 633

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)+\frac{a\sqrt{1-\frac{bx^2}{a}}\int}{}\right.}{2ac}\right.$$

$$\overline{\qquad d^2 \qquad}$$

$\downarrow$ 632

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}\,d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}\,d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}\,d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}\,d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)+\frac{a\sqrt{1-\frac{bx^2}{a}}\int}{}\right.}{2ac}\right.}{d^2}$$

$\downarrow$ 186

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c(bc^2-ad^2)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2(bc^2-ad^2)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.}{}$$

$$\frac{C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)-\frac{2a\sqrt{1-\frac{bx^2}{a}}}{}\right.}{2ac}\right.}{d^2}$$

$\downarrow$ 413

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}\,d^3}{c^3\left(bc^2-ad^2\right)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}\,d^3}{3c\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}\,d^3}{3c^2\left(bc^2-ad^2\right)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{\cdots}}{\cdots}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}\,d^3}{c^2\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\cdots\right.}{c^2\sqrt{c+dx}\sqrt{\cdots}}\right.$$

$$\frac{C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)-\frac{2a\sqrt{1-\frac{bx^2}{a}}\cdots}{\cdots}\right)}{2ac}\cdots\right.}{d^2}$$

↓ 412

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(\frac{4\sqrt{a-bx^2}d^3}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{8b\sqrt{a-bx^2}d^3}{3c(bc^2-ad^2)^2\sqrt{c+dx}}+\frac{2\sqrt{a-bx^2}d^3}{3c^2(bc^2-ad^2)(c+dx)^{3/2}}-\frac{4\sqrt{a}\sqrt{}}{}\right.$$

$$(2cC-Bd)\left(\frac{2\sqrt{a-bx^2}d^3}{c^2(bc^2-ad^2)\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^2}{c^2(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(2,\right.}{c^2\sqrt{c+dx}\sqrt{}}\right.$$

$$\frac{C\left(-\frac{d\left(b\left(\frac{2\sqrt{a}c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right)-\frac{2a\sqrt{1-\frac{bx^2}{a}}}{}\right)}{2ac}\right.}{d^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(A + (c*(c*C - B*d))/d^2)*((2*d^3*Sqrt[a - b*x^2])/(3*c^2*(b*c^2 - a*d^2)*
(c + d*x)^(3/2)) + (8*b*d^3*Sqrt[a - b*x^2])/(3*c*(b*c^2 - a*d^2)^2*Sqrt[c
 + d*x]) + (4*d^3*Sqrt[a - b*x^2])/(c^3*(b*c^2 - a*d^2)*Sqrt[c + d*x]) - (
Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c^3*x) + (Sqrt[b]*Sqrt[c + d*x]*Sqrt[1 -
 (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)
/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[a]*c^3*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b
]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (8*Sqrt[a]*b^(3/2)*d^2*Sqrt[c + d*x]*
Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]
], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(3*c*(b*c^2 - a*d^2)^2*Sqrt[(Sqrt[b]*
(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (4*Sqrt[a]*Sqrt[b]*
d^2*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x
)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(c^3*(b*c^2 - a*d^2
)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (Sq
rt[b]*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a
]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]
*c)/Sqrt[a] + d)])/(Sqrt[a]*c^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]) + (2*Sqrt[a
]*Sqrt[b]*d^2*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (
b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/(
(Sqrt[b]*c)/Sqrt[a] + d)])/(3*c^2*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a - b
*x^2]) + (5*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 636
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_))/Sqrt[(a_) + (b_.)*(x_)^2],
 x_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]
/(a*e*(m + 1))), x] - Simp[1/(2*a*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c +
 d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2
)*d*(2*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x
^3 - 2*a*(m + 1)*(c + d*x)^(n + 1/2))/x, x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 9.64 (sec) , antiderivative size = 1145, normalized size of antiderivative = 1.68

method	result	size
elliptic	Expression too large to display	1145
risch	Expression too large to display	1554
default	Expression too large to display	11676

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/(a*d^2-b*c
^2)/d*(A*d^2-B*c*d+C*c^2)/c^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2
-2/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2*(6*A*a*d^4-10*A*b*c^2*d^2-3*B*a*c*d^3+
7*B*b*c^3*d-4*C*b*c^4)/c^3/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-A/c^3/a/x*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(1/3*b*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)/c^
2-1/3*b/c^2*(6*A*a*d^4-10*A*b*c^2*d^2-3*B*a*c*d^3+7*B*b*c^3*d-4*C*b*c^4)/(
a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/
(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF
(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2))+2*(-1/3*b*d*(6*A*a*d^4-10*A*b*c^2*d^2-3*B*a*c*d^3+7*B*
b*c^3*d-4*C*b*c^4)/(a*d^2-b*c^2)^2/c^3-1/2*A*b*d/a/c^3)*(c/d-1/b*(a*b)^(1/
2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),
((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+(5*A*d-2*B*c)/c^4*
(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x^2\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{a-b\,x^2}\,{(c+d\,x)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{x^2\,(dx+c)^{\frac{5}{2}}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.205 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 786

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = -\frac{2d^2(c^2C-Bcd+Ad^2)\sqrt{a-bx^2}}{3c^3(bc^2-ad^2)(c+dx)^{3/2}}$$

$$+\frac{2d^2(3ad^2(c^2C-2Bcd+3Ad^2)-bc^2(7c^2C-10Bcd+13Ad^2))\sqrt{a-bx^2}}{3c^4(bc^2-ad^2)^2\sqrt{c+dx}}$$

$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2ac^3x^2}-\frac{(4Bc-11Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4ac^4x}$$

$$+\frac{\sqrt{b}(3b^2c^4(4Bc-11Ad)-3a^2d^3(8c^2C-20Bcd+35Ad^2)+2abc^2d(28c^2C-52Bcd+85Ad^2))\sqrt{c+dx}\,\sqrt{}\cdots}{12\sqrt{a}c^4(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{b}(3bc^2(4Bc-9Ad)+ad(8c^2C-20Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2}{\sqrt{b}c}\cdots\right.}{12\sqrt{a}c^3(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(4Abc^2+8ac^2C-20aBcd+35aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4ac^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*d^2*(A*d^2-B*c*d+C*c^2)*(-b*x^2+a)^(1/2)/c^3/(-a*d^2+b*c^2)/(d*x+c)^(
3/2)+2/3*d^2*(3*a*d^2*(3*A*d^2-2*B*c*d+C*c^2)-b*c^2*(13*A*d^2-10*B*c*d+7*C
*c^2))*(-b*x^2+a)^(1/2)/c^4/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-1/2*A*(d*x+c)^(
1/2)*(-b*x^2+a)^(1/2)/a/c^3/x^2-1/4*(-11*A*d+4*B*c)*(d*x+c)^(1/2)*(-b*x^2+
a)^(1/2)/a/c^4/x+1/12*b^(1/2)*(3*b^2*c^4*(-11*A*d+4*B*c)-3*a^2*d^3*(35*A*d
^2-20*B*c*d+8*C*c^2)+2*a*b*c^2*d*(85*A*d^2-52*B*c*d+28*C*c^2))*(d*x+c)^(1/
2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c^4/(-a*d^2+b*c^2
)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/12*b^(1/2)*(3
*b*c^2*(-9*A*d+4*B*c)+a*d*(35*A*d^2-20*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2)
)^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/c
^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(35*A*a*d^2+4*A*b*c^2
-20*B*a*c*d+8*C*a*c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a
)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1
/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.41 (sec) , antiderivative size = 3781, normalized size of antiderivative = 4.81

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output

```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-1/2*A/(a*c^3*x^2) + (-4*B*c + 11*A*d)/(4*a
*c^4*x) - (2*d^2*(c^2*C - B*c*d + A*d^2))/(3*c^3*(b*c^2 - a*d^2)*(c + d*x)
^2) - (2*d^2*(7*b*c^4*C - 10*b*B*c^3*d + 13*A*b*c^2*d^2 - 3*a*c^2*C*d^2 +
6*a*B*c*d^3 - 9*a*A*d^4))/(3*c^4*(b*c^2 - a*d^2)^2*(c + d*x))) + (d*Sqrt[a
 - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(12*b^3*B*c^6*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]] - 33*A*b^3*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 56*a*b^2
*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 104*a*b^2*B*c^4*d^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]] + 170*a*A*b^2*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] -
24*a^2*b*c^3*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 60*a^2*b*B*c^2*d^4*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]] - 105*a^2*A*b*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]] + (12*b^3*B*c^8*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (33*A*b
^3*c^7*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (56*a*b^2*c^7*C*d*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (116*a*b^2*B*c^6*d^2*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (203*a*A*b^2*c^5*d^3*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]])/(c + d*x)^2 - (80*a^2*b*c^5*C*d^3*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]])/(c + d*x)^2 + (164*a^2*b*B*c^4*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
)/(c + d*x)^2 - (275*a^2*A*b*c^3*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c +
d*x)^2 + (24*a^3*c^3*C*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (
60*a^3*B*c^2*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (105*a^3*A*
c*d^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (24*b^3*B*c^7*Sqrt[...
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2545 vs. $2(786) = 1572$.

Time = 15.07 (sec) , antiderivative size = 2545, normalized size of antiderivative = 3.24, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.657$, Rules used = {2355, 637, 2009, 2355, 27, 636, 25, 637, 2009, 2352, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 637$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2}\sqrt{a-bx^2}}-\frac{d^3}{c^3(c+dx)^{5/2}\sqrt{a-bx^2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{}{c^3x^2\sqrt{c}}\right.$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 2009$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 2355$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\int \frac{C}{d^2x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 27$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d^2}$$

$$\Big\downarrow \ 636$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\left(\frac{\int-\frac{-\frac{bdx^2}{c}-2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{d^2}+\frac{C\left(-\frac{\int\frac{-\frac{bdx^2}{c}-2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{C\left(-\frac{\int\frac{-\frac{bdx^2}{c}-2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}-$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\sqrt{a-bx^2}}+\frac{d^2}{c^3x\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{d}{c^2x^2\sqrt{c+dx}\sqrt{a-bx^2}}+\frac{1}{cx^3\sqrt{c+dx}\sqrt{a-bx^2}}\right)dx}{d^2}$$

$\downarrow$ 2009

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\right.}{c^3\sqrt{c+}}\right.}{}$$

$$\frac{C\left(-\frac{\int\frac{-\frac{bdx^2}{c}-2bx+\frac{3ad}{c}}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$\Big\downarrow$ 2352

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\right.}{c^3\sqrt{c+}}\right.}{}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{\int\frac{\frac{3abd^2x^2}{c}+2abdx+a\left(\frac{3ad^2}{c}+4bc\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$\Big\downarrow$ 2351

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.}{\cdots}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{a\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int\frac{\frac{3abxd^2}{c}+2abd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$$\Big\downarrow \; 600$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.}{\cdots}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+a\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{3abd\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{c}}{2ac}}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$$\Big\downarrow \; 509$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{}}{}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\right.}{c^3\sqrt{c+}}\right.}{}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-abd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+a\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{3abd\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{c\sqrt{a-bx^2}}}{2ac}}{4a}-\frac{\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}\right)}{d^2}$$

$\downarrow$ 508

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-bd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dxa+\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dxa}{2ac}}{4a}\right.}{d^2}$$

$\downarrow$ 327

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\cdots\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\cdots\right.}{\cdots}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-bd\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dxa+\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dxa\cdots}{2ac}}{4a}\cdots\right)}{d^2}$$

$$\Big\downarrow \text{ 512}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\cdots\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\cdots\right.}{\cdots}$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\left(\frac{3ad^2}{c}+4bc\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dxa-\frac{bd\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{c+dx}\sqrt{\cdots}}\cdots}{\sqrt{a-bx^2}}\cdots}{2ac}}{4a}\cdots\right)}{d^2}$$

$\downarrow$ 511

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{a}\cdots}{\cdots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+d\cdots}}\right.$$

$$C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)\cdots}}}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}\cdots}{4a}\cdots\right.$$

$$\cdots\Big/ d^2$$

$\downarrow$ 321

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\cdots}\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4a}\right.}{d^2}$$

$\downarrow$ 633

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.$$

$$\frac{C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\cdots}\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4a}\right.}{d^2}$$

$$\downarrow 632$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\cdots\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\cdots\right.$$

$$C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\cdots\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}\cdots}{4a}\cdots\right.$$

$$\overline{\hspace{20em}}$$

$$d^2$$

$$\downarrow 186$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$\frac{(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+d\cdots}}\right.\cdots}{\cdots}$$

$$C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\cdots\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{2ac}\cdots}{4a}\cdots\right.$$

$$\overline{\qquad\qquad\qquad d^2\qquad\qquad\qquad}$$

↓ 413

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\ldots}}{\ldots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticPi}\left(\ldots\right.}{c^3\sqrt{c+\ldots}}\right.$$

$$C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\ldots}{\frac{\sqrt{b}\ldots}{\sqrt{a}}}\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4a}\right.$$

$$\overline{\qquad\qquad\qquad d^2\qquad\qquad\qquad}$$

$\downarrow$ 412

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\left(-\frac{6\sqrt{a-bx^2}d^4}{c^4\left(bc^2-ad^2\right)\sqrt{c+dx}}-\frac{8b\sqrt{a-bx^2}d^4}{3c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}-\frac{2\sqrt{a-bx^2}d^4}{3c^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{6\sqrt{\cdots}}{\cdots}\right.$$

$$(2cC-Bd)\left(-\frac{2\sqrt{a-bx^2}d^4}{c^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)d^3}{c^3(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{3\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticPi}\left(\cdots\right.}{c^3\sqrt{c+\cdots}}\right.$$

$$C\left(-\frac{-\frac{3\sqrt{c+dx}\sqrt{a-bx^2}d}{c^2x}-\frac{-\frac{6\sqrt{b}d\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{3/2}}{c\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}+\frac{2\sqrt{b}d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\cdots}{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4a}\right.$$

$$\overline{\qquad d^2 \qquad}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^(5/2)*Sqrt[a - b*x^2]),x]
```

output
```
(A + (c*(c*C - B*d))/d^2)*((-2*d^4*Sqrt[a - b*x^2])/(3*c^3*(b*c^2 - a*d^2)
*(c + d*x)^(3/2)) - (8*b*d^4*Sqrt[a - b*x^2])/(3*c^2*(b*c^2 - a*d^2)^2*Sqr
t[c + d*x]) - (6*d^4*Sqrt[a - b*x^2])/(c^4*(b*c^2 - a*d^2)*Sqrt[c + d*x])
- (Sqrt[c + d*x]*Sqrt[a - b*x^2])/(2*a*c^3*x^2) + (11*d*Sqrt[c + d*x]*Sqrt
[a - b*x^2])/(4*a*c^4*x) - (11*Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]
*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*
c)/Sqrt[a] + d)])/(4*Sqrt[a]*c^4*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqr
t[a]*d)]*Sqrt[a - b*x^2]) + (8*Sqrt[a]*b^(3/2)*d^3*Sqrt[c + d*x]*Sqrt[1 -
(b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/
((Sqrt[b]*c)/Sqrt[a] + d)])/(3*c^2*(b*c^2 - a*d^2)^2*Sqrt[(Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (6*Sqrt[a]*Sqrt[b]*d^3*Sqr
t[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(c^4*(b*c^2 - a*d^2)*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (9*Sqrt[b]
*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)
/Sqrt[a] + d)])/(4*Sqrt[a]*c^3*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*Sqrt[a]
*Sqrt[b]*d^3*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b
*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((
Sqrt[b]*c)/Sqrt[a] + d)])/(3*c^3*(b*c^2 - a*d^2)*Sqrt[c + d*x]*Sqrt[a -...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 636
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_))/Sqrt[(a_) + (b_.)*(x_)^2],
 x_Symbol] :> Simp[c^(n - 1/2)*(e*x)^(m + 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]
/(a*e*(m + 1))), x] - Simp[1/(2*a*e*(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c +
 d*x]*Sqrt[a + b*x^2]))*ExpandToSum[(2*a*c^(n + 1/2)*(m + 1) + a*c^(n - 1/2
)*d*(2*m + 3)*x + 2*b*c^(n + 1/2)*(m + 2)*x^2 + b*c^(n - 1/2)*d*(2*m + 5)*x
^3 - 2*a*(m + 1)*(c + d*x)^(n + 1/2))/x, x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && IGtQ[n + 3/2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [A] (verified)

Time = 10.81 (sec) , antiderivative size = 1263, normalized size of antiderivative = 1.61

method	result	size
elliptic	Expression too large to display	1263
risch	Expression too large to display	2004
default	Expression too large to display	16172

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/(a*d^2-b*c^
2)*(A*d^2-B*c*d+C*c^2)/c^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2/
3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^2*d*(9*A*a*d^4-13*A*b*c^2*d^2-6*B*a*c*d^3+1
0*B*b*c^3*d+3*C*a*c^2*d^2-7*C*b*c^4)/c^4/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-1/
2*A/c^3/a/x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4/a*(11*A*d-4*B*c)/c^4*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(-1/3*b*d*(A*d^2-B*c*d+C*c^2)/(a*d^
2-b*c^2)/c^3+1/3*b/c^3*d*(9*A*a*d^4-13*A*b*c^2*d^2-6*B*a*c*d^3+10*B*b*c^3*
d+3*C*a*c^2*d^2-7*C*b*c^4)/(a*d^2-b*c^2)^2+1/4*A*b*d/a/c^3)*(c/d-1/b*(a*b)
^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/3*b*d
^2*(9*A*a*d^4-13*A*b*c^2*d^2-6*B*a*c*d^3+10*B*b*c^3*d+3*C*a*c^2*d^2-7*C*b*
c^4)/(a*d^2-b*c^2)^2/c^4+1/8*(11*A*d-4*B*c)/a*b*d/c^4)*(c/d-1/b*(a*b)^(1/2
))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(
(-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4*(35*A*a*d^2+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^3\sqrt{a-bx^2}\,(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(5/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(a - b*x**2)*(c + d*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}(dx+c)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{a-b\,x^2}\,{\left(c+d\,x\right)}^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\,(dx+c)^{\frac{5}{2}}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(1/2),x)
```

3.206 $\int \frac{Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 270

$$\int \frac{Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{a}C\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(cC-Bd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output

```
-2*a^(1/2)*C*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x
/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b
^(1/2)/d/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+2*a^(1/2)*
(-B*d+C*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Elli
pticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/b^(1/2)/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.84 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.50

$$\int \frac{Bx+Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2\left(Cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(a-bx^2)+i\sqrt{b}C\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.\right.}{bd^2\sqrt{\cdots}}$$

input
```
Integrate[(B*x + C*x^2)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*(C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2) + I*Sqrt[b]*C*(Sqrt[
b]*c - Sqrt[a]*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[
a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]
*c - Sqrt[a]*d)] - I*Sqrt[b]*(Sqrt[b]*B - Sqrt[a]*C)*d*Sqrt[(d*(Sqrt[a]/Sq
rt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
 d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*
x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(b*d^2*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]]*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {9, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Bx+Cx^2}{x\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{B+Cx}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{600}$$

$$\frac{C\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{509}$$

$$\frac{C\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}$$

$$\Big\downarrow \text{508}$$

$$-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{512}$$

$$-\frac{\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{511}$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

input
```
Int[(B*x + C*x^2)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*Sqrt[a]*C*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(c*C - B*d)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(220) = 440$.

Time = 2.84 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.84

method	result
default	$\frac{2\left(B\,\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)bcd-B\sqrt{ab}\;\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)d^2-C\,\mathrm{EllipticF}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\sqrt{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2B\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\;\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}+\frac{2C\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{a\cdots}}{b}}}\cdots}{\cdots}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\cdots}}$

input
```
int((C*x^2+B*x)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(B*EllipticF((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c
)/((a*b)^(1/2)*d+b*c))^(1/2))*b*c*d-B*(a*b)^(1/2)*EllipticF((-(d*x+c)*b/((
a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))
*d^2-C*EllipticF((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b
*c)/((a*b)^(1/2)*d+b*c))^(1/2))*a*d^2+C*(a*b)^(1/2)*EllipticF((-(d*x+c)*b/
((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2
))*c*d+C*EllipticE((-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d
-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*a*d^2-C*EllipticE((-(d*x+c)*b/((a*b)^(1/
2)*d-b*c))^(1/2),(-((a*b)^(1/2)*d-b*c)/((a*b)^(1/2)*d+b*c))^(1/2))*b*c^2)*
((b*x+(a*b)^(1/2))*d/((a*b)^(1/2)*d-b*c))^(1/2)*((-b*x+(a*b)^(1/2))*d/((a*
b)^(1/2)*d+b*c))^(1/2)*(-(d*x+c)*b/((a*b)^(1/2)*d-b*c))^(1/2)*(-b*x^2+a)^(
1/2)*(d*x+c)^(1/2)/b/d^2/(-b*d*x^3-b*c*x^2+a*d*x+a*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.67

$$\int \frac{Bx + Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \frac{2\left(3\sqrt{-bd}Cd\text{weierstrassZeta}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2}, -\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3}, \text{weierstrassPInverse}\left(\frac{4\,(bc^2+3\,ad^2)}{3\,bd^2}, -\frac{8\,(bc^3-9\,acd^2)}{27\,bd^3},\right.\right.\right.}{3\,bd^2}$$

input
```
integrate((C*x^2+B*x)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
2/3*(3*sqrt(-b*d)*C*d*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + (C*c - 3*B
*d)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d))/(b*d^2)
```

Sympy [F]

$$\int \frac{Bx + Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{B + Cx}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((B + C*x)/(sqrt(a - b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{Bx + Cx^2}{x\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \int \frac{Cx^2 + Bx}{\sqrt{-bx^2 + a}\sqrt{dx + c}x}\,dx$$

input
```
integrate((C*x^2+B*x)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{Bx + Cx^2}{x\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \int \frac{Cx^2 + Bx}{\sqrt{-bx^2 + a}\sqrt{dx + c}x}\,dx$$

input
```
integrate((C*x^2+B*x)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Bx + Cx^2}{x\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \int \frac{C\,x^2 + B\,x}{x\,\sqrt{a - b\,x^2}\,\sqrt{c + d\,x}}\,dx$$

input
```
int((B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((B*x + C*x^2)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{Bx + Cx^2}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,x}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)c + \left(\int \frac{\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)b$$

input
```
int((C*x^2+B*x)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((sqrt(c + d*x)*sqrt(a - b*x**2)*x)/(a*c + a*d*x - b*c*x**2 - b*d*x**3)
,x)*c + int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d
*x**3),x)*b
```

3.207 $\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 615

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{a\sqrt{c+dx}(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{b^3\,(bc^2-ad^2)\sqrt{a-bx^2}}$$

$$+\frac{2(60aCd^2+b(57c^2C-49Bcd+35Ad^2))\sqrt{c+dx}\sqrt{a-bx^2}}{105b^3d^3}$$

$$-\frac{2(16cC-7Bd)(c+dx)^{3/2}\sqrt{a-bx^2}}{35b^2d^3}+\frac{2C(c+dx)^{5/2}\sqrt{a-bx^2}}{7b^2d^3}$$

$$+\frac{\sqrt{a}(9a^2d^4(37cC-49Bd)-abcd^2(132c^2C-224Bcd-245Ad^2)-4b^2c^3(24c^2C-28Bcd+35Ad^2))\sqrt{c-}}{105b^{5/2}d^4\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{\sqrt{a}(225a^2Cd^4+4b^2c^2(24c^2C-28Bcd+35Ad^2)+abd^2(204c^2C-308Bcd+175Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\ \mathrm{I}}{105b^{7/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^3/(-a*d
^2+b*c^2)/(-b*x^2+a)^(1/2)+2/105*(60*a*C*d^2+b*(35*A*d^2-49*B*c*d+57*C*c^2
))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^3/d^3-2/35*(-7*B*d+16*C*c)*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/b^2/d^3+2/7*C*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b^2/d^3+1
/105*a^(1/2)*(9*a^2*d^4*(-49*B*d+37*C*c)-a*b*c*d^2*(-245*A*d^2-224*B*c*d+1
32*C*c^2)-4*b^2*c^3*(35*A*d^2-28*B*c*d+24*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a
)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1
/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^4/(-a*d^2+b*c^2)/((d*x+c)/(c
+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/105*a^(1/2)*(225*a^2*C*d^4+4
*b^2*c^2*(35*A*d^2-28*B*c*d+24*C*c^2)+a*b*d^2*(175*A*d^2-308*B*c*d+204*C*c
^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(7/2)/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.21 (sec) , antiderivative size = 818, normalized size of antiderivative = 1.33

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(\frac{48bc^2C-56bBcd+70Abd^2+120aCd^2}{d^3}+\frac{6b(-6cC+7Bd)x}{d^2}+\frac{30bCx^2}{d}+\frac{105a\left(a^2Cd-Ab^2cx\cdots\right.}{\left(bc^2-\cdots\right.}\right.}{b^3}\right.}{\cdots}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*((48*b*c^2*C - 56*b*B*c*d + 70*A*b*d^2 + 120*
a*C*d^2)/d^3 + (6*b*(-6*c*C + 7*B*d)*x)/d^2 + (30*b*C*x^2)/d + (105*a*(a^2
*C*d - A*b^2*c*x + a*b*(-(B*c) + A*d - c*C*x + B*d*x)))/((b*c^2 - a*d^2)*(
-a + b*x^2))))/b^3 - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(9*a^2*d^4*(-37*c
*C + 49*B*d) + a*b*c*d^2*(132*c^2*C - 224*B*c*d - 245*A*d^2) + 4*b^2*c^3*(
24*c^2*C - 28*B*c*d + 35*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt
[a]*d)*(9*a^2*d^4*(-37*c*C + 49*B*d) + a*b*c*d^2*(132*c^2*C - 224*B*c*d -
245*A*d^2) + 4*b^2*c^3*(24*c^2*C - 28*B*c*d + 35*A*d^2))*Sqrt[(d*(Sqrt[a]/
Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
 + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c +
d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqr
t[b]*c - Sqrt[a]*d)*(225*a^2*C*d^4 + 9*a^(3/2)*Sqrt[b]*d^3*(62*c*C - 49*B*
d) + 4*b^2*c^2*(24*c^2*C - 28*B*c*d + 35*A*d^2) + 12*Sqrt[a]*b^(3/2)*c*d*(
6*c^2*C - 7*B*c*d + 35*A*d^2) + a*b*d^2*(204*c^2*C - 308*B*c*d + 175*A*d^2
))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
 d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)
])/(b^3*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-(b*c^2) + a*d^2)*(-a + b*x^2)
)))/(105*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 4.45 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2180, 27, 2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2} \sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc+Ad)+ad(cC-3Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^3}}{2\sqrt{c+dx}\sqrt{a-bx^2}}d}{a\left(bc^2-ad^2\right)}$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} - \frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc+Ad)+ad(cC-3Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^3}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 2185$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} - \frac{-\frac{2\int \frac{2ad^3(16cC-7Bd)\left(bc^2-ad^2\right)x^3+\frac{2ad^2\left(bc^2-ad^2\right)\left(11bCc^2-7Abd^2-12aCd^2\right)x^2}{b}+\frac{ad\left(4b^2Cc^5-abd^2\left(24Cc^2+14Bdc+7Ad^2\right)c+a^2d^4(13cC+21Bd)\right)x}{b}+\frac{a^2}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} - \frac{-\frac{\int \frac{2ad^3(16cC-7Bd)\left(bc^2-ad^2\right)x^3+\frac{2ad^2\left(bc^2-ad^2\right)\left(11bCc^2-7Abd^2-12aCd^2\right)x^2}{b}+\frac{ad\left(4b^2Cc^5-abd^2\left(24Cc^2+14Bdc+7Ad^2\right)c+a^2d^4(13cC+21Bd)\right)x}{b}+\frac{a^2d}{}}{\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 2185$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} - \frac{-\frac{-\frac{2\int -\frac{-2a\left(bc^2-ad^2\right)\left(60aCd^2+b\left(57Cc^2-49Bdc+35Ad^2\right)\right)x^2d^5+\frac{a^2\left(35a^2Cd^4-ab\left(116Cc^2-77Bdc-35Ad^2\right)d^2+2b^2c^2\left(23Cc^2-21Bdc-35Ad^2\right)\right)d^5}{b}-a\left(4b\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4}}{2a\,(}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} - \frac{-\frac{\frac{\int \frac{-2a\left(bc^2-ad^2\right)\left(60aCd^2+b\left(57Cc^2-49Bdc+35Ad^2\right)\right)x^2d^5+a^2\left(\frac{35a^2Cd^4}{b}-a\left(116Cc^2-77Bdc-35Ad^2\right)d^2+2bc^2\left(23Cc^2-21Bdc-35Ad^2\right)\right)d^5-a\left(4b^2(11cC\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4}}{2a\,(bc}$$

$$\Big\downarrow \; 2185$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{\frac{\frac{4ad^4\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)\left(60aCd^2+b\left(35Ad^2-49Bcd+57c^2C\right)\right)}{3b}-\frac{2\int -\frac{ad^6\left(ad\left(225a^2Cd^4-ab\left(354Cc^2-133Bdc-175Ad^2\right)d^2+4b^2c^2\left(6Cc^2-7Bdc-70\right.\right.\right.}{}}{}}{5bd^3}}{7b}$$

$\Big\downarrow$ 27

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{\frac{\frac{ad^4\int \frac{ad\left(225a^2Cd^4-ab\left(354Cc^2-133Bdc-175Ad^2\right)d^2+4b^2c^2\left(6Cc^2-7Bdc-70Ad^2\right)\right)-b\left(9a^2(37cC-49Bd)d^4-abc\left(132Cc^2-224Bdc-245Ad^2\right)d^2-4b^2c^3\left(2\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{3b}}{5bd^3}}{7bd^4}$$

$\Big\downarrow$ 600

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{\frac{ad^4\left(-\frac{\left(bc^2-ad^2\right)\left(225a^2Cd^4+abd^2\left(175Ad^2-308Bcd+204c^2C\right)+4b^2c^2\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(9a^2d^4(37cC-49Bd)-abcd^2\left(-2\right.\right.}{}\right.}{3b}}{5bd^3}$$

$\Big\downarrow$ 509

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{\frac{ad^4\left(-\frac{\left(bc^2-ad^2\right)\left(225a^2Cd^4+abd^2\left(175Ad^2-308Bcd+204c^2C\right)+4b^2c^2\left(35Ad^2-28Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(9a^2d^4(37cC-49Bd)-\right.}{}\right.}{3b}}{5}$$

$\Big\downarrow$ 508

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)}-$$

$$- \frac{ad^4\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2d^4(37cC-49Bd)-abcd^2\left(-245Ad^2-224Bcd+132c^2C\right)-4b^2c^3\left(35Ad^2-28Bcd+24c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}} - \cdots\right.}{3b}$$

$\downarrow$ 327

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)}-$$

$$- \frac{ad^4\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2d^4(37cC-49Bd)-abcd^2\left(-245Ad^2-224Bcd+132c^2C\right)-4b^2c^3\left(35Ad^2-28Bcd+24c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}} \cdots\right.}{3b}$$

$\downarrow$ 512

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\,(bc^2-ad^2)}-$$

$$- \frac{ad^4\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9a^2d^4(37cC-49Bd)-abcd^2\left(-245Ad^2-224Bcd+132c^2C\right)-4b^2c^3\left(35Ad^2-28Bcd+24c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}} \cdots\right.}{3b}$$

$\downarrow$ 511

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$ad^4\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(225a^2Cd^4+abd^2\left(175Ad^2-308Bcd+204c^2C\right)+4b^2c^2\left(35Ad^2-28Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$\downarrow 321$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$ad^4\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(225a^2Cd^4+abd^2\left(175Ad^2-308Bcd+204c^2C\right)+4b^2c^2\left(35Ad^2-28Bcd+24c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(a*Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x)
)/(b^3*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-4*a*C*(b*c^2 - a*d^2)*(c + d*
x)^(5/2)*Sqrt[a - b*x^2])/(7*b^2*d^3) - ((-4*a*d*(16*c*C - 7*B*d)*(b*c^2 -
 a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b) + ((4*a*d^4*(b*c^2 - a*d^2)
*(60*a*C*d^2 + b*(57*c^2*C - 49*B*c*d + 35*A*d^2))*Sqrt[c + d*x]*Sqrt[a -
b*x^2])/(3*b) + (a*d^4*((2*Sqrt[a]*Sqrt[b]*(9*a^2*d^4*(37*c*C - 49*B*d) -
a*b*c*d^2*(132*c^2*C - 224*B*c*d - 245*A*d^2) - 4*b^2*c^3*(24*c^2*C - 28*B
*c*d + 35*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[
1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt
[a]*(b*c^2 - a*d^2)*(225*a^2*C*d^4 + 4*b^2*c^2*(24*c^2*C - 28*B*c*d + 35*A
*d^2) + a*b*d^2*(204*c^2*C - 308*B*c*d + 175*A*d^2))*Sqrt[(Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
 (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]
*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*b))/(5*b*d^3))/(7*b*d^4))/(2*a*(b*c
^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 10.73 (sec) , antiderivative size = 1079, normalized size of antiderivative = 1.75

method	result	size
elliptic	Expression too large to display	1079
risch	Expression too large to display	1693
default	Expression too large to display	4795

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(-1/2*a*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b^3*x+1/2*a^2*(A*b*d-B*b*c+C*a*
d)/(a*d^2-b*c^2)/b^4)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/7*C/b^2/d*x^2*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(-B/b+6/7*C/b/d*c)/b/d*x*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)-2/3*(-1/b^2*(A*b+C*a)-5/7*C/b^2*a-4/5*(-B/b+6/7*C/b/d*
c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-1/2/b^3*d*a^2*(A*b*d-B*
b*c+C*a*d)/(a*d^2-b*c^2)+1/b^2*c*a*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)+2/5*(
-B/b+6/7*C/b/d*c)/b/d*a*c+1/3*(-1/b^2*(A*b+C*a)-5/7*C/b^2*a-4/5*(-B/b+6/7*
C/b/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2
))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Ellipt
icF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2))+2*(-B*a/b^2+1/2*a*d*(A*b*c-B*a*d+C*a*c)/b^2/(a*d^2-
b*c^2)-4/7*C/b^2/d*a*c+3/5*(-B/b+6/7*C/b/d*c)/b*a-2/3*(-1/b^2*(A*b+C*a)-5/
7*C/b^2*a-4/5*(-B/b+6/7*C/b/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b
)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b
*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 981, normalized size of antiderivative = 1.60

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
1/315*((96*C*a*b^3*c^6 - 112*B*a*b^3*c^5*d - 140*B*a^2*b^2*c^3*d^3 + 42*B*
a^3*b*c*d^5 + 20*(3*C*a^2*b^2 + 7*A*a*b^3)*c^4*d^2 + (729*C*a^3*b + 595*A*
a^2*b^2)*c^2*d^4 - 75*(9*C*a^4 + 7*A*a^3*b)*d^6 - (96*C*b^4*c^6 - 112*B*b^
4*c^5*d - 140*B*a*b^3*c^3*d^3 + 42*B*a^2*b^2*c*d^5 + 20*(3*C*a*b^3 + 7*A*b
^4)*c^4*d^2 + (729*C*a^2*b^2 + 595*A*a*b^3)*c^2*d^4 - 75*(9*C*a^3*b + 7*A*
a^2*b^2)*d^6)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b
*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(96*C*a*b
^3*c^5*d - 112*B*a*b^3*c^4*d^2 - 224*B*a^2*b^2*c^2*d^4 + 441*B*a^3*b*d^6 +
 4*(33*C*a^2*b^2 + 35*A*a*b^3)*c^3*d^3 - (333*C*a^3*b + 245*A*a^2*b^2)*c*d
^5 - (96*C*b^4*c^5*d - 112*B*b^4*c^4*d^2 - 224*B*a*b^3*c^2*d^4 + 441*B*a^2
*b^2*d^6 + 4*(33*C*a*b^3 + 35*A*b^4)*c^3*d^3 - (333*C*a^2*b^2 + 245*A*a*b^
3)*c*d^5)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -
8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2
)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(48*
C*a*b^3*c^4*d^2 - 56*B*a*b^3*c^3*d^3 + 161*B*a^2*b^2*c*d^5 + 2*(36*C*a^2*b
^2 + 35*A*a*b^3)*c^2*d^4 - 25*(9*C*a^3*b + 7*A*a^2*b^2)*d^6 - 30*(C*b^4*c^
2*d^4 - C*a*b^3*d^6)*x^4 + 6*(6*C*b^4*c^3*d^3 - 7*B*b^4*c^2*d^4 - 6*C*a*b^
3*c*d^5 + 7*B*a*b^3*d^6)*x^3 - 2*(24*C*b^4*c^4*d^2 - 28*B*b^4*c^3*d^3 + 28
*B*a*b^3*c*d^5 + 7*(3*C*a*b^3 + 5*A*b^4)*c^2*d^4 - 5*(9*C*a^2*b^2 + 7*A*a*
b^3)*d^6)*x^2 - 3*(12*C*a*b^3*c^3*d^3 - 14*B*a*b^3*c^2*d^4 + 49*B*a^2*b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^4\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x^4(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.208 $\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 520

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{a\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{b^2\,(bc^2-ad^2)\,\sqrt{a-bx^2}}$$

$$-\frac{2(7cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15b^2d^2}+\frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5b^2d^2}$$

$$-\frac{\sqrt{a}(63a^2Cd^4-abd^2(32c^2C+35Bcd-45Ad^2)-2b^2c^2(8c^2C-10Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\Big(\text{ar}}{15b^{5/2}d^3\,(bc^2-ad^2)\,\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{a}(ad^2(44cC-25Bd)+2bc(8c^2C-10Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\right.}{15b^{5/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/b^2/(-a*d^2+b*c
^2)/(-b*x^2+a)^(1/2)-2/15*(-5*B*d+7*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^
2/d^2+2/5*C*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d^2-1/15*a^(1/2)*(63*a^2*C*
d^4-a*b*d^2*(-45*A*d^2+35*B*c*d+32*C*c^2)-2*b^2*c^2*(15*A*d^2-10*B*c*d+8*C
*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)
/d^3/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)
-1/15*a^(1/2)*(a*d^2*(-25*B*d+44*C*c)+2*b*c*(15*A*d^2-10*B*c*d+8*C*c^2))*(
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1
-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d)
)^(1/2))/b^(5/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.85 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.40

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(\frac{2(-4cC+5Bd)}{d^2}+\frac{6Cx}{d}-\frac{15a(acC+bBcx-ad(B+Cx)+Ab(c-dx))}{(bc^2-ad^2)(-a+bx^2)}\right)}{b^2}-\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*((2*(-4*c*C + 5*B*d))/d^2 + (6*C*x)/d - (15*a
*(a*c*C + b*B*c*x - a*d*(B + C*x) + A*b*(c - d*x)))/((b*c^2 - a*d^2)*(-a +
 b*x^2))))/b^2 - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(63*a^2*C*d^4 - 2*b^2
*c^2*(8*c^2*C - 10*B*c*d + 15*A*d^2) + a*b*d^2*(-32*c^2*C - 35*B*c*d + 45*
A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(63*a^2*C*d^4 - 2*
b^2*c^2*(8*c^2*C - 10*B*c*d + 15*A*d^2) + a*b*d^2*(-32*c^2*C - 35*B*c*d +
45*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/S
qrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sq
rt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(63*a^(3/2)*C*d^3
+ a*Sqrt[b]*d^2*(44*c*C - 25*B*d) + 3*Sqrt[a]*b*d*(4*c^2*C - 20*B*c*d + 15
*A*d^2) + 2*b^(3/2)*c*(8*c^2*C - 10*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sq
rt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
 d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*
x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(b^3*d^4*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(-(b*c^2) + a*d^2)*(-a + b*x^2))))/(15*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2180, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx+Cx^2)}{(a-bx^2)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2-3ad^2\right)-a\left(3aCd^2-bc(2cC+Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))}{b^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2-3ad^2\right)-a\left(3aCd^2-bc(2cC+Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))}{b^2}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{2\int \frac{2a(7cC-5Bd)\left(bc^2-ad^2\right)x^2d^2+\frac{a^2\left(ad^2(11cC+5Bd)-bc\left(6Cc^2+10Bdc-5Ad^2\right)\right)d^2}{b}+\frac{a\left(21a^2Cd^4-5ab\left(4Cc^2+Bdc-3Ad^2\right)d^2+2b^2\left(2c^4C-5Ac^2d^2\right)\right)xd}{b}}{2\sqrt{c+dx}\sqrt{a-bx^2}}d}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\int \frac{2a(7cC-5Bd)\left(bc^2-ad^2\right)x^2d^2+\frac{a^2\left(ad^2(11cC+5Bd)-bc\left(6Cc^2+10Bdc-5Ad^2\right)\right)d^2}{b}+\frac{a\left(21a^2Cd^4-5ab\left(4Cc^2+Bdc-3Ad^2\right)d^2+2b^2\left(2c^4C-5Ac^2d^2\right)\right)xd}{b}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{-\frac{2\int -\frac{ad^3\left(ad\left(ad^2(19cC+25Bd)-bc\left(4Cc^2+40Bdc-15Ad^2\right)\right)+\left(63a^2Cd^4-ab\left(32Cc^2+35Bdc-45Ad^2\right)d^2-2b^2c^2\left(8Cc^2-10Bdc+15Ad^2\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{4ad\sqrt{a-b}}{}}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{ad\int \frac{ad\left(ad^2(19cC+25Bd)-bc\left(4Cc^2+40Bdc-15Ad^2\right)\right)+\left(63a^2Cd^4-ab\left(32Cc^2+35Bdc-45Ad^2\right)d^2-2b^2c^2\left(8Cc^2-10Bdc+15Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b}-\frac{4ad\sqrt{a-bx^2}\sqrt{c+d}}{}}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{ad\left(\frac{\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32c^2C\right)-2b^2c^2\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}+\frac{\left(bc^2-ad^2\right)\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bcd+8c^2C\right)\right.}{d}\right.}{3b}}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\downarrow$ 509

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{ad\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32c^2C\right)-2b^2c^2\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\left(bc^2-ad^2\right)\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bc\right.\right.}{d}\right.}{3b}}{5bd^3}}{2a\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{ad\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32c^2C\right)-2b\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\cdots}}\right.}{3b}}{5bd^3}}{2a\left(bc^2-\cdots\right.}$$

$\downarrow$ 327

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{ad\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32c^2C\right)-2b\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\cdots}}\right.}{3b}}{5bd^3}}{2a\left(bc^2-\cdots\right.}$$

$\downarrow$ 512

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{ad\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32\right.\right.}{\sqrt{b}d\sqrt{}}\right.}{3b}$$

$$5bd^3 \qquad 2a\,(bc$$

$\downarrow$ 511

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{ad\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(44cC-25Bd)+2bc\left(15Ad^2-10Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-b}}{}\right.}{3b}$$

$\downarrow$ 321

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)} -$$

$$-\frac{ad\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^2Cd^4-abd^2\left(-45Ad^2+35Bcd+32c^2C\right)-2b^2c^2\left(15Ad^2-10Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc\right.}{}\right.}{3b}$$

$$5$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(a*Sqrt[c + d*x]*(A*b*c + a*c*C - a*B*d + (b*B*c - A*b*d - a*C*d)*x))/(b^2
*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-4*a*C*(b*c^2 - a*d^2)*(c + d*x)^(3/
2)*Sqrt[a - b*x^2])/(5*b^2*d^2) - ((-4*a*d*(7*c*C - 5*B*d)*(b*c^2 - a*d^2)
*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + (a*d*((-2*Sqrt[a]*(63*a^2*C*d^4 -
a*b*d^2*(32*c^2*C + 35*B*c*d - 45*A*d^2) - 2*b^2*c^2*(8*c^2*C - 10*B*c*d +
 15*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (S
qrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*
Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sq
rt[a]*(b*c^2 - a*d^2)*(a*d^2*(44*c*C - 25*B*d) + 2*b*c*(8*c^2*C - 10*B*c*d
 + 15*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (
b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/(
(Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*
b))/(5*b*d^3))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(450) = 900$.

Time = 10.26 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.75

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(\frac{a(Abd-Bbc+aCd)x}{2\left(a\,d^2-b\,c^2\right)b^3}-\frac{a(Abc-Bad+Cac)}{2\left(a\,d^2-b\,c^2\right)b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2Cx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5b^2d}-\frac{2\left(-\frac{B}{b}+\frac{4Cc}{5bd}\right)\sqrt{-bd\,x^3-}}{3bd}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*a*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/b^3*x-1/2*a*(A*b*c-B*a*d+C*a*c)/
(a*d^2-b*c^2)/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/5*C/b^2/d*x*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-B/b+4/5*C/b/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)+2*(1/2*a*d*(A*b*c-B*a*d+C*a*c)/b^2/(a*d^2-b*c^2)-1/b^2*c*a*(A
*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)-2/5*C/b^2/d*a*c+1/3*(-B/b+4/5*C/b/d*c)/b*a
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+2*(-1/b^2*(A*b+C*a)-1/2*d*a*(A*b*d-B*b*c+C*a*d)/b^2/(a*d^2-b*c^2)-3/
5*C/b^2*a-2/3*(-B/b+4/5*C/b/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*
x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/
2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b
)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.42

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/45*((16*C*a*b^2*c^5 - 20*B*a*b^2*c^4*d - 85*B*a^2*b*c^2*d^3 - 6*C*a^3*c
*d^4 + 75*B*a^3*d^5 + 10*(2*C*a^2*b + 3*A*a*b^2)*c^3*d^2 - (16*C*b^3*c^5 -
 20*B*b^3*c^4*d - 85*B*a*b^2*c^2*d^3 - 6*C*a^2*b*c*d^4 + 75*B*a^2*b*d^5 +
10*(2*C*a*b^2 + 3*A*b^3)*c^3*d^2)*x^2)*sqrt(-b*d)*weierstrassPInverse(4/3*
(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x +
 c)/d) + 3*(16*C*a*b^2*c^4*d - 20*B*a*b^2*c^3*d^2 + 35*B*a^2*b*c*d^4 + 2*(
16*C*a^2*b + 15*A*a*b^2)*c^2*d^3 - 9*(7*C*a^3 + 5*A*a^2*b)*d^5 - (16*C*b^3
*c^4*d - 20*B*b^3*c^3*d^2 + 35*B*a*b^2*c*d^4 + 2*(16*C*a*b^2 + 15*A*b^3)*c
^2*d^3 - 9*(7*C*a^2*b + 5*A*a*b^2)*d^5)*x^2)*sqrt(-b*d)*weierstrassZeta(4/
3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstras
sPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3)
, 1/3*(3*d*x + c)/d)) + 3*(8*C*a*b^2*c^3*d^2 - 10*B*a*b^2*c^2*d^3 + 25*B*a
^2*b*d^5 - (23*C*a^2*b + 15*A*a*b^2)*c*d^4 + 6*(C*b^3*c^2*d^3 - C*a*b^2*d^
5)*x^3 - 2*(4*C*b^3*c^3*d^2 - 5*B*b^3*c^2*d^3 - 4*C*a*b^2*c*d^4 + 5*B*a*b^
2*d^5)*x^2 - 3*(2*C*a*b^2*c^2*d^3 + 5*B*a*b^2*c*d^4 - (7*C*a^2*b + 5*A*a*b
^2)*d^5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a*b^4*c^2*d^4 - a^2*b^3*d^6 -
 (b^5*c^2*d^4 - a*b^4*d^6)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^3\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^3(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.209 $\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 450

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{c+dx}(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{b^2(bc^2-ad^2)\sqrt{a-bx^2}} + \frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3b^2d} + \frac{\sqrt{a}(ad^2(7cC-9Bd)-bc(4c^2C-6Bcd-3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{3/2}d^2(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} + \frac{\sqrt{a}(5aCd^2+b(4c^2C-6Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3b^{5/2}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^2/(-a*d^2
+b*c^2)/(-b*x^2+a)^(1/2)+2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d+1/3*a^
(1/2)*(a*d^2*(-9*B*d+7*C*c)-b*c*(-3*A*d^2-6*B*c*d+4*C*c^2))*(d*x+c)^(1/2)*
((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(
1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^2/(-a*d^2+b*c^2)/(
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/3*a^(1/2)*(5*a*C*d
^2+b*(3*A*d^2-6*B*c*d+4*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b
*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^2/(d*x+c)^(1/2)/(-b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.12 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.43

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(\frac{2C}{d}+\frac{3\left(a^2Cd-Ab^2cx+ab(-Bc+Ad-cCx+Bdx)\right)}{\left(bc^2-ad^2\right)\left(-a+bx^2\right)}\right)}{b^2}-\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(ad^2(-7cC+\right.}{}\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*((2*C)/d + (3*(a^2*C*d - A*b^2*c*x + a*b*(-(B
*c) + A*d - c*C*x + B*d*x)))/((b*c^2 - a*d^2)*(-a + b*x^2))))/b^2 - (d^2*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a*d^2*(-7*c*C + 9*B*d) + b*c*(4*c^2*C - 6*B
*c*d - 3*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^2*(-
7*c*C + 9*B*d) + b*c*(4*c^2*C - 6*B*c*d - 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[
b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*
x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]]
, (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*d*(Sqrt[b]*c - Sqrt
[a]*d)*(6*A*b^(3/2)*c*d + 5*a^(3/2)*C*d^2 + 3*a*Sqrt[b]*d*(4*c*C - 3*B*d)
+ Sqrt[a]*b*(4*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/
(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*
EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b
]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(b^2*d^3*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]*(-(b*c^2) + a*d^2)*(-a + b*x^2))))/(3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2180, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2} \sqrt{c+dx}}dx$$

$$\Big\downarrow \textbf{2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc+Ad)+ad(cC-3Bd))x}{b}+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc+Ad)+ad(cC-3Bd))x}{b}+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^2}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2185

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{2\int-\frac{ad\left(d\left(3Ab\left(2bc^2-ad^2\right)-a\left(5aCd^2-bc(8cC-3Bd)\right)\right)+b\left(ad^2(7cC-9Bd)-bc\left(4Cc^2-6Bdc-3Ad^2\right)\right)x\right)}{2b\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{4aC\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}{3b^2d}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{a\int\frac{d\left(3Ab\left(2bc^2-ad^2\right)-a\left(5aCd^2-bc(8cC-3Bd)\right)\right)+b\left(ad^2(7cC-9Bd)-bc\left(4Cc^2-6Bdc-3Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b^2d}-\frac{4aC\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}{3b^2d}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{a\left(\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\left(ad^2(7cC-9Bd)-bc\left(-3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)}{3b^2d}-\frac{4aC\sqrt{a-bx^2}\sqrt{c+}}{3b^2}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{a\left(\frac{\left(bc^2-ad^2\right)\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(7cC-9Bd)-bc\left(-3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{3b^2d}-4aC\sqrt{a-}}{2a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)}-\frac{a\left(\frac{(bc^2-ad^2)\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(7cC-9Bd)-bc\left(-3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d\cdots}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)\cdots}}\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3b^2d}\cdots}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 327$$

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)}-\frac{a\left(\frac{(bc^2-ad^2)\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(7cC-9Bd)-bc\left(-3Ad^2-6Bcd+4c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{2}}\cdots\right)\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3b^2d}\cdots}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 512$$

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)}-\frac{a\left(\frac{\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(7cC-9Bd)-bc\left(-3Ad^2-6Bcd+4c^2C\right)\right)E\left(\mathrm{arcsi}\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{3b^2d}\cdots}{2a\,(bc^2-ad^2)}$$

$$\Big\downarrow \; 511$$

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2\cdots\right)\cdots}{\cdots}\right)}{3b^2d}\cdots}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 321

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^2\sqrt{a-bx^2}\,(bc^2-ad^2)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5aCd^2+b\left(3Ad^2-6Bcd+4c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(7cC-9B\dots\right.}{\dots}\right)}{3b^2d}\cdots}{2a\,(bc^2-ad^2)}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x))/
(b^2*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-4*a*C*(b*c^2 - a*d^2)*Sqrt[c +
d*x]*Sqrt[a - b*x^2])/(3*b^2*d) + (a*((-2*Sqrt[a]*Sqrt[b]*(a*d^2*(7*c*C -
9*B*d) - b*c*(4*c^2*C - 6*B*c*d - 3*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)
/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*
Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(5*a*C*d^2 + b*(4*c^2*C - 6*
B*c*d + 3*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1
 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*
d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))
/(3*b^2*d))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(386) = 772$.

Time = 8.48 (sec), antiderivative size = 801, normalized size of antiderivative = 1.78

method	result
elliptic	$\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(-\frac{(Abc-Bad+Cac)x}{2\left(a\,d^2-b\,c^2\right)b^2}+\frac{a(Abd-Bbc+aCd)}{2\left(a\,d^2-b\,c^2\right)b^3}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2C\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3b^2d}+\frac{2\left(-\frac{da(Abd-Bbc+aCd)}{2b^2\left(a\,d^2-b\,c^2\right)}\right.}{}\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(-1/2*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b^2*x+1/2*a*(A*b*d-B*b*c+C*a*d)/(
a*d^2-b*c^2)/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/3*C/b^2/d*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)+2*(-1/2*d*a*(A*b*d-B*b*c+C*a*d)/b^2/(a*d^2-b*c^2)+1/
b*c*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)-1/3*C/b^2*a)*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),
((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-B/b+1/2*d*(A*b*
c-B*a*d+C*a*c)/b/(a*d^2-b*c^2)+2/3*C/b/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/
b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.29

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{(4\,Cab^2c^4 - 6\,Bab^2c^3d + (17\,Ca^2b + 15\,Aab^2)c^2d^2 - 3\,(5\,Ca^3 + 3\,Aa^2b)d^4 -}{}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
1/9*((4*C*a*b^2*c^4 - 6*B*a*b^2*c^3*d + (17*C*a^2*b + 15*A*a*b^2)*c^2*d^2
- 3*(5*C*a^3 + 3*A*a^2*b)*d^4 - (4*C*b^3*c^4 - 6*B*b^3*c^3*d + (17*C*a*b^2
 + 15*A*b^3)*c^2*d^2 - 3*(5*C*a^2*b + 3*A*a*b^2)*d^4)*x^2)*sqrt(-b*d)*weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d) + 3*(4*C*a*b^2*c^3*d - 6*B*a*b^2*c^2*d^2 + 9*B*
a^2*b*d^4 - (7*C*a^2*b + 3*A*a*b^2)*c*d^3 - (4*C*b^3*c^3*d - 6*B*b^3*c^2*d
^2 + 9*B*a*b^2*d^4 - (7*C*a*b^2 + 3*A*b^3)*c*d^3)*x^2)*sqrt(-b*d)*weierstr
assZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(2*C*a*b^2*c^2*d^2 + 3*B*a*b^2*c*d^3 -
 (5*C*a^2*b + 3*A*a*b^2)*d^4 - 2*(C*b^3*c^2*d^2 - C*a*b^2*d^4)*x^2 - 3*(B*
a*b^2*d^4 - (C*a*b^2 + A*b^3)*c*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a
*b^4*c^2*d^3 - a^2*b^3*d^5 - (b^5*c^2*d^3 - a*b^4*d^5)*x^2)
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x^2\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x^2(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.210 $\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 383

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{c+dx}\left(Abc+acC-aBd+b\left(Bc-\left(A+\frac{aC}{b}\right)d\right)x\right)}{b\,(bc^2-ad^2)\,\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{a}(3aCd^2-b(2c^2C+Bcd-Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{b^{3/2}d\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{a}(2cC-Bd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{b^{3/2}d\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+b*(B*c-(A+a*C/b)*d)*x)/b/(-a*d^2+b*c^2)/(
-b*x^2+a)^(1/2)-a^(1/2)*(3*a*C*d^2-b*(-A*d^2+B*c*d+2*C*c^2))*(d*x+c)^(1/2)
*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^
(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d/(-a*d^2+b*c^2)/((
d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-a^(1/2)*(-B*d+2*C*c)*
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(
1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d
))^(1/2))/b^(3/2)/d/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.06 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.48

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-\frac{b(c+dx)(acC+bBcx-ad(B+Cx)+Ab(c-dx))}{-a+bx^2}+\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\,(-3aCd^2+b(2c^2C+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(-((b*(c + d*x)*(a*c*C + b*B*c*x - a*d*(B + C*x) + A*b*(c
 - d*x)))/(-a + b*x^2)) + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-3*a*C*d^2
+ b*(2*c^2*C + B*c*d - A*d^2))*(-a + b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[
a]*d)*(3*a*C*d^2 + b*(-2*c^2*C - B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]*d*(Sqrt[b]*c -
 Sqrt[a]*d)*(3*a*C*d + b*(-2*B*c + A*d) + Sqrt[a]*Sqrt[b]*(2*c*C - B*d))*S
qrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x
)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(
d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(b^2*(b*c^2 - a*d^2)*Sq
rt[c + d*x])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(bc(2Bc-Ad)-ad(cC+Bd)+b\left(2Cc^2+Bdc-\frac{(Ab+3aC)d^2}{b}\right)x\right)}{2b\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int \frac{bc(2Bc-Ad)-ad(cC+Bd)-\left(3aCd^2-b\left(2Cc^2+Bdc-Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{600}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{509}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{508}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(2cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 512$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(2cC-Bd)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(2cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3aCd^2-b\left(-Ad^2+Bcd+\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{}}}{2b\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{b\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3aCd^2-b\left(-Ad^2+Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(2cC-Bd)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\mathrm{Ellip}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+}}}{2b\left(bc^2-ad^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[c + d*x]*(A*b*c + a*c*C - a*B*d + b*(B*c - (A + (a*C)/b)*d)*x))/(b*(
b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((2*Sqrt[a]*(3*a*C*d^2 - b*(2*c^2*C + B*
c*d - A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*
Sqrt[a]*(2*c*C - B*d)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
+ Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sq
rt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x
]*Sqrt[a - b*x^2]))/(2*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(329) = 658$.

Time = 4.91 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.96

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(\frac{(Abd-Bbc+aCd)x}{2\left(a d^2-b c^2\right)b^2}-\frac{Abc-Bad+Cac}{2\left(a d^2-b c^2\right)b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(\frac{d(Abc-Bad+Cac)}{2b\left(a d^2-b c^2\right)}-\frac{c(Abd-Bbc+aCd)}{b\left(a d^2-b c^2\right)}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\ldots}}{\sqrt{-bd\ldots}}\ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/b^2*x-1/2*(A*b*c-B*a*d+C*a*c)/(a*d
^2-b*c^2)/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1/2*d*(A*b*c-B*a*d+C*a*c)
/b/(a*d^2-b*c^2)-1/b*c*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-C/b-1/2*
d*(A*b*d-B*b*c+C*a*d)/b/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*
b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.15

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = -\frac{(2\,Cabc^3-5\,Babc^2d+2\,Aabcd^2+3\,Ba^2d^3-(2\,Cb^2c^3-5\,Bb^2c^2d+2\,Ab^2cd^2+3\,Babd^3)x^2)\sqrt{-bd}\mathrm{weiers}}{}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/3*((2*C*a*b*c^3 - 5*B*a*b*c^2*d + 2*A*a*b*c*d^2 + 3*B*a^2*d^3 - (2*C*b^
2*c^3 - 5*B*b^2*c^2*d + 2*A*b^2*c*d^2 + 3*B*a*b*d^3)*x^2)*sqrt(-b*d)*weier
strassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b
*d^3), 1/3*(3*d*x + c)/d) + 3*(2*C*a*b*c^2*d + B*a*b*c*d^2 - (3*C*a^2 + A*
a*b)*d^3 - (2*C*b^2*c^2*d + B*b^2*c*d^2 - (3*C*a*b + A*b^2)*d^3)*x^2)*sqrt
(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c
*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(B*a*b*d^3 - (C*a*b + A
*b^2)*c*d^2 - (B*b^2*c*d^2 - (C*a*b + A*b^2)*d^3)*x)*sqrt(-b*x^2 + a)*sqrt
(d*x + c))/(a*b^3*c^2*d^2 - a^2*b^2*d^4 - (b^4*c^2*d^2 - a*b^3*d^4)*x^2)
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.211 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 363

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{c+dx}\left(a\left(Bc-\left(A+\frac{aC}{b}\right)d\right)+(Abc+acC-aBd)x\right)}{a\,(bc^2-ad^2)\sqrt{a-bx^2}} + \frac{(Abc+acC-aBd)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\,(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{(Ab-aC)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}b^{3/2}\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(a*(B*c-(A+a*C/b)*d)+(A*b*c-B*a*d+C*a*c)*x)/a/(-a*d^2+b*c^2)
/(-b*x^2+a)^(1/2)+(A*b*c-B*a*d+C*a*c)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*E
llipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)
*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-(A*b-C*a)*((d*x+c)/(c+a^(1/2)*d/b^(1/2
)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2
^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/(d
*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.64 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.43

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{-((c+dx)\,(-a^2Cd+Ab^2cx+ab(-Ad+cCx+B(c-dx))))+\frac{d^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}}{\ldots}}{\ldots}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(-((c + d*x)*(-(a^2*C*d) + A*b^2*c*x + a*b*(-(A*d) + c*C*x + B*(c - d*x)))
) + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(A*b*c + a*c*C - a*B*d)*(-a + b*x^
2) - I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*c + a*c*C - a*B*d)*Sqrt[(d*(Sq
rt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x
))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqr
t[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*
(Sqrt[b]*c - Sqrt[a]*d)*(A*b*d - a*C*d + Sqrt[a]*Sqrt[b]*(-2*c*C + B*d))*S
qrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x
)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]))/(a*b*(-(b*c^2) + a*d^2)*Sqrt[c + d*x]*S
qrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a-bx^2)^{3/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 2180$$

$$\frac{\int \frac{a\left(aCd^2-b\left(2Cc^2-Bdc+Ad^2\right)\right)-bd(Abc+aCc-aBd)x}{2b\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a\left(aCd^2-b\left(2Cc^2-Bdc+Ad^2\right)\right)-bd(Abc+aCc-aBd)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 600$$

$$\frac{(Ab-aC)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-b(-aBd+acC+Abc)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 509$$

$$\frac{(Ab-aC)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{b\sqrt{1-\frac{bx^2}{a}}(-aBd+acC+Abc)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 508$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(-aBd+acC+Abc)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+(Ab-aC)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 327$$

$$\frac{(Ab-aC)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(-aBd+acC+Abc)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 512

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}(Ab-aC)\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(-aBd+acC+Abc)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 511

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(-aBd+acC+Abc)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(Ab-aC)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\cdots}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(-aBd+acC+Abc)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(Ab-aC)\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\text{arcsi}\cdots\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}}{2ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[c + d*x]*(a*(B*c - (A + (a*C)/b)*d) + (A*b*c + a*c*C - a*B*d)*x))/(a
*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) + ((2*Sqrt[a]*Sqrt[b]*(A*b*c + a*c*C - a
*B*d)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]
*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c
 + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(A*b - a*C
)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1
 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*
d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(2
*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(309) = 618$.

Time = 4.45 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.10

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(-\frac{(Abc-Bad+Cac)x}{2b\left(a d^2-b c^2\right)a}+\frac{Abd-Bbc+aCd}{2\left(a d^2-b c^2\right)b^2}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(-\frac{C}{b}+\frac{Ab+aC}{ba}-\frac{d(Abd-Bbc+aCd)}{2b\left(a d^2-b c^2\right)}+\frac{c(Abc-Bad+Cac)}{\left(a d^2-b c^2\right)a}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(-1/2/b*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/a*x+1/2*(A*b*d-B*b*c+C*a*d)/(a*
d^2-b*c^2)/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-C/b+(A*b+C*a)/b/a-1/2*d
*(A*b*d-B*b*c+C*a*d)/b/(a*d^2-b*c^2)+c*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/a
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+d*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/a*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/
b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx =$$
$$-\frac{(2\,Ba^2bcd-(5\,Ca^2b-Aab^2)c^2+3\,(Ca^3-Aa^2b)d^2-(2\,Bab^2cd-(5\,Cab^2-Ab^3)c^2+3\,(Ca^2b-Aab^2)}{}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="frica
s")
```

output
```
-1/3*((2*B*a^2*b*c*d - (5*C*a^2*b - A*a*b^2)*c^2 + 3*(C*a^3 - A*a^2*b)*d^2
 - (2*B*a*b^2*c*d - (5*C*a*b^2 - A*b^3)*c^2 + 3*(C*a^2*b - A*a*b^2)*d^2)*x
^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 3*(B*a^2*b*d^2 - (C*a^2*b
+ A*a*b^2)*c*d - (B*a*b^2*d^2 - (C*a*b^2 + A*b^3)*c*d)*x^2)*sqrt(-b*d)*wei
erstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a
*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(B*a*b^2*c*d - (C*a^2*b + A*a*b^2
)*d^2 - (B*a*b^2*d^2 - (C*a*b^2 + A*b^3)*c*d)*x)*sqrt(-b*x^2 + a)*sqrt(d*x
 + c))/(a^2*b^3*c^2*d - a^3*b^2*d^3 - (a*b^4*c^2*d - a^2*b^3*d^3)*x^2)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2}{(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.212 $\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 476

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx=\frac{\sqrt{c+dx}(Abc+acC-aBd+(bBc-(Ab+aC)d)x)}{a\left(bc^2-ad^2\right)\sqrt{a-bx^2}}$$
$$+\frac{(bBc-Abd-aCd)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\left(bc^2-ad^2\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{B\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(B*b*c-(A*b+C*a)*d)*x)/a/(-a*d^2+b*c^2)/(
-b*x^2+a)^(1/2)+(-A*b*d+B*b*c-C*a*d)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*El
lipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*
d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-B*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)
*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^
(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(d*x+c)^(1/
2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.48 (sec) , antiderivative size = 1223, normalized size of antiderivative = 2.57

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(-(b^2*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]) + A*b^2*c^3*d*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]] + a*b*c^3*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + a*b*B*c^2*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - a*A*b*c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]] - a^2*c*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 2*b^2*B*c^3*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 2*A*b^2*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]*(c + d*x) - 2*a*b*c^2*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - b
^2*B*c^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + A*b^2*c*d*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + a*b*c*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*(c + d*x)^2 + b*c*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)*(a*c*C + b*
B*c*x - a*d*(B + C*x) + A*b*(c - d*x)) - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*
d)*(-(b*B*c) + A*b*d + a*C*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*c
- Sqrt[a]*Sqrt[b]*B*c - a*c*C + 2*Sqrt[a]*A*Sqrt[b]*d)*Sqrt[(d*(Sqrt[a]/Sq
rt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c +
 d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*
x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + (2*I)*A*b^2*c^2*d*
Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*
x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{B+Cx}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 638}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{B+Cx}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-\frac{\int -\frac{bd(a(cC-Bd)-(bBc-aCd)x)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 27}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\int \frac{a(cC-Bd)-(bBc-aCd)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 600}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\left(\frac{B\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{(bBc-aCd)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 509}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\left(\frac{B\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}(bBc-aCd)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{ 508}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\left(\frac{B\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(bBc-aCd)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{327}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\left(\frac{B\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(bBc-aCd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{512}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{d\left(\frac{B\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(bBc-aCd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{511}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(bBc-aCd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{2a\left(bc^2-ad^2\right)}$$

$$\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(bBc-aCd)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{2a\left(bc^2-ad^2\right)}$$

$$\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{a\sqrt{a-bx^2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(401) = 802$.

Time = 5.14 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.07

method	result
elliptic	$\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2(-bdx-bc)\left(\frac{(Abd-Bbc+aCd)x}{2b\left(a\,d^2-b\,c^2\right)a}-\frac{Abc-Bad+Cac}{2b\left(a\,d^2-b\,c^2\right)a}\right)}{\sqrt{\left(x^2-\frac{a}{b}\right)(-bdx-bc)}}+\frac{2\left(\frac{B}{a}+\frac{d(Abc-Bad+Cac)}{2\left(a\,d^2-b\,c^2\right)a}-\frac{c(Abd-Bbc+aCd)}{\left(a\,d^2-b\,c^2\right)a}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)}{\sqrt{-b}}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(A*b*d-B*b*c+C*a*d)/b/(a*d^2-b*c^2)/a*x-1/2*(A*b*c-B*a*d+C*a*c)/b/(a
*d^2-b*c^2)/a)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(B/a+1/2*d*(A*b*c-B*a*d+C*
a*c)/(a*d^2-b*c^2)/a-c*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/a)*(c/d-1/b*(a*b)
^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-d/a*(A*b*d
-B*b*c+C*a*d)/(a*d^2-b*c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2)))-2/a*A*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*
c)^(1/2)/c*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(
a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b^2*d*x^6 + b^2
*c*x^5 - 2*a*b*d*x^4 - 2*a*b*c*x^3 + a^2*d*x^2 + a^2*c*x), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{x\,(a-bx^2)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*(a - b*x**2)**(3/2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.213 $\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 545

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{c+dx}(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{a^2(bc^2-ad^2)\sqrt{a-bx^2}} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{a^2cx} + \frac{\sqrt{b}(ac(cC-Bd)+A(2bc^2-ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a^{3/2}c(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{(2Ab+aC)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a^{3/2}\sqrt{b}\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{\left(2B-\frac{Ad}{c}\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/a^2/(-a*d^2
+b*c^2)/(-b*x^2+a)^(1/2)-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c/x+b^(1/2)*
(a*c*(-B*d+C*c)+A*(-a*d^2+2*b*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ell
ipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)
*c+a^(1/2)*d))^(1/2))/a^(3/2)/c/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(1/2)*d/b^(1/
2)))^(1/2)/(-b*x^2+a)^(1/2)-(2*A*b+C*a)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1
/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2)
,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(d*x+c)^
(1/2)/(-b*x^2+a)^(1/2)-(2*B-A*d/c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*(
(-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2
^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/(d*x+c)^(1/2)/(-b*x^2+a)
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.39 (sec) , antiderivative size = 1739, normalized size of antiderivative = 3.19

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(-(A/(c*x)) + (a^2*C*d - A*b^2*c*x + a*b*(-(B
*c) + A*d - c*C*x + B*d*x))/((b*c^2 - a*d^2)*(-a + b*x^2))))/a^2 + (2*A*b^
2*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + a*b*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]] - a*b*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*a*A*b*c^3*d^2*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]] - a^2*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] +
 a^2*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + a^2*A*c*d^4*Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]] - 4*A*b^2*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) -
2*a*b*c^4*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 2*a*b*B*c^3*d*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 2*a*A*b*c^2*d^2*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(c + d*x) + 2*A*b^2*c^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x
)^2 + a*b*c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - a*b*B*c^2*d*S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - a*A*b*c*d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(a*c*(c*C -
 B*d) + A*(2*b*c^2 - a*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqr
t[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*Ar
cSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*
d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(Sqrt[a]
*Sqrt[b]*B*c^2 + a*c*(-(c*C) + 2*B*d) - A*(2*b*c^2 + 2*Sqrt[a]*Sqrt[b]*c*d
 + a*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/S
qrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c +...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2 (a-bx^2)^{3/2} \sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^2\,(a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^2\,(a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \left(\frac{\sqrt{c+dx}C}{dx\left(a-bx^2\right)^{3/2}}+\frac{(Bd-cC)\sqrt{c+dx}}{d^2x^2\left(a-bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \textbf{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \left(\frac{C\sqrt{c+dx}}{dx\left(a-bx^2\right)^{3/2}}-\frac{(cC-Bd)\sqrt{c+dx}}{d^2x^2\left(a-bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \textbf{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)}{d^2x^2\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \textbf{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \textbf{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}-\frac{Cd\sqrt{c+dx}}{x(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \textbf{7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+2d\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \textbf{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-2d\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2(a-bx^2)^{3/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx-$$

$$\frac{2\int \left(-\frac{B}{ax\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{bCc^2-aCd^2-bBd(c+dx)}{a(-bc^2+2b(c+dx)c+ad^2-b(c+dx)^2)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{c(cC}{ad^2x^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(}{}}}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx-$$

$$2\left(-\frac{(cC-Bd)\sqrt[4]{bc^2-ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2-ad^2}}+1\right)\sqrt{\frac{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}{\left(a-\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2-ad^2}}+1\right)^2}}\text{EllipticPi}\left(\frac{\left(\sqrt{b}c+\sqrt{bc^2-ad^2}\right)^2}{4\sqrt{b}c\sqrt{bc^2-ad^2}},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2-ad^2}}\right),\frac{1}{2}\right.}{8a^2\sqrt[4]{b}cd^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(466) = 932$.

Time = 8.60 (sec) , antiderivative size = 1045, normalized size of antiderivative = 1.92

method	result	size
elliptic	Expression too large to display	1045
risch	Expression too large to display	1100
default	Expression too large to display	3767

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(-1/2*(A*b*c-B*a*d+C*a*c)/a^2/(a*d^2-b*c^2)*x+1/2*(A*b*d-B*b*c+C*a*d)/(a*
d^2-b*c^2)/b/a)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-A/c/a^2/x*(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)+2*((A*b+C*a)/a^2-1/2*d/a*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^
2)+b*c*(A*b*c-B*a*d+C*a*c)/a^2/(a*d^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/
d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/2*b*d*(A*b*c-B*a*d+
C*a*c)/a^2/(a*d^2-b*c^2)-1/2*A*b*d/c/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)
^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*
(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+1/a*(A*d-2*B*c)/c^2*(c/d-1/b*(a*
b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d
-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2
)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}\,x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b^2*d*x^7 + b^2
*c*x^6 - 2*a*b*d*x^5 - 2*a*b*c*x^4 + a^2*d*x^3 + a^2*c*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.214 $\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 623

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \frac{b\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{a^2(bc^2-ad^2)\sqrt{a-bx^2}}$$
$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2a^2cx^2}-\frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4a^2c^2x}$$
$$+\frac{\sqrt{b}(bc^2(8Bc-7Ad)-ad(4c^2C+4Bcd-3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^{3/2}c^2(bc^2-ad^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(8Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^{3/2}c\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{\left(\frac{12Ab}{a}+8C-\frac{d(4Bc-3Ad)}{c^2}\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
b*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a^2/(-a*d^2+b*c
^2)/(-b*x^2+a)^(1/2)-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c/x^2-1/4*(-
3*A*d+4*B*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^2/x+1/4*b^(1/2)*(b*c^2*(
-7*A*d+8*B*c)-a*d*(-3*A*d^2+4*B*c*d+4*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)
^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^2/(-a*d^2+b*c^2)/((d*x+c)/(c+a^(
1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/4*b^(1/2)*(-A*d+8*B*c)*((d*x+c)/
(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)
*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))
/a^(3/2)/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(12*A*b/a+8*C-d*(-3*A*d+4*B*
c)/c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipti
cPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*
c+a^(1/2)*d))^(1/2))/a/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.36 (sec) , antiderivative size = 2670, normalized size of antiderivative = 4.29

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-1/2*A/(a^2*c*x^2) + (-4*B*c + 3*A*d)/(4*a^
2*c^2*x) + (A*b^2*c + a*b*c*C - a*b*B*d + b^2*B*c*x - A*b^2*d*x - a*b*C*d*
x)/(a^2*(-(b*c^2) + a*d^2)*(-a + b*x^2))) - (d*Sqrt[a - (b*(c + d*x)^2*(-1
 + c/(c + d*x))^2)/d^2]*(-8*b^2*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 7*A
*b^2*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 4*a*b*c^3*C*d*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]] + 4*a*b*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*a*A*b*
c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - (8*b^2*B*c^6*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]])/(c + d*x)^2 + (7*A*b^2*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c
+ d*x)^2 + (4*a*b*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (1
2*a*b*B*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (10*a*A*b*c^
3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (4*a^2*c^3*C*d^3*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (4*a^2*B*c^2*d^4*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]])/(c + d*x)^2 + (3*a^2*A*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
])/(c + d*x)^2 + (16*b^2*B*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) -
 (14*A*b^2*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (8*a*b*c^4*C*
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (8*a*b*B*c^3*d^2*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (6*a*A*b*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]])/(c + d*x) + (I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(b*c^2*(8*B*c - 7
*A*d) + a*d*(-4*c^2*C - 4*B*c*d + 3*A*d^2))*Sqrt[1 - c/(c + d*x) - (Sqrt[a
]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a-bx^2\right)^{3/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^3\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^3\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \left(\frac{\sqrt{c+dx}C}{dx^2\left(a-bx^2\right)^{3/2}}+\frac{(Bd-cC)\sqrt{c+dx}}{d^2x^3\left(a-bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \left(\frac{C\sqrt{c+dx}}{dx^2\left(a-bx^2\right)^{3/2}}-\frac{(cC-Bd)\sqrt{c+dx}}{d^2x^3\left(a-bx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)}{d^2x^3\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^3\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^3\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}}{x^3\left(a-bx^2\right)^{3/2}}-\frac{Cd\sqrt{c+dx}}{x^2\left(a-bx^2\right)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+2d^2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(a-bx^2\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+ 2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+ 2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+ 2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\big(-\big((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\big(-\big((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b\big(-\big((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{3/2}}d\sqrt{c+dx}$$

$\Big\downarrow$ 2248

$$2\int \left(\frac{B}{adx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{bc(cC-Bd)-aCd^2}{a^2d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^2d^2\left(bc^2-2b(c+dx\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx$$

input
```
Int[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(530) = 1060$.

Time = 8.64 (sec) , antiderivative size = 1123, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1123
risch	Expression too large to display	1530
default	Expression too large to display	5530

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2*(-b*d*x-b*c)
*(1/2*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/a^2*x-1/2*(A*b*c-B*a*d+C*a*c)/(a*d
^2-b*c^2)/a^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-1/2*A/c/a^2/x^2*(-b*d*x^3-b*
c*x^2+a*d*x+a*c)^(1/2)+1/4*(3*A*d-4*B*c)/a^2/c^2*(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)/x+2*(B/a^2*b+1/2*b*d*(A*b*c-B*a*d+C*a*c)/a^2/(a*d^2-b*c^2)-b*c*(
A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/a^2+1/4*A*b*d/c/a^2)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/2*b*d/a^2*
(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)+1/8*(3*A*d-4*B*c)/a^2*b*d/c^2)*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/4*(3*A*
a*d^2+12*A*b*c^2-4*B*a*c*d+8*C*a*c^2)/a^2/c^3*(c/d-1/b*(a*b)^(1/2))*((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(1/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,(a-b\,x^2)^{3/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(3/2),x)
```

3.215 $\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 713

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{a(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{b^3\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(aAb^2c^2d^4+a^3Cd^6+a^2bd^4(c^2C-2Bcd+Ad^2)+2b^3c^4(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{b^3d^3\,(bc^2-ad^2)^2\sqrt{c+dx}}$$
$$-\frac{2(12cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15b^2d^3}+\frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5b^2d^3}$$
$$+\frac{\sqrt{a}(63a^3Cd^6-5a^2bd^4(3c^2C+16Bcd-9Ad^2)+4b^3c^4(24c^2C-20Bcd+15Ad^2)-ab^2c^2d^2(84c^2C-10}{15b^{5/2}d^4\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{\sqrt{a}(a^2d^4(69cC-25Bd)-4b^2c^3(24c^2C-20Bcd+15Ad^2)+abcd^2(12c^2C-40Bcd+45Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}}{15b^{5/2}d^4\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^3/(-a*d^2+b*c^2)/(d*x
+c)^(1/2)/(-b*x^2+a)^(1/2)-(a*A*b^2*c^2*d^4+a^3*C*d^6+a^2*b*d^4*(A*d^2-2*B
*c*d+C*c^2)+2*b^3*c^4*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/b^3/d^3/(-a*d^
2+b*c^2)^2/(d*x+c)^(1/2)-2/15*(-5*B*d+12*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/
2)/b^2/d^3+2/5*C*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d^3+1/15*a^(1/2)*(63*a
^3*C*d^6-5*a^2*b*d^4*(-9*A*d^2+16*B*c*d+3*C*c^2)+4*b^3*c^4*(15*A*d^2-20*B*
c*d+24*C*c^2)-a*b^2*c^2*d^2*(45*A*d^2-100*B*c*d+84*C*c^2))*(d*x+c)^(1/2)*(
(-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1
/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^4/(-a*d^2+b*c^2)^2/
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/15*a^(1/2)*(a^2*d
^4*(-25*B*d+69*C*c)-4*b^2*c^3*(15*A*d^2-20*B*c*d+24*C*c^2)+a*b*c*d^2*(45*A
*d^2-40*B*c*d+12*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)
/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^4/(-a*d^2+b*c^2)/(d*x+c)^(1/2
)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.99 (sec) , antiderivative size = 935, normalized size of antiderivative = 1.31

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(15(c+dx)\left(\frac{2(-9cC+5Bd)}{15b^2d^3}+\frac{2Cx}{5b^2d^2}-\frac{2c^4(c^2C-Bcd+Ad^2)}{d^3(bc^2-ad^2)^2(c+dx)}-\frac{a(Ab^2c^2x+\right.\right.}{}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(15*(c + d*x)*((2*(-9*c*C + 5*B*d))/(15*b^2*d^3) + (2*C*x
)/(5*b^2*d^2) - (2*c^4*(c^2*C - B*c*d + A*d^2))/(d^3*(b*c^2 - a*d^2)^2*(c
+ d*x)) - (a*(A*b^2*c^2*x + a^2*d*(-2*c*C + d*(B + C*x)) + a*b*(c^2*C*x +
B*c*(c - 2*d*x) + A*d*(-2*c + d*x))))/(b^2*(b*c^2 - a*d^2)^2*(-a + b*x^2))
) - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(63*a^3*C*d^6 + a*b^2*c^2*d^2*(-84
*c^2*C + 100*B*c*d - 45*A*d^2) + 5*a^2*b*d^4*(-3*c^2*C - 16*B*c*d + 9*A*d^
2) + 4*b^3*c^4*(24*c^2*C - 20*B*c*d + 15*A*d^2))*(a - b*x^2) + I*Sqrt[b]*(
Sqrt[b]*c - Sqrt[a]*d)*(63*a^3*C*d^6 + a*b^2*c^2*d^2*(-84*c^2*C + 100*B*c*
d - 45*A*d^2) + 5*a^2*b*d^4*(-3*c^2*C - 16*B*c*d + 9*A*d^2) + 4*b^3*c^4*(2
4*c^2*C - 20*B*c*d + 15*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*
Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I
*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[
a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*
d)*(63*a^(5/2)*C*d^5 + a^2*Sqrt[b]*d^4*(69*c*C - 25*B*d) + 12*Sqrt[a]*b^2*
c^3*d*(-6*c*C + 5*B*d) + 3*a^(3/2)*b*d^3*(18*c^2*C - 35*B*c*d + 15*A*d^2)
- 4*b^(5/2)*c^3*(24*c^2*C - 20*B*c*d + 15*A*d^2) + a*b^(3/2)*c*d^2*(12*c^2
*C - 40*B*c*d + 45*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcS
inh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)
/(Sqrt[b]*c - Sqrt[a]*d)])/(b^3*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c...
```

Rubi [A] (verified)

Time = 4.96 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2180, 27, 2182, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a-bx^2)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)}{b^3}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}d}{a\left(bc^2-ad^2\right)}$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}-$$

$$\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)}{b^3}}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}$$

$\downarrow$ 2182

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2\int \frac{\frac{2aC\left(bc^2-ad^2\right)^2x^3}{bd}-\frac{2a(cC-Bd)\left(bc^2-ad^2\right)^2x^2}{bd^2}+\frac{a\left(3a^3Cd^6-a^2b\left(Cc^2+4Bdc-3Ad^2\right)d^4-ab^2c^2\left(2Cc^2-4Bdc+3Ad^2\right)d^2+4b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)x}{b^2d^3}+\frac{a^2}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{2a\left(bc^2\right.}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\int \frac{\frac{2aC\left(bc^2-ad^2\right)^2x^3}{bd}-\frac{2a(cC-Bd)\left(bc^2-ad^2\right)^2x^2}{bd^2}+\frac{a\left(3a^3Cd^6-a^2b\left(Cc^2+4Bdc-3Ad^2\right)d^4-ab^2c^2\left(2Cc^2-4Bdc+3Ad^2\right)d^2+4b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)x}{b^2d^3}+a^2\left(\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}}{2a\left(bc^2-a\right.}$$

$\downarrow$ 2185

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}-$$

$$\frac{-\frac{2\int -\frac{\frac{d\left(a^2(6cC+5Bd)d^4-abc^2(2cC+15Bd)d^2+2b^2c^3\left(8Cc^2-5Bdc+10Ad^2\right)\right)a^2}{b}-2d(12cC-5Bd)\left(bc^2-ad^2\right)^2x^2a+\frac{\left(21a^3Cd^6-a^2b\left(21Cc^2+20Bdc-15Ad^2\right)\right.}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{bc^2-ad^2}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{\int \frac{d\left(\frac{a^2(6cC+5Bd)d^4}{b}-ac^2(2cC+15Bd)d^2+2bc^3\left(8Cc^2-5Bdc+10Ad^2\right)\right)a^2-2d(12cC-5Bd)\left(bc^2-ad^2\right)^2x^2a+\frac{\left(21a^3Cd^6-a^2b\left(21Cc^2+20Bdc-15Ad^2\right)d^4+ab^2 \cdots\right.}{\cdots}}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{5bd^3}}{bc^2-ad^2}$$

$$\Big\downarrow \; 2185$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{\frac{4a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)^2(12cC-5Bd)}{3b}-\frac{2\int -\frac{ad^2\left(ad\left(-a^2(6cC-25Bd)d^4+abc^2(42cC-65Bd)d^2+4b^2c^3\left(6Cc^2-5Bdc+15Ad^2\right)\right)+\left(63a^3Cd^6-5a^2b\left(3Cc^2 \cdots\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{3bd^2}}{5bd^3}}{bc^2-ad^2}$$

$$\Big\downarrow \; 27$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{\frac{a\int \frac{ad\left(-a^2(6cC-25Bd)d^4+abc^2(42cC-65Bd)d^2+4b^2c^3\left(6Cc^2-5Bdc+15Ad^2\right)\right)+\left(63a^3Cd^6-5a^2b\left(3Cc^2+16Bdc-9Ad^2\right)d^4-ab^2c^2\left(84Cc^2-100Bdc+45Ad^2\right) \cdots\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{3b}}{5bd^3}}{bc^2-ad^2}$$

$$\Big\downarrow \; 600$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{\frac{a\left(\frac{\left(bc^2-ad^2\right)\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\left(63a^3Cd^6-5a^2bd^4\left(-9Ad^2+16Bc \cdots\right.\right.}{\cdots}\right.}{3b}}{5bd^3}}{\cdots}$$

$$\Big\downarrow \; 509$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{\frac{a\left(\frac{\left(bc^2-ad^2\right)\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}\left(63a^3Cd^6-5a^2bd^4\left(-9A \cdots\right.\right.}{\cdots}\right.}{3b}}{5bd^3}}{\cdots}$$

$$\Big\downarrow \; 508$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\left(bc^2-ad^2\right)\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^3Cd^6-5\right.}{}\right.$$

$$3b$$

$\downarrow$ 327

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\left(bc^2-ad^2\right)\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63a^3Cd^6-5\right.}{}\right.$$

$$3b$$

$\downarrow$ 512

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(63\right.}{}\right.$$

$$3b$$

$\downarrow$ 511

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}-$$

$$\frac{2a\sqrt{a-bx^2}\left(a^3Cd^6+aAb^2c^2d^4+a^2b(Cc^2-2Bdc+Ad^2)d^4+2b^3c^4(Cc^2-Bdc+Ad^2)\right)}{b^3d^3(bc^2-ad^2)\sqrt{c+dx}}+\frac{\frac{4a(12cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}\left(bc^2-ad^2\right)^2}{3b}+\frac{a\left(-\frac{2\sqrt{a}\left(63a^3\right.}{}\right.}{}}{}$$

$\downarrow$ 321

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(a^2d^4(69cC-25Bd)+abcd^2\left(45Ad^2-40Bcd+12c^2C\right)-4b^2c^3\left(15Ad^2-20Bcd+24c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

input
```
Int[(x^4*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x))/(b^3*(b*c^2
- a*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((2*a*(a*A*b^2*c^2*d^4 + a^3*C*d
^6 + a^2*b*d^4*(c^2*C - 2*B*c*d + A*d^2) + 2*b^3*c^4*(c^2*C - B*c*d + A*d^
2))*Sqrt[a - b*x^2])/(b^3*d^3*(b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((-4*a*C*(b
*c^2 - a*d^2)^2*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b^2*d^3) + ((4*a*(12*c
*C - 5*B*d)*(b*c^2 - a*d^2)^2*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b) + (a*((
-2*Sqrt[a]*(63*a^3*C*d^6 - 5*a^2*b*d^4*(3*c^2*C + 16*B*c*d - 9*A*d^2) + 4*
b^3*c^4*(24*c^2*C - 20*B*c*d + 15*A*d^2) - a*b^2*c^2*d^2*(84*c^2*C - 100*B
*c*d + 45*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[
1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt
[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) -
 (2*Sqrt[a]*(b*c^2 - a*d^2)*(a^2*d^4*(69*c*C - 25*B*d) - 4*b^2*c^3*(24*c^2
*C - 20*B*c*d + 15*A*d^2) + a*b*c*d^2*(12*c^2*C - 40*B*c*d + 45*A*d^2))*Sq
rt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ellipt
icF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(3*b))/(5*b*d^3))/(
b*c^2 - a*d^2))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1482 vs. $2(639) = 1278$.

Time = 12.90 (sec) , antiderivative size = 1483, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	1483
risch	Expression too large to display	1825
default	Expression too large to display	5390

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(1/2*(A*a
^2*b*d^6+A*a*b^2*c^2*d^4+2*A*b^3*c^4*d^2-2*B*a^2*b*c*d^5-2*B*b^3*c^5*d+C*a
^3*d^6+C*a^2*b*c^2*d^4+2*C*b^3*c^6)/(a*d^2-b*c^2)^2/b^3/d^4*x^2-1/2*(A*b*c
-B*a*d+C*a*c)*a/d/(a*d^2-b*c^2)/b^3*x-1/2*a*c*(2*A*a*b*c*d^4+2*A*b^2*c^3*d
^2-B*a^2*d^5-B*a*b*c^2*d^3-2*B*b^2*c^4*d+2*C*a^2*c*d^4+2*C*b^2*c^5)/d^4/b^
3/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4))/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(1/2
)+2/5*C/b^2/d^2*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(-1/d^2/b*(B*d-C*
c)+4/5*C/b/d^2*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*((A*b*c*d^2-B*a
*d^3-B*b*c^2*d+C*a*c*d^2+C*b*c^3)/b^2/d^4-1/2*(4*A*a^2*b*c*d^6-2*A*a*b^2*c
^3*d^4+2*A*b^3*c^5*d^2-3*B*a^3*d^7+B*a^2*b*c^2*d^5-2*B*b^3*c^6*d+4*C*a^3*c
*d^6-2*C*a^2*b*c^3*d^4+2*C*b^3*c^7)/d^4/b^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4
)+1/b^2*(A*b*c-B*a*d+C*a*c)*a/(a*d^2-b*c^2)-2/5*C/b^2/d^2*a*c+1/3*(-1/d^2/
b*(B*d-C*c)+4/5*C/b/d^2*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a
*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1
/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)
^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1
/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/d^3*(A*b*d^2-B*b*c*d+C*a*d^2+C*b
*c^2)/b^2+3/2*(A*a^2*b*d^6+A*a*b^2*c^2*d^4+2*A*b^3*c^4*d^2-2*B*a^2*b*c*d^5
-2*B*b^3*c^5*d+C*a^3*d^6+C*a^2*b*c^2*d^4+2*C*b^3*c^6)/d^3/b^2/(a^2*d^4-2*a
*b*c^2*d^2+b^2*c^4)-2/b^2/d^3*(A*a^2*b*d^6+A*a*b^2*c^2*d^4+2*A*b^3*c^4*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1822 vs. $2(643) = 1286$.

Time = 0.17 (sec) , antiderivative size = 1822, normalized size of antiderivative = 2.56

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/45*((96*C*a*b^3*c^8 - 80*B*a*b^3*c^7*d + 160*B*a^2*b^2*c^5*d^3 + 115*B*
a^3*b*c^3*d^5 - 75*B*a^4*c*d^7 - 12*(13*C*a^2*b^2 - 5*A*a*b^3)*c^6*d^2 - 3
*(47*C*a^3*b + 75*A*a^2*b^2)*c^4*d^4 + 9*(9*C*a^4 + 5*A*a^3*b)*c^2*d^6 - (
96*C*b^4*c^7*d - 80*B*b^4*c^6*d^2 + 160*B*a*b^3*c^4*d^4 + 115*B*a^2*b^2*c^
2*d^6 - 75*B*a^3*b*d^8 - 12*(13*C*a*b^3 - 5*A*b^4)*c^5*d^3 - 3*(47*C*a^2*b
^2 + 75*A*a*b^3)*c^3*d^5 + 9*(9*C*a^3*b + 5*A*a^2*b^2)*c*d^7)*x^3 - (96*C*
b^4*c^8 - 80*B*b^4*c^7*d + 160*B*a*b^3*c^5*d^3 + 115*B*a^2*b^2*c^3*d^5 - 7
5*B*a^3*b*c*d^7 - 12*(13*C*a*b^3 - 5*A*b^4)*c^6*d^2 - 3*(47*C*a^2*b^2 + 75
*A*a*b^3)*c^4*d^4 + 9*(9*C*a^3*b + 5*A*a^2*b^2)*c^2*d^6)*x^2 + (96*C*a*b^3
*c^7*d - 80*B*a*b^3*c^6*d^2 + 160*B*a^2*b^2*c^4*d^4 + 115*B*a^3*b*c^2*d^6
- 75*B*a^4*d^8 - 12*(13*C*a^2*b^2 - 5*A*a*b^3)*c^5*d^3 - 3*(47*C*a^3*b + 7
5*A*a^2*b^2)*c^3*d^5 + 9*(9*C*a^4 + 5*A*a^3*b)*c*d^7)*x)*sqrt(-b*d)*weiers
trassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*
d^3), 1/3*(3*d*x + c)/d) + 3*(96*C*a*b^3*c^7*d - 80*B*a*b^3*c^6*d^2 + 100*
B*a^2*b^2*c^4*d^4 - 80*B*a^3*b*c^2*d^6 - 12*(7*C*a^2*b^2 - 5*A*a*b^3)*c^5*
d^3 - 15*(C*a^3*b + 3*A*a^2*b^2)*c^3*d^5 + 9*(7*C*a^4 + 5*A*a^3*b)*c*d^7 -
 (96*C*b^4*c^6*d^2 - 80*B*b^4*c^5*d^3 + 100*B*a*b^3*c^3*d^5 - 80*B*a^2*b^2
*c*d^7 - 12*(7*C*a*b^3 - 5*A*b^4)*c^4*d^4 - 15*(C*a^2*b^2 + 3*A*a*b^3)*c^2
*d^6 + 9*(7*C*a^3*b + 5*A*a^2*b^2)*d^8)*x^3 - (96*C*b^4*c^7*d - 80*B*b^4*c
^6*d^2 + 100*B*a*b^3*c^4*d^4 - 80*B*a^2*b^2*c^2*d^6 - 12*(7*C*a*b^3 - 5...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^4\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^4(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.216 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 625

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \frac{a(Abc+acC-aBd+(bBc-Abd-aCd)x)}{b^2(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(abcd^3(Bc-2Ad)-a^2d^4(2cC-Bd)-2b^2c^3(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{b^2d^2(bc^2-ad^2)^2\sqrt{c+dx}}$$
$$+\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3b^2d^2}$$
$$-\frac{\sqrt{a}(a^2d^4(16cC-9Bd)-abcd^2(20c^2C-9Bcd-6Ad^2)+2b^2c^3(8c^2C-6Bcd+3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}}{3b^{3/2}d^3(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{a}(5a^2Cd^4+abd^2(8c^2C-9Bcd+3Ad^2)-2b^2c^2(8c^2C-6Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\text{a}\right.}{3b^{5/2}d^3(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/b^2/(-a*d^2+b*c^2)/(d*x+c)^(1
/2)/(-b*x^2+a)^(1/2)-(a*b*c*d^3*(-2*A*d+B*c)-a^2*d^4*(-B*d+2*C*c)-2*b^2*c^
3*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/b^2/d^2/(-a*d^2+b*c^2)^2/(d*x+c)^(
1/2)+2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^2-1/3*a^(1/2)*(a^2*d^4*(-9
*B*d+16*C*c)-a*b*c*d^2*(-6*A*d^2-9*B*c*d+20*C*c^2)+2*b^2*c^3*(3*A*d^2-6*B*
c*d+8*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
b^(3/2)/d^3/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2
+a)^(1/2)-1/3*a^(1/2)*(5*a^2*C*d^4+a*b*d^2*(3*A*d^2-9*B*c*d+8*C*c^2)-2*b^2
*c^2*(3*A*d^2-6*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b
*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)
*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^3/(-a*d^2+b*c^2)/(d*x+
c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.95 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.33

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(3(c+dx)\left(\frac{2C}{3b^2d^2}+\frac{2c^3(c^2C-Bcd+Ad^2)}{(bc^2d-ad^3)^2(c+dx)}-\frac{a(a^2Cd^2+b^2Bc^2x+ab(c^2C+Bd^2}{b^2(bc^2-}\right.\right.}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(3*(c + d*x)*((2*C)/(3*b^2*d^2) + (2*c^3*(c^2*C - B*c*d +
 A*d^2))/((b*c^2*d - a*d^3)^2*(c + d*x)) - (a*(a^2*C*d^2 + b^2*B*c^2*x + a
*b*(c^2*C + B*d^2*x - 2*c*d*(B + C*x)) + A*b*(a*d^2 + b*c*(c - 2*d*x))))/(
b^2*(b*c^2 - a*d^2)^2*(-a + b*x^2))) - (-(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*(a^2*d^4*(16*c*C - 9*B*d) + 2*b^2*c^3*(8*c^2*C - 6*B*c*d + 3*A*d^2) + a
*b*c*d^2*(-20*c^2*C + 9*B*c*d + 6*A*d^2))*(a - b*x^2)) - I*Sqrt[b]*(Sqrt[b
]*c - Sqrt[a]*d)*(a^2*d^4*(16*c*C - 9*B*d) + 2*b^2*c^3*(8*c^2*C - 6*B*c*d
+ 3*A*d^2) + a*b*c*d^2*(-20*c^2*C + 9*B*c*d + 6*A*d^2))*Sqrt[(d*(Sqrt[a]/S
qrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
+ d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(Sqrt
[b]*c - Sqrt[a]*d)*(5*a^2*C*d^4 + 3*a^(3/2)*Sqrt[b]*d^3*(7*c*C - 3*B*d) +
3*Sqrt[a]*b^(3/2)*c*d*(-4*c^2*C + 3*A*d^2) + a*b*d^2*(8*c^2*C - 9*B*c*d +
3*A*d^2) - 2*b^2*c^2*(8*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[
b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*
x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]]
, (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(b^2*d^4*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)^2*(-a + b*x^2))))/(3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 3.51 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2180, 27, 2182, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2} (c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-3Ad)-ad(3cC-Bd))}{b^2}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-3Ad)-ad(3cC-Bd))}{b^2}}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 2182

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$
$$\frac{2\int \frac{\frac{\left(a^2Cd^4-b^2c^2\left(2Cc^2-4Bdc+5Ad^2\right)-ab\left(3c^2Cd^2-Ad^4\right)\right)a^2}{b^2d}+\frac{2C\left(bc^2-ad^2\right)^2x^2a}{bd}-\frac{\left(a^2(4cC-3Bd)d^4-abc\left(4Cc^2-3Bdc-2Ad^2\right)d^2+2b^2c^3\left(2Cc^2-2Bdc+\right.\right.}{bd^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}$$
$$2a\,(bc^2-ad^2)$$

$\Big\downarrow$ 27

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$
$$\frac{\int \frac{\frac{\left(a^2Cd^4-b^2c^2\left(2Cc^2-4Bdc+5Ad^2\right)-ab\left(3c^2Cd^2-Ad^4\right)\right)a^2}{b^2d}+\frac{2C\left(bc^2-ad^2\right)^2x^2a}{bd}-\frac{\left(a^2(4cC-3Bd)d^4-abc\left(4Cc^2-3Bdc-2Ad^2\right)d^2+2b^2c^3\left(2Cc^2-2Bdc+Ad\right.\right.}{bd^2}}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}$$
$$2a\,(bc^2-ad^2)$$

$\Big\downarrow$ 2185

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$
$$\frac{-\frac{2\int -\frac{a\left(ad\left(5a^2Cd^4-b^2c^2\left(4Cc^2-12Bdc+15Ad^2\right)-ab\left(13c^2Cd^2-3Ad^4\right)\right)-b\left(a^2(16cC-9Bd)d^4-abc\left(20Cc^2-9Bdc-6Ad^2\right)d^2+2b^2c^3\left(8Cc^2-6Bdc+3Ad^2\right)\right)x\right.}{2b\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}}{bc^2-ad^2}$$
$$2a\,(bc^2-ad^2)$$

$\Big\downarrow$ 27

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$
$$\frac{\frac{a\int \frac{ad\left(5a^2Cd^4-b^2c^2\left(4Cc^2-12Bdc+15Ad^2\right)-ab\left(13c^2Cd^2-3Ad^4\right)\right)-b\left(a^2(16cC-9Bd)d^4-abc\left(20Cc^2-9Bdc-6Ad^2\right)d^2+2b^2c^3\left(8Cc^2-6Bdc+3Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3b^2d^2}-}{bc^2-ad^2}$$
$$2a\,(bc^2-ad^2)$$

$\Big\downarrow$ 600

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{a\left(-\frac{(bc^2-ad^2)\left(5a^2Cd^4+abd^2(3Ad^2-9Bcd+8c^2C)-2b^2c^2(3Ad^2-6Bcd+8c^2C)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(a^2d^4(16cC-9Bd)-abcd^2(-6Ad^2-9Bcd+20c^2C\right)}{d}\right.}{3b^2d^2}}{bc^2-ad^2}$$

$2a($

$\downarrow$ 509

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{a\left(-\frac{(bc^2-ad^2)\left(5a^2Cd^4+abd^2(3Ad^2-9Bcd+8c^2C)-2b^2c^2(3Ad^2-6Bcd+8c^2C)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(a^2d^4(16cC-9Bd)-abcd^2(-6Ad^2-9Bc\right.}{d\sqrt{}}\right.}{3b^2d^2}}{bc^2-ad^2}$$

$\downarrow$ 508

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{a\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(16cC-9Bd)-abcd^2(-6Ad^2-9Bcd+20c^2C)+2b^2c^3(3Ad^2-6Bcd+8c^2C)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{(bc^2-ad^2)(5a^2}{}\right.}{3b^2d^2}}{bc^2-ad^2}$$

$\downarrow$ 327

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{a\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(16cC-9Bd)-abcd^2(-6Ad^2-9Bcd+20c^2C)+2b^2c^3(3Ad^2-6Bcd+8c^2C)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{(bc^2-ad^2)(5}{}\right.}{3b^2d^2}}{bc^2-ad^2}$$

$\downarrow$ 512

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(a^2d^4(16cC-9Bd)-abcd^2\left(-6Ad^2-9Bcd+20c^2C\right)+2b^2c^3\left(3Ad^2-6Bcd+8c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2\right.}{}\right.$$

$$\overline{3b^2d^2}$$

$$\overline{bc^2-ad^2}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5a^2Cd^4+abd^2\left(3Ad^2-9Bcd+8c^2C\right)-2b^2c^2\left(3Ad^2-6Bcd+8c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.$$

$$\overline{3b^2d^2}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(5a^2Cd^4+abd^2\left(3Ad^2-9Bcd+8c^2C\right)-2b^2c^2\left(3Ad^2-6Bcd+8c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{}}{}\right.$$

$$\overline{3b^2d^2}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(A*b*c + a*c*C - a*B*d + (b*B*c - A*b*d - a*C*d)*x))/(b^2*(b*c^2 - a*d^
2)*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((2*a*(a*b*c*d^3*(B*c - 2*A*d) - a^2*d
^4*(2*c*C - B*d) - 2*b^2*c^3*(c^2*C - B*c*d + A*d^2))*Sqrt[a - b*x^2])/(b^
2*d^2*(b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((-4*a*C*(b*c^2 - a*d^2)^2*Sqrt[c +
 d*x]*Sqrt[a - b*x^2])/(3*b^2*d^2) + (a*((2*Sqrt[a]*Sqrt[b]*(a^2*d^4*(16*c
*C - 9*B*d) - a*b*c*d^2*(20*c^2*C - 9*B*c*d - 6*A*d^2) + 2*b^2*c^3*(8*c^2*
C - 6*B*c*d + 3*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin
[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])
/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (
2*Sqrt[a]*(b*c^2 - a*d^2)*(5*a^2*C*d^4 + a*b*d^2*(8*c^2*C - 9*B*c*d + 3*A*
d^2) - 2*b^2*c^2*(8*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(
Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqr
t[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sq
rt[c + d*x]*Sqrt[a - b*x^2])))/(3*b^2*d^2))/(b*c^2 - a*d^2))/(2*a*(b*c^2 -
 a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1286 vs. $2(557) = 1114$.

Time = 12.39 (sec) , antiderivative size = 1287, normalized size of antiderivative = 2.06

method	result	size
elliptic	Expression too large to display	1287
risch	Expression too large to display	1746
default	Expression too large to display	4362

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(-1/2*(2*
A*a*b*c*d^4+2*A*b^2*c^3*d^2-B*a^2*d^5-B*a*b*c^2*d^3-2*B*b^2*c^4*d+2*C*a^2*
c*d^4+2*C*b^2*c^5)/b^2/d^3/(a*d^2-b*c^2)^2*x^2+1/2*(A*b*d-B*b*c+C*a*d)/d*a
/(a*d^2-b*c^2)/b^3*x+1/2*a*c*(A*a*b*d^4+3*A*b^2*c^2*d^2-2*B*a*b*c*d^3-2*B*
b^2*c^3*d+C*a^2*d^4+C*a*b*c^2*d^2+2*C*b^2*c^4)/d^3/(a^2*d^4-2*a*b*c^2*d^2+
b^2*c^4)/b^3)/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(1/2)+2/3*C/b^2/d^2*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(-1/d^3*(A*b*d^2-B*b*c*d+C*a*d^2+C*b*c^2)/
b^2+1/2*(3*A*a^2*b*d^6-A*a*b^2*c^2*d^4+2*A*b^3*c^4*d^2-4*B*a^2*b*c*d^5+2*B
*a*b^2*c^3*d^3-2*B*b^3*c^5*d+3*C*a^3*d^6-C*a^2*b*c^2*d^4+2*C*b^3*c^6)/d^3/
b^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)-1/b^2*(A*b*d-B*b*c+C*a*d)*a/(a*d^2-b*c
^2)-1/3*C/b^2/d*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2
))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Ellipt
icF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2))+2*(-1/d^2/b*(B*d-C*c)-3/2*(2*A*a*b*c*d^4+2*A*b^2*c^
3*d^2-B*a^2*d^5-B*a*b*c^2*d^3-2*B*b^2*c^4*d+2*C*a^2*c*d^4+2*C*b^2*c^5)/d^2
/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b+2/b/d^2*(2*A*a*b*c*d^4+2*A*b^2*c^3*d^2-
B*a^2*d^5-B*a*b*c^2*d^3-2*B*b^2*c^4*d+2*C*a^2*c*d^4+2*C*b^2*c^5)/(a*d^2-b*
c^2)^2+2/3*C/b/d^2*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1548 vs. $2(561) = 1122$.

Time = 0.13 (sec) , antiderivative size = 1548, normalized size of antiderivative = 2.48

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
1/9*((16*C*a*b^3*c^7 - 12*B*a*b^3*c^6*d + 45*B*a^2*b^2*c^4*d^3 - 9*B*a^3*b
*c^2*d^5 - 2*(16*C*a^2*b^2 - 3*A*a*b^3)*c^5*d^2 - (23*C*a^3*b + 39*A*a^2*b
^2)*c^3*d^4 + 3*(5*C*a^4 + 3*A*a^3*b)*c*d^6 - (16*C*b^4*c^6*d - 12*B*b^4*c
^5*d^2 + 45*B*a*b^3*c^3*d^4 - 9*B*a^2*b^2*c*d^6 - 2*(16*C*a*b^3 - 3*A*b^4)
*c^4*d^3 - (23*C*a^2*b^2 + 39*A*a*b^3)*c^2*d^5 + 3*(5*C*a^3*b + 3*A*a^2*b^
2)*d^7)*x^3 - (16*C*b^4*c^7 - 12*B*b^4*c^6*d + 45*B*a*b^3*c^4*d^3 - 9*B*a^
2*b^2*c^2*d^5 - 2*(16*C*a*b^3 - 3*A*b^4)*c^5*d^2 - (23*C*a^2*b^2 + 39*A*a*
b^3)*c^3*d^4 + 3*(5*C*a^3*b + 3*A*a^2*b^2)*c*d^6)*x^2 + (16*C*a*b^3*c^6*d
- 12*B*a*b^3*c^5*d^2 + 45*B*a^2*b^2*c^3*d^4 - 9*B*a^3*b*c*d^6 - 2*(16*C*a^
2*b^2 - 3*A*a*b^3)*c^4*d^3 - (23*C*a^3*b + 39*A*a^2*b^2)*c^2*d^5 + 3*(5*C*
a^4 + 3*A*a^3*b)*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d
^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(16
*C*a*b^3*c^6*d - 12*B*a*b^3*c^5*d^2 + 9*B*a^2*b^2*c^3*d^4 - 9*B*a^3*b*c*d^
6 - 2*(10*C*a^2*b^2 - 3*A*a*b^3)*c^4*d^3 + 2*(8*C*a^3*b + 3*A*a^2*b^2)*c^2
*d^5 - (16*C*b^4*c^5*d^2 - 12*B*b^4*c^4*d^3 + 9*B*a*b^3*c^2*d^5 - 9*B*a^2*
b^2*d^7 - 2*(10*C*a*b^3 - 3*A*b^4)*c^3*d^4 + 2*(8*C*a^2*b^2 + 3*A*a*b^3)*c
*d^6)*x^3 - (16*C*b^4*c^6*d - 12*B*b^4*c^5*d^2 + 9*B*a*b^3*c^3*d^4 - 9*B*a
^2*b^2*c*d^6 - 2*(10*C*a*b^3 - 3*A*b^4)*c^4*d^3 + 2*(8*C*a^2*b^2 + 3*A*a*b
^3)*c^2*d^5)*x^2 + (16*C*a*b^3*c^5*d^2 - 12*B*a*b^3*c^4*d^3 + 9*B*a^2*b^2*
c^2*d^5 - 9*B*a^3*b*d^7 - 2*(10*C*a^2*b^2 - 3*A*a*b^3)*c^3*d^4 + 2*(8*C...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x^3\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.217 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 556

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \frac{a(bBc-Abd-aCd)+b(Abc+acC-aBd)x}{b^2(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(a^2Cd^4+abd^2(c^2C-2Bcd+Ad^2)+b^2c^2(2c^2C-2Bcd+3Ad^2))\sqrt{a-bx^2}}{b^2d(bc^2-ad^2)^2\sqrt{c+dx}}$$

$$+\frac{\sqrt{a}(3a^2Cd^4-abd^2(3c^2C+2Bcd-Ad^2)+b^2c^2(4c^2C-2Bcd+3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}\right)\cdots\right)}{b^{3/2}d^2(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}\cdots}$$

$$+\frac{\sqrt{a}(ad^2(3cC-Bd)-bc(4c^2C-2Bcd+Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\cdots\right)}{b^{3/2}d^2(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}\cdots}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^2/(-a*d^2+b*c^2)/(d*x+c
)^(1/2)/(-b*x^2+a)^(1/2)-(a^2*C*d^4+a*b*d^2*(A*d^2-2*B*c*d+C*c^2)+b^2*c^2*
(3*A*d^2-2*B*c*d+2*C*c^2))*(-b*x^2+a)^(1/2)/b^2/d/(-a*d^2+b*c^2)^2/(d*x+c)
^(1/2)+a^(1/2)*(3*a^2*C*d^4-a*b*d^2*(-A*d^2+2*B*c*d+3*C*c^2)+b^2*c^2*(3*A*
d^2-2*B*c*d+4*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-
b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))
^(1/2))/b^(3/2)/d^2/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)
/(-b*x^2+a)^(1/2)+a^(1/2)*(a*d^2*(-B*d+3*C*c)-b*c*(A*d^2-2*B*c*d+4*C*c^2))
*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*
(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*
d))^(1/2))/b^(3/2)/d^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.77 (sec) , antiderivative size = 733, normalized size of antiderivative = 1.32

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(3a^2Cd^4+abd^2(-3c^2C-2Bcd+Ad^2)-2b^2c^2(c^2C-Bcd+\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(3*a^2*C*d^4 + a*b*d^2*(-3*c^2*C - 2*B*c*d + A*d^2) - 2*b
^2*c^2*(c^2*C - B*c*d + A*d^2) + b^2*c^2*(4*c^2*C - 2*B*c*d + 3*A*d^2) - (
b*d*(c + d*x)*(A*b^2*c^2*x + a^2*d*(-2*c*C + d*(B + C*x)) + a*b*(c^2*C*x +
 B*c*(c - 2*d*x) + A*d*(-2*c + d*x))))/(-a + b*x^2) - (I*Sqrt[b]*(Sqrt[b]*
c - Sqrt[a]*d)*(3*a^2*C*d^4 + a*b*d^2*(-3*c^2*C - 2*B*c*d + A*d^2) + b^2*c
^2*(4*c^2*C - 2*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)
]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE
[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqr
t[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a
+ b*x^2)) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(2*A*b^2*c^2*d + 3*a^2*C*d^
3 + a*b*d^2*(-3*B*c + A*d) + a^(3/2)*Sqrt[b]*d^2*(3*c*C - B*d) - Sqrt[a]*b
^(3/2)*c*(4*c^2*C - 2*B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c +
d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellip
ticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c +
 Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-
a + b*x^2))))/(b^2*d*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2180, 27, 2182, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A+Bx+Cx^2)}{(a-bx^2)^{3/2} (c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \ 2180$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc-Ad)-ad(cC+Bd))x}{b}+\frac{a\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)}{b^2}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc-Ad)-ad(cC+Bd))x}{b}+\frac{a\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)}{b^2}}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 2182

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{2\int \frac{a\left(bd\left(\frac{a^2Bd^3}{b}+ac^2(4cC-5Bd)+2Ac\left(bc^2+ad^2\right)\right)\right)+\left(3a^2Cd^4-ab\left(3Cc^2+2Bdc-Ad^2\right)d^2+b^2c^2\left(4Cc^2-2Bdc+3Ad^2\right)\right)x\right)}{2bd\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2a\sqrt{a-bx^2}\left(a^2Cd^4+abd^2\cdots\right.}{\cdots}}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 27

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{a\int \frac{d\left(2Abc\left(bc^2+ad^2\right)+a\left(aBd^3+bc^2(4cC-5Bd)\right)\right)+\left(3a^2Cd^4-ab\left(3Cc^2+2Bdc-Ad^2\right)d^2+b^2c^2\left(4Cc^2-2Bdc+3Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bd(bc^2-ad^2)}+\frac{2a\sqrt{a-bx^2}\left(a^2Cd^4+abd^2(A\cdots\right.}{b^2\cdots}}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 600

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{a\left(\frac{\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3Ad^2-2Bcd+4c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}+\frac{\left(bc^2-ad^2\right)\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{d}\cdots\right.}{bd(bc^2-ad^2)}\cdots}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 509

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{a\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3Ad^2-2Bcd+4c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{\left(bc^2-ad^2\right)\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int \frac{\cdots}{\sqrt{c+dx}\cdots}}{d}\cdots\right.}{bd(bc^2-ad^2)}\cdots}{2a\,(bc^2-ad^2)}$$

$\Big\downarrow$ 508

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3Ad^2-2Bcd+\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.$$

$$\frac{\quad}{bd(bc^2-ad^2)}$$

$$2a\left(bc^2-ad^2\right)$$

$\downarrow$ 327

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\left(bc^2-ad^2\right)\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3Ad^2-2Bcd+\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.$$

$$\frac{\quad}{bd(bc^2-ad^2)}$$

$$2a\left(bc^2-ad^2\right)$$

$\downarrow$ 512

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$a\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3A\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+}{\sqrt{a}d+}}}\right.$$

$$\frac{\quad}{bd(bc^2-ad^2)}$$

$$2a\left(bc^2-ad^2\right.$$

$\downarrow$ 511

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(3cC-Bd)-bc\left(Ad^2-2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(\cdots\right.}{\cdots}\right)}{bd(bc^2-ad^2)}$$

$\Big\downarrow$ 321

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4-abd^2\left(-Ad^2+2Bcd+3c^2C\right)+b^2c^2\left(3Ad^2-2Bcd+4c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\cdots}}{\cdots}\right)}{bd(bc^2-ad^2)}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x)/(b^2*(b*c^2 - a*
d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((2*a*(a^2*C*d^4 + a*b*d^2*(c^2*C -
2*B*c*d + A*d^2) + b^2*c^2*(2*c^2*C - 2*B*c*d + 3*A*d^2))*Sqrt[a - b*x^2])
/(b^2*d*(b*c^2 - a*d^2)*Sqrt[c + d*x]) + (a*((-2*Sqrt[a]*(3*a^2*C*d^4 - a*
b*d^2*(3*c^2*C + 2*B*c*d - A*d^2) + b^2*c^2*(4*c^2*C - 2*B*c*d + 3*A*d^2))
*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[
b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^
2 - a*d^2)*(a*d^2*(3*c*C - B*d) - b*c*(4*c^2*C - 2*B*c*d + A*d^2))*Sqrt[(S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b*d*(b*c^2 - a*d^2)))/(
2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1057 vs. $2(498) = 996$.

Time = 6.03 (sec) , antiderivative size = 1058, normalized size of antiderivative = 1.90

method	result	size
elliptic	Expression too large to display	1058
default	Expression too large to display	3766

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(1/2*(A*a
*b*d^4+3*A*b^2*c^2*d^2-2*B*a*b*c*d^3-2*B*b^2*c^3*d+C*a^2*d^4+C*a*b*c^2*d^2
+2*C*b^2*c^4)/d^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b^2*x^2-1/2/d*(A*b*c-B*a
*d+C*a*c)/b^2/(a*d^2-b*c^2)*x-1/2*a*c*(4*A*b*c*d^2-B*a*d^3-3*B*b*c^2*d+2*C
*a*c*d^2+2*C*b*c^3)/d^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b^2)/(-(x^3+c/d*x^
2-a*x/b-a*c/b/d)*b*d)^(1/2)+2*(-1/d^2/b*(B*d-C*c)-1/2*(4*A*a*b*c*d^4-3*B*a
^2*d^5+B*a*b*c^2*d^3-2*B*b^2*c^4*d+4*C*a^2*c*d^4-2*C*a*b*c^3*d^2+2*C*b^2*c
^5)/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b/d^2+(A*b*c-B*a*d+C*a*c)/b/(a*d^2-b*c
^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(
a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2))
)^(1/2))+2*(-C/b/d-1/2*(A*a*b*d^4+3*A*b^2*c^2*d^2-2*B*a*b*c*d^3-2*B*b^2*c^
3*d+C*a^2*d^4+C*a*b*c^2*d^2+2*C*b^2*c^4)/d/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)
/b)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a
*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2
))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1220 vs. $2(502) = 1004$.

Time = 0.13 (sec) , antiderivative size = 1220, normalized size of antiderivative = 2.19

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/3*((4*C*a*b^2*c^6 - 2*B*a*b^2*c^5*d + 13*B*a^2*b*c^3*d^3 - 3*B*a^3*c*d^
5 - 3*(5*C*a^2*b + A*a*b^2)*c^4*d^2 + (3*C*a^3 - 5*A*a^2*b)*c^2*d^4 - (4*C
*b^3*c^5*d - 2*B*b^3*c^4*d^2 + 13*B*a*b^2*c^2*d^4 - 3*B*a^2*b*d^6 - 3*(5*C
*a*b^2 + A*b^3)*c^3*d^3 + (3*C*a^2*b - 5*A*a*b^2)*c*d^5)*x^3 - (4*C*b^3*c^
6 - 2*B*b^3*c^5*d + 13*B*a*b^2*c^3*d^3 - 3*B*a^2*b*c*d^5 - 3*(5*C*a*b^2 +
A*b^3)*c^4*d^2 + (3*C*a^2*b - 5*A*a*b^2)*c^2*d^4)*x^2 + (4*C*a*b^2*c^5*d -
 2*B*a*b^2*c^4*d^2 + 13*B*a^2*b*c^2*d^4 - 3*B*a^3*d^6 - 3*(5*C*a^2*b + A*a
*b^2)*c^3*d^3 + (3*C*a^3 - 5*A*a^2*b)*c*d^5)*x)*sqrt(-b*d)*weierstrassPInv
erse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3
*(3*d*x + c)/d) + 3*(4*C*a*b^2*c^5*d - 2*B*a*b^2*c^4*d^2 - 2*B*a^2*b*c^2*d
^4 - 3*(C*a^2*b - A*a*b^2)*c^3*d^3 + (3*C*a^3 + A*a^2*b)*c*d^5 - (4*C*b^3*
c^4*d^2 - 2*B*b^3*c^3*d^3 - 2*B*a*b^2*c*d^5 - 3*(C*a*b^2 - A*b^3)*c^2*d^4
+ (3*C*a^2*b + A*a*b^2)*d^6)*x^3 - (4*C*b^3*c^5*d - 2*B*b^3*c^4*d^2 - 2*B*
a*b^2*c^2*d^4 - 3*(C*a*b^2 - A*b^3)*c^3*d^3 + (3*C*a^2*b + A*a*b^2)*c*d^5)
*x^2 + (4*C*a*b^2*c^4*d^2 - 2*B*a*b^2*c^3*d^3 - 2*B*a^2*b*c*d^5 - 3*(C*a^2
*b - A*a*b^2)*c^2*d^4 + (3*C*a^3 + A*a^2*b)*d^6)*x)*sqrt(-b*d)*weierstrass
Zeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), wei
erstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/
(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(2*C*a*b^2*c^4*d^2 - 3*B*a*b^2*c^3*d^3 -
B*a^2*b*c*d^5 + 2*(C*a^2*b + 2*A*a*b^2)*c^2*d^4 - (2*C*b^3*c^4*d^2 - 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^2\left(C\,x^2+B\,x+A\right)}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.218 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 498

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \frac{Abc+acC-aBd+b\left(Bc-\left(A+\frac{aC}{b}\right)d\right)x}{b\,(bc^2-ad^2)\,\sqrt{c+dx}\sqrt{a-bx^2}} + \frac{(ad^2(2cC-Bd)+bc(2c^2C-3Bcd+4Ad^2))\sqrt{a-bx^2}}{b\,(bc^2-ad^2)^2\sqrt{c+dx}} - \frac{\sqrt{a}(ad^2(2cC-Bd)+bc(2c^2C-3Bcd+4Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}d\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{\sqrt{a}(aCd^2-b(2c^2C-Bcd+Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{b^{3/2}d\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(A*b*c+C*a*c-B*a*d+b*(B*c-(A+a*C/b)*d)*x)/b/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(
-b*x^2+a)^(1/2)+(a*d^2*(-B*d+2*C*c)+b*c*(4*A*d^2-3*B*c*d+2*C*c^2))*(-b*x^2
+a)^(1/2)/b/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)-a^(1/2)*(a*d^2*(-B*d+2*C*c)+b*c
*(4*A*d^2-3*B*c*d+2*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1
/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/b^(1/2)/d/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(
1/2)/(-b*x^2+a)^(1/2)-a^(1/2)*(a*C*d^2-b*(A*d^2-B*c*d+2*C*c^2))*((d*x+c)/(
c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
b^(3/2)/d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.40 (sec) , antiderivative size = 660, normalized size of antiderivative = 1.33

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\Bigg(ad^2(-2cC+Bd)+2bc(c^2C-Bcd+Ad^2)-bc(2c^2C-3Bcd}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(a*d^2*(-2*c*C + B*d) + 2*b*c*(c^2*C - B*c*d + A*d^2) - b
*c*(2*c^2*C - 3*B*c*d + 4*A*d^2) - ((c + d*x)*(a^2*C*d^2 + b^2*B*c^2*x + a
*b*(c^2*C + B*d^2*x - 2*c*d*(B + C*x)) + A*b*(a*d^2 + b*c*(c - 2*d*x))))/(
-a + b*x^2) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a*d^2*(2*c*C - B*d) + b*
c*(2*c^2*C - 3*B*c*d + 4*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]
*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[
I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt
[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a +
 b*x^2)) + (I*(Sqrt[b]*c - Sqrt[a]*d)*(-(a^(3/2)*C*d^2) + b^(3/2)*c*(2*B*c
 - 3*A*d) + a*Sqrt[b]*d*(-3*c*C + B*d) + Sqrt[a]*b*(2*c^2*C - B*c*d + A*d^
2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b]
- d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d
)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(b*(b*c^2 - a*d^2)^2
*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2180, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(bc(2Bc-3Ad)-ad(3cC-Bd)+b\left(2Cc^2-Bdc+\frac{(Ab-aC)d^2}{b}\right)x\right)}{2b(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\int \frac{bc(2Bc-3Ad)-ad(3cC-Bd)-\left(aCd^2-b\left(2Cc^2-Bdc+Ad^2\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2b\,(bc^2-ad^2)}$$

$\big\downarrow$ 688

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{2\int \frac{a^2Cd^3-ab\left(5Cc^2-2Bdc+Ad^2\right)d+b^2c^2(2Bc-3Ad)-b\left(a(2cC-Bd)d^2+bc\left(2Cc^2-3Bdc+4Ad^2\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2\cdots\right.\right.}{\sqrt{c+dx}\,(bc^2-ad^2)}}{2b\,(bc^2-ad^2)}$$

$\big\downarrow$ 27

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\int \frac{a^2Cd^3-ab\left(5Cc^2-2Bdc+Ad^2\right)d+b^2c^2(2Bc-3Ad)-b\left(a(2cC-Bd)d^2+bc\left(2Cc^2-3Bdc+4Ad^2\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2\cdots\right.\right.}{\sqrt{c+dx}\,(bc^2-ad^2)}}{2b\,(bc^2-ad^2)}$$

$\big\downarrow$ 600

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{-\frac{(bc^2-ad^2)\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(ad^2(2cC-Bd)+\cdots\right.}{\sqrt{c+dx}\,(b\cdots}}{2b\,(bc^2-ad^2)}$$

$\big\downarrow$ 509

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{-\frac{(bc^2-ad^2)\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2C\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(ad^2(2c\cdots\right.}{\cdots}}{2b\,(bc^2-ad^2)}$$

$\big\downarrow$ 508

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2C\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx}\cdots}}{d}}{bc^2-ad^2}}{2b\,(bc^2-ad^2)}$$

$\Big\downarrow$ 327

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-}\cdots}}{d}}{bc^2-ad^2}}{2b\,(bc^2-ad^2)}$$

$\Big\downarrow$ 512

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(2cC-Bd)+bc\left(4Ad^2-3Bcd+2c^2C\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int\frac{\cdots}{\sqrt{c}\cdots}}{d\sqrt{a-bx^2}}}{bc^2-ad^2}}{2b\,(bc^2-ad^2)}$$

$\Big\downarrow$ 511

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(2cC-Bd\cdots\right.}{\cdots}}{bc^2-ad^2}}{2b\,(bc^2-ad^2)}$$

$\Big\downarrow$ 321

$$\frac{\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(aCd^2-b\left(Ad^2-Bcd+2c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(2cC-Bd)+bc\left(4A\right.\right.}{d\sqrt{a-bx}}}{bc^2-ad^2}}{2b\,(bc^2-ad^2)}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(A*b*c + a*c*C - a*B*d + b*(B*c - (A + (a*C)/b)*d)*x)/(b*(b*c^2 - a*d^2)*S
qrt[c + d*x]*Sqrt[a - b*x^2]) - ((-2*(a*d^2*(2*c*C - B*d) + b*c*(2*c^2*C -
 3*B*c*d + 4*A*d^2))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((
2*Sqrt[a]*Sqrt[b]*(a*d^2*(2*c*C - B*d) + b*c*(2*c^2*C - 3*B*c*d + 4*A*d^2)
)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/
Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c +
 d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d
^2)*(a*C*d^2 - b*(2*c^2*C - B*c*d + A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt
[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]
*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c
 + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2))/(2*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(440) = 880$.

Time = 6.21 (sec) , antiderivative size = 968, normalized size of antiderivative = 1.94

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(\frac{2bd\left(-\frac{(4Abc\,d^2-Ba\,d^3-3Bb\,c^2d+2Cac\,d^2+2Cb\,c^3)x^2}{2bd(a^2d^4-2ab\,c^2d^2+b^2c^4)}+\frac{(Abd-Bbc+aCd)x}{2d\,b^2(a\,d^2-b\,c^2)}+\frac{c(3Aab\,d^2+A\,b^2c^2-4Bacdb+a^2C\,d^2+3}{2(a^2d^4-2ab\,c^2d^2+b^2c^4)b^2d}\right.}{\sqrt{-\left(x^3+\frac{c\,x^2}{d}-\frac{ax}{b}-\frac{ac}{bd}\right)bd}}\right.}{}$
default	Expression too large to display

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(-1/2*(4*
A*b*c*d^2-B*a*d^3-3*B*b*c^2*d+2*C*a*c*d^2+2*C*b*c^3)/b/d/(a^2*d^4-2*a*b*c^
2*d^2+b^2*c^4)*x^2+1/2/d*(A*b*d-B*b*c+C*a*d)/b^2/(a*d^2-b*c^2)*x+1/2*c*(3*
A*a*b*d^2+A*b^2*c^2-4*B*a*b*c*d+C*a^2*d^2+3*C*a*b*c^2)/(a^2*d^4-2*a*b*c^2*
d^2+b^2*c^4)/b^2/d)/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(1/2)+2*(-C/b/d+1/2
*(3*A*a*b*d^4+A*b^2*c^2*d^2-4*B*a*b*c*d^3+3*C*a^2*d^4-C*a*b*c^2*d^2+2*C*b^
2*c^4)/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b/d-(A*b*d-B*b*c+C*a*d)/b/(a*d^2-b*
c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*
(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2))+(4*A*b*c*d^2-B*a*d^3-3*B*b*c^2*d+2*C*a*c*d^2+2*C*b*c^3)/(a^2*d^4
-2*a*b*c^2*d^2+b^2*c^4)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)
^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*(
(-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-
c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*Ellipt
icF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(444) = 888$.

Time = 0.11 (sec) , antiderivative size = 1067, normalized size of antiderivative = 2.14

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
1/3*((2*C*a*b^2*c^5 + 3*B*a*b^2*c^4*d + 5*B*a^2*b*c^2*d^3 - (13*C*a^2*b +
5*A*a*b^2)*c^3*d^2 + 3*(C*a^3 - A*a^2*b)*c*d^4 - (2*C*b^3*c^4*d + 3*B*b^3*
c^3*d^2 + 5*B*a*b^2*c*d^4 - (13*C*a*b^2 + 5*A*b^3)*c^2*d^3 + 3*(C*a^2*b -
A*a*b^2)*d^5)*x^3 - (2*C*b^3*c^5 + 3*B*b^3*c^4*d + 5*B*a*b^2*c^2*d^3 - (13
*C*a*b^2 + 5*A*b^3)*c^3*d^2 + 3*(C*a^2*b - A*a*b^2)*c*d^4)*x^2 + (2*C*a*b^
2*c^4*d + 3*B*a*b^2*c^3*d^2 + 5*B*a^2*b*c*d^4 - (13*C*a^2*b + 5*A*a*b^2)*c
^2*d^3 + 3*(C*a^3 - A*a^2*b)*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b
*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c
)/d) + 3*(2*C*a*b^2*c^4*d - 3*B*a*b^2*c^3*d^2 - B*a^2*b*c*d^4 + 2*(C*a^2*b
 + 2*A*a*b^2)*c^2*d^3 - (2*C*b^3*c^3*d^2 - 3*B*b^3*c^2*d^3 - B*a*b^2*d^5 +
 2*(C*a*b^2 + 2*A*b^3)*c*d^4)*x^3 - (2*C*b^3*c^4*d - 3*B*b^3*c^3*d^2 - B*a
*b^2*c*d^4 + 2*(C*a*b^2 + 2*A*b^3)*c^2*d^3)*x^2 + (2*C*a*b^2*c^3*d^2 - 3*B
*a*b^2*c^2*d^3 - B*a^2*b*d^5 + 2*(C*a^2*b + 2*A*a*b^2)*c*d^4)*x)*sqrt(-b*d
)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)
/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3
- 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(4*B*a*b^2*c^2*d^3 - (3*C*a*
b^2 + A*b^3)*c^3*d^2 - (C*a^2*b + 3*A*a*b^2)*c*d^4 + (2*C*b^3*c^3*d^2 - 3*
B*b^3*c^2*d^3 - B*a*b^2*d^5 + 2*(C*a*b^2 + 2*A*b^3)*c*d^4)*x^2 - (B*b^3*c^
3*d^2 - B*a*b^2*c*d^4 - (C*a*b^2 + A*b^3)*c^2*d^3 + (C*a^2*b + A*a*b^2)*d^
5)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(a*b^4*c^5*d^2 - 2*a^2*b^3*c^3*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \int \frac{x(C\,x^2+Bx+A)}{(dx+c)^{\frac{3}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.219 $\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 487

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx = \frac{a\left(Bc-\left(A+\frac{aC}{b}\right)d\right)+(Abc+acC-aBd)x}{a(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{d(Ab(bc^2+3ad^2)+a(aCd^2+bc(3cC-4Bd)))\sqrt{a-bx^2}}{ab(bc^2-ad^2)^2\sqrt{c+dx}} + \frac{(Ab(bc^2+3ad^2)+a(aCd^2+bc(3cC-4Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{(Abc+acC-aBd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(B*c-(A+a*C/b)*d)+(A*b*c-B*a*d+C*a*c)*x)/a/(-a*d^2+b*c^2)/(d*x+c)^(1/2)
/(-b*x^2+a)^(1/2)-d*(A*b*(3*a*d^2+b*c^2)+a*(a*C*d^2+b*c*(-4*B*d+3*C*c)))*(
-b*x^2+a)^(1/2)/a/b/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)+(A*b*(3*a*d^2+b*c^2)+a*
(a*C*d^2+b*c*(-4*B*d+3*C*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE
(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(
1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(
1/2)))^(1/2)/(-b*x^2+a)^(1/2)-(A*b*c-B*a*d+C*a*c)*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2
)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.02 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{a^2Cd^3}{b}+acd(3cC-4Bd)+Ad(bc^2+3ad^2)-2ad(c^2C-Bcd\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*((a^2*C*d^3)/b + a*c*d*(3*c*C - 4*B*d) + A*d*(b*c^2 + 3*a
*d^2) - 2*a*d*(c^2*C - B*c*d + A*d^2) - ((c + d*x)*(A*b^2*c^2*x + a^2*d*(-
2*c*C + d*(B + C*x)) + a*b*(c^2*C*x + B*c*(c - 2*d*x) + A*d*(-2*c + d*x)))
)/(-a + b*x^2) - (I*(Sqrt[b]*c - Sqrt[a]*d)*(A*b*(b*c^2 + 3*a*d^2) + a*(a*
C*d^2 + b*c*(3*c*C - 4*B*d)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(Sqrt[b]*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-
a + b*x^2)) - (I*Sqrt[a]*(Sqrt[b]*c - Sqrt[a]*d)*(A*b^(3/2)*c*d - a^(3/2)*
C*d^2 + a*Sqrt[b]*d*(c*C - B*d) + Sqrt[a]*b*(-2*c^2*C + 3*B*c*d - 3*A*d^2)
)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] -
d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]
)/(Sqrt[b]*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(a*(b*c^2 - a*
d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2180, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a-bx^2)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(aCd^2+b\left(2Cc^2-3Bdc+3Ad^2\right)\right)-bd(Abc+aCc-aBd)x}{2b(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\int\frac{a\left(aCd^2+b\left(2Cc^2-3Bdc+3Ad^2\right)\right)-bd(Abc+aCc-aBd)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 688

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{2\int\frac{b\left(a\left(a(2cC-Bd)d^2+bc\left(2Cc^2-3Bdc+4Ad^2\right)\right)+d\left(Ab\left(bc^2+3ad^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\,(Ab(3ad^2+bc^2)+a(aCd^2+bc(3cC-}{\sqrt{c+dx}(bc^2-ad^2)}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 27

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{b\int\frac{a\left(a(2cC-Bd)d^2+bc\left(2Cc^2-3Bdc+4Ad^2\right)\right)+d\left(Ab\left(bc^2+3ad^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\,(Ab(3ad^2+bc^2)+a(aCd^2+bc(3cC-4B}{\sqrt{c+dx}(bc^2-ad^2)}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 600

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{b\left((Ab(3ad^2+bc^2)+a(aCd^2+bc(3cC-4Bd)))\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-(bc^2-ad^2)(-aBd+acC+Abc)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\,(Ab(3ad^2+bc^2)}{\sqrt{c+dx}(}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 509

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-$$

$$\frac{\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-(bc^2-ad^2)(-aBd+acC+Abc)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)}{bc^2-ad^2}+\frac{2d\sqrt{a-bx^2}\,(Ab(3ad^2}{\sqrt{}}}{2ab\,(bc^2-ad^2)}$$

$\Big\downarrow$ 508

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{b\left(-\left((bc^2-ad^2)(-aBd+acC+Abc)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\cdots}}{\sqrt{\cdots}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{bc^2-ad^2}\cdots}{2ab\,(bc^2-ad^2)}$$

$\downarrow$ 327

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{b\left(-(bc^2-ad^2)(-aBd+acC+Abc)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\cdots}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{bc^2-ad^2}\cdots}{2ab\,(bc^2-ad^2)}$$

$\downarrow$ 512

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{\frac{b\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(-aBd+acC+Abc)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2+bc(3cC-4Bd)\right)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\cdots\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots\right)}{bc^2-ad^2}\cdots}{2ab\,(bc^2-ad^2)}$$

$\downarrow$ 511

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-aBd+acC+Abc)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2\right.\right.\cdots}{\sqrt{b}\sqrt{a-b\cdots}}\right)}{bc^2-ad^2}}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}-\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-aBd+acC+Abc)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\left(3ad^2+bc^2\right)+a\left(aCd^2+bc(3cC\cdots\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}\cdots}{\sqrt{a}\cdots}}}\right)}{bc^2-ad^2}}{2ab\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(B*c - (A + (a*C)/b)*d) + (A*b*c + a*c*C - a*B*d)*x)/(a*(b*c^2 - a*d^2)
*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - ((2*d*(A*b*(b*c^2 + 3*a*d^2) + a*(a*C*d^
2 + b*c*(3*c*C - 4*B*d)))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x])
 + (b*((-2*Sqrt[a]*(A*b*(b*c^2 + 3*a*d^2) + a*(a*C*d^2 + b*c*(3*c*C - 4*B*
d)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*
x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[(Sqr
t[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(A*
b*c + a*c*C - a*B*d)*(b*c^2 - a*d^2)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
 Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqr
t[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*S
qrt[a - b*x^2])))/(b*c^2 - a*d^2))/(2*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 951 vs. $2(429) = 858$.

Time = 6.13 (sec) , antiderivative size = 952, normalized size of antiderivative = 1.95

method	result
elliptic	$\sqrt{(-b\,x^2+a)(dx+c)}\left(\frac{2bd\left(\frac{\left(3Aab\,d^2+A\,b^2c^2-4Bacdb+a^2C\,d^2+3ab\,c^2C\right)x^2}{2a\left(a^2d^4-2ab\,c^2d^2+b^2c^4\right)b}-\frac{(Abc-Bad+Cac)x}{2db\left(a\,d^2-b\,c^2\right)a}-\frac{2Aa\,d^3+2Ab\,c^2d-3Bac\,d^2-Bb\,c^3+4Ca\,c}{2\left(a^2d^4-2ab\,c^2d^2+b^2c^4\right)bd}\right.}{\sqrt{-\left(x^3+\frac{c\,x^2}{d}-\frac{ax}{b}-\frac{ac}{bd}\right)bd}}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(1/2*(3*A
*a*b*d^2+A*b^2*c^2-4*B*a*b*c*d+C*a^2*d^2+3*C*a*b*c^2)/a/(a^2*d^4-2*a*b*c^2
*d^2+b^2*c^4)/b*x^2-1/2*(A*b*c-B*a*d+C*a*c)/d/b/(a*d^2-b*c^2)/a*x-1/2*(2*A
*a*d^3+2*A*b*c^2*d-3*B*a*c*d^2-B*b*c^3+4*C*a*c^2*d)/(a^2*d^4-2*a*b*c^2*d^2
+b^2*c^4)/b/d)/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(1/2)+2*(-1/2*(6*A*a*b*c
*d^2-2*A*b^2*c^3-3*B*a^2*d^3-B*a*b*c^2*d+4*C*a^2*c*d^2)/a/(a^2*d^4-2*a*b*c
^2*d^2+b^2*c^4)+(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/a)*(c/d-1/b*(a*b)^(1/2))
*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b
)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x
^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2
),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-d*(3*A*a*b*d^2+A*
b^2*c^2-4*B*a*b*c*d+C*a^2*d^2+3*C*a*b*c^2)/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^
4)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*
b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*
b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2)
)*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(433) = 866$.

Time = 0.11 (sec) , antiderivative size = 1006, normalized size of antiderivative = 2.07

$$\int \frac{A + Bx + Cx^2}{(c + dx)^{3/2} (a - bx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="frica
s")
```

output
```
-1/3*((5*B*a^2*b*c^3*d + 3*B*a^3*c*d^3 - (3*C*a^2*b - A*a*b^2)*c^4 - (5*C*
a^3 + 9*A*a^2*b)*c^2*d^2 - (5*B*a*b^2*c^2*d^2 + 3*B*a^2*b*d^4 - (3*C*a*b^2
 - A*b^3)*c^3*d - (5*C*a^2*b + 9*A*a*b^2)*c*d^3)*x^3 - (5*B*a*b^2*c^3*d +
3*B*a^2*b*c*d^3 - (3*C*a*b^2 - A*b^3)*c^4 - (5*C*a^2*b + 9*A*a*b^2)*c^2*d^
2)*x^2 + (5*B*a^2*b*c^2*d^2 + 3*B*a^3*d^4 - (3*C*a^2*b - A*a*b^2)*c^3*d -
(5*C*a^3 + 9*A*a^2*b)*c*d^3)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2
+ 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)
- 3*(4*B*a^2*b*c^2*d^2 - (3*C*a^2*b + A*a*b^2)*c^3*d - (C*a^3 + 3*A*a^2*b)
*c*d^3 - (4*B*a*b^2*c*d^3 - (3*C*a*b^2 + A*b^3)*c^2*d^2 - (C*a^2*b + 3*A*a
*b^2)*d^4)*x^3 - (4*B*a*b^2*c^2*d^2 - (3*C*a*b^2 + A*b^3)*c^3*d - (C*a^2*b
 + 3*A*a*b^2)*c*d^3)*x^2 + (4*B*a^2*b*c*d^3 - (3*C*a^2*b + A*a*b^2)*c^2*d^
2 - (C*a^3 + 3*A*a^2*b)*d^4)*x)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*
a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3
*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x
+ c)/d)) - 3*(B*a*b^2*c^3*d + 3*B*a^2*b*c*d^3 - 2*A*a^2*b*d^4 - 2*(2*C*a^2
*b + A*a*b^2)*c^2*d^2 - (4*B*a*b^2*c*d^3 - (3*C*a*b^2 + A*b^3)*c^2*d^2 - (
C*a^2*b + 3*A*a*b^2)*d^4)*x^2 - (B*a*b^2*c^2*d^2 - B*a^2*b*d^4 - (C*a*b^2
+ A*b^3)*c^3*d + (C*a^2*b + A*a*b^2)*c*d^3)*x)*sqrt(-b*x^2 + a)*sqrt(d*x +
 c))/(a^2*b^3*c^5*d - 2*a^3*b^2*c^3*d^3 + a^4*b*c*d^5 - (a*b^4*c^4*d^2 - 2
*a^2*b^3*c^2*d^4 + a^3*b^2*d^6)*x^3 - (a*b^4*c^5*d - 2*a^2*b^3*c^3*d^3 ...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{A+Bx+Cx^2}{\left(a-bx^2\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((a - b*x**2)**(3/2)*(c + d*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output

```
( - 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(a**2*c**
2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c*d*x**3 - 2*a
*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x**6),x)*b**2*
c*d**2 - 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(a**
2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c*d*x**3
- 2*a*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x**6),x)*
b**2*d**3*x - sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3)/(
a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c*d*x*
*3 - 2*a*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x**6),
x)*b*c**2*d**2 - sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**3
)/(a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c*d
*x**3 - 2*a*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x**
6),x)*b*c*d**3*x - 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*
x)/(a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c*
d*x**3 - 2*a*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x*
*6),x)*a*b*c*d**2 - 3*sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)
*x)/(a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b*c
*d*x**3 - 2*a*b*d**2*x**4 + b**2*c**2*x**4 + 2*b**2*c*d*x**5 + b**2*d**2*x
**6),x)*a*b*d**3*x - sqrt(a - b*x**2)*int((sqrt(c + d*x)*sqrt(a - b*x**2)*
x)/(a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - 2*a*b*c**2*x**2 - 4*a*b...
```

3.220 $\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 611

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\frac{Abc+acC-aBd+(bBc-Abd-aCd)x}{a\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{d(bc^2(Bc-2Ad)-ad(4c^2C-3Bcd+2Ad^2))\sqrt{a-bx^2}}{ac\left(bc^2-ad^2\right)^2\sqrt{c+dx}}+\frac{\sqrt{b}(bc^2(Bc-2Ad)-ad(4c^2C-3Bcd+2Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}c\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}-\frac{(bBc-Abd-aCd)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a}\sqrt{b}\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}-\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{ac\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/
(-b*x^2+a)^(1/2)-d*(b*c^2*(-2*A*d+B*c)-a*d*(2*A*d^2-3*B*c*d+4*C*c^2))*(-b*
x^2+a)^(1/2)/a/c/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)+b^(1/2)*(b*c^2*(-2*A*d+B*c
)-a*d*(2*A*d^2-3*B*c*d+4*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipt
icE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+
a^(1/2)*d))^(1/2))/a^(1/2)/c/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2
)))^(1/2)/(-b*x^2+a)^(1/2)-(-A*b*d+B*b*c-C*a*d)*((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/
(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c/(d*x+c)
^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.15 (sec) , antiderivative size = 1424, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(2*A*b^2*c^5 + 4*a*b*c^5*C - (b^2*B*c^6)/d - 2*a*b*B*c^4*d - 4*a^2*c^3*C*d
^2 + 3*a^2*B*c^2*d^3 - 2*a^2*A*c*d^4 - 4*A*b^2*c^4*(c + d*x) - 8*a*b*c^4*C
*(c + d*x) + (2*b^2*B*c^5*(c + d*x))/d + 6*a*b*B*c^3*d*(c + d*x) - 4*a*A*b
*c^2*d^2*(c + d*x) + 2*A*b^2*c^3*(c + d*x)^2 + 4*a*b*c^3*C*(c + d*x)^2 - (
b^2*B*c^4*(c + d*x)^2)/d - 3*a*b*B*c^2*d*(c + d*x)^2 + 2*a*A*b*c*d^2*(c +
d*x)^2 + c*(2*a*d^2*(c^2*C - B*c*d + A*d^2)*(a - b*x^2) + c*(c + d*x)*(a^2
*C*d^2 + b^2*B*c^2*x + a*b*(c^2*C + B*d^2*x - 2*c*d*(B + C*x)) + A*b*(a*d^
2 + b*c*(c - 2*d*x)))) + (I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(b*c^2*(B*c
- 2*A*d) + a*d*(-4*c^2*C + 3*B*c*d - 2*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] +
x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3
/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sq
rt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]]) - (I*(Sqrt[b]*c - Sqrt[a]*d)*(A*(b^(3/2)*c^3 + 3*Sqrt[a]*b*c^2*d -
 4*a*Sqrt[b]*c*d^2 - 2*a^(3/2)*d^3) + Sqrt[a]*c^2*(-(b*B*c) + a*C*d + 3*Sq
rt[a]*Sqrt[b]*(-(c*C) + B*d)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*S
qrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*
ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a
]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + ((2*I)*A*b
^2*c^4*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[
b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a-bx^2\right)^{3/2}(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx+\int \frac{B+Cx}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+\int \frac{B+Cx}{(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{686}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx-\frac{\int -\frac{bd(3a(cC-Bd)+(bBc-aCd)x)}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{ab\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\int \frac{3a(cC-Bd)+(bBc-aCd)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{688}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\left(\frac{2\int \frac{a\left(aCd^2+bc(3cC-4Bd)\right)-b\left(bBc^2-ad(4cC-3Bd)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-ad(4cC-3Bd)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\left(\frac{\int \frac{a\left(aCd^2+bc(3cC-4Bd)\right)-b\left(bBc^2-ad(4cC-3Bd)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-ad(4cC-3Bd)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{600}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\,(a-bx^2)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\right)dx+\frac{d\left(\frac{\frac{\left(bc^2-ad^2\right)(bBc-aCd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(bBc^2-ad(4cC-3Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-ad(4cC-3Bd)\right)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\,(a-bx^2)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\right)dx+\frac{d\left(\frac{\frac{\left(bc^2-ad^2\right)(bBc-aCd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(bBc^2-ad(4cC-3Bd)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-ad(4cC-3Bd)\right)}{\sqrt{c+dx}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\,(a-bx^2)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\right)dx+\frac{d\left(\frac{\frac{\left(bc^2-ad^2\right)(bBc-aCd)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-ad(\right.}{\sqrt{c+dx}(bc^2-}\right.}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$A\int\left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{\left(bc^2-ad^2\right)(bBc-aCd)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(bBc^2-a\right.}{\sqrt{c+dx}\left(bc^2\right.}\right.}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{512}$$

$$A\int\left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(bBc-aCd)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}}{\sqrt{c}}\right.}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{511}$$

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\cdots}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\cdots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 321

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\cdots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\downarrow$ 7239

$$A\int \frac{1}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\cdots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 637

$$A\int \left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\cdots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 7239

$$A\int \frac{1}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\,\text{EllipticF}\left(\arcsin\left(\sqrt{\cdots}\right)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\cdots\right)}{2a\left(bc^2-ad^2\right)}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 637

$$A\int\left(\frac{1}{cx\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bBc^2-ad(4cC-3Bd)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bBc-aCd)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{}}{}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{bc^2-ad^2}\right.}{2a\left(bc^2-ad^2\right)}$$

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$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 637

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$\Big\downarrow$ 7239

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$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}\sqrt{c+dx}\,(bc^2-ad^2)}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1280 vs. 2(532) = 1064.

Time = 6.27 (sec) , antiderivative size = 1281, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	1281
default	Expression too large to display	4185

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(-1/2/a*(
2*A*a*d^3+2*A*b*c^2*d-3*B*a*c*d^2-B*b*c^3+4*C*a*c^2*d)/c/(a*d^2-b*c^2)^2*x
^2+1/2*(A*b*d-B*b*c+C*a*d)/a/(a*d^2-b*c^2)/b/d*x+1/2*(2*A*a^2*d^4+A*a*b*c^
2*d^2+A*b^2*c^4-2*B*a^2*c*d^3-2*B*a*b*c^3*d+3*C*a^2*c^2*d^2+C*a*b*c^4)/a/(
a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/b/d/c)/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(
1/2)+2*(1/2*(5*A*a*b*d^3-A*b^2*c^2*d-6*B*a*b*c*d^2+2*B*b^2*c^3+3*C*a^2*d^3
+C*a*b*c^2*d)/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)-(A*b*d-B*b*c+C*a*d)/a/(a*d
^2-b*c^2))*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+
c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2))+2*(-3/2*b*d/a*(2*A*a*d^3+2*A*b*c^2*d-3*B*a*c*d^2-B*b*c^3+4*
C*a*c^2*d)/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/c+2*b*d/a*(2*A*a*d^3+2*A*b*c^2*
d-3*B*a*c*d^2-B*b*c^3+4*C*a*c^2*d)/c/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((
-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-2/a/c^2*A*(c/d-1/...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b^2*d^2*x^7 + 2
*b^2*c*d*x^6 - 4*a*b*c*d*x^4 + 2*a^2*c*d*x^2 + (b^2*c^2 - 2*a*b*d^2)*x^5 +
 a^2*c^2*x - (2*a*b*c^2 - a^2*d^2)*x^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\,(dx+c)^{\frac{3}{2}}\,\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.221 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 719

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{a(bBc-Abd-aCd)+b(Abc+acC-aBd)x}{a^2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{d\left(A\left(b^2c^4+abc^2d^2+2a^2d^4\right)+ac\left(bc^2(cC-2Bd)+ad^2(3cC-2Bd)\right)\right)\sqrt{a-bx^2}}{a^2c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}}$$

$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{a^2c^2x}$$

$$+\frac{\sqrt{b}\left(A\left(2b^2c^4-abc^2d^2+3a^2d^4\right)+ac\left(bc^2(cC-2Bd)+ad^2(3cC-2Bd)\right)\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E\left(\arcsin\left(\sqrt{1\cdots}\right.\right.}{a^{3/2}c^2\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{b}\left(ac(cC-Bd)+A\left(2bc^2-ad^2\right)\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a^{3/2}c\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(2Bc-3Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{ac^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/a^2/(-a*d^2+b*c^2)/(d*x+c
)^(1/2)/(-b*x^2+a)^(1/2)-d*(A*(2*a^2*d^4+a*b*c^2*d^2+b^2*c^4)+a*c*(b*c^2*(
-2*B*d+C*c)+a*d^2*(-2*B*d+3*C*c)))*(-b*x^2+a)^(1/2)/a^2/c^2/(-a*d^2+b*c^2)
^2/(d*x+c)^(1/2)-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^2/x+b^(1/2)*(A*(3*
a^2*d^4-a*b*c^2*d^2+2*b^2*c^4)+a*c*(b*c^2*(-2*B*d+C*c)+a*d^2*(-2*B*d+3*C*c
)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^
2/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-
b^(1/2)*(a*c*(-B*d+C*c)+A*(-a*d^2+2*b*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2))
)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c/(-a*d^2+b*
c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(-3*A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c^2/(d*x
+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.77 (sec) , antiderivative size = 2824, normalized size of antiderivative = 3.93

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-(A/(a^2*c^2*x)) - (2*d^3*(c^2*C - B*c*d +
A*d^2))/(c^2*(b*c^2 - a*d^2)^2*(c + d*x)) + (-(a*b^2*B*c^2) + 2*a*A*b^2*c*
d + 2*a^2*b*c*C*d - a^2*b*B*d^2 - A*b^3*c^2*x - a*b^2*c^2*C*x + 2*a*b^2*B*
c*d*x - a*A*b^2*d^2*x - a^2*b*C*d^2*x)/(a^2*(-(b*c^2) + a*d^2)^2*(-a + b*x
^2))) - (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-2*A*b^3*c^
5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - a*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]] + 2*a*b^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + a*A*b^2*c^3*d^2*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*a^2*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]] + 2*a^2*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*a^2*A*b*c*d^4*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - (2*A*b^3*c^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]])/(c + d*x)^2 - (a*b^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2
 + (2*a*b^2*B*c^6*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (3*a*A*b
^2*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (2*a^2*b*c^5*C*d^
2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (4*a^2*A*b*c^3*d^4*Sqrt[-c
 + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (3*a^3*c^3*C*d^4*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]])/(c + d*x)^2 - (2*a^3*B*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
])/(c + d*x)^2 + (3*a^3*A*c*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^
2 + (4*A*b^3*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (2*a*b^2*c^6*
C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (4*a*b^2*B*c^5*d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (2*a*A*b^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2 (a-bx^2)^{3/2} (c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{3/2} (a-bx^2)^{3/2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2} (a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\,(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}\right)dx$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2\sqrt{c+dx}\,(a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\int \frac{C\sqrt{c+dx}}{d^2x^2\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}-\frac{d}{c^2 x\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}-\frac{d}{c^2 x\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}-\frac{d}{c^2 x\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx} \left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2 (c+dx)^{3/2} \left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^2(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1435 vs. $2(636) = 1272$.

Time = 11.00 (sec) , antiderivative size = 1436, normalized size of antiderivative = 2.00

method	result	size
elliptic	Expression too large to display	1436
risch	Expression too large to display	1743
default	Expression too large to display	5829

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-A/c^2/a^2/x*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*b*d*(1/2*(2*A*a^2*d^4+A*a*b*c^2*d^2+A*b
^2*c^4-2*B*a^2*c*d^3-2*B*a*b*c^3*d+3*C*a^2*c^2*d^2+C*a*b*c^4)/(a*d^2-b*c^2
)^2/c^2/a^2*x^2-1/2*(A*b*c-B*a*d+C*a*c)/d/(a*d^2-b*c^2)/a^2*x-1/2*(2*A*a^2
*d^5+2*A*b^2*c^4*d-2*B*a^2*c*d^4-B*a*b*c^3*d^2-B*b^2*c^5+2*C*a^2*c^2*d^3+2
*C*a*b*c^4*d)/a/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/d/b/c^2)/(-(x^3+c/d*x^2-a*
x/b-a*c/b/d)*b*d)^(1/2)+2*(-1/2*b*(2*A*a^2*d^4+4*A*a*b*c^2*d^2-2*A*b^2*c^4
-5*B*a^2*c*d^3+B*a*b*c^3*d+6*C*a^2*c^2*d^2-2*C*a*b*c^4)/a^2/(a^2*d^4-2*a*b
*c^2*d^2+b^2*c^4)/c+b*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/a^2)*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)
/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/2*A
*b*d/a^2/c^2+3/2*b*d*(2*A*a^2*d^4+A*a*b*c^2*d^2+A*b^2*c^4-2*B*a^2*c*d^3-2*
B*a*b*c^3*d+3*C*a^2*c^2*d^2+C*a*b*c^4)/a^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)
/c^2-2*b*d*(2*A*a^2*d^4+A*a*b*c^2*d^2+A*b^2*c^4-2*B*a^2*c*d^3-2*B*a*b*c^3*
d+3*C*a^2*c^2*d^2+C*a*b*c^4)/(a*d^2-b*c^2)^2/c^2/a^2)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\,(dx+c)^{\frac{3}{2}}\,\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.222 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 803

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{b(Abc+acC-aBd+(bBc-Abd-aCd)x)}{a^2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{d(b^2c^4(Bc-2Ad)-abc^3d(2cC-Bd)-2a^2d^3(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{a^2c^3\left(bc^2-ad^2\right)^2\sqrt{c+dx}}$$
$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2a^2c^2x^2}-\frac{(4Bc-7Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4a^2c^3x}$$
$$+\frac{\sqrt{b}(b^2c^4(8Bc-15Ad)-2abc^2d(4c^2C+2Bcd-7Ad^2)-a^2d^3(8c^2C-12Bcd+15Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}}{4a^{3/2}c^3\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(bc^2(8Bc-9Ad)-ad(4c^2C+4Bcd-5Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^{3/2}c^2\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{(12Abc^2+8ac^2C-12aBcd+15aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^2c^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
b*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a^2/(-a*d^2+b*c^2)/(d*x+c)^(1
/2)/(-b*x^2+a)^(1/2)-d*(b^2*c^4*(-2*A*d+B*c)-a*b*c^3*d*(-B*d+2*C*c)-2*a^2*
d^3*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/a^2/c^3/(-a*d^2+b*c^2)^2/(d*x+c)
^(1/2)-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^2/x^2-1/4*(-7*A*d+4*B*c)
*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^3/x+1/4*b^(1/2)*(b^2*c^4*(-15*A*d+8*
B*c)-2*a*b*c^2*d*(-7*A*d^2+2*B*c*d+4*C*c^2)-a^2*d^3*(15*A*d^2-12*B*c*d+8*C
*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/
2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)
/c^3/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/
2)-1/4*b^(1/2)*(b*c^2*(-9*A*d+8*B*c)-a*d*(-5*A*d^2+4*B*c*d+4*C*c^2))*((d*x
+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(
1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1
/2))/a^(3/2)/c^2/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4*(15*A*a
*d^2+12*A*b*c^2-12*B*a*c*d+8*C*a*c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2
)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),
2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^2/c^3/(d*x+c)^(1/2)/(
-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.10 (sec) , antiderivative size = 3770, normalized size of antiderivative = 4.69

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-1/2*A/(a^2*c^2*x^2) + (-4*B*c + 7*A*d)/(4*
a^2*c^3*x) + (2*d^4*(c^2*C - B*c*d + A*d^2))/(c^3*(b*c^2 - a*d^2)^2*(c + d
*x)) + (-(A*b^3*c^2) - a*b^2*c^2*C + 2*a*b^2*B*c*d - a*A*b^2*d^2 - a^2*b*C
*d^2 - b^3*B*c^2*x + 2*A*b^3*c*d*x + 2*a*b^2*c*C*d*x - a*b^2*B*d^2*x)/(a^2
*(-(b*c^2) + a*d^2)^2*(-a + b*x^2))) + (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/
(c + d*x))^2)/d^2]*(8*b^3*B*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 15*A*b^3*
c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*a*b^2*c^5*C*d*Sqrt[-c + (Sqrt[a]*
d)/Sqrt[b]] - 4*a*b^2*B*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 14*a*A*b^
2*c^3*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*a^2*b*c^3*C*d^3*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]] + 12*a^2*b*B*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 15
*a^2*A*b*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + (8*b^3*B*c^8*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (15*A*b^3*c^7*d*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]])/(c + d*x)^2 - (8*a*b^2*c^7*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c +
 d*x)^2 - (12*a*b^2*B*c^6*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2
+ (29*a*A*b^2*c^5*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (16*a^
2*b*B*c^4*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (29*a^2*A*b*c^
3*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (8*a^3*c^3*C*d^5*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (12*a^3*B*c^2*d^6*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]])/(c + d*x)^2 + (15*a^3*A*c*d^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]])/(c + d*x)^2 - (16*b^3*B*c^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3 \left(a-bx^2\right)^{3/2} (c+dx)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+ \right.$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\int \frac{C\sqrt{c+dx}}{d^2x^3\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int \frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\Big\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\Big\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\Big\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\Big\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{\sqrt{c+dx}}{x^3(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\left(\frac{c}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{d}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^(3/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1561 vs. $2(706) = 1412$.

Time = 12.06 (sec) , antiderivative size = 1562, normalized size of antiderivative = 1.95

method	result	size
elliptic	Expression too large to display	1562
risch	Expression too large to display	2153
default	Expression too large to display	8022

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*b*d*(-1/2/a^2
*(2*A*a^2*d^5+2*A*b^2*c^4*d-2*B*a^2*c*d^4-B*a*b*c^3*d^2-B*b^2*c^5+2*C*a^2*
c^2*d^3+2*C*a*b*c^4*d)/(a*d^2-b*c^2)^2/c^3*x^2+1/2*(A*b*d-B*b*c+C*a*d)/a^2
/d/(a*d^2-b*c^2)*x+1/2*(2*A*a^3*d^6+A*a*b^2*c^4*d^2+A*b^3*c^6-2*B*a^3*c*d^
5-2*B*a*b^2*c^5*d+2*C*a^3*c^2*d^4+C*a^2*b*c^4*d^2+C*a*b^2*c^6)/a^2/(a^2*d^
4-2*a*b*c^2*d^2+b^2*c^4)/c^3/d/b)/(-(x^3+c/d*x^2-a*x/b-a*c/b/d)*b*d)^(1/2)
-1/2*A/c^2/a^2/x^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4*(7*A*d-4*B*c)/a^
2/c^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(1/2*b*(2*A*a^2*d^5+3*A*a*b*c
^2*d^3-A*b^2*c^4*d-2*B*a^2*c*d^4-4*B*a*b*c^3*d^2+2*B*b^2*c^5+5*C*a^2*c^2*d
^3-C*a*b*c^4*d)/a^2/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/c^2-b*(A*b*d-B*b*c+C*a
*d)/a^2/(a*d^2-b*c^2)+1/4*A*b*d/a^2/c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-3/2*b*d/a^2*(2*A*a^2*d^5+
2*A*b^2*c^4*d-2*B*a^2*c*d^4-B*a*b*c^3*d^2-B*b^2*c^5+2*C*a^2*c^2*d^3+2*C*a*
b*c^4*d)/(a^2*d^4-2*a*b*c^2*d^2+b^2*c^4)/c^3+2*b*d/a^2*(2*A*a^2*d^5+2*A*b^
2*c^4*d-2*B*a^2*c*d^4-B*a*b*c^3*d^2-B*b^2*c^5+2*C*a^2*c^2*d^3+2*C*a*b*c^4*
d)/(a*d^2-b*c^2)^2/c^3+1/8*(7*A*d-4*B*c)/a^2*b*d/c^3)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x^3(c + dx)^{3/2} \left(a - bx^2\right)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(3/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(3/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\,(dx+c)^{\frac{3}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(3/2)/(-b*x^2+a)^(3/2),x)
```

3.223 $\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 854

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{a(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{b^3(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}$$

$$-\frac{(3aAb^2c^2d^4+3a^3Cd^6+3a^2bd^4(c^2C-2Bcd+Ad^2)+2b^3c^4(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{3b^3d^3(bc^2-ad^2)^2(c+dx)^{3/2}}$$

$$-\frac{(3a^3d^6(3cC-Bd)-2b^3c^5(8c^2C-5Bcd+2Ad^2)+3a^2bcd^4(c^2C-3Bcd+3Ad^2)+3ab^2c^3d^2(12c^2C-}{3b^2d^3(bc^2-ad^2)^3\sqrt{c+dx}}$$

$$+\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3b^2d^3}$$

$$+\frac{\sqrt{a}(a^3d^6(25cC-9Bd)-9a^2bcd^4(5c^2C-Bcd-Ad^2)-4b^3c^5(8c^2C-4Bcd+Ad^2)+3ab^2c^3d^2(28c^2C-}{3b^{3/2}d^4(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{\sqrt{a}(5a^3Cd^6+3a^2bd^4(5c^2C-4Bcd+Ad^2)+4b^3c^4(8c^2C-4Bcd+Ad^2)-3ab^2c^2d^2(20c^2C-12Bcd+}{3b^{5/2}d^4(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^3/(-a*d^2+b*c^2)/(d*x
+c)^(3/2)/(-b*x^2+a)^(1/2)-1/3*(3*a*A*b^2*c^2*d^4+3*a^3*C*d^6+3*a^2*b*d^4*
(A*d^2-2*B*c*d+C*c^2)+2*b^3*c^4*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/b^3/
d^3/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)-1/3*(3*a^3*d^6*(-B*d+3*C*c)-2*b^3*c^5*(
2*A*d^2-5*B*c*d+8*C*c^2)+3*a^2*b*c*d^4*(3*A*d^2-3*B*c*d+C*c^2)+3*a*b^2*c^3
*d^2*(9*A*d^2-10*B*c*d+12*C*c^2))*(-b*x^2+a)^(1/2)/b^2/d^3/(-a*d^2+b*c^2)^
3/(d*x+c)^(1/2)+2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^3+1/3*a^(1/2)*(
a^3*d^6*(-9*B*d+25*C*c)-9*a^2*b*c*d^4*(-A*d^2-B*c*d+5*C*c^2)-4*b^3*c^5*(A*
d^2-4*B*c*d+8*C*c^2)+3*a*b^2*c^3*d^2*(9*A*d^2-16*B*c*d+28*C*c^2))*(d*x+c)^
(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/
2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^4/(-a*d^2+b*
c^2)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/3*a^(1/2)*
(5*a^3*C*d^6+3*a^2*b*d^4*(A*d^2-4*B*c*d+5*C*c^2)+4*b^3*c^4*(A*d^2-4*B*c*d+
8*C*c^2)-3*a*b^2*c^2*d^2*(5*A*d^2-12*B*c*d+20*C*c^2))*((d*x+c)/(c+a^(1/2)*
d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))
^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^
4/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.55 (sec) , antiderivative size = 1232, normalized size of antiderivative = 1.44

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((2*C)/(3*b^2*d^3) - (2*c^4*(c^2*C - B*c*d +
 A*d^2))/(3*d^3*(-(b*c^2) + a*d^2)^2*(c + d*x)^2) - (2*c^3*(8*b*c^4*C - 5*
b*B*c^3*d + 2*A*b*c^2*d^2 - 18*a*c^2*C*d^2 + 15*a*B*c*d^3 - 12*a*A*d^4))/(
3*d^3*(-(b*c^2) + a*d^2)^3*(c + d*x)) + (-(a^2*b^2*B*c^3) + 3*a^2*A*b^2*c^
2*d + 3*a^3*b*c^2*C*d - 3*a^3*b*B*c*d^2 + a^3*A*b*d^3 + a^4*C*d^3 - a*A*b^
3*c^3*x - a^2*b^2*c^3*C*x + 3*a^2*b^2*B*c^2*d*x - 3*a^2*A*b^2*c*d^2*x - 3*
a^3*b*c*C*d^2*x + a^3*b*B*d^3*x)/(b^2*(b*c^2 - a*d^2)^3*(-a + b*x^2))) - (
Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-(Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]]*(a^3*d^6*(-25*c*C + 9*B*d) + 9*a^2*b*c*d^4*(5*c^2*C - B*c*d - A
*d^2) + 4*b^3*c^5*(8*c^2*C - 4*B*c*d + A*d^2) - 3*a*b^2*c^3*d^2*(28*c^2*C
- 16*B*c*d + 9*A*d^2))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c + d*x))^2))
+ (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(a^3*d^6*(-25*c*C + 9*B*d) + 9*a^2*b*
c*d^4*(5*c^2*C - B*c*d - A*d^2) + 4*b^3*c^5*(8*c^2*C - 4*B*c*d + A*d^2) -
3*a*b^2*c^3*d^2*(28*c^2*C - 16*B*c*d + 9*A*d^2))*Sqrt[1 - c/(c + d*x) - (S
qrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]
*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] + (I
*Sqrt[a]*d*(Sqrt[b]*c - Sqrt[a]*d)*(5*a^3*C*d^6 + 3*a^(5/2)*Sqrt[b]*d^5*(1
0*c*C - 3*B*d) - 3*a^(3/2)*b^(3/2)*c*d^3*(10*c^2*C + B*c*d - 4*A*d^2) + 3*
a^2*b*d^4*(5*c^2*C - 4*B*c*d + A*d^2) + 4*b^3*c^4*(8*c^2*C - 4*B*c*d + ...
```

Rubi [A] (warning: unable to verify)

Time = 5.56 (sec) , antiderivative size = 912, normalized size of antiderivative = 1.07, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2180, 27, 2182, 27, 2182, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx+Cx^2)}{(a-bx^2)^{3/2} (c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \; 2180$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc-3Ad)-ad(3cC-Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2+3ad^2\right)+a\left(3aCd^2+bc(2cC-5Bd)\right.\right.}{b^3}}{2(c+dx)^{5/2}\sqrt{a-bx^2}}}{a\left(bc^2-ad^2\right)}$$

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^4+2aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{2a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{a^2(bc(2Bc-3Ad)-ad(3cC-Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2+3ad^2\right)+a\left(3aCd^2+bc(2cC-5Bd)\right)\right)}{b^3}}{(c+dx)^{5/2}\sqrt{a-bx^2}}}{2a\left(bc^2-ad^2\right)}$$

$\downarrow$ 2182

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2\int \frac{\frac{6aC\left(bc^2-ad^2\right)^2x^3}{bd}-\frac{6a(cC-Bd)\left(bc^2-ad^2\right)^2x^2}{bd^2}+\frac{a\left(3a^3Cd^6-3a^2b\left(3c^2C-Ad^2\right)d^4-3ab^2c^2\left(2Cc^2-4Bdc+5Ad^2\right)d^2+4b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)x}{b^2d^3}+\frac{3a^2\left(a^2\right.}{}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}}{3(bc^2-ad^2)}}{2a}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\int \frac{\frac{6aC\left(bc^2-ad^2\right)^2x^3}{bd}-\frac{6a(cC-Bd)\left(bc^2-ad^2\right)^2x^2}{bd^2}+\frac{a\left(3a^3Cd^6-3a^2b\left(3c^2C-Ad^2\right)d^4-3ab^2c^2\left(2Cc^2-4Bdc+5Ad^2\right)d^2+4b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)x}{b^2d^3}+3a^2\left(\frac{2(C}{}\right.}{(c+dx)^{3/2}\sqrt{a-bx^2}}}{3(bc^2-ad^2)}}{2a\,(}$$

$\downarrow$ 2182

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{2\int -\frac{-\frac{6aCx^2\left(bc^2-ad^2\right)^3}{bd^2}+a^2\left(\frac{2b\left(5Cc^2-2Bdc-4Ad^2\right)c^4}{d^2}-3a\left(12Cc^2-13Bdc+9Ad^2\right)c^2+\frac{3a^3Cd^4}{b^2}-\frac{3a^2d^2\left(3Cc^2+Bdc-Ad^2\right)}{b}\right)-\frac{a\left(3a^3(7cC-3Bd)d^6-3a^2b\right.}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}\left(3a^3d^6(3cC-Bd)+3a^2bcd^4\left(3Ad^2-3Bcd+c^2C\right)+3ab^2c^3d^2\left(9Ad^2-10Bcd+12c^2C\right)-2b^3c^5\left(2Ad^2-5Bcd+8c^2C\right)\right)}{b^2d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\int\frac{-\frac{6aCx^2\left(bc^2-ad^2\right)^3}{bd^2}+a^2\left(2\right.}{}}{}$$

$\downarrow$ 2185

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}\left(3a^3d^6(3cC-Bd)+3a^2bcd^4\left(3Ad^2-3Bcd+c^2C\right)+3ab^2c^3d^2\left(9Ad^2-10Bcd+12c^2C\right)-2b^3c^5\left(2Ad^2-5Bcd+8c^2C\right)\right)}{b^2d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{\frac{4aC\sqrt{a-bx^2}\sqrt{c+dx}\left(bc^2-ad^2\right)}{b^2d^3}}{}$$

$\downarrow$ 27

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}\left(3a^3d^6(3cC-Bd)+3a^2bcd^4\left(3Ad^2-3Bcd+c^2C\right)+3ab^2c^3d^2\left(9Ad^2-10Bcd+12c^2C\right)-2b^3c^5\left(2Ad^2-5Bcd+8c^2C\right)\right)}{b^2d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{a\int\frac{ad\left(5a^3Cd^6-3a^2b\left(5Cc^2+B\right.\right.}{}}{}$$

$\downarrow$ 600

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}\left(3a^3d^6(3cC-Bd)+3a^2bcd^4\left(3Ad^2-3Bcd+c^2C\right)+3ab^2c^3d^2\left(9Ad^2-10Bcd+12c^2C\right)-2b^3c^5\left(2Ad^2-5Bcd+8c^2C\right)\right)}{b^2d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{a\left(-\frac{\left(bc^2-ad^2\right)\left(5a^3Cd^6+3a^2\right.}{}\right.}{}$$

$\downarrow$ 509

$$\frac{a(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{b^3\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}\left(3a^3d^6(3cC-Bd)+3a^2bcd^4\left(3Ad^2-3Bcd+c^2C\right)+3ab^2c^3d^2\left(9Ad^2-10Bcd+12c^2C\right)-2b^3c^5\left(2Ad^2-5Bcd+8c^2C\right)\right)}{b^2d^3\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{a\left(-\frac{\left(bc^2-ad^2\right)\left(5a^3Cd^6+3a^2\right.}{}\right.}{}$$

$\downarrow$ 508

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}(3a^3Cd^6+3aAb^2c^2d^4+3a^2b(Cc^2-2Bdc+Ad^2)d^4+2b^3c^4(Cc^2-Bdc+Ad^2))}{3b^3d^3(bc^2-ad^2)(c+dx)^{3/2}}+\frac{2a\left(3a^3(3cC-Bd)d^6+3a^2bc\left(Cc^2-3Bdc+3Ad^2\right)d^4+3ab^2c\right.}{b^2d^3\left(b\right.}}{}$$

$\downarrow$ 327

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}(3a^3Cd^6+3aAb^2c^2d^4+3a^2b(Cc^2-2Bdc+Ad^2)d^4+2b^3c^4(Cc^2-Bdc+Ad^2))}{3b^3d^3(bc^2-ad^2)(c+dx)^{3/2}}+\frac{2a\left(3a^3(3cC-Bd)d^6+3a^2bc\left(Cc^2-3Bdc+3Ad^2\right)d^4+3ab^2c\right.}{b^2d^3\left(b\right.}}{}$$

$\downarrow$ 512

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3aAb^2c^2d^4+3a^2b\left(Cc^2-2Bdc+Ad^2\right)d^4+2b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)}{3b^3d^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2a\left(3a^3(3cC-Bd)d^6+3a^2bc\left(Cc^2-3Bdc+3Ad^2\right)d^4+3ab^2c\cdots\right.}{b^2d^3\left(b\cdots\right.}}{}$$

$\Big\downarrow$ 511

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3aAb^2c^2d^4+3a^2b\left(Cc^2-2Bdc+Ad^2\right)d^4+2b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)}{3b^3d^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2a\left(3a^3(3cC-Bd)d^6+3a^2bc\left(Cc^2-3Bdc+3Ad^2\right)d^4+3ab^2c\cdots\right.}{b^2d^3\left(b\cdots\right.}}{}$$

$\Big\downarrow$ 321

$$\frac{a(a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x)}{b^3\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3aAb^2c^2d^4+3a^2b\left(Cc^2-2Bdc+Ad^2\right)d^4+2b^3c^4\left(Cc^2-Bdc+Ad^2\right)\right)}{3b^3d^3\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{2a\left(3a^3(3cC-Bd)d^6+3a^2bc\left(Cc^2-3Bdc+3Ad^2\right)d^4+3ab^2c\cdots\right.}{b^2d^3\left(b\cdots\right.}}{}$$

input

```
Int[(x^4*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x))/(b^3*(b*c^2
- a*d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((2*a*(3*a*A*b^2*c^2*d^4 + 3*a
^3*C*d^6 + 3*a^2*b*d^4*(c^2*C - 2*B*c*d + A*d^2) + 2*b^3*c^4*(c^2*C - B*c*
d + A*d^2))*Sqrt[a - b*x^2])/(3*b^3*d^3*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) +
 ((2*a*(3*a^3*d^6*(3*c*C - B*d) - 2*b^3*c^5*(8*c^2*C - 5*B*c*d + 2*A*d^2)
+ 3*a^2*b*c*d^4*(c^2*C - 3*B*c*d + 3*A*d^2) + 3*a*b^2*c^3*d^2*(12*c^2*C -
10*B*c*d + 9*A*d^2))*Sqrt[a - b*x^2])/(b^2*d^3*(b*c^2 - a*d^2)*Sqrt[c + d*
x]) - ((4*a*C*(b*c^2 - a*d^2)^3*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(b^2*d^3) +
 (a*((2*Sqrt[a]*Sqrt[b]*(a^3*d^6*(25*c*C - 9*B*d) - 9*a^2*b*c*d^4*(5*c^2*C
 - B*c*d - A*d^2) - 4*b^3*c^5*(8*c^2*C - 4*B*c*d + A*d^2) + 3*a*b^2*c^3*d^
2*(28*c^2*C - 16*B*c*d + 9*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellip
ticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqr
t[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a -
b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(5*a^3*C*d^6 + 3*a^2*b*d^4*(5*c^2*C -
 4*B*c*d + A*d^2) + 4*b^3*c^4*(8*c^2*C - 4*B*c*d + A*d^2) - 3*a*b^2*c^2*d^
2*(20*c^2*C - 12*B*c*d + 5*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + S
qrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[
a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*S
qrt[a - b*x^2])))/(b^2*d^3))/(b*c^2 - a*d^2))/(3*(b*c^2 - a*d^2)))/(2*a*(b
*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 15.46 (sec) , antiderivative size = 1470, normalized size of antiderivative = 1.72

method	result	size
elliptic	Expression too large to display	1470
risch	Expression too large to display	2532
default	Expression too large to display	11680

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/d^5/(a*d^2
-b*c^2)^2*c^4*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/
d)^2+2/3*(-b*d*x^2+a*d)/d^4/(a*d^2-b*c^2)^3*c^3*(12*A*a*d^4-2*A*b*c^2*d^2-
15*B*a*c*d^3+5*B*b*c^3*d+18*C*a*c^2*d^2-8*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d)
)^(1/2)-2*(-b*d*x-b*c)*(-1/2/b^2*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*
b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)*a/(a*d^2-b*c^2)^3*x+1/2*a^2*(A*a*b*d^3+3*
A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)
^3/b^3)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2/3*C/b^2/d^3*(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)+2*(-(A*b*d^2-2*B*b*c*d+C*a*d^2+3*C*b*c^2)/d^4/b^2+1/3*b/d^4*
c^4*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2+1/3*b/d^4*c^4*(12*A*a*d^4-2*A*b*c^
2*d^2-15*B*a*c*d^3+5*B*b*c^3*d+18*C*a*c^2*d^2-8*C*b*c^4)/(a*d^2-b*c^2)^3+1
/b^2/(a*d^2-b*c^2)^2*(A*a*b*d^2+A*b^2*c^2-2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2)
*a-1/2/b^2*d*a^2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^
3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3+1/b*c*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-
3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)*a/(a*d^2-b*c^2)^3-1/3*C/b^2/d^2*a)*
(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^
(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1
/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2))+2*(-1/b/d^3*(B*d-2*C*c)+1/3*b/d^3*c^3*(12*A*a*d^4-2*A*b*c^2*d^2-15*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. $2(782) = 1564$.

Time = 0.34 (sec) , antiderivative size = 2719, normalized size of antiderivative = 3.18

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
1/9*((32*C*a*b^4*c^10 - 16*B*a*b^4*c^9*d + 60*B*a^2*b^3*c^7*d^3 - 126*B*a^
3*b^2*c^5*d^5 + 18*B*a^4*b*c^3*d^7 - 4*(27*C*a^2*b^3 - A*a*b^4)*c^8*d^2 +
3*(45*C*a^3*b^2 - A*a^2*b^3)*c^6*d^4 + 4*(5*C*a^4*b + 18*A*a^3*b^2)*c^4*d^
6 - 3*(5*C*a^5 + 3*A*a^4*b)*c^2*d^8 - (32*C*b^5*c^8*d^2 - 16*B*b^5*c^7*d^3
 + 60*B*a*b^4*c^5*d^5 - 126*B*a^2*b^3*c^3*d^7 + 18*B*a^3*b^2*c*d^9 - 4*(27
*C*a*b^4 - A*b^5)*c^6*d^4 + 3*(45*C*a^2*b^3 - A*a*b^4)*c^4*d^6 + 4*(5*C*a^
3*b^2 + 18*A*a^2*b^3)*c^2*d^8 - 3*(5*C*a^4*b + 3*A*a^3*b^2)*d^10)*x^4 - 2*
(32*C*b^5*c^9*d - 16*B*b^5*c^8*d^2 + 60*B*a*b^4*c^6*d^4 - 126*B*a^2*b^3*c^
4*d^6 + 18*B*a^3*b^2*c^2*d^8 - 4*(27*C*a*b^4 - A*b^5)*c^7*d^3 + 3*(45*C*a^
2*b^3 - A*a*b^4)*c^5*d^5 + 4*(5*C*a^3*b^2 + 18*A*a^2*b^3)*c^3*d^7 - 3*(5*C
*a^4*b + 3*A*a^3*b^2)*c*d^9)*x^3 - (32*C*b^5*c^10 - 16*B*b^5*c^9*d + 76*B*
a*b^4*c^7*d^3 - 186*B*a^2*b^3*c^5*d^5 + 144*B*a^3*b^2*c^3*d^7 - 18*B*a^4*b
*c*d^9 - 4*(35*C*a*b^4 - A*b^5)*c^8*d^2 + (243*C*a^2*b^3 - 7*A*a*b^4)*c^6*
d^4 - 5*(23*C*a^3*b^2 - 15*A*a^2*b^3)*c^4*d^6 - (35*C*a^4*b + 81*A*a^3*b^2
)*c^2*d^8 + 3*(5*C*a^5 + 3*A*a^4*b)*d^10)*x^2 + 2*(32*C*a*b^4*c^9*d - 16*B
*a*b^4*c^8*d^2 + 60*B*a^2*b^3*c^6*d^4 - 126*B*a^3*b^2*c^4*d^6 + 18*B*a^4*b
*c^2*d^8 - 4*(27*C*a^2*b^3 - A*a*b^4)*c^7*d^3 + 3*(45*C*a^3*b^2 - A*a^2*b^
3)*c^5*d^5 + 4*(5*C*a^4*b + 18*A*a^3*b^2)*c^3*d^7 - 3*(5*C*a^5 + 3*A*a^4*b
)*c*d^9)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2),
-8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(32*C*a*b^4*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \int \frac{x^4\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^4(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.224 $\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 767

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{a(Abc+acC-aBd+(bBc-Abd-aCd)x)}{b^2(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}$$

$$-\frac{(3abcd^3(Bc-2Ad)-3a^2d^4(2cC-Bd)-2b^2c^3(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{3b^2d^2(bc^2-ad^2)^2(c+dx)^{3/2}}$$

$$+\frac{(3a^3Cd^6-2b^3c^4(5c^2C-2Bcd-Ad^2)+3a^2bd^4(3c^2C-3Bcd+Ad^2)+3ab^2c^2d^2(10c^2C-9Bcd+9Ad^2}{3b^2d^2(bc^2-ad^2)^3\sqrt{c+dx}}$$

$$-\frac{\sqrt{a}(9a^3Cd^6-2b^3c^4(8c^2C-2Bcd-Ad^2)-3a^2bd^4(3c^2C+3Bcd-Ad^2)+3ab^2c^2d^2(16c^2C-9Bcd+9}{3b^{3/2}d^3(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{a}(3a^2d^4(4cC-Bd)+2b^2c^3(8c^2C-2Bcd-Ad^2)-3abcd^2(12c^2C-5Bcd+2Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}}{3b^{3/2}d^3(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
a*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/b^2/(-a*d^2+b*c^2)/(d*x+c)^(3
/2)/(-b*x^2+a)^(1/2)-1/3*(3*a*b*c*d^3*(-2*A*d+B*c)-3*a^2*d^4*(-B*d+2*C*c)-
2*b^2*c^3*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/b^2/d^2/(-a*d^2+b*c^2)^2/(
d*x+c)^(3/2)+1/3*(3*a^3*C*d^6-2*b^3*c^4*(-A*d^2-2*B*c*d+5*C*c^2)+3*a^2*b*d
^4*(A*d^2-3*B*c*d+3*C*c^2)+3*a*b^2*c^2*d^2*(9*A*d^2-9*B*c*d+10*C*c^2))*(-b
*x^2+a)^(1/2)/b^2/d^2/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)-1/3*a^(1/2)*(9*a^3*C*
d^6-2*b^3*c^4*(-A*d^2-2*B*c*d+8*C*c^2)-3*a^2*b*d^4*(-A*d^2+3*B*c*d+3*C*c^2
)+3*a*b^2*c^2*d^2*(9*A*d^2-9*B*c*d+16*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)
^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^3/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a
^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/3*a^(1/2)*(3*a^2*d^4*(-B*d+4*C
*c)+2*b^2*c^3*(-A*d^2-2*B*c*d+8*C*c^2)-3*a*b*c*d^2*(2*A*d^2-5*B*c*d+12*C*c
^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/b^(3/2)/d^3/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.06 (sec) , antiderivative size = 1167, normalized size of antiderivative = 1.52

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((2*c^3*(c^2*C - B*c*d + A*d^2))/(3*d^2*(-(b
*c^2) + a*d^2)^2*(c + d*x)^2) + (2*c^2*(5*b*c^4*C - 2*b*B*c^3*d - A*b*c^2*
d^2 - 15*a*c^2*C*d^2 + 12*a*B*c*d^3 - 9*a*A*d^4))/(3*d^2*(-(b*c^2) + a*d^2
)^3*(c + d*x)) + (-(a*A*b^2*c^3) - a^2*b*c^3*C + 3*a^2*b*B*c^2*d - 3*a^2*A
*b*c*d^2 - 3*a^3*c*C*d^2 + a^3*B*d^3 - a*b^2*B*c^3*x + 3*a*A*b^2*c^2*d*x +
 3*a^2*b*c^2*C*d*x - 3*a^2*b*B*c*d^2*x + a^2*A*b*d^3*x + a^3*C*d^3*x)/(b*(
b*c^2 - a*d^2)^3*(-a + b*x^2))) - (Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*
x))^2)/d^2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-9*a^3*C*d^6 + 2*b^3*c^4*(8*c
^2*C - 2*B*c*d - A*d^2) + 3*a^2*b*d^4*(3*c^2*C + 3*B*c*d - A*d^2) - 3*a*b^
2*c^2*d^2*(16*c^2*C - 9*B*c*d + 9*A*d^2))*(-((a*d^2)/(c + d*x)^2) + b*(-1
+ c/(c + d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(-9*a^3*C*d^6 + 2*b
^3*c^4*(8*c^2*C - 2*B*c*d - A*d^2) + 3*a^2*b*d^4*(3*c^2*C + 3*B*c*d - A*d^
2) - 3*a*b^2*c^2*d^2*(16*c^2*C - 9*B*c*d + 9*A*d^2))*Sqrt[1 - c/(c + d*x)
- (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqr
t[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x]
+ (I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(-9*a^(5/2)*C*d^5 + 3*a^2*S
qrt[b]*d^4*(-4*c*C + B*d) - 3*a^(3/2)*b*d^3*(c^2*C - 4*B*c*d + A*d^2) + 2*
b^(5/2)*c^3*(-8*c^2*C + 2*B*c*d + A*d^2) + 3*a*b^(3/2)*c*d^2*(12*c^2*C - 5
*B*c*d + 2*A*d^2) - 3*Sqrt[a]*b^2*c^2*d*(4*c^2*C - 4*B*c*d + 7*A*d^2))*...
```

Rubi [A] (verified)

Time = 3.70 (sec) , antiderivative size = 810, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2180, 27, 2182, 27, 2182, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+B x+C x^2\right)}{\left(a-b x^2\right)^{3/2} (c+d x)^{5/2}} \, dx$$

$$\Big\downarrow \; 2180$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-5Ad)-ad(5cC-3Bd))}{b^2}}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^3+2aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a\left(Ab\left(2bc^2+ad^2\right)+a\left(aCd^2+bc(2cC-3Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-5Ad)-ad(5cC-3Bd))}{b^2}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2\int -\frac{\frac{3\left(a^2Cd^4+ab\left(5Cc^2-4Bdc+Ad^2\right)d^2+b^2c^2\left(2Cc^2-4Bdc+7Ad^2\right)\right)a^2}{b^2d}-\frac{6C\left(bc^2-ad^2\right)^2x^2a}{bd}-\frac{\left(3a^2Bd^5+3abc\left(4Cc^2-5Bdc+2Ad^2\right)d^2-2b^2c^3\left(2Cc^2-2Bdc\right.\right.}{bd^2}}{2(c+dx)^{3/2}\sqrt{a-bx^2}}}{3(bc^2-ad^2)}}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2a\sqrt{a-bx^2}\left(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{\int \frac{\frac{3\left(a^2Cd^4+ab\left(5Cc^2-4Bdc+Ad^2\right)d^2+b^2c^2\left(2Cc^2-4Bdc+7Ad^2\right)\right)a^{\cdots}}{b^2d}}{\cdots}}{\cdots}}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2182$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2a\sqrt{a-bx^2}\left(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{2\int -\frac{a\left(ad\left(3a^2(cC-Bd)d^4-3abc\left(13Cc^2-9Bdc+3Ad^2\right)d^2+b^2c^3\left(4C\cdots\right.\right.\right.}{\cdots}}{\cdots}}{\cdots}$$

$$\Big\downarrow \; 27$$

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3(Ad^2-Bcd+c^2C))}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 600

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3(Ad^2-Bcd+c^2C))}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 509

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3(Ad^2-Bcd+c^2C))}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 508

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}(bc^2-ad^2)}-$$

$$\frac{2a\sqrt{a-bx^2}(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3(Ad^2-Bcd+c^2C))}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 327

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-9\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 512

$$\frac{a(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(-3a^2d^4(2cC-Bd)+3abcd^3(Bc-2Ad)-2b^2c^3\left(Ad^2-Bcd+c^2C\right)\right)}{3b^2d^2(c+dx)^{3/2}(bc^2-ad^2)}-\frac{2a\sqrt{a-bx^2}\left(3a^3Cd^6+3a^2bd^4\left(Ad^2-3Bcd+3c^2C\right)+3ab^2c^2d^2\left(9Ad^2-9\right.\right.}{b^2d^2\sqrt{c+dx}\left(bc^2-ad^2\right)}}{}$$

$\downarrow$ 511

$$\frac{a(Abc+aCc-aBd+(bBc-Abd-aCd)x)}{b^2\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\left(-3a^2(2cC-Bd)d^4+3abc(Bc-2Ad)d^3-2b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)\sqrt{a-bx^2}}{3b^2d^2(bc^2-ad^2)(c+dx)^{3/2}}-\frac{2a\left(3a^3Cd^6+3a^2b\left(3Cc^2-3Bdc+Ad^2\right)d^4+3ab^2c^2\left(10Cc^2-9Bdc+9Ad\right.\right.}{b^2d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}}{}$$

$\downarrow$ 321

$$\frac{a(Abc+aCc-aBd+(bBc-Abd-aCd)x)}{b^2(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\left(-3a^2(2cC-Bd)d^4+3abc(Bc-2Ad)d^3-2b^2c^3\left(Cc^2-Bdc+Ad^2\right)\right)\sqrt{a-bx^2}}{3b^2d^2(bc^2-ad^2)(c+dx)^{3/2}}-\frac{\frac{2a\left(3a^3Cd^6+3a^2b\left(3Cc^2-3Bdc+Ad^2\right)d^4+3ab^2c^2\left(10Cc^2-9Bdc+9Ad\right.\right.}{b^2d^2\left(bc^2-ad^2\right)\sqrt{c+dx}}}{}}{}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(A*b*c + a*c*C - a*B*d + (b*B*c - A*b*d - a*C*d)*x))/(b^2*(b*c^2 - a*d^
2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((2*a*(3*a*b*c*d^3*(B*c - 2*A*d) - 3
*a^2*d^4*(2*c*C - B*d) - 2*b^2*c^3*(c^2*C - B*c*d + A*d^2))*Sqrt[a - b*x^2
])/(3*b^2*d^2*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) - ((2*a*(3*a^3*C*d^6 - 2*b^
3*c^4*(5*c^2*C - 2*B*c*d - A*d^2) + 3*a^2*b*d^4*(3*c^2*C - 3*B*c*d + A*d^2
) + 3*a*b^2*c^2*d^2*(10*c^2*C - 9*B*c*d + 9*A*d^2))*Sqrt[a - b*x^2])/(b^2*
d^2*(b*c^2 - a*d^2)*Sqrt[c + d*x]) - (a*((2*Sqrt[a]*(9*a^3*C*d^6 - 2*b^3*c
^4*(8*c^2*C - 2*B*c*d - A*d^2) - 3*a^2*b*d^4*(3*c^2*C + 3*B*c*d - A*d^2) +
 3*a*b^2*c^2*d^2*(16*c^2*C - 9*B*c*d + 9*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b
*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((
Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
 Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a^2*d^4*(4*c
*C - B*d) + 2*b^2*c^3*(8*c^2*C - 2*B*c*d - A*d^2) - 3*a*b*c*d^2*(12*c^2*C
- 5*B*c*d + 2*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sq
rt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
 (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2
])))/(b*d^2*(b*c^2 - a*d^2)))/(3*(b*c^2 - a*d^2)))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 9.10 (sec) , antiderivative size = 1376, normalized size of antiderivative = 1.79

method	result	size
elliptic	Expression too large to display	1376
default	Expression too large to display	10264

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/d^4/(a*d^2-
b*c^2)^2*c^3*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d
)^2-2/3*(-b*d*x^2+a*d)/d^3/(a*d^2-b*c^2)^3*c^2*(9*A*a*d^4+A*b*c^2*d^2-12*B
*a*c*d^3+2*B*b*c^3*d+15*C*a*c^2*d^2-5*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1
/2)-2*(-b*d*x-b*c)*(1/2*a*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3
+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3/b^2*x-1/2*a*(3*A*a*b*c*d^2+A*b^2
*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/b^2)
/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-1/b/d^3*(B*d-2*C*c)-1/3*b/d^3*c^3*(A*d
^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2-1/3*b/d^3*c^3*(9*A*a*d^4+A*b*c^2*d^2-12*B*
a*c*d^3+2*B*b*c^3*d+15*C*a*c^2*d^2-5*C*b*c^4)/(a*d^2-b*c^2)^3-a*(2*A*b*c*d
-B*a*d^2-B*b*c^2+2*C*a*c*d)/(a*d^2-b*c^2)^2/b+1/2*a*d*(3*A*a*b*c*d^2+A*b^2
*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/b/(a*d^2-b*c^2)^3-1/
b*c*a*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c
^2*d)/(a*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^
(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+2*(-C/d^2/b-1/3*b*c^2/d^2*(9*A*a*d^4+A*b*c^2*d^
2-12*B*a*c*d^3+2*B*b*c^3*d+15*C*a*c^2*d^2-5*C*b*c^4)/(a*d^2-b*c^2)^3-1/2*d
*a*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2290 vs. $2(701) = 1402$.

Time = 0.24 (sec) , antiderivative size = 2290, normalized size of antiderivative = 2.99

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/9*((16*C*a*b^3*c^9 - 4*B*a*b^3*c^8*d + 3*B*a^2*b^2*c^6*d^3 - 72*B*a^3*b
*c^4*d^5 + 9*B*a^4*c^2*d^7 - 2*(30*C*a^2*b^2 + A*a*b^3)*c^7*d^2 + 42*(3*C*
a^3*b + A*a^2*b^2)*c^5*d^4 - 6*(3*C*a^4 - 4*A*a^3*b)*c^3*d^6 - (16*C*b^4*c
^7*d^2 - 4*B*b^4*c^6*d^3 + 3*B*a*b^3*c^4*d^5 - 72*B*a^2*b^2*c^2*d^7 + 9*B*
a^3*b*d^9 - 2*(30*C*a*b^3 + A*b^4)*c^5*d^4 + 42*(3*C*a^2*b^2 + A*a*b^3)*c^
3*d^6 - 6*(3*C*a^3*b - 4*A*a^2*b^2)*c*d^8)*x^4 - 2*(16*C*b^4*c^8*d - 4*B*b
^4*c^7*d^2 + 3*B*a*b^3*c^5*d^4 - 72*B*a^2*b^2*c^3*d^6 + 9*B*a^3*b*c*d^8 -
2*(30*C*a*b^3 + A*b^4)*c^6*d^3 + 42*(3*C*a^2*b^2 + A*a*b^3)*c^4*d^5 - 6*(3
*C*a^3*b - 4*A*a^2*b^2)*c^2*d^7)*x^3 - (16*C*b^4*c^9 - 4*B*b^4*c^8*d + 7*B
*a*b^3*c^6*d^3 - 75*B*a^2*b^2*c^4*d^5 + 81*B*a^3*b*c^2*d^7 - 9*B*a^4*d^9 -
 2*(38*C*a*b^3 + A*b^4)*c^7*d^2 + 2*(93*C*a^2*b^2 + 22*A*a*b^3)*c^5*d^4 -
18*(8*C*a^3*b + A*a^2*b^2)*c^3*d^6 + 6*(3*C*a^4 - 4*A*a^3*b)*c*d^8)*x^2 +
2*(16*C*a*b^3*c^8*d - 4*B*a*b^3*c^7*d^2 + 3*B*a^2*b^2*c^5*d^4 - 72*B*a^3*b
*c^3*d^6 + 9*B*a^4*c*d^8 - 2*(30*C*a^2*b^2 + A*a*b^3)*c^6*d^3 + 42*(3*C*a^
3*b + A*a^2*b^2)*c^4*d^5 - 6*(3*C*a^4 - 4*A*a^3*b)*c^2*d^7)*x)*sqrt(-b*d)*
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(16*C*a*b^3*c^8*d - 4*B*a*b^3*c^7*d^2 +
 27*B*a^2*b^2*c^5*d^4 + 9*B*a^3*b*c^3*d^6 - 2*(24*C*a^2*b^2 + A*a*b^3)*c^6
*d^3 + 9*(C*a^3*b - 3*A*a^2*b^2)*c^4*d^5 - 3*(3*C*a^4 + A*a^3*b)*c^2*d^7 -
 (16*C*b^4*c^6*d^3 - 4*B*b^4*c^5*d^4 + 27*B*a*b^3*c^3*d^6 + 9*B*a^2*b^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^3\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^3(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.225 $\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 714

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{a(bBc-Abd-aCd)+b(Abc+acC-aBd)x}{b^2(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}$$

$$-\frac{(3a^2Cd^4+3abd^2(c^2C-2Bcd+Ad^2)+b^2c^2(2c^2C-2Bcd+5Ad^2))\sqrt{a-bx^2}}{3b^2d(bc^2-ad^2)^2(c+dx)^{3/2}}$$

$$-\frac{(3a^2d^4(3cC-Bd)-b^2c^3(4c^2C+2Bcd-11Ad^2)+3abcd^2(9c^2C-9Bcd+7Ad^2))\sqrt{a-bx^2}}{3bd(bc^2-ad^2)^3\sqrt{c+dx}}$$

$$+\frac{\sqrt{a}(3a^2d^4(3cC-Bd)-b^2c^3(4c^2C+2Bcd-11Ad^2)+3abcd^2(9c^2C-9Bcd+7Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}\,E}{3\sqrt{b}d^2(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{\sqrt{a}(3a^2Cd^4+b^2c^2(4c^2C+2Bcd-5Ad^2)-3abd^2(5c^2C-2Bcd+Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\text{arc}\right.}{3b^{3/2}d^2(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^2/(-a*d^2+b*c^2)/(d*x+c
)^(3/2)/(-b*x^2+a)^(1/2)-1/3*(3*a^2*C*d^4+3*a*b*d^2*(A*d^2-2*B*c*d+C*c^2)+
b^2*c^2*(5*A*d^2-2*B*c*d+2*C*c^2))*(-b*x^2+a)^(1/2)/b^2/d/(-a*d^2+b*c^2)^2
/(d*x+c)^(3/2)-1/3*(3*a^2*d^4*(-B*d+3*C*c)-b^2*c^3*(-11*A*d^2+2*B*c*d+4*C*
c^2)+3*a*b*c*d^2*(7*A*d^2-9*B*c*d+9*C*c^2))*(-b*x^2+a)^(1/2)/b/d/(-a*d^2+b
*c^2)^3/(d*x+c)^(1/2)+1/3*a^(1/2)*(3*a^2*d^4*(-B*d+3*C*c)-b^2*c^3*(-11*A*d
^2+2*B*c*d+4*C*c^2)+3*a*b*c*d^2*(7*A*d^2-9*B*c*d+9*C*c^2))*(d*x+c)^(1/2)*(
(-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1
/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d^2/(-a*d^2+b*c^2)^3/
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/3*a^(1/2)*(3*a^2*
C*d^4+b^2*c^2*(-5*A*d^2+2*B*c*d+4*C*c^2)-3*a*b*d^2*(A*d^2-2*B*c*d+5*C*c^2)
)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)
*d))^(1/2))/b^(3/2)/d^2/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 31.21 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.25

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(-3a^2d^4(-3cC+Bd)-b^2c^3(4c^2C+2Bcd-11Ad^2)+3abcd^2\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(-3*a^2*d^4*(-3*c*C + B*d) - b^2*c^3*(4*c^2*C + 2*B*c*d -
 11*A*d^2) + 3*a*b*c*d^2*(9*c^2*C - 9*B*c*d + 7*A*d^2) + 2*b*c*(b*c^2*(2*c
^2*C + B*c*d - 4*A*d^2) - 3*a*d^2*(4*c^2*C - 3*B*c*d + 2*A*d^2)) - (2*b*c^
2*(b*c^2 - a*d^2)*(c^2*C - B*c*d + A*d^2))/(c + d*x) + (3*d*(c + d*x)*(a^3
*C*d^3 - A*b^3*c^3*x + a^2*b*d*(3*c^2*C + d^2*(A + B*x) - 3*c*d*(B + C*x))
 - a*b^2*c*(c^2*C*x + B*c*(c - 3*d*x) + 3*A*d*(-c + d*x))))/(-a + b*x^2) +
 (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*(-3*c*C + B*d) + b^2*c^3*(4
*c^2*C + 2*B*c*d - 11*A*d^2) - 3*a*b*c*d^2*(9*c^2*C - 9*B*c*d + 7*A*d^2))*
Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*
x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/
(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) + (I*(Sqrt[b]*c - Sqrt[a
]*d)*(6*A*b^(5/2)*c^3*d + 3*a^(5/2)*C*d^4 - 3*a^2*Sqrt[b]*d^3*(-4*c*C + B*
d) + Sqrt[a]*b^2*c^2*(4*c^2*C + 2*B*c*d - 5*A*d^2) - 3*a^(3/2)*b*d^2*(5*c^
2*C - 2*B*c*d + A*d^2) + 3*a*b^(3/2)*c*d*(4*c^2*C - 7*B*c*d + 6*A*d^2))*Sq
rt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)
/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(3*b*d*(b*c^2 - a*d^2)^3*S
qrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2180, 27, 2182, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2} (c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc-3Ad)-ad(3cC-Bd))x}{b}+\frac{a\left(Ab\left(2bc^2+3ad^2\right)+a\left(3aCd^2+bc(2cC-5Bd)\right)\right)}{b^2}}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{2aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{a(bc(2Bc-3Ad)-ad(3cC-Bd))x}{b}+\frac{a\left(Ab\left(2bc^2+3ad^2\right)+a\left(3aCd^2+bc(2cC-5Bd)\right)\right)}{b^2}}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 2182$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{2\int \frac{a\left(3d\left(2Abc\left(bc^2+3ad^2\right)+a\left(b(4cC-7Bd)c^2+ad^2(4cC-Bd)\right)\right)+\left(3a^2Cd^4-3ab\left(5Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(4Cc^2+2Bdc-5Ad^2\right)\right)x\right)}{2bd(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2a\sqrt{a-bx^2}(3a}{}}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\int \frac{3d\left(2Abc\left(bc^2+3ad^2\right)+a\left(b(4cC-7Bd)c^2+ad^2(4cC-Bd)\right)\right)+\left(3a^2Cd^4-3ab\left(5Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(4Cc^2+2Bdc-5Ad^2\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3bd(bc^2-ad^2)}+\frac{2a\sqrt{a-bx^2}(3a^2C}{}}{2a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \ 688$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\left(\frac{2\int \frac{d\left(Ab\left(6b^2c^4+23abd^2c^2+3a^2d^4\right)-a\left(3a^2Cd^4-9abc(3cC-Bd)d^2-b^2c^3(8cC-23Bd)\right)\right)+b\left(3a^2(3cC-Bd)d^4+3abc\left(9Cc^2-9Bdc+7Ad^2\right)d^2-b^2c^3\left(4Cc^2+2\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}\right.}{3bd(bc^2-ad^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\left(\frac{\int \frac{d\left(Ab\left(6b^2c^4+23abd^2c^2+3a^2d^4\right)-a\left(3a^2Cd^4-9abc(3cC-Bd)d^2-b^2c^3(8cC-23Bd)\right)\right)+b\left(3a^2(3cC-Bd)d^4+3abc\left(9Cc^2-9Bdc+7Ad^2\right)d^2-b^2c^3\left(4Cc^2+2B\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{bc^2-ad^2}\right.}{3bd(bc^2-ad^2)}$$

$$\Big\downarrow \ 600$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\frac{\left(bc^2-ad^2\right)\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\left(3a^2d^4(3cC-Bd)+3abcd^2\left(7Ad^2-9Bcd+9c^2C\right)-b\right.}{d}}{bc^2-ad^2}\right.}{3bd\left(bc^2-ad^2\right)}$$

$\downarrow$ 509

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\frac{\left(bc^2-ad^2\right)\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(3cC-Bd)+3abcd^2\left(7Ad^2-9Bcd+\right.\right.}{d\sqrt{a}}}{bc^2-ad^2}\right.}{3bd\left(bc^2-ad\right.}$$

$\downarrow$ 508

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\frac{\left(bc^2-ad^2\right)\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(3cC-Bd)+3abcd^2\left(7\right.\right.}{}}{bc^2-ad^2}\right.}{}$$

$\downarrow$ 327

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$a\left(\frac{\frac{\left(bc^2-ad^2\right)\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(3cC-Bd)+3abcd^2\left(7\ldots\right.\right.}{\ldots}}{bc^2-ad^2}\ldots\right.$$

$$\Big\downarrow \text{512}$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$a\left(\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2d^4(3cC-Bd)\ldots\right.}{\ldots}}{bc^2-ad^2}\ldots\right.$$

$$\Big\downarrow \text{511}$$

$$\frac{a(bBc-Abd-aCd)+b(Abc+aCc-aBd)x}{b^2\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}-$$

$$\frac{\frac{2a\sqrt{a-bx^2}\left(3a^2Cd^4+3ab\left(Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(2Cc^2-2Bdc+5Ad^2\right)\right)}{3b^2d\left(bc^2-ad^2\right)(c+dx)^{3/2}}+\frac{a\left(\frac{2\sqrt{a-bx^2}\left(3a^2(3cC-Bd)d^4+3abc\left(9Cc^2-9Bdc+7Ad^2\right)d^2-b^2c^3\left(4\ldots\right.\right)}{\left(bc^2-ad^2\right)\sqrt{c+dx}}\ldots\right.}{\ldots}}{\ldots}$$

$$\Big\downarrow \text{321}$$

$$\frac{bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc)}{b^2\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$a\left(\frac{-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3a^2Cd^4-3abd^2\left(Ad^2-2Bcd+5c^2C\right)+b^2c^2\left(-5Ad^2+2Bcd+4c^2C\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{\cdots}}{\cdots}}{bc^2-ad^2}\cdots\right.$$

input
```
Int[(x^2*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x)/(b^2*(b*c^2 - a*
d^2)*(c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((2*a*(3*a^2*C*d^4 + 3*a*b*d^2*(c^
2*C - 2*B*c*d + A*d^2) + b^2*c^2*(2*c^2*C - 2*B*c*d + 5*A*d^2))*Sqrt[a - b
*x^2])/(3*b^2*d*(b*c^2 - a*d^2)*(c + d*x)^(3/2)) + (a*((2*(3*a^2*d^4*(3*c*
C - B*d) - b^2*c^3*(4*c^2*C + 2*B*c*d - 11*A*d^2) + 3*a*b*c*d^2*(9*c^2*C -
 9*B*c*d + 7*A*d^2))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c + d*x]) + ((
-2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(3*c*C - B*d) - b^2*c^3*(4*c^2*C + 2*B*c*d -
 11*A*d^2) + 3*a*b*c*d^2*(9*c^2*C - 9*B*c*d + 7*A*d^2))*Sqrt[c + d*x]*Sqrt
[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (
2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a^2*C*d^4 + b
^2*c^2*(4*c^2*C + 2*B*c*d - 5*A*d^2) - 3*a*b*d^2*(5*c^2*C - 2*B*c*d + A*d^
2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*
EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c
)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(b*c^2 - a*d^2
)))/(3*b*d*(b*c^2 - a*d^2)))/(2*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2182
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
 d + e*x, x]}, Simp[e*R*(d + e*x)^(m + 1)*((a + b*x^2)^(p + 1)/((m + 1)*(b*
d^2 + a*e^2))), x] + Simp[1/((m + 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^(m +
1)*(a + b*x^2)^p*ExpandToSum[(m + 1)*(b*d^2 + a*e^2)*Qx + b*d*R*(m + 1) - b
*e*R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, d, e, p}, x] && PolyQ[Pq,
 x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(644) = 1288$.

Time = 7.77 (sec) , antiderivative size = 1358, normalized size of antiderivative = 1.90

method	result	size
elliptic	Expression too large to display	1358
default	Expression too large to display	9032

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/d^3/(a*d^2
-b*c^2)^2*c^2*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/
d)^2+2/3*(-b*d*x^2+a*d)/d^2/(a*d^2-b*c^2)^3*c*(6*A*a*d^4+4*A*b*c^2*d^2-9*B
*a*c*d^3-B*b*c^3*d+12*C*a*c^2*d^2-2*C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1/2
)-2*(-b*d*x-b*c)*(-1/2/b*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+
3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3*x+1/2*(A*a*b*d^3+3*A*b^2*c^2*d-3*
B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)*a/(a*d^2-b*c^2)^3/b^2)/((x^
2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-C/d^2/b+1/3*b*c^2/d^2*(A*d^2-B*c*d+C*c^2)/(
a*d^2-b*c^2)^2+1/3*b*c^2/d^2*(6*A*a*d^4+4*A*b*c^2*d^2-9*B*a*c*d^3-B*b*c^3*
d+12*C*a*c^2*d^2-2*C*b*c^4)/(a*d^2-b*c^2)^3+1/(a*d^2-b*c^2)^2/b*(A*a*b*d^2
+A*b^2*c^2-2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2)-1/2*d*a*(A*a*b*d^3+3*A*b^2*c^2
*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/b/(a*d^2-b*c^2)^3+c*(3
*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a
*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(
-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2))+2*(1/3*b*c/d*(6*A*a*d^4+4*A*b*c^2*d^2-9*B*a*c*d^3-B*b*c
^3*d+12*C*a*c^2*d^2-2*C*b*c^4)/(a*d^2-b*c^2)^3+1/2*d*(3*A*a*b*c*d^2+A*b^2*
c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3)*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2078 vs. $2(648) = 1296$.

Time = 0.26 (sec) , antiderivative size = 2078, normalized size of antiderivative = 2.91

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
1/9*((4*C*a*b^3*c^8 + 2*B*a*b^3*c^7*d - 42*B*a^2*b^2*c^5*d^3 - 24*B*a^3*b*
c^3*d^5 - (3*C*a^2*b^2 - 7*A*a*b^3)*c^6*d^2 + 24*(3*C*a^3*b + 2*A*a^2*b^2)
*c^4*d^4 - 9*(C*a^4 - A*a^3*b)*c^2*d^6 - (4*C*b^4*c^6*d^2 + 2*B*b^4*c^5*d^
3 - 42*B*a*b^3*c^3*d^5 - 24*B*a^2*b^2*c*d^7 - (3*C*a*b^3 - 7*A*b^4)*c^4*d^
4 + 24*(3*C*a^2*b^2 + 2*A*a*b^3)*c^2*d^6 - 9*(C*a^3*b - A*a^2*b^2)*d^8)*x^
4 - 2*(4*C*b^4*c^7*d + 2*B*b^4*c^6*d^2 - 42*B*a*b^3*c^4*d^4 - 24*B*a^2*b^2
*c^2*d^6 - (3*C*a*b^3 - 7*A*b^4)*c^5*d^3 + 24*(3*C*a^2*b^2 + 2*A*a*b^3)*c^
3*d^5 - 9*(C*a^3*b - A*a^2*b^2)*c*d^7)*x^3 - (4*C*b^4*c^8 + 2*B*b^4*c^7*d
- 44*B*a*b^3*c^5*d^3 + 18*B*a^2*b^2*c^3*d^5 + 24*B*a^3*b*c*d^7 - 7*(C*a*b^
3 - A*b^4)*c^6*d^2 + (75*C*a^2*b^2 + 41*A*a*b^3)*c^4*d^4 - 3*(27*C*a^3*b +
 13*A*a^2*b^2)*c^2*d^6 + 9*(C*a^4 - A*a^3*b)*d^8)*x^2 + 2*(4*C*a*b^3*c^7*d
 + 2*B*a*b^3*c^6*d^2 - 42*B*a^2*b^2*c^4*d^4 - 24*B*a^3*b*c^2*d^6 - (3*C*a^
2*b^2 - 7*A*a*b^3)*c^5*d^3 + 24*(3*C*a^3*b + 2*A*a^2*b^2)*c^3*d^5 - 9*(C*a
^4 - A*a^3*b)*c*d^7)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^
2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(4*C
*a*b^3*c^7*d + 2*B*a*b^3*c^6*d^2 + 27*B*a^2*b^2*c^4*d^4 + 3*B*a^3*b*c^2*d^
6 - (27*C*a^2*b^2 + 11*A*a*b^3)*c^5*d^3 - 3*(3*C*a^3*b + 7*A*a^2*b^2)*c^3*
d^5 - (4*C*b^4*c^5*d^3 + 2*B*b^4*c^4*d^4 + 27*B*a*b^3*c^2*d^6 + 3*B*a^2*b^
2*d^8 - (27*C*a*b^3 + 11*A*b^4)*c^3*d^5 - 3*(3*C*a^2*b^2 + 7*A*a*b^3)*c*d^
7)*x^4 - 2*(4*C*b^4*c^6*d^2 + 2*B*b^4*c^5*d^3 + 27*B*a*b^3*c^3*d^5 + 3*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{x^2\,\left(C\,x^2+B\,x+A\right)}{\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.226 $\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 642

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{Abc+acC-aBd+b\left(Bc-\left(A+\frac{aC}{b}\right)d\right)x}{b(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}$$
$$+\frac{(3ad^2(2cC-Bd)+bc(2c^2C-5Bcd+8Ad^2))\sqrt{a-bx^2}}{3b(bc^2-ad^2)^2(c+dx)^{3/2}}$$
$$+\frac{(3a^2Cd^4+3abd^2(9c^2C-7Bcd+3Ad^2)+b^2c^2(2c^2C-11Bcd+23Ad^2))\sqrt{a-bx^2}}{3b(bc^2-ad^2)^3\sqrt{c+dx}}$$
$$-\frac{\sqrt{a}(3a^2Cd^4+3abd^2(9c^2C-7Bcd+3Ad^2)+b^2c^2(2c^2C-11Bcd+23Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\Bigg(\arcsin}{3\sqrt{b}d(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{\sqrt{a}(3ad^2(2cC-Bd)+bc(2c^2C-5Bcd+8Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}}\right.}{3\sqrt{b}d(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(A*b*c+C*a*c-B*a*d+b*(B*c-(A+a*C/b)*d)*x)/b/(-a*d^2+b*c^2)/(d*x+c)^(3/2)/(
-b*x^2+a)^(1/2)+1/3*(3*a*d^2*(-B*d+2*C*c)+b*c*(8*A*d^2-5*B*c*d+2*C*c^2))*(
-b*x^2+a)^(1/2)/b/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)+1/3*(3*a^2*C*d^4+3*a*b*d^
2*(3*A*d^2-7*B*c*d+9*C*c^2)+b^2*c^2*(23*A*d^2-11*B*c*d+2*C*c^2))*(-b*x^2+a
)^(1/2)/b/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)-1/3*a^(1/2)*(3*a^2*C*d^4+3*a*b*d^
2*(3*A*d^2-7*B*c*d+9*C*c^2)+b^2*c^2*(23*A*d^2-11*B*c*d+2*C*c^2))*(d*x+c)^(
1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d/(-a*d^2+b*c^2
)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/3*a^(1/2)*(3*
a*d^2*(-B*d+2*C*c)+b*c*(8*A*d^2-5*B*c*d+2*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/
2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d/(-a*
d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.06 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.35

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(\frac{2c\left(-bc^2+ad^2\right)\left(c^2C-Bcd+Ad^2\right)}{(c+dx)^2}-\frac{2\left(3ad^2\left(3c^2C-2Bcd+Ad^2\right)+bc^2\left(c^2C-4Bcd+7\cdots\right.\right.}{c+dx}\cdots\right)}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*((2*c*(-(b*c^2) + a*d^2)*(c^2*C - B*c*d + A*d
^2))/(c + d*x)^2 - (2*(3*a*d^2*(3*c^2*C - 2*B*c*d + A*d^2) + b*c^2*(c^2*C
- 4*B*c*d + 7*A*d^2)))/(c + d*x) + (3*(-(b^2*B*c^3*x) + a^2*d^2*(-3*c*C +
d*(B + C*x)) + a*b*c*(-(c^2*C) - 3*B*d^2*x + 3*c*d*(B + C*x)) + A*b*(-(b*c
^2*(c - 3*d*x)) + a*d^2*(-3*c + d*x))))/(a - b*x^2)))/(-(b*c^2) + a*d^2)^3
 + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*C*d^4 + 3*a*b*d^2*(9*c^2*C -
 7*B*c*d + 3*A*d^2) + b^2*c^2*(2*c^2*C - 11*B*c*d + 23*A*d^2))*(a - b*x^2)
 + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*C*d^4 + 3*a*b*d^2*(9*c^2*C - 7
*B*c*d + 3*A*d^2) + b^2*c^2*(2*c^2*C - 11*B*c*d + 23*A*d^2))*Sqrt[(d*(Sqrt
[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))
]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]*d*
(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*C*d^3 + 3*b^2*c^2*(-2*B*c + 5*A*d) + 3*a^(3
/2)*Sqrt[b]*d^2*(-2*c*C + B*d) + Sqrt[a]*b^(3/2)*c*(-2*c^2*C + 5*B*c*d - 8
*A*d^2) + 3*a*b*d*(7*c^2*C - 6*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(b*d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(b*c^2 - a*d^2)^3*(-a + b*x^2))))/(3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$, Rules used = {2180, 27, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(bc(2Bc-5Ad)-ad(5cC-3Bd)+b\left(2Cc^2-3Bdc+\frac{(3Ab+aC)d^2}{b}\right)x\right)}{2b(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{bc(2Bc-5Ad)-ad(5cC-3Bd)+\left(aCd^2+b\left(2Cc^2-3Bdc+3Ad^2\right)\right)x}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2b\left(bc^2-ad^2\right)}$$

$\downarrow$ 688

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2\int -\frac{3\left(a^2Cd^3+ab\left(7Cc^2-6Bdc+3Ad^2\right)d-b^2c^2(2Bc-5Ad)\right)-b\left(3a(2cC-Bd)d^2+bc\left(2Cc^2-5Bdc+8Ad^2\right)\right)x}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-\right.\right.}{3(c+dx)^{3/2}(bc^2-ad^2)}}{2b\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\int \frac{3\left(a^2Cd^3+ab\left(7Cc^2-6Bdc+3Ad^2\right)d-b^2c^2(2Bc-5Ad)\right)-b\left(3a(2cC-Bd)d^2+bc\left(2Cc^2-5Bdc+8Ad^2\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-5\right.\right.}{3(c+dx)^{3/2}(bc^2-ad^2)}}{2b\left(bc^2-ad^2\right)}$$

$\downarrow$ 688

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\int -\frac{b\left(3b^2(2Bc-5Ad)c^3-abd\left(23Cc^2-23Bdc+17Ad^2\right)c-3a^2d^3(3cC-Bd)-\left(3a^2Cd^4+3ab\left(9Cc^2-7Bdc+3Ad^2\right)d^2+b^2c^2\left(2Cc^2-11Bdc+23Ad^2\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2\sqrt{}}{}}{3(bc^2-ad^2)}}{2b\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\int \frac{3b^2(2Bc-5Ad)c^3-abd\left(23Cc^2-23Bdc+17Ad^2\right)c-3a^2d^3(3cC-Bd)-\left(3\right.}{\sqrt{c+dx}\sqrt{}}}{bc^2-}}{3(bc^2-ad^2)}}{2b\left(bc^2-ad^2\right)}$$

$\downarrow$ 600

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-5Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{d}\cdots\right)}{\cdots}}{3(bc^2-ad^2)}}{2b\left(bc^2-\cdots\right.}$$

$$\Big\downarrow \; 509$$

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-5Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{d}\cdots\right)}{\cdots}}{3(bc^2-ad^2)}}{2b\left(\cdots\right.}$$

$$\Big\downarrow \; 508$$

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23A\cdots\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+\cdots}{\sqrt{a}d+\cdots}}}\cdots\right)}{\cdots}}{3(bc^2-ad^2)}}{\cdots}$$

$$\Big\downarrow \; 327$$

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-5Bcd+2c^2C\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\cdots}{d}\cdots\right)}{\cdots}}{3(bc^2-ad^2)}}{\cdots}$$

$\big\downarrow$ 512

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3ad^2(2cC-Bd)+bc\left(8Ad^2-5Bcd+2c^2C\right)\right)\int \frac{}{\sqrt{c+}}}{d\sqrt{a-bx^2}}\right.}{}}{3(bc^2-ad^2)}$$

$\big\downarrow$ 511

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23A\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+}{\sqrt{a}d+}}}\right.}{}}{}$$

$\big\downarrow$ 321

$$\frac{bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc}{b\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{\frac{2\sqrt{a-bx^2}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23Ad^2-11Bcd+2c^2C\right)\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^4+3abd^2\left(3Ad^2-7Bcd+9c^2C\right)+b^2c^2\left(23A\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+}{\sqrt{a}d+}}}\right.}{}}{}$$

input
```
Int[(x*(A + B*x + C*x^2))/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(A*b*c + a*c*C - a*B*d + b*(B*c - (A + (a*C)/b)*d)*x)/(b*(b*c^2 - a*d^2)*(
c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((-2*(3*a*d^2*(2*c*C - B*d) + b*c*(2*c^2
*C - 5*B*c*d + 8*A*d^2))*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d*x)^(3/
2)) - ((2*(3*a^2*C*d^4 + 3*a*b*d^2*(9*c^2*C - 7*B*c*d + 3*A*d^2) + b^2*c^2
*(2*c^2*C - 11*B*c*d + 23*A*d^2))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c
 + d*x]) - (b*((2*Sqrt[a]*(3*a^2*C*d^4 + 3*a*b*d^2*(9*c^2*C - 7*B*c*d + 3*
A*d^2) + b^2*c^2*(2*c^2*C - 11*B*c*d + 23*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (
b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/(
(Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
+ Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a*d^2*(2*c*
C - B*d) + b*c*(2*c^2*C - 5*B*c*d + 8*A*d^2))*Sqrt[(Sqrt[b]*(c + d*x))/(Sq
rt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[
b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt
[c + d*x]*Sqrt[a - b*x^2])))/(b*c^2 - a*d^2))/(3*(b*c^2 - a*d^2)))/(2*b*(b
*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(572) = 1144$.

Time = 8.00 (sec) , antiderivative size = 1318, normalized size of antiderivative = 2.05

method	result	size
elliptic	Expression too large to display	1318
default	Expression too large to display	8308

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/d^2/(a*d^2-
b*c^2)^2*c*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^
2-2/3*(-b*d*x^2+a*d)/d/(a*d^2-b*c^2)^3*(3*A*a*d^4+7*A*b*c^2*d^2-6*B*a*c*d^
3-4*B*b*c^3*d+9*C*a*c^2*d^2+C*b*c^4)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2*(-b*
d*x-b*c)*(1/2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3
*C*a*b*c^2*d)/(a*d^2-b*c^2)^3/b*x-1/2*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3
*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/b)/((x^2-a/b)*(-b*d*
x-b*c))^(1/2)+2*(-1/3*b*c/d*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2-1/3*b*c/d*
(3*A*a*d^4+7*A*b*c^2*d^2-6*B*a*c*d^3-4*B*b*c^3*d+9*C*a*c^2*d^2+C*b*c^4)/(a
*d^2-b*c^2)^3-(2*A*b*c*d-B*a*d^2-B*b*c^2+2*C*a*c*d)/(a*d^2-b*c^2)^2+1/2*d*
(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/
(a*d^2-b*c^2)^3-c*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d
^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/3*b*(3*A*a*d^4+7*A*b*c^2*d^2-
6*B*a*c*d^3-4*B*b*c^3*d+9*C*a*c^2*d^2+C*b*c^4)/(a*d^2-b*c^2)^3-1/2*d*(A*a*
b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^
2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(576) = 1152$.

Time = 0.31 (sec) , antiderivative size = 1759, normalized size of antiderivative = 2.74

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
1/9*((2*C*a*b^2*c^7 + 7*B*a*b^2*c^6*d + 48*B*a^2*b*c^4*d^3 + 9*B*a^3*c^2*d
^5 - 2*(21*C*a^2*b + 11*A*a*b^2)*c^5*d^2 - 6*(4*C*a^3 + 7*A*a^2*b)*c^3*d^4
 - (2*C*b^3*c^5*d^2 + 7*B*b^3*c^4*d^3 + 48*B*a*b^2*c^2*d^5 + 9*B*a^2*b*d^7
 - 2*(21*C*a*b^2 + 11*A*b^3)*c^3*d^4 - 6*(4*C*a^2*b + 7*A*a*b^2)*c*d^6)*x^
4 - 2*(2*C*b^3*c^6*d + 7*B*b^3*c^5*d^2 + 48*B*a*b^2*c^3*d^4 + 9*B*a^2*b*c*
d^6 - 2*(21*C*a*b^2 + 11*A*b^3)*c^4*d^3 - 6*(4*C*a^2*b + 7*A*a*b^2)*c^2*d^
5)*x^3 - (2*C*b^3*c^7 + 7*B*b^3*c^6*d + 41*B*a*b^2*c^4*d^3 - 39*B*a^2*b*c^
2*d^5 - 9*B*a^3*d^7 - 22*(2*C*a*b^2 + A*b^3)*c^5*d^2 + 2*(9*C*a^2*b - 10*A
*a*b^2)*c^3*d^4 + 6*(4*C*a^3 + 7*A*a^2*b)*c*d^6)*x^2 + 2*(2*C*a*b^2*c^6*d
+ 7*B*a*b^2*c^5*d^2 + 48*B*a^2*b*c^3*d^4 + 9*B*a^3*c*d^6 - 2*(21*C*a^2*b +
 11*A*a*b^2)*c^4*d^3 - 6*(4*C*a^3 + 7*A*a^2*b)*c^2*d^5)*x)*sqrt(-b*d)*weie
rstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*C*a*b^2*c^6*d - 11*B*a*b^2*c^5*d^2 - 21*
B*a^2*b*c^3*d^4 + (27*C*a^2*b + 23*A*a*b^2)*c^4*d^3 + 3*(C*a^3 + 3*A*a^2*b
)*c^2*d^5 - (2*C*b^3*c^4*d^3 - 11*B*b^3*c^3*d^4 - 21*B*a*b^2*c*d^6 + (27*C
*a*b^2 + 23*A*b^3)*c^2*d^5 + 3*(C*a^2*b + 3*A*a*b^2)*d^7)*x^4 - 2*(2*C*b^3
*c^5*d^2 - 11*B*b^3*c^4*d^3 - 21*B*a*b^2*c^2*d^5 + (27*C*a*b^2 + 23*A*b^3)
*c^3*d^4 + 3*(C*a^2*b + 3*A*a*b^2)*c*d^6)*x^3 - (2*C*b^3*c^6*d - 11*B*b^3*
c^5*d^2 - 10*B*a*b^2*c^3*d^4 + 21*B*a^2*b*c*d^6 + (25*C*a*b^2 + 23*A*b^3)*
c^4*d^3 - 2*(12*C*a^2*b + 7*A*a*b^2)*c^2*d^5 - 3*(C*a^3 + 3*A*a^2*b)*d^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{3/2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx=\int \frac{x(C\,x^2+Bx+A)}{(dx+c)^{\frac{5}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input

```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output

```
int(x*(C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.227 $\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 627

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{a\left(Bc-\left(A+\frac{aC}{b}\right)d\right)+(Abc+acC-aBd)x}{a\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}$$
$$-\frac{d(Ab(3bc^2+5ad^2)+a(3aCd^2+bc(5cC-8Bd)))\sqrt{a-bx^2}}{3ab\left(bc^2-ad^2\right)^2(c+dx)^{3/2}}$$
$$-\frac{d(Abc(3bc^2+29ad^2)+a(bc^2(11cC-23Bd)+3ad^2(7cC-3Bd)))\sqrt{a-bx^2}}{3a\left(bc^2-ad^2\right)^3\sqrt{c+dx}}$$
$$+\frac{\sqrt{b}(Abc(3bc^2+29ad^2)+a(bc^2(11cC-23Bd)+3ad^2(7cC-3Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)\right.}{3\sqrt{a}\left(bc^2-ad^2\right)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(Ab(3bc^2+5ad^2)+a(3aCd^2+bc(5cC-8Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}\sqrt{b}\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(B*c-(A+a*C/b)*d)+(A*b*c-B*a*d+C*a*c)*x)/a/(-a*d^2+b*c^2)/(d*x+c)^(3/2)
/(-b*x^2+a)^(1/2)-1/3*d*(A*b*(5*a*d^2+3*b*c^2)+a*(3*a*C*d^2+b*c*(-8*B*d+5*
C*c)))*(-b*x^2+a)^(1/2)/a/b/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)-1/3*d*(A*b*c*(2
9*a*d^2+3*b*c^2)+a*(b*c^2*(-23*B*d+11*C*c)+3*a*d^2*(-3*B*d+7*C*c)))*(-b*x^
2+a)^(1/2)/a/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)+1/3*b^(1/2)*(A*b*c*(29*a*d^2+3
*b*c^2)+a*(b*c^2*(-23*B*d+11*C*c)+3*a*d^2*(-3*B*d+7*C*c)))*(d*x+c)^(1/2)*(
(-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1
/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/(-a*d^2+b*c^2)^3/((d*
x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/3*(A*b*(5*a*d^2+3*b*c
^2)+a*(3*a*C*d^2+b*c*(-8*B*d+5*C*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2
)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2
^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/(-a*d^2+b*
c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.01 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(-\frac{2d\left(-bc^2+ad^2\right)\left(c^2C-Bcd+Ad^2\right)}{(c+dx)^2}-\frac{2d\left(3ad^2(-2cC+Bd)+bc\left(-4c^2C+7Bcd-10A\cdots\right.\right.}{c+dx}\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*((-2*d*(-(b*c^2) + a*d^2)*(c^2*C - B*c*d + A*
d^2))/(c + d*x)^2 - (2*d*(3*a*d^2*(-2*c*C + B*d) + b*c*(-4*c^2*C + 7*B*c*d
 - 10*A*d^2)))/(c + d*x) + (3*(a^3*C*d^3 - A*b^3*c^3*x + a^2*b*d*(3*c^2*C
+ d^2*(A + B*x) - 3*c*d*(B + C*x)) - a*b^2*c*(c^2*C*x + B*c*(c - 3*d*x) +
3*A*d*(-c + d*x))))/(a*(a - b*x^2))))/(-(b*c^2) + a*d^2)^3 - (d^2*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]]*(A*b*c*(3*b*c^2 + 29*a*d^2) + a*(b*c^2*(11*c*C - 23
*B*d) + 3*a*d^2*(7*c*C - 3*B*d)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqr
t[a]*d)*(A*b*c*(3*b*c^2 + 29*a*d^2) + a*(b*c^2*(11*c*C - 23*B*d) + 3*a*d^2
*(7*c*C - 3*B*d)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt
[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b
]*c - Sqrt[a]*d)] + I*Sqrt[a]*(Sqrt[b]*c - Sqrt[a]*d)*(3*A*b^2*c^2*d + 3*a
^2*C*d^3 + 9*a^(3/2)*Sqrt[b]*d^2*(-2*c*C + B*d) + a*b*d*(5*c^2*C - 8*B*c*d
 + 5*A*d^2) - 3*Sqrt[a]*b^(3/2)*c*(2*c^2*C - 5*B*c*d + 8*A*d^2))*Sqrt[(d*(
Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d
*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/S
qrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a*d*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*(b*c^2 - a*d^2)^3*(-a + b*x^2))))/(3*Sqrt[c + d
*x])
```

Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$, Rules used = {2180, 27, 688, 27, 688, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a-bx^2)^{3/2}(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(3aCd^2+b\left(2Cc^2-5Bdc+5Ad^2\right)\right)-3bd(Abc+aCc-aBd)x}{2b(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int\frac{a\left(3aCd^2+b\left(2Cc^2-5Bdc+5Ad^2\right)\right)-3bd(Abc+aCc-aBd)x}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 688

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{2\int\frac{b\left(3a\left(3a(2cC-Bd)d^2+bc\left(2Cc^2-5Bdc+8Ad^2\right)\right)-d\left(Ab\left(3bc^2+5ad^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)x\right)}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc\right.\right.}{3(c+dx)^{3/2}(bc^2-ad^2)}}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{b\int\frac{3a\left(3a(2cC-Bd)d^2+bc\left(2Cc^2-5Bdc+8Ad^2\right)\right)-d\left(Ab\left(3bc^2+5ad^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}+\frac{2d\sqrt{a-bx^2}\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc(5c\right.\right.}{3(c+dx)^{3/2}(bc^2-ad^2)}}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 688

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\frac{2\int\frac{a\left(3a^2Cd^4+ab\left(23Cc^2-17Bdc+5Ad^2\right)d^2+3b^2c^2\left(2Cc^2-5Bdc+9Ad^2\right)\right)+bd\left(Abc\left(3bc^2+29ad^2\right)+a\left(b(11cC-23Bd)c^2+3ad^2(7cC-3Bd)\right)\right)x}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-b}}{}}{3(bc^2-ad^2)}\right.}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-\frac{b\left(\frac{\frac{\int\frac{a\left(3a^2Cd^4+ab\left(23Cc^2-17Bdc+5Ad^2\right)d^2+3b^2c^2\left(2Cc^2-5Bdc+9Ad^2\right)\right)+bd\left(Abc\left(3bc^2+29ad^2\right)+a\left(b(11cC-23Bd)c^2+3ad^2(7cC-3Bd)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}+\frac{2d\sqrt{a-bx}}{}}{3(bc^2-ad^2)}\right.}{2ab\left(bc^2-ad^2\right)}$$

$\downarrow$ 600

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(\frac{b\left(Abc\left(29ad^2+3bc^2\right)+a\left(3ad^2(7cC-3Bd)+bc^2(11cC-23Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx-\left(bc^2-ad^2\right)\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}d}{bc^2-ad^2}}{3(bc^2-ad^2)}}{2ab\left(bc^2-a\right.}$$

$$\Big\downarrow \text{509}$$

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(\frac{\frac{b\sqrt{1-\frac{bx^2}{a}}\left(Abc\left(29ad^2+3bc^2\right)+a\left(3ad^2(7cC-3Bd)+bc^2(11cC-23Bd)\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\left(bc^2-ad^2\right)\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)\int\frac{}{\sqrt{c+d}}}{bc^2-ad^2}\right.}{3(bc^2-ad^2)}}{2ab\left(bc^2\right.}$$

$$\Big\downarrow \text{508}$$

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{b\left(\frac{-\left(\left(bc^2-ad^2\right)\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx\right)-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Abc\left(29ad^2+3bc^2\right)+a\left(3ad^2(7cC-3Bd)+bc^2(\right.\right.}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{bc}}}}}{bc^2-ad^2}\right.}{3(bc^2-ad^2)}$$

$$\Big\downarrow \text{327}$$

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\,(bc^2-ad^2)}-$$

$$\frac{b\,\frac{-(bc^2-ad^2)\,(Ab(5ad^2+3bc^2)+a(3aCd^2+bc(5cC-8Bd)))\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,(Abc(29ad^2+3bc^2)+a(3ad^2(7cC-3Bd)+bc^2(11c}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}}{3(bc^2-ad^2)}$$

$\downarrow$ 512

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\,(bc^2-ad^2)}-$$

$$\frac{b\,\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\,(bc^2-ad^2)\,(Ab(5ad^2+3bc^2)+a(3aCd^2+bc(5cC-8Bd)))\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\,(Abc(29ad^2+3bc^2)+a(3ad^2(7cC-3Bd}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{}}{\sqrt{}}}}}{bc^2-ad^2}}{3(bc^2-ad^2)}$$

$\downarrow$ 511

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\,(bc^2-ad^2)}-$$

$$b\,\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\,(bc^2-ad^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,(Ab(5ad^2+3bc^2)+a(3aCd^2+bc(5cC-8Bd)))\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c}}{}}{bc^2-ad^2}$$

$\downarrow$ 321

$$\frac{x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}-$$

$$b\left(\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(Ab\left(5ad^2+3bc^2\right)+a\left(3aCd^2+bc(5cC-8Bd)\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(Ab\right.}{}}{bc^2-ad^2}\right.$$

input
```
Int[(A + B*x + C*x^2)/((c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(a*(B*c - (A + (a*C)/b)*d) + (A*b*c + a*c*C - a*B*d)*x)/(a*(b*c^2 - a*d^2)
*(c + d*x)^(3/2)*Sqrt[a - b*x^2]) - ((2*d*(A*b*(3*b*c^2 + 5*a*d^2) + a*(3*
a*C*d^2 + b*c*(5*c*C - 8*B*d)))*Sqrt[a - b*x^2])/(3*(b*c^2 - a*d^2)*(c + d
*x)^(3/2)) + (b*((2*d*(A*b*c*(3*b*c^2 + 29*a*d^2) + a*(b*c^2*(11*c*C - 23*
B*d) + 3*a*d^2*(7*c*C - 3*B*d)))*Sqrt[a - b*x^2])/((b*c^2 - a*d^2)*Sqrt[c
 + d*x]) + ((-2*Sqrt[a]*Sqrt[b]*(A*b*c*(3*b*c^2 + 29*a*d^2) + a*(b*c^2*(11*
c*C - 23*B*d) + 3*a*d^2*(7*c*C - 3*B*d)))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a
]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]
*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt
[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(A*b*(3*b*c^2 + 5*a*d^2) + a*(3*
a*C*d^2 + b*c*(5*c*C - 8*B*d)))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt
[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sqrt[a
 - b*x^2]))/(b*c^2 - a*d^2)))/(3*(b*c^2 - a*d^2)))/(2*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1294 vs. 2(557) = 1114.

Time = 7.87 (sec) , antiderivative size = 1295, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	1295
default	Expression too large to display	7804

input
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3/d/(a*d^2-b
*c^2)^2*(A*d^2-B*c*d+C*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2
/3*(-b*d*x^2+a*d)/(a*d^2-b*c^2)^3*(10*A*b*c*d^2-3*B*a*d^3-7*B*b*c^2*d+6*C*
a*c*d^2+4*C*b*c^3)/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2*(-b*d*x-b*c)*(-1/2*(3*
A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*
d^2-b*c^2)^3/a*x+1/2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^
2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3/b)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(
1/3*b*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2+1/3*b*c*(10*A*b*c*d^2-3*B*a*d^3-
7*B*b*c^2*d+6*C*a*c*d^2+4*C*b*c^3)/(a*d^2-b*c^2)^3+(A*a*b*d^2+A*b^2*c^2-2*
B*a*b*c*d+C*a^2*d^2+C*a*b*c^2)/(a*d^2-b*c^2)^2/a-1/2*d*(A*a*b*d^3+3*A*b^2*
c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3+b*c
*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)
/(a*d^2-b*c^2)^3/a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/
2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Ellip
ticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1
/b*(a*b)^(1/2)))^(1/2))+2*(1/3*b*d*(10*A*b*c*d^2-3*B*a*d^3-7*B*b*c^2*d+6*C
*a*c*d^2+4*C*b*c^3)/(a*d^2-b*c^2)^3+1/2*b*d*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2
*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/a/(a*d^2-b*c^2)^3)*(c/d-1/b*(a
*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1804 vs. $2(561) = 1122$.

Time = 0.21 (sec) , antiderivative size = 1804, normalized size of antiderivative = 2.88

$$\int \frac{A + Bx + Cx^2}{(c + dx)^{5/2}\,(a - bx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="frica
s")
```

output
```
-1/9*((22*B*a^2*b^2*c^5*d + 42*B*a^3*b*c^3*d^3 - (7*C*a^2*b^2 - 3*A*a*b^3)
*c^6 - 4*(12*C*a^3*b + 13*A*a^2*b^2)*c^4*d^2 - 3*(3*C*a^4 + 5*A*a^3*b)*c^2
*d^4 - (22*B*a*b^3*c^3*d^3 + 42*B*a^2*b^2*c*d^5 - (7*C*a*b^3 - 3*A*b^4)*c^
4*d^2 - 4*(12*C*a^2*b^2 + 13*A*a*b^3)*c^2*d^4 - 3*(3*C*a^3*b + 5*A*a^2*b^2
)*d^6)*x^4 - 2*(22*B*a*b^3*c^4*d^2 + 42*B*a^2*b^2*c^2*d^4 - (7*C*a*b^3 - 3
*A*b^4)*c^5*d - 4*(12*C*a^2*b^2 + 13*A*a*b^3)*c^3*d^3 - 3*(3*C*a^3*b + 5*A
*a^2*b^2)*c*d^5)*x^3 - (22*B*a*b^3*c^5*d + 20*B*a^2*b^2*c^3*d^3 - 42*B*a^3
*b*c*d^5 - (7*C*a*b^3 - 3*A*b^4)*c^6 - (41*C*a^2*b^2 + 55*A*a*b^3)*c^4*d^2
 + (39*C*a^3*b + 37*A*a^2*b^2)*c^2*d^4 + 3*(3*C*a^4 + 5*A*a^3*b)*d^6)*x^2
+ 2*(22*B*a^2*b^2*c^4*d^2 + 42*B*a^3*b*c^2*d^4 - (7*C*a^2*b^2 - 3*A*a*b^3)
*c^5*d - 4*(12*C*a^3*b + 13*A*a^2*b^2)*c^3*d^3 - 3*(3*C*a^4 + 5*A*a^3*b)*c
*d^5)*x)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/
27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 3*(23*B*a^2*b^2*c^4*d
^2 + 9*B*a^3*b*c^2*d^4 - (11*C*a^2*b^2 + 3*A*a*b^3)*c^5*d - (21*C*a^3*b +
29*A*a^2*b^2)*c^3*d^3 - (23*B*a*b^3*c^2*d^4 + 9*B*a^2*b^2*d^6 - (11*C*a*b^
3 + 3*A*b^4)*c^3*d^3 - (21*C*a^2*b^2 + 29*A*a*b^3)*c*d^5)*x^4 - 2*(23*B*a*
b^3*c^3*d^3 + 9*B*a^2*b^2*c*d^5 - (11*C*a*b^3 + 3*A*b^4)*c^4*d^2 - (21*C*a
^2*b^2 + 29*A*a*b^3)*c^2*d^4)*x^3 - (23*B*a*b^3*c^4*d^2 - 14*B*a^2*b^2*c^2
*d^4 - 9*B*a^3*b*d^6 - (11*C*a*b^3 + 3*A*b^4)*c^5*d - 2*(5*C*a^2*b^2 + 13*
A*a*b^3)*c^3*d^3 + (21*C*a^3*b + 29*A*a^2*b^2)*c*d^5)*x^2 + 2*(23*B*a^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(a-b\,x^2\right)^{3/2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{C\,x^2+Bx+A}{(dx+c)^{\frac{5}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.228 $\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 782

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\frac{Abc+acC-aBd+(bBc-Abd-aCd)x}{a\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}$$
$$-\frac{d(3bc^2(Bc-2Ad)-ad(8c^2C-5Bcd+2Ad^2))\sqrt{a-bx^2}}{3ac\left(bc^2-ad^2\right)^2(c+dx)^{3/2}}$$
$$-\frac{d(3b^2c^4(Bc-3Ad)-3a^2d^3(3c^2C-2Ad^2)-abc^2d(23c^2C-29Bcd+29Ad^2))\sqrt{a-bx^2}}{3ac^2\left(bc^2-ad^2\right)^3\sqrt{c+dx}}$$
$$+\frac{\sqrt{b}(3b^2c^4(Bc-3Ad)-3a^2d^3(3c^2C-2Ad^2)-abc^2d(23c^2C-29Bcd+29Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\text{arcs}\right.}{3\sqrt{a}c^2\left(bc^2-ad^2\right)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(3bc^2(Bc-2Ad)-ad(8c^2C-5Bcd+2Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{c}}\right.}{3\sqrt{a}c\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{ac^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a/(-a*d^2+b*c^2)/(d*x+c)^(3/2)/
(-b*x^2+a)^(1/2)-1/3*d*(3*b*c^2*(-2*A*d+B*c)-a*d*(2*A*d^2-5*B*c*d+8*C*c^2)
)*(-b*x^2+a)^(1/2)/a/c/(-a*d^2+b*c^2)^2/(d*x+c)^(3/2)-1/3*d*(3*b^2*c^4*(-3
*A*d+B*c)-3*a^2*d^3*(-2*A*d^2+3*C*c^2)-a*b*c^2*d*(29*A*d^2-29*B*c*d+23*C*c
^2))*(-b*x^2+a)^(1/2)/a/c^2/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)+1/3*b^(1/2)*(3*
b^2*c^4*(-3*A*d+B*c)-3*a^2*d^3*(-2*A*d^2+3*C*c^2)-a*b*c^2*d*(29*A*d^2-29*B
*c*d+23*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2
)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2)
)/a^(1/2)/c^2/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x
^2+a)^(1/2)-1/3*b^(1/2)*(3*b*c^2*(-2*A*d+B*c)-a*d*(2*A*d^2-5*B*c*d+8*C*c^2
))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/
2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2
)*d))^(1/2))/a^(1/2)/c/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A
*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/a/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.51 (sec) , antiderivative size = 2384, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
(2*a*c*d^2*(b*c^2 - a*d^2)*(c^2*C - B*c*d + A*d^2)*(a - b*x^2) + 2*a*d^2*(
3*a*d^2*(c^2*C - A*d^2) + b*c^2*(7*c^2*C - 10*B*c*d + 13*A*d^2))*(c + d*x)
*(a - b*x^2) + 3*b*c^2*(c + d*x)^2*(b^2*B*c^3*x - a^2*d^2*(-3*c*C + d*(B +
 C*x)) + a*b*c*(c^2*C + 3*B*d^2*x - 3*c*d*(B + C*x)) + A*b*(b*c^2*(c - 3*d
*x) + a*d^2*(3*c - d*x))))/(3*a*c^2*(b*c^2 - a*d^2)^3*(c + d*x)^(3/2)*Sqrt
[a - b*x^2]) - (3*b^3*B*c^8*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 9*A*b^3*c^7*d
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 23*a*b^2*c^7*C*d*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]] + 26*a*b^2*B*c^6*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 20*a*A*b^2*c
^5*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 14*a^2*b*c^5*C*d^3*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]] - 29*a^2*b*B*c^4*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 35*a
^2*A*b*c^3*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 9*a^3*c^3*C*d^5*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]] - 6*a^3*A*c*d^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 6*b^
3*B*c^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 18*A*b^3*c^6*d*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 46*a*b^2*c^6*C*d*Sqrt[-c + (Sqrt[a]*d)/
Sqrt[b]]*(c + d*x) - 58*a*b^2*B*c^5*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c
+ d*x) + 58*a*A*b^2*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 18*
a^2*b*c^4*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 12*a^2*A*b*c^2*
d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 3*b^3*B*c^6*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]*(c + d*x)^2 - 9*A*b^3*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(c + d*x)^2 - 23*a*b^2*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+\int \frac{B+Cx}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\int \frac{B+Cx}{(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \; 686$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx-\frac{\int -\frac{bd(5a(cC-Bd)+3(bBc-aCd)x)}{2(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{ab\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\int \frac{5a(cC-Bd)+3(bBc-aCd)x}{(c+dx)^{5/2}\sqrt{a-bx^2}}dx}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 688$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\left(\frac{2\int \frac{3a\left(3aCd^2+bc(5cC-8Bd)\right)+b\left(3bBc^2-ad(8cC-5Bd)\right)x}{2(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(3bBc^2-ad(8cC-5Bd)\right)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{d\left(\frac{\int \frac{3a\left(3aCd^2+bc(5cC-8Bd)\right)+b\left(3bBc^2-ad(8cC-5Bd)\right)x}{(c+dx)^{3/2}\sqrt{a-bx^2}}dx}{3(bc^2-ad^2)}-\frac{2\sqrt{a-bx^2}\left(3bBc^2-ad(8cC-5Bd)\right)}{3(c+dx)^{3/2}(bc^2-ad^2)}\right)}{2a\left(bc^2-ad^2\right)}+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 688$$

$$d\left(\frac{\frac{2\int \frac{b\left(a\left(3b(5cC-9Bd)c^2+ad^2(17cC-5Bd)\right)-\left(3b^2Bc^3-abd(23cC-29Bd)c-9a^2Cd^3\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2\sqrt{a}}{}\right.$$

$$\overline{\qquad 2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$d\left(\frac{\frac{b\int \frac{a\left(3b(5cC-9Bd)c^2+ad^2(17cC-5Bd)\right)-\left(3b^2Bc^3-abd(23cC-29Bd)c-9a^2Cd^3\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)}{\sqrt{c+dx}\left(bc^2-ad^2\right)}}{3(bc^2-ad^2)}-\frac{2\sqrt{a-b}}{3}\right.$$

$$\overline{\qquad 2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 600

$$d\left(\frac{\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3bBc^2-ad(8cC-5Bd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-9a^2Cd^3-abcd(23c\right.}{\sqrt{c+dx}\left(bc^2-a\right.}}{3(bc^2-ad^2)}\right.$$

$$\overline{\qquad 2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 509

$$d\left(\frac{\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3bBc^2-ad(8cC-5Bd)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{\sqrt{1-\frac{bx^2}{a}}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}\right)}{bc^2-ad^2}-\frac{2\sqrt{a-bx^2}\left(-9a^2Cd^3\ldots\right.}{\sqrt{c+\ldots}}}{3(bc^2-ad^2)}\right.$$

$$\overline{\qquad 2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+\frac{\left(bc^2-ad^2\right)\left(3bBc^2-ad(8cC-5Bd)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}\ldots}{d}\ldots\right.}{bc^2-ad^2}\ldots}{3(bc^2-ad^2)}\right.$$

$$\overline{\qquad 2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$d\left(\frac{\frac{b\left(\frac{\left(bc^2-ad^2\right)\left(3bBc^2-ad(8cC-5Bd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}}\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$\overline{2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$d\left(\frac{\frac{b\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(3bBc^2-ad(8cC-5Bd)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$\overline{2a\left(bc^2-ad^2\right)}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 511

$$\frac{d\left(\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\right)}{2a\left(bc\right.}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 321

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\right)}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 7239

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{637}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{7239}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\cdots\right.}{\sqrt{b}d\sqrt{a-b\cdots}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-\cdots\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{637}$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\cdots\right.}{\sqrt{b}d\sqrt{a-b\cdots}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\cdots\right)}{2a\left(bc^2-\cdots\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{7239}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{637}$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{7239}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$2a\left(bc^2-\right.$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{637}$$

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right)}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$2a\left(bc^2-\right.$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{7239}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\cdots\right.}{\sqrt{b}d\sqrt{a-b\cdots}}\right)}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}\cdots\right.}{2a\left(bc^2-\cdots\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 637

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\cdots\right.}{\sqrt{b}d\sqrt{a-b\cdots}}\right)}{bc^2-ad^2}}{3\left(bc^2-ad^2\right)}\cdots\right.}{2a\left(bc^2-\cdots\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 7239

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$\overline{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{637}$$

$$A\int\left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.$$

$$\overline{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{7239}$$

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 637

$$A\int \left(-\frac{d}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{1}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx+$$

$$\frac{d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\right.}{\sqrt{b}d\sqrt{a-b}}\right.}{bc^2-ad^2}}{3(bc^2-ad^2)}\right.}{2a\left(bc^2-\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 7239

$$A\int \frac{1}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+$$

$$d\left(\frac{\frac{b\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(-9a^2Cd^3-abcd(23cC-29Bd)+3b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3bBc^2-ad(8c\cdots\right.}{\sqrt{b}d\sqrt{a-b\cdots}}\cdots}{bc^2-ad^2}}{3(bc^2-ad^2)}\cdots\right.$$

$$\overline{2a\left(bc^2-\cdots\right.}$$

$$\frac{x(bBc-aCd)+a(cC-Bd)}{a\sqrt{a-bx^2}(c+dx)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(x*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 688
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 + a*e^2))   Int[(d +
 e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
 + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && LtQ[m, -1]
&& (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(691) = 1382$.

Time = 8.19 (sec) , antiderivative size = 1552, normalized size of antiderivative = 1.98

method	result	size
elliptic	Expression too large to display	1552
default	Expression too large to display	11155

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3/(a*d^2-b*c^
2)^2*(A*d^2-B*c*d+C*c^2)/c*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)^2+2/
3*(-b*d*x^2+a*d)*d/(a*d^2-b*c^2)^3*(3*A*a*d^4-13*A*b*c^2*d^2+10*B*b*c^3*d-
3*C*a*c^2*d^2-7*C*b*c^4)/c^2/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-2*(-b*d*x-b*c)
*(1/2/a*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b
*c^2*d)/(a*d^2-b*c^2)^3*x-1/2*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c
^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/a)/((x^2-a/b)*(-b*d*x-b*c))^
(1/2)+2*(-1/3*b*d*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2/c+1/3*b/c*d*(3*A*a*d
^4-13*A*b*c^2*d^2+10*B*b*c^3*d-3*C*a*c^2*d^2-7*C*b*c^4)/(a*d^2-b*c^2)^3-b/
a*(2*A*b*c*d-B*a*d^2-B*b*c^2+2*C*a*c*d)/(a*d^2-b*c^2)^2+1/2*b*d*(3*A*a*b*c
*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/a/(a*d^2-b
*c^2)^3-b*c/a*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3
*C*a*b*c^2*d)/(a*d^2-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^
(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(1/3*b*d^2*(3*A*a*d^4-13*A*b*c^2*d^2+
10*B*b*c^3*d-3*C*a*c^2*d^2-7*C*b*c^4)/(a*d^2-b*c^2)^3/c^2-1/2*b*d/a*(A*a*b
*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2
-b*c^2)^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\,(a-bx^2)^{3/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\,(dx+c)^{\frac{5}{2}}\,\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.229 $\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 919

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \frac{a(bBc-Abd-aCd)+b(Abc+acC-aBd)x}{a^2\left(bc^2-ad^2\right)(c+dx)^{3/2}\sqrt{a-bx^2}}$$

$$-\frac{d(A(3b^2c^4+3abc^2d^2+2a^2d^4)+ac(3bc^2(cC-2Bd)+ad^2(5cC-2Bd)))\sqrt{a-bx^2}}{3a^2c^2\left(bc^2-ad^2\right)^2(c+dx)^{3/2}}$$

$$-\frac{d(A(3b^3c^6+9ab^2c^4d^2+32a^2bc^2d^4-12a^3d^6)+ac(6a^2Bd^5+3b^2c^4(cC-3Bd)+29abc^2d^2(cC-Bd)))\sqrt{\cdots}}{3a^2c^3\left(bc^2-ad^2\right)^3\sqrt{c+dx}}$$

$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{a^2c^3x}$$

$$+\frac{\sqrt{b}(A(6b^3c^6+41a^2bc^2d^4-15a^3d^6)+ac(6a^2Bd^5+3b^2c^4(cC-3Bd)+29abc^2d^2(cC-Bd)))\sqrt{c+dx}\sqrt{\cdots}}{3a^{3/2}c^3\left(bc^2-ad^2\right)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{b}(A(6b^2c^4-3abc^2d^2+5a^2d^4)+ac(3bc^2(cC-2Bd)+ad^2(5cC-2Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\cdots\right)}{3a^{3/2}c^2\left(bc^2-ad^2\right)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(2Bc-5Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{ac^3\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/a^2/(-a*d^2+b*c^2)/(d*x+c
)^(3/2)/(-b*x^2+a)^(1/2)-1/3*d*(A*(2*a^2*d^4+3*a*b*c^2*d^2+3*b^2*c^4)+a*c*
(3*b*c^2*(-2*B*d+C*c)+a*d^2*(-2*B*d+5*C*c)))*(-b*x^2+a)^(1/2)/a^2/c^2/(-a*
d^2+b*c^2)^2/(d*x+c)^(3/2)-1/3*d*(A*(-12*a^3*d^6+32*a^2*b*c^2*d^4+9*a*b^2*
c^4*d^2+3*b^3*c^6)+a*c*(6*a^2*B*d^5+3*b^2*c^4*(-3*B*d+C*c)+29*a*b*c^2*d^2*
(-B*d+C*c)))*(-b*x^2+a)^(1/2)/a^2/c^3/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)-A*(d*
x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^3/x+1/3*b^(1/2)*(A*(-15*a^3*d^6+41*a^2*b
*c^2*d^4+6*b^3*c^6)+a*c*(6*a^2*B*d^5+3*b^2*c^4*(-3*B*d+C*c)+29*a*b*c^2*d^2
*(-B*d+C*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*
x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/
a^(3/2)/c^3/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2
+a)^(1/2)-1/3*b^(1/2)*(A*(5*a^2*d^4-3*a*b*c^2*d^2+6*b^2*c^4)+a*c*(3*b*c^2*
(-2*B*d+C*c)+a*d^2*(-2*B*d+5*C*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*
((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(
1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^2/(-a*d^2+b*c^2)^2
/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(-5*A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/
2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)
*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c^3/(d*x+c)^
(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.83 (sec) , antiderivative size = 9484, normalized size of antiderivative = 10.32

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a-bx^2\right)^{3/2}(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\int \frac{C}{d^2x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 638

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$
$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$
$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$
$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$
$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$
$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\cdots\right)dx-\frac{(2cC-Bd)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\cdots\right)dx-\frac{(2cC-Bd)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^2(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\cdots\right.$$
$$\frac{(2cC-Bd)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\cdots\right.$$
$$\frac{(2cC-Bd)\int \left(\frac{d^2}{c^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^2(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(\frac{2d^2}{c^3(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}+\frac{d^2}{c^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}-\frac{2d}{c^3x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}+\right.$$

$$\frac{(2cC-Bd)\int\left(\frac{d^2}{c^2(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d}{c^2x\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^2\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 13.64 (sec) , antiderivative size = 1621, normalized size of antiderivative = 1.76

method	result	size
elliptic	Expression too large to display	1621
risch	Expression too large to display	2485
default	Expression too large to display	15714

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3*d/(a*d^2-b
*c^2)^2*(A*d^2-B*c*d+C*c^2)/c^2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d)
^2-2/3*(-b*d*x^2+a*d)*d^2/(a*d^2-b*c^2)^3*(6*A*a*d^4-16*A*b*c^2*d^2-3*B*a*
c*d^3+13*B*b*c^3*d-10*C*b*c^4)/c^3/((x+c/d)*(-b*d*x^2+a*d))^(1/2)-A/c^3/a^
2/x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2*(-b*d*x-b*c)*(-1/2*b*(3*A*a*b*c*d
^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2
)^3/a^2*x+1/2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3
*C*a*b*c^2*d)/(a*d^2-b*c^2)^3/a)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1/3*d^2
*b*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2/c^2-1/3*b/c^2*d^2*(6*A*a*d^4-16*A*b
*c^2*d^2-3*B*a*c*d^3+13*B*b*c^3*d-10*C*b*c^4)/(a*d^2-b*c^2)^3+b/(a*d^2-b*c
^2)^2*(A*a*b*d^2+A*b^2*c^2-2*B*a*b*c*d+C*a^2*d^2+C*a*b*c^2)/a^2-1/2*b*d/a*
(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/
(a*d^2-b*c^2)^3+b^2*c*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C
*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/a^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(-1/3*b*d^3*(6*A*a*d^4-16
*A*b*c^2*d^2-3*B*a*c*d^3+13*B*b*c^3*d-10*C*b*c^4)/(a*d^2-b*c^2)^3/c^3-1/2*
A*b*d/a^2/c^3+1/2*b^2*d*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b^2*d^3*x^9 + 3
*b^2*c*d^2*x^8 + 3*a^2*c^2*d*x^3 + (3*b^2*c^2*d - 2*a*b*d^3)*x^7 + a^2*c^3
*x^2 + (b^2*c^3 - 6*a*b*c*d^2)*x^6 - (6*a*b*c^2*d - a^2*d^3)*x^5 - (2*a*b*
c^3 - 3*a^2*c*d^2)*x^4), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\left(a-b\,x^2\right)^{3/2}\,\left(c+d\,x\right)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\,(dx+c)^{\frac{5}{2}}\,\left(-b\,x^2+a\right)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.230 $\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1012

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx = \frac{b(Abc+acC-aBd+(bBc-Abd-aCd)x)}{a^2(bc^2-ad^2)(c+dx)^{3/2}\sqrt{a-bx^2}}$$

$$-\frac{d(3b^2c^4(Bc-2Ad)-3abc^3d(2cC-Bd)-2a^2d^3(c^2C-Bcd+Ad^2))\sqrt{a-bx^2}}{3a^2c^3(bc^2-ad^2)^2(c+dx)^{3/2}}$$

$$-\frac{d(3b^3c^6(Bc-3Ad)-3ab^2c^4d(3c^2C-3Bcd+Ad^2)+6a^3d^5(c^2C-2Bcd+3Ad^2)-a^2bc^2d^3(29c^2C-3}{3a^2c^4(bc^2-ad^2)^3\sqrt{c+dx}}$$

$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2a^2c^3x^2}-\frac{(4Bc-11Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4a^2c^4x}$$

$$+\frac{\sqrt{b}(3b^3c^6(8Bc-23Ad)+3a^3d^5(8c^2C-20Bcd+35Ad^2)-a^2bc^2d^3(116c^2C-164Bcd+251Ad^2)-ab^2(}{12a^{3/2}c^4(bc^2-ad^2)^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-b}}$$

$$-\frac{\sqrt{b}(3b^2c^4(8Bc-17Ad)-6abc^2d(4c^2C+2Bcd-9Ad^2)-a^2d^3(8c^2C-20Bcd+35Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-b}{a}}}{12a^{3/2}c^3(bc^2-ad^2)^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(12Abc^2+8ac^2C-20aBcd+35aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^2c^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
b*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a^2/(-a*d^2+b*c^2)/(d*x+c)^(3
/2)/(-b*x^2+a)^(1/2)-1/3*d*(3*b^2*c^4*(-2*A*d+B*c)-3*a*b*c^3*d*(-B*d+2*C*c
)-2*a^2*d^3*(A*d^2-B*c*d+C*c^2))*(-b*x^2+a)^(1/2)/a^2/c^3/(-a*d^2+b*c^2)^2
/(d*x+c)^(3/2)-1/3*d*(3*b^3*c^6*(-3*A*d+B*c)-3*a*b^2*c^4*d*(A*d^2-3*B*c*d+
3*C*c^2)+6*a^3*d^5*(3*A*d^2-2*B*c*d+C*c^2)-a^2*b*c^2*d^3*(38*A*d^2-32*B*c*
d+29*C*c^2))*(-b*x^2+a)^(1/2)/a^2/c^4/(-a*d^2+b*c^2)^3/(d*x+c)^(1/2)-1/2*A
*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^3/x^2-1/4*(-11*A*d+4*B*c)*(d*x+c)^(1
/2)*(-b*x^2+a)^(1/2)/a^2/c^4/x+1/12*b^(1/2)*(3*b^3*c^6*(-23*A*d+8*B*c)+3*a
^3*d^5*(35*A*d^2-20*B*c*d+8*C*c^2)-a^2*b*c^2*d^3*(251*A*d^2-164*B*c*d+116*
C*c^2)-a*b^2*(-87*A*c^4*d^3+36*C*c^6*d))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2
)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^
(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^4/(-a*d^2+b*c^2)^3/((d*x+c)/(c+a^(1/2
)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/12*b^(1/2)*(3*b^2*c^4*(-17*A*d+8*B*
c)-6*a*b*c^2*d*(-9*A*d^2+2*B*c*d+4*C*c^2)-a^2*d^3*(35*A*d^2-20*B*c*d+8*C*c
^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/a^(3/2)/c^3/(-a*d^2+b*c^2)^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
-1/4*(35*A*a*d^2+12*A*b*c^2-20*B*a*c*d+8*C*a*c^2)*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^2/c^4/...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.56 (sec) , antiderivative size = 12354, normalized size of antiderivative = 12.21

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3 (a-bx^2)^{3/2} (c+dx)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2} (a-bx^2)^{3/2}}dx+\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3(c+dx)^{3/2} (a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2} (a-bx^2)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2} (a-bx^2)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx} (a-bx^2)^{3/2}}-\right.$$
$$\int \frac{\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}}{x^3(c+dx)^{3/2} (a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2} (a-bx^2)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2} (a-bx^2)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx} (a-bx^2)^{3/2}}-\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\int \frac{C}{d^2x^3\sqrt{c+dx} (a-bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2} (a-bx^2)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2} (a-bx^2)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx} (a-bx^2)^{3/2}}-\right.$$
$$\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{637}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}(a-bx^2)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{(2cC-Bd)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2}(a-bx^2)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}(a-bx^2)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{(2cC-Bd)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 7239$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 637$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 7239$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int\frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 637$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 7239$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$
$$\frac{(2cC-Bd)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$
$$\frac{(2cC-Bd)\int \left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}dx-\frac{(2cC-Bd)\int \frac{1}{x^3(c+dx)^{3/2}(a-bx^2)^{3/2}}dx}{d^2}+\frac{C\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

$\downarrow$ 637

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\left(-\frac{3d^3}{c^4(c+dx)^{3/2}\left(a-bx^2\right)^{3/2}}-\frac{d^3}{c^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}+\frac{3d^2}{c^4x\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}-\right.$$

$$\frac{(2cC-Bd)\int\left(-\frac{d^3}{c^3(c+dx)^{3/2}(a-bx^2)^{3/2}}+\frac{d^2}{c^3x\sqrt{c+dx}(a-bx^2)^{3/2}}-\frac{d}{c^2x^2\sqrt{c+dx}(a-bx^2)^{3/2}}+\frac{1}{cx^3\sqrt{c+dx}(a-bx^2)^{3/2}}\right)dx}{d^2}+$$

$$\frac{C\int\frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{d^2}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(c + d*x)^(5/2)*(a - b*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 637
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> Int[ExpandIntegrand[(a + b*x^2)^p/Sqrt[c + d*x], x^m*(c + d*x)^(n + 1
/2), x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[p + 1/2] && IntegerQ[n
 + 1/2] && IntegerQ[m]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 16.14 (sec) , antiderivative size = 1732, normalized size of antiderivative = 1.71

method	result	size
elliptic	Expression too large to display	1732
risch	Expression too large to display	2916
default	Expression too large to display	21237

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2/3*d^2/(a*d^2-
b*c^2)^2*(A*d^2-B*c*d+C*c^2)/c^3*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x+c/d
)^2+2/3*(-b*d*x^2+a*d)*d^3/(a*d^2-b*c^2)^3*(9*A*a*d^4-19*A*b*c^2*d^2-6*B*a
*c*d^3+16*B*b*c^3*d+3*C*a*c^2*d^2-13*C*b*c^4)/c^4/((x+c/d)*(-b*d*x^2+a*d))
^(1/2)-2*(-b*d*x-b*c)*(1/2/a^2*b*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*a*b*c*d^2-B*
b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3*x-1/2*b*(3*A*a*b*c*d^2+A*
b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/a
^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-1/2*A/c^3/a^2/x^2*(-b*d*x^3-b*c*x^2+a*d
*x+a*c)^(1/2)+1/4/a^2*(11*A*d-4*B*c)/c^4*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2
)/x+2*(-1/3*b*d^3*(A*d^2-B*c*d+C*c^2)/(a*d^2-b*c^2)^2/c^3+1/3/c^3*d^3*b*(9
*A*a*d^4-19*A*b*c^2*d^2-6*B*a*c*d^3+16*B*b*c^3*d+3*C*a*c^2*d^2-13*C*b*c^4)
/(a*d^2-b*c^2)^3-b^2/a^2*(2*A*b*c*d-B*a*d^2-B*b*c^2+2*C*a*c*d)/(a*d^2-b*c^
2)^2+1/2*b^2*d*(3*A*a*b*c*d^2+A*b^2*c^3-B*a^2*d^3-3*B*a*b*c^2*d+3*C*a^2*c*
d^2+C*a*b*c^3)/(a*d^2-b*c^2)^3/a^2-b^2*c/a^2*(A*a*b*d^3+3*A*b^2*c^2*d-3*B*
a*b*c*d^2-B*b^2*c^3+C*a^2*d^3+3*C*a*b*c^2*d)/(a*d^2-b*c^2)^3+1/4*A*b*d/a^2
/c^3)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*
(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2))+2*(1/3*b*d^4*(9*A*a*d^4-19*A*b*c^2*d^2-6*B*a*c*d^3+16*B*b*c^3...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(5/2)/(-b*x**2+a)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}\left(a-bx^2\right)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{\left(-bx^2+a\right)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(3/2)*(d*x + c)^(5/2)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^3\,(a-b\,x^2)^{3/2}\,(c+d\,x)^{5/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(3/2)*(c + d*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3(c+dx)^{5/2}(a-bx^2)^{3/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^3\,(dx+c)^{\frac{5}{2}}\,(-b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(5/2)/(-b*x^2+a)^(3/2),x)
```

3.231 $\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 748

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{a^2\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{3b^3\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}$$

$$-\frac{a\sqrt{c+dx}(4Abc(3bc^2-2ad^2)+a(bc^2(18cC-17Bd)-ad^2(14cC-13Bd))+(15a^2Cd^3+b^2c^2(14Bc-}{6b^3\left(bc^2-ad^2\right)^2\sqrt{a-bx^2}}$$

$$+\frac{2(7cC-5Bd)\sqrt{c+dx}\sqrt{a-bx^2}}{15b^3d^2}-\frac{2C(c+dx)^{3/2}\sqrt{a-bx^2}}{5b^3d^2}$$

$$-\frac{\sqrt{a}(231a^3Cd^6+ab^2c^2d^2(92c^2C+150Bcd-185Ad^2)-15a^2bd^4(25c^2C+6Bcd-7Ad^2)+4b^3c^4(8c^2C-}{30b^{7/2}d^3\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-b}}$$

$$-\frac{\sqrt{a}(3a^2d^4(51cC-25Bd)-abcd^2(116c^2C-30Bcd-65Ad^2)-4b^2c^3(8c^2C-10Bcd+15Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}}{\sqrt{b}}}}}{30b^{7/2}d^3\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*a^2*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/b^3/(-a*d
^2+b*c^2)/(-b*x^2+a)^(3/2)-1/6*a*(d*x+c)^(1/2)*(4*A*b*c*(-2*a*d^2+3*b*c^2)
+a*(b*c^2*(-17*B*d+18*C*c)-a*d^2*(-13*B*d+14*C*c))+(15*a^2*C*d^3+b^2*c^2*(
-13*A*d+14*B*c)-a*b*d*(-9*A*d^2+10*B*c*d+19*C*c^2))*x)/b^3/(-a*d^2+b*c^2)^
2/(-b*x^2+a)^(1/2)+2/15*(-5*B*d+7*C*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^3/
d^2-2/5*C*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^3/d^2-1/30*a^(1/2)*(231*a^3*C*d
^6+a*b^2*c^2*d^2*(-185*A*d^2+150*B*c*d+92*C*c^2)-15*a^2*b*d^4*(-7*A*d^2+6*
B*c*d+25*C*c^2)+4*b^3*c^4*(15*A*d^2-10*B*c*d+8*C*c^2))*(d*x+c)^(1/2)*((-b*
x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*
(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(7/2)/d^3/(-a*d^2+b*c^2)^2/((d*
x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/30*a^(1/2)*(3*a^2*d^4
*(-25*B*d+51*C*c)-a*b*c*d^2*(-65*A*d^2-30*B*c*d+116*C*c^2)-4*b^2*c^3*(15*A
*d^2-10*B*c*d+8*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2
)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(7/2)/d^3/(-a*d^2+b*c^2)/(d*x+c)^(1/2)
/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 33.28 (sec) , antiderivative size = 1155, normalized size of antiderivative = 1.54

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^5*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*(-4*c*C + 5*B*d))/(15*b^3*d^2) - (2*C*x
)/(5*b^3*d) + (a^2*A*b*c + a^3*c*C - a^3*B*d + a^2*b*B*c*x - a^2*A*b*d*x -
 a^3*C*d*x)/(3*b^3*(b*c^2 - a*d^2)*(-a + b*x^2)^2) + (12*a*A*b^2*c^3 + 18*
a^2*b*c^3*C - 17*a^2*b*B*c^2*d - 8*a^2*A*b*c*d^2 - 14*a^3*c*C*d^2 + 13*a^3
*B*d^3 + 14*a*b^2*B*c^3*x - 13*a*A*b^2*c^2*d*x - 19*a^2*b*c^2*C*d*x - 10*a
^2*b*B*c*d^2*x + 9*a^2*A*b*d^3*x + 15*a^3*C*d^3*x)/(6*b^3*(b*c^2 - a*d^2)^
2*(-a + b*x^2))) + (Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-(
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(231*a^3*C*d^6 + a*b^2*c^2*d^2*(92*c^2*C +
150*B*c*d - 185*A*d^2) + 15*a^2*b*d^4*(-25*c^2*C - 6*B*c*d + 7*A*d^2) + 4*
b^3*c^4*(8*c^2*C - 10*B*c*d + 15*A*d^2))*(-((a*d^2)/(c + d*x)^2) + b*(-1 +
 c/(c + d*x))^2)) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(231*a^3*C*d^6 + a*
b^2*c^2*d^2*(92*c^2*C + 150*B*c*d - 185*A*d^2) + 15*a^2*b*d^4*(-25*c^2*C -
 6*B*c*d + 7*A*d^2) + 4*b^3*c^4*(8*c^2*C - 10*B*c*d + 15*A*d^2))*Sqrt[1 -
c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqr
t[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sq
rt[c + d*x] + (I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(-231*a^(5/2)*C
*d^5 + 3*a^2*Sqrt[b]*d^4*(-51*c*C + 25*B*d) + a*b^(3/2)*c*d^2*(116*c^2*C -
 30*B*c*d - 65*A*d^2) + 3*a^(3/2)*b*d^3*(74*c^2*C + 55*B*c*d - 35*A*d^2) +
 12*Sqrt[a]*b^2*c^2*d*(2*c^2*C - 15*B*c*d + 10*A*d^2) + 4*b^(5/2)*c^3*(...
```

Rubi [A] (verified)

Time = 4.95 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2180, 27, 2180, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx+Cx^2)}{(a-bx^2)^{5/2} \sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^5+6aB\left(c^2-\frac{ad^2}{b}\right)x^4+\frac{6a(Ab+aC)\left(bc^2-ad^2\right)x^3}{b^2}+\frac{6a^2B\left(bc^2-ad^2\right)x^2}{b^2}+\frac{3a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^3}+\frac{a^3(bc(2B}{}}{2\sqrt{c+dx}(a-bx^2)^{3/2}}}{3a\left(bc^2-ad^2\right)}$$

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 27

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\int \frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^5+6aB\left(c^2-\frac{ad^2}{b}\right)x^4+\frac{6a(Ab+aC)\left(bc^2-ad^2\right)x^3}{b^2}+\frac{6a^2B\left(bc^2-ad^2\right)x^2}{b^2}+\frac{3a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^3}+\frac{a^3(bc(2Bc-}{}}{\sqrt{c+dx}(a-bx^2)^{3/2}}}{6a\left(bc^2-ad^2\right)}$$

$\downarrow$ 2180

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\int -\frac{\frac{\left(12b^2(2Bc-Ad)c^3-abd\left(18Cc^2+31Bdc-8Ad^2\right)c+a^2d^3(14cC+11Bd)\right)a^3}{b^3}+\frac{12C\left(bc^2-ad^2\right)^2x^3a^2}{b^2}+\frac{12B\left(bc^2-ad^2\right)^2x^2a^2}{b^2}+\frac{\left(Ab\left(12b^2c^4-37abd^2c^2+21a^2d\right.\right.}{}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{a(bc^2-ad^2)}$$

$\downarrow$ 27

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int}{}$$

$\downarrow$ 2185

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 27

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\big\downarrow$ 2185

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\big\downarrow$ 27

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\big\downarrow$ 600

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\big\downarrow$ 509

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\Big\downarrow$ 508

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\Big\downarrow$ 327

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\Big\downarrow$ 512

$$\frac{a^2\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(x\left(15a^2Cd^3-abd\left(-9Ad^2+10Bcd+19c^2C\right)+b^2c^2(14Bc-13Ad)\right)+4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-}{}$$

$\Big\downarrow$ 511

$$\frac{a^2\sqrt{c+dx}(Abc+aCc-aBd+(bBc-Abd-aCd)x)}{3b^3\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}-$$

$$d$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)+\left(15a^2Cd^3-ab\left(19Cc^2+10Bdc-9Ad^2\right)d+b^2c^2(14Bc-13Ad)\right)x\right)}{b^3\left(bc^2-ad^2\right)\sqrt{a-bx^2}}-}{}$$

$$\Big\downarrow \ 321$$

$$\frac{a^2\sqrt{c+dx}(Abc+aCc-aBd+(bBc-Abd-aCd)x)}{3b^3\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}-$$

$$d$$

$$\frac{\frac{a^2\sqrt{c+dx}\left(4Abc\left(3bc^2-2ad^2\right)+a\left(bc^2(18cC-17Bd)-ad^2(14cC-13Bd)\right)+\left(15a^2Cd^3-ab\left(19Cc^2+10Bdc-9Ad^2\right)d+b^2c^2(14Bc-13Ad)\right)x\right)}{b^3\left(bc^2-ad^2\right)\sqrt{a-bx^2}}-}{}$$

input
```
Int[(x^5*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(a^2*Sqrt[c + d*x]*(A*b*c + a*c*C - a*B*d + (b*B*c - A*b*d - a*C*d)*x))/(3
*b^3*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) - ((a^2*Sqrt[c + d*x]*(4*A*b*c*(3*
b*c^2 - 2*a*d^2) + a*(b*c^2*(18*c*C - 17*B*d) - a*d^2*(14*c*C - 13*B*d)) +
 (15*a^2*C*d^3 + b^2*c^2*(14*B*c - 13*A*d) - a*b*d*(19*c^2*C + 10*B*c*d -
9*A*d^2))*x))/(b^3*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-24*a^2*C*(b*c^2 -
 a*d^2)^2*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b^3*d^2) + ((8*a^2*d*(7*c*C
- 5*B*d)*(b*c^2 - a*d^2)^2*Sqrt[c + d*x]*Sqrt[a - b*x^2])/b^2 + (a^2*d*((-
2*Sqrt[a]*(231*a^3*C*d^6 + a*b^2*c^2*d^2*(92*c^2*C + 150*B*c*d - 185*A*d^2
) - 15*a^2*b*d^4*(25*c^2*C + 6*B*c*d - 7*A*d^2) + 4*b^3*c^4*(8*c^2*C - 10*
B*c*d + 15*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt
[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqr
t[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
- (2*Sqrt[a]*(b*c^2 - a*d^2)*(3*a^2*d^4*(51*c*C - 25*B*d) - a*b*c*d^2*(116
*c^2*C - 30*B*c*d - 65*A*d^2) - 4*b^2*c^3*(8*c^2*C - 10*B*c*d + 15*A*d^2))
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/S
qrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/b^2)/(5*b*d^3))/
(2*a*(b*c^2 - a*d^2)))/(6*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 15.64 (sec) , antiderivative size = 1273, normalized size of antiderivative = 1.70

method	result	size
elliptic	Expression too large to display	1273
risch	Expression too large to display	2218
default	Expression too large to display	10909

input
```
int(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((1/3*a^2*(A*b*d
-B*b*c+C*a*d)/(a*d^2-b*c^2)/b^5*x-1/3*a^2*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2
)/b^5)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/1
2*a*(9*A*a*b*d^3-13*A*b^2*c^2*d-10*B*a*b*c*d^2+14*B*b^2*c^3+15*C*a^2*d^3-1
9*C*a*b*c^2*d)/b^4/(a*d^2-b*c^2)^2*x+1/12*a*(8*A*a*b*c*d^2-12*A*b^2*c^3-13
*B*a^2*d^3+17*B*a*b*c^2*d+14*C*a^2*c*d^2-18*C*a*b*c^3)/b^4/(a*d^2-b*c^2)^2
)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)-2/5*C/b^3/d*x*(-b*d*x^3-b*c*x^2+a*d*x+a*c
)^(1/2)-2/3*(B/b^2-4/5*C/b^2/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2
*(2*B*a/b^3-1/6*a*(A*b*c*d+13*B*a*d^2-14*B*b*c^2+C*a*c*d)/b^3/(a*d^2-b*c^2
)-1/12/b^3*d*a*(8*A*a*b*c*d^2-12*A*b^2*c^3-13*B*a^2*d^3+17*B*a*b*c^2*d+14*
C*a^2*c*d^2-18*C*a*b*c^3)/(a*d^2-b*c^2)^2+1/6/b^3*c*a*(9*A*a*b*d^3-13*A*b^
2*c^2*d-10*B*a*b*c*d^2+14*B*b^2*c^3+15*C*a^2*d^3-19*C*a*b*c^2*d)/(a*d^2-b*
c^2)^2+2/5*C/b^3/d*a*c+1/3*(B/b^2-4/5*C/b^2/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2)
)*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*((A*b+2*C*a)/b
^3+1/12*d*a*(9*A*a*b*d^3-13*A*b^2*c^2*d-10*B*a*b*c*d^2+14*B*b^2*c^3+15*C*a
^2*d^3-19*C*a*b*c^2*d)/b^3/(a*d^2-b*c^2)^2+3/5*C/b^3*a-2/3*(B/b^2-4/5*C/b^
2/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1621 vs. $2(674) = 1348$.

Time = 0.15 (sec) , antiderivative size = 1621, normalized size of antiderivative = 2.17

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output
```
1/90*((32*C*a^2*b^3*c^7 - 40*B*a^2*b^3*c^6*d - 270*B*a^3*b^2*c^4*d^3 + 495
*B*a^4*b*c^2*d^5 - 225*B*a^5*d^7 + 4*(17*C*a^3*b^2 + 15*A*a^2*b^3)*c^5*d^2
 - (57*C*a^4*b + 5*A*a^3*b^2)*c^3*d^4 - 3*(C*a^5 + 5*A*a^4*b)*c*d^6 + (32*
C*b^5*c^7 - 40*B*b^5*c^6*d - 270*B*a*b^4*c^4*d^3 + 495*B*a^2*b^3*c^2*d^5 -
 225*B*a^3*b^2*d^7 + 4*(17*C*a*b^4 + 15*A*b^5)*c^5*d^2 - (57*C*a^2*b^3 + 5
*A*a*b^4)*c^3*d^4 - 3*(C*a^3*b^2 + 5*A*a^2*b^3)*c*d^6)*x^4 - 2*(32*C*a*b^4
*c^7 - 40*B*a*b^4*c^6*d - 270*B*a^2*b^3*c^4*d^3 + 495*B*a^3*b^2*c^2*d^5 -
225*B*a^4*b*d^7 + 4*(17*C*a^2*b^3 + 15*A*a*b^4)*c^5*d^2 - (57*C*a^3*b^2 +
5*A*a^2*b^3)*c^3*d^4 - 3*(C*a^4*b + 5*A*a^3*b^2)*c*d^6)*x^2)*sqrt(-b*d)*we
ierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)
/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(32*C*a^2*b^3*c^6*d - 40*B*a^2*b^3*c^5*d^
2 + 150*B*a^3*b^2*c^3*d^4 - 90*B*a^4*b*c*d^6 + 4*(23*C*a^3*b^2 + 15*A*a^2*
b^3)*c^4*d^3 - 5*(75*C*a^4*b + 37*A*a^3*b^2)*c^2*d^5 + 21*(11*C*a^5 + 5*A*
a^4*b)*d^7 + (32*C*b^5*c^6*d - 40*B*b^5*c^5*d^2 + 150*B*a*b^4*c^3*d^4 - 90
*B*a^2*b^3*c*d^6 + 4*(23*C*a*b^4 + 15*A*b^5)*c^4*d^3 - 5*(75*C*a^2*b^3 + 3
7*A*a*b^4)*c^2*d^5 + 21*(11*C*a^3*b^2 + 5*A*a^2*b^3)*d^7)*x^4 - 2*(32*C*a*
b^4*c^6*d - 40*B*a*b^4*c^5*d^2 + 150*B*a^2*b^3*c^3*d^4 - 90*B*a^3*b^2*c*d^
6 + 4*(23*C*a^2*b^3 + 15*A*a*b^4)*c^4*d^3 - 5*(75*C*a^3*b^2 + 37*A*a^2*b^3
)*c^2*d^5 + 21*(11*C*a^4*b + 5*A*a^3*b^2)*d^7)*x^2)*sqrt(-b*d)*weierstrass
Zeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^5}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^5/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^5}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^5/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^5\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^5*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^5*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^5(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^5(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int(x^5*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.232 $\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 681

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{a\sqrt{c+dx}(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{3b^3(bc^2-ad^2)(a-bx^2)^{3/2}}$$
$$-\frac{\sqrt{c+dx}(a(13a^2Cd^3+b^2c^2(12Bc-11Ad)-abd(17c^2C+8Bcd-7Ad^2))+b(4Abc(2bc^2-ad^2)+a(bc}{6b^3(bc^2-ad^2)^2\sqrt{a-bx^2}}$$
$$-\frac{2C\sqrt{c+dx}\sqrt{a-bx^2}}{3b^3d}$$
$$+\frac{\sqrt{a}(3a^2d^4(6cC-7Bd)-abcd^2(30c^2C-37Bcd-4Ad^2)+4b^2c^3(2c^2C-3Bcd-2Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx}{a}}}{6b^{5/2}d^2(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{\sqrt{a}(15a^2Cd^4-abd^2(6c^2C+13Bcd-5Ad^2)-4b^2c^2(2c^2C-3Bcd+Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\Bigg(a}{6b^{7/2}d^2(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*a*(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^3/(
-a*d^2+b*c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(a*(13*a^2*C*d^3+b^2*c^2*
(-11*A*d+12*B*c)-a*b*d*(-7*A*d^2+8*B*c*d+17*C*c^2))+b*(4*A*b*c*(-a*d^2+2*b
*c^2)+a*(b*c^2*(-13*B*d+14*C*c)-a*d^2*(-9*B*d+10*C*c)))*x)/b^3/(-a*d^2+b*c
^2)^2/(-b*x^2+a)^(1/2)-2/3*C*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^3/d+1/6*a^(1
/2)*(3*a^2*d^4*(-7*B*d+6*C*c)-a*b*c*d^2*(-4*A*d^2-37*B*c*d+30*C*c^2)+4*b^2
*c^3*(-2*A*d^2-3*B*c*d+2*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipt
icE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+
a^(1/2)*d))^(1/2))/b^(5/2)/d^2/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1
/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/6*a^(1/2)*(15*a^2*C*d^4-a*b*d^2*(-5*A*d^2+1
3*B*c*d+6*C*c^2)-4*b^2*c^2*(A*d^2-3*B*c*d+2*C*c^2))*((d*x+c)/(c+a^(1/2)*d/
b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(7/2)/d^2/
(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.21 (sec) , antiderivative size = 1061, normalized size of antiderivative = 1.56

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*((-2*C)/(3*b^3*d) + (a^2*b*B*c - a^2*A*b*d -
 a^3*C*d + a*A*b^2*c*x + a^2*b*c*C*x - a^2*b*B*d*x)/(3*b^3*(b*c^2 - a*d^2)
*(-a + b*x^2)^2) + (12*a*b^2*B*c^3 - 11*a*A*b^2*c^2*d - 17*a^2*b*c^2*C*d -
 8*a^2*b*B*c*d^2 + 7*a^2*A*b*d^3 + 13*a^3*C*d^3 + 8*A*b^3*c^3*x + 14*a*b^2
*c^3*C*x - 13*a*b^2*B*c^2*d*x - 4*a*A*b^2*c*d^2*x - 10*a^2*b*c*C*d^2*x + 9
*a^2*b*B*d^3*x)/(6*b^3*(b*c^2 - a*d^2)^2*(-a + b*x^2))) + (Sqrt[a - (b*(c
+ d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2
*d^4*(6*c*C - 7*B*d) + 4*b^2*c^3*(2*c^2*C - 3*B*c*d - 2*A*d^2) + a*b*c*d^2
*(-30*c^2*C + 37*B*c*d + 4*A*d^2))*(-((a*d^2)/(c + d*x)^2) + b*(-1 + c/(c
+ d*x))^2) - (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*(6*c*C - 7*B*d)
 + 4*b^2*c^3*(2*c^2*C - 3*B*c*d - 2*A*d^2) + a*b*c*d^2*(-30*c^2*C + 37*B*c
*d + 4*A*d^2))*Sqrt[1 - c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqr
t[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticE[I*ArcSinh[S
qrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqr
t[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] - (I*d*(Sqrt[b]*c - Sqrt[a]*d)*(12*A*b
^(5/2)*c^3*d - 15*a^(5/2)*C*d^4 + 3*a^2*Sqrt[b]*d^3*(-11*c*C + 7*B*d) + a^
(3/2)*b*d^2*(6*c^2*C + 13*B*c*d - 5*A*d^2) + 3*a*b^(3/2)*c*d*(12*c^2*C - 8
*B*c*d - 3*A*d^2) + 4*Sqrt[a]*b^2*c^2*(2*c^2*C - 3*B*c*d + A*d^2))*Sqrt[1
- c/(c + d*x) - (Sqrt[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (S
qrt[a]*d)/(Sqrt[b]*(c + d*x))]*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d...
```

Rubi [A] (verified)

Time = 3.72 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2180, 27, 2180, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2)}{(a-bx^2)^{5/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 2180$$

$$\frac{\int -\frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^4+6aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{6a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{3a^2(bc(2Bc-Ad)-ad(cC+Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^3}}{2\sqrt{c+dx}(a-bx^2)^{3/2}}d}{3a\left(bc^2-ad^2\right)}$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\int \frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^4+6aB\left(c^2-\frac{ad^2}{b}\right)x^3+\frac{6a(Ab+aC)\left(bc^2-ad^2\right)x^2}{b^2}+\frac{3a^2(bc(2Bc-Ad)-ad(cC+Bd))x}{b^2}+\frac{a^2\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^3}}{\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{6a\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 2180$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\int -\frac{\frac{12C\left(bc^2-ad^2\right)^2x^2a^2}{b^2}+\frac{\left(Ab\left(12b^2c^4-13abd^2c^2+5a^2d^4\right)+a\left(11a^2Cd^4-abc(31cC-8Bd)d^2+12b^2c^3(2cC-Bd)\right)\right)a^2}{b^3}+\frac{\left(4b^2(3Bc+2Ad)c^3+abd\left(14Cc^2-37Bd\right.\right.}{b^2}}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{a(bc^2-ad^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd(-7Ad^2+8Bcd+17c^2C)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc(2bc^2-ad^2)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}(bc^2-ad^2)}-\int}{}$$

$$\Big\downarrow \; 2185$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd(-7Ad^2+8Bcd+17c^2C)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc(2bc^2-ad^2)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}(bc^2-ad^2)}-}{}$$

$$\Big\downarrow \; 27$$

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 600

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 509

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 508

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 327

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 512

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 511

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

$\downarrow$ 321

$$\frac{a\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^3\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{a\sqrt{c+dx}\left(a\left(13a^2Cd^3-abd\left(-7Ad^2+8Bcd+17c^2C\right)+b^2c^2(12Bc-11Ad)\right)+bx\left(4Abc\left(2bc^2-ad^2\right)+a\left(bc^2(14cC-13Bd)-ad^2(10cC-9Bd)\right)\right)\right)}{b^3\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(a*Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x)
)/(3*b^3*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) - ((a*Sqrt[c + d*x]*(a*(13*a^2
*C*d^3 + b^2*c^2*(12*B*c - 11*A*d) - a*b*d*(17*c^2*C + 8*B*c*d - 7*A*d^2))
 + b*(4*A*b*c*(2*b*c^2 - a*d^2) + a*(b*c^2*(14*c*C - 13*B*d) - a*d^2*(10*c
*C - 9*B*d)))*x))/(b^3*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-8*a^2*C*(b*c^
2 - a*d^2)^2*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(b^3*d) + (a^2*((2*Sqrt[a]*Sqr
t[b]*(3*a^2*d^4*(6*c*C - 7*B*d) - a*b*c*d^2*(30*c^2*C - 37*B*c*d - 4*A*d^2
) + 4*b^2*c^3*(2*c^2*C - 3*B*c*d - 2*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2
)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt
[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]
*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(15*a^2*C*d^4 - a*b*d^2*(6*
c^2*C + 13*B*c*d - 5*A*d^2) - 4*b^2*c^2*(2*c^2*C - 3*B*c*d + A*d^2))*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF
[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a]
 + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(b^3*d))/(2*a*(b*c^2 -
 a*d^2)))/(6*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 14.95 (sec) , antiderivative size = 1187, normalized size of antiderivative = 1.74

method	result	size
elliptic	Expression too large to display	1187
risch	Expression too large to display	2483
default	Expression too large to display	9545

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((-1/3*a*(A*b*c-
B*a*d+C*a*c)/b^4/(a*d^2-b*c^2)*x+1/3*a^2*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)
/b^5)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12/
b^3*(4*A*a*b*c*d^2-8*A*b^2*c^3-9*B*a^2*d^3+13*B*a*b*c^2*d+10*C*a^2*c*d^2-1
4*C*a*b*c^3)/(a*d^2-b*c^2)^2*x-1/12*a*(7*A*a*b*d^3-11*A*b^2*c^2*d-8*B*a*b*
c*d^2+12*B*b^2*c^3+13*C*a^2*d^3-17*C*a*b*c^2*d)/(a*d^2-b*c^2)^2/b^4)/((x^2
-a/b)*(-b*d*x-b*c))^(1/2)-2/3*C/b^3/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2
*((A*b+2*C*a)/b^3-1/6/b^3/(a*d^2-b*c^2)*(7*A*a*b*d^2-8*A*b^2*c^2+B*a*b*c*d
+13*C*a^2*d^2-14*C*a*b*c^2)+1/12/b^3*d*a*(7*A*a*b*d^3-11*A*b^2*c^2*d-8*B*a
*b*c*d^2+12*B*b^2*c^3+13*C*a^2*d^3-17*C*a*b*c^2*d)/(a*d^2-b*c^2)^2-1/6/b^2
*c*(4*A*a*b*c*d^2-8*A*b^2*c^3-9*B*a^2*d^3+13*B*a*b*c^2*d+10*C*a^2*c*d^2-14
*C*a*b*c^3)/(a*d^2-b*c^2)^2+1/3*C/b^3*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b
*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B/b^2-1/12*d*(4*A*a*b*c*d^
2-8*A*b^2*c^3-9*B*a^2*d^3+13*B*a*b*c^2*d+10*C*a^2*c*d^2-14*C*a*b*c^3)/b^2/
(a*d^2-b*c^2)^2-2/3*C/b^2/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*
b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/
b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1357 vs. $2(613) = 1226$.

Time = 0.13 (sec) , antiderivative size = 1357, normalized size of antiderivative = 1.99

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output
```
-1/18*((8*C*a^2*b^3*c^6 - 12*B*a^2*b^3*c^5*d + B*a^3*b^2*c^3*d^3 + 3*B*a^4
*b*c*d^5 + 2*(27*C*a^3*b^2 + 14*A*a^2*b^3)*c^4*d^2 - (99*C*a^4*b + 35*A*a^
3*b^2)*c^2*d^4 + 15*(3*C*a^5 + A*a^4*b)*d^6 + (8*C*b^5*c^6 - 12*B*b^5*c^5*
d + B*a*b^4*c^3*d^3 + 3*B*a^2*b^3*c*d^5 + 2*(27*C*a*b^4 + 14*A*b^5)*c^4*d^
2 - (99*C*a^2*b^3 + 35*A*a*b^4)*c^2*d^4 + 15*(3*C*a^3*b^2 + A*a^2*b^3)*d^6
)*x^4 - 2*(8*C*a*b^4*c^6 - 12*B*a*b^4*c^5*d + B*a^2*b^3*c^3*d^3 + 3*B*a^3*
b^2*c*d^5 + 2*(27*C*a^2*b^3 + 14*A*a*b^4)*c^4*d^2 - (99*C*a^3*b^2 + 35*A*a
^2*b^3)*c^2*d^4 + 15*(3*C*a^4*b + A*a^3*b^2)*d^6)*x^2)*sqrt(-b*d)*weierstr
assPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^
3), 1/3*(3*d*x + c)/d) + 3*(8*C*a^2*b^3*c^5*d - 12*B*a^2*b^3*c^4*d^2 + 37*
B*a^3*b^2*c^2*d^4 - 21*B*a^4*b*d^6 - 2*(15*C*a^3*b^2 + 4*A*a^2*b^3)*c^3*d^
3 + 2*(9*C*a^4*b + 2*A*a^3*b^2)*c*d^5 + (8*C*b^5*c^5*d - 12*B*b^5*c^4*d^2
+ 37*B*a*b^4*c^2*d^4 - 21*B*a^2*b^3*d^6 - 2*(15*C*a*b^4 + 4*A*b^5)*c^3*d^3
 + 2*(9*C*a^2*b^3 + 2*A*a*b^4)*c*d^5)*x^4 - 2*(8*C*a*b^4*c^5*d - 12*B*a*b^
4*c^4*d^2 + 37*B*a^2*b^3*c^2*d^4 - 21*B*a^3*b^2*d^6 - 2*(15*C*a^2*b^3 + 4*
A*a*b^4)*c^3*d^3 + 2*(9*C*a^3*b^2 + 2*A*a^2*b^3)*c*d^5)*x^2)*sqrt(-b*d)*we
ierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*
d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*
a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(4*C*a^2*b^3*c^4*d^2 + 10*B*a^2*
b^3*c^3*d^3 - 6*B*a^3*b^2*c*d^5 - (23*C*a^3*b^2 + 9*A*a^2*b^3)*c^2*d^4 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{(Cx^2+Bx+A)x^4}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^4/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{x^4\,(C\,x^2+B\,x+A)}{{(a-b\,x^2)}^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^4*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^4(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^4(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int(x^4*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.233 $\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 608

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{a\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{3b^2\,(bc^2-ad^2)\,(a-bx^2)^{3/2}}$$

$$-\frac{\sqrt{c+dx}(2Abc(3bc^2-ad^2)+a(bc^2(12cC-11Bd)-ad^2(8cC-7Bd))+(9a^2Cd^3+b^2c^2(8Bc-7Ad)-}{6b^2\,(bc^2-ad^2)^2\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{a}(21a^2Cd^4+b^2c^2(12c^2C+8Bcd-7Ad^2)-abd^2(37c^2C+4Bcd-3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\right.}{6b^{5/2}d\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{\sqrt{a}(ad^2(13cC-5Bd)-bc(12c^2C-4Bcd-Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{c}}{\sqrt{bc+}}\right.}{6b^{5/2}d\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*a*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/b^2/(-a*d^2
+b*c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(2*A*b*c*(-a*d^2+3*b*c^2)+a*(b*
c^2*(-11*B*d+12*C*c)-a*d^2*(-7*B*d+8*C*c))+(9*a^2*C*d^3+b^2*c^2*(-7*A*d+8*
B*c)-a*b*d*(-3*A*d^2+4*B*c*d+13*C*c^2))*x)/b^2/(-a*d^2+b*c^2)^2/(-b*x^2+a)
^(1/2)-1/6*a^(1/2)*(21*a^2*C*d^4+b^2*c^2*(-7*A*d^2+8*B*c*d+12*C*c^2)-a*b*d
^2*(-3*A*d^2+4*B*c*d+37*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellipti
cE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a
^(1/2)*d))^(1/2))/b^(5/2)/d/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)
))^(1/2)/(-b*x^2+a)^(1/2)-1/6*a^(1/2)*(a*d^2*(-5*B*d+13*C*c)-b*c*(-A*d^2-4
*B*c*d+12*C*c^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b
^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.59 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.34

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{b(c+dx)\left(2a\left(bc^2-ad^2\right)(acC+bBcx-ad(B+Cx)+Ab(c-dx))+\left(-a+bx^2\right)\left(8b^2Bc^3x+a^2d^2\right.\right.}{}\right.}{}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - b*x^2]*((b*(c + d*x)*(2*a*(b*c^2 - a*d^2)*(a*c*C + b*B*c*x - a*d
*(B + C*x) + A*b*(c - d*x)) + (-a + b*x^2)*(8*b^2*B*c^3*x + a^2*d^2*(-8*c*
C + 7*B*d + 9*C*d*x) + a*b*c*(12*c^2*C - 11*B*c*d - 13*c*C*d*x - 4*B*d^2*x
) + A*b*(b*c^2*(6*c - 7*d*x) + a*d^2*(-2*c + 3*d*x)))))/(a - b*x^2)^2 - (d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(21*a^2*C*d^4 + b^2*c^2*(12*c^2*C + 8*B*
c*d - 7*A*d^2) + a*b*d^2*(-37*c^2*C - 4*B*c*d + 3*A*d^2))*(a - b*x^2) + I*
Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(21*a^2*C*d^4 + b^2*c^2*(12*c^2*C + 8*B*c*
d - 7*A*d^2) + a*b*d^2*(-37*c^2*C - 4*B*c*d + 3*A*d^2))*Sqrt[(d*(Sqrt[a]/S
qrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c
+ d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d
*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]*d*(Sqrt
[b]*c - Sqrt[a]*d)*(21*a^2*C*d^3 + 6*b^2*c^2*(2*B*c - A*d) + a^(3/2)*Sqrt[
b]*d^2*(13*c*C - 5*B*d) + 3*a*b*d*(-8*c^2*C - 3*B*c*d + A*d^2) + Sqrt[a]*b
^(3/2)*c*(-12*c^2*C + 4*B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
+ d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
 + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*(a - b*x^2))))/(6*b^3*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2180, 27, 2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx+Cx^2)}{(a-bx^2)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^3+6aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{3a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))}{b^2}}{2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{3a\left(bc^2-ad^2\right)}+\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int\frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^3+6aB\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{3a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)x}{b^2}+\frac{a^2(bc(2Bc-Ad)-ad(cC+Bd))}{b^2}}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 2180

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\int-\frac{a^2\left(6b^2(2Bc-Ad)c^3-abd\left(12Cc^2+13Bdc-2Ad^2\right)c+a^2d^3(8cC+5Bd)+\left(21a^2Cd^4-ab\left(37Cc^2+4Bdc-3Ad^2\right)d^2+b^2c^2\left(12Cc^2+8Bdc-7Ad^2\right)\right)x\right)}{2b^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a(bc^2-ad^2)}+\frac{a\sqrt{c}\cdots}{\cdots}}{6a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\int\frac{6b^2(2B\cdots}{\cdots}}{\cdots}}{6a\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\left(21a^2C\cdots\right.}{\cdots}\cdots\right.}{\cdots}}{6a\left(bc\cdots\right.}$$

$\Big\downarrow$ 509

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\sqrt{1-\frac{bx\cdots}{a}}\cdots}{\cdots}\cdots\right.}{\cdots}}{6a\cdots}$$

$\Big\downarrow$ 508

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\left(bc^2-a\right.}{}\right.}{}}{}$$

$\big\downarrow$ 327

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\left(bc^2-a\right.}{}\right.}{}}{}$$

$\big\downarrow$ 512

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(\frac{\sqrt{1-\frac{bx}{a}}}{}\right.}{}}{}$$

$\big\downarrow$ 511

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{\ldots}}{\ldots}\right.}{\ldots}}{\ldots}$$

$\downarrow$ 321

$$\frac{a\sqrt{c+dx}(x(-aCd-Abd+bBc)-aBd+acC+Abc)}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{a\sqrt{c+dx}\left(x\left(9a^2Cd^3-abd\left(-3Ad^2+4Bcd+13c^2C\right)+b^2c^2(8Bc-7Ad)\right)+2Abc\left(3bc^2-ad^2\right)+a\left(bc^2(12cC-11Bd)-ad^2(8cC-7Bd)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{a\left(-\frac{2\sqrt{a}\sqrt{\ldots}}{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(a*Sqrt[c + d*x]*(A*b*c + a*c*C - a*B*d + (b*B*c - A*b*d - a*C*d)*x))/(3*b
^2*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) - ((a*Sqrt[c + d*x]*(2*A*b*c*(3*b*c^
2 - a*d^2) + a*(b*c^2*(12*c*C - 11*B*d) - a*d^2*(8*c*C - 7*B*d)) + (9*a^2*
C*d^3 + b^2*c^2*(8*B*c - 7*A*d) - a*b*d*(13*c^2*C + 4*B*c*d - 3*A*d^2))*x)
)/(b^2*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - (a*((-2*Sqrt[a]*(21*a^2*C*d^4 +
b^2*c^2*(12*c^2*C + 8*B*c*d - 7*A*d^2) - a*b*d^2*(37*c^2*C + 4*B*c*d - 3*A
*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]
*(b*c^2 - a*d^2)*(a*d^2*(13*c*C - 5*B*d) - b*c*(12*c^2*C - 4*B*c*d - A*d^2
))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*E
llipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)
/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*b^2*(b*c^2
- a*d^2)))/(6*a*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1109 vs. $2(542) = 1084$.

Time = 7.51 (sec) , antiderivative size = 1110, normalized size of antiderivative = 1.83

method	result	size
elliptic	Expression too large to display	1110
default	Expression too large to display	8457

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((1/3*a*(A*b*d-B
*b*c+C*a*d)/(a*d^2-b*c^2)/b^4*x-1/3*a*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b^
4)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12*(3
*A*a*b*d^3-7*A*b^2*c^2*d-4*B*a*b*c*d^2+8*B*b^2*c^3+9*C*a^2*d^3-13*C*a*b*c^
2*d)/(a*d^2-b*c^2)^2/b^3*x+1/12*(2*A*a*b*c*d^2-6*A*b^2*c^3-7*B*a^2*d^3+11*
B*a*b*c^2*d+8*C*a^2*c*d^2-12*C*a*b*c^3)/b^3/(a*d^2-b*c^2)^2)/((x^2-a/b)*(-
b*d*x-b*c))^(1/2)+2*(B/b^2-1/6*(A*b*c*d+7*B*a*d^2-8*B*b*c^2+C*a*c*d)/b^2/(
a*d^2-b*c^2)-1/12/b^2*d*(2*A*a*b*c*d^2-6*A*b^2*c^3-7*B*a^2*d^3+11*B*a*b*c^
2*d+8*C*a^2*c*d^2-12*C*a*b*c^3)/(a*d^2-b*c^2)^2+1/6/b^2*c*(3*A*a*b*d^3-7*A
*b^2*c^2*d-4*B*a*b*c*d^2+8*B*b^2*c^3+9*C*a^2*d^3-13*C*a*b*c^2*d)/(a*d^2-b*
c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/
b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))+2*(C/b^2+1/12*d*(3*A*a*b*d^3-7*A*b^2*c^2*d-4*B*a*b*c*d^2+8*B*b
^2*c^3+9*C*a^2*d^3-13*C*a*b*c^2*d)/b^2/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/
2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(
a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(546) = 1092$.

Time = 0.12 (sec) , antiderivative size = 1116, normalized size of antiderivative = 1.84

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output
```
1/18*((12*C*a^2*b^2*c^5 - 28*B*a^2*b^2*c^4*d + 35*B*a^3*b*c^2*d^3 - 15*B*a
^4*d^5 - (C*a^3*b - 11*A*a^2*b^2)*c^3*d^2 - 3*(C*a^4 + A*a^3*b)*c*d^4 + (1
2*C*b^4*c^5 - 28*B*b^4*c^4*d + 35*B*a*b^3*c^2*d^3 - 15*B*a^2*b^2*d^5 - (C*
a*b^3 - 11*A*b^4)*c^3*d^2 - 3*(C*a^2*b^2 + A*a*b^3)*c*d^4)*x^4 - 2*(12*C*a
*b^3*c^5 - 28*B*a*b^3*c^4*d + 35*B*a^2*b^2*c^2*d^3 - 15*B*a^3*b*d^5 - (C*a
^2*b^2 - 11*A*a*b^3)*c^3*d^2 - 3*(C*a^3*b + A*a^2*b^2)*c*d^4)*x^2)*sqrt(-b
*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*
c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(12*C*a^2*b^2*c^4*d + 8*B*a^2*b^2*c
^3*d^2 - 4*B*a^3*b*c*d^4 - (37*C*a^3*b + 7*A*a^2*b^2)*c^2*d^3 + 3*(7*C*a^4
 + A*a^3*b)*d^5 + (12*C*b^4*c^4*d + 8*B*b^4*c^3*d^2 - 4*B*a*b^3*c*d^4 - (3
7*C*a*b^3 + 7*A*b^4)*c^2*d^3 + 3*(7*C*a^2*b^2 + A*a*b^3)*d^5)*x^4 - 2*(12*
C*a*b^3*c^4*d + 8*B*a*b^3*c^3*d^2 - 4*B*a^2*b^2*c*d^4 - (37*C*a^2*b^2 + 7*
A*a*b^3)*c^2*d^3 + 3*(7*C*a^3*b + A*a^2*b^2)*d^5)*x^2)*sqrt(-b*d)*weierstr
assZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3),
weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^
2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(9*B*a^2*b^2*c^2*d^3 + 6*C*a^3*b*c*d^4
 - 5*B*a^3*b*d^5 - 2*(5*C*a^2*b^2 + 2*A*a*b^3)*c^3*d^2 + (8*B*b^4*c^3*d^2
- 4*B*a*b^3*c*d^4 - (13*C*a*b^3 + 7*A*b^4)*c^2*d^3 + 3*(3*C*a^2*b^2 + A*a*
b^3)*d^5)*x^3 - (11*B*a*b^3*c^2*d^3 - 7*B*a^2*b^2*d^5 - 6*(2*C*a*b^3 + A*b
^4)*c^3*d^2 + 2*(4*C*a^2*b^2 + A*a*b^3)*c*d^4)*x^2 - (6*B*a*b^3*c^3*d^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^3}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^3/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^3\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^3*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{x^3(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int(x^3*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.234 $\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 601

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{c+dx}(a(bBc-Abd-aCd)+b(Abc+acC-aBd)x)}{3b^2(bc^2-ad^2)(a-bx^2)^{3/2}}$$
$$-\frac{\sqrt{c+dx}(a(7a^2Cd^3+b^2c^2(6Bc-5Ad)-abd(11c^2C+2Bcd-Ad^2))+b(2Abc(bc^2+ad^2)+a(bc^2(8cC$$
$$-\frac{(2Abc(bc^2+ad^2)+a(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{}}{\sqrt{bc+}}\right.}{6\sqrt{a}b^{3/2}(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$+\frac{(Ab(2bc^2-ad^2)+a(5aCd^2-bc(4cC+Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6\sqrt{a}b^{5/2}(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(a*(-A*b*d+B*b*c-C*a*d)+b*(A*b*c-B*a*d+C*a*c)*x)/b^2/(-a
*d^2+b*c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(a*(7*a^2*C*d^3+b^2*c^2*(-5
*A*d+6*B*c)-a*b*d*(-A*d^2+2*B*c*d+11*C*c^2))+b*(2*A*b*c*(a*d^2+b*c^2)+a*(b
*c^2*(-7*B*d+8*C*c)-a*d^2*(-3*B*d+4*C*c)))*x)/a/b^2/(-a*d^2+b*c^2)^2/(-b*x
^2+a)^(1/2)-1/6*(2*A*b*c*(a*d^2+b*c^2)+a*(b*c^2*(-7*B*d+8*C*c)-a*d^2*(-3*B
*d+4*C*c)))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/
a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^
(1/2)/b^(3/2)/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x
^2+a)^(1/2)+1/6*(A*b*(-a*d^2+2*b*c^2)+a*(5*a*C*d^2-b*c*(B*d+4*C*c)))*((d*x
+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(
1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1
/2))/a^(1/2)/b^(5/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.66 (sec) , antiderivative size = 821, normalized size of antiderivative = 1.37

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(-5a^4Cd^3+2Ab^4c^3x^3+ab^3cx^2\left(8c^2Cx+Bc(6c-7dx)+Ad(-5c+2dx)\right)+a^2b^2\left(Ad\right.\right.}{}\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output

```
(Sqrt[a - b*x^2]*(((c + d*x)*(-5*a^4*C*d^3 + 2*A*b^4*c^3*x^3 + a*b^3*c*x^2
*(8*c^2*C*x + B*c*(6*c - 7*d*x) + A*d*(-5*c + 2*d*x)) + a^2*b^2*(A*d*(3*c^
2 - 4*c*d*x + d^2*x^2) - c*C*x*(6*c^2 + 11*c*d*x + 4*d^2*x^2) + B*(-4*c^3
+ 5*c^2*d*x - 2*c*d^2*x^2 + 3*d^3*x^3)) + a^3*b*d*(9*c^2*C + 2*c*C*d*x + d
^2*(A + x*(-B + 7*C*x)))))/(a - b*x^2)^2 - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]*(2*A*b*c*(b*c^2 + a*d^2) + a*(b*c^2*(8*c*C - 7*B*d) + a*d^2*(-4*c*C +
 3*B*d)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(2*A*b*c*(b*c^2
+ a*d^2) + a*(b*c^2*(8*c*C - 7*B*d) + a*d^2*(-4*c*C + 3*B*d)))*Sqrt[(d*(Sq
rt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x
))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqr
t[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*
(Sqrt[b]*c - Sqrt[a]*d)*(2*A*b^2*c^2*d + 5*a^2*C*d^3 - 3*a^(3/2)*Sqrt[b]*d
^2*(-3*c*C + B*d) - 3*Sqrt[a]*b^(3/2)*c*(4*c^2*C - 2*B*c*d + A*d^2) - a*b*
d*(4*c^2*C + B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqr
t[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*Ar
cSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*
d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2)
)))/(6*a*b^2*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2180, 27, 2180, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a-bx^2\right)^{5/2} \sqrt{c+dx}} \, dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{3a(bc(2Bc-Ad)-ad(cC+Bd))x}{b}+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^2}}{2\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{3a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{6aC\left(c^2-\frac{ad^2}{b}\right)x^2+\frac{3a(bc(2Bc-Ad)-ad(cC+Bd))x}{b}+\frac{a\left(Ab\left(2bc^2-ad^2\right)-a\left(aCd^2-bc(2cC-Bd)\right)\right)}{b^2}}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6a\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 2180

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\int -\frac{a\left(a\left(5a^2Cd^4-ab\left(13Cc^2-2Bdc+Ad^2\right)d^2+b^2c^2\left(12Cc^2-6Bdc+5Ad^2\right)\right)+bd\left(2Abc\left(bc^2+ad^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)x\right)}{2b^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}\left(a(7\right.}{}}{6a\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 27

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\int \frac{a\left(5a^2Cd^4\right.}{}}{}}{6a\left(bc^2-ad^2\right)}$$

$\big\downarrow$ 600

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{b\left(2Abc\left(ad^2\right.\right.}{}}{6a\left(bc^2-a\right.}$$

$\big\downarrow$ 509

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(2\right.}{}}{6a\left(bc^2-\right.}$$

$\big\downarrow$ 508

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\left(\left(bc^2-ad\right.\right.}{}}{}$$

$\downarrow$ 327

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\left(bc^2-ad^2\right)}{}}{}$$

$\downarrow$ 512

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{-\frac{\sqrt{1-\frac{bx^2}{a}}\left(\right.}{}}{}}{}$$

$\downarrow$ 511

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}}{}}{}}{}$$

$\downarrow$ 321

$$\frac{\sqrt{c+dx}(bx(-aBd+acC+Abc)+a(-aCd-Abd+bBc))}{3b^2\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$\frac{\sqrt{c+dx}\left(a\left(7a^2Cd^3-abd\left(-Ad^2+2Bcd+11c^2C\right)+b^2c^2(6Bc-5Ad)\right)+bx\left(2Abc\left(ad^2+bc^2\right)+a\left(bc^2(8cC-7Bd)-ad^2(4cC-3Bd)\right)\right)\right)}{b^2\sqrt{a-bx^2}\left(bc^2-ad^2\right)}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}}{}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[c + d*x]*(a*(b*B*c - A*b*d - a*C*d) + b*(A*b*c + a*c*C - a*B*d)*x))/
(3*b^2*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) - ((Sqrt[c + d*x]*(a*(7*a^2*C*d^
3 + b^2*c^2*(6*B*c - 5*A*d) - a*b*d*(11*c^2*C + 2*B*c*d - A*d^2)) + b*(2*A
*b*c*(b*c^2 + a*d^2) + a*(b*c^2*(8*c*C - 7*B*d) - a*d^2*(4*c*C - 3*B*d)))*
x))/(b^2*(b*c^2 - a*d^2)*Sqrt[a - b*x^2]) - ((-2*Sqrt[a]*Sqrt[b]*(2*A*b*c*
(b*c^2 + a*d^2) + a*(b*c^2*(8*c*C - 7*B*d) - a*d^2*(4*c*C - 3*B*d)))*Sqrt[
c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]
]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(S
qrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(A*b*
(2*b*c^2 - a*d^2) + a*(5*a*C*d^2 - b*c*(4*c*C + B*d)))*Sqrt[(Sqrt[b]*(c +
d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1
 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[
b]*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(2*b^2*(b*c^2 - a*d^2)))/(6*a*(b*c^2 -
a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1125 vs. $2(535) = 1070$.

Time = 6.51 (sec) , antiderivative size = 1126, normalized size of antiderivative = 1.87

method	result	size
elliptic	Expression too large to display	1126
default	Expression too large to display	7616

input
```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((-1/3*(A*b*c-B*
a*d+C*a*c)/b^3/(a*d^2-b*c^2)*x+1/3*(A*b*d-B*b*c+C*a*d)*a/b^4/(a*d^2-b*c^2)
)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12/b^2
*(2*A*a*b*c*d^2+2*A*b^2*c^3+3*B*a^2*d^3-7*B*a*b*c^2*d-4*C*a^2*c*d^2+8*C*a*
b*c^3)/a/(a*d^2-b*c^2)^2*x-1/12*(A*a*b*d^3-5*A*b^2*c^2*d-2*B*a*b*c*d^2+6*B
*b^2*c^3+7*C*a^2*d^3-11*C*a*b*c^2*d)/(a*d^2-b*c^2)^2/b^3)/((x^2-a/b)*(-b*d
*x-b*c))^(1/2)+2*(C/b^2-1/6/b^2/(a*d^2-b*c^2)*(A*a*b*d^2-2*A*b^2*c^2+B*a*b
*c*d+7*C*a^2*d^2-8*C*a*b*c^2)/a+1/12/b^2*d*(A*a*b*d^3-5*A*b^2*c^2*d-2*B*a*
b*c*d^2+6*B*b^2*c^3+7*C*a^2*d^3-11*C*a*b*c^2*d)/(a*d^2-b*c^2)^2+1/6/b*c*(2
*A*a*b*c*d^2+2*A*b^2*c^3+3*B*a^2*d^3-7*B*a*b*c^2*d-4*C*a^2*c*d^2+8*C*a*b*c
^3)/a/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^
(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))+1/6*d*(2*A*a*b*c*d^2+2*A*b^2*c^3+3*B*a^2*d^3-7*
B*a*b*c^2*d-4*C*a^2*c*d^2+8*C*a*b*c^3)/b/a/(a*d^2-b*c^2)^2*(c/d-1/b*(a*b)^
(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/
b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1093 vs. $2(539) = 1078$.

Time = 0.13 (sec) , antiderivative size = 1093, normalized size of antiderivative = 1.82

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output

```
1/18*((11*B*a^3*b^2*c^3*d - 3*B*a^4*b*c*d^3 - 2*(14*C*a^3*b^2 - A*a^2*b^3)
*c^4 + (35*C*a^4*b - 13*A*a^3*b^2)*c^2*d^2 - 3*(5*C*a^5 - A*a^4*b)*d^4 + (
11*B*a*b^4*c^3*d - 3*B*a^2*b^3*c*d^3 - 2*(14*C*a*b^4 - A*b^5)*c^4 + (35*C*
a^2*b^3 - 13*A*a*b^4)*c^2*d^2 - 3*(5*C*a^3*b^2 - A*a^2*b^3)*d^4)*x^4 - 2*(
11*B*a^2*b^3*c^3*d - 3*B*a^3*b^2*c*d^3 - 2*(14*C*a^2*b^3 - A*a*b^4)*c^4 +
(35*C*a^3*b^2 - 13*A*a^2*b^3)*c^2*d^2 - 3*(5*C*a^4*b - A*a^3*b^2)*d^4)*x^2
)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c
^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) - 3*(7*B*a^3*b^2*c^2*d^2 - 3*B
*a^4*b*d^4 - 2*(4*C*a^3*b^2 + A*a^2*b^3)*c^3*d + 2*(2*C*a^4*b - A*a^3*b^2)
*c*d^3 + (7*B*a*b^4*c^2*d^2 - 3*B*a^2*b^3*d^4 - 2*(4*C*a*b^4 + A*b^5)*c^3*
d + 2*(2*C*a^2*b^3 - A*a*b^4)*c*d^3)*x^4 - 2*(7*B*a^2*b^3*c^2*d^2 - 3*B*a^
3*b^2*d^4 - 2*(4*C*a^2*b^3 + A*a*b^4)*c^3*d + 2*(2*C*a^3*b^2 - A*a^2*b^3)*
c*d^3)*x^2)*sqrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/2
7*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(
b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(4*B*a^
2*b^3*c^3*d - 3*(3*C*a^3*b^2 + A*a^2*b^3)*c^2*d^2 + (5*C*a^4*b - A*a^3*b^2
)*d^4 + (7*B*a*b^4*c^2*d^2 - 3*B*a^2*b^3*d^4 - 2*(4*C*a*b^4 + A*b^5)*c^3*d
 + 2*(2*C*a^2*b^3 - A*a*b^4)*c*d^3)*x^3 - (6*B*a*b^4*c^3*d - 2*B*a^2*b^3*c
*d^3 - (11*C*a^2*b^3 + 5*A*a*b^4)*c^2*d^2 + (7*C*a^3*b^2 + A*a^2*b^3)*d^4)
*x^2 + (6*C*a^2*b^3*c^3*d - 5*B*a^2*b^3*c^2*d^2 + B*a^3*b^2*d^4 - 2*(C*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input

```
integrate(x**2*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output

```
Timed out
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^2\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input

```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output

```
int(x^2*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.235 $\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 571

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{c+dx}\left(Abc+acC-aBd+b\left(Bc-\left(A+\frac{aC}{b}\right)d\right)x\right)}{3b\,(bc^2-ad^2)\,(a-bx^2)^{3/2}}$$

$$+\frac{\sqrt{c+dx}(a(ad^2(2cC-Bd)-bc(6c^2C-5Bcd+4Ad^2))-(3a^2Cd^3+b^2c^2(2Bc-Ad)-abd(7c^2C-2B}{6ab\,(bc^2-ad^2)^2\sqrt{a-bx^2}}$$

$$-\frac{(3a^2Cd^3+b^2c^2(2Bc-Ad)-abd(7c^2C-2Bcd+3Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{c}}\right.}{6\sqrt{a}b^{3/2}\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$+\frac{(bc(2Bc-Ad)-ad(cC+Bd))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6\sqrt{a}b^{3/2}\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+b*(B*c-(A+a*C/b)*d)*x)/b/(-a*d^2+b*c^
2)/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(1/2)*(a*(a*d^2*(-B*d+2*C*c)-b*c*(4*A*d^2-
5*B*c*d+6*C*c^2))-(3*a^2*C*d^3+b^2*c^2*(-A*d+2*B*c)-a*b*d*(3*A*d^2-2*B*c*d
+7*C*c^2))*x)/a/b/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(1/2)-1/6*(3*a^2*C*d^3+b^2*c
^2*(-A*d+2*B*c)-a*b*d*(3*A*d^2-2*B*c*d+7*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)
/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/
2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/(-a*d^2+b*c^2)^2/((d*x+
c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)+1/6*(b*c*(-A*d+2*B*c)-a*d
*(B*d+C*c))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ell
ipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)
*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.24 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.37

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{b(c+dx)(b^3c^2(2Bc-Ad)x^3+a^3d^3(B-Cx)+ab^2(cx^2(6c^2C-5Bcd-7cCdx+2Bd^2x)+A(2\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - b*x^2]*((b*(c + d*x)*(b^3*c^2*(2*B*c - A*d)*x^3 + a^3*d^3*(B - C
*x) + a*b^2*(c*x^2*(6*c^2*C - 5*B*c*d - 7*c*C*d*x + 2*B*d^2*x) + A*(2*c^3
- c^2*d*x + 4*c*d^2*x^2 - 3*d^3*x^3)) + a^2*b*(-4*c^3*C + c^2*d*(3*B + 5*C
*x) - 2*c*d^2*(3*A + x*(2*B + C*x)) + d^3*x*(5*A + x*(B + 3*C*x)))))/(a -
b*x^2)^2 - (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(3*a^2*C*d^3 + b^2*c^2*(2*B
*c - A*d) + a*b*d*(-7*c^2*C + 2*B*c*d - 3*A*d^2))*(a - b*x^2) + I*Sqrt[b]*
(Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*C*d^3 + b^2*c^2*(2*B*c - A*d) + a*b*d*(-7*c
^2*C + 2*B*c*d - 3*A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[
-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcS
inh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)
/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[a]*Sqrt[b]*d*(Sqrt[b]*c - Sqrt[a]*d)*(3
*a^(3/2)*C*d^2 + b^(3/2)*c*(-2*B*c + A*d) + a*Sqrt[b]*d*(c*C + B*d) - 3*Sq
rt[a]*b*(2*c^2*C - B*c*d + A*d^2))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x
)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Elliptic
F[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sq
rt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a -
b*x^2))))/(6*a*b^2*(b*c^2 - a*d^2)^2*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2180, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{(a-bx^2)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2180}$$

$$\frac{\int -\frac{a\left(bc(2Bc-Ad)-ad(cC+Bd)+3b\left(2Cc^2-Bdc+\frac{(Ab-aC)d^2}{b}\right)x\right)}{2b\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{3a(bc^2-ad^2)}+\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{\int \frac{bc(2Bc-Ad)-ad(cC+Bd)-3\left(aCd^2-b\left(2Cc^2-Bdc+Ad^2\right)\right)x}{\sqrt{c+dx}\left(a-bx^2\right)^{3/2}}dx}{6b\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 686

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{\int \frac{bd\left(a\left(ad^2(2cC-Bd)-bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)+\left(3a^2Cd^3-ab\left(7Cc^2-2Bdc+3Ad^2\right)d+b^2c^2(2Bc-Ad)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\left(ad^2(2cC-Bd)-bc\left(4Ad^2-\right.\right.\right.}{}}{6b\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{d\int \frac{a\left(ad^2(2cC-Bd)-bc\left(6Cc^2-5Bdc+4Ad^2\right)\right)+\left(3a^2Cd^3-ab\left(7Cc^2-2Bdc+3Ad^2\right)d+b^2c^2(2Bc-Ad)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\left(ad^2(2cC-Bd)-bc\left(4Ad^2-5B\right.\right.\right.}{}}{6b\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{d\left(\frac{\left(3a^2Cd^3-abd\left(3Ad^2-2Bcd+7c^2C\right)+b^2c^2(2Bc-Ad)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)(bc(2Bc-Ad)-ad(Bd+cC))\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}\left(a\left(a\right.\right.}{}}{6b\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-\frac{-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2Cd^3-abd\left(3Ad^2-2Bcd+7c^2C\right)+b^2c^2(2Bc-Ad)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)(bc(2Bc-Ad)-ad(Bd+cC))\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+\right.}{}}{6b\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 508

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3-abd\left(3Ad^2-2Bcd+7c^2C\right)+b^2c^2(2Bc-Ad)\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)(bc(2Bc-Ad)-ad(Bd+cC))\int}{d}\right.}{2a(bc^2-ad^2)}$$

$$6b\left(bc^2-a\right.$$

↓ 327

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)(bc(2Bc-Ad)-ad(Bd+cC))\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3-abd\left(3Ad^2-2Bcd+7c^2C\right)+b^2c^2(2Bc-Ad)\right)E\left(\arcsin\left(\frac{\sqrt{}}{}\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}$$

$$6b\left(bc^2-a\right.$$

↓ 512

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)(bc(2Bc-Ad)-ad(Bd+cC))\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3-abd\left(3Ad^2-2Bcd+7c^2C\right)+b^2c^2(2Bc-Ad)\right)E\left(\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}$$

$$6b\left(bc^2\right.$$

↓ 511

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bc(2Bc-Ad)-ad(Bd+cC))\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3-abd\ldots\right.}{\ldots}\right)}{2a(bc^2-ad^2)}$$

$\downarrow$ 321

$$\frac{\sqrt{c+dx}\left(bx\left(Bc-d\left(\frac{aC}{b}+A\right)\right)-aBd+acC+Abc\right)}{3b\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}-$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(bc(2Bc-Ad)-ad(Bd+cC))\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3-abd\left(3Ad^2-\ldots\right.\right.}{\ldots}\right)}{2a(bc^2-ad^2)}$$

input
```
Int[(x*(A + B*x + C*x^2))/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[c + d*x]*(A*b*c + a*c*C - a*B*d + b*(B*c - (A + (a*C)/b)*d)*x))/(3*b
*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) - (-((Sqrt[c + d*x]*(a*(a*d^2*(2*c*C -
 B*d) - b*c*(6*c^2*C - 5*B*c*d + 4*A*d^2)) - (3*a^2*C*d^3 + b^2*c^2*(2*B*c
 - A*d) - a*b*d*(7*c^2*C - 2*B*c*d + 3*A*d^2))*x))/(a*(b*c^2 - a*d^2)*Sqrt
[a - b*x^2])) - (d*((-2*Sqrt[a]*(3*a^2*C*d^3 + b^2*c^2*(2*B*c - A*d) - a*b
*d*(7*c^2*C - 2*B*c*d + 3*A*d^2))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*Ellipt
icE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt
[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sq
rt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(b*c*(2*B*c - A*d) - a*d*(c*C
+ B*d))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[
b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/(2*a*(b*c
^2 - a*d^2)))/(6*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(505) = 1010$.

Time = 6.52 (sec) , antiderivative size = 1069, normalized size of antiderivative = 1.87

method	result	size
elliptic	Expression too large to display	1069
default	Expression too large to display	6942

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((1/3*(A*b*d-B*b
*c+C*a*d)/(a*d^2-b*c^2)/b^3*x-1/3*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b^3)*(
-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12*(3*A*a*
b*d^3+A*b^2*c^2*d-2*B*a*b*c*d^2-2*B*b^2*c^3-3*C*a^2*d^3+7*C*a*b*c^2*d)/a/(
a*d^2-b*c^2)^2/b^2*x-1/12*(4*A*b*c*d^2+B*a*d^3-5*B*b*c^2*d-2*C*a*c*d^2+6*C
*b*c^3)/(a*d^2-b*c^2)^2/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(-1/6/b/(a*d
^2-b*c^2)*(A*b*c*d+B*a*d^2-2*B*b*c^2+C*a*c*d)/a+1/12/b*d*(4*A*b*c*d^2+B*a*
d^3-5*B*b*c^2*d-2*C*a*c*d^2+6*C*b*c^3)/(a*d^2-b*c^2)^2-1/6/b*c*(3*A*a*b*d^
3+A*b^2*c^2*d-2*B*a*b*c*d^2-2*B*b^2*c^3-3*C*a^2*d^3+7*C*a*b*c^2*d)/a/(a*d^
2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/
d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2))-1/6*d*(3*A*a*b*d^3+A*b^2*c^2*d-2*B*a*b*c*d^2-2*B*b^2*c^3-3
*C*a^2*d^3+7*C*a*b*c^2*d)/b/a/(a*d^2-b*c^2)^2*(c/d-1/b*(a*b)^(1/2))*((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+
1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 1006, normalized size of antiderivative = 1.76

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="fri
cas")
```

output
```
1/18*((2*B*a^2*b^2*c^4 - 13*B*a^3*b*c^2*d^2 + 3*B*a^4*d^4 + (11*C*a^3*b -
A*a^2*b^2)*c^3*d - 3*(C*a^4 - 3*A*a^3*b)*c*d^3 + (2*B*b^4*c^4 - 13*B*a*b^3
*c^2*d^2 + 3*B*a^2*b^2*d^4 + (11*C*a*b^3 - A*b^4)*c^3*d - 3*(C*a^2*b^2 - 3
*A*a*b^3)*c*d^3)*x^4 - 2*(2*B*a*b^3*c^4 - 13*B*a^2*b^2*c^2*d^2 + 3*B*a^3*b
*d^4 + (11*C*a^2*b^2 - A*a*b^3)*c^3*d - 3*(C*a^3*b - 3*A*a^2*b^2)*c*d^3)*x
^2)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b
*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(2*B*a^2*b^2*c^3*d + 2*B
*a^3*b*c*d^3 - (7*C*a^3*b + A*a^2*b^2)*c^2*d^2 + 3*(C*a^4 - A*a^3*b)*d^4 +
 (2*B*b^4*c^3*d + 2*B*a*b^3*c*d^3 - (7*C*a*b^3 + A*b^4)*c^2*d^2 + 3*(C*a^2
*b^2 - A*a*b^3)*d^4)*x^4 - 2*(2*B*a*b^3*c^3*d + 2*B*a^2*b^2*c*d^3 - (7*C*a
^2*b^2 + A*a*b^3)*c^2*d^2 + 3*(C*a^3*b - A*a^2*b^2)*d^4)*x^2)*sqrt(-b*d)*w
eierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b
*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9
*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(3*B*a^2*b^2*c^2*d^2 - 6*A*a^2*
b^2*c*d^3 + B*a^3*b*d^4 - 2*(2*C*a^2*b^2 - A*a*b^3)*c^3*d + (2*B*b^4*c^3*d
 + 2*B*a*b^3*c*d^3 - (7*C*a*b^3 + A*b^4)*c^2*d^2 + 3*(C*a^2*b^2 - A*a*b^3)
*d^4)*x^3 + (6*C*a*b^3*c^3*d - 5*B*a*b^3*c^2*d^2 + B*a^2*b^2*d^4 - 2*(C*a^
2*b^2 - 2*A*a*b^3)*c*d^3)*x^2 - (4*B*a^2*b^2*c*d^3 - (5*C*a^2*b^2 - A*a*b^
3)*c^2*d^2 + (C*a^3*b - 5*A*a^2*b^2)*d^4)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c
))/(a^3*b^4*c^4*d - 2*a^4*b^3*c^2*d^3 + a^5*b^2*d^5 + (a*b^6*c^4*d - 2*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{(Cx^2+Bx+A)x}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)*x/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{x\,(C\,x^2+B\,x+A)}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{x(C\,x^2+Bx+A)}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.236 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 593

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx = \frac{\sqrt{c+dx}\left(a\left(Bc-\left(A+\frac{aC}{b}\right)d\right)+(Abc+acC-aBd)x\right)}{3a\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}$$
$$-\frac{\sqrt{c+dx}(ad(Ab(bc^2-5ad^2)+a(aCd^2-bc(5cC-4Bd)))-b(4Abc(bc^2-2ad^2)-a(ad^2(2cC-3Bd)+}{6a^2b\left(bc^2-ad^2\right)^2\sqrt{a-bx^2}}$$
$$+\frac{(4Abc(bc^2-2ad^2)-a(ad^2(2cC-3Bd)+bc^2(2cC-Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}}{\sqrt{bc+}}\right.}{6a^{3/2}\sqrt{b}\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(Ab(4bc^2-5ad^2)+a(aCd^2-bc(2cC-Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}b^{3/2}\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(a*(B*c-(A+a*C/b)*d)+(A*b*c-B*a*d+C*a*c)*x)/a/(-a*d^2+b*
c^2)/(-b*x^2+a)^(3/2)-1/6*(d*x+c)^(1/2)*(a*d*(A*b*(-5*a*d^2+b*c^2)+a*(a*C*
d^2-b*c*(-4*B*d+5*C*c)))-b*(4*A*b*c*(-2*a*d^2+b*c^2)-a*(a*d^2*(-3*B*d+2*C*
c)+b*c^2*(-B*d+2*C*c)))*x)/a^2/b/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(1/2)+1/6*(4*
A*b*c*(-2*a*d^2+b*c^2)-a*(a*d^2*(-3*B*d+2*C*c)+b*c^2*(-B*d+2*C*c)))*(d*x+c
)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(-a*
d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/6*(A
*b*(-5*a*d^2+4*b*c^2)+a*(a*C*d^2-b*c*(-B*d+2*C*c)))*((d*x+c)/(c+a^(1/2)*d/
b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(3
/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.25 (sec) , antiderivative size = 802, normalized size of antiderivative = 1.35

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{a-bx^2}\left(\frac{(c+dx)\left(a^4Cd^3-4Ab^4c^3x^3+a^3bd\left(3c^2C-2cd(3B+2Cx)+d^2\left(7A+5Bx+Cx^2\right)\right)+ab^3cx(c\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(Sqrt[a - b*x^2]*(((c + d*x)*(a^4*C*d^3 - 4*A*b^4*c^3*x^3 + a^3*b*d*(3*c^2
*C - 2*c*d*(3*B + 2*C*x) + d^2*(7*A + 5*B*x + C*x^2)) + a*b^3*c*x*(c*(2*c*
C - B*d)*x^2 + A*(6*c^2 + c*d*x + 8*d^2*x^2)) + a^2*b^2*(B*(2*c^3 - c^2*d*
x + 4*c*d^2*x^2 - 3*d^3*x^3) - d*(c*C*x^2*(5*c - 2*d*x) + A*(3*c^2 + 10*c*
d*x + 5*d^2*x^2)))))/(a - b*x^2)^2 + (d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(
4*A*b*c*(b*c^2 - 2*a*d^2) + a*(b*c^2*(-2*c*C + B*d) + a*d^2*(-2*c*C + 3*B*
d)))*(a - b*x^2) + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(4*A*b*c*(b*c^2 - 2*a
*d^2) + a*(b*c^2*(-2*c*C + B*d) + a*d^2*(-2*c*C + 3*B*d)))*Sqrt[(d*(Sqrt[a
]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*
(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
+ d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*Sqrt[a]*d*(S
qrt[b]*c - Sqrt[a]*d)*(A*b*(4*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d - 5*a*d^2) + a
*(a*C*d^2 + 3*Sqrt[a]*Sqrt[b]*d*(c*C - B*d) + b*c*(-2*c*C + B*d)))*Sqrt[(d
*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(6*a^2*b*(b*c^2 - a*d^2)^2*Sqrt[
c + d*x])
```

Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2180, 27, 686, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a-bx^2)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2180}$$

$$\frac{\int \frac{Ab(4bc^2-5ad^2)+a(aCd^2-bc(2cC-Bd))+3bd(Abc+aCc-aBd)x}{2b\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{3a(bc^2-ad^2)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a(a-bx^2)^{3/2}(bc^2-ad^2)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{Ab(4bc^2-5ad^2)+a(aCd^2-bc(2cC-Bd))+3bd(Abc+aCc-aBd)x}{\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{6ab\,(bc^2-ad^2)}+\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\,(a-bx^2)^{3/2}\,(bc^2-ad^2)}$$

$\downarrow$ 686

$$\frac{-\frac{\int \frac{bd\left(ad\left(Ab\left(bc^2-5ad^2\right)+a\left(aCd^2-bc(5cC-4Bd)\right)\right)+b\left(4Abc\left(bc^2-2ad^2\right)-a\left(b(2cC-Bd)c^2+ad^2(2cC-3Bd)\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(Ab(bc^2-5ad^2)+a(a}{}}{6ab\,(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\,(a-bx^2)^{3/2}\,(bc^2-ad^2)}$$

$\downarrow$ 27

$$\frac{-\frac{d\int \frac{ad\left(Ab\left(bc^2-5ad^2\right)+a\left(aCd^2-bc(5cC-4Bd)\right)\right)+b\left(4Abc\left(bc^2-2ad^2\right)-a\left(b(2cC-Bd)c^2+ad^2(2cC-3Bd)\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(Ab(bc^2-5ad^2)+a(aC}{}}{6ab\,(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\,(a-bx^2)^{3/2}\,(bc^2-ad^2)}$$

$\downarrow$ 600

$$\frac{-\frac{d\left(\frac{b\left(4Abc\left(bc^2-2ad^2\right)-a\left(ad^2(2cC-3Bd)+bc^2(2cC-Bd)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}}{6ab\,(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\,(a-bx^2)^{3/2}\,(bc^2-ad^2)}$$

$\downarrow$ 509

$$\frac{-\frac{d\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(4Abc\left(bc^2-2ad^2\right)-a\left(ad^2(2cC-3Bd)+bc^2(2cC-Bd)\right)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-}}}{d}\right.}{2a(bc^2-ad^2)}}{6ab\,(bc^2-ad}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\,(a-bx^2)^{3/2}\,(bc^2-ad^2)}$$

$\downarrow$ 508

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4Abc\left(bc^2-2ad^2\right)-a\left(ad^2(2cC-3Bd)+bc^2(2cC\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}$$

6a

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 327

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4Abc\left(bc^2-2ad^2\right)-a\left(ad^2(2cC-3Bd)+bc^2(2cC\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right)}{2a(bc^2-ad^2)}$$

6

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 512

$$-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4Abc\left(bc^2-2ad^2\right)-a\left(ad^2(2cC-3Bd)\right.\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(}{\sqrt{a}}}}\right)}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 511

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{}}{}\right)}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(Ab\left(4bc^2-5ad^2\right)+a\left(aCd^2-bc(2cC-Bd)\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(4A}{}\right)}{2a(bc^2-ad^2)}$$

$$\frac{\sqrt{c+dx}\left(x(-aBd+acC+Abc)+a\left(Bc-d\left(\frac{aC}{b}+A\right)\right)\right)}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

input

```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output

```
(Sqrt[c + d*x]*(a*(B*c - (A + (a*C)/b)*d) + (A*b*c + a*c*C - a*B*d)*x))/(3
*a*(b*c^2 - a*d^2)*(a - b*x^2)^(3/2)) + (-((Sqrt[c + d*x]*(a*d*(A*b*(b*c^2
 - 5*a*d^2) + a*(a*C*d^2 - b*c*(5*c*C - 4*B*d))) - b*(4*A*b*c*(b*c^2 - 2*a
*d^2) - a*(a*d^2*(2*c*C - 3*B*d) + b*c^2*(2*c*C - B*d)))*x))/(a*(b*c^2 - a
*d^2)*Sqrt[a - b*x^2])) - (d*((-2*Sqrt[a]*Sqrt[b]*(4*A*b*c*(b*c^2 - 2*a*d^
2) - a*(a*d^2*(2*c*C - 3*B*d) + b*c^2*(2*c*C - B*d)))*Sqrt[c + d*x]*Sqrt[1
 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*
d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[a - b*x^2]) + (2*Sqrt[a]*(b*c^2 - a*d^2)*(A*b*(4*b*c^2 - 5*
a*d^2) + a*(a*C*d^2 - b*c*(2*c*C - B*d)))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b
]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x
)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c +
 d*x]*Sqrt[a - b*x^2])))/(2*a*(b*c^2 - a*d^2)))/(6*a*b*(b*c^2 - a*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2180
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a + b*x^2, x], R = Coeff[PolynomialRema
inder[Pq, a + b*x^2, x], x, 0], S = Coeff[PolynomialRemainder[Pq, a + b*x^2
, x], x, 1]}, Simp[(-(d + e*x)^(m + 1))*(a + b*x^2)^(p + 1)*((a*(e*R - d*S)
 + (b*d*R + a*e*S)*x)/(2*a*(p + 1)*(b*d^2 + a*e^2))), x] + Simp[1/(2*a*(p +
 1)*(b*d^2 + a*e^2))   Int[(d + e*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*
(p + 1)*(b*d^2 + a*e^2)*Qx + b*d^2*R*(2*p + 3) - a*e*(d*S*m - e*R*(m + 2*p
+ 3)) + e*(b*d*R + a*e*S)*(m + 2*p + 4)*x, x], x], x]] /; FreeQ[{a, b, d, e
, m}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] &&  !(IGtQ[
m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(527) = 1054$.

Time = 6.31 (sec) , antiderivative size = 1104, normalized size of antiderivative = 1.86

method	result	size
elliptic	Expression too large to display	1104
default	Expression too large to display	7032

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((-1/3*(A*b*c-B*
a*d+C*a*c)/a/(a*d^2-b*c^2)/b^2*x+1/3*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/b^3
)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12/b*(
8*A*a*b*c*d^2-4*A*b^2*c^3-3*B*a^2*d^3-B*a*b*c^2*d+2*C*a^2*c*d^2+2*C*a*b*c^
3)/a^2/(a*d^2-b*c^2)^2*x+1/12*d*(5*A*a*b*d^2-A*b^2*c^2-4*B*a*b*c*d-C*a^2*d
^2+5*C*a*b*c^2)/(a*d^2-b*c^2)^2/a/b^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1
/6/b/(a*d^2-b*c^2)*(5*A*a*b*d^2-4*A*b^2*c^2-B*a*b*c*d-C*a^2*d^2+2*C*a*b*c^
2)/a^2-1/12/b*d^2*(5*A*a*b*d^2-A*b^2*c^2-4*B*a*b*c*d-C*a^2*d^2+5*C*a*b*c^2
)/(a*d^2-b*c^2)^2/a+1/6*c*(8*A*a*b*c*d^2-4*A*b^2*c^3-3*B*a^2*d^3-B*a*b*c^2
*d+2*C*a^2*c*d^2+2*C*a*b*c^3)/a^2/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*(
(x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3
-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),
((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/6*d*(8*A*a*b*c*d^
2-4*A*b^2*c^3-3*B*a^2*d^3-B*a*b*c^2*d+2*C*a^2*c*d^2+2*C*a*b*c^3)/a^2/(a*d^
2-b*c^2)^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*
b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(531) = 1062$.

Time = 0.12 (sec) , antiderivative size = 1079, normalized size of antiderivative = 1.82

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}\,(a - bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="frica
s")
```

output
```
-1/18*((B*a^3*b^2*c^3*d - 9*B*a^4*b*c*d^3 - 2*(C*a^3*b^2 - 2*A*a^2*b^3)*c^
4 + (13*C*a^4*b - 11*A*a^3*b^2)*c^2*d^2 - 3*(C*a^5 - 5*A*a^4*b)*d^4 + (B*a
*b^4*c^3*d - 9*B*a^2*b^3*c*d^3 - 2*(C*a*b^4 - 2*A*b^5)*c^4 + (13*C*a^2*b^3
 - 11*A*a*b^4)*c^2*d^2 - 3*(C*a^3*b^2 - 5*A*a^2*b^3)*d^4)*x^4 - 2*(B*a^2*b
^3*c^3*d - 9*B*a^3*b^2*c*d^3 - 2*(C*a^2*b^3 - 2*A*a*b^4)*c^4 + (13*C*a^3*b
^2 - 11*A*a^2*b^3)*c^2*d^2 - 3*(C*a^4*b - 5*A*a^3*b^2)*d^4)*x^2)*sqrt(-b*d
)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*
d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(B*a^3*b^2*c^2*d^2 + 3*B*a^4*b*d^4 -
2*(C*a^3*b^2 - 2*A*a^2*b^3)*c^3*d - 2*(C*a^4*b + 4*A*a^3*b^2)*c*d^3 + (B*a
*b^4*c^2*d^2 + 3*B*a^2*b^3*d^4 - 2*(C*a*b^4 - 2*A*b^5)*c^3*d - 2*(C*a^2*b^
3 + 4*A*a*b^4)*c*d^3)*x^4 - 2*(B*a^2*b^3*c^2*d^2 + 3*B*a^3*b^2*d^4 - 2*(C*
a^2*b^3 - 2*A*a*b^4)*c^3*d - 2*(C*a^3*b^2 + 4*A*a^2*b^3)*c*d^3)*x^2)*sqrt(
-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*
d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*
c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(2*B*a^2*b^3*c^3*d - 6*B
*a^3*b^2*c*d^3 + 3*(C*a^3*b^2 - A*a^2*b^3)*c^2*d^2 + (C*a^4*b + 7*A*a^3*b^
2)*d^4 - (B*a*b^4*c^2*d^2 + 3*B*a^2*b^3*d^4 - 2*(C*a*b^4 - 2*A*b^5)*c^3*d
- 2*(C*a^2*b^3 + 4*A*a*b^4)*c*d^3)*x^3 + (4*B*a^2*b^3*c*d^3 - (5*C*a^2*b^3
 - A*a*b^4)*c^2*d^2 + (C*a^3*b^2 - 5*A*a^2*b^3)*d^4)*x^2 + (6*A*a*b^4*c^3*
d - B*a^2*b^3*c^2*d^2 + 5*B*a^3*b^2*d^4 - 2*(2*C*a^3*b^2 + 5*A*a^2*b^3)...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}\,(a - bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="maxim
a")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="giac"
)
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{C\,x^2+Bx+A}{\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.237 $\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}(a-bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 685

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{3a\,(bc^2-ad^2)\,(a-bx^2)^{3/2}}$$

$$+\frac{\sqrt{c+dx}(2Abc(3bc^2-5ad^2)-ad(bBc^2+ad(4cC-5Bd))+(3a^2Cd^3+b^2c^2(4Bc-5Ad)+abd(c^2C-8}{6a^2\,(bc^2-ad^2)^2\sqrt{a-bx^2}}$$

$$+\frac{(3a^2Cd^3+b^2c^2(4Bc-5Ad)+abd(c^2C-8Bcd+9Ad^2))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}}\right.}{6a^{3/2}\sqrt{b}\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{(bc(4Bc+Ad)+ad(cC-5Bd))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{6a^{3/2}\sqrt{b}\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a/(-a*d^2+b*c
^2)/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(1/2)*(2*A*b*c*(-5*a*d^2+3*b*c^2)-a*d*(b*
B*c^2+a*d*(-5*B*d+4*C*c))+(3*a^2*C*d^3+b^2*c^2*(-5*A*d+4*B*c)+a*b*d*(9*A*d
^2-8*B*c*d+C*c^2))*x)/a^2/(-a*d^2+b*c^2)^2/(-b*x^2+a)^(1/2)+1/6*(3*a^2*C*d
^3+b^2*c^2*(-5*A*d+4*B*c)+a*b*d*(9*A*d^2-8*B*c*d+C*c^2))*(d*x+c)^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(-a*d^2+b*c^2)^
2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/6*(b*c*(A*d+4*B
*c)+a*d*(-5*B*d+C*c))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)
^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*
d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/b^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/
2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(
1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.40 (sec) , antiderivative size = 1968, normalized size of antiderivative = 2.87

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
(b*c*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)*(2*a*(b*c^2 - a*d^2)*(a*c*
C + b*B*c*x - a*d*(B + C*x) + A*b*(c - d*x)) + (a - b*x^2)*(4*b^2*B*c^3*x
+ a^2*d^2*(-4*c*C + 5*B*d + 3*C*d*x) + a*b*c*d*(c*C*x - B*(c + 8*d*x)) + A
*b*(b*c^2*(6*c - 5*d*x) + a*d^2*(-10*c + 9*d*x)))) - (a - b*x^2)*(4*b^3*B*
c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 5*A*b^3*c^5*d*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]] + a*b^2*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 12*a*b^2*B*c^4*d^
2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 14*a*A*b^2*c^3*d^3*Sqrt[-c + (Sqrt[a]*d
)/Sqrt[b]] + 2*a^2*b*c^3*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 8*a^2*b*B*
c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 9*a^2*A*b*c*d^5*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]] - 3*a^3*c*C*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*b^3*B*c^5
*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 10*A*b^3*c^4*d*Sqrt[-c + (Sqrt
[a]*d)/Sqrt[b]]*(c + d*x) - 2*a*b^2*c^4*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*(c + d*x) + 16*a*b^2*B*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) -
 18*a*A*b^2*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) - 6*a^2*b*c^2
*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 4*b^3*B*c^4*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 5*A*b^3*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[
b]]*(c + d*x)^2 + a*b^2*c^3*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2
 - 8*a*b^2*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 9*a*A*b^
2*c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 + 3*a^2*b*c*C*d^3*Sqrt[
-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 2351}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \frac{B+Cx}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 638}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \frac{B+Cx}{\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{\int -\frac{b(4bBc^2+ad(cC-5Bd)+3d(bBc-aCd)x)}{2\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{3ab\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\int \frac{4bBc^2+ad(cC-5Bd)+3d(bBc-aCd)x}{\sqrt{c+dx}(a-bx^2)^{3/2}}dx}{6a\left(bc^2-ad^2\right)}+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 686

$$\frac{-\frac{\int \frac{bd\left(ad\left(bBc^2+ad(4cC-5Bd)\right)+\left(4b^2Bc^3+abd(cC-8Bd)c+3a^2Cd^3\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{ab(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(ad(4cC-5Bd)+bBc^2)-x(3a^2Cd^3+abcd(cC-8Bd)+4b^2}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 27

$$\frac{-\frac{d\int \frac{ad\left(bBc^2+ad(4cC-5Bd)\right)+\left(4b^2Bc^3+abd(cC-8Bd)c+3a^2Cd^3\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(ad(4cC-5Bd)+bBc^2)-x(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc}{a\sqrt{a-bx^2}(bc^2-ad^2)}}{6a\left(bc^2-ad^2\right)}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 600

$$\frac{-\frac{d\left(\frac{\left(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc^3\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}-\frac{\left(bc^2-ad^2\right)\left(ad(cC-5Bd)+4bBc^2\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(ad(4cC-5Bd)+bBc^2}{a\sqrt{a-}}}{6a\left(bc^2-ad^2\right)}$$

$$A\int \frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\Big\downarrow$ 509

$$-\frac{d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc^3\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{\left(bc^2-ad^2\right)\left(ad(cC-5Bd)+4bBc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}-\frac{\sqrt{c+dx}(ad(ad(4cC-5B}{}$$

$$6a\left(bc^2-ad^2\right)$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 508

$$-\frac{d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc^3\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{bc}}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{\left(bc^2-ad^2\right)\left(ad(cC-5Bd)+4bBc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}\right)}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 327

$$-\frac{d\left(-\frac{\left(bc^2-ad^2\right)\left(ad(cC-5Bd)+4bBc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{bc}}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{2a(bc^2-ad^2)}$$

$$6a\left(bc^2-ad^2\right)$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$\downarrow$ 512

$$-\frac{-\frac{d\left(-\frac{\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\left(ad(cC-5Bd)+4bBc^2\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3+abcd(cC-8Bd)+4b^2Bc^3\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\ldots\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\ldots\right)}{2a(bc^2-ad^2)}\ldots}{6a\left(bc^2-ad^2\right)}$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 511$$

$$-\frac{-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad(cC-5Bd)+4bBc^2\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3+abcd(cC\ldots\right.}{\sqrt{b}d\sqrt{\ldots}}\ldots\right)}{2a(bc^2-ad^2)}\ldots}{6a\left(bc^2-ad^2\right)}$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

$$\Big\downarrow \; 321$$

$$A\int\frac{1}{x\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+$$

$$-\frac{-\frac{d\left(\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\left(bc^2-ad^2\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad(cC-5Bd)+4bBc^2\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3a^2Cd^3+abcd(cC-8Bd)+\ldots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\ldots}}\ldots\right)}{2a(bc^2-ad^2)}\ldots}{6a\left(bc^2-ad^2\right)}$$

$$\frac{\sqrt{c+dx}(x(bBc-aCd)+a(cC-Bd))}{3a\left(a-bx^2\right)^{3/2}\left(bc^2-ad^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/(x*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1318 vs. $2(598) = 1196$.

Time = 7.66 (sec) , antiderivative size = 1319, normalized size of antiderivative = 1.93

method	result	size
elliptic	Expression too large to display	1319
default	Expression too large to display	9313

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output

```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((1/3*(A*b*d-B*b
*c+C*a*d)/(a*d^2-b*c^2)/b^2/a*x-1/3*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b^2/
a)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12/a^2
*(9*A*a*b*d^3-5*A*b^2*c^2*d-8*B*a*b*c*d^2+4*B*b^2*c^3+3*C*a^2*d^3+C*a*b*c^
2*d)/(a*d^2-b*c^2)^2/b*x-1/12*(10*A*a*b*c*d^2-6*A*b^2*c^3-5*B*a^2*d^3+B*a*
b*c^2*d+4*C*a^2*c*d^2)/(a*d^2-b*c^2)^2/a^2/b)/((x^2-a/b)*(-b*d*x-b*c))^(1/
2)+2*(-1/6/a^2*(A*b*c*d-5*B*a*d^2+4*B*b*c^2+C*a*c*d)/(a*d^2-b*c^2)+1/12*d*
(10*A*a*b*c*d^2-6*A*b^2*c^3-5*B*a^2*d^3+B*a*b*c^2*d+4*C*a^2*c*d^2)/(a*d^2-
b*c^2)^2/a^2-1/6*c/a^2*(9*A*a*b*d^3-5*A*b^2*c^2*d-8*B*a*b*c*d^2+4*B*b^2*c^
3+3*C*a^2*d^3+C*a*b*c^2*d)/(a*d^2-b*c^2)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)
/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^
2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))-1/6*d/a^2*(9*A*a*b*d^3-5*A
*b^2*c^2*d-8*B*a*b*c*d^2+4*B*b^2*c^3+3*C*a^2*d^3+C*a*b*c^2*d)/(a*d^2-b*c^2
)^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a
*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2
))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}x}\,dx$$

input

```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="fri
cas")
```

output

```
integral(-(C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)/(b^3*d*x^8 + b^
3*c*x^7 - 3*a*b^2*d*x^6 - 3*a*b^2*c*x^5 + 3*a^2*b*d*x^4 + 3*a^2*b*c*x^3 -
a^3*d*x^2 - a^3*c*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}\,x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}\,x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="gia
c")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.238 $\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 783

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \frac{\sqrt{c+dx}\left(bBc-Abd-aCd+b\left(\frac{Abc}{a}+cC-Bd\right)x\right)}{3a\,(bc^2-ad^2)\,(a-bx^2)^{3/2}}$$
$$+\frac{\sqrt{c+dx}(a(5a^2Cd^3+b^2c^2(6Bc-7Ad)-abd(c^2C+10Bcd-11Ad^2))+b(2Abc(5bc^2-7ad^2)-a(ad^2(8\ldots}{6a^3\,(bc^2-ad^2)^2\sqrt{a-bx^2}}$$
$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{a^3cx}$$
$$+\frac{\sqrt{b}(2A(8b^2c^4-13abc^2d^2+3a^2d^4)-ac(ad^2(8cC-9Bd)-bc^2(4cC-5Bd)))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\ldots\right.}{6a^{5/2}c\,(bc^2-ad^2)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{(Ab(16bc^2-17ad^2)-a(5aCd^2-bc(4cC+Bd)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{\ldots}}\right.}{6a^{5/2}\sqrt{b}\,(bc^2-ad^2)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{\left(2B-\frac{Ad}{c}\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{a^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*(d*x+c)^(1/2)*(B*b*c-A*b*d-a*C*d+b*(A*b*c/a+C*c-B*d)*x)/a/(-a*d^2+b*c^
2)/(-b*x^2+a)^(3/2)+1/6*(d*x+c)^(1/2)*(a*(5*a^2*C*d^3+b^2*c^2*(-7*A*d+6*B*
c)-a*b*d*(-11*A*d^2+10*B*c*d+C*c^2))+b*(2*A*b*c*(-7*a*d^2+5*b*c^2)-a*(a*d^
2*(-9*B*d+8*C*c)-b*c^2*(-5*B*d+4*C*c)))*x)/a^3/(-a*d^2+b*c^2)^2/(-b*x^2+a)
^(1/2)-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^3/c/x+1/6*b^(1/2)*(2*A*(3*a^2*d^
4-13*a*b*c^2*d^2+8*b^2*c^4)-a*c*(a*d^2*(-9*B*d+8*C*c)-b*c^2*(-5*B*d+4*C*c)
))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(5/2)/c/(
-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/6
*(A*b*(-17*a*d^2+16*b*c^2)-a*(5*a*C*d^2-b*c*(B*d+4*C*c)))*((d*x+c)/(c+a^(1
/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(5/2
)/b^(1/2)/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-(2*B-A*d/c)*((d*x+
c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(
1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^
(1/2))/a^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.45 (sec) , antiderivative size = 2943, normalized size of antiderivative = 3.76

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
Sqrt[c + d*x]*Sqrt[a - b*x^2]*(-(A/(a^3*c*x)) + (-(a*b*B*c) + a*A*b*d + a^
2*C*d - A*b^2*c*x - a*b*c*C*x + a*b*B*d*x)/(3*a^2*(-(b*c^2) + a*d^2)*(-a +
 b*x^2)^2) + (-6*a*b^2*B*c^3 + 7*a*A*b^2*c^2*d + a^2*b*c^2*C*d + 10*a^2*b*
B*c*d^2 - 11*a^2*A*b*d^3 - 5*a^3*C*d^3 - 10*A*b^3*c^3*x - 4*a*b^2*c^3*C*x
+ 5*a*b^2*B*c^2*d*x + 14*a*A*b^2*c*d^2*x + 8*a^2*b*c*C*d^2*x - 9*a^2*b*B*d
^3*x)/(6*a^3*(-(b*c^2) + a*d^2)^2*(-a + b*x^2))) - (d*Sqrt[a - (b*(c + d*x
)^2*(-1 + c/(c + d*x))^2)/d^2]*(-16*A*b^3*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]] - 4*a*b^2*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 5*a*b^2*B*c^4*d*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]] + 26*a*A*b^2*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]] + 8*a^2*b*c^3*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 9*a^2*b*B*c^2*d^3*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 6*a^2*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqr
t[b]] - (16*A*b^3*c^7*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (4*a*b
^2*c^7*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (5*a*b^2*B*c^6*d*Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (42*a*A*b^2*c^5*d^2*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (12*a^2*b*c^5*C*d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]])/(c + d*x)^2 - (14*a^2*b*B*c^4*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]])/(c + d*x)^2 - (32*a^2*A*b*c^3*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c
 + d*x)^2 - (8*a^3*c^3*C*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 +
 (9*a^3*B*c^2*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (6*a^3*A*c
*d^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (32*A*b^3*c^6*Sqrt[-...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a-bx^2\right)^{5/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \; 2355$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^2\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 638$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^2\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 7293$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \left(\frac{\sqrt{c+dx}C}{dx\left(a-bx^2\right)^{5/2}}+\frac{(Bd-cC)\sqrt{c+dx}}{d^2x^2\left(a-bx^2\right)^{5/2}}\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \left(\frac{C\sqrt{c+dx}}{dx\left(a-bx^2\right)^{5/2}}-\frac{(cC-Bd)\sqrt{c+dx}}{d^2x^2\left(a-bx^2\right)^{5/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)}{d^2x^2\left(a-bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^2\left(a-bx^2\right)^{5/2}}dx}{d^2}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^2\left(a-bx^2\right)^{5/2}}dx}{d^2}$$

$$\Big\downarrow \text{7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}}{x^2\left(a-bx^2\right)^{5/2}}-\frac{Cd\sqrt{c+dx}}{x\left(a-bx^2\right)^{5/2}}\right)dx}{d^2}$$

$$\Big\downarrow \text{7296}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+2d^3\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-2d^3\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\sqrt{a-bx^2}\left(ad^2-bd^2x^2\right)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2011}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2(a-bx^2)^{5/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2091}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-$$

$$\frac{2\int \left(-\frac{(-bCc^2+aCd^2+bBd(c+dx))d^2}{a(-bc^2+2b(c+dx)c+ad^2-b(c+dx)^2)^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{B}{a^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{\cdots}{a^2(-bc^2+2b(c+\cdots}\right.}{d}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-\frac{2\int \frac{d^4(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^2(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2)^3}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-$$

$$2d^5\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$

$$2d^5\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2019}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+2d^5\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^8x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-2d^5\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^8x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-$$

$$\frac{2\int \left(-\frac{\left(-bCc^2+aCd^2+bBd(c+dx)\right)d^2}{a\left(-bc^2+2b(c+dx)c+ad^2-b(c+dx)^2\right)^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{B}{a^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{\cdots}{a^2\left(-bc^2+2b(c+\cdots\right.}\right.}{d}$$

$$\Big\downarrow \text{7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{2\int \frac{d^4(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-2d^5\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+2d^5\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 2019}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+2d^5\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^8x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-2d^5\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^8x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{2\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^2x^2\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 2248}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{2\int \left(-\frac{\left(-bCc^2+aCd^2+bBd(c+dx)\right)d^2}{a\left(-bc^2+2b(c+dx)c+ad^2-b(c+dx)^2\right)^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{B}{a^2x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{}{a^2(-bc^2+2b(c+}\right.}{d}$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^2\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-\frac{2\int \frac{d^4(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-$$
$$2d^5\int-\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int\frac{1}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$
$$2d^5\int\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^2x^2\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. 2(692) = 1384.

Time = 13.26 (sec) , antiderivative size = 1412, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	1412
risch	Expression too large to display	2044
default	Expression too large to display	12500

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*((-1/3*(A*b*c-B*
a*d+C*a*c)/b/a^2/(a*d^2-b*c^2)*x+1/3*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/b^2
/a)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(-1/12*(
14*A*a*b*c*d^2-10*A*b^2*c^3-9*B*a^2*d^3+5*B*a*b*c^2*d+8*C*a^2*c*d^2-4*C*a*
b*c^3)/a^3/(a*d^2-b*c^2)^2*x+1/12*(11*A*a*b*d^3-7*A*b^2*c^2*d-10*B*a*b*c*d
^2+6*B*b^2*c^3+5*C*a^2*d^3-C*a*b*c^2*d)/(a*d^2-b*c^2)^2/b/a^2)/((x^2-a/b)*
(-b*d*x-b*c))^(1/2)-A/c/a^3/x*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*(1/6/(a
*d^2-b*c^2)*(11*A*a*b*d^2-10*A*b^2*c^2-B*a*b*c*d+5*C*a^2*d^2-4*C*a*b*c^2)/
a^3-1/12*d*(11*A*a*b*d^3-7*A*b^2*c^2*d-10*B*a*b*c*d^2+6*B*b^2*c^3+5*C*a^2*
d^3-C*a*b*c^2*d)/(a*d^2-b*c^2)^2/a^2+1/6*b*c*(14*A*a*b*c*d^2-10*A*b^2*c^3-
9*B*a^2*d^3+5*B*a*b*c^2*d+8*C*a^2*c*d^2-4*C*a*b*c^3)/a^3/(a*d^2-b*c^2)^2)*
(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^
(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^
(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1
/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/
2))+2*(1/12*b*d*(14*A*a*b*c*d^2-10*A*b^2*c^3-9*B*a^2*d^3+5*B*a*b*c^2*d+8*C
*a^2*c*d^2-4*C*a*b*c^3)/(a*d^2-b*c^2)^2/a^3-1/2*A*b*d/c/a^3)*(c/d-1/b*(a*b
)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)
/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE((...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}\,x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.239 $\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 845

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}\,dx = \frac{b\sqrt{c+dx}(Abc+acC-aBd+(bBc-Abd-aCd)x)}{3a^2\left(bc^2-ad^2\right)\left(a-bx^2\right)^{3/2}}$$
$$+\frac{b\sqrt{c+dx}(4Abc(3bc^2-4ad^2)-a(ad^2(10cC-11Bd)-bc^2(6cC-7Bd))+(9a^2Cd^3+b^2c^2(10Bc-11A}{6a^3\left(bc^2-ad^2\right)^2\sqrt{a-bx^2}}$$
$$-\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2a^3cx^2}-\frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4a^3c^2x}$$
$$+\frac{\sqrt{b}(b^2c^4(32Bc-31Ad)-2abc^2d(5c^2C+26Bcd-24Ad^2)+3a^2d^3(6c^2C+4Bcd-3Ad^2))\sqrt{c+dx}\sqrt{a-}}{12a^{5/2}c^2\left(bc^2-ad^2\right)^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{\sqrt{b}(bc^2(32Bc-Ad)+ad(2c^2C-34Bcd+3Ad^2))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}}{\sqrt{bc}+\sqrt{}}\right.}{12a^{5/2}c\left(bc^2-ad^2\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$-\frac{\left(\frac{20Ab}{a}+8C-\frac{d(4Bc-3Ad)}{c^2}\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4a^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
1/3*b*(d*x+c)^(1/2)*(A*b*c+C*a*c-B*a*d+(-A*b*d+B*b*c-C*a*d)*x)/a^2/(-a*d^2
+b*c^2)/(-b*x^2+a)^(3/2)+1/6*b*(d*x+c)^(1/2)*(4*A*b*c*(-4*a*d^2+3*b*c^2)-a
*(a*d^2*(-11*B*d+10*C*c)-b*c^2*(-7*B*d+6*C*c))+(9*a^2*C*d^3+b^2*c^2*(-11*A
*d+10*B*c)-a*b*d*(-15*A*d^2+14*B*c*d+5*C*c^2))*x)/a^3/(-a*d^2+b*c^2)^2/(-b
*x^2+a)^(1/2)-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^3/c/x^2-1/4*(-3*A*d+4
*B*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^3/c^2/x+1/12*b^(1/2)*(b^2*c^4*(-31*
A*d+32*B*c)-2*a*b*c^2*d*(-24*A*d^2+26*B*c*d+5*C*c^2)+3*a^2*d^3*(-3*A*d^2+4
*B*c*d+6*C*c^2))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/a^(5/2)/c^2/(-a*d^2+b*c^2)^2/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*
x^2+a)^(1/2)-1/12*b^(1/2)*(b*c^2*(-A*d+32*B*c)+a*d*(3*A*d^2-34*B*c*d+2*C*c
^2))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(
1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1
/2)*d))^(1/2))/a^(5/2)/c/(-a*d^2+b*c^2)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/4
*(20*A*b/a+8*C-d*(-3*A*d+4*B*c)/c^2)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)
*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2
,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^2/(d*x+c)^(1/2)/(-b*x^
2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.74 (sec) , antiderivative size = 9327, normalized size of antiderivative = 11.04

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3(a-bx^2)^{5/2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 2355}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^3(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 638}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\int \frac{\left(\frac{B}{d}+\frac{Cx}{d}-\frac{cC}{d^2}\right)\sqrt{c+dx}}{x^3(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\int \left(\frac{\sqrt{c+dx}C}{dx^2(a-bx^2)^{5/2}}+\frac{(Bd-cC)\sqrt{c+dx}}{d^2x^3(a-bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 7293}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\int \left(\frac{C\sqrt{c+dx}}{dx^2(a-bx^2)^{5/2}}-\frac{(cC-Bd)\sqrt{c+dx}}{d^2x^3(a-bx^2)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 7239}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\int \frac{\sqrt{c+dx}(-cC+dxC+Bd)}{d^2x^3(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}(a-bx^2)^{5/2}}dx+\frac{\int -\frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^3(a-bx^2)^{5/2}}dx}{d^2}$$

$$\Big\downarrow \text{ 25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-\frac{\int \frac{\sqrt{c+dx}(cC-dxC-Bd)}{x^3(a-bx^2)^{5/2}}dx}{d^2}$$

$$\Big\downarrow \; 7293$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-\frac{\int \left(\frac{(cC-Bd)\sqrt{c+dx}}{x^3(a-bx^2)^{5/2}}-\frac{Cd\sqrt{c+dx}}{x^2(a-bx^2)^{5/2}}\right)dx}{d^2}$$

$$\Big\downarrow \; 7296$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+ 2d^4\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\sqrt{a-bx^2}\,(ad^2-bd^2x^2)^2}d\sqrt{c+dx}$$

$$\Big\downarrow \; 2011$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+ 2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\,(a-bx^2)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \; 2091$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+ 2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \; 2248$$

$$2\int \left(\frac{B}{a^2dx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bc(cC-Bd)-aCd^2}{a^3d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^3d^2\,(bc^2-2b(c+d}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx$$

$$\Big\downarrow \; 7239$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+ 2\int -\frac{d^3(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \; 27$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+$$
$$2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-$$
$$2d^6\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$\downarrow$ 2019

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx-$$
$$2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^9x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+$$
$$2d^6\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^9x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$\downarrow$ 27

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$\downarrow$ 2248

$$2\int \left(\frac{B}{a^2dx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bc(cC-Bd)-aCd^2}{a^3d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^3d^2\left(bc^2-2b(c+d\right.}\right.$$
$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\left(a-bx^2\right)^{5/2}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$
$$2\int -\frac{d^3(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$
$$2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-$$
$$2d^6\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2019}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-$$
$$2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^9x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$
$$2d^6\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^9x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+$$
$$2\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^3x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

$$\Big\downarrow \text{2248}$$

$$2\int \left(\frac{B}{a^2dx^2\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}-\frac{2bc(cC-Bd)-aCd^2}{a^3d^3x\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}+\frac{b(-((cC-Bd}{a^3d^2\left(bc^2-2b(c+d\right.}\right.$$

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx$$

$\downarrow$ 7239

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+2\int -\frac{d^3(c+dx)(-2cC+(c+dx)C+Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$\downarrow$ 27

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx+2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$\downarrow$ 25

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-2d^6\int -\frac{(c+dx)(2cC-(c+dx)C-Bd)\sqrt{-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a}}{d^3x^3\left(bc^2-2b(c+dx)c-ad^2+b(c+dx)^2\right)^3}d\sqrt{c+dx}$$

$\downarrow$ 2019

$$\left(A+\frac{c(cC-Bd)}{d^2}\right)\int \frac{1}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}dx-2d^6\int \frac{(c+dx)(2cC-(c+dx)C-Bd)}{d^9x^3\left(-\frac{bc^2}{d^2}+\frac{2b(c+dx)c}{d^2}-\frac{b(c+dx)^2}{d^2}+a\right)^{5/2}}d\sqrt{c+dx}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*Sqrt[c + d*x]*(a - b*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 638
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Unintegrable[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x] /; FreeQ
[{a, b, c, d, e, m, n, p}, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2248
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)
^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + b*x^2
+ c*x^4], Px*(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^(p + 1/2), x], x] /;
 FreeQ[{a, b, c, d, e, f, m}, x] && PolyQ[Px, x] && IntegerQ[p + 1/2] && In
tegerQ[q]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

rule 7296
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 14.02 (sec) , antiderivative size = 1478, normalized size of antiderivative = 1.75

method	result	size
elliptic	Expression too large to display	1478
risch	Expression too large to display	2445
default	Expression too large to display	16351

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x,method=_RETURNVERBO
SE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/2*A/c/a^3/x^
2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4/a^3*(3*A*d-4*B*c)/c^2*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)/x+(1/3*(A*b*d-B*b*c+C*a*d)/(a*d^2-b*c^2)/b/a^2*x-1
/3*(A*b*c-B*a*d+C*a*c)/(a*d^2-b*c^2)/b/a^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(
1/2)/(x^2-a/b)^2-2*(-b*d*x-b*c)*(1/12/a^3*(15*A*a*b*d^3-11*A*b^2*c^2*d-14*
B*a*b*c*d^2+10*B*b^2*c^3+9*C*a^2*d^3-5*C*a*b*c^2*d)/(a*d^2-b*c^2)^2*x-1/12
*(16*A*a*b*c*d^2-12*A*b^2*c^3-11*B*a^2*d^3+7*B*a*b*c^2*d+10*C*a^2*c*d^2-6*
C*a*b*c^3)/a^3/(a*d^2-b*c^2)^2)/((x^2-a/b)*(-b*d*x-b*c))^(1/2)+2*(1/4*A*b*
d/c/a^3-1/6*b/a^3*(A*b*c*d-11*B*a*d^2+10*B*b*c^2+C*a*c*d)/(a*d^2-b*c^2)+1/
12*b*d*(16*A*a*b*c*d^2-12*A*b^2*c^3-11*B*a^2*d^3+7*B*a*b*c^2*d+10*C*a^2*c*
d^2-6*C*a*b*c^3)/a^3/(a*d^2-b*c^2)^2-1/6*b*c/a^3*(15*A*a*b*d^3-11*A*b^2*c^
2*d-14*B*a*b*c*d^2+10*B*b^2*c^3+9*C*a^2*d^3-5*C*a*b*c^2*d)/(a*d^2-b*c^2)^2
)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b
)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b
)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+2*(1/8*(3*A*d-4*B*c)/a^3*b*d/c^2-1/12*b*d/a^3*(15*A*a*b*d^3-11*A*b^2
*c^2*d-14*B*a*b*c*d^2+10*B*b^2*c^3+9*C*a^2*d^3-5*C*a*b*c^2*d)/(a*d^2-b*c^2
)^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(
a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(d*x+c)**(1/2)/(-b*x**2+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="m
axima")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{Cx^2+Bx+A}{(-bx^2+a)^{\frac{5}{2}}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x, algorithm="g
iac")
```

output
```
integrate((C*x^2 + B*x + A)/((-b*x^2 + a)^(5/2)*sqrt(d*x + c)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^3\,(a-b\,x^2)^{5/2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(a - b*x^2)^(5/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{c+dx}\,(a-bx^2)^{5/2}}\,dx = \int \frac{C\,x^2+Bx+A}{x^3\sqrt{dx+c}\,(-b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(5/2),x)
```

3.240 $\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 800

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2(675a^2d^4F+b^2(1881c^2Cd^2-1617Bcd^3+1155Ad^4-2057c^3dD+2185c^4F)+3abd^2(275Cd^2-c(4}{3465b^3d^5}$$
$$-\frac{2(ad^2(539dD-1282cF)-b(1584cCd^2-693Bd^3-2563c^2dD+3590c^3F))(c+dx)^{3/2}\sqrt{a-bx^2}}{3465b^2d^5}$$
$$-\frac{2(81ad^2F+b(99Cd^2-c(319dD-668cF)))(c+dx)^{5/2}\sqrt{a-bx^2}}{693b^2d^5}$$
$$-\frac{2(11dD-46cF)(c+dx)^{7/2}\sqrt{a-bx^2}}{99bd^5}-\frac{2F(c+dx)^{9/2}\sqrt{a-bx^2}}{11bd^5}$$
$$-\frac{2\sqrt{a}(3a^2d^4(539dD-382cF)-3abd^2(484cCd^2-693Bd^3-396c^2dD+344c^3F)-2b^2c(792c^2Cd^2-}{3465b^{5/2}d^6\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}}{\sqrt{b}}}}}$$
$$-\frac{2\sqrt{a}(675a^3d^6F+2b^3c^2(792c^2Cd^2-924Bcd^3+1155Ad^4-704c^3dD+640c^4F)+ab^2d^2(1056c^2Cd^2-}{}$$

output
```
-2/3465*(675*a^2*d^4*F+b^2*(1155*A*d^4-1617*B*c*d^3+1881*C*c^2*d^2-2057*D*
c^3*d+2185*F*c^4)+3*a*b*d^2*(275*C*d^2-c*(407*D*d-491*F*c)))*(d*x+c)^(1/2)
*(-b*x^2+a)^(1/2)/b^3/d^5-2/3465*(a*d^2*(539*D*d-1282*F*c)-b*(-693*B*d^3+1
584*C*c*d^2-2563*D*c^2*d+3590*F*c^3))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d
^5-2/693*(81*a*d^2*F+b*(99*C*d^2-c*(319*D*d-668*F*c)))*(d*x+c)^(5/2)*(-b*x
^2+a)^(1/2)/b^2/d^5-2/99*(11*D*d-46*F*c)*(d*x+c)^(7/2)*(-b*x^2+a)^(1/2)/b/
d^5-2/11*F*(d*x+c)^(9/2)*(-b*x^2+a)^(1/2)/b/d^5-2/3465*a^(1/2)*(3*a^2*d^4*
(539*D*d-382*F*c)-3*a*b*d^2*(-693*B*d^3+484*C*c*d^2-396*D*c^2*d+344*F*c^3)
-2*b^2*c*(1155*A*d^4-924*B*c*d^3+792*C*c^2*d^2-704*D*c^3*d+640*F*c^4))*(d*
x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^6/((d*x
+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/3465*a^(1/2)*(675*a^3*
d^6*F+2*b^3*c^2*(1155*A*d^4-924*B*c*d^3+792*C*c^2*d^2-704*D*c^3*d+640*F*c^
4)+a*b^2*d^2*(1155*A*d^4-1617*B*c*d^3+1056*C*c^2*d^2-836*D*c^3*d+712*F*c^4
)+3*a^2*b*d^4*(275*C*d^2-c*(407*D*d-266*F*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/
2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*
2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(7/2)/d^6/(d*x+
c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.87 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.19

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(3a^2d^4(-539dD+382cF)+3abd^2(484cCd^2-693Bd^3-396c^2dD+344c^3F)+2b^2c(792c^2C\right.}{}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3 + F*x^4))/(Sqrt[c + d*x]*Sqrt[a -
b*x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(3*a^2*d^4*(-539*d*D + 382*c*F) + 3*a*b*d^2*(484*c*C*d^
2 - 693*B*d^3 - 396*c^2*d*D + 344*c^3*F) + 2*b^2*c*(792*c^2*C*d^2 - 924*B*
c*d^3 + 1155*A*d^4 - 704*c^3*d*D + 640*c^4*F) - (c + d*x)*(675*a^2*d^4*F +
 a*b*d^2*(825*C*d^2 + 596*c^2*F - 2*c*d*(341*D + 236*F*x) + d^2*x*(539*D +
 405*F*x)) + b^2*(640*c^4*F - 32*c^3*d*(22*D + 15*F*x) + 8*c^2*d^2*(99*C +
 66*D*x + 50*F*x^2) - 2*c*d^3*(462*B + x*(297*C + 5*x*(44*D + 35*F*x))) +
d^4*(1155*A + x*(693*B + 5*x*(99*C + 77*D*x + 63*F*x^2)))))) - (I*Sqrt[b]*(
Sqrt[b]*c - Sqrt[a]*d)*(3*a^2*d^4*(-539*d*D + 382*c*F) + 3*a*b*d^2*(484*c*
C*d^2 - 693*B*d^3 - 396*c^2*d*D + 344*c^3*F) + 2*b^2*c*(792*c^2*C*d^2 - 92
4*B*c*d^3 + 1155*A*d^4 - 704*c^3*d*D + 640*c^4*F))*Sqrt[(d*(Sqrt[a]/Sqrt[b
] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x
)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]],
 (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*(-a + b*x^2)) - (I*Sqrt[a]*(675*a^(5/2)*d^5*F - 3*a^2*Sqrt[b]
*d^4*(539*d*D - 382*c*F) + 3*a*b^(3/2)*d^2*(484*c*C*d^2 - 693*B*d^3 - 396*
c^2*d*D + 344*c^3*F) + Sqrt[a]*b^2*d*(-396*c^2*C*d^2 + 462*B*c*d^3 + 1155*
A*d^4 + 352*c^3*d*D - 320*c^4*F) + 2*b^(5/2)*c*(792*c^2*C*d^2 - 924*B*c*d^
3 + 1155*A*d^4 - 704*c^3*d*D + 640*c^4*F) + 3*a^(3/2)*b*d^3*(275*C*d^2 + 4
*c*(33*d*D - 29*c*F)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((
Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSi...
```

Rubi [A] (verified)

Time = 5.29 (sec) , antiderivative size = 834, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2185, 27, 2185, 27, 2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2+Dx^3+Fx^4\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{bd^5(11dD-46cF)x^5+d^4\left(-74bFc^2+11bCd^2+9ad^2F\right)x^4+d^3\left(-56bFc^3+36ad^2Fc+11bBd^3\right)x^3+d^2\left(-19bFc^4+54ad^2Fc^2+11Abd^4\right)x^2-}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{11bd^6}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{bd^5(11dD-46cF)x^5+d^4\left(-74bFc^2+11bCd^2+9ad^2F\right)x^4+d^3\left(-56bFc^3+36ad^2Fc+11bBd^3\right)x^3+d^2\left(-19bFc^4+54ad^2Fc^2+11Abd^4\right)x^2-2c^3d(}{\sqrt{c+dx}\sqrt{a-bx^2}}}{11bd^6}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 2185

$$-\frac{\frac{2\int -\frac{b\left(99bCd^2+81aFd^2-bc(319dD-668cF)\right)x^4d^9+b\left(99bBd^3+a(77dD+2cF)d^2-3bc^2(121dD-338cF)\right)x^3d^8+3b\left(33Abd^4+ac(77dD-160cF)d^2-bc^3(55dD-\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{9bd^5}}{11bd^6}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{b\left(99bCd^2+81aFd^2-bc(319dD-668cF)\right)x^4d^9+b\left(99bBd^3+a(77dD+2cF)d^2-3bc^2(121dD-338cF)\right)x^3d^8+3b\left(33Abd^4+ac(77dD-160cF)d^2-bc^3(55dD-173cF\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{9bd^5}}{11bd^6}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 2185

$$-\frac{2\int -\frac{b^2\left(ad^2(539dD-1282cF)-b\left(3590Fc^3-2563dDc^2+1584Cd^2c-693Bd^3\right)\right)x^3d^{12}+b\left(405a^2Fd^4+ab\left(495Cd^2+c(22dD-911cF)\right)d^2-b^2\left(3715Fc^4-2354dDc^3\right.\right.}{}}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{b^2\left(ad^2(539dD-1282cF)-b\left(3590Fc^3-2563dDc^2+1584Cd^2c-693Bd^3\right)\right)x^3d^{12}+b\left(405a^2Fd^4+ab\left(495Cd^2+c(22dD-911cF)\right)d^2-b^2\left(3715Fc^4-2354dDc^3+10\right.\right.}{}}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$$\Big\downarrow \; 2185$$

$$-\frac{2\int -\frac{3\left(b^2\left(675a^2Fd^4+3ab\left(275Cd^2-c(407dD-491cF)\right)d^2+b^2\left(2185Fc^4-2057dDc^3+1881Cd^2c^2-1617Bd^3c+1155Ad^4\right)\right)x^2d^{14}+ab^2c\left(ad^2(539dD-607cF)-b\right.}{}}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$$\Big\downarrow \; 27$$

$$\frac{3\int \frac{b^2\left(675a^2Fd^4+3ab\left(275Cd^2-c(407dD-491cF)\right)d^2+b^2\left(2185Fc^4-2057dDc^3+1881Cd^2c^2-1617Bd^3c+1155Ad^4\right)\right)x^2d^{14}+ab^2c\left(ad^2(539dD-607cF)-b\left(835\right.\right.}{\sqrt{c+}}}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$$\Big\downarrow \; 2185$$

$$3\left(-\frac{2\int -\frac{b^2d^{15}\left(ad\left(675a^2Fd^4+3ab\left(275Cd^2+4c(33dD-29cF)\right)d^2-b^2\left(320Fc^4-352dDc^3+396Cd^2c^2-462Bd^3c-1155Ad^4\right)\right)+b\left(3a^2(539dD-382cF)d^4-3ab\left(3\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$$\Big\downarrow \; 27$$

$$3\left(\frac{1}{3}bd^{13}\int \frac{ad\left(675a^2Fd^4+3ab\left(275Cd^2+4c(33dD-29cF)\right)d^2-b^2\left(320Fc^4-352dDc^3+396Cd^2c^2-462Bd^3c-1155Ad^4\right)\right)+b\left(3a^2(539dD-382cF)d^4-3ab\left(344Fc\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$$\Big\downarrow \; 600$$

$$3\left(\frac{1}{3}bd^{13}\left(\frac{b\left(3a^2d^4(539dD-382cF)-3abd^2\left(-693Bd^3+344c^3F-396c^2dD+484cCd^2\right)-2b^2c\left(1155Ad^4-924Bcd^3+640c^4F-704c^3dD+792c^2Cd^2\right)\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}d}{d}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\downarrow$ 509

$$3\left(\frac{1}{3}bd^{13}\left(\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(539dD-382cF)-3abd^2\left(-693Bd^3+344c^3F-396c^2dD+484cCd^2\right)-2b^2c\left(1155Ad^4-924Bcd^3+640c^4F-704c^3dD+792c^2Cd^2\right)\right)\int}{d\sqrt{a-bx^2}}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\downarrow$ 508

$$3\left(\frac{1}{3}bd^{13}\left(\frac{\left(675a^3d^6F+3a^2bd^4\left(275Cd^2-c(407dD-266cF)\right)+ab^2d^2\left(1155Ad^4-1617Bcd^3+712c^4F-836c^3dD+1056c^2Cd^2\right)+2b^3c^2\left(1155Ad^4-924Bcd^3+640c^4\right.\right.}{d}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\downarrow$ 327

$$3\left(\frac{1}{3}bd^{13}\left(\frac{\left(675a^3d^6F+3a^2bd^4\left(275Cd^2-c(407dD-266cF)\right)+ab^2d^2\left(1155Ad^4-1617Bcd^3+712c^4F-836c^3dD+1056c^2Cd^2\right)+2b^3c^2\left(1155Ad^4-924Bcd^3+640c^4\right.\right.}{d}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 512

$$3\left(\frac{1}{3}bd^{13}\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(675a^3d^6F+3a^2bd^4\left(275Cd^2-c(407dD-266cF)\right)+ab^2d^2\left(1155Ad^4-1617Bcd^3+712c^4F-836c^3dD+1056c^2Cd^2\right)+2b^3c^2\left(1155Ad^4-924Bc\right.\right.}{d\sqrt{a-bx^2}}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{9/2}}{11bd^5}$$

$\Big\downarrow$ 511

$$3\left(\frac{1}{3}bd^{13}\left(-\frac{2\sqrt{a}\sqrt{b}\left(3a^2(539dD-382cF)d^4-3ab\left(344Fc^3-396dDc^2+484Cd^2c-693Bd^3\right)d^2-2b^2c\left(640Fc^4-704dDc^3+792Cd^2c^2-924Bd^3c+1155Ad^4\right)\right)\sqrt{c+d}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.\right.$$

$$\frac{2F(c+dx)^{9/2}\sqrt{a-bx^2}}{11bd^5}$$

$\Big\downarrow$ 321

$$3\left(\frac{1}{3}bd^{13}\left(-\frac{2\sqrt{a}\sqrt{b}\left(3a^2(539dD-382cF)d^4-3ab\left(344Fc^3-396dDc^2+484Cd^2c-693Bd^3\right)d^2-2b^2c\left(640Fc^4-704dDc^3+792Cd^2c^2-924Bd^3c+1155Ad^4\right)\right)\sqrt{c+d}}{d\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}\right.\right.$$

$$\frac{2F(c+dx)^{9/2}\sqrt{a-bx^2}}{11bd^5}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3 + F*x^4))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]
),x]
```

output
```
(-2*F*(c + d*x)^(9/2)*Sqrt[a - b*x^2])/(11*b*d^5) + ((-2*d*(11*d*D - 46*c*
F)*(c + d*x)^(7/2)*Sqrt[a - b*x^2])/9 + ((-2*d^6*(99*b*C*d^2 + 81*a*d^2*F
- b*c*(319*d*D - 668*c*F))*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/7 + ((-2*b*d^1
0*(a*d^2*(539*d*D - 1282*c*F) - b*(1584*c*C*d^2 - 693*B*d^3 - 2563*c^2*d*D
 + 3590*c^3*F))*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + (3*((-2*b*d^13*(675*a
^2*d^4*F + b^2*(1881*c^2*C*d^2 - 1617*B*c*d^3 + 1155*A*d^4 - 2057*c^3*d*D
+ 2185*c^4*F) + 3*a*b*d^2*(275*C*d^2 - c*(407*d*D - 491*c*F)))*Sqrt[c + d*
x]*Sqrt[a - b*x^2])/3 + (b*d^13*((-2*Sqrt[a]*Sqrt[b]*(3*a^2*d^4*(539*d*D -
 382*c*F) - 3*a*b*d^2*(484*c*C*d^2 - 693*B*d^3 - 396*c^2*d*D + 344*c^3*F)
- 2*b^2*c*(792*c^2*C*d^2 - 924*B*c*d^3 + 1155*A*d^4 - 704*c^3*d*D + 640*c^
4*F))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]
*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]*
(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(675*a^3
*d^6*F + 2*b^3*c^2*(792*c^2*C*d^2 - 924*B*c*d^3 + 1155*A*d^4 - 704*c^3*d*D
 + 640*c^4*F) + a*b^2*d^2*(1056*c^2*C*d^2 - 1617*B*c*d^3 + 1155*A*d^4 - 83
6*c^3*d*D + 712*c^4*F) + 3*a^2*b*d^4*(275*C*d^2 - c*(407*d*D - 266*c*F)))*
Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Elli
pticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sq
rt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3))/(5*b*d^3))/(7
*b*d^4))/(9*b*d^5))/(11*b*d^6)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 8.10 (sec) , antiderivative size = 1324, normalized size of antiderivative = 1.66

method	result	size
elliptic	Expression too large to display	1324
default	Expression too large to display	6726

input
```
int(x^2*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/11*F/b/d*x^4
*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/9*(D-10/11*F/d*c)/b/d*x^3*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/b/d*x^
2*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(B+8/11*F/b/d*a*c+7/9*(D-10/11*F/
d*c)/b*a-6/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/d*c)/b/d*x*(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)-2/3*(A+2/3*(D-10/11*F/d*c)/b/d*a*c+5/7*(C+9/11*F/b
*a-8/9*(D-10/11*F/d*c)/d*c)/b*a-4/5*(B+8/11*F/b/d*a*c+7/9*(D-10/11*F/d*c)/
b*a-6/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)+2*(2/5*(B+8/11*F/b/d*a*c+7/9*(D-10/11*F/d*c)/b*a-6/7
*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/d*c)/b/d*a*c+1/3*(A+2/3*(D-10/11*F
/d*c)/b/d*a*c+5/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/b*a-4/5*(B+8/11*F
/b/d*a*c+7/9*(D-10/11*F/d*c)/b*a-6/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c
)/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/
2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))
/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elliptic
F(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*
(a*b)^(1/2)))^(1/2))+2*(4/7*(C+9/11*F/b*a-8/9*(D-10/11*F/d*c)/d*c)/b/d*a*c
+3/5*(B+8/11*F/b/d*a*c+7/9*(D-10/11*F/d*c)/b*a-6/7*(C+9/11*F/b*a-8/9*(D-10
/11*F/d*c)/d*c)/d*c)/b*a-2/3*(A+2/3*(D-10/11*F/d*c)/b/d*a*c+5/7*(C+9/11*F/
b*a-8/9*(D-10/11*F/d*c)/d*c)/b*a-4/5*(B+8/11*F/b/d*a*c+7/9*(D-10/11*F/d...
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 687, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="fricas")
```

output
```
-2/10395*((1280*F*b^3*c^6 - 1408*D*b^3*c^5*d + 72*(F*a*b^2 + 22*C*b^3)*c^4
*d^2 - 132*(D*a*b^2 + 14*B*b^3)*c^3*d^3 + 6*(17*F*a^2*b + 44*C*a*b^2 + 385
*A*b^3)*c^2*d^4 - 33*(13*D*a^2*b + 21*B*a*b^2)*c*d^5 + 45*(45*F*a^3 + 55*C
*a^2*b + 77*A*a*b^2)*d^6)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*
d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(1
280*F*b^3*c^5*d - 1408*D*b^3*c^4*d^2 + 24*(43*F*a*b^2 + 66*C*b^3)*c^3*d^3
- 132*(9*D*a*b^2 + 14*B*b^3)*c^2*d^4 + 6*(191*F*a^2*b + 242*C*a*b^2 + 385*
A*b^3)*c*d^5 - 231*(7*D*a^2*b + 9*B*a*b^2)*d^6)*sqrt(-b*d)*weierstrassZeta
(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), weierst
rassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d
^3), 1/3*(3*d*x + c)/d)) + 3*(315*F*b^3*d^6*x^4 + 640*F*b^3*c^4*d^2 - 704*
D*b^3*c^3*d^3 + 4*(149*F*a*b^2 + 198*C*b^3)*c^2*d^4 - 22*(31*D*a*b^2 + 42*
B*b^3)*c*d^5 + 15*(45*F*a^2*b + 55*C*a*b^2 + 77*A*b^3)*d^6 - 35*(10*F*b^3*
c*d^5 - 11*D*b^3*d^6)*x^3 + 5*(80*F*b^3*c^2*d^4 - 88*D*b^3*c*d^5 + 9*(9*F*
a*b^2 + 11*C*b^3)*d^6)*x^2 - (480*F*b^3*c^3*d^3 - 528*D*b^3*c^2*d^4 + 2*(2
36*F*a*b^2 + 297*C*b^3)*c*d^5 - 77*(7*D*a*b^2 + 9*B*b^3)*d^6)*x)*sqrt(-b*x
^2 + a)*sqrt(d*x + c))/(b^4*d^7)
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate(x**2*(F*x**4+D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1
/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2 + D*x**3 + F*x**4)/(sqrt(a - b*x**2)*sqrt(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Fx^4+Dx^3+Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x
 + c)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Fx^4+Dx^3+Cx^2+Bx+A)x^2}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x^2*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)*x^2/(sqrt(-b*x^2 + a)*sqrt(d*x
 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{x^2\,\left(A+B\,x+C\,x^2+F\,x^4+x^3\,D\right)}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + F*x^4 + x^3*D))/((a - b*x^2)^(1/2)*(c + d*x)^(
1/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + F*x^4 + x^3*D))/((a - b*x^2)^(1/2)*(c + d*x)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\text{Too large to display}$$

input
```
int(x^2*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 408*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*c*d**3*f - 3234*sqrt(c + d*x)*
sqrt(a - b*x**2)*a**2*d**5 - 4158*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b**2*d*
*4 - 320*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**3*d*f - 44*sqrt(c + d*x)*sq
rt(a - b*x**2)*a*b*c**2*d**3 + 1888*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**
2*d**2*f*x - 2156*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**4*x - 1620*sqrt(
c + d*x)*sqrt(a - b*x**2)*a*b*c*d**3*f*x**2 - 2772*sqrt(c + d*x)*sqrt(a -
b*x**2)*b**3*c*d**3*x + 1920*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**4*f*x
+ 264*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*d**2*x - 1600*sqrt(c + d*x)
*sqrt(a - b*x**2)*b**2*c**3*d*f*x**2 - 220*sqrt(c + d*x)*sqrt(a - b*x**2)*
b**2*c**2*d**3*x**2 + 1400*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d**2*f
*x**3 - 1540*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**4*x**3 - 1260*sqrt(c
 + d*x)*sqrt(a - b*x**2)*b**2*c*d**3*f*x**4 + 3438*int((sqrt(c + d*x)*sqrt
(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*b*c*d**4*f
- 4851*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 -
 b*d*x**3),x)*a**2*b*d**6 + 6930*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)
/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*c*d**4 - 6237*int((sqrt(c +
 d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3
*d**5 + 3096*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*
x**2 - b*d*x**3),x)*a*b**2*c**3*d**2*f + 792*int((sqrt(c + d*x)*sqrt(a - b
*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c**2*d**4 - ...
```

3.241 $\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 644

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2(3ad^2(25dD-37cF)-b(147cCd^2-105Bd^3-171c^2dD+187c^3F))\sqrt{c+dx}\sqrt{a-bx^2}}{315b^2d^4} - \frac{2(49ad^2F+b(63Cd^2-c(144dD-233cF)))(c+dx)^{3/2}\sqrt{a-bx^2}}{315b^2d^4} - \frac{2(9dD-29cF)(c+dx)^{5/2}\sqrt{a-bx^2}}{63bd^4} - \frac{2F(c+dx)^{7/2}\sqrt{a-bx^2}}{9bd^4} - \frac{2\sqrt{a}(147a^2d^4F+b^2(168c^2Cd^2-210Bcd^3+315Ad^4-144c^3dD+128c^4F)+3abd^2(63Cd^2-4c(11d}{315b^{5/2}d^5\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} - \frac{2\sqrt{a}(3a^2d^4(25dD-37cF)-abd^2(147cCd^2-105Bd^3-96c^2dD+76c^3F)-b^2c(168c^2Cd^2-210Bc}{315b^{5/2}d^5\sqrt{c+dx}}$$

output
```
-2/315*(3*a*d^2*(25*D*d-37*F*c)-b*(-105*B*d^3+147*C*c*d^2-171*D*c^2*d+187*
F*c^3))*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b^2/d^4-2/315*(49*a*d^2*F+b*(63*C*d
^2-c*(144*D*d-233*F*c)))*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b^2/d^4-2/63*(9*D*
d-29*F*c)*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^4-2/9*F*(d*x+c)^(7/2)*(-b*x^2
+a)^(1/2)/b/d^4-2/315*a^(1/2)*(147*a^2*d^4*F+b^2*(315*A*d^4-210*B*c*d^3+16
8*C*c^2*d^2-144*D*c^3*d+128*F*c^4)+3*a*b*d^2*(63*C*d^2-4*c*(11*D*d-9*F*c))
)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(
1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(5/2)/d^5/
((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/315*a^(1/2)*(3*a^
2*d^4*(25*D*d-37*F*c)-a*b*d^2*(-105*B*d^3+147*C*c*d^2-96*D*c^2*d+76*F*c^3)
-b^2*c*(315*A*d^4-210*B*c*d^3+168*C*c^2*d^2-144*D*c^3*d+128*F*c^4))*((d*x+
c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1
/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/
2))/b^(5/2)/d^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.31 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.24

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(-147a^2d^4F-b^2(168c^2Cd^2-210Bcd^3+315Ad^4-144c^3dD+128c^4F)-3abd^2(63Cd^2+\right.}{}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3 + F*x^4))/(Sqrt[c + d*x]*Sqrt[a - b*
x^2]),x]
```

output
```
(2*Sqrt[a - b*x^2]*(-147*a^2*d^4*F - b^2*(168*c^2*C*d^2 - 210*B*c*d^3 + 31
5*A*d^4 - 144*c^3*d*D + 128*c^4*F) - 3*a*b*d^2*(63*C*d^2 + 4*c*(-11*d*D +
9*c*F)) + b*(c + d*x)*(a*d^2*(-75*d*D + 62*c*F - 49*d*F*x) + b*(64*c^3*F -
 24*c^2*d*(3*D + 2*F*x) + 2*c*d^2*(42*C + x*(27*D + 20*F*x)) - d^3*(105*B
+ x*(63*C + 5*x*(9*D + 7*F*x))))) + (I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*(14
7*a^2*d^4*F + b^2*(168*c^2*C*d^2 - 210*B*c*d^3 + 315*A*d^4 - 144*c^3*d*D +
 128*c^4*F) + 3*a*b*d^2*(63*C*d^2 + 4*c*(-11*d*D + 9*c*F)))*Sqrt[(d*(Sqrt[
a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]
*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) + (I*Sqrt[a]*Sqrt[b]*(147*a^2*d^4*F -
3*a^(3/2)*Sqrt[b]*d^3*(25*d*D + 12*c*F) - Sqrt[a]*b^(3/2)*d*(42*c*C*d^2 +
105*B*d^3 - 36*c^2*d*D + 32*c^3*F) + b^2*(168*c^2*C*d^2 - 210*B*c*d^3 + 31
5*A*d^4 - 144*c^3*d*D + 128*c^4*F) + 3*a*b*d^2*(63*C*d^2 + 4*c*(-11*d*D +
9*c*F)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqr
t[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt
[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2))))/(315*b^3*d^4*Sq
rt[c + d*x])
```

Rubi [A] (verified)

Time = 3.46 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.04, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.349$, Rules used = {2185, 27, 2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2+Dx^3+Fx^4\right)}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{bd^4(9dD-29cF)x^4+d^3\left(-33bFc^2+9bCd^2+7ad^2F\right)x^3+3d^2\left(-5bFc^3+7ad^2Fc+3bBd^3\right)x^2+d\left(-2bFc^4+21ad^2Fc^2+9Abd^4\right)x+7ac^3d^2F}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{9bd^5}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bd^4(9dD-29cF)x^4+d^3(-33bFc^2+9bCd^2+7ad^2F)x^3+3d^2(-5bFc^3+7ad^2Fc+3bBd^3)x^2+d(-2bFc^4+21ad^2Fc^2+9Abd^4)x+7ac^3d^2F}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{9bd^5} - \frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 2185

$$-\frac{\frac{2\int -\frac{b\left(63bCd^2+49aFd^2-bc(144dD-233cF)\right)x^3d^7+b\left(63bBd^3+a(45dD+2cF)d^2-bc^2(99dD-214cF)\right)x^2d^6+3abc^2(15dD-32cF)d^6+b\left(63Abd^4+ac(90dD-14\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{9bd^5} - \frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{b\left(63bCd^2+49aFd^2-bc(144dD-233cF)\right)x^3d^7+b\left(63bBd^3+a(45dD+2cF)d^2-bc^2(99dD-214cF)\right)x^2d^6+3abc^2(15dD-32cF)d^6+b\left(63Abd^4+ac(90dD-143cF)d\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{7bd^4}}{9bd^5} - \frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 2185

$$-\frac{\frac{2\int -\frac{3\left(b^2\left(3ad^2(25dD-37cF)-b\left(187Fc^3-171dDc^2+147Cd^2c-105Bd^3\right)\right)x^2d^9+abc\left(63bCd^2+49aFd^2-bc(69dD-73cF)\right)d^9+b\left(49a^2Fd^4+ab\left(-38Fc^2+6dDc\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4} \quad \frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{b^2\left(3ad^2(25dD-37cF)-b\left(187Fc^3-171dDc^2+147Cd^2c-105Bd^3\right)\right)x^2d^9+abc\left(63bCd^2+49aFd^2-bc(69dD-73cF)\right)d^9+b\left(49a^2Fd^4+ab\left(-38Fc^2+6dDc+63C\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}}{5bd^3}}{7bd^4} \quad \frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 2185

$$3\left(-\frac{2\int -\frac{b^2d^{10}\left(ad\left(3a(25dD+12cF)d^2+b\left(32Fc^3-36dDc^2+42Cd^2c+105Bd^3\right)\right)+\left(147a^2Fd^4+3ab\left(63Cd^2-4c(11dD-9cF)\right)d^2+b^2\left(128Fc^4-144dDc^3+168Cd^2\right.\right.\right.}{2\sqrt{c+dx}\sqrt{a-bx^2}}}{3bd^2}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 27

$$3\left(\frac{1}{3}bd^8\int\frac{ad\left(3a(25dD+12cF)d^2+b\left(32Fc^3-36dDc^2+42Cd^2c+105Bd^3\right)\right)+\left(147a^2Fd^4+3ab\left(63Cd^2-4c(11dD-9cF)\right)d^2+b^2\left(128Fc^4-144dDc^3+168Cd^2c^2-21\right.\right.}{\sqrt{c+dx}\sqrt{a-bx^2}}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 600

$$3\left(\frac{1}{3}bd^8\left(\frac{\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 509

$$3\left(\frac{1}{3}bd^8\left(\frac{\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 508

$$3\left(\frac{1}{3}bd^8\left(\frac{\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 327

$$3\left(\frac{1}{3}bd^8\left(\frac{\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 512

$$3\left(\frac{1}{3}bd^8\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-}}}{d\sqrt{a-bx^2}}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\downarrow$ 511

$$3\left(\frac{1}{3}bd^8\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315Ad^4-210Bcd^3+128c^4F-144c^3dD+168c^2C\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

$\Big\downarrow$ 321

$$3\left(\frac{1}{3}bd^8\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(3a^2d^4(25dD-37cF)-abd^2\left(-105Bd^3+76c^3F-96c^2dD+147cCd^2\right)-b^2c\left(315A\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.\right.$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{7/2}}{9bd^4}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3 + F*x^4))/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),
x]
```

output
```
(-2*F*(c + d*x)^(7/2)*Sqrt[a - b*x^2])/(9*b*d^4) + ((-2*d*(9*d*D - 29*c*F)
*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/7 + ((-2*d^5*(63*b*C*d^2 + 49*a*d^2*F -
b*c*(144*d*D - 233*c*F))*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + (3*((-2*b*d^
8*(3*a*d^2*(25*d*D - 37*c*F) - b*(147*c*C*d^2 - 105*B*d^3 - 171*c^2*d*D +
187*c^3*F))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + (b*d^8*((-2*Sqrt[a]*(147*a^
2*d^4*F + b^2*(168*c^2*C*d^2 - 210*B*c*d^3 + 315*A*d^4 - 144*c^3*d*D + 128
*c^4*F) + 3*a*b*d^2*(63*C*d^2 - 4*c*(11*d*D - 9*c*F)))*Sqrt[c + d*x]*Sqrt[
1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2
*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[
b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*(3*a^2*d^4*(25*d*D - 37*c
*F) - a*b*d^2*(147*c*C*d^2 - 105*B*d^3 - 96*c^2*d*D + 76*c^3*F) - b^2*c*(1
68*c^2*C*d^2 - 210*B*c*d^3 + 315*A*d^4 - 144*c^3*d*D + 128*c^4*F))*Sqrt[(S
qrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] +
 d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3))/(5*b*d^3))/(7*b*d^4)
)/(9*b*d^5)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 6.13 (sec) , antiderivative size = 978, normalized size of antiderivative = 1.52

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2F\,x^3\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{9bd}-\frac{2\left(D-\frac{8Fc}{9d}\right)x^2\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{7bd}-\frac{2\left(C+\frac{7Fa}{9b}-\frac{6\left(D-\frac{8Fc}{9d}\right)c}{7d}\right)x\sqrt{-bd\,x^3-}}{5bd}\right.}{}$
default	Expression too large to display

input
```
int(x*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/9*F/b/d*x^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/7*(D-8/9*F/d*c)/b/d*x^2*(-b*d*x^3-b*c
*x^2+a*d*x+a*c)^(1/2)-2/5*(C+7/9*F/b*a-6/7*(D-8/9*F/d*c)/d*c)/b/d*x*(-b*d*
x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(B+2/3*F/b/d*a*c+5/7*(D-8/9*F/d*c)/b*a-4/
5*(C+7/9*F/b*a-6/7*(D-8/9*F/d*c)/d*c)/d*c)/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c
)^(1/2)+2*(2/5*(C+7/9*F/b*a-6/7*(D-8/9*F/d*c)/d*c)/b/d*a*c+1/3*(B+2/3*F/b/
d*a*c+5/7*(D-8/9*F/d*c)/b*a-4/5*(C+7/9*F/b*a-6/7*(D-8/9*F/d*c)/d*c)/d*c)/b
*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a
*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c
/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))
^(1/2))+2*(A+4/7*(D-8/9*F/d*c)/b/d*a*c+3/5*(C+7/9*F/b*a-6/7*(D-8/9*F/d*c)/
d*c)/b*a-2/3*(B+2/3*F/b/d*a*c+5/7*(D-8/9*F/d*c)/b*a-4/5*(C+7/9*F/b*a-6/7*(
D-8/9*F/d*c)/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(
a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/
2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)
,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*El
lipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/
d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.80

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= \frac{2\left((128\,Fb^2c^5-144\,Db^2c^4d+12\,(Fab+14\,Cb^2)c^3d^2-6\,(4\,Dab+35\,Bb^2)c^2d^3+3\,(13\,Fa^2+21\,Cab+\right.}{}$$

input
```
integrate(x*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="fricas")
```

output
```
2/945*((128*F*b^2*c^5 - 144*D*b^2*c^4*d + 12*(F*a*b + 14*C*b^2)*c^3*d^2 -
6*(4*D*a*b + 35*B*b^2)*c^2*d^3 + 3*(13*F*a^2 + 21*C*a*b + 105*A*b^2)*c*d^4
 - 45*(5*D*a^2 + 7*B*a*b)*d^5)*sqrt(-b*d)*weierstrassPInverse(4/3*(b*c^2 +
 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) +
 3*(128*F*b^2*c^4*d - 144*D*b^2*c^3*d^2 + 12*(9*F*a*b + 14*C*b^2)*c^2*d^3
- 6*(22*D*a*b + 35*B*b^2)*c*d^4 + 21*(7*F*a^2 + 9*C*a*b + 15*A*b^2)*d^5)*s
qrt(-b*d)*weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*
a*c*d^2)/(b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27
*(b*c^3 - 9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) - 3*(35*F*b^2*d^5*x^3 -
64*F*b^2*c^3*d^2 + 72*D*b^2*c^2*d^3 - 2*(31*F*a*b + 42*C*b^2)*c*d^4 + 15*(
5*D*a*b + 7*B*b^2)*d^5 - 5*(8*F*b^2*c*d^4 - 9*D*b^2*d^5)*x^2 + (48*F*b^2*c
^2*d^3 - 54*D*b^2*c*d^4 + 7*(7*F*a*b + 9*C*b^2)*d^5)*x)*sqrt(-b*x^2 + a)*s
qrt(d*x + c))/(b^3*d^6)
```

Sympy [F]

$$\int \frac{x(A + Bx + Cx^2 + Dx^3 + Fx^4)}{\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \int \frac{x(A + Bx + Cx^2 + Dx^3 + Fx^4)}{\sqrt{a - bx^2}\sqrt{c + dx}}\,dx$$

input
```
integrate(x*(F*x**4+D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2)
,x)
```

output
```
Integral(x*(A + B*x + C*x**2 + D*x**3 + F*x**4)/(sqrt(a - b*x**2)*sqrt(c +
 d*x)), x)
```

Maxima [F]

$$\int \frac{x(A + Bx + Cx^2 + Dx^3 + Fx^4)}{\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \int \frac{(Fx^4 + Dx^3 + Cx^2 + Bx + A)x}{\sqrt{-bx^2 + a}\sqrt{dx + c}}\,dx$$

input
```
integrate(x*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*sqrt(d*x +
 c)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{(Fx^4+Dx^3+Cx^2+Bx+A)x}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate(x*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)*x/(sqrt(-b*x^2 + a)*sqrt(d*x +
 c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+F\,x^4+x^3\,D)}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + F*x^4 + x^3*D))/((a - b*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((x*(A + B*x + C*x^2 + F*x^4 + x^3*D))/((a - b*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2+Dx^3+Fx^4)}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
int(x*(F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 294*sqrt(c + d*x)*sqrt(a - b*x**2)*a**2*d**3*f - 630*sqrt(c + d*x)*sqr
t(a - b*x**2)*a*b**2*d**3 + 32*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c**2*d*f
 - 414*sqrt(c + d*x)*sqrt(a - b*x**2)*a*b*c*d**3 - 196*sqrt(c + d*x)*sqrt(
a - b*x**2)*a*b*c*d**2*f*x - 192*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**3*
f*x - 36*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c**2*d**2*x + 160*sqrt(c + d*
x)*sqrt(a - b*x**2)*b**2*c**2*d*f*x**2 - 180*sqrt(c + d*x)*sqrt(a - b*x**2
)*b**2*c*d**3*x**2 - 140*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*c*d**2*f*x**3
 - 441*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 -
 b*d*x**3),x)*a**2*b*d**4*f - 945*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2
)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**3*d**4 - 324*int((sqrt(c + d
*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b**2*c
**2*d**2*f - 171*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x -
b*c*x**2 - b*d*x**3),x)*a*b**2*c*d**4 + 630*int((sqrt(c + d*x)*sqrt(a - b*
x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**4*c*d**3 - 384*int((
sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x
)*b**3*c**4*f - 72*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x
- b*c*x**2 - b*d*x**3),x)*b**3*c**3*d**2 + 147*int((sqrt(c + d*x)*sqrt(a -
 b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**3*d**4*f + 315*int((sq
rt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*
b**2*d**4 + 180*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c...
```

3.242 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 517

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= -\frac{2(25ad^2F+b(35Cd^2-c(49dD-57cF)))\sqrt{c+dx}\sqrt{a-bx^2}}{105b^2d^3}$$
$$-\frac{2(7dD-16cF)(c+dx)^{3/2}\sqrt{a-bx^2}}{35bd^3}-\frac{2F(c+dx)^{5/2}\sqrt{a-bx^2}}{7bd^3}$$
$$-\frac{2\sqrt{a}(ad^2(63dD-44cF)-b(70cCd^2-105Bd^3-56c^2dD+48c^3F))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{105b^{3/2}d^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$-\frac{2\sqrt{a}(25a^2d^4F+b^2(70c^2Cd^2-105Bcd^3+105Ad^4-56c^3dD+48c^4F)+abd^2(35Cd^2-c(49dD-32}{105b^{5/2}d^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/105*(25*a*d^2*F+b*(35*C*d^2-c*(49*D*d-57*F*c)))*(d*x+c)^(1/2)*(-b*x^2+a
)^(1/2)/b^2/d^3-2/35*(7*D*d-16*F*c)*(d*x+c)^(3/2)*(-b*x^2+a)^(1/2)/b/d^3-2
/7*F*(d*x+c)^(5/2)*(-b*x^2+a)^(1/2)/b/d^3-2/105*a^(1/2)*(a*d^2*(63*D*d-44*
F*c)-b*(-105*B*d^3+70*C*c*d^2-56*D*c^2*d+48*F*c^3))*(d*x+c)^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^
(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d^4/((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/105*a^(1/2)*(25*a^2*d^4*F+b^2*(105*A*d^4-
105*B*c*d^3+70*C*c^2*d^2-56*D*c^3*d+48*F*c^4)+a*b*d^2*(35*C*d^2-c*(49*D*d-
32*F*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*Ellip
ticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c
+a^(1/2)*d))^(1/2))/b^(5/2)/d^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.84 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{2\sqrt{a-bx^2}\left(ad^2(-63dD+44cF)+b(70cCd^2-105Bd^3-56c^2dD+48c^3F)-(c+dx)\left(25ad^2F+b(35\right.\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]
),x]
```

output
```
(2*Sqrt[a - b*x^2]*(a*d^2*(-63*d*D + 44*c*F) + b*(70*c*C*d^2 - 105*B*d^3 -
 56*c^2*d*D + 48*c^3*F) - (c + d*x)*(25*a*d^2*F + b*(35*C*d^2 + 24*c^2*F +
 3*d^2*x*(7*D + 5*F*x) - 2*c*d*(14*D + 9*F*x))) + (I*Sqrt[b]*(Sqrt[b]*c -
Sqrt[a]*d)*(a*d^2*(63*d*D - 44*c*F) + b*(-70*c*C*d^2 + 105*B*d^3 + 56*c^2*
d*D - 48*c^3*F))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a
]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)])/(d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^2)) - (I*(1
05*A*b^2*d^3 + 25*a^2*d^3*F + a^(3/2)*Sqrt[b]*d^2*(-63*d*D + 44*c*F) + Sqr
t[a]*b^(3/2)*(70*c*C*d^2 - 105*B*d^3 - 56*c^2*d*D + 48*c^3*F) + a*b*d*(35*
C*d^2 + 2*c*(7*d*D - 6*c*F)))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sq
rt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*A
rcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]
*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(-a + b*x^
2))))/(105*b^2*d^3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {2185, 27, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{bd^3(7dD-16cF)x^3+d^2\left(-11bFc^2+7bCd^2+5ad^2F\right)x^2+d\left(-2bFc^3+10ad^2Fc+7bBd^3\right)x+d^2\left(5aFc^2+7Abd^2\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bd^3(7dD-16cF)x^3+d^2(-11bFc^2+7bCd^2+5ad^2F)x^2+d(-2bFc^3+10ad^2Fc+7bBd^3)x+d^2(5aFc^2+7Abd^2)}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 2185

$$\frac{-\frac{2\int -\frac{b\left(35bCd^2+25aFd^2-bc(49dD-57cF)\right)x^2d^5+b\left(35Abd^2+ac(21dD-23cF)\right)d^5+b\left(35bBd^3+a(21dD+2cF)d^2-2bc^2(7dD-11cF)\right)xd^4}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2}{5}d\sqrt{a-\ldots}}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{b\left(35bCd^2+25aFd^2-bc(49dD-57cF)\right)x^2d^5+b\left(35Abd^2+ac(21dD-23cF)\right)d^5+b\left(35bBd^3+a(21dD+2cF)d^2-2bc^2(7dD-11cF)\right)xd^4}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2}{5}d\sqrt{a-bx^2}(\ldots}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 2185

$$\frac{\frac{-\frac{2\int -\frac{bd^6\left(d\left(105Ab^2d^2+a\left(35bCd^2+25aFd^2+2bc(7dD-6cF)\right)\right)+b\left(ad^2(63dD-44cF)-b\left(48Fc^3-56dDc^2+70Cd^2c-105Bd^3\right)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}(\ldots}{5bd^3}}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{1}{3}d^4\int \frac{d\left(105Ab^2d^2+a\left(35bCd^2+25aFd^2+2bc(7dD-6cF)\right)\right)+b\left(ad^2(63dD-44cF)-b\left(48Fc^3-56dDc^2+70Cd^2c-105Bd^3\right)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}d^4\sqrt{a-bx^2}\sqrt{c+dx}(25ad^2\ldots}{5bd^3}}{7bd^4}-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$\downarrow$ 600

$$\frac{\frac{1}{3}d^4\left(\frac{\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\left(ad^2(63dD-44cF)-b\left(-105Bd\ldots\right.\right.}{\ldots}\right.}{5bd^3}\ldots-\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 509$$

$$\frac{\frac{1}{3}d^4\left(\frac{\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{b\sqrt{1-\frac{bx^2}{a}}\left(ad^2(63dD-44cF)-b\right.}{}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 508$$

$$\frac{\frac{1}{3}d^4\left(\frac{\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(ad^2(63c\right.}{}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 327$$

$$\frac{\frac{1}{3}d^4\left(\frac{\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\text{arcsin}\right.}{}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 512$$

$$\frac{\frac{1}{3}d^4\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}}{}\right.}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 511$$

$$\frac{\frac{1}{3}d^4\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD+70c^2Cd^2\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\sqrt{\right.}\right.}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

$$\Big\downarrow \ 321$$

$$\frac{\frac{1}{3}d^4\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(25a^2d^4F+abd^2\left(35Cd^2-c(49dD-32cF)\right)+b^2\left(105Ad^4-105Bcd^3+48c^4F-56c^3dD\right.\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}\right.}{}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{5/2}}{7bd^3}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output

```
(-2*F*(c + d*x)^(5/2)*Sqrt[a - b*x^2])/(7*b*d^3) + ((-2*d*(7*d*D - 16*c*F)
*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/5 + ((-2*d^4*(35*b*C*d^2 + 25*a*d^2*F -
b*c*(49*d*D - 57*c*F))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + (d^4*((-2*Sqrt[a
]*Sqrt[b]*(a*d^2*(63*d*D - 44*c*F) - b*(70*c*C*d^2 - 105*B*d^3 - 56*c^2*d*
D + 48*c^3*F))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
 (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[
(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]
*(25*a^2*d^4*F + b^2*(70*c^2*C*d^2 - 105*B*c*d^3 + 105*A*d^4 - 56*c^3*d*D
+ 48*c^4*F) + a*b*d^2*(35*C*d^2 - c*(49*d*D - 32*c*F)))*Sqrt[(Sqrt[b]*(c +
 d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[
1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt
[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2])))/3)/(5*b*d^3))/(7*b*d^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.51

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(-\frac{2F x^2\sqrt{-bd x^3-bc x^2+adx+ac}}{7bd}-\frac{2\left(D-\frac{6Fc}{7d}\right)x\sqrt{-bd x^3-bc x^2+adx+ac}}{5bd}-\frac{2\left(C+\frac{5Fa}{7b}-\frac{4\left(D-\frac{6Fc}{7d}\right)c}{5d}\right)\sqrt{-bd x^3-bc \cdots}}{3bd}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/7*F/b/d*x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/5*(D-6/7*F/d*c)/b/d*x*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)-2/3*(C+5/7*F/b*a-4/5*(D-6/7*F/d*c)/d*c)/b/d*(-b*d*x^3-
b*c*x^2+a*d*x+a*c)^(1/2)+2*(A+2/5*(D-6/7*F/d*c)/b/d*a*c+1/3*(C+5/7*F/b*a-4
/5*(D-6/7*F/d*c)/d*c)/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(
1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a
*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2
)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/
(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(B+4/7*F/b/d*a*c+3/5*(D-6/7*F/d*c)/b*a-2/
3*(C+5/7*F/b*a-4/5*(D-6/7*F/d*c)/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^
(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2
+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b
)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b
*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a
*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2\left((48\,Fb^2c^4-56\,Db^2c^3d+2\,(4\,Fab+35\,Cb^2)c^2d^2-21\,(Dab+5\,Bb^2)cd^3+15\,(5\,Fa^2+7\,Cab+21\,\right.}{}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algo
rithm="fricas")
```

output
```
-2/315*((48*F*b^2*c^4 - 56*D*b^2*c^3*d + 2*(4*F*a*b + 35*C*b^2)*c^2*d^2 -
21*(D*a*b + 5*B*b^2)*c*d^3 + 15*(5*F*a^2 + 7*C*a*b + 21*A*b^2)*d^4)*sqrt(-
b*d)*weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a
*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d) + 3*(48*F*b^2*c^3*d - 56*D*b^2*c^2*d^2
 + 2*(22*F*a*b + 35*C*b^2)*c*d^3 - 21*(3*D*a*b + 5*B*b^2)*d^4)*sqrt(-b*d)*
weierstrassZeta(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 - 9*a*c*d^2)/(
b*d^3), weierstrassPInverse(4/3*(b*c^2 + 3*a*d^2)/(b*d^2), -8/27*(b*c^3 -
9*a*c*d^2)/(b*d^3), 1/3*(3*d*x + c)/d)) + 3*(15*F*b^2*d^4*x^2 + 24*F*b^2*c
^2*d^2 - 28*D*b^2*c*d^3 + 5*(5*F*a*b + 7*C*b^2)*d^4 - 3*(6*F*b^2*c*d^3 - 7
*D*b^2*d^4)*x)*sqrt(-b*x^2 + a)*sqrt(d*x + c))/(b^3*d^5)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2),x
)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(sqrt(a - b*x**2)*sqrt(c + d
*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),
x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/((a - b*x^2)^(1/2)*(c + d*x)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$=\frac{-4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,acdf-42\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,a\,d^3-70\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,b^2d^2+24\sqrt{dx+c}}{}$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 4*sqrt(c + d*x)*sqrt(a - b*x**2)*a*c*d*f - 42*sqrt(c + d*x)*sqrt(a - b
*x**2)*a*d**3 - 70*sqrt(c + d*x)*sqrt(a - b*x**2)*b**2*d**2 + 24*sqrt(c +
d*x)*sqrt(a - b*x**2)*b*c**2*f*x - 28*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d
**2*x - 20*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*d*f*x**2 + 44*int((sqrt(c +
d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c*d
**2*f - 63*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x*
*2 - b*d*x**3),x)*a*b*d**4 - 105*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)
/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*d**3 + 48*int((sqrt(c + d*x)*
sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c**3*f
+ 14*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b
*d*x**3),x)*b**2*c**2*d**2 + 2*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c +
 a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*c*d**2*f + 21*int((sqrt(c + d*x)*sqr
t(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a**2*d**4 + 70*int((
sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b
**2*c*d**2 + 35*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x*
*2 - b*d*x**3),x)*a*b**2*d**3 - 24*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a
*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**3*f + 28*int((sqrt(c + d*x)*sq
rt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c**2*d**2)/(70*
b**2*c*d**2)
```

3.243 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 529

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx \\
&= -\frac{2(5dD-7cF)\sqrt{c+dx}\sqrt{a-bx^2}}{15bd^2}-\frac{2F(c+dx)^{3/2}\sqrt{a-bx^2}}{5bd^2} \\
&\quad -\frac{2\sqrt{a}\left(15bCd^2-10bcdD+8bc^2F+9ad^2F\right)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{15b^{3/2}d^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad -\frac{2\sqrt{a}\left(ad^2(5dD-7cF)-b(15cCd^2-15Bd^3-10c^2dD+8c^3F)\right)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\Bigg(\arcsin\Bigg(}{15b^{3/2}d^3\sqrt{c+dx}\sqrt{a-bx^2}} \\
&\quad -\frac{2A\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-2/15*(5*D*d-7*F*c)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d^2-2/5*F*(d*x+c)^(3/
2)*(-b*x^2+a)^(1/2)/b/d^2-2/15*a^(1/2)*(15*C*b*d^2-10*D*b*c*d+9*F*a*d^2+8*
F*b*c^2)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(
1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(3/
2)/d^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-2/15*a^(1/2)
*(a*d^2*(5*D*d-7*F*c)-b*(-15*B*d^3+15*C*c*d^2-10*D*c^2*d+8*F*c^3))*((d*x+c
)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/
2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2
))/b^(3/2)/d^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-2*A*((d*x+c)/(c+a^(1/2)*d/b^
(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1
/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/(d*x+c)^(1/
2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.43 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx =$$

$$-\frac{2\left(cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(15bCd^2+9ad^2F+2bc(-5dD+4cF))\,(a-bx^2)+bcd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}(c+dx)(5dD\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^
2]),x]
```

output
```
(-2*(c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(15*b*C*d^2 + 9*a*d^2*F + 2*b*c*
(-5*d*D + 4*c*F))*(a - b*x^2) + b*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c
+ d*x)*(5*d*D - 4*c*F + 3*d*F*x)*(a - b*x^2) + I*c*(b*c - Sqrt[a]*Sqrt[b]*
d)*(15*b*C*d^2 + 9*a*d^2*F + 2*b*c*(-5*d*D + 4*c*F))*Sqrt[(d*(Sqrt[a]/Sqrt
[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d
*x)^(3/2)*(EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - EllipticF[I*ArcSinh
[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(S
qrt[b]*c - Sqrt[a]*d)]) + I*b*(a*c*d^2*(-5*d*D + 7*c*F) + b*(15*c^2*C*d^2
- 15*B*c*d^3 + 15*A*d^4 - 10*c^3*d*D + 8*c^4*F))*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - (15*I)*A*b^2*d^4*Sqrt[(d
*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*c)/(Sqrt[b]*c - Sqrt[a]*d), I*
ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a
]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(15*b^2*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*Sqrt[c + d*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2351, 633, 632, 186, 413, 412, 2185, 27, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{2351}$$

$$A\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{633}$$

$$\frac{A\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{632}$$

$$\frac{A\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{186}$$

$$\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2A\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{413}$$

$$\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\Big\downarrow \text{412}$$

$$\int \frac{Fx^3+Dx^2+Cx+B}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{b(5dD-7cF)x^2d^2+(5bBd+3acF)d^2+\left(-2bFc^2+5bCd^2+3ad^2F\right)xd}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{b(5dD-7cF)x^2d^2+(5bBd+3acF)d^2+\left(-2bFc^2+5bCd^2+3ad^2F\right)xd}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \; 2185$$

$$-\frac{-\frac{2\int -\frac{bd^3\left(d(15bBd+5aDd+2acF)+\left(15bCd^2+9aFd^2-2bc(5dD-4cF)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(5dD-7cF)}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}d\int \frac{d(15bBd+5aDd+2acF)+\left(15bCd^2+9aFd^2-2bc(5dD-4cF)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2}{3}d\sqrt{a-bx^2}\sqrt{c+dx}(5dD-7cF)}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{600}$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\left(9ad^2F-2bc(5dD-4cF)+15bCd^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}\right)-\frac{2}{3}}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{509}$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{\sqrt{1-\frac{bx^2}{a}}\left(9ad^2F-2bc(5dD-4cF)+15bCd^2\right)\int\frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}}{d\sqrt{a-bx^2}}\right.}{5bd^3}-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{508}$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(9ad^2F-2bc(5dD-4cF)+15bCd^2\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5bd^3}$$

$$\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{1}{3}d\left(\frac{\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)(9}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\right.}{5bd^3}$$

$$\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{512}$$

$$\frac{\frac{1}{3}d\left(\frac{\sqrt{1-\frac{bx^2}{a}}\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\ldots\right)}{\sqrt{b}d\sqrt{a-bx^2}\ldots}\right)}{5bd^3}$$

$$\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{\frac{1}{3}d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(ad^2(5dD-7cF)-b\left(-15Bd^3+8c^3F-10c^2dD+15cCd^2\right)\right)\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\ldots}{\ldots}\right)}{5bd^3}$$

$$\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

$$\Big\downarrow \text{ 321}$$

$$-\frac{2A\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+$$

$$\frac{\frac{1}{3}d\left(-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)(ad^2(5dD-7cF)-b(-15Bd^3+8c^3F-10c^2dD+15cCd^2))}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}\sqrt{}}{}\right.}{5bd^3}$$

$$\frac{2F\sqrt{a-bx^2}(c+dx)^{3/2}}{5bd^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x*Sqrt[c + d*x]*Sqrt[a - b*x^2]),x]
```

output
```
(-2*F*(c + d*x)^(3/2)*Sqrt[a - b*x^2])/(5*b*d^2) + ((-2*d*(5*d*D - 7*c*F)*
Sqrt[c + d*x]*Sqrt[a - b*x^2])/3 + (d*((-2*Sqrt[a]*(15*b*C*d^2 + 9*a*d^2*F
 - 2*b*c*(5*d*D - 4*c*F))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcS
in[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)
])/(Sqrt[b]*d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b
*x^2]) - (2*Sqrt[a]*(a*d^2*(5*d*D - 7*c*F) - b*(15*c*C*d^2 - 15*B*d^3 - 10
*c^2*d*D + 8*c^3*F))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqr
t[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
(2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]
)))/3)/(5*b*d^3) - (2*A*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt
[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (S
qrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt
[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqr
t[a]))/Sqrt[b]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2351

```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. $2(442) = 884$.

Time = 3.14 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.69

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2Fx\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{5bd}-\frac{2\left(D-\frac{4Fc}{5d}\right)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(B+\frac{2Fac}{5bd}+\frac{\left(D-\frac{4Fc}{5d}\right)a}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\cdots}}{\cdots}\cdots\right)}{\cdots}$
default	Expression too large to display

input

```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/5*F/b/d*x*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*(D-4/5*F/d*c)/b/d*(-b*d*x^3-b*c*x^2+a
*d*x+a*c)^(1/2)+2*(B+2/5*F/b/d*a*c+1/3*(D-4/5*F/d*c)/b*a)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(C+3/5*F/b
*a-2/3*(D-4/5*F/d*c)/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*
b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)
*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(
(-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*Elli
pticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2)))-2*A*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b
)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b
*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(
1/2)/c*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)
^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{a-bx^2}\,\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/x/(d*x+c)**(1/2)/(-b*x**2+a)**(1/2)
,x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(x*sqrt(a - b*x**2)*sqrt(c +
 d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{x\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$=\frac{-6\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,adf-10\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcd-4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}\,bcfx-9\Big(\int \frac{\sqrt{dx+c}\,\sqrt{-}}{-bd\,x^3-bc\,x}}{}$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
( - 6*sqrt(c + d*x)*sqrt(a - b*x**2)*a*d*f - 10*sqrt(c + d*x)*sqrt(a - b*x
**2)*b*c*d - 4*sqrt(c + d*x)*sqrt(a - b*x**2)*b*c*f*x - 9*int((sqrt(c + d*
x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d**2*
f - 8*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 -
b*d*x**3),x)*b**2*c**2*f - 5*int((sqrt(c + d*x)*sqrt(a - b*x**2)*x**2)/(a*
c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c*d**2 + 10*int((sqrt(c + d*x)*sq
rt(a - b*x**2))/(a*c*x + a*d*x**2 - b*c*x**3 - b*d*x**4),x)*a*b**2*c*d + 3
*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),
x)*a**2*d**2*f + 4*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c
*x**2 - b*d*x**3),x)*a*b*c**2*f + 5*int((sqrt(c + d*x)*sqrt(a - b*x**2))/(
a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*c*d**2 + 10*int((sqrt(c + d*x)*s
qrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**3*c*d)/(10*b**2
*c*d)
```

3.244 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 522

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = -\frac{2F\sqrt{c+dx}\sqrt{a-bx^2}}{3bd} - \frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{acx}$$

$$+\frac{(3Abd^2-2ac(3dD-2cF))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{a}\sqrt{b}cd^2\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$

$$-\frac{(2ad^2(3bC+aF)+b(3Abd^2-2ac(3dD-2cF)))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{}}{\sqrt{b}c}\right.}{3\sqrt{a}b^{3/2}d^2\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$-\frac{(2Bc-Ad)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{c\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*F*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d-A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)
/a/c/x+1/3*(3*A*b*d^2-2*a*c*(3*D*d-2*F*c))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1
/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/c/d^2/((d*x+c)/(c+a^(1/2)*d/b
^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/3*(2*a*d^2*(3*C*b+F*a)+b*(3*A*b*d^2-2*a*
c*(3*D*d-2*F*c)))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b
^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(3/2)/d^2/(d*x+c)^(1/2)/(-b*x^2+a)^(
1/2)-(-A*d+2*B*c)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/
2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d
/(b^(1/2)*c+a^(1/2)*d))^(1/2))/c/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.26 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$

$$= \frac{\sqrt{a-bx^2}\left(3\left(-\frac{2F}{3bd}-\frac{A}{acx}\right)(c+dx)-\frac{cd^2\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}\left(3Abd^2+2ac(-3dD+2cF)\right)\left(-a+bx^2\right)-ic\left(bc-\sqrt{a}\sqrt{b}d\right)\left(3Abd^2+2ac(-3dD\right.}{}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^2*Sqrt[c + d*x]*Sqrt[a - b*
x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*(3*((-2*F)/(3*b*d) - A/(a*c*x))*(c + d*x) - (c*d^2*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*(3*A*b*d^2 + 2*a*c*(-3*d*D + 2*c*F))*(-a + b*x^2)
 - I*c*(b*c - Sqrt[a]*Sqrt[b]*d)*(3*A*b*d^2 + 2*a*c*(-3*d*D + 2*c*F))*Sqrt
[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(
c + d*x))]*(c + d*x)^(3/2)*(EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - El
lipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*
c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]) - I*(3*A*b*d^2*(b*c^2 + a*d^2) +
2*a*c*(a*c*d^2*F + b*(3*c*C*d^2 - 3*B*d^3 - 3*c^2*d*D + 2*c^3*F)))*Sqrt[(d
*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c +
 d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + (3*I)*
a*b*d^3*(-2*B*c + A*d)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((
Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*
c)/(Sqrt[b]*c - Sqrt[a]*d), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[
c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/(a*b*c^2*d^3*
Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(a - b*x^2))))/(3*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 2.57 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.13, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.378, Rules used = {2352, 25, 2351, 633, 632, 186, 413, 412, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \ 2352$$

$$-\frac{\int -\frac{2acFx^3-(Abd-2acD)x^2+2acCx+a(2Bc-Ad)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{2acFx^3-(Abd-2acD)x^2+2acCx+a(2Bc-Ad)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{2351}$$

$$\frac{a(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{633}$$

$$\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{632}$$

$$\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{186}$$

$$\frac{\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{413}$$

$$\frac{\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \text{412}$$

$$\frac{\int \frac{2acFx^2+(2acD-Abd)x+2acC}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 2185

$$\frac{-\frac{2\int -\frac{d\left(2acd(3bC+aF)-b\left(3Abd^2-2ac(3dD-2cF)\right)x\right)}{2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\cdots}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{2acd(3bC+aF)-b\left(3Abd^2-2ac(3dD-2cF)\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd}-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-4\cdots}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 600

$$\frac{\frac{\frac{c\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\left(3Abd^2-2ac(3dD-2cF)\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{3bd}-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\cdots\right)}{\sqrt{a}d+\sqrt{\cdots}}}\cdots}{\sqrt{a-bx^2}\sqrt{-\cdots}}}{2ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 509

$$\frac{\frac{\frac{c\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{b\sqrt{1-\frac{bx^2}{a}}\left(3Abd^2-2ac(3dD-2cF)\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3bd}-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\cdots}}{\sqrt{a}\cdots}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 508

$$\frac{\frac{\frac{c\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3Abd^2-2ac(3dD-2cF)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}-\frac{2\cdots}{\cdots}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 327

$$\frac{\frac{\frac{c\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3Abd^2-2ac(3dD-2cF)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}-\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$\downarrow$ 512

$$\frac{\frac{\frac{c\sqrt{1-\frac{bx^2}{a}}\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3Abd^2-2ac(3dD-2cF)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}\cdots}\cdots\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}}{3bd}\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \; 511$$

$$\frac{\dfrac{\dfrac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3Abd^2-2ac(3dD-2cF)\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\dfrac{2\sqrt{a}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(b\left(3Abd^2-2ac(3dD-2cF)\right)+2ad^2(aF+3bC)\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}}{3bd}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

$$\Big\downarrow \; 321$$

$$-\frac{2a\sqrt{1-\frac{bx^2}{a}}(2Bc-Ad)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(3Abd^2-2ac(3dD-2cF)\right)E\left(\text{a}\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{acx}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^2*Sqrt[c + d*x]*Sqrt[a - b*x^2]),
x]
```

output
```
-((A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x)) + ((-4*a*c*F*Sqrt[c + d*x]*Sq
rt[a - b*x^2])/(3*b*d) + ((2*Sqrt[a]*Sqrt[b]*(3*A*b*d^2 - 2*a*c*(3*d*D - 2
*c*F))*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(d*Sqrt[(Sqrt[b]
*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*Sqrt[a]*c*(2*a*
d^2*(3*b*C + a*F) + b*(3*A*b*d^2 - 2*a*c*(3*d*D - 2*c*F)))*Sqrt[(Sqrt[b]*(
c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sq
rt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(S
qrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^2]))/(3*b*d) - (2*a*(2*B*c - A*d)*Sqrt
[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c
+ Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]],
 (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a
]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]]))/(2*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.67

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{A\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{acx}-\frac{2F\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3bd}+\frac{2\left(C+\frac{Fa}{3b}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\sqrt{\ldots}}{-\frac{c}{d}+\ldots}}}{\sqrt{-bd\,x^3-bc\,x^2+a\ldots}}\right.}{\ldots}$
default	Expression too large to display

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/a*A/c/x*(-b*
d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)-2/3*F/b/d*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2
)+2*(C+1/3*F/b*a)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1
/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2)
)/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Ellipti
cF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2))+2*(D-1/2*b*d/a/c*A-2/3*F/d*c)*(c/d-1/b*(a*b)^(1/2))*
((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)
^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^
3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d
-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c
/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+(A*d-2*B*c)/c^2*(c/d-1
/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))
/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2))
)^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*
(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/x**2/(d*x+c)**(1/2)/(-b*x**2+a)**(1
/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(x**2*sqrt(a - b*x**2)*sqrt(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^2}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+cx^2}}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{x^2\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^2*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{F\,x^4+Dx^3+C\,x^2+Bx+A}{x^2\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.245 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 548

$$\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx \\
&= -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{2acx^2}-\frac{(4Bc-3Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{4ac^2x} \\
&\quad +\frac{(bd(4Bc-3Ad)-8ac^2F)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}\sqrt{b}c^2d\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad -\frac{(bd(4Bc-Ad)+8ac(dD-cF))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4\sqrt{a}\sqrt{b}cd\sqrt{c+dx}\sqrt{a-bx^2}} \\
&\quad -\frac{(4Abc^2+8ac^2C-4aBcd+3aAd^2)\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{4ac^2\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}$$

output
```
-1/2*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x^2-1/4*(-3*A*d+4*B*c)*(d*x+c)^(
1/2)*(-b*x^2+a)^(1/2)/a/c^2/x+1/4*(b*d*(-3*A*d+4*B*c)-8*a*c^2*F)*(d*x+c)^(
1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2
),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/c^2/d/(
(d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-b*x^2+a)^(1/2)-1/4*(b*d*(-A*d+4*B*c
)+8*a*c*(D*d-F*c))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1
/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(
b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(1/2)/b^(1/2)/c/d/(d*x+c)^(1/2)/(-b*x^2+a)^
(1/2)-1/4*(3*A*a*d^2+4*A*b*c^2-4*B*a*c*d+8*C*a*c^2)*((d*x+c)/(c+a^(1/2)*d/
b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c^2/(d*
x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 29.75 (sec) , antiderivative size = 1598, normalized size of antiderivative = 2.92

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^3*Sqrt[c + d*x]*Sqrt[a - b*
x^2]),x]
```

output
```
(Sqrt[a - b*x^2]*(-(((c + d*x)*(2*A*c + 4*B*c*x - 3*A*d*x))/(a*c^2*x^2)) -
 (4*b^2*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 3*A*b^2*c^3*d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]] - 4*a*b*B*c^2*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 3
*a*A*b*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 8*a*b*c^5*Sqrt[-c + (Sqrt[a]
*d)/Sqrt[b]]*F + 8*a^2*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*F - 8*b^2*B*
c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 6*A*b^2*c^2*d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*(c + d*x) + 16*a*b*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]
]*F*(c + d*x) + 4*b^2*B*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 -
 3*A*b^2*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*(c + d*x)^2 - 8*a*b*c^3*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*F*(c + d*x)^2 + I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a
]*d)*(b*d*(-4*B*c + 3*A*d) + 8*a*c^2*F)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c
+ d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ell
ipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c
 + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - I*Sqrt[b]*d*(-(A*Sqrt[b]*d*(2*b*c
^2 + 3*Sqrt[a]*Sqrt[b]*c*d + 3*a*d^2)) + 4*Sqrt[a]*c*(b*B*c*d + Sqrt[a]*Sq
rt[b]*(-2*c*C*d + B*d^2 + 2*c^2*D) - 2*a*c^2*F))*Sqrt[(d*(Sqrt[a]/Sqrt[b]
+ x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (
Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] - (4*I)*A*b^2*c^2*d^2*Sqrt
[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x...
```

Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 603, normalized size of antiderivative = 1.10, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2352, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2352}$$

$$-\frac{\int -\frac{4acFx^3+(Abd+4acD)x^2+2c(Ab+2aC)x+a(4Bc-3Ad)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4acFx^3+(Abd+4acD)x^2+2c(Ab+2aC)x+a(4Bc-3Ad)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 2352

$$\frac{-\frac{\int -\frac{-a\left(bd(4Bc-3Ad)-8ac^2F\right)x^2+2ac(Abd+4acD)x+a\left(4Abc^2+8aCc^2-4aBdc+3aAd^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{-a\left(bd(4Bc-3Ad)-8ac^2F\right)x^2+2ac(Abd+4acD)x+a\left(4Abc^2+8aCc^2-4aBdc+3aAd^2\right)}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 2351

$$\frac{\frac{\int \frac{8a^2Dc^2+2aAbdc-a\left(bd(4Bc-3Ad)-8ac^2F\right)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(4Bc-3Ad)}{cx}}{4ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 600

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{a\left(bd(4Bc-3Ad)-8ac^2F\right)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}+\frac{ac(8ac(dD-cF)+bd(4Bc-Ad))\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 509

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{a\sqrt{1-\frac{bx^2}{a}}\left(bd(4Bc-3Ad)-8ac^2F\right)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{ac(8ac(dD-cF)+bd(4Bc-Ad))\int \frac{1}{\sqrt{c+dx}\sqrt{\cdots}}}{d}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 508

$$\frac{\frac{\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)\int\frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ac(8ac(dD-c}{}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 327

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ac(8ac(dD-cF)+bd(4Bc-Ad))\int\frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8\right.}{\sqrt{b}d\sqrt{a-bx^2}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 512

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ac\sqrt{1-\frac{bx^2}{a}}(8ac(dD-cF)+bd(4Bc-Ad))\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc\right.}{\sqrt{b}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 511

$$\frac{\frac{-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(8ac(dD-cF)+bd(4Bc-Ad))\int\frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}+a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{}{x\sqrt{c+}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 321

$$\frac{\frac{a\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2a^{3/2}c\sqrt{1-\cdots}}{\cdots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 633

$$\frac{\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2a^{3/2}\cdots}{\cdots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 632

$$\frac{\frac{\frac{a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 186

$$\frac{-\frac{2a\sqrt{1-\frac{bx^2}{a}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)E\left(\arcsin\cdots\right.}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}\cdots}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$\downarrow$ 413

$$-\frac{2a\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(3aAd^2-4aBcd+8ac^2C+4Abc^2\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8\right.}{\sqrt{b}d\sqrt{a-bx}}$$

$$2ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

$$\Big\downarrow \; 412$$

$$\frac{2a^{3/2}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(bd(4Bc-3Ad)-8ac^2F\right)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)(8ac}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$2ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{2acx^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^3*Sqrt[c + d*x]*Sqrt[a - b*x^2]),
x]
```

output
```
-1/2*(A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x^2) + (-(((4*B*c - 3*A*d)*Sqr
t[c + d*x]*Sqrt[a - b*x^2])/(c*x)) + ((2*a^(3/2)*(b*d*(4*B*c - 3*A*d) - 8*
a*c^2*F)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt
[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqr
t[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*a^(3/
2)*c*(b*d*(4*B*c - A*d) + 8*a*c*(d*D - c*F))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqr
t[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b
]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[
c + d*x]*Sqrt[a - b*x^2]) - (2*a*(4*A*b*c^2 + 8*a*c^2*C - 4*a*B*c*d + 3*a*
A*d^2)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/
(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]
]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[a - b*x^2]*Sqrt[
c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]]))
/(2*a*c))/(4*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{A\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{2ca\,x^2}+\frac{(3Ad-4Bc)\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{4a\,c^2x}+\frac{2\left(D+\frac{bdA}{4ac}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\cdots}{\sqrt{-bd\,x^3\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/2*A/c/a/x^2*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/4*(3*A*d-4*B*c)/a/c^2*(-b*d*x^3-b*c*x
^2+a*d*x+a*c)^(1/2)/x+2*(D+1/4*b*d/a/c*A)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(
1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+
a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(F+1/8*b*d*(3*A*d-4*B*c)/a
/c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*
(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1
/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2)))-1/4/c^3/a*(3*A*a*d^2+4*A*b*c^2-4*B*a*c*d+8*C*a*c^2)*(c/d-1/b*(a*b)^(
1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2
)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a
*b)^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="fricas")
```

output
```
integral(-(F*x^4 + D*x^3 + C*x^2 + B*x + A)*sqrt(-b*x^2 + a)*sqrt(d*x + c)
/(b*d*x^6 + b*c*x^5 - a*d*x^4 - a*c*x^3), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/x**3/(d*x+c)**(1/2)/(-b*x**2+a)**(1
/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(x**3*sqrt(a - b*x**2)*sqrt(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^3}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{x^3\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^3*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{F\,x^4+Dx^3+C\,x^2+Bx+A}{x^3\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.246 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 642

$$
\begin{aligned}
&\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx \\
&= -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{3acx^3} - \frac{(6Bc-5Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{12ac^2x^2} \\
&\quad - \frac{(16Abc^2+24ac^2C-18aBcd+15aAd^2)\sqrt{c+dx}\sqrt{a-bx^2}}{24a^2c^3x} \\
&\quad + \frac{\sqrt{b}(16Abc^2+24ac^2C-18aBcd+15aAd^2)\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{24a^{3/2}c^3\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}} \\
&\quad - \frac{(Ab(16bc^2+5ad^2)+6ac(4bcC-bBd+8acF))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{24a^{3/2}\sqrt{b}c^2\sqrt{c+dx}\sqrt{a-bx^2}} \\
&\quad - \frac{(4bc^2(2Bc-Ad)-a(8c^2Cd-6Bcd^2+5Ad^3-16c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(2,\arcsin\left(\frac{\sqrt{1-\cdots}}{\sqrt{2}}\right.\right.}{8ac^3\sqrt{c+dx}\sqrt{a-bx^2}}
\end{aligned}
$$

output
```
-1/3*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x^3-1/12*(-5*A*d+6*B*c)*(d*x+c)^
(1/2)*(-b*x^2+a)^(1/2)/a/c^2/x^2-1/24*(15*A*a*d^2+16*A*b*c^2-18*B*a*c*d+24
*C*a*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^3/x+1/24*b^(1/2)*(15*A*a*d^
2+16*A*b*c^2-18*B*a*c*d+24*C*a*c^2)*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*Ell
ipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)
*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^3/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/(-
b*x^2+a)^(1/2)-1/24*(A*b*(5*a*d^2+16*b*c^2)+6*a*c*(-B*b*d+4*C*b*c+8*F*a*c)
)*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(1/2
*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)
*d))^(1/2))/a^(3/2)/b^(1/2)/c^2/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)-1/8*(4*b*c^
2*(-A*d+2*B*c)-a*(5*A*d^3-6*B*c*d^2+8*C*c^2*d-16*D*c^3))*((d*x+c)/(c+a^(1/
2)*d/b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1
/2))^(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a/c^
3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 32.43 (sec) , antiderivative size = 2328, normalized size of antiderivative = 3.63

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^4*Sqrt[c + d*x]*Sqrt[a - b*
x^2]),x]
```

output
```
(-1/3*A/(a*c*x^3) + (-6*B*c + 5*A*d)/(12*a*c^2*x^2) + (-16*A*b*c^2 - 24*a*
c^2*C + 18*a*B*c*d - 15*a*A*d^2)/(24*a^2*c^3*x))*Sqrt[c + d*x]*Sqrt[a - b*
x^2] - (d*Sqrt[a - (b*(c + d*x)^2*(-1 + c/(c + d*x))^2)/d^2]*(-16*A*b^2*c^
3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 24*a*b*c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]] + 18*a*b*B*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 15*a*A*b*c*d^2*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]] - (16*A*b^2*c^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])
/(c + d*x)^2 - (24*a*b*c^5*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 +
 (18*a*b*B*c^4*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (a*A*b*c^3*
d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (24*a^2*c^3*C*d^2*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (18*a^2*B*c^2*d^3*Sqrt[-c + (Sqrt[
a]*d)/Sqrt[b]])/(c + d*x)^2 + (15*a^2*A*c*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]])/(c + d*x)^2 + (32*A*b^2*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)
+ (48*a*b*c^4*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) - (36*a*b*B*c^3*
d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (30*a*A*b*c^2*d^2*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (I*Sqrt[b]*c*(Sqrt[b]*c - Sqrt[a]*d)*(6*
a*c*(4*c*C - 3*B*d) + A*(16*b*c^2 + 15*a*d^2))*Sqrt[1 - c/(c + d*x) - (Sqr
t[a]*d)/(Sqrt[b]*(c + d*x))]*Sqrt[1 - c/(c + d*x) + (Sqrt[a]*d)/(Sqrt[b]*(
c + d*x))]*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x
]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)])/Sqrt[c + d*x] + (I*S
qrt[a]*(A*d*(16*b^(3/2)*c^3 + 2*Sqrt[a]*b*c^2*d + 15*a*Sqrt[b]*c*d^2 + ...
```

Rubi [A] (verified)

Time = 4.05 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.10, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.422$, Rules used = {2352, 25, 2352, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{a-bx^2}\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2352}$$

$$-\frac{\int -\frac{6acFx^3+3(Abd+2acD)x^2+2c(2Ab+3aC)x+a(6Bc-5Ad)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6acFx^3+3(Abd+2acD)x^2+2c(2Ab+3aC)x+a(6Bc-5Ad)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 2352

$$-\frac{\frac{\int -\frac{a\left(24aFc^2+bd(6Bc-5Ad)\right)x^2+2ac(6bBc+12aDc+Abd)x+a\left(16Abc^2+24aCc^2-18aBdc+15aAd^2\right)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(6Bc-5Ad)}{2cx^2}}{6ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a\left(24aFc^2+bd(6Bc-5Ad)\right)x^2+2ac(6bBc+12aDc+Abd)x+a\left(16Abc^2+24aCc^2-18aBdc+15aAd^2\right)}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}dx}{4ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(6Bc-5Ad)}{2cx^2}}{6ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 2352

$$-\frac{\frac{\frac{\int -\frac{3\left(4bc^2(2Bc-Ad)-a\left(-16Dc^3+8Cdc^2-6Bd^2c+5Ad^3\right)\right)a^2+2c\left(24aFc^2+bd(6Bc-5Ad)\right)xa^2-bd\left(16Abc^2+24aCc^2-18aBdc+15aAd^2\right)x^2a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\sqrt{a-bx^2}\sqrt{}}{4ac}}{6ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{3\left(4bc^2(2Bc-Ad)-a\left(-16Dc^3+8Cdc^2-6Bd^2c+5Ad^3\right)\right)a^2+2c\left(24aFc^2+bd(6Bc-5Ad)\right)xa^2-bd\left(16Abc^2+24aCc^2-18aBdc+15aAd^2\right)x^2a}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx}{2ac}-\sqrt{a-bx^2}\sqrt{c+d}}{4ac}}{6ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 2351

$$\frac{\frac{\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{48a^3Fc^3+12a^2bBdc^2-10a^2Abd^2c-abd\left(16Abc^2+24aCc^2-18aBdc+15aAd^2\right)}{\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}}{4ac}}{6ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\Big\downarrow$ 600

$$\frac{\frac{\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+ac\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-ab\left(15aA\ldots\right.}{2ac}}{4ac}}{6ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 509

$$\frac{\frac{\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+ac\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{ab\sqrt{1-\frac{b\ldots}{}}}{\ldots}}{2ac}}{4ac}}{6ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 508

$$\frac{\frac{\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(15aAd^2-18aBcd+24ac^2C+16Abc^2\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{\ldots}{x\ldots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 327

$$\frac{\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+ac\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2a^{3/2}\sqrt{b}\ldots}{\ldots}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 512

$$\frac{\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{ac\sqrt{1-\frac{bx^2}{a}}\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}}{2ac}}{4ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 511

$$\frac{-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-\right.\right.}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 321

$$\frac{3a^2\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(Ab\left(5a\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 633

$$\frac{\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+\right.\right.}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}}{2ac}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 632

$$\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right.\right.}{\sqrt{b}\sqrt{a-b}}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 186

$$-\frac{6a^2\sqrt{1-\frac{bx^2}{a}}\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{Elli}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 413

$$-\frac{6a^2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(4bc^2(2Bc-Ad)-a\left(5Ad^3-6Bcd^2-16c^3D+8c^2Cd\right)\right)\int\frac{\sqrt{a}}{\sqrt{b}x\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

$\downarrow$ 412

$$-\frac{2a^{3/2}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)\left(Ab\left(5ad^2+16bc^2\right)+6ac(8acF-bBd+4bcC)\right)}{\sqrt{b}\sqrt{a-bx^2}\sqrt{c+dx}}+\frac{2a^{3/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(15aAd^2-18a\right.}{\sqrt{}}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{3acx^3}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^4*Sqrt[c + d*x]*Sqrt[a - b*x^2]),
x]
```

output
```
-1/3*(A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x^3) + (-1/2*((6*B*c - 5*A*d)*
Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c*x^2) + (-(((16*A*b*c^2 + 24*a*c^2*C - 18
*a*B*c*d + 15*a*A*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c*x)) + ((2*a^(3/2)
*Sqrt[b]*(16*A*b*c^2 + 24*a*c^2*C - 18*a*B*c*d + 15*a*A*d^2)*Sqrt[c + d*x]
*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2
]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c
 + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*a^(3/2)*c*(A*b*(16*b*c^2 + 5*a*d^2) +
 6*a*c*(4*b*c*C - b*B*d + 8*a*c*F))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c +
Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt
[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*Sqrt[c + d*x]*Sq
rt[a - b*x^2]) - (6*a^2*(4*b*c^2*(2*B*c - A*d) - a*(8*c^2*C*d - 6*B*c*d^2
+ 5*A*d^3 - 16*c^3*D))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[
b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[Sqrt[1 - (Sq
rt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(Sqrt[
a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt
[a]))/Sqrt[b]]))/(2*a*c))/(4*a*c))/(6*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 6.06 (sec) , antiderivative size = 1015, normalized size of antiderivative = 1.58

method	result	size
elliptic	Expression too large to display	1015
default	Expression too large to display	5024

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/3*A/c/a/x^3*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/12/a*(5*A*d-6*B*c)/c^2*(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)/x^2-1/24/a^2/c^3*(15*A*a*d^2+16*A*b*c^2-18*B*a*c*d+24
*C*a*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+2*(F-1/24*b*d/a*(5*A*d-6*B*
c)/c^2)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/
b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/
b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d
)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/
2)))^(1/2))-1/24*b*d*(15*A*a*d^2+16*A*b*c^2-18*B*a*c*d+24*C*a*c^2)/c^3/a^2
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)
^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*
EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(
a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
+1/8/a*(5*A*a*d^3+4*A*b*c^2*d-6*B*a*c*d^2-8*B*b*c^3+8*C*a*c^2*d-16*D*a*c^3
)/c^4*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*
(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*
(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*d*EllipticPi(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),-(-c/d+1/b*(a*b)^(1/2))/c*d,((-c/d+1/b*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{x^4\sqrt{c + dx}\sqrt{a - bx^2}}\, dx = \text{Timed out}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/x**4/(d*x+c)**(1/2)/(-b*x**2+a)**(1
/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(x**4*sqrt(a - b*x**2)*sqrt(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^4), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^4}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{x^4\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^4*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^4*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{F\,x^4+Dx^3+C\,x^2+Bx+A}{x^4\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^4/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.247 $\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 782

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx$$
$$= -\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{4acx^4} - \frac{(8Bc-7Ad)\sqrt{c+dx}\sqrt{a-bx^2}}{24ac^2x^3}$$
$$- \frac{(36Abc^2+48ac^2C-40aBcd+35aAd^2)\sqrt{c+dx}\sqrt{a-bx^2}}{96a^2c^3x^2}$$
$$- \frac{(4bc^2(32Bc-25Ad)-3a(48c^2Cd-40Bcd^2+35Ad^3-64c^3D))\sqrt{c+dx}\sqrt{a-bx^2}}{192a^2c^4x}$$
$$+ \frac{\sqrt{b}(4bc^2(32Bc-25Ad)-3a(48c^2Cd-40Bcd^2+35Ad^3-64c^3D))\sqrt{c+dx}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-}}{\sqrt{}}\right.\right.}{192a^{3/2}c^4\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{a-bx^2}}$$
$$- \frac{\sqrt{b}(4bc^2(32Bc-7Ad)-a(48c^2Cd-40Bcd^2+35Ad^3-192c^3D))\sqrt{\frac{c+dx}{c+\frac{\sqrt{a}d}{\sqrt{b}}}}\sqrt{\frac{a-bx^2}{a}}\,\mathrm{EllipticF}\left(\arcsin\right.}{192a^{3/2}c^3\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$- \frac{(A(48b^2c^4+24abc^2d^2+35a^2d^4)+8ac(4bc^2(2cC-Bd)+a(6cCd^2-5Bd^3-8c^2dD+16c^3F)))\sqrt{\frac{c-}{c+}}}{64a^2c^4\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-1/4*A*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a/c/x^4-1/24*(-7*A*d+8*B*c)*(d*x+c)^
(1/2)*(-b*x^2+a)^(1/2)/a/c^2/x^3-1/96*(35*A*a*d^2+36*A*b*c^2-40*B*a*c*d+48
*C*a*c^2)*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/a^2/c^3/x^2-1/192*(4*b*c^2*(-25*A
*d+32*B*c)-3*a*(35*A*d^3-40*B*c*d^2+48*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*(-
b*x^2+a)^(1/2)/a^2/c^4/x+1/192*b^(1/2)*(4*b*c^2*(-25*A*d+32*B*c)-3*a*(35*A
*d^3-40*B*c*d^2+48*C*c^2*d-64*D*c^3))*(d*x+c)^(1/2)*((-b*x^2+a)/a)^(1/2)*E
llipticE(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/
2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^4/((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)/
(-b*x^2+a)^(1/2)-1/192*b^(1/2)*(4*b*c^2*(-7*A*d+32*B*c)-a*(35*A*d^3-40*B*c
*d^2+48*C*c^2*d-192*D*c^3))*((d*x+c)/(c+a^(1/2)*d/b^(1/2)))^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^
(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^(3/2)/c^3/(d*x+c)^(1/2)/(-b*x^2+a)
^(1/2)-1/64*(A*(35*a^2*d^4+24*a*b*c^2*d^2+48*b^2*c^4)+8*a*c*(4*b*c^2*(-B*d
+2*C*c)+a*(-5*B*d^3+6*C*c*d^2-8*D*c^2*d+16*F*c^3)))*((d*x+c)/(c+a^(1/2)*d/
b^(1/2)))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2,2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/a^2/c^4/(
d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 34.37 (sec) , antiderivative size = 3260, normalized size of antiderivative = 4.17

$$\int \frac{A + Bx + Cx^2 + Dx^3 + Fx^4}{x^5\sqrt{c + dx}\sqrt{a - bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^5*Sqrt[c + d*x]*Sqrt[a - b*
x^2]),x]
```

output
```
(-1/4*A/(a*c*x^4) + (-8*B*c + 7*A*d)/(24*a*c^2*x^3) + (-36*A*b*c^2 - 48*a*
c^2*C + 40*a*B*c*d - 35*a*A*d^2)/(96*a^2*c^3*x^2) + (-128*b*B*c^3 + 100*A*
b*c^2*d + 144*a*c^2*C*d - 120*a*B*c*d^2 + 105*a*A*d^3 - 192*a*c^3*D)/(192*
a^2*c^4*x))*Sqrt[c + d*x]*Sqrt[a - b*x^2] + (d*Sqrt[a - (b*(c + d*x)^2*(-1
 + c/(c + d*x))^2)/d^2]*(128*b^2*B*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 10
0*A*b^2*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 144*a*b*c^3*C*d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]] + 120*a*b*B*c^2*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] - 1
05*a*A*b*c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]] + 192*a*b*c^4*Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]*D + (128*b^2*B*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d
*x)^2 - (100*A*b^2*c^5*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (14
4*a*b*c^5*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (8*a*b*B*c^4*d
^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 - (5*a*A*b*c^3*d^3*Sqrt[-c
+ (Sqrt[a]*d)/Sqrt[b]])/(c + d*x)^2 + (144*a^2*c^3*C*d^3*Sqrt[-c + (Sqrt[a
]*d)/Sqrt[b]])/(c + d*x)^2 - (120*a^2*B*c^2*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt
[b]])/(c + d*x)^2 + (105*a^2*A*c*d^5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c +
d*x)^2 + (192*a*b*c^6*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D)/(c + d*x)^2 - (192
*a^2*c^4*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D)/(c + d*x)^2 - (256*b^2*B*c^
5*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (200*A*b^2*c^4*d*Sqrt[-c + (
Sqrt[a]*d)/Sqrt[b]])/(c + d*x) + (288*a*b*c^4*C*d*Sqrt[-c + (Sqrt[a]*d)/Sq
rt[b]])/(c + d*x) - (240*a*b*B*c^3*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]])/...
```

Rubi [A] (verified)

Time = 5.49 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.09, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2352, 25, 2352, 25, 2352, 25, 2352, 25, 2351, 600, 509, 508, 327, 512, 511, 321, 633, 632, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \ 2352$$

$$-\frac{\int -\frac{8acFx^3+(5Abd+8acD)x^2+2c(3Ab+4aC)x+a(8Bc-7Ad)}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}dx}{8ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{8acFx^3+(5Abd+8acD)x^2+2c(3Ab+4aC)x+a(8Bc-7Ad)}{x^4\sqrt{c+dx}\sqrt{a-bx^2}}dx}{8ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 2352$$

$$\frac{-\frac{\int -\frac{3a\left(16aFc^2+bd(8Bc-7Ad)\right)x^2+2ac(16bBc+24aDc+Abd)x+a\left(36Abc^2+48aCc^2-40aBdc+35aAd^2\right)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(8Bc-7Ad)}{3cx^3}}{8ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{3a\left(16aFc^2+bd(8Bc-7Ad)\right)x^2+2ac(16bBc+24aDc+Abd)x+a\left(36Abc^2+48aCc^2-40aBdc+35aAd^2\right)}{x^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{6ac}-\frac{\sqrt{a-bx^2}\sqrt{c+dx}(8Bc-7Ad)}{3cx^3}}{8ac}-\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 2352$$

$$\frac{\frac{-\frac{\int -\frac{\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)a^2+bd\left(36Abc^2+48aCc^2-40aBdc+35aAd^2\right)x^2a+2c\left(Ab\left(36bc^2-7ad^2\right)+8ac(6bcC+bBd+12\cdots\right.}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}}{4ac}\cdots}{6ac}\cdots}{8ac}\cdots$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)a^2+bd\left(36Abc^2+48aCc^2-40aBdc+35aAd^2\right)x^2a+2c\left(Ab\left(36bc^2-7ad^2\right)+8ac(6bcC+bBd+12acF\cdots\right.}{x^2\sqrt{c+dx}\sqrt{a-bx^2}}}{4ac}\cdots}{6ac}\cdots}{8ac}\cdots$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 2352$$

$$-\frac{\frac{\int -\frac{-bd\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)x^2a^2+3\left(A\left(48b^2c^4+24abd^2c^2+35a^2d^4\right)+8ac\left(4b(2cC-Bd)c^2+a\left(16Fc^3-8dDc^2+6Cd^2\cdots\right.\right.\right.}{x\sqrt{c+dx}\sqrt{a-bx^2}}}{2ac}\cdots}{4ac}\cdots$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \ 25$$

$$\int \frac{-bd\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)x^2a^2+3\left(A\left(48b^2c^4+24abd^2c^2+35a^2d^4\right)+8ac\left(4b(2cC-Bd)c^2+a\left(16Fc^3-8dDc^2+6Cd^2c-\right.\right.\right.}{x\sqrt{c+dx}\sqrt{a-bx^2}}$$

$$2ac$$

$$4ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 2351

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\int \frac{70Abcd^3a^3-80bBc^2d^2a^3+96bc^3Cd}{}$$

$$2ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 600

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+a^2bc\left(4bc^2(32Bc-7Ad)-a\left(35Ad^3-40\right.\right.$$

$$2ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 509

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+a^2bc\left(4bc^2(32Bc-7Ad)-a\left(35Ad^3-40\right.\right.$$

$$2ac$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 508

$$\frac{2a^{5/2}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\left(4bc^2(32Bc-25Ad)-3a\left(35Ad^3-40Bcd^2-64c^3D+48c^2Cd\right)\right)\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}+3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+\right.$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \; 327$$

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+a^2bc\left(4bc^2(32Bc-7Ad)-a\left(35Ad^3-40\right.\right.$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \; 512$$

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int \frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{a^2bc\sqrt{1-\frac{bx^2}{a}}\left(4bc^2(32Bc-7Ad)-a\left(3\right.\right.}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$$\Big\downarrow \; 511$$

$$-\frac{2a^{5/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(4bc^2(32Bc-7Ad)-a\left(35Ad^3-40Bcd^2-192c^3D+48c^2Cd\right)\right)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{a-bx^2}\sqrt{c+dx}}+3a^2\left(A\left(35a^2\right.\right.$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 321

$$3a^2\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{2a^{5/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{Ellipt}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 633

$$\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int\frac{1}{x\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{\sqrt{a-bx^2}}-\frac{2a^{5/2}\sqrt{b}c\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c}{\sqrt{a}d}}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 632

$$\frac{3a^2\sqrt{1-\frac{bx^2}{a}}\left(A\left(35a^2d^4+24abc^2d^2+48b^2c^4\right)+8ac\left(a\left(-5Bd^3+16c^3F-8c^2dD+6cCd^2\right)+4bc^2(2cC-Bd)\right)\right)\int\frac{1}{x\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}}dx}{\sqrt{a-bx^2}}-\frac{2a^{5/2}\sqrt{b}c\sqrt{1-\frac{b}{}}}{}$$

$$\frac{A\sqrt{a-bx^2}\sqrt{c+dx}}{4acx^4}$$

$\downarrow$ 186

$$\frac{2\sqrt{b}\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{5/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}-\frac{2\sqrt{b}c\left(4bc^2(32Bc-7Ad)-a\left(-192L\right.\right.}{}$$

$$\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{4acx^4}$$

$\downarrow$ 413

$$\frac{2\sqrt{b}\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{5/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2\sqrt{b}c\left(4bc^2(32Bc-7Ad)-a\left(-192I\right.\right.}{}$$

$$\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{4acx^4}$$

$\downarrow$ 412

$$\frac{2\sqrt{b}\left(4bc^2(32Bc-25Ad)-3a\left(-64Dc^3+48Cdc^2-40Bd^2c+35Ad^3\right)\right)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)a^{5/2}}{\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} - \frac{2\sqrt{b}c\left(4bc^2(32Bc-7Ad)-a\left(-192I\right.\right.}{}$$

$$\frac{A\sqrt{c+dx}\sqrt{a-bx^2}}{4acx^4}$$

input

```
Int[(A + B*x + C*x^2 + D*x^3 + F*x^4)/(x^5*Sqrt[c + d*x]*Sqrt[a - b*x^2]),
x]
```

output
```
-1/4*(A*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(a*c*x^4) + (-1/3*((8*B*c - 7*A*d)*
Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c*x^3) + (-1/2*((36*A*b*c^2 + 48*a*c^2*C -
 40*a*B*c*d + 35*a*A*d^2)*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c*x^2) + (-((a*(
4*b*c^2*(32*B*c - 25*A*d) - 3*a*(48*c^2*C*d - 40*B*c*d^2 + 35*A*d^3 - 64*c
^3*D))*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(c*x)) + ((2*a^(5/2)*Sqrt[b]*(4*b*c^
2*(32*B*c - 25*A*d) - 3*a*(48*c^2*C*d - 40*B*c*d^2 + 35*A*d^3 - 64*c^3*D))
*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[b]*x)/S
qrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[(Sqrt[b]*(c + d*
x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) - (2*a^(5/2)*Sqrt[b]*c*(4*b*
c^2*(32*B*c - 7*A*d) - a*(48*c^2*C*d - 40*B*c*d^2 + 35*A*d^3 - 192*c^3*D))
*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (b*x^2)/a]*Ell
ipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/S
qrt[a] + d)])/(Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (6*a^2*(A*(48*b^2*c^4 + 24
*a*b*c^2*d^2 + 35*a^2*d^4) + 8*a*c*(4*b*c^2*(2*c*C - B*d) + a*(6*c*C*d^2 -
 5*B*d^3 - 8*c^2*d*D + 16*c^3*F)))*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d
*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[2, ArcSin[
Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]
*d)])/(Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqr
t[b]*x)/Sqrt[a]))/Sqrt[b]]))/(2*a*c))/(4*a*c))/(6*a*c))/(8*a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 632
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1 -
q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 633
```
Int[1/((x_)*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :
> Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(x*Sqrt[c + d*x]*Sqrt[1
+ b*(x^2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2351
```
Int[((Px_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.))/(x_), x_S
ymbol] :> Int[PolynomialQuotient[Px, x, x]*(c + d*x)^n*(a + b*x^2)^p, x] +
Simp[PolynomialRemainder[Px, x, x]   Int[(c + d*x)^n*((a + b*x^2)^p/x), x],
 x] /; FreeQ[{a, b, c, d, n, p}, x] && PolynomialQ[Px, x]
```

rule 2352
```
Int[((Px_)*((e_.)*(x_))^(m_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x
_)^2]), x_Symbol] :> With[{Px0 = Coefficient[Px, x, 0]}, Simp[Px0*(e*x)^(m
+ 1)*Sqrt[c + d*x]*(Sqrt[a + b*x^2]/(a*c*e*(m + 1))), x] + Simp[1/(2*a*c*e*
(m + 1))   Int[((e*x)^(m + 1)/(Sqrt[c + d*x]*Sqrt[a + b*x^2]))*ExpandToSum[
2*a*c*(m + 1)*((Px - Px0)/x) - Px0*(a*d*(2*m + 3) + 2*b*c*(m + 2)*x + b*d*(
2*m + 5)*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[Px,
x] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 7.03 (sec) , antiderivative size = 1171, normalized size of antiderivative = 1.50

method	result	size
elliptic	Expression too large to display	1171
default	Expression too large to display	7124

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-1/4/a/c*A/x^4*
(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+1/24/a*(7*A*d-8*B*c)/c^2*(-b*d*x^3-b*c*
x^2+a*d*x+a*c)^(1/2)/x^3-1/96/a^2/c^3*(35*A*a*d^2+36*A*b*c^2-40*B*a*c*d+48
*C*a*c^2)*(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x^2+1/192/a^2*(105*A*a*d^3+10
0*A*b*c^2*d-120*B*a*c*d^2-128*B*b*c^3+144*C*a*c^2*d-192*D*a*c^3)/c^4*(-b*d
*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/x+1/96*b*d*(35*A*a*d^2+36*A*b*c^2-40*B*a*c*d
+48*C*a*c^2)/a^2/c^3*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))
^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1
/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*Elli
pticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-
1/b*(a*b)^(1/2)))^(1/2))+1/192*b*d/a^2*(105*A*a*d^3+100*A*b*c^2*d-120*B*a*
c*d^2-128*B*b*c^3+144*C*a*c^2*d-192*D*a*c^3)/c^4*(c/d-1/b*(a*b)^(1/2))*((x
+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b
*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+
1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))-1/64*(35*A*a^2*d^4+24*A*a
*b*c^2*d^2+48*A*b^2*c^4-40*B*a^2*c*d^3-32*B*a*b*c^3*d+48*C*a^2*c^2*d^2+64*
C*a*b*c^4-64*D*a^2*c^3*d+128*F*a^2*c^4)/a^2/c^5*(c/d-1/b*(a*b)^(1/2))*(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{a-bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((F*x**4+D*x**3+C*x**2+B*x+A)/x**5/(d*x+c)**(1/2)/(-b*x**2+a)**(1
/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3 + F*x**4)/(x**5*sqrt(a - b*x**2)*sqrt(
c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^5}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^5), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{Fx^4+Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}x^5}\,dx$$

input
```
integrate((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x,
algorithm="giac")
```

output
```
integrate((F*x^4 + D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c
)*x^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+F\,x^4+x^3\,D}{x^5\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^5*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2 + F*x^4 + x^3*D)/(x^5*(a - b*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3+Fx^4}{x^5\sqrt{c+dx}\sqrt{a-bx^2}}\,dx=\int \frac{F\,x^4+Dx^3+C\,x^2+Bx+A}{x^5\sqrt{dx+c}\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

output
```
int((F*x^4+D*x^3+C*x^2+B*x+A)/x^5/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2),x)
```

3.248 $\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 329

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = -\frac{2Be\sqrt{ex}\sqrt{a-bx^2}}{3bd} - \frac{2a^{3/4}(Bc-Ad)e^{3/2}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{b^{3/4}d^2\sqrt{a-bx^2}} + \frac{2\sqrt[4]{a}\left(aBd^2+3bc(Bc-Ad)+3\sqrt{a}\sqrt{b}d(Bc-Ad)\right)e^{3/2}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{3b^{5/4}d^3\sqrt{a-bx^2}} - \frac{2\sqrt[4]{a}c(Bc-Ad)e^{3/2}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d^3\sqrt{a-bx^2}}$$

output
```
-2/3*B*e*(e*x)^(1/2)*(-b*x^2+a)^(1/2)/b/d-2*a^(3/4)*(-A*d+B*c)*e^(3/2)*((-
b*x^2+a)/a)^(1/2)*EllipticE(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),I)/b^(3/4)
/d^2/(-b*x^2+a)^(1/2)+2/3*a^(1/4)*(a*B*d^2+3*b*c*(-A*d+B*c)+3*a^(1/2)*b^(1
/2)*d*(-A*d+B*c))*e^(3/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(b^(1/4)*(e*x)^(1/
2)/a^(1/4)/e^(1/2),I)/b^(5/4)/d^3/(-b*x^2+a)^(1/2)-2*a^(1/4)*c*(-A*d+B*c)*
e^(3/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2
),-a^(1/2)*d/b^(1/2)/c,I)/b^(1/4)/d^3/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.01

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2(ex)^{3/2}\sqrt{a-bx^2}\left(-Bd^2x + \frac{\frac{3i\sqrt{a}\sqrt{b}d(Bc-Ad)\sqrt{1-\frac{a}{bx^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{x}} + \frac{i\sqrt{a}d\left(\sqrt{a}B \cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x))/((c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(e*x)^(3/2)*Sqrt[a - b*x^2]*(-(B*d^2*x) + (((3*I)*Sqrt[a]*Sqrt[b]*d*(B*
c - A*d)*Sqrt[1 - a/(b*x^2)]*EllipticE[I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/
Sqrt[x]], -1])/Sqrt[x] + (I*Sqrt[a]*d*(Sqrt[a]*B*d + Sqrt[b]*(-3*B*c + 3*A
*d))*Sqrt[1 - a/(b*x^2)]*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/Sqrt
[x]], -1])/Sqrt[x] + 3*(B*c - A*d)*(Sqrt[-(Sqrt[a]/Sqrt[b])]*d*(-b + a/x^2
) + (I*b*c*Sqrt[1 - a/(b*x^2)]*EllipticPi[-((Sqrt[b]*c)/(Sqrt[a]*d)), I*Ar
cSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1])/Sqrt[x]))/(Sqrt[-(Sqrt[a]/Sq
rt[b])]*(-b + a/x^2))))/(3*b*d^3*x^2)
```

Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.05, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$, Rules used = {2354, 2237, 25, 2235, 25, 27, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx)}{\sqrt{a-bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{e^2x^2(Ae+Bxe)}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2237}$$

$$\frac{2\left(-\frac{e^2\int -\frac{3bd(Ae+Bxe)x^2+\frac{B(ce+dxe)\left(ae^2-3be^2x^2\right)}{e^2}}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{3bd}-\frac{Be^2\sqrt{ex}\sqrt{a-bx^2}}{3bd}\right)}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{e^2\int \frac{3bd(Ae+Bxe)x^2+\frac{B(ce+dxe)\left(ae^2-3be^2x^2\right)}{e^2}}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{3bd}-\frac{Be^2\sqrt{ex}\sqrt{a-bx^2}}{3bd}\right)}{e}$$

$$\Big\downarrow \text{2235}$$

$$\frac{2\left(\frac{e^2\left(-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{\int -\frac{\left(aBd^2+3bc(Bc-Ad)\right)e-3bd(Bc-Ad)ex}{e\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{3bd}-\frac{Be^2\sqrt{ex}\sqrt{a-bx^2}}{3bd}\right)}{e}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(\frac{e^2\left(\frac{\int \frac{\left(aBd^2+3bc(Bc-Ad)\right)e-3bd(Bc-Ad)ex}{e\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{3bd}-\frac{Be^2\sqrt{ex}\sqrt{a-bx^2}}{3bd}\right)}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{e^2\left(\frac{\int \frac{\left(aBd^2+3bc(Bc-Ad)\right)e-3bd(Bc-Ad)ex}{\sqrt{a-bx^2}}d\sqrt{ex}}{d^2e}-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{3bd}-\frac{Be^2\sqrt{ex}\sqrt{a-bx^2}}{3bd}\right)}{e}$$

$\downarrow$ 1513

$$\frac{2\left(\frac{e^2\left(\frac{e\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\int \frac{1}{\sqrt{a-bx^2}}d\sqrt{ex}-3\sqrt{a}\sqrt{b}de(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a}e\sqrt{a-bx^2}}d\sqrt{ex}}{d^2e}-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{3bd}-\dots\right)}{e}$$

$\downarrow$ 27

$$\frac{2\left(\frac{e^2\left(\frac{e\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\int \frac{1}{\sqrt{a-bx^2}}d\sqrt{ex}-3\sqrt{b}d(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{d^2e}-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{3bd}-Be\dots\right)}{e}$$

$\downarrow$ 765

$$\frac{2\left(\frac{e^2\left(\frac{\frac{e\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{\sqrt{a-bx^2}}-3\sqrt{b}d(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{d^2e}-\frac{3bc^2e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\dots\right.}{3bd}\dots\right.}{e}$$

$\downarrow$ 762

$$\frac{2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-3\sqrt{b}d(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{d^2e}-\frac{3bc^2e(Bc-Ad)}{\cdots}}{3bd}\cdots\right)}{e}$$

$\big\downarrow$ 1390

$$\frac{2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-\frac{3\sqrt{b}d\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{\sqrt{a-bx^2}}}{d^2e}-\frac{3bc^2\cdots}{\cdots}}{3bd}\cdots\right)}{e}$$

$\big\downarrow$ 1389

$$\frac{2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-\frac{3\sqrt{a}\sqrt{b}de\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\int\frac{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}d\sqrt{ex}}{\sqrt{a-bx^2}}}{d^2e}-\frac{3b\cdots}{\cdots}}{3bd}\cdots\right)}{e}$$

$$\Big\downarrow \; 327$$

$$2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-\frac{3a^{3/4}\sqrt[4]{b}de^{3/2}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)E\left(\arcsin\left(\cdots\right.\right.}{\sqrt{a-bx^2}}\cdots}{d^2e}\cdots\right)}{3bd}\cdots\right.}{e}\cdots$$

$$\Big\downarrow \; 1543$$

$$2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-\frac{3a^{3/4}\sqrt[4]{b}de^{3/2}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)E\left(\arcsin\left(\cdots\right.\right.}{\sqrt{a-bx^2}}\cdots}{d^2e}\cdots\right)}{3bd}\cdots\right.}{e}\cdots$$

$$\Big\downarrow \; 1542$$

$$2\left(\frac{e^2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(3\sqrt{a}\sqrt{b}d(Bc-Ad)+aBd^2+3bc(Bc-Ad)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}-\frac{3a^{3/4}\sqrt[4]{b}de^{3/2}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)E\left(\arcsin\left(\cdots\right.\right.}{\sqrt{a-bx^2}}\cdots}{d^2e}\cdots\right)}{3bd}\cdots\right.}{e}\cdots$$

input
```
Int[((e*x)^(3/2)*(A + B*x))/((c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(-1/3*(B*e^2*Sqrt[e*x]*Sqrt[a - b*x^2])/(b*d) + (e^2*(((-3*a^(3/4)*b^(1
/4)*d*(B*c - A*d)*e^(3/2)*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], -1])/Sqrt[a - b*x^2] + (a^(1/4)*(a*B*d^2 + 3*
b*c*(B*c - A*d) + 3*Sqrt[a]*Sqrt[b]*d*(B*c - A*d))*e^(3/2)*Sqrt[1 - (b*x^2
)/a]*EllipticF[ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1/4
)*Sqrt[a - b*x^2]))/(d^2*e) - (3*a^(1/4)*b^(3/4)*c*(B*c - A*d)*Sqrt[e]*Sqr
t[1 - (b*x^2)/a]*EllipticPi[-((Sqrt[a]*d)/(Sqrt[b]*c)), ArcSin[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(d^2*Sqrt[a - b*x^2])))/(3*b*d)))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. $2(259) = 518$.

Time = 3.07 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{xe(-b\,x^2+a)}\left(-\frac{2Be\sqrt{-be\,x^3+aex}}{3db}+\frac{\left(-\frac{c(Ad-Bc)e^2}{d^3}+\frac{B\,e^2 a}{3db}\right)\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{a}}{b}\right.}{\sqrt{ab}}}\right.}{b\sqrt{-be\,x^3+aex}}\right.}{\left(\frac{3d(Ad-Bc)\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\left(-\frac{2\sqrt{ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{ab}\,\text{Ellipt}}{}\right.}{\sqrt{-be\,x^3+aex}}\right.}$
risch	$-\frac{2Bx\sqrt{-b\,x^2+a}\,e^2}{3db\sqrt{ex}}+$
default	Expression too large to display

input
```
int((e*x)^(3/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(-b*x^2+a)^(1/2)*(x*e*(-b*x^2+a))^(1/2)*(-2/3*B/d*e/b*(-
b*e*x^3+a*e*x)^(1/2)+(-c*(A*d-B*c)*e^2/d^3+1/3*B/d*e^2/b*a)/b*(a*b)^(1/2)*
((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^
(1/2))^(1/2)*(-b/(a*b)^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)*EllipticF(((x
+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/d^2*(A*d-B*c)*e^2/b*
(a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/
2))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)*(
-2/b*(a*b)^(1/2)*EllipticE(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2
^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2
),1/2*2^(1/2)))+c^2*(A*d-B*c)*e^2/d^4/b*(a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b
/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)
^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)/(c/d-1/b*(a*b)^(1/2))*EllipticPi(((
x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),-1/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1
/2)),1/2*2^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}\,(A+Bx)}{\sqrt{a-bx^2}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(3/2)*(B*x+A)/(d*x+c)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((e*x)**(3/2)*(A + B*x)/(sqrt(a - b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(Bx+A)(ex)^{\frac{3}{2}}}{\sqrt{-bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*(e*x)^(3/2)/(sqrt(-b*x^2 + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(Bx+A)(ex)^{\frac{3}{2}}}{\sqrt{-bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*(e*x)^(3/2)/(sqrt(-b*x^2 + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(e\,x)^{3/2}\,(A+B\,x)}{\sqrt{a-b\,x^2}\,(c+d\,x)}\,dx$$

input
```
int(((e*x)^(3/2)*(A + B*x))/((a - b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(((e*x)^(3/2)*(A + B*x))/((a - b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \frac{\sqrt{e}\,e\left(-2\sqrt{x}\,\sqrt{-b\,x^2+a}\,a-3\left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}\,x^2}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)abd+3\left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+a}\right.\right.}{3b}$$

input
```
int((e*x)^(3/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*e*( - 2*sqrt(x)*sqrt(a - b*x**2)*a - 3*int((sqrt(x)*sqrt(a - b*x*
*2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*a*b*d + 3*int((sqrt(x)*sq
rt(a - b*x**2)*x**2)/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)*b**2*c + int((
sqrt(x)*sqrt(a - b*x**2))/(a*c*x + a*d*x**2 - b*c*x**3 - b*d*x**4),x)*a**2
*c + int((sqrt(x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x**3),x)
*a**2*d))/(3*b*c)
```

3.249 $\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 269

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2a^{3/4}B\sqrt{e}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{b^{3/4}d\sqrt{a-bx^2}} + \frac{2\sqrt[4]{a}\left(Ad-B\left(c+\frac{\sqrt{a}d}{\sqrt{b}}\right)\right)\sqrt{e}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d^2\sqrt{a-bx^2}} + \frac{2\sqrt[4]{a}(Bc-Ad)\sqrt{e}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d^2\sqrt{a-bx^2}}$$

output
```
2*a^(3/4)*B*e^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(b^(1/4)*(e*x)^(1/2)/a^(
1/4)/e^(1/2),I)/b^(3/4)/d/(-b*x^2+a)^(1/2)+2*a^(1/4)*(A*d-B*(c+a^(1/2)*d/b
^(1/2)))*e^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(b^(1/4)*(e*x)^(1/2)/a^(1/4
)/e^(1/2),I)/b^(1/4)/d^2/(-b*x^2+a)^(1/2)+2*a^(1/4)*(-A*d+B*c)*e^(1/2)*((-
b*x^2+a)/a)^(1/2)*EllipticPi(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),-a^(1/2)*
d/b^(1/2)/c,I)/b^(1/4)/d^2/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx =$$
$$-\frac{2i\sqrt{ex}\sqrt{1-\frac{bx^2}{a}}\left(\sqrt{a}BdE\left(i\text{arcsinh}\left(\sqrt{-\frac{\sqrt{b}x}{\sqrt{a}}}\right)\middle|-1\right)-\left(\sqrt{a}Bd+\sqrt{b}(Bc-Ad)\right)\text{EllipticF}\left(i\text{arcsinh}\right.\right.}{\sqrt{b}d^2\sqrt{-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x))/((c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
((-2*I)*Sqrt[e*x]*Sqrt[1 - (b*x^2)/a]*(Sqrt[a]*B*d*EllipticE[I*ArcSinh[Sqr
t[-((Sqrt[b]*x)/Sqrt[a])]], -1] - (Sqrt[a]*B*d + Sqrt[b]*(B*c - A*d))*Elli
pticF[I*ArcSinh[Sqrt[-((Sqrt[b]*x)/Sqrt[a])]], -1] + Sqrt[b]*(B*c - A*d)*E
llipticPi[-((Sqrt[a]*d)/(Sqrt[b]*c)), I*ArcSinh[Sqrt[-((Sqrt[b]*x)/Sqrt[a]
)]], -1]))/(Sqrt[b]*d^2*Sqrt[-((Sqrt[b]*x)/Sqrt[a])]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2354, 2235, 1513, 27, 765, 762, 1390, 1389, 327, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx)}{\sqrt{a-bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{ 2354}$$

$$\frac{2\int \frac{ex(Ae+Bxe)}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 2235}$$

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{\int \frac{(Bc-Ad)e-Bdex}{\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}\right)}{e}$$

$\downarrow$ 1513

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\left(e\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\int \frac{1}{\sqrt{a-bx^2}}d\sqrt{ex}\right)-\frac{\sqrt{a}Bde\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a}e\sqrt{a-bx^2}}d\sqrt{ex}}{\sqrt{b}}}{d^2}\right)}{e}$$

$\downarrow$ 27

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\left(e\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\int \frac{1}{\sqrt{a-bx^2}}d\sqrt{ex}\right)-\frac{Bd\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{\sqrt{b}}}{d^2}\right)}{e}$$

$\downarrow$ 765

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\frac{e\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{\sqrt{a-bx^2}}-\frac{Bd\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{\sqrt{b}}}{d^2}\right)}{e}$$

$\downarrow$ 762

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\frac{Bd\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a-bx^2}}d\sqrt{ex}}{\sqrt{b}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}\sqrt{a-bx^2}}}{d^2}\right)}{e}$$

$\downarrow$ 1390

$$\frac{2\left(\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\frac{Bd\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{\sqrt{b}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-\right.}{\sqrt[4]{b}\sqrt{a-bx^2}}}{d^2}\right.}{e}$$

$\downarrow$ 1389

$$2\left(\frac{\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\frac{\sqrt{a}Bde\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}d\sqrt{ex}}{\sqrt{b}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\right.}{\sqrt[4]{b}\sqrt{a-bx^2}}}{d^2}}{e}\right.$$

$\downarrow$ 327

$$2\left(\frac{\frac{ce^2(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d^2}-\frac{-\frac{a^{3/4}Bde^{3/2}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\text{EllipticF}\left(\text{arc}\right.}{\sqrt[4]{b}\sqrt{a-bx^2}}}{d^2}}{e}\right.$$

$\downarrow$ 1543

$$2\left(\frac{\frac{ce^2\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{d^2\sqrt{a-bx^2}}-\frac{-\frac{a^{3/4}Bde^{3/2}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}\left(Ad-B\left(\frac{\sqrt{a}d}{\sqrt{b}}+c\right)\right)\text{El}}{\sqrt[4]{b}\sqrt{a-b}}}{d^2}}{e}\right.$$

$\downarrow$ 1542

$$2\left(\frac{\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d^2\sqrt{a-bx^2}}-\frac{-\frac{a^{3/4}Bde^{3/2}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{b^{3/4}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}e^{3/2}\sqrt{1-b}}{}}{d^2}}{e}\right.$$

input
```
Int[(Sqrt[e*x]*(A + B*x))/((c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(-((-((a^(3/4)*B*d*e^(3/2)*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[(b^(1/4
)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(3/4)*Sqrt[a - b*x^2])) - (a^(1/4
)*(A*d - B*(c + (Sqrt[a]*d)/Sqrt[b]))*e^(3/2)*Sqrt[1 - (b*x^2)/a]*Elliptic
F[ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1/4)*Sqrt[a - b*
x^2]))/d^2) + (a^(1/4)*(B*c - A*d)*e^(3/2)*Sqrt[1 - (b*x^2)/a]*EllipticPi[
-((Sqrt[a]*d)/(Sqrt[b]*c)), ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
 -1])/(b^(1/4)*d^2*Sqrt[a - b*x^2])))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1513
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[-c/a, 2]}, Simp[(d*q - e)/q   Int[1/Sqrt[a + c*x^4], x], x] + Simp[e/q
  Int[(1 + q*x^2)/Sqrt[a + c*x^4], x], x]] /; FreeQ[{a, c, d, e}, x] && Neg
Q[c/a] && NeQ[c*d^2 + a*e^2, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2235
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff[P4x, x, 4]}, Si
mp[-(e^2)^(-1)   Int[(C*d - B*e - C*e*x^2)/Sqrt[a + c*x^4], x], x] + Simp[(
C*d^2 - B*d*e + A*e^2)/e^2   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /
; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[c*d^2 - a*e^2, 0]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.54

method	result
default	$\dfrac{\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)bcd-A\sqrt{ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)d^{2}-A\,\mathrm{EllipticPi}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{ab}\,d}{\sqrt{ab}\,d-bc},\frac{\sqrt{2}}{2}\right)bcd-B\,\mathrm{EllipticF}\right.}{}$
elliptic	$\dfrac{\sqrt{ex}\,\sqrt{xe(-b\,x^{2}+a)}\left(\frac{(Ad-Bc)e\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{d^{2}b\sqrt{-be\,x^{3}+aex}}+\frac{Be\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-}}{}\right.}{}$

input
```
int((e*x)^(1/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(A*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*c*d-A*(a
*b)^(1/2)*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),1/2*2^(1/2))*d^2
-A*EllipticPi(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),(a*b)^(1/2)*d/((a*b)^(
1/2)*d-b*c),1/2*2^(1/2))*b*c*d-B*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1/2))
^(1/2),1/2*2^(1/2))*a*d^2-B*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2
),1/2*2^(1/2))*b*c^2+2*B*(a*b)^(1/2)*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1
/2))^(1/2),1/2*2^(1/2))*c*d+B*EllipticPi(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(
1/2),(a*b)^(1/2)*d/((a*b)^(1/2)*d-b*c),1/2*2^(1/2))*b*c^2+2*B*EllipticE(((
b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*d^2-2*B*(a*b)^(1/2)*Ell
ipticE(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),1/2*2^(1/2))*c*d)*(-b/(a*b)^(
1/2)*x)^(1/2)*((-b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2)*2^(1/2)*((b*x+(a*b)^(
1/2))/(a*b)^(1/2))^(1/2)*(a*b)^(1/2)*(e*x)^(1/2)/b/(-b*x^2+a)^(1/2)/d^2/(-
(a*b)^(1/2)*d+b*c)/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{ex}\,(A+Bx)}{\sqrt{a-bx^2}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(1/2)*(B*x+A)/(d*x+c)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(e*x)*(A + B*x)/(sqrt(a - b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex}}{\sqrt{-bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)*sqrt(e*x)/(sqrt(-b*x^2 + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex}}{\sqrt{-bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)*sqrt(e*x)/(sqrt(-b*x^2 + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{e\,x}\,(A+B\,x)}{\sqrt{a-b\,x^2}\,(c+d\,x)}\,dx$$

input
```
int(((e*x)^(1/2)*(A + B*x))/((a - b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(((e*x)^(1/2)*(A + B*x))/((a - b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a-bx^2}}\,dx = \sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}\,x}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)b + \left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)a\right)$$

input
```
int((e*x)^(1/2)*(B*x+A)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

output
```
sqrt(e)*(int((sqrt(x)*sqrt(a - b*x**2)*x)/(a*c + a*d*x - b*c*x**2 - b*d*x*
*3),x)*b + int((sqrt(x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*d*x*
*3),x)*a)
```

3.250 $\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 177

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2\sqrt[4]{a}B\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d\sqrt{e}\sqrt{a-bx^2}} - \frac{2\sqrt[4]{a}(Bc-Ad)\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}cd\sqrt{e}\sqrt{a-bx^2}}$$

output
```
2*a^(1/4)*B*((-b*x^2+a)/a)^(1/2)*EllipticF(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(
1/2),I)/b^(1/4)/d/e^(1/2)/(-b*x^2+a)^(1/2)-2*a^(1/4)*(-A*d+B*c)*((-b*x^2+a
)/a)^(1/2)*EllipticPi(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),-a^(1/2)*d/b^(1/
2)/c,I)/b^(1/4)/c/d/e^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.90 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2i\sqrt{1-\frac{a}{bx^2}}x^{3/2}\left(Ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right),-1\right)+(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{b}c}{\sqrt{a}d},i\text{arcsinh}\left(\frac{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right),-1\right)\right)}{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}cd\sqrt{ex}\sqrt{a-bx^2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
((2*I)*Sqrt[1 - a/(b*x^2)]*x^(3/2)*(A*d*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[a]
/Sqrt[b])]/Sqrt[x]], -1] + (B*c - A*d)*EllipticPi[-((Sqrt[b]*c)/(Sqrt[a]*d
)), I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1]))/(Sqrt[-(Sqrt[a]/Sqr
t[b])]*c*d*Sqrt[e*x]*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2354, 2229, 765, 762, 1543, 1542}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{Ae+Bxe}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2229}$$

$$\frac{2\left(\frac{B\int \frac{1}{\sqrt{a-bx^2}}d\sqrt{ex}}{d}-\frac{e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d}\right)}{e}$$

$\Big\downarrow$ 765

$$\frac{2\left(\frac{B\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{d\sqrt{a-bx^2}}-\frac{e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d}\right)}{e}$$

$\Big\downarrow$ 762

$$\frac{2\left(\frac{\sqrt[4]{a}B\sqrt{e}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d\sqrt{a-bx^2}}-\frac{e(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{d}\right)}{e}$$

$\Big\downarrow$ 1543

$$\frac{2\left(\frac{\sqrt[4]{a}B\sqrt{e}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d\sqrt{a-bx^2}}-\frac{e\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\int \frac{1}{(ce+dxe)\sqrt{1-\frac{bx^2}{a}}}d\sqrt{ex}}{d\sqrt{a-bx^2}}\right)}{e}$$

$\Big\downarrow$ 1542

$$\frac{2\left(\frac{\sqrt[4]{a}B\sqrt{e}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}d\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}\sqrt{e}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}cd\sqrt{a-bx^2}}\right)}{e}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*((a^(1/4)*B*Sqrt[e]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[(b^(1/4)*Sqrt[
e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1/4)*d*Sqrt[a - b*x^2]) - (a^(1/4)*(B*c
 - A*d)*Sqrt[e]*Sqrt[1 - (b*x^2)/a]*EllipticPi[-((Sqrt[a]*d)/(Sqrt[b]*c)),
 ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1/4)*c*d*Sqrt[a -
 b*x^2])))/e
```

Defintions of rubi rules used

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1542
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-c/a, 4]}, Simp[(1/(d*Sqrt[a]*q))*EllipticPi[-e/(d*q^2), ArcSin[q*x
], -1], x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

rule 1543
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Simp[
Sqrt[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[1/((d + e*x^2)*Sqrt[1 + c*(x^4/a)
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

rule 2229
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Simp[B/e   Int[1/Sqrt[a + c*x^4], x], x] + Simp[(e*A - d*B)/
e   Int[1/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e, A, B}
, x] && NeQ[c*d^2 - a*e^2, 0] && NegQ[c/a]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(A\,\text{EllipticPi}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{ab}\,d}{\sqrt{ab}\,d-bc},\frac{\sqrt{2}}{2}\right)bd-B\sqrt{ab}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)d+B\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)bc-B\,\text{EllipticPi}\left(\cdots\right.}{\sqrt{-b\,x^{2}+a}\,db\left(-\sqrt{ab}\,d+bc\right)\sqrt{ex}}$
elliptic	$\frac{\sqrt{xe\left(-b\,x^{2}+a\right)}\left(\frac{B\sqrt{ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{-\frac{bx}{\sqrt{ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{db\sqrt{-be\,x^{3}+aex}}+\frac{\left(Ad-Bc\right)\sqrt{ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\,\sqrt{-\frac{2\left(x-\cdots\right.}{\cdots}}\cdots}{d^{2}\cdots}\right.}{\sqrt{ex}\,\sqrt{-b\,x^{2}+a}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(d*x+c)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(A*EllipticPi(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2),(a*b)^(1/2)*d/((a*b)^(
1/2)*d-b*c),1/2*2^(1/2))*b*d-B*(a*b)^(1/2)*EllipticF(((b*x+(a*b)^(1/2))/(a
*b)^(1/2))^(1/2),1/2*2^(1/2))*d+B*EllipticF(((b*x+(a*b)^(1/2))/(a*b)^(1/2)
)^(1/2),1/2*2^(1/2))*b*c-B*EllipticPi(((b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2
),(a*b)^(1/2)*d/((a*b)^(1/2)*d-b*c),1/2*2^(1/2))*b*c)*(-b/(a*b)^(1/2)*x)^(
1/2)*((-b*x+(a*b)^(1/2))/(a*b)^(1/2))^(1/2)*2^(1/2)*((b*x+(a*b)^(1/2))/(a*
b)^(1/2))^(1/2)*(a*b)^(1/2)/(-b*x^2+a)^(1/2)/d/b/(-(a*b)^(1/2)*d+b*c)/(e*x
)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx}{\sqrt{ex}\sqrt{a-bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(d*x+c)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a - b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx=\int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx=\int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{a-b\,x^2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a - b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{\sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}}{-bd\,x^4-bc\,x^3+ad\,x^2+acx}dx\right)a+\left(\int \frac{\sqrt{x}\,\sqrt{-b\,x^2+a}}{-bd\,x^3-bc\,x^2+adx+ac}dx\right)b\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*(int((sqrt(x)*sqrt(a - b*x**2))/(a*c*x + a*d*x**2 - b*c*x**3 - b*
d*x**4),x)*a + int((sqrt(x)*sqrt(a - b*x**2))/(a*c + a*d*x - b*c*x**2 - b*
d*x**3),x)*b))/e
```

3.251 $\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}} dx$

Optimal result

Integrand size = 32, antiderivative size = 280

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}} dx = -\frac{2A\sqrt{a-bx^2}}{ace\sqrt{ex}} - \frac{2A\sqrt[4]{b}\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{a}ce^{3/2}\sqrt{a-bx^2}} + \frac{2A\sqrt[4]{b}\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{a}ce^{3/2}\sqrt{a-bx^2}} + \frac{2\sqrt[4]{a}(Bc-Ad)\sqrt{\frac{a-bx^2}{a}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}c^2e^{3/2}\sqrt{a-bx^2}}$$

output
```
-2*A*(-b*x^2+a)^(1/2)/a/c/e/(e*x)^(1/2)-2*A*b^(1/4)*((-b*x^2+a)/a)^(1/2)*E
llipticE(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),I)/a^(1/4)/c/e^(3/2)/(-b*x^2+
a)^(1/2)+2*A*b^(1/4)*((-b*x^2+a)/a)^(1/2)*EllipticF(b^(1/4)*(e*x)^(1/2)/a^
(1/4)/e^(1/2),I)/a^(1/4)/c/e^(3/2)/(-b*x^2+a)^(1/2)+2*a^(1/4)*(-A*d+B*c)*(
(-b*x^2+a)/a)^(1/2)*EllipticPi(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),-a^(1/2
)*d/b^(1/2)/c,I)/b^(1/4)/c^2/e^(3/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.76 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2i\sqrt{1-\frac{a}{bx^2}}x^{5/2}\left(A\sqrt{b}cE\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\left(A\sqrt{b}c-\sqrt{a}Bc\right.\right.}{}$$

input
```
Integrate[(A + B*x)/((e*x)^(3/2)*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
((2*I)*Sqrt[1 - a/(b*x^2)]*x^(5/2)*(A*Sqrt[b]*c*EllipticE[I*ArcSinh[Sqrt[-
(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1] - (A*Sqrt[b]*c - Sqrt[a]*B*c + Sqrt[a]*A*
d)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1] + Sqrt[a]*(-
(B*c) + A*d)*EllipticPi[-((Sqrt[b]*c)/(Sqrt[a]*d)), I*ArcSinh[Sqrt[-(Sqrt[
a]/Sqrt[b])]/Sqrt[x]], -1]))/(Sqrt[a]*Sqrt[-(Sqrt[a]/Sqrt[b])]*c^2*(e*x)^(
3/2)*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2354, 2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(ex)^{3/2}\sqrt{a-bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{Ae+Bxe}{ex(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2249}$$

$$\frac{2\int \left(\frac{A}{cex\sqrt{a-bx^2}}+\frac{Bc-Ad}{c(ce+dxe)\sqrt{a-bx^2}}\right)d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}c^2\sqrt{e}\sqrt{a-bx^2}}+\frac{A\sqrt[4]{b}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{a}c\sqrt{e}\sqrt{a-bx^2}}-\frac{A\sqrt[4]{b}\sqrt{1-\frac{bx^2}{a}}}{}\right.}{e}$$

input
```
Int[(A + B*x)/((e*x)^(3/2)*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(-((A*Sqrt[a - b*x^2])/(a*c*Sqrt[e*x])) - (A*b^(1/4)*Sqrt[1 - (b*x^2)/a
]*EllipticE[ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(a^(1/4)*c
*Sqrt[e]*Sqrt[a - b*x^2]) + (A*b^(1/4)*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSi
n[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(a^(1/4)*c*Sqrt[e]*Sqrt[a -
 b*x^2]) + (a^(1/4)*(B*c - A*d)*Sqrt[1 - (b*x^2)/a]*EllipticPi[-((Sqrt[a]*
d)/(Sqrt[b]*c)), ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1
/4)*c^2*Sqrt[e]*Sqrt[a - b*x^2])))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{2A\sqrt{-b\,x^2+a}}{ace\sqrt{ex}}-\frac{\left(\frac{A\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\left(-\frac{2\sqrt{ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{ab}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}} \cdots\right.}{b}\cdots\right.}{\sqrt{-be\,x^3+aex}}\cdots\right)}{\cdots}$
elliptic	$\frac{\sqrt{xe\left(-b\,x^2+a\right)}\left(-\frac{2\left(-be\,x^2+ae\right)A}{e^2ac\sqrt{x\left(-be\,x^2+ae\right)}}-\frac{A\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\left(-\frac{2\sqrt{ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{ab}\cdots}{\cdots}\cdots\right.}{aec\sqrt{-be\,x^3+aex}}\cdots\right)}{\sqrt{ex}\,\sqrt{-\cdots}}$
default	$\frac{2A\sqrt{2}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)abc\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}}\sqrt{\frac{-bx+\sqrt{ab}}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}-2A\sqrt{2}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{\frac{bx+\sqrt{ab}}{\sqrt{ab}}}\sqrt{\frac{-bx+\sqrt{ab}}{\sqrt{ab}}}\cdots}{\cdots}$

input
```
int((B*x+A)/(e*x)^(3/2)/(d*x+c)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(-b*x^2+a)^(1/2)/a/c/e/(e*x)^(1/2)-1/c/a*(A*(a*b)^(1/2)*((x+1/b*(a*b)
^(1/2))*b/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*
(-b/(a*b)^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)*(-2/b*(a*b)^(1/2)*Elliptic
E(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(a*b)^(1/2)*E
llipticF(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2^(1/2)))+a*(A*d-B*
c)/d/b*(a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a
*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^
(1/2)/(c/d-1/b*(a*b)^(1/2))*EllipticPi(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))
^(1/2),-1/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)),1/2*2^(1/2)))/e*(x*e*(-b*x^2
+a))^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx}{(ex)^{\frac{3}{2}}\sqrt{a-bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(3/2)/(d*x+c)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((e*x)**(3/2)*sqrt(a - b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x}{{(e\,x)}^{3/2}\,\sqrt{a-b\,x^2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((e*x)^(3/2)*(a - b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(3/2)*(a - b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{x}\,\sqrt{-b\,x^2+a}-\left(\int \frac{\sqrt{-b\,x^2+a}}{\sqrt{x}\,ac+\sqrt{x}\,adx-\sqrt{x}\,bc\,x^2-\sqrt{x}\,bd\,x^3}dx\right)adx-\left(\int \right.\right.}{}$$

input
```
int((B*x+A)/(e*x)^(3/2)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(x)*sqrt(a - b*x**2) - int(sqrt(a - b*x**2)/(sqrt(x)*a*
c + sqrt(x)*a*d*x - sqrt(x)*b*c*x**2 - sqrt(x)*b*d*x**3),x)*a*d*x - int((s
qrt(a - b*x**2)*x**2)/(sqrt(x)*a*c + sqrt(x)*a*d*x - sqrt(x)*b*c*x**2 - sq
rt(x)*b*d*x**3),x)*b*d*x - int((sqrt(a - b*x**2)*x)/(sqrt(x)*a*c + sqrt(x)
*a*d*x - sqrt(x)*b*c*x**2 - sqrt(x)*b*d*x**3),x)*b*c*x + int((sqrt(x)*sqrt
(a - b*x**2))/(a*c*x + a*d*x**2 - b*c*x**3 - b*d*x**4),x)*b*c*x))/(c*e**2*
x)
```

3.252 $\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 356

$$\begin{aligned}\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx =& -\frac{2A\sqrt{a-bx^2}}{3ace(ex)^{3/2}}-\frac{2(Bc-Ad)\sqrt{a-bx^2}}{ac^2e^2\sqrt{ex}}\\ &-\frac{2\sqrt[4]{b}(Bc-Ad)\sqrt{\frac{a-bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)\middle|-1\right)}{\sqrt[4]{a}c^2e^{5/2}\sqrt{a-bx^2}}\\ &+\frac{2\sqrt[4]{b}\left(A\sqrt{b}c+3\sqrt{a}Bc-3\sqrt{a}Ad\right)\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{3a^{3/4}c^2e^{5/2}\sqrt{a-bx^2}}\\ &-\frac{2\sqrt[4]{a}d(Bc-Ad)\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}c^3e^{5/2}\sqrt{a-bx^2}}\end{aligned}$$

output
```
-2/3*A*(-b*x^2+a)^(1/2)/a/c/e/(e*x)^(3/2)-2*(-A*d+B*c)*(-b*x^2+a)^(1/2)/a/
c^2/e^2/(e*x)^(1/2)-2*b^(1/4)*(-A*d+B*c)*((-b*x^2+a)/a)^(1/2)*EllipticE(b^
(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),I)/a^(1/4)/c^2/e^(5/2)/(-b*x^2+a)^(1/2)+
2/3*b^(1/4)*(A*b^(1/2)*c+3*a^(1/2)*B*c-3*a^(1/2)*A*d)*((-b*x^2+a)/a)^(1/2)
*EllipticF(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2),I)/a^(3/4)/c^2/e^(5/2)/(-b*
x^2+a)^(1/2)-2*a^(1/4)*d*(-A*d+B*c)*((-b*x^2+a)/a)^(1/2)*EllipticPi(b^(1/4
)*(e*x)^(1/2)/a^(1/4)/e^(1/2),-a^(1/2)*d/b^(1/2)/c,I)/b^(1/4)/c^3/e^(5/2)/
(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.18 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \frac{2x^2\sqrt{a-bx^2}\left(-\frac{c(3Bcx+A(c-3dx))}{x}+\frac{3i\sqrt{a}\sqrt{b}c(Bc-Ad)\sqrt{1-\frac{a}{bx^2}}x^{3/2}E\left(i\operatorname{arcsinh}\left(\cdots\right.\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(A + B*x)/((e*x)^(5/2)*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*x^2*Sqrt[a - b*x^2]*(-((c*(3*B*c*x + A*(c - 3*d*x)))/x) + ((3*I)*Sqrt[a
]*Sqrt[b]*c*(B*c - A*d)*Sqrt[1 - a/(b*x^2)]*x^(3/2)*EllipticE[I*ArcSinh[Sq
rt[-(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1] + I*(-3*(Sqrt[a]*Sqrt[b]*B*c^2 + a*B*
c*d) + A*(b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d + 3*a*d^2))*Sqrt[1 - a/(b*x^2)]*x^
(3/2)*EllipticF[I*ArcSinh[Sqrt[-(Sqrt[a]/Sqrt[b])]/Sqrt[x]], -1] + 3*(B*c
- A*d)*(Sqrt[-(Sqrt[a]/Sqrt[b])]*c*(a - b*x^2) + I*a*d*Sqrt[1 - a/(b*x^2)]
*x^(3/2)*EllipticPi[-((Sqrt[b]*c)/(Sqrt[a]*d)), I*ArcSinh[Sqrt[-(Sqrt[a]/S
qrt[b])]/Sqrt[x]], -1]))/(Sqrt[-(Sqrt[a]/Sqrt[b])]*(a - b*x^2))))/(3*a*c^3
*(e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2354, 2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(ex)^{5/2}\sqrt{a-bx^2}(c+dx)}dx$$

$$\Big\downarrow \ 2354$$

$$\frac{2\int \frac{Ae+Bxe}{e^2x^2(ce+dxe)\sqrt{a-bx^2}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2249}$$

$$\frac{2\int \left(\frac{A}{ce^2x^2\sqrt{a-bx^2}}+\frac{Bc-Ad}{c^2e^2x\sqrt{a-bx^2}}-\frac{d(Bc-Ad)}{c^2e(ce+dxe)\sqrt{a-bx^2}}\right)d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{Ab^{3/4}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{3a^{3/4}ce^{3/2}\sqrt{a-bx^2}}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^2}{a}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\sqrt{a}d}{\sqrt{b}c},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),-1\right)}{\sqrt[4]{b}c^3e^{3/2}\sqrt{a-bx^2}}+\frac{\sqrt[4]{b}\sqrt{1-\frac{bx^2}{a}}}{}\right.}{e}$$

input
```
Int[(A + B*x)/((e*x)^(5/2)*(c + d*x)*Sqrt[a - b*x^2]),x]
```

output
```
(2*(-1/3*(A*Sqrt[a - b*x^2])/(a*c*(e*x)^(3/2)) - ((B*c - A*d)*Sqrt[a - b*x
^2])/(a*c^2*e*Sqrt[e*x]) - (b^(1/4)*(B*c - A*d)*Sqrt[1 - (b*x^2)/a]*Ellipt
icE[ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(a^(1/4)*c^2*e^(3/
2)*Sqrt[a - b*x^2]) + (A*b^(3/4)*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[(b^(
1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(3*a^(3/4)*c*e^(3/2)*Sqrt[a - b*x
^2]) + (b^(1/4)*(B*c - A*d)*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[(b^(1/4)*
Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(a^(1/4)*c^2*e^(3/2)*Sqrt[a - b*x^2])
- (a^(1/4)*d*(B*c - A*d)*Sqrt[1 - (b*x^2)/a]*EllipticPi[-((Sqrt[a]*d)/(Sqr
t[b]*c)), ArcSin[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], -1])/(b^(1/4)*c^3
*e^(3/2)*Sqrt[a - b*x^2])))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.51

method	result
elliptic	$\frac{\sqrt{xe(-b\,x^2+a)}\left(-\frac{2A\sqrt{-be\,x^3+aex}}{3e^3ac\,x^2}+\frac{2\left(-be\,x^2+ae\right)(Ad-Bc)}{e^3a\,c^2\sqrt{x\left(-be\,x^2+ae\right)}}+\frac{A\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\right.}{3a\,e^2c\sqrt{-be\,x^3+aex}}\right.}{}$
risch	$-\frac{2\sqrt{-b\,x^2+a}\,(-3Adx+3Bcx+Ac)}{3a\,c^2x\,e^2\sqrt{ex}}+\frac{\left(\frac{Ac\sqrt{ab}\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}}\sqrt{-\frac{bx}{\sqrt{ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{ab}}{b}\right)b}{\sqrt{ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{-be\,x^3+aex}}+\frac{3Ad\sqrt{ab}\sqrt{}}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*e*(-b*x^2+a))^(1/2)/(e*x)^(1/2)/(-b*x^2+a)^(1/2)*(-2/3/e^3/a*A/c*(-b*e*
x^3+a*e*x)^(1/2)/x^2+2*(-b*e*x^2+a*e)/e^3/a*(A*d-B*c)/c^2/(x*(-b*e*x^2+a*e
))^(1/2)+1/3/a/e^2*A/c*(a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/
2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)^(1/2)*x)^(1/2)/(
-b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),
1/2*2^(1/2))+1/a/e^2*(A*d-B*c)/c^2*(a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b
)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)^(1/2
)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)*(-2/b*(a*b)^(1/2)*EllipticE(((x+1/b*(a*b
)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+1
/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),1/2*2^(1/2)))+1/e^2*(A*d-B*c)/c^2/b*(
a*b)^(1/2)*((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2)*(-2*(x-1/b*(a*b)^(1/2
))*b/(a*b)^(1/2))^(1/2)*(-b/(a*b)^(1/2)*x)^(1/2)/(-b*e*x^3+a*e*x)^(1/2)/(c
/d-1/b*(a*b)^(1/2))*EllipticPi(((x+1/b*(a*b)^(1/2))*b/(a*b)^(1/2))^(1/2),-
1/b*(a*b)^(1/2)/(c/d-1/b*(a*b)^(1/2)),1/2*2^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="frica
s")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx}{(ex)^{\frac{5}{2}}\sqrt{a-bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(5/2)/(d*x+c)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((e*x)**(5/2)*sqrt(a - b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*(e*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}(dx+c)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x, algorithm="giac"
)
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*(d*x + c)*(e*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x}{(e\,x)^{5/2}\,\sqrt{a-b\,x^2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x)/((e*x)^(5/2)*(a - b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(5/2)*(a - b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{(ex)^{\frac{5}{2}}\,(dx+c)\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(-b*x^2+a)^(1/2),x)
```

3.253 $\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 626

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2Be\sqrt{ex}\sqrt{a+bx^2}}{3bd}$$

$$-\frac{2(Bc-Ad)e\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{b}d^2\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{c^{3/2}(Bc-Ad)e^{3/2}\arctan\left(\frac{\sqrt{bc^2+ad^2}\sqrt{ex}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{d^{5/2}\sqrt{bc^2+ad^2}}$$

$$+\frac{2\sqrt[4]{a}(Bc-Ad)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{b^{3/4}d^2\sqrt{a+bx^2}}$$

$$+\frac{\sqrt[4]{a}\left(aBd^2+\sqrt{a}\sqrt{b}d(2Bc-3Ad)-6bc(Bc-Ad)\right)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{3b^{5/4}d^2\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{a+bx^2}}$$

$$+\frac{c\left(\sqrt{b}c+\sqrt{a}d\right)(Bc-Ad)e^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right.}{2\sqrt[4]{a}\sqrt[4]{b}d^3\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{a+bx^2}}$$

output
```
2/3*B*e*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b/d-2*(-A*d+B*c)*e*(e*x)^(1/2)*(b*x^2+
a)^(1/2)/b^(1/2)/d^2/(a^(1/2)+b^(1/2)*x)-c^(3/2)*(-A*d+B*c)*e^(3/2)*arctan
((a*d^2+b*c^2)^(1/2)*(e*x)^(1/2)/c^(1/2)/d^(1/2)/e^(1/2)/(b*x^2+a)^(1/2))/
d^(5/2)/(a*d^2+b*c^2)^(1/2)+2*a^(1/4)*(-A*d+B*c)*e^(3/2)*(a^(1/2)+b^(1/2)*
x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*
(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))/b^(3/4)/d^2/(b*x^2+a)^(1/2)+1/3
*a^(1/4)*(a*B*d^2+a^(1/2)*b^(1/2)*d*(-3*A*d+2*B*c)-6*b*c*(-A*d+B*c))*e^(3/
2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJaco
biAM(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)),1/2*2^(1/2))/b^(5/4)/d^
2/(b^(1/2)*c-a^(1/2)*d)/(b*x^2+a)^(1/2)+1/2*c*(b^(1/2)*c+a^(1/2)*d)*(-A*d+
B*c)*e^(3/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*E
llipticPi(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),-1/4*(b^(1/2)
*c-a^(1/2)*d)^2/a^(1/2)/b^(1/2)/c/d,1/2*2^(1/2))/a^(1/4)/b^(1/4)/d^3/(b^(1
/2)*c-a^(1/2)*d)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.79 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.54

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2(ex)^{3/2}\left(\frac{Bd^2(a+bx^2)}{x}+\frac{-\frac{3\sqrt{a}\sqrt{b}d(-Bc+Ad)\sqrt{1+\frac{a}{bx^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{x}}+\frac{\sqrt{a}d\left(-i\sqrt{a}Bd+\sqrt{b}\cdots\right.}{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
Integrate[((e*x)^(3/2)*(A + B*x))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*(e*x)^(3/2)*((B*d^2*(a + b*x^2))/x + ((-3*Sqrt[a]*Sqrt[b]*d*(-(B*c) + A
*d)*Sqrt[1 + a/(b*x^2)]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt
[x]], -1])/Sqrt[x] + (Sqrt[a]*d*((-I)*Sqrt[a]*B*d + Sqrt[b]*(-3*B*c + 3*A*
d))*Sqrt[1 + a/(b*x^2)]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt
[x]], -1])/Sqrt[x] + ((3*I)*(B*c - A*d)*(I*Sqrt[(I*Sqrt[a])/Sqrt[b]]*d*(a
+ b*x^2) + b*c*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticPi[((-I)*Sqrt[b]*c)/(Sq
rt[a]*d), I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/x^2)/Sqrt[(I
*Sqrt[a])/Sqrt[b]]))/(3*b*d^3*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 3.31 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {2354, 2237, 2233, 25, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(A+Bx)}{\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \ 2354$$

$$\frac{2\int \frac{e^2x^2(Ae+Bxe)}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \ 2237$$

$$\frac{2\left(\frac{e^2\int \frac{3bdx^2(Ae+Bxe)-\frac{B(ce+dxe)\left(3bx^2e^2+ae^2\right)}{e^2}}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{3bd}+\frac{Be^2\sqrt{ex}\sqrt{a+bx^2}}{3bd}\right)}{e}$$

$$\Big\downarrow \ 2233$$

$$\frac{2\left(\frac{e^2\left(\frac{e^2\int -\frac{b\left(\sqrt{a}c\left(\sqrt{a}Bd+3\sqrt{b}(Bc-Ad)\right)e+\left(aBd^2+3\sqrt{a}\sqrt{b}(Bc-Ad)d-3bc(Bc-Ad)\right)xe\right)}{e^2(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{bd}+\frac{3\sqrt{a}\sqrt{b}(Bc-Ad)\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{d}\right)}{3bd}+\frac{Be^2\sqrt{ex}\sqrt{\ldots}}{3bd}\ldots\right)}{e}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{e^2\left(\frac{3\sqrt{a}\sqrt{b}(Bc-Ad)\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{d}-\frac{e^2\int \frac{b\left(\sqrt{a}c\left(\sqrt{a}Bd+3\sqrt{b}(Bc-Ad)\right)e+\left(aBd^2+3\sqrt{a}\sqrt{b}(Bc-Ad)d-3bc(Bc-Ad)\right)xe\right)}{e^2(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{bd}\right)}{3bd}+\frac{Be^2\sqrt{ex}\sqrt{a\ldots}}{3bd}\ldots\right)}{e}$$

$\downarrow$ 27

$$\frac{2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{de}-\frac{\int \frac{\sqrt{a}c\left(\sqrt{a}Bd+3\sqrt{b}(Bc-Ad)\right)e+\left(aBd^2+3\sqrt{a}\sqrt{b}(Bc-Ad)d-3bc(Bc-Ad)\right)xe}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{d}\right)}{3bd}+\frac{Be^2\sqrt{ex}\sqrt{a+bx^2}}{3bd}\right)}{e}$$

$\downarrow$ 1510

$$\frac{2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{de}-\frac{\int \frac{\sqrt{a}c\left(\sqrt{a}Bd+3\sqrt{b}(Bc-Ad)\right)e+\left(aBd^2+3\cdots\right.}{(ce+dxe)\sqrt{b\cdots}}}{d}\cdots}{3bd}\cdots\right.}{e}$$

$\downarrow$ 2227

$$2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{de}-\frac{-\frac{3\sqrt{a}bc^2e(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a}e(ce+dxe)\sqrt{bx^2+\cdots}}}{\sqrt{b}c-\sqrt{a}d}\cdots}{\cdots}\right)}{3bd}\cdots}{e}\cdots\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{de}-\frac{-\frac{3bc^2(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}\cdots}{\cdots}\right)}{3bd}\cdots}{e}\cdots\right)$$

$$\Big\downarrow \; 761$$

$$2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{de}-\frac{-\frac{3bc^2(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}}{}\right)}{3bd}}{e}\right.$$

$\downarrow$ 2221

$$2\left(\frac{e^2\left(\frac{3\sqrt{b}(Bc-Ad)\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{de}-\frac{-\frac{3bc^2(Bc-Ad)\left(\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\left(\sqrt{a}e+\sqrt{b}ex\right.}{}\right.}{}}{}\right)}{}\right.$$

input
```
Int[((e*x)^(3/2)*(A + B*x))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*((B*e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(3*b*d) + (e^2*((3*Sqrt[b]*(B*c - A*
d)*(-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (a^(1/4
)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e +
Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])],
 1/2])/(b^(1/4)*Sqrt[a + b*x^2])))/(d*e) - (-1/2*(a^(1/4)*(a*B*d^2 + Sqrt[
a]*Sqrt[b]*d*(2*B*c - 3*A*d) - 6*b*c*(B*c - A*d))*(Sqrt[a]*e + Sqrt[b]*e*x
)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan
[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*(Sqrt[b]*c - Sqrt[
a]*d)*Sqrt[e]*Sqrt[a + b*x^2]) - (3*b*c^2*(B*c - A*d)*(-1/2*((Sqrt[b]*c -
Sqrt[a]*d)*Sqrt[e]*ArcTan[(Sqrt[b*c^2 + a*d^2]*Sqrt[e*x])/(Sqrt[c]*Sqrt[d]
*Sqrt[e]*Sqrt[a + b*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[b*c^2 + a*d^2]) + ((Sqrt
[b]*c + Sqrt[a]*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqr
t[a]*e + Sqrt[b]*e*x)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*c)/Sqrt[a] - d
)^2)/(Sqrt[b]*c*d), 2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])
/(4*a^(1/4)*b^(1/4)*c*d*Sqrt[e]*Sqrt[a + b*x^2])))/(Sqrt[b]*c - Sqrt[a]*d)
)/d))/(3*b*d)))/e
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2237
```
Int[(Px_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> W
ith[{q = Expon[Px, x]}, Simp[Coeff[Px, x, q]*x^(q - 5)*(Sqrt[a + c*x^4]/(c*
e*(q - 3))), x] + Simp[1/(c*e*(q - 3))   Int[(c*e*(q - 3)*Px - Coeff[Px, x,
 q]*x^(q - 6)*(d + e*x^2)*(a*(q - 5) + c*(q - 3)*x^4))/((d + e*x^2)*Sqrt[a
+ c*x^4]), x], x] /; GtQ[q, 4]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Px, x]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.87

method	result
elliptic	$\frac{\sqrt{ex}\,\sqrt{xe(b\,x^2+a)}\left(\frac{2Be\sqrt{be\,x^3+aex}}{3db}+\frac{\left(-\frac{c(Ad-Bc)e^2}{d^3}-\frac{B\,e^2a}{3db}\right)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\sqrt{\ldots}\right.}{\sqrt{-\ldots}}}\right.}{b\sqrt{be\,x^3+aex}}\right.}{\ldots}$
risch	$\frac{2Bx\sqrt{b\,x^2+a}\,e^2}{3db\sqrt{ex}}+\frac{\left(\frac{3d(Ad-Bc)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-a\ldots}}{\ldots}\right.}{\sqrt{be\,x^3+aex}}\right.}{\ldots}$
default	Expression too large to display

input
```
int((e*x)^(3/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/e/x*(e*x)^(1/2)/(b*x^2+a)^(1/2)*(x*e*(b*x^2+a))^(1/2)*(2/3*B/d*e/b*(b*e*
x^3+a*e*x)^(1/2)+(-c*(A*d-B*c)*e^2/d^3-1/3*B/d*e^2/b*a)*(-a*b)^(1/2)/b*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*EllipticF(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+1/d^2*(A*d-B*c)*e^2*(-a*b
)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2
*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),
1/2*2^(1/2)))+c^2*(A*d-B*c)*e^2/d^4*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)
^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)/(-(-a*b)^(1/2)/b+c/d)*EllipticPi(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),-(-a*b)^(1/2)/b/(-(-a*b)^(1/2)/b+c/
d),1/2*2^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}\,(A+Bx)}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(3/2)*(B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((e*x)**(3/2)*(A + B*x)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(Bx+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)*(e*x)^(3/2)/(sqrt(b*x^2 + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(Bx+A)(ex)^{\frac{3}{2}}}{\sqrt{bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(3/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(e*x)^(3/2)/(sqrt(b*x^2 + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(e\,x)^{3/2}\,(A+B\,x)}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int(((e*x)^(3/2)*(A + B*x))/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(((e*x)^(3/2)*(A + B*x))/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{(ex)^{3/2}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e}\,e\left(2\sqrt{x}\,\sqrt{b\,x^2+a}\,a-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^3+bc\,x^2+adx+ac}dx\right)abd+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^3+bc\,x^2+adx+ac}d\right.\right.}{3bc}$$

input
```
int((e*x)^(3/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*e*(2*sqrt(x)*sqrt(a + b*x**2)*a - 3*int((sqrt(x)*sqrt(a + b*x**2)
*x**2)/(a*c + a*d*x + b*c*x**2 + b*d*x**3),x)*a*b*d + 3*int((sqrt(x)*sqrt(
a + b*x**2)*x**2)/(a*c + a*d*x + b*c*x**2 + b*d*x**3),x)*b**2*c - int((sqr
t(x)*sqrt(a + b*x**2))/(a*c*x + a*d*x**2 + b*c*x**3 + b*d*x**4),x)*a**2*c
- int((sqrt(x)*sqrt(a + b*x**2))/(a*c + a*d*x + b*c*x**2 + b*d*x**3),x)*a*
*2*d))/(3*b*c)
```

3.254 $\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 562

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2B\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{b}d\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}(Bc-Ad)\sqrt{e}\arctan\left(\frac{\sqrt{bc^2+ad^2}\sqrt{ex}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{d^{3/2}\sqrt{bc^2+ad^2}}$$
$$- \frac{2\sqrt[4]{a}B\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{b^{3/4}d\sqrt{a+bx^2}}$$
$$- \frac{\sqrt[4]{a}\left(\sqrt{a}Bd-\sqrt{b}(2Bc-Ad)\right)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{b^{3/4}d\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{a+bx^2}}$$
$$- \frac{\left(\sqrt{b}c+\sqrt{a}d\right)(Bc-Ad)\sqrt{e}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\right.}{2\sqrt[4]{a}\sqrt[4]{b}d^2\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{a+bx^2}}$$

output
```
2*B*(e*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/d/(a^(1/2)+b^(1/2)*x)+c^(1/2)*(-A*
d+B*c)*e^(1/2)*arctan((a*d^2+b*c^2)^(1/2)*(e*x)^(1/2)/c^(1/2)/d^(1/2)/e^(1
/2)/(b*x^2+a)^(1/2))/d^(3/2)/(a*d^2+b*c^2)^(1/2)-2*a^(1/4)*B*e^(1/2)*(a^(1
/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arc
tan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),1/2*2^(1/2))/b^(3/4)/d/(b*x^2+a)
^(1/2)-a^(1/4)*(a^(1/2)*B*d-b^(1/2)*(-A*d+2*B*c))*e^(1/2)*(a^(1/2)+b^(1/2)
*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/
4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)),1/2*2^(1/2))/b^(3/4)/d/(b^(1/2)*c-a^(1/2)*
d)/(b*x^2+a)^(1/2)-1/2*(b^(1/2)*c+a^(1/2)*d)*(-A*d+B*c)*e^(1/2)*(a^(1/2)+b
^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticPi(sin(2*arctan(
b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),-1/4*(b^(1/2)*c-a^(1/2)*d)^2/a^(1/2)
/b^(1/2)/c/d,1/2*2^(1/2))/a^(1/4)/b^(1/4)/d^2/(b^(1/2)*c-a^(1/2)*d)/(b*x^2
+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{ex}\sqrt{1+\frac{bx^2}{a}}\Big(\sqrt{a}BdE\Big(i\text{arcsinh}\Big(\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\Big)\Big|-1\Big)+i\Big(\Big(i\sqrt{a}Bd+\sqrt{b}(Bc-Ad)\Big)\text{EllipticF}\Big(i\text{arcsinh}\Big(}{\sqrt{b}d^2\sqrt{\frac{i\sqrt{b}x}{\sqrt{a}}}\sqrt{a+bx^2}}$$

input
```
Integrate[(Sqrt[e*x]*(A + B*x))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*Sqrt[e*x]*Sqrt[1 + (b*x^2)/a]*(Sqrt[a]*B*d*EllipticE[I*ArcSinh[Sqrt[(I*
Sqrt[b]*x)/Sqrt[a]]], -1] + I*((I*Sqrt[a]*B*d + Sqrt[b]*(B*c - A*d))*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]], -1] + Sqrt[b]*(-(B*c) + A*d)*
EllipticPi[((-I)*Sqrt[a]*d)/(Sqrt[b]*c), I*ArcSinh[Sqrt[(I*Sqrt[b]*x)/Sqrt
[a]]], -1])))/(Sqrt[b]*d^2*Sqrt[(I*Sqrt[b]*x)/Sqrt[a]]*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 2.35 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {2354, 2233, 27, 1510, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(A+Bx)}{\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{ex(Ae+Bxe)}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2233}$$

$$\frac{2\left(\frac{e^2\int \frac{\sqrt{b}\left(\sqrt{a}Bce+\left(\sqrt{a}Bd-\sqrt{b}(Bc-Ad)\right)xe\right)}{e(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{bd}-\frac{\sqrt{a}Be\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{a}e\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}d}\right)}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{e\int \frac{\sqrt{a}Bce+\left(\sqrt{a}Bd-\sqrt{b}(Bc-Ad)\right)xe}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}d}-\frac{B\int \frac{\sqrt{a}e-\sqrt{b}ex}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}d}\right)}{e}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2\left(\frac{e\int \frac{\sqrt{a}Bce+\left(\sqrt{a}Bd-\sqrt{b}(Bc-Ad)\right)xe}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}d}-\frac{B\left(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\frac{e^2\sqrt{ex}\sqrt{a+bx^2}}{\sqrt{a}e+\sqrt{b}ex}\right)}{\sqrt{b}d}\right)}{e}$$

$$\Big\downarrow \text{2227}$$

$$2\Bigg(\frac{e\left(-\frac{\sqrt{a}\left(\sqrt{a}Bd-\sqrt{b}(2Bc-Ad)\right)\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}-\frac{\sqrt{a}\sqrt{b}ce(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a}e(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}\right)}{\sqrt{b}d}-\frac{B\Bigg(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\Big(}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}$$

$$e$$

$\Big\downarrow$ 27

$$2\Bigg(\frac{e\left(-\frac{\sqrt{a}\left(\sqrt{a}Bd-\sqrt{b}(2Bc-Ad)\right)\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}-\frac{\sqrt{b}c(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}\right)}{\sqrt{b}d}-\frac{B\Bigg(\frac{\sqrt[4]{a}\sqrt{e}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}E\Big(2\arctan}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}d}$$

$$e$$

$\Big\downarrow$ 761

$$2\Bigg(\frac{e\left(-\frac{\sqrt{b}c(Bc-Ad)\int\frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}-\frac{\sqrt[4]{a}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\left(\sqrt{a}Bd-\sqrt{b}(2Bc-Ad)\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{b}\sqrt{e}\sqrt{a+bx^2}\left(\sqrt{b}c-\sqrt{a}d\right)}\right)}{\sqrt{b}d}-\frac{B}{}$$

$$e$$

$\Big\downarrow$ 2221

$$2\left(\frac{e\left(-\frac{\sqrt{b}c(Bc-Ad)\left(\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\operatorname{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)^2}{4\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}cd\sqrt{e}\sqrt{a+bx^2}}-\frac{\sqrt{e}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{ex}\sqrt{\cdots}}{\sqrt{c}\sqrt{d}\sqrt{\cdots}}\right.\cdots}{2\sqrt{c}\sqrt{d}\sqrt{ad^2+bc^2}}\cdots\right)}{\sqrt{b}c-\sqrt{a}d}\cdots\right)}{\sqrt{b}d}\cdots\right.$$

input
```
Int[(Sqrt[e*x]*(A + B*x))/((c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*(-((B*(-((e^2*Sqrt[e*x]*Sqrt[a + b*x^2])/(Sqrt[a]*e + Sqrt[b]*e*x)) + (
a^(1/4)*Sqrt[e]*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a
]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2])))/(Sqrt[b]*d)) + (e*(-1/2*(a^(1/4)*
(Sqrt[a]*B*d - Sqrt[b]*(2*B*c - A*d))*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^
2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(b^(1/4)*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[e
]*Sqrt[a + b*x^2]) - (Sqrt[b]*c*(B*c - A*d)*(-1/2*((Sqrt[b]*c - Sqrt[a]*d)
*Sqrt[e]*ArcTan[(Sqrt[b*c^2 + a*d^2]*Sqrt[e*x])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*S
qrt[a + b*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[b*c^2 + a*d^2]) + ((Sqrt[b]*c + Sq
rt[a]*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + S
qrt[b]*e*x)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*c)/Sqrt[a] - d)^2)/(Sqrt
[b]*c*d), 2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*a^(1/4
)*b^(1/4)*c*d*Sqrt[e]*Sqrt[a + b*x^2])))/(Sqrt[b]*c - Sqrt[a]*d)))/(Sqrt[b
]*d)))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2233
```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x, 2], C = Coeff
[P4x, x, 4]}, Simp[-C/(e*q)   Int[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] + Sim
p[1/(c*e)   Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a*e*q))*x^2)/((d + e*x
^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[P4x, x^2,
2] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.79

method	result
default	$\frac{\left(A\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bcd-A\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)d^{2}-A\,\mathrm{EllipticPi}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{-ab}\,d}{d\sqrt{-ab}-bc},\frac{\sqrt{2}}{2}\right)bcd+B\,\mathrm{E}\right.}{}$
elliptic	$\frac{\sqrt{ex}\,\sqrt{xe(b\,x^{2}+a)}\left(\frac{(Ad-Bc)e\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{d^{2}b\sqrt{be\,x^{3}+aex}}+\frac{Be\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-}}{}\right.}{\sqrt{-}}}}{}\right.}{}$

input
```
int((e*x)^(1/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(A*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*b*c*d-A*
(-a*b)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*d^2-A*EllipticPi(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),(-a*b)^(1/2)*d/
(d*(-a*b)^(1/2)-b*c),1/2*2^(1/2))*b*c*d+B*EllipticF(((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*d^2-B*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))*b*c^2+2*B*(-a*b)^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*c*d+B*EllipticPi(((b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),(-a*b)^(1/2)*d/(d*(-a*b)^(1/2)-b*c),1/2*2^(1/2))*b*
c^2-2*B*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*d
^2-2*B*(-a*b)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*
2^(1/2))*c*d)*(-b/(-a*b)^(1/2)*x)^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)*(e*x)^
(1/2)/b/(b*x^2+a)^(1/2)/d^2/(b*c-d*(-a*b)^(1/2))/x
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{ex}\,(A+Bx)}{\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((e*x)**(1/2)*(B*x+A)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(e*x)*(A + B*x)/(sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex}}{\sqrt{bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)*sqrt(e*x)/(sqrt(b*x^2 + a)*(d*x + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{(Bx+A)\sqrt{ex}}{\sqrt{bx^2+a}(dx+c)}\,dx$$

input
```
integrate((e*x)^(1/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*sqrt(e*x)/(sqrt(b*x^2 + a)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{e\,x}\,(A+B\,x)}{\sqrt{b\,x^2+a}\,(c+d\,x)}\,dx$$

input
```
int(((e*x)^(1/2)*(A + B*x))/((a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int(((e*x)^(1/2)*(A + B*x))/((a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{\sqrt{ex}(A+Bx)}{(c+dx)\sqrt{a+bx^2}}\,dx = \sqrt{e}\left(\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}\,x}{bd\,x^3+bc\,x^2+adx+ac}dx\right)b + \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{bd\,x^3+bc\,x^2+adx+ac}dx\right)a\right)$$

input
```
int((e*x)^(1/2)*(B*x+A)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
sqrt(e)*(int((sqrt(x)*sqrt(a + b*x**2)*x)/(a*c + a*d*x + b*c*x**2 + b*d*x*
*3),x)*b + int((sqrt(x)*sqrt(a + b*x**2))/(a*c + a*d*x + b*c*x**2 + b*d*x*
*3),x)*a)
```

3.255 $\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 407

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{(Bc-Ad)\arctan\left(\frac{\sqrt{bc^2+ad^2}\sqrt{ex}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{\sqrt{c}\sqrt{d}\sqrt{bc^2+ad^2}\sqrt{e}} + \frac{\left(A\sqrt{b}-\sqrt{a}B\right)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{e}\sqrt{a+bx^2}} + \frac{\left(\sqrt{b}c+\sqrt{a}d\right)(Bc-Ad)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}cd\left(\sqrt{b}c-\sqrt{a}d\right)\sqrt{e}\sqrt{a+bx^2}}$$

output
```
-(-A*d+B*c)*arctan((a*d^2+b*c^2)^(1/2)*(e*x)^(1/2)/c^(1/2)/d^(1/2)/e^(1/2)
/(b*x^2+a)^(1/2))/c^(1/2)/d^(1/2)/(a*d^2+b*c^2)^(1/2)/e^(1/2)+(A*b^(1/2)-a
^(1/2)*B)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)),1/2*2^(1/2))/a^(
1/4)/b^(1/4)/(b^(1/2)*c-a^(1/2)*d)/e^(1/2)/(b*x^2+a)^(1/2)+1/2*(b^(1/2)*c+
a^(1/2)*d)*(-A*d+B*c)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2
)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),-1/4
*(b^(1/2)*c-a^(1/2)*d)^2/a^(1/2)/b^(1/2)/c/d,1/2*2^(1/2))/a^(1/4)/b^(1/4)/
c/d/(b^(1/2)*c-a^(1/2)*d)/e^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.89 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = \frac{2i\sqrt{1+\frac{a}{bx^2}}x^{3/2}\left(Ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right),-1\right)+(Bc-Ad)\,\text{EllipticPi}\left(-\frac{i\sqrt{b}c}{\sqrt{a}d},i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right.\right.\right.}{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}cd\sqrt{ex}\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[e*x]*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
((2*I)*Sqrt[1 + a/(b*x^2)]*x^(3/2)*(A*d*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a
])/Sqrt[b]]/Sqrt[x]], -1] + (B*c - A*d)*EllipticPi[((-I)*Sqrt[b]*c)/(Sqrt[
a]*d), I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(Sqrt[(I*Sqrt[a
])/Sqrt[b]]*c*d*Sqrt[e*x]*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2354, 2227, 27, 761, 2221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx^2}(c+dx)}\,dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{Ae+Bxe}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2227}$$

$$\frac{2\left(\frac{\left(A\sqrt{b}-\sqrt{a}B\right)\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}+\frac{\sqrt{a}e(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{\sqrt{a}e(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}\right)}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\left(A\sqrt{b}-\sqrt{a}B\right)\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}+\frac{(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}\right)}{e}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\left(\frac{(Bc-Ad)\int \frac{\sqrt{b}xe+\sqrt{a}e}{(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{\sqrt{b}c-\sqrt{a}d}+\frac{\left(A\sqrt{b}-\sqrt{a}B\right)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}\sqrt{e}\sqrt{a+bx^2}\left(\sqrt{b}c-\sqrt{a}d\right)}\right)}{e}$$

$$\Big\downarrow \text{2221}$$

$$\frac{2\left(\frac{(Bc-Ad)\left(\frac{\left(\sqrt{a}d+\sqrt{b}c\right)\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{b}c}{\sqrt{a}}-d\right)^2}{4\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt[4]{b}cd\sqrt{e}\sqrt{a+bx^2}}-\frac{\sqrt{e}\left(\sqrt{b}c-\sqrt{a}d\right)\arctan\left(\frac{\sqrt{ex}\sqrt{ad^2+bc^2}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{2\sqrt{c}\sqrt{d}\sqrt{ad^2+bc^2}}\right)}{\sqrt{b}c-\sqrt{a}d}\cdots\right)}{e}$$

input
```
Int[(A + B*x)/(Sqrt[e*x]*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*(((A*Sqrt[b] - Sqrt[a]*B)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2
*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/
(a^(1/4)*Sqrt[e])], 1/2])/(2*a^(1/4)*b^(1/4)*(Sqrt[b]*c - Sqrt[a]*d)*Sqrt[
e]*Sqrt[a + b*x^2]) + ((B*c - A*d)*(-1/2*((Sqrt[b]*c - Sqrt[a]*d)*Sqrt[e]*
ArcTan[(Sqrt[b*c^2 + a*d^2]*Sqrt[e*x])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a + b
*x^2])])/(Sqrt[c]*Sqrt[d]*Sqrt[b*c^2 + a*d^2]) + ((Sqrt[b]*c + Sqrt[a]*d)*
(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*
x)^2]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[b]*c)/Sqrt[a] - d)^2)/(Sqrt[b]*c*d),
 2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(4*a^(1/4)*b^(1/4)
*c*d*Sqrt[e]*Sqrt[a + b*x^2])))/(Sqrt[b]*c - Sqrt[a]*d)))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 2221
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(ArcTan[Rt[c*(d/e
) + a*(e/d), 2]*(x/Sqrt[a + c*x^4])]/(2*d*e*Rt[c*(d/e) + a*(e/d), 2])), x]
+ Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*
d*e*q*Sqrt[a + c*x^4]))*EllipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x
], 1/2], x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && Po
sQ[c/a] && EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[c*(d/e) + a*(e/d)]
```

rule 2227
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q) - a*B*(e + d*q)
)/(c*d^2 - a*e^2)   Int[1/Sqrt[a + c*x^4], x], x] + Simp[a*(B*d - A*e)*((e
+ d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x]
, x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
 && NeQ[c*A^2 - a*B^2, 0]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.66

method	result
default	$\frac{\left(A\,\mathrm{EllipticPi}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{-ab}\,d}{d\sqrt{-ab}-bc},\frac{\sqrt{2}}{2}\right)bd-B\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)d+B\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)bc-B\,\mathrm{Ellip}\right.}{\sqrt{b\,x^{2}+a}\,db\left(bc-d\sqrt{-ab}\right)\sqrt{ex}}$
elliptic	$\frac{\sqrt{xe\left(b\,x^{2}+a\right)}\left(\frac{B\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{db\sqrt{be\,x^{3}+aex}}+\frac{(Ad-Bc)\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{\sqrt{ex}\,\sqrt{b\,x^{2}+a}}$

input
```
int((B*x+A)/(e*x)^(1/2)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(A*EllipticPi(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),(-a*b)^(1/2)*d/(d*(-
a*b)^(1/2)-b*c),1/2*2^(1/2))*b*d-B*(-a*b)^(1/2)*EllipticF(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*d+B*EllipticF(((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))*b*c-B*EllipticPi(((b*x+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),(-a*b)^(1/2)*d/(d*(-a*b)^(1/2)-b*c),1/2*2^(1/2))*b*c)*(-b/(-a
*b)^(1/2)*x)^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((b*x+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-a*b)^(1/2)/(b*x^2+a)^(1/2)/d/b/(b*c-d*
(-a*b)^(1/2))/(e*x)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx}{\sqrt{ex}\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(1/2)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(e*x)*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\sqrt{ex}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(1/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{e\,x}\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(1/2)*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{ex}(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e}\,\left(\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b d\,x^4+b c\,x^3+a d\,x^2+a c x}d x\right) a+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b d\,x^3+b c\,x^2+a d x+a c}d x\right) b\right)}{e}$$

input
```
int((B*x+A)/(e*x)^(1/2)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*(int((sqrt(x)*sqrt(a + b*x**2))/(a*c*x + a*d*x**2 + b*c*x**3 + b*
d*x**4),x)*a + int((sqrt(x)*sqrt(a + b*x**2))/(a*c + a*d*x + b*c*x**2 + b*
d*x**3),x)*b))/e
```

3.256 $\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 598

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx=-\frac{2A\sqrt{a+bx^2}}{ace\sqrt{ex}}+\frac{2A\sqrt{b}\sqrt{ex}\sqrt{a+bx^2}}{ace^2\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{\sqrt{d}(Bc-Ad)\arctan\left(\frac{\sqrt{bc^2+ad^2}\sqrt{ex}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{c^{3/2}\sqrt{bc^2+ad^2}e^{3/2}}-\frac{2A\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}ce^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt[4]{b}\left(A\sqrt{b}c+\sqrt{a}Bc-2\sqrt{a}Ad\right)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{a^{3/4}c\left(\sqrt{b}c-\sqrt{a}d\right)e^{3/2}\sqrt{a+bx^2}}-\frac{\left(\sqrt{b}c+\sqrt{a}d\right)(Bc-Ad)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c^2\left(\sqrt{b}c-\sqrt{a}d\right)e^{3/2}\sqrt{a+bx^2}}$$

output
```
-2*A*(b*x^2+a)^(1/2)/a/c/e/(e*x)^(1/2)+2*A*b^(1/2)*(e*x)^(1/2)*(b*x^2+a)^(
1/2)/a/c/e^2/(a^(1/2)+b^(1/2)*x)+d^(1/2)*(-A*d+B*c)*arctan((a*d^2+b*c^2)^(
1/2)*(e*x)^(1/2)/c^(1/2)/d^(1/2)/e^(1/2)/(b*x^2+a)^(1/2))/c^(3/2)/(a*d^2+b
*c^2)^(1/2)/e^(3/2)-2*A*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/
2))),1/2*2^(1/2))/a^(3/4)/c/e^(3/2)/(b*x^2+a)^(1/2)+b^(1/4)*(A*b^(1/2)*c+a
^(1/2)*B*c-2*a^(1/2)*A*d)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*
x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)),
1/2*2^(1/2))/a^(3/4)/c/(b^(1/2)*c-a^(1/2)*d)/e^(3/2)/(b*x^2+a)^(1/2)-1/2*(
b^(1/2)*c+a^(1/2)*d)*(-A*d+B*c)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*EllipticPi(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1
/2))),-1/4*(b^(1/2)*c-a^(1/2)*d)^2/a^(1/2)/b^(1/2)/c/d,1/2*2^(1/2))/a^(1/4
)/b^(1/4)/c^2/(b^(1/2)*c-a^(1/2)*d)/e^(3/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.57 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.39

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{1+\frac{a}{bx^2}}x^{5/2}\left(A\sqrt{b}cE\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-1\right)-\left(A\sqrt{b}c+i\sqrt{a}Bc-i\sqrt{a}Ad\right)\text{EllipticF}\left(i\text{arcsinh}\left(\right.\right.\right.}{\sqrt{a}\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}c^2(ex)^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[(A + B*x)/((e*x)^(3/2)*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*Sqrt[1 + a/(b*x^2)]*x^(5/2)*(A*Sqrt[b]*c*EllipticE[I*ArcSinh[Sqrt[(I*S
qrt[a])/Sqrt[b]]/Sqrt[x]], -1] - (A*Sqrt[b]*c + I*Sqrt[a]*B*c - I*Sqrt[a]*
A*d)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + I*Sqrt[
a]*(B*c - A*d)*EllipticPi[((-I)*Sqrt[b]*c)/(Sqrt[a]*d), I*ArcSinh[Sqrt[(I*
Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1]))/(Sqrt[a]*Sqrt[(I*Sqrt[a])/Sqrt[b]]*c^2*(
e*x)^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 740, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2354, 2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(ex)^{3/2}\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{Ae+Bxe}{ex(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2249}$$

$$\frac{2\int \left(\frac{A}{cex\sqrt{bx^2+a}}+\frac{Bc-Ad}{c(ce+dxe)\sqrt{bx^2+a}}\right)d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\Bigg(-\frac{a^{3/4}\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)^2\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\frac{ae^2+be^2x^2}{\left(\sqrt{a}e+\sqrt{b}ex\right)^2}}(Bc-Ad)\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{4\sqrt[4]{b}c^2e^{3/2}\sqrt{a+bx^2}(bc^2-ad^2)}+\frac{A\sqrt[4]{b}\left(\sqrt{a}e+\sqrt{b}ex\right)\sqrt{\cdots}}{\cdots}$$

input
```
Int[(A + B*x)/((e*x)^(3/2)*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*(-((A*Sqrt[a + b*x^2])/(a*c*Sqrt[e*x])) + (A*Sqrt[b]*Sqrt[e*x]*Sqrt[a +
 b*x^2])/(a*c*(Sqrt[a]*e + Sqrt[b]*e*x)) + (Sqrt[d]*(B*c - A*d)*ArcTan[(Sq
rt[b*c^2 + a*d^2]*Sqrt[e*x])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2])])/(
2*c^(3/2)*Sqrt[b*c^2 + a*d^2]*Sqrt[e]) - (A*b^(1/4)*(Sqrt[a]*e + Sqrt[b]*e
*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[2*ArcT
an[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*c*e^(3/2)*Sqrt[a
 + b*x^2]) + (A*b^(1/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)
/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1
/4)*Sqrt[e])], 1/2])/(2*a^(3/4)*c*e^(3/2)*Sqrt[a + b*x^2]) + (b^(1/4)*(B*c
 - A*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sq
rt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1
/2])/(2*a^(1/4)*c*(Sqrt[b]*c - Sqrt[a]*d)*e^(3/2)*Sqrt[a + b*x^2]) - (a^(3
/4)*((Sqrt[b]*c)/Sqrt[a] + d)^2*(B*c - A*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt
[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticPi[-1/4*(Sqrt[b]
*c - Sqrt[a]*d)^2/(Sqrt[a]*Sqrt[b]*c*d), 2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(
1/4)*Sqrt[e])], 1/2])/(4*b^(1/4)*c^2*(b*c^2 - a*d^2)*e^(3/2)*Sqrt[a + b*x^
2])))/e
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{2A\sqrt{b\,x^{2}+a}}{ace\sqrt{ex}}+\frac{\left(\frac{A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\left(\right.}{}\right.}{\sqrt{be\,x^{3}+aex}}\right.}{}$
elliptic	$\frac{\sqrt{xe\left(b\,x^{2}+a\right)}\left(-\frac{2\left(be\,x^{2}+ae\right)A}{e^{2}ac\sqrt{x\left(be\,x^{2}+ae\right)}}+\frac{A\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right.}{aec\sqrt{be\,x^{3}+aex}}\right.}{}$
default	$\frac{2A\sqrt{2}\ \mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)abc\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}-2A\sqrt{2}\ \mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ad\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}}{}$

input
```
int((B*x+A)/(e*x)^(3/2)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*A*(b*x^2+a)^(1/2)/a/c/e/(e*x)^(1/2)+1/c/a*(A*(-a*b)^(1/2)*((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*
(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*Ellipti
cE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*E
llipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))-a*(A*d-B*
c)/d*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)
^(1/2)/(-(-a*b)^(1/2)/b+c/d)*EllipticPi(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),-(-a*b)^(1/2)/b/(-(-a*b)^(1/2)/b+c/d),1/2*2^(1/2)))/e*(x*e*(b*x^2+
a))^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx}{(ex)^{\frac{3}{2}}\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(3/2)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((e*x)**(3/2)*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*(e*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(3/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*(e*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x}{{(e\,x)}^{3/2}\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x)/((e*x)^(3/2)*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(3/2)*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(ex)^{3/2}(c+dx)\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{x}\,\sqrt{b\,x^2+a}-\left(\int \frac{\sqrt{b\,x^2+a}}{\sqrt{x}\,ac+\sqrt{x}\,adx+\sqrt{x}\,bc\,x^2+\sqrt{x}\,bd\,x^3}dx\right)adx+\left(\int \frac{}{\sqrt{x}}\right.\right.}{}$$

input
```
int((B*x+A)/(e*x)^(3/2)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(x)*sqrt(a + b*x**2) - int(sqrt(a + b*x**2)/(sqrt(x)*a*
c + sqrt(x)*a*d*x + sqrt(x)*b*c*x**2 + sqrt(x)*b*d*x**3),x)*a*d*x + int((s
qrt(a + b*x**2)*x**2)/(sqrt(x)*a*c + sqrt(x)*a*d*x + sqrt(x)*b*c*x**2 + sq
rt(x)*b*d*x**3),x)*b*d*x + int((sqrt(a + b*x**2)*x)/(sqrt(x)*a*c + sqrt(x)
*a*d*x + sqrt(x)*b*c*x**2 + sqrt(x)*b*d*x**3),x)*b*c*x + int((sqrt(x)*sqrt
(a + b*x**2))/(a*c*x + a*d*x**2 + b*c*x**3 + b*d*x**4),x)*b*c*x))/(c*e**2*
x)
```

3.257 $\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 682

$$
\begin{aligned}
\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx &= -\frac{2A\sqrt{a+bx^2}}{3ace(ex)^{3/2}} - \frac{2(Bc-Ad)\sqrt{a+bx^2}}{ac^2e^2\sqrt{ex}} \\
&+ \frac{2\sqrt{b}(Bc-Ad)\sqrt{ex}\sqrt{a+bx^2}}{ac^2e^3\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{d^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{bc^2+ad^2}\sqrt{ex}}{\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a+bx^2}}\right)}{c^{5/2}\sqrt{bc^2+ad^2}e^{5/2}} \\
&- \frac{2\sqrt[4]{b}(Bc-Ad)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right)|\frac{1}{2}\right)}{a^{3/4}c^2e^{5/2}\sqrt{a+bx^2}} \\
&+ \frac{\sqrt[4]{b}\left(3\sqrt{a}\sqrt{b}Bc^2-6aBcd-A\left(bc^2+2\sqrt{a}\sqrt{b}cd-6ad^2\right)\right)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\right.}{3a^{5/4}c^2\left(\sqrt{b}c-\sqrt{a}d\right)e^{5/2}\sqrt{a+bx^2}} \\
&+ \frac{d\left(\sqrt{b}c+\sqrt{a}d\right)(Bc-Ad)\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c^3\left(\sqrt{b}c-\sqrt{a}d\right)e^{5/2}\sqrt{a+bx^2}}
\end{aligned}
$$

output
```
-2/3*A*(b*x^2+a)^(1/2)/a/c/e/(e*x)^(3/2)-2*(-A*d+B*c)*(b*x^2+a)^(1/2)/a/c^
2/e^2/(e*x)^(1/2)+2*b^(1/2)*(-A*d+B*c)*(e*x)^(1/2)*(b*x^2+a)^(1/2)/a/c^2/e
^3/(a^(1/2)+b^(1/2)*x)-d^(3/2)*(-A*d+B*c)*arctan((a*d^2+b*c^2)^(1/2)*(e*x)
^(1/2)/c^(1/2)/d^(1/2)/e^(1/2)/(b*x^2+a)^(1/2))/c^(5/2)/(a*d^2+b*c^2)^(1/2
)/e^(5/2)-2*b^(1/4)*(-A*d+B*c)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(
1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2
))),1/2*2^(1/2))/a^(3/4)/c^2/e^(5/2)/(b*x^2+a)^(1/2)+1/3*b^(1/4)*(3*a^(1/2
)*b^(1/2)*B*c^2-6*B*a*c*d-A*(b*c^2+2*a^(1/2)*b^(1/2)*c*d-6*a*d^2))*(a^(1/2
)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2)),1/2*2^(1/2))/a^(5/4)/c^2/(b^(1/2)
*c-a^(1/2)*d)/e^(5/2)/(b*x^2+a)^(1/2)+1/2*d*(b^(1/2)*c+a^(1/2)*d)*(-A*d+B*
c)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticPi(
sin(2*arctan(b^(1/4)*(e*x)^(1/2)/a^(1/4)/e^(1/2))),-1/4*(b^(1/2)*c-a^(1/2)
*d)^2/a^(1/2)/b^(1/2)/c/d,1/2*2^(1/2))/a^(1/4)/b^(1/4)/c^3/(b^(1/2)*c-a^(1
/2)*d)/e^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.66 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.56

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2\left(\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}cx(a+bx^2)\left(3Bcx+A(c-3dx)\right)+3\sqrt{a}\sqrt{b}c(Bc-Ad)\sqrt{1+\frac{a}{bx^2}}x^{7/2}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{a}}{\sqrt{b}}}}{\sqrt{x}}\right)\middle|-\right.\right.}{}$$

input
```
Integrate[(A + B*x)/((e*x)^(5/2)*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*c*x*(a + b*x^2)*(3*B*c*x + A*(c - 3*d*x)) +
 3*Sqrt[a]*Sqrt[b]*c*(B*c - A*d)*Sqrt[1 + a/(b*x^2)]*x^(7/2)*EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1] + (-3*Sqrt[a]*Sqrt[b]*B*c^2
 + (3*I)*a*B*c*d + A*(I*b*c^2 + 3*Sqrt[a]*Sqrt[b]*c*d - (3*I)*a*d^2))*Sqrt
[1 + a/(b*x^2)]*x^(7/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt
[x]], -1] - 3*(B*c - A*d)*x^2*(Sqrt[(I*Sqrt[a])/Sqrt[b]]*c*(a + b*x^2) + I
*a*d*Sqrt[1 + a/(b*x^2)]*x^(3/2)*EllipticPi[((-I)*Sqrt[b]*c)/(Sqrt[a]*d),
I*ArcSinh[Sqrt[(I*Sqrt[a])/Sqrt[b]]/Sqrt[x]], -1])))/(3*a*Sqrt[(I*Sqrt[a])
/Sqrt[b]]*c^3*(e*x)^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 922, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2354, 2249, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(ex)^{5/2}\sqrt{a+bx^2}(c+dx)}dx$$

$$\Big\downarrow \text{2354}$$

$$\frac{2\int \frac{Ae+Bxe}{e^2x^2(ce+dxe)\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2249}$$

$$\frac{2\int \left(\frac{A}{ce^2x^2\sqrt{bx^2+a}}+\frac{Bc-Ad}{c^2e^2x\sqrt{bx^2+a}}-\frac{d(Bc-Ad)}{c^2e(ce+dxe)\sqrt{bx^2+a}}\right)d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{a^{3/4}d(Bc-Ad)\left(\sqrt{b}xe+\sqrt{a}e\right)\sqrt{\frac{bx^2e^2+ae^2}{\left(\sqrt{b}xe+\sqrt{a}e\right)^2}}\,\text{EllipticPi}\left(-\frac{\left(\sqrt{b}c-\sqrt{a}d\right)^2}{4\sqrt{a}\sqrt{b}cd},2\arctan\left(\frac{\sqrt[4]{b}\sqrt{ex}}{\sqrt[4]{a}\sqrt{e}}\right),\frac{1}{2}\right)\left(\frac{\sqrt{b}c}{\sqrt{a}}+d\right)^2}{4\sqrt[4]{b}c^3(bc^2-ad^2)e^{5/2}\sqrt{bx^2+a}}-\frac{d^{3/2}(Bc-Ad)\arctan\left(\frac{\sqrt{}}{\sqrt{c}}\right.}{2c^{5/2}\sqrt{bc^2+ad^2}}\right.$$

input
```
Int[(A + B*x)/((e*x)^(5/2)*(c + d*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*(-1/3*(A*Sqrt[a + b*x^2])/(a*c*(e*x)^(3/2)) - ((B*c - A*d)*Sqrt[a + b*x
^2])/(a*c^2*e*Sqrt[e*x]) + (Sqrt[b]*(B*c - A*d)*Sqrt[e*x]*Sqrt[a + b*x^2])
/(a*c^2*e*(Sqrt[a]*e + Sqrt[b]*e*x)) - (d^(3/2)*(B*c - A*d)*ArcTan[(Sqrt[b
*c^2 + a*d^2]*Sqrt[e*x])/(Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2])])/(2*c^
(5/2)*Sqrt[b*c^2 + a*d^2]*e^(3/2)) - (b^(1/4)*(B*c - A*d)*(Sqrt[a]*e + Sqr
t[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])/(a^(3/4)*c^2*e^(5/2
)*Sqrt[a + b*x^2]) - (A*b^(3/4)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*
e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x
])/(a^(1/4)*Sqrt[e])], 1/2])/(6*a^(5/4)*c*e^(5/2)*Sqrt[a + b*x^2]) + (b^(1
/4)*(B*c - A*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a
]*e + Sqrt[b]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt
[e])], 1/2])/(2*a^(3/4)*c^2*e^(5/2)*Sqrt[a + b*x^2]) - (b^(1/4)*d*(B*c - A
*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b
]*e*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(1/4)*Sqrt[e])], 1/2])
/(2*a^(1/4)*c^2*(Sqrt[b]*c - Sqrt[a]*d)*e^(5/2)*Sqrt[a + b*x^2]) + (a^(3/4
)*d*((Sqrt[b]*c)/Sqrt[a] + d)^2*(B*c - A*d)*(Sqrt[a]*e + Sqrt[b]*e*x)*Sqrt
[(a*e^2 + b*e^2*x^2)/(Sqrt[a]*e + Sqrt[b]*e*x)^2]*EllipticPi[-1/4*(Sqrt[b]
*c - Sqrt[a]*d)^2/(Sqrt[a]*Sqrt[b]*c*d), 2*ArcTan[(b^(1/4)*Sqrt[e*x])/(a^(
1/4)*Sqrt[e])], 1/2])/(4*b^(1/4)*c^3*(b*c^2 - a*d^2)*e^(5/2)*Sqrt[a + b...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2249
```
Int[(Px_)*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)
^4)^(p_), x_Symbol] :> Int[ExpandIntegrand[1/Sqrt[a + c*x^4], Px*(f*x)^m*(d
 + e*x^2)^q*(a + c*x^4)^(p + 1/2), x], x] /; FreeQ[{a, c, d, e, f, m}, x] &
& PolyQ[Px, x] && IntegerQ[p + 1/2] && IntegerQ[q]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.83

method	result
elliptic	$\frac{\sqrt{xe(b\,x^2+a)}\left(-\frac{2A\sqrt{be\,x^3+aex}}{3e^3ac\,x^2}+\frac{2\left(be\,x^2+ae\right)(Ad-Bc)}{e^3a\,c^2\sqrt{x\left(be\,x^2+ae\right)}}-\frac{A\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{3a\,e^2c\sqrt{be\,x^3+aex}}\right.}{}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3Adx+3Bcx+Ac)}{3a\,c^2x\,e^2\sqrt{ex}}-\frac{\left(\frac{Ac\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{be\,x^3+aex}}+\frac{3Ad\sqrt{}}{}\right.}{}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*e*(b*x^2+a))^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)*(-2/3/e^3/a*A/c*(b*e*x^3
+a*e*x)^(1/2)/x^2+2*(b*e*x^2+a*e)/e^3/a*(A*d-B*c)/c^2/(x*(b*e*x^2+a*e))^(1
/2)-1/3/a/e^2*A/c*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(
-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e
*x^3+a*e*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*
2^(1/2))-1/a/e^2*(A*d-B*c)/c^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)^(1/2)*
x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))+1/e^2*(A*d-B*c)/c^2*(-a*b)
^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*e*x^3+a*e*x)^(1/2)/(-(-
a*b)^(1/2)/b+c/d)*EllipticPi(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),-(-
a*b)^(1/2)/b/(-(-a*b)^(1/2)/b+c/d),1/2*2^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx}{(ex)^{\frac{5}{2}}\sqrt{a+bx^2}\,(c+dx)}\,dx$$

input
```
integrate((B*x+A)/(e*x)**(5/2)/(d*x+c)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((e*x)**(5/2)*sqrt(a + b*x**2)*(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*(e*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}(dx+c)\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*(d*x + c)*(e*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{A+B\,x}{(e\,x)^{5/2}\,\sqrt{b\,x^2+a}\,\left(c+d\,x\right)}\,dx$$

input
```
int((A + B*x)/((e*x)^(5/2)*(a + b*x^2)^(1/2)*(c + d*x)),x)
```

output
```
int((A + B*x)/((e*x)^(5/2)*(a + b*x^2)^(1/2)*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(ex)^{5/2}(c+dx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{(ex)^{\frac{5}{2}}\,(dx+c)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x)^(5/2)/(d*x+c)/(b*x^2+a)^(1/2),x)
```

3.258 $\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 1090

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}} \, dx = \text{Too large to display}$$

output
```
-a^(1/2)*c*C*(b+a/x^2)*(d+c/x)*(e*x)^(3/2)/b/d/(a*d^2+b*c^2)^(1/2)/e^2/(1+
a^(1/2)*(d+c/x)/(a*d^2+b*c^2)^(1/2))/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)+C*(b+a/
x^2)*(d+c/x)*x*(e*x)^(3/2)/b/d/e^2/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)-1/2*(-2*B
*d+C*c)*(b+a/x^2)^(1/2)*(d+c/x)^(1/2)*(e*x)^(3/2)*arctanh(b^(1/2)*(d+c/x)^
(1/2)/d^(1/2)/(b+a/x^2)^(1/2))/b^(1/2)/d^(3/2)/e^2/(d*x+c)^(1/2)/(b*x^2+a)
^(1/2)+a^(1/4)*C*(a*d^2+b*c^2)^(3/4)*(1+a^(1/2)*(d+c/x)/(a*d^2+b*c^2)^(1/2
))*((b+a/x^2)/(b+a*d^2/c^2)/(1+a^(1/2)*(d+c/x)/(a*d^2+b*c^2)^(1/2))^2)^(1/
2)*(d+c/x)^(1/2)*(e*x)^(3/2)*EllipticE(sin(2*arctan(a^(1/4)*(d+c/x)^(1/2)/
(a*d^2+b*c^2)^(1/4))),1/2*(2+2*a^(1/2)*d/(a*d^2+b*c^2)^(1/2))^(1/2))/b/c/d
/e^2/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)-(a^(1/2)*b*c^2*(-B*d+C*c)+a^(3/2)*d^2*(
-B*d+C*c)+d*(A*b*c+B*a*d-C*a*c)*(a*d^2+b*c^2)^(1/2))*(1+a^(1/2)*(d+c/x)/(a
*d^2+b*c^2)^(1/2))*((b+a/x^2)/(b+a*d^2/c^2)/(1+a^(1/2)*(d+c/x)/(a*d^2+b*c^
2)^(1/2))^2)^(1/2)*(d+c/x)^(1/2)*(e*x)^(3/2)*InverseJacobiAM(2*arctan(a^(1
/4)*(d+c/x)^(1/2)/(a*d^2+b*c^2)^(1/4)),1/2*(2+2*a^(1/2)*d/(a*d^2+b*c^2)^(1
/2))^(1/2))/a^(1/4)/b/c^2/d/(a*d^2+b*c^2)^(1/4)/e^2/(d*x+c)^(1/2)/(b*x^2+a
)^(1/2)-1/4*(-2*B*d+C*c)*(a*d^2+b*c^2)^(1/4)*(a^(1/2)*d-(a*d^2+b*c^2)^(1/2
))^2*(1+a^(1/2)*(d+c/x)/(a*d^2+b*c^2)^(1/2))*((b+a/x^2)/(b+a*d^2/c^2)/(1+a
^(1/2)*(d+c/x)/(a*d^2+b*c^2)^(1/2))^2)^(1/2)*(d+c/x)^(1/2)*(e*x)^(3/2)*Ell
ipticPi(sin(2*arctan(a^(1/4)*(d+c/x)^(1/2)/(a*d^2+b*c^2)^(1/4))),1/4*(a^(1
/2)*d+(a*d^2+b*c^2)^(1/2))^2/a^(1/2)/d/(a*d^2+b*c^2)^(1/2),1/2*(2+2*a^(...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.04 (sec) , antiderivative size = 1004, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[e*x]*Sqrt[c + d*x]*Sqrt[a + b*x^2]),x]
```

output
```
-((x^3*(-((C*(c + d*x)*(a + b*x^2))/x^3) + (A*b*d*Sqrt[2 + ((2*I)*Sqrt[a])
/(Sqrt[b]*x)]*(Sqrt[a] + I*Sqrt[b]*x)*Sqrt[(Sqrt[a]*(c + d*x))/(((-I)*Sqrt
[b]*c + Sqrt[a]*d)*x)]*EllipticF[ArcSin[Sqrt[(c*(Sqrt[b] - (I*Sqrt[a])/x))
/(Sqrt[b]*c + I*Sqrt[a]*d)]], (c + (I*Sqrt[a]*d)/Sqrt[b])/(2*c)])/(Sqrt[a]
*x^2*Sqrt[(c*((-I)*Sqrt[a] + Sqrt[b]*x))/((Sqrt[b]*c + I*Sqrt[a]*d)*x)]) +
 (C*Sqrt[(c*(Sqrt[b] + (I*Sqrt[a])/x))/(Sqrt[b]*c - I*Sqrt[a]*d)]*(I*Sqrt[
b] + Sqrt[a]/x)*Sqrt[(Sqrt[a]*(d + c/x))/((-I)*Sqrt[b]*c + Sqrt[a]*d)]*((I
*Sqrt[b]*c + Sqrt[a]*d)*EllipticE[ArcSin[Sqrt[(Sqrt[a]*(d + c/x))/((-I)*Sq
rt[b]*c + Sqrt[a]*d)]], ((-I)*Sqrt[b]*c + Sqrt[a]*d)/(I*Sqrt[b]*c + Sqrt[a
]*d)] - I*Sqrt[b]*c*EllipticF[ArcSin[Sqrt[(Sqrt[a]*(d + c/x))/((-I)*Sqrt[b
]*c + Sqrt[a]*d)]], ((-I)*Sqrt[b]*c + Sqrt[a]*d)/(I*Sqrt[b]*c + Sqrt[a]*d)
]))/(Sqrt[(c*(Sqrt[b] - (I*Sqrt[a])/x))/(Sqrt[b]*c + I*Sqrt[a]*d)]*x) + (S
qrt[b]*C*(Sqrt[b]*c + I*Sqrt[a]*d)*Sqrt[1 + (I*Sqrt[a])/(Sqrt[b]*x)]*Sqrt[
(Sqrt[a]*c*(Sqrt[a] + I*Sqrt[b]*x)*(c + d*x))/((Sqrt[b]*c + I*Sqrt[a]*d)^2
*x^2)]*EllipticPi[1 + (I*Sqrt[a]*d)/(Sqrt[b]*c), ArcSin[Sqrt[(c*(Sqrt[b] -
 (I*Sqrt[a])/x))/(Sqrt[b]*c + I*Sqrt[a]*d)]], (c + (I*Sqrt[a]*d)/Sqrt[b])/
(2*c)])/(Sqrt[2]*x) - (Sqrt[2]*Sqrt[b]*B*d*(Sqrt[b]*c + I*Sqrt[a]*d)*Sqrt[
1 + (I*Sqrt[a])/(Sqrt[b]*x)]*Sqrt[(Sqrt[a]*c*(Sqrt[a] + I*Sqrt[b]*x)*(c +
d*x))/((Sqrt[b]*c + I*Sqrt[a]*d)^2*x^2)]*EllipticPi[1 + (I*Sqrt[a]*d)/(Sqr
t[b]*c), ArcSin[Sqrt[(c*(Sqrt[b] - (I*Sqrt[a])/x))/(Sqrt[b]*c + I*Sqrt[...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{a+bx^2}\sqrt{c+dx}}\,dx$$

$$\Big\downarrow \text{ 2354}$$

$$\frac{2\int \frac{Cx^2+Bx+A}{\sqrt{c+dx}\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

$$\Big\downarrow \text{ 2261}$$

$$\frac{2\int \frac{Cx^2+Bx+A}{\sqrt{c+dx}\sqrt{bx^2+a}}d\sqrt{ex}}{e}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[e*x]*Sqrt[c + d*x]*Sqrt[a + b*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2261
```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Unintegrable[Px*(d + e*x^2)^q*(a + c*x^4)^p, x] /; FreeQ[{a, c, d, e,
p, q}, x] && PolyQ[Px, x]
```

rule 2354
```
Int[(Px_)*((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> With[{k = Denominator[m]}, Simp[k/e   Subst[Int[(Px /.
 x -> x^k/e)*x^(k*(m + 1) - 1)*(c + d*(x^k/e))^n*(a + b*(x^(2*k)/e^2))^p, x
], x, (e*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, n, p}, x] && PolyQ[Px, x]
&& FractionQ[m]
```

Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.11

method	result	size
elliptic	Expression too large to display	1215
default	Expression too large to display	4067

input
```
int((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
(e*x*(b*x^2+a)*(d*x+c))^(1/2)/(e*x)^(1/2)/(b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2
*A*c/d*(-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)/b))^(1/2)*(x-(-a*b)^(
1/2)/b)^2*(-(x+(-a*b)^(1/2)/b)/(x-(-a*b)^(1/2)/b))^(1/2)*(-(-a*b)^(1/2)/b*
(x+c/d)/c*d/(x-(-a*b)^(1/2)/b))^(1/2)/(-c/d-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
/(b*d*e*x*(x-(-a*b)^(1/2)/b)*(x+(-a*b)^(1/2)/b)*(x+c/d))^(1/2)*EllipticF((
-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)/b))^(1/2),2^(1/2)*(c/d/(c/d+(
-a*b)^(1/2)/b))^(1/2))+2*B*c/d*(-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/
2)/b))^(1/2)*(x-(-a*b)^(1/2)/b)^2*(-(x+(-a*b)^(1/2)/b)/(x-(-a*b)^(1/2)/b))
^(1/2)*(-(-a*b)^(1/2)/b*(x+c/d)/c*d/(x-(-a*b)^(1/2)/b))^(1/2)/(-c/d-(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b/(b*d*e*x*(x-(-a*b)^(1/2)/b)*(x+(-a*b)^(1/2)/b)*(x
+c/d))^(1/2)*((-a*b)^(1/2)/b*EllipticF((-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-
a*b)^(1/2)/b))^(1/2),2^(1/2)*(c/d/(c/d+(-a*b)^(1/2)/b))^(1/2))-(-a*b)^(1/2
)/b*EllipticPi((-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)/b))^(1/2),-c/
d/(-c/d-(-a*b)^(1/2)/b),2^(1/2)*(c/d/(c/d+(-a*b)^(1/2)/b))^(1/2)))+C*(x*(x
+(-a*b)^(1/2)/b)*(x+c/d)+c/d*(-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)
/b))^(1/2)*(x-(-a*b)^(1/2)/b)^2*(-(x+(-a*b)^(1/2)/b)/(x-(-a*b)^(1/2)/b))^(
1/2)*(-(-a*b)^(1/2)/b*(x+c/d)/c*d/(x-(-a*b)^(1/2)/b))^(1/2)*((-c/d*(-a*b)^
(1/2)/b-a/b)/(-c/d-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b*EllipticF((-(-c/d-(-a*b)
^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)/b))^(1/2),2^(1/2)*(c/d/(c/d+(-a*b)^(1/2)/b
))^(1/2))+EllipticE((-(-c/d-(-a*b)^(1/2)/b)*x/c*d/(x-(-a*b)^(1/2)/b))^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{a+bx^2}\sqrt{c+dx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x)**(1/2)/(d*x+c)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(e*x)*sqrt(a + b*x**2)*sqrt(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}\sqrt{ex}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*sqrt(e*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{e\,x}\,\sqrt{b\,x^2+a}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e*x)^(1/2)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{ex}\sqrt{c+dx}\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{ex}\,\sqrt{dx+c}\,\sqrt{b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x)^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2),x)
```

3.259 $\int (gx)^m(c+dx)^{-p}(c(1+m)-d(2+m+p)x)\left(c^2-d^2x^2\right)^p\,dx$

Optimal result

Integrand size = 43, antiderivative size = 38

$$\int (gx)^m(c+dx)^{-p}(c(1+m)-d(2+m+p)x)\left(c^2-d^2x^2\right)^p\,dx = \frac{(gx)^{1+m}(c+dx)^{-1-p}\left(c^2-d^2x^2\right)^{1+p}}{g}$$

output
```
(g*x)^(1+m)*(d*x+c)^(-1-p)*(-d^2*x^2+c^2)^(p+1)/g
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int (gx)^m(c+dx)^{-p}(c(1+m)-d(2+m+p)x)\left(c^2-d^2x^2\right)^p\,dx =$$

$$-\frac{x(gx)^m(c+dx)^{-p}\left(1-\frac{dx}{c}\right)^{-p}\left(c^2-d^2x^2\right)^p\left(-c(2+m)\,\text{Hypergeometric2F1}\left(1+m,-p,2+m,\frac{dx}{c}\right)+\right.}{2+m}$$

input
```
Integrate[((g*x)^m*(c*(1 + m) - d*(2 + m + p)*x)*(c^2 - d^2*x^2)^p)/(c + d
*x)^p,x]
```

output
```
-((x*(g*x)^m*(c^2 - d^2*x^2)^p*(-(c*(2 + m)*Hypergeometric2F1[1 + m, -p, 2
 + m, (d*x)/c]) + d*(2 + m + p)*x*Hypergeometric2F1[2 + m, -p, 3 + m, (d*x
)/c]))/((2 + m)*(c + d*x)^p*(1 - (d*x)/c)^p))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.66, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (gx)^m (c+dx)^{-p} \left(c^2-d^2x^2\right)^p (c(m+1)-dx(m+p+2)) \, dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(c(m+1)(gx)^m (c+dx)^{-p} \left(c^2-d^2x^2\right)^p - \frac{d(m+p+2)(gx)^{m+1} (c+dx)^{-p} \left(c^2-d^2x^2\right)^p}{g}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{c(gx)^{m+1}(c+dx)^{-p} \left(1-\frac{dx}{c}\right)^{-p} \left(c^2-d^2x^2\right)^p \text{Hypergeometric2F1}\left(m+1,-p,m+2,\frac{dx}{c}\right)}{g} - \frac{d(m+p+2)(gx)^{m+2}(c+dx)^{-p} \left(1-\frac{dx}{c}\right)^{-p} \left(c^2-d^2x^2\right)^p \text{Hypergeometric2F1}\left(m+2,-p,m+3,\frac{dx}{c}\right)}{g^2(m+2)}$$

input
```
Int[((g*x)^m*(c*(1 + m) - d*(2 + m + p)*x)*(c^2 - d^2*x^2)^p)/(c + d*x)^p,
x]
```

output
```
(c*(g*x)^(1 + m)*(c^2 - d^2*x^2)^p*Hypergeometric2F1[1 + m, -p, 2 + m, (d*
x)/c])/(g*(c + d*x)^p*(1 - (d*x)/c)^p) - (d*(2 + m + p)*(g*x)^(2 + m)*(c^2
 - d^2*x^2)^p*Hypergeometric2F1[2 + m, -p, 3 + m, (d*x)/c])/(g^2*(2 + m)*(
c + d*x)^p*(1 - (d*x)/c)^p)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 17.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

method	result
gosper	$(-d^2x^2+c^2)^p (gx)^m (-dx+c)\, x(dx+c)^{-p}$
parallelrisch	$\frac{\left(-x^2(gx)^m\left(-d^2x^2+c^2\right)^p d^3+x(gx)^m\left(-d^2x^2+c^2\right)^p c\, d^2\right)(dx+c)^{-p}}{d^2}$
orering	$\frac{(-dx+c)x(gx)^m(c(1+m)-d(2+m+p)x)\left(-d^2x^2+c^2\right)^p(dx+c)^{-p}}{-dmx-dpx+cm-2dx+c}$
risch	$(-dx+c)^p x^m g^m (-dx+c)\, x\, \mathrm{e}^{\frac{i\pi\left(-\operatorname{csgn}(igx)^3 m+\operatorname{csgn}(igx)^2 \operatorname{csgn}(ig)m+\operatorname{csgn}(igx)^2 \operatorname{csgn}(ix)m-\operatorname{csgn}(igx) \operatorname{csgn}(ig) \operatorname{csgn}(i}{}}$

input
```
int((g*x)^m*(c*(1+m)-d*(2+m+p)*x)*(-d^2*x^2+c^2)^p/((d*x+c)^p),x,method=_R
ETURNVERBOSE)
```

output
```
(-d^2*x^2+c^2)^p*(g*x)^m*(-d*x+c)*x/((d*x+c)^p)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (gx)^m (c+dx)^{-p} (c(1+m)-d(2+m+p)x) \left(c^2-d^2x^2\right)^p \, dx = -\frac{(dx^2-cx)(-d^2x^2+c^2)^p (gx)^m}{(dx+c)^p}$$

input
```
integrate((g*x)^m*(c*(1+m)-d*(2+m+p)*x)*(-d^2*x^2+c^2)^p/((d*x+c)^p),x, al
gorithm="fricas")
```

output
```
-(d*x^2 - c*x)*(-d^2*x^2 + c^2)^p*(g*x)^m/(d*x + c)^p
```

Sympy [A] (verification not implemented)

Time = 92.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int (gx)^m (c+dx)^{-p} (c(1+m)-d(2+m+p)x) \left(c^2-d^2x^2\right)^p \, dx = cx(gx)^m (c+dx)^{-p} \left(c^2-d^2x^2\right)^p - dx^2(gx)^m (c+dx)^{-p} \left(c^2-d^2x^2\right)^p$$

input
```
integrate((g*x)**m*(c*(1+m)-d*(2+m+p)*x)*(-d**2*x**2+c**2)**p/((d*x+c)**p)
,x)
```

output
```
c*x*(g*x)**m*(c**2 - d**2*x**2)**p/(c + d*x)**p - d*x**2*(g*x)**m*(c**2 -
d**2*x**2)**p/(c + d*x)**p
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int (gx)^m (c+dx)^{-p} (c(1+m)-d(2+m+p)x) \left(c^2-d^2x^2\right)^p \, dx = -\left(dg^m x^2 - cg^m x\right) e^{(p\log(-dx+c)+m\log(x))}$$

input
```
integrate((g*x)^m*(c*(1+m)-d*(2+m+p)*x)*(-d^2*x^2+c^2)^p/((d*x+c)^p),x, al
gorithm="maxima")
```

output
```
-(d*g^m*x^2 - c*g^m*x)*e^(p*log(-d*x + c) + m*log(x))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.00

$$\int (gx)^m (c+dx)^{-p} (c(1+m)-d(2+m+p)x) \left(c^2-d^2x^2\right)^p \, dx = -\frac{(gx)^m dx^2 e^{(p\log(dx+c)+p\log(-dx+c))}}{(dx+c)^p} + \frac{(gx)^m cx e^{(p\log(dx+c)+p\log(-dx+c))}}{(dx+c)^p}$$

input
```
integrate((g*x)^m*(c*(1+m)-d*(2+m+p)*x)*(-d^2*x^2+c^2)^p/((d*x+c)^p),x, al
gorithm="giac")
```

output
```
-(g*x)^m*d*x^2*e^(p*log(d*x + c) + p*log(-d*x + c))/(d*x + c)^p + (g*x)^m*
c*x*e^(p*log(d*x + c) + p*log(-d*x + c))/(d*x + c)^p
```

Mupad [B] (verification not implemented)

Time = 17.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int (gx)^m(c+dx)^{-p}(c(1+m)-d(2+m+p)x)\left(c^2-d^2x^2\right)^p\,dx = \frac{x\,\left(c^2-d^2\,x^2\right)^p\,{\left(g\,x\right)}^m\,\left(c-d\,x\right)}{{\left(c+d\,x\right)}^p}$$

input
```
int(((c^2 - d^2*x^2)^p*(g*x)^m*(c*(m + 1) - d*x*(m + p + 2)))/(c + d*x)^p,
x)
```

output
```
(x*(c^2 - d^2*x^2)^p*(g*x)^m*(c - d*x))/(c + d*x)^p
```

Reduce [F]

$$\int (gx)^m(c+dx)^{-p}(c(1+m)-d(2+m+p)x)\left(c^2-d^2x^2\right)^p\,dx = g^m\left(-\left(\int \frac{x^m(-d^2x^2+c^2)^p\,x}{(dx+c)^p}dx\right)dm-\left(\int \frac{x^m(-d^2x^2+c^2)^p\,x}{(dx+c)^p}dx\right)dp - 2\left(\int \frac{x^m(-d^2x^2+c^2)^p\,x}{(dx+c)^p}dx\right)d+\left(\int \frac{x^m(-d^2x^2+c^2)^p}{(dx+c)^p}dx\right)cm + \left(\int \frac{x^m(-d^2x^2+c^2)^p}{(dx+c)^p}dx\right)c\right)$$

input
```
int((g*x)^m*(c*(1+m)-d*(2+m+p)*x)*(-d^2*x^2+c^2)^p/((d*x+c)^p),x)
```

output
```
g**m*( - int((x**m*(c**2 - d**2*x**2)**p*x)/(c + d*x)**p,x)*d*m - int((x**
m*(c**2 - d**2*x**2)**p*x)/(c + d*x)**p,x)*d*p - 2*int((x**m*(c**2 - d**2*
x**2)**p*x)/(c + d*x)**p,x)*d + int((x**m*(c**2 - d**2*x**2)**p)/(c + d*x)
**p,x)*c*m + int((x**m*(c**2 - d**2*x**2)**p)/(c + d*x)**p,x)*c)
```

3.260 $\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 248

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \frac{(de-cf)(gx)^{1+m}\sqrt{c^2-d^2x^2}}{cdg(c+dx)^{3/2}} - \frac{(cf(5+4m)-d(e+4em))\left(\frac{dx}{c}\right)^{-m}(gx)^m(c-dx)\sqrt{c+dx}\,\text{AppellF1}\left(\frac{1}{2},-m,1,\frac{3}{2},1-\frac{dx}{c},\frac{c-dx}{2c}\right)}{2cd^2\sqrt{c^2-d^2x^2}} + \frac{(cf(3+2m)-d(e+2em))\left(\frac{dx}{c}\right)^{-m}(gx)^m(c-dx)\sqrt{c+dx}\,\text{Hypergeometric2F1}\left(\frac{1}{2},-m,\frac{3}{2},1-\frac{dx}{c}\right)}{cd^2\sqrt{c^2-d^2x^2}}$$

output
```
(-c*f+d*e)*(g*x)^(1+m)*(-d^2*x^2+c^2)^(1/2)/c/d/g/(d*x+c)^(3/2)-1/2*(c*f*(
5+4*m)-d*(4*e*m+e))*(g*x)^m*(-d*x+c)*(d*x+c)^(1/2)*AppellF1(1/2,1,-m,3/2,1
/2*(-d*x+c)/c,1-d*x/c)/c/d^2/((d*x/c)^m)/(-d^2*x^2+c^2)^(1/2)+(c*f*(3+2*m)
-d*(2*e*m+e))*(g*x)^m*(-d*x+c)*(d*x+c)^(1/2)*hypergeom([1/2, -m],[3/2],1-d
*x/c)/c/d^2/((d*x/c)^m)/(-d^2*x^2+c^2)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 6.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \frac{x(gx)^m\sqrt{c-dx}\left(cf\,\text{AppellF1}\left(1+m,-\frac{1}{2},1,2+m,\frac{dx}{c},-\frac{dx}{c}\right)+(de-c\ldots\right.}{c^2d(1+m)\sqrt{1-\frac{dx}{c}}}$$

input
```
Integrate[((g*x)^m*(e + f*x)*Sqrt[c^2 - d^2*x^2])/(c + d*x)^(5/2),x]
```

output
```
(x*(g*x)^m*Sqrt[c - d*x]*(c*f*AppellF1[1 + m, -1/2, 1, 2 + m, (d*x)/c, -((
d*x)/c)] + (d*e - c*f)*AppellF1[1 + m, -1/2, 2, 2 + m, (d*x)/c, -((d*x)/c)
]))/(c^2*d*(1 + m)*Sqrt[1 - (d*x)/c])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {2355, 27, 586, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c^2-d^2x^2}(e+fx)(gx)^m}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{2355}$$

$$\left(e-\frac{cf}{d}\right)\int \frac{(gx)^m\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}dx+\int \frac{f(gx)^m\sqrt{c^2-d^2x^2}}{d(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(e-\frac{cf}{d}\right)\int \frac{(gx)^m\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}dx+\frac{f\int \frac{(gx)^m\sqrt{c^2-d^2x^2}}{(c+dx)^{3/2}}dx}{d}$$

$$\Big\downarrow \text{586}$$

$$\frac{\sqrt{c^2-d^2x^2}\left(e-\frac{cf}{d}\right)\int \frac{(gx)^m\sqrt{c-dx}}{(c+dx)^2}dx}{\sqrt{c-dx}\sqrt{c+dx}}+\frac{f\sqrt{c^2-d^2x^2}\int \frac{(gx)^m\sqrt{c-dx}}{c+dx}dx}{d\sqrt{c-dx}\sqrt{c+dx}}$$

$$\Big\downarrow \text{152}$$

$$\frac{\sqrt{c^2-d^2x^2}\left(e-\frac{cf}{d}\right)\int\frac{(gx)^m\sqrt{1-\frac{dx}{c}}}{(c+dx)^2}dx}{\sqrt{c+dx}\sqrt{1-\frac{dx}{c}}}+\frac{f\sqrt{c^2-d^2x^2}\int\frac{(gx)^m\sqrt{1-\frac{dx}{c}}}{c+dx}dx}{d\sqrt{c+dx}\sqrt{1-\frac{dx}{c}}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\sqrt{c^2-d^2x^2}(gx)^{m+1}\left(e-\frac{cf}{d}\right)\text{AppellF1}\left(m+1,-\frac{1}{2},2,m+2,\frac{dx}{c},-\frac{dx}{c}\right)}{c^2g(m+1)\sqrt{c+dx}\sqrt{1-\frac{dx}{c}}}+\frac{f\sqrt{c^2-d^2x^2}(gx)^{m+1}\text{AppellF1}\left(m+1,-\frac{1}{2},1,m+2,\frac{dx}{c},-\frac{dx}{c}\right)}{cdg(m+1)\sqrt{c+dx}\sqrt{1-\frac{dx}{c}}}$$

input
```
Int[((g*x)^m*(e + f*x)*Sqrt[c^2 - d^2*x^2])/(c + d*x)^(5/2),x]
```

output
```
(f*(g*x)^(1 + m)*Sqrt[c^2 - d^2*x^2]*AppellF1[1 + m, -1/2, 1, 2 + m, (d*x)
/c, -((d*x)/c)])/(c*d*g*(1 + m)*Sqrt[c + d*x]*Sqrt[1 - (d*x)/c]) + ((e - (
c*f)/d)*(g*x)^(1 + m)*Sqrt[c^2 - d^2*x^2]*AppellF1[1 + m, -1/2, 2, 2 + m,
(d*x)/c, -((d*x)/c)])/(c^2*g*(1 + m)*Sqrt[c + d*x]*Sqrt[1 - (d*x)/c])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 586
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_)
, x_Symbol] :> Simp[(a + b*x^2)^FracPart[p]/((c + d*x)^FracPart[p]*(a/c + (
b*x)/d)^FracPart[p])   Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 2355
```
Int[(Px_)*((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)
^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(e*x)^m*(c + d*
x)^(n + 1)*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c + d*x, x]   I
nt[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}
, x] && PolynomialQ[Px, x] && LtQ[n, 0]
```

Maple [F]

$$\int \frac{(gx)^{m} (fx+e) \sqrt{-d^{2} x^{2}+c^{2}}}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x)
```

output
```
int((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x)
```

Fricas [F]

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{-d^{2} x^{2}+c^{2}}(fx+e)(gx)^{m}}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x, algorithm=
"fricas")
```

output
```
integral(sqrt(-d^2*x^2 + c^2)*sqrt(d*x + c)*(f*x + e)*(g*x)^m/(d^3*x^3 + 3
*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Sympy [F]

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \int \frac{(gx)^m\sqrt{-(-c+dx)(c+dx)}(e+fx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x)**m*(f*x+e)*(-d**2*x**2+c**2)**(1/2)/(d*x+c)**(5/2),x)
```

output
```
Integral((g*x)**m*sqrt(-(-c + d*x)*(c + d*x))*(e + f*x)/(c + d*x)**(5/2),
x)
```

Maxima [F]

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{-d^2x^2+c^2}(fx+e)(gx)^m}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x, algorithm=
"maxima")
```

output
```
integrate(sqrt(-d^2*x^2 + c^2)*(f*x + e)*(g*x)^m/(d*x + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \int \frac{\sqrt{-d^2x^2+c^2}(fx+e)(gx)^m}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(-d^2*x^2 + c^2)*(f*x + e)*(g*x)^m/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = \int \frac{(e+f\,x)\,\sqrt{c^2-d^2\,x^2}\,(g\,x)^m}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(((e + f*x)*(c^2 - d^2*x^2)^(1/2)*(g*x)^m)/(c + d*x)^(5/2),x)
```

output
```
int(((e + f*x)*(c^2 - d^2*x^2)^(1/2)*(g*x)^m)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(gx)^m(e+fx)\sqrt{c^2-d^2x^2}}{(c+dx)^{5/2}}\,dx = g^m\left(\left(\int \frac{x^m\sqrt{-d^2x^2+c^2}\,x}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx\right)f + \left(\int \frac{x^m\sqrt{-d^2x^2+c^2}}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx\right)e\right)$$

input
```
int((g*x)^m*(f*x+e)*(-d^2*x^2+c^2)^(1/2)/(d*x+c)^(5/2),x)
```

output
```
g**m*(int((x**m*sqrt(c**2 - d**2*x**2)*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c +
 d*x)*c*d*x + sqrt(c + d*x)*d**2*x**2),x)*f + int((x**m*sqrt(c**2 - d**2*x
**2))/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d*x)*d**2*x**
2),x)*e)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
              finalresult={"A",""}
            ,(*ELSE*)
              finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
             ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file