Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.8/111-1.2.1.8-b

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CHAPTER 1

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**50**]. This is test number [111].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (50)	0.00 (0)
Maple	100.00 (50)	0.00 (0)
Rubi	66.00 (33)	34.00 (17)
Fricas	42.00 (21)	58.00 (29)
Reduce	40.00 (20)	60.00 (30)
Giac	16.00 (8)	84.00 (42)
Maxima	10.00 (5)	90.00 (45)
Sympy	4.00 (2)	96.00 (48)
Mupad	0.00 (0)	100.00 (50)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

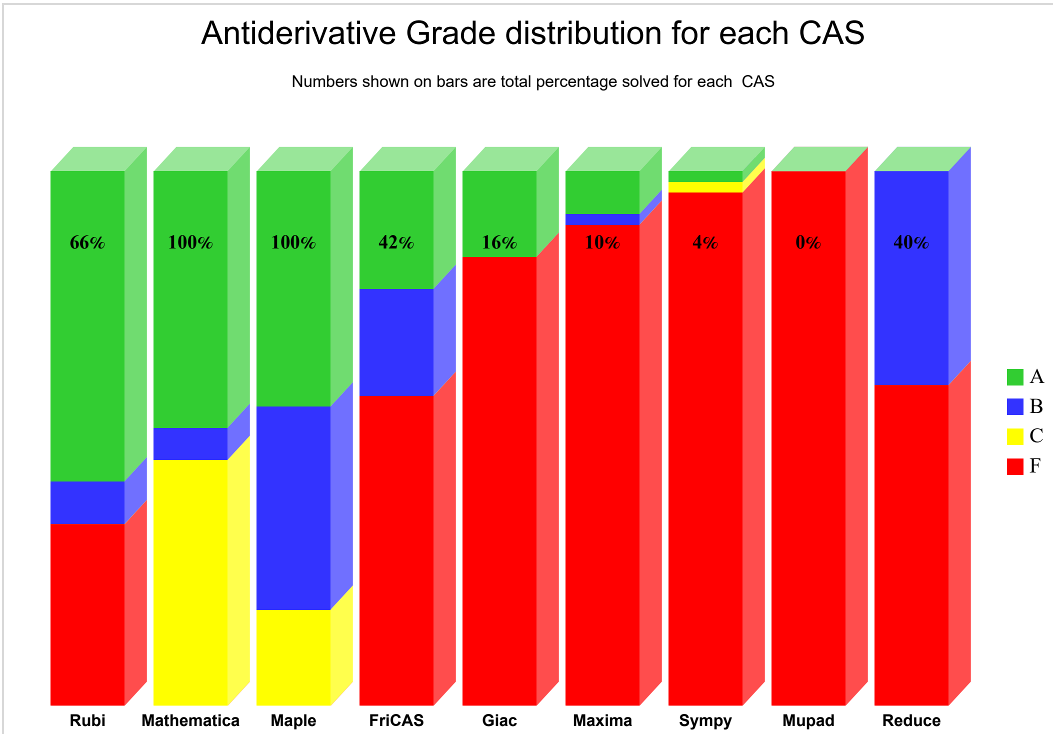
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

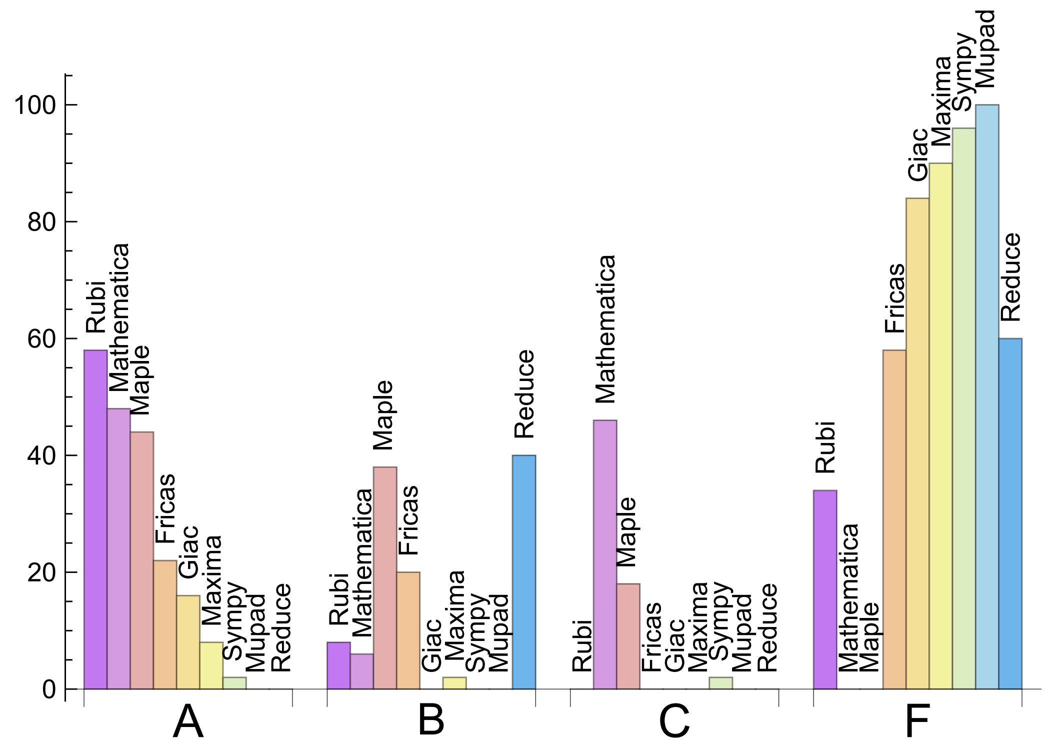
System	% A grade	% B grade	% C grade	% F grade
Rubi	58.000	8.000	0.000	34.000
Mathematica	48.000	6.000	46.000	0.000
Maple	44.000	38.000	18.000	0.000
Fricas	22.000	20.000	0.000	58.000
Giac	16.000	0.000	0.000	84.000
Maxima	8.000	2.000	0.000	90.000
Sympy	2.000	0.000	2.000	96.000
Mupad	0.000	0.000	0.000	100.000
Reduce	0.000	40.000	0.000	60.000

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Rubi	17	100.00	0.00	0.00
Fricas	29	62.07	37.93	0.00
Reduce	30	100.00	0.00	0.00
Giac	42	95.24	0.00	4.76
Maxima	45	100.00	0.00	0.00
Sympy	48	77.08	22.92	0.00
Mupad	50	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.12
Giac	0.16
Reduce	0.47
Rubi	1.61
Maple	2.37
Fricas	8.67
Sympy	9.27
Mathematica	17.12
Mupad	-nan(ind)

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	233.40	1.50	286.00	1.60
Giac	329.50	1.09	226.50	1.09
Rubi	394.85	1.34	248.00	1.31
Mathematica	623.28	1.44	298.00	1.01
Reduce	727.35	2.64	545.50	2.10
Maple	769.94	1.92	468.00	1.64
Fricas	977.76	3.30	366.00	2.04
Sympy	1405.50	8.55	1405.50	8.55
Mupad	-nan(ind)	-nan(ind)	nan	nan

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

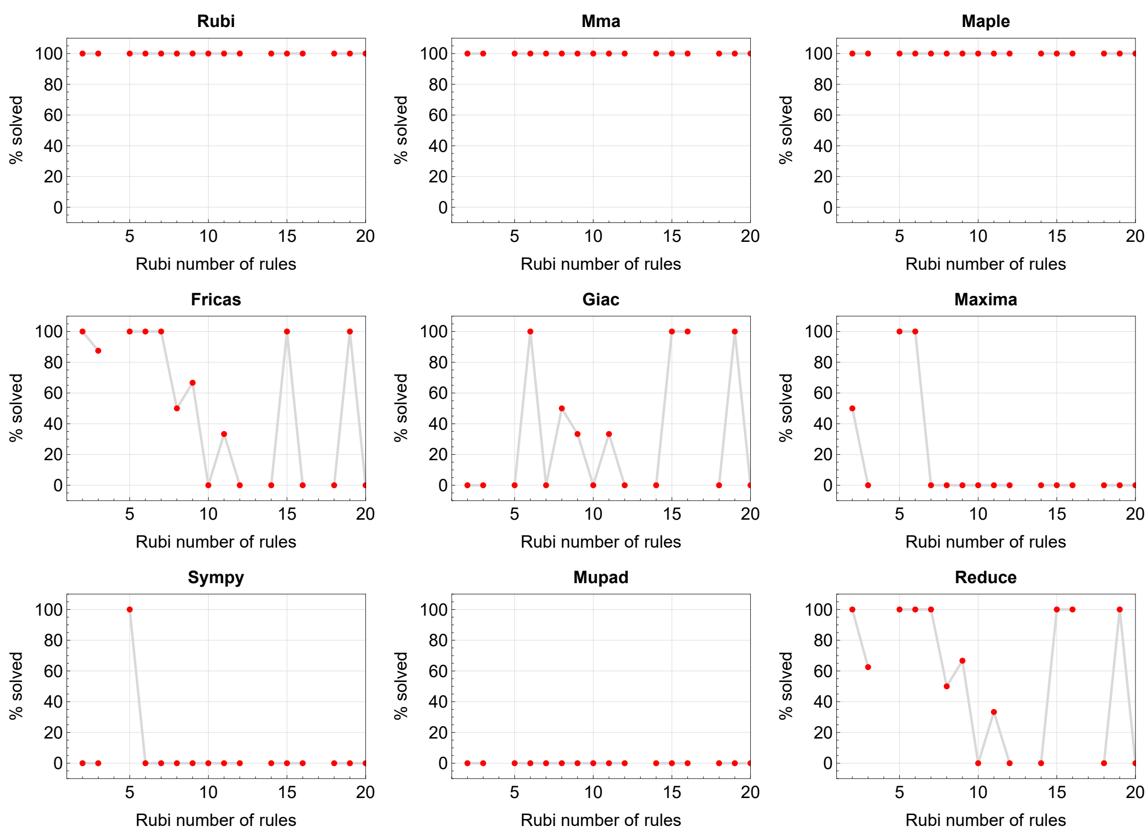


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

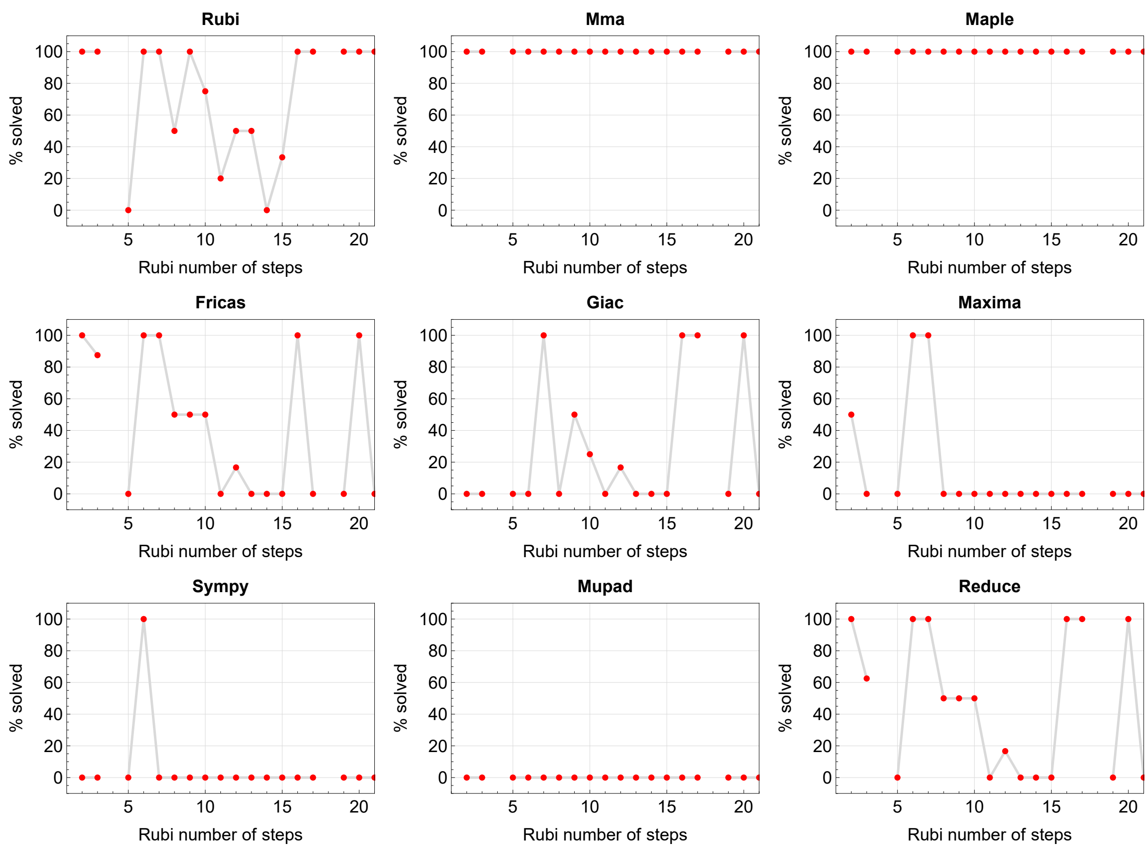


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

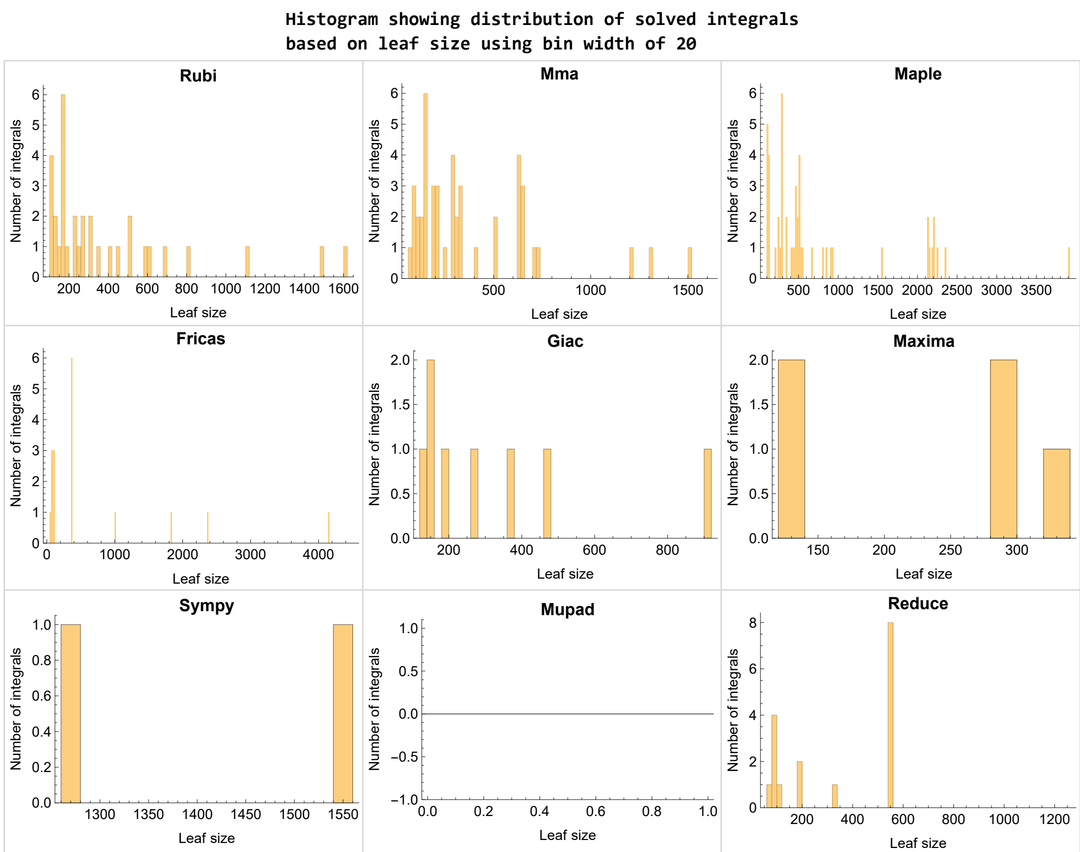


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

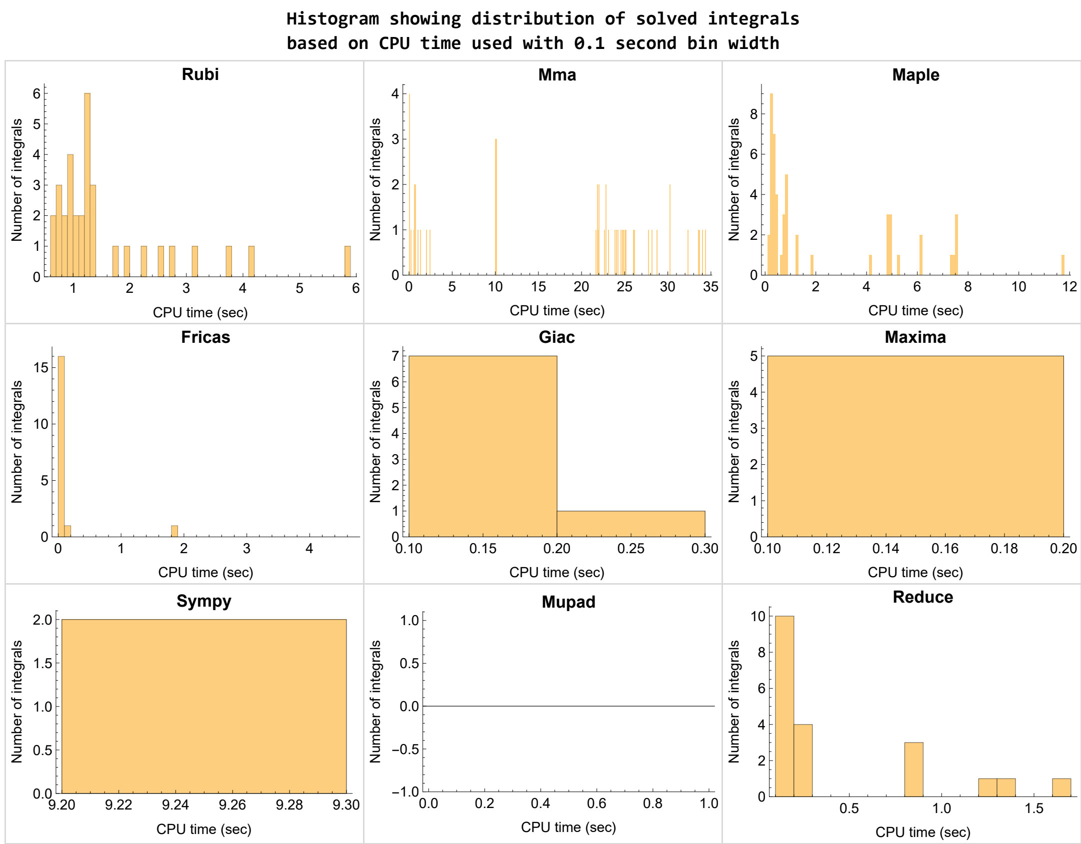


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

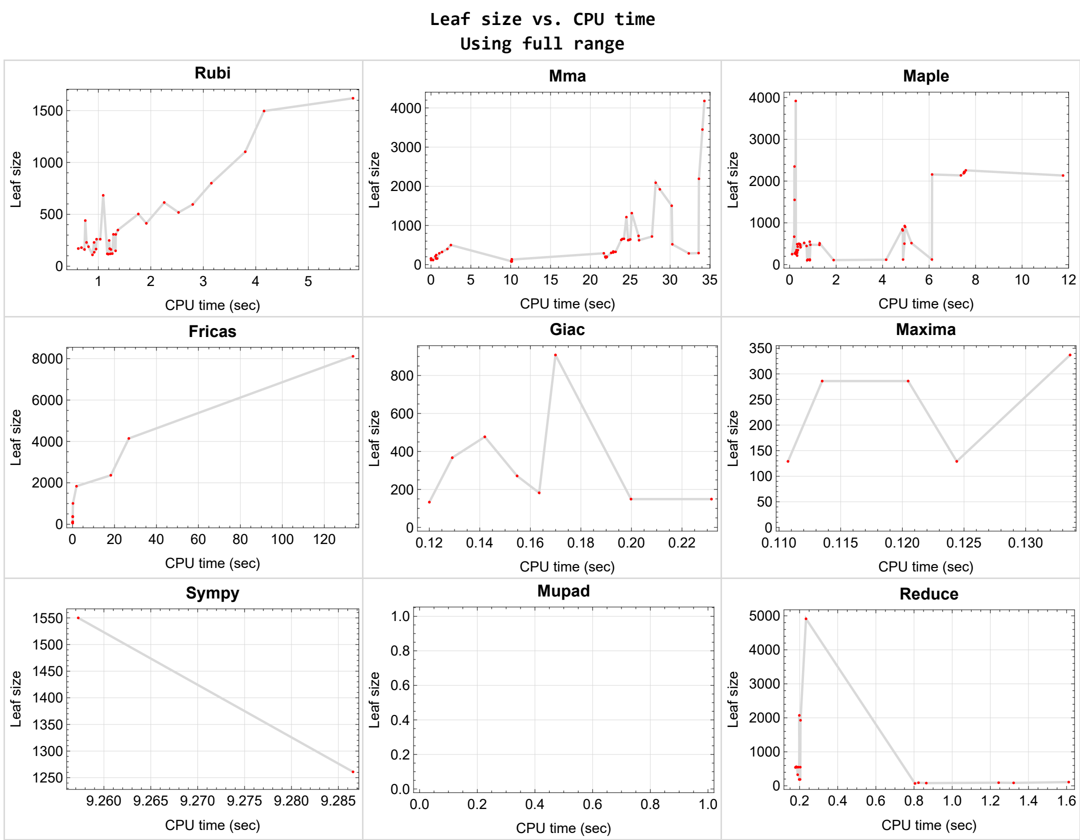


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{3, 4, 41, 42, 43\}$

Mathematica $\{3, 4, 19, 23, 24, 25, 44, 46, 48, 49, 50\}$

Maple $\{48, 49, 50\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

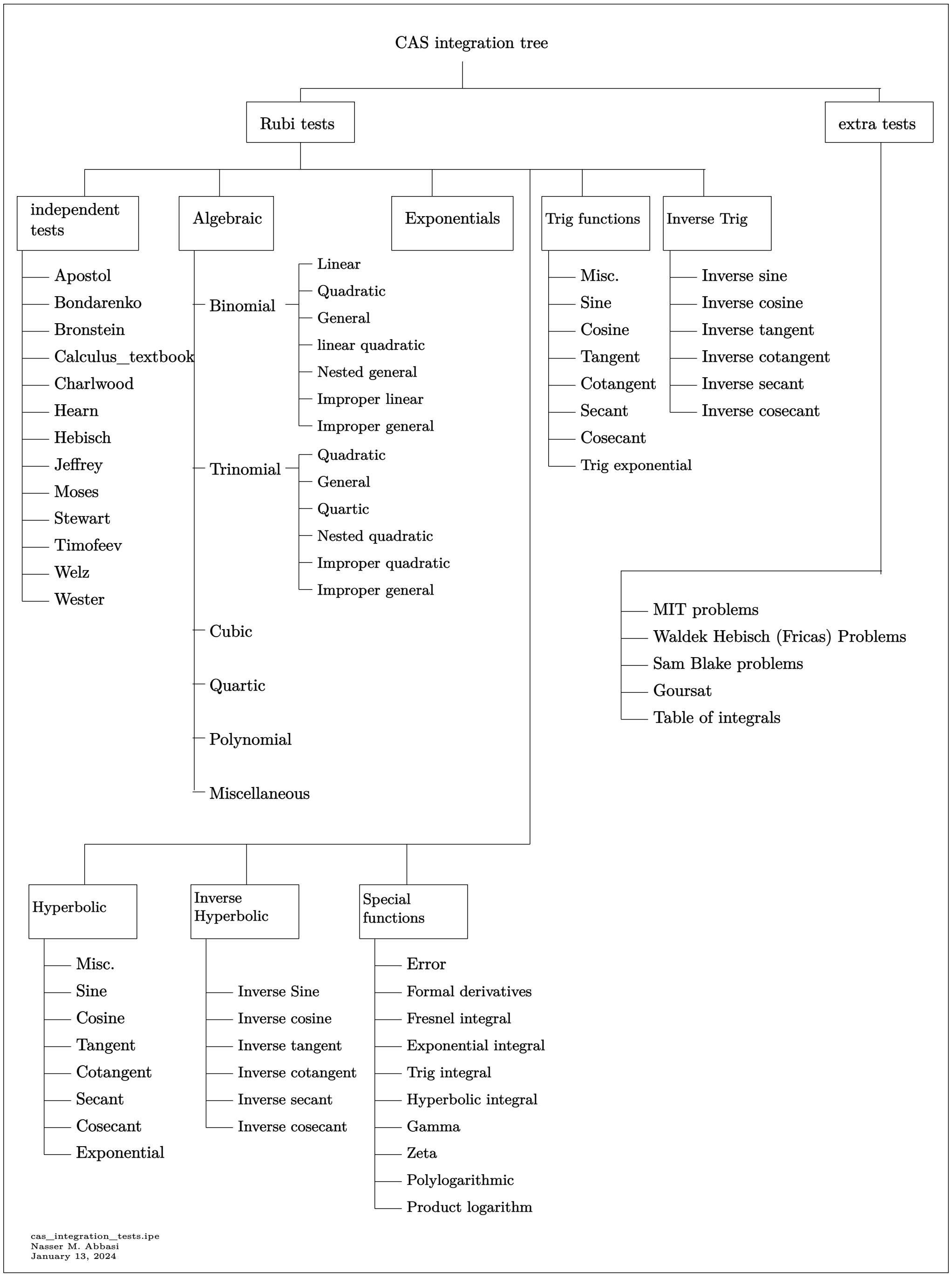
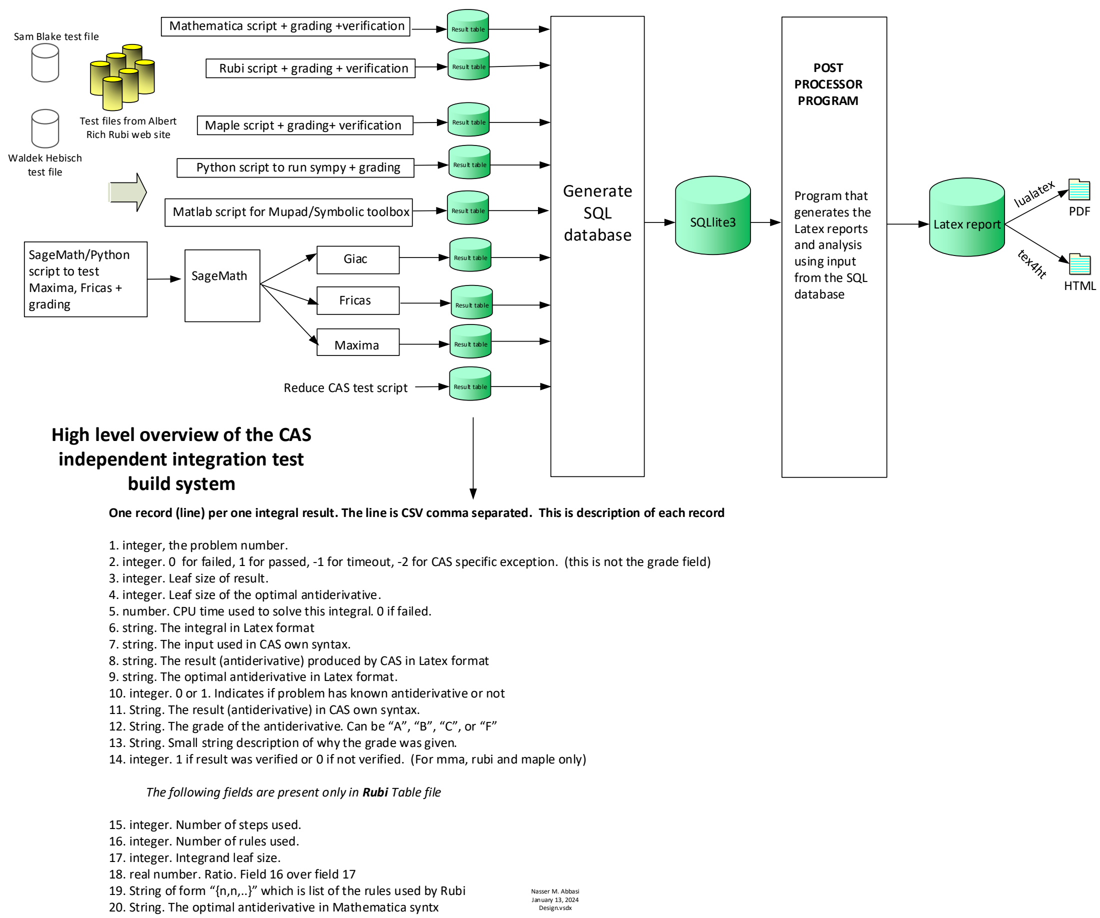


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 25
Mma 25
Maple 26
Fricas 26
Maxima 26
Giac 27
Mupad 27
Sympy 27
Reduce 28

Rubi

A grade { 3, 4, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40 }

B grade { 10, 41, 42, 43 }

C grade { }

F normal fail { 1, 2, 5, 6, 7, 8, 9, 12, 13, 14, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 44, 45, 46, 47 }

B grade { 48, 49, 50 }

C grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 38, 39, 40, 41, 42, 43 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43 }

B grade { 1, 2, 3, 4, 5, 6, 7, 28, 29, 30, 31, 39, 44, 45, 46, 47, 48, 49, 50 }

C grade { 8, 9, 10, 11, 12, 13, 14, 15, 16 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26 }

B grade { 27, 28, 29, 30, 32, 33, 34, 35, 36, 37 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 25, 44, 45, 47 }

F(-1) timedout fail { 31, 38, 39, 40, 41, 42, 43, 46, 48, 49, 50 }

F(-2) exception fail { }

Maxima

A grade { 15, 16, 33, 35 }

B grade { 32 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 15, 16, 26, 27, 28, 29, 30, 31 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 32, 33, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { }

F(-2) exception fail { 34, 37 }

Mupad

A grade { }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-2) exception fail { }

Sympy

A grade { 35 }

B grade { }

C grade { 33 }

F normal fail { 2, 3, 4, 5, 9, 10, 11, 12, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 30, 31, 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { 1, 6, 7, 8, 13, 14, 15, 16, 21, 22, 29 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 23, 24, 25, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	0	203	473	0	0	0	0	237	0
N.S.	1	0.00	0.84	1.95	0.00	0.00	0.00	0.00	0.98	0.00
time (sec)	N/A	0.000	21.885	1.283	0.000	0.000	0.000	0.000	37.602	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	0	193	463	0	0	0	0	199	0
N.S.	1	0.00	0.90	2.16	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.000	21.869	0.458	0.000	0.000	0.000	0.000	30.661	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	307	326	430	0	0	0	0	161	0
N.S.	1	1.66	1.76	2.32	0.00	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	1.288	23.162	0.346	0.000	0.000	0.000	0.000	24.687	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	165	301	346	0	0	0	0	128	0
N.S.	1	1.06	1.93	2.22	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.957	22.606	0.355	0.000	0.000	0.000	0.000	20.916	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	0	183	416	0	0	0	0	158	0
N.S.	1	0.00	0.99	2.25	0.00	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.000	21.951	0.472	0.000	0.000	0.000	0.000	25.749	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	0	193	446	0	0	0	0	36	0
N.S.	1	0.00	0.90	2.08	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.000	22.055	0.730	0.000	0.000	0.000	0.000	200.031	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	B	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	0	203	476	0	0	0	0	420	0
N.S.	1	0.00	0.84	1.96	0.00	0.00	0.00	0.00	1.73	0.00
time (sec)	N/A	0.000	22.038	0.893	0.000	0.000	0.000	0.000	22.664	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	0	632	510	0	0	0	0	36	0
N.S.	1	0.00	1.76	1.42	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	24.866	1.292	0.000	0.000	0.000	0.000	200.037	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	0	625	500	0	0	0	0	36	0
N.S.	1	0.00	1.89	1.51	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	24.735	0.400	0.000	0.000	0.000	0.000	200.031	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	683	654	490	0	0	0	0	36	0
N.S.	1	2.26	2.17	1.62	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.093	24.003	0.326	0.000	0.000	0.000	0.000	200.027	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	440	632	346	0	0	0	0	36	0
N.S.	1	1.62	2.33	1.28	0.00	0.00	0.00	0.00	0.13	0.00
time (sec)	N/A	0.752	23.868	0.294	0.000	0.000	0.000	0.000	200.027	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	0	659	490	0	0	0	0	36	0
N.S.	1	0.00	2.18	1.62	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	24.211	0.446	0.000	0.000	0.000	0.000	200.030	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	0	623	520	0	0	0	0	36	0
N.S.	1	0.00	1.88	1.57	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	26.104	0.625	0.000	0.000	0.000	0.000	200.027	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	0	643	550	0	0	0	0	36	0
N.S.	1	0.00	1.79	1.53	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	25.004	0.865	0.000	0.000	0.000	0.000	200.029	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	170	117	251	129	116	0	149	187	0
N.S.	1	1.24	0.85	1.83	0.94	0.85	0.00	1.09	1.36	0.00
time (sec)	N/A	0.618	0.254	0.274	0.124	0.082	0.000	0.232	0.203	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	179	117	251	129	116	0	149	187	0
N.S.	1	1.31	0.85	1.83	0.94	0.85	0.00	1.09	1.36	0.00
time (sec)	N/A	0.679	0.005	0.106	0.111	0.082	0.000	0.200	0.198	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	119	82	109	0	69	0	0	92	0
N.S.	1	0.82	0.57	0.75	0.00	0.48	0.00	0.00	0.63	0.00
time (sec)	N/A	1.230	10.144	1.893	0.000	0.065	0.000	0.000	0.823	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	109	77	104	0	64	0	0	70	0
N.S.	1	0.96	0.68	0.91	0.00	0.56	0.00	0.00	0.61	0.00
time (sec)	N/A	0.889	10.083	0.754	0.000	0.067	0.000	0.000	0.805	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	119	137	120	0	97	0	0	81	0
N.S.	1	0.82	0.94	0.82	0.00	0.66	0.00	0.00	0.55	0.00
time (sec)	N/A	1.171	10.132	0.783	0.000	0.071	0.000	0.000	0.864	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	116	130	123	0	114	0	0	93	0
N.S.	1	0.77	0.86	0.81	0.00	0.75	0.00	0.00	0.62	0.00
time (sec)	N/A	1.190	10.146	0.869	0.000	0.077	0.000	0.000	1.244	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	121	87	106	0	85	0	0	87	0
N.S.	1	0.90	0.65	0.79	0.00	0.63	0.00	0.00	0.65	0.00
time (sec)	N/A	1.267	10.070	0.868	0.000	0.069	0.000	0.000	1.322	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	148	92	111	0	90	0	0	110	0
N.S.	1	0.88	0.55	0.66	0.00	0.54	0.00	0.00	0.65	0.00
time (sec)	N/A	1.328	10.071	0.869	0.000	0.071	0.000	0.000	1.611	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	161	295	123	0	63	0	0	122	0
N.S.	1	1.01	1.86	0.77	0.00	0.40	0.00	0.00	0.77	0.00
time (sec)	N/A	1.241	33.578	6.121	0.000	0.068	0.000	0.000	1.172	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	287	118	0	58	0	0	99	0
N.S.	1	1.07	2.30	0.94	0.00	0.46	0.00	0.00	0.79	0.00
time (sec)	N/A	0.922	32.345	4.153	0.000	0.069	0.000	0.000	0.879	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	169	290	121	0	0	0	0	106	0
N.S.	1	1.33	2.28	0.95	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	1.217	21.683	4.880	0.000	0.000	0.000	0.000	1.543	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	248	211	215	0	1006	0	133	327	0
N.S.	1	1.42	1.21	1.23	0.00	5.75	0.00	0.76	1.87	0.00
time (sec)	N/A	1.207	0.546	0.322	0.000	0.199	0.000	0.120	0.190	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	306	246	266	0	1836	0	182	541	0
N.S.	1	1.32	1.06	1.15	0.00	7.91	0.00	0.78	2.33	0.00
time (sec)	N/A	1.332	0.656	0.225	0.000	1.887	0.000	0.163	0.178	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	518	322	1550	0	4145	0	367	2073	0
N.S.	1	1.63	1.01	4.87	0.00	13.03	0.00	1.15	6.52	0.00
time (sec)	N/A	2.528	1.381	0.214	0.000	26.840	0.000	0.129	0.199	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	800	500	3918	0	8113	0	908	4913	0
N.S.	1	1.57	0.98	7.68	0.00	15.91	0.00	1.78	9.63	0.00
time (sec)	N/A	3.154	2.494	0.263	0.000	133.717	0.000	0.170	0.234	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	413	289	669	0	2365	0	271	558	0
N.S.	1	1.33	0.93	2.16	0.00	7.63	0.00	0.87	1.80	0.00
time (sec)	N/A	1.915	1.038	0.197	0.000	18.249	0.000	0.155	0.182	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	595	400	2349	0	0	0	477	1928	0
N.S.	1	1.48	1.00	5.84	0.00	0.00	0.00	1.19	4.80	0.00
time (sec)	N/A	2.798	2.057	0.210	0.000	0.000	0.000	0.142	0.205	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	229	156	280	337	366	0	0	550	0
N.S.	1	1.48	1.01	1.81	2.17	2.36	0.00	0.00	3.55	0.00
time (sec)	N/A	0.775	0.777	0.306	0.134	0.090	0.000	0.000	0.204	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	161	156	289	286	366	1550	0	550	0
N.S.	1	1.05	1.01	1.88	1.86	2.38	10.06	0.00	3.57	0.00
time (sec)	N/A	0.737	0.636	0.272	0.113	0.095	9.257	0.000	0.180	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	260	156	297	0	366	0	0	550	0
N.S.	1	1.45	0.87	1.66	0.00	2.04	0.00	0.00	3.07	0.00
time (sec)	N/A	0.966	0.717	0.270	0.000	0.090	0.000	0.000	0.182	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	188	156	289	286	366	1261	0	550	0
N.S.	1	1.05	0.87	1.61	1.60	2.04	7.04	0.00	3.07	0.00
time (sec)	N/A	0.813	0.009	0.253	0.120	0.093	9.287	0.000	0.183	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	229	156	280	0	366	0	0	550	0
N.S.	1	1.28	0.87	1.56	0.00	2.04	0.00	0.00	3.07	0.00
time (sec)	N/A	0.919	0.005	0.315	0.000	0.092	0.000	0.000	0.194	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	260	156	297	0	366	0	0	550	0
N.S.	1	1.45	0.87	1.66	0.00	2.04	0.00	0.00	3.07	0.00
time (sec)	N/A	1.037	0.005	0.321	0.000	0.100	0.000	0.000	0.187	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	348	310	502	0	0	0	0	32	0
N.S.	1	1.09	0.97	1.58	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	1.373	22.861	4.938	0.000	0.000	0.000	0.000	200.021	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	481	503	1214	819	0	0	0	0	37	0
N.S.	1	1.05	2.52	1.70	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.762	24.518	4.867	0.000	0.000	0.000	0.000	200.028	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	614	1922	903	0	0	0	0	42	0
N.S.	1	1.06	3.31	1.55	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	2.255	28.715	4.970	0.000	0.000	0.000	0.000	200.032	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	1103	336	515	0	0	0	0	31	0
N.S.	1	2.52	0.77	1.18	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	3.799	22.869	5.242	0.000	0.000	0.000	0.000	200.036	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	1496	1315	844	0	0	0	0	36	0
N.S.	1	2.18	1.92	1.23	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	4.157	25.194	4.849	0.000	0.000	0.000	0.000	200.030	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	783	1619	2091	926	0	0	0	0	41	0
N.S.	1	2.07	2.67	1.18	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	5.852	28.192	4.950	0.000	0.000	0.000	0.000	200.029	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	0	519	2133	0	0	0	0	27	0
N.S.	1	0.00	0.79	3.24	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	30.279	11.763	0.000	0.000	0.000	0.000	200.036	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	659	0	718	2136	0	0	0	0	28	0
N.S.	1	0.00	1.09	3.24	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	27.725	7.368	0.000	0.000	0.000	0.000	200.033	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1042	0	1503	2158	0	0	0	0	30	0
N.S.	1	0.00	1.44	2.07	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	30.206	6.129	0.000	0.000	0.000	0.000	200.030	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	677	0	734	2187	0	0	0	0	32	0
N.S.	1	0.00	1.08	3.23	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	0.000	26.044	7.491	0.000	0.000	0.000	0.000	200.031	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	0	2191	2217	0	0	0	0	34	0
N.S.	1	0.00	2.06	2.08	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	33.620	7.506	0.000	0.000	0.000	0.000	200.032	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1060	0	3448	2208	0	0	0	0	36	0
N.S.	1	0.00	3.25	2.08	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	34.059	7.527	0.000	0.000	0.000	0.000	200.029	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1075	0	4178	2257	0	0	0	0	37	0
N.S.	1	0.00	3.89	2.10	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	34.305	7.579	0.000	0.000	0.000	0.000	200.028	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [3] had the largest ratio of [.5263160000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	F	0	0	N/A	0.000	N/A
2	F	0	0	N/A	0.000	N/A
3	A	21	20	1.66	38	0.526
4	A	12	11	1.06	38	0.289
5	F	0	0	N/A	0.000	N/A
6	F	0	0	N/A	0.000	N/A
7	F	0	0	N/A	0.000	N/A
8	F	0	0	N/A	0.000	N/A
9	F	0	0	N/A	0.000	N/A
10	B	19	18	2.26	38	0.474
11	A	13	12	1.62	38	0.316
12	F	0	0	N/A	0.000	N/A
13	F	0	0	N/A	0.000	N/A
14	F	0	0	N/A	0.000	N/A
15	A	7	6	1.24	41	0.146
16	A	7	6	1.31	34	0.176
17	A	8	7	0.82	43	0.163
18	A	8	7	0.96	41	0.171
19	A	10	9	0.82	43	0.209
20	A	3	3	0.77	43	0.070

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	3	0.90	43	0.070
22	A	3	3	0.88	43	0.070
23	A	3	3	1.01	43	0.070
24	A	3	3	1.07	40	0.075
25	A	3	3	1.33	43	0.070
26	A	9	8	1.42	41	0.195
27	A	10	9	1.32	46	0.196
28	A	16	15	1.63	46	0.326
29	A	20	19	1.57	46	0.413
30	A	12	11	1.33	51	0.216
31	A	17	16	1.48	51	0.314
32	A	2	2	1.48	39	0.051
33	A	6	5	1.05	38	0.132
34	A	2	2	1.45	47	0.043
35	A	6	5	1.05	63	0.079
36	A	3	3	1.28	78	0.038
37	A	3	3	1.45	78	0.038
38	A	10	9	1.09	34	0.265
39	A	13	12	1.05	39	0.308
40	A	15	14	1.06	44	0.318
41	B	9	8	2.52	33	0.242
42	B	11	10	2.18	38	0.263
43	B	12	11	2.07	43	0.256
44	F	0	0	N/A	0.000	N/A
45	F	0	0	N/A	0.000	N/A
46	F	0	0	N/A	0.000	N/A
47	F	0	0	N/A	0.000	N/A
48	F	0	0	N/A	0.000	N/A
49	F	0	0	N/A	0.000	N/A
50	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

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3.46 $\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$ . . . 473
3.47 $\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$ . . . 484
3.48 $\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$ . . . 493
3.49 $\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$ . . . 504
3.50 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$ . . . 515

3.1 $\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result 46
Mathematica [C] (verified) 47
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Fricas [F] 53
Sympy [F(-1)] 54
Maxima [F] 54
Giac [F] 54
Mupad [F(-1)] 55
Reduce [F] 55

Optimal result

Integrand size = 38, antiderivative size = 243

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{64\sqrt{1+2x}(5232011+1719891x)\sqrt{3-4x^2}}{10945935} + \frac{2\sqrt{1+2x}(200681+143542x)\left(3-4x^2\right)^{3/2}}{243243} - \frac{2(70-33x)\sqrt{1+2x}\left(3-4x^2\right)^{5/2}}{1287} - \frac{610834712\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2189187} - \frac{24360445832\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{32837805\sqrt{1+\sqrt{3}}} + \frac{405004\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{6561}$$

output

```
-64/10945935*(1+2*x)^(1/2)*(5232011+1719891*x)*(-4*x^2+3)^(1/2)+2/243243*(
1+2*x)^(1/2)*(200681+143542*x)*(-4*x^2+3)^(3/2)-2/1287*(70-33*x)*(1+2*x)^(
1/2)*(-4*x^2+3)^(5/2)-610834712/2189187*(1+3^(1/2))^(1/2)*EllipticE(1/6*(3
-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))-24360445832/32837805*Ellipt
icF(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/2)
+405004/6561*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*
6^(1/2),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.88 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2\left(9\sqrt{1+2x}\sqrt{3-4x^2}(-145690567-33132357x-21834180x^2-32\right.}{}$$

input
```
Integrate[((3 - 4*x^2)^(5/2)*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
(2*(9*Sqrt[1 + 2*x]*Sqrt[3 - 4*x^2]*(-145690567 - 33132357*x - 21834180*x^
2 - 32573520*x^3 - 9525600*x^4 + 4490640*x^5) + ((12*I)*(1145315085*(1 + S
qrt[3])*EllipticE[I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3
]] - (4190370814 + 1145315085*Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[1 + 2*x]/S
qrt[-1 + Sqrt[3]]], -2 + Sqrt[3]] + 2845659355*EllipticPi[(-3*(-1 + Sqrt[3
]))/13, I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]]))/Sqrt[
1 + Sqrt[3]]))/98513415
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3-4x^2)^{5/2}(-2x^2+6x+4)}{(5-3x)\sqrt{2x+1}}\,dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{(3-4x^2)^{5/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(3-4x^2)^{5/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{682}$$

$$-\frac{5}{572}\int \frac{8(82x+321)\left(3-4x^2\right)^{3/2}}{9\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 27

$$-\frac{10\int \frac{(82x+321)\left(3-4x^2\right)^{3/2}}{\sqrt{2x+1}}dx}{1287}+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 682

$$-\frac{10\left(\frac{1}{63}\sqrt{2x+1}(574x+2561)\left(3-4x^2\right)^{3/2}-\frac{1}{84}\int -\frac{32(1711x+2136)\sqrt{3-4x^2}}{\sqrt{2x+1}}dx\right)}{1287}+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 27

$$-\frac{10\left(\frac{8}{21}\int \frac{(1711x+2136)\sqrt{3-4x^2}}{\sqrt{2x+1}}dx+\frac{1}{63}\sqrt{2x+1}(574x+2561)\left(3-4x^2\right)^{3/2}\right)}{1287}+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 682

$$-\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\sqrt{2x+1}(5133x+7258)\sqrt{3-4x^2}-\frac{1}{60}\int -\frac{4(59830x+58947)}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)+\frac{1}{63}\sqrt{2x+1}(574x+2561)\left(3-4x^2\right)^{3/2}\right)}{1287}+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 27

$$-\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\int \frac{59830x+58947}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(5133x+7258)\right)+\frac{1}{63}\sqrt{2x+1}(574x+2561)\left(3-4x^2\right)^{3/2}\right)}{1287}+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\downarrow$ 600

$$-\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(29032\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+29915\int\frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(5133x+7258)\right)+\frac{1}{63}\sqrt{2x+}\right.}{1287}$$

$$\frac{76}{9}\int\frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\Big\downarrow$ 508

$$\frac{76}{9}\int\frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(29032\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{29915\sqrt{3+\sqrt{3}}\int\frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(5133x+72\right.\right.}{1287}$$

$$\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\Big\downarrow$ 327

$$-\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(29032\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{29915\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(5133x+\right.\right.}{1287}$$

$$\frac{76}{9}\int\frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\Big\downarrow$ 511

$$-\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(-\frac{29032\sqrt[4]{3}\int\frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt{3+\sqrt{3}}}-\frac{29915\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+}\right.\right.}{1287}$$

$$\frac{76}{9}\int\frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$\Big\downarrow$ 321

$$\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(-\frac{29032\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{29915\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3}\right.\right.}{1287}$$

$$\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{744}$$

$$\frac{76}{9}\int \frac{\left(3-4x^2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{10\left(\frac{8}{21}\left(\frac{1}{15}\left(-\frac{29032\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{29915\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3}\right.\right.}{1287}$$

$$\frac{2(70-33x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}{1287}$$

input
```
Int[((3 - 4*x^2)^(5/2)*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349

```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. 2(181) = 362.

Time = 1.28 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{2\left(4490640x^5-9525600x^4-32573520x^3-21834180x^2-33132357x-145690567\right)\left(4x^2-3\right)\sqrt{1+2x}\,\sqrt{\left(-4x^2+3\right)\left(1+2x\right)}}{10945935\sqrt{-\left(4x^2-3\right)\left(1+2x\right)}\,\sqrt{-4x^2+3}} - \frac{2\left(\ldots 4\ldots - - \ldots\right)}{\ldots}$
elliptic	$\frac{\sqrt{-\left(4x^2-3\right)\left(1+2x\right)}\left(\frac{32x^5\sqrt{-8x^3-4x^2+6x+3}}{39}-\frac{2240x^4\sqrt{-8x^3-4x^2+6x+3}}{1287}-\frac{206816x^3\sqrt{-8x^3-4x^2+6x+3}}{34749}-\frac{970408x^2\sqrt{-8x^3-4x^2+6x+3}}{243243}\ldots\right)}{\ldots}$
default	$-\frac{2\sqrt{-4x^2+3}\,\sqrt{1+2x}\left(3933645309+8761864257x-2351572560x^5-27535517163\sqrt{3}-1155398904x^4-524180160x^7+1096220062\ldots\right)}{\ldots}$

input
```
int((-4*x^2+3)^(5/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
-2/10945935*(4490640*x^5-9525600*x^4-32573520*x^3-21834180*x^2-33132357*x-
145690567)*(4*x^2-3)*(1+2*x)^(1/2)/(-(4*x^2-3)*(1+2*x))^(1/2)*((-4*x^2+3)*
(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)-2*(-4788995216/14073345*((x+1/2*3^(1/2))*3
^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/
2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2)
)*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-1221669424/6567561*((x
+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2
*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*3^(1/2)+1/2)*El
lipticE(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-
1/2))^(1/2))-1/2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^
(1/2)/(1/2*3^(1/2)-1/2))^(1/2)))-29565292/59049*((x+1/2*3^(1/2))*3^(1/2))^
(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2
)/(-8*x^3-4*x^2+6*x+3)^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3*3^(1/2)*((x
+1/2*3^(1/2))*3^(1/2))^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1/2)/(1/2*3^(
1/2)-1/2))^(1/2)))*((-4*x^2+3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)/(1+2*x)^(1/
2)
```

Fricas [F]

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{5}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(5/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(2*(16*x^6 - 48*x^5 - 56*x^4 + 72*x^3 + 57*x^2 - 27*x - 18)*sqrt(-
4*x^2 + 3)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \text{Timed out}$$

input
```
integrate((-4*x**2+3)**(5/2)*(-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{5}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(5/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)*(-4*x^2 + 3)^(5/2)/((3*x - 5)*sqrt(2*x + 1)),
x)
```

Giac [F]

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{5}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(5/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)*(-4*x^2 + 3)^(5/2)/((3*x - 5)*sqrt(2*x + 1)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx=\int -\frac{(3-4\,x^2)^{5/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((3 - 4*x^2)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((3 - 4*x^2)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(3-4x^2)^{5/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx&=\frac{32\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^5}{39}\\&-\frac{2240\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^4}{1287}-\frac{206816\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^3}{34749}\\&-\frac{970408\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^2}{243243}-\frac{7362746\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{1216215}\\&+\frac{30130187\sqrt{2x+1}\,\sqrt{-4x^2+3}}{3648645}-\frac{305417356\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^3}{24x^4-28x^3-38x^2+21x+15}dx\right)}{243243}\\&+\frac{2696716081\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{24x^4-28x^3-38x^2+21x+15}dx\right)}{3648645}\\&+\frac{18228295\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{24x^4-28x^3-38x^2+21x+15}dx\right)}{243243}\end{aligned}$$

input
```
int((-4*x^2+3)^(5/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
(2993760*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**5 - 6350400*sqrt(2*x + 1)*sq
rt( - 4*x**2 + 3)*x**4 - 21715680*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**3 -
 14556120*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**2 - 22088238*sqrt(2*x + 1)*
sqrt( - 4*x**2 + 3)*x + 30130187*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3) - 45812
60340*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**3)/(24*x**4 - 28*x**3 - 38
*x**2 + 21*x + 15),x) + 2696716081*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*
x)/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x) + 273424425*int((sqrt(2*x
+ 1)*sqrt( - 4*x**2 + 3))/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x))/36
48645
```

3.2 $\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result 57
Mathematica [C] (verified) 58
Rubi [F] 58
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Fricas [F] 64
Sympy [F] 65
Maxima [F] 65
Giac [F] 66
Mupad [F(-1)] 66
Reduce [F] 66

Optimal result

Integrand size = 38, antiderivative size = 214

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2\sqrt{1+2x}(17237+8982x)\sqrt{3-4x^2}}{8505} - \frac{2}{189}(16-7x)\sqrt{1+2x}(3-4x^2)^{3/2} + \frac{53104\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{1701} + \frac{2304832\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{25515\sqrt{1+\sqrt{3}}} - \frac{5548}{729}\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)$$

output
```
2/8505*(1+2*x)^(1/2)*(17237+8982*x)*(-4*x^2+3)^(1/2)-2/189*(16-7*x)*(1+2*x
)^(1/2)*(-4*x^2+3)^(3/2)+53104/1701*(1+3^(1/2))^(1/2)*EllipticE(1/6*(3-2*x
*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))+2304832/25515*EllipticF(1/6*(3-
2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/2)-5548/729*(
106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),-54/73
-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.87 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2\left(-117\sqrt{1+2x}\sqrt{3-4x^2}(-15077-9927x-2880x^2+1260x^3)-\right.}{}$$

input
```
Integrate[((3 - 4*x^2)^(3/2)*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
(2*(-117*Sqrt[1 + 2*x]*Sqrt[3 - 4*x^2]*(-15077 - 9927*x - 2880*x^2 + 1260*
x^3) - ((12*I)*(1294410*(1 + Sqrt[3])*EllipticE[I*ArcSinh[Sqrt[1 + 2*x]/Sq
rt[-1 + Sqrt[3]]], -2 + Sqrt[3]] - 26*(193837 + 49785*Sqrt[3])*EllipticF[I
*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]] + 3543785*Ellipt
icPi[(-3*(-1 + Sqrt[3]))/13, I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]],
-2 + Sqrt[3]]))/Sqrt[1 + Sqrt[3]]))/995085
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(3-4x^2)^{3/2}(-2x^2+6x+4)}{(5-3x)\sqrt{2x+1}}\,dx$$

$$\Big\downarrow \; 2349$$

$$\frac{76}{9}\int \frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(3-4x^2)^{3/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \; 682$$

$$-\frac{1}{84}\int \frac{8(22x+75)\sqrt{3-4x^2}}{3\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}(3-4x^2)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{63}\int \frac{(22x+75)\sqrt{3-4x^2}}{\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{682}$$

$$-\frac{2}{63}\left(\frac{1}{15}\sqrt{2x+1}(66x+331)\sqrt{3-4x^2}-\frac{1}{60}\int -\frac{32(215x+273)}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{63}\left(\frac{8}{15}\int \frac{215x+273}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(66x+331)\right)+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{600}$$

$$-\frac{2}{63}\left(\frac{8}{15}\left(\frac{331}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{215}{2}\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(66x+331)\right)+\frac{76}{9}\int \frac{\left(3-4x^2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{508}$$

$$\frac{76}{9}\int \frac{\left(3-4x^2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{63}\left(\frac{8}{15}\left(\frac{331}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{215\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{2\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x^2}(66x+331)\right)-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$-\frac{2}{63}\left(\frac{8}{15}\left(\frac{331}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{215\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)+\frac{1}{15}\sqrt{2x+1}\sqrt{3-4x\right.$$
$$\frac{76}{9}\int\frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}(3-4x^2)^{3/2}$$

$\downarrow$ 511

$$-\frac{2}{63}\left(\frac{8}{15}\left(-\frac{331\sqrt[4]{3}\int\frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{2\sqrt{3+\sqrt{3}}}-\frac{215\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)+\right.$$
$$\frac{76}{9}\int\frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}(3-4x^2)^{3/2}$$

$\downarrow$ 321

$$\frac{76}{9}\int\frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{63}\left(\frac{8}{15}\left(-\frac{331\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{215\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)+\right.$$
$$\frac{2}{189}(16-7x)\sqrt{2x+1}(3-4x^2)^{3/2}$$

$\downarrow$ 744

$$\frac{76}{9}\int\frac{(3-4x^2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{63}\left(\frac{8}{15}\left(-\frac{331\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{215\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)+\right.$$
$$\frac{2}{189}(16-7x)\sqrt{2x+1}(3-4x^2)^{3/2}$$

input
```
Int[((3 - 4*x^2)^(3/2)*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(158) = 316$.

Time = 0.46 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.16

method	result
risch	$\frac{2(1260x^3-2880x^2-9927x-15077)(4x^2-3)\sqrt{1+2x}\sqrt{(-4x^2+3)(1+2x)}}{8505\sqrt{-(4x^2-3)(1+2x)}\sqrt{-4x^2+3}}+\frac{2\left(-\frac{443056\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right)\sqrt{3}}}{10935\sqrt{-8x^3-4x^2}}\right.}{}$
elliptic	$\frac{\sqrt{-(4x^2-3)(1+2x)}\left(-\frac{8x^3\sqrt{-8x^3-4x^2+6x+3}}{27}+\frac{128x^2\sqrt{-8x^3-4x^2+6x+3}}{189}+\frac{2206x\sqrt{-8x^3-4x^2+6x+3}}{945}+\frac{30154\sqrt{-8x^3-4x^2+6x+3}}{8505}-\frac{8861}{}\right.}{}$
default	$\frac{2\sqrt{-4x^2+3}\sqrt{1+2x}\left(407079+1082187x-162000x^5-2849553\sqrt{3}-886464x^4+71046x^2-1321416x^3+90720x^6-7575309\sqrt{3}\,x-836\right.}{}$

input
```
int((-4*x^2+3)^(3/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
2/8505*(1260*x^3-2880*x^2-9927*x-15077)*(4*x^2-3)*(1+2*x)^(1/2)/(-(4*x^2-3
)*(1+2*x))^(1/2)*((-4*x^2+3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)+2*(-443056/10
935*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3
*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3
^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-
106208/5103*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(
1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*
3^(1/2)+1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2
)/(1/2*3^(1/2)-1/2))^(1/2))-1/2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(
1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2)))-405004/6561*((x+1/2*3^(1/2
))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3
^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3
*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1
/2)/(1/2*3^(1/2)-1/2))^(1/2)))*((-4*x^2+3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)
/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{3}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(3/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-2*(4*x^4 - 12*x^3 - 11*x^2 + 9*x + 6)*sqrt(-4*x^2 + 3)*sqrt(2*x
+ 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F]

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{6\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{9x\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{11x^2\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx + \int \frac{12x^3\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx + \int \left(-\frac{4x^4\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx\right)$$

input
```
integrate((-4*x**2+3)**(3/2)*(-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-6*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
+ Integral(-9*x*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
 + Integral(11*x**2*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1))
, x) + Integral(12*x**3*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x +
 1)), x) + Integral(-4*x**4*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2
*x + 1)), x))
```

Maxima [F]

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{3}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(3/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)*(-4*x^2 + 3)^(3/2)/((3*x - 5)*sqrt(2*x + 1)),
x)
```

Giac [F]

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)(-4\,x^2+3)^{\frac{3}{2}}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(3/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)*(-4*x^2 + 3)^(3/2)/((3*x - 5)*sqrt(2*x + 1)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(3-4\,x^2)^{3/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((3 - 4*x^2)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((3 - 4*x^2)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{(3-4x^2)^{3/2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx &= -\frac{8\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^3}{27} \\ &+ \frac{128\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^2}{189} + \frac{2206\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{945} \\ &- \frac{1012\sqrt{2x+1}\,\sqrt{-4x^2+3}}{2835} + \frac{26552\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^3}{24x^4-28x^3-38x^2+21x+15}dx\right)}{189} \\ &- \frac{172106\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{24x^4-28x^3-38x^2+21x+15}dx\right)}{2835} + \frac{1198\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{24x^4-28x^3-38x^2+21x+15}dx\right)}{189}\end{aligned}$$

input
```
int((-4*x^2+3)^(3/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
(2*( - 420*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**3 + 960*sqrt(2*x + 1)*sqrt
( - 4*x**2 + 3)*x**2 + 3309*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x - 506*sqrt
(2*x + 1)*sqrt( - 4*x**2 + 3) + 199140*int((sqrt(2*x + 1)*sqrt( - 4*x**2 +
 3)*x**3)/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x) - 86053*int((sqrt(2
*x + 1)*sqrt( - 4*x**2 + 3)*x)/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x
) + 8985*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3))/(24*x**4 - 28*x**3 - 38*x
**2 + 21*x + 15),x)))/2835
```

3.3 $\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result . . . 68
Mathematica [C] (warning: unable to verify) . . . 69
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Optimal result

Integrand size = 38, antiderivative size = 185

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{2}{135}(26-9x)\sqrt{1+2x}\sqrt{3-4x^2} - \frac{142}{27}\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right) - \frac{4696\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{405\sqrt{1+\sqrt{3}}} + \frac{76}{81}\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)$$

output

```
-2/135*(26-9*x)*(1+2*x)^(1/2)*(-4*x^2+3)^(1/2)-142/27*(1+3^(1/2))^(1/2)*El
lipticE(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))-4696/405*Elli
pticF(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/
2)+76/81*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1
/2),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.16 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2(1+2x)\sqrt{3-4x^2}\left(39(-26+9x) - \frac{(1+2x)\left(-\frac{13845i(-1+\sqrt{3})\sqrt{\frac{-3+4x^2}{(1+2x)^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-1+\sqrt{3}}}{\sqrt{1+2x}}\right)|-2-\sqrt{3}\right)}{\sqrt{1+2x}} + \frac{3i(-5543+4615\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(Sqrt[3 - 4*x^2]*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
(2*(1 + 2*x)*Sqrt[3 - 4*x^2]*(39*(-26 + 9*x) - ((1 + 2*x)*(((-13845*I)*(-1
 + Sqrt[3])*Sqrt[(-3 + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*ArcSinh[Sqrt[-1 + S
qrt[3]]/Sqrt[1 + 2*x]], -2 - Sqrt[3]])/Sqrt[1 + 2*x] + ((3*I)*(-5543 + 461
5*Sqrt[3])*Sqrt[(-3 + 4*x^2)/(1 + 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[-1 + Sq
rt[3]]/Sqrt[1 + 2*x]], -2 - Sqrt[3]])/Sqrt[1 + 2*x] - (5*(2769*Sqrt[-1 + S
qrt[3]]*(-3 + 4*x^2) + (5548*I)*(1 + 2*x)^(3/2)*Sqrt[(-3 + 4*x^2)/(1 + 2*x
)^2]*EllipticPi[(-13*(1 + Sqrt[3]))/6, I*ArcSinh[Sqrt[-1 + Sqrt[3]]/Sqrt[1
 + 2*x]], -2 - Sqrt[3]]))/(1 + 2*x)^2))/(Sqrt[-1 + Sqrt[3]]*(3 - 4*x^2))))
/(1755*Sqrt[9 + 18*x])
```

Rubi [A] (warning: unable to verify)

Time = 1.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.66, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {2349, 682, 27, 600, 508, 327, 511, 321, 724, 27, 600, 508, 327, 511, 321, 730, 27, 186, 25, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3-4x^2}\left(-2x^2+6x+4\right)}{(5-3x)\sqrt{2x+1}}\,dx$$

$\Big\downarrow$ 2349

$$\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\sqrt{3-4x^2}}{\sqrt{2x+1}}dx$$

$\Big\downarrow$ 682

$$-\frac{1}{60}\int \frac{8(50x+129)}{9\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\Big\downarrow$ 27

$$-\frac{2}{135}\int \frac{50x+129}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\Big\downarrow$ 600

$$-\frac{2}{135}\left(104\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+25\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)+\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\Big\downarrow$ 508

$$\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\left(104\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{25\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt[4]{3}}\right)-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\Big\downarrow$ 327

$$-\frac{2}{135}\left(104\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\Big\downarrow$ 511

$$-\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\int \frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+$$
$$\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\downarrow$ 321

$$\frac{76}{9}\int \frac{\sqrt{3-4x^2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\downarrow$ 724

$$\frac{76}{9}\left(-\frac{73}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{1}{9}\int -\frac{4(3x+5)}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\downarrow$ 27

$$\frac{76}{9}\left(\frac{4}{9}\int \frac{3x+5}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{73}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$\downarrow$ 600

$$\frac{76}{9}\left(\frac{4}{9}\left(\frac{7}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{3}{2}\int\frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)-\frac{73}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{508}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(\frac{7}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}\int\frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)-\frac{73}{9}\int\frac{}{(5-3x)\sqrt{}}\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{327}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(\frac{7}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)-\frac{73}{9}\int\frac{}{(5-3x)\sqrt{2x}}\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{511}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\int\frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)-\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{321}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{730}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \text{186}$$

$$\frac{76}{9}\left(\frac{146\int -\frac{1}{(13-3(2x+1))\sqrt{-\sqrt{3}(2x+1)+\sqrt{3}+3}\sqrt{\sqrt{3}(2x+1)-\sqrt{3}+3}}d\sqrt{2x+1}}{3\sqrt{3}}+\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),\right.}{2\sqrt{3+\sqrt{3}}}\right.\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \ 25$$

$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)\right.$$
$$\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

$$\Big\downarrow \ 412$$

$$-\frac{2}{135}\left(-\frac{104\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{25\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+$$
$$\frac{76}{9}\left(\frac{4}{9}\left(-\frac{7\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}-\frac{1}{2}3^{3/4}\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)\right)\right)$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{3-4x^2}(26-9x)$$

input
```
Int[(Sqrt[3 - 4*x^2]*(4 + 6*x - 2*x^2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
(-2*(26 - 9*x)*Sqrt[1 + 2*x]*Sqrt[3 - 4*x^2])/135 - (2*((-25*Sqrt[3 + Sqrt
[3]]*EllipticE[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]], 3 - Sqrt[3]])/3^(1/4
) - (104*3^(1/4)*EllipticF[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]], 3 - Sqrt
[3]])/Sqrt[3 + Sqrt[3]]))/135 + (76*((4*(-1/2*(3^(3/4)*Sqrt[3 + Sqrt[3]]*E
llipticE[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]], 3 - Sqrt[3]]) - (7*3^(1/4)
*EllipticF[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]], 3 - Sqrt[3]])/(2*Sqrt[3
+ Sqrt[3]])))/9 - (146*EllipticPi[(3*(1 + Sqrt[3]))/13, ArcSin[Sqrt[(-1 +
Sqrt[3])/2]*Sqrt[1 + 2*x]], -2 - Sqrt[3]])/(117*Sqrt[-1 + Sqrt[3]])))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 724
```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Simp[(c*d^2 + a*e^2)/e^2   Int[1/((d + e*x)*Sqrt[f + g*x]*
Sqrt[a + c*x^2]), x], x] - Simp[1/e^2   Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sq
rt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 730
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/((e + f*x
)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 2349

```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(135) = 270$.

Time = 0.35 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.32

method	result
elliptic	$\frac{\sqrt{-(4x^2-3)(1+2x)}\left(\frac{2x\sqrt{-8x^3-4x^2+6x+3}}{15}-\frac{52\sqrt{-8x^3-4x^2+6x+3}}{135}+\frac{13652\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\cdots}}{\cdots}\right.}{1215\sqrt{-8x^3-4x^2+6x+3}}\right.}{\cdots}$
risch	$-\frac{2(-26+9x)(4x^2-3)\sqrt{1+2x}\sqrt{(-4x^2+3)(1+2x)}}{135\sqrt{-(4x^2-3)(1+2x)}\sqrt{-4x^2+3}}-\frac{2\left(-\frac{6826\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\cdots}}{3}\right.}{1215\sqrt{-8x^3-4x^2+6x+3}}\right.}{\cdots}$
default	$-\frac{2\sqrt{-4x^2+3}\sqrt{1+2x}\left(648x^4+23480\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}\sqrt{-(1+2x)\left(\sqrt{3}-1\right)}\sqrt{3}\sqrt{\left(-2x+\sqrt{3}\right)\sqrt{3}}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}}{6},\cdots\right.\right.}{\cdots}$

input
```
int((-4*x^2+3)^(1/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(4*x^2-3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)/(1+2*x)^(1/2)*(2/15*x*(-8*x^3-
4*x^2+6*x+3)^(1/2)-52/135*(-8*x^3-4*x^2+6*x+3)^(1/2)+13652/1215*((x+1/2*3^
(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2
))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2
*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))+568/81*((x+1/2
*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(
1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*3^(1/2)+1/2)*Ellipt
icE(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2)
)^(1/2))-1/2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2
)/(1/2*3^(1/2)-1/2))^(1/2)))+11096/729*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(
x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3
-4*x^2+6*x+3)^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3*3^(1/2)*((x+1/2*3^(1
/2))*3^(1/2))^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1/2)/(1/2*3^(1/2)-1/2)
)^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)\sqrt{-4\,x^2+3}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(1/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5
), x)
```

Sympy [F]

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{2\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{3x\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{x^2\sqrt{3-4x^2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-4*x**2+3)**(1/2)*(-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-2*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
+ Integral(-3*x*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
 + Integral(x**2*sqrt(3 - 4*x**2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x
))
```

Maxima [F]

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)\sqrt{-4\,x^2+3}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(1/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)/((3*x - 5)*sqrt(2*x + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(x^2-3\,x-2)\sqrt{-4\,x^2+3}}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-4*x^2+3)^(1/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)/((3*x - 5)*sqrt(2*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{\sqrt{3-4\,x^2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((3 - 4*x^2)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((3 - 4*x^2)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\begin{aligned}\int \frac{\sqrt{3-4x^2}(4+6x-2x^2)}{(5-3x)\sqrt{1+2x}}\,dx &= \frac{2\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{15} + \frac{49\sqrt{2x+1}\,\sqrt{-4x^2+3}}{180} \\ &\quad - \frac{71\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^3}{24x^4-28x^3-38x^2+21x+15}dx\right)}{3} \\ &\quad + \frac{3797\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{24x^4-28x^3-38x^2+21x+15}dx\right)}{180} \\ &\quad + \frac{71\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{24x^4-28x^3-38x^2+21x+15}dx\right)}{12}\end{aligned}$$

input
```
int((-4*x^2+3)^(1/2)*(-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
(24*sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x + 49*sqrt(2*x + 1)*sqrt( - 4*x**2
+ 3) - 4260*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**3)/(24*x**4 - 28*x**
3 - 38*x**2 + 21*x + 15),x) + 3797*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*
x)/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x) + 1065*int((sqrt(2*x + 1)*
sqrt( - 4*x**2 + 3))/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x))/180
```

3.4 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx$

Optimal result 82
Mathematica [C] (warning: unable to verify) 83
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Giac [F] 90
Mupad [F(-1)] 90
Reduce [F] 91

Optimal result

Integrand size = 38, antiderivative size = 156

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx = -\frac{1}{3}\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right) + \frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{9\sqrt{1+\sqrt{3}}} - \frac{76}{657}\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)$$

output
```
-1/3*(1+3^(1/2))^(1/2)*EllipticE(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1
/2))^(1/2))+11/9*EllipticF(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(
1/2))/(1+3^(1/2))^(1/2)-76/657*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2
*x*3^(1/2))^(1/2)*6^(1/2),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 22.61 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.93

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx =$$

$$-\frac{(3+6x)^{3/2}\left(-78+\frac{156}{(1+2x)^2}+\frac{156}{1+2x}-\frac{78i\sqrt{-1+\sqrt{3}}\sqrt{\frac{-3+4x^2}{(1+2x)^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-1+\sqrt{3}}}{\sqrt{1+2x}}\right)|-2-\sqrt{3}\right)}{\sqrt{1+2x}}+\frac{6i\left(-16+13\sqrt{3}\right)\sqrt{\frac{-}{(1}}}{}\right.}{702\sqrt{9-}}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[3 - 4*x^2]),x]
```

output
```
-1/702*((3 + 6*x)^(3/2)*(-78 + 156/(1 + 2*x)^2 + 156/(1 + 2*x) - ((78*I)*S
qrt[-1 + Sqrt[3]]*Sqrt[(-3 + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*ArcSinh[Sqrt[
-1 + Sqrt[3]]/Sqrt[1 + 2*x]], -2 - Sqrt[3]])/Sqrt[1 + 2*x] + ((6*I)*(-16 +
 13*Sqrt[3])*Sqrt[(-3 + 4*x^2)/(1 + 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[-1 +
Sqrt[3]]/Sqrt[1 + 2*x]], -2 - Sqrt[3]])/(Sqrt[-1 + Sqrt[3]]*Sqrt[1 + 2*x])
 + ((304*I)*Sqrt[(-3 + 4*x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(1 + Sqrt[3]))/
6, I*ArcSinh[Sqrt[-1 + Sqrt[3]]/Sqrt[1 + 2*x]], -2 - Sqrt[3]])/(Sqrt[-1 +
Sqrt[3]]*Sqrt[1 + 2*x])))/Sqrt[9 - 12*x^2]
```

Rubi [A] (warning: unable to verify)

Time = 0.96 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.289, Rules used = {2349, 600, 508, 327, 511, 321, 730, 27, 186, 25, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx$$

$$\Big\downarrow \text{ 2349}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\sqrt{3-4x^2}}dx$$

$$\Big\downarrow \text{ 600}$$

$$-\frac{11}{9}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{1}{3}\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx$$

$$\Big\downarrow \text{ 508}$$

$$-\frac{11}{9}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{3\sqrt[4]{3}}$$

$$\Big\downarrow \text{ 327}$$

$$-\frac{11}{9}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$$\Big\downarrow \text{ 511}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{11\int \frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$$\Big\downarrow \text{ 730}$$

$$\frac{76\int \frac{3}{(5-3x)\sqrt{2x+1}\sqrt{3-2\sqrt{3}x}\sqrt{2\sqrt{3}x+3}}dx}{9\sqrt{3}}+\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$\Big\downarrow$ 27

$$\frac{76\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{3-2\sqrt{3}x}\sqrt{2\sqrt{3}x+3}}dx}{3\sqrt{3}}+\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$\Big\downarrow$ 186

$$-\frac{152\int -\frac{1}{(13-3(2x+1))\sqrt{-\sqrt{3}(2x+1)+\sqrt{3}+3}\sqrt{\sqrt{3}(2x+1)-\sqrt{3}+3}}d\sqrt{2x+1}}{3\sqrt{3}}+\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$\Big\downarrow$ 25

$$\frac{152\int \frac{1}{(13-3(2x+1))\sqrt{-\sqrt{3}(2x+1)+\sqrt{3}+3}\sqrt{\sqrt{3}(2x+1)-\sqrt{3}+3}}d\sqrt{2x+1}}{3\sqrt{3}}+\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}$$

$\Big\downarrow$ 412

$$\frac{11\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{3\ 3^{3/4}\sqrt{3+\sqrt{3}}}-\frac{\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{3\sqrt[4]{3}}+\frac{152\,\text{EllipticPi}\left(\frac{3}{13}\left(1+\sqrt{3}\right),\arcsin\left(\sqrt{\frac{1}{2}\left(-1+\sqrt{3}\right)}\sqrt{2x+1}\right),-2-\sqrt{3}\right)}{117\sqrt{\sqrt{3}-1}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[3 - 4*x^2]),x]
```

output
```
-1/3*(Sqrt[3 + Sqrt[3]]*EllipticE[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]], 3
 - Sqrt[3]])/3^(1/4) + (11*EllipticF[ArcSin[Sqrt[3 - 2*Sqrt[3]*x]/Sqrt[6]]
, 3 - Sqrt[3]])/(3*3^(3/4)*Sqrt[3 + Sqrt[3]]) + (152*EllipticPi[(3*(1 + Sq
rt[3]))/13, ArcSin[Sqrt[(-1 + Sqrt[3])/2]*Sqrt[1 + 2*x]], -2 - Sqrt[3]])/(
117*Sqrt[-1 + Sqrt[3]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 730
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[1/Sqrt[a]   Int[1/((e + f*x
)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, b, c, d,
 e, f}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(112) = 224$.

Time = 0.36 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{1+2x}\,\sqrt{-4x^2+3}\,\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}\,\sqrt{-(1+2x)\left(\sqrt{3}-1\right)}\,\sqrt{3}\,\sqrt{\left(-2x+\sqrt{3}\right)\sqrt{3}}\left(-3\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}}{6},\frac{\sqrt{2}\,\sqrt{\sqrt{3}\left(\sqrt{3}-1\right)}}{\sqrt{3}-1}\right.\right.}{}$
elliptic	$\frac{\sqrt{-(4x^2-3)(1+2x)}\left(-\frac{16\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\,\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right)\sqrt{3}}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{\sqrt{3}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\right)}{27\sqrt{-8x^3-4x^2+6x+3}}+\frac{4\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\,\sqrt{-\frac{x}{\ldots}}}{}\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
1/27*(1+2*x)^(1/2)*(-4*x^2+3)^(1/2)*((2*x+3^(1/2))*3^(1/2))^(1/2)/(3^(1/2)
-1)*(-(1+2*x)*(3^(1/2)-1))^(1/2)*3^(1/2)*((-2*x+3^(1/2))*3^(1/2))^(1/2)*(-
3*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2))*3^(1/2))^(1/2),2^(1/2)/(3^(
1/2)-1)*(3^(1/2)*(3^(1/2)-1))^(1/2))+33*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*
x+3^(1/2))*3^(1/2))^(1/2),2^(1/2)/(3^(1/2)-1)*(3^(1/2)*(3^(1/2)-1))^(1/2))
*3^(1/2)+21*EllipticE(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2))*3^(1/2))^(1/2),2^
(1/2)/(3^(1/2)-1)*(3^(1/2)*(3^(1/2)-1))^(1/2))*3^(1/2)+110*EllipticF(1/6*3
^(1/2)*2^(1/2)*((2*x+3^(1/2))*3^(1/2))^(1/2),2^(1/2)/(3^(1/2)-1)*(3^(1/2)*
(3^(1/2)-1))^(1/2))-152*EllipticPi(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2))*3^(1
/2))^(1/2),6*3^(1/2)/(10+3*3^(1/2)),2^(1/2)/(3^(1/2)-1)*(3^(1/2)*(3^(1/2)-
1))^(1/2)))/(8*x^3+4*x^2-6*x-3)/(10+3*3^(1/2))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{\sqrt{-4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)*sqrt(2*x + 1)/(24*x^4 - 28*x^
3 - 38*x^2 + 21*x + 15), x)
```

Sympy [F]

$$\begin{aligned}&\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx\\&=2\left(\int \left(-\frac{3x}{3x\sqrt{3-4x^2}\sqrt{2x+1}-5\sqrt{3-4x^2}\sqrt{2x+1}}\right)dx\right.\\&\quad+\int \frac{x^2}{3x\sqrt{3-4x^2}\sqrt{2x+1}-5\sqrt{3-4x^2}\sqrt{2x+1}}\,dx\\&\quad\left.+\int \left(-\frac{2}{3x\sqrt{3-4x^2}\sqrt{2x+1}-5\sqrt{3-4x^2}\sqrt{2x+1}}\right)dx\right)\end{aligned}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(-4*x**2+3)**(1/2),x)
```

output
```
2*(Integral(-3*x/(3*x*sqrt(3 - 4*x**2)*sqrt(2*x + 1) - 5*sqrt(3 - 4*x**2)*
sqrt(2*x + 1)), x) + Integral(x**2/(3*x*sqrt(3 - 4*x**2)*sqrt(2*x + 1) - 5
*sqrt(3 - 4*x**2)*sqrt(2*x + 1)), x) + Integral(-2/(3*x*sqrt(3 - 4*x**2)*s
qrt(2*x + 1) - 5*sqrt(3 - 4*x**2)*sqrt(2*x + 1)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{-4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(1/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/(sqrt(-4*x^2 + 3)*(3*x - 5)*sqrt(2*x + 1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{-4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/(sqrt(-4*x^2 + 3)*(3*x - 5)*sqrt(2*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,(3\,x-5)\,\sqrt{3-4\,x^2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(1/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3-4x^2}}\,dx = -2\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^2}{24x^4-28x^3-38x^2+21x+15}dx\right) + 6\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{24x^4-28x^3-38x^2+21x+15}dx\right) + 4\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{24x^4-28x^3-38x^2+21x+15}dx\right)$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(1/2),x)
```

output
```
2*( - int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**2)/(24*x**4 - 28*x**3 - 38
*x**2 + 21*x + 15),x) + 3*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x)/(24*x*
*4 - 28*x**3 - 38*x**2 + 21*x + 15),x) + 2*int((sqrt(2*x + 1)*sqrt( - 4*x*
*2 + 3))/(24*x**4 - 28*x**3 - 38*x**2 + 21*x + 15),x))
```

3.5 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(3-4x^2\right)^{3/2}}\,dx$

Optimal result 92
Mathematica [C] (verified) 93
Rubi [F] 93
Maple [B] (verified) 97
Fricas [F] 98
Sympy [F] 98
Maxima [F] 99
Giac [F] 99
Mupad [F(-1)] 99
Reduce [F] 100

Optimal result

Integrand size = 38, antiderivative size = 185

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(3-4x^2\right)^{3/2}}\,dx = \frac{\sqrt{1+2x}(87+242x)}{876\sqrt{3-4x^2}} + \frac{121}{876}\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right) - \frac{52\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{219\sqrt{1+\sqrt{3}}} + \frac{76\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{5329}$$

output
```
1/876*(1+2*x)^(1/2)*(87+242*x)/(-4*x^2+3)^(1/2)+121/876*(1+3^(1/2))^(1/2)*
EllipticE(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))-52/219*Elli
pticF(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/
2)+76/5329*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*6^
(1/2),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.95 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = \frac{\frac{13\sqrt{1+2x}(87+242x)}{\sqrt{3-4x^2}} - \frac{i\left(1573\left(1+\sqrt{3}\right)E\left(i\text{arcsinh}\left(\frac{\sqrt{1+2x}}{\sqrt{-1+\sqrt{3}}}\right)|-2+\sqrt{3}\right)-13\left(-87+1\right.\right.}{}}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(3/2)),x]
```

output
```
((13*Sqrt[1 + 2*x]*(87 + 242*x))/Sqrt[3 - 4*x^2] - (I*(1573*(1 + Sqrt[3])*
EllipticE[I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]] - 13*
(-87 + 121*Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]],
 -2 + Sqrt[3]] - 1824*EllipticPi[(-3*(-1 + Sqrt[3]))/13, I*ArcSinh[Sqrt[1
+ 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]]))/Sqrt[1 + Sqrt[3]])/11388
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 2349}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx + \int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx + \frac{1}{96}\int -\frac{8(34x+33)}{9\sqrt{2x+1}\sqrt{3-4x^2}}dx - \frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx - \frac{1}{108}\int \frac{34x+33}{\sqrt{2x+1}\sqrt{3-4x^2}}dx - \frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \; 600$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx+\frac{1}{108}\left(-16\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-17\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)-\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \; 508$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx+\frac{1}{108}\left(\frac{17\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt[4]{3}}-16\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \; 327$$

$$\frac{1}{108}\left(\frac{17\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}-16\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \; 511$$

$$\frac{1}{108}\left(\frac{16\sqrt[4]{3}\int \frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt{3+\sqrt{3}}}+\frac{17\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

$$\Big\downarrow \; 321$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx+$$
$$\frac{1}{108}\left(\frac{16\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}+\frac{17\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$
$$\Big\downarrow \; 744$$
$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx+$$
$$\frac{1}{108}\left(\frac{16\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}+\frac{17\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)-$$
$$\frac{\sqrt{2x+1}(33-34x)}{108\sqrt{3-4x^2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(135) = 270$.

Time = 0.47 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.25

method	result
elliptic	$\frac{\sqrt{-(4x^2-3)(1+2x)}\left(-\frac{2(-4-8x)\left(\frac{29}{2336}+\frac{121x}{3504}\right)}{\sqrt{\left(x^2-\frac{3}{4}\right)(-4-8x)}}+\frac{29\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}}{3},\sqrt{\frac{\sqrt{3}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\right.}{438\sqrt{-8x^3-4x^2+6x+3}}\right.}{}$
default	$-\frac{\sqrt{1+2x}\,\sqrt{-4x^2+3}\left(2080\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}\sqrt{-(1+2x)\left(\sqrt{3}-1\right)}\sqrt{3}\sqrt{\left(-2x+\sqrt{3}\right)\sqrt{3}}\,\text{EllipticF}\left(\frac{\sqrt{3}\sqrt{2}\sqrt{\left(2x+\sqrt{3}\right)\sqrt{3}}}{6},\frac{\sqrt{2}\sqrt{\sqrt{3}\left(\sqrt{}\right.}}{\sqrt{3}-1}\right.\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(4*x^2-3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)/(1+2*x)^(1/2)*(-2*(-4-8*x)*(29
/2336+121/3504*x)/((x^2-3/4)*(-4-8*x))^(1/2)+29/438*((x+1/2*3^(1/2))*3^(1/
2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^
(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^
(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-121/657*((x+1/2*3^(1/2))*3
^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/
2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*3^(1/2)+1/2)*EllipticE(1/3*3^(
1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-1/
2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1
/2)-1/2))^(1/2)))+152/657*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3
^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)
^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))
^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)*sqrt(2*x + 1)/(96*x^6 - 112*x^
5 - 224*x^4 + 168*x^3 + 174*x^2 - 63*x - 45), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = 2\Bigg(\int \Bigg(-\frac{3x}{-12x^3\sqrt{3-4x^2}\sqrt{2x+1}+20x^2\sqrt{3-4x^2}\sqrt{2x+1}+9x\sqrt{}}$$
$$+\int \frac{x^2}{-12x^3\sqrt{3-4x^2}\sqrt{2x+1}+20x^2\sqrt{3-4x^2}\sqrt{2x+1}+9x\sqrt{3-4x^2}\sqrt{2x+1}-15\sqrt{3-4x^2}\sqrt{2x+1}}\,dx$$
$$+\int \Bigg(-\frac{2}{-12x^3\sqrt{3-4x^2}\sqrt{2x+1}+20x^2\sqrt{3-4x^2}\sqrt{2x+1}+9x\sqrt{3-4x^2}\sqrt{2x+1}-15\sqrt{3-4x^2}\sqrt{2x+1}}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(-4*x**2+3)**(3/2),x)
```

output
```
2*(Integral(-3*x/(-12*x**3*sqrt(3 - 4*x**2)*sqrt(2*x + 1) + 20*x**2*sqrt(3
 - 4*x**2)*sqrt(2*x + 1) + 9*x*sqrt(3 - 4*x**2)*sqrt(2*x + 1) - 15*sqrt(3
- 4*x**2)*sqrt(2*x + 1)), x) + Integral(x**2/(-12*x**3*sqrt(3 - 4*x**2)*sq
rt(2*x + 1) + 20*x**2*sqrt(3 - 4*x**2)*sqrt(2*x + 1) + 9*x*sqrt(3 - 4*x**2
)*sqrt(2*x + 1) - 15*sqrt(3 - 4*x**2)*sqrt(2*x + 1)), x) + Integral(-2/(-1
2*x**3*sqrt(3 - 4*x**2)*sqrt(2*x + 1) + 20*x**2*sqrt(3 - 4*x**2)*sqrt(2*x
+ 1) + 9*x*sqrt(3 - 4*x**2)*sqrt(2*x + 1) - 15*sqrt(3 - 4*x**2)*sqrt(2*x +
 1)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(3/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((-4*x^2 + 3)^(3/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((-4*x^2 + 3)^(3/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,{\left(3-4\,x^2\right)}^{3/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(3/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{3/2}}\,dx = 2\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x^2}{96x^6-112x^5-224x^4+168x^3+174x^2-63x-45}dx\right) - 6\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{96x^6-112x^5-224x^4+168x^3+174x^2-63x-45}dx\right) - 4\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{96x^6-112x^5-224x^4+168x^3+174x^2-63x-45}dx\right)$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(3/2),x)
```

output
```
2*(int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x**2)/(96*x**6 - 112*x**5 - 224*
x**4 + 168*x**3 + 174*x**2 - 63*x - 45),x) - 3*int((sqrt(2*x + 1)*sqrt( -
4*x**2 + 3)*x)/(96*x**6 - 112*x**5 - 224*x**4 + 168*x**3 + 174*x**2 - 63*x
 - 45),x) - 2*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3))/(96*x**6 - 112*x**5
- 224*x**4 + 168*x**3 + 174*x**2 - 63*x - 45),x))
```

3.6 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 214

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \frac{\sqrt{1+2x}(87+242x)}{2628\,(3-4x^2)^{3/2}} + \frac{\sqrt{1+2x}(1713+8941x)}{575532\sqrt{3-4x^2}} + \frac{8941\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{1151064} - \frac{12367\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{1151064\sqrt{1+\sqrt{3}}} - \frac{684\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{389017}$$

output
```
1/2628*(1+2*x)^(1/2)*(87+242*x)/(-4*x^2+3)^(3/2)+1/575532*(1+2*x)^(1/2)*(1
713+8941*x)/(-4*x^2+3)^(1/2)+8941/1151064*(1+3^(1/2))^(1/2)*EllipticE(1/6*
(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))-12367/1151064*EllipticF(1
/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/2)-684/
389017*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2
),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.06 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.90

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \frac{\frac{26\sqrt{1+2x}(24192+79821x-6852x^2-35764x^3)}{(3-4x^2)^{3/2}} - \frac{i\left(116233\left(1+\sqrt{3}\right)E\left(i\text{arcsinh}\left(\frac{\sqrt{1+2}}{\sqrt{-1+}}\right.\right.\right.}{}}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(5/2)),x]
```

output
```
((26*Sqrt[1 + 2*x]*(24192 + 79821*x - 6852*x^2 - 35764*x^3))/(3 - 4*x^2)^(
3/2) - (I*(116233*(1 + Sqrt[3])*EllipticE[I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1
+ Sqrt[3]]], -2 + Sqrt[3]] - 13*(-3426 + 8941*Sqrt[3])*EllipticF[I*ArcSinh
[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]] + 295488*EllipticPi[(-3*
(-1 + Sqrt[3]))/13, I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt
[3]]))/Sqrt[1 + Sqrt[3]])/14963832
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{5/2}}dx$$

$$\Big\downarrow \; 2349$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{5/2}}dx + \int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(3-4x^2)^{5/2}}dx$$

$$\Big\downarrow \; 686$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{5/2}}dx + \frac{1}{288}\int -\frac{8(97-102x)}{9\sqrt{2x+1}\,(3-4x^2)^{3/2}}dx - \frac{\sqrt{2x+1}(33-34x)}{324\,(3-4x^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx-\frac{1}{324}\int \frac{97-102x}{\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+\frac{1}{324}\left(-\frac{1}{96}\int \frac{32(125x+111)}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{\sqrt{2x+1}(111-125x)}{3\sqrt{3-4x^2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+\frac{1}{324}\left(-\frac{1}{3}\int \frac{125x+111}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{\sqrt{2x+1}(111-125x)}{3\sqrt{3-4x^2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$$\Big\downarrow \text{600}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+\frac{1}{324}\left(\frac{1}{3}\left(-\frac{97}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{125}{2}\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)-\frac{(111-125x)\sqrt{2x+1}}{3\sqrt{3-4x^2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$$\Big\downarrow \text{508}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+\frac{1}{324}\left(\frac{1}{3}\left(\frac{125\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{2\sqrt[4]{3}}-\frac{97}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-\frac{(111-125x)\sqrt{2x+1}}{3\sqrt{3-4x^2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{1}{324}\left(\frac{1}{3}\left(\frac{125\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}-\frac{97}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx\right)-\frac{(111-125x)\sqrt{2x+1}}{3\sqrt{3-4x^2}}\right.$$
$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$\downarrow$ 511

$$\frac{1}{324}\left(\frac{1}{3}\left(\frac{97\sqrt[4]{3}\int\frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{2\sqrt{3+\sqrt{3}}}+\frac{125\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)-\frac{(111}{}\right.$$
$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$\downarrow$ 321

$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+$$
$$\frac{1}{324}\left(\frac{1}{3}\left(\frac{97\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}+\frac{125\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)-\frac{(11}{}\right.$$
$$\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

$\downarrow$ 744

$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx+$$
$$\frac{1}{324}\left(\frac{1}{3}\left(\frac{97\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{2\sqrt{3+\sqrt{3}}}+\frac{125\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{2\sqrt[4]{3}}\right)-\frac{(11}{}\right.$$
$$\frac{\sqrt{2x+1}(33-34x)}{324\left(3-4x^2\right)^{3/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(158) = 316$.

Time = 0.73 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.08

method	result
elliptic	$\sqrt{-(4x^2-3)(1+2x)}\left(\frac{\left(\frac{29}{14016}+\frac{121x}{21024}\right)\sqrt{-8x^3-4x^2+6x+3}}{\left(x^2-\frac{3}{4}\right)^2}-\frac{2(-4-8x)\left(\frac{571}{1534752}+\frac{8941x}{4604256}\right)}{\sqrt{\left(x^2-\frac{3}{4}\right)(-4-8x)}}+\frac{571\sqrt{\left(x+\frac{\sqrt{3}}{2}\right)\sqrt{3}}\sqrt{-\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}}}\sqrt{-3\left(x-\frac{\sqrt{3}}{2}\right.}}{287766\sqrt{-8}}\right.$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(4*x^2-3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)/(1+2*x)^(1/2)*((29/14016+121/2
1024*x)*(-8*x^3-4*x^2+6*x+3)^(1/2)/(x^2-3/4)^2-2*(-4-8*x)*(571/1534752+894
1/4604256*x)/((x^2-3/4)*(-4-8*x))^(1/2)+571/287766*((x+1/2*3^(1/2))*3^(1/2
))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(
1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(
1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-8941/863298*((x+1/2*3^(1/2)
)*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^
(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*3^(1/2)+1/2)*EllipticE(1/3*
3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))
-1/2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3
^(1/2)-1/2))^(1/2)))-152/5329*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1
/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*
x+3)^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1
/2))^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x, algorit
hm="fricas")
```

output
```
integral(-2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)*sqrt(2*x + 1)/(384*x^8 - 448*
x^7 - 1184*x^6 + 1008*x^5 + 1368*x^4 - 756*x^3 - 702*x^2 + 189*x + 135), x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(-4*x**2+3)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((-4*x^2 + 3)^(5/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((-4*x^2 + 3)^(5/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(3-4\,x^2\right)^{5/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(5/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{5/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(-4x^2+3)^{\frac{5}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(5/2),x)
```

3.7 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 243

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx = \frac{\sqrt{1+2x}(87+242x)}{4380\,(3-4x^2)^{5/2}} + \frac{\sqrt{1+2x}(1614+33395x)}{2877660\,(3-4x^2)^{3/2}} + \frac{\sqrt{1+2x}(8374881+3813406x)}{1008332064\sqrt{3-4x^2}} + \frac{1906703\sqrt{1+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{1008332064} - \frac{642599\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{63020754\sqrt{1+\sqrt{3}}} + \frac{6156\sqrt{2\left(53+67\sqrt{3}\right)}\,\text{EllipticPi}\left(-\frac{6}{73}\left(9+10\sqrt{3}\right),\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{28398241}$$

output
```
1/4380*(1+2*x)^(1/2)*(87+242*x)/(-4*x^2+3)^(5/2)+1/2877660*(1+2*x)^(1/2)*(
1614+33395*x)/(-4*x^2+3)^(3/2)+1/1008332064*(1+2*x)^(1/2)*(8374881+3813406
*x)/(-4*x^2+3)^(1/2)+1906703/1008332064*(1+3^(1/2))^(1/2)*EllipticE(1/6*(3
-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))-642599/63020754*EllipticF(1
/6*(3-2*x*3^(1/2))^(1/2)*6^(1/2),(3-3^(1/2))^(1/2))/(1+3^(1/2))^(1/2)+6156
/28398241*(106+134*3^(1/2))^(1/2)*EllipticPi(1/6*(3-2*x*3^(1/2))^(1/2)*6^(
1/2),-54/73-60/73*3^(1/2),(3-3^(1/2))^(1/2))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.04 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx = \frac{\frac{13\sqrt{1+2x}(485495397+625684878x-1016296632x^2-691640880x^3+669990480x^4+30507248}{(3-4x^2)^{5/2}}}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(7/2)),x]
```

output
```
((13*Sqrt[1 + 2*x]*(485495397 + 625684878*x - 1016296632*x^2 - 691640880*x
^3 + 669990480*x^4 + 305072480*x^5))/(3 - 4*x^2)^(5/2) - ((5*I)*(24787139*
(1 + Sqrt[3])*EllipticE[I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 +
Sqrt[3]] - 13*(-8374881 + 1906703*Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[1 + 2*
x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]] - 31912704*EllipticPi[(-3*(-1 + Sqrt
[3]))/13, I*ArcSinh[Sqrt[1 + 2*x]/Sqrt[-1 + Sqrt[3]]], -2 + Sqrt[3]]))/Sqr
t[1 + Sqrt[3]])/65541584160
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{7/2}}dx$$

$$\Big\downarrow \; 2349$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{7/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(3-4x^2)^{7/2}}dx$$

$$\Big\downarrow \; 686$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{7/2}}dx+\frac{1}{480}\int -\frac{56(23-34x)}{9\sqrt{2x+1}\,(3-4x^2)^{5/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{540\,(3-4x^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\int \frac{23-34x}{\sqrt{2x+1}\left(3-4x^2\right)^{5/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\left(\frac{1}{288}\int \frac{32(76-111x)}{\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx+\frac{\sqrt{2x+1}(30-37x)}{9\left(3-4x^2\right)^{3/2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\left(\frac{1}{9}\int \frac{76-111x}{\sqrt{2x+1}\left(3-4x^2\right)^{3/2}}dx+\frac{\sqrt{2x+1}(30-37x)}{9\left(3-4x^2\right)^{3/2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{96}\int \frac{4(970x+789)}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{\sqrt{2x+1}(789-970x)}{24\sqrt{3-4x^2}}\right)+\frac{\sqrt{2x+1}(30-37x)}{9\left(3-4x^2\right)^{3/2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\int \frac{970x+789}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+\frac{\sqrt{2x+1}(789-970x)}{24\sqrt{3-4x^2}}\right)+\frac{\sqrt{2x+1}(30-37x)}{9\left(3-4x^2\right)^{3/2}}\right)-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{600}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(304\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx+485\int \frac{\sqrt{2x+1}}{\sqrt{3-4x^2}}dx\right)+\frac{\sqrt{2x+1}(789-970x)}{24\sqrt{3-4x^2}}\right)+\frac{\sqrt{2x+1}(30-37}{9\left(3-4x^2\right)^{3/2}}\right.\ \frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{508}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-$$

$$\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(304\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{485\sqrt{3+\sqrt{3}}\int \frac{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}}{\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt[4]{3}}\right)+\frac{\sqrt{2x+1}(789-970x}{24\sqrt{3-4x^2}}\right.\right.$$

$$\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{327}$$

$$-\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(304\int \frac{1}{\sqrt{2x+1}\sqrt{3-4x^2}}dx-\frac{485\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)+\frac{\sqrt{2x+1}(789-}{24\sqrt{3-4}}\right.\right.$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{511}$$

$$-\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(-\frac{304\sqrt[4]{3}\int \frac{1}{\sqrt{1-\frac{3-2\sqrt{3}x}{3+\sqrt{3}}}\sqrt{\frac{1}{6}\left(2\sqrt{3}x-3\right)+1}}d\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}}{\sqrt{3+\sqrt{3}}}-\frac{485\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)\right.\right.$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(3-4x^2\right)^{7/2}}dx-$$

$$\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(-\frac{304\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{485\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right)\right.\right.$$

$$\frac{\sqrt{2x+1}(33-34x)}{540\left(3-4x^2\right)^{5/2}}$$

$$\Big\downarrow \text{744}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(3-4x^2)^{7/2}}dx -$$

$$\frac{7}{540}\left(\frac{1}{9}\left(\frac{1}{24}\left(-\frac{304\sqrt[4]{3}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right),3-\sqrt{3}\right)}{\sqrt{3+\sqrt{3}}}-\frac{485\sqrt{3+\sqrt{3}}E\left(\arcsin\left(\frac{\sqrt{3-2\sqrt{3}x}}{\sqrt{6}}\right)|3-\sqrt{3}\right)}{\sqrt[4]{3}}\right.\right.\right.$$

$$\frac{\sqrt{2x+1}(33-34x)}{540\,(3-4x^2)^{5/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 - 4*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 475 vs. $2(181) = 362$.

Time = 0.89 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.96

method	result
elliptic	$\sqrt{-(4x^2-3)(1+2x)}\left(\frac{\left(-\frac{29}{93440}-\frac{121x}{140160}\right)\sqrt{-8x^3-4x^2+6x+3}}{\left(x^2-\frac{3}{4}\right)^3}+\frac{\left(\frac{269}{7673760}+\frac{6679x}{9208512}\right)\sqrt{-8x^3-4x^2+6x+3}}{\left(x^2-\frac{3}{4}\right)^2}-\frac{2(-4-8x)\left(\frac{2791627}{2688885504}+\frac{19067}{403332}\right.}{\sqrt{\left(x^2-\frac{3}{4}\right)(-4-8x)}}\right.$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(7/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(4*x^2-3)*(1+2*x))^(1/2)/(-4*x^2+3)^(1/2)/(1+2*x)^(1/2)*((-29/93440-121/
140160*x)*(-8*x^3-4*x^2+6*x+3)^(1/2)/(x^2-3/4)^3+(269/7673760+6679/9208512
*x)*(-8*x^3-4*x^2+6*x+3)^(1/2)/(x^2-3/4)^2-2*(-4-8*x)*(2791627/2688885504+
1906703/4033328256*x)/((x^2-3/4)*(-4-8*x))^(1/2)+2791627/504166032*((x+1/2
*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(
1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*EllipticF(1/3*3^(1/2)*((x+
1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2))-1906703/756
249048*((x+1/2*3^(1/2))*3^(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*
(-3*(x-1/2*3^(1/2))*3^(1/2))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)*((-1/2*3^(1/
2)+1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),(3^(1/2)/(1/
2*3^(1/2)-1/2))^(1/2))-1/2*EllipticF(1/3*3^(1/2)*((x+1/2*3^(1/2))*3^(1/2))
^(1/2),(3^(1/2)/(1/2*3^(1/2)-1/2))^(1/2)))+1368/389017*((x+1/2*3^(1/2))*3^
(1/2))^(1/2)*(-(x+1/2)/(1/2*3^(1/2)-1/2))^(1/2)*(-3*(x-1/2*3^(1/2))*3^(1/2
))^(1/2)/(-8*x^3-4*x^2+6*x+3)^(1/2)/(-1/2*3^(1/2)-5/3)*EllipticPi(1/3*3^(1
/2)*((x+1/2*3^(1/2))*3^(1/2))^(1/2),-3^(1/2)/(-1/2*3^(1/2)-5/3),(3^(1/2)/(
1/2*3^(1/2)-1/2))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(7/2),x, algorit
hm="fricas")
```

output
```
integral(2*(x^2 - 3*x - 2)*sqrt(-4*x^2 + 3)*sqrt(2*x + 1)/(1536*x^10 - 179
2*x^9 - 5888*x^8 + 5376*x^7 + 9024*x^6 - 6048*x^5 - 6912*x^4 + 3024*x^3 +
2646*x^2 - 567*x - 405), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx=\text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(-4*x**2+3)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(7/2),x, algorit
hm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((-4*x^2 + 3)^(7/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(-4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(7/2),x, algorit
hm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((-4*x^2 + 3)^(7/2)*(3*x - 5)*sqrt(2*x + 1)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,(3\,x-5)\,(3-4\,x^2)^{7/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(7/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(3 - 4*x^2)^(7/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3-4x^2)^{7/2}}\,dx =$$

$$-12\left(\int \frac{\sqrt{2x+1}\,x^4}{2304\sqrt{-4x^2+3}\,x^{10}-12160\sqrt{-4x^2+3}\,x^8+21184\sqrt{-4x^2+3}\,x^6-16344\sqrt{-4x^2+3}\,x^4+5643\sqrt{}\right.$$

$$-14\left(\int \frac{\sqrt{2x+1}\,x^3}{2304\sqrt{-4x^2+3}\,x^{10}-12160\sqrt{-4x^2+3}\,x^8+21184\sqrt{-4x^2+3}\,x^6-16344\sqrt{-4x^2+3}\,x^4+5643\sqrt{}\right.$$

$$+10\left(\int \frac{\sqrt{2x+1}\,x^2}{2304\sqrt{-4x^2+3}\,x^{10}-12160\sqrt{-4x^2+3}\,x^8+21184\sqrt{-4x^2+3}\,x^6-16344\sqrt{-4x^2+3}\,x^4+5643\sqrt{}\right.$$

$$-6\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}\,x}{1536x^{10}-1792x^9-5888x^8+5376x^7+9024x^6-6048x^5-6912x^4+3024x^3+2646x^2-567x-}\right.$$

$$-4\left(\int \frac{\sqrt{2x+1}\,\sqrt{-4x^2+3}}{1536x^{10}-1792x^9-5888x^8+5376x^7+9024x^6-6048x^5-6912x^4+3024x^3+2646x^2-567x-}\right.$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(-4*x^2+3)^(7/2),x)
```

output
```
2*( - 6*int((sqrt(2*x + 1)*x**4)/(2304*sqrt( - 4*x**2 + 3)*x**10 - 12160*s
qrt( - 4*x**2 + 3)*x**8 + 21184*sqrt( - 4*x**2 + 3)*x**6 - 16344*sqrt( - 4
*x**2 + 3)*x**4 + 5643*sqrt( - 4*x**2 + 3)*x**2 - 675*sqrt( - 4*x**2 + 3))
,x) - 7*int((sqrt(2*x + 1)*x**3)/(2304*sqrt( - 4*x**2 + 3)*x**10 - 12160*s
qrt( - 4*x**2 + 3)*x**8 + 21184*sqrt( - 4*x**2 + 3)*x**6 - 16344*sqrt( - 4
*x**2 + 3)*x**4 + 5643*sqrt( - 4*x**2 + 3)*x**2 - 675*sqrt( - 4*x**2 + 3))
,x) + 5*int((sqrt(2*x + 1)*x**2)/(2304*sqrt( - 4*x**2 + 3)*x**10 - 12160*s
qrt( - 4*x**2 + 3)*x**8 + 21184*sqrt( - 4*x**2 + 3)*x**6 - 16344*sqrt( - 4
*x**2 + 3)*x**4 + 5643*sqrt( - 4*x**2 + 3)*x**2 - 675*sqrt( - 4*x**2 + 3))
,x) - 3*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3)*x)/(1536*x**10 - 1792*x**9
- 5888*x**8 + 5376*x**7 + 9024*x**6 - 6048*x**5 - 6912*x**4 + 3024*x**3 +
2646*x**2 - 567*x - 405),x) - 2*int((sqrt(2*x + 1)*sqrt( - 4*x**2 + 3))/(1
536*x**10 - 1792*x**9 - 5888*x**8 + 5376*x**7 + 9024*x**6 - 6048*x**5 - 69
12*x**4 + 3024*x**3 + 2646*x**2 - 567*x - 405),x))
```

3.8 $\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result . . . 120
Mathematica [C] (verified) . . . 121
Rubi [F] . . . 122
Maple [C] (verified) . . . 127
Fricas [F] . . . 128
Sympy [F(-1)] . . . 129
Maxima [F] . . . 129
Giac [F] . . . 129
Mupad [F(-1)] . . . 130
Reduce [F] . . . 130

Optimal result

Integrand size = 38, antiderivative size = 360

$$
\begin{aligned}
\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx &= -\frac{1615154392\sqrt{1+2x}\sqrt{3+4x^2}}{1563705(3+2x)} \\
&- \frac{16\sqrt{1+2x}(59536964+19603449x)\sqrt{3+4x^2}}{10945935} \\
&- \frac{2\sqrt{1+2x}(311111+101962x)(3+4x^2)^{3/2}}{243243} \\
&- \frac{2(70-33x)\sqrt{1+2x}(3+4x^2)^{5/2}}{1287} + \frac{1225804\sqrt{\frac{127}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right)}{2187} \\
&+ \frac{1615154392\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{1563705\sqrt{3+4x^2}} \\
&- \frac{1576225160\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2189187\sqrt{3+4x^2}} \\
&- \frac{14338681\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{85293\sqrt{3+4x^2}}
\end{aligned}
$$

output
```
-1615154392*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)/(4691115+3127410*x)-16/10945935*
(1+2*x)^(1/2)*(59536964+19603449*x)*(4*x^2+3)^(1/2)-2/243243*(1+2*x)^(1/2)
*(311111+101962*x)*(4*x^2+3)^(3/2)-2/1287*(70-33*x)*(1+2*x)^(1/2)*(4*x^2+3
)^(5/2)+1225804/85293*4953^(1/2)*arctanh(1/39*4953^(1/2)*(1+2*x)^(1/2)/(4*
x^2+3)^(1/2))+1615154392/1563705*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/
2)*EllipticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2))/(4*x^2+
3)^(1/2)-1576225160/2189187*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*In
verseJacobiAM(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(
1/2)-14338681/85293*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi
(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.87 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.76

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2\sqrt{1+2x}\left(3(3+4x^2)(-523653847-168066477x-70288380x^2-1\right.}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(3 + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
(2*Sqrt[1 + 2*x]*(3*(3 + 4*x^2)*(-523653847 - 168066477*x - 70288380*x^2 -
 11617200*x^3 - 9525600*x^4 + 4490640*x^5) - (2*(1 + 2*x)*((8479560558*Sqr
t[(-I)/(I + Sqrt[3])]*(3 + 4*x^2))/(1 + 2*x)^2 + (4239780279*(-I + Sqrt[3]
)*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sq
rt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*Ellipti
cE[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I
 - Sqrt[3])])/Sqrt[1 + 2*x] - (3*(-950047897*I + 1413260093*Sqrt[3])*Sqrt[
(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*
I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticF[I*Ar
cSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt
[3])])/Sqrt[1 + 2*x] + ((14983921645*I)*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt
[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])
*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticPi[(13*(1 - I*Sqrt[3]))/12, I*ArcSi
nh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3]
)])/Sqrt[1 + 2*x]))/Sqrt[(-I)/(I + Sqrt[3])]))/(32837805*Sqrt[3 + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)(4x^2+3)^{5/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{(4x^2+3)^{5/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(4x^2+3)^{5/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{5}{572}\int -\frac{8(321-478x)(4x^2+3)^{3/2}}{9\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(4x^2+3)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}(4x^2+3)^{5/2}}{1287}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10\int \frac{(321-478x)(4x^2+3)^{3/2}}{\sqrt{2x+1}}dx}{1287}+\frac{76}{9}\int \frac{(4x^2+3)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}(4x^2+3)^{5/2}}{1287}$$

$$\Big\downarrow \text{682}$$

$$-\frac{10\left(\frac{1}{84}\int \frac{64(1173-2455x)\sqrt{4x^2+3}}{\sqrt{2x+1}}dx+\frac{1}{63}(4801-3346x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}\right)}{1287}+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10\left(\frac{16}{21}\int \frac{(1173-2455x)\sqrt{4x^2+3}}{\sqrt{2x+1}}dx+\frac{1}{63}(4801-3346x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}\right)}{1287}+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{682}$$

$$-\frac{10\left(\frac{16}{21}\left(\frac{1}{60}\int \frac{20(8511-17458x)}{\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(2155-1473x)\right)+\frac{1}{63}(4801-3346x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}{1287}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10\left(\frac{16}{21}\left(\frac{1}{3}\int \frac{8511-17458x}{\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(2155-1473x)\right)+\frac{1}{63}(4801-3346x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}\right)}{1287}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{599}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{10\left(\frac{16}{21}\left(\frac{1}{3}(2155-1473x)\sqrt{2x+1}\sqrt{4x^2+3}-\frac{1}{6}\int -\frac{2(17240-8729(2x+1))}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{1}{63}(4801-3346x)\sqrt{2x+1}\left(\right.}{1287}$$

$$\frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx - \frac{10\left(\frac{16}{21}\left(\frac{1}{3}\int \frac{17240-8729(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(2155-1473x)\right)+\frac{1}{63}(4801-3346x)\sqrt{2x+1}(4x\right.}{1287} - \frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$\downarrow$ 744

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx - \frac{10\left(\frac{16}{21}\left(\frac{1}{3}\int \frac{17240-8729(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(2155-1473x)\right)+\frac{1}{63}(4801-3346x)\sqrt{2x+1}(4x\right.}{1287} - \frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$\downarrow$ 1511

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx - \frac{10\left(\frac{16}{21}\left(\frac{1}{3}\left(17458\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-218\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(2\right.\right.\right.}{1287} - \frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$\downarrow$ 27

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx - \frac{10\left(\frac{16}{21}\left(\frac{1}{3}\left(8729\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-218\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{1}{3}\sqrt{2x+1}\sqrt{4x^2+3}(215\right.\right.\right.}{1287} - \frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$\downarrow$ 1416

$$-\frac{10\left(\frac{16}{21}\left(\frac{1}{3}\left(8729\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{109(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right)+\frac{1}{3}\sqrt{\right.\right.}{1287}$$
$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx - \frac{2(70-33x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}{1287}$$

$\downarrow$ 1509

$$\frac{76}{9}\int \frac{(4x^2+3)^{5/2}}{(5-3x)\sqrt{2x+1}}dx -$$

$$\frac{10\left(\frac{16}{21}\left(\frac{1}{3}\left(8729\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{\sqrt{(2x+1)^2-2(2x+1)+4}}-\frac{\sqrt{2x+1}\sqrt{(2x+1)^2-2(2x+1)+4}}{2x+3}\right)-\frac{109(2x+3)\sqrt{\frac{(2x+1}{}}}{}\right.\right.\right.}{}$$

$$\frac{2(70-33x)\sqrt{2x+1}(4x^2+3)^{5/2}}{1287}$$

input
```
Int[((4 + 6*x - 2*x^2)*(3 + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.42

method	result
risch	$\frac{2(4490640x^5-9525600x^4-11617200x^3-70288380x^2-168066477x-523653847)\sqrt{4x^2+3}\,\sqrt{1+2x}}{10945935}+\frac{2\left(-\frac{142672522688\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{}}{}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(\frac{32x^5\sqrt{8x^3+4x^2+6x+3}}{39}-\frac{2240x^4\sqrt{8x^3+4x^2+6x+3}}{1287}-\frac{73760x^3\sqrt{8x^3+4x^2+6x+3}}{34749}-\frac{3123928x^2\sqrt{8x^3+4x^2+6x+3}}{243243}-\frac{373481}{}\right.}{}$
default	$\frac{2\sqrt{4x^2+3}\,\sqrt{1+2x}\left(107775360x^8-174726720x^7+54377140228i\sqrt{3}\,\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x}{1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x}{-1+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}},\sqrt{-}\right.\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/10945935*(4490640*x^5-9525600*x^4-11617200*x^3-70288380*x^2-168066477*x-
523653847)*(4*x^2+3)^(1/2)*(1+2*x)^(1/2)+2*(-142672522688/32837805*(1/2-1/
2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-
1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^
3+4*x^2+6*x+3)^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+
1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))-3230308784/1563705*(1/2-1/2*I*
3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*
I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*
x^2+6*x+3)^(1/2)*((-1/2-1/2*I*3^(1/2))*EllipticE(((x+1/2)/(1/2-1/2*I*3^(1/
2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*3^(1/2
)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/
2-1/2*I*3^(1/2)))^(1/2)))+311354216/85293*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/
2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x
+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*Elli
pticPi(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2),((-1/2+1/2*
I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))*((4*x^2+3)*(1+2*x))^(1/2)/(4*x^2+
3)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\left(3+4x^2\right)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="fricas")
```

output
```
integral(2*(16*x^6 - 48*x^5 - 8*x^4 - 72*x^3 - 39*x^2 - 27*x - 18)*sqrt(4*
x^2 + 3)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+3)**(5/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="maxima")
```

output
```
2*integrate((4*x^2 + 3)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x
)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate(2*(4*x^2 + 3)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\,(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(4\,x^2+3)^{5/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((4*x^2 + 3)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((4*x^2 + 3)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)\,(3+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)\,(4x^2+3)^{\frac{5}{2}}}{(5-3x)\,\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.9 $\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result 131
Mathematica [C] (verified) 132
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Maxima [F] 139
Giac [F] 140
Mupad [F(-1)] 140
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Optimal result

Integrand size = 38, antiderivative size = 331

$$\begin{aligned}\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx &= -\frac{87704\sqrt{1+2x}\sqrt{3+4x^2}}{1215(3+2x)} \\ &- \frac{2\sqrt{1+2x}(25499+6714x)\sqrt{3+4x^2}}{8505} \\ &- \frac{2}{189}(16-7x)\sqrt{1+2x}(3+4x^2)^{3/2} + \frac{9652}{243}\sqrt{\frac{127}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right) \\ &+ \frac{87704\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{1215\sqrt{3+4x^2}} \\ &- \frac{85348\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{1701\sqrt{3+4x^2}} \\ &- \frac{112903\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{9477\sqrt{3+4x^2}}\end{aligned}$$

output
```
-87704*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)/(3645+2430*x)-2/8505*(1+2*x)^(1/2)*(2
5499+6714*x)*(4*x^2+3)^(1/2)-2/189*(16-7*x)*(1+2*x)^(1/2)*(4*x^2+3)^(3/2)+
9652/9477*4953^(1/2)*arctanh(1/39*4953^(1/2)*(1+2*x)^(1/2)/(4*x^2+3)^(1/2)
)+87704/1215*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticE(sin(2*a
rctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2))/(4*x^2+3)^(1/2)-85348/1701*
2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*(
1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(1/2)-112903/9477*2^(1/2)*(3+
2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi(sin(2*arctan(1/2*(1+2*x)^(1/2)
*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.73 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.89

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \frac{2\sqrt{1+2x}\left(39(3+4x^2)(-27659-5769x-2880x^2+1260x^3) - \frac{2(1+\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(3 + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
(2*Sqrt[1 + 2*x]*(39*(3 + 4*x^2)*(-27659 - 5769*x - 2880*x^2 + 1260*x^3) -
 (2*(1 + 2*x)*((2992899*(-I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[
3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*
x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticE[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3
])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] - (3*(-64
7147*I + 997633*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I +
Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[
3])*(1 + 2*x))]*EllipticF[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 +
2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] + 133*((45006*Sqrt[(-I)
/(I + Sqrt[3])]*(3 + 4*x^2))/(1 + 2*x)^2 + ((80645*I)*Sqrt[(3*I + Sqrt[3]
+ 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2
*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticPi[(13*(1 - I*Sqrt[3]
))/12, I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3]
)/(I - Sqrt[3])])/Sqrt[1 + 2*x])))/Sqrt[(-I)/(I + Sqrt[3])]))/(331695*Sqrt
[3 + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-2x^2+6x+4\right)\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\left(4x^2+3\right)^{3/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{84}\int -\frac{8(75-106x)\sqrt{4x^2+3}}{3\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{63}\int \frac{(75-106x)\sqrt{4x^2+3}}{\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{682}$$

$$-\frac{2}{63}\left(\frac{1}{60}\int \frac{32(321-532x)}{\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{1}{15}\sqrt{2x+1}\sqrt{4x^2+3}(587-318x)\right)+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{63}\left(\frac{8}{15}\int \frac{321-532x}{\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{1}{15}\sqrt{2x+1}\sqrt{4x^2+3}(587-318x)\right)+\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{599}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{63}\left(\frac{1}{15}(587-318x)\sqrt{2x+1}\sqrt{4x^2+3}-\frac{4}{15}\int -\frac{2(587-266(2x+1))}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{63}\left(\frac{8}{15}\int \frac{587-266(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{1}{15}\sqrt{2x+1}\sqrt{4x^2+3}(587-318x)\right)-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{744}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{63}\left(\frac{8}{15}\int \frac{587-266(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{1}{15}\sqrt{2x+1}\sqrt{4x^2+3}(587-318x)\right)-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$$\Big\downarrow \text{1511}$$

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{63}\left(\frac{8}{15}\left(55\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+532\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{1}{15}\sqrt{\right.$$
$$\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$\Big\downarrow$ 27

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{63}\left(\frac{8}{15}\left(55\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+266\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{1}{15}\sqrt{2\right.$$
$$\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$\Big\downarrow$ 1416

$$-\frac{2}{63}\left(\frac{8}{15}\left(266\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{55(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\right.\right.}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.$$
$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

$\Big\downarrow$ 1509

$$\frac{76}{9}\int \frac{\left(4x^2+3\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-$$
$$\frac{2}{63}\left(\frac{8}{15}\left(\frac{55(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+266\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)-}{(2x+3)^2}}}{\sqrt{(2x+1)^2-2}}\right.\right.\right.$$
$$\frac{2}{189}(16-7x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}$$

input
```
Int[((4 + 6*x - 2*x^2)*(3 + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.51

method	result
risch	$\frac{2(1260x^3-2880x^2-5769x-27659)\sqrt{4x^2+3}\sqrt{1+2x}}{8505}+\frac{2\left(-\frac{7795216\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+}{\frac{1}{2}-i}}\right.}{25515\sqrt{8x^3+4x^2+6x+3}}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(\frac{8x^3\sqrt{8x^3+4x^2+6x+3}}{27}-\frac{128x^2\sqrt{8x^3+4x^2+6x+3}}{189}-\frac{1282x\sqrt{8x^3+4x^2+6x+3}}{945}-\frac{55318\sqrt{8x^3+4x^2+6x+3}}{8505}-\frac{15590432\left(\frac{1}{2}-i\right.}{}\right.}{}$
default	$\frac{2\sqrt{4x^2+3}\sqrt{1+2x}\left(38697308i\sqrt{3}\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x}{1+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x}{-1+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}},\sqrt{-\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}}\right)-42903140i\sqrt{3}\sqrt{-\frac{1+}{-1-}}\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/8505*(1260*x^3-2880*x^2-5769*x-27659)*(4*x^2+3)^(1/2)*(1+2*x)^(1/2)+2*(-
7795216/25515*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-
1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*
3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^
(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))-175408/12
15*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/
2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(
1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*((-1/2-1/2*I*3^(1/2))*EllipticE(((x+1/2)/(1
/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)
)+1/2*I*3^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I
*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))+2451608/9477*(1/2-1/2*I*3^(1/2))*(
(x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2))
)^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)
^(1/2)*EllipticPi(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2),
((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))*((4*x^2+3)*(1+2*x))^(1
/2)/(4*x^2+3)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="fricas")
```

output
```
integral(2*(4*x^4 - 12*x^3 - 5*x^2 - 9*x - 6)*sqrt(4*x^2 + 3)*sqrt(2*x + 1
)/(6*x^2 - 7*x - 5), x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{6\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{9x\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{5x^2\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{12x^3\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{4x^4\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+3)**(3/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-6*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
+ Integral(-9*x*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
 + Integral(-5*x**2*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1))
, x) + Integral(-12*x**3*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x
+ 1)), x) + Integral(4*x**4*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2
*x + 1)), x))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="maxima")
```

output
```
2*integrate((4*x^2 + 3)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x
)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+3)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate(2*(4*x^2 + 3)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(4\,x^2+3)^{3/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((4*x^2 + 3)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((4*x^2 + 3)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)(3+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)\,(4x^2+3)^{\frac{3}{2}}}{(5-3x)\,\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.10 $\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 302

$$\begin{aligned}
&\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx \\
&= -\frac{2}{135}(26-9x)\sqrt{1+2x}\sqrt{3+4x^2}-\frac{602\sqrt{1+2x}\sqrt{3+4x^2}}{135(3+2x)} \\
&\quad +\frac{76}{27}\sqrt{\frac{127}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right) \\
&\quad +\frac{602\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{135\sqrt{3+4x^2}} \\
&\quad -\frac{85\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{27\sqrt{3+4x^2}} \\
&\quad -\frac{889\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{1053\sqrt{3+4x^2}}
\end{aligned}$$

output
```
-2/135*(26-9*x)*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)-602*(1+2*x)^(1/2)*(4*x^2+3)^
(1/2)/(405+270*x)+76/1053*4953^(1/2)*arctanh(1/39*4953^(1/2)*(1+2*x)^(1/2)
/(4*x^2+3)^(1/2))+602/135*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*Elli
pticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2))/(4*x^2+3)^(1/2
)-85/27*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*InverseJacobiAM(2*arct
an(1/2*(1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(1/2)-889/1053*2^(1/2
)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi(sin(2*arctan(1/2*(1+2*x)^
(1/2)*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.00 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.17

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx$$

$$= \frac{\sqrt{1+2x}\left(78(-26+9x)(3+4x^2) - \frac{(1+2x)\left(23478\sqrt{-\frac{i}{i+\sqrt{3}}} + \frac{93912\sqrt{-\frac{i}{i+\sqrt{3}}}}{(1+2x)^2} - \frac{46956\sqrt{-\frac{i}{i+\sqrt{3}}}}{1+2x} + \frac{11739\left(-i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}+2\left(-i+\cdots\right.}{\left(-i+\sqrt{3}\right)(1+\cdots}}}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*Sqrt[3 + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
(Sqrt[1 + 2*x]*(78*(-26 + 9*x)*(3 + 4*x^2) - ((1 + 2*x)*(23478*Sqrt[(-I)/(
I + Sqrt[3])] + (93912*Sqrt[(-I)/(I + Sqrt[3])])/(1 + 2*x)^2 - (46956*Sqrt
[(-I)/(I + Sqrt[3])])/(1 + 2*x) + (11739*(-I + Sqrt[3])*Sqrt[(3*I + Sqrt[3
] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] +
 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticE[I*ArcSinh[(2*Sqrt
[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1
 + 2*x] - (3*(-2057*I + 3913*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3
])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x
)/((I + Sqrt[3])*(1 + 2*x))]*EllipticF[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3]
)])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] + ((48260*
I)*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*S
qrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*Ellipt
icPi[(13*(1 - I*Sqrt[3]))/12, I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[
1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x]))/Sqrt[(-I)/(I + Sq
rt[3])]))/(5265*Sqrt[3 + 4*x^2])
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 683 vs. $2(302) = 604$.

Time = 1.09 (sec) , antiderivative size = 683, normalized size of antiderivative = 2.26, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {2349, 682, 27, 599, 27, 724, 27, 599, 25, 729, 1511, 27, 1416, 1509, 1540, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\sqrt{4x^2+3}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{60}\int -\frac{8(129-158x)}{9\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{135}\int \frac{129-158x}{\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 599$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx+\frac{1}{135}\int -\frac{2(208-79(2x+1))}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 724$$

$$\frac{76}{9}\left(\frac{127}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{1}{9}\int \frac{4(3x+5)}{\sqrt{2x+1}\sqrt{4x^2+3}}dx\right)-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\left(\frac{127}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{4}{9}\int \frac{3x+5}{\sqrt{2x+1}\sqrt{4x^2+3}}dx\right)-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 599$$

$$\frac{76}{9}\left(\frac{127}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx+\frac{2}{9}\int -\frac{3(2x+1)+7}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 25$$

$$\frac{76}{9}\left(\frac{127}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{2}{9}\int \frac{3(2x+1)+7}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \; 729$$

$$-\frac{2}{135}\int \frac{208-79(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+$$
$$\frac{76}{9}\left(\frac{254}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{2}{9}\int \frac{3(2x+1)+7}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+\right.$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{1511}$$

$$-\frac{2}{135}\left(50\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+158\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+$$
$$\frac{76}{9}\left(\frac{2}{9}\left(6\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-13\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{254}{9}\int \frac{}{(}\right.$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{2}{135}\left(50\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+79\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+$$
$$\frac{76}{9}\left(\frac{2}{9}\left(3\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-13\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{254}{9}\int \frac{}{(1}\right.$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{1416}$$

$$-\frac{2}{135}\left(79\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+}}{\sqrt{2}}\right.\right.}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.$$
$$\frac{76}{9}\left(\frac{2}{9}\left(3\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{13(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right.\right.}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.$$
$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{1509}$$

$$\frac{76}{9}\left(\frac{254}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{2}{9}\left(3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x+\right.}\right.\right.\right.$$

$$\frac{2}{135}\left(\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+79\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x-\right.}\right.\right.$$

$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \ 1540$$

$$\frac{76}{9}\left(\frac{254}{9}\left(\frac{1}{19}\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{6}{19}\int \frac{2x+3}{2(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\right.\right.$$

$$\frac{2}{135}\left(\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+79\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x-\right.}\right.\right.$$

$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \ 27$$

$$\frac{76}{9}\left(\frac{254}{9}\left(\frac{1}{19}\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{3}{19}\int \frac{2x+3}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{\right.\right.$$

$$\frac{2}{135}\left(\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+79\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x-\right.}\right.\right.$$

$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \ 1416$$

$$\frac{76}{9}\left(\frac{254}{9}\left(\frac{3}{19}\int \frac{2x+3}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{Elliptic}}{38\sqrt{2}\sqrt{(2x+1)^2-2(2}\right.\right.$$

$$\frac{2}{135}\left(\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+79\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x-\right.}\right.\right.$$

$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

$$\Big\downarrow \text{2222}$$

$$\frac{76}{9}\left(\frac{254}{9}\left(\frac{3}{19}\left(\frac{19\text{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{2x+1}}{\sqrt{(2x+1)^2-2(2x+1)+4}}\right)}{2\sqrt{4953}}-\frac{7(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{2x}}{\sqrt{}}\right.\right.}{156\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.\right.$$

$$\frac{2}{135}\left(\frac{25(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+79\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(\right.}{\sqrt{(2x+1)^2-2(2x}}\right.\right.$$

$$\frac{2}{135}\sqrt{2x+1}\sqrt{4x^2+3}(26-9x)$$

input

```
Int[((4 + 6*x - 2*x^2)*Sqrt[3 + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output

```
(-2*(26 - 9*x)*Sqrt[1 + 2*x]*Sqrt[3 + 4*x^2])/135 - (2*(79*(-((Sqrt[1 + 2*
x]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2])/(3 + 2*x)) + (Sqrt[2]*(3 + 2*x)*Sq
rt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*EllipticE[2*ArcTan[Sqrt[1
+ 2*x]/Sqrt[2]], 3/4])/Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2]) + (25*(3 + 2*x
)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*EllipticF[2*ArcTan[Sqr
t[1 + 2*x]/Sqrt[2]], 3/4])/(Sqrt[2]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2])))
/135 + (76*((2*(3*(-((Sqrt[1 + 2*x]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2])/(
3 + 2*x)) + (Sqrt[2]*(3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2
*x)^2]*EllipticE[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/Sqrt[4 - 2*(1 + 2*
x) + (1 + 2*x)^2]) - (13*(3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3
 + 2*x)^2]*EllipticF[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/(2*Sqrt[2]*Sqr
t[4 - 2*(1 + 2*x) + (1 + 2*x)^2])))/9 + (254*(((3 + 2*x)*Sqrt[(4 - 2*(1 +
2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*EllipticF[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]]
, 3/4])/(38*Sqrt[2]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2]) + (3*((19*ArcTanh
[(Sqrt[127/39]*Sqrt[1 + 2*x])/Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2]])/(2*Sqr
t[4953]) - (7*(3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*
EllipticPi[361/312, 2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/(156*Sqrt[2]*Sq
rt[4 - 2*(1 + 2*x) + (1 + 2*x)^2])))/19))/9))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 724
```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)
]), x_Symbol] :> Simp[(c*d^2 + a*e^2)/e^2   Int[1/((d + e*x)*Sqrt[f + g*x]*
Sqrt[a + c*x^2]), x], x] - Simp[1/e^2   Int[(c*d - c*e*x)/(Sqrt[f + g*x]*Sq
rt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349

```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.62

method	result
risch	$\frac{2(-26+9x)\sqrt{4x^2+3}\sqrt{1+2x}}{135}+\frac{2\left(-\frac{8374\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\right)}{405\sqrt{8x^3+4x^2+6x+3}}-\frac{1204\left(\frac{1}{2}\right.}{}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(\frac{2x\sqrt{8x^3+4x^2+6x+3}}{15}-\frac{52\sqrt{8x^3+4x^2+6x+3}}{135}-\frac{16748\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\right.}{405\sqrt{8x^3+4x^2+6x+3}}\right.}{}$
default	$\frac{2\sqrt{4x^2+3}\sqrt{1+2x}\left(42692i\sqrt{3}\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x}{1+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}+2x}{-1+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}},\sqrt{-\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}}\right)-48260i\sqrt{3}\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\right.}{}$

input

```
int((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2/135*(-26+9*x)*(4*x^2+3)^(1/2)*(1+2*x)^(1/2)+2*(-8374/405*(1/2-1/2*I*3^(1
/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^
(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+
6*x+3)^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^
(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))-1204/135*(1/2-1/2*I*3^(1/2))*((x+1/2)/
(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*
((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*(
(-1/2-1/2*I*3^(1/2))*EllipticE(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+
1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*3^(1/2)*EllipticF(((x+1/
2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^
(1/2)))+19304/1053*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)
*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1
/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*EllipticPi(((x+1/2)/(1/2-1/
2*I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*
3^(1/2)))^(1/2)))*((4*x^2+3)*(1+2*x))^(1/2)/(4*x^2+3)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4\,x^2+3}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5)
, x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{2\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{3x\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{x^2\sqrt{4x^2+3}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+3)**(1/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-2*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
+ Integral(-3*x*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x)
 + Integral(x**2*sqrt(4*x**2 + 3)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x
))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4x^2+3}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="maxima")
```

output
```
2*integrate(sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int \frac{2\sqrt{4x^2+3}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algorith
m="giac")
```

output
```
integrate(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int -\frac{\sqrt{4x^2+3}\,(-2x^2+6x+4)}{\sqrt{2x+1}\,(3x-5)}\,dx$$

input
```
int(-((4*x^2 + 3)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)),x)
```

output
```
int(-((4*x^2 + 3)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)\sqrt{3+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int \frac{(-2x^2+6x+4)\sqrt{4x^2+3}}{(5-3x)\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+3)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.11 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}} dx$

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Optimal result

Integrand size = 38, antiderivative size = 271

$$\begin{aligned}
\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx
&= \frac{\sqrt{1+2x}\sqrt{3+4x^2}}{3(3+2x)} + \frac{76\text{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right)}{3\sqrt{4953}} \\
&\quad - \frac{\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{3\sqrt{3+4x^2}} \\
&\quad + \frac{(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{6\sqrt{2}\sqrt{3+4x^2}} \\
&\quad - \frac{7\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\,\text{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{117\sqrt{3+4x^2}}
\end{aligned}$$

output
```
(1+2*x)^(1/2)*(4*x^2+3)^(1/2)/(9+6*x)+76/14859*4953^(1/2)*arctanh(1/39*495
3^(1/2)*(1+2*x)^(1/2)/(4*x^2+3)^(1/2))-1/3*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2
*x)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2
))/(4*x^2+3)^(1/2)+1/12*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(1/2)
-7/117*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi(sin(2*arctan
(1/2*(1+2*x)^(1/2)*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.87 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.33

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx$$

$$= \frac{(3+6x)^{3/2}\left(78\sqrt{-\frac{i}{i+\sqrt{3}}}+\frac{312\sqrt{-\frac{i}{i+\sqrt{3}}}}{(1+2x)^2}-\frac{156\sqrt{-\frac{i}{i+\sqrt{3}}}}{1+2x}+\frac{39\left(-i+\sqrt{3}\right)\sqrt{\frac{3i+\sqrt{3}+2\left(-i+\sqrt{3}\right)x}{\left(-i+\sqrt{3}\right)(1+2x)}}\sqrt{\frac{-3i+\sqrt{3}+2\left(i+\sqrt{3}\right)x}{\left(i+\sqrt{3}\right)(1+2x)}}E\left(i\text{arcsi}\right.}{\sqrt{1+2x}}\right.}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[3 + 4*x^2]),x]
```

output
```
((3 + 6*x)^(3/2)*(78*Sqrt[(-I)/(I + Sqrt[3])] + (312*Sqrt[(-I)/(I + Sqrt[3
])])/(1 + 2*x)^2 - (156*Sqrt[(-I)/(I + Sqrt[3])])/(1 + 2*x) + (39*(-I + Sq
rt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x)
)]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*El
lipticE[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3
])/(I - Sqrt[3])])/Sqrt[1 + 2*x] - (3*(-16*I + 13*Sqrt[3])*Sqrt[(3*I + Sqr
t[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3
] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticF[I*ArcSinh[(2*S
qrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqr
t[1 + 2*x] - ((152*I)*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqr
t[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])
*(1 + 2*x))]*EllipticPi[(13*(1 - I*Sqrt[3]))/12, I*ArcSinh[(2*Sqrt[(-I)/(I
 + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x])
)/(702*Sqrt[(-3*I)/(I + Sqrt[3])]*Sqrt[3 + 4*x^2])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.62, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2349, 599, 27, 729, 1511, 27, 1416, 1509, 1540, 27, 1416, 2222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\sqrt{4x^2+3}}dx$$

$$\Big\downarrow \text{599}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{1}{2}\int \frac{2(11-3(2x+1))}{9\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{1}{9}\int \frac{11-3(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{729}$$

$$\frac{152}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{1}{9}\int \frac{11-3(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{1511}$$

$$\frac{1}{9}\left(-5\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-6\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{152}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{9}\left(-5\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-3\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)+\frac{152}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{1416}$$

$$\frac{1}{9}\left(-3\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right)+\frac{152}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}$$

$$\Big\downarrow \text{1509}$$

$$\frac{152}{9}\int \frac{1}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{1}{9}\left(-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\text{ a}\right.}{\sqrt{(2x+1)^2-2(2x+1}}\right.\right.$$

$$\Big\downarrow \text{1540}$$

$$\frac{152}{9}\left(\frac{1}{19}\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{6}{19}\int \frac{2x+3}{2(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x\right.$$
$$\frac{1}{9}\left(-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\,a\right.}{\sqrt{(2x+1)^2-2(2x+1}}\right.\right.$$

$\downarrow$ 27

$$\frac{152}{9}\left(\frac{1}{19}\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{3}{19}\int \frac{2x+3}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+\right.$$
$$\frac{1}{9}\left(-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\,a\right.}{\sqrt{(2x+1)^2-2(2x+1}}\right.\right.$$

$\downarrow$ 1416

$$\frac{152}{9}\left(\frac{3}{19}\int \frac{2x+3}{(13-3(2x+1))\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}+\frac{(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\,a\right.}{38\sqrt{2}\sqrt{(2x+1)^2-2(2x+}}\right.$$
$$\frac{1}{9}\left(-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\,a\right.}{\sqrt{(2x+1)^2-2(2x+1}}\right.\right.$$

$\downarrow$ 2222

$$\frac{152}{9}\left(\frac{3}{19}\left(\frac{19\text{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{2x+1}}{\sqrt{(2x+1)^2-2(2x+1)+4}}\right)}{2\sqrt{4953}}-\frac{7(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\right.}{156\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.$$
$$\frac{1}{9}\left(-\frac{5(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-3\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\,a\right.}{\sqrt{(2x+1)^2-2(2x+1}}\right.\right.$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[3 + 4*x^2]),x]
```

output
```
(-3*(-((Sqrt[1 + 2*x]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2])/(3 + 2*x)) + (S
qrt[2]*(3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*Ellipti
cE[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)
^2]) - (5*(3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*Elli
pticF[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/(2*Sqrt[2]*Sqrt[4 - 2*(1 + 2*
x) + (1 + 2*x)^2]))/9 + (152*(((3 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)
^2)/(3 + 2*x)^2]*EllipticF[2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/(38*Sqrt
[2]*Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2]) + (3*((19*ArcTanh[(Sqrt[127/39]*S
qrt[1 + 2*x])/Sqrt[4 - 2*(1 + 2*x) + (1 + 2*x)^2]])/(2*Sqrt[4953]) - (7*(3
 + 2*x)*Sqrt[(4 - 2*(1 + 2*x) + (1 + 2*x)^2)/(3 + 2*x)^2]*EllipticPi[361/3
12, 2*ArcTan[Sqrt[1 + 2*x]/Sqrt[2]], 3/4])/(156*Sqrt[2]*Sqrt[4 - 2*(1 + 2*
x) + (1 + 2*x)^2])))/19))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2222
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTanh[Rt[b - c*(d/e) - a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
b - c*(d/e) - a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a +
 b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*Ell
ipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
/; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&
 EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && NegQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{1+2x}\,\sqrt{4x^2+3}\left(-1+i\sqrt{3}\right)\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x}{1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x}{-1+i\sqrt{3}}}\left(39i\sqrt{3}\,\text{EllipticF}\left(\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}},\sqrt{-\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}}\right)-39i\,\text{EllipticE}\left(\sqrt{\cdots}\right.\right.}{\cdots}$
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(-\frac{16\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\frac{x+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\text{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\right)}{9\sqrt{8x^3+4x^2+6x+3}}+\frac{4\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\,\sqrt{\cdots}}{\cdots}\right.}{\cdots}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/117*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)*(-1+I*3^(1/2))*(-(1+2*x)/(-1+I*3^(1/2
)))^(1/2)*((I*3^(1/2)-2*x)/(1+I*3^(1/2)))^(1/2)*((I*3^(1/2)+2*x)/(-1+I*3^(
1/2)))^(1/2)*(39*I*3^(1/2)*EllipticF((-(1+2*x)/(-1+I*3^(1/2)))^(1/2),(-(-1
+I*3^(1/2))/(1+I*3^(1/2)))^(1/2))-39*I*EllipticE((-(1+2*x)/(-1+I*3^(1/2)))
^(1/2),(-(-1+I*3^(1/2))/(1+I*3^(1/2)))^(1/2))*3^(1/2)-104*EllipticF((-(1+2
*x)/(-1+I*3^(1/2)))^(1/2),(-(-1+I*3^(1/2))/(1+I*3^(1/2)))^(1/2))-39*Ellipt
icE((-(1+2*x)/(-1+I*3^(1/2)))^(1/2),(-(-1+I*3^(1/2))/(1+I*3^(1/2)))^(1/2))
+152*EllipticPi((-(1+2*x)/(-1+I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2),(-(-1+
I*3^(1/2))/(1+I*3^(1/2)))^(1/2)))/(8*x^3+4*x^2+6*x+3)
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x, algorith
m="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(24*x^4 - 28*x^3
- 2*x^2 - 21*x - 15), x)
```

Sympy [F]

$$\begin{aligned}\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx &= 2\Bigg(\int\left(-\frac{3x}{3x\sqrt{2x+1}\sqrt{4x^2+3}-5\sqrt{2x+1}\sqrt{4x^2+3}}\right)dx \\ &\quad+\int\frac{x^2}{3x\sqrt{2x+1}\sqrt{4x^2+3}-5\sqrt{2x+1}\sqrt{4x^2+3}}\,dx \\ &\quad+\int\left(-\frac{2}{3x\sqrt{2x+1}\sqrt{4x^2+3}-5\sqrt{2x+1}\sqrt{4x^2+3}}\right)dx\Bigg)\end{aligned}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+3)**(1/2),x)
```

output
```
2*(Integral(-3*x/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 5*sqrt(2*x + 1)*sqr
t(4*x**2 + 3)), x) + Integral(x**2/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 5
*sqrt(2*x + 1)*sqrt(4*x**2 + 3)), x) + Integral(-2/(3*x*sqrt(2*x + 1)*sqrt
(4*x**2 + 3) - 5*sqrt(2*x + 1)*sqrt(4*x**2 + 3)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x, algorith
m="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/(sqrt(4*x^2 + 3)*(3*x - 5)*sqrt(2*x + 1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+3}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x, algorith
m="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/(sqrt(4*x^2 + 3)*(3*x - 5)*sqrt(2*x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,(3\,x-5)\,\sqrt{4\,x^2+3}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(1/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(1/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{3+4x^2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,\sqrt{4x^2+3}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(1/2),x)
```

3.12 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 302

$$\begin{aligned}\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx &= -\frac{(111-670x)\sqrt{1+2x}}{3048\sqrt{3+4x^2}} \\ &- \frac{335\sqrt{1+2x}\sqrt{3+4x^2}}{3048(3+2x)} + \frac{76}{127}\sqrt{\frac{3}{1651}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right) \\ &+ \frac{335(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{1524\sqrt{2}\sqrt{3+4x^2}} \\ &- \frac{(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{24\sqrt{2}\sqrt{3+4x^2}} \\ &- \frac{7\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{1651\sqrt{3+4x^2}}\end{aligned}$$

output
```
-1/3048*(111-670*x)*(1+2*x)^(1/2)/(4*x^2+3)^(1/2)-335*(1+2*x)^(1/2)*(4*x^2
+3)^(1/2)/(9144+6096*x)+76/209677*4953^(1/2)*arctanh(1/39*4953^(1/2)*(1+2*
x)^(1/2)/(4*x^2+3)^(1/2))+335/3048*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(
1/2)*EllipticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2))/(4*x^
2+3)^(1/2)-1/48*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*InverseJacobiA
M(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(1/2)-7/1651*
2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi(sin(2*arctan(1/2*(1
+2*x)^(1/2)*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.21 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.18

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \frac{\sqrt{1+2x}\left(78\sqrt{3}(-111+670x) - \frac{3(1+2x)\left(8710\sqrt{-\frac{i}{i+\sqrt{3}}}+\frac{34840\sqrt{-\frac{i}{i+\sqrt{3}}}}{(1+2x)^2}-\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + 2*x]*(78*Sqrt[3]*(-111 + 670*x) - (3*(1 + 2*x)*(8710*Sqrt[(-I)/(
I + Sqrt[3])] + (34840*Sqrt[(-I)/(I + Sqrt[3])])/(1 + 2*x)^2 - (17420*Sqrt
[(-I)/(I + Sqrt[3])])/(1 + 2*x) + (4355*(-I + Sqrt[3])*Sqrt[(3*I + Sqrt[3]
 + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] +
2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticE[I*ArcSinh[(2*Sqrt[
(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1
+ 2*x] - (5*(441*I + 871*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x
)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((
I + Sqrt[3])*(1 + 2*x))]*EllipticF[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/
Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] + ((3648*I)*Sq
rt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(
-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticPi[
(13*(1 - I*Sqrt[3]))/12, I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2
*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x]))/Sqrt[(-1/3*I)/(I + Sqr
t[3])]))/(237744*Sqrt[3]*Sqrt[3 + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx$$

$$\Big\downarrow \text{686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx-\frac{1}{192}\int \frac{8(2x+33)}{9\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx-\frac{1}{216}\int \frac{2x+33}{\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \text{599}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx+\frac{1}{432}\int -\frac{2(2x+33)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx-\frac{1}{216}\int \frac{2x+33}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 744$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx-\frac{1}{216}\int \frac{2x+33}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 1511$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx+\frac{1}{216}\left(2\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-34\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx+\frac{1}{216}\left(\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-34\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 1416$$

$$\frac{1}{216}\left(\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{17(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right.}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.$$
$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{3/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

$$\Big\downarrow \; 1509$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx+$$

$$\frac{1}{216}\left(-\frac{17(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}+\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\Big(2\,\text{a}}{\sqrt{(2x+1)^2-2(2x+1}}\right.$$

$$\frac{\sqrt{2x+1}(33-2x)}{216\sqrt{4x^2+3}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(-\frac{2(4+8x)\left(\frac{37}{8128}-\frac{335x}{12192}\right)}{\sqrt{\left(x^2+\frac{3}{4}\right)(4+8x)}}-\frac{37\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{-\frac{1}{2}+\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\right)}{508\sqrt{8x^3+4x^2+6x+3}}-\right.}{}$
default	$-\frac{\sqrt{1+2x}\,\sqrt{4x^2+3}\left(2912i\sqrt{3}\,\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}-2x}{1+i\sqrt{3}}}\,\sqrt{\frac{i\sqrt{3}+2x}{-1+i\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}},\sqrt{-\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}}\right)+3648i\sqrt{3}\,\sqrt{-\frac{1+2x}{-1+i\sqrt{3}}}\right.}{}$

input

```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
((4*x^2+3)*(1+2*x))^(1/2)/(4*x^2+3)^(1/2)/(1+2*x)^(1/2)*(-2*(4+8*x)*(37/81
28-335/12192*x)/((x^2+3/4)*(4+8*x))^(1/2)-37/508*(1/2-1/2*I*3^(1/2))*((x+1
/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1
/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/
2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1
/2-1/2*I*3^(1/2)))^(1/2))-335/762*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*
3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3
^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*((-1/2-1/2*I
*3^(1/2))*EllipticE(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/
2))/(-1/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*3^(1/2)*EllipticF(((x+1/2)/(1/2-1/2
*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))+304
/1651*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^
(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2))
)^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*EllipticPi(((x+1/2)/(1/2-1/2*I*3^(1/2)))
^(1/2),3/13-3/13*I*3^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/
2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input

```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x, algorith
m="fricas")
```

output

```
integral(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(96*x^6 - 112*x^5
 + 64*x^4 - 168*x^3 - 66*x^2 - 63*x - 45), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = 2\left(\int \left(-\frac{3x}{12x^3\sqrt{2x+1}\sqrt{4x^2+3}-20x^2\sqrt{2x+1}\sqrt{4x^2+3}+9x\sqrt{2x}\right.\right.$$
$$+\int \frac{x^2}{12x^3\sqrt{2x+1}\sqrt{4x^2+3}-20x^2\sqrt{2x+1}\sqrt{4x^2+3}+9x\sqrt{2x+1}\sqrt{4x^2+3}-15\sqrt{2x+1}\sqrt{4x^2+3}}\,dx$$
$$\left.+\int \left(-\frac{2}{12x^3\sqrt{2x+1}\sqrt{4x^2+3}-20x^2\sqrt{2x+1}\sqrt{4x^2+3}+9x\sqrt{2x+1}\sqrt{4x^2+3}-15\sqrt{2x+1}\sqrt{4x^2+3}}\right)\right.$$

input

```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+3)**(3/2),x)
```

output

```
2*(Integral(-3*x/(12*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 20*x**2*sqrt(2*
x + 1)*sqrt(4*x**2 + 3) + 9*x*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 15*sqrt(2*x
 + 1)*sqrt(4*x**2 + 3)), x) + Integral(x**2/(12*x**3*sqrt(2*x + 1)*sqrt(4*
x**2 + 3) - 20*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 3) + 9*x*sqrt(2*x + 1)*sqr
t(4*x**2 + 3) - 15*sqrt(2*x + 1)*sqrt(4*x**2 + 3)), x) + Integral(-2/(12*x
**3*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 20*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 3
) + 9*x*sqrt(2*x + 1)*sqrt(4*x**2 + 3) - 15*sqrt(2*x + 1)*sqrt(4*x**2 + 3)
), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x, algorith
m="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 3)^(3/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x, algorith
m="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 3)^(3/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,{\left(4\,x^2+3\right)}^{3/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(3/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(3/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{3/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+3)^{\frac{3}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(3/2),x)
```

3.13 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(3+4x^2\right)^{5/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 331

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(3+4x^2\right)^{5/2}}\,dx = -\frac{(111-670x)\sqrt{1+2x}}{9144\left(3+4x^2\right)^{3/2}}+\frac{\sqrt{1+2x}(36501+345268x)}{6967728\sqrt{3+4x^2}} - \frac{86317\sqrt{1+2x}\sqrt{3+4x^2}}{3483864(3+2x)}+\frac{684\sqrt{\frac{3}{1651}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right)}{16129} + \frac{86317(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{1741932\sqrt{2}\sqrt{3+4x^2}} - \frac{827(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{109728\sqrt{2}\sqrt{3+4x^2}} - \frac{63\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\operatorname{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{209677\sqrt{3+4x^2}}$$

output
```
-1/9144*(111-670*x)*(1+2*x)^(1/2)/(4*x^2+3)^(3/2)+1/6967728*(1+2*x)^(1/2)*
(36501+345268*x)/(4*x^2+3)^(1/2)-86317*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)/(1045
1592+6967728*x)+684/26628979*4953^(1/2)*arctanh(1/39*4953^(1/2)*(1+2*x)^(1
/2)/(4*x^2+3)^(1/2))+86317/3483864*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(
1/2)*EllipticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),1/2*3^(1/2))/(4*x^
2+3)^(1/2)-827/219456*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*InverseJ
acobiAM(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2)),1/2*3^(1/2))/(4*x^2+3)^(1/2)-6
3/209677*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticPi(sin(2*arct
an(1/2*(1+2*x)^(1/2)*2^(1/2))),361/312,1/2*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.10 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.88

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \frac{\sqrt{1+2x}\left(26(24921+1546344x+146004x^2+1381072x^3)+\frac{(1+2x)\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(5/2)),x]
```

output
```
(Sqrt[1 + 2*x]*(26*(24921 + 1546344*x + 146004*x^2 + 1381072*x^3) + ((1 +
2*x)*(3 + 4*x^2)*((-2244242*(-I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + S
qrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[
3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticE[I*ArcSinh[(2*Sqrt[(-I)/(I + Sq
rt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] + ((1
065489*I + 2244242*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I
 + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sq
rt[3])*(1 + 2*x))]*EllipticF[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1
 + 2*x]], (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] - 76*((59059*Sqrt[(-
I)/(I + Sqrt[3])]*(3 + 4*x^2))/(1 + 2*x)^2 + ((7776*I)*Sqrt[(3*I + Sqrt[3]
 + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] +
2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticPi[(13*(1 - I*Sqrt[3
]))/12, I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I + Sqrt[3
])/(I - Sqrt[3])])/Sqrt[1 + 2*x])))/Sqrt[(-I)/(I + Sqrt[3])]))/(181160928*
(3 + 4*x^2)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx$$

$$\Big\downarrow \; 2349$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx$$

$$\Big\downarrow \; 686$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx-\frac{1}{576}\int \frac{8(161-6x)}{9\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{648\,(4x^2+3)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx-\frac{1}{648}\int \frac{161-6x}{\sqrt{2x+1}\,(4x^2+3)^{3/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{648\,(4x^2+3)^{3/2}}$$

$$\Big\downarrow \; 686$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+\frac{1}{648}\left(\frac{1}{192}\int -\frac{32(123-76x)}{\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(76x+123)}{6\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+\frac{1}{648}\left(-\frac{1}{6}\int \frac{123-76x}{\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(76x+123)}{6\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 599$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+\frac{1}{648}\left(\frac{1}{12}\int -\frac{2(161-38(2x+1))}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(76x+123)}{6\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+\frac{1}{648}\left(-\frac{1}{6}\int \frac{161-38(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(76x+123)}{6\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 744$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+\frac{1}{648}\left(-\frac{1}{6}\int \frac{161-38(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(76x+123)}{6\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$$\Big\downarrow \; 1511$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+$$
$$\frac{1}{648}\left(\frac{1}{6}\left(-85\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-76\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x}\cdots}{\cdots}\right.$$
$$\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+$$
$$\frac{1}{648}\left(\frac{1}{6}\left(-85\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-38\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x}\cdots}{\cdots}\right.$$
$$\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$\Big\downarrow$ 1416

$$\frac{1}{648}\left(\frac{1}{6}\left(-38\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{85(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.$$
$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

$\Big\downarrow$ 1509

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{5/2}}dx+$$
$$\frac{1}{648}\left(\frac{1}{6}\left(-\frac{85(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{2\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-38\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)\cdots}{(2x+3)^2}}\cdots}{\sqrt{(2x+1)^2-\cdots}}\right.\right.\right.$$
$$\frac{\sqrt{2x+1}(33-2x)}{648\left(4x^2+3\right)^{3/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.57

method	result
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(\frac{\left(-\frac{37}{48768}+\frac{335x}{73152}\right)\sqrt{8x^3+4x^2+6x+3}}{\left(x^2+\frac{3}{4}\right)^2}-\frac{2(4+8x)\left(-\frac{12167}{18580608}-\frac{86317x}{13935456}\right)}{\sqrt{\left(x^2+\frac{3}{4}\right)(4+8x)}}+\frac{12167\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x+\frac{1}{2}}{\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{i\sqrt{3}}{2}}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\cdots}}{1161288\sqrt{8\cdots}}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
((4*x^2+3)*(1+2*x))^(1/2)/(4*x^2+3)^(1/2)/(1+2*x)^(1/2)*((-37/48768+335/73
152*x)*(8*x^3+4*x^2+6*x+3)^(1/2)/(x^2+3/4)^2-2*(4+8*x)*(-12167/18580608-86
317/13935456*x)/((x^2+3/4)*(4+8*x))^(1/2)+12167/1161288*(1/2-1/2*I*3^(1/2)
)*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/
2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x
+3)^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/
2))/(-1/2-1/2*I*3^(1/2)))^(1/2))-86317/870966*(1/2-1/2*I*3^(1/2))*((x+1/2)
/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)
*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*
((-1/2-1/2*I*3^(1/2))*EllipticE(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2
+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))+1/2*I*3^(1/2)*EllipticF(((x+1
/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))
^(1/2)))+2736/209677*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/
2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2
+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*EllipticPi(((x+1/2)/(1/2-
1/2*I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*
I*3^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x, algorith
m="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(384*x^8 - 448*x^
7 + 544*x^6 - 1008*x^5 - 72*x^4 - 756*x^3 - 378*x^2 - 189*x - 135), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+3)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x, algorith
m="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 3)^(5/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x, algorith
m="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 3)^(5/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+3\right)^{5/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(5/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(5/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{5/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,\left(4x^2+3\right)^{\frac{5}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(5/2),x)
```

3.14 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 360

$$\begin{aligned}\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx &= -\frac{(111-670x)\sqrt{1+2x}}{15240\,(3+4x^2)^{5/2}} \\ &+ \frac{\sqrt{1+2x}(16257+363373x)}{17419320\,(3+4x^2)^{3/2}} + \frac{\sqrt{1+2x}(262436763+1353435866x)}{106188174720\sqrt{3+4x^2}} \\ &- \frac{676717933\sqrt{1+2x}\sqrt{3+4x^2}}{106188174720(3+2x)} + \frac{6156\sqrt{\frac{3}{1651}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{127}{39}}\sqrt{1+2x}}{\sqrt{3+4x^2}}\right)}{2048383} \\ &+ \frac{676717933(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}E\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right)|\frac{3}{4}\right)}{53094087360\sqrt{2}\sqrt{3+4x^2}} \\ &- \frac{299755(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{167225472\sqrt{2}\sqrt{3+4x^2}} \\ &- \frac{567\sqrt{2}(3+2x)\sqrt{\frac{3+4x^2}{(3+2x)^2}}\,\text{EllipticPi}\left(\frac{361}{312},2\arctan\left(\frac{\sqrt{1+2x}}{\sqrt{2}}\right),\frac{3}{4}\right)}{26628979\sqrt{3+4x^2}}\end{aligned}$$

output
```
-1/15240*(111-670*x)*(1+2*x)^(1/2)/(4*x^2+3)^(5/2)+1/17419320*(1+2*x)^(1/2
)*(16257+363373*x)/(4*x^2+3)^(3/2)+1/106188174720*(1+2*x)^(1/2)*(262436763
+1353435866*x)/(4*x^2+3)^(1/2)-676717933*(1+2*x)^(1/2)*(4*x^2+3)^(1/2)/(31
8564524160+212376349440*x)+6156/3381880333*4953^(1/2)*arctanh(1/39*4953^(1
/2)*(1+2*x)^(1/2)/(4*x^2+3)^(1/2))+676717933/106188174720*2^(1/2)*(3+2*x)*
((4*x^2+3)/(3+2*x)^2)^(1/2)*EllipticE(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/
2))),1/2*3^(1/2))/(4*x^2+3)^(1/2)-299755/334450944*2^(1/2)*(3+2*x)*((4*x^2
+3)/(3+2*x)^2)^(1/2)*InverseJacobiAM(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2)),1
/2*3^(1/2))/(4*x^2+3)^(1/2)-567/26628979*2^(1/2)*(3+2*x)*((4*x^2+3)/(3+2*x
)^2)^(1/2)*EllipticPi(sin(2*arctan(1/2*(1+2*x)^(1/2)*2^(1/2))),361/312,1/2
*3^(1/2))/(4*x^2+3)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.00 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.79

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \frac{\sqrt{1+2x}\left(26(1885821075+23494665978x+6694893000x^2+4134\right.}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(7/2)),x]
```

output
```
(Sqrt[1 + 2*x]*(26*(1885821075 + 23494665978*x + 6694893000*x^2 + 41342948
016*x^3 + 4198988208*x^4 + 21654973856*x^5) - ((1 + 2*x)*(3 + 4*x^2)^2*((1
7594666258*Sqrt[(-I)/(I + Sqrt[3])]*(3 + 4*x^2))/(1 + 2*x)^2 + (8797333129
*(3 - I*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])
*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 +
 2*x))]*EllipticE[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (
I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[3 + 6*x] - (I*Sqrt[3]*(-8797333129*I + 1
349977333*Sqrt[3])*Sqrt[(3*I + Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3
])*(1 + 2*x))]*Sqrt[(-3*I + Sqrt[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1
 + 2*x))]*EllipticF[I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]],
 (I + Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x] + ((638254080*I)*Sqrt[(3*I +
Sqrt[3] + 2*(-I + Sqrt[3])*x)/((-I + Sqrt[3])*(1 + 2*x))]*Sqrt[(-3*I + Sqr
t[3] + 2*(I + Sqrt[3])*x)/((I + Sqrt[3])*(1 + 2*x))]*EllipticPi[(13*(1 - I
*Sqrt[3]))/12, I*ArcSinh[(2*Sqrt[(-I)/(I + Sqrt[3])])/Sqrt[1 + 2*x]], (I +
 Sqrt[3])/(I - Sqrt[3])])/Sqrt[1 + 2*x]))/Sqrt[(-I)/(I + Sqrt[3])]))/(2760
892542720*(3 + 4*x^2)^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 2349}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{7/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+3)^{7/2}}dx$$

$$\Big\downarrow \text{ 686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{7/2}}dx-\frac{1}{960}\int \frac{8(289-14x)}{9\sqrt{2x+1}\,(4x^2+3)^{5/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{1080\,(4x^2+3)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{7/2}}dx-\frac{\int \frac{289-14x}{\sqrt{2x+1}(4x^2+3)^{5/2}}dx}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\,(4x^2+3)^{5/2}}$$

$$\Big\downarrow \text{ 686}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{576}\int -\frac{64(201x+689)}{\sqrt{2x+1}(4x^2+3)^{3/2}}dx-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{-\frac{1}{9}\int \frac{201x+689}{\sqrt{2x+1}(4x^2+3)^{3/2}}dx-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \; 686$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(\frac{1}{192}\int -\frac{4(3531-3962x)}{\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(-\frac{1}{48}\int \frac{3531-3962x}{\sqrt{2x+1}\sqrt{4x^2+3}}dx-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \; 599$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(\frac{1}{96}\int -\frac{2(5512-1981(2x+1))}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(-\frac{1}{48}\int \frac{5512-1981(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{744}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(-\frac{1}{48}\int \frac{5512-1981(2x+1)}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\frac{\sqrt{2x+1}(67x+111)}{9(4x^2+3)^{3/2}}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{1511}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(\frac{1}{48}\left(-1550\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-3962\int \frac{1-2x}{2\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\sqrt{\ldots}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx+\frac{\frac{1}{9}\left(\frac{1}{48}\left(-1550\int \frac{1}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-1981\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}\right)-\frac{\sqrt{2x+1}(3962x+3531)}{48\sqrt{4x^2+3}}\right)-\sqrt{\ldots}}{1080}-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{\frac{1}{9}\left(\frac{1}{48}\left(-1981\int \frac{1-2x}{\sqrt{(2x+1)^2-2(2x+1)+4}}d\sqrt{2x+1}-\frac{775(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}\right)-\frac{\sqrt{2x+1}(\ldots}{48\ldots}\right.}{1080}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+3\right)^{7/2}}dx-\frac{\sqrt{2x+1}(33-2x)}{1080\left(4x^2+3\right)^{5/2}}$$

$$\Big\downarrow \text{1509}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+3)^{7/2}}dx+$$

$$\frac{\frac{1}{9}\left(\frac{1}{48}\left(-\frac{775(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right),\frac{3}{4}\right)}{\sqrt{2}\sqrt{(2x+1)^2-2(2x+1)+4}}-1981\left(\frac{\sqrt{2}(2x+3)\sqrt{\frac{(2x+1)^2-2(2x+1)+4}{(2x+3)^2}}E\left(2\arctan\left(\frac{\sqrt{2x+1}}{\sqrt{2}}\right.\right.}{\sqrt{(2x+1)^2-2(2x+1)+4}}\right.\right.\right.}{1080}$$

$$\frac{\sqrt{2x+1}(33-2x)}{1080\,(4x^2+3)^{5/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(3 + 4*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.53

method	result
elliptic	$\frac{\sqrt{(4x^2+3)(1+2x)}\left(\frac{\left(-\frac{37}{325120}+\frac{67x}{97536}\right)\sqrt{8x^3+4x^2+6x+3}}{\left(x^2+\frac{3}{4}\right)^3}+\frac{\left(\frac{5419}{92903040}+\frac{363373x}{278709120}\right)\sqrt{8x^3+4x^2+6x+3}}{\left(x^2+\frac{3}{4}\right)^2}-\frac{2(4+8x)\left(-\frac{87478921}{283168465920}-\frac{67671}{42475}\right.}{\sqrt{\left(x^2+\frac{3}{4}\right)(4+8x)}}\right.}{}$
default	Expression too large to display

input

```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x,method=_RETURNV
ERBOSE)
```

output

```
((4*x^2+3)*(1+2*x))^(1/2)/(4*x^2+3)^(1/2)/(1+2*x)^(1/2)*((-37/325120+67/97
536*x)*(8*x^3+4*x^2+6*x+3)^(1/2)/(x^2+3/4)^3+(5419/92903040+363373/2787091
20*x)*(8*x^3+4*x^2+6*x+3)^(1/2)/(x^2+3/4)^2-2*(4+8*x)*(-87478921/283168465
920-676717933/424752698880*x)/((x^2+3/4)*(4+8*x))^(1/2)+87478921/176980291
20*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/
2))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(
1/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/
2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))-676717933/2654704368
0*(1/2-1/2*I*3^(1/2))*((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2
))/(-1/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1
/2)/(8*x^3+4*x^2+6*x+3)^(1/2)*((-1/2-1/2*I*3^(1/2))*EllipticE(((x+1/2)/(1/
2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2))
+1/2*I*3^(1/2)*EllipticF(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),((-1/2+1/2*I*
3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))+24624/26628979*(1/2-1/2*I*3^(1/2))*
((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)
))^(1/2)*((x+1/2*I*3^(1/2))/(-1/2+1/2*I*3^(1/2)))^(1/2)/(8*x^3+4*x^2+6*x+3
)^(1/2)*EllipticPi(((x+1/2)/(1/2-1/2*I*3^(1/2)))^(1/2),3/13-3/13*I*3^(1/2)
,((-1/2+1/2*I*3^(1/2))/(-1/2-1/2*I*3^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x, algorith
m="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 3)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(1536*x^10 - 1792
*x^9 + 3328*x^8 - 5376*x^7 + 1344*x^6 - 6048*x^5 - 1728*x^4 - 3024*x^3 - 1
674*x^2 - 567*x - 405), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+3)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x, algorith
m="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 3)^(7/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+3)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x, algorith
m="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 3)^(7/2)*(3*x - 5)*sqrt(2*x + 1)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+3\right)^{7/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(7/2)),x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(4*x^2 + 3)^(7/2)), x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(3+4x^2)^{7/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+3)^{\frac{7}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+3)^(7/2),x)
```

3.15 $\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx$

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Optimal result

Integrand size = 41, antiderivative size = 137

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = -\frac{(3a^2BD+4b^2(BC+AD))\,x\sqrt{a-bx}\sqrt{a+bx}}{8b^4} - \frac{BDx^3\sqrt{a-bx}\sqrt{a+bx}}{4b^2} + \frac{(8Ab^4C+3a^4BD+4a^2b^2(BC+AD))\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{4b^5}$$

output
```
-1/8*(3*a^2*B*D+4*b^2*(A*D+B*C))*x*(-b*x+a)^(1/2)*(b*x+a)^(1/2)/b^4-1/4*B*
D*x^3*(-b*x+a)^(1/2)*(b*x+a)^(1/2)/b^2+1/4*(8*A*b^4*C+3*a^4*B*D+4*a^2*b^2*
(A*D+B*C))*arctan((b*x+a)^(1/2)/(-b*x+a)^(1/2))/b^5
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = \frac{-bx\sqrt{a-bx}\sqrt{a+bx}(3a^2BD+2b^2(2BC+2AD+BDx^2))+2(8Ab^4C+4a^2b^2BC+4a^2Ab^2D+3a^4B}{8b^5}$$

input
```
Integrate[(A*C + (B*C + A*D)*x^2 + B*D*x^4)/(Sqrt[a - b*x]*Sqrt[a + b*x]),
x]
```

output
```
(-(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(3*a^2*B*D + 2*b^2*(2*B*C + 2*A*D + B*D
*x^2))) + 2*(8*A*b^4*C + 4*a^2*b^2*B*C + 4*a^2*A*b^2*D + 3*a^4*B*D)*ArcTan
[Sqrt[a + b*x]/Sqrt[a - b*x]])/(8*b^5)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.146$, Rules used = {1789, 1473, 25, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(AD+BC)+AC+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \ 1789$$

$$\frac{\sqrt{a^2-b^2x^2}\int \frac{BDx^4+(BC+AD)x^2+AC}{\sqrt{a^2-b^2x^2}}dx}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \ 1473$$

$$\frac{\sqrt{a^2-b^2x^2}\left(-\frac{\int -\frac{4ACb^2+\left(3BDa^2+4b^2(BC+AD)\right)x^2}{\sqrt{a^2-b^2x^2}}dx}{4b^2}-\frac{BDx^3\sqrt{a^2-b^2x^2}}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\int \frac{4ACb^2+\left(3BDa^2+4b^2(BC+AD)\right)x^2}{\sqrt{a^2-b^2x^2}}dx}{4b^2}-\frac{BDx^3\sqrt{a^2-b^2x^2}}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \ 299$$

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\left(3a^4BD+4a^2b^2(AD+BC)+8Ab^4C\right)\int \frac{1}{\sqrt{a^2-b^2x^2}}dx}{2b^2}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+4AD+4BC\right)}{4b^2}-\frac{BDx^3\sqrt{a^2-b^2x^2}}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$\Big\downarrow$ 224

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\left(3a^4BD+4a^2b^2(AD+BC)+8Ab^4C\right)\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}}{2b^2}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+4AD+4BC\right)}{4b^2}-\frac{BDx^3\sqrt{a^2-b^2x^2}}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)\left(3a^4BD+4a^2b^2(AD+BC)+8Ab^4C\right)}{2b^3}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+4AD+4BC\right)}{4b^2}-\frac{BDx^3\sqrt{a^2-b^2x^2}}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

input
```
Int[(A*C + (B*C + A*D)*x^2 + B*D*x^4)/(Sqrt[a - b*x]*Sqrt[a + b*x]),x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-1/4*(B*D*x^3*Sqrt[a^2 - b^2*x^2])/b^2 + (-1/2*((4*B
*C + 4*A*D + (3*a^2*B*D)/b^2)*x*Sqrt[a^2 - b^2*x^2]) + ((8*A*b^4*C + 3*a^4
*B*D + 4*a^2*b^2*(B*C + A*D))*ArcTan[(b*x)/Sqrt[a^2 - b^2*x^2]])/(2*b^3))/
(4*b^2)))/(Sqrt[a - b*x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 1789
```
Int[((d1_) + (e1_.)*(x_)^(non2_.))^(q_.)*((d2_) + (e2_.)*(x_)^(non2_.))^(q_
.)*((a_.) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(p_.), x_Symbol] :> Simp[(d
1 + e1*x^(n/2))^FracPart[q]*((d2 + e2*x^(n/2))^FracPart[q]/(d1*d2 + e1*e2*x
^n)^FracPart[q])   Int[(d1*d2 + e1*e2*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x],
 x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[
non2, n/2] && EqQ[d2*e1 + d1*e2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{-bx+a}\,\sqrt{bx+a}\left(-2BD\ \mathrm{csgn}(b)b^{3}x^{3}\sqrt{-b^{2}x^{2}+a^{2}}-4AD\ \mathrm{csgn}(b)b^{3}\sqrt{-b^{2}x^{2}+a^{2}}\,x-4BC\ \mathrm{csgn}(b)b^{3}\sqrt{-b^{2}x^{2}+a^{2}}\,x-3BD\ \mathrm{csgn}(b)b\right.}{}$

input
```
int((A*C+(A*D+B*C)*x^2+B*D*x^4)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/8*(-b*x+a)^(1/2)*(b*x+a)^(1/2)*(-2*B*D*csgn(b)*b^3*x^3*(-b^2*x^2+a^2)^(1
/2)-4*A*D*csgn(b)*b^3*(-b^2*x^2+a^2)^(1/2)*x-4*B*C*csgn(b)*b^3*(-b^2*x^2+a
^2)^(1/2)*x-3*B*D*csgn(b)*b*(-b^2*x^2+a^2)^(1/2)*a^2*x+8*A*C*arctan(csgn(b
)*b*x/(-b^2*x^2+a^2)^(1/2))*b^4+4*A*D*arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1
/2))*a^2*b^2+4*B*C*arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1/2))*a^2*b^2+3*B*D*
arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1/2))*a^4)*csgn(b)/b^5/(-b^2*x^2+a^2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = -\frac{(2\,BDb^3x^3+(3\,BDa^2b+4\,(BC+AD)b^3)x)\sqrt{bx+a}\sqrt{-bx+a}+2\,(3\,BDa^4+8\,ACb^4+4\,(BC+A}{8\,b^5}$$

input
```
integrate((A*C+(A*D+B*C)*x^2+B*D*x^4)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/8*((2*B*D*b^3*x^3 + (3*B*D*a^2*b + 4*(B*C + A*D)*b^3)*x)*sqrt(b*x + a)*
sqrt(-b*x + a) + 2*(3*B*D*a^4 + 8*A*C*b^4 + 4*(B*C + A*D)*a^2*b^2)*arctan(
(sqrt(b*x + a)*sqrt(-b*x + a) - a)/(b*x)))/b^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = \text{Timed out}$$

input
```
integrate((A*C+(A*D+B*C)*x**2+B*D*x**4)/(-b*x+a)**(1/2)/(b*x+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = -\frac{\sqrt{-b^2x^2+a^2}BDx^3}{4\,b^2}+\frac{3\,BDa^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5}+\frac{AC\arcsin\left(\frac{bx}{a}\right)}{b}-\frac{3\,\sqrt{-b^2x^2+a^2}BDa^2x}{8\,b^4}+\frac{(BC+AD)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3}-\frac{\sqrt{-b^2x^2+a^2}(BC+AD)x}{2\,b^2}$$

input
```
integrate((A*C+(A*D+B*C)*x^2+B*D*x^4)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/4*sqrt(-b^2*x^2 + a^2)*B*D*x^3/b^2 + 3/8*B*D*a^4*arcsin(b*x/a)/b^5 + A*
C*arcsin(b*x/a)/b - 3/8*sqrt(-b^2*x^2 + a^2)*B*D*a^2*x/b^4 + 1/2*(B*C + A*
D)*a^2*arcsin(b*x/a)/b^3 - 1/2*sqrt(-b^2*x^2 + a^2)*(B*C + A*D)*x/b^2
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{AC+(BC+AD)x^2+BDx^4}{\sqrt{a-bx}\sqrt{a+bx}}\,dx$$

$$= \frac{(5\,BDa^3+4\,BCab^2+4\,ADab^2-(9\,BDa^2+4\,BCb^2+4\,ADb^2+2\,((bx+a)BD-3\,BDa)(bx+a))(b}{8\,b^5}$$

input
```
integrate((A*C+(A*D+B*C)*x^2+B*D*x^4)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algo
rithm="giac")
```

output
```
1/8*((5*B*D*a^3 + 4*B*C*a*b^2 + 4*A*D*a*b^2 - (9*B*D*a^2 + 4*B*C*b^2 + 4*A
*D*b^2 + 2*((b*x + a)*B*D - 3*B*D*a)*(b*x + a))*(b*x + a))*sqrt(b*x + a)*s
qrt(-b*x + a) + 2*(3*B*D*a^4 + 4*B*C*a^2*b^2 + 4*A*D*a^2*b^2 + 8*A*C*b^4)*
arcsin(1/2*sqrt(2)*sqrt(b*x + a)/sqrt(a)))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{AC + (BC + AD)x^2 + BDx^4}{\sqrt{a - bx}\sqrt{a + bx}}\,dx = \int \frac{x^2\,(A\,D + B\,C) + A\,C + B\,x^4\,D}{\sqrt{a + b\,x}\,\sqrt{a - b\,x}}\,dx$$

input
```
int((x^2*(A*D + B*C) + A*C + B*x^4*D)/((a + b*x)^(1/2)*(a - b*x)^(1/2)),x)
```

output
```
int((x^2*(A*D + B*C) + A*C + B*x^4*D)/((a + b*x)^(1/2)*(a - b*x)^(1/2)), x
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.36

$$\int \frac{AC + (BC + AD)x^2 + BDx^4}{\sqrt{a - bx}\sqrt{a + bx}}\,dx = \frac{-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4d - 8asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^3bd - 8asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^2b^2c - 16asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a\,b^3c - 3\sqrt{bx +}}{}$$

input
```
int((A*C+(A*D+B*C)*x^2+B*D*x^4)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x)
```

output
```
( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*d - 8*asin(sqrt(a - b*x)/
(sqrt(a)*sqrt(2)))*a**3*b*d - 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2
*b**2*c - 16*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**3*c - 3*sqrt(a + b
*x)*sqrt(a - b*x)*a**2*b*d*x - 4*sqrt(a + b*x)*sqrt(a - b*x)*a*b**2*d*x -
4*sqrt(a + b*x)*sqrt(a - b*x)*b**3*c*x - 2*sqrt(a + b*x)*sqrt(a - b*x)*b**
3*d*x**3)/(8*b**4)
```

3.16 $\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}} dx$

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Optimal result

Integrand size = 34, antiderivative size = 137

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}} dx = -\frac{(3a^2BD+4b^2(BC+AD))x\sqrt{a-bx}\sqrt{a+bx}}{8b^4} - \frac{BDx^3\sqrt{a-bx}\sqrt{a+bx}}{4b^2} + \frac{(8Ab^4C+3a^4BD+4a^2b^2(BC+AD))\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{a-bx}}\right)}{4b^5}$$

output
```
-1/8*(3*a^2*B*D+4*b^2*(A*D+B*C))*x*(-b*x+a)^(1/2)*(b*x+a)^(1/2)/b^4-1/4*B*
D*x^3*(-b*x+a)^(1/2)*(b*x+a)^(1/2)/b^2+1/4*(8*A*b^4*C+3*a^4*B*D+4*a^2*b^2*
(A*D+B*C))*arctan((b*x+a)^(1/2)/(-b*x+a)^(1/2))/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}} dx$$
$$= \frac{-bx\sqrt{a-bx}\sqrt{a+bx}(3a^2BD+2b^2(2BC+2AD+BDx^2))+2(8Ab^4C+4a^2b^2BC+4a^2Ab^2D+3a^4B}{8b^5}$$

input
```
Integrate[((A + B*x^2)*(C + D*x^2))/(Sqrt[a - b*x]*Sqrt[a + b*x]),x]
```

output
```
(-(b*x*Sqrt[a - b*x]*Sqrt[a + b*x]*(3*a^2*B*D + 2*b^2*(2*B*C + 2*A*D + B*D
*x^2))) + 2*(8*A*b^4*C + 4*a^2*b^2*B*C + 4*a^2*A*b^2*D + 3*a^4*B*D)*ArcTan
[Sqrt[a + b*x]/Sqrt[a - b*x]])/(8*b^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2041, 403, 25, 299, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx$$

$$\Big\downarrow \text{2041}$$

$$\frac{\sqrt{a^2-b^2x^2}\int \frac{(Bx^2+A)(Dx^2+C)}{\sqrt{a^2-b^2x^2}}dx}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \text{403}$$

$$\frac{\sqrt{a^2-b^2x^2}\left(-\frac{\int -\frac{(3BDa^2+2b^2(2BC+AD))x^2+A(Da^2+4b^2C)}{\sqrt{a^2-b^2x^2}}dx}{4b^2}-\frac{Dx\sqrt{a^2-b^2x^2}(A+Bx^2)}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\int \frac{(3BDa^2+2b^2(2BC+AD))x^2+A(Da^2+4b^2C)}{\sqrt{a^2-b^2x^2}}dx}{4b^2}-\frac{Dx\sqrt{a^2-b^2x^2}(A+Bx^2)}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$$\Big\downarrow \text{299}$$

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\left(3a^4BD+4a^2Ab^2D+4a^2b^2BC+8Ab^4C\right)\int \frac{1}{\sqrt{a^2-b^2x^2}}dx}{2b^2}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+2AD+4BC\right)}{4b^2}-\frac{Dx\sqrt{a^2-b^2x^2}(A+Bx^2)}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$\Big\downarrow$ 224

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\left(3a^4BD+4a^2Ab^2D+4a^2b^2BC+8Ab^4C\right)\int \frac{1}{\frac{b^2x^2}{a^2-b^2x^2}+1}d\frac{x}{\sqrt{a^2-b^2x^2}}}{2b^2}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+2AD+4BC\right)}{4b^2}-\frac{Dx\sqrt{a^2-b^2x^2}(A+Bx^2)}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{a^2-b^2x^2}\left(\frac{\frac{\arctan\left(\frac{bx}{\sqrt{a^2-b^2x^2}}\right)\left(3a^4BD+4a^2Ab^2D+4a^2b^2BC+8Ab^4C\right)}{2b^3}-\frac{1}{2}x\sqrt{a^2-b^2x^2}\left(\frac{3a^2BD}{b^2}+2AD+4BC\right)}{4b^2}-\frac{Dx\sqrt{a^2-b^2x^2}(A+Bx^2)}{4b^2}\right)}{\sqrt{a-bx}\sqrt{a+bx}}$$

input
```
Int[((A + B*x^2)*(C + D*x^2))/(Sqrt[a - b*x]*Sqrt[a + b*x]),x]
```

output
```
(Sqrt[a^2 - b^2*x^2]*(-1/4*(D*x*Sqrt[a^2 - b^2*x^2]*(A + B*x^2))/b^2 + (-1
/2*((4*B*C + 2*A*D + (3*a^2*B*D)/b^2)*x*Sqrt[a^2 - b^2*x^2]) + ((8*A*b^4*C
 + 4*a^2*b^2*B*C + 4*a^2*A*b^2*D + 3*a^4*B*D)*ArcTan[(b*x)/Sqrt[a^2 - b^2*
x^2]])/(2*b^3))/(4*b^2)))/(Sqrt[a - b*x]*Sqrt[a + b*x])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 2041
```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Simp[(e1 + f1*x^(n/2))^FracPart[r]*((e2 + f2*x^(n/2))^FracPart[r]/(e1*e2 +
f1*f2*x^n)^FracPart[r])   Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^
n)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n
2, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.11 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

method	result
default	$\frac{\sqrt{-bx+a}\,\sqrt{bx+a}\,\left(-2BD\,\mathrm{csgn}(b)b^3x^3\sqrt{-b^2x^2+a^2}-4AD\,\mathrm{csgn}(b)b^3\sqrt{-b^2x^2+a^2}\,x-4BC\,\mathrm{csgn}(b)b^3\sqrt{-b^2x^2+a^2}\,x-3BD\,\mathrm{csgn}(b)b\right.}{}$

input
```
int((B*x^2+A)*(D*x^2+C)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/8*(-b*x+a)^(1/2)*(b*x+a)^(1/2)*(-2*B*D*csgn(b)*b^3*x^3*(-b^2*x^2+a^2)^(1
/2)-4*A*D*csgn(b)*b^3*(-b^2*x^2+a^2)^(1/2)*x-4*B*C*csgn(b)*b^3*(-b^2*x^2+a
^2)^(1/2)*x-3*B*D*csgn(b)*b*(-b^2*x^2+a^2)^(1/2)*a^2*x+8*A*C*arctan(csgn(b
)*b*x/(-b^2*x^2+a^2)^(1/2))*b^4+4*A*D*arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1
/2))*a^2*b^2+4*B*C*arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1/2))*a^2*b^2+3*B*D*
arctan(csgn(b)*b*x/(-b^2*x^2+a^2)^(1/2))*a^4)*csgn(b)/b^5/(-b^2*x^2+a^2)^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = -\frac{(2\,BDb^3x^3+(3\,BDa^2b+4\,(BC+AD)b^3)x)\sqrt{bx+a}\sqrt{-bx+a}+2\,(3\,BDa^4+8\,ACb^4+4\,(BC+A}{8\,b^5}$$

input
```
integrate((B*x^2+A)*(D*x^2+C)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/8*((2*B*D*b^3*x^3 + (3*B*D*a^2*b + 4*(B*C + A*D)*b^3)*x)*sqrt(b*x + a)*
sqrt(-b*x + a) + 2*(3*B*D*a^4 + 8*A*C*b^4 + 4*(B*C + A*D)*a^2*b^2)*arctan(
(sqrt(b*x + a)*sqrt(-b*x + a) - a)/(b*x)))/b^5
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = \text{Timed out}$$

input
```
integrate((B*x**2+A)*(D*x**2+C)/(-b*x+a)**(1/2)/(b*x+a)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = -\frac{\sqrt{-b^2x^2+a^2}BDx^3}{4\,b^2} + \frac{3\,BDa^4\arcsin\left(\frac{bx}{a}\right)}{8\,b^5} + \frac{AC\arcsin\left(\frac{bx}{a}\right)}{b} - \frac{3\,\sqrt{-b^2x^2+a^2}BDa^2x}{8\,b^4} + \frac{(BC+AD)a^2\arcsin\left(\frac{bx}{a}\right)}{2\,b^3} - \frac{\sqrt{-b^2x^2+a^2}(BC+AD)x}{2\,b^2}$$

input
```
integrate((B*x^2+A)*(D*x^2+C)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algorithm="m
axima")
```

output
```
-1/4*sqrt(-b^2*x^2 + a^2)*B*D*x^3/b^2 + 3/8*B*D*a^4*arcsin(b*x/a)/b^5 + A*
C*arcsin(b*x/a)/b - 3/8*sqrt(-b^2*x^2 + a^2)*B*D*a^2*x/b^4 + 1/2*(B*C + A*
D)*a^2*arcsin(b*x/a)/b^3 - 1/2*sqrt(-b^2*x^2 + a^2)*(B*C + A*D)*x/b^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = \frac{(5\,BDa^3+4\,BCab^2+4\,ADab^2-(9\,BDa^2+4\,BCb^2+4\,ADb^2+2\,((bx+a)BD-3\,BDa)(bx+a))(b}{8\,b^5}$$

input
```
integrate((B*x^2+A)*(D*x^2+C)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x, algorithm="g
iac")
```

output
```
1/8*((5*B*D*a^3 + 4*B*C*a*b^2 + 4*A*D*a*b^2 - (9*B*D*a^2 + 4*B*C*b^2 + 4*A
*D*b^2 + 2*((b*x + a)*B*D - 3*B*D*a)*(b*x + a))*(b*x + a))*sqrt(b*x + a)*s
qrt(-b*x + a) + 2*(3*B*D*a^4 + 4*B*C*a^2*b^2 + 4*A*D*a^2*b^2 + 8*A*C*b^4)*
arcsin(1/2*sqrt(2)*sqrt(b*x + a)/sqrt(a)))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx = \int \frac{(B\,x^2+A)\,(C+x^2\,D)}{\sqrt{a+b\,x}\,\sqrt{a-b\,x}}\,dx$$

input
```
int(((A + B*x^2)*(C + x^2*D))/((a + b*x)^(1/2)*(a - b*x)^(1/2)),x)
```

output
```
int(((A + B*x^2)*(C + x^2*D))/((a + b*x)^(1/2)*(a - b*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx^2)(C+Dx^2)}{\sqrt{a-bx}\sqrt{a+bx}}\,dx$$
$$= \frac{-6asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^4d-8asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^3bd-8asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a^2b^2c-16asin\left(\frac{\sqrt{-bx+a}}{\sqrt{a}\,\sqrt{2}}\right)a\,b^3c-3\sqrt{bx+}}{}$$

input
```
int((B*x^2+A)*(D*x^2+C)/(-b*x+a)^(1/2)/(b*x+a)^(1/2),x)
```

output
```
( - 6*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**4*d - 8*asin(sqrt(a - b*x)/
(sqrt(a)*sqrt(2)))*a**3*b*d - 8*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a**2
*b**2*c - 16*asin(sqrt(a - b*x)/(sqrt(a)*sqrt(2)))*a*b**3*c - 3*sqrt(a + b
*x)*sqrt(a - b*x)*a**2*b*d*x - 4*sqrt(a + b*x)*sqrt(a - b*x)*a*b**2*d*x -
4*sqrt(a + b*x)*sqrt(a - b*x)*b**3*c*x - 2*sqrt(a + b*x)*sqrt(a - b*x)*b**
3*d*x**3)/(8*b**4)
```

3.17 $\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Reduce [B] (verification not implemented) 216

Optimal result

Integrand size = 43, antiderivative size = 145

$$\begin{aligned}\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx &= \frac{13723\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{11664} \\ &\quad - \frac{983\sqrt{-1+3x}\sqrt{1+3x}(2+x^2)^{3/2}}{1944} \\ &\quad + \frac{5}{54}\sqrt{-1+3x}\sqrt{1+3x}(2+x^2)^{5/2} \\ &\quad - \frac{112511\sqrt{-1+9x^2}\operatorname{arcsinh}\left(\frac{\sqrt{-1+9x^2}}{\sqrt{19}}\right)}{34992\sqrt{-1+3x}\sqrt{1+3x}}\end{aligned}$$

output
```
13723/11664*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)-983/1944*(-1+3*x)^(
1/2)*(1+3*x)^(1/2)*(x^2+2)^(3/2)+5/54*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)
^(5/2)-112511/34992*(9*x^2-1)^(1/2)*arcsinh(1/19*(9*x^2-1)^(1/2)*19^(1/2))
/(-1+3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 10.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.57

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{3\sqrt{2+x^2}(-6247+57801x^2-15282x^4+9720x^6)+112511\sqrt{1-9x^2}\arcsin\left(\frac{\sqrt{1-9x^2}}{\sqrt{19}}\right)}{34992\sqrt{-1+3x}\sqrt{1+3x}}$$

input
```
Integrate[(x^3*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 +
 x^2]),x]
```

output
```
(3*Sqrt[2 + x^2]*(-6247 + 57801*x^2 - 15282*x^4 + 9720*x^6) + 112511*Sqrt[
1 - 9*x^2]*ArcSin[Sqrt[1 - 9*x^2]/Sqrt[19]])/(34992*Sqrt[-1 + 3*x]*Sqrt[1
+ 3*x])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2038, 7283, 2118, 27, 164, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(5x^4+3x^2+2)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{x^3(5x^4+3x^2+2)}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{7283}$$

$$\frac{\sqrt{9x^2-1}\int \frac{x^2(5x^4+3x^2+2)}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{2118}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{27}\int \frac{x^2(148-263x^2)}{2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x^4\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{54}\int \frac{x^2(148-263x^2)}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x^4\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 164$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{54}\left(\frac{1}{108}(6247-1578x^2)\sqrt{x^2+2}\sqrt{9x^2-1}-\frac{112511}{216}\int \frac{1}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2\right)+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x^4\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 64$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{54}\left(\frac{1}{108}(6247-1578x^2)\sqrt{x^2+2}\sqrt{9x^2-1}-\frac{112511}{972}\int \frac{1}{\sqrt{\frac{x^4}{9}+\frac{19}{9}}}d\sqrt{9x^2-1}\right)+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x^4\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{54}\left(\frac{1}{108}(6247-1578x^2)\sqrt{x^2+2}\sqrt{9x^2-1}-\frac{112511}{324}\text{arcsinh}\left(\frac{\sqrt{9x^2-1}}{\sqrt{19}}\right)\right)+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x^4\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(x^3*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2])
,x]
```

output
```
(Sqrt[-1 + 9*x^2]*((5*x^4*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/27 + (((6247 - 1
578*x^2)*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/108 - (112511*ArcSinh[Sqrt[-1 + 9
*x^2]/Sqrt[19]])/324)/54))/(2*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 7283
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> With[{lst = PowerVariableExpn[u, m + 1, x
]}, Simp[1/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x]
, x], x, (lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], m + 1
]] /; IntegerQ[m] && NeQ[m, -1] && NonsumQ[u] && (GtQ[m, 0] ||  !AlgebraicF
unctionQ[u, x])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

method	result
risch	$\frac{(1080x^4-1578x^2+6247)\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{11664}-\frac{112511\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}\,\sqrt{(-1+3x)(1+3x)(x^2+2)}}{209952\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(\frac{6247\sqrt{9x^4+17x^2-2}}{11664}-\frac{112511\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}}{209952}-\frac{263\sqrt{9x^4+17x^2-2}\,x^2}{1944}+\frac{5x^4\sqrt{9x^4+17x^2-2}}{54}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
default	$-\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\,\big(-6480x^4\sqrt{9x^4+17x^2-2}+9468\sqrt{9x^4+17x^2-2}\,x^2+114063\ln\left(\frac{17}{6}+3x^2+\sqrt{9x^4+17x^2-2}\right)-1552\ln\big(6x}{69984\sqrt{9x^4+17x^2-2}}$

input
```
int(x^3*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/11664*(1080*x^4-1578*x^2+6247)*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2
)-112511/209952*ln(1/9*(17/2+9*x^2)*9^(1/2)+(9*x^4+17*x^2-2)^(1/2))*9^(1/2
)*((-1+3*x)*(1+3*x)*(x^2+2))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{1}{11664}\left(1080\,x^4-1578\,x^2+6247\right)\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1} + \frac{112511}{69984}\log\left(-18\,x^2+6\,\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}-17\right)$$

input
```
integrate(x^3*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
1/11664*(1080*x^4 - 1578*x^2 + 6247)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x
- 1) + 112511/69984*log(-18*x^2 + 6*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x -
 1) - 17)
```

Sympy [F]

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x^3\cdot(5x^4+3x^2+2)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate(x**3*(5*x**4+3*x**2+2)/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Integral(x**3*(5*x**4 + 3*x**2 + 2)/(sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2
 + 2)), x)
```

Maxima [F]

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x^3}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x^3*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x^3/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x -
1)), x)
```

Giac [F]

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x^3}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x^3*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x^3/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x -
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x^3\,(5\,x^4+3\,x^2+2)}{\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((x^3*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(
1/2)),x)
```

output
```
int((x^3*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(
1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.63

$$\int \frac{x^3(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^4}{54} - \frac{263\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^2}{1944} + \frac{6247\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{11664} + \frac{112511\log\left(-3\sqrt{x^2+2}+\sqrt{3x+1}\,\sqrt{3x-1}\right)}{34992}$$

input
```
int(x^3*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
(3240*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**4 - 4734*sqrt(3*x + 1)
*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**2 + 18741*sqrt(3*x + 1)*sqrt(3*x - 1)*sqr
t(x**2 + 2) + 112511*log( - 3*sqrt(x**2 + 2) + sqrt(3*x + 1)*sqrt(3*x - 1)
))/34992
```

3.18 $\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Optimal result

Integrand size = 41, antiderivative size = 114

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = -\frac{109}{216}\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2} + \frac{5}{36}\sqrt{-1+3x}\sqrt{1+3x}\left(2+x^2\right)^{3/2} + \frac{1385\sqrt{-1+9x^2}\operatorname{arcsinh}\left(\frac{\sqrt{-1+9x^2}}{\sqrt{19}}\right)}{648\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
-109/216*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)+5/36*(-1+3*x)^(1/2)*(1
+3*x)^(1/2)*(x^2+2)^(3/2)+1385/648*(9*x^2-1)^(1/2)*arcsinh(1/19*(9*x^2-1)^
(1/2)*19^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 10.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.68

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{3\sqrt{2+x^2}(49-471x^2+270x^4)-1385\sqrt{1-9x^2}\arcsin\left(\frac{\sqrt{1-9x^2}}{\sqrt{19}}\right)}{648\sqrt{-1+3x}\sqrt{1+3x}}$$

input
```
Integrate[(x*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x
^2]),x]
```

output
```
(3*Sqrt[2 + x^2]*(49 - 471*x^2 + 270*x^4) - 1385*Sqrt[1 - 9*x^2]*ArcSin[Sq
rt[1 - 9*x^2]/Sqrt[19]])/(648*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {2038, 7266, 1194, 27, 90, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(5x^4+3x^2+2\right)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{ 2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{x\left(5x^4+3x^2+2\right)}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 1194}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\int -\frac{3\left(109x^2+26\right)}{2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2+\frac{5}{18}\sqrt{9x^2-1}\left(x^2+2\right)^{3/2}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{5}{18}\left(x^2+2\right)^{3/2}\sqrt{9x^2-1}-\frac{1}{12}\int \frac{109x^2+26}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{12}\left(\frac{1385}{18}\int\frac{1}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2-\frac{109}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)+\frac{5}{18}\sqrt{9x^2-1}\left(x^2+2\right)^{3/2}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 64$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{12}\left(\frac{1385}{81}\int\frac{1}{\sqrt{\frac{x^4}{9}+\frac{19}{9}}}d\sqrt{9x^2-1}-\frac{109}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)+\frac{5}{18}\sqrt{9x^2-1}\left(x^2+2\right)^{3/2}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \; 222$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{12}\left(\frac{1385}{27}\operatorname{arcsinh}\left(\frac{\sqrt{9x^2-1}}{\sqrt{19}}\right)-\frac{109}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)+\frac{5}{18}\sqrt{9x^2-1}\left(x^2+2\right)^{3/2}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

input

```
Int[(x*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x
]
```

output

```
(Sqrt[-1 + 9*x^2]*((5*(2 + x^2)^(3/2)*Sqrt[-1 + 9*x^2])/18 + ((-109*Sqrt[2
 + x^2]*Sqrt[-1 + 9*x^2])/9 + (1385*ArcSinh[Sqrt[-1 + 9*x^2]/Sqrt[19]])/27
)/12))/(2*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1194
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[c^p*(d + e*x)^(m + 2*p)*((f + g*x
)^(n + 1)/(g*e^(2*p)*(m + n + 2*p + 1))), x] + Simp[1/(g*e^(2*p)*(m + n + 2
*p + 1))   Int[(d + e*x)^m*(f + g*x)^n*ExpandToSum[g*(m + n + 2*p + 1)*(e^(
2*p)*(a + b*x + c*x^2)^p - c^p*(d + e*x)^(2*p)) - c^p*(e*f - d*g)*(m + 2*p)
*(d + e*x)^(2*p - 1), x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ
[p, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && NeQ[m + n + 2*p + 1, 0]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.91

method	result
risch	$\frac{(30x^2-49)\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{216}+\frac{1385\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}\,\sqrt{(-1+3x)(1+3x)(x^2+2)}}{3888\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(\frac{1385\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}}{3888}-\frac{49\sqrt{9x^4+17x^2-2}}{216}+\frac{5\sqrt{9x^4+17x^2-2}\,x^2}{36}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
default	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\left(540\sqrt{9x^4+17x^2-2}\,x^2-882\sqrt{9x^4+17x^2-2}+3379\ln\left(\frac{17}{6}+3x^2+\sqrt{9x^4+17x^2-2}\right)+776\ln\left(6x^2+\frac{17}{3}+2\sqrt{9\ldots}\right.\right.}{3888\sqrt{9x^4+17x^2-2}}$

input
```
int(x*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
1/216*(30*x^2-49)*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)+1385/3888*ln(
1/9*(17/2+9*x^2)*9^(1/2)+(9*x^4+17*x^2-2)^(1/2))*9^(1/2)*((-1+3*x)*(1+3*x)
*(x^2+2))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.56

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\frac{1}{216}\left(30\,x^2-49\right)\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}-\frac{1385}{1296}\,\log\left(-18\,x^2+6\,\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}-17\right)$$

input
```
integrate(x*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="fricas")
```

output
```
1/216*(30*x^2 - 49)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1) - 1385/1296*
log(-18*x^2 + 6*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1) - 17)
```

Sympy [F]

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x(5x^4+3x^2+2)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate(x*(5*x**4+3*x**2+2)/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**(1/
2),x)
```

output
```
Integral(x*(5*x**4 + 3*x**2 + 2)/(sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2 +
2)), x)
```

Maxima [F]

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)
), x)
```

Giac [F]

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x\,(5\,x^4+3\,x^2+2)}{\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((x*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)),x)
```

output
```
int((x*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.61

$$\int \frac{x(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^2}{36} - \frac{49\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{216} + \frac{1385\log\left(-3\sqrt{x^2+2}-\sqrt{3x+1}\,\sqrt{3x-1}\right)}{648}$$

input
```
int(x*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
(90*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**2 - 147*sqrt(3*x + 1)*sq
rt(3*x - 1)*sqrt(x**2 + 2) + 1385*log( - 3*sqrt(x**2 + 2) - sqrt(3*x + 1)*
sqrt(3*x - 1)))/648
```

3.19 $\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Optimal result

Integrand size = 43, antiderivative size = 146

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5}{18}\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2} - \frac{31\sqrt{-1+9x^2}\operatorname{arcsinh}\left(\frac{\sqrt{-1+9x^2}}{\sqrt{19}}\right)}{54\sqrt{-1+3x}\sqrt{1+3x}} - \frac{\sqrt{2}\sqrt{-1+9x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{2}\sqrt{-1+9x^2}}\right)}{\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
5/18*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)-31/54*(9*x^2-1)^(1/2)*arcs
inh(1/19*(9*x^2-1)^(1/2)*19^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)-2^(1/2)*(9
*x^2-1)^(1/2)*arctan(1/2*(x^2+2)^(1/2)*2^(1/2)/(9*x^2-1)^(1/2))/(-1+3*x)^(
1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 10.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$
$$= \frac{\sqrt{-1+9x^2}\left(15\sqrt{2+x^2}\sqrt{-\left(-1+9x^2\right)^2}-31\sqrt{-1+9x^2}\arcsin\left(\frac{\sqrt{1-9x^2}}{\sqrt{19}}\right)-54\sqrt{2-18x^2}\arctan\left(\frac{\sqrt{2-}}{\sqrt{-2+}}\right.\right.}{54\sqrt{-1+3x}\sqrt{1+3x}\sqrt{1-9x^2}}$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(x*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2
]),x]
```

output
```
(Sqrt[-1 + 9*x^2]*(15*Sqrt[2 + x^2]*Sqrt[-(-1 + 9*x^2)^2] - 31*Sqrt[-1 + 9
*x^2]*ArcSin[Sqrt[1 - 9*x^2]/Sqrt[19]] - 54*Sqrt[2 - 18*x^2]*ArcTan[Sqrt[2
 + x^2]/Sqrt[-2 + 18*x^2]]))/(54*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[1 - 9*x
^2])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2038, 7282, 2118, 27, 175, 64, 104, 217, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{x\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$
$$\Big\downarrow \text{ 2038}$$
$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$
$$\Big\downarrow \text{ 7282}$$
$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{2118}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{9}\int \frac{36-31x^2}{2x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\int \frac{36-31x^2}{x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{175}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\left(36\int \frac{1}{x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2-31\int \frac{1}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2\right)+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{64}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\left(36\int \frac{1}{x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx^2-\frac{62}{9}\int \frac{1}{\sqrt{\frac{x^4}{9}+\frac{19}{9}}}d\sqrt{9x^2-1}\right)+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{104}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\left(72\int \frac{1}{-x^4-2}d\frac{\sqrt{x^2+2}}{\sqrt{9x^2-1}}-\frac{62}{9}\int \frac{1}{\sqrt{\frac{x^4}{9}+\frac{19}{9}}}d\sqrt{9x^2-1}\right)+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\left(-\frac{62}{9}\int \frac{1}{\sqrt{\frac{x^4}{9}+\frac{19}{9}}}d\sqrt{9x^2-1}-36\sqrt{2}\arctan\left(\frac{\sqrt{x^2+2}}{\sqrt{2}\sqrt{9x^2-1}}\right)\right)+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{222}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{1}{18}\left(-\frac{62}{3}\text{arcsinh}\left(\frac{\sqrt{9x^2-1}}{\sqrt{19}}\right)-36\sqrt{2}\arctan\left(\frac{\sqrt{x^2+2}}{\sqrt{2}\sqrt{9x^2-1}}\right)\right)+\frac{5}{9}\sqrt{x^2+2}\sqrt{9x^2-1}\right)}{2\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(x*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[-1 + 9*x^2]*((5*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/9 + ((-62*ArcSinh[Sq
rt[-1 + 9*x^2]/Sqrt[19]])/3 - 36*Sqrt[2]*ArcTan[Sqrt[2 + x^2]/(Sqrt[2]*Sqr
t[-1 + 9*x^2])])/18))/(2*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 175
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 2118
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Simp[1/(d*f*b^q*(m + n + p +
q + 1))   Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q
- 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
 c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
 + q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

rule 7282
```
Int[(u_)/(x_), x_Symbol] :> With[{lst = PowerVariableExpn[u, 0, x]}, Simp[1
/lst[[2]]   Subst[Int[NormalizeIntegrand[Simplify[lst[[1]]/x], x], x], x, (
lst[[3]]*x)^lst[[2]]], x] /;  !FalseQ[lst] && NeQ[lst[[2]], 0]] /; NonsumQ[
u] &&  !RationalFunctionQ[u, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.82

method	result
elliptic	$\frac{\sqrt{\left(x^{2}+2\right)\left(9x^{2}-1\right)}\,\left(-\frac{31\ln\left(\frac{\left(\frac{17}{2}+9x^{2}\right)\sqrt{9}}{9}+\sqrt{9x^{4}+17x^{2}-2}\right)\sqrt{9}}{324}+\frac{\sqrt{2}\,\arctan\left(\frac{\left(17x^{2}-4\right)\sqrt{2}}{4\sqrt{9x^{4}+17x^{2}-2}}\right)}{2}+\frac{5\sqrt{9x^{4}+17x^{2}-2}}{18}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^{2}+2}}$
default	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^{2}+2}\,\left(54\sqrt{2}\,\arctan\left(\frac{\left(17x^{2}-4\right)\sqrt{2}}{4\sqrt{9x^{4}+17x^{2}-2}}\right)-225\ln\left(\frac{17}{6}+3x^{2}+\sqrt{9x^{4}+17x^{2}-2}\right)+194\ln\left(6x^{2}+\frac{17}{3}+2\sqrt{9x^{4}+17x^{2}-2}\right.\right.}{108\sqrt{9x^{4}+17x^{2}-2}}$

input
```
int((5*x^4+3*x^2+2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
((x^2+2)*(9*x^2-1))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2)*(-31/
324*ln(1/9*(17/2+9*x^2)*9^(1/2)+(9*x^4+17*x^2-2)^(1/2))*9^(1/2)+1/2*2^(1/2
)*arctan(1/4*(17*x^2-4)*2^(1/2)/(9*x^4+17*x^2-2)^(1/2))+5/18*(9*x^4+17*x^2
-2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.66

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5}{18}\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1} + \sqrt{2}\arctan\left(-\frac{3}{2}\sqrt{2}x^2 + \frac{1}{2}\sqrt{2}\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}\right) + \frac{31}{108}\log\left(-18\,x^2+6\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1} - 17\right)$$

input
```
integrate((5*x^4+3*x^2+2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="fricas")
```

output
```
5/18*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1) + sqrt(2)*arctan(-3/2*sqrt(
2)*x^2 + 1/2*sqrt(2)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)) + 31/108*l
og(-18*x^2 + 6*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1) - 17)
```

Sympy [F]

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5x^4+3x^2+2}{x\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate((5*x**4+3*x**2+2)/x/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**(1/
2),x)
```

output
```
Integral((5*x**4 + 3*x**2 + 2)/(x*sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2 +
2)), x)
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
), x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,
algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{x\,\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/(x*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/2)
),x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/(x*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2+5x^4}{x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=-\sqrt{2}\,atan\left(\frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,\sqrt{2}}{18x^2-2}\right)+\frac{5\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{18}+\frac{31\log\left(-3\sqrt{x^2+2}+\sqrt{3x+1}\,\sqrt{3x-1}\right)}{54}$$

input
```
int((5*x^4+3*x^2+2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
( - 54*sqrt(2)*atan((sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*sqrt(2))/(
18*x**2 - 2)) + 15*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2) + 31*log( -
3*sqrt(x**2 + 2) + sqrt(3*x + 1)*sqrt(3*x - 1)))/54
```

3.20 $\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Reduce [B] (verification not implemented) 237

Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{2x^2} + \frac{5\sqrt{-1+9x^2}\operatorname{arcsinh}\left(\frac{\sqrt{-1+9x^2}}{\sqrt{19}}\right)}{3\sqrt{-1+3x}\sqrt{1+3x}} - \frac{23\sqrt{-1+9x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{2}\sqrt{-1+9x^2}}\right)}{2\sqrt{2}\sqrt{-1+3x}\sqrt{1+3x}}$$

output

```
1/2*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)/x^2+5/3*(9*x^2-1)^(1/2)*arc
sinh(1/19*(9*x^2-1)^(1/2)*19^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)-23/4*2^(1
/2)*(9*x^2-1)^(1/2)*arctan(1/2*(x^2+2)^(1/2)*2^(1/2)/(9*x^2-1)^(1/2))/(-1+
3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 10.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{\sqrt{-1+9x^2}\left(\frac{6\sqrt{2+x^2}\sqrt{-1+9x^2}}{x^2}+\frac{20\sqrt{-1+9x^2}\arcsin\left(\frac{\sqrt{1-9x^2}}{\sqrt{19}}\right)}{\sqrt{1-9x^2}}-69\sqrt{2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{-2+18x^2}}\right)\right)}{12\sqrt{-1+3x}\sqrt{1+3x}}$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(x^3*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x
^2]),x]
```

output
```
(Sqrt[-1 + 9*x^2]*((6*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/x^2 + (20*Sqrt[-1 +
9*x^2]*ArcSin[Sqrt[1 - 9*x^2]/Sqrt[19]])/Sqrt[1 - 9*x^2] - 69*Sqrt[2]*ArcT
an[Sqrt[2 + x^2]/Sqrt[-2 + 18*x^2]]))/(12*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{x^3\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

$$\Big\downarrow \text{2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x^3\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5x}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3}{\sqrt{x^2+2}\sqrt{9x^2-1}x}+\frac{2}{\sqrt{x^2+2}\sqrt{9x^2-1}x^3}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{5}{3}\operatorname{arcsinh}\left(\frac{\sqrt{9x^2-1}}{\sqrt{19}}\right)-\frac{23\arctan\left(\frac{\sqrt{x^2+2}}{\sqrt{2}\sqrt{9x^2-1}}\right)}{2\sqrt{2}}+\frac{\sqrt{x^2+2}\sqrt{9x^2-1}}{2x^2}\right)}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(x^3*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x
]
```

output
```
(Sqrt[-1 + 9*x^2]*((Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/(2*x^2) + (5*ArcSinh[S
qrt[-1 + 9*x^2]/Sqrt[19]])/3 - (23*ArcTan[Sqrt[2 + x^2]/(Sqrt[2]*Sqrt[-1 +
 9*x^2])])/(2*Sqrt[2])))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

method	result
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(\frac{5\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}}{18}+\frac{\sqrt{9x^4+17x^2-2}}{2x^2}+\frac{23\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)}{8}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
risch	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{2x^2}+\frac{\left(\frac{23\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)}{8}+\frac{5\ln\left(\frac{\left(\frac{17}{2}+9x^2\right)\sqrt{9}}{9}+\sqrt{9x^4+17x^2-2}\right)\sqrt{9}}{18}\right)\sqrt{(-1+3x)(1+3x)(x^2+2)}}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
default	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\left(69\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)x^2-368\ln\left(\frac{17}{6}+3x^2+\sqrt{9x^4+17x^2-2}\right)x^2+388\ln\left(6x^2+\frac{17}{3}+2\sqrt{9x^4+17\cdots\right.\right.}{24\sqrt{9x^4+17x^2-2}\,x^2}$

input
```
int((5*x^4+3*x^2+2)/x^3/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
((x^2+2)*(9*x^2-1))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2)*(5/18
*ln(1/9*(17/2+9*x^2)*9^(1/2)+(9*x^4+17*x^2-2)^(1/2))*9^(1/2)+1/2*(9*x^4+17
*x^2-2)^(1/2)/x^2+23/8*2^(1/2)*arctan(1/4*(17*x^2-4)*2^(1/2)/(9*x^4+17*x^2
-2)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.75

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{69\sqrt{2}x^2\arctan\left(-\frac{3}{2}\sqrt{2}x^2+\frac{1}{2}\sqrt{2}\sqrt{x^2+2}\sqrt{3x+1}\sqrt{3x-1}\right)-10x^2\log\left(-18x^2+6\sqrt{x^2+2}\sqrt{3x+1}\cdots\right.}{12x^2}$$

input
```
integrate((5*x^4+3*x^2+2)/x^3/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
1/12*(69*sqrt(2)*x^2*arctan(-3/2*sqrt(2)*x^2 + 1/2*sqrt(2)*sqrt(x^2 + 2)*s
qrt(3*x + 1)*sqrt(3*x - 1)) - 10*x^2*log(-18*x^2 + 6*sqrt(x^2 + 2)*sqrt(3*
x + 1)*sqrt(3*x - 1) - 17) + 18*x^2 + 6*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3
*x - 1))/x^2
```

Sympy [F]

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5x^4+3x^2+2}{x^3\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate((5*x**4+3*x**2+2)/x**3/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Integral((5*x**4 + 3*x**2 + 2)/(x**3*sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2
 + 2)), x)
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^3}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^3/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^3), x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^3}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^3/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{x^3\,\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/(x^3*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)),x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/(x^3*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{2+3x^2+5x^4}{x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$
$$=\frac{-69\sqrt{2}\,atan\left(\frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,\sqrt{2}}{18x^2-2}\right)x^2+6\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}+20\log\left(-3\sqrt{x^2+2}-\sqrt{3x+1}\,\sqrt{\right.}{12x^2}$$

input
```
int((5*x^4+3*x^2+2)/x^3/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output

```
( - 69*sqrt(2)*atan((sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*sqrt(2))/(
18*x**2 - 2))*x**2 + 6*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2) + 20*log
( - 3*sqrt(x**2 + 2) - sqrt(3*x + 1)*sqrt(3*x - 1))*x**2)/(12*x**2)
```

3.21 $\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Rubi [A] (verified) 240
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Fricas [A] (verification not implemented) 242
Sympy [F(-1)] 243
Maxima [F] 243
Giac [F] 243
Mupad [F(-1)] 244
Reduce [B] (verification not implemented) 244

Optimal result

Integrand size = 43, antiderivative size = 134

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{4x^4} + \frac{63\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{16x^2} - \frac{1223\sqrt{-1+9x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{2}\sqrt{-1+9x^2}}\right)}{16\sqrt{2}\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
1/4*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)/x^4+63/16*(-1+3*x)^(1/2)*(1
+3*x)^(1/2)*(x^2+2)^(1/2)/x^2-1223/32*2^(1/2)*(9*x^2-1)^(1/2)*arctan(1/2*(
x^2+2)^(1/2)*2^(1/2)/(9*x^2-1)^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{2\sqrt{2+x^2}(-4-27x^2+567x^4)-1223x^4\sqrt{-2+18x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{-2+18x^2}}\right)}{32x^4\sqrt{-1+3x}\sqrt{1+3x}}$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(x^5*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x
^2]),x]
```

output
```
(2*Sqrt[2 + x^2]*(-4 - 27*x^2 + 567*x^4) - 1223*x^4*Sqrt[-2 + 18*x^2]*ArcT
an[Sqrt[2 + x^2]/Sqrt[-2 + 18*x^2]])/(32*x^4*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{x^5\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

$$\Big\downarrow \text{ 2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x^5\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5}{x\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3}{x^3\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{2}{x^5\sqrt{x^2+2}\sqrt{9x^2-1}}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{9x^2-1}\left(-\frac{1223\arctan\left(\frac{\sqrt{x^2+2}}{\sqrt{2}\sqrt{9x^2-1}}\right)}{16\sqrt{2}}+\frac{63\sqrt{x^2+2}\sqrt{9x^2-1}}{16x^2}+\frac{\sqrt{x^2+2}\sqrt{9x^2-1}}{4x^4}\right)}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(x^5*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x
]
```

output
```
(Sqrt[-1 + 9*x^2]*((Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/(4*x^4) + (63*Sqrt[2 +
 x^2]*Sqrt[-1 + 9*x^2])/(16*x^2) - (1223*ArcTan[Sqrt[2 + x^2]/(Sqrt[2]*Sqr
t[-1 + 9*x^2])])/(16*Sqrt[2])))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.79

method	result
risch	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\,(63x^2+4)}{16x^4}+\frac{1223\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)\sqrt{(-1+3x)(1+3x)(x^2+2)}}{64\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(\frac{\sqrt{9x^4+17x^2-2}}{4x^4}+\frac{63\sqrt{9x^4+17x^2-2}}{16x^2}+\frac{1223\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)}{64}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
default	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\left(1223\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)x^4-9312\ln\left(\frac{17}{6}+3x^2+\sqrt{9x^4+17x^2-2}\right)x^4+9312\ln\left(6x^2+\frac{17}{3}+2\sqrt{9x^4\right.\right.}{64x^4\sqrt{9x^4+17x^2-2}}$

input
```
int((5*x^4+3*x^2+2)/x^5/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/16*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)*(63*x^2+4)/x^4+1223/64*2^(
1/2)*arctan(1/4*(17*x^2-4)*2^(1/2)/(9*x^4+17*x^2-2)^(1/2))*((-1+3*x)*(1+3*
x)*(x^2+2))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.63

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{1223\sqrt{2}x^4\arctan\left(-\frac{3}{2}\sqrt{2}x^2+\frac{1}{2}\sqrt{2}\sqrt{x^2+2}\sqrt{3x+1}\sqrt{3x-1}\right)+378x^4+2(63x^2+4)\sqrt{x^2+2}\sqrt{3x+}}{32x^4}$$

input
```
integrate((5*x^4+3*x^2+2)/x^5/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
1/32*(1223*sqrt(2)*x^4*arctan(-3/2*sqrt(2)*x^2 + 1/2*sqrt(2)*sqrt(x^2 + 2)
*sqrt(3*x + 1)*sqrt(3*x - 1)) + 378*x^4 + 2*(63*x^2 + 4)*sqrt(x^2 + 2)*sqr
t(3*x + 1)*sqrt(3*x - 1))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \text{Timed out}$$

input
```
integrate((5*x**4+3*x**2+2)/x**5/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^5}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^5/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^5), x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^5}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^5/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^5), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{x^5\,\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/(x^5*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)),x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/(x^5*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int \frac{2+3x^2+5x^4}{x^5\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{-1223\sqrt{2}\,atan\left(\frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,\sqrt{2}}{18x^2-2}\right)x^4+126\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^2+8\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{32x^4}$$

input
```
int((5*x^4+3*x^2+2)/x^5/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
( - 1223*sqrt(2)*atan((sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*sqrt(2))
/(18*x**2 - 2))*x**4 + 126*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**2
 + 8*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2))/(32*x**4)
```

3.22 $\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

Optimal result 245
Mathematica [A] (verified) 246
Rubi [A] (verified) 246
Maple [A] (verified) 248
Fricas [A] (verification not implemented) 248
Sympy [F(-1)] 249
Maxima [F] 249
Giac [F] 249
Mupad [F(-1)] 250
Reduce [B] (verification not implemented) 250

Optimal result

Integrand size = 43, antiderivative size = 168

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{6x^6} + \frac{103\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{48x^4} + \frac{1927\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}{64x^2} - \frac{35231\sqrt{-1+9x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{2}\sqrt{-1+9x^2}}\right)}{64\sqrt{2}\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
1/6*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)/x^6+103/48*(-1+3*x)^(1/2)*(
1+3*x)^(1/2)*(x^2+2)^(1/2)/x^4+1927/64*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2
)^(1/2)/x^2-35231/128*2^(1/2)*(9*x^2-1)^(1/2)*arctan(1/2*(x^2+2)^(1/2)*2^(
1/2)/(9*x^2-1)^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 10.07 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.55

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{2\sqrt{2+x^2}(-32-124x^2-2073x^4+52029x^6)-105693x^6\sqrt{-2+18x^2}\arctan\left(\frac{\sqrt{2+x^2}}{\sqrt{-2+18x^2}}\right)}{384x^6\sqrt{-1+3x}\sqrt{1+3x}}$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(x^7*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x
^2]),x]
```

output
```
(2*Sqrt[2 + x^2]*(-32 - 124*x^2 - 2073*x^4 + 52029*x^6) - 105693*x^6*Sqrt[
-2 + 18*x^2]*ArcTan[Sqrt[2 + x^2]/Sqrt[-2 + 18*x^2]])/(384*x^6*Sqrt[-1 + 3
*x]*Sqrt[1 + 3*x])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{x^7\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

$$\Big\downarrow \text{2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x^7\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5}{x^3\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3}{x^5\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{2}{x^7\sqrt{x^2+2}\sqrt{9x^2-1}}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{9x^2-1}\left(-\frac{35231\arctan\left(\frac{\sqrt{x^2+2}}{\sqrt{2}\sqrt{9x^2-1}}\right)}{64\sqrt{2}}+\frac{1927\sqrt{x^2+2}\sqrt{9x^2-1}}{64x^2}+\frac{\sqrt{x^2+2}\sqrt{9x^2-1}}{6x^6}+\frac{103\sqrt{x^2+2}\sqrt{9x^2-1}}{48x^4}\right)}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(x^7*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x
]
```

output
```
(Sqrt[-1 + 9*x^2]*((Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/(6*x^6) + (103*Sqrt[2
+ x^2]*Sqrt[-1 + 9*x^2])/(48*x^4) + (1927*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/
(64*x^2) - (35231*ArcTan[Sqrt[2 + x^2]/(Sqrt[2]*Sqrt[-1 + 9*x^2])])/(64*Sq
rt[2])))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

method	result
risch	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\,(5781x^4+412x^2+32)}{192x^6}+\frac{35231\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)\sqrt{(-1+3x)(1+3x)(x^2+2)}}{256\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(\frac{\sqrt{9x^4+17x^2-2}}{6x^6}+\frac{103\sqrt{9x^4+17x^2-2}}{48x^4}+\frac{1927\sqrt{9x^4+17x^2-2}}{64x^2}+\frac{35231\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)}{256}\right)}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
default	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\left(105693\sqrt{2}\,\arctan\left(\frac{(17x^2-4)\sqrt{2}}{4\sqrt{9x^4+17x^2-2}}\right)x^6-1005696\ln\left(\frac{17}{6}+3x^2+\sqrt{9x^4+17x^2-2}\right)x^6+1005696\ln\left(6x^2+\right.\right.}{768\sqrt{9x^4+17x^2-2}\,x^6}$

input
```
int((5*x^4+3*x^2+2)/x^7/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/192*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)*(5781*x^4+412*x^2+32)/x^6
+35231/256*2^(1/2)*arctan(1/4*(17*x^2-4)*2^(1/2)/(9*x^4+17*x^2-2)^(1/2))*(
(-1+3*x)*(1+3*x)*(x^2+2))^(1/2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{105693\,\sqrt{2}x^6\arctan\left(-\frac{3}{2}\,\sqrt{2}x^2+\frac{1}{2}\,\sqrt{2}\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}\right)+34686\,x^6+2\,(5781\,x^4+412\,x^2+3}{384\,x^6}$$

input
```
integrate((5*x^4+3*x^2+2)/x^7/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
1/384*(105693*sqrt(2)*x^6*arctan(-3/2*sqrt(2)*x^2 + 1/2*sqrt(2)*sqrt(x^2 +
 2)*sqrt(3*x + 1)*sqrt(3*x - 1)) + 34686*x^6 + 2*(5781*x^4 + 412*x^2 + 32)
*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1))/x^6
```

Sympy [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \text{Timed out}$$

input
```
integrate((5*x**4+3*x**2+2)/x**7/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^7}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^7/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^7), x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^7}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^7/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^7), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{x^7\,\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/(x^7*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)),x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/(x^7*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{2+3x^2+5x^4}{x^7\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$
$$= \frac{-105693\sqrt{2}\,atan\left(\frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,\sqrt{2}}{18x^2-2}\right)x^6+11562\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^4+824\sqrt{3x+1}\,\sqrt{3x-}}{384x^6}$$

input
```
int((5*x^4+3*x^2+2)/x^7/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
( - 105693*sqrt(2)*atan((sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*sqrt(2
))/(18*x**2 - 2))*x**6 + 11562*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*
x**4 + 824*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**2 + 64*sqrt(3*x +
 1)*sqrt(3*x - 1)*sqrt(x**2 + 2))/(384*x**6)
```

3.23 $\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

Optimal result . . . 251
Mathematica [C] (warning: unable to verify) . . . 252
Rubi [A] (verified) . . . 252
Maple [A] (verified) . . . 254
Fricas [A] (verification not implemented) . . . 254
Sympy [F] . . . 255
Maxima [F] . . . 255
Giac [F] . . . 255
Mupad [F(-1)] . . . 256
Reduce [F] . . . 256

Optimal result

Integrand size = 43, antiderivative size = 159

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = -\frac{41}{243}x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2} + \frac{1}{9}x^3\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2} + \frac{2042\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)\left|-\frac{1}{18}\right.\right)}{729\sqrt{-1+3x}\sqrt{1+3x}} - \frac{2083\sqrt{2}\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{729\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
-41/243*x*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)+1/9*x^3*(-1+3*x)^(1/2
)*(1+3*x)^(1/2)*(x^2+2)^(1/2)+2042/729*2^(1/2)*(-9*x^2+1)^(1/2)*EllipticE(
3*x,1/6*I*2^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)-2083/729*2^(1/2)*(-9*x^2+1
)^(1/2)*EllipticF(3*x,1/6*I*2^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.58 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.86

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$

$$= \frac{\sqrt{\frac{-1+3x}{1+3x}}\left(9x(1+3x)\left(2+x^2\right)\left(-41+27x^2\right)+6126\left(2+x^2\right)\left(1+\frac{2}{-1+3x}\right)-\frac{2042i(1+3x)\sqrt{\frac{2+x^2}{(1-3x)^2}}E\left(\arcsin\left(\frac{\sqrt{-}}{}\right.\right.}{\sqrt{\frac{i(1+3x)}{\left(17i+6\sqrt{2}\right)(-1}}}\right.}{2}$$

input
```
Integrate[(x^2*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 +
 x^2]),x]
```

output
```
(Sqrt[(-1 + 3*x)/(1 + 3*x)]*(9*x*(1 + 3*x)*(2 + x^2)*(-41 + 27*x^2) + 6126
*(2 + x^2)*(1 + 2/(-1 + 3*x)) - ((2042*I)*(1 + 3*x)*Sqrt[(2 + x^2)/(1 - 3*
x)^2]*EllipticE[ArcSin[Sqrt[-1/3*I + Sqrt[2] + (19*I)/(3 - 9*x)]/2^(3/4)],
 (12*Sqrt[2])/(17*I + 6*Sqrt[2])])/Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])*
(-1 + 3*x))] + (36190*I)*(-1 + 3*x)*Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])
*(-1 + 3*x))]*Sqrt[(2 + x^2)/(1 - 3*x)^2]*EllipticF[ArcSin[Sqrt[-1/3*I + S
qrt[2] + (19*I)/(3 - 9*x)]/2^(3/4)], (12*Sqrt[2])/(17*I + 6*Sqrt[2])]))/(2
187*Sqrt[2 + x^2])
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(5x^4+3x^2+2\right)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

$$\Big\downarrow \; 2038$$

$$\frac{\sqrt{9x^2-1}\int \frac{x^2(5x^4+3x^2+2)}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \ 7293$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5x^6}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3x^4}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{2x^2}{\sqrt{x^2+2}\sqrt{9x^2-1}}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{9x^2-1}\left(-\frac{2083\sqrt{2}\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{729\sqrt{9x^2-1}}+\frac{2042\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)\middle|-\frac{1}{18}\right)}{729\sqrt{9x^2-1}}-\frac{41}{243}\sqrt{x^2+2}\sqrt{9x^2-1}x+\frac{1}{9}\sqrt{x}\right.}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(x^2*(2 + 3*x^2 + 5*x^4))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2])
,x]
```

output
```
(Sqrt[-1 + 9*x^2]*((-41*x*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/243 + (x^3*Sqrt[
2 + x^2]*Sqrt[-1 + 9*x^2])/9 + (2042*Sqrt[2]*Sqrt[1 - 9*x^2]*EllipticE[Arc
Sin[3*x], -1/18])/(729*Sqrt[-1 + 9*x^2]) - (2083*Sqrt[2]*Sqrt[1 - 9*x^2]*E
llipticF[ArcSin[3*x], -1/18])/(729*Sqrt[-1 + 9*x^2])))/(Sqrt[-1 + 3*x]*Sqr
t[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

method	result
default	$-\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\,\left(-2187x^7+1304i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)-2042i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\,\operatorname{EllipticE}\left(\frac{ix\sqrt{2}}{2}\right.\right.}{2187(9x^4+17x^2-2)}$
risch	$\frac{x(27x^2-41)\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{243}+\frac{\left(\frac{41i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)}{243\sqrt{9x^4+17x^2-2}}-\frac{1021i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,\left(\operatorname{EllipticF}\left(\right.\right.}{2187\sqrt{9x^4+}}\right.}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\,\left(-\frac{41x\sqrt{9x^4+17x^2-2}}{243}+\frac{41i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,\operatorname{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)}{243\sqrt{9x^4+17x^2-2}}-\frac{1021i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,\left(\operatorname{EllipticF}\left(\frac{ix\sqrt{}}{2}\right.\right.}{2187\sqrt{9x^4+17}}\right.}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$

input
```
int(x^2*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2187*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)*(-2187*x^7+1304*I*(x^2+
2)^(1/2)*(-9*x^2+1)^(1/2)*EllipticF(1/2*I*x*2^(1/2),3*I*2^(1/2))-2042*I*(x
^2+2)^(1/2)*(-9*x^2+1)^(1/2)*EllipticE(1/2*I*x*2^(1/2),3*I*2^(1/2))-810*x^
5+6759*x^3-738*x)/(9*x^4+17*x^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{9\,(243\,x^4-369\,x^2+2042)\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}+2042\,xE(\arcsin\left(\frac{1}{3\,x}\right)\,|-18)+4600\,xF(\arcsin\left(\right.}{19683\,x}$$

input
```
integrate(x^2*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
1/19683*(9*(243*x^4 - 369*x^2 + 2042)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x
 - 1) + 2042*x*elliptic_e(arcsin(1/3/x), -18) + 4600*x*elliptic_f(arcsin(1
/3/x), -18))/x
```

Sympy [F]

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x^2\cdot(5x^4+3x^2+2)}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate(x**2*(5*x**4+3*x**2+2)/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Integral(x**2*(5*x**4 + 3*x**2 + 2)/(sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2
 + 2)), x)
```

Maxima [F]

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x^2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x^2*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x^2/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x -
1)), x)
```

Giac [F]

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{(5\,x^4+3\,x^2+2)x^2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate(x^2*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)*x^2/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x -
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{x^2\,(5\,x^4+3\,x^2+2)}{\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((x^2*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(
1/2)),x)
```

output
```
int((x^2*(3*x^2 + 5*x^4 + 2))/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(2+3x^2+5x^4)}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^3}{9} - \frac{41\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x}{243} + \frac{2042\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^2}{9x^4+17x^2-2}dx\right)}{243} - \frac{82\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{9x^4+17x^2-2}dx\right)}{243}$$

input
```
int(x^2*(5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
(27*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x**3 - 41*sqrt(3*x + 1)*sqr
t(3*x - 1)*sqrt(x**2 + 2)*x + 2042*int((sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x
**2 + 2)*x**2)/(9*x**4 + 17*x**2 - 2),x) - 82*int((sqrt(3*x + 1)*sqrt(3*x
- 1)*sqrt(x**2 + 2))/(9*x**4 + 17*x**2 - 2),x))/243
```

3.24 $\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

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Rubi [A] (verified) 259
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Mupad [F(-1)] 262
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Optimal result

Integrand size = 40, antiderivative size = 125

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5}{27}x\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2} - \frac{89\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)\left|-\frac{1}{18}\right.\right)}{81\sqrt{-1+3x}\sqrt{1+3x}} + \frac{121\sqrt{2}\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{81\sqrt{-1+3x}\sqrt{1+3x}}$$

output
```
5/27*x*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)-89/81*2^(1/2)*(-9*x^2+1)
^(1/2)*EllipticE(3*x,1/6*I*2^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)+121/81*2^
(1/2)*(-9*x^2+1)^(1/2)*EllipticF(3*x,1/6*I*2^(1/2))/(-1+3*x)^(1/2)/(1+3*x)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.34 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.30

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$

$$= \frac{1}{243}\sqrt{\frac{-1+3x}{1+3x}}\sqrt{2+x^2}\left(45x(1+3x)-267\left(1+\frac{2}{-1+3x}\right)\right.$$
$$+\frac{89i(1+3x)\sqrt{\frac{2+x^2}{(1-3x)^2}}E\left(\arcsin\left(\frac{\sqrt{-\frac{i}{3}+\sqrt{2}+\frac{19i}{3-9x}}}{2^{3/4}}\right)|\frac{12\sqrt{2}}{17i+6\sqrt{2}}\right)}{\sqrt{\frac{i(1+3x)}{(17i+6\sqrt{2})(-1+3x)}}(2+x^2)}$$
$$\left.-\frac{2665i\sqrt{\frac{i(1+3x)}{(17i+6\sqrt{2})(-1+3x)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-\frac{i}{3}+\sqrt{2}+\frac{19i}{3-9x}}}{2^{3/4}}\right),\frac{12\sqrt{2}}{17i+6\sqrt{2}}\right)}{(-1+3x)\sqrt{\frac{2+x^2}{(1-3x)^2}}}\right)$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2])
,x]
```

output
```
(Sqrt[(-1 + 3*x)/(1 + 3*x)]*Sqrt[2 + x^2]*(45*x*(1 + 3*x) - 267*(1 + 2/(-1
 + 3*x)) + ((89*I)*(1 + 3*x)*Sqrt[(2 + x^2)/(1 - 3*x)^2]*EllipticE[ArcSin[
Sqrt[-1/3*I + Sqrt[2] + (19*I)/(3 - 9*x)]/2^(3/4)], (12*Sqrt[2])/(17*I + 6
*Sqrt[2])])/(Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])*(-1 + 3*x))]*(2 + x^2)
) - ((2665*I)*Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])*(-1 + 3*x))]*Elliptic
F[ArcSin[Sqrt[-1/3*I + Sqrt[2] + (19*I)/(3 - 9*x)]/2^(3/4)], (12*Sqrt[2])/
(17*I + 6*Sqrt[2])])/((-1 + 3*x)*Sqrt[(2 + x^2)/(1 - 3*x)^2])))/243
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{ 2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5x^4}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3x^2}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{2}{\sqrt{x^2+2}\sqrt{9x^2-1}}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{121\sqrt{2}\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{81\sqrt{9x^2-1}}-\frac{89\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)|-\frac{1}{18}\right)}{81\sqrt{9x^2-1}}+\frac{5}{27}\sqrt{x^2+2}\sqrt{9x^2-1}x\right)}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x]
```

output
```
(Sqrt[-1 + 9*x^2]*((5*x*Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/27 - (89*Sqrt[2]*S
qrt[1 - 9*x^2]*EllipticE[ArcSin[3*x], -1/18])/(81*Sqrt[-1 + 9*x^2]) + (121
*Sqrt[2]*Sqrt[1 - 9*x^2]*EllipticF[ArcSin[3*x], -1/18])/(81*Sqrt[-1 + 9*x^
2])))/(Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 4.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\,(-405x^5+487i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}(\frac{ix\sqrt{2}}{2},3i\sqrt{2})+89i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\ \mathrm{EllipticE}(\frac{ix\sqrt{2}}{2},3i\sqrt{}}{243(9x^4+17x^2-2)}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\,(\frac{5x\sqrt{9x^4+17x^2-2}}{27}-\frac{32i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}(\frac{ix\sqrt{2}}{2},3i\sqrt{2})}{27\sqrt{9x^4+17x^2-2}}+\frac{89i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,(\mathrm{EllipticF}(\frac{ix\sqrt{2}}{2},3i\sqrt{}}{486\sqrt{9x^4+17x^2-2}}}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
risch	$\frac{5x\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{27}+\frac{(-\frac{32i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}(\frac{ix\sqrt{2}}{2},3i\sqrt{2})}{27\sqrt{9x^4+17x^2-2}}+\frac{89i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\,(\mathrm{EllipticF}(\frac{ix\sqrt{2}}{2},3i\sqrt{2})}{486\sqrt{9x^4+17x^2-2}}}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$

input
```
int((5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
-1/243*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)*(-405*x^5+487*I*(x^2+2)^
(1/2)*(-9*x^2+1)^(1/2)*EllipticF(1/2*I*x*2^(1/2),3*I*2^(1/2))+89*I*(x^2+2)
^(1/2)*(-9*x^2+1)^(1/2)*EllipticE(1/2*I*x*2^(1/2),3*I*2^(1/2))-765*x^3+90*
x)/(9*x^4+17*x^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{9\,(45\,x^2-89)\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}-89\,xE(\arcsin\left(\frac{1}{3\,x}\right)\,|-18)-5095\,xF(\arcsin\left(\frac{1}{3\,x}\right)\,|-18)}{2187\,x}$$

input
```
integrate((5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x, al
gorithm="fricas")
```

output
```
1/2187*(9*(45*x^2 - 89)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1) - 89*x*e
lliptic_e(arcsin(1/3/x), -18) - 5095*x*elliptic_f(arcsin(1/3/x), -18))/x
```

Sympy [F]

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5x^4+3x^2+2}{\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate((5*x**4+3*x**2+2)/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**(1/2)
,x)
```

output
```
Integral((5*x**4 + 3*x**2 + 2)/(sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2 + 2)
), x)
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)),
 x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx=\int \frac{5\,x^4+3\,x^2+2}{\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/2)),
x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/((3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{2+3x^2+5x^4}{\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x}{27} - \frac{89\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}\,x^2}{9x^4+17x^2-2}dx\right)}{27} + \frac{64\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{9x^4+17x^2-2}dx\right)}{27}$$

input

```
int((5*x^4+3*x^2+2)/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output

```
(5*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2)*x - 89*int((sqrt(3*x + 1)*sq
rt(3*x - 1)*sqrt(x**2 + 2)*x**2)/(9*x**4 + 17*x**2 - 2),x) + 64*int((sqrt(
3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2))/(9*x**4 + 17*x**2 - 2),x))/27
```

3.25 $\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$

Optimal result 264
Mathematica [C] (warning: unable to verify) 265
Rubi [A] (verified) 265
Maple [A] (verified) 267
Fricas [F] 267
Sympy [F] 268
Maxima [F] 268
Giac [F] 268
Mupad [F(-1)] 269
Reduce [F] 269

Optimal result

Integrand size = 43, antiderivative size = 127

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = -\frac{(1-9x^2)\sqrt{2+x^2}}{x\sqrt{-1+3x}\sqrt{1+3x}} - \frac{4\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)\left|-\frac{1}{18}\right.\right)}{3\sqrt{-1+3x}\sqrt{1+3x}} + \frac{11\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{3\sqrt{1+3x}\sqrt{-2+6x}}$$

output
```
-(-9*x^2+1)*(x^2+2)^(1/2)/x/(-1+3*x)^(1/2)/(1+3*x)^(1/2)-4/3*2^(1/2)*(-9*x
^2+1)^(1/2)*EllipticE(3*x,1/6*I*2^(1/2))/(-1+3*x)^(1/2)/(1+3*x)^(1/2)+11/3
*(-9*x^2+1)^(1/2)*EllipticF(3*x,1/6*I*2^(1/2))/(1+3*x)^(1/2)/(-2+6*x)^(1/2
)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.68 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.28

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx$$
$$= \frac{\sqrt{\frac{-1+3x}{1+3x}}\left(\frac{9(1+3x)(2+x^2)}{x}-12(2+x^2)\left(1+\frac{2}{-1+3x}\right)+\frac{4i(1+3x)\sqrt{\frac{2+x^2}{(1-3x)^2}}E\left(\arcsin\left(\frac{\sqrt{-\frac{i}{3}+\sqrt{2}+\frac{19i}{3-9x}}}{2^{3/4}}\right)|\frac{12\sqrt{2}}{17i+6\sqrt{2}}\right)}{\sqrt{\frac{i(1+3x)}{(17i+6\sqrt{2})(-1+3x)}}}-122\ldots\right.}{9\sqrt{2+x^2}}$$

input
```
Integrate[(2 + 3*x^2 + 5*x^4)/(x^2*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x
^2]),x]
```

output
```
(Sqrt[(-1 + 3*x)/(1 + 3*x)]*((9*(1 + 3*x)*(2 + x^2))/x - 12*(2 + x^2)*(1 +
 2/(-1 + 3*x)) + ((4*I)*(1 + 3*x)*Sqrt[(2 + x^2)/(1 - 3*x)^2]*EllipticE[Ar
cSin[Sqrt[-1/3*I + Sqrt[2] + (19*I)/(3 - 9*x)]/2^(3/4)], (12*Sqrt[2])/(17*
I + 6*Sqrt[2])])/Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])*(-1 + 3*x))] - (12
2*I)*(-1 + 3*x)*Sqrt[(I*(1 + 3*x))/((17*I + 6*Sqrt[2])*(-1 + 3*x))]*Sqrt[(
2 + x^2)/(1 - 3*x)^2]*EllipticF[ArcSin[Sqrt[-1/3*I + Sqrt[2] + (19*I)/(3 -
 9*x)]/2^(3/4)], (12*Sqrt[2])/(17*I + 6*Sqrt[2])]))/(9*Sqrt[2 + x^2])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2038, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^4+3x^2+2}{x^2\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$
$$\Big\downarrow \text{2038}$$

$$\frac{\sqrt{9x^2-1}\int \frac{5x^4+3x^2+2}{x^2\sqrt{x^2+2}\sqrt{9x^2-1}}dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{\sqrt{9x^2-1}\int \left(\frac{5x^2}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{3}{\sqrt{x^2+2}\sqrt{9x^2-1}}+\frac{2}{\sqrt{x^2+2}\sqrt{9x^2-1}x^2}\right)dx}{\sqrt{3x-1}\sqrt{3x+1}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{9x^2-1}\left(\frac{4\sqrt{2}\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{3\sqrt{9x^2-1}}+\frac{\sqrt{1-9x^2}\,\text{EllipticF}\left(\arcsin(3x),-\frac{1}{18}\right)}{\sqrt{2}\sqrt{9x^2-1}}-\frac{4\sqrt{2}\sqrt{1-9x^2}E\left(\arcsin(3x)|-\frac{1}{18}\right)}{3\sqrt{9x^2-1}}+\sqrt{x^2+2}\right.}{\sqrt{3x-1}\sqrt{3x+1}}$$

input
```
Int[(2 + 3*x^2 + 5*x^4)/(x^2*Sqrt[-1 + 3*x]*Sqrt[1 + 3*x]*Sqrt[2 + x^2]),x
]
```

output
```
(Sqrt[-1 + 9*x^2]*((Sqrt[2 + x^2]*Sqrt[-1 + 9*x^2])/x - (4*Sqrt[2]*Sqrt[1
- 9*x^2]*EllipticE[ArcSin[3*x], -1/18])/(3*Sqrt[-1 + 9*x^2]) + (Sqrt[1 - 9
*x^2]*EllipticF[ArcSin[3*x], -1/18])/(Sqrt[2]*Sqrt[-1 + 9*x^2]) + (4*Sqrt[
2]*Sqrt[1 - 9*x^2]*EllipticF[ArcSin[3*x], -1/18])/(3*Sqrt[-1 + 9*x^2])))/(
Sqrt[-1 + 3*x]*Sqrt[1 + 3*x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2038
```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Simp[(a1 + b1*x^(n/2))
^FracPart[p]*((a2 + b2*x^(n/2))^FracPart[p]/(a1*a2 + b1*b2*x^n)^FracPart[p]
)   Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
 b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(Eq
Q[n, 2] && IGtQ[q, 0])
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 4.88 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
default	$-\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}\left(23i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)x+4i\sqrt{x^2+2}\,\sqrt{-9x^2+1}\,x\,\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)-81x^4 \cdots\right.}{9x(9x^4+17x^2-2)}$
elliptic	$\frac{\sqrt{(x^2+2)(9x^2-1)}\left(-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)}{2\sqrt{9x^4+17x^2-2}}+\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)-\mathrm{EllipticE}\left(\frac{ix\sqrt{2}}{2} \cdots\right.\right.}{9\sqrt{9x^4+17x^2-2}}\right.}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$
risch	$\frac{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}{x}+\frac{\left(-\frac{3i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\ \mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)}{2\sqrt{9x^4+17x^2-2}}+\frac{2i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{-9x^2+1}\left(\mathrm{EllipticF}\left(\frac{ix\sqrt{2}}{2},3i\sqrt{2}\right)-\mathrm{Elli}\cdots\right.}{9\sqrt{9x^4+17x^2-2}}\right.}{\sqrt{-1+3x}\,\sqrt{1+3x}\,\sqrt{x^2+2}}$

input
```
int((5*x^4+3*x^2+2)/x^2/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/9*(-1+3*x)^(1/2)*(1+3*x)^(1/2)*(x^2+2)^(1/2)*(23*I*(x^2+2)^(1/2)*(-9*x^
2+1)^(1/2)*EllipticF(1/2*I*x*2^(1/2),3*I*2^(1/2))*x+4*I*(x^2+2)^(1/2)*(-9*
x^2+1)^(1/2)*x*EllipticE(1/2*I*x*2^(1/2),3*I*2^(1/2))-81*x^4-153*x^2+18)/x
/(9*x^4+17*x^2-2)
```

Fricas [F]

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^2}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^2/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="fricas")
```

output
```
integral((5*x^4 + 3*x^2 + 2)*sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)/(9*
x^6 + 17*x^4 - 2*x^2), x)
```

Sympy [F]

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5x^4+3x^2+2}{x^2\sqrt{3x-1}\sqrt{3x+1}\sqrt{x^2+2}}\,dx$$

input
```
integrate((5*x**4+3*x**2+2)/x**2/(-1+3*x)**(1/2)/(1+3*x)**(1/2)/(x**2+2)**
(1/2),x)
```

output
```
Integral((5*x**4 + 3*x**2 + 2)/(x**2*sqrt(3*x - 1)*sqrt(3*x + 1)*sqrt(x**2
 + 2)), x)
```

Maxima [F]

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^2}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^2/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^2), x)
```

Giac [F]

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{\sqrt{x^2+2}\sqrt{3\,x+1}\sqrt{3\,x-1}x^2}\,dx$$

input
```
integrate((5*x^4+3*x^2+2)/x^2/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x
, algorithm="giac")
```

output
```
integrate((5*x^4 + 3*x^2 + 2)/(sqrt(x^2 + 2)*sqrt(3*x + 1)*sqrt(3*x - 1)*x
^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \int \frac{5\,x^4+3\,x^2+2}{x^2\,\sqrt{3\,x-1}\,\sqrt{3\,x+1}\,\sqrt{x^2+2}}\,dx$$

input
```
int((3*x^2 + 5*x^4 + 2)/(x^2*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)),x)
```

output
```
int((3*x^2 + 5*x^4 + 2)/(x^2*(3*x - 1)^(1/2)*(3*x + 1)^(1/2)*(x^2 + 2)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{2+3x^2+5x^4}{x^2\sqrt{-1+3x}\sqrt{1+3x}\sqrt{2+x^2}}\,dx = \frac{5\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}+8\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{9x^6+17x^4-2x^2}dx\right)x+27\left(\int \frac{\sqrt{3x+1}\,\sqrt{3x-1}\,\sqrt{x^2+2}}{9x^4+17x^2-2}dx\right)x}{9x}$$

input
```
int((5*x^4+3*x^2+2)/x^2/(-1+3*x)^(1/2)/(1+3*x)^(1/2)/(x^2+2)^(1/2),x)
```

output
```
(5*sqrt(3*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2) + 8*int((sqrt(3*x + 1)*sqrt(
3*x - 1)*sqrt(x**2 + 2))/(9*x**6 + 17*x**4 - 2*x**2),x)*x + 27*int((sqrt(3
*x + 1)*sqrt(3*x - 1)*sqrt(x**2 + 2))/(9*x**4 + 17*x**2 - 2),x)*x)/(9*x)
```

3.26 $\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$

Optimal result 270
Mathematica [A] (verified) 270
Rubi [A] (verified) 271
Maple [A] (verified) 274
Fricas [A] (verification not implemented) 274
Sympy [F] 275
Maxima [F] 276
Giac [A] (verification not implemented) 276
Mupad [F(-1)] 277
Reduce [B] (verification not implemented) 277

Optimal result

Integrand size = 41, antiderivative size = 175

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\sqrt{2}(Bc-Ad)\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}d(de-cf)} - \frac{2(Be-Af)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc^2-bd^2x^2}}{\sqrt{b}\sqrt{de+cf}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{f}(de-cf)\sqrt{de+cf}}$$

output
```
2^(1/2)*(-A*d+B*c)*arctanh(1/2*(-b*d^2*x^2+b*c^2)^(1/2)*2^(1/2)/b^(1/2)/c^
(1/2)/(d*x+c)^(1/2))/b^(1/2)/c^(1/2)/d/(-c*f+d*e)-2*(-A*f+B*e)*arctanh(f^(
1/2)*(-b*d^2*x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)^(1/2)/(d*x+c)^(1/2))/b^(1/
2)/f^(1/2)/(-c*f+d*e)/(c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = -\frac{\sqrt{c^2-d^2x^2}\left(2\sqrt{c}d(-Be+Af)\arctan\left(\frac{\sqrt{-de-cf}\sqrt{c^2-d^2x^2}}{\sqrt{f}(c-dx)\sqrt{c+dx}}\right)+\sqrt{2}(Bc-Ad)\sqrt{f}\sqrt{-de-cf}\text{arctanh}\left(\frac{\sqrt{2}\cdots}{\sqrt{\cdots}}\right.\right.}{\sqrt{c}d\sqrt{f}\sqrt{-de-cf}(-de+cf)\sqrt{b(c^2-d^2x^2)}}$$

input
```
Integrate[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 - b*d^2*x^2]),x]
```

output
```
-((Sqrt[c^2 - d^2*x^2]*(2*Sqrt[c]*d*(-(B*e) + A*f)*ArcTan[(Sqrt[-(d*e) - c
*f]*Sqrt[c^2 - d^2*x^2])/(Sqrt[f]*(c - d*x)*Sqrt[c + d*x])] + Sqrt[2]*(B*c
 - A*d)*Sqrt[f]*Sqrt[-(d*e) - c*f]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[c + d*x])
/Sqrt[c^2 - d^2*x^2]]))/(Sqrt[c]*d*Sqrt[f]*Sqrt[-(d*e) - c*f]*(-(d*e) + c*
f)*Sqrt[b*(c^2 - d^2*x^2)]))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {2349, 27, 471, 221, 718, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+\int \frac{B}{f\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+\frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{f}$$

$$\Big\downarrow \text{471}$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+\frac{2Bd\int \frac{1}{\frac{d^2\left(bc^2-bd^2x^2\right)}{c+dx}-2bcd^2}d\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{c+dx}}}{f}$$

$$\Big\downarrow \text{221}$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{2}B\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df}$$

$$\Big\downarrow \text{718}$$

$$\frac{\sqrt{c+dx}\left(A-\frac{Be}{f}\right)\sqrt{bc-bdx}\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{2}B\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df}$$

$$\Big\downarrow \text{97}$$

$$\frac{\sqrt{c+dx}\left(A-\frac{Be}{f}\right)\sqrt{bc-bdx}\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{2}B\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{c+dx}\left(A-\frac{Be}{f}\right)\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{2}B\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{c+dx}\left(A-\frac{Be}{f}\right)\sqrt{bc-bdx}\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{2}B\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df}$$

input
```
Int[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 - b*d^2*x^2]),x]
```

output
```
((A - (B*e)/f)*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[Sqrt[b*
c - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f))) + (2
*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])/(
Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/Sqrt[b*c^2 - b*d^2*x^2] - (Sqrt[2]*
B*ArcTanh[Sqrt[b*c^2 - b*d^2*x^2]/(Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[c + d*x])]
)/(Sqrt[b]*Sqrt[c]*d*f)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.23

method	result
default	$\frac{\sqrt{b(-d^2x^2+c^2)}\left(A\operatorname{arctanh}\left(\frac{\sqrt{(-dx+c)b}\,\sqrt{2}}{2\sqrt{bc}}\right)\sqrt{b(cf+de)f}\,\sqrt{2}\,d-2A\operatorname{arctanh}\left(\frac{f\sqrt{(-dx+c)b}}{\sqrt{b(cf+de)f}}\right)\sqrt{bc}\,df-B\operatorname{arctanh}\left(\frac{\sqrt{(-dx+c)b}\,\sqrt{2}}{2\sqrt{bc}}\right.\right.}{\sqrt{dx+c}\,\sqrt{(-dx+c)b}\,d(cf-de)\sqrt{b(cf+de)f}\,\sqrt{bc}}$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
(b*(-d^2*x^2+c^2))^(1/2)*(A*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(
1/2))*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*d-2*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*
(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*d*f-B*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/
2)/(b*c)^(1/2))*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*c+2*B*arctanh(f*((-d*x+c)*b)
^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*d*e)/(d*x+c)^(1/2)/((-d*x+c)*b)^
(1/2)/d/(c*f-d*e)/(b*(c*f+d*e)*f)^(1/2)/(b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1006, normalized size of antiderivative = 5.75

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x, algori
thm="fricas")
```

output
```
[1/2*(sqrt(2)*((B*b*c*d - A*b*d^2)*e*f + (B*b*c^2 - A*b*c*d)*f^2)*sqrt(1/(
b*c))*log(-(d^2*x^2 - 2*c*d*x - 2*sqrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*
x + c)*c*sqrt(1/(b*c)) - 3*c^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*sqrt(b*d*e*
f + b*c*f^2)*(B*d*e - A*d*f)*log(-(b*d^2*f*x^2 - b*c*d*e - 2*b*c^2*f - (b*
d^2*e + b*c*d*f)*x + 2*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(b*d*e*f + b*c*f^2)*sq
rt(d*x + c))/(d*f*x^2 + c*e + (d*e + c*f)*x)))/(b*d^3*e^2*f - b*c^2*d*f^3)
, (sqrt(2)*((B*b*c*d - A*b*d^2)*e*f + (B*b*c^2 - A*b*c*d)*f^2)*sqrt(-1/(b*
c))*arctan(sqrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(-1/(b*c))
/(d^2*x^2 - c^2)) + sqrt(b*d*e*f + b*c*f^2)*(B*d*e - A*d*f)*log(-(b*d^2*f*
x^2 - b*c*d*e - 2*b*c^2*f - (b*d^2*e + b*c*d*f)*x + 2*sqrt(-b*d^2*x^2 + b*
c^2)*sqrt(b*d*e*f + b*c*f^2)*sqrt(d*x + c))/(d*f*x^2 + c*e + (d*e + c*f)*x
)))/(b*d^3*e^2*f - b*c^2*d*f^3), 1/2*(sqrt(2)*((B*b*c*d - A*b*d^2)*e*f + (
B*b*c^2 - A*b*c*d)*f^2)*sqrt(1/(b*c))*log(-(d^2*x^2 - 2*c*d*x - 2*sqrt(2)*
sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(1/(b*c)) - 3*c^2)/(d^2*x^2 +
 2*c*d*x + c^2)) + 4*sqrt(-b*d*e*f - b*c*f^2)*(B*d*e - A*d*f)*arctan(sqrt(
-b*d^2*x^2 + b*c^2)*sqrt(-b*d*e*f - b*c*f^2)*sqrt(d*x + c)/(b*c*d*e + b*c^
2*f + (b*d^2*e + b*c*d*f)*x)))/(b*d^3*e^2*f - b*c^2*d*f^3), (sqrt(2)*((B*b
*c*d - A*b*d^2)*e*f + (B*b*c^2 - A*b*c*d)*f^2)*sqrt(-1/(b*c))*arctan(sqrt(
2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(-1/(b*c))/(d^2*x^2 - c^2)
) + 2*sqrt(-b*d*e*f - b*c*f^2)*(B*d*e - A*d*f)*arctan(sqrt(-b*d^2*x^2 +...
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{A+Bx}{\sqrt{-b\left(-c+dx\right)\left(c+dx\right)}\sqrt{c+dx}\left(e+fx\right)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*d**2*x**2+b*c**2)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(-b*(-c + d*x)*(c + d*x))*sqrt(c + d*x)*(e + f*x))
, x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*(f*x + e)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = -\frac{\frac{\sqrt{2}(Bc-Ad)\arctan\left(\frac{\sqrt{2}\sqrt{-(dx+c)b+2\,bc}}{2\,\sqrt{-bc}}\right)}{\sqrt{-bc}(de-cf)} - \frac{2\,(Bde-Adf)\arctan\left(\frac{\sqrt{-(dx+c)b+2\,bcf}}{\sqrt{-bdef-bcf^2}}\right)}{\sqrt{-bdef-bcf^2}(de-cf)}}{d}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x, algori
thm="giac")
```

output
```
-(sqrt(2)*(B*c - A*d)*arctan(1/2*sqrt(2)*sqrt(-(d*x + c)*b + 2*b*c)/sqrt(-
b*c))/(sqrt(-b*c)*(d*e - c*f)) - 2*(B*d*e - A*d*f)*arctan(sqrt(-(d*x + c)*
b + 2*b*c)*f/sqrt(-b*d*e*f - b*c*f^2))/(sqrt(-b*d*e*f - b*c*f^2)*(d*e - c*
f)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{A+B\,x}{(e+f\,x)\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.87

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\sqrt{b}\left(4\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)acdfi-4\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)bcdei-\sqrt{c}\,\sqrt{2}\log\left(\sqrt{-dx}\right.\right.}{}$$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x)
```

output
```
(sqrt(b)*(4*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt
(c*f + d*e)))*a*c*d*f*i - 4*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*
i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*d*e*i - sqrt(c)*sqrt(2)*log(sqrt(c - d*x
) - sqrt(c)*sqrt(2))*a*c*d*f**2 - sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt
(c)*sqrt(2))*a*d**2*e*f + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt
(2))*b*c**2*f**2 + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*b*
c*d*e*f + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*a*c*d*f**2
+ sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*a*d**2*e*f - sqrt(c
)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*b*c**2*f**2 - sqrt(c)*sqrt(
2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*b*c*d*e*f))/(2*b*c*d*f*(c**2*f**2
- d**2*e**2))
```

3.27 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$

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Optimal result

Integrand size = 46, antiderivative size = 232

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = -\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}} - \frac{\sqrt{2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}d^2(de-cf)} + \frac{2(Ce^2-Bef+Af^2)\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{bc^2-bd^2x^2}}{\sqrt{b}\sqrt{de+cf}\sqrt{c+dx}}\right)}{\sqrt{b}f^{3/2}(de-cf)\sqrt{de+cf}}$$

output
```
-2*C*(-b*d^2*x^2+b*c^2)^(1/2)/b/d^2/f/(d*x+c)^(1/2)-2^(1/2)*(A*d^2-B*c*d+C
*c^2)*arctanh(1/2*(-b*d^2*x^2+b*c^2)^(1/2)*2^(1/2)/b^(1/2)/c^(1/2)/(d*x+c)
^(1/2))/b^(1/2)/c^(1/2)/d^2/(-c*f+d*e)+2*(A*f^2-B*e*f+C*e^2)*arctanh(f^(1/
2)*(-b*d^2*x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)^(1/2)/(d*x+c)^(1/2))/b^(1/2)
/f^(3/2)/(-c*f+d*e)/(c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\sqrt{c^2-d^2x^2}\left(-\frac{2C\sqrt{c^2-d^2x^2}}{d^2f\sqrt{c+dx}}+\frac{2(Ce^2+f(-Be+Af))\arctan\left(\frac{\sqrt{-de-cf}\sqrt{c^2-d^2x^2}}{\sqrt{f}(c-dx)\sqrt{c+dx}}\right)}{f^{3/2}\sqrt{-de-cf}(de-cf)}+\frac{\sqrt{2}(c^2C-Bcd+Ad^2)\text{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{c+dx}}{\sqrt{c^2-d^2x^2}}\right.}{\sqrt{c}d^2(-de+cf)}}{\sqrt{b(c^2-d^2x^2)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 - b*d^2*x^
2]),x]
```

output
```
(Sqrt[c^2 - d^2*x^2]*((-2*C*Sqrt[c^2 - d^2*x^2])/(d^2*f*Sqrt[c + d*x]) + (
2*(C*e^2 + f*(-(B*e) + A*f))*ArcTan[(Sqrt[-(d*e) - c*f]*Sqrt[c^2 - d^2*x^2
])/(Sqrt[f]*(c - d*x)*Sqrt[c + d*x])])/(f^(3/2)*Sqrt[-(d*e) - c*f]*(d*e -
c*f)) + (Sqrt[2]*(c^2*C - B*c*d + A*d^2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[c +
 d*x])/Sqrt[c^2 - d^2*x^2]])/(Sqrt[c]*d^2*(-(d*e) + c*f))))/Sqrt[b*(c^2 -
d^2*x^2)]
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.196$, Rules used = {2349, 600, 458, 471, 221, 718, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{600}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{df^2}+\frac{C\int \frac{\sqrt{c+dx}}{\sqrt{bc^2-bd^2x^2}}dx}{df}$$

$$\Big\downarrow \text{458}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{df^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{471}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{2(-Bdf+cCf+Cde)\int \frac{1}{\frac{d^2\left(bc^2-bd^2x^2\right)}{c+dx}-2bcd^2}d\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{c+dx}}}{f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{221}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(-Bdf+cCf+Cde)}{\sqrt{b}\sqrt{c}d^2f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{718}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(-Bdf+cCf+Cde)}{\sqrt{b}\sqrt{c}d^2f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{97}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(-Bdf+cCf+Cde)}{\sqrt{b}\sqrt{c}d^2f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(-Bdf+cCf+Cde)}{\sqrt{b}\sqrt{c}d^2f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(-Bdf+cCf+Cde)}{\sqrt{b}\sqrt{c}d^2f^2}-\frac{2C\sqrt{bc^2-bd^2x^2}}{bd^2f\sqrt{c+dx}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 - b*d^2*x^2]),x]
```

output
```
(-2*C*Sqrt[b*c^2 - b*d^2*x^2])/(b*d^2*f*Sqrt[c + d*x]) + ((A + (e*(C*e - B
*f))/f^2)*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[Sqrt[b*c - b
*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f))) + (2*Sqrt
[f]*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])/(Sqrt[
b]*(d*e - c*f)*Sqrt[d*e + c*f])))/Sqrt[b*c^2 - b*d^2*x^2] + (Sqrt[2]*(C*d*
e + c*C*f - B*d*f)*ArcTanh[Sqrt[b*c^2 - b*d^2*x^2]/(Sqrt[2]*Sqrt[b]*Sqrt[c
]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[c]*d^2*f^2)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] && !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{2C(-dx+c)\sqrt{-\frac{b\left(d^2x^2-c^2\right)}{dx+c}}\,\sqrt{dx+c}}{d^2f\sqrt{-b(dx-c)}\,\sqrt{-b(d^2x^2-c^2)}}+\frac{2\left(-\frac{d^2\left(Af^2-Bef+Ce^2\right)\operatorname{arctanh}\left(\frac{f\sqrt{-bdx+bc}}{\sqrt{b(cf+de)f}}\right)}{(cf-de)\sqrt{b(cf+de)f}}+\frac{f\left(Ad^2-Bcd+Cc^2\right)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-bdx}}{2\sqrt{}}\right.}{2(cf-de)\sqrt{bc}}\right.}{d^2f\sqrt{-b(d^2x^2-c^2)}}$
default	$\frac{\sqrt{b(-d^2x^2+c^2)}\left(A\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{(-dx+c)b}\,\sqrt{2}}{2\sqrt{bc}}\right)\sqrt{b(cf+de)f}\,bd^2f-2A\sqrt{bc}\operatorname{arctanh}\left(\frac{f\sqrt{(-dx+c)b}}{\sqrt{b(cf+de)f}}\right)bd^2f^2-B\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{(-}}{}\right.\right.}{}$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,method=
_RETURNVERBOSE)
```

output
```
-2*C*(-d*x+c)/d^2/f/(-b*(d*x-c))^(1/2)*(-1/(d*x+c)*b*(d^2*x^2-c^2))^(1/2)*
(d*x+c)^(1/2)/(-b*(d^2*x^2-c^2))^(1/2)+2/d^2/f*(-d^2*(A*f^2-B*e*f+C*e^2)/(
c*f-d*e)/(b*(c*f+d*e)*f)^(1/2)*arctanh(f*(-b*d*x+b*c)^(1/2)/(b*(c*f+d*e)*f
)^(1/2))+1/2*f*(A*d^2-B*c*d+C*c^2)/(c*f-d*e)*2^(1/2)/(b*c)^(1/2)*arctanh(1
/2*(-b*d*x+b*c)^(1/2)*2^(1/2)/(b*c)^(1/2)))*(-1/(d*x+c)*b*(d^2*x^2-c^2))^(
1/2)*(d*x+c)^(1/2)/(-b*(d^2*x^2-c^2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. 2(197) = 394.

Time = 1.89 (sec) , antiderivative size = 1836, normalized size of antiderivative = 7.91

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,
algorithm="fricas")
```

output
```
[-1/2*(sqrt(2)*((C*b*c^3*d - B*b*c^2*d^2 + A*b*c*d^3)*e*f^2 + (C*b*c^4 - B
*b*c^3*d + A*b*c^2*d^2)*f^3 + ((C*b*c^2*d^2 - B*b*c*d^3 + A*b*d^4)*e*f^2 +
 (C*b*c^3*d - B*b*c^2*d^2 + A*b*c*d^3)*f^3)*x)*sqrt(1/(b*c))*log(-(d^2*x^2
 - 2*c*d*x - 2*sqrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(1/(b*
c)) - 3*c^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 2*(C*c*d^2*e^2 - B*c*d^2*e*f + A
*c*d^2*f^2 + (C*d^3*e^2 - B*d^3*e*f + A*d^3*f^2)*x)*sqrt(b*d*e*f + b*c*f^2
)*log(-(b*d^2*f*x^2 - b*c*d*e - 2*b*c^2*f - (b*d^2*e + b*c*d*f)*x + 2*sqrt
(-b*d^2*x^2 + b*c^2)*sqrt(b*d*e*f + b*c*f^2)*sqrt(d*x + c))/(d*f*x^2 + c*e
 + (d*e + c*f)*x)) + 4*(C*d^2*e^2*f - C*c^2*f^3)*sqrt(-b*d^2*x^2 + b*c^2)*
sqrt(d*x + c))/(b*c*d^4*e^2*f^2 - b*c^3*d^2*f^4 + (b*d^5*e^2*f^2 - b*c^2*d
^3*f^4)*x), -(sqrt(2)*((C*b*c^3*d - B*b*c^2*d^2 + A*b*c*d^3)*e*f^2 + (C*b*
c^4 - B*b*c^3*d + A*b*c^2*d^2)*f^3 + ((C*b*c^2*d^2 - B*b*c*d^3 + A*b*d^4)*
e*f^2 + (C*b*c^3*d - B*b*c^2*d^2 + A*b*c*d^3)*f^3)*x)*sqrt(-1/(b*c))*arcta
n(sqrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(-1/(b*c))/(d^2*x^2
 - c^2)) + (C*c*d^2*e^2 - B*c*d^2*e*f + A*c*d^2*f^2 + (C*d^3*e^2 - B*d^3*e
*f + A*d^3*f^2)*x)*sqrt(b*d*e*f + b*c*f^2)*log(-(b*d^2*f*x^2 - b*c*d*e - 2
*b*c^2*f - (b*d^2*e + b*c*d*f)*x + 2*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(b*d*e*f
 + b*c*f^2)*sqrt(d*x + c))/(d*f*x^2 + c*e + (d*e + c*f)*x)) + 2*(C*d^2*e^2
*f - C*c^2*f^3)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c))/(b*c*d^4*e^2*f^2 -
 b*c^3*d^2*f^4 + (b*d^5*e^2*f^2 - b*c^2*d^3*f^4)*x), -1/2*(sqrt(2)*((C*...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{-b\,(-c+dx)\,(c+dx)}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*d**2*x**2+b*c**2)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-b*(-c + d*x)*(c + d*x))*sqrt(c + d*x)*(
e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,
algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*(f*x +
 e)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\frac{\sqrt{2}(Cc^2-Bcd+Ad^2)\arctan\left(\frac{\sqrt{2}\sqrt{-(dx+c)b+2\,bc}}{2\,\sqrt{-bc}}\right)}{(d^2e-cdf)\sqrt{-bc}}-\frac{2\,(Cde^2-Bdef+Adf^2)\arctan\left(\frac{\sqrt{-(dx+c)b+2\,bcf}}{\sqrt{-bdef-bcf^2}}\right)}{\sqrt{-bdef-bcf^2}(def-cf^2)}-\frac{2\,\sqrt{-(dx+c)b+2\,bc}C}{bdf}}{d}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,
algorithm="giac")
```

output
```
(sqrt(2)*(C*c^2 - B*c*d + A*d^2)*arctan(1/2*sqrt(2)*sqrt(-(d*x + c)*b + 2*
b*c)/sqrt(-b*c))/((d^2*e - c*d*f)*sqrt(-b*c)) - 2*(C*d*e^2 - B*d*e*f + A*d
*f^2)*arctan(sqrt(-(d*x + c)*b + 2*b*c)*f/sqrt(-b*d*e*f - b*c*f^2))/(sqrt(
-b*d*e*f - b*c*f^2)*(d*e*f - c*f^2)) - 2*sqrt(-(d*x + c)*b + 2*b*c)*C/(b*d
*f))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(e+f\,x)\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\sqrt{b}\left(4\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)ac\,d^2f^2i-4\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)bc\,d^2efi+4\sqrt{f}\,\sqrt{cf+}\right.}{}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x)
```

output

```
(sqrt(b)*(4*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt
(c*f + d*e)))*a*c*d**2*f**2*i - 4*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d
*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*d**2*e*f*i + 4*sqrt(f)*sqrt(c*f +
d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**2*d**2*e**2*i
- 4*sqrt(c - d*x)*c**4*f**3 + 4*sqrt(c - d*x)*c**2*d**2*e**2*f - sqrt(c)*s
qrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*a*c*d**2*f**3 - sqrt(c)*sqrt(2
)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*a*d**3*e*f**2 + sqrt(c)*sqrt(2)*log
(sqrt(c - d*x) - sqrt(c)*sqrt(2))*b*c**2*d*f**3 + sqrt(c)*sqrt(2)*log(sqrt
(c - d*x) - sqrt(c)*sqrt(2))*b*c*d**2*e*f**2 - sqrt(c)*sqrt(2)*log(sqrt(c
- d*x) - sqrt(c)*sqrt(2))*c**4*f**3 - sqrt(c)*sqrt(2)*log(sqrt(c - d*x) -
sqrt(c)*sqrt(2))*c**3*d*e*f**2 + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(
c)*sqrt(2))*a*c*d**2*f**3 + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sq
rt(2))*a*d**3*e*f**2 - sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2)
)*b*c**2*d*f**3 - sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*b*c
*d**2*e*f**2 + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*c**4*f
**3 + sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*c**3*d*e*f**2))
/(2*b*c*d**2*f**2*(c**2*f**2 - d**2*e**2))
```

3.28 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$

Optimal result 288
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Optimal result

Integrand size = 46, antiderivative size = 318

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \frac{(Ce^2-Bef+Af^2)\sqrt{bc^2-bd^2x^2}}{bf\left(d^2e^2-c^2f^2\right)\sqrt{c+dx}(e+fx)} - \frac{\sqrt{2}(c^2C-Bcd+Ad^2)\operatorname{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}d(de-cf)^2} - \frac{(Ce(d^2e^2-cdef-4c^2f^2)-f(Adf(3de+cf)-B(d^2e^2+cdef+2c^2f^2)))\operatorname{arctanh}\left(\frac{\sqrt{f}\sqrt{bc^2-bd^2x^2}}{\sqrt{b}\sqrt{de+cf}\sqrt{c+dx}}\right)}{\sqrt{b}f^{3/2}(de-cf)^2(de+cf)^{3/2}}$$

output
```
(A*f^2-B*e*f+C*e^2)*(-b*d^2*x^2+b*c^2)^(1/2)/b/f/(-c^2*f^2+d^2*e^2)/(d*x+c
)^(1/2)/(f*x+e)-2^(1/2)*(A*d^2-B*c*d+C*c^2)*arctanh(1/2*(-b*d^2*x^2+b*c^2)
^(1/2)*2^(1/2)/b^(1/2)/c^(1/2)/(d*x+c)^(1/2))/b^(1/2)/c^(1/2)/d/(-c*f+d*e)
^2-(C*e*(-4*c^2*f^2-c*d*e*f+d^2*e^2)-f*(A*d*f*(c*f+3*d*e)-B*(2*c^2*f^2+c*d
*e*f+d^2*e^2)))*arctanh(f^(1/2)*(-b*d^2*x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)
^(1/2)/(d*x+c)^(1/2))/b^(1/2)/f^(3/2)/(-c*f+d*e)^2/(c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$
$$= \frac{\sqrt{c^2-d^2x^2}\left(\frac{(de-cf)(Ce^2+f(-Be+Af))\sqrt{c^2-d^2x^2}}{f(de+cf)\sqrt{c+dx}(e+fx)} - \frac{(Ce(d^2e^2-cdef-4c^2f^2)+f(-Adf(3de+cf)+B(d^2e^2+cdef+2c^2f^2)))\arctan(}{f^{3/2}(-de-cf)^{3/2}}\right.}{(de-cf)^2\sqrt{b(c^2-d^2x^2)}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)^2*Sqrt[b*c^2 - b*d^2*
x^2]),x]
```

output
```
(Sqrt[c^2 - d^2*x^2]*(((d*e - c*f)*(C*e^2 + f*(-(B*e) + A*f))*Sqrt[c^2 - d
^2*x^2])/(f*(d*e + c*f)*Sqrt[c + d*x]*(e + f*x)) - ((C*e*(d^2*e^2 - c*d*e*
f - 4*c^2*f^2) + f*(-(A*d*f*(3*d*e + c*f)) + B*(d^2*e^2 + c*d*e*f + 2*c^2*
f^2)))*ArcTan[(Sqrt[-(d*e) - c*f]*Sqrt[c^2 - d^2*x^2])/(Sqrt[f]*(-c + d*x)
*Sqrt[c + d*x])])/(f^(3/2)*(-(d*e) - c*f)^(3/2)) - (Sqrt[2]*(c^2*C - B*c*d
 + A*d^2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[c + d*x])/Sqrt[c^2 - d^2*x^2]])/(S
qrt[c]*d)))/((d*e - c*f)^2*Sqrt[b*(c^2 - d^2*x^2)])
```

Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.63, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.326$, Rules used = {2349, 718, 114, 27, 174, 73, 221, 2349, 27, 471, 221, 718, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$
$$\Big\downarrow \ 2349$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{ 718}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{ 114}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{\int \frac{bd(2de+cf-dfx)}{2(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{b(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\int \frac{2de+cf-dfx}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{ 174}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2d(cf+de)\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f(cf+3de)\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2f(cf+3de)\int\frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{4(cf+de)\int\frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-\cdots}}{b(e+fx)(d^2\cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\int\frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx$$

$\Big\downarrow$ 221

$$\int\frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-\cdots}}{b(e+fx)(d^2e\cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 2349

$$-\frac{(2Ce-Bf)\int\frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^2}+\int\frac{C}{f^2\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-\cdots}}{b(e+fx)(d^2e\cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 27

$$-\frac{(2Ce-Bf)\int\frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^2}+\frac{C\int\frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{f^2}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-\cdots}}{b(e+fx)(d^2e\cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 471

$$-\frac{(2Ce-Bf)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^2}+\frac{2Cd\int \frac{1}{\frac{d^2\left(bc^2-bd^2x^2\right)}{c+dx}-2bcd^2}d\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{c+dx}}}{f^2}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 221

$$-\frac{(2Ce-Bf)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^2}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}C\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df^2}$$

$\downarrow$ 718

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{f^2\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}C\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df^2}$$

$\downarrow$ 97

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}C\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}C\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df^2}$$

$$\Big\downarrow \; 221$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-}}{b(e+fx)(d^2e}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}-$$

$$\frac{\sqrt{2}C\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}df^2}$$

input

```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)^2*Sqrt[b*c^2 - b*d^2*x^2]),
x]
```

output
```
-(((2*C*e - B*f)*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[Sqrt[
b*c - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f))) +
(2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])
/(Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(f^2*Sqrt[b*c^2 - b*d^2*x^2])) +
((A + (e*(C*e - B*f))/f^2)*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*((f*Sqrt[b*c -
b*d*x])/(b*(d^2*e^2 - c^2*f^2)*(e + f*x)) + (d*((-2*Sqrt[2]*(d*e + c*f)*Ar
cTanh[Sqrt[b*c - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e
- c*f)) + (2*Sqrt[f]*(3*d*e + c*f)*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sq
rt[b]*Sqrt[d*e + c*f])])/(Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(2*(d^2*e
^2 - c^2*f^2))))/Sqrt[b*c^2 - b*d^2*x^2] - (Sqrt[2]*C*ArcTanh[Sqrt[b*c^2 -
 b*d^2*x^2]/(Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[c]*d*f
^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1549 vs. $2(283) = 566$.

Time = 0.21 (sec) , antiderivative size = 1550, normalized size of antiderivative = 4.87

method	result	size
default	Expression too large to display	1550

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
(b*(-d^2*x^2+c^2))^(1/2)*(4*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)
^(1/2))*(b*c)^(1/2)*b*c^2*d*e^2*f^2+C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f
+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^2*e^3*f+B*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e
)*f)^(1/2)*(b*c)^(1/2)*c*d*e*f^2-C*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2
)*(b*c)^(1/2)*c*d*e^2*f-C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/
2))*(b*c)^(1/2)*b*d^3*e^3*f*x-A*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*
((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*d^3*e^2*f+A*arctanh(f*((-d*x+c)*
b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^2*e*f^3-2*B*arctanh(f*((
-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^2*d*e*f^3-B*arctan
h(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^2*e^2*f^2-
C*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*
c)^(1/2))*b*c^3*e*f^2+A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2)
)*(b*c)^(1/2)*b*c*d^2*f^4*x+3*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*
f)^(1/2))*(b*c)^(1/2)*b*d^3*e*f^3*x-2*B*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c
*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^2*d*f^4*x-B*arctanh(f*((-d*x+c)*b)^(1/2)
/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^3*e^2*f^2*x-C*(b*(c*f+d*e)*f)^(1/2
)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^3*f^3*x+
B*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*
c)^(1/2))*b*c*d^2*e*f^2*x-C*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d
*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^2*d*e*f^2*x+3*A*arctanh(f*((-d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(283) = 566$.

Time = 26.84 (sec) , antiderivative size = 4145, normalized size of antiderivative = 13.03

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$
$$= \int \frac{A+Bx+Cx^2}{\sqrt{-b\left(-c+dx\right)\left(c+dx\right)}\sqrt{c+dx}\left(e+fx\right)^2}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**2/(-b*d**2*x**2+b*c**2)**
(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(-b*(-c + d*x)*(c + d*x))*sqrt(c + d*x)*(
e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*(f*x +
 e)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$
$$=\frac{\frac{\sqrt{2}(Cc^2-Bcd+Ad^2)\arctan\left(\frac{\sqrt{2}\sqrt{-(dx+c)b+2bc}}{2\sqrt{-bc}}\right)}{(d^2e^2-2cdef+c^2f^2)\sqrt{-bc}}+\frac{(Cd^3e^3-Ccd^2e^2f+Bd^3e^2f-4Cc^2def^2+Bcd^2ef^2-3Ad^3ef^2+2Bc^2df^3-Acd^2f^3)\,\mathrm{arct}}{(d^3e^3f-cd^2e^2f^2-c^2def^3+c^3f^4)\sqrt{-bdef-bcf^2}}}{d}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="giac")
```

output
```
(sqrt(2)*(C*c^2 - B*c*d + A*d^2)*arctan(1/2*sqrt(2)*sqrt(-(d*x + c)*b + 2*
b*c)/sqrt(-b*c))/((d^2*e^2 - 2*c*d*e*f + c^2*f^2)*sqrt(-b*c)) + (C*d^3*e^3
 - C*c*d^2*e^2*f + B*d^3*e^2*f - 4*C*c^2*d*e*f^2 + B*c*d^2*e*f^2 - 3*A*d^3
*e*f^2 + 2*B*c^2*d*f^3 - A*c*d^2*f^3)*arctan(sqrt(-(d*x + c)*b + 2*b*c)*f/
sqrt(-b*d*e*f - b*c*f^2))/((d^3*e^3*f - c*d^2*e^2*f^2 - c^2*d*e*f^3 + c^3*
f^4)*sqrt(-b*d*e*f - b*c*f^2)) + (sqrt(-(d*x + c)*b + 2*b*c)*C*d^2*e^2 - s
qrt(-(d*x + c)*b + 2*b*c)*B*d^2*e*f + sqrt(-(d*x + c)*b + 2*b*c)*A*d^2*f^2
)/((d^2*e^2*f - c^2*f^3)*(b*d*e + b*c*f + ((d*x + c)*b - 2*b*c)*f)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(e+f\,x)^2\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^2*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^2*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2073, normalized size of antiderivative = 6.52

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x)
```

output

```
(sqrt(b)*( - 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*s
qrt(c*f + d*e)))*a*c**2*d**2*e*f**3*i - 2*sqrt(f)*sqrt(c*f + d*e)*atan((sq
rt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c**2*d**2*f**4*i*x - 6*sqrt(
f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c
*d**3*e**2*f**2*i - 6*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sq
rt(f)*sqrt(c*f + d*e)))*a*c*d**3*e*f**3*i*x + 4*sqrt(f)*sqrt(c*f + d*e)*at
an((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**3*d*e*f**3*i + 4*sq
rt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*
b*c**3*d*f**4*i*x + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sq
rt(f)*sqrt(c*f + d*e)))*b*c**2*d**2*e**2*f**2*i + 2*sqrt(f)*sqrt(c*f + d*e
)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**2*d**2*e*f**3*i
*x + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f
+ d*e)))*b*c*d**3*e**3*f*i + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)
*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*d**3*e**2*f**2*i*x - 8*sqrt(f)*sqrt(c
*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**4*d*e**2*
f**2*i - 8*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(
c*f + d*e)))*c**4*d*e*f**3*i*x - 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c -
d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**3*d**2*e**3*f*i - 2*sqrt(f)*sqrt(c
*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**3*d**2*e*
*2*f**2*i*x + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(...
```

3.29 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx$

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Optimal result

Integrand size = 46, antiderivative size = 510

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx &= \frac{(Ce^2-Bef+Af^2)\sqrt{bc^2-bd^2x^2}}{2bf(d^2e^2-c^2f^2)\sqrt{c+dx}(e+fx)^2} \\
&- \frac{(Ce(d^2e^2-cdef-8c^2f^2)-f(Adf(7de+cf)-B(3d^2e^2+cdef+4c^2f^2)))\sqrt{bc^2-bd^2x^2}}{4bf(de-cf)^2(de+cf)^2\sqrt{c+dx}(e+fx)} \\
&- \frac{\sqrt{2}(c^2C-Bcd+Ad^2)\,\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}(de-cf)^3} \\
&- \frac{(C(d^4e^4-2cd^3e^3f-15c^2d^2e^2f^2-8c^3def^3-8c^4f^4)-df(Adf(15d^2e^2+10cdef+7c^2f^2)-B(3d^3e^3}{4\sqrt{b}f^{3/2}(de-cf)^3(de+cf)^{5/2}}
\end{aligned}
$$

output

```
1/2*(A*f^2-B*e*f+C*e^2)*(-b*d^2*x^2+b*c^2)^(1/2)/b/f/(-c^2*f^2+d^2*e^2)/(d
*x+c)^(1/2)/(f*x+e)^2-1/4*(C*e*(-8*c^2*f^2-c*d*e*f+d^2*e^2)-f*(A*d*f*(c*f+
7*d*e)-B*(4*c^2*f^2+c*d*e*f+3*d^2*e^2)))*(-b*d^2*x^2+b*c^2)^(1/2)/b/f/(-c*
f+d*e)^2/(c*f+d*e)^2/(d*x+c)^(1/2)/(f*x+e)-2^(1/2)*(A*d^2-B*c*d+C*c^2)*arc
tanh(1/2*(-b*d^2*x^2+b*c^2)^(1/2)*2^(1/2)/b^(1/2)/c^(1/2)/(d*x+c)^(1/2))/b
^(1/2)/c^(1/2)/(-c*f+d*e)^3-1/4*(C*(-8*c^4*f^4-8*c^3*d*e*f^3-15*c^2*d^2*e^
2*f^2-2*c*d^3*e^3*f+d^4*e^4)-d*f*(A*d*f*(7*c^2*f^2+10*c*d*e*f+15*d^2*e^2)-
B*(4*c^3*f^3+19*c^2*d*e*f^2+6*c*d^2*e^2*f+3*d^3*e^3)))*arctanh(f^(1/2)*(-b
*d^2*x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)^(1/2)/(d*x+c)^(1/2))/b^(1/2)/f^(3/
2)/(-c*f+d*e)^3/(c*f+d*e)^(5/2)
```

Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 500, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx$$

$$= \frac{\sqrt{c^2-d^2x^2}\left(\frac{\sqrt{c^2-d^2x^2}(Ce(d^2e^2(e-fx)+cdef(e+fx)+2c^2f^2(3e+4fx))+f(-B(cdef(e+fx)+2c^2f^2(e+2fx)+d^2e^2(5e+3fx))+Af(-2}{f(de-cf)^2(de+cf)^2\sqrt{c+dx}(e+fx)^2}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)^3*Sqrt[b*c^2 - b*d^2*
x^2]),x]
```

output
```
(Sqrt[c^2 - d^2*x^2]*((Sqrt[c^2 - d^2*x^2]*(C*e*(d^2*e^2*(e - f*x) + c*d*e
*f*(e + f*x) + 2*c^2*f^2*(3*e + 4*f*x)) + f*(-(B*(c*d*e*f*(e + f*x) + 2*c^
2*f^2*(e + 2*f*x) + d^2*e^2*(5*e + 3*f*x))) + A*f*(-2*c^2*f^2 + c*d*f*(e +
 f*x) + d^2*e*(9*e + 7*f*x)))))/(f*(d*e - c*f)^2*(d*e + c*f)^2*Sqrt[c + d*
x]*(e + f*x)^2) + ((C*(d^4*e^4 - 2*c*d^3*e^3*f - 15*c^2*d^2*e^2*f^2 - 8*c^
3*d*e*f^3 - 8*c^4*f^4) + d*f*(-(A*d*f*(15*d^2*e^2 + 10*c*d*e*f + 7*c^2*f^2
)) + B*(3*d^3*e^3 + 6*c*d^2*e^2*f + 19*c^2*d*e*f^2 + 4*c^3*f^3)))*ArcTan[(
Sqrt[-(d*e) - c*f]*Sqrt[c^2 - d^2*x^2])/(Sqrt[f]*(-c + d*x)*Sqrt[c + d*x])
])/(f^(3/2)*(-(d*e) - c*f)^(5/2)*(d*e - c*f)^3) + (4*Sqrt[2]*(c^2*C - B*c*
d + A*d^2)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[c + d*x])/Sqrt[c^2 - d^2*x^2]])/(
Sqrt[c]*(-(d*e) + c*f)^3)))/(4*Sqrt[b*(c^2 - d^2*x^2)])
```

Rubi [A] (verified)

Time = 3.15 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.57, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.413$, Rules used = {2349, 718, 114, 27, 168, 27, 174, 73, 221, 2349, 27, 718, 97, 73, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \; 2349$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}dx+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \; 718$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^3}dx}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \; 114$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{\int \frac{bd(4de+cf-3dfx)}{2(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{2b(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{2b(e+fx)^2(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\int \frac{4de+cf-3dfx}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{4(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{2b(e+fx)^2(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow \; 168$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{\int \frac{bd\left(8d^2e^2+9cdfe+7c^2f^2-df(7de+cf)x\right)}{2(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{b\left(d^2e^2-c^2f^2\right)}+\frac{f\sqrt{bc-bdx}(cf+7de)}{b(e+fx)(de-cf)(cf+de)}\right)}{4(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{2b(e+fx)^2(d^2e^2-c^{\cdots}}\right)}{\sqrt{bc^2-bd^2x^2}}+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow 27$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\int \frac{8d^2e^2+9cdfe+7c^2f^2-df(7de+cf)x}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{2\left(d^2e^2-c^2f^2\right)}+\frac{f\sqrt{bc-bdx}(cf+7de)}{b(e+fx)(de-cf)(cf+de)}\right)}{4(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{2b(e+fx)^2(d^2e^2-c^2f^2 \cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow 174$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{8d(cf+de)^2\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\left(7c^2f^2+10cdef+15d^2e^2\right)\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{2\left(d^2e^2-c^2f^2\right)}+\frac{f\sqrt{bc- \cdots}}{b(e+fx) \cdots}\right)}{4(d^2e^2-c^2f^2) \cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow 73$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2f\left(7c^2f^2+10cdef+15d^2e^2\right)\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{16(cf+de)^2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bd \cdots}}{b(de-cf) \cdots}\right.}{2\left(d^2e^2-c^2f^2\right) \cdots}\right.}{4(d^2e^2-c^2f^2) \cdots}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx$$

$$\Big\downarrow 221$$

$$\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2\left(d^2e^2-c^2f^2\right)}\right.}{4(d^2e^2-c^2f^2)}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 2349

$$-\frac{(2Ce-Bf)\int \frac{1}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx}{f^2}+\int \frac{C}{f^2\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2\left(d^2e^2-c^2f^2\right)}\right.}{4(d^2e^2-c^2f^2)}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 27

$$-\frac{(2Ce-Bf)\int \frac{1}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx}{f^2}+\frac{C\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^2}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2\left(d^2e^2-c^2f^2\right)}\right.}{4(d^2e^2-c^2f^2)}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 718

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2\left(d^2e^2-c^2f^2\right)}\right)}{4(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \; 97$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{2\left(d^2e^2-c^2f^2\right)}\right)}{4(d^2e^2-c^2f^2)}\right)}{\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \; 73$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\cdots\right)}{2\left(d^2e^2-c^2f^2\right)}\cdots\right)}{4(d^2e^2-c^2f^2)}\cdots\right)}{\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \; 114$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{\int \frac{bd(2de+cf-dfx)}{2(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{b(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\cdots\right)}{2\left(d^2e^2-c^2f^2\right)}\cdots\right)}{4(d^2e^2-c^2f^2)}\cdots\right)}{\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{d\int \frac{2de+cf-dfx}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}(7c^2f^2+10cdef+15d^2e^2)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}}{4(d^2e^2-c^2f^2)}}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 174

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{d\left(\frac{2d(cf+de)\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f(cf+3de)\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}(7c^2f^2+10cdef+15d^2e^2)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}}{4(d^2e^2-c^2f^2)}}{\sqrt{bc^2-bd^2x^2}}$$

$\downarrow$ 73

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}(2Ce-Bf)\left(\frac{d\left(\frac{2f(cf+3de)\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{4(cf+de)\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bd}}{b(e+fx)(d^2e^2-}\right.}{f^2\sqrt{bc^2-bd^2x^2}}$$

$$\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{d\left(\frac{d\left(\frac{2\sqrt{f}\left(7c^2f^2+10cdef+15d^2e^2\right)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{8\sqrt{2}(cf+de)^2\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2\left(d^2e^2-c^2f^2\right)}\right.}{4(d^2e^2-c^2f^2)}\right.}{\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \; 221$$

$$\frac{C\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{de+cf}}\right)}{\sqrt{b}(de-cf)\sqrt{de+cf}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^2\sqrt{bc^2-bd^2x^2}}-$$

$$\frac{(2Ce-Bf)\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{\sqrt{bc-bdx}f}{b(d^2e^2-c^2f^2)(e+fx)}+\frac{d\left(\frac{2\sqrt{f}(3de+cf)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{de+cf}}\right)}{\sqrt{b}(de-cf)\sqrt{de+cf}}-\frac{2\sqrt{2}(de+cf)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^2\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\left(A+\frac{e(Ce-Bf)}{f^2}\right)\sqrt{c+dx}\sqrt{bc-bdx}\left(\frac{\sqrt{bc-bdx}f}{2b(d^2e^2-c^2f^2)(e+fx)^2}+\frac{d\left(\frac{f\sqrt{bc-bdx}(7de+cf)}{b(de-cf)(de+cf)(e+fx)}+\frac{d\left(\frac{2\sqrt{f}\left(15d^2e^2+10cdfe+7c^2f^2\right)\text{arctan}}{\sqrt{b}(de-cf)\sqrt{de+cf}}\right.}{}\right.}{4(d^2e^2-c^2f}\right.}{\sqrt{bc^2-bd^2x^2}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)^3*Sqrt[b*c^2 - b*d^2*x^2]),
x]
```

output
```
(C*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[Sqrt[b*c - b*d*x]/(
Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f))) + (2*Sqrt[f]*Arc
Tanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])/(Sqrt[b]*(d*e
 - c*f)*Sqrt[d*e + c*f])))/(f^2*Sqrt[b*c^2 - b*d^2*x^2]) - ((2*C*e - B*f)*
Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*((f*Sqrt[b*c - b*d*x])/(b*(d^2*e^2 - c^2*f
^2)*(e + f*x)) + (d*((-2*Sqrt[2]*(d*e + c*f)*ArcTanh[Sqrt[b*c - b*d*x]/(Sq
rt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f)) + (2*Sqrt[f]*(3*d*e
 + c*f)*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])/(S
qrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(2*(d^2*e^2 - c^2*f^2))))/(f^2*Sqrt[
b*c^2 - b*d^2*x^2]) + ((A + (e*(C*e - B*f))/f^2)*Sqrt[c + d*x]*Sqrt[b*c -
b*d*x]*((f*Sqrt[b*c - b*d*x])/(2*b*(d^2*e^2 - c^2*f^2)*(e + f*x)^2) + (d*(
(f*(7*d*e + c*f)*Sqrt[b*c - b*d*x])/(b*(d*e - c*f)*(d*e + c*f)*(e + f*x))
+ (d*((-8*Sqrt[2]*(d*e + c*f)^2*ArcTanh[Sqrt[b*c - b*d*x]/(Sqrt[2]*Sqrt[b]
*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f)) + (2*Sqrt[f]*(15*d^2*e^2 + 10*c*
d*e*f + 7*c^2*f^2)*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e +
 c*f])])/(Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(2*(d^2*e^2 - c^2*f^2))))
/(4*(d^2*e^2 - c^2*f^2))))/Sqrt[b*c^2 - b*d^2*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 168
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3917 vs. $2(465) = 930$.

Time = 0.26 (sec) , antiderivative size = 3918, normalized size of antiderivative = 7.68

method	result	size
default	Expression too large to display	3918

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^3/(-b*d^2*x^2+b*c^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/4*(b*(-d^2*x^2+c^2))^(1/2)/b*(-4*C*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctan
h(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^2*d^2*e^4*f+8*C*arctanh(
f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^3*d*e*f^5*x^2-
4*A*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(
b*c)^(1/2))*b*c^2*d^2*f^5*x^2+15*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*
e)*f)^(1/2))*(b*c)^(1/2)*b*c^2*d^2*e^2*f^4*x^2+2*C*arctanh(f*((-d*x+c)*b)^
(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*e^3*f^3*x^2-8*A*(b*(c*f+d
*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b
*d^4*e^3*f^2*x+14*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b
*c)^(1/2)*b*c^2*d^2*e*f^5*x+20*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)
*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*e^2*f^4*x-8*B*arctanh(f*((-d*x+c)*b)^(1/2)/
(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^3*d*e*f^5*x-38*B*arctanh(f*((-d*x+c
)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^2*d^2*e^2*f^4*x-12*B*arc
tanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*e^3*f
^3*x+16*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*
b*c^3*d*e^2*f^4*x+2*A*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)
*c^3*f^5+C*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*d^3*e^5-C*
arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^4*e^6+
15*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^2
*d^2*e^4*f^2+2*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2002 vs. $2(465) = 930$.

Time = 133.72 (sec) , antiderivative size = 8113, normalized size of antiderivative = 15.91

$$\int \frac{A + Bx + Cx^2}{\sqrt{c + dx}(e + fx)^3\sqrt{bc^2 - bd^2x^2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^3/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**3/(-b*d**2*x**2+b*c**2)**
(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^3/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*(f*x +
 e)^3), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 908, normalized size of antiderivative = 1.78

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^3/(-b*d^2*x^2+b*c^2)^(1/2),x
, algorithm="giac")
```

output
```
1/4*(4*sqrt(2)*(C*c^2*d - B*c*d^2 + A*d^3)*arctan(1/2*sqrt(2)*sqrt(-(d*x +
 c)*b + 2*b*c)/sqrt(-b*c))/((d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3
*f^3)*sqrt(-b*c)) + (C*d^5*e^4 - 2*C*c*d^4*e^3*f + 3*B*d^5*e^3*f - 15*C*c^
2*d^3*e^2*f^2 + 6*B*c*d^4*e^2*f^2 - 15*A*d^5*e^2*f^2 - 8*C*c^3*d^2*e*f^3 +
 19*B*c^2*d^3*e*f^3 - 10*A*c*d^4*e*f^3 - 8*C*c^4*d*f^4 + 4*B*c^3*d^2*f^4 -
 7*A*c^2*d^3*f^4)*arctan(sqrt(-(d*x + c)*b + 2*b*c)*f/sqrt(-b*d*e*f - b*c*
f^2))/((d^5*e^5*f - c*d^4*e^4*f^2 - 2*c^2*d^3*e^3*f^3 + 2*c^3*d^2*e^2*f^4
+ c^4*d*e*f^5 - c^5*f^6)*sqrt(-b*d*e*f - b*c*f^2)) + (sqrt(-(d*x + c)*b +
2*b*c)*C*b*d^5*e^4 - 5*sqrt(-(d*x + c)*b + 2*b*c)*B*b*d^5*e^3*f + 7*sqrt(-
(d*x + c)*b + 2*b*c)*C*b*c^2*d^3*e^2*f^2 - 4*sqrt(-(d*x + c)*b + 2*b*c)*B*
b*c*d^4*e^2*f^2 + 9*sqrt(-(d*x + c)*b + 2*b*c)*A*b*d^5*e^2*f^2 + 8*sqrt(-(
d*x + c)*b + 2*b*c)*C*b*c^3*d^2*e*f^3 - 3*sqrt(-(d*x + c)*b + 2*b*c)*B*b*c
^2*d^3*e*f^3 + 8*sqrt(-(d*x + c)*b + 2*b*c)*A*b*c*d^4*e*f^3 - 4*sqrt(-(d*x
 + c)*b + 2*b*c)*B*b*c^3*d^2*f^4 - sqrt(-(d*x + c)*b + 2*b*c)*A*b*c^2*d^3*
f^4 + (-(d*x + c)*b + 2*b*c)^(3/2)*C*d^4*e^3*f - (-(d*x + c)*b + 2*b*c)^(3
/2)*C*c*d^3*e^2*f^2 + 3*(-(d*x + c)*b + 2*b*c)^(3/2)*B*d^4*e^2*f^2 - 8*(-(
d*x + c)*b + 2*b*c)^(3/2)*C*c^2*d^2*e*f^3 + (-(d*x + c)*b + 2*b*c)^(3/2)*B
*c*d^3*e*f^3 - 7*(-(d*x + c)*b + 2*b*c)^(3/2)*A*d^4*e*f^3 + 4*(-(d*x + c)*
b + 2*b*c)^(3/2)*B*c^2*d^2*f^4 - (-(d*x + c)*b + 2*b*c)^(3/2)*A*c*d^3*f^4)
/((d^4*e^4*f - 2*c^2*d^2*e^2*f^3 + c^4*f^5)*(b*d*e + b*c*f + ((d*x + c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{\left(e+f\,x\right)}^3\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)^3*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/
2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)^3*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*x)^(1/
2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4913, normalized size of antiderivative = 9.63

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)^3\sqrt{bc^2-bd^2x^2}}\,dx = \text{Too large to display}$$

input

```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^3/(-b*d^2*x^2+b*c^2)^(1/2),x)
```

output

```
(sqrt(b)*(7*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt
(c*f + d*e)))*a*c**3*d**2*e**2*f**4*i + 14*sqrt(f)*sqrt(c*f + d*e)*atan((s
qrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c**3*d**2*e*f**5*i*x + 7*sq
rt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*
a*c**3*d**2*f**6*i*x**2 + 10*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f
*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c**2*d**3*e**3*f**3*i + 20*sqrt(f)*sqrt(c
*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c**2*d**3*
e**2*f**4*i*x + 10*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(
f)*sqrt(c*f + d*e)))*a*c**2*d**3*e*f**5*i*x**2 + 15*sqrt(f)*sqrt(c*f + d*e
)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c*d**4*e**4*f**2*i
 + 30*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f +
 d*e)))*a*c*d**4*e**3*f**3*i*x + 15*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c -
 d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c*d**4*e**2*f**4*i*x**2 - 4*sqrt(f
)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*
*4*d*e**2*f**4*i - 8*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqr
t(f)*sqrt(c*f + d*e)))*b*c**4*d*e*f**5*i*x - 4*sqrt(f)*sqrt(c*f + d*e)*ata
n((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**4*d*f**6*i*x**2 - 19
*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)
))*b*c**3*d**2*e**3*f**3*i - 38*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x
)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**3*d**2*e**2*f**4*i*x - 19*sqrt(f...
```

3.30 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$

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Optimal result

Integrand size = 51, antiderivative size = 310

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx
&= \frac{2(3dDe-3Cdf+2cDf)\sqrt{bc^2-bd^2x^2}}{3bd^3f^2\sqrt{c+dx}} - \frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f} \\
&\quad - \frac{\sqrt{2}(c^2Cd-Bcd^2+Ad^3-c^3D)\,\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}d^3(de-cf)} \\
&\quad - \frac{2(De^3-f(Ce^2-f(Be-Af)))\,\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc^2-bd^2x^2}}{\sqrt{b}\sqrt{de+cf}\sqrt{c+dx}}\right)}{\sqrt{b}f^{5/2}(de-cf)\sqrt{de+cf}}
\end{aligned}
$$

output

```
2/3*(-3*C*d*f+2*D*c*f+3*D*d*e)*(-b*d^2*x^2+b*c^2)^(1/2)/b/d^3/f^2/(d*x+c)^
(1/2)-2/3*D*(d*x+c)^(1/2)*(-b*d^2*x^2+b*c^2)^(1/2)/b/d^3/f-2^(1/2)*(A*d^3-
B*c*d^2+C*c^2*d-D*c^3)*arctanh(1/2*(-b*d^2*x^2+b*c^2)^(1/2)*2^(1/2)/b^(1/2
)/c^(1/2)/(d*x+c)^(1/2))/b^(1/2)/c^(1/2)/d^3/(-c*f+d*e)-2*(D*e^3-f*(C*e^2-
f*(-A*f+B*e)))*arctanh(f^(1/2)*(-b*d^2*x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)^
(1/2)/(d*x+c)^(1/2))/b^(1/2)/f^(5/2)/(-c*f+d*e)/(c*f+d*e)^(1/2)
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$$
$$= \frac{\sqrt{c^2-d^2x^2}\left(\frac{2\sqrt{c^2-d^2x^2}(cDf+d(3De-3Cf-Dfx))}{d^3f^2\sqrt{c+dx}}+\frac{6(De^3-f(Ce^2+f(-Be+Af)))\arctan\left(\frac{\sqrt{-de-cf}\sqrt{c^2-d^2x^2}}{\sqrt{f}(-c+dx)\sqrt{c+dx}}\right)}{f^{5/2}\sqrt{-de-cf}(de-cf)}-\frac{3\sqrt{2}(-c^2Cd}{}\right.}{3\sqrt{b(c^2-d^2x^2)}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 -
b*d^2*x^2]),x]
```

output
```
(Sqrt[c^2 - d^2*x^2]*((2*Sqrt[c^2 - d^2*x^2]*(c*D*f + d*(3*D*e - 3*C*f - D
*f*x)))/(d^3*f^2*Sqrt[c + d*x]) + (6*(D*e^3 - f*(C*e^2 + f*(-(B*e) + A*f))
)*ArcTan[(Sqrt[-(d*e) - c*f]*Sqrt[c^2 - d^2*x^2])/(Sqrt[f]*(-c + d*x)*Sqrt
[c + d*x])])/(f^(5/2)*Sqrt[-(d*e) - c*f]*(d*e - c*f)) - (3*Sqrt[2]*(-(c^2*
C*d) + B*c*d^2 - A*d^3 + c^3*D)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[c + d*x])/Sq
rt[c^2 - d^2*x^2]])/(Sqrt[c]*d^3*(-(d*e) + c*f))))/(3*Sqrt[b*(c^2 - d^2*x^
2)])
```

Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.216$, Rules used = {2349, 718, 97, 73, 221, 2170, 27, 600, 458, 471, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$$
$$\Big\downarrow \ 2349$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}$$

$$\Big\downarrow \text{718}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{97}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{221}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{2170}$$

$$-\frac{2\int -\frac{bd^2\left(3\left(De^2-f(Ce-Bf)\right)d^2-f(3dDe-3Cdf+2cDf)xd+c^2Df^2\right)}{2f^3\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{3bd^4}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

$\downarrow$ 27

$$\frac{\int \frac{3\left(De^2-f(Ce-Bf)\right)d^2-f(3dDe-3Cdf+2cDf)xd+c^2Df^2}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{3d^2f^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

$\downarrow$ 600

$$\frac{3\left(d^2\left(De^2-f(Ce-Bf)\right)+c^2Df^2+cdf(De-Cf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-f(2cDf-3Cdf+3dDe)\int \frac{\sqrt{c+}}{\sqrt{bc^2-b}}}{3d^2f^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

$\downarrow$ 458

$$\frac{3\left(d^2\left(De^2-f(Ce-Bf)\right)+c^2Df^2+cdf(De-Cf)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx+\frac{2f\sqrt{bc^2-bd^2x^2}(2cDf-3Cdf+3dDe)}{bd\sqrt{c+dx}}}{3d^2f^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

$\downarrow$ 471

$$\frac{6d\left(d^2\left(De^2-f(Ce-Bf)\right)+c^2Df^2+cdf(De-Cf)\right)\int \frac{1}{\frac{d^2\left(bc^2-bd^2x^2\right)}{c+dx}-2bcd^2}d\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{c+dx}}+\frac{2f\sqrt{bc^2-bd^2x^2}(2cDf-3Cd}{bd\sqrt{c+dx}}}{3d^2f^3}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-$$

$$\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\frac{2f\sqrt{bc^2-bd^2x^2}(2cDf-3Cdf+3dDe)}{bd\sqrt{c+dx}}-\frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)\left(d^2\left(De^2-f(Ce-Bf)\right)+c^2Df^2+cdf(De-Cf)\right)}{\sqrt{b}\sqrt{c}d}}{3d^2f^3}-$$

$$\frac{2D\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}{3bd^3f}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[b*c^2 - b*d^2*
x^2]),x]
```

output
```
(-2*D*Sqrt[c + d*x]*Sqrt[b*c^2 - b*d^2*x^2])/(3*b*d^3*f) - ((D*e^3 - f*(C*
e^2 - f*(B*e - A*f)))*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[
Sqrt[b*c - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f)
)) + (2*Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*
f])])/(Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(f^3*Sqrt[b*c^2 - b*d^2*x^2]
) + ((2*f*(3*d*D*e - 3*C*d*f + 2*c*D*f)*Sqrt[b*c^2 - b*d^2*x^2])/(b*d*Sqrt
[c + d*x]) - (3*Sqrt[2]*(c^2*D*f^2 + c*d*f*(D*e - C*f) + d^2*(D*e^2 - f*(C
*e - B*f)))*ArcTanh[Sqrt[b*c^2 - b*d^2*x^2]/(Sqrt[2]*Sqrt[b]*Sqrt[c]*Sqrt[
c + d*x])])/(Sqrt[b]*Sqrt[c]*d))/(3*d^2*f^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2170
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - 2*e*f*(m +
 p + q)*(d + e*x)^(q - 2)*(a*e - b*d*x), x], x], x] /; NeQ[m + q + 2*p + 1,
 0]] /; FreeQ[{a, b, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2,
0] &&  !IGtQ[m, 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(267) = 534$.

Time = 0.20 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.16

method	result
default	$-\frac{\sqrt{b(-d^2x^2+c^2)}\left(6A\,\mathrm{arctanh}\left(\frac{f\sqrt{(-dx+c)b}}{\sqrt{b(cf+de)f}}\right)\sqrt{bc}\,b\,d^3f^3-3A\sqrt{b(cf+de)f}\,\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{(-dx+c)b}\,\sqrt{2}}{2\sqrt{bc}}\right)b\,d^3f^2-6B\,\mathrm{arctanh}\left(\frac{f}{\sqrt{}}\right.\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x,m
ethod=_RETURNVERBOSE)
```

output
```
-1/3*(b*(-d^2*x^2+c^2))^(1/2)/b*(6*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+
d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^3*f^3-3*A*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arc
tanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*d^3*f^2-6*B*arctanh(f*(
(-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^3*e*f^2+3*B*(b*(c
*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2
))*b*c*d^2*f^2+6*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*
c)^(1/2)*b*d^3*e^2*f-3*C*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctanh(1/2*((-d*x+
c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^2*d*f^2-6*D*arctanh(f*((-d*x+c)*b)^(1
/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^3*e^3+3*D*(b*(c*f+d*e)*f)^(1/2)
*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^3*f^2+2*D
*c*d*f^2*x*(b*c)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*((-d*x+c)*b)^(1/2)-2*D*d^2*e*
f*x*(b*c)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*((-d*x+c)*b)^(1/2)+6*C*((-d*x+c)*b)^
(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*c*d*f^2-6*C*((-d*x+c)*b)^(1/2)*(b*
(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*d^2*e*f-2*D*c^2*f^2*(b*c)^(1/2)*(b*(c*f+d*e
)*f)^(1/2)*((-d*x+c)*b)^(1/2)-4*D*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)
*(b*c)^(1/2)*c*d*e*f+6*D*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1
/2)*d^2*e^2)/(d*x+c)^(1/2)/((-d*x+c)*b)^(1/2)/d^3/f^2/(c*f-d*e)/(b*(c*f+d*
e)*f)^(1/2)/(b*c)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(266) = 532$.

Time = 18.25 (sec) , antiderivative size = 2365, normalized size of antiderivative = 7.63

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{c + dx}(e + fx)\sqrt{bc^2 - bd^2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/
2),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(2)*((D*b*c^4*d - C*b*c^3*d^2 + B*b*c^2*d^3 - A*b*c*d^4)*e*f^
3 + (D*b*c^5 - C*b*c^4*d + B*b*c^3*d^2 - A*b*c^2*d^3)*f^4 + ((D*b*c^3*d^2
- C*b*c^2*d^3 + B*b*c*d^4 - A*b*d^5)*e*f^3 + (D*b*c^4*d - C*b*c^3*d^2 + B*
b*c^2*d^3 - A*b*c*d^4)*f^4)*x)*sqrt(1/(b*c))*log(-(d^2*x^2 - 2*c*d*x + 2*s
qrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c)*c*sqrt(1/(b*c)) - 3*c^2)/(d^
2*x^2 + 2*c*d*x + c^2)) - 6*(D*c*d^3*e^3 - C*c*d^3*e^2*f + B*c*d^3*e*f^2 -
 A*c*d^3*f^3 + (D*d^4*e^3 - C*d^4*e^2*f + B*d^4*e*f^2 - A*d^4*f^3)*x)*sqrt
(b*d*e*f + b*c*f^2)*log(-(b*d^2*f*x^2 - b*c*d*e - 2*b*c^2*f - (b*d^2*e + b
*c*d*f)*x + 2*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(b*d*e*f + b*c*f^2)*sqrt(d*x +
c))/(d*f*x^2 + c*e + (d*e + c*f)*x)) - 4*(3*D*d^3*e^3*f - 3*D*c^2*d*e*f^3
+ (D*c*d^2 - 3*C*d^3)*e^2*f^2 - (D*c^3 - 3*C*c^2*d)*f^4 - (D*d^3*e^2*f^2 -
 D*c^2*d*f^4)*x)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c))/(b*c*d^5*e^2*f^3
- b*c^3*d^3*f^5 + (b*d^6*e^2*f^3 - b*c^2*d^4*f^5)*x), 1/3*(3*sqrt(2)*((D*b
*c^4*d - C*b*c^3*d^2 + B*b*c^2*d^3 - A*b*c*d^4)*e*f^3 + (D*b*c^5 - C*b*c^4
*d + B*b*c^3*d^2 - A*b*c^2*d^3)*f^4 + ((D*b*c^3*d^2 - C*b*c^2*d^3 + B*b*c*
d^4 - A*b*d^5)*e*f^3 + (D*b*c^4*d - C*b*c^3*d^2 + B*b*c^2*d^3 - A*b*c*d^4)
*f^4)*x)*sqrt(-1/(b*c))*arctan(sqrt(2)*sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x +
 c)*c*sqrt(-1/(b*c))/(d^2*x^2 - c^2)) + 3*(D*c*d^3*e^3 - C*c*d^3*e^2*f + B
*c*d^3*e*f^2 - A*c*d^3*f^3 + (D*d^4*e^3 - C*d^4*e^2*f + B*d^4*e*f^2 - A*d^
4*f^3)*x)*sqrt(b*d*e*f + b*c*f^2)*log(-(b*d^2*f*x^2 - b*c*d*e - 2*b*c^2...
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{-b\left(-c+dx\right)\left(c+dx\right)}\sqrt{c+dx}\left(e+fx\right)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*d**2*x**2+b*c**
2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(-b*(-c + d*x)*(c + d*x))*sqrt(c
 + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/
2),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c
)*(f*x + e)), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx =$$

$$-\frac{\frac{3\sqrt{2}(Dc^3-Cc^2d+Bcd^2-Ad^3)\arctan\left(\frac{\sqrt{2}\sqrt{-(dx+c)b+2bc}}{2\sqrt{-bc}}\right)}{(d^3e-cd^2f)\sqrt{-bc}} - \frac{6(Dde^3-Cde^2f+Bdef^2-Adf^3)\arctan\left(\frac{\sqrt{-(dx+c)b+2bcf}}{\sqrt{-bdef-bcf^2}}\right)}{\sqrt{-bdef-bcf^2}(def^2-cf^3)} - \frac{2(3\sqrt{-}\cdots}{\cdots}}{3d}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/
2),x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(2)*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*arctan(1/2*sqrt(2)*sqr
t(-(d*x + c)*b + 2*b*c)/sqrt(-b*c))/((d^3*e - c*d^2*f)*sqrt(-b*c)) - 6*(D*
d*e^3 - C*d*e^2*f + B*d*e*f^2 - A*d*f^3)*arctan(sqrt(-(d*x + c)*b + 2*b*c)
*f/sqrt(-b*d*e*f - b*c*f^2))/(sqrt(-b*d*e*f - b*c*f^2)*(d*e*f^2 - c*f^3))
 - 2*(3*sqrt(-(d*x + c)*b + 2*b*c)*D*b^5*d^5*e*f - 3*sqrt(-(d*x + c)*b + 2*
b*c)*C*b^5*d^5*f^2 + (-(d*x + c)*b + 2*b*c)^(3/2)*D*b^4*d^4*f^2)/(b^6*d^6*
f^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(e+f\,x)\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*
x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(b*c^2 - b*d^2*x^2)^(1/2)*(c + d*
x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 558, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}\,dx$$
$$= \frac{\sqrt{b}\left(12\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)ac\,d^2f^3i-12\sqrt{f}\,\sqrt{cf+de}\,atan\left(\frac{\sqrt{-dx+c}\,fi}{\sqrt{f}\,\sqrt{cf+de}}\right)bc\,d^2e\,f^2i+12\sqrt{f}\,\sqrt{c}\right.}{}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*d^2*x^2+b*c^2)^(1/2),x)
```

output
```
(sqrt(b)*(12*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqr
t(c*f + d*e)))*a*c*d**2*f**3*i - 12*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c -
 d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*d**2*e*f**2*i + 12*sqrt(f)*sqrt(
c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**2*d**2*e
**2*f*i - 12*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqr
t(c*f + d*e)))*c*d**3*e**3*i - 8*sqrt(c - d*x)*c**4*f**4 + 12*sqrt(c - d*x
)*c**3*d*e*f**3 - 4*sqrt(c - d*x)*c**3*d*f**4*x + 8*sqrt(c - d*x)*c**2*d**
2*e**2*f**2 - 12*sqrt(c - d*x)*c*d**3*e**3*f + 4*sqrt(c - d*x)*c*d**3*e**2
*f**2*x - 3*sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*a*c*d**2*
f**4 - 3*sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*a*d**3*e*f**
3 + 3*sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*b*c**2*d*f**4 +
 3*sqrt(c)*sqrt(2)*log(sqrt(c - d*x) - sqrt(c)*sqrt(2))*b*c*d**2*e*f**3 +
3*sqrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*a*c*d**2*f**4 + 3*s
qrt(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*a*d**3*e*f**3 - 3*sqrt
(c)*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*b*c**2*d*f**4 - 3*sqrt(c)
*sqrt(2)*log(sqrt(c - d*x) + sqrt(c)*sqrt(2))*b*c*d**2*e*f**3))/(6*b*c*d**
2*f**3*(c**2*f**2 - d**2*e**2))
```

3.31 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$

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Optimal result

Integrand size = 51, antiderivative size = 402

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$
$$= -\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}} - \frac{(De^3-f(Ce^2-f(Be-Af)))\sqrt{bc^2-bd^2x^2}}{bf^2(de-cf)(de+cf)\sqrt{c+dx}(e+fx)}$$
$$-\frac{\sqrt{2}(c^2Cd-Bcd^2+Ad^3-c^3D)\operatorname{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{c}d^2(de-cf)^2}$$
$$-\frac{(2c^2f^2(3De^2-f(2Ce-Bf))-d^2(3De^4-ef(Ce^2+Bef-3Af^2))+cdf(De^3-f(Ce^2-Bef+A}{\sqrt{b}f^{5/2}(de-cf)^2(de+cf)^{3/2}}$$

output

```
-2*D*(-b*d^2*x^2+b*c^2)^(1/2)/b/d^2/f^2/(d*x+c)^(1/2)-(D*e^3-f*(C*e^2-f*(-
A*f+B*e)))*(-b*d^2*x^2+b*c^2)^(1/2)/b/f^2/(-c*f+d*e)/(c*f+d*e)/(d*x+c)^(1/
2)/(f*x+e)-2^(1/2)*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*arctanh(1/2*(-b*d^2*x^2+b
*c^2)^(1/2)*2^(1/2)/b^(1/2)/c^(1/2)/(d*x+c)^(1/2))/b^(1/2)/c^(1/2)/d^2/(-c
*f+d*e)^2-(2*c^2*f^2*(3*D*e^2-f*(-B*f+2*C*e))-d^2*(3*D*e^4-e*f*(-3*A*f^2+B
*e*f+C*e^2))+c*d*f*(D*e^3-f*(A*f^2-B*e*f+C*e^2)))*arctanh(f^(1/2)*(-b*d^2*
x^2+b*c^2)^(1/2)/b^(1/2)/(c*f+d*e)^(1/2)/(d*x+c)^(1/2))/b^(1/2)/f^(5/2)/(-
c*f+d*e)^2/(c*f+d*e)^(3/2)
```

Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \frac{\sqrt{c^2-d^2x^2}\left(\frac{\sqrt{c^2-d^2x^2}(2c^2Df^2(e+fx)+d^2(f(Ce^2-Bef+Af^2)-De^2(3e+2fx)))}{d^2f^2(d^2e^2-c^2f^2)\sqrt{c+dx}(e+fx)}+\frac{(-2c^2f^2(3De^2+f(-2Ce+Bf))+d^2(3De^4-ef(\cdots}{\cdots}\right.}{\sqrt{b(\cdots}}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)^2*Sqrt[b*c^2
- b*d^2*x^2]),x]
```

output
```
(Sqrt[c^2 - d^2*x^2]*((Sqrt[c^2 - d^2*x^2]*(2*c^2*D*f^2*(e + f*x) + d^2*(f
*(C*e^2 - B*e*f + A*f^2) - D*e^2*(3*e + 2*f*x))))/(d^2*f^2*(d^2*e^2 - c^2*
f^2)*Sqrt[c + d*x]*(e + f*x)) + ((-2*c^2*f^2*(3*D*e^2 + f*(-2*C*e + B*f))
+ d^2*(3*D*e^4 - e*f*(C*e^2 + B*e*f - 3*A*f^2)) + c*d*f*(-(D*e^3) + f*(C*e
^2 - B*e*f + A*f^2)))*ArcTan[(Sqrt[-(d*e) - c*f]*Sqrt[c^2 - d^2*x^2])/(Sqr
t[f]*(-c + d*x)*Sqrt[c + d*x])])/(f^(5/2)*(-(d*e) - c*f)^(3/2)*(d*e - c*f)
^2) + (Sqrt[2]*(-(c^2*C*d) + B*c*d^2 - A*d^3 + c^3*D)*ArcTanh[(Sqrt[2]*Sqr
t[c]*Sqrt[c + d*x])/Sqrt[c^2 - d^2*x^2]])/(Sqrt[c]*d^2*(d*e - c*f)^2)))/Sq
rt[b*(c^2 - d^2*x^2)]
```

Rubi [A] (verified)

Time = 2.80 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.48, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2349, 718, 114, 27, 174, 73, 221, 2349, 600, 458, 471, 221, 718, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx$$

$$\Big\downarrow \; 2349$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}dx}{f^3}$$

$$\Big\downarrow \text{718}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)^2}dx}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{114}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{\int \frac{bd(2de+cf-dfx)}{2(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{b(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\int \frac{2de+cf-dfx}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{2(d^2e^2-c^2f^2)}+\frac{f\sqrt{bc-bdx}}{b(e+fx)(d^2e^2-c^2f^2)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{174}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2d(cf+de)\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f(cf+3de)\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{2(d^2e^2-c^2f^2)}+\frac{}{b(e+}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\Big\downarrow \text{73}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2f(cf+3de)\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{4(cf+de)\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bd}}{b(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 221

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 2349

$$\frac{\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}+\int \frac{\frac{C}{f^2}+\frac{Dx}{f^2}-\frac{2De}{f^3}}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 600

$$\frac{\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}-\frac{(cDf-Cdf+2dDe)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{df^3}+\frac{D\int \frac{\sqrt{c+dx}}{\sqrt{bc^2-bd^2x^2}}dx}{df^2}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$\Big\downarrow$ 458

$$\frac{\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}-\frac{(cDf-Cdf+2dDe)\int \frac{1}{\sqrt{c+dx}\sqrt{bc^2-bd^2x^2}}dx}{df^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$\downarrow$ 471

$$\frac{\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}-\frac{2(cDf-Cdf+2dDe)\int \frac{1}{\frac{d^2\left(bc^2-bd^2x^2\right)}{c+dx}-2bcd^2}d\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{c+dx}}}{f^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$\downarrow$ 221

$$\frac{\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bc^2-bd^2x^2}}dx}{f^3}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(cDf-Cdf+2dDe)}{\sqrt{b}\sqrt{c}d^2f^3}-\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$\downarrow$ 718

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(3De^2-f(2Ce-Bf)\right)\int \frac{1}{(c+dx)\sqrt{bc-bdx}(e+fx)}dx}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(cDf-Cdf+2dDe)}{\sqrt{b}\sqrt{c}d^2f^3}-\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$$\Big\downarrow \; 97$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(3De^2-f(2Ce-Bf)\right)\left(\frac{d\int \frac{1}{(c+dx)\sqrt{bc-bdx}}dx}{de-cf}-\frac{f\int \frac{1}{\sqrt{bc-bdx}(e+fx)}dx}{de-cf}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(cDf-Cdf+2dDe)}{\sqrt{b}\sqrt{c}d^2f^3}-\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(3De^2-f(2Ce-Bf)\right)\left(\frac{2f\int \frac{1}{e+\frac{cf}{d}-\frac{f(bc-bdx)}{bd}}d\sqrt{bc-bdx}}{bd(de-cf)}-\frac{2\int \frac{1}{2c-\frac{bc-bdx}{b}}d\sqrt{bc-bdx}}{b(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right.}{2(d^2e^2-c^2f^2)}\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(cDf-Cdf+2dDe)}{\sqrt{b}\sqrt{c}d^2f^3}-\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

$$\Big\downarrow \; 221$$

$$-\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{d\left(\frac{2\sqrt{f}(cf+3de)\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{2\sqrt{2}(cf+de)\text{arctanh}\left(\frac{\sqrt{bc-bd}}{\sqrt{2}\sqrt{b}\sqrt{}}\right.}{\sqrt{b}\sqrt{c}(de-cf)}\cdots\right.}{2(d^2e^2-c^2f^2)}\cdots\right.}{f^3\sqrt{bc^2-bd^2x^2}}$$

$$\frac{\sqrt{c+dx}\sqrt{bc-bdx}\left(3De^2-f(2Ce-Bf)\right)\left(\frac{2\sqrt{f}\text{arctanh}\left(\frac{\sqrt{f}\sqrt{bc-bdx}}{\sqrt{b}\sqrt{cf+de}}\right)}{\sqrt{b}(de-cf)\sqrt{cf+de}}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc-bdx}}{\sqrt{2}\sqrt{b}\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}(de-cf)}\right)}{f^3\sqrt{bc^2-bd^2x^2}}+$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{bc^2-bd^2x^2}}{\sqrt{2}\sqrt{b}\sqrt{c}\sqrt{c+dx}}\right)(cDf-Cdf+2dDe)}{\sqrt{b}\sqrt{c}d^2f^3}-\frac{2D\sqrt{bc^2-bd^2x^2}}{bd^2f^2\sqrt{c+dx}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)^2*Sqrt[b*c^2 - b*d^
2*x^2]),x]
```

output
```
(-2*D*Sqrt[b*c^2 - b*d^2*x^2])/(b*d^2*f^2*Sqrt[c + d*x]) + ((3*D*e^2 - f*(
2*C*e - B*f))*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*(-((Sqrt[2]*ArcTanh[Sqrt[b*c
 - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]*(d*e - c*f))) + (2*
Sqrt[f]*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x])/(Sqrt[b]*Sqrt[d*e + c*f])])/(S
qrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(f^3*Sqrt[b*c^2 - b*d^2*x^2]) - ((D*
e^3 - f*(C*e^2 - f*(B*e - A*f)))*Sqrt[c + d*x]*Sqrt[b*c - b*d*x]*((f*Sqrt[
b*c - b*d*x])/(b*(d^2*e^2 - c^2*f^2)*(e + f*x)) + (d*((-2*Sqrt[2]*(d*e + c
*f)*ArcTanh[Sqrt[b*c - b*d*x]/(Sqrt[2]*Sqrt[b]*Sqrt[c])])/(Sqrt[b]*Sqrt[c]
*(d*e - c*f)) + (2*Sqrt[f]*(3*d*e + c*f)*ArcTanh[(Sqrt[f]*Sqrt[b*c - b*d*x
])/(Sqrt[b]*Sqrt[d*e + c*f])])/(Sqrt[b]*(d*e - c*f)*Sqrt[d*e + c*f])))/(2*
(d^2*e^2 - c^2*f^2))))/(f^3*Sqrt[b*c^2 - b*d^2*x^2]) + (Sqrt[2]*(2*d*D*e -
 C*d*f + c*D*f)*ArcTanh[Sqrt[b*c^2 - b*d^2*x^2]/(Sqrt[2]*Sqrt[b]*Sqrt[c]*S
qrt[c + d*x])])/(Sqrt[b]*Sqrt[c]*d^2*f^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 458
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c
, d, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && EqQ[n + p, 0]
```

rule 471
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2*d   Subst[Int[1/(2*b*c + d^2*x^2), x], x, Sqrt[a + b*x^2]/Sqrt[c + d*x]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^2 + a*d^2, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 718
```
Int[((d_) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_)
^2)^(p_), x_Symbol] :> Simp[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*
(a/d + (c*x)/e)^FracPart[p])   Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/
e)*x)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && EqQ[c*d^2 + a*e^2,
 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2348 vs. $2(363) = 726$.

Time = 0.21 (sec) , antiderivative size = 2349, normalized size of antiderivative = 5.84

method	result	size
default	Expression too large to display	2349

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x
,method=_RETURNVERBOSE)
```

output
```
(b*(-d^2*x^2+c^2))^(1/2)*(-B*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^
(1/2))*(b*c)^(1/2)*b*c*d^3*e^2*f^3+4*C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*
f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c^2*d^2*e^2*f^3+C*arctanh(f*((-d*x+c)*b)^(1
/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*e^3*f^2+D*(b*(c*f+d*e)*f)^(
1/2)*2^(1/2)*arctanh(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^4*e*f
^3-6*D*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c
^2*d^2*e^3*f^2-D*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)
^(1/2)*b*c*d^3*e^4*f-2*D*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1
/2)*d^3*e^3*f*x+B*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*c*d
^2*e*f^3-C*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*c*d^2*e^2*
f^2+2*D*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*c^2*d*e^2*f^2
+3*D*((-d*x+c)*b)^(1/2)*(b*(c*f+d*e)*f)^(1/2)*(b*c)^(1/2)*c*d^2*e^3*f+A*ar
ctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*f^5*
x+3*A*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*d^
4*e*f^4*x-2*B*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1
/2)*b*c^2*d^2*f^5*x-B*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*
(b*c)^(1/2)*b*d^4*e^2*f^3*x-C*arctanh(f*((-d*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)
^(1/2))*(b*c)^(1/2)*b*d^4*e^3*f^2*x+D*(b*(c*f+d*e)*f)^(1/2)*2^(1/2)*arctan
h(1/2*((-d*x+c)*b)^(1/2)*2^(1/2)/(b*c)^(1/2))*b*c^4*f^4*x+A*arctanh(f*((-d
*x+c)*b)^(1/2)/(b*(c*f+d*e)*f)^(1/2))*(b*c)^(1/2)*b*c*d^3*e*f^4-2*B*arc...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(
1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{-b\left(-c+dx\right)\left(c+dx\right)}\sqrt{c+dx}\left(e+fx\right)^2}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)**2/(-b*d**2*x**2+b*
c**2)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(-b*(-c + d*x)*(c + d*x))*sqrt(c
 + d*x)*(e + f*x)**2), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bd^2x^2+bc^2}\sqrt{dx+c}(fx+e)^2}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(
1/2),x, algorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*d^2*x^2 + b*c^2)*sqrt(d*x + c
)*(f*x + e)^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx =$$

$$-\frac{\frac{\sqrt{2}\left(Dc^3-Cc^2d+Bcd^2-Ad^3\right)\arctan\left(\frac{\sqrt{2}\sqrt{-(dx+c)b+2\,bc}}{2\,\sqrt{-bc}}\right)}{\left(d^3e^2-2\,cd^2ef+c^2df^2\right)\sqrt{-bc}}+\frac{\left(3\,Dd^3e^4-Dcd^2e^3f-Cd^3e^3f-6\,Dc^2de^2f^2+Ccd^2e^2f^2-Bd^3e^2f^2+4\,Cc^2de\right.}{\left(d^3e^3f^2-cd^2e^2f^3-c^2def\right.}}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(
1/2),x, algorithm="giac")
```

output
```
-(sqrt(2)*(D*c^3 - C*c^2*d + B*c*d^2 - A*d^3)*arctan(1/2*sqrt(2)*sqrt(-(d*
x + c)*b + 2*b*c)/sqrt(-b*c))/((d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sqrt(-b
*c)) + (3*D*d^3*e^4 - D*c*d^2*e^3*f - C*d^3*e^3*f - 6*D*c^2*d*e^2*f^2 + C*
c*d^2*e^2*f^2 - B*d^3*e^2*f^2 + 4*C*c^2*d*e*f^3 - B*c*d^2*e*f^3 + 3*A*d^3*
e*f^3 - 2*B*c^2*d*f^4 + A*c*d^2*f^4)*arctan(sqrt(-(d*x + c)*b + 2*b*c)*f/s
qrt(-b*d*e*f - b*c*f^2))/((d^3*e^3*f^2 - c*d^2*e^2*f^3 - c^2*d*e*f^4 + c^3
*f^5)*sqrt(-b*d*e*f - b*c*f^2)) + (sqrt(-(d*x + c)*b + 2*b*c)*D*d^2*e^3 -
sqrt(-(d*x + c)*b + 2*b*c)*C*d^2*e^2*f + sqrt(-(d*x + c)*b + 2*b*c)*B*d^2*
e*f^2 - sqrt(-(d*x + c)*b + 2*b*c)*A*d^2*f^3)/((d^2*e^2*f^2 - c^2*f^4)*(b*
d*e + b*c*f + ((d*x + c)*b - 2*b*c)*f)) + 2*sqrt(-(d*x + c)*b + 2*b*c)*D/(
b*d*f^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)^2\sqrt{bc^2-bd^2x^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(e+f\,x\right)^2\,\sqrt{b\,c^2-b\,d^2\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)^2*(b*c^2 - b*d^2*x^2)^(1/2)*(c +
d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)^2*(b*c^2 - b*d^2*x^2)^(1/2)*(c +
d*x)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1928, normalized size of antiderivative = 4.80

$$\int \frac{A + Bx + Cx^2 + Dx^3}{\sqrt{c + dx}(e + fx)^2 \sqrt{bc^2 - bd^2x^2}}\, dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)^2/(-b*d^2*x^2+b*c^2)^(1/2),x
)
```

output
```
(sqrt(b)*( - 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*s
qrt(c*f + d*e)))*a*c**2*d**2*e*f**4*i - 2*sqrt(f)*sqrt(c*f + d*e)*atan((sq
rt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c**2*d**2*f**5*i*x - 6*sqrt(
f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*a*c
*d**3*e**2*f**3*i - 6*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sq
rt(f)*sqrt(c*f + d*e)))*a*c*d**3*e*f**4*i*x + 4*sqrt(f)*sqrt(c*f + d*e)*at
an((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**3*d*e*f**4*i + 4*sq
rt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*
b*c**3*d*f**5*i*x + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sq
rt(f)*sqrt(c*f + d*e)))*b*c**2*d**2*e**2*f**3*i + 2*sqrt(f)*sqrt(c*f + d*e
)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c**2*d**2*e*f**4*i
*x + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f
+ d*e)))*b*c*d**3*e**3*f**2*i + 2*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d
*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*b*c*d**3*e**2*f**3*i*x - 8*sqrt(f)*sqr
t(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**4*d*e*
*2*f**3*i - 8*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sq
rt(c*f + d*e)))*c**4*d*e*f**4*i*x + 10*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(
c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**3*d**2*e**3*f**2*i + 10*sqrt(f
)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i)/(sqrt(f)*sqrt(c*f + d*e)))*c**3
*d**2*e**2*f**3*i*x + 4*sqrt(f)*sqrt(c*f + d*e)*atan((sqrt(c - d*x)*f*i...
```

3.32 $\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx$

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Optimal result

Integrand size = 39, antiderivative size = 155

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = -\frac{c^2Cd-Bcd^2+Ad^3-c^3D}{3cd^4(c+dx)\sqrt{c^2-d^2x^2}} + \frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}} - \frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/c/d^4/(d*x+c)/(-d^2*x^2+c^2)^(1/2)+1/3*
(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*D*c^3)*x)/c^3/d^4/(-d^2*x^2
+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{c^2-d^2x^2}\left(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2C+Dx)-c^2d^3(A+x(-B+Cx))+c^3d^2(B+2x(C+\right.}{c^3(c-dx)(c+dx)^2}}{3d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(c^2 - d^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.48, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2168}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)\left(c^2-d^2x^2\right)^{3/2}}+\frac{Bd^2+c^2D-cCd}{d^3\left(c^2-d^2x^2\right)^{3/2}}+\frac{x(Cd-cD)}{d^2\left(c^2-d^2x^2\right)^{3/2}}+\frac{Dx^2}{d\left(c^2-d^2x^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3c^3d^3\sqrt{c^2-d^2x^2}}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3cd^4(c+dx)\sqrt{c^2-d^2x^2}}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}-\frac{x\left(-Bd^2+c^2(-D)+cCd\right)}{c^2d^3\sqrt{c^2-d^2x^2}}+\frac{Cd-cD}{d^4\sqrt{c^2-d^2x^2}}+\frac{Dx}{d^3\sqrt{c^2-d^2x^2}}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c + d*x)*(c^2 - d^2*x^2)^(3/2)),x]
```

output
```
(C*d - c*D)/(d^4*Sqrt[c^2 - d^2*x^2]) + (D*x)/(d^3*Sqrt[c^2 - d^2*x^2]) -
((c*C*d - B*d^2 - c^2*D)*x)/(c^2*d^3*Sqrt[c^2 - d^2*x^2]) + (2*(c^2*C*d -
B*c*d^2 + A*d^3 - c^3*D)*x)/(3*c^3*d^3*Sqrt[c^2 - d^2*x^2]) - (c^2*C*d - B
*c*d^2 + A*d^3 - c^3*D)/(3*c*d^4*(c + d*x)*Sqrt[c^2 - d^2*x^2]) - (D*ArcTa
n[(d*x)/Sqrt[c^2 - d^2*x^2]])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2168
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + b*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, b,
d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.81

method	result
default	$\frac{\frac{B\,d^2 x}{c^2\sqrt{-d^2x^2+c^2}}+\frac{Dx}{\sqrt{-d^2x^2+c^2}}+\frac{Cd-Dc}{d\sqrt{-d^2x^2+c^2}}+Dd^2\left(\frac{x}{d^2\sqrt{-d^2x^2+c^2}}-\frac{\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{d^2\sqrt{d^2}}\right)-\frac{Cdx}{c\sqrt{-d^2x^2+c^2}}}{d^3}+\frac{(A\,d^3-Bc\,d^2+C\,c}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/d^3*(B*d^2*x/c^2/(-d^2*x^2+c^2)^(1/2)+D*x/(-d^2*x^2+c^2)^(1/2)+1/d*(C*d-
D*c)/(-d^2*x^2+c^2)^(1/2)+D*d^2*(x/d^2/(-d^2*x^2+c^2)^(1/2)-1/d^2/(d^2)^(1
/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2)))-C/c*d*x/(-d^2*x^2+c^2)^(1/
2))+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4*(-1/3/c/d/(x+c/d)/(-d^2*(x+c/d)^2+2*
c*d*(x+c/d))^(1/2)-1/3/d/c^3*(-2*d^2*(x+c/d)+2*c*d)/(-d^2*(x+c/d)^2+2*c*d*
(x+c/d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(145) = 290$.

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.36

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\,(c^2 - d^2x^2)^{3/2}}\,dx = \frac{2\,Dc^6 - 2\,Cc^5d - Bc^4d^2 + Ac^3d^3 - (2\,Dc^3d^3 - 2\,Cc^2d^4 - Bcd^5 + Ad^6)x^3}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="f
ricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)\,(c^2 - d^2x^2)^{3/2}}\,dx = \int \frac{A + Bx + Cx^2 + Dx^3}{\left(-\left(-c + dx\right)\left(c + dx\right)\right)^{\frac{3}{2}}\left(c + dx\right)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)/(-d**2*x**2+c**2)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((-(-c + d*x)*(c + d*x))**(3/2)*(c +
d*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(145) = 290$.

Time = 0.13 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.17

$$\int \frac{A + Bx + Cx^2 + Dx^3}{(c + dx)(c^2 - d^2x^2)^{3/2}}\,dx = \frac{Dc^3}{3\left(\sqrt{-d^2x^2 + c^2}cd^5x + \sqrt{-d^2x^2 + c^2}c^2d^4\right)} - \frac{Cc^2}{3\left(\sqrt{-d^2x^2 + c^2}cd^4x + \sqrt{-d^2x^2 + c^2}c^2d^3\right)} + \frac{Bc}{3\left(\sqrt{-d^2x^2 + c^2}cd^3x + \sqrt{-d^2x^2 + c^2}c^2d^2\right)} - \frac{A}{3\left(\sqrt{-d^2x^2 + c^2}cd^2x + \sqrt{-d^2x^2 + c^2}c^2d\right)} + \frac{2\,Ax}{3\sqrt{-d^2x^2 + c^2}c^3} + \frac{4\,Dx}{3\sqrt{-d^2x^2 + c^2}d^3} - \frac{Cx}{3\sqrt{-d^2x^2 + c^2}cd^2} + \frac{Bx}{3\sqrt{-d^2x^2 + c^2}c^2d} - \frac{D\arcsin\left(\frac{dx}{c}\right)}{d^4} - \frac{Dc}{\sqrt{-d^2x^2 + c^2}d^4} + \frac{C}{\sqrt{-d^2x^2 + c^2}d^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="m
axima")
```

output
```
1/3*D*c^3/(sqrt(-d^2*x^2 + c^2)*c*d^5*x + sqrt(-d^2*x^2 + c^2)*c^2*d^4) -
1/3*C*c^2/(sqrt(-d^2*x^2 + c^2)*c*d^4*x + sqrt(-d^2*x^2 + c^2)*c^2*d^3) +
1/3*B*c/(sqrt(-d^2*x^2 + c^2)*c*d^3*x + sqrt(-d^2*x^2 + c^2)*c^2*d^2) - 1/
3*A/(sqrt(-d^2*x^2 + c^2)*c*d^2*x + sqrt(-d^2*x^2 + c^2)*c^2*d) + 2/3*A*x/
(sqrt(-d^2*x^2 + c^2)*c^3) + 4/3*D*x/(sqrt(-d^2*x^2 + c^2)*d^3) - 1/3*C*x/
(sqrt(-d^2*x^2 + c^2)*c*d^2) + 1/3*B*x/(sqrt(-d^2*x^2 + c^2)*c^2*d) - D*ar
csin(d*x/c)/d^4 - D*c/(sqrt(-d^2*x^2 + c^2)*d^4) + C/(sqrt(-d^2*x^2 + c^2)
*d^3)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\left(-d^2x^2+c^2\right)^{\frac{3}{2}}(dx+c)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="g
iac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/((-d^2*x^2 + c^2)^(3/2)*(d*x + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{\left(c^2-d^2\,x^2\right)^{3/2}\,(c+d\,x)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c^2 - d^2*x^2)^(3/2)*(c + d*x)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c^2 - d^2*x^2)^(3/2)*(c + d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.55

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.33 $\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx$

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Optimal result

Integrand size = 38, antiderivative size = 154

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx=-\frac{\left(c^2Cd-Bcd^2+Ad^3-c^3D\right)(c-dx)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}+\frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
-1/3*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*(-d*x+c)/c/d^4/(-d^2*x^2+c^2)^(3/2)+1/3
*(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*D*c^3)*x)/c^3/d^4/(-d^2*x^
2+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/d^4
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx=\frac{\frac{\sqrt{c^2-d^2x^2}\left(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2C+Dx)-c^2d^3(A+x(-B+Cx))+c^3\right.}{c^3(c-dx)(c+dx)^2}}{3d^4}$$

input
```
Integrate[((c - d*x)*(A + B*x + C*x^2 + D*x^3))/(c^2 - d^2*x^2)^(5/2),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2166, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{2166}$$

$$-\frac{\int \frac{-\frac{3cDx^2}{d}-\frac{3c(Cd-cD)x}{d^2}+\frac{-Dc^3+Cdc^2-Bd^2c-2Ad^3}{d^3}}{\left(c^2-d^2x^2\right)^{3/2}}dx}{3c}-\frac{(c-dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{2345}$$

$$-\frac{-\frac{\int -\frac{3c^3D}{d^3\sqrt{c^2-d^2x^2}}dx}{c^2}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{c^2d^4\sqrt{c^2-d^2x^2}}}{3c}-\frac{(c-dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3cD\int \frac{1}{\sqrt{c^2-d^2x^2}}dx}{d^3}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{c^2d^4\sqrt{c^2-d^2x^2}}}{3c}-\frac{(c-dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{3cD\int \frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}}{d^3}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{c^2d^4\sqrt{c^2-d^2x^2}}}{3c}-\frac{(c-dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{3cD\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{c^2d^4\sqrt{c^2-d^2x^2}}}{3c}-\frac{(c-dx)\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^4\left(c^2-d^2x^2\right)^{3/2}}$$

input
```
Int[((c - d*x)*(A + B*x + C*x^2 + D*x^3))/(c^2 - d^2*x^2)^(5/2),x]
```

output
```
-1/3*((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*(c - d*x))/(c*d^4*(c^2 - d^2*x^2
)^(3/2)) - (-((3*c^3*(C*d - c*D) - d*(c^2*C*d - B*c*d^2 - 2*A*d^3 - 4*c^3*
D)*x)/(c^2*d^4*Sqrt[c^2 - d^2*x^2])) + (3*c*D*ArcTan[(d*x)/Sqrt[c^2 - d^2*
x^2]])/d^4)/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2166
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[Pq, a*e + b*d*x, x], R = PolynomialRemainde
r[Pq, a*e + b*d*x, x]}, Simp[(-d)*R*(d + e*x)^m*((a + b*x^2)^(p + 1)/(2*a*e
*(p + 1))), x] + Simp[d/(2*a*(p + 1))   Int[(d + e*x)^(m - 1)*(a + b*x^2)^(
p + 1)*ExpandToSum[2*a*e*(p + 1)*Qx + R*(m + 2*p + 2), x], x], x]] /; FreeQ
[{a, b, d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && ILtQ[p + 1/2,
 0] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.88

method	result
default	$Ac\left(\frac{x}{3c^2(-d^2x^2+c^2)^{\frac{3}{2}}}+\frac{2x}{3c^4\sqrt{-d^2x^2+c^2}}\right)+\frac{-Ad+Bc}{3d^2(-d^2x^2+c^2)^{\frac{3}{2}}}+(-Bd+Cc)\left(\frac{x}{2d^2(-d^2x^2+c^2)^{\frac{3}{2}}}-\frac{c^2\left(\frac{}{3c^2(-d^2\cdots}\right.}{}\right.$

input
```
int((-d*x+c)*(D*x^3+C*x^2+B*x+A)/(-d^2*x^2+c^2)^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
A*c*(1/3*x/c^2/(-d^2*x^2+c^2)^(3/2)+2/3/c^4*x/(-d^2*x^2+c^2)^(1/2))+1/3*(-
A*d+B*c)/d^2/(-d^2*x^2+c^2)^(3/2)+(-B*d+C*c)*(1/2*x/d^2/(-d^2*x^2+c^2)^(3/
2)-1/2*c^2/d^2*(1/3*x/c^2/(-d^2*x^2+c^2)^(3/2)+2/3/c^4*x/(-d^2*x^2+c^2)^(1
/2)))+(-C*d+D*c)*(x^2/d^2/(-d^2*x^2+c^2)^(3/2)-2/3*c^2/d^4/(-d^2*x^2+c^2)^
(3/2))-D*d*(1/3*x^3/d^2/(-d^2*x^2+c^2)^(3/2)-1/d^2*(x/d^2/(-d^2*x^2+c^2)^(
1/2)-1/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(145) = 290$.

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.38

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{(c^2-d^2x^2)^{5/2}}\,dx = \frac{2\,Dc^6-2\,Cc^5d-Bc^4d^2+Ac^3d^3-\left(2\,Dc^3d^3-2\,Cc^2d^4-Bcd^5\right.}{}$$

input
```
integrate((-d*x+c)*(D*x^3+C*x^2+B*x+A)/(-d^2*x^2+c^2)^(5/2),x, algorithm="
fricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.26 (sec) , antiderivative size = 1550, normalized size of antiderivative = 10.06

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{(c^2-d^2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((-d*x+c)*(D*x**3+C*x**2+B*x+A)/(-d**2*x**2+c**2)**(5/2),x)
```

output
```
A*c*Piecewise((3*I*c**2*x/(-3*c**7*sqrt(-1 + d**2*x**2/c**2) + 3*c**5*d**2
*x**2*sqrt(-1 + d**2*x**2/c**2)) - 2*I*d**2*x**3/(-3*c**7*sqrt(-1 + d**2*x
**2/c**2) + 3*c**5*d**2*x**2*sqrt(-1 + d**2*x**2/c**2)), Abs(d**2*x**2/c**
2) > 1), (-3*c**2*x/(-3*c**7*sqrt(1 - d**2*x**2/c**2) + 3*c**5*d**2*x**2*s
qrt(1 - d**2*x**2/c**2)) + 2*d**2*x**3/(-3*c**7*sqrt(1 - d**2*x**2/c**2) +
 3*c**5*d**2*x**2*sqrt(1 - d**2*x**2/c**2)), True)) - A*d*Piecewise((-1/(-
3*c**2*d**2*sqrt(c**2 - d**2*x**2) + 3*d**4*x**2*sqrt(c**2 - d**2*x**2)),
Ne(d, 0)), (x**2/(2*(c**2)**(5/2)), True)) + B*c*Piecewise((-1/(-3*c**2*d*
*2*sqrt(c**2 - d**2*x**2) + 3*d**4*x**2*sqrt(c**2 - d**2*x**2)), Ne(d, 0))
, (x**2/(2*(c**2)**(5/2)), True)) - B*d*Piecewise((I*x**3/(-3*c**5*sqrt(-1
 + d**2*x**2/c**2) + 3*c**3*d**2*x**2*sqrt(-1 + d**2*x**2/c**2)), Abs(d**2
*x**2/c**2) > 1), (-x**3/(-3*c**5*sqrt(1 - d**2*x**2/c**2) + 3*c**3*d**2*x
**2*sqrt(1 - d**2*x**2/c**2)), True)) + C*c*Piecewise((I*x**3/(-3*c**5*sqr
t(-1 + d**2*x**2/c**2) + 3*c**3*d**2*x**2*sqrt(-1 + d**2*x**2/c**2)), Abs(
d**2*x**2/c**2) > 1), (-x**3/(-3*c**5*sqrt(1 - d**2*x**2/c**2) + 3*c**3*d*
*2*x**2*sqrt(1 - d**2*x**2/c**2)), True)) - C*d*Piecewise((2*c**2/(-3*c**2
*d**4*sqrt(c**2 - d**2*x**2) + 3*d**6*x**2*sqrt(c**2 - d**2*x**2)) - 3*d**
2*x**2/(-3*c**2*d**4*sqrt(c**2 - d**2*x**2) + 3*d**6*x**2*sqrt(c**2 - d**2
*x**2)), Ne(d, 0)), (x**4/(4*(c**2)**(5/2)), True)) + D*c*Piecewise((2*c**
2/(-3*c**2*d**4*sqrt(c**2 - d**2*x**2) + 3*d**6*x**2*sqrt(c**2 - d**2*x...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.86

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx = -\frac{1}{3}\,Ddx\left(\frac{3\,x^2}{\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^2}-\frac{2\,c^2}{\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^4}\right)+\frac{Ax}{3\left(-d^2x^2+c^2\right)^{\frac{3}{2}}c}+\frac{(Dc-Cd)x^2}{\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^2}+\frac{Bc}{3\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^2}-\frac{A}{3\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d}+\frac{2\,Ax}{3\sqrt{-d^2x^2+c^2}c^3}+\frac{Dx}{3\sqrt{-d^2x^2+c^2}d^3}+\frac{(Cc-Bd)x}{3\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^2}-\frac{D\arcsin\left(\frac{dx}{c}\right)}{d^4}-\frac{2\,(Dc-Cd)c^2}{3\left(-d^2x^2+c^2\right)^{\frac{3}{2}}d^4}-\frac{(Cc-Bd)x}{3\sqrt{-d^2x^2+c^2}c^2d^2}$$

input
```
integrate((-d*x+c)*(D*x^3+C*x^2+B*x+A)/(-d^2*x^2+c^2)^(5/2),x, algorithm="
maxima")
```

output
```
-1/3*D*d*x*(3*x^2/((-d^2*x^2 + c^2)^(3/2)*d^2) - 2*c^2/((-d^2*x^2 + c^2)^(
3/2)*d^4)) + 1/3*A*x/((-d^2*x^2 + c^2)^(3/2)*c) + (D*c - C*d)*x^2/((-d^2*x
^2 + c^2)^(3/2)*d^2) + 1/3*B*c/((-d^2*x^2 + c^2)^(3/2)*d^2) - 1/3*A/((-d^2
*x^2 + c^2)^(3/2)*d) + 2/3*A*x/(sqrt(-d^2*x^2 + c^2)*c^3) + 1/3*D*x/(sqrt(
-d^2*x^2 + c^2)*d^3) + 1/3*(C*c - B*d)*x/((-d^2*x^2 + c^2)^(3/2)*d^2) - D*
arcsin(d*x/c)/d^4 - 2/3*(D*c - C*d)*c^2/((-d^2*x^2 + c^2)^(3/2)*d^4) - 1/3
*(C*c - B*d)*x/(sqrt(-d^2*x^2 + c^2)*c^2*d^2)
```

Giac [F]

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx=\int -\frac{(Dx^3+Cx^2+Bx+A)(dx-c)}{(-d^2x^2+c^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-d*x+c)*(D*x^3+C*x^2+B*x+A)/(-d^2*x^2+c^2)^(5/2),x, algorithm="
giac")
```

output
```
integrate(-(D*x^3 + C*x^2 + B*x + A)*(d*x - c)/(-d^2*x^2 + c^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx=\int \frac{(c-d\,x)\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(c^2-d^2\,x^2\right)}^{5/2}}\,dx$$

input
```
int(((c - d*x)*(A + B*x + C*x^2 + x^3*D))/(c^2 - d^2*x^2)^(5/2),x)
```

output
```
int(((c - d*x)*(A + B*x + C*x^2 + x^3*D))/(c^2 - d^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.57

$$\int \frac{(c-dx)\left(A+Bx+Cx^2+Dx^3\right)}{\left(c^2-d^2x^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int((-d*x+c)*(D*x^3+C*x^2+B*x+A)/(-d^2*x^2+c^2)^(5/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.34 $\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$

Optimal result 357
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Reduce [B] (verification not implemented) 363

Optimal result

Integrand size = 47, antiderivative size = 179

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = -\frac{c\left(c^2C-Bcd+Ad^2-\frac{c^3D}{d}\right)-(c^2Cd-Bcd^2+Ad^3-c^3D)\,x}{3cd^3\,(c^2-d^2x^2)^{3/2}} + \frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}} - \frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
-1/3*(c*(C*c^2-B*c*d+A*d^2-c^3*D/d)-(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*x)/c/d^3
/(-d^2*x^2+c^2)^(3/2)+1/3*(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*D
*c^3)*x)/c^3/d^4/(-d^2*x^2+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/d
^4
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$
$$= \frac{\frac{\sqrt{c^2-d^2x^2}(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2C+Dx)-c^2d^3(A+x(-B+Cx))+c^3d^2(B+2x(C+2Dx)))}{c^3(c-dx)(c+dx)^2}+6D\arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-\cdots}}\right.}{3d^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/((c - d*x)*(c + d*x)^2*Sqrt[c^2 - d^2*
x^2]),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$

$$\Big\downarrow \text{2348}$$

$$\int \left(\frac{Ad^3+Bcd^2+c^3D+c^2Cd}{4c^2d^3(c-dx)\sqrt{c^2-d^2x^2}}+\frac{Ad^3+Bcd^2+5c^3D-3c^2Cd}{4c^2d^3(c+dx)\sqrt{c^2-d^2x^2}}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2cd^3(c+dx)^2\sqrt{c^2-d^2x^2}}-\frac{D}{d^3\sqrt{c^2-d\cdots}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{c^2-d^2x^2}(-Ad^3-Bcd^2-5c^3D+3c^2Cd)}{4c^3d^4(c+dx)}+\frac{\sqrt{c^2-d^2x^2}(Ad^3+Bcd^2+c^3D+c^2Cd)}{4c^3d^4(c-dx)}-\frac{\sqrt{c^2-d^2x^2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{6c^3d^4(c+dx)}-\frac{\sqrt{c^2-d^2x^2}(Ad^3-Bcd^2+c^3(-D)+c^2Cd)}{6c^2d^4(c+dx)^2}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/((c - d*x)*(c + d*x)^2*Sqrt[c^2 - d^2*x^2]),
x]
```

output
```
((c^2*C*d + B*c*d^2 + A*d^3 + c^3*D)*Sqrt[c^2 - d^2*x^2])/(4*c^3*d^4*(c -
d*x)) - ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c^2 - d^2*x^2])/(6*c^2*d
^4*(c + d*x)^2) + ((3*c^2*C*d - B*c*d^2 - A*d^3 - 5*c^3*D)*Sqrt[c^2 - d^2*
x^2])/(4*c^3*d^4*(c + d*x)) - ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c^
2 - d^2*x^2])/(6*c^3*d^4*(c + d*x)) - (D*ArcTan[(d*x)/Sqrt[c^2 - d^2*x^2]]
)/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.66

method	result
default	$-\frac{D\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{d^3\sqrt{d^2}}-\frac{(A\,d^3+Bc\,d^2+C\,c^2d+Dc^3)\sqrt{-d^2\left(x-\frac{c}{d}\right)^2-2cd\left(x-\frac{c}{d}\right)}}{4d^5c^3\left(x-\frac{c}{d}\right)}+\frac{(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3)\left(-\frac{\sqrt{-d^2(}}{}\right.}{}$

input
```
int((D*x^3+C*x^2+B*x+A)/(-d*x+c)/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
-D/d^3/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2))-1/4/d^5*(A*d
^3+B*c*d^2+C*c^2*d+D*c^3)/c^3/(x-c/d)*(-d^2*(x-c/d)^2-2*c*d*(x-c/d))^(1/2)
+1/2/d^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/c*(-1/3/c/d/(x+c/d)^2*(-d^2*(x+c/d)
^2+2*c*d*(x+c/d))^(1/2)-1/3/c^2/(x+c/d)*(-d^2*(x+c/d)^2+2*c*d*(x+c/d))^(1/
2))-1/4/d^5*(A*d^3+B*c*d^2-3*C*c^2*d+5*D*c^3)/c^3/(x+c/d)*(-d^2*(x+c/d)^2+
2*c*d*(x+c/d))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(168) = 336$.

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \frac{2\,Dc^6-2\,Cc^5d-Bc^4d^2+Ac^3d^3-(2\,Dc^3d^3-2\,Cc^2d^4-Bcd^5+Ad^6)x^3-(2\,Dc^4d^2-2\,Cc^3d^3-Bc^2d}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(-d*x+c)/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, a
lgorithm="fricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$
$$= -\int \frac{A}{-c^3\sqrt{c^2-d^2x^2}-c^2dx\sqrt{c^2-d^2x^2}+cd^2x^2\sqrt{c^2-d^2x^2}+d^3x^3\sqrt{c^2-d^2x^2}}\,dx$$
$$-\int \frac{Bx}{-c^3\sqrt{c^2-d^2x^2}-c^2dx\sqrt{c^2-d^2x^2}+cd^2x^2\sqrt{c^2-d^2x^2}+d^3x^3\sqrt{c^2-d^2x^2}}\,dx$$
$$-\int \frac{Cx^2}{-c^3\sqrt{c^2-d^2x^2}-c^2dx\sqrt{c^2-d^2x^2}+cd^2x^2\sqrt{c^2-d^2x^2}+d^3x^3\sqrt{c^2-d^2x^2}}\,dx$$
$$-\int \frac{Dx^3}{-c^3\sqrt{c^2-d^2x^2}-c^2dx\sqrt{c^2-d^2x^2}+cd^2x^2\sqrt{c^2-d^2x^2}+d^3x^3\sqrt{c^2-d^2x^2}}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(-d*x+c)/(d*x+c)**2/(-d**2*x**2+c**2)**(1/
2),x)
```

output
```
-Integral(A/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sqrt(c**2 - d**2*x**2
) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt(c**2 - d**2*x**2))
, x) - Integral(B*x/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sqrt(c**2 - d
**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt(c**2 - d**
2*x**2)), x) - Integral(C*x**2/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sq
rt(c**2 - d**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt
(c**2 - d**2*x**2)), x) - Integral(D*x**3/(-c**3*sqrt(c**2 - d**2*x**2) -
c**2*d*x*sqrt(c**2 - d**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**
3*x**3*sqrt(c**2 - d**2*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \int -\frac{Dx^3+Cx^2+Bx+A}{\sqrt{-d^2x^2+c^2}(dx+c)^2(dx-c)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(-d*x+c)/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, a
lgorithm="maxima")
```

output
```
-integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-d^2*x^2 + c^2)*(d*x + c)^2*(d*
x - c)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(-d*x+c)/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, a
lgorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -1/a
bs(sageVARd)*(-1/4*(sageVARA*sageVARd^3+sageVARD*sageVARc^3+sageVARB*sageV
ARc*sageVARd^2+sageVARC*sageVARc^2*sageVARd)/sageVARc^3/sageVARd^3/sqrt(2*
sageVARc*sageVARd*(
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{\sqrt{c^2-d^2\,x^2}\,{\left(c+d\,x\right)}^2\,\left(c-d\,x\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((c^2 - d^2*x^2)^(1/2)*(c + d*x)^2*(c - d*x)
),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((c^2 - d^2*x^2)^(1/2)*(c + d*x)^2*(c - d*x)
), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.07

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \text{Too large to display}$$

input
```
int((D*x^3+C*x^2+B*x+A)/(-d*x+c)/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.35 $\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c^2-d^2x^2)^{5/2}}\,dx$

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Optimal result

Integrand size = 63, antiderivative size = 179

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c^2-d^2x^2)^{5/2}}\,dx = \frac{cd\left(Bc-\frac{c^2C}{d}-Ad+\frac{c^3D}{d^2}\right)+(c^2Cd-}{3cd^3\,(c^2-d^2x^2)^{3/}}$$
$$+\frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
1/3*(c*d*(B*c-c^2*C/d-A*d+c^3*D/d^2)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*x)/c/d^
3/(-d^2*x^2+c^2)^(3/2)+1/3*(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*
D*c^3)*x)/c^3/d^4/(-d^2*x^2+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/
d^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c^2-d^2x^2)^{5/2}}\,dx = \frac{\frac{\sqrt{c^2-d^2x^2}(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2}{c^3(c-d}}{}$$

input
```
Integrate[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x
^4)/(c^2 - d^2*x^2)^(5/2),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2345, 2345, 27, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(Bc-Ad)+Ac+x^2(cC-Bd)-x^3(Cd-cD)-dDx^4}{(c^2-d^2x^2)^{5/2}}dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{cd\left(-Ad+Bc+\frac{c^3D}{d^2}-\frac{c^2C}{d}\right)+x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^3\left(c^2-d^2x^2\right)^{3/2}}-\frac{\int \frac{-\frac{3Dx^2c^2}{d}-\frac{3(Cd-cD)xc^2}{d^2}+\frac{\left(-Dc^3+Cdc^2-Bd^2c-2Ad^3\right)c}{d^3}}{(c^2-d^2x^2)^{3/2}}dx}{3c^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{cd\left(-Ad+Bc+\frac{c^3D}{d^2}-\frac{c^2C}{d}\right)+x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^3\left(c^2-d^2x^2\right)^{3/2}}-\frac{-\frac{\int -\frac{3c^4D}{d^3\sqrt{c^2-d^2x^2}}dx}{c^2}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{cd^4\sqrt{c^2-d^2x^2}}}{3c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{cd\left(-Ad+Bc+\frac{c^3D}{d^2}-\frac{c^2C}{d}\right)+x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^3\left(c^2-d^2x^2\right)^{3/2}}-\frac{\frac{3c^2D\int\frac{1}{\sqrt{c^2-d^2x^2}}dx}{d^3}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{cd^4\sqrt{c^2-d^2x^2}}}{3c^2}$$

$$\Big\downarrow \ 224$$

$$\frac{cd\left(-Ad+Bc+\frac{c^3D}{d^2}-\frac{c^2C}{d}\right)+x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^3\left(c^2-d^2x^2\right)^{3/2}}-\frac{\frac{3c^2D\int\frac{1}{\frac{d^2x^2}{c^2-d^2x^2}+1}d\frac{x}{\sqrt{c^2-d^2x^2}}}{d^3}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{cd^4\sqrt{c^2-d^2x^2}}}{3c^2}$$

$$\Big\downarrow \ 216$$

$$\frac{cd\left(-Ad+Bc+\frac{c^3D}{d^2}-\frac{c^2C}{d}\right)+x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3cd^3\left(c^2-d^2x^2\right)^{3/2}}-\frac{\frac{3c^2D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}-\frac{3c^3(Cd-cD)-dx\left(-2Ad^3-Bcd^2-4c^3D+c^2Cd\right)}{cd^4\sqrt{c^2-d^2x^2}}}{3c^2}$$

input
```
Int[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x^4)/(c
^2 - d^2*x^2)^(5/2),x]
```

output
```
(c*d*(B*c - (c^2*C)/d - A*d + (c^3*D)/d^2) + (c^2*C*d - B*c*d^2 + A*d^3 -
c^3*D)*x)/(3*c*d^3*(c^2 - d^2*x^2)^(3/2)) - (-((3*c^3*(C*d - c*D) - d*(c^2
*C*d - B*c*d^2 - 2*A*d^3 - 4*c^3*D)*x)/(c*d^4*Sqrt[c^2 - d^2*x^2])) + (3*c
^2*D*ArcTan[(d*x)/Sqrt[c^2 - d^2*x^2]])/d^4)/(3*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.61

method	result
default	$Ac\left(\frac{x}{3c^{2}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{3}{2}}}+\frac{2x}{3c^{4}\sqrt{-d^{2}x^{2}+c^{2}}}\right)+\frac{-Ad+Bc}{3d^{2}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{3}{2}}}+\left(-Bd+Cc\right)\left(\frac{x}{2d^{2}\left(-d^{2}x^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{c^{2}\left(\frac{}{3c^{2}\left(-d^{2}\right.}\right.}{}\right.$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d^2*x^2+c^2)
^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*c*(1/3*x/c^2/(-d^2*x^2+c^2)^(3/2)+2/3/c^4*x/(-d^2*x^2+c^2)^(1/2))+1/3*(-
A*d+B*c)/d^2/(-d^2*x^2+c^2)^(3/2)+(-B*d+C*c)*(1/2*x/d^2/(-d^2*x^2+c^2)^(3/
2)-1/2*c^2/d^2*(1/3*x/c^2/(-d^2*x^2+c^2)^(3/2)+2/3/c^4*x/(-d^2*x^2+c^2)^(1
/2)))+(-C*d+D*c)*(x^2/d^2/(-d^2*x^2+c^2)^(3/2)-2/3*c^2/d^4/(-d^2*x^2+c^2)^
(3/2))-D*d*(1/3*x^3/d^2/(-d^2*x^2+c^2)^(3/2)-1/d^2*(x/d^2/(-d^2*x^2+c^2)^(
1/2)-1/d^2/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(170) = 340$.

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.04

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c^2 - d^2x^2)^{5/2}} \, dx = \frac{2\,Dc^6 - 2\,Cc^5d - Bc^4d^2 + Ac^3d^3 - (2}{}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d^2*x^
2+c^2)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [A] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 1261, normalized size of antiderivative = 7.04

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c^2 - d^2x^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x**2-(C*d-D*c)*x**3-d*D*x**4)/(-d**
2*x**2+c**2)**(5/2),x)
```

output
```
A*c*Piecewise((3*I*c**2*x/(-3*c**7*sqrt(-1 + d**2*x**2/c**2) + 3*c**5*d**2
*x**2*sqrt(-1 + d**2*x**2/c**2)) - 2*I*d**2*x**3/(-3*c**7*sqrt(-1 + d**2*x
**2/c**2) + 3*c**5*d**2*x**2*sqrt(-1 + d**2*x**2/c**2)), Abs(d**2*x**2/c**
2) > 1), (-3*c**2*x/(-3*c**7*sqrt(1 - d**2*x**2/c**2) + 3*c**5*d**2*x**2*s
qrt(1 - d**2*x**2/c**2)) + 2*d**2*x**3/(-3*c**7*sqrt(1 - d**2*x**2/c**2) +
 3*c**5*d**2*x**2*sqrt(1 - d**2*x**2/c**2)), True)) - D*d*Piecewise((6*I*c
**3*sqrt(-1 + d**2*x**2/c**2)*acosh(d*x/c)/(-6*c**3*d**5*sqrt(-1 + d**2*x*
*2/c**2) + 6*c*d**7*x**2*sqrt(-1 + d**2*x**2/c**2)) - 3*pi*c**3*sqrt(-1 +
d**2*x**2/c**2)/(-6*c**3*d**5*sqrt(-1 + d**2*x**2/c**2) + 6*c*d**7*x**2*sq
rt(-1 + d**2*x**2/c**2)) - 6*I*c**2*d*x/(-6*c**3*d**5*sqrt(-1 + d**2*x**2/
c**2) + 6*c*d**7*x**2*sqrt(-1 + d**2*x**2/c**2)) - 6*I*c*d**2*x**2*sqrt(-1
 + d**2*x**2/c**2)*acosh(d*x/c)/(-6*c**3*d**5*sqrt(-1 + d**2*x**2/c**2) +
6*c*d**7*x**2*sqrt(-1 + d**2*x**2/c**2)) + 3*pi*c*d**2*x**2*sqrt(-1 + d**2
*x**2/c**2)/(-6*c**3*d**5*sqrt(-1 + d**2*x**2/c**2) + 6*c*d**7*x**2*sqrt(-
1 + d**2*x**2/c**2)) + 8*I*d**3*x**3/(-6*c**3*d**5*sqrt(-1 + d**2*x**2/c**
2) + 6*c*d**7*x**2*sqrt(-1 + d**2*x**2/c**2)), Abs(d**2*x**2/c**2) > 1), (
-3*c**3*sqrt(1 - d**2*x**2/c**2)*asin(d*x/c)/(-3*c**3*d**5*sqrt(1 - d**2*x
**2/c**2) + 3*c*d**7*x**2*sqrt(1 - d**2*x**2/c**2)) + 3*c**2*d*x/(-3*c**3*
d**5*sqrt(1 - d**2*x**2/c**2) + 3*c*d**7*x**2*sqrt(1 - d**2*x**2/c**2)) +
3*c*d**2*x**2*sqrt(1 - d**2*x**2/c**2)*asin(d*x/c)/(-3*c**3*d**5*sqrt(1...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.60

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c^2-d^2x^2)^{5/2}}\,dx = -\frac{1}{3}\,Ddx\left(\frac{3\,x^2}{(-d^2x^2+c^2)^{\frac{3}{2}}d^2}-\frac{2\,c^2}{(-d^2x^2+c^2)^{\frac{3}{2}}d^4}\right)+\frac{Ax}{3\,(-d^2x^2+c^2)^{\frac{3}{2}}c} + \frac{(Dc-Cd)x^2}{(-d^2x^2+c^2)^{\frac{3}{2}}d^2}+\frac{Bc}{3\,(-d^2x^2+c^2)^{\frac{3}{2}}d^2}-\frac{A}{3\,(-d^2x^2+c^2)^{\frac{3}{2}}d} + \frac{2\,Ax}{3\,\sqrt{-d^2x^2+c^2}c^3}+\frac{Dx}{3\,\sqrt{-d^2x^2+c^2}d^3}+\frac{(Cc-Bd)x}{3\,(-d^2x^2+c^2)^{\frac{3}{2}}d^2} - \frac{D\arcsin\left(\frac{dx}{c}\right)}{d^4}-\frac{2\,(Dc-Cd)c^2}{3\,(-d^2x^2+c^2)^{\frac{3}{2}}d^4}-\frac{(Cc-Bd)x}{3\,\sqrt{-d^2x^2+c^2}c^2d^2}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d^2*x^
2+c^2)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*D*d*x*(3*x^2/((-d^2*x^2 + c^2)^(3/2)*d^2) - 2*c^2/((-d^2*x^2 + c^2)^(
3/2)*d^4)) + 1/3*A*x/((-d^2*x^2 + c^2)^(3/2)*c) + (D*c - C*d)*x^2/((-d^2*x
^2 + c^2)^(3/2)*d^2) + 1/3*B*c/((-d^2*x^2 + c^2)^(3/2)*d^2) - 1/3*A/((-d^2
*x^2 + c^2)^(3/2)*d) + 2/3*A*x/(sqrt(-d^2*x^2 + c^2)*c^3) + 1/3*D*x/(sqrt(
-d^2*x^2 + c^2)*d^3) + 1/3*(C*c - B*d)*x/((-d^2*x^2 + c^2)^(3/2)*d^2) - D*
arcsin(d*x/c)/d^4 - 2/3*(D*c - C*d)*c^2/((-d^2*x^2 + c^2)^(3/2)*d^4) - 1/3
*(C*c - B*d)*x/(sqrt(-d^2*x^2 + c^2)*c^2*d^2)
```

Giac [F]

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c^2 - d^2x^2)^{5/2}}\,dx = \int -\frac{Ddx^4 - (Dc - Cd)x^3 - (Cc - Bc}{(-d^2x^2 + c^2}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d^2*x^
2+c^2)^(5/2),x, algorithm="giac")
```

output
```
integrate(-(D*d*x^4 - (D*c - C*d)*x^3 - (C*c - B*d)*x^2 - A*c - (B*c - A*d
)*x)/(-d^2*x^2 + c^2)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c^2 - d^2x^2)^{5/2}}\,dx = -\int \frac{x^3\,(C\,d - c\,D) - A\,c + x\,(A\,d - B\,c) + x^2\,(B\,d - C\,c) + d\,x^4\,D}{(c^2 - d^2\,x^2)^{5/2}}\,dx$$

input
```
int(-(x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
c^2 - d^2*x^2)^(5/2),x)
```

output
```
-int((x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
c^2 - d^2*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.07

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c^2 - d^2x^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d^2*x^2+c^2)
^(5/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.36 $\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx$

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Optimal result

Integrand size = 78, antiderivative size = 179

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = \frac{cd\left(Bc-\frac{c^2C}{d}-Ad+\frac{c^3D}{d^2}\right)+(c^2Cd-}{3cd^3\left(c^2-d^2x^2\right)^{3/}}$$

$$+\frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
1/3*(c*d*(B*c-c^2*C/d-A*d+c^3*D/d^2)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*x)/c/d^
3/(-d^2*x^2+c^2)^(3/2)+1/3*(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*
D*c^3)*x)/c^3/d^4/(-d^2*x^2+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/
d^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = \frac{\frac{\sqrt{c^2-d^2x^2}\left(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2\right.}{c^3(c-d}}{}$$

input
```
Integrate[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x
^4)/((c - d*x)*(c + d*x)*(c^2 - d^2*x^2)^(3/2)),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {2019, 2168, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(Bc-Ad)+Ac+x^2(cC-Bd)-x^3(Cd-cD)-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c+dx)\left(c^2-d^2x^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{2168}$$

$$\int \left(\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{d^3(c+dx)\left(c^2-d^2x^2\right)^{3/2}}+\frac{Bd^2+c^2D-cCd}{d^3\left(c^2-d^2x^2\right)^{3/2}}+\frac{x(Cd-cD)}{d^2\left(c^2-d^2x^2\right)^{3/2}}+\frac{Dx^2}{d\left(c^2-d^2x^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{3c^3d^3\sqrt{c^2-d^2x^2}}-\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{3cd^4(c+dx)\sqrt{c^2-d^2x^2}}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}-\frac{x\left(-Bd^2+c^2(-D)+cCd\right)}{c^2d^3\sqrt{c^2-d^2x^2}}+\frac{Cd-cD}{d^4\sqrt{c^2-d^2x^2}}+\frac{Dx}{d^3\sqrt{c^2-d^2x^2}}$$

input
```
Int[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x^4)/((
c - d*x)*(c + d*x)*(c^2 - d^2*x^2)^(3/2)),x]
```

output
```
(C*d - c*D)/(d^4*Sqrt[c^2 - d^2*x^2]) + (D*x)/(d^3*Sqrt[c^2 - d^2*x^2]) -
((c*C*d - B*d^2 - c^2*D)*x)/(c^2*d^3*Sqrt[c^2 - d^2*x^2]) + (2*(c^2*C*d -
B*c*d^2 + A*d^3 - c^3*D)*x)/(3*c^3*d^3*Sqrt[c^2 - d^2*x^2]) - (c^2*C*d - B
*c*d^2 + A*d^3 - c^3*D)/(3*c*d^4*(c + d*x)*Sqrt[c^2 - d^2*x^2]) - (D*ArcTa
n[(d*x)/Sqrt[c^2 - d^2*x^2]])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2168
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Int[ExpandIntegrand[(a + b*x^2)^p, (d + e*x)^m*Pq, x], x] /; FreeQ[{a, b,
d, e}, x] && PolyQ[Pq, x] && EqQ[b*d^2 + a*e^2, 0] && EqQ[m + Expon[Pq, x]
+ 2*p + 1, 0] && ILtQ[m, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.56

method	result
default	$\frac{\frac{B\,d^{2} x}{c^{2} \sqrt{-d^{2} x^{2}+c^{2}}}+\frac{D x}{\sqrt{-d^{2} x^{2}+c^{2}}}+\frac{C d -D c}{d \sqrt{-d^{2} x^{2}+c^{2}}}+D d^{2} \left(\frac{x}{d^{2} \sqrt{-d^{2} x^{2}+c^{2}}}-\frac{\arctan\left(\frac{\sqrt{d^{2}}\, x}{\sqrt{-d^{2} x^{2}+c^{2}}}\right)}{d^{2} \sqrt{d^{2}}}\right)-\frac{C d x}{c \sqrt{-d^{2} x^{2}+c^{2}}}}{d^{3}}+\frac{\left(A\,d^{3}-B c\,d^{2}+C c}{}$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)/(d*x+
c)/(-d^2*x^2+c^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/d^3*(B*d^2*x/c^2/(-d^2*x^2+c^2)^(1/2)+D*x/(-d^2*x^2+c^2)^(1/2)+1/d*(C*d-
D*c)/(-d^2*x^2+c^2)^(1/2)+D*d^2*(x/d^2/(-d^2*x^2+c^2)^(1/2)-1/d^2/(d^2)^(1
/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2)))-C/c*d*x/(-d^2*x^2+c^2)^(1/
2))+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/d^4*(-1/3/c/d/(x+c/d)/(-d^2*(x+c/d)^2+2*
c*d*(x+c/d))^(1/2)-1/3/d/c^3*(-2*d^2*(x+c/d)+2*c*d)/(-d^2*(x+c/d)^2+2*c*d*
(x+c/d))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(170) = 340$.

Time = 0.09 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.04

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = \frac{2\,Dc^6-2\,Cc^5d-Bc^4d^2+Ac^3d^3-(2$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [F]

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\left(-\left(-c+dx\right)\left(c+dx\right)\right)^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x**2-(C*d-D*c)*x**3-d*D*x**4)/(-d*x
+c)/(d*x+c)/(-d**2*x**2+c**2)**(3/2),x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/((-(-c + d*x)*(c + d*x))**(3/2)*(c +
d*x)), x)
```

Maxima [F]

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx=\int \frac{Ddx^4-(Dc-Cd)x^3-(Cc-Bd)}{(-d^2x^2+c^2)^{\frac{3}{2}}(dx+c}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((D*d*x^4 - (D*c - C*d)*x^3 - (C*c - B*d)*x^2 - A*c - (B*c - A*d)
*x)/((-d^2*x^2 + c^2)^(3/2)*(d*x + c)*(d*x - c)), x)
```

Giac [F]

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)(c+dx)\left(c^2-d^2x^2\right)^{3/2}}\,dx=\int \frac{Ddx^4-(Dc-Cd)x^3-(Cc-Bd)}{(-d^2x^2+c^2)^{\frac{3}{2}}(dx+c}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
/(d*x+c)/(-d^2*x^2+c^2)^(3/2),x, algorithm="giac")
```

output
```
integrate((D*d*x^4 - (D*c - C*d)*x^3 - (C*c - B*d)*x^2 - A*c - (B*c - A*d)
*x)/((-d^2*x^2 + c^2)^(3/2)*(d*x + c)*(d*x - c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c - dx)(c + dx)\left(c^2 - d^2x^2\right)^{3/2}}\,dx = -\int \frac{x^3\,(C\,d - c\,D) - A\,c + x\,(A\,d - B\,c) + x^2\,(B\,d - C\,c) + d\,x^4\,D}{\left(c^2 - d^2\,x^2\right)^{3/2}\,(c + d\,x)\,(c - d\,x)}\,dx$$

input
```
int(-(x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
(c^2 - d^2*x^2)^(3/2)*(c + d*x)*(c - d*x)),x)
```

output
```
-int((x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
(c^2 - d^2*x^2)^(3/2)*(c + d*x)*(c - d*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.07

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c - dx)(c + dx)\left(c^2 - d^2x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)/(d*x+
c)/(-d^2*x^2+c^2)^(3/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.37 $\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$

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Optimal result

Integrand size = 78, antiderivative size = 179

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx = \frac{cd\left(Bc-\frac{c^2C}{d}-Ad+\frac{c^3D}{d^2}\right)+(c^2Cd-Bcd^2+Ad^3-c^3D)\,x}{3cd^3\,(c^2-d^2x^2)^{3/2}} + \frac{3c^3(Cd-cD)-d(c^2Cd-Bcd^2-2Ad^3-4c^3D)\,x}{3c^3d^4\sqrt{c^2-d^2x^2}} - \frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

output
```
1/3*(c*d*(B*c-c^2*C/d-A*d+c^3*D/d^2)+(A*d^3-B*c*d^2+C*c^2*d-D*c^3)*x)/c/d^
3/(-d^2*x^2+c^2)^(3/2)+1/3*(3*c^3*(C*d-D*c)-d*(-2*A*d^3-B*c*d^2+C*c^2*d-4*
D*c^3)*x)/c^3/d^4/(-d^2*x^2+c^2)^(1/2)-D*arctan(d*x/(-d^2*x^2+c^2)^(1/2))/
d^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.87

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$
$$=\frac{\frac{\sqrt{c^2-d^2x^2}\left(-2c^5D+2Ad^5x^2+cd^4x(2A+Bx)+c^4d(2C+Dx)-c^2d^3(A+x(-B+Cx))+c^3d^2(B+2x(C+2Dx))\right)}{c^3(c-dx)(c+dx)^2}+6D\arctan\left(\frac{dx}{\sqrt{c^2}-\sqrt{c^2-}}\right.}{3d^4}$$

input
```
Integrate[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x
^4)/((c - d*x)^2*(c + d*x)^2*Sqrt[c^2 - d^2*x^2]),x]
```

output
```
((Sqrt[c^2 - d^2*x^2]*(-2*c^5*D + 2*A*d^5*x^2 + c*d^4*x*(2*A + B*x) + c^4*
d*(2*C + D*x) - c^2*d^3*(A + x*(-B + C*x)) + c^3*d^2*(B + 2*x*(C + 2*D*x))
))/(c^3*(c - d*x)*(c + d*x)^2) + 6*D*ArcTan[(d*x)/(Sqrt[c^2] - Sqrt[c^2 -
d^2*x^2])])/(3*d^4)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.45, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2019, 2348, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(Bc-Ad)+Ac+x^2(cC-Bd)-x^3(Cd-cD)-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$

$$\Big\downarrow \text{2019}$$

$$\int \frac{A+Bx+Cx^2+Dx^3}{(c-dx)(c+dx)^2\sqrt{c^2-d^2x^2}}dx$$

$$\Big\downarrow \text{2348}$$

$$\int \left(\frac{Ad^3+Bcd^2+c^3D+c^2Cd}{4c^2d^3(c-dx)\sqrt{c^2-d^2x^2}}+\frac{Ad^3+Bcd^2+5c^3D-3c^2Cd}{4c^2d^3(c+dx)\sqrt{c^2-d^2x^2}}+\frac{Ad^3-Bcd^2+c^3(-D)+c^2Cd}{2cd^3(c+dx)^2\sqrt{c^2-d^2x^2}}-\frac{D}{d^3\sqrt{c^2-d}}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{c^2-d^2x^2}\left(-Ad^3-Bcd^2-5c^3D+3c^2Cd\right)}{4c^3d^4(c+dx)}+\frac{\sqrt{c^2-d^2x^2}\left(Ad^3+Bcd^2+c^3D+c^2Cd\right)}{4c^3d^4(c-dx)}-\frac{\sqrt{c^2-d^2x^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6c^3d^4(c+dx)}-\frac{\sqrt{c^2-d^2x^2}\left(Ad^3-Bcd^2+c^3(-D)+c^2Cd\right)}{6c^2d^4(c+dx)^2}-\frac{D\arctan\left(\frac{dx}{\sqrt{c^2-d^2x^2}}\right)}{d^4}$$

input
```
Int[(A*c + (B*c - A*d)*x + (c*C - B*d)*x^2 - (C*d - c*D)*x^3 - d*D*x^4)/((
c - d*x)^2*(c + d*x)^2*Sqrt[c^2 - d^2*x^2]),x]
```

output
```
((c^2*C*d + B*c*d^2 + A*d^3 + c^3*D)*Sqrt[c^2 - d^2*x^2])/(4*c^3*d^4*(c -
d*x)) - ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c^2 - d^2*x^2])/(6*c^2*d
^4*(c + d*x)^2) + ((3*c^2*C*d - B*c*d^2 - A*d^3 - 5*c^3*D)*Sqrt[c^2 - d^2*
x^2])/(4*c^3*d^4*(c + d*x)) - ((c^2*C*d - B*c*d^2 + A*d^3 - c^3*D)*Sqrt[c^
2 - d^2*x^2])/(6*c^3*d^4*(c + d*x)) - (D*ArcTan[(d*x)/Sqrt[c^2 - d^2*x^2]]
)/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2019
```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

rule 2348
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(c + d*x)^m*(e + f*x)^
n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[P
x, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ[m] && ILtQ[n, 0])) &&
!(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.66

method	result
default	$-\frac{D\arctan\left(\frac{\sqrt{d^2}\,x}{\sqrt{-d^2x^2+c^2}}\right)}{d^3\sqrt{d^2}}-\frac{(A\,d^3+Bc\,d^2+C\,c^2d+Dc^3)\sqrt{-d^2\left(x-\frac{c}{d}\right)^2-2cd\left(x-\frac{c}{d}\right)}}{4d^5c^3\left(x-\frac{c}{d}\right)}+\frac{(A\,d^3-Bc\,d^2+C\,c^2d-Dc^3)\left(-\frac{\sqrt{-d^2(}}{}\right.}{}$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)^2/(d*
x+c)^2/(-d^2*x^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-D/d^3/(d^2)^(1/2)*arctan((d^2)^(1/2)*x/(-d^2*x^2+c^2)^(1/2))-1/4/d^5*(A*d
^3+B*c*d^2+C*c^2*d+D*c^3)/c^3/(x-c/d)*(-d^2*(x-c/d)^2-2*c*d*(x-c/d))^(1/2)
+1/2/d^5*(A*d^3-B*c*d^2+C*c^2*d-D*c^3)/c*(-1/3/c/d/(x+c/d)^2*(-d^2*(x+c/d)
^2+2*c*d*(x+c/d))^(1/2)-1/3/c^2/(x+c/d)*(-d^2*(x+c/d)^2+2*c*d*(x+c/d))^(1/
2))-1/4/d^5*(A*d^3+B*c*d^2-3*C*c^2*d+5*D*c^3)/c^3/(x+c/d)*(-d^2*(x+c/d)^2+
2*c*d*(x+c/d))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(170) = 340$.

Time = 0.10 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.04

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$
$$=\frac{2\,Dc^6-2\,Cc^5d-Bc^4d^2+Ac^3d^3-(2\,Dc^3d^3-2\,Cc^2d^4-Bcd^5+Ad^6)x^3-(2\,Dc^4d^2-2\,Cc^3d^3-Bc^2d}{}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
^2/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*D*c^6 - 2*C*c^5*d - B*c^4*d^2 + A*c^3*d^3 - (2*D*c^3*d^3 - 2*C*c^2*
d^4 - B*c*d^5 + A*d^6)*x^3 - (2*D*c^4*d^2 - 2*C*c^3*d^3 - B*c^2*d^4 + A*c*
d^5)*x^2 + (2*D*c^5*d - 2*C*c^4*d^2 - B*c^3*d^3 + A*c^2*d^4)*x + 6*(D*c^3*
d^3*x^3 + D*c^4*d^2*x^2 - D*c^5*d*x - D*c^6)*arctan(-(c - sqrt(-d^2*x^2 +
c^2))/(d*x)) + (2*D*c^5 - 2*C*c^4*d - B*c^3*d^2 + A*c^2*d^3 - (4*D*c^3*d^2
 - C*c^2*d^3 + B*c*d^4 + 2*A*d^5)*x^2 - (D*c^4*d + 2*C*c^3*d^2 + B*c^2*d^3
 + 2*A*c*d^4)*x)*sqrt(-d^2*x^2 + c^2))/(c^3*d^7*x^3 + c^4*d^6*x^2 - c^5*d^
5*x - c^6*d^4)
```

Sympy [F]

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c - dx)^2(c + dx)^2\sqrt{c^2 - d^2x^2}}\,dx$$
$$= -\int \frac{A}{-c^3\sqrt{c^2 - d^2x^2} - c^2dx\sqrt{c^2 - d^2x^2} + cd^2x^2\sqrt{c^2 - d^2x^2} + d^3x^3\sqrt{c^2 - d^2x^2}}\,dx$$
$$- \int \frac{Bx}{-c^3\sqrt{c^2 - d^2x^2} - c^2dx\sqrt{c^2 - d^2x^2} + cd^2x^2\sqrt{c^2 - d^2x^2} + d^3x^3\sqrt{c^2 - d^2x^2}}\,dx$$
$$- \int \frac{Cx^2}{-c^3\sqrt{c^2 - d^2x^2} - c^2dx\sqrt{c^2 - d^2x^2} + cd^2x^2\sqrt{c^2 - d^2x^2} + d^3x^3\sqrt{c^2 - d^2x^2}}\,dx$$
$$- \int \frac{Dx^3}{-c^3\sqrt{c^2 - d^2x^2} - c^2dx\sqrt{c^2 - d^2x^2} + cd^2x^2\sqrt{c^2 - d^2x^2} + d^3x^3\sqrt{c^2 - d^2x^2}}\,dx$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x**2-(C*d-D*c)*x**3-d*D*x**4)/(-d*x
+c)**2/(d*x+c)**2/(-d**2*x**2+c**2)**(1/2),x)
```

output
```
-Integral(A/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sqrt(c**2 - d**2*x**2
) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt(c**2 - d**2*x**2))
, x) - Integral(B*x/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sqrt(c**2 - d
**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt(c**2 - d**
2*x**2)), x) - Integral(C*x**2/(-c**3*sqrt(c**2 - d**2*x**2) - c**2*d*x*sq
rt(c**2 - d**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**3*x**3*sqrt
(c**2 - d**2*x**2)), x) - Integral(D*x**3/(-c**3*sqrt(c**2 - d**2*x**2) -
c**2*d*x*sqrt(c**2 - d**2*x**2) + c*d**2*x**2*sqrt(c**2 - d**2*x**2) + d**
3*x**3*sqrt(c**2 - d**2*x**2)), x)
```

Maxima [F]

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c - dx)^2(c + dx)^2\sqrt{c^2 - d^2x^2}}\,dx$$
$$= \int -\frac{Ddx^4 - (Dc - Cd)x^3 - (Cc - Bd)x^2 - Ac - (Bc - Ad)x}{\sqrt{-d^2x^2 + c^2}\,(dx + c)^2\,(dx - c)^2}\,dx$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
^2/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((D*d*x^4 - (D*c - C*d)*x^3 - (C*c - B*d)*x^2 - A*c - (B*c - A*d
)*x)/(sqrt(-d^2*x^2 + c^2)*(d*x + c)^2*(d*x - c)^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{Ac + (Bc - Ad)x + (cC - Bd)x^2 - (Cd - cD)x^3 - dDx^4}{(c - dx)^2(c + dx)^2\sqrt{c^2 - d^2x^2}}\,dx$$
$$= \text{Exception raised: NotImplementedError}$$

input
```
integrate((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)
^2/(d*x+c)^2/(-d^2*x^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: -1/a
bs(sageVARd)*(-1/4*(sageVARA*sageVARd^3+sageVARD*sageVARc^3+sageVARB*sageV
ARc*sageVARd^2+sageVARC*sageVARc^2*sageVARd)/sageVARc^3/sageVARd^3/sqrt(2*
sageVARc*sageVARd*(
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx$$
$$=-\int \frac{x^3\,(C\,d-c\,D)-A\,c+x\,(A\,d-B\,c)+x^2\,(B\,d-C\,c)+d\,x^4\,D}{\sqrt{c^2-d^2\,x^2}\,{(c+d\,x)}^2\,{(c-d\,x)}^2}\,dx$$

input
```
int(-(x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
(c^2 - d^2*x^2)^(1/2)*(c + d*x)^2*(c - d*x)^2),x)
```

output
```
-int((x^3*(C*d - c*D) - A*c + x*(A*d - B*c) + x^2*(B*d - C*c) + d*x^4*D)/(
(c^2 - d^2*x^2)^(1/2)*(c + d*x)^2*(c - d*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.07

$$\int \frac{Ac+(Bc-Ad)x+(cC-Bd)x^2-(Cd-cD)x^3-dDx^4}{(c-dx)^2(c+dx)^2\sqrt{c^2-d^2x^2}}\,dx=\text{Too large to display}$$

input
```
int((A*c+(-A*d+B*c)*x+(-B*d+C*c)*x^2-(C*d-D*c)*x^3-d*D*x^4)/(-d*x+c)^2/(d*
x+c)^2/(-d^2*x^2+c^2)^(1/2),x)
```

output
```
( - 3*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**4*c**3 -
6*sqrt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)**3*c**3 + 6*sq
rt(c**2 - d**2*x**2)*asin((d*x)/c)*tan(asin((d*x)/c)/2)*c**3 + 3*sqrt(c**2
 - d**2*x**2)*asin((d*x)/c)*c**3 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)
/c)/2)**4*a*d**2 + 3*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**4*c**3 -
 6*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)**2*a*d**2 - 6*sqrt(c**2 - d
**2*x**2)*tan(asin((d*x)/c)/2)**2*c**3 - 8*sqrt(c**2 - d**2*x**2)*tan(asin
((d*x)/c)/2)*a*d**2 + 8*sqrt(c**2 - d**2*x**2)*tan(asin((d*x)/c)/2)*b*c*d
- sqrt(c**2 - d**2*x**2)*a*d**2 + 4*sqrt(c**2 - d**2*x**2)*b*c*d + 3*sqrt(
c**2 - d**2*x**2)*c**3 + 3*tan(asin((d*x)/c)/2)**4*b*c**2*d + 3*tan(asin((
d*x)/c)/2)**4*c**4 + 6*tan(asin((d*x)/c)/2)**3*b*c**2*d + 6*tan(asin((d*x)
/c)/2)**3*c**4 - 6*tan(asin((d*x)/c)/2)*b*c**2*d - 6*tan(asin((d*x)/c)/2)*
c**4 - 3*b*c**2*d - 3*c**4)/(3*sqrt(c**2 - d**2*x**2)*c**3*d**3*(tan(asin(
(d*x)/c)/2)**4 + 2*tan(asin((d*x)/c)/2)**3 - 2*tan(asin((d*x)/c)/2) - 1))
```

3.38 $\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}} dx$

Optimal result 387
Mathematica [C] (verified) 388
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Optimal result

Integrand size = 34, antiderivative size = 318

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}} dx = -\frac{2\sqrt{a}B\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}f\sqrt{c+dx}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(Be-Af)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(\frac{2\sqrt{a}f}{\sqrt{b}e+\sqrt{a}f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f\left(\sqrt{b}e+\sqrt{a}f\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*a^(1/2)*B*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/
2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b
^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/f/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+2*a^(
1/2)*(-A*f+B*e)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*((-b*x^2+a)/
a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2*a^(1/2)*f/(b
^(1/2)*e+a^(1/2)*f),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/f/(b^
(1/2)*e+a^(1/2)*f)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.86 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx$$
$$= \frac{2i\sqrt{\frac{d\left(\frac{\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)\left((-Bcf+Adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}}{\sqrt{c+dx}}\right),\frac{\sqrt{b}c+\sqrt{a}d}{\sqrt{b}c-\sqrt{a}d}\right)+d(Be\ldots}{d\sqrt{-c+\frac{\sqrt{a}d}{\sqrt{b}}}f(de-cf)\sqrt{a-bx^2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^2]),x]
```

output
```
((2*I)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[
b] - d*x)/(c + d*x))]*(c + d*x)*((-(B*c*f) + A*d*f)*EllipticF[I*ArcSinh[Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt
[b]*c - Sqrt[a]*d)] + d*(B*e - A*f)*EllipticPi[(Sqrt[b]*(-(d*e) + c*f))/((
Sqrt[b]*c - Sqrt[a]*d)*f), I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)]))/(d*Sqrt[-c +
(Sqrt[a]*d)/Sqrt[b]]*f*(d*e - c*f)*Sqrt[a - b*x^2])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {2349, 27, 512, 511, 321, 731, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a-bx^2}\sqrt{c+dx}(e+fx)}\,dx$$

$$\Big\downarrow \; 2349$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\int \frac{B}{f\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \ 27$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{f}$$

$$\Big\downarrow \ 512$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\frac{B\sqrt{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{f\sqrt{a-bx^2}}$$

$$\Big\downarrow \ 511$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$\Big\downarrow \ 321$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$\Big\downarrow \ 731$$

$$\frac{\sqrt{1-\frac{bx^2}{a}}\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}(e+fx)}dx}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

$$\Big\downarrow \ 186$$

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

$\Big\downarrow$ 413

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\left(A-\frac{Be}{f}\right)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\int \frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

$\Big\downarrow$ 412

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\left(A-\frac{Be}{f}\right)\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}-\frac{2\sqrt{a}B\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}f\sqrt{a-bx^2}\sqrt{c+dx}}$$

input
```
Int[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*Sqrt[a]*B*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[1 - (
b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/(
(Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*f*Sqrt[c + d*x]*Sqrt[a - b*x^2]) - (2*
(A - (B*e)/f)*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqr
t[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*EllipticPi[(2*f)/((Sqrt[b]*e)/Sqrt[a] + f)
, ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c
+ Sqrt[a]*d)])/(((Sqrt[b]*e)/Sqrt[a] + f)*Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a
]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 731
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a
+ b*x^2]   Int[1/((e + f*x)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
 x]] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(-b x^2+a)(dx+c)}\left(\frac{2B\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{f\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}+\frac{2(Af-Be)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\cdots}}{\cdots}\cdots\right.}{\sqrt{-b\,x^2+a}\,\sqrt{dx+c}}$
default	$-\frac{2\left(A\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\frac{\left(-\sqrt{ab}\,d+bc\right)f}{b(cf-de)},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)bcdf-A\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{\sqrt{ab}\,d-bc}},\frac{\left(-\sqrt{ab}\,d+bc\right)f}{b(cf-de)},\sqrt{-\frac{\sqrt{ab}\,d-bc}{\sqrt{ab}\,d+bc}}\right)\sqrt{\cdots}\cdots\right.}{\cdots}$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*B/f*(c/d-1/b*
(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-
c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(
1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^
(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+2*(A*
f-B*e)/f^2*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(-c/d+e/f)*Ell
ipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(-c/d+1/b*(a*b)^(1/2))/(-c/d
+e/f),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a-bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a - b*x**2)*sqrt(c + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x}{(e+f\,x)\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

3.39 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx$

Optimal result 396
Mathematica [C] (verified) 397
Rubi [A] (verified) 398
Maple [B] (verified) 405
Fricas [F(-1)] 406
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Maxima [F] 407
Giac [F] 407
Mupad [F(-1)] 408
Reduce [F] 408

Optimal result

Integrand size = 39, antiderivative size = 481

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = -\frac{2\sqrt{a}C\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}df\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}} + \frac{2\sqrt{a}(Cde+cCf-Bdf)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{b}df^2\sqrt{c+dx}\sqrt{a-bx^2}} - \frac{2\sqrt{a}(Ce^2-Bef+Af^2)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(\frac{2\sqrt{a}f}{\sqrt{b}e+\sqrt{a}f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^2\left(\sqrt{b}e+\sqrt{a}f\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2*a^(1/2)*C*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^
(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1
/2)/d/f/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)+2*a
^(1/2)*(-B*d*f+C*c*f+C*d*e)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*
(1-b*x^2/a)^(1/2)*EllipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2
)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/d/f^2/(d*x+c)^(1/2)/(-b
*x^2+a)^(1/2)-2*a^(1/2)*(A*f^2-B*e*f+C*e^2)*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^
(1/2)*d))^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^
(1/2)*2^(1/2),2*a^(1/2)*f/(b^(1/2)*e+a^(1/2)*f),2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/f^2/(b^(1/2)*e+a^(1/2)*f)/(d*x+c)^(1/2)/(-b*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.52 (sec) , antiderivative size = 1214, normalized size of antiderivative = 2.52

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*(b*c^2*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f - a*C*d^3*Sqrt[-c + (Sqr
t[a]*d)/Sqrt[b]]*e*f - b*c^3*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*f^2 + a*c*C*
d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*f^2 - 2*b*c*C*d*Sqrt[-c + (Sqrt[a]*d)/S
qrt[b]]*e*f*(c + d*x) + 2*b*c^2*C*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*f^2*(c +
d*x) + b*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f*(c + d*x)^2 - b*c*C*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*f^2*(c + d*x)^2 - I*Sqrt[b]*C*(Sqrt[b]*c - Sqrt[a
]*d)*f*(d*e - c*f)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sqrt
[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b
]*c - Sqrt[a]*d)] + I*Sqrt[b]*d*f*(Sqrt[a]*C*(-(d*e) + c*f) + Sqrt[b]*(c*C
*e - B*c*f + A*d*f))*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((Sq
rt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sq
rt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt
[b]*c - Sqrt[a]*d)] - I*b*C*d^2*e^2*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*
x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*Ellipti
cPi[(Sqrt[b]*(-(d*e) + c*f))/((Sqrt[b]*c - Sqrt[a]*d)*f), I*ArcSinh[Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c + Sqrt[a]*d)/(Sqrt[b]*
c - Sqrt[a]*d)] + I*b*B*d^2*e*f*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x))/(c + d*x)]*
Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[
(Sqrt[b]*(-(d*e) + c*f))/((Sqrt[b]*c - Sqrt[a]*d)*f), I*ArcSinh[Sqrt[-c...
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2349, 600, 509, 508, 327, 512, 511, 321, 731, 186, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}\sqrt{c+dx}(e+fx)}\,dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{600}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{df^2}+\frac{C\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{df}$$

$$\Big\downarrow \text{509}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{df^2}+\frac{C\sqrt{1-\frac{bx^2}{a}}\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{df\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{508}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{df^2}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}\int \frac{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}}{\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{327}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{df^2}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{512}$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx-\frac{\sqrt{1-\frac{bx^2}{a}}(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{df^2\sqrt{a-bx^2}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 511

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\int \frac{1}{\sqrt{1-\frac{d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\frac{\sqrt{b}c}{\sqrt{a}}+d}}\sqrt{\frac{1}{2}\left(\frac{\sqrt{b}x}{\sqrt{a}}-1\right)+1}}d\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 321

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$\downarrow$ 731

$$\frac{\sqrt{1-\frac{bx^2}{a}}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}(e+fx)}dx}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \ 186$$

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \ 413$$

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int\frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

$$\Big\downarrow \text{ 412}$$

$$-\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(A+\frac{e(Ce-Bf)}{f^2}\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}+\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}(-Bdf+cCf+Cde)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df^2\sqrt{a-bx^2}\sqrt{c+dx}}-\frac{2\sqrt{a}C\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{\sqrt{b}df\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*Sqrt[a]*C*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[ArcSin[Sqrt[1 -
(Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*
d*f*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2]) + (
2*Sqrt[a]*(C*d*e + c*C*f - B*d*f)*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sq
rt[a]*d)]*Sqrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a
]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*f^2*Sqrt[c + d*x
]*Sqrt[a - b*x^2]) - (2*(A + (e*(C*e - B*f))/f^2)*Sqrt[1 - (b*x^2)/a]*Sqrt
[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)]*Ellipt
icPi[(2*f)/((Sqrt[b]*e)/Sqrt[a] + f), ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]
/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(((Sqrt[b]*e)/Sqrt[a] +
 f)*Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*(1 - (Sqrt[b
]*x)/Sqrt[a]))/Sqrt[b]])
```

Defintions of rubi rules used

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 731
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a
+ b*x^2]   Int[1/((e + f*x)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
 x]] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(392) = 784$.

Time = 4.87 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.70

method	result
elliptic	$\sqrt{(-b x^2+a)(dx+c)}\left(\frac{2(Bf-Ce)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\text{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\right)}{f^2\sqrt{-bdx^3-bcx^2+adx+ac}}+\frac{2C\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{}}{}\right.$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(2*(B*f-C*e)/f^2
*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)
^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-
1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1
/2))+2*C/f*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x
-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d
+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-1/b*(a*
b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)
^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(1/2)*EllipticF(((x+c/d)/
(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)
))^(1/2)))+2*(A*f^2-B*e*f+C*e^2)/f^3*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1
/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*
((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x
+a*c)^(1/2)/(-c/d+e/f)*EllipticPi(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(-
c/d+1/b*(a*b)^(1/2))/(-c/d+e/f),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1
/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorith
m="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{a-bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a - b*x**2)*sqrt(c + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(e+f\,x)\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

3.40 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx$

Optimal result 409
Mathematica [C] (verified) 410
Rubi [A] (verified) 411
Maple [A] (verified) 419
Fricas [F(-1)] 420
Sympy [F] 420
Maxima [F] 420
Giac [F] 421
Mupad [F(-1)] 421
Reduce [F] 421

Optimal result

Integrand size = 44, antiderivative size = 581

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx=-\frac{2D\sqrt{c+dx}\sqrt{a-bx^2}}{3bdf}$$
$$+\frac{2\sqrt{a}(3dDe-3Cdf+2cDf)\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{3\sqrt{b}d^2f^2\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{a-bx^2}}$$
$$-\frac{2\sqrt{a}\left(3bdDe^2+adDf^2-3bdf(Ce-Bf)+bcf\left(3De-3Cf+\frac{2cDf}{d}\right)\right)\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{1-\frac{bx^2}{a}}\,\text{EllipticF}\left(\cdots\right.}{3b^{3/2}df^3\sqrt{c+dx}\sqrt{a-bx^2}}$$
$$+\frac{2\sqrt{a}(De^3-f(Ce^2-f(Be-Af)))\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{a}d}}\sqrt{\frac{a-bx^2}{a}}\,\text{EllipticPi}\left(\frac{2\sqrt{a}f}{\sqrt{b}e+\sqrt{a}f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}\cdots}{\sqrt{b}c+\cdots}\right.}{f^3\left(\sqrt{b}e+\sqrt{a}f\right)\sqrt{c+dx}\sqrt{a-bx^2}}$$

output
```
-2/3*D*(d*x+c)^(1/2)*(-b*x^2+a)^(1/2)/b/d/f+2/3*a^(1/2)*(-3*C*d*f+2*D*c*f+
3*D*d*e)*(d*x+c)^(1/2)*(1-b*x^2/a)^(1/2)*EllipticE(1/2*(1-b^(1/2)*x/a^(1/2
))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/2)*d))^(1/2))/b^(1/2)/
d^2/f^2/(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)/(-b*x^2+a)^(1/2)-2/3
*a^(1/2)*(3*b*d*D*e^2+a*d*D*f^2-3*b*d*f*(-B*f+C*e)+b*c*f*(3*D*e-3*C*f+2*c*
D*f/d))*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))^(1/2)*(1-b*x^2/a)^(1/2)*El
lipticF(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(1/2),2^(1/2)*(a^(1/2)*d/(b^(1/2
)*c+a^(1/2)*d))^(1/2))/b^(3/2)/d/f^3/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)+2*a^(1
/2)*(D*e^3-f*(C*e^2-f*(-A*f+B*e)))*(b^(1/2)*(d*x+c)/(b^(1/2)*c+a^(1/2)*d))
^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticPi(1/2*(1-b^(1/2)*x/a^(1/2))^(1/2)*2^(
1/2),2*a^(1/2)*f/(b^(1/2)*e+a^(1/2)*f),2^(1/2)*(a^(1/2)*d/(b^(1/2)*c+a^(1/
2)*d))^(1/2))/f^3/(b^(1/2)*e+a^(1/2)*f)/(d*x+c)^(1/2)/(-b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.72 (sec) , antiderivative size = 1922, normalized size of antiderivative = 3.31

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^
2]),x]
```

output
```
(-2*D*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b*d*f) - (2*(3*b*c^2*d^2*Sqrt[-c +
 (Sqrt[a]*d)/Sqrt[b]]*D*e^2*f - 3*a*d^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D*e
^2*f - 3*b*c^2*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f^2 + 3*a*C*d^4*Sqrt
[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f^2 - b*c^3*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*
D*e*f^2 + a*c*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D*e*f^2 + 3*b*c^3*C*d*Sqr
t[-c + (Sqrt[a]*d)/Sqrt[b]]*f^3 - 3*a*c*C*d^3*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b
]]*f^3 - 2*b*c^4*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D*f^3 + 2*a*c^2*d^2*Sqrt[-
c + (Sqrt[a]*d)/Sqrt[b]]*D*f^3 - 6*b*c*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*
D*e^2*f*(c + d*x) + 6*b*c*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f^2*(c +
d*x) + 2*b*c^2*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D*e*f^2*(c + d*x) - 6*b*c^
2*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*f^3*(c + d*x) + 4*b*c^3*Sqrt[-c + (Sq
rt[a]*d)/Sqrt[b]]*D*f^3*(c + d*x) + 3*b*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]
*D*e^2*f*(c + d*x)^2 - 3*b*C*d^2*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*e*f^2*(c +
 d*x)^2 - b*c*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*D*e*f^2*(c + d*x)^2 + 3*b*c
*C*d*Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]*f^3*(c + d*x)^2 - 2*b*c^2*Sqrt[-c + (S
qrt[a]*d)/Sqrt[b]]*D*f^3*(c + d*x)^2 + I*Sqrt[b]*(Sqrt[b]*c - Sqrt[a]*d)*f
*(-(d*e) + c*f)*(3*d*D*e - 3*C*d*f + 2*c*D*f)*Sqrt[(d*(Sqrt[a]/Sqrt[b] + x
))/(c + d*x)]*Sqrt[-(((Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/
2)*EllipticE[I*ArcSinh[Sqrt[-c + (Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqr
t[b]*c + Sqrt[a]*d)/(Sqrt[b]*c - Sqrt[a]*d)] + I*d*f*(a*d*D*f*(-(d*e) +...
```

Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2349, 731, 186, 413, 412, 2185, 27, 600, 509, 508, 327, 512, 511, 321}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\sqrt{c+dx}(e+fx)}\,dx$$

$$\Big\downarrow \; 2349$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}dx}{f^3}$$

$$\Big\downarrow \text{731}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx-\frac{\sqrt{1-\frac{bx^2}{a}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+dx}(e+fx)}dx}{f^3\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{186}$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{c+\frac{\sqrt{a}d}{\sqrt{b}}-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{f^3\sqrt{a-bx^2}}+\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{413}$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{\frac{\sqrt{b}x}{\sqrt{a}}+1}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}c+\sqrt{a}d}}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f-f\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}d\sqrt{1-}}{f^3\sqrt{a-bx^2}\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{412}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{a-bx^2}}dx+\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\Big\downarrow \text{2185}$$

$$-\frac{2\int -\frac{d\left(d\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)-bf(3dDe-3Cdf+2cDf)x\right)}{2f^3\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bd^2}+\frac{\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}}{}\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{d\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)-bf(3dDe-3Cdf+2cDf)x}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{3bdf^3}+\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \text{ 600}$$

$$\frac{\frac{\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)+bcf(2cDf-3Cdf+3dDe)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{bf(2cDf-3Cdf+3dDe)\int \frac{\sqrt{c+dx}}{\sqrt{a-bx^2}}dx}{d}}{3bdf^3}+\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \text{ 509}$$

$$\frac{\frac{\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)+bcf(2cDf-3Cdf+3dDe)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}-\frac{bf\sqrt{1-\frac{bx^2}{a}}(2cDf-3Cdf+3dDe)\int \frac{\sqrt{c+dx}}{\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}}{3bdf^3}+$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \; 508$$

$$\frac{\frac{\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)+bcf(2cDf-3Cdf+3dDe)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cDf-3Cdf+3dDe)\int \frac{\sqrt{1-\frac{d\left(\cdots\right.}{\cdots}}}{\sqrt{\frac{1}{2}\left(\cdots\right.}}\cdots}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}}{3bdf^3}\cdots$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \; 327$$

$$\frac{\frac{\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)+bcf(2cDf-3Cdf+3dDe)\right)\int \frac{1}{\sqrt{c+dx}\sqrt{a-bx^2}}dx}{d}+\frac{2\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cDf-3Cdf+3dDe)E\left(\arcsin\cdots\right.}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\cdots}}{3bdf^3}\cdots$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$$\Big\downarrow \; 512$$

$$\frac{\frac{\sqrt{1-\frac{bx^2}{a}}\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\right)+bcf(2cDf-3Cdf+3dDe)\right)\int\frac{1}{\sqrt{c+dx}\sqrt{1-\frac{bx^2}{a}}}dx}{d\sqrt{a-bx^2}}+\frac{2\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cDf-3Cdf+3dDe)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c\ldots}{\sqrt{a}d+\ldots}}}}{3bdf^3}$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$\downarrow$ 511

$$\frac{\frac{2\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cDf-3Cdf+3dDe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\left(d^2\left(aDf^2+3b\left(De^2-f(Ce-Bf)\right)\ldots\right.\right.}{\sqrt{\ldots}}}{3bdf^3}$$

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

$\downarrow$ 321

$$\frac{2\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}d+\sqrt{b}c}}\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\text{EllipticPi}\left(\frac{2f}{\frac{\sqrt{b}e}{\sqrt{a}}+f},\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\frac{2\sqrt{a}d}{\sqrt{b}c+\sqrt{a}d}\right)}{f^3\sqrt{a-bx^2}\left(\frac{\sqrt{b}e}{\sqrt{a}}+f\right)\sqrt{-\frac{\sqrt{a}d\left(1-\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}d}{\sqrt{b}}+c}}$$

$$\frac{\frac{2\sqrt{a}\sqrt{b}f\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx}(2cDf-3Cdf+3dDe)E\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right)|\frac{2d}{\frac{\sqrt{b}c}{\sqrt{a}}+d}\right)}{d\sqrt{a-bx^2}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}}-\frac{2\sqrt{a}\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{\sqrt{b}(c+dx)}{\sqrt{a}d+\sqrt{b}c}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1-\frac{\sqrt{b}x}{\sqrt{a}}}}{\sqrt{2}}\right),\ldots\right.}{\sqrt{b}c\ldots}}{3bdf^3}$$

$$\frac{2D\sqrt{a-bx^2}\sqrt{c+dx}}{3bdf}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a - b*x^2]),x]
```

output
```
(-2*D*Sqrt[c + d*x]*Sqrt[a - b*x^2])/(3*b*d*f) + ((2*Sqrt[a]*Sqrt[b]*f*(3*
d*D*e - 3*C*d*f + 2*c*D*f)*Sqrt[c + d*x]*Sqrt[1 - (b*x^2)/a]*EllipticE[Arc
Sin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]], (2*d)/((Sqrt[b]*c)/Sqrt[a] + d
)])/(d*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*Sqrt[a - b*x^2])
- (2*Sqrt[a]*(b*c*f*(3*d*D*e - 3*C*d*f + 2*c*D*f) + d^2*(a*D*f^2 + 3*b*(D*
e^2 - f*(C*e - B*f))))*Sqrt[(Sqrt[b]*(c + d*x))/(Sqrt[b]*c + Sqrt[a]*d)]*S
qrt[1 - (b*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[b]*x)/Sqrt[a]]/Sqrt[2]]
, (2*d)/((Sqrt[b]*c)/Sqrt[a] + d)])/(Sqrt[b]*d*Sqrt[c + d*x]*Sqrt[a - b*x^
2]))/(3*b*d*f^3) + (2*(D*e^3 - f*(C*e^2 - f*(B*e - A*f)))*Sqrt[1 - (b*x^2)
/a]*Sqrt[1 - (Sqrt[a]*d*(1 - (Sqrt[b]*x)/Sqrt[a]))/(Sqrt[b]*c + Sqrt[a]*d)
]*EllipticPi[(2*f)/((Sqrt[b]*e)/Sqrt[a] + f), ArcSin[Sqrt[1 - (Sqrt[b]*x)/
Sqrt[a]]/Sqrt[2]], (2*Sqrt[a]*d)/(Sqrt[b]*c + Sqrt[a]*d)])/(f^3*((Sqrt[b]*
e)/Sqrt[a] + f)*Sqrt[a - b*x^2]*Sqrt[c + (Sqrt[a]*d)/Sqrt[b] - (Sqrt[a]*d*
(1 - (Sqrt[b]*x)/Sqrt[a]))/Sqrt[b]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 508
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> With[{q
 = Rt[-b/a, 2]}, Simp[-2*(Sqrt[c + d*x]/(Sqrt[a]*q*Sqrt[q*((c + d*x)/(d + c
*q))]))   Subst[Int[Sqrt[1 - 2*d*(x^2/(d + c*q))]/Sqrt[1 - x^2], x], x, Sqr
t[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[a, 0]
```

rule 509
```
Int[Sqrt[(c_) + (d_.)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sq
rt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[Sqrt[c + d*x]/Sqrt[1 + b*(x^2/a)],
x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 511
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Wit
h[{q = Rt[-b/a, 2]}, Simp[-2*(Sqrt[q*((c + d*x)/(d + c*q))]/(Sqrt[a]*q*Sqrt
[c + d*x]))   Subst[Int[1/(Sqrt[1 - 2*d*(x^2/(d + c*q))]*Sqrt[1 - x^2]), x]
, x, Sqrt[(1 - q*x)/2]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] && GtQ[
a, 0]
```

rule 512
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[Sqrt[1 + b*(x^2/a)]/Sqrt[a + b*x^2]   Int[1/(Sqrt[c + d*x]*Sqrt[1 + b*(x^
2/a)]), x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 600
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[B/d   Int[Sqrt[c + d*x]/Sqrt[a + b*x^2], x], x] - Simp
[(B*c - A*d)/d   Int[1/(Sqrt[c + d*x]*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a,
 b, c, d, A, B}, x] && NegQ[b/a]
```

rule 731
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-b/a, 2]}, Simp[Sqrt[1 + b*(x^2/a)]/Sqrt[a
+ b*x^2]   Int[1/((e + f*x)*Sqrt[c + d*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
 x]] /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[b/a] &&  !GtQ[a, 0]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.97 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(-b\,x^2+a)(dx+c)}\left(-\frac{2D\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}{3fbd}+\frac{2\left(\frac{B\,f^2-Cef+De^2}{f^3}+\frac{Da}{3fb}\right)\left(\frac{c}{d}-\frac{\sqrt{ab}}{b}\right)\sqrt{\frac{x+\frac{c}{d}}{\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x-\frac{\sqrt{ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{ab}}{b}}}\,\text{Elliptic}}{\sqrt{-bd\,x^3-bc\,x^2+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
((-b*x^2+a)*(d*x+c))^(1/2)/(-b*x^2+a)^(1/2)/(d*x+c)^(1/2)*(-2/3*D/f/b/d*(-
b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)+2*((B*f^2-C*e*f+D*e^2)/f^3+1/3*D/f/b*a)*(
c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(
1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(
1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(c/d-1/
b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2
))+2*(1/f^2*(C*f-D*e)-2/3*D/f/d*c)*(c/d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b
*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((
x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a
*c)^(1/2)*((-c/d-1/b*(a*b)^(1/2))*EllipticE(((x+c/d)/(c/d-1/b*(a*b)^(1/2))
)^(1/2),((-c/d+1/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2))+1/b*(a*b)^(
1/2)*EllipticF(((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),((-c/d+1/b*(a*b)^(1/2
))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))+2*(A*f^3-B*e*f^2+C*e^2*f-D*e^3)/f^4*(c/
d-1/b*(a*b)^(1/2))*((x+c/d)/(c/d-1/b*(a*b)^(1/2)))^(1/2)*((x-1/b*(a*b)^(1/
2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)*((x+1/b*(a*b)^(1/2))/(-c/d+1/b*(a*b)^(1/
2)))^(1/2)/(-b*d*x^3-b*c*x^2+a*d*x+a*c)^(1/2)/(-c/d+e/f)*EllipticPi(((x+c/
d)/(c/d-1/b*(a*b)^(1/2)))^(1/2),(-c/d+1/b*(a*b)^(1/2))/(-c/d+e/f),((-c/d+1
/b*(a*b)^(1/2))/(-c/d-1/b*(a*b)^(1/2)))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a-bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(-b*x**2+a)**(1/2),
x)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a - b*x**2)*sqrt(c + d*x)*(e +
f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x +
 e)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{-bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(-b*x^2 + a)*sqrt(d*x + c)*(f*x +
 e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{(e+f\,x)\,\sqrt{a-b\,x^2}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(a - b*x^2)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a-bx^2}}\,dx = \int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{-b\,x^2+a}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(-b*x^2+a)^(1/2),x)
```

3.41 $\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx$

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Optimal result

Integrand size = 33, antiderivative size = 438

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \frac{2B\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt{\sqrt{b}c+\sqrt{-a}d}}\right),\frac{\sqrt{b}c+\sqrt{-a}d}{\sqrt{b}c-\sqrt{-a}d}\right)}{\sqrt[4]{b}df\sqrt{a+bx^2}} - \frac{2\sqrt{\sqrt{b}c+\sqrt{-a}d}(Be-Af)\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}\,\text{EllipticPi}\left(-\frac{\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)f}{de-cf},\arcsin\left(\frac{\sqrt[4]{b}\sqrt{c+}}{\sqrt{\sqrt{b}c+\sqrt{-}}}\right.\right.}{\sqrt[4]{b}f(de-cf)\sqrt{a+bx^2}}$$

output
```
2*B*(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2
)*d))^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))^(1/2)*EllipticF(b
^(1/4)*(d*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),((b^(1/2)*c+(-a)^(1/2)
*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(1/4)/d/f/(b*x^2+a)^(1/2)-2*(b^(1/2
)*c+(-a)^(1/2)*d)^(1/2)*(-A*f+B*e)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2
)*d))^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))^(1/2)*EllipticPi(
b^(1/4)*(d*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),-(c+(-a)^(1/2)*d/b^(1
/2))*f/(-c*f+d*e),((b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2
))/b^(1/4)/f/(-c*f+d*e)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.87 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \frac{2i\sqrt{\frac{d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)}{c+dx}}\sqrt{-\frac{\frac{i\sqrt{a}d}{\sqrt{b}}-dx}{c+dx}}(c+dx)\left((-Bcf+Adf)\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}}{\sqrt{c+dx}}\right),\frac{\sqrt{b}c-i\sqrt{a}d}{\sqrt{b}c+i\sqrt{a}d}\right)+d(B}{d\sqrt{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}f(de-cf)\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^2]),x]
```

output
```
((2*I)*Sqrt[(d*((I*Sqrt[a])/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[a]*d)
/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)*((-(B*c*f) + A*d*f)*EllipticF[I*ArcS
inh[Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c - I*Sqrt[a
]*d)/(Sqrt[b]*c + I*Sqrt[a]*d)] + d*(B*e - A*f)*EllipticPi[(Sqrt[b]*(-(d*e
) + c*f))/((Sqrt[b]*c + I*Sqrt[a]*d)*f), I*ArcSinh[Sqrt[-c - (I*Sqrt[a]*d)
/Sqrt[b]]/Sqrt[c + d*x]], (Sqrt[b]*c - I*Sqrt[a]*d)/(Sqrt[b]*c + I*Sqrt[a]
*d)]))/(d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f*(d*e - c*f)*Sqrt[a + b*x^2])
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1103 vs. 2(438) = 876.

Time = 3.80 (sec) , antiderivative size = 1103, normalized size of antiderivative = 2.52, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2349, 27, 510, 729, 1416, 1540, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}\sqrt{c+dx}(e+fx)}\,dx$$

$$\Big\downarrow \; 2349$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx+\int \frac{B}{f\sqrt{c+dx}\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \; 27$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx+\frac{B\int \frac{1}{\sqrt{c+dx}\sqrt{bx^2+a}}dx}{f}$$

$$\Big\downarrow \; 510$$

$$\left(A-\frac{Be}{f}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx+\frac{2B\int \frac{1}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{df}$$

$$\Big\downarrow \; 729$$

$$2\left(A-\frac{Be}{f}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}+\frac{2B\int \frac{1}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{df}$$

$$\Big\downarrow \; 1416$$

$$2\left(A-\frac{Be}{f}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}+\frac{B\sqrt[4]{ad^2+bc^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)\sqrt{\frac{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}{\left(a+\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right),\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}df\sqrt{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}}$$

$$\Big\downarrow \; 1540$$

$$2\left(A-\frac{Be}{f}\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d\left(adf^2-be(de-2cf)\right)}-\right.$$

$$\frac{B\sqrt[4]{ad^2+bc^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)\sqrt{\frac{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}{\left(a+\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right),\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}df\sqrt{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}}$$

$$\Big\downarrow \; 1416$$

$$2\left(A-\frac{Be}{f}\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d\left(adf^2-be(de-2cf)\right)}\right.$$

$$\frac{B\sqrt[4]{ad^2+bc^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)\sqrt{\frac{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}{\left(a+\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right),\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}df\sqrt{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}}$$

$$\Big\downarrow \text{2220}$$

$$\frac{B\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right),\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}df\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}$$

$$2\left(A-\frac{Be}{f}\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\left(\frac{\left(f-\frac{\sqrt{b}(de-cf)}{\sqrt{bc^2+ad^2}}\right)\arctan\left(\frac{\sqrt{be^2+af^2}\sqrt{c+dx}}{\sqrt{f}\sqrt{de-cf}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}}}\right.}{2\sqrt{f}\sqrt{de-cf}\sqrt{be^2+af^2}}\right.}{}\right.$$

input
```
Int[(A + B*x)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^2]),x]
```

output
```
(B*(b*c^2 + a*d^2)^(1/4)*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqr
t[(a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b
*c^2)/d^2)*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])^2)]*EllipticF[2*A
rcTan[(b^(1/4)*Sqrt[c + d*x])/(b*c^2 + a*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqr
t[b*c^2 + a*d^2])/2])/(b^(1/4)*d*f*Sqrt[a + (b*c^2)/d^2 - (2*b*c*(c + d*x)
)/d^2 + (b*(c + d*x)^2)/d^2]) + 2*(A - (B*e)/f)*(-1/2*(b^(1/4)*(b*c^2 + a*
d^2)^(1/4)*(Sqrt[b*c^2 + a*d^2]*f + Sqrt[b]*(d*e - c*f))*(1 + (Sqrt[b]*(c
+ d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2
 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b
*c^2 + a*d^2])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c + d*x])/(b*c^2 + a*d
^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c^2 + a*d^2])/2])/(d*(a*d*f^2 - b*e*(d
*e - 2*c*f))*Sqrt[a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2
)/d^2]) + (Sqrt[b*c^2 + a*d^2]*f*(Sqrt[b*c^2 + a*d^2]*f + Sqrt[b]*(d*e - c
*f))*(((f - (Sqrt[b]*(d*e - c*f))/Sqrt[b*c^2 + a*d^2])*ArcTan[(Sqrt[b*e^2
+ a*f^2]*Sqrt[c + d*x])/(Sqrt[f]*Sqrt[d*e - c*f]*Sqrt[a + (b*c^2)/d^2 - (2
*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2])])/(2*Sqrt[f]*Sqrt[d*e - c*f]*S
qrt[b*e^2 + a*f^2]) + ((Sqrt[b]/f + Sqrt[b*c^2 + a*d^2]/(d*e - c*f))*(1 +
(Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 - (2*b*c*(c
 + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqrt[b]*(c +
d*x))/Sqrt[b*c^2 + a*d^2])^2)]*EllipticPi[-1/4*(Sqrt[b*c^2 + a*d^2]*f -...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 510
```
Int[1/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Sim
p[2/d   Subst[Int[1/Sqrt[(b*c^2 + a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2
)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{\sqrt{(dx+c)(b\,x^2+a)}\left(\frac{2B\left(-\frac{\sqrt{-ab}}{b}+\frac{c}{d}\right)\sqrt{\frac{x+\frac{c}{d}}{-\frac{\sqrt{-ab}}{b}+\frac{c}{d}}}\,\sqrt{\frac{x-\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{b}}}\,\sqrt{\frac{x+\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{-\frac{\sqrt{-ab}}{b}+\frac{c}{d}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{b}}}\right)}{f\sqrt{bd\,x^3+bc\,x^2+adx+ac}}+\frac{2(Af-Be)\cdots}{\cdots}\right)}{\sqrt{dx+c}\,\sqrt{b\,x^2+a}}$
default	$-\frac{2\left(A\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{-ab}-bc}},\frac{(bc-d\sqrt{-ab})f}{b(cf-de)},\sqrt{-\frac{d\sqrt{-ab}-bc}{d\sqrt{-ab}+bc}}\right)bcdf-A\sqrt{-ab}\,\mathrm{EllipticPi}\left(\sqrt{-\frac{(dx+c)b}{d\sqrt{-ab}-bc}},\frac{(bc-d\sqrt{-ab})f}{b(cf-de)},\sqrt{-\frac{d\sqrt{-\cdots}}{d\sqrt{-\cdots}}}\cdots\right.\right.}{\cdots}$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*(b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)*(2*B/f*(-(-a*b)^(1
/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(1/2)/b)/(-c/d
-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2)/b))^(1/2)/(
b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(-(-a*b)^(1/2)/b+c/d))
^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2))+2*(A*f-B*e)/f^
2*(-(-a*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(
1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2
)/b))^(1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)/(-c/d+e/f)*EllipticPi(((x+c/
d)/(-(-a*b)^(1/2)/b+c/d))^(1/2),(-c/d+(-a*b)^(1/2)/b)/(-c/d+e/f),((-c/d+(-
a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a+bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a + b*x**2)*sqrt(c + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \int \frac{Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x}{(e+f\,x)\,\sqrt{b\,x^2+a}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

3.42 $\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx$

Optimal result 431
Mathematica [C] (verified) 432
Rubi [B] (warning: unable to verify) 433
Maple [A] (verified) 439
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Giac [F] 441
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Reduce [F] 442

Optimal result

Integrand size = 38, antiderivative size = 685

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2C\left(\sqrt{b}c-\sqrt{-a}d\right)\sqrt{\sqrt{b}c+\sqrt{-a}d}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}E\left(\arcsin\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt{\sqrt{b}c+\sqrt{-a}d}}\right)|\frac{\sqrt{b}c+\sqrt{-a}d}{\sqrt{b}c-\sqrt{-a}d}\cdots\right)}{b^{3/4}d^2f\sqrt{a+bx^2}}$$

$$-\frac{2\sqrt{\sqrt{b}c+\sqrt{-a}d}\left(\sqrt{-a}Cf+\sqrt{b}(Ce-Bf)\right)\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}\,\mathrm{EllipticF}\left(\arcsin\left(\cdots\right)\right)}{b^{3/4}df^2\sqrt{a+bx^2}}$$

$$+\frac{2\sqrt{\sqrt{b}c+\sqrt{-a}d}(Ce^2-Bef+Af^2)\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}\,\mathrm{EllipticPi}\left(-\frac{\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)f}{de-cf},\arcsin\cdots\right)}{\sqrt[4]{b}f^2(de-cf)\sqrt{a+bx^2}}$$

output
```
-2*C*(b^(1/2)*c-(-a)^(1/2)*d)*(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)*(1-b^(1/2)*(d
*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(
1/2)*d))^(1/2)*EllipticE(b^(1/4)*(d*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1
/2),((b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(3/4)/d^2
/f/(b*x^2+a)^(1/2)-2*(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)*((-a)^(1/2)*C*f+b^(1/2
)*(-B*f+C*e))*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2)*(1-b^(1/2
)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))^(1/2)*EllipticF(b^(1/4)*(d*x+c)^(1/2)/
(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),((b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(
1/2)*d))^(1/2))/b^(3/4)/d/f^2/(b*x^2+a)^(1/2)+2*(b^(1/2)*c+(-a)^(1/2)*d)^(
1/2)*(A*f^2-B*e*f+C*e^2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2
)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))^(1/2)*EllipticPi(b^(1/4)*(d
*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),-(c+(-a)^(1/2)*d/b^(1/2))*f/(-c
*f+d*e),((b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(1/4)
/f^2/(-c*f+d*e)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.19 (sec) , antiderivative size = 1315, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^2]),x]
```

output

```
(-2*(b*c^2*C*d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*e*f + a*C*d^3*Sqrt[-c - (I
*Sqrt[a]*d)/Sqrt[b]]*e*f - b*c^3*C*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f^2 -
a*c*C*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f^2 - 2*b*c*C*d*Sqrt[-c - (I*Sq
rt[a]*d)/Sqrt[b]]*e*f*(c + d*x) + 2*b*c^2*C*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b
]]*f^2*(c + d*x) + b*C*d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*e*f*(c + d*x)^2
- b*c*C*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f^2*(c + d*x)^2 + Sqrt[b]*C*((-I)
*Sqrt[b]*c + Sqrt[a]*d)*f*(d*e - c*f)*Sqrt[(d*((I*Sqrt[a])/Sqrt[b] + x))/(
c + d*x)]*Sqrt[-(((I*Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)
*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + d*x]], (Sqr
t[b]*c - I*Sqrt[a]*d)/(Sqrt[b]*c + I*Sqrt[a]*d)] + Sqrt[b]*d*f*(Sqrt[a]*C*
(-(d*e) + c*f) + I*Sqrt[b]*(c*C*e - B*c*f + A*d*f))*Sqrt[(d*((I*Sqrt[a])/S
qrt[b] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(
c + d*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]/Sqrt[c
 + d*x]], (Sqrt[b]*c - I*Sqrt[a]*d)/(Sqrt[b]*c + I*Sqrt[a]*d)] - I*b*C*d^2
*e^2*Sqrt[(d*((I*Sqrt[a])/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[a]*d)/S
qrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]*(-(d*e) + c*
f))/((Sqrt[b]*c + I*Sqrt[a]*d)*f), I*ArcSinh[Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[
b]]/Sqrt[c + d*x]], (Sqrt[b]*c - I*Sqrt[a]*d)/(Sqrt[b]*c + I*Sqrt[a]*d)] +
 I*b*B*d^2*e*f*Sqrt[(d*((I*Sqrt[a])/Sqrt[b] + x))/(c + d*x)]*Sqrt[-(((I*Sq
rt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*x)^(3/2)*EllipticPi[(Sqrt[b]...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1496 vs. $2(685) = 1370$.

Time = 4.16 (sec) , antiderivative size = 1496, normalized size of antiderivative = 2.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2349, 599, 27, 729, 1511, 1416, 1509, 1540, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}\sqrt{c+dx}(e+fx)}\,dx$$

$$\Big\downarrow \; 2349$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx+\int \frac{\frac{B}{f}+\frac{Cx}{f}-\frac{Ce}{f^2}}{\sqrt{c+dx}\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \; 599$$

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx-\frac{2\int \frac{Cde+cCf-Bdf-Cf(c+dx)}{f^2\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d^2}$$

$\big\downarrow$ 27

$$\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx-\frac{2\int \frac{Cde+cCf-Bdf-Cf(c+dx)}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d^2f^2}$$

$\big\downarrow$ 729

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}-\frac{2\int \frac{Cde+cCf-Bdf-Cf(c+dx)}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d^2f^2}$$

$\big\downarrow$ 1511

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}-$$

$$\frac{2\left(\left(Cf\left(c-\frac{\sqrt{ad^2+bc^2}}{\sqrt{b}}\right)-Bdf+Cde\right)\int \frac{1}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}+\frac{Cf\sqrt{ad^2+bc^2}\int \frac{1-\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)}{d^2}}}}{\sqrt{b}}\right.}{d^2f^2}$$

$\big\downarrow$ 1416

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}-$$

$$\frac{2\left(\frac{Cf\sqrt{ad^2+bc^2}\int \frac{1-\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{\sqrt{b}}+\frac{\sqrt[4]{ad^2+bc^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)\sqrt{\frac{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}{\left(a+\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)^2}}\text{EllipticF}\left(2\,\text{arc}\right.}{2\sqrt[4]{b}\sqrt{a+\frac{bc^2}{d^2}-\frac{2}{}}}\right.}{d^2f^2}$$

$\big\downarrow$ 1509

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}-$$

$$2\left(\frac{\sqrt[4]{ad^2+bc^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)\sqrt{\frac{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}{\left(a+\frac{bc^2}{d^2}\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{ad^2+bc^2}}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right),\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)\left(Cf\left(c-\frac{\sqrt{ad^2}}{\sqrt{}}\right.\right.}{2\sqrt[4]{b}\sqrt{a+\frac{bc^2}{d^2}-\frac{2bc(c+dx)}{d^2}+\frac{b(c+dx)^2}{d^2}}}\right.$$

$\downarrow$ 1540

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\int\frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{}}{d\left(adf^2-be(de-2cf)\right)}\right.$$

$$2\left(\frac{C\sqrt{bc^2+ad^2}f\left(\frac{\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}-\frac{\sqrt{c+dx}\sqrt{}}{\left(\frac{bc}{d}\right.}\right.}{\sqrt{b}}\right.$$

$\downarrow$ 1416

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{}}{d\left(adf^2-be(de-2cf)\right)}\right.$$

$$2\left(\frac{C\sqrt{bc^2+ad^2}f\left(\frac{\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}-\frac{\sqrt{c+dx}\sqrt{}}{\left(\frac{bc}{d}\right.}\right.}{\sqrt{b}}\right.$$

$\downarrow$ 2220

$$2\left(A+\frac{e(Ce-Bf)}{f^2}\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\left(\frac{\left(f-\frac{\sqrt{b}(de-cf)}{\sqrt{bc^2+ad^2}}\right)\arctan\left(\frac{\sqrt{be^2+af^2}\sqrt{c+}}{\sqrt{f}\sqrt{de-cf}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}}}\right.}{2\sqrt{f}\sqrt{de-cf}\sqrt{be^2+af^2}}\right.}{}\right.$$

$$2\left(\frac{C\sqrt{bc^2+ad^2}f\left(\frac{\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c+dx}}{\sqrt[4]{bc^2+ad^2}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}c}{\sqrt{bc^2+ad^2}}+1\right)\right)}{\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}-\frac{\sqrt{c+dx}\sqrt{}}{\left(\frac{bc}{d}\right.}\right.}{\sqrt{b}}\right.$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*((C*Sqrt[b*c^2 + a*d^2]*f*(-((Sqrt[c + d*x]*Sqrt[a + (b*c^2)/d^2 - (2*
b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2])/((a + (b*c^2)/d^2)*(1 + (Sqrt[b
]*(c + d*x))/Sqrt[b*c^2 + a*d^2]))) + ((b*c^2 + a*d^2)^(1/4)*(1 + (Sqrt[b]
*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 - (2*b*c*(c + d*x))
/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqrt[b]*(c + d*x))/Sq
rt[b*c^2 + a*d^2])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c + d*x])/(b*c^2 +
 a*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c^2 + a*d^2])/2])/(b^(1/4)*Sqrt[a
+ (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2])))/Sqrt[b] +
((b*c^2 + a*d^2)^(1/4)*(C*d*e - B*d*f + C*(c - Sqrt[b*c^2 + a*d^2]/Sqrt[b]
)*f)*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 -
 (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqr
t[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
c + d*x])/(b*c^2 + a*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c^2 + a*d^2])/2]
)/(2*b^(1/4)*Sqrt[a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2
)/d^2])))/(d^2*f^2) + 2*(A + (e*(C*e - B*f))/f^2)*(-1/2*(b^(1/4)*(b*c^2 +
a*d^2)^(1/4)*(Sqrt[b*c^2 + a*d^2]*f + Sqrt[b]*(d*e - c*f))*(1 + (Sqrt[b]*(
c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d
^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqrt[b]*(c + d*x))/Sqrt
[b*c^2 + a*d^2])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c + d*x])/(b*c^2 + a
*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c^2 + a*d^2])/2])/(d*(a*d*f^2 - b...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.23

method	result
elliptic	$\frac{\sqrt{(dx+c)(b\,x^2+a)}\left(\frac{2(Bf-Ce)\left(-\frac{\sqrt{-ab}}{b}+\frac{c}{d}\right)\sqrt{\frac{x+\frac{c}{d}}{-\frac{\sqrt{-ab}}{b}+\frac{c}{d}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{b}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{c}{d}}{-\frac{\sqrt{-ab}}{b}+\frac{c}{d}}},\sqrt{\frac{-\frac{c}{d}+\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{b}}}\right)}{f^2\sqrt{bd\,x^3+bc\,x^2+adx+ac}}+\frac{2C\cdots}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((d*x+c)*(b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)*(2*(B*f-C*e)/f^2*(
-(-a*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(1/2
)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2)/b
))^(1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(-(-a*b)^(1/
2)/b+c/d))^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2))+2*C/
f*(-(-a*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(
1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2
)/b))^(1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d-(-a*b)^(1/2)/b)*Ellip
ticE(((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-
a*b)^(1/2)/b))^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+c/d)/(-(-a*b)^(1/2)/b+c
/d))^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)))+2*(A*f^2-
B*e*f+C*e^2)/f^3*(-(-a*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/
2)*((x-(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-
c/d+(-a*b)^(1/2)/b))^(1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)/(-c/d+e/f)*El
lipticPi(((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2),(-c/d+(-a*b)^(1/2)/b)/(-c/d
+e/f),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a+bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(b*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a + b*x**2)*sqrt(c + d*x)*(e + f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(e+f\,x)\,\sqrt{b\,x^2+a}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

3.43 $\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx$

Optimal result 443
Mathematica [C] (verified) 444
Rubi [B] (warning: unable to verify) 445
Maple [A] (verified) 452
Fricas [F(-1)] 453
Sympy [F] 454
Maxima [F] 454
Giac [F] 454
Mupad [F(-1)] 455
Reduce [F] 455

Optimal result

Integrand size = 43, antiderivative size = 783

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \frac{2D\sqrt{c+dx}\sqrt{a+bx^2}}{3bdf}$$

$$+\frac{2\left(\sqrt{b}c-\sqrt{-a}d\right)\sqrt{\sqrt{b}c+\sqrt{-a}d}(3dDe-3Cdf+2cDf)\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}E\left(\arcsin\left(\right.\right.}{3b^{3/4}d^3f^2\sqrt{a+bx^2}}$$

$$-\frac{2\sqrt{\sqrt{b}c+\sqrt{-a}d}\left(adDf^2-\sqrt{-a}\sqrt{b}f(3dDe-3Cdf+2cDf)-3bd(De^2-f(Ce-Bf))\right)\sqrt{1-\frac{\sqrt{b}}{\sqrt{b}c}}}{3b^{5/4}d^2f^3\sqrt{a+bx^2}}$$

$$-\frac{2\sqrt{\sqrt{b}c+\sqrt{-a}d}(De^3-f(Ce^2-f(Be-Af)))\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c-\sqrt{-a}d}}\sqrt{1-\frac{\sqrt{b}(c+dx)}{\sqrt{b}c+\sqrt{-a}d}}\text{EllipticPi}\left(-\frac{\left(c+\frac{\sqrt{-a}}{\sqrt{b}}\right.}{de-c}\right.}{\sqrt[4]{b}f^3(de-cf)\sqrt{a+bx^2}}$$

output
```
2/3*D*(d*x+c)^(1/2)*(b*x^2+a)^(1/2)/b/d/f+2/3*(b^(1/2)*c-(-a)^(1/2)*d)*(b^
(1/2)*c+(-a)^(1/2)*d)^(1/2)*(-3*C*d*f+2*D*c*f+3*D*d*e)*(1-b^(1/2)*(d*x+c)/
(b^(1/2)*c-(-a)^(1/2)*d))^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d
))^(1/2)*EllipticE(b^(1/4)*(d*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),((
b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(3/4)/d^3/f^2/(
b*x^2+a)^(1/2)-2/3*(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)*(a*d*D*f^2-(-a)^(1/2)*b^
(1/2)*f*(-3*C*d*f+2*D*c*f+3*D*d*e)-3*b*d*(D*e^2-f*(-B*f+C*e)))*(1-b^(1/2)*
(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2)*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c+(-a)
^(1/2)*d))^(1/2)*EllipticF(b^(1/4)*(d*x+c)^(1/2)/(b^(1/2)*c+(-a)^(1/2)*d)^
(1/2),((b^(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(5/4)/d
^2/f^3/(b*x^2+a)^(1/2)-2*(b^(1/2)*c+(-a)^(1/2)*d)^(1/2)*(D*e^3-f*(C*e^2-f*
(-A*f+B*e)))*(1-b^(1/2)*(d*x+c)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2)*(1-b^(1/2)
*(d*x+c)/(b^(1/2)*c+(-a)^(1/2)*d))^(1/2)*EllipticPi(b^(1/4)*(d*x+c)^(1/2)/
(b^(1/2)*c+(-a)^(1/2)*d)^(1/2),-(c+(-a)^(1/2)*d/b^(1/2))*f/(-c*f+d*e),((b^
(1/2)*c+(-a)^(1/2)*d)/(b^(1/2)*c-(-a)^(1/2)*d))^(1/2))/b^(1/4)/f^3/(-c*f+d
*e)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.19 (sec) , antiderivative size = 2091, normalized size of antiderivative = 2.67

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^
2]),x]
```

output
```
(2*(-3*b*c^2*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e^2*f - 3*a*d^4*Sqrt[-
c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e^2*f + 3*b*c^2*C*d^2*Sqrt[-c - (I*Sqrt[a]*d)
/Sqrt[b]]*e*f^2 + 3*a*C*d^4*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*e*f^2 + b*c^3
*d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e*f^2 + a*c*d^3*Sqrt[-c - (I*Sqrt[a]
*d)/Sqrt[b]]*D*e*f^2 - 3*b*c^3*C*d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f^3 -
3*a*c*C*d^3*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*f^3 + 2*b*c^4*Sqrt[-c - (I*Sq
rt[a]*d)/Sqrt[b]]*D*f^3 + 2*a*c^2*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*f
^3 + 6*b*c*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e^2*f*(c + d*x) - 6*b*c*
C*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*e*f^2*(c + d*x) - 2*b*c^2*d*Sqrt[-c
 - (I*Sqrt[a]*d)/Sqrt[b]]*D*e*f^2*(c + d*x) + 6*b*c^2*C*d*Sqrt[-c - (I*Sqr
t[a]*d)/Sqrt[b]]*f^3*(c + d*x) - 4*b*c^3*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*
D*f^3*(c + d*x) - 3*b*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e^2*f*(c + d*
x)^2 + 3*b*C*d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*e*f^2*(c + d*x)^2 + b*c*
d*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*e*f^2*(c + d*x)^2 - 3*b*c*C*d*Sqrt[-c
 - (I*Sqrt[a]*d)/Sqrt[b]]*f^3*(c + d*x)^2 + 2*b*c^2*Sqrt[-c - (I*Sqrt[a]*d
)/Sqrt[b]]*D*f^3*(c + d*x)^2 + d^2*Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]*D*f^2*
(d*e - c*f)*(c + d*x)*(a + b*x^2) + Sqrt[b]*((-I)*Sqrt[b]*c + Sqrt[a]*d)*f
*(-(d*e) + c*f)*(3*d*D*e - 3*C*d*f + 2*c*D*f)*Sqrt[(d*((I*Sqrt[a])/Sqrt[b]
 + x))/(c + d*x)]*Sqrt[-(((I*Sqrt[a]*d)/Sqrt[b] - d*x)/(c + d*x))]*(c + d*
x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-c - (I*Sqrt[a]*d)/Sqrt[b]]/Sqrt[c + ...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1619 vs. $2(783) = 1566$.

Time = 5.85 (sec) , antiderivative size = 1619, normalized size of antiderivative = 2.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2349, 729, 1540, 1416, 2185, 27, 599, 1511, 1416, 1509, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}\sqrt{c+dx}(e+fx)}dx$$

$$\Big\downarrow \text{2349}$$

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bx^2+a}}dx-\frac{\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{\sqrt{c+dx}(e+fx)\sqrt{bx^2+a}}dx}{f^3}$$

$\Big\downarrow$ 729

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bx^2+a}}dx-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\int \frac{1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{f^3}$$

$\Big\downarrow$ 1540

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bx^2+a}}dx-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.}{f^3}$$

$\Big\downarrow$ 1416

$$\int \frac{\frac{De^2}{f^3}-\frac{Ce}{f^2}+\frac{Dx^2}{f}+\left(\frac{C}{f}-\frac{De}{f^2}\right)x+\frac{B}{f}}{\sqrt{c+dx}\sqrt{bx^2+a}}dx-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.}{}$$

$\Big\downarrow$ 2185

$$-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.}{}$$

$$\frac{2\int -\frac{d(d(aDf^2-3b(De^2-f(Ce-Bf)))+bf(3dDe-3Cdf+2cDf)x)}{2f^3\sqrt{c+dx}\sqrt{bx^2+a}}dx}{3bd^2}+\frac{2D\sqrt{a+bx^2}\sqrt{c+dx}}{3bdf}$$

$$\downarrow 27$$

$$-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.}{}$$

$$\frac{\int \frac{d(aDf^2-3b(De^2-f(Ce-Bf)))+bf(3dDe-3Cdf+2cDf)x}{\sqrt{c+dx}\sqrt{bx^2+a}}dx}{3bdf^3}+\frac{2D\sqrt{a+bx^2}\sqrt{c+dx}}{3bdf}$$

$$\downarrow 599$$

$$-\frac{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{f\sqrt{ad^2+bc^2}\left(f\sqrt{ad^2+bc^2}+\sqrt{b}(de-cf)\right)\int \frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.}{}$$

$$\frac{2\int \frac{-((aDf^2-3b(De^2-f(Ce-Bf)))d^2)+bcf(3dDe-3Cdf+2cDf)-bf(3dDe-3Cdf+2cDf)(c+dx)}{\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{3bd^3f^3}+$$

$$\frac{2D\sqrt{a+bx^2}\sqrt{c+dx}}{3bdf}$$

$$\downarrow 1511$$

$$\frac{2\sqrt{c+dx}\sqrt{bx^2+a}D}{3bdf}+$$

$$2\left(\left(-\left(\left(aDf^2-3b\left(De^2-f(Ce-Bf)\right)\right)d^2\right)+bcf(3dDe-3Cdf+2cDf)-\sqrt{b}\sqrt{bc^2+ad^2}f(3dDe-3Cdf+\right.\right.$$

$$2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\int\frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.$$

$$\Big\downarrow \textbf{1416}$$

$$\frac{2\sqrt{c+dx}\sqrt{bx^2+a}D}{3bdf}+$$

$$2\left(\frac{\sqrt[4]{bc^2+ad^2}\left(-\left(\left(aDf^2-3b\left(De^2-f(Ce-Bf)\right)\right)d^2\right)+bcf(3dDe-3Cdf+2cDf)-\sqrt{b}\sqrt{bc^2+ad^2}f(3dDe-3Cdf+2cDf)\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)}{2\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}\right.$$

$$2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\int\frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.$$

$$\Big\downarrow \textbf{1509}$$

$$\frac{2\sqrt{c+dx}\sqrt{bx^2+a}D}{3bdf}+$$

$$2\left(\sqrt{b}\sqrt{bc^2+ad^2}f(3dDe-3Cdf+2cDf)\left(\frac{\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}}{\sqrt[4]{bc^2}}\right.\right.}{\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}\right.\right.$$

$$2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\int\frac{\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1}{(de-cf+f(c+dx))\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}d\sqrt{c+dx}}{d(adf^2-be(de-2cf))}\right.$$

$\downarrow$ 2220

$$\frac{2\sqrt{c+dx}\sqrt{bx^2+a}D}{3bdf}+$$

$$\frac{2\left(\sqrt{b}\sqrt{bc^2+ad^2}f(3dDe-3Cdf+2cDf)\left(\frac{\sqrt[4]{bc^2+ad^2}\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)\sqrt{\frac{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}{\left(\frac{bc^2}{d^2}+a\right)\left(\frac{\sqrt{b}(c+dx)}{\sqrt{bc^2+ad^2}}+1\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\cdots}{\sqrt[4]{bc^2\cdots}}\right.\right.}{\sqrt[4]{b}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c+dx)^2}{d^2}+a}}\cdots\right.\right.}{2\left(De^3-f\left(Ce^2-f(Be-Af)\right)\right)\left(\frac{\sqrt{bc^2+ad^2}f\left(\sqrt{bc^2+ad^2}f+\sqrt{b}(de-cf)\right)\left(\frac{\left(f-\frac{\sqrt{b}(de-cf)}{\sqrt{bc^2+ad^2}}\right)\arctan\left(\frac{\sqrt{be^2+af^2}\sqrt{c+dx}}{\sqrt{f}\sqrt{de-cf}\sqrt{\frac{bc^2}{d^2}-\frac{2b(c+dx)c}{d^2}+\frac{b(c\cdots}{}}}\right.}{2\sqrt{f}\sqrt{de-cf}\sqrt{be^2+af^2}}\cdots\right.}{\cdots}\right.\cdots}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(Sqrt[c + d*x]*(e + f*x)*Sqrt[a + b*x^2]),x]
```

output
```
(2*D*Sqrt[c + d*x]*Sqrt[a + b*x^2])/(3*b*d*f) + (2*(Sqrt[b]*Sqrt[b*c^2 + a
*d^2]*f*(3*d*D*e - 3*C*d*f + 2*c*D*f)*(-((Sqrt[c + d*x]*Sqrt[a + (b*c^2)/d
^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2])/((a + (b*c^2)/d^2)*(1 +
 (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2]))) + ((b*c^2 + a*d^2)^(1/4)*(1 +
(Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*c^2)/d^2 - (2*b*c*(c
 + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d^2)*(1 + (Sqrt[b]*(c +
d*x))/Sqrt[b*c^2 + a*d^2])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c + d*x])/
(b*c^2 + a*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c^2 + a*d^2])/2])/(b^(1/4)
*Sqrt[a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2])) + (
(b*c^2 + a*d^2)^(1/4)*(b*c*f*(3*d*D*e - 3*C*d*f + 2*c*D*f) - Sqrt[b]*Sqrt[
b*c^2 + a*d^2]*f*(3*d*D*e - 3*C*d*f + 2*c*D*f) - d^2*(a*D*f^2 - 3*b*(D*e^2
 - f*(C*e - B*f))))*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a
+ (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)
/d^2)*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])^2)]*EllipticF[2*ArcTan
[(b^(1/4)*Sqrt[c + d*x])/(b*c^2 + a*d^2)^(1/4)], (1 + (Sqrt[b]*c)/Sqrt[b*c
^2 + a*d^2])/2])/(2*b^(1/4)*Sqrt[a + (b*c^2)/d^2 - (2*b*c*(c + d*x))/d^2 +
 (b*(c + d*x)^2)/d^2])))/(3*b*d^3*f^3) - (2*(D*e^3 - f*(C*e^2 - f*(B*e - A
*f)))*(-1/2*(b^(1/4)*(b*c^2 + a*d^2)^(1/4)*(Sqrt[b*c^2 + a*d^2]*f + Sqrt[b
]*(d*e - c*f))*(1 + (Sqrt[b]*(c + d*x))/Sqrt[b*c^2 + a*d^2])*Sqrt[(a + (b*
c^2)/d^2 - (2*b*c*(c + d*x))/d^2 + (b*(c + d*x)^2)/d^2)/((a + (b*c^2)/d...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 599
```
Int[((A_.) + (B_.)*(x_))/(Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]
), x_Symbol] :> Simp[-2/d^2   Subst[Int[(B*c - A*d - B*x^2)/Sqrt[(b*c^2 + a
*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)], x], x, Sqrt[c + d*x]], x] /; Fr
eeQ[{a, b, c, d, A, B}, x] && PosQ[b/a]
```

rule 729
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)
^2]), x_Symbol] :> Simp[2   Subst[Int[1/((d*e - c*f + f*x^2)*Sqrt[(b*c^2 +
a*d^2)/d^2 - 2*b*c*(x^2/d^2) + b*(x^4/d^2)]), x], x, Sqrt[c + d*x]], x] /;
FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 1509
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

rule 1511
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Simp[(e + d*q)/q   Int[1/Sqrt[a + b*x^2 + c*x^
4], x], x] - Simp[e/q   Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /;
NeQ[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Pos
Q[c/a]
```

rule 1540
```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Simp[(c*d + a*e*q)/(c*d^2 - a*e^2)   Int[1
/Sqrt[a + b*x^2 + c*x^4], x], x] - Simp[(a*e*(e + d*q))/(c*d^2 - a*e^2)   I
nt[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

rule 2185
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + b*x^2)^(p + 1)/(b*e^(q - 1)*(m + q + 2*p + 1))), x] + Si
mp[1/(b*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a + b*x^2)^p*ExpandToSum[
b*e^q*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x
)^(q - 2)*(a*e^2*(m + q - 1) - b*d^2*(m + q + 2*p + 1) - 2*b*d*e*(m + q + p
)*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, d
, e, m, p}, x] && PolyQ[Pq, x] && NeQ[b*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] &&
True) &&  !(IGtQ[m, 0] && RationalQ[a, b, d, e] && (IntegerQ[p] || ILtQ[p +
 1/2, 0]))
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 926, normalized size of antiderivative = 1.18

method	result
elliptic	$\frac{\sqrt{(dx+c)(b\,x^2+a)}\left(\frac{2D\sqrt{bd\,x^3+bc\,x^2+adx+ac}}{3fbd}+\frac{2\left(\frac{B\,f^2-Cef+De^2}{f^3}-\frac{Da}{3fb}\right)\left(-\frac{\sqrt{-ab}}{b}+\frac{c}{d}\right)\sqrt{\frac{x+\frac{c}{d}}{-\frac{\sqrt{-ab}}{b}+\frac{c}{d}}}\sqrt{\frac{x-\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}-\frac{\sqrt{-ab}}{b}}}\sqrt{\frac{x+\frac{\sqrt{-ab}}{b}}{-\frac{c}{d}+\frac{\sqrt{-ab}}{b}}}\,\mathrm{El}}{\sqrt{bd\,x^3+bc\,x^2+adx+ac}}\right.}{}$
default	Expression too large to display

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((d*x+c)*(b*x^2+a))^(1/2)/(d*x+c)^(1/2)/(b*x^2+a)^(1/2)*(2/3*D/f/b/d*(b*d*
x^3+b*c*x^2+a*d*x+a*c)^(1/2)+2*((B*f^2-C*e*f+D*e^2)/f^3-1/3*D/f/b*a)*(-(-a
*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(1/2)/b)
/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2)/b))^(
1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)*EllipticF(((x+c/d)/(-(-a*b)^(1/2)/b
+c/d))^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2))+2*(1/f^2
*(C*f-D*e)-2/3*D/f/d*c)*(-(-a*b)^(1/2)/b+c/d)*((x+c/d)/(-(-a*b)^(1/2)/b+c/
d))^(1/2)*((x-(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2)*((x+(-a*b)^(1/2
)/b)/(-c/d+(-a*b)^(1/2)/b))^(1/2)/(b*d*x^3+b*c*x^2+a*d*x+a*c)^(1/2)*((-c/d
-(-a*b)^(1/2)/b)*EllipticE(((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2),((-c/d+(-
a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/b))^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+c
/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)
/b))^(1/2)))+2*(A*f^3-B*e*f^2+C*e^2*f-D*e^3)/f^4*(-(-a*b)^(1/2)/b+c/d)*((x
+c/d)/(-(-a*b)^(1/2)/b+c/d))^(1/2)*((x-(-a*b)^(1/2)/b)/(-c/d-(-a*b)^(1/2)/
b))^(1/2)*((x+(-a*b)^(1/2)/b)/(-c/d+(-a*b)^(1/2)/b))^(1/2)/(b*d*x^3+b*c*x^
2+a*d*x+a*c)^(1/2)/(-c/d+e/f)*EllipticPi(((x+c/d)/(-(-a*b)^(1/2)/b+c/d))^(
1/2),(-c/d+(-a*b)^(1/2)/b)/(-c/d+e/f),((-c/d+(-a*b)^(1/2)/b)/(-c/d-(-a*b)^
(1/2)/b))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\text{Timed out}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{a+bx^2}\sqrt{c+dx}\,(e+fx)}\,dx$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(d*x+c)**(1/2)/(f*x+e)/(b*x**2+a)**(1/2),x
)
```

output
```
Integral((A + B*x + C*x**2 + D*x**3)/(sqrt(a + b*x**2)*sqrt(c + d*x)*(e +
f*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x +
e)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Dx^3+Cx^2+Bx+A}{\sqrt{bx^2+a}\sqrt{dx+c}(fx+e)}\,dx$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((D*x^3 + C*x^2 + B*x + A)/(sqrt(b*x^2 + a)*sqrt(d*x + c)*(f*x +
e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{(e+f\,x)\,\sqrt{b\,x^2+a}\,\sqrt{c+d\,x}}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/((e + f*x)*(a + b*x^2)^(1/2)*(c + d*x)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2+Dx^3}{\sqrt{c+dx}(e+fx)\sqrt{a+bx^2}}\,dx=\int \frac{Dx^3+C\,x^2+Bx+A}{\sqrt{dx+c}\,(fx+e)\,\sqrt{b\,x^2+a}}dx$$

input
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

output
```
int((D*x^3+C*x^2+B*x+A)/(d*x+c)^(1/2)/(f*x+e)/(b*x^2+a)^(1/2),x)
```

3.44 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 456
Mathematica [A] (warning: unable to verify) 457
Rubi [F] 458
Maple [B] (verified) 460
Fricas [F] 461
Sympy [F] 462
Maxima [F] 462
Giac [F] 462
Mupad [F(-1)] 463
Reduce [F] 463

Optimal result

Integrand size = 31, antiderivative size = 659

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \frac{2e\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,E\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\cdots}}{\sqrt{\sqrt{c}f+\cdots}}\cdots\right.\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}(ef-dg)\sqrt{\frac{(ef-dg)(\sqrt{a}+\cdots}{(\sqrt{c}f-\sqrt{a}g)(\cdots}}\cdots}$$

$$-\frac{2\sqrt{c}\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right),\frac{(\sqrt{c}d-\sqrt{a}e)(\sqrt{c}f+\cdots}{(\sqrt{c}d+\sqrt{a}e)(\sqrt{c}f-\cdots}\cdots\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\left(\sqrt{c}f-\sqrt{a}g\right)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}}$$

output
```
2*e*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-
c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1/2)*d+a^(1/
2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1
/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^
(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(-d*g+e
*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(
-c*x^2+a)^(1/2)-2*c^(1/2)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*
(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*Elli
pticF((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2
)/(e*x+d)^(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^
(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^
(1/2)*e)^(1/2)/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1
/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 30.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 0.79

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx =$$

$$-\frac{2\sqrt{a-cx^2}\left(\sqrt{a}\sqrt{c}e(ef-dg)\sqrt{-\frac{(-cf^2+ag^2)(a-cx^2)}{ac(f+gx)^2}}-e\left(\sqrt{c}d+\sqrt{a}e\right)\left(\sqrt{c}f-\sqrt{a}g\right)\sqrt{\frac{(\sqrt{c}f+\sqrt{a}g)(d+ex)}{(\sqrt{c}d+\sqrt{a}e)(f+gx)}}E\Bigg(\right.}{\sqrt{a}\sqrt{c}\,(cd^2-}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*Sqrt[a - c*x^2]*(Sqrt[a]*Sqrt[c]*e*(e*f - d*g)*Sqrt[-(((-(c*f^2) + a*g
^2)*(a - c*x^2))/(a*c*(f + g*x)^2))] - e*(Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*
f - Sqrt[a]*g)*Sqrt[((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))/((Sqrt[c]*d + Sqrt
[a]*e)*(f + g*x))]*EllipticE[ArcSin[Sqrt[(f - (Sqrt[a]*g)/Sqrt[c] - (Sqrt[
c]*f*x)/Sqrt[a] + g*x)/(2*f + 2*g*x)]], (2*Sqrt[a]*Sqrt[c]*(e*f - d*g))/((
Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))] + (c*d^2 - a*e^2)*g*Sqrt[
((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))/((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))]*E
llipticF[ArcSin[Sqrt[(f - (Sqrt[a]*g)/Sqrt[c] - (Sqrt[c]*f*x)/Sqrt[a] + g*
x)/(2*f + 2*g*x)]], (2*Sqrt[a]*Sqrt[c]*(e*f - d*g))/((Sqrt[c]*d + Sqrt[a]*
e)*(Sqrt[c]*f - Sqrt[a]*g))]))/(Sqrt[a]*Sqrt[c]*(c*d^2 - a*e^2)*(-(e*f) +
d*g)*Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[-(((-(c*f^2) + a*g^2)*(a - c*x^2))/(
a*c*(f + g*x)^2))])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \ 733$$

$$\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}$$

$$\Big\downarrow \ 732$$

$$\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}$$

$$\Big\downarrow \ 744$$

$$\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(cd^2-ae^2\right)}{(d+ex)^2\left(cf^2-ag^2\right)}-\frac{2(f+gx)(cdf-aeg)}{(d+ex)\left(cf^2-ag^2\right)}+1}{\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)^2}}\text{EllipticF}\left(2\right.}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f+gx)^2(cd^2-ae^2)}{(d+ex)^2(cf^2-ag^2)}-\frac{2(f+gx)(cdf}{(d+ex)(cf^2}}}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. $2(507) = 1014$.

Time = 11.76 (sec) , antiderivative size = 2133, normalized size of antiderivative = 3.24

method	result	size
elliptic	Expression too large to display	2133
default	Expression too large to display	7582

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)*e/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2*((a*e^2
*g-c*d^2*g+c*d*e*f)/(a*e^2-c*d^2)/(d*g-e*f)-a*e^2*g/(a*d*e^2*g-a*e^3*f-c*d
^3*g+c*d^2*e*f))*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1
/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2
))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-
1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*
e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*Ellipti
cF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),(
(-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(
a*c)^(1/2)-f/g))^(1/2))+2*(c*e/(a*e^2-c*d^2)+2*c*e^2*f/(a*d*e^2*g-a*e^3*f-
c*d^3*g+c*d^2*e*f))*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/
(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(
1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))
/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(
-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*(-f/
g*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g)
)^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/
e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+(f/g-d/e)*EllipticPi(((-1/c*(a*c)^(1/2...
```

Fricas [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{1}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-sqrt(-c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^2*g*x^5 + (c*e
^2*f + 2*c*d*e*g)*x^4 - a*d^2*f + (2*c*d*e*f + (c*d^2 - a*e^2)*g)*x^3 - (2
*a*d*e*g - (c*d^2 - a*e^2)*f)*x^2 - (2*a*d*e*f + a*d^2*g)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{1}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{1}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(1/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{1}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(1/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,{(d+e\,x)}^{3/2}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{1}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.45 $\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 464
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Sympy [F] 471
Maxima [F] 471
Giac [F] 471
Mupad [F(-1)] 472
Reduce [F] 472

Optimal result

Integrand size = 32, antiderivative size = 659

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx =$$

$$-\frac{2d\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}E\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right)|\frac{(\sqrt{c}d-\sqrt{a}e)(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)(\sqrt{c}f-\sqrt{a}g)}\right)}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}(ef-dg)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}}$$

$$+\frac{2\sqrt{a}\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right),\frac{(\sqrt{c}d-\sqrt{a}e)(\sqrt{c}f+\sqrt{\ }}{(\sqrt{c}d+\sqrt{a}e)(\sqrt{c}f-\sqrt{\ }}\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\left(\sqrt{c}f-\sqrt{a}g\right)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}}$$

output
```
-2*d*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)
-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1/2)*d+a^(1
/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(
1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a
^(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(-d*g+
e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/
(-c*x^2+a)^(1/2)+2*a^(1/2)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)
*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*Ell
ipticF((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/
2)/(e*x+d)^(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a
^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a
^(1/2)*e)^(1/2)/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(
1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 27.73 (sec) , antiderivative size = 718, normalized size of antiderivative = 1.09

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx =$$

$$-\frac{\sqrt{d+ex}\sqrt{\frac{(\sqrt{c}d+\sqrt{a}e)(f+gx)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}\left(2de(\sqrt{c}f-\sqrt{a}g)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}E\left(\arcsin\left(\sqrt{\frac{(\sqrt{c}d+\sqrt{a}e)(f+gx)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\right)\right.\right.}{}$$

input
```
Integrate[x/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
-((Sqrt[d + e*x]*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sq
rt[a]*g)*(d + e*x))]*Sqrt[a - c*x^2]*(2*d*e*(Sqrt[c]*f - Sqrt[a]*g)*Sqrt[(
(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt[a]*g)*(d + e*x))]*El
lipticE[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt
[a]*g)*(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((S
qrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))] + (-(e*f) + d*g)*(Sqrt[2]*
(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(d + (Sqrt[a]*e)/Sqrt[c] + (Sqrt[c]*d*x)/Sqrt
[a] + e*x)/(d + e*x)]*EllipticF[ArcSin[Sqrt[((e*f - d*g)*(-Sqrt[a] + Sqrt[
c]*x))/((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]], -(((Sqrt[c]*d*f)/Sqrt[a] - e
*f + d*g - (Sqrt[a]*e*g)/Sqrt[c])/(2*e*f - 2*d*g))] + 2*Sqrt[c]*d*Sqrt[((e
*f - d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt[a]*g)*(d + e*x))]*Elli
pticF[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt[a
]*g)*(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqr
t[c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))])))/(e*(-(c*d^2) + a*e^2)*(e*
f - d*g)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[((e*f - d*g)*(-Sqrt[a] + Sqrt[c]*x))/(
(Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2349}$$

$$\int \frac{1}{e\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e}-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \text{732}$$

$$-\frac{2(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \; 733$$

$$-\frac{2(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}-\frac{d\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \; 732$$

$$-\frac{2(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}-\frac{d\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \; 744$$

$$-\frac{2(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}-\frac{d\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \; 1416$$

$$-\frac{\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\cdots\right.}{e\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)\cdots}{(cf^2-ag^2)\cdots}\cdots}}$$

$$\frac{d\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\cdots\right.\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}\cdots\right.}{e}$$

input
```
Int[x/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. $2(507) = 1014$.

Time = 7.37 (sec) , antiderivative size = 2136, normalized size of antiderivative = 3.24

method	result	size
elliptic	Expression too large to display	2136
default	Expression too large to display	7632

input
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(-2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+
c*d^2*e*f)*d/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2*(1/e-(
a*e^2*g-c*d^2*g+c*d*e*f)*d/e/(a*e^2-c*d^2)/(d*g-e*f)+a*e*g/(a*d*e^2*g-a*e^
3*f-c*d^3*g+c*d^2*e*f)*d)*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x
+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(
a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^
(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+
d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2
)*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g)
)^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/
e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+2*(-c*d/(a*e^2-c*d^2)-2*c*e*f/(a*d*e^2*g-
a*e^3*f-c*d^3*g+c*d^2*e*f)*d)*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g
)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1
/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a
*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-
f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^
(1/2)*(-f/g*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/
e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c
)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+(f/g-d/e)*EllipticPi(((-1/c*...
```

Fricas [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{x}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-sqrt(-c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)*x/(c*e^2*g*x^5 + (c
*e^2*f + 2*c*d*e*g)*x^4 - a*d^2*f + (2*c*d*e*f + (c*d^2 - a*e^2)*g)*x^3 -
(2*a*d*e*g - (c*d^2 - a*e^2)*f)*x^2 - (2*a*d*e*f + a*d^2*g)*x), x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate(x/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(x/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="gia
c")
```

output
```
integrate(x/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(x/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int(x/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.46 $\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

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Rubi [F] 475
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Reduce [F] 483

Optimal result

Integrand size = 34, antiderivative size = 1042

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Too large to display}$$

output
```
2*d^2*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2
)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1/2)*d+a^(
1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^
(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-
a^(1/2)*g))^(1/2))/e/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(-d
*g+e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/
2)/(-c*x^2+a)^(1/2)+2*d*(c^(1/2)*d-2*a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)^(1/2
)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*
g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(
1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^
(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/e^2/(c^(1/2)*
d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)
*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)
+2*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(-d*g+e*f)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)
*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticPi((c^(1
/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(
1/2),e*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/g,((c^(1/2)*d-a^(1/2)*e
)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2)
)/e^2/(c^(1/2)*d+a^(1/2)*e)^(1/2)/g/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(a^(
1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 30.21 (sec) , antiderivative size = 1503, normalized size of antiderivative = 1.44

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[x^2/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*d^2*Sqrt[f + g*x]*Sqrt[a - c*x^2])/((-(c*d^2) + a*e^2)*(e*f - d*g)*Sqr
t[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a - (c*(d + e*x)^2*(-1 + d/(d + e*x))^
2)/e^2]*(-(d^2*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x))*(-((a*e^2)/(d + e*x
)^2) + c*(-1 + d/(d + e*x))^2)) + (d^2*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(g +
(e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]*Sqrt[((e*
f - d*g)*((Sqrt[a]*e)/(d + e*x) + Sqrt[c]*(1 - d/(d + e*x))))/(e*(Sqrt[c]*
f - Sqrt[a]*g))]*((Sqrt[a]*e)/(d + e*x) + Sqrt[c]*(-1 + d/(d + e*x)))*(e*(
Sqrt[c]*f - Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(g +
 (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]], ((Sqrt
[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt
[c]*f - Sqrt[a]*g))] + Sqrt[c]*(-(e*f) + d*g)*EllipticF[ArcSin[Sqrt[((Sqrt
[c]*d + Sqrt[a]*e)*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f
+ Sqrt[a]*g))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[
c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))]))/((d + e*x)*Sqrt[-(((e*f - d*
g)*((Sqrt[a]*e)/(d + e*x) + Sqrt[c]*(-1 + d/(d + e*x))))/(e*(Sqrt[c]*f + S
qrt[a]*g)))]) + (d*(e*f - d*g)*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(g + (e*f)/(d
 + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]*Sqrt[2 + (2*Sqrt[
a]*e)/(Sqrt[c]*(d + e*x)) - (2*Sqrt[c]*d*(-1 + d/(d + e*x)))/(Sqrt[a]*e)]*
(-(Sqrt[a]*Sqrt[c]*e) + (a*e^2)/(d + e*x) + c*(d - d^2/(d + e*x)))*Ellipti
cF[ArcSin[Sqrt[-(((e*f - d*g)*((Sqrt[a]*e)/(d + e*x) + Sqrt[c]*(-1 + d/...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \; 2349$$

$$\frac{d^2\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 733$$

$$\frac{d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 732$$

$$\frac{d^2\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 744

$$\frac{d^2\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 1416

$$\frac{d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(cd^2-ae^2\right)}{(d+ex)^2\left(cf^2-ag^2\right)}-\frac{2(f+gx)(cdf-ae}{(d+ex)\left(cf^2-ag^2\right.}}{\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f+gx)^2\left(cd^2-ae\right.}{(d+ex)^2\left(cf^2-ag\right.}}}}{e^2}$$

$$\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 2349

$$\frac{d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(cd^2-ae^2)}{(d+ex)^2(cf^2-ag^2)}-\frac{2(f+gx)(cdf-ae}{(d+ex)(cf^2-ag^2}}{\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f+gx)^2(cd^2-ae}{(d+ex)^2(cf^2-ag}}}\right.}{e^2}$$

$$\frac{2d\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int \frac{\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \ 27$$

$$\frac{d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(cd^2-ae^2)}{(d+ex)^2(cf^2-ag^2)}-\frac{2(f+gx)(cdf-ae}{(d+ex)(cf^2-ag^2}}{\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f+gx)^2(cd^2-ae}{(d+ex)^2(cf^2-ag}}}\right.}{e^2}$$

$$\frac{2d\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\frac{\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}$$

$$\Big\downarrow \ 726$$

$$\frac{\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{c}}\right.\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}\right.}{e^2}$$

$$\frac{2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dxd}{e^2}+$$

$$\frac{2\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{f}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{d}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}g\sqrt{a-cx^2}}$$

$$\Big\downarrow \ 732$$

$$\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2\left(a-cx^2\right)}{\left(cf^2-ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{c}}\right.\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}\right.$$

$$e^2$$

$$\frac{4(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}d}{e^2(ef-dg)\sqrt{a-cx^2}}+$$

$$\frac{2\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{f}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{d}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

$$\Big\downarrow \text{1416}$$

$$\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2\left(a-cx^2\right)}{\left(cf^2-ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}}{\sqrt[4]{c}}\right.\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}\right.$$

$$e^2$$

$$\frac{2\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\text{EllipticF}\left(2\right.}{e^2\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)}{(cf^2-ag^2)}}}$$

$$\frac{2\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\sqrt{f}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{d}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

input
```
Int[x^2/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2157 vs. $2(807) = 1614$.

Time = 6.13 (sec) , antiderivative size = 2158, normalized size of antiderivative = 2.07

method	result	size
elliptic	Expression too large to display	2158
default	Expression too large to display	21234

input
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)/e*d^2/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2*(-d
/e^2+d^2*(a*e^2*g-c*d^2*g+c*d*e*f)/e^2/(a*e^2-c*d^2)/(d*g-e*f)-a*g/(a*d*e^
2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*d^2)*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/
2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e
)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+
1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f
/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1
/2)))^(1/2)*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/
e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c
)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+2*(1/e+d^2/e*c/(a*e^2-c*d^2)+2*
c*f/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*d^2)*(1/c*(a*c)^(1/2)-d/e)*((-1/
c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2
*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f
/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a
*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1
/c*(a*c)^(1/2)))^(1/2)*(-f/g*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1
/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2
)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+(f/g-d/e)*E...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="f
ricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x^2}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate(x**2/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x^2}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="m
axima")
```

output
```
integrate(x^2/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x^2}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(x^2/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x^2}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(x^2/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int(x^2/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{x^2}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.47 $\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 484
Mathematica [A] (verified) 485
Rubi [F] 486
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Fricas [F] 490
Sympy [F] 491
Maxima [F] 491
Giac [F] 491
Mupad [F(-1)] 492
Reduce [F] 492

Optimal result

Integrand size = 36, antiderivative size = 677

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx =$$

$$-\frac{2(Bd-Ae)\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,E\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right)\middle|\frac{(\sqrt{c}d-\sqrt{a}e)(\sqrt{c}f+\sqrt{\cdots}}{(\sqrt{c}d+\sqrt{a}e)(\sqrt{c}f-\sqrt{\cdots}}\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}(ef-dg)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}}$$

$$+\frac{2\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\sqrt{c}f+\sqrt{a}g}\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\sqrt{c}d+\sqrt{a}e}\sqrt{f+gx}}{\sqrt{\sqrt{c}f+\sqrt{a}g}\sqrt{d+ex}}\right),\frac{(\sqrt{c}\cdots}{(\sqrt{c}\cdots}\right.}{\left(\sqrt{c}d-\sqrt{a}e\right)\sqrt{\sqrt{c}d+\sqrt{a}e}\left(\sqrt{c}f-\sqrt{a}g\right)\sqrt{\frac{(ef-dg)(\sqrt{a}+\sqrt{c}x)}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}}$$

output
```
-2*(-A*e+B*d)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)
*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1/
2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1
/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^
(1/2)*f-a^(1/2)*g))^(1/2))/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/
2)/(-d*g+e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d
))^(1/2)/(-c*x^2+a)^(1/2)+2*(a^(1/2)*B-A*c^(1/2))*(c^(1/2)*f+a^(1/2)*g)^(1
/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2
)*g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c
^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+
a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/(c^(1/2)*d-
a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(
a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 26.04 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \frac{\sqrt{d+ex}\sqrt{\frac{(\sqrt{c}d+\sqrt{a}e)(f+gx)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{a-cx^2}\left(2e(-Bd+Ae)\left(\sqrt{c}f-\sqrt{a}g\right)\right.}{}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output

```
(Sqrt[d + e*x]*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[c]*f + Sqrt
[a]*g)*(d + e*x))]*Sqrt[a - c*x^2]*(2*e*(-(B*d) + A*e)*(Sqrt[c]*f - Sqrt[a
]*g)*Sqrt[((e*f - d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt[a]*g)*(d
+ e*x))]*EllipticE[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x))/((Sqrt[
c]*f + Sqrt[a]*g)*(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt
[a]*g))/((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))] - (e*f - d*g)*(
-(Sqrt[2]*B*(Sqrt[c]*d - Sqrt[a]*e)*Sqrt[(d + (Sqrt[a]*e)/Sqrt[c] + (Sqrt[
c]*d*x)/Sqrt[a] + e*x)/(d + e*x)]*EllipticF[ArcSin[Sqrt[((e*f - d*g)*(-Sqr
t[a] + Sqrt[c]*x))/((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]], -(((Sqrt[c]*d*f)
/Sqrt[a] - e*f + d*g - (Sqrt[a]*e*g)/Sqrt[c])/(2*e*f - 2*d*g))]) - 2*Sqrt[
c]*(B*d - A*e)*Sqrt[((e*f - d*g)*(Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f - Sqrt
[a]*g)*(d + e*x))]*EllipticF[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(f + g*x
))/((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c
]*f + Sqrt[a]*g))/((Sqrt[c]*d + Sqrt[a]*e)*(Sqrt[c]*f - Sqrt[a]*g))])))/(e
*(-(c*d^2) + a*e^2)*(e*f - d*g)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[((e*f - d*g)*(-
Sqrt[a] + Sqrt[c]*x))/((Sqrt[c]*f + Sqrt[a]*g)*(d + e*x))]*Sqrt[f + g*x])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{ 2349}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx+\int \frac{B}{e\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx+\frac{B\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e}$$

$$\Big\downarrow \text{ 732}$$

$$\frac{\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx-2B(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}$$

$\downarrow$ 733

$$\left(A-\frac{Bd}{e}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)-\frac{2B(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}$$

$\downarrow$ 732

$$\left(A-\frac{Bd}{e}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}d}{ef-dg}\right.$$
$$\frac{2B(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}$$

$\downarrow$ 744

$$\left(A-\frac{Bd}{e}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}d}{ef-dg}\right.$$
$$\frac{2B(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a-cx^2}(ef-dg)}$$

$\downarrow$ 1416

$$\left(A-\frac{Bd}{e}\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)}{(cf^2-ag^2)(d+ex)}}}\right.$$

$$\frac{B\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{EllipticF}\left(2\right.}{e\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(}{(cf^2-ag^2)(}}}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2186 vs. $2(523) = 1046$.

Time = 7.49 (sec) , antiderivative size = 2187, normalized size of antiderivative = 3.23

method	result	size
elliptic	Expression too large to display	2187
default	Expression too large to display	15225

input
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)*(A*e-B*d)/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1/2)+2
*(B/e+(A*e-B*d)*(a*e^2*g-c*d^2*g+c*d*e*f)/e/(a*e^2-c*d^2)/(d*g-e*f)-a*e*g/
(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*(A*e-B*d))*(1/c*(a*c)^(1/2)-d/e)*((-
1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)
^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((
-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*
(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x
+1/c*(a*c)^(1/2)))^(1/2)*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(
a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/
e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))+2*(c*(A*e-B*d)/(a*
e^2-c*d^2)+2*c*e*f/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*(A*e-B*d))*(1/c*(
a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+
f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e
)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x
+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-
1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*(-f/g*EllipticF(((-1/c*(a*c)^(
1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(
1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g...
```

Fricas [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-sqrt(-c*x^2 + a)*(B*x + A)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^2*g*
x^5 + (c*e^2*f + 2*c*d*e*g)*x^4 - a*d^2*f + (2*c*d*e*f + (c*d^2 - a*e^2)*g
)*x^3 - (2*a*d*e*g - (c*d^2 - a*e^2)*f)*x^2 - (2*a*d*e*f + a*d^2*g)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate((B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,{(d+e\,x)}^{3/2}}\,dx$$

input
```
int((A + B*x)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((A + B*x)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.48 $\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 493
Mathematica [B] (warning: unable to verify) 494
Rubi [F] 495
Maple [B] (warning: unable to verify) 500
Fricas [F(-1)] 501
Sympy [F] 502
Maxima [F] 502
Giac [F] 502
Mupad [F(-1)] 503
Reduce [F] 503

Optimal result

Integrand size = 38, antiderivative size = 1064

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Too large to display}$$

output
```
2*(A*e^2+C*d^2)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*
f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(
1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^
(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(
c^(1/2)*f-a^(1/2)*g))^(1/2))/e/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)
^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e
*x+d))^(1/2)/(-c*x^2+a)^(1/2)-2*(2*a^(1/2)*C*d*e-c^(1/2)*(-A*e^2+C*d^2))*(
c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/
2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)*d+a^(1/2)*e)
^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1/2)*d
-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)
*g))^(1/2))/e^2/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(c^(1/2)
*f-a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d
))^(1/2)/(-c*x^2+a)^(1/2)+2*C*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(-d*g+e*f)*(a^(1
/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x
+d))^(1/2)*EllipticPi((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f
+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),e*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)
*e)/g,((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(
c^(1/2)*f-a^(1/2)*g))^(1/2))/e^2/(c^(1/2)*d+a^(1/2)*e)^(1/2)/g/(c^(1/2)*f-
a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2191 vs. $2(1064) = 2128$.

Time = 33.62 (sec) , antiderivative size = 2191, normalized size of antiderivative = 2.06

$$\int \frac{A + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*(C*d^2 + A*e^2)*Sqrt[f + g*x]*Sqrt[a - c*x^2])/((-(c*d^2) + a*e^2)*(e*
f - d*g)*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*Sqrt[a - (c*(d + e*x)^2*(-1 + d
/(d + e*x))^2)/e^2]*(-((C*d^2 + A*e^2)*(g + (e*f)/(d + e*x) - (d*g)/(d + e
*x))*(-((a*e^2)/(d + e*x)^2) + c*(-1 + d/(d + e*x))^2)) + (-2*C*d^2*(c*d -
 Sqrt[a]*Sqrt[c]*e)*Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(g + (e*f)/(d + e*x) - (
d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]*Sqrt[((e*f - d*g)*((Sqrt[a]*
e)/(d + e*x) + Sqrt[c]*(1 - d/(d + e*x))))/(e*(Sqrt[c]*f - Sqrt[a]*g))]*(-
(Sqrt[a]*Sqrt[c]*e) + (a*e^2)/(d + e*x) + c*(d - d^2/(d + e*x)))*(e*(c*d^2
 - a*e^2)*(Sqrt[c]*f - Sqrt[a]*g)*EllipticE[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt
[a]*e)*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g)
)]], ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[c]*d + Sqrt[
a]*e)*(Sqrt[c]*f - Sqrt[a]*g))] + (Sqrt[c]*d - Sqrt[a]*e)*(c*d + Sqrt[a]*S
qrt[c]*e)*(-(e*f) + d*g)*EllipticF[ArcSin[Sqrt[((Sqrt[c]*d + Sqrt[a]*e)*(g
 + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]], ((Sq
rt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/((Sqrt[c]*d + Sqrt[a]*e)*(Sq
rt[c]*f - Sqrt[a]*g))]) - 2*A*e^2*(c*d - Sqrt[a]*Sqrt[c]*e)*Sqrt[((Sqrt[c]
*d + Sqrt[a]*e)*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(e*(Sqrt[c]*f + S
qrt[a]*g))]*Sqrt[((e*f - d*g)*((Sqrt[a]*e)/(d + e*x) + Sqrt[c]*(1 - d/(d +
 e*x))))/(e*(Sqrt[c]*f - Sqrt[a]*g))]*(-(Sqrt[a]*Sqrt[c]*e) + (a*e^2)/(d +
 e*x) + c*(d - d^2/(d + e*x)))*(e*(c*d^2 - a*e^2)*(Sqrt[c]*f - Sqrt[a]*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}\,dx$$

$$\Big\downarrow \; 2349$$

$$\left(A+\frac{Cd^2}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx+\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 733$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)+\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{732}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{744}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{(f+gx)}{(d+ex)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(e}\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{(f+gx)}{(d+ex)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(e}\right.$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 27

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{(f+gx)}{(d+ex)^2}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(e}\right.$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}$$

$\downarrow$ 726

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\operatorname{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx}{(cf^2-ag^2)(d+ex}}}\right.$$

$\downarrow$ 732

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\operatorname{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)}{(cf^2-ag^2)(d+ex)}}}\right.$$

$$\frac{4Cd(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{a-cx^2}}$$

$$\Big\downarrow \text{1416}$$

$$\frac{2Cd\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\operatorname{EllipticF}}{e^2\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg}{(cf^2-ag^2}}}$$

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\operatorname{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)}{(cf^2-ag^2)(d+ex)}}}\right.$$

input

```
Int[(A + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2216 vs. $2(829) = 1658$.

Time = 7.51 (sec) , antiderivative size = 2217, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	2217
default	Expression too large to display	28952

input
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)/e*(A*e^2+C*d^2)/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(
1/2)+2*(-C*d/e^2+(A*e^2+C*d^2)*(a*e^2*g-c*d^2*g+c*d*e*f)/e^2/(a*e^2-c*d^2)
/(d*g-e*f)-a*g/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*(A*e^2+C*d^2))*(1/c*(
a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+
f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e
)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x
+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-
1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*EllipticF(((-1/c*(a*c)^(1/2)+f
/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*
(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))
+2*(C/e+1/e*c*(A*e^2+C*d^2)/(a*e^2-c*d^2)+2*c*f/(a*d*e^2*g-a*e^3*f-c*d^3*g
+c*d^2*e*f)*(A*e^2+C*d^2))*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(
x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*
(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)
^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g
+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/
2)*(-f/g*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/
(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{A+Cx^2}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate((C*x**2+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral((A + C*x**2)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x
)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Cx^2+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Cx^2+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{C\,x^2+A}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((A + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((A + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{C\,x^2+A}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.49 $\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 504
Mathematica [B] (warning: unable to verify) 505
Rubi [F] 506
Maple [B] (warning: unable to verify) 511
Fricas [F(-1)] 512
Sympy [F] 513
Maxima [F] 513
Giac [F] 513
Mupad [F(-1)] 514
Reduce [F] 514

Optimal result

Integrand size = 40, antiderivative size = 1060

$$\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Too large to display}$$

output
```
2*d*(-B*e+C*d)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f
)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticE((c^(1
/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(
1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c
^(1/2)*f-a^(1/2)*g))^(1/2))/e/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^
(1/2)/(-d*g+e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*
x+d))^(1/2)/(-c*x^2+a)^(1/2)+2*(c^(1/2)*C*d^2-a^(1/2)*e*(-B*e+2*C*d))*(c^(
1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*
x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)*d+a^(1/2)*e)^(1
/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),((c^(1/2)*d-a^
(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g)
)^(1/2))/e^2/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(1/2)/(c^(1/2)*f-
a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^
(1/2)/(-c*x^2+a)^(1/2)+2*C*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(-d*g+e*f)*(a^(1/2)
+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d)
)^(1/2)*EllipticPi((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^
(1/2)*g)^(1/2)/(e*x+d)^(1/2),e*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)
/g,((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(
1/2)*f-a^(1/2)*g))^(1/2))/e^2/(c^(1/2)*d+a^(1/2)*e)^(1/2)/g/(c^(1/2)*f-a^(
1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)*g)/(e*x+d))^...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3448 vs. $2(1060) = 2120$.

Time = 34.06 (sec) , antiderivative size = 3448, normalized size of antiderivative = 3.25

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
(-2*d*(C*d - B*e)*Sqrt[f + g*x]*Sqrt[a - c*x^2])/((-(c*d^2) + a*e^2)*(e*f
- d*g)*Sqrt[d + e*x]) - (2*(-((d*(C*d - B*e)*Sqrt[d + e*x]*(-c - (c*d^2)/(
d + e*x)^2 + (a*e^2)/(d + e*x)^2 + (2*c*d)/(d + e*x))*(g + (e*f)/(d + e*x)
 - (d*g)/(d + e*x))*Sqrt[a - (c*(d + e*x)^2*(-1 + d/(d + e*x))^2)/e^2])/((
(a*e^2)/(d + e*x)^2 - c*(-1 + d/(d + e*x))^2)*Sqrt[f + ((d + e*x)*(g - (d*
g)/(d + e*x)))/e])) + ((c*d^2 - a*e^2)*(-(e*f) + d*g)*Sqrt[(-c - (c*d^2)/(
d + e*x)^2 + (a*e^2)/(d + e*x)^2 + (2*c*d)/(d + e*x))*(g + (e*f)/(d + e*x)
 - (d*g)/(d + e*x))]*Sqrt[a - (c*(d + e*x)^2*(-1 + d/(d + e*x))^2)/e^2]*((
-2*C*d*Sqrt[(-((-(c*d) - Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + (d + e*x
)^(-1))/(-((-(c*d) - Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + (-(c*d) + Sq
rt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2))]*(-((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c
*d^2) + a*e^2)) + (d + e*x)^(-1))*Sqrt[(-(g/(-(e*f) + d*g)) + (d + e*x)^(-
1))/((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2) - g/(-(e*f) + d*g))]*
EllipticF[ArcSin[Sqrt[((-(e*f) + d*g)*(-Sqrt[c] + (Sqrt[c]*d)/(d + e*x) +
(Sqrt[a]*e)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]], ((Sqrt[c]*d - Sqrt[
a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/(2*Sqrt[a]*Sqrt[c]*(-(e*f) + d*g))])/(Sqrt[
(-((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + (d + e*x)^(-1))/(-((
-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + g/(-(e*f) + d*g))]*Sqrt[
(c + (c*d^2 - a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x))*(-g + (-(e*f) + d*g)
/(d + e*x))]) + (B*e*Sqrt[(-((-(c*d) - Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Bx+Cx^2}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x(B+Cx)}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2349}$$

$$\frac{d(Cd-Be)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{733}$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx+\frac{d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}$$

$$\Big\downarrow \; 732$$

$$\frac{d(Cd-Be)\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 744$$

$$\frac{d(Cd-Be)\left(\frac{2g(d+ex)\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 1416$$

$$\frac{d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(cd^2-ae^2\right)}{(d+ex)^2\left(cf^2-ag^2\right)}-\frac{2(\cdots}{(d\cdots}}{\left(\frac{(f+gx)\sqrt{cd^2-\cdots}}{(d+ex)\sqrt{cf^2-\cdots}}\right.}}\cdots}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f\cdots}{(d\cdots}}}\cdots\right)}{e^2}$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \; 2349$$

$$\frac{d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(cd^2-ae^2\right)}{(d+ex)^2\left(cf^2-ag^2\right)}-\frac{2(}{(d}}{\left(\frac{(f+gx)\sqrt{cd^2-}}{(d+ex)\sqrt{cf^2-}}\right.}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f}{(d}}}\right.}{e^2}$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 27

$$\frac{d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{\left(a-cx^2\right)(ef-dg)^2}{(d+ex)^2\left(cf^2-ag^2\right)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(cd^2-ae^2\right)}{(d+ex)^2\left(cf^2-ag^2\right)}-\frac{2(}{(d}}{\left(\frac{(f+gx)\sqrt{cd^2-}}{(d+ex)\sqrt{cf^2-}}\right.}}}{\sqrt{a-cx^2}\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{\frac{(f}{(d}}}\right.}{e^2}$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}$$

$\downarrow$ 726

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}eg}\sqrt{a-cx^2}}$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+$$

$$\frac{d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2\left(a-cx^2\right)}{\left(cf^2-ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{EllipticF}\left(\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)}{\left(cf^2-ag^2\right)}}}\right.}{e^2}$$

$\downarrow$ 732

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\cdots}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}\cdots}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}\,g\sqrt{a-cx^2}}$$

$$\frac{d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{EllipticF}\left(\cdots\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)\cdots}{(cf^2-ag^2)\cdots}\cdots}}\right.}{e^2\cdots}$$

$$\frac{2(2Cd-Be)(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{a-cx^2}}$$

$$\Big\downarrow\ 1416$$

$$\frac{(2Cd-Be)\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{El}\cdots}{e^2\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf\cdots}{(cf^2\cdots}\cdots}}$$

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}\cdots}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}\cdots}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}\,g\sqrt{a-cx^2}}$$

$$\frac{d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{EllipticF}\left(\cdots\right.}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)\cdots}{(cf^2-ag^2)\cdots}\cdots}}\right.}{e^2\cdots}$$

input
```
Int[(B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2207 vs. 2(825) = 1650.

Time = 7.53 (sec) , antiderivative size = 2208, normalized size of antiderivative = 2.08

method	result	size
elliptic	Expression too large to display	2208
default	Expression too large to display	29044

input
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(-2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+
c*d^2*e*f)/e*d*(B*e-C*d)/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f))^(1
/2)+2*((B*e-C*d)/e^2-d*(B*e-C*d)*(a*e^2*g-c*d^2*g+c*d*e*f)/e^2/(a*e^2-c*d^
2)/(d*g-e*f)+a*g/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*d*(B*e-C*d))*(1/c*(
a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+
f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e
)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x
+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-
1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*EllipticF(((-1/c*(a*c)^(1/2)+f
/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*
(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(a*c)^(1/2)-f/g))^(1/2))
+2*(C/e-1/e*c*d*(B*e-C*d)/(a*e^2-c*d^2)-2*c*f/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)*d*(B*e-C*d))*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/
e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c
)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/
2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e
)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*(
-f/g*EllipticF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f
/g))^(1/2),((-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algo
rithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \int \frac{x(B + Cx)}{\sqrt{a - cx^2}\,(d + ex)^{\frac{3}{2}}\,\sqrt{f + gx}}\,dx$$

input
```
integrate((C*x**2+B*x)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),x)
```

output
```
Integral(x*(B + C*x)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

Maxima [F]

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \int \frac{Cx^2 + Bx}{\sqrt{-cx^2 + a}(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algo
rithm="maxima")
```

output
```
integrate((C*x^2 + B*x)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),
x)
```

Giac [F]

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\,dx = \int \frac{Cx^2 + Bx}{\sqrt{-cx^2 + a}(ex + d)^{\frac{3}{2}}\sqrt{gx + f}}\,dx$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate((C*x^2 + B*x)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),
x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\, dx = \int \frac{C\, x^2 + B\, x}{\sqrt{f + g\, x}\, \sqrt{a - c\, x^2}\, {(d + e\, x)}^{3/2}}\, dx$$

input
```
int((B*x + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int((B*x + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a - cx^2}}\, dx = \int \frac{C\, x^2 + Bx}{(ex + d)^{\frac{3}{2}}\, \sqrt{gx + f}\, \sqrt{-c\, x^2 + a}} dx$$

input
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

3.50 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx$

Optimal result 515
Mathematica [B] (warning: unable to verify) 516
Rubi [F] 517
Maple [B] (warning: unable to verify) 522
Fricas [F(-1)] 523
Sympy [F] 524
Maxima [F] 524
Giac [F] 524
Mupad [F(-1)] 525
Reduce [F] 525

Optimal result

Integrand size = 41, antiderivative size = 1075

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Too large to display}$$

output
```
2*(A*e^2-B*d*e+C*d^2)*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-
d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*Elliptic
E((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e
*x+d)^(1/2),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2
)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/e/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1
/2)*e)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1/2)
*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)-2*(a^(1/2)*e*(-B*e+2*C*d)-c^(1/2)*(-A*
e^2+C*d^2))*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(
a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^(1/2)*g)/(e*x+d))^(1/2)*EllipticF((c^(1/2)
*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2
),((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d+a^(1/2)*e)/(c^(1
/2)*f-a^(1/2)*g))^(1/2))/e^2/(c^(1/2)*d-a^(1/2)*e)/(c^(1/2)*d+a^(1/2)*e)^(
1/2)/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1
/2)*g)/(e*x+d))^(1/2)/(-c*x^2+a)^(1/2)+2*C*(c^(1/2)*f+a^(1/2)*g)^(1/2)*(-d
*g+e*f)*(a^(1/2)+c^(1/2)*x)*(-(-d*g+e*f)*(a^(1/2)-c^(1/2)*x)/(c^(1/2)*f+a^
(1/2)*g)/(e*x+d))^(1/2)*EllipticPi((c^(1/2)*d+a^(1/2)*e)^(1/2)*(g*x+f)^(1/
2)/(c^(1/2)*f+a^(1/2)*g)^(1/2)/(e*x+d)^(1/2),e*(c^(1/2)*f+a^(1/2)*g)/(c^(1
/2)*d+a^(1/2)*e)/g,((c^(1/2)*d-a^(1/2)*e)*(c^(1/2)*f+a^(1/2)*g)/(c^(1/2)*d
+a^(1/2)*e)/(c^(1/2)*f-a^(1/2)*g))^(1/2))/e^2/(c^(1/2)*d+a^(1/2)*e)^(1/2)/
g/(c^(1/2)*f-a^(1/2)*g)/((-d*g+e*f)*(a^(1/2)+c^(1/2)*x)/(c^(1/2)*f-a^(1...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4178 vs. $2(1075) = 2150$.

Time = 34.30 (sec) , antiderivative size = 4178, normalized size of antiderivative = 3.89

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]
),x]
```

output
```
(-2*(C*d^2 - B*d*e + A*e^2)*Sqrt[f + g*x]*Sqrt[a - c*x^2])/((-(c*d^2) + a*
e^2)*(e*f - d*g)*Sqrt[d + e*x]) - (2*(((-(C*d^2) + B*d*e - A*e^2)*Sqrt[d +
 e*x]*(-c - (c*d^2)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 + (2*c*d)/(d + e*x))
*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x))*Sqrt[a - (c*(d + e*x)^2*(-1 + d/(
d + e*x))^2)/e^2])/(((a*e^2)/(d + e*x)^2 - c*(-1 + d/(d + e*x))^2)*Sqrt[f
+ ((d + e*x)*(g - (d*g)/(d + e*x)))/e]) + ((c*d^2 - a*e^2)*(-(e*f) + d*g)*
Sqrt[(-c - (c*d^2)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 + (2*c*d)/(d + e*x))*
(g + (e*f)/(d + e*x) - (d*g)/(d + e*x))]*Sqrt[a - (c*(d + e*x)^2*(-1 + d/(
d + e*x))^2)/e^2]*((-2*C*d*Sqrt[(-((-(c*d) - Sqrt[a]*Sqrt[c]*e)/(-(c*d^2)
+ a*e^2)) + (d + e*x)^(-1))/(-((-(c*d) - Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*
e^2)) + (-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2))]*(-((-(c*d) + Sqr
t[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + (d + e*x)^(-1))*Sqrt[(-(g/(-(e*f) +
d*g)) + (d + e*x)^(-1))/((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2) -
 g/(-(e*f) + d*g))]*EllipticF[ArcSin[Sqrt[((-(e*f) + d*g)*(-Sqrt[c] + (Sqr
t[c]*d)/(d + e*x) + (Sqrt[a]*e)/(d + e*x)))/(e*(Sqrt[c]*f + Sqrt[a]*g))]],
 ((Sqrt[c]*d - Sqrt[a]*e)*(Sqrt[c]*f + Sqrt[a]*g))/(2*Sqrt[a]*Sqrt[c]*(-(e
*f) + d*g))])/(Sqrt[(-((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) +
(d + e*x)^(-1))/(-((-(c*d) + Sqrt[a]*Sqrt[c]*e)/(-(c*d^2) + a*e^2)) + g/(-
(e*f) + d*g))]*Sqrt[(c + (c*d^2 - a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x))*
(-g + (-(e*f) + d*g)/(d + e*x))]) + (B*e*Sqrt[(-((-(c*d) - Sqrt[a]*Sqrt...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a-cx^2}(d+ex)^{3/2}\sqrt{f+gx}}dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{733}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{ef-dg}\right)+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{732}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f}}{(d+ex)^{3/}}}{ef-}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{744}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-ae^2\right)(f+gx)^2}{\left(cf^2-ag^2\right)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{\left(cf^2-ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a-cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f}}{(d+ex)^{3/}}}{ef-}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{1416}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)}{\sqrt{a-cx^2}\sqrt[4]{cd^2}}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)}{\sqrt{a-cx^2}\sqrt[4]{cd^2}}\right.$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\int\frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{a-cx^2}}dx$$

$\downarrow$ 27

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{a-cx^2}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-ag^2}\sqrt{-\frac{(a-cx^2)(ef-dg)^2}{(d+ex)^2(cf^2-ag^2)}}\left(\frac{(f+gx)\sqrt{cd^2-ae^2}}{(d+ex)\sqrt{cf^2-ag^2}}+1\right)}{\sqrt{a-cx^2}\sqrt[4]{cd^2}}\right.$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+\frac{C\int\frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}$$

$\downarrow$ 726

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\operatorname{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}g\sqrt{a-cx^2}}$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a-cx^2}}dx}{e^2}+$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-ae}{(cf^2-ag}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+\right.}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-}{(cf^2-}}}\right.$$

$\downarrow$ 732

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e\cdots}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g\cdots}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}g\sqrt{a-cx^2}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-ae\cdots}{(cf^2-ag\cdots}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+\cdots\right.}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-\cdots}{(cf^2-\cdots}}}\right.$$

$$\frac{2(2Cd-Be)(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{a-cx^2}}$$

$\big\downarrow$ 1416

$$\frac{(2Cd-Be)\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-aeg)(f+gx)}{(cf^2-ag^2)(d+ex)}+1}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)^2}}\,\text{El}\cdots}{e^2\sqrt[4]{cd^2-ae^2}(ef-dg)\sqrt{a-cx^2}\sqrt{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf\cdots}{(cf^2\cdots}}}$$

$$\frac{2C\sqrt{-cf-\sqrt{a}\sqrt{c}g}\sqrt{-\frac{(ef-dg)(\sqrt{a}-\sqrt{c}x)}{(\sqrt{c}f+\sqrt{a}g)(d+ex)}}\sqrt{\frac{(ef-dg)(\sqrt{c}x+\sqrt{a})}{(\sqrt{c}f-\sqrt{a}g)(d+ex)}}(d+ex)\,\text{EllipticPi}\left(\frac{e(\sqrt{c}f+\sqrt{a}g)}{(\sqrt{c}d+\sqrt{a}e)g},\arcsin\left(\frac{\sqrt{-cd-\sqrt{a}\sqrt{c}e\cdots}}{\sqrt{-cf-\sqrt{a}\sqrt{c}g\cdots}}\right.\right.}{e^2\sqrt{-cd-\sqrt{a}\sqrt{c}e}g\sqrt{a-cx^2}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-ag^2}(d+ex)\sqrt{-\frac{(ef-dg)^2(a-cx^2)}{(cf^2-ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-ae^2)(f+gx)^2}{(cf^2-ag^2)(d+ex)^2}-\frac{2(cdf-ae\cdots}{(cf^2-ag\cdots}}{\left(\frac{\sqrt{cd^2-ae^2}(f+gx)}{\sqrt{cf^2-ag^2}(d+ex)}+\cdots\right.}}}{\sqrt[4]{cd^2-ae^2}(ef-dg)^2\sqrt{a-cx^2}\sqrt{\frac{(cd^2-\cdots}{(cf^2-\cdots}}}\right.$$

input

```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a - c*x^2]),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 732
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(a_) + (b_.)*
(x_)^2]), x_Symbol] :> Simp[-2*(c + d*x)*(Sqrt[(d*e - c*f)^2*((a + b*x^2)/(
(b*e^2 + a*f^2)*(c + d*x)^2))]/((d*e - c*f)*Sqrt[a + b*x^2]))   Subst[Int[1
/Sqrt[Simp[1 - (2*b*c*e + 2*a*d*f)*(x^2/(b*e^2 + a*f^2)) + (b*c^2 + a*d^2)*
(x^4/(b*e^2 + a*f^2)), x]], x], x, Sqrt[e + f*x]/Sqrt[c + d*x]], x] /; Free
Q[{a, b, c, d, e, f}, x]
```

rule 733
```
Int[1/(Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(3/2)*Sqrt[(a_) + (b_.
)*(x_)^2]), x_Symbol] :> Simp[d/(d*e - c*f)   Int[1/(Sqrt[c + d*x]*Sqrt[e +
 f*x]*Sqrt[a + b*x^2]), x], x] - Simp[f/(d*e - c*f)   Int[Sqrt[c + d*x]/((e
 + f*x)^(3/2)*Sqrt[a + b*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2349

```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2256 vs. $2(840) = 1680$.

Time = 7.58 (sec) , antiderivative size = 2257, normalized size of antiderivative = 2.10

method	result	size
elliptic	Expression too large to display	2257
default	Expression too large to display	36658

input

```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x,method=_R
ETURNVERBOSE)
```

output
```
((e*x+d)*(g*x+f)*(-c*x^2+a))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(-c*x^2+a)^
(1/2)*(2*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e*f)/(a*d*e^2*g-a*e^3*f-c*d^3*g+c
*d^2*e*f)/e*(A*e^2-B*d*e+C*d^2)/((x+d/e)*(-c*e*g*x^3-c*e*f*x^2+a*e*g*x+a*e
*f))^(1/2)+2*((B*e-C*d)/e^2+(A*e^2-B*d*e+C*d^2)*(a*e^2*g-c*d^2*g+c*d*e*f)/
e^2/(a*e^2-c*d^2)/(d*g-e*f)-a*g/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*(A*e
^2-B*d*e+C*d^2))*(1/c*(a*c)^(1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1
/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2
))/(1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-
1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*
e*g*(x+d/e)*(x+f/g)*(x-1/c*(a*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*Ellipti
cF(((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),(
(-f/g-1/c*(a*c)^(1/2))*(1/c*(a*c)^(1/2)-d/e)/(-1/c*(a*c)^(1/2)-d/e)/(1/c*(
a*c)^(1/2)-f/g))^(1/2))+2*(C/e+1/e*c*(A*e^2-B*d*e+C*d^2)/(a*e^2-c*d^2)+2*c
*f/(a*d*e^2*g-a*e^3*f-c*d^3*g+c*d^2*e*f)*(A*e^2-B*d*e+C*d^2))*(1/c*(a*c)^(
1/2)-d/e)*((-1/c*(a*c)^(1/2)+f/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^
(1/2)*(x+f/g)^2*((-f/g+d/e)*(x-1/c*(a*c)^(1/2))/(1/c*(a*c)^(1/2)+d/e)/(x+f
/g))^(1/2)*((-f/g+d/e)*(x+1/c*(a*c)^(1/2))/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))
^(1/2)/(-1/c*(a*c)^(1/2)+f/g)/(-f/g+d/e)/(-c*e*g*(x+d/e)*(x+f/g)*(x-1/c*(a
*c)^(1/2))*(x+1/c*(a*c)^(1/2)))^(1/2)*(-f/g*EllipticF(((-1/c*(a*c)^(1/2)+f
/g)*(x+d/e)/(-1/c*(a*c)^(1/2)+d/e)/(x+f/g))^(1/2),((-f/g-1/c*(a*c)^(1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, al
gorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{a-cx^2}\,(d+ex)^{\frac{3}{2}}\,\sqrt{f+gx}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(-c*x**2+a)**(1/2),
x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(a - c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*
x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, al
gorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f
)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{-cx^2+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x, al
gorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(-c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f
)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{f+g\,x}\,\sqrt{a-c\,x^2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),
x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(a - c*x^2)^(1/2)*(d + e*x)^(3/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a-cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{-c\,x^2+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(-c*x^2+a)^(1/2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 526
4.2 Links to plain text integration problems used in this report for each CAS . 544

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
      return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
    elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file