Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/1.2.1.8/112-1.2.1.8-c

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**75**]. This is test number [112].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (75)	0.00 (0)
Maple	100.00 (75)	0.00 (0)
Rubi	68.00 (51)	32.00 (24)
Reduce	37.33 (28)	62.67 (47)
Fricas	26.67 (20)	73.33 (55)
Giac	25.33 (19)	74.67 (56)
Mupad	14.67 (11)	85.33 (64)
Maxima	10.67 (8)	89.33 (67)
Sympy	10.67 (8)	89.33 (67)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

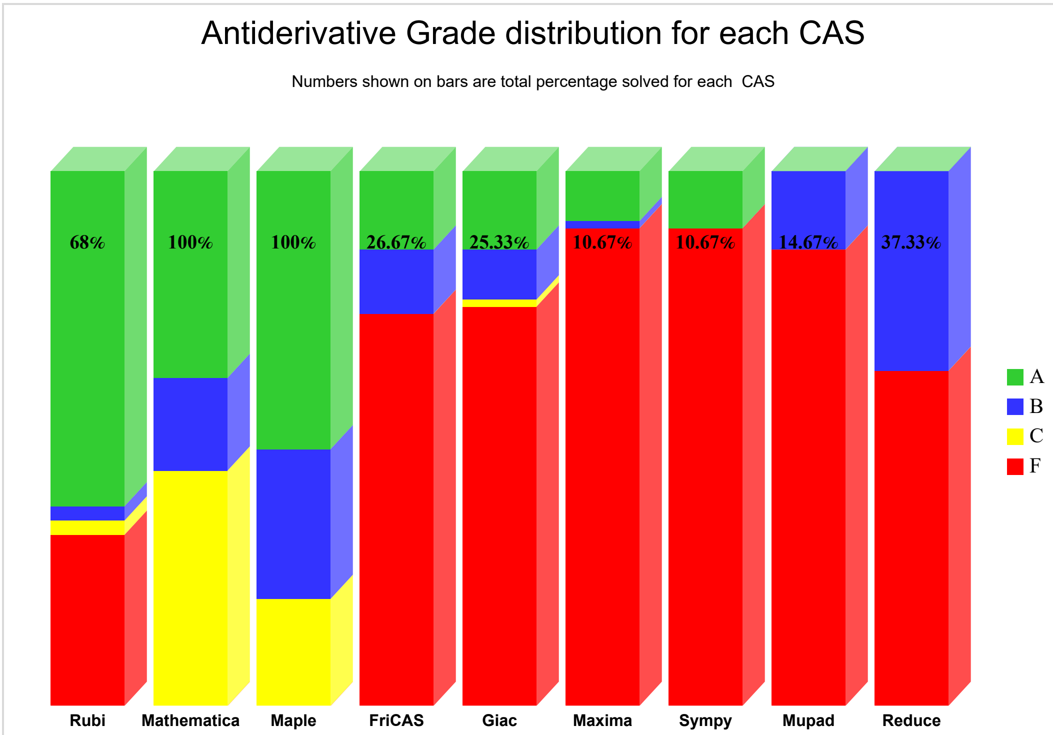
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

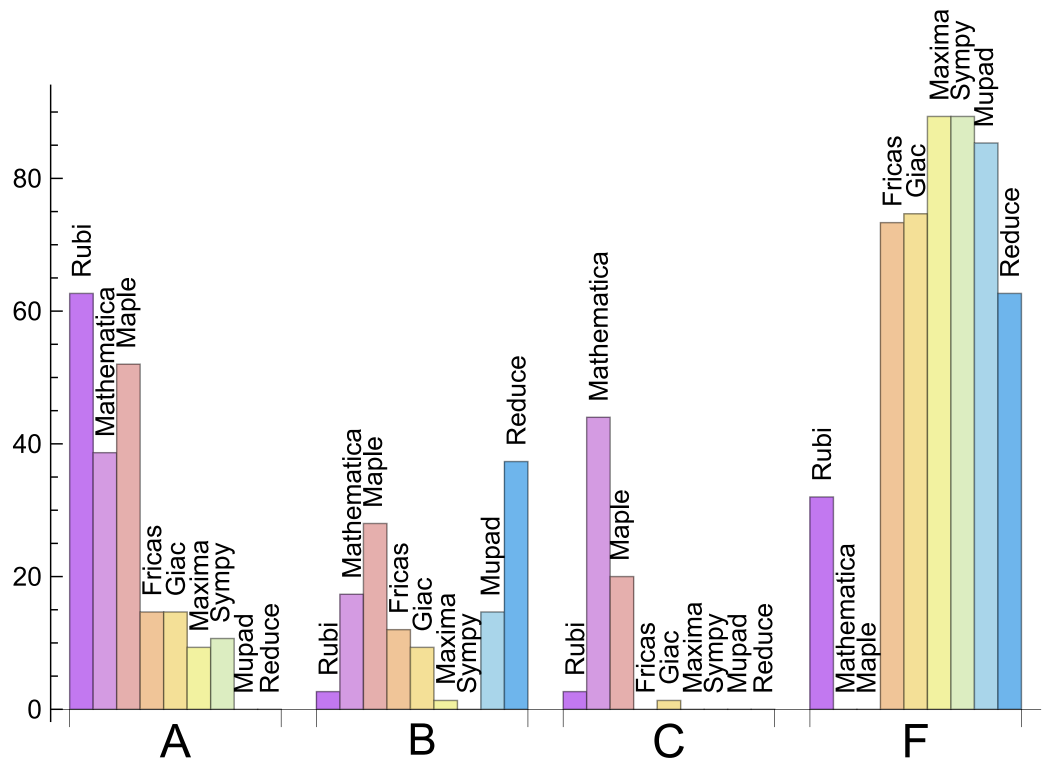
System	% A grade	% B grade	% C grade	% F grade
Rubi	62.667	2.667	2.667	32.000
Maple	52.000	28.000	20.000	0.000
Mathematica	38.667	17.333	44.000	0.000
Fricas	14.667	12.000	0.000	73.333
Giac	14.667	9.333	1.333	74.667
Sympy	10.667	0.000	0.000	89.333
Maxima	9.333	1.333	0.000	89.333
Mupad	0.000	14.667	0.000	85.333
Reduce	0.000	37.333	0.000	62.667

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Rubi	24	100.00	0.00	0.00
Reduce	47	100.00	0.00	0.00
Fricas	55	38.18	61.82	0.00
Giac	56	80.36	3.57	16.07
Mupad	64	0.00	100.00	0.00
Maxima	67	77.61	0.00	22.39
Sympy	67	83.58	16.42	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Giac	0.22
Reduce	0.51
Rubi	2.04
Fricas	14.16
Maple	15.68
Mathematica	18.65
Mupad	31.82
Sympy	85.90

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	377.75	1.08	310.00	1.13
Maxima	381.00	1.46	229.50	1.42
Rubi	527.92	1.21	441.00	1.06
Fricas	835.75	3.06	697.00	1.74
Giac	983.84	3.28	298.00	1.36
Maple	3278.01	31.28	563.00	1.51
Mathematica	6282.97	101.39	622.00	1.33
Reduce	6528.32	18.53	1897.00	5.35
Mupad	31627.82	97.28	359.00	1.47

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

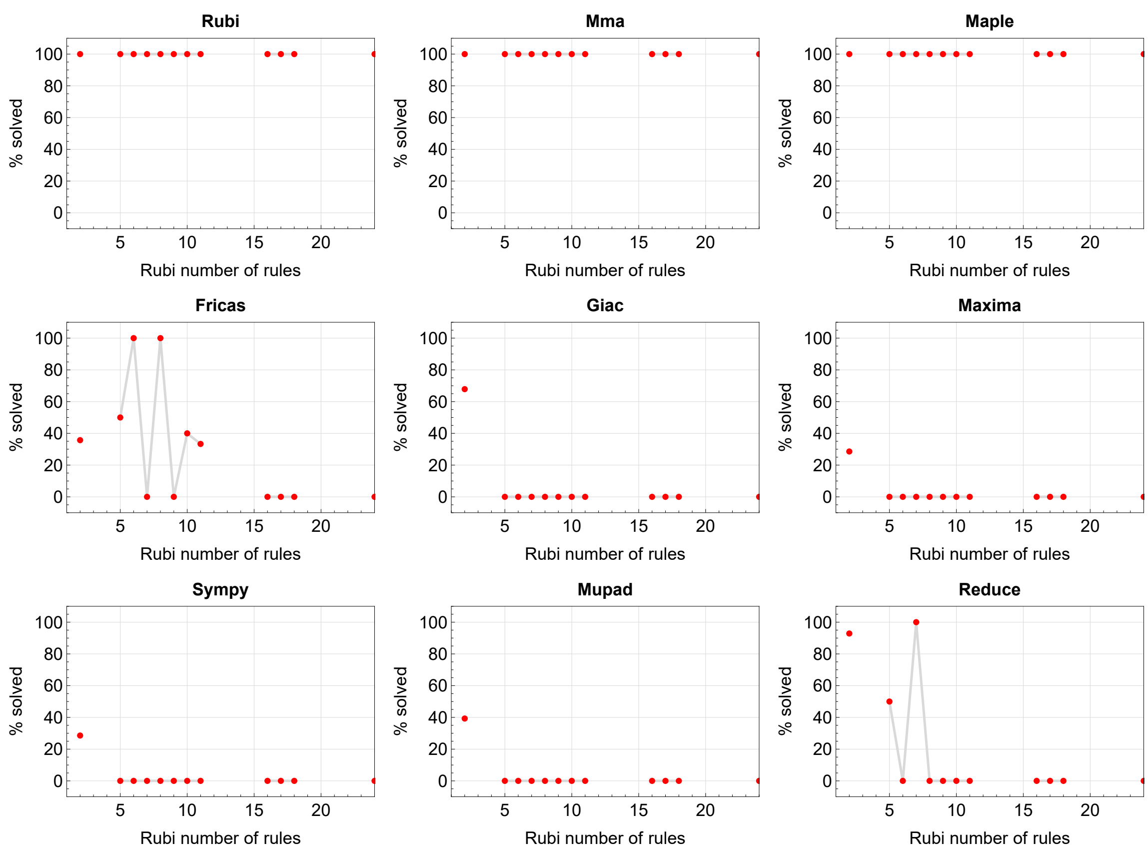


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

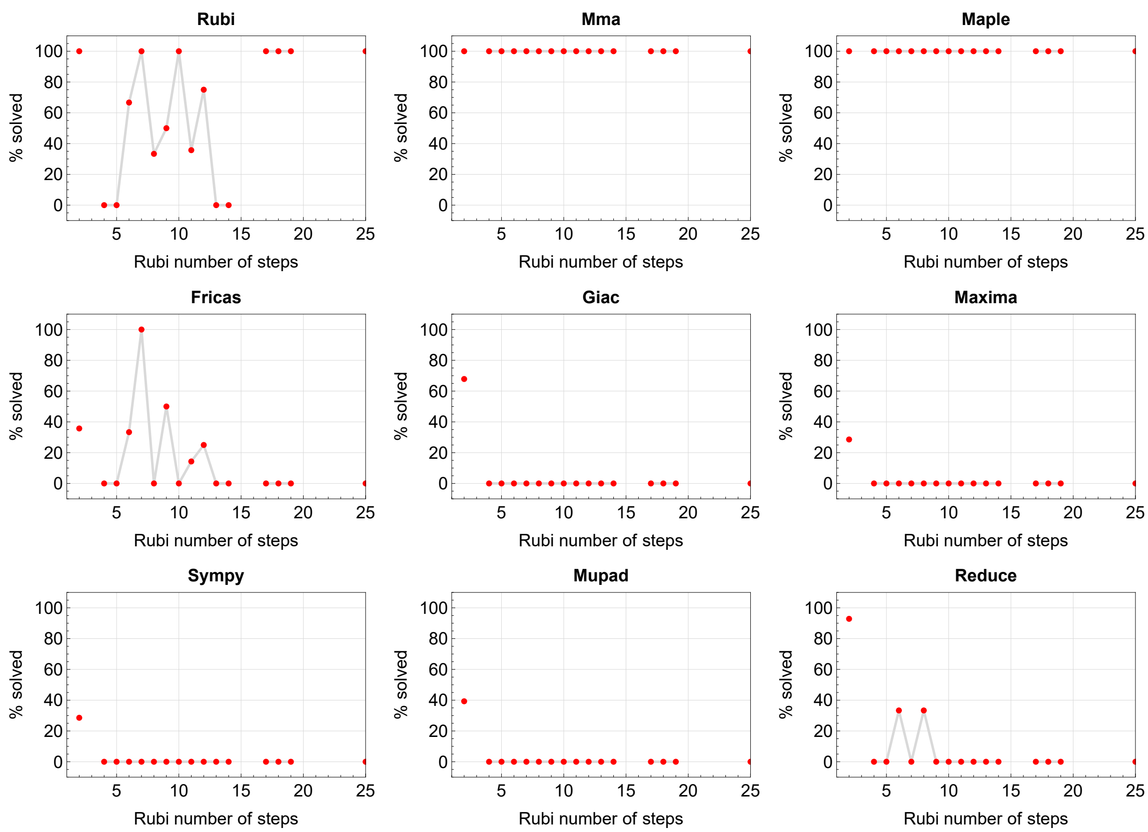


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

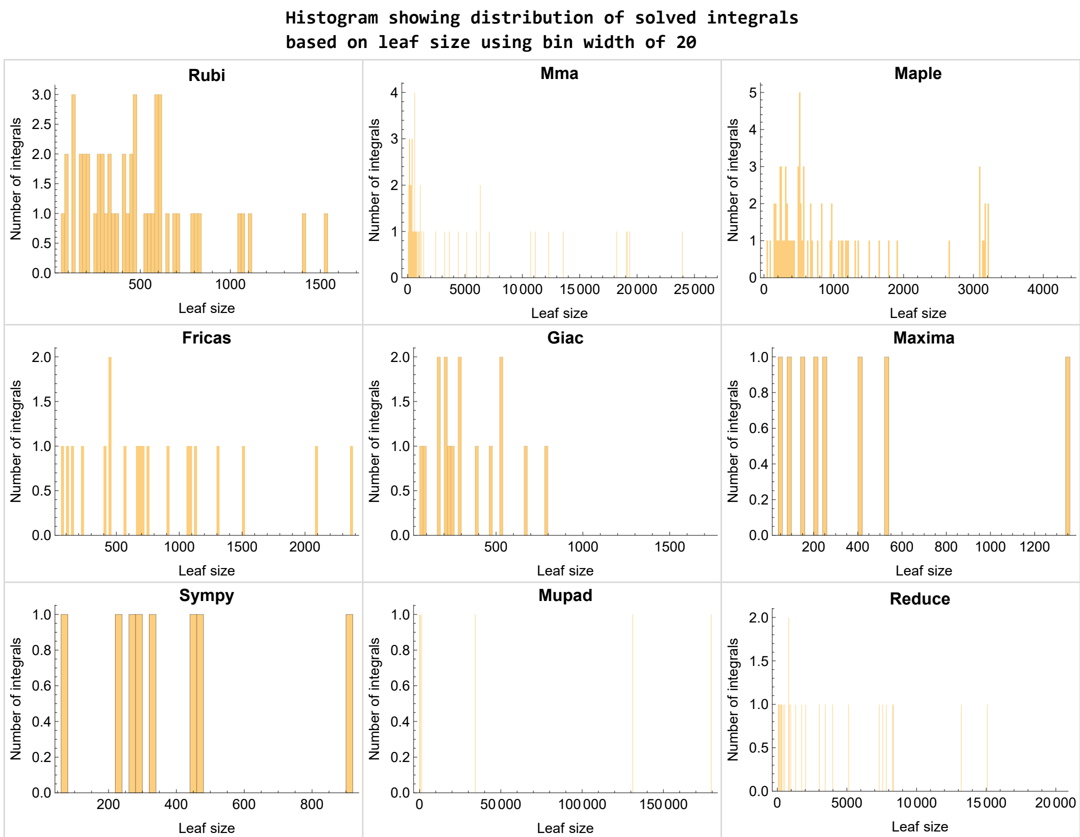


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

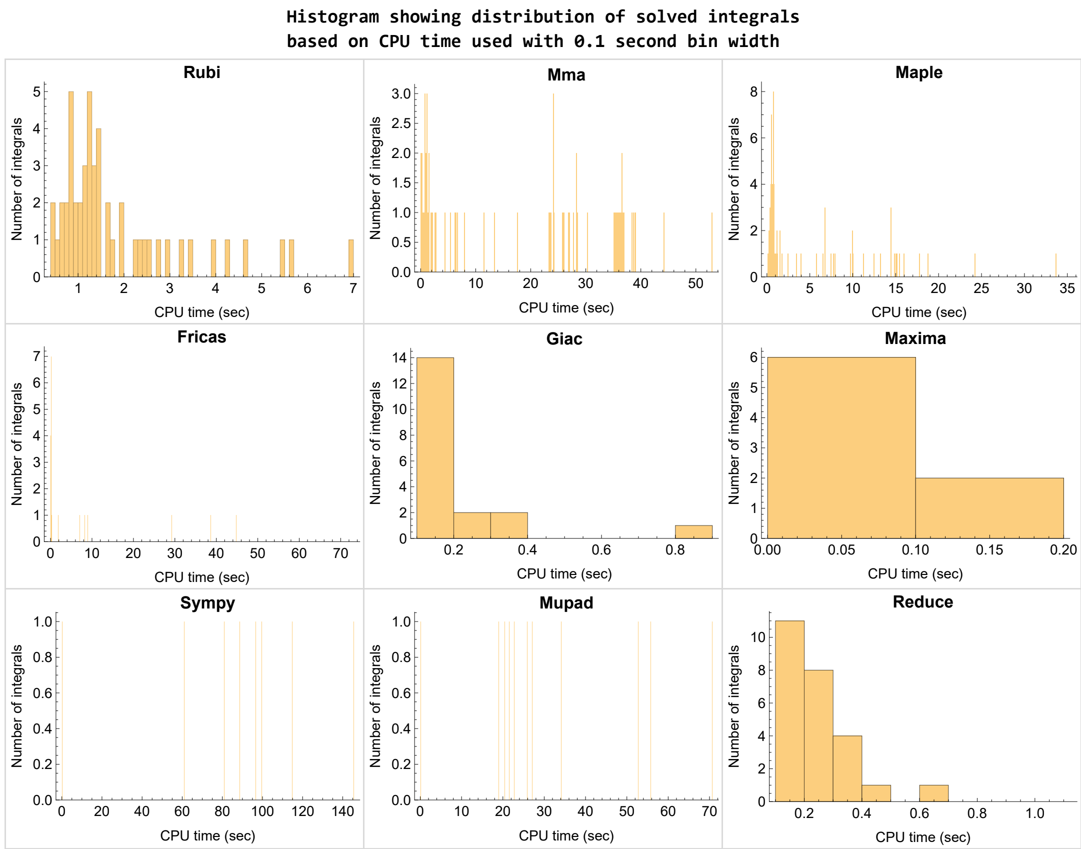


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

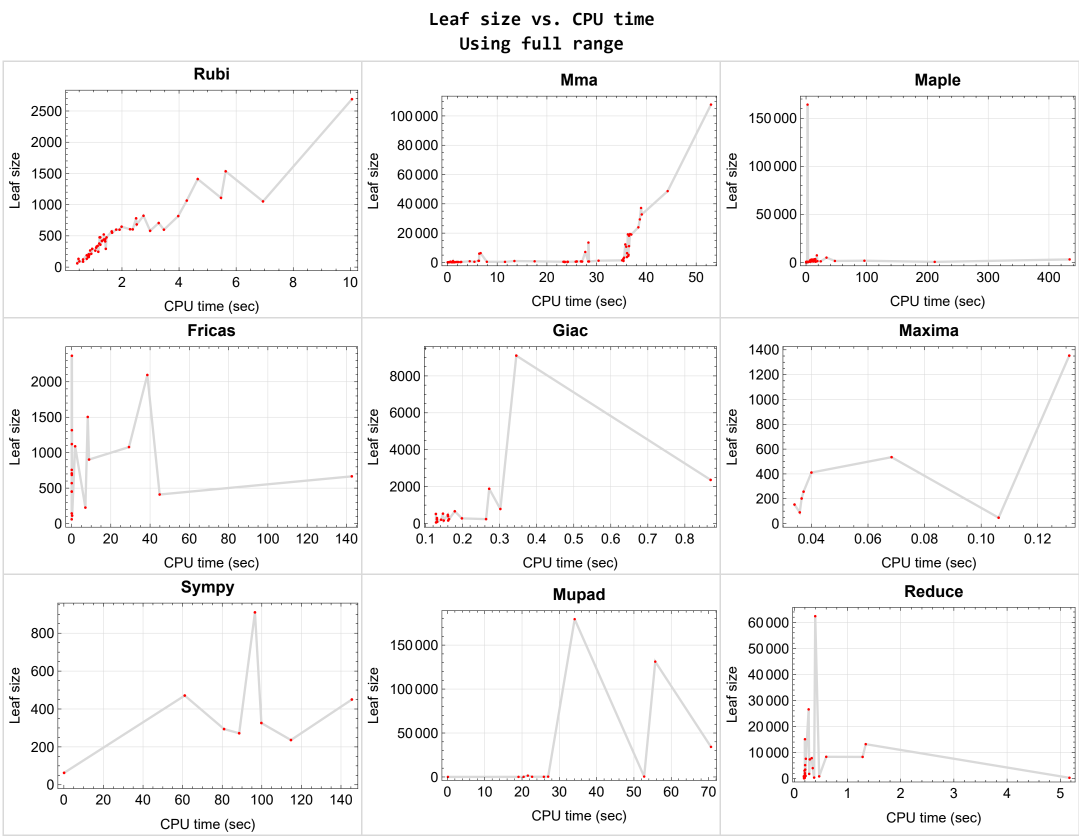


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{7, 10, 23, 24, 25, 49, 52, 53, 60, 61, 64, 65\}$

Mathematica $\{1, 2, 3, 4, 5, 6, 7, 11, 50, 51, 52, 53, 54, 55, 56, 57, 61, 69, 70, 71, 72, 73, 74, 75\}$

Maple $\{1, 2, 3, 4, 5, 6, 69, 70, 71, 72, 73, 74, 75\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

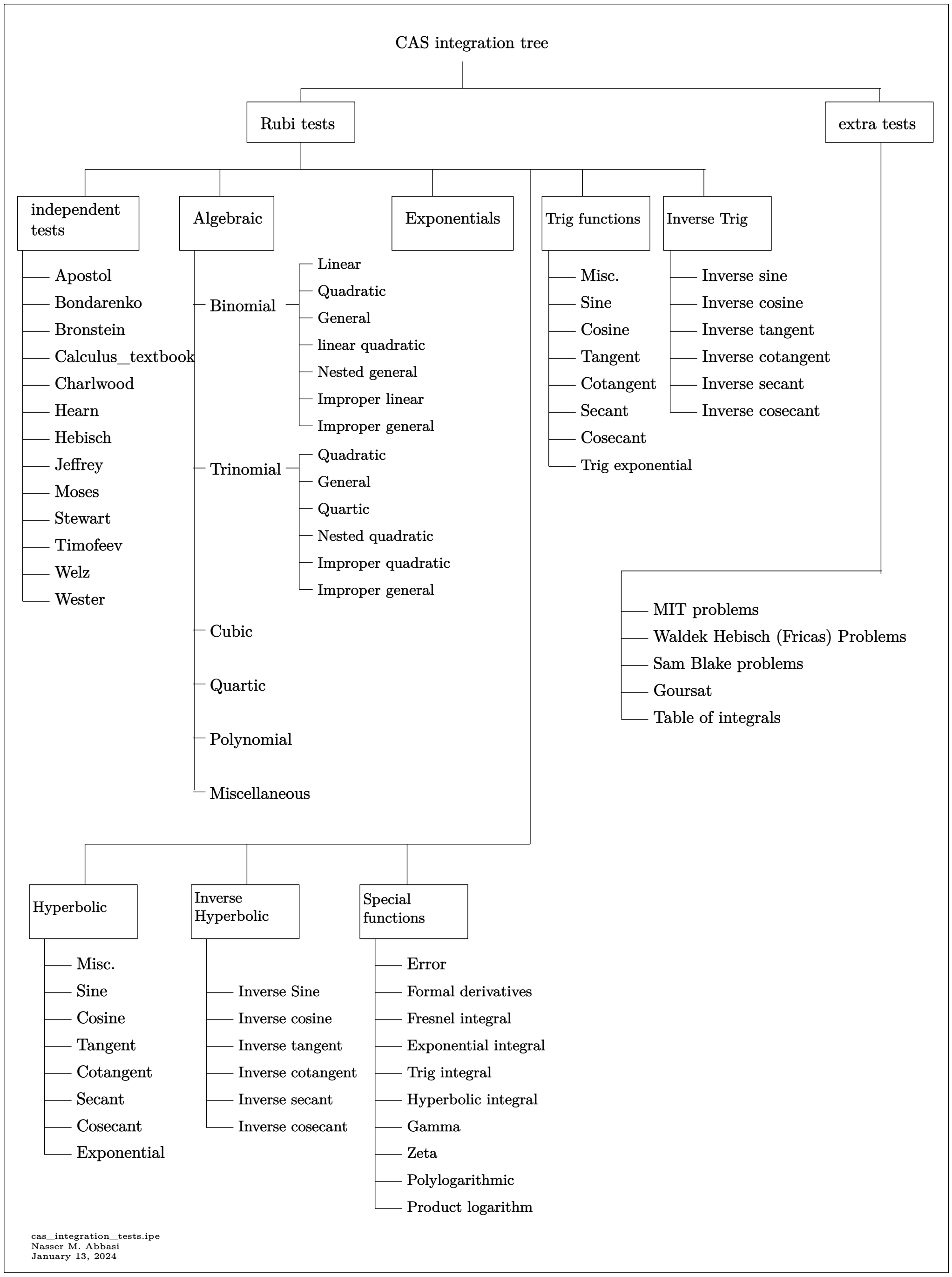
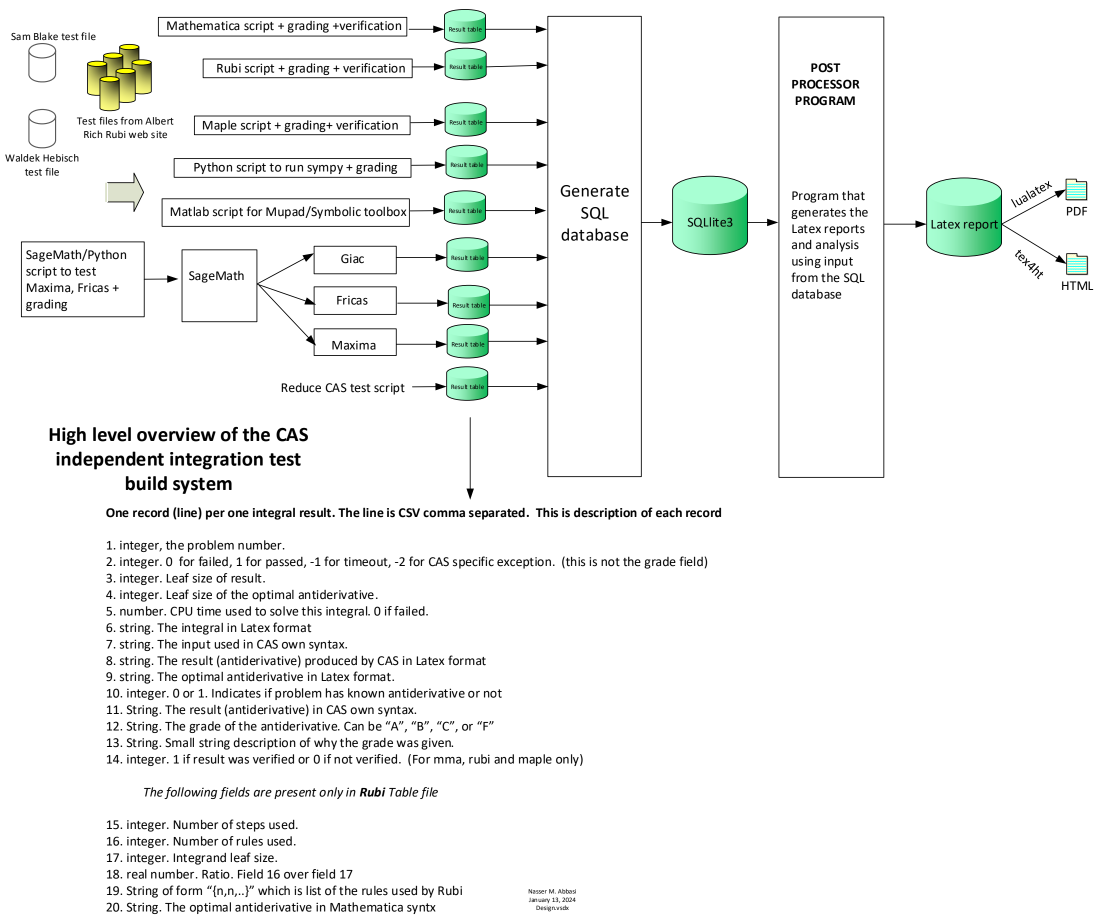


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 57, 58, 59, 60, 61 }

B grade { 24, 25 }

C grade { 64, 65 }

F normal fail { 1, 2, 3, 4, 5, 6, 11, 50, 51, 54, 55, 56, 62, 63, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 12, 13, 14, 15, 16, 17, 18, 19, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48 }

B grade { 2, 3, 4, 5, 6, 28, 69, 70, 71, 72, 73, 74, 75 }

C grade { 1, 7, 8, 9, 10, 11, 20, 21, 22, 23, 24, 25, 26, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 57, 58, 60, 61 }

B grade { 2, 3, 4, 5, 6, 8, 9, 10, 22, 23, 28, 42, 43, 59, 69, 70, 71, 72, 73, 74, 75 }

C grade { 1, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 9, 12, 20, 21, 22, 27, 30, 31, 57, 58, 59 }

B grade { 7, 8, 16, 17, 18, 19, 28, 29, 42 }

C grade { }

F normal fail { 4, 5, 6, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 75 }

F(-1) timedout fail { 1, 2, 3, 10, 11, 13, 14, 15, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 60, 61, 69, 73, 74 }

F(-2) exception fail { }

Maxima

A grade { 27, 28, 29, 30, 31, 32, 33 }

B grade { 34 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 42, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { 12, 13, 14, 15, 16, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46 }

Giac

A grade { 18, 27, 29, 30, 31, 32, 35, 36, 37, 38, 39 }

B grade { 12, 19, 28, 33, 34, 40, 41 }

C grade { 42 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 23, 24, 25, 26, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-1) timedout fail { 43, 48 }

F(-2) exception fail { 13, 14, 15, 16, 17, 44, 45, 46, 47 }

Mupad

A grade { }

B grade { 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 42 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-2) exception fail { }

Sympy

A grade { 27, 35, 36, 37, 38, 39, 40, 41 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-1) timedout fail { 11, 28, 29, 30, 31, 32, 33, 34, 50, 56, 62 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 12, 14, 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 20, 21, 22, 23, 24, 25, 26, 42, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1	0	7130	1903	0	0	0	0	31	0
N.S.	1	0.00	7130.00	1903.00	0.00	0.00	0.00	0.00	31.00	0.00
time (sec)	N/A	0.000	27.713	6.734	0.000	0.000	0.000	0.000	200.033	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1474	0	13559	2659	0	0	0	0	34	0
N.S.	1	0.00	9.20	1.80	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	28.346	7.411	0.000	0.000	0.000	0.000	200.027	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1585	0	19147	3091	0	0	0	0	39	0
N.S.	1	0.00	12.08	1.95	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	36.276	9.785	0.000	0.000	0.000	0.000	200.032	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1396	0	6336	3212	0	0	0	0	39	0
N.S.	1	0.00	4.54	2.30	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	36.113	13.263	0.000	0.000	0.000	0.000	200.031	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1265	0	29355	7328	0	0	0	0	39	0
N.S.	1	0.00	23.21	5.79	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	38.678	17.791	0.000	0.000	0.000	0.000	200.038	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1780	0	107718	4998	0	0	0	0	39	0
N.S.	1	0.00	60.52	2.81	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	52.980	33.690	0.000	0.000	0.000	0.000	200.029	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	995	1052	23948	1643	0	2367	0	0	39	0
N.S.	1	1.06	24.07	1.65	0.00	2.38	0.00	0.00	0.04	0.00
time (sec)	N/A	6.941	38.384	11.271	0.000	0.174	0.000	0.000	200.029	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	646	706	11109	1192	0	1317	0	0	37	0
N.S.	1	1.09	17.20	1.85	0.00	2.04	0.00	0.00	0.06	0.00
time (sec)	N/A	3.287	36.508	10.046	0.000	0.160	0.000	0.000	200.025	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	522	772	965	0	708	0	0	32	0
N.S.	1	1.07	1.59	1.99	0.00	1.46	0.00	0.00	0.07	0.00
time (sec)	N/A	1.363	4.487	5.760	0.000	0.108	0.000	0.000	200.069	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	748	1065	980	1353	0	0	0	0	39	0
N.S.	1	1.42	1.31	1.81	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	4.271	35.401	7.762	0.000	0.000	0.000	0.000	200.031	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	A	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1097	0	48666	1780	0	0	0	0	39	0
N.S.	1	0.00	44.36	1.62	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	44.267	96.130	0.000	0.000	0.000	0.000	200.048	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	266	187	161	0	903	0	471	235	0
N.S.	1	1.45	1.02	0.88	0.00	4.91	0.00	2.56	1.28	0.00
time (sec)	N/A	0.887	1.171	0.536	0.000	8.928	0.000	0.160	0.198	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	475	321	425	0	0	0	0	30	0
N.S.	1	1.31	0.89	1.17	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.460	2.784	0.729	0.000	0.000	0.000	0.000	200.031	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	329	231	305	0	0	0	0	8291	0
N.S.	1	1.32	0.93	1.22	0.00	0.00	0.00	0.00	33.30	0.00
time (sec)	N/A	1.115	1.540	0.631	0.000	0.000	0.000	0.000	1.285	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	206	161	245	0	0	0	0	7805	0
N.S.	1	1.20	0.94	1.43	0.00	0.00	0.00	0.00	45.64	0.00
time (sec)	N/A	0.838	1.426	0.539	0.000	0.000	0.000	0.000	0.328	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	142	199	0	1079	0	0	7325	0
N.S.	1	1.00	1.08	1.51	0.00	8.17	0.00	0.00	55.49	0.00
time (sec)	N/A	0.477	0.791	0.464	0.000	29.260	0.000	0.000	0.292	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	142	206	0	1090	0	0	336	0
N.S.	1	1.00	1.07	1.55	0.00	8.20	0.00	0.00	2.53	0.00
time (sec)	N/A	0.767	0.753	0.534	0.000	1.838	0.000	0.000	0.374	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	213	213	252	0	1504	0	208	535	0
N.S.	1	1.20	1.20	1.42	0.00	8.50	0.00	1.18	3.02	0.00
time (sec)	N/A	0.925	1.500	0.626	0.000	8.253	0.000	0.142	0.184	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	339	265	313	0	2096	0	536	816	0
N.S.	1	1.37	1.07	1.26	0.00	8.45	0.00	2.16	3.29	0.00
time (sec)	N/A	1.149	1.981	0.729	0.000	38.604	0.000	0.147	0.468	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	775	817	19047	1172	0	757	0	0	35	0
N.S.	1	1.05	24.58	1.51	0.00	0.98	0.00	0.00	0.05	0.00
time (sec)	N/A	3.973	36.968	9.945	0.000	0.102	0.000	0.000	200.016	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	606	12291	950	0	570	0	0	33	0
N.S.	1	1.06	21.45	1.66	0.00	0.99	0.00	0.00	0.06	0.00
time (sec)	N/A	2.283	35.756	7.944	0.000	0.096	0.000	0.000	200.031	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	476	1080	823	0	451	0	0	32	0
N.S.	1	1.06	2.40	1.83	0.00	1.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.241	6.246	6.786	0.000	0.085	0.000	0.000	200.035	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	781	1189	1064	0	0	0	0	35	0
N.S.	1	1.31	1.99	1.79	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.496	30.385	6.530	0.000	0.000	0.000	0.000	200.030	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	1410	1405	1122	0	0	0	0	35	0
N.S.	1	2.17	2.16	1.73	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	4.654	35.120	9.997	0.000	0.000	0.000	0.000	200.035	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	769	2689	10711	1205	0	0	0	0	35	0
N.S.	1	3.50	13.93	1.57	0.00	0.00	0.00	0.00	0.05	0.00
time (sec)	N/A	10.057	35.943	14.411	0.000	0.000	0.000	0.000	200.030	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	445	286	628	0	0	0	0	30	0
N.S.	1	1.15	0.74	1.63	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	1.411	23.615	3.905	0.000	0.000	0.000	0.000	200.026	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	47	60	63	60	92	61
N.S.	1	1.00	0.90	0.80	0.78	1.00	1.05	1.00	1.53	1.02
time (sec)	N/A	0.434	0.065	0.418	0.106	0.076	0.113	0.128	0.189	0.139

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	492	483	411	666	0	520	1325	359
N.S.	1	1.00	2.01	1.97	1.68	2.72	0.00	2.12	5.41	1.47
time (sec)	N/A	1.171	0.629	0.342	0.040	142.749	0.000	0.128	0.200	27.009

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	159	286	257	410	0	298	815	220
N.S.	1	1.00	0.98	1.77	1.59	2.53	0.00	1.84	5.03	1.36
time (sec)	N/A	0.821	0.222	0.262	0.037	44.883	0.000	0.130	0.190	22.704

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	156	153	225	0	163	431	137
N.S.	1	1.00	1.00	1.28	1.25	1.84	0.00	1.34	3.53	1.12
time (sec)	N/A	0.638	0.130	0.208	0.034	7.058	0.000	0.149	0.191	20.425

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	93	91	90	111	0	93	184	87
N.S.	1	1.00	1.02	1.00	0.99	1.22	0.00	1.02	2.02	0.96
time (sec)	N/A	0.509	0.072	0.160	0.036	0.313	0.000	0.131	0.186	19.056

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	165	164	202	0	0	219	851	213
N.S.	1	1.00	1.02	1.02	1.25	0.00	0.00	1.36	5.29	1.32
time (sec)	N/A	0.837	0.226	0.333	0.037	0.000	0.000	0.130	0.181	25.856

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	514	310	535	0	0	9098	5126	436
N.S.	1	1.00	1.76	1.06	1.83	0.00	0.00	31.16	17.55	1.49
time (sec)	N/A	1.434	0.584	0.683	0.068	0.000	0.000	0.345	0.204	52.741

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	F(-1)	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	899	687	1353	0	0	1885	15095	131190
N.S.	1	1.00	1.55	1.18	2.33	0.00	0.00	3.24	25.98	225.80
time (sec)	N/A	2.986	1.116	1.751	0.131	0.000	0.000	0.272	0.201	55.718

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	599	550	300	327	0	0	450	665	3422	0
N.S.	1	0.92	0.50	0.55	0.00	0.00	0.75	1.11	5.71	0.00
time (sec)	N/A	1.655	1.890	1.576	0.000	0.000	145.492	0.179	0.206	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	418	238	278	0	0	326	399	2036	0
N.S.	1	1.01	0.57	0.67	0.00	0.00	0.79	0.96	4.92	0.00
time (sec)	N/A	1.304	1.214	0.724	0.000	0.000	99.775	0.160	0.200	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	292	216	232	0	0	272	239	951	179516
N.S.	1	1.04	0.77	0.82	0.00	0.00	0.96	0.85	3.37	636.58
time (sec)	N/A	0.959	0.977	0.559	0.000	0.000	88.564	0.163	0.213	34.112

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	188	156	0	0	236	164	3015	34205
N.S.	1	1.00	1.00	0.83	0.00	0.00	1.26	0.87	16.04	181.94
time (sec)	N/A	0.850	0.740	0.424	0.000	0.000	114.721	0.161	0.193	70.656

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	311	245	227	0	0	294	281	7571	0
N.S.	1	1.29	1.01	0.94	0.00	0.00	1.21	1.16	31.29	0.00
time (sec)	N/A	1.096	2.611	0.952	0.000	0.000	80.934	0.198	0.217	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	441	426	338	0	0	471	789	26571	0
N.S.	1	1.17	1.13	0.90	0.00	0.00	1.25	2.10	70.67	0.00
time (sec)	N/A	1.362	7.929	3.424	0.000	0.000	61.065	0.302	0.270	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	667	571	755	563	0	0	910	2365	62380	0
N.S.	1	0.86	1.13	0.84	0.00	0.00	1.36	3.55	93.52	0.00
time (sec)	N/A	1.641	17.529	211.903	0.000	0.000	96.529	0.870	0.394	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	224	0	143	0	240	42	1482
N.S.	1	1.00	0.99	2.57	0.00	1.64	0.00	2.76	0.48	17.03
time (sec)	N/A	0.638	0.328	0.391	0.000	0.099	0.000	0.263	0.208	21.558

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	599	945	164167	0	0	0	0	60	0
N.S.	1	1.19	1.88	325.73	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	3.470	13.469	2.490	0.000	0.000	0.000	0.000	0.184	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	600	455	768	0	0	0	0	13191	0
N.S.	1	1.19	0.90	1.52	0.00	0.00	0.00	0.00	26.17	0.00
time (sec)	N/A	1.798	5.428	1.146	0.000	0.000	0.000	0.000	1.343	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	377	276	392	0	0	0	0	8335	0
N.S.	1	1.27	0.93	1.32	0.00	0.00	0.00	0.00	28.16	0.00
time (sec)	N/A	1.205	2.194	0.794	0.000	0.000	0.000	0.000	0.601	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	188	187	252	0	0	0	0	3976	0
N.S.	1	1.05	1.04	1.41	0.00	0.00	0.00	0.00	22.21	0.00
time (sec)	N/A	0.766	0.943	0.587	0.000	0.000	0.000	0.000	0.347	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	287	401	0	0	0	0	265	0
N.S.	1	1.00	1.09	1.52	0.00	0.00	0.00	0.00	1.01	0.00
time (sec)	N/A	1.066	1.170	0.711	0.000	0.000	0.000	0.000	5.172	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	411	392	672	0	0	0	0	1758	0
N.S.	1	1.12	1.07	1.83	0.00	0.00	0.00	0.00	4.79	0.00
time (sec)	N/A	1.415	11.574	0.822	0.000	0.000	0.000	0.000	0.281	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	604	521	674	0	0	0	0	34	0
N.S.	1	1.39	1.20	1.55	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	2.378	25.782	15.943	0.000	0.000	0.000	0.000	200.028	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	0	356	517	0	0	0	0	39	0
N.S.	1	0.00	0.85	1.24	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	23.447	1.296	0.000	0.000	0.000	0.000	200.028	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	346	507	0	0	0	0	39	0
N.S.	1	0.00	0.90	1.31	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	23.596	0.865	0.000	0.000	0.000	0.000	200.032	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	599	439	497	0	0	0	0	39	0
N.S.	1	1.69	1.24	1.40	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	1.915	23.312	0.722	0.000	0.000	0.000	0.000	200.031	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	358	342	358	0	0	0	0	39	0
N.S.	1	1.11	1.06	1.11	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.243	24.157	0.417	0.000	0.000	0.000	0.000	200.028	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	0	421	497	0	0	0	0	39	0
N.S.	1	0.00	1.19	1.40	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.000	24.211	0.577	0.000	0.000	0.000	0.000	200.026	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	361	530	0	0	0	0	39	0
N.S.	1	0.00	0.94	1.37	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.000	24.135	0.757	0.000	0.000	0.000	0.000	200.026	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	0	373	563	0	0	0	0	39	0
N.S.	1	0.00	0.89	1.35	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	24.127	1.005	0.000	0.000	0.000	0.000	200.026	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1068	1109	37177	1316	0	1121	0	0	39	0
N.S.	1	1.04	34.81	1.23	0.00	1.05	0.00	0.00	0.04	0.00
time (sec)	N/A	5.469	38.923	18.717	0.000	0.143	0.000	0.000	200.023	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	683	18202	979	0	686	0	0	37	0
N.S.	1	1.05	28.00	1.51	0.00	1.06	0.00	0.00	0.06	0.00
time (sec)	N/A	2.518	36.502	12.449	0.000	0.133	0.000	0.000	200.029	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	476	1080	823	0	451	0	0	32	0
N.S.	1	1.06	2.40	1.83	0.00	1.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.217	6.337	6.760	0.000	0.102	0.000	0.000	200.028	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	822	19347	1114	0	0	0	0	39	0
N.S.	1	1.28	30.09	1.73	0.00	0.00	0.00	0.00	0.06	0.00
time (sec)	N/A	2.755	36.717	24.258	0.000	0.000	0.000	0.000	200.029	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F	F(-1)
verified	N/A	No	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	916	1534	32746	1508	0	0	0	0	39	0
N.S.	1	1.67	35.75	1.65	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	5.633	39.068	47.528	0.000	0.000	0.000	0.000	200.032	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	642	528	0	0	0	0	39	0
N.S.	1	0.00	1.25	1.03	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	28.469	1.550	0.000	0.000	0.000	0.000	200.043	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	0	638	518	0	0	0	0	39	0
N.S.	1	0.00	1.33	1.08	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	26.003	0.848	0.000	0.000	0.000	0.000	200.029	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	647	631	508	0	0	0	0	39	0
N.S.	1	1.44	1.41	1.13	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.987	25.992	0.730	0.000	0.000	0.000	0.000	200.029	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	420	593	361	0	0	0	0	39	0
N.S.	1	1.01	1.43	0.87	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	1.299	27.085	0.542	0.000	0.000	0.000	0.000	200.029	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	0	622	508	0	0	0	0	39	0
N.S.	1	0.00	1.39	1.13	0.00	0.00	0.00	0.00	0.09	0.00
time (sec)	N/A	0.000	28.306	0.697	0.000	0.000	0.000	0.000	200.027	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	480	0	638	541	0	0	0	0	39	0
N.S.	1	0.00	1.33	1.13	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	26.906	0.878	0.000	0.000	0.000	0.000	200.031	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	C	C	F	F	F	F	F	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	0	687	574	0	0	0	0	39	0
N.S.	1	0.00	1.34	1.12	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.000	26.837	1.130	0.000	0.000	0.000	0.000	200.029	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	0	3222	3126	0	0	0	0	32	0
N.S.	1	0.00	2.36	2.29	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	0.000	35.508	434.050	0.000	0.000	0.000	0.000	200.032	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	882	0	2445	3097	0	0	0	0	30	0
N.S.	1	0.00	2.77	3.51	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	35.364	14.477	0.000	0.000	0.000	0.000	200.032	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	869	0	5981	3091	0	0	0	0	29	0
N.S.	1	0.00	6.88	3.56	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.403	14.471	0.000	0.000	0.000	0.000	200.030	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	897	0	3628	3141	0	0	0	0	34	0
N.S.	1	0.00	4.04	3.50	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.000	36.088	14.858	0.000	0.000	0.000	0.000	200.025	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1384	0	4412	3177	0	0	0	0	36	0
N.S.	1	0.00	3.19	2.30	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	36.359	15.029	0.000	0.000	0.000	0.000	200.026	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F(-1)	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1382	0	5152	3170	0	0	0	0	38	0
N.S.	1	0.00	3.73	2.29	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	36.416	15.102	0.000	0.000	0.000	0.000	200.026	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	B	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1396	0	6336	3212	0	0	0	0	39	0
N.S.	1	0.00	4.54	2.30	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	0.000	6.674	15.448	0.000	0.000	0.000	0.000	200.023	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [25] had the largest ratio of [.648649000000000031]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	F	0	0	N/A	0.000	N/A
2	F	0	0	N/A	0.000	N/A
3	F	0	0	N/A	0.000	N/A
4	F	0	0	N/A	0.000	N/A
5	F	0	0	N/A	0.000	N/A
6	F	0	0	N/A	0.000	N/A
7	A	12	11	1.06	41	0.268
8	A	9	8	1.09	39	0.205
9	A	7	6	1.07	34	0.176
10	A	10	9	1.42	41	0.220
11	F	0	0	N/A	0.000	N/A
12	A	2	2	1.45	33	0.061
13	A	2	2	1.31	30	0.067
14	A	2	2	1.32	30	0.067
15	A	2	2	1.20	28	0.071
16	A	6	5	1.00	27	0.185
17	A	2	2	1.00	30	0.067
18	A	2	2	1.20	30	0.067
19	A	2	2	1.37	30	0.067
20	A	11	10	1.05	37	0.270

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	9	8	1.06	35	0.229
22	A	7	6	1.06	34	0.176
23	A	11	10	1.31	37	0.270
24	B	17	16	2.17	37	0.432
25	B	25	24	3.50	37	0.649
26	A	6	5	1.15	32	0.156
27	A	2	2	1.00	26	0.077
28	A	2	2	1.00	37	0.054
29	A	2	2	1.00	37	0.054
30	A	2	2	1.00	35	0.057
31	A	2	2	1.00	30	0.067
32	A	2	2	1.00	37	0.054
33	A	2	2	1.00	37	0.054
34	A	2	2	1.00	37	0.054
35	A	2	2	0.92	39	0.051
36	A	2	2	1.01	39	0.051
37	A	2	2	1.04	39	0.051
38	A	2	2	1.00	39	0.051
39	A	2	2	1.29	39	0.051
40	A	2	2	1.17	39	0.051
41	A	2	2	0.86	39	0.051
42	A	2	2	1.00	26	0.077
43	A	2	2	1.19	41	0.049
44	A	2	2	1.19	39	0.051
45	A	2	2	1.27	37	0.054
46	A	8	7	1.05	32	0.219
47	A	2	2	1.00	39	0.051
48	A	2	2	1.12	39	0.051
49	A	11	10	1.39	36	0.278
50	F	0	0	N/A	0.000	N/A
51	F	0	0	N/A	0.000	N/A
52	A	18	17	1.69	41	0.415

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	11	10	1.11	41	0.244
54	F	0	0	N/A	0.000	N/A
55	F	0	0	N/A	0.000	N/A
56	F	0	0	N/A	0.000	N/A
57	A	11	10	1.04	41	0.244
58	A	9	8	1.05	39	0.205
59	A	7	6	1.06	34	0.176
60	A	12	11	1.28	41	0.268
61	A	17	16	1.67	41	0.390
62	F	0	0	N/A	0.000	N/A
63	F	0	0	N/A	0.000	N/A
64	C	19	18	1.44	41	0.439
65	C	12	11	1.01	41	0.268
66	F	0	0	N/A	0.000	N/A
67	F	0	0	N/A	0.000	N/A
68	F	0	0	N/A	0.000	N/A
69	F	0	0	N/A	0.000	N/A
70	F	0	0	N/A	0.000	N/A
71	F	0	0	N/A	0.000	N/A
72	F	0	0	N/A	0.000	N/A
73	F	0	0	N/A	0.000	N/A
74	F	0	0	N/A	0.000	N/A
75	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 1

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx=0$$

output
```
0
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 1 in optimal.

Time = 27.71 (sec) , antiderivative size = 7130, normalized size of antiderivative = 7130.00

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx=\text{Result too large to show}$$

input
```
Integrate[((A + B*x)*Sqrt[f + g*x])/(Sqrt[d + e*x]*Sqrt[a + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{a+cx^2}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{2349}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+a}}dx+\int \frac{B\sqrt{d+ex}\sqrt{f+gx}}{e\sqrt{cx^2+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+a}}dx+\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+a}}dx}{e}$$

$$\Big\downarrow \text{726}$$

$$\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+a}}dx}{e}+\frac{2(f+gx)\sqrt{cd-\sqrt{-a}\sqrt{c}e}\left(A-\frac{Bd}{e}\right)\sqrt{-\frac{(\sqrt{-a}+\sqrt{c}x)(ef-dg)}{(f+gx)(\sqrt{c}d-\sqrt{-a}e)}}\sqrt{-\frac{(\sqrt{-a}\sqrt{c}x+a)(ef-dg)}{(f+gx)(\sqrt{-a}\sqrt{c}d-ae)}}\text{EllipticPi}\left(\frac{(\sqrt{c}d-\sqrt{-a}e)g}{e(\sqrt{c}f-\sqrt{-a}g)},\text{arcsin}\right.}{e\sqrt{a+cx^2}\sqrt{cf-\sqrt{-a}\sqrt{c}g}}$$

$$\Big\downarrow \text{744}$$

$$\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+a}}dx}{e}+\frac{2(f+gx)\sqrt{cd-\sqrt{-a}\sqrt{c}e}\left(A-\frac{Bd}{e}\right)\sqrt{-\frac{(\sqrt{-a}+\sqrt{c}x)(ef-dg)}{(f+gx)(\sqrt{c}d-\sqrt{-a}e)}}\sqrt{-\frac{(\sqrt{-a}\sqrt{c}x+a)(ef-dg)}{(f+gx)(\sqrt{-a}\sqrt{c}d-ae)}}\text{EllipticPi}\left(\frac{(\sqrt{c}d-\sqrt{-a}e)g}{e(\sqrt{c}f-\sqrt{-a}g)},\text{arcsin}\right.}{e\sqrt{a+cx^2}\sqrt{cf-\sqrt{-a}\sqrt{c}g}}$$

input
```
Int[((A + B*x)*Sqrt[f + g*x])/(Sqrt[d + e*x]*Sqrt[a + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 726
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x
_)^2]), x_Symbol] :> With[{q = Rt[-4*a*c, 2]}, Simp[Sqrt[2]*Sqrt[2*c*f - g*
q]*Sqrt[-q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*((q + 2*c*x)/((2*c*f - g*q)*
(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a + q*x)/((q*f - 2*a*g)*(d + e*x)))]/(g*
Sqrt[2*c*d - e*q]*Sqrt[2*a*(c/q) + c*x]*Sqrt[a + c*x^2]))*EllipticPi[e*((2*
c*f - g*q)/(g*(2*c*d - e*q))), ArcSin[Sqrt[2*c*d - e*q]*(Sqrt[f + g*x]/(Sqr
t[2*c*f - g*q]*Sqrt[d + e*x]))], (q*d - 2*a*e)*((2*c*f - g*q)/((q*f - 2*a*g
)*(2*c*d - e*q)))], x]] /; FreeQ[{a, c, d, e, f, g}, x]
```

rule 744
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_) + (c_.)*(x_
)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p,
 x] /; FreeQ[{a, c, d, e, f, g, m, n, p}, x]
```

rule 2349
```
Int[(Px_)*((c_) + (d_.)*(x_))^(m_.)*((e_) + (f_.)*(x_))^(n_.)*((a_.) + (b_.
)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, c + d*x, x]*(c + d
*x)^(m + 1)*(e + f*x)^n*(a + b*x^2)^p, x] + Simp[PolynomialRemainder[Px, c
+ d*x, x]   Int[(c + d*x)^m*(e + f*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a,
b, c, d, e, f, n, p}, x] && PolynomialQ[Px, x] && LtQ[m, 0] &&  !IntegerQ[n
] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 1.

Time = 6.73 (sec) , antiderivative size = 1903, normalized size of antiderivative = 1903.00

method	result	size
elliptic	Expression too large to display	1903
default	Expression too large to display	22386

input
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((g*x+f)*(c*x^2+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2)/(e*x+d)^(1
/2)*(2*A*f*(-f/g+d/e)*((-d/e-1/c*(-a*c)^(1/2))*(x+f/g)/(f/g-d/e)/(x-1/c*(-
a*c)^(1/2)))^(1/2)*(x-1/c*(-a*c)^(1/2))^2*((1/c*(-a*c)^(1/2)+f/g)*(x+1/c*(
-a*c)^(1/2))/(-1/c*(-a*c)^(1/2)+f/g)/(x-1/c*(-a*c)^(1/2)))^(1/2)*((1/c*(-a
*c)^(1/2)+f/g)*(x+d/e)/(f/g-d/e)/(x-1/c*(-a*c)^(1/2)))^(1/2)/(-d/e-1/c*(-a
*c)^(1/2))/(1/c*(-a*c)^(1/2)+f/g)/(c*e*g*(x+f/g)*(x-1/c*(-a*c)^(1/2))*(x+1
/c*(-a*c)^(1/2))*(x+d/e))^(1/2)*EllipticF(((-d/e-1/c*(-a*c)^(1/2))*(x+f/g)
/(f/g-d/e)/(x-1/c*(-a*c)^(1/2)))^(1/2),2^(1/2)*(1/c*(-a*c)^(1/2)*(-f/g+d/e
)/(1/c*(-a*c)^(1/2)-f/g)/(d/e+1/c*(-a*c)^(1/2)))^(1/2))+2*(A*g+B*f)*(-f/g+
d/e)*((-d/e-1/c*(-a*c)^(1/2))*(x+f/g)/(f/g-d/e)/(x-1/c*(-a*c)^(1/2)))^(1/2
)*(x-1/c*(-a*c)^(1/2))^2*((1/c*(-a*c)^(1/2)+f/g)*(x+1/c*(-a*c)^(1/2))/(-1/
c*(-a*c)^(1/2)+f/g)/(x-1/c*(-a*c)^(1/2)))^(1/2)*((1/c*(-a*c)^(1/2)+f/g)*(x
+d/e)/(f/g-d/e)/(x-1/c*(-a*c)^(1/2)))^(1/2)/(-d/e-1/c*(-a*c)^(1/2))/(1/c*(
-a*c)^(1/2)+f/g)/(c*e*g*(x+f/g)*(x-1/c*(-a*c)^(1/2))*(x+1/c*(-a*c)^(1/2))*
(x+d/e))^(1/2)*(1/c*(-a*c)^(1/2)*EllipticF(((-d/e-1/c*(-a*c)^(1/2))*(x+f/g
)/(f/g-d/e)/(x-1/c*(-a*c)^(1/2)))^(1/2),2^(1/2)*(1/c*(-a*c)^(1/2)*(-f/g+d/
e)/(1/c*(-a*c)^(1/2)-f/g)/(d/e+1/c*(-a*c)^(1/2)))^(1/2))+(-1/c*(-a*c)^(1/2
)-f/g)*EllipticPi(((-d/e-1/c*(-a*c)^(1/2))*(x+f/g)/(f/g-d/e)/(x-1/c*(-a*c)
^(1/2)))^(1/2),(f/g-d/e)/(-d/e-1/c*(-a*c)^(1/2)),2^(1/2)*(1/c*(-a*c)^(1/2)
*(-f/g+d/e)/(1/c*(-a*c)^(1/2)-f/g)/(d/e+1/c*(-a*c)^(1/2)))^(1/2))+B*g*...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx = \int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{a+cx^2}\sqrt{d+ex}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+a)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(f + g*x)/(sqrt(a + c*x**2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(A+B\,x)}{\sqrt{c\,x^2+a}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(A + B*x))/((a + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(((f + g*x)^(1/2)*(A + B*x))/((a + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+cx^2}}\,dx=\int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{ex+d}\,\sqrt{c\,x^2+a}}dx$$

input
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x)
```

output
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+a)^(1/2),x)
```

3.2 $\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 1474

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
B*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/(e*x+d)^(1/2)+B*(a*e^2-b*d*e+c*d^2)*
(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*
(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/
2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)/(2*c*f-(
b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^(1/2)
)*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*
c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/c/e/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)
^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(
1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2)
)*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*(A*e*(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e)-B*(c*d^2-e*((-4*a*c+b^2)^(1/2)*d+a*e)))*(-d*g+e*f)*(b-(-4*a*c+b^2)^(
1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2
)^(1/2))*g)/(e*x+d))^(1/2)*EllipticF((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2
)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c
*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-
4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e^2/(2*c*d-(
b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-
(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c
*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+(-d*g+e*f)
*(2*A*c*e*g+B*(-b*e*g-c*d*g+c*e*f))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 13559 vs. $2(1474) = 2948$.

Time = 28.35 (sec) , antiderivative size = 13559, normalized size of antiderivative = 9.20

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*x)*Sqrt[f + g*x])/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{B\sqrt{d+ex}\sqrt{f+gx}}{e\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1276}$$

$$\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e}+$$

$$\frac{\sqrt{2}(f+gx)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bd}{e}\right)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{-\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(f+gx)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{-\frac{(x(\sqrt{}}{(f+g}}}{e\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx\sqrt{}}}$$

$$\Big\downarrow \text{1292}$$

$$\frac{B\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e}+$$

$$\frac{\sqrt{2}(f+gx)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bd}{e}\right)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{-\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(f+gx)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{-\frac{(x(\sqrt{}}{(f+g}}}{e\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx\sqrt{}}}$$

input
```
Int[((A + B*x)*Sqrt[f + g*x])/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2658 vs. $2(1325) = 2650$.

Time = 7.41 (sec) , antiderivative size = 2659, normalized size of antiderivative = 1.80

method	result	size
elliptic	Expression too large to display	2659
default	Expression too large to display	85419

input
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b
*x+a)^(1/2)*(2*A*f*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g
)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(
1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2
)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)*(-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2))+2*(A*g+B*f)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^
(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(
f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(
-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)**(1/2)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)*sqrt(f + g*x)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x
)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x
)
```

Giac [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+g\,x}\,(A+B\,x)}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(A + B*x))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),
x)
```

output
```
int(((f + g*x)^(1/2)*(A + B*x))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{(A+Bx)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Bx+A)\sqrt{gx+f}}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((B*x+A)*(g*x+f)^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.3 $\int \frac{\sqrt{f+gx}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 1585

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
(3*c*C*d^2-C*e*(-a*e+b*d)-2*c*e*(-A*e+B*d))*g*(e*x+d)^(1/2)*(c*x^2+b*x+a)^
(1/2)/c/e^2/(a*e^2-b*d*e+c*d^2)/(g*x+f)^(1/2)+C*(g*x+f)^(1/2)*(c*x^2+b*x+a
)^(1/2)/c/e/(e*x+d)^(1/2)-(3*c*C*d^2-C*e*(-a*e+b*d)-2*c*e*(-A*e+B*d))*(g*x
+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)+(-4*a*
c+b^2)^(1/4)*(3*c*C*d^2-C*e*(-a*e+b*d)-2*c*e*(-A*e+B*d))*(c*f^2-g*(-a*g+b*
f))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)*(e*x+d)^(1/2)*((2*c*f-(b-(-4*a*c+b^
2)^(1/2))*g)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(-4*a*c+b^2)^(1/2)/(g*x+f))^(1
/2)*EllipticE(1/2*(-2*c*f+(b+(-4*a*c+b^2)^(1/2))*g)^(1/2)*(b-(-4*a*c+b^2)^
(1/2)+2*c*x)^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/4)/(g*x+f)^(1/2),2*(c*(-4*a*c+b
^2)^(1/2)*(-d*g+e*f)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^
2)^(1/2))*g))^(1/2))/c^(1/2)/e^2/(a*e^2-b*d*e+c*d^2)/(2*c*f-(b-(-4*a*c+b^2
)^(1/2))*g)/(-2*c*f+(b+(-4*a*c+b^2)^(1/2))*g)^(1/2)/((2*c*f-(b-(-4*a*c+b^2
)^(1/2))*g)*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(g*x+f))^(1/2)/(c*x^2
+b*x+a)^(1/2)+1/2*(C*e*(-a*e+b*d)*g-c*C*d*(-3*d*g+4*e*f)+2*B*c*e*(-d*g+e*f
))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)*(((b+(-4*a*c+b^2)^(1/2))*d-2*a*e+(2*
c*d-(b-(-4*a*c+b^2)^(1/2))*e)*x)/(-4*a*c+b^2)^(1/2)/(e*x+d))^(1/2)*(g*x+f)
^(1/2)*EllipticF((-d*g+e*f)^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)/(2*c*
f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),1/2*(-2*(2*c*d*f+2*a*e*g-(
-4*a*c+b^2)^(1/2)*(-d*g+e*f)-b*(d*g+e*f))/(-4*a*c+b^2)^(1/2)/(-d*g+e*f))^(
1/2))*2^(1/2)/c/e^3/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(-d*g+e*f)^(...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 19147 vs. $2(1585) = 3170$.

Time = 36.28 (sec) , antiderivative size = 19147, normalized size of antiderivative = 12.08

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[f + g*x]*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x
+ c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{f+gx}\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\left(\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}\right)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\left(\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}\right)\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}\sqrt{f+gx}}{e^2\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{\sqrt{f+gx}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{ 1276}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx+\frac{C\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e^2}-\frac{\sqrt{2}(f+gx)\sqrt{-\sqrt{b^2-4ac}+b+2cx}(2Cd-Be)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{-\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(f+gx)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{-\frac{(x}{(f}}}{e^3\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}}$$

$$\Big\downarrow \text{ 1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx+\frac{C\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{cx^2+bx+a}}dx}{e^2}-$$

$$\frac{\sqrt{2}(f+gx)\sqrt{-\sqrt{b^2-4ac}+b+2cx}(2Cd-Be)\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\sqrt{-\frac{\left(\sqrt{b^2-4ac}+b+2cx\right)(ef-dg)}{(f+gx)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}}\sqrt{-\frac{(x}{(f}}}{e^3\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}+cx}}$$

input
```
Int[(Sqrt[f + g*x]*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3090 vs. $2(1426) = 2852$.

Time = 9.78 (sec) , antiderivative size = 3091, normalized size of antiderivative = 1.95

method	result	size
elliptic	Expression too large to display	3091
default	Expression too large to display	261664

input
```
int((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b
*x+a)^(1/2)*(-2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)*(A*e
^2-B*d*e+C*d^2)/e^2/(a*e^2-b*d*e+c*d^2)/((x+d/e)*(c*e*g*x^3+b*e*g*x^2+c*e*
f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*((A*e^2*g-B*d*e*g+B*e^2*f+C*d^2*g-C*
d*e*f)/e^3-(A*e^2-B*d*e+C*d^2)/e^3*(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*
f)/(a*e^2-b*d*e+c*d^2)+(a*e*g+b*e*f)*(A*e^2-B*d*e+C*d^2)/e^2/(a*e^2-b*d*e+
c*d^2))*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)
/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/
(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(
-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(
x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^
(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+
d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(1/2))
))^(1/2))+2*(1/e^2*(B*e*g-C*d*g+C*e*f)-(A*e^2-B*d*e+C*d^2)/e^2*(b*e*g-c*d*
g+c*e*f)/(a*e^2-b*d*e+c*d^2)+(2*b*e*g+2*c*e*f)*(A*e^2-B*d*e+C*d^2)/e^2/(a*
e^2-b*d*e+c*d^2))*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{\sqrt{f+gx}\,(A+Bx+Cx^2)}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**(1/2)*(C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/
2),x)
```

output
```
Integral(sqrt(f + g*x)*(A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x +
 c*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)
^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)
^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{f+g\,x}\,(C\,x^2+B\,x+A)}{(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^(1/2)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)
^(1/2)),x)
```

output
```
int(((f + g*x)^(1/2)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)
^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{f+gx}(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{\sqrt{gx+f}\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x+f)^(1/2)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.4 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 1396

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*(A*e^2-B*d*e+C*d^2)*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/
2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^
(1/2))*g)/(e*x+d))^(1/2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*
(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*
a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e/(2*c*d-(b-(-
4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*
a*c+b^2)^(1/2))*g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*
x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*(
(b+(-4*a*c+b^2)^(1/2))*e*(-B*e+2*C*d)-2*c*(-A*e^2+C*d^2))*(b-(-4*a*c+b^2)^
(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^
2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticF((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/
2)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*
c*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e^2/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b
-(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*
c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*C*(-d*g
+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x
)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticPi((2*c*d-(b-...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6336 vs. $2(1396) = 2792$.

Time = 36.11 (sec) , antiderivative size = 6336, normalized size of antiderivative = 4.54

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a + bx + cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x +
c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1281}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1280}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 1416$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{\ldots}}{(d+ex)\sqrt{\ldots}}\right.\cdots}{\sqrt{a}\cdots}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 2154$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{\ldots}}{(d+ex)\sqrt{\ldots}}\right.\cdots}{\sqrt{a}\cdots}\right.$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 27$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{\ldots}}{(d+ex)\sqrt{\ldots}}\right.\cdots}{\sqrt{a}\cdots}\right.$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\downarrow 1276$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed\cdots\right.}{\left(cf^2-g(bf\cdots\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c\cdots}}\right.$$

$\downarrow$ 1280

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed\cdots\right.}{\left(cf^2-g(bf\cdots\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c\cdots}}\right.$$

$$\frac{2(2Cd-Be)(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{cx^2+bx+a}}$$

$\downarrow$ 1416

$$\frac{(2Cd-Be)\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-}{\left(\frac{\sqrt{cd^2-bed}}{\sqrt{cf^2-g(bf}}\right.}}}{e^2\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd}{(cf}}}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed-\right.}{\left(cf^2-g(bf\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c}}\right.$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3211 vs. 2(1255) = 2510.

Time = 13.26 (sec) , antiderivative size = 3212, normalized size of antiderivative = 2.30

method	result	size
elliptic	Expression too large to display	3212
default	Expression too large to display	176190

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output

```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b
*x+a)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2-B*d*e+C*d^2)
/((x+d/e)*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(
(B*e-C*d)/e^2+1/e^2*(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*(A*e^2-B*d*e
+C*d^2)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(a*e*g+b
*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2-B
*d*e+C*d^2))*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g
-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*
(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(
-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+2*(C/e+1/e*(b*e*g-c*d*g+c*e*f)*(A*e^2-B*d*e+C*d^2)/(a*d*e^2
*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e*f)/(a*...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input

```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="fricas")
```

output

```
integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x +
f)/(c*e^2*g*x^5 + (c*e^2*f + (2*c*d*e + b*e^2)*g)*x^4 + a*d^2*f + ((2*c*d*
e + b*e^2)*f + (c*d^2 + 2*b*d*e + a*e^2)*g)*x^3 + ((c*d^2 + 2*b*d*e + a*e^
2)*f + (b*d^2 + 2*a*d*e)*g)*x^2 + (a*d^2*g + (b*d^2 + 2*a*d*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x +
 c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*
x + f)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*
x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.5 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 1265

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*c^(1/2)*(-4*a*c+b^2)^(1/4)*(A*e^2-B*d*e+C*d^2)*(b-(-4*a*c+b^2)^(1/2)+2*c
*x)^(1/2)*((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c
/(-4*a*c+b^2)^(1/2)/(e*x+d))^(1/2)*(g*x+f)^(1/2)*EllipticE(1/2*(-2*c*d+(b+
(-4*a*c+b^2)^(1/2))*e)^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)/c^(1/2)/(-
4*a*c+b^2)^(1/4)/(e*x+d)^(1/2),2*(-c*(-4*a*c+b^2)^(1/2)*(-d*g+e*f)/(2*c*d-
(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e)/(-2*c*d+(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)/(-d*g+e*
f)^2/((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)*(g*x+f)/(2*c*f-(b-(-4*a*c+b^2)^(1/2
))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*c^(1/2)*(-4*a*c+b^2)^(1/4)*(A*g
^2-B*f*g+C*f^2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)*(e*x+d)^(1/2)*((2*c*f-(
b-(-4*a*c+b^2)^(1/2))*g)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c/(-4*a*c+b^2)^(1/2)
/(g*x+f))^(1/2)*EllipticE(1/2*(-2*c*f+(b+(-4*a*c+b^2)^(1/2))*g)^(1/2)*(b-(
-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/4)/(g*x+f)^(1/2),2*
(c*(-4*a*c+b^2)^(1/2)*(-d*g+e*f)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(
b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(-2*c*f+
(b+(-4*a*c+b^2)^(1/2))*g)^(1/2)/(-d*g+e*f)^2/((2*c*f-(b-(-4*a*c+b^2)^(1/2)
)*g)*(e*x+d)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(g*x+f))^(1/2)/(c*x^2+b*x+a)
^(1/2)-2^(1/2)*(2*C*d*f+2*A*e*g-B*(d*g+e*f))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^
(1/2)*(((b+(-4*a*c+b^2)^(1/2))*d-2*a*e+(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)*x)
/(-4*a*c+b^2)^(1/2)/(e*x+d))^(1/2)*(g*x+f)^(1/2)*EllipticF((-d*g+e*f)^(...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 29355 vs. $2(1265) = 2530$.

Time = 38.68 (sec) , antiderivative size = 29355, normalized size of antiderivative = 23.21

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^(3/2)*Sqrt[a + b*x
+ c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1281}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1280}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{(2Cd-Be)\left(-\frac{2e(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}-\frac{g\int \frac{\sqrt{\cdot}}{(f+gx)^{3/2}\cdots}}{ef\cdots}\right.}{e^2}$$

$$\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{(2Cd-Be)\left(-\frac{2e(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}-\frac{g\int \frac{\sqrt{\cdot}}{(f+gx)^{3/2}\cdots}}{ef\cdots}\right.}{e^2}$$

$$\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1416}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{(2Cd-Be)\left(-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{e(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\cdots}}{\sqrt{a+bx+cx^2}(ef-dg)\cdots}\right.}{\cdots}$$

$$\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 7327 vs. 2(1117) = 2234.

Time = 17.79 (sec) , antiderivative size = 7328, normalized size of antiderivative = 5.79

method	result	size
elliptic	Expression too large to display	7328
default	Expression too large to display	571362

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x +
f)/(c*e^2*g^2*x^6 + (2*c*e^2*f*g + (2*c*d*e + b*e^2)*g^2)*x^5 + a*d^2*f^2
+ (c*e^2*f^2 + 2*(2*c*d*e + b*e^2)*f*g + (c*d^2 + 2*b*d*e + a*e^2)*g^2)*x^
4 + ((2*c*d*e + b*e^2)*f^2 + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*g + (b*d^2 + 2*
a*d*e)*g^2)*x^3 + (a*d^2*g^2 + (c*d^2 + 2*b*d*e + a*e^2)*f^2 + 2*(b*d^2 +
2*a*d*e)*f*g)*x^2 + (2*a*d^2*f*g + (b*d^2 + 2*a*d*e)*f^2)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{3}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*(f + g*x)**(3/2)*sqrt(a + b*
x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{{(f+g\,x)}^{3/2}\,{(d+e\,x)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(3/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(3/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,(gx+f)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.6 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 1780

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)
/(e*x+d)^(1/2)/(g*x+f)^(3/2)+2/3*g*(b*(C*d*f*(3*d*g+e*f)-e*g*(-A*d*g-3*A*e
*f+4*B*d*f))-c*(4*C*d^2*f^2-B*d*f*(d*g+3*e*f)+A*(d^2*g^2+3*e^2*f^2))-a*(C*
(3*d^2*g^2+e^2*f^2)+e*g*(4*A*e*g-B*(3*d*g+e*f))))*(e*x+d)^(1/2)*(c*x^2+b*x
+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)^2/(c*f^2-g*(-a*g+b*f))/(g*x+f)^(3
/2)+2/3*c^(1/2)*(-4*a*c+b^2)^(1/4)*(c^2*f*(2*C*d^2*f^2*(d*g+3*e*f)-B*d*f*(
-d^2*g^2+6*d*e*f*g+3*e^2*f^2)+A*(-4*d^3*g^3+9*d^2*e*f*g^2+3*e^3*f^3))+c*(b
*B*d*f*g*(d^2*g^2+3*d*e*f*g+12*e^2*f^2)-b*C*d*f^2*(4*d^2*g^2+9*d*e*f*g+3*e
^2*f^2)-A*b*g*(-2*d^3*g^3+3*d^2*e*f*g^2+9*d*e^2*f^2*g+6*e^3*f^3)+a*C*f*(6*
d^3*g^3+5*d^2*e*f*g^2+2*d*e^2*f^2*g+3*e^3*f^3)+a*g*(A*e*g*(5*d^2*g^2-4*d*e
*f*g+15*e^2*f^2)-B*(3*d^3*g^3+2*d^2*e*f*g^2+5*d*e^2*f^2*g+6*e^3*f^3)))-e*g
*(a^2*g*(2*e*g*(-4*A*e*g+3*B*d*g+B*e*f)+C*(-3*d^2*g^2-6*d*e*f*g+e^2*f^2))-
b^2*(C*d*f^2*(7*d*g+e*f)-g*(B*d*f*(d*g+7*e*f)-A*(-2*d^2*g^2+7*d*e*f*g+3*e^
2*f^2)))+a*b*(C*f*(12*d^2*g^2+3*d*e*f*g+e^2*f^2)+g*(A*e*g*(3*d*g+13*e*f)-B
*(3*d^2*g^2+9*d*e*f*g+4*e^2*f^2)))))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)*(e
*x+d)^(1/2)*((2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)
/c/(-4*a*c+b^2)^(1/2)/(g*x+f))^(1/2)*EllipticE(1/2*(-2*c*f+(b+(-4*a*c+b^2)
^(1/2))*g)^(1/2)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)^(1/2)/c^(1/2)/(-4*a*c+b^2)^(
1/4)/(g*x+f)^(1/2),2*(c*(-4*a*c+b^2)^(1/2)*(-d*g+e*f)/(2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(a*e^2-b*d*e+c*d^...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 107718 vs. $2(1780) = 3560$.

Time = 52.98 (sec) , antiderivative size = 107718, normalized size of antiderivative = 60.52

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{3/2}(f + gx)^{5/2}\sqrt{a + bx + cx^2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^(5/2)*Sqrt[a + b*x
+ c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \ 2154$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \ 1292$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \ 2154$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \ 27$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \ 1282$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{\int \frac{3cf(ef-dg)-g(3bef-2bdg-2aeg)-g(3cef-cdg-beg)x}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ef-dg)(ag^2-bfg+cf^2)}+\frac{2g^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ef-dg)(ag^2-bfg+cf^2)}\right)}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1285}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{\int \frac{3cf(ef-dg)-g(3bef-2bdg-2aeg)-g(3cef-cdg-beg)x}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ef-dg)(ag^2-bfg+cf^2)}+\frac{2g^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ef-dg)(ag^2-bfg+cf^2)}\right)}{e^2}+\frac{C\left(-\frac{\int -\frac{3cdf-2bdg+aeg+(3cef-cdg-beg)x}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ag^2-bfg+cf^2)}-\frac{2g\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ag^2-bfg+cf^2)}\right)}{e^2}$$

$$\Big\downarrow \text{25}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{\int \frac{3cf(ef-dg)-g(3bef-2bdg-2aeg)-g(3cef-cdg-beg)x}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ef-dg)(ag^2-bfg+cf^2)}+\frac{2g^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ef-dg)(ag^2-bfg+cf^2)}\right)}{e^2}+\frac{C\left(\frac{\int \frac{3cdf-2bdg+aeg+(3cef-cdg-beg)x}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ag^2-bfg+cf^2)}-\frac{2g\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ag^2-bfg+cf^2)}\right)}{e^2}$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{\int \frac{\left(bg^2+\frac{cdg^2}{e}-3cfg\right)\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(g(-2aeg-bdg+3bef)-c\left(3ef^2-\frac{d^2g^2}{e}\right)\right)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ef-dg)(ag^2-bfg+cf^2)}+\frac{2g^2\sqrt{d\cdots}}{3(f+gx)^{3/2}(\cdots}\right.}{e^2}$$

$$\frac{C\left(\frac{\int \frac{\left(3cf-bg-\frac{cdg}{e}\right)\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-g\left(-ae+bd-\frac{cd^2}{e}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ag^2-bfg+cf^2)}-\frac{2g\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ag^2-bfg+cf^2)}\right)}{e^2}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{-\left(\left(g(-2aeg-bdg+3bef)-c\left(3ef^2-\frac{d^2g^2}{e}\right)\right)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx\right)-g\left(-bg-\frac{cdg}{e}+3cf\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+}}}{3(ef-dg)(ag^2-bfg+cf^2)}}{e^2}$$

$$\frac{C\left(\frac{\left(-bg-\frac{cdg}{e}+3cf\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-g\left(-ae+bd-\frac{cd^2}{e}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{3(ag^2-bfg+cf^2)}-\frac{2g\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3(f+gx)^{3/2}(ag^2-bfg+cf^2)}\right)}{e^2}$$

$\downarrow$ 1281

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(\frac{-\left(g(-2aeg-bdg+3bef)-c\left(3ef^2-\frac{d^2g^2}{e}\right)\right)\left(\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)-g\left(-bg-\frac{cdg}{e}+\right.}{3(ef-dg)(ag^2-bfg+cf^2)}}{e^2}$$

$$\frac{C\left(\frac{\left(-bg-\frac{cdg}{e}+3cf\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-g\left(-ae+bd-\frac{cd^2}{e}\right)\left(\frac{e\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{3(ag^2-bfg+cf^2)}-\frac{2g}{3(f+}\right.}{e^2}$$

$\downarrow$ 1280

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx+$$

$$\frac{C\left(\frac{\left(3cf-bg-\frac{cdg}{e}\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(-\frac{cd^2}{e}+bd-ae\right)g\left(-\frac{g\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{2e(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\int}{}\right.}{3(cf^2-bgf+ag^2)}\right.}{e^2}$$

$$(2Cd-Be)\left(\frac{2\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{3(ef-dg)(cf^2-bgf+ag^2)(f+gx)^{3/2}}+\frac{-g\left(3cf-bg-\frac{cdg}{e}\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(g(3bef-bdg-2aeg)-c\left(3ef^2-\frac{d^2g^2}{e}\right.\right.}{}\right.$$

$\downarrow$ 1292

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int\frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx+$$

$$C\left(\frac{\left(3cf-bg-\frac{cdg}{e}\right)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(-\frac{cd^2}{e}+bd-ae\right)g\left(-\frac{\frac{g\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{2e(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\int}{}\right.}{3(cf^2-bgf+ag^2)}\right.$$

$$\overline{\qquad e^2\qquad}$$

$$(2Cd-Be)\left(\frac{2\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{3(ef-dg)(cf^2-bgf+ag^2)(f+gx)^{3/2}}+\frac{-g\left(3cf-bg-\frac{cdg}{e}\right)\int\frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(g(3bef-bdg-2aeg)-c\left(3ef^2-\frac{d^2g^2}{e}\right.\right.}{}\right.$$

$\downarrow$ 1416

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{cx^2+bx+a}}dx+$$

$$C\left(\frac{\left(3cf-bg-\frac{cdg}{e}\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(-\frac{cd^2}{e}+bd-ae\right)g\left(-\frac{e\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+\cdots}}{\sqrt{cf^2-g(bf-\cdots}}\right.}{\sqrt[4]{c\cdots}}\right.}{\cdots}\right.$$

$$(2Cd-Be)\left(\frac{2\sqrt{d+ex}\sqrt{cx^2+bx+a}\,g^2}{3(ef-dg)(cf^2-bgf+ag^2)(f+gx)^{3/2}}+\frac{-g\left(3cf-bg-\frac{cdg}{e}\right)\int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{cx^2+bx+a}}dx-\left(g(3bef-bdg-2aeg)-c\left(3ef^2-\frac{d^2g^2}{e}\cdots\right.\right.}{\cdots}\right.$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1285
```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[e*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
(Sqrt[a + b*x + c*x^2]/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/(2*(
m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^(m + 1)/(Sqrt[f + g*x]*Sqr
t[a + b*x + c*x^2]))*Simp[2*c*d*f*(m + 1) - e*(a*g + b*f*(2*m + 3)) - 2*(b*
e*g*(2 + m) - c*(d*g*(m + 1) - e*f*(m + 2)))*x - c*e*g*(2*m + 5)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4997 vs. 2(1665) = 3330.

Time = 33.69 (sec) , antiderivative size = 4998, normalized size of antiderivative = 2.81

method	result	size
elliptic	Expression too large to display	4998
default	Expression too large to display	1786054

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b
*x+a)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*e*(A*e^2-B*d*e+C*d^2)
/(d*g-e*f)^2/((x+d/e)*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f
))^(1/2)-2/3/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*f^3)/g*(
A*g^2-B*f*g+C*f^2)/(d*g-e*f)*(c*e*g*x^4+b*e*g*x^3+c*d*g*x^3+c*e*f*x^3+a*e*
g*x^2+b*d*g*x^2+b*e*f*x^2+c*d*f*x^2+a*d*g*x+a*e*f*x+b*d*f*x+a*d*f)^(1/2)/(
x+f/g)^2+2/3*(c*e*g*x^3+b*e*g*x^2+c*d*g*x^2+a*e*g*x+b*d*g*x+a*d*g)/(a*d*g^
3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*f^3)^2*(5*A*a*e*g^4+2*A*b*d*
g^4-7*A*b*e*f*g^3-4*A*c*d*f*g^3+9*A*c*e*f^2*g^2-3*B*a*d*g^4-2*B*a*e*f*g^3+
B*b*d*f*g^3+4*B*b*e*f^2*g^2+B*c*d*f^2*g^2-6*B*c*e*f^3*g+6*C*a*d*f*g^3-C*a*
e*f^2*g^2-4*C*b*d*f^2*g^2-C*b*e*f^3*g+2*C*c*d*f^3*g+3*C*c*e*f^4)/(d*g-e*f)
/((x+f/g)*(c*e*g*x^3+b*e*g*x^2+c*d*g*x^2+a*e*g*x+b*d*g*x+a*d*g))^(1/2)+2*(
(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*(A*e^2-B*d*e+C*d^2)/(a*d*e^2*g-a
*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/(d*g-e*f)^2-(a*e*g+b*e*f)/(a
*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*e*(A*e^2-B*d*e+C*d
^2)/(d*g-e*f)^2-1/3*(A*b*e*g^3+A*c*d*g^3-3*A*c*e*f*g^2-B*b*e*f*g^2-B*c*d*f
*g^2+3*B*c*e*f^2*g+C*b*e*f^2*g+C*c*d*f^2*g-3*C*c*e*f^3)/g/(a*d*g^3-a*e*f*g
^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*f^3)/(d*g-e*f)+1/3/g*(a*e*g^2+b*d*g^2
-b*e*f*g-c*d*f*g+c*e*f^2)*(5*A*a*e*g^4+2*A*b*d*g^4-7*A*b*e*f*g^3-4*A*c*...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x +
f)/(c*e^2*g^3*x^7 + (3*c*e^2*f*g^2 + (2*c*d*e + b*e^2)*g^3)*x^6 + a*d^2*f^
3 + (3*c*e^2*f^2*g + 3*(2*c*d*e + b*e^2)*f*g^2 + (c*d^2 + 2*b*d*e + a*e^2)
*g^3)*x^5 + (c*e^2*f^3 + 3*(2*c*d*e + b*e^2)*f^2*g + 3*(c*d^2 + 2*b*d*e +
a*e^2)*f*g^2 + (b*d^2 + 2*a*d*e)*g^3)*x^4 + (a*d^2*g^3 + (2*c*d*e + b*e^2)
*f^3 + 3*(c*d^2 + 2*b*d*e + a*e^2)*f^2*g + 3*(b*d^2 + 2*a*d*e)*f*g^2)*x^3
+ (3*a*d^2*f*g^2 + (c*d^2 + 2*b*d*e + a*e^2)*f^3 + 3*(b*d^2 + 2*a*d*e)*f^2
*g)*x^2 + (3*a*d^2*f^2*g + (b*d^2 + 2*a*d*e)*f^3)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\,(f+gx)^{\frac{5}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*(f + g*x)**(5/2)*sqrt(a + b*
x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^(5/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^{\frac{5}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(f+g\,x)^{5/2}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(5/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(5/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^{5/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,(gx+f)^{\frac{5}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.7 $\int \frac{(f+gx)^2(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 995

$$\int \frac{(f+gx)^2\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2(6cCd^2-Ce(bd-ae)-5ce(Bd-Ae))\,(ef-dg)^2\sqrt{a+bx+cx^2}}{5ce^3\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$-\frac{2g(4bCeg-c(4Cef-6Cdg+5Beg))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e^3}$$

$$+\frac{2C(f+gx)^2\sqrt{a+bx+cx^2}}{5ce\sqrt{d+ex}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2Ce^3(bd-ae)g^2+ce^2g(9a^2Ce^2g+2abe(10Cef-13Cdg+5Beg)-b^2d(20Cef-9C}{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(4bCe^2(bd-ae)g^2+ceg(ae(10Cef-12Cdg+5Beg)-bd(10Cef-8Cdg+5Beg))+}{}$$

output
```
-2/5*(6*c*C*d^2-C*e*(-a*e+b*d)-5*c*e*(-A*e+B*d))*(-d*g+e*f)^2*(c*x^2+b*x+a
)^(1/2)/c/e^3/(a*e^2-b*d*e+c*d^2)/(e*x+d)^(1/2)-2/15*g*(4*b*C*e*g-c*(5*B*e
*g-6*C*d*g+4*C*e*f))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+2/5*C*(g*x+
f)^2*(c*x^2+b*x+a)^(1/2)/c/e/(e*x+d)^(1/2)-1/15*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(8*b^2*C*e^3*(-a*e+b*d)*g^2+c*e^2*g*(9*a^2*C*e^2*g+2*a*b*e*(5*B*e*g-13*C*
d*g+10*C*e*f)-b^2*d*(10*B*e*g-9*C*d*g+20*C*e*f))-c^3*(2*C*d^2*(24*d^2*g^2-
40*d*e*f*g+15*e^2*f^2)+5*e*(3*A*e*(2*d^2*g^2-2*d*e*f*g+e^2*f^2)-B*d*(8*d^2
*g^2-12*d*e*f*g+3*e^2*f^2)))+c^2*e*(b*d*(15*e*g*(A*e*g-B*d*g+2*B*e*f)+C*(1
6*d^2*g^2-30*d*e*f*g+15*e^2*f^2))-a*e*(5*e*g*(3*A*e*g-5*B*d*g+6*B*e*f)+C*(
24*d^2*g^2-50*d*e*f*g+15*e^2*f^2))))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a
*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2
),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^
4/(a*e^2-b*d*e+c*d^2)/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(
c*x^2+b*x+a)^(1/2)-2/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(4*b*C*e^2*(-a*e+b*d)*g
^2+c*e*g*(a*e*(5*B*e*g-12*C*d*g+10*C*e*f)-b*d*(5*B*e*g-8*C*d*g+10*C*e*f))+
c^2*(2*C*d*(24*d^2*g^2-40*d*e*f*g+15*e^2*f^2)-5*e*(6*A*e*g*(-d*g+e*f)+B*(8
*d^2*g^2-12*d*e*f*g+3*e^2*f^2))))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))
*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b
)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^4/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 38.38 (sec) , antiderivative size = 23948, normalized size of antiderivative = 24.07

$$\int \frac{(f+gx)^2\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x +
c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 6.94 (sec) , antiderivative size = 1052, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.268, Rules used = {2181, 27, 25, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int \frac{C\left(-\frac{cd^2}{e}+bd-ae\right)g^2x^3-\frac{\left(cd^2-bed+ae^2\right)g(2Cef-Cdg+Beg)x^2}{e^2}-\frac{\left(2cCd^2(ef-dg)^2-e(bd-ae)\left(C(ef-dg)^2+eg(2Bef-Bdg+Aeg)\right)-ce\left(Bd\left(e^2f^2-4\right.\right.\right.}{e^3}}{2\sqrt{d+ex}\sqrt{c}}}{ae^2-bde}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{-C\left(-\frac{cd^2}{e}+bd-ae\right)g^2x^3+\frac{\left(cd^2-bed+ae^2\right)g(2Cef-Cdg+Beg)x^2}{e^2}-\frac{\left(e(bd-ae)\left(C(ef-dg)^2+eg(2Bef-Bdg+Aeg)\right)-c\left(2Cd^2(ef-dg)^2-e\left(Bd\left(e^2f^2\right.\right.\right.\right.}{e^3}}{\sqrt{d+ex}\sqrt{c}}}{ae^2-bd}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-C\left(-\frac{cd^2}{e}+bd-ae\right)g^2x^3+\frac{\left(cd^2-bed+ae^2\right)g(2Cef-Cdg+Beg)x^2}{e^2}+\frac{\left(2cCd^2(ef-dg)^2-e(bd-ae)\left(C(ef-dg)^2+eg(2Bef-Bdg+Aeg)\right)-ce\left(Bd\left(e^2f^2-4de\right.\right.\right.}{e^3}}{\sqrt{d+ex}\sqrt{cx^2+b}}}{ae^2-bde+}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{2184}$$

$$2\int \frac{b^2Ceg^2d^3+b\left(2aCde^2g^2+cCd\left(5e^2f^2-10degf+4d^2g^2\right)-5ce\left(B(ef-dg)^2+Aeg(2ef-dg)\right)\right)d+e\left(cd^2-bed+ae^2\right)g(10cCef-12cCdg+5Bceg-4bCeg)x^2+5Ace}{}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 27

$$\int \frac{b^2Ceg^2d^3+b\left(2aCde^2g^2+cCd\left(5e^2f^2-10degf+4d^2g^2\right)-5ce\left(B(ef-dg)^2+Aeg(2ef-dg)\right)\right)d+e\left(cd^2-bed+ae^2\right)g(10cCef-12cCdg+5Bceg-4bCeg)x^2+5Ace^3}{}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 2184

$$\frac{2C\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}\sqrt{cx^2+bx+a}g^2}{5ce^3}+\frac{2\left(cd^2-bed+ae^2\right)g\sqrt{d+ex}\sqrt{cx^2+bx+a}(5Bceg-4bCeg+2cC(5ef-6dg))}{3c}+2\int -\frac{e^2\left(4Cd^2e^2g^2b^3-5cd^2eg(\right.}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)(ef-dg)^2\sqrt{cx^2+bx+a}}{e^3\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$\downarrow$ 27

$$\frac{2C\left(cd^2-bed+ae^2\right)(d+ex)^{3/2}\sqrt{cx^2+bx+a}g^2}{5ce^3}+\frac{2\left(cd^2-bed+ae^2\right)g(5Bceg-4bCeg+2cC(5ef-6dg))\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\int \frac{4Cd^2e^2g^2b^3-5cd^2eg(2Cef-}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)(ef-dg)^2\sqrt{cx^2+bx+a}}{e^3\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$\downarrow$ 1269

$$\frac{2g\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ae^2-bde+cd^2\right)(-4bCeg+5Bceg+2cC(5ef-6dg))}{3c}-\frac{\left(ce^2g\left(9a^2Ce^2g+2abe(5Beg-13Cdg+10Cef)+b^2(-d)(10Beg-9Cdg+20Cef)\right)+c^2e\right.}{}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)}{e^3\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$\frac{\frac{2C(cd^2-bed+ae^2)(d+ex)^{3/2}\sqrt{cx^2+bx+a}g^2}{5ce^3}+\frac{\frac{2\left(cd^2-bed+ae^2\right)g(5Bceg-4bCeg+2cC(5ef-6dg))\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\Big(}{}}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)(ef-dg)^2\sqrt{cx^2+bx+a}}{e^3\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$\big\downarrow$ 321

$$\frac{\frac{2C(cd^2-bed+ae^2)(d+ex)^{3/2}\sqrt{cx^2+bx+a}g^2}{5ce^3}+\frac{\frac{2\left(cd^2-bed+ae^2\right)g(5Bceg-4bCeg+2cC(5ef-6dg))\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)\Big(}{}}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)(ef-dg)^2\sqrt{cx^2+bx+a}}{e^3\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

$\big\downarrow$ 327

$$\frac{\frac{2C(cd^2-bed+ae^2)(d+ex)^{3/2}\sqrt{cx^2+bx+a}g^2}{5ce^3}+\frac{\frac{2\left(cd^2-bed+ae^2\right)g(5Bceg-4bCeg+2cC(5ef-6dg))\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{\sqrt{2}\sqrt{b^2-4ac}\Big(-\Big(\Big(2C\Big(15e^2f^2-}{}}{}}{}$$

$$\frac{2\left(Cd^2-e(Bd-Ae)\right)(ef-dg)^2\sqrt{cx^2+bx+a}}{e^3\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}$$

input

```
Int[((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])/(e^3*(c*d
^2 - b*d*e + a*e^2)*Sqrt[d + e*x]) + ((2*C*(c*d^2 - b*d*e + a*e^2)*g^2*(d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e^3) + ((2*(c*d^2 - b*d*e + a*e^2
)*g*(5*B*c*e*g - 4*b*C*e*g + 2*c*C*(5*e*f - 6*d*g))*Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])/(3*c) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^2*C*e^3*(b*d - a*e
)*g^2 + c*e^2*g*(9*a^2*C*e^2*g + 2*a*b*e*(10*C*e*f - 13*C*d*g + 5*B*e*g) -
 b^2*d*(20*C*e*f - 9*C*d*g + 10*B*e*g)) - c^3*(2*C*d^2*(15*e^2*f^2 - 40*d*
e*f*g + 24*d^2*g^2) + 5*e*(3*A*e*(e^2*f^2 - 2*d*e*f*g + 2*d^2*g^2) - B*d*(
3*e^2*f^2 - 12*d*e*f*g + 8*d^2*g^2))) + c^2*e*(b*d*(15*e*g*(2*B*e*f - B*d*
g + A*e*g) + C*(15*e^2*f^2 - 30*d*e*f*g + 16*d^2*g^2)) - a*e*(5*e*g*(6*B*e
*f - 5*B*d*g + 3*A*e*g) + C*(15*e^2*f^2 - 50*d*e*f*g + 24*d^2*g^2))))*Sqrt
[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b
^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x
))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(4*b*C*e^2*(b*d - a*e)*g^2 + c
*e*g*(a*e*(10*C*e*f - 12*C*d*g + 5*B*e*g) - b*d*(10*C*e*f - 8*C*d*g + 5*B*
e*g)) + c^2*(2*C*d*(15*e^2*f^2 - 40*d*e*f*g + 24*d^2*g^2) - 5*e*(6*A*e*g*(
e*f - d*g) + B*(3*e^2*f^2 - 12*d*e*f*g + 8*d^2*g^2))))*Sqrt[(c*(d + e*x))/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 ...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 11.27 (sec) , antiderivative size = 1643, normalized size of antiderivative = 1.65

method	result	size
elliptic	Expression too large to display	1643
risch	Expression too large to display	2058
default	Expression too large to display	31952

input
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/(a*e^2-b*d*e+c*d^2)/e^4*(A*d^2*e^2*g^2-2*A*d*e^3*f*g+A*e^4*f
^2-B*d^3*e*g^2+2*B*d^2*e^2*f*g-B*d*e^3*f^2+C*d^4*g^2-2*C*d^3*e*f*g+C*d^2*e
^2*f^2)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2/5*C*g^2/c/e^2*x*(c*e*x^3+b*e
*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(1/e^2*g*(B*e*g-C*d*g+2*C*e*f)-2/5
/c/e^2*(2*b*e+2*c*d)*C*g^2)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^
(1/2)+2*(-(A*d*e^2*g^2-2*A*e^3*f*g-B*d^2*e*g^2+2*B*d*e^2*f*g-B*e^3*f^2+C*d
^3*g^2-2*C*d^2*e*f*g+C*d*e^2*f^2)/e^4-1/e^4*(b*e-c*d)*(A*d^2*e^2*g^2-2*A*d
*e^3*f*g+A*e^4*f^2-B*d^3*e*g^2+2*B*d^2*e^2*f*g-B*d*e^3*f^2+C*d^4*g^2-2*C*d
^3*e*f*g+C*d^2*e^2*f^2)/(a*e^2-b*d*e+c*d^2)+b/e^3/(a*e^2-b*d*e+c*d^2)*(A*d
^2*e^2*g^2-2*A*d*e^3*f*g+A*e^4*f^2-B*d^3*e*g^2+2*B*d^2*e^2*f*g-B*d*e^3*f^2
+C*d^4*g^2-2*C*d^3*e*f*g+C*d^2*e^2*f^2)-2/5*a/c*d/e^2*C*g^2-2/3*(1/e^2*g*(
B*e*g-C*d*g+2*C*e*f)-2/5/c/e^2*(2*b*e+2*c*d)*C*g^2)/c/e*(1/2*a*e+1/2*b*d))
*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*Ellip
ticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*
a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(1/e^3*(
A*e^2*g^2-B*d*e*g^2+2*B*e^2*f*g+C*d^2*g^2-2*C*d*e*f*g+C*e^2*f^2)+1/e^3*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2367 vs. $2(937) = 1874$.

Time = 0.17 (sec) , antiderivative size = 2367, normalized size of antiderivative = 2.38

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="fricas")
```

output
```
-2/45*((15*(2*C*c^4*d^4*e^2 - (2*C*b*c^3 + B*c^4)*d^3*e^3 - (C*b^2*c^2 + 2
*A*c^4 - 2*(2*C*a + B*b)*c^3)*d^2*e^4 + (C*a*b*c^2 - (3*B*a - A*b)*c^3)*d*
e^5)*f^2 - 10*(8*C*c^4*d^5*e - (7*C*b*c^3 + 6*B*c^4)*d^4*e^2 - (2*C*b^2*c^
2 - 3*A*c^4 - (11*C*a + 6*B*b)*c^3)*d^3*e^3 - (2*C*b^3*c + 6*(2*B*a + A*b)
*c^3 - (7*C*a*b + 3*B*b^2)*c^2)*d^2*e^4 + (2*C*a*b^2*c + 9*A*a*c^3 - 3*(C*
a^2 + B*a*b)*c^2)*d*e^5)*f*g + (48*C*c^4*d^6 - 40*(C*b*c^3 + B*c^4)*d^5*e
- 5*(2*C*b^2*c^2 - 6*A*c^4 - (12*C*a + 7*B*b)*c^3)*d^4*e^2 - 5*(C*b^3*c +
(11*B*a + 6*A*b)*c^3 - 2*(2*C*a*b + B*b^2)*c^2)*d^3*e^3 - (8*C*b^4 - 60*A*
a*c^3 + (18*C*a^2 + 35*B*a*b + 15*A*b^2)*c^2 - 2*(17*C*a*b^2 + 5*B*b^3)*c)
*d^2*e^4 + (8*C*a*b^3 + 15*(B*a^2 + A*a*b)*c^2 - (21*C*a^2*b + 10*B*a*b^2)
*c)*d*e^5)*g^2 + (15*(2*C*c^4*d^3*e^3 - (2*C*b*c^3 + B*c^4)*d^2*e^4 - (C*b
^2*c^2 + 2*A*c^4 - 2*(2*C*a + B*b)*c^3)*d*e^5 + (C*a*b*c^2 - (3*B*a - A*b)
*c^3)*e^6)*f^2 - 10*(8*C*c^4*d^4*e^2 - (7*C*b*c^3 + 6*B*c^4)*d^3*e^3 - (2*
C*b^2*c^2 - 3*A*c^4 - (11*C*a + 6*B*b)*c^3)*d^2*e^4 - (2*C*b^3*c + 6*(2*B*
a + A*b)*c^3 - (7*C*a*b + 3*B*b^2)*c^2)*d*e^5 + (2*C*a*b^2*c + 9*A*a*c^3 -
 3*(C*a^2 + B*a*b)*c^2)*e^6)*f*g + (48*C*c^4*d^5*e - 40*(C*b*c^3 + B*c^4)*
d^4*e^2 - 5*(2*C*b^2*c^2 - 6*A*c^4 - (12*C*a + 7*B*b)*c^3)*d^3*e^3 - 5*(C*
b^3*c + (11*B*a + 6*A*b)*c^3 - 2*(2*C*a*b + B*b^2)*c^2)*d^2*e^4 - (8*C*b^4
 - 60*A*a*c^3 + (18*C*a^2 + 35*B*a*b + 15*A*b^2)*c^2 - 2*(17*C*a*b^2 + 5*B
*b^3)*c)*d*e^5 + (8*C*a*b^3 + 15*(B*a^2 + A*a*b)*c^2 - (21*C*a^2*b + 10...
```

Sympy [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**2*(C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x
)
```

output
```
Integral((f + g*x)**2*(A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x +
c*x**2)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)^2}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)^2/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(
3/2)), x)
```

Giac [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)^2}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)^2/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(
3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(f+g\,x)^2\,(C\,x^2+B\,x+A)}{(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/
2)),x)
```

output
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{(f+gx)^2(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)^2\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.8 $\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 646

$$\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2(Cd^2-e(Bd-Ae))(ef-dg)\sqrt{a+bx+cx^2}}{e^2(cd^2-bde+ae^2)\sqrt{d+ex}}+\frac{2Cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce^2}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2bCe^2(bd-ae)g-ce(3bd(Cef-Cdg+Beg)-ae(3Cef-5Cdg+3Beg))+c^2(C(6d^2e}{3c^2e^3(cd^2-bde+ae^2}$$

$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(Ce(bd-ae)g-2cCd(3ef-4dg)+3ce(Bef-2Bdg+Aeg))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+}{b}}}{3c^2e^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(-d*g+e*f)*(c*x^2+b*x+a)^(1/2)/e^2/(a*e^2-b*d*e+c*
d^2)/(e*x+d)^(1/2)+2/3*C*g*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/3*2^(
1/2)*(-4*a*c+b^2)^(1/2)*(2*b*C*e^2*(-a*e+b*d)*g-c*e*(3*b*d*(B*e*g-C*d*g+C*
e*f)-a*e*(3*B*e*g-5*C*d*g+3*C*e*f))+c^2*(C*(-8*d^3*g+6*d^2*e*f)-3*e*(B*d*(
-2*d*g+e*f)-A*e*(-d*g+e*f))))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2)
)^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(
-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^3/(a*e^
2-b*d*e+c*d^2)/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b
*x+a)^(1/2)+2/3*2^(1/2)*(-4*a*c+b^2)^(1/2)*(C*e*(-a*e+b*d)*g-2*c*C*d*(-4*d
*g+3*e*f)+3*c*e*(A*e*g-2*B*d*g+B*e*f))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(
b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.51 (sec) , antiderivative size = 11109, normalized size of antiderivative = 17.20

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x + c*
x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2181, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 2181$$

$$-\frac{2\int \frac{C\left(-\frac{cd^2}{e}+bd-ae\right)gx^2-\frac{\left(2cC(ef-dg)d^2-e(bd-ae)(Cef-Cdg+Beg)-ce(Bd(ef-2dg)-Ae(ef-dg))\right)x}{e^2}+\frac{e(a(Cd-Be)(ef-dg)-Ae(cdf+aeg))-bd(C\cdots}{e^2}\cdots}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}\cdots}{ae^2-bde+cd^2\cdots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{C\left(-\frac{cd^2}{e}+bd-ae\right)gx^2-\frac{\left(2cC(ef-dg)d^2-e(bd-ae)(Cef-Cdg+Beg)-ce(Bd(ef-2dg)-Ae(ef-dg))\right)x}{e^2}+\frac{e(a(Cd-Be)(ef-dg)-Ae(cdf+aeg))-bd(Cd(e\cdots}{e^2}\cdots}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}\cdots}{ae^2-bde+cd^2\cdots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 2184$$

$$-\frac{\frac{2\int -\frac{b^2Cegd^2+bc(Cd(3ef-4dg)-3e(Bef-Bdg+Aeg))d+e\left(3Ace(cdf+aeg)-a\left(aCge^2-3Bc(ef-dg)e+cCd(3ef-2dg)\right)\right)+\left(\left(2Cd^2(3ef-4dg)-3e(Bd(ef-2dg\cdots}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}\cdots}{3ce^2}\cdots}{ae^2-bde+\cdots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{b^2Cegd^2+bc(Cd(3ef-4dg)-3e(Bef-Bdg+Aeg))d+e\left(3Ace(cdf+aeg)-a\left(aCge^2-3Bc(ef-dg)e+cCd(3ef-2dg)\right)\right)+\left(\left(C\left(6d^2ef-8d^3g\right)-3e(Bd(ef-2dg\cdots}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}\cdots}{3ce^2}\cdots}{ae^2-bde+\cdots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 1269$$

$$-\frac{-\frac{\frac{\left(-ce(3bd(Beg-Cdg+Cef)-ae(3Beg-5Cdg+3Cef))+2bCe^2g(bd-ae)+c^2\left(2Cd^2(3ef-4dg)-3e(Bd(ef-2dg)-Ae(ef-dg))\right)\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}+\frac{\left(ae^2-b\cdots\right.}{\cdots}}{3ce^2}\cdots}{ae^2-bde+c\cdots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 1172

$$-\frac{2\left(Cd^2-e(Bd-Ae)\right)\sqrt{cx^2+bx+a}(ef-dg)}{e^2\left(cd^2-bed+ae^2\right)\sqrt{d+ex}}-\frac{-\frac{2C\left(cd^2-bed+ae^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+ag}}{3ce^2}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\left(cd^2-bed+ae^2\right)(Ce(bd-ae)g-2cCd(3ef-4dg)+3ce(Bef-2Bdg+Aeg))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)\dots}}\dots}{ce\sqrt{d+ex}\sqrt{cx^2+\dots}}}{\dots}$$

$\downarrow$ 321

$$-\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(3bd(Beg-Cdg+Cef)-ae(3Beg-5Cdg+3Cef))+2bCe^2g(bd-ae)+c^2\left(2Cd^2(3ef-4dg)-3e(Bd(ef-2dg)-Ae(e\dots\right.\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\dots}{\dots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$\downarrow$ 327

$$-\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(-ce(3bd(Beg-Cdg+Cef)-ae(3Beg-5Cdg+3Cef))+\dots\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\dots}{\dots}$$

$$\frac{2\sqrt{a+bx+cx^2}(ef-dg)\left(Cd^2-e(Bd-Ae)\right)}{e^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input

```
Int[((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*(e*f - d*g)*Sqrt[a + b*x + c*x^2])/(e^2*(c*d^2
 - b*d*e + a*e^2)*Sqrt[d + e*x]) - ((-2*C*(c*d^2 - b*d*e + a*e^2)*g*Sqrt[d
 + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e^2) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*
b*C*e^2*(b*d - a*e)*g - c*e*(3*b*d*(C*e*f - C*d*g + B*e*g) - a*e*(3*C*e*f
- 5*C*d*g + 3*B*e*g)) + c^2*(2*C*d^2*(3*e*f - 4*d*g) - 3*e*(B*d*(e*f - 2*d
*g) - A*e*(e*f - d*g))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)*(
C*e*(b*d - a*e)*g - 2*c*C*d*(3*e*f - 4*d*g) + 3*c*e*(B*e*f - 2*B*d*g + A*e
*g))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))/(
3*c*e^2))/(c*d^2 - b*d*e + a*e^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1191 vs. 2(588) = 1176.

Time = 10.05 (sec) , antiderivative size = 1192, normalized size of antiderivative = 1.85

method	result	size
elliptic	Expression too large to display	1192
risch	Expression too large to display	1746
default	Expression too large to display	15929

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*(c*e*x^
2+b*e*x+a*e)/(a*e^2-b*d*e+c*d^2)/e^3*(A*d*e^2*g-A*e^3*f-B*d^2*e*g+B*d*e^2*
f+C*d^3*g-C*d^2*e*f)/((x+d/e)*(c*e*x^2+b*e*x+a*e))^(1/2)+2/3*C*g/e^2/c*(c*
e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*((A*e^2*g-B*d*e*g+B*e^2*f+C
*d^2*g-C*d*e*f)/e^3+1/e^3*(b*e-c*d)*(A*d*e^2*g-A*e^3*f-B*d^2*e*g+B*d*e^2*f
+C*d^3*g-C*d^2*e*f)/(a*e^2-b*d*e+c*d^2)-b/e^2/(a*e^2-b*d*e+c*d^2)*(A*d*e^2
*g-A*e^3*f-B*d^2*e*g+B*d*e^2*f+C*d^3*g-C*d^2*e*f)-2/3*C*g/e^2/c*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(1/e^2*(B*e*g-C*d*g+C*e*f)-1/e^2*c*(A*d*e^2*g-A*e^3*f-B*d^2*e*g+B*d*e^2*f+
C*d^3*g-C*d^2*e*f)/(a*e^2-b*d*e+c*d^2)-2/3*C*g/e^2/c*(b*e+c*d))*(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(598) = 1196$.

Time = 0.16 (sec) , antiderivative size = 1317, normalized size of antiderivative = 2.04

$$\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
-2/9*(sqrt(c*e)*(3*(2*C*c^3*d^4*e - (2*C*b*c^2 + B*c^3)*d^3*e^2 - (C*b^2*c
 + 2*A*c^3 - 2*(2*C*a + B*b)*c^2)*d^2*e^3 + (C*a*b*c - (3*B*a - A*b)*c^2)*
d*e^4)*f - (8*C*c^3*d^5 - (7*C*b*c^2 + 6*B*c^3)*d^4*e - (2*C*b^2*c - 3*A*c
^3 - (11*C*a + 6*B*b)*c^2)*d^3*e^2 - (2*C*b^3 + 6*(2*B*a + A*b)*c^2 - (7*C
*a*b + 3*B*b^2)*c)*d^2*e^3 + (2*C*a*b^2 + 9*A*a*c^2 - 3*(C*a^2 + B*a*b)*c)
*d*e^4)*g + (3*(2*C*c^3*d^3*e^2 - (2*C*b*c^2 + B*c^3)*d^2*e^3 - (C*b^2*c +
 2*A*c^3 - 2*(2*C*a + B*b)*c^2)*d*e^4 + (C*a*b*c - (3*B*a - A*b)*c^2)*e^5)
*f - (8*C*c^3*d^4*e - (7*C*b*c^2 + 6*B*c^3)*d^3*e^2 - (2*C*b^2*c - 3*A*c^3
 - (11*C*a + 6*B*b)*c^2)*d^2*e^3 - (2*C*b^3 + 6*(2*B*a + A*b)*c^2 - (7*C*a
*b + 3*B*b^2)*c)*d*e^4 + (2*C*a*b^2 + 9*A*a*c^2 - 3*(C*a^2 + B*a*b)*c)*e^5
)*g)*x)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c
^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2
*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*sqrt(
c*e)*(3*(2*C*c^3*d^3*e^2 - (C*b*c^2 + B*c^3)*d^2*e^3 + (C*a*c^2 + A*c^3)*d
*e^4)*f - (8*C*c^3*d^4*e - 3*(C*b*c^2 + 2*B*c^3)*d^3*e^2 - (2*C*b^2*c - 3*
A*c^3 - (5*C*a + 3*B*b)*c^2)*d^2*e^3 + (2*C*a*b*c - 3*B*a*c^2)*d*e^4)*g +
(3*(2*C*c^3*d^2*e^3 - (C*b*c^2 + B*c^3)*d*e^4 + (C*a*c^2 + A*c^3)*e^5)*f -
 (8*C*c^3*d^3*e^2 - 3*(C*b*c^2 + 2*B*c^3)*d^2*e^3 - (2*C*b^2*c - 3*A*c^3 -
 (5*C*a + 3*B*b)*c^2)*d*e^4 + (2*C*a*b*c - 3*B*a*c^2)*e^5)*g)*x)*weierstra
ssZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*...
```

Sympy [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{\frac{3}{2}}\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)*(C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)*(A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x
**2)), x)
```

Maxima [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/
2)), x)
```

Giac [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/
2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(f+g\,x)\,\left(C\,x^2+B\,x+A\right)}{{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
),x)
```

output
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)\,(C\,x^2+Bx+A)}{(ex+d)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.9 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 486

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd^2-Ce(bd-ae)-ce(Bd-Ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\right.}{ce^2\left(cd^2-bde+ae^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(2Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2\cdots}}{2cd-\left(b+\sqrt{\cdots}\right.}\right.}{ce^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/e/(a*e^2-b*d*e+c*d^2)/(e*x+d)^
(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2-C*e*(-a*e+b*d)-c*e*(-A*e+B*d))
*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c
*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b
+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e^2/(a*e^2-b*d*e+c*d^2)/(c*(e*x+d)/(2*c*
d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a*c+b
^2)^(1/2)*(-B*e+2*C*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*
(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b
^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2))/c/e^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 772, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(-e^2(Cd^2+e(-Bd+Ae))\,(a+x(b+cx))+\frac{e^2(2cCd^2+Ce(-bd+ae)+ce(-}{c}\right.}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(-(e^2*(C*d^2 + e*(-(B*d) + A*e))*(a + x*(b + c*x))) + (e^2*(2*c*C*d^2
+ C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))*(a + x*(b + c*x)))/c - ((I/2)*(
d + e*x)^(3/2)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[1 + (2*(c*d^2 + e*(-(b*d) + a*e)))/
((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2])*(2*c*C*d^2 + C*e*(-(b*d) + a*e) + c*e*(-(B*d) + A*e))
*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 -
4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + (-(b^2*C*d*e^2) +
 2*a*c*C*d*e^2 - 2*a*B*c*e^3 - 2*c*C*d^2*Sqrt[(b^2 - 4*a*c)*e^2] + B*c*d*e
*Sqrt[(b^2 - 4*a*c)*e^2] - a*C*e^2*Sqrt[(b^2 - 4*a*c)*e^2] - A*c*e^2*(2*c*
d + Sqrt[(b^2 - 4*a*c)*e^2]) + b*(B*c*d*e^2 + A*c*e^3 + a*C*e^3 + C*d*e*Sq
rt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e +
 a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]
))]))/(Sqrt[2]*c*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])])))/(e^3*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[d + e*x]*Sqrt[a +
 x*(b + c*x)])
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2181, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2181}$$

$$-\frac{2\int -\frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{2e\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{bd(Cd-Be)+e(Acd-aCd+aBe)-e\left(-\frac{2cCd^2}{e}+Bcd+bCd-Ace-aCe\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{-\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx-\frac{(2Cd-Be)\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{e\left(ae^2-bde+cd^2\right)}-\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\cdots}}{\cdots}}{e\left(ae^2-bde+\cdots\right)}\cdots$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 321$$

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\cdots}}{\cdots}}{e\left(ae^2-bde+c\cdots\right)}\cdots$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \; 327$$

$$\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-aCe-Ace+bCd+Bcd-\frac{2cCd^2}{e}\right)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2\cdots}}{\cdots}}{e\left(ae^2-bde\cdots\right)}\cdots$$

$$\frac{2\sqrt{a+bx+cx^2}\left(Cd^2-e(Bd-Ae)\right)}{e\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*(C*d^2 - e*(B*d - A*e))*Sqrt[a + b*x + c*x^2])/(e*(c*d^2 - b*d*e + a*e
^2)*Sqrt[d + e*x]) + (-((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(B*c*d + b*C*d - (2*c*C
*d^2)/e - A*c*e - a*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)])/(c*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a
+ b*x + c*x^2])) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*C*d - B*e)*(c*d^2 - b*d
*e + a*e^2)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x
^2]))/(e*(c*d^2 - b*d*e + a*e^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2181
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> With[{Qx = PolynomialQuotient[Pq, d + e*x, x], R = Polynomi
alRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + b*x + c*x^2)
^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Simp[1/((m + 1)*(c*d^2 -
b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*ExpandToSum[(m
+ 1)*(c*d^2 - b*d*e + a*e^2)*Qx + c*d*R*(m + 1) - b*e*R*(m + p + 2) - c*e*R
*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d, e, p}, x] && PolyQ[Pq,
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(438) = 876$.

Time = 5.76 (sec) , antiderivative size = 965, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(-\frac{2\left(ce\,x^2+bxe+ae\right)\left(A\,e^2-Bde+C\,d^2\right)}{\left(a\,e^2-bde+c\,d^2\right)e^2\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bxe+ae\right)}}+\frac{2\left(\frac{Be-Cd}{e^2}-\frac{(be-cd)\left(A\,e^2-Bde+C\,d^2\right)}{e^2\left(a\,e^2-bde+c\,d^2\right)}+\frac{b\left(A\,e^2-Bde+C\,d^2\right)}{e\left(a\,e^2-bde+c\,d^2\right)}\right)}{\cdots}\right.}{\cdots}$
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/(a*e^2-b*d*e+c*d^2)/e^2*(A*e^2-B*d*e+C*d^2)/((x+d/e)*(c*e*x^
2+b*e*x+a*e))^(1/2)+2*((B*e-C*d)/e^2-1/e^2*(b*e-c*d)*(A*e^2-B*d*e+C*d^2)/(
a*e^2-b*d*e+c*d^2)+b/e/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2))*(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e+c/e*(A*e^2-B*d
*e+C*d^2)/(a*e^2-b*d*e+c*d^2))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1
/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^
2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*
c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
-2/3*((2*C*c^2*d^4 - (2*C*b*c + B*c^2)*d^3*e - (C*b^2 + 2*A*c^2 - 2*(2*C*a
 + B*b)*c)*d^2*e^2 + (C*a*b - (3*B*a - A*b)*c)*d*e^3 + (2*C*c^2*d^3*e - (2
*C*b*c + B*c^2)*d^2*e^2 - (C*b^2 + 2*A*c^2 - 2*(2*C*a + B*b)*c)*d*e^3 + (C
*a*b - (3*B*a - A*b)*c)*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c
*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d^3*e - (C*b*c + B*c^2)*d^2*e^2 + (C
*a*c + A*c^2)*d*e^3 + (2*C*c^2*d^2*e^2 - (C*b*c + B*c^2)*d*e^3 + (C*a*c +
A*c^2)*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3
*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^
2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(
3*c*e*x + c*d + b*e)/(c*e))) + 3*(C*c^2*d^2*e^2 - B*c^2*d*e^3 + A*c^2*e^4)
*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2
*d*e^5 + (c^3*d^2*e^4 - b*c^2*d*e^5 + a*c^2*e^6)*x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.10 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 748

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx=-\frac{2(Cd^2-e(Bd-Ae))\sqrt{a+bx+cx^2}}{(cd^2-bde+ae^2)(ef-dg)\sqrt{d+ex}}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(Cd^2-e(Bd-Ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e(cd^2-bde+ae^2)(ef-dg)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$-\frac{4\sqrt{2}\sqrt{b^2-4ac}(Cf^2-Bfg+Ag^2)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g},\operatorname{arcs}\right.}{g\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(ef-dg)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-2*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)
/(e*x+d)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(C*d^2-e*(-A*e+B*d))*(e*x+d)^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*
c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2
)^(1/2))*e))^(1/2))/e/(a*e^2-b*d*e+c*d^2)/(-d*g+e*f)/(c*(e*x+d)/(2*c*d-(b+
(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(
1/2)*C*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a
)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)
*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))
/c/e/g/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*g
^2-B*f*g+C*f^2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*g/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g
),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/g/(2*c
*f-(b+(-4*a*c+b^2)^(1/2))*g)/(-d*g+e*f)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.40 (sec) , antiderivative size = 980, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{i(d+ex)\sqrt{1-\frac{2(cd^2+e(-bd+ae))}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{2+\frac{4(cd^2+e(-bd}{\left(-2cd+be+\sqrt{(b^2-4a}\right.}}}{}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
((I/2)*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + S
qrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e))
)/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*c*d - b*e + Sq
rt[(b^2 - 4*a*c)*e^2])*(C*d^2 + e*(-(B*d) + A*e))*g*(-(e*f) + d*g)*Ellipti
cE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] - g*(4*a*C*d*e^3*f - 2*a*B*
e^4*f - C*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2]*f + B*d*e^2*Sqrt[(b^2 - 4*a*c)*e^2
]*f - A*e^3*Sqrt[(b^2 - 4*a*c)*e^2]*f - 2*a*C*d^2*e^2*g + 2*a*A*e^4*g + C*
d^3*Sqrt[(b^2 - 4*a*c)*e^2]*g - B*d^2*e*Sqrt[(b^2 - 4*a*c)*e^2]*g + A*d*e^
2*Sqrt[(b^2 - 4*a*c)*e^2]*g + 2*c*d*e*(C*d^2*f - B*d^2*g + A*e*(-(e*f) + 2
*d*g)) + b*e*(C*d^2*(-3*e*f + d*g) + e*(A*e*(e*f - 3*d*g) + B*d*(e*f + d*g
))))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^
2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + 2*e^2*(c*d^2
+ e*(-(b*d) + a*e))*(C*f^2 + g*(-(B*f) + A*g))*EllipticPi[((-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])*(e*f - d*g))/(2*(c*d^2 + e*(-(b*d) + a*e))*g), I
*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(e^2*(c*d^2 + e*(-(b*d) + ...
```

Rubi [A] (warning: unable to verify)

Time = 4.27 (sec) , antiderivative size = 1065, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {2154, 1237, 27, 1269, 1172, 321, 327, 1288, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1237}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-\frac{2\int \frac{cCdf-Bcdg+bCdg-aCeg+c(Cef+Cdg-Beg)x}{2g^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}+\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-\frac{\int \frac{C(bd-ae)g+cd(Cf-Bg)+c(Cef+Cdg-Beg)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{g^2\left(ae^2-bde+cd^2\right)}+\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1269}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-\frac{\frac{c(-Beg+Cdg+Cef)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{Cg\left(ae^2-bde+cd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{g^2\left(ae^2-bde+cd^2\right)}+\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{1172}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-Beg+Cdg+Cef)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}Cg\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{}}{g^2\left(ae^2-bde+cd^2\right)}$$

$$\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{321}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-$$
$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-Beg+Cdg+Cef)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}Cg\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}}{}}{g^2\left(ae^2-bde+cd^2\right)}$$
$$\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 327}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx-$$
$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-Beg+Cdg+Cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}Cg\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a\right.}{b}}}{}}{g^2\left(ae^2-bde+cd^2\right)}$$
$$\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 1288}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \left(\frac{e}{(ef-dg)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}-\frac{g}{(ef-dg)\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}\right)dx$$
$$\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(-Beg+Cdg+Cef)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}Cg\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a\right.}{b}}}{}}{g^2\left(ae^2-bde+cd^2\right)}$$
$$\frac{2\sqrt{a+bx+cx^2}(-Beg+Cdg+Cef)}{g^2\sqrt{d+ex}\left(ae^2-bde+cd^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2\sqrt{cx^2+bx+a}(Cef+Cdg-Beg)}{(cd^2-bed+ae^2)\,g^2\sqrt{d+ex}}-\frac{\sqrt{2}\sqrt{b^2-4ac}(Cef+Cdg-Beg)\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}C\left(cd^2-bed+\right.}{}}{(cd^2-bed+ae^2)\,g^2}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(-\frac{2\sqrt{cx^2+bx+a}e^2}{(cd^2-bed+ae^2)(ef-dg)\sqrt{d+ex}}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\right.}{(cd^2-bed+ae^2)(ef-dg)\sqrt{\frac{}{2cd}}}\right.$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(C*e*f + C*d*g - B*e*g)*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*e + a*e^2)
*g^2*Sqrt[d + e*x]) - ((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(C*e*f + C*d*g - B*e*g)*
Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSi
n[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(e*Sqrt[(c*(d + e
*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt
[2]*Sqrt[b^2 - 4*a*c]*C*(c*d^2 - b*d*e + a*e^2)*g*Sqrt[(c*(d + e*x))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])
/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))/((c*d^2 - b*d*e + a*e^2)*g^2)
+ (A + (f*(C*f - B*g))/g^2)*((-2*e^2*Sqrt[a + b*x + c*x^2])/((c*d^2 - b*d*
e + a*e^2)*(e*f - d*g)*Sqrt[d + e*x]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*Sqrt[
d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((c*d^2 - b*d*e + a*e^
2)*(e*f - d*g)*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*g*S
qrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[-1/2*((2*c*...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1352 vs. $2(672) = 1344$.

Time = 7.76 (sec) , antiderivative size = 1353, normalized size of antiderivative = 1.81

method	result	size
elliptic	Expression too large to display	1353
default	Expression too large to display	17980

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*(c*e*x^
2+b*e*x+a*e)/(a*e^2-b*d*e+c*d^2)/e*(A*e^2-B*d*e+C*d^2)/(d*g-e*f)/((x+d/e)*
(c*e*x^2+b*e*x+a*e))^(1/2)+2*(C/e/g+1/e*(b*e-c*d)*(A*e^2-B*d*e+C*d^2)/(d*g
-e*f)/(a*e^2-b*d*e+c*d^2)-b/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/(d*g-e
*f))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*E
llipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2*(A*e
^2-B*d*e+C*d^2)*c/(d*g-e*f)/(a*e^2-b*d*e+c*d^2)*(d/e-1/2*(b+(-4*a*c+b^2)^(
1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x
^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)))+2*(A*g^2-B*f*g+C*f^2)/(d*g-e*f)/g^2*(d/e-1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{3/2}(f + gx)\sqrt{a + bx + cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*(f + g*x)*sqrt(a + b*x + c*x
**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(f+g\,x)\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),
x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,(gx+f)\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.11 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 1097

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
-2*e*(C*d^2-e*(-A*e+B*d))*(c*x^2+b*x+a)^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e*
f)^2/(e*x+d)^(1/2)-g*(C*f^2-g*(-A*g+B*f))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2
)/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)/(g*x+f)-1/2*(-4*a*c+b^2)^(1/2)*(b*C*d*f
*(2*d*g+e*f)-b*e*g*(-A*d*g-2*A*e*f+3*B*d*f)+a*e*g*(-3*A*e*g+2*B*d*g+B*e*f)
-a*C*(2*d^2*g^2+e^2*f^2)-c*(3*C*d^2*f^2-B*d*f*(d*g+2*e*f)+A*(d^2*g^2+2*e^2
*f^2)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*
(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2
*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/(a*e^2-b*d*e+c*d^2)/(-d*g+e
*f)^2/(c*f^2-g*(-a*g+b*f))/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1
/2)/(c*x^2+b*x+a)^(1/2)+2^(1/2)*(-4*a*c+b^2)^(1/2)*(C*f^2-g*(-A*g+B*f))*(c
*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c
+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),
(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/g/(-d*g+
e*f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a
*c+b^2)^(1/2)*(c*(C*f^3*(2*d*g+e*f)-f*g*(3*B*e*f^2-A*g*(-2*d*g+5*e*f)))-g^
2*(b*(3*C*d*f^2+A*g*(-d*g+4*e*f)-B*f*(d*g+2*e*f))+a*(C*f*(-4*d*g+e*f)+g*(-
3*A*e*g+2*B*d*g+B*e*f))))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/
2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*g/(2*c*f-(b+(-4*a*c+b^2
)^(1/2))*g),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 44.27 (sec) , antiderivative size = 48666, normalized size of antiderivative = 44.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^2*Sqrt[a + b*x + c*
x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1292}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\int \frac{C}{g^2(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\frac{C\int \frac{1}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{g^2}$$

$$\Big\downarrow \text{ 1167}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\frac{C\left(-\frac{2\int -\frac{c\sqrt{d+ex}}{2\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}(ae^2-bde+cd^2)}\right)}{g^2}$$

$$\Big\downarrow \text{ 27}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\frac{C\left(\frac{c\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{ae^2-bde+cd^2}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}(ae^2-bde+cd^2)}\right)}{g^2}$$

$$\Big\downarrow \text{ 1172}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx+\frac{C\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}(ae^2-bde+cd^2)}\right)}{g^2}-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}$$

$$\Big\downarrow \text{ 327}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \frac{1}{(d+ex)^{3/2}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\frac{C\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}(ae^2-bde+cd^2)}\right)}{g^2}$$

$$\Big\downarrow \text{ 1288}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx-\frac{(2Cf-Bg)\int \left(\frac{e}{(ef-dg)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}-\frac{g}{(ef-dg)\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}\right)dx}{g^2}+\frac{C\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2e\sqrt{a+bx+cx^2}}{\sqrt{d+ex}(ae^2-bde+cd^2)}\right)}{g^2}$$

$\downarrow$ 2009

$$\frac{C\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(cd^2-bed+ae^2)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2e\sqrt{cx^2+bx+a}}{(cd^2-bed+ae^2)\sqrt{d+ex}}\right)}{g^2}-$$

$$\frac{(2Cf-Bg)\left(-\frac{2\sqrt{cx^2+bx+a}e^2}{(cd^2-bed+ae^2)(ef-dg)\sqrt{d+ex}}+\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2-}\right.}\right.}{(cd^2-bed+ae^2)(ef-dg)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}\right.}{}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{(d+ex)^{3/2}(f+gx)^2\sqrt{cx^2+bx+a}}dx$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1288
```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a
 + b*x + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x] && IntegerQ[n + 1/2]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 96.13 (sec) , antiderivative size = 1780, normalized size of antiderivative = 1.62

method	result	size
elliptic	Expression too large to display	1780
default	Expression too large to display	121389

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-2*(c*e*x
^2+b*e*x+a*e)/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/(d*g-e*f)^2/((x+d/e)
*(c*e*x^2+b*e*x+a*e))^(1/2)-g/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f
^2*g-c*e*f^3)*(A*g^2-B*f*g+C*f^2)/(d*g-e*f)*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)/(g*x+f)+2*(-(b*e-c*d)*(A*e^2-B*d*e+C*d^2)/(a*e^2-b*d*e+c
*d^2)/(d*g-e*f)^2+b*e/(a*e^2-b*d*e+c*d^2)*(A*e^2-B*d*e+C*d^2)/(d*g-e*f)^2+
1/2*c*e*f*(A*g^2-B*f*g+C*f^2)/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f
^2*g-c*e*f^3)/(d*g-e*f)/g)*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/
(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*
e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^
(1/2))))^(1/2))+2*(c*e*(A*e^2-B*d*e+C*d^2)/(a*e^2-b*d*e+c*d^2)/(d*g-e*f)^2
+1/2*e*c*(A*g^2-B*f*g+C*f^2)/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^
2*g-c*e*f^3)/(d*g-e*f))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x
)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}(gx+f)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*(g*x +
f)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\left(f+g\,x\right)^2\,\left(d+e\,x\right)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\left(ex+d\right)^{\frac{3}{2}}\left(gx+f\right)^2\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

3.12 $\int \frac{(d+ex)(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 184

$$\begin{aligned}\int \frac{(d+ex)(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx &= -\frac{Ad\sqrt{a+bx+cx^2}}{2ax^2}+\frac{(3Abd-4aBd-4aAe)\sqrt{a+bx+cx^2}}{4a^2x}\\ &\quad+\frac{(4abBd-A(3b^2d-4acd-4abe)-8a^2(Cd+Be))\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}}\\ &\quad+\frac{Ce\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\end{aligned}$$

output
```
-1/2*A*d*(c*x^2+b*x+a)^(1/2)/a/x^2+1/4*(-4*A*a*e+3*A*b*d-4*B*a*d)*(c*x^2+b
*x+a)^(1/2)/a^2/x+1/8*(4*a*b*B*d-A*(-4*a*b*e-4*a*c*d+3*b^2*d)-8*a^2*(B*e+C
*d))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)+C*e*arctan
h(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx+cx^2}}\,dx = \frac{1}{4}\left(\frac{\sqrt{a+x(b+cx)}(3Abdx-2a(2Bdx+A(d+2ex)))}{a^2x^2} + \frac{3Ab^2d\,\mathrm{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{4(Acd+b(Bd+Ae)-2a(Cd+Be))\mathrm{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{4Ce\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}\right)$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((Sqrt[a + x*(b + c*x)]*(3*A*b*d*x - 2*a*(2*B*d*x + A*(d + 2*e*x))))/(a^2*
x^2) + (3*A*b^2*d*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^
(5/2) + (4*(A*c*d + b*(B*d + A*e) - 2*a*(C*d + B*e))*ArcTanh[(-(Sqrt[c]*x)
 + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) - (4*C*e*Log[b + 2*c*x - 2*Sqr
t[c]*Sqrt[a + x*(b + c*x)]])/Sqrt[c])/4
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Ae+Bd}{x^2\sqrt{a+bx+cx^2}}+\frac{Ad}{x^3\sqrt{a+bx+cx^2}}+\frac{Be+Cd}{x\sqrt{a+bx+cx^2}}+\frac{Ce}{\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ad(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}}+\frac{b(Ae+Bd)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}}+\frac{3Abd\sqrt{a+bx+cx^2}}{4a^2x}-\frac{\sqrt{a+bx+cx^2}(Ae+Bd)}{ax}-\frac{Ad\sqrt{a+bx+cx^2}}{2ax^2}-\frac{(Be+Cd)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}+\frac{Ce\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/(x^3*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(A*d*Sqrt[a + b*x + c*x^2])/(a*x^2) + (3*A*b*d*Sqrt[a + b*x + c*x^2])
/(4*a^2*x) - ((B*d + A*e)*Sqrt[a + b*x + c*x^2])/(a*x) - (A*(3*b^2 - 4*a*c
)*d*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)) +
(b*(B*d + A*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*
a^(3/2)) - ((C*d + B*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^
2])])/Sqrt[a] + (C*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])
])/Sqrt[c]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(4Aaex-3xAbd+4Badx+2Aad)}{4a^2x^2}-\frac{-\frac{\left(4Aabe+4Aacd-3A\,b^2d-8B\,a^2e+4Badb-8C\,a^2d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}}{8a^2}$
default	$\frac{Ce\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+(Ae+Bd)\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)-\frac{(Be+Cd)\ln\left(\frac{2a+b}{}\right.}{}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(4*A*a*e*x-3*A*b*d*x+4*B*a*d*x+2*A*a*d)/a^2/x^2-1
/8/a^2*(-(4*A*a*b*e+4*A*a*c*d-3*A*b^2*d-8*B*a^2*e+4*B*a*b*d-8*C*a^2*d)/a^(
1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-8*C*a^2*e*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 903, normalized size of antiderivative = 4.91

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fric
as")
```

output
```
[1/16*(8*C*a^3*sqrt(c)*e*x^2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (4*(2*B*a^2 - A*a*b)*c*e - (4*A
*a*c^2 - (8*C*a^2 - 4*B*a*b + 3*A*b^2)*c)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (
b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^
2) - 4*(2*A*a^2*c*d + (4*A*a^2*c*e + (4*B*a^2 - 3*A*a*b)*c*d)*x)*sqrt(c*x^
2 + b*x + a))/(a^3*c*x^2), -1/16*(16*C*a^3*sqrt(-c)*e*x^2*arctan(1/2*sqrt(
c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + (4*(2*B*a
^2 - A*a*b)*c*e - (4*A*a*c^2 - (8*C*a^2 - 4*B*a*b + 3*A*b^2)*c)*d)*sqrt(a)
*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*A*a^2*c*d + (4*A*a^2*c*e + (4*B*a^2 - 3*A*
a*b)*c*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), 1/8*(4*C*a^3*sqrt(c)*e*x^
2*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqr
t(c) - 4*a*c) + (4*(2*B*a^2 - A*a*b)*c*e - (4*A*a*c^2 - (8*C*a^2 - 4*B*a*b
 + 3*A*b^2)*c)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - 2*(2*A*a^2*c*d + (4*A*a^2*c*e + (4*B
*a^2 - 3*A*a*b)*c*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*c*x^2), -1/8*(8*C*a^3*
sqrt(-c)*e*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*
x^2 + b*c*x + a*c)) - (4*(2*B*a^2 - A*a*b)*c*e - (4*A*a*c^2 - (8*C*a^2 - 4
*B*a*b + 3*A*b^2)*c)*d)*sqrt(-a)*x^2*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x
 + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) + 2*(2*A*a^2*c*d + (4*A*a^2*c...
```

Sympy [F]

$$\int \frac{(d+ex)\,(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx=\int \frac{(d+ex)\,(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/x**3/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((d + e*x)*(A + B*x + C*x**2)/(x**3*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(158) = 316$.

Time = 0.16 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.56

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{x^3\sqrt{a+bx+cx^2}}\,dx = -\frac{Ce\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}} + \frac{(8\,Ca^2d-4\,Babd+3\,Ab^2d-4\,Aacd+8\,Ba^2e-4\,Aabe)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\sqrt{-a}a^2} + \frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Babd-3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ab^2d+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aacd}{}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac
")
```

output
```
-C*e*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/sqrt(c)
+ 1/4*(8*C*a^2*d - 4*B*a*b*d + 3*A*b^2*d - 4*A*a*c*d + 8*B*a^2*e - 4*A*a*b
*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a^2) +
 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a*b*d - 3*(sqrt(c)*x - sqr
t(c*x^2 + b*x + a))^3*A*b^2*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*
a*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*b*e + 8*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))^2*B*a^2*sqrt(c)*d + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x
+ a))^2*A*a^2*sqrt(c)*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*B*a^2*b*d
+ 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a*b^2*d + 4*(sqrt(c)*x - sqrt(c*
x^2 + b*x + a))*A*a^2*c*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a^2*b*
e - 8*B*a^3*sqrt(c)*d + 8*A*a^2*b*sqrt(c)*d - 8*A*a^3*sqrt(c)*e)/(((sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx+cx^2}}\,dx = \int \frac{(d+e\,x)\,\left(C\,x^2+B\,x+A\right)}{x^3\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/(x^3*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(((d + e*x)*(A + B*x + C*x^2))/(x^3*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{x^3\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{-4\sqrt{c\,x^2+bx+a}\,a^2d-8\sqrt{c\,x^2+bx+a}\,a^2ex-2\sqrt{c\,x^2+bx+a}\,abdx+4\sqrt{a}\log\left(2\sqrt{a}\,\sqrt{c\,x^2+bx+a}\right.}{}$$

input
```
int((e*x+d)*(C*x^2+B*x+A)/x^3/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x + c*x**2)*a**2*d - 8*sqrt(a + b*x + c*x**2)*a**2*e*x -
2*sqrt(a + b*x + c*x**2)*a*b*d*x + 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x +
c*x**2) - 2*a - b*x)*a*b*e*x**2 + 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c
*x**2) - 2*a - b*x)*a*c*d*x**2 - sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x*
*2) - 2*a - b*x)*b**2*d*x**2 - 4*sqrt(a)*log(x)*a*b*e*x**2 - 4*sqrt(a)*log
(x)*a*c*d*x**2 + sqrt(a)*log(x)*b**2*d*x**2 + 8*sqrt(c)*log( - 2*sqrt(c)*s
qrt(a + b*x + c*x**2) - b - 2*c*x)*a**2*e*x**2)/(8*a**2*x**2)
```

3.13 $\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 362

$$\begin{aligned}
&\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{(6Ace(4cd+3be)-B(24c^2d^2+15b^2e^2+2ce(9bd-8ae)))\sqrt{a+bx+cx^2}}{24c^3e^3} \\
&\quad -\frac{(6Bcd+5bBe-6Ace)x\sqrt{a+bx+cx^2}}{12c^2e^2}+\frac{Bx^2\sqrt{a+bx+cx^2}}{3ce} \\
&\quad +\frac{(2Ace(8c^2d^2+3b^2e^2+4ce(bd-ae))-B(16c^3d^3+5b^3e^3+6bce^2(bd-2ae)+8c^2de(bd-ae)))\,\text{arcta}}{16c^{7/2}e^4} \\
&\quad +\frac{d^3(Bd-Ae)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

output

```
-1/24*(6*A*c*e*(3*b*e+4*c*d)-B*(24*c^2*d^2+15*b^2*e^2+2*c*e*(-8*a*e+9*b*d)
))*(c*x^2+b*x+a)^(1/2)/c^3/e^3-1/12*(-6*A*c*e+5*B*b*e+6*B*c*d)*x*(c*x^2+b*
x+a)^(1/2)/c^2/e^2+1/3*B*x^2*(c*x^2+b*x+a)^(1/2)/c/e+1/16*(2*A*c*e*(8*c^2*
d^2+3*b^2*e^2+4*c*e*(-a*e+b*d))-B*(16*c^3*d^3+5*b^3*e^3+6*b*c*e^2*(-2*a*e+
b*d)+8*c^2*d*e*(-a*e+b*d)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(7/2)/e^4+d^3*(-A*e+B*d)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*
e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{2e\sqrt{a+x(b+cx)}(6Ace(-4cd-3be+2cex)+B(15b^2e^2-2ce(-9bd+8ae+5bex)+4c^2(6d^2-3dex+2e^2x^2)))}{c^3}+\frac{96d^3(Bd-Ae)\sqrt{-cd^2+bde-ae^2}\,\mathrm{ar}}{cd^2+e(-}}{}$$

input
```
Integrate[(x^3*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(6*A*c*e*(-4*c*d - 3*b*e + 2*c*e*x) + B*(15*b^
2*e^2 - 2*c*e*(-9*b*d + 8*a*e + 5*b*e*x) + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*
x^2))))/c^3 + (96*d^3*(B*d - A*e)*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(S
qrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]
])/(c*d^2 + e*(-(b*d) + a*e)) + (3*(-2*A*c*e*(8*c^2*d^2 + 3*b^2*e^2 + 4*c*
e*(b*d - a*e)) + B*(16*c^3*d^3 + 5*b^3*e^3 + 6*b*c*e^2*(b*d - 2*a*e) + 8*c
^2*d*e*(b*d - a*e)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(
7/2))/(48*e^4)
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.31, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2153}$$

$$\int \left(\frac{d^3(Bd-Ae)}{e^4(d+ex)\sqrt{a+bx+cx^2}}-\frac{d^2(Bd-Ae)}{e^4\sqrt{a+bx+cx^2}}+\frac{dx(Bd-Ae)}{e^3\sqrt{a+bx+cx^2}}+\frac{x^2(Ae-Bd)}{e^2\sqrt{a+bx+cx^2}}+\frac{Bx^3}{e\sqrt{a+bx+cx^2}}\right)$$

$$\downarrow \text{2009}$$

$$-\frac{(3b^2-4ac)(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^2}-\frac{bd(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^3}-\frac{d^2(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e^4}+\frac{d^3(Bd-Ae)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}}+\frac{3b\sqrt{a+bx+cx^2}(Bd-Ae)}{4c^2e^2}+\frac{d\sqrt{a+bx+cx^2}(Bd-Ae)}{ce^3}-\frac{x\sqrt{a+bx+cx^2}(Bd-Ae)}{2ce^2}-\frac{bB(5b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}e}+\frac{B(-16ac+15b^2-10bcx)\sqrt{a+bx+cx^2}}{24c^3e}+\frac{Bx^2\sqrt{a+bx+cx^2}}{3ce}$$

input
```
Int[(x^3*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(d*(B*d - A*e)*Sqrt[a + b*x + c*x^2])/(c*e^3) + (3*b*(B*d - A*e)*Sqrt[a +
b*x + c*x^2])/(4*c^2*e^2) - ((B*d - A*e)*x*Sqrt[a + b*x + c*x^2])/(2*c*e^2
) + (B*x^2*Sqrt[a + b*x + c*x^2])/(3*c*e) + (B*(15*b^2 - 16*a*c - 10*b*c*x
)*Sqrt[a + b*x + c*x^2])/(24*c^3*e) - (b*B*(5*b^2 - 12*a*c)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2)*e) - (d^2*(B*d - A*e
)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e^4) -
(b*d*(B*d - A*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
2*c^(3/2)*e^3) - ((3*b^2 - 4*a*c)*(B*d - A*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e^2) + (d^3*(B*d - A*e)*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x +
 c*x^2])])/(e^4*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{\left(-8B\,e^{2}c^{2}x^{2}-12A\,c^{2}e^{2}x+10Bbc\,e^{2}x+12B\,c^{2}dex+18Abc\,e^{2}+24A\,c^{2}de+16Bac\,e^{2}-15B\,b^{2}e^{2}-18Bbcde-24B\,c^{2}d^{2}\right)\sqrt{c\,x^{2}+bx+}}{24c^{3}e^{3}}$
	$e^{2}(Ae-Bd)\left(\frac{x\sqrt{c\,x^{2}+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-de(Ae-Bd)\left(\frac{\sqrt{c\,x^{2}+b}}{c}\right.$
default	

input
```
int(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*B*c^2*e^2*x^2-12*A*c^2*e^2*x+10*B*b*c*e^2*x+12*B*c^2*d*e*x+18*A*
b*c*e^2+24*A*c^2*d*e+16*B*a*c*e^2-15*B*b^2*e^2-18*B*b*c*d*e-24*B*c^2*d^2)*
(c*x^2+b*x+a)^(1/2)/c^3/e^3-1/16/c^3/e^3*((8*A*a*c^2*e^3-6*A*b^2*c*e^3-8*A
*b*c^2*d*e^2-16*A*c^3*d^2*e-12*B*a*b*c*e^3-8*B*a*c^2*d*e^2+5*B*b^3*e^3+6*B
*b^2*c*d*e^2+8*B*b*c^2*d^2*e+16*B*c^3*d^3)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))/c^(1/2)-16*d^3*(A*e-B*d)*c^3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**3*(A + B*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^3*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((x^3*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^3(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^3(Bx+A)}{(ex+d)\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^3*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.14 $\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 249

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{(4Bcd+3bBe-4Ace)\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{Bx\sqrt{a+bx+cx^2}}{2ce} - \frac{(4Ace(2cd+be)-B(8c^2d^2+3b^2e^2+4ce(bd-ae)))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e^3} - \frac{d^2(Bd-Ae)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}$$

output
```
-1/4*(-4*A*c*e+3*B*b*e+4*B*c*d)*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*B*x*(c*x^2
+b*x+a)^(1/2)/c/e-1/8*(4*A*c*e*(b*e+2*c*d)-B*(8*c^2*d^2+3*b^2*e^2+4*c*e*(-
a*e+b*d)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3-
d^2*(-A*e+B*d)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^
(1/2)/(c*x^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{2e\sqrt{a+x(b+cx)}(4Ace+B(-4cd-3be+2cex))}{c^2} - \frac{16d^2(Bd-Ae)\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{(4Ace(2cd+be)-B(8\cdots}{\cdots}}{8e^3}$$

input
```
Integrate[(x^2*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(4*A*c*e + B*(-4*c*d - 3*b*e + 2*c*e*x)))/c^2
- (16*d^2*(B*d - A*e)*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d +
e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 +
e*(-(b*d) + a*e)) + ((4*A*c*e*(2*c*d + b*e) - B*(8*c^2*d^2 + 3*b^2*e^2 + 4
*c*e*(b*d - a*e)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^(5/
2))/(8*e^3)
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.32, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2153}$$
$$\int \left(-\frac{d^2(Bd-Ae)}{e^3(d+ex)\sqrt{a+bx+cx^2}}+\frac{d(Bd-Ae)}{e^3\sqrt{a+bx+cx^2}}+\frac{x(Ae-Bd)}{e^2\sqrt{a+bx+cx^2}}+\frac{Bx^2}{e\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{b(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2}-\frac{d^2(Bd-Ae)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}}+\frac{d(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e^3}-\frac{\sqrt{a+bx+cx^2}(Bd-Ae)}{ce^2}+\frac{B(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e}-\frac{3bB\sqrt{a+bx+cx^2}}{4c^2e}+\frac{Bx\sqrt{a+bx+cx^2}}{2ce}$$

input
```
Int[(x^2*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-3*b*B*Sqrt[a + b*x + c*x^2])/(4*c^2*e) - ((B*d - A*e)*Sqrt[a + b*x + c*x
^2])/(c*e^2) + (B*x*Sqrt[a + b*x + c*x^2])/(2*c*e) + (B*(3*b^2 - 4*a*c)*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*e) + (d*(
B*d - A*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c
]*e^3) + (b*(B*d - A*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(2*c^(3/2)*e^2) - (d^2*(B*d - A*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b
*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^3*Sqrt[c
*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.22

method	result
risch	$\frac{(2Bcex+4Ace-3Bbe-4Bcd)\sqrt{c\,x^2+bx+a}}{4c^2e^2}-\frac{\frac{\left(4Abc\,e^2+8A\,c^2de+4Bac\,e^2-3B\,b^2e^2-4Bbcde-8B\,c^2d^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{8d^2}{}}{}$
default	$-\frac{d^2(Ae-Bd)\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+\frac{a\,e^2-bde+c\,d^2}{e^2}}}{x+\frac{d}{e}}\right)}{e^4\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}-\frac{-e(Ae-}{}$

input
```
int(x^2*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*B*c*e*x+4*A*c*e-3*B*b*e-4*B*c*d)*(c*x^2+b*x+a)^(1/2)/c^2/e^2-1/8/c^
2/e^2*((4*A*b*c*e^2+8*A*c^2*d*e+4*B*a*c*e^2-3*B*b^2*e^2-4*B*b*c*d*e-8*B*c^
2*d^2)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+8*d^2*(A*e-B*
d)*c^2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2
*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\,(A+B\,x)}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((x^2*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 8291, normalized size of antiderivative = 33.30

$$\int \frac{x^2(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output

```
( - 8*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*a*b*c**3*d**2*e**2 + 16*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*
d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e*
*2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*at
an((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqr
t(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c
*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**4*d**3*e + 8*
sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 -
 b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*s
qrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*
e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*
sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8
*c**2*d**2))*b**2*c**3*d**3*e - 16*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*
d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**
2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e...
```

3.15 $\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 171

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{B\sqrt{a+bx+cx^2}}{ce} - \frac{(2Bcd+bBe-2Ace)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{d(Bd-Ae)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

output
```
B*(c*x^2+b*x+a)^(1/2)/c/e-1/2*(-2*A*c*e+B*b*e+2*B*c*d)*arctanh(1/2*(2*c*x+
b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^2+d*(-A*e+B*d)*arctanh(1/2*(b*d-
2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/e^2/(
a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{-\frac{2Be\sqrt{a+x(b+cx)}}{c}+\frac{4d(Bd-Ae)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\sqrt{-cd^2+e(bd-ae)}}+\frac{(2Bcd+bBe-2Ace)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}}{2e^2}$$

input
```
Integrate[(x*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((-2*B*e*Sqrt[a + x*(b + c*x)])/c + (4*d*(B*d - A*e)*ArcTan[(Sqrt[c]*
(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/Sqrt
[-(c*d^2) + e*(b*d - a*e)] + ((2*B*c*d + b*B*e - 2*A*c*e)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2))/e^2
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{d(Bd-Ae)}{e^2(d+ex)\sqrt{a+bx+cx^2}}+\frac{Ae-Bd}{e^2\sqrt{a+bx+cx^2}}+\frac{Bx}{e\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(Bd-Ae)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2-bde+cd^2}}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e^2}-\frac{bB\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e}+\frac{B\sqrt{a+bx+cx^2}}{ce}$$

input
```
Int[(x*(A + B*x))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(B*Sqrt[a + b*x + c*x^2])/(c*e) - (b*B*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(2*c^(3/2)*e) - ((B*d - A*e)*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e^2) + (d*(B*d - A*e)*ArcTanh[(b*
d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x +
 c*x^2])])/(e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.43

method	result
risch	$\frac{B\sqrt{c\,x^2+bx+a}}{ce}+\frac{\frac{(2Ace-Bbe-2Bcd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}+\frac{2d(Ae-Bd)c\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-bde+c\,}{e^2}}}{x+\frac{d}{e}}\right)}{e^2\sqrt{\frac{a\,e^2-bde+c\,d^2}{e^2}}}}{2ce}$
default	$\frac{\frac{Ae\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+Be\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{Bd\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}}{e^2}+\frac{d(Ae-Bd)\ln\left(\frac{2a\,e}{}\right.}{}$

input
```
int(x*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*(c*x^2+b*x+a)^(1/2)/c/e+1/2/c/e*((2*A*c*e-B*b*e-2*B*c*d)/e*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+2*d*(A*e-B*d)*c/e^2/((a*e^2-b*d*e+c
*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d
*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x*(B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{x\,(A+B\,x)}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((x*(A + B*x))/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 7805, normalized size of antiderivative = 45.64

$$\int \frac{x(A+Bx)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(x*(B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2
)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
 b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2))*a*b*c**2*d*e**2 - 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c
*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b*
*2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*s
qrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*
a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**3*d**2*e - 2*sqrt(4*
sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e
+ c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e
**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c
*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2))*b**2*c**2*d**2*e + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*
e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 +
 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sq
rt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*...
```

3.16 $\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 132

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{B\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e} - \frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}}$$

output
```
B*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/e-(-A*e+B*d)*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*
x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{\frac{2(Bd-Ae)\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{B\log\left(e\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}}{e}$$

input
```
Integrate[(A + B*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-(((2*(B*d - A*e)*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x)
 - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-
(b*d) + a*e)) + (B*Log[e*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/S
qrt[c])/e)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1269}$$

$$\frac{B\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{(Bd-Ae)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1092}$$

$$\frac{2B\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{(Bd-Ae)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{(Bd-Ae)\int \frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2(Bd-Ae)\int \frac{1}{4(cd^2-bed+ae^2)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}e}-\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}$$

input
```
Int[(A + B*x)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(B*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (
(B*d - A*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e
+ a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.51

method	result
default	$\frac{B\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{e\sqrt{c}}-\frac{(Ae-Bd)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+}}{x+\frac{d}{e}}\right.}{e^{2}\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(A*e-B*d)/e^2/((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(116) = 232.

Time = 29.26 (sec) , antiderivative size = 1079, normalized size of antiderivative = 8.17

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((B*c*d^2 - B*b*d*e + B*a*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2
 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - (B*c*d - A*c*e)*
sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2
 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e
+ a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*
d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)))/(c^2
*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(B*c*d^2 - B*b*d*e + B*a*e^2)*sqrt(
-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x
 + a*c)) + (B*c*d - A*c*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*
a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*
x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (
2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*
x^2 + 2*d*e*x + d^2)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), -1/2*(2*(B*c*d -
 A*c*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e
^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d
*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*
b*e^2)*x)) - (B*c*d^2 - B*b*d*e + B*a*e^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*
x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e
 - b*c*d*e^2 + a*c*e^3), -((B*c*d - A*c*e)*sqrt(-c*d^2 + b*d*e - a*e^2)*ar
ctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a...
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+B\,x}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7325, normalized size of antiderivative = 55.49

$$\int \frac{A + Bx}{(d + ex)\sqrt{a + bx + cx^2}} \, dx = \text{Too large to display}$$

input
```
int((B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*a*b*c*e**2 + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d
**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2
*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqr
t(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2
- b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*
c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c**2*d*e + 2*sqrt(4*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d
**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 -
 b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)
/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2
- b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))
*b**2*c*d*e - 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x
+ c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**...
```

3.17 $\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 133

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{A\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{(Bd-Ae)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d\sqrt{cd^2-bde+ae^2}}$$

output
```
-A*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)/d+(-A*e+B*d)
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b
*x+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\left(\frac{(Bd-Ae)\sqrt{-cd^2+bde-ae^2}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{A\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

input
```
Integrate[(A + B*x)/(x*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*(((B*d - A*e)*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x)
- e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(
b*d) + a*e)) + (A*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sq
rt[a]))/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Bd-Ae}{d(d+ex)\sqrt{a+bx+cx^2}}+\frac{A}{dx\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Bd-Ae)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d\sqrt{ae^2-bde+cd^2}}-\frac{A\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

input
```
Int[(A + B*x)/(x*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d))
+ ((B*d - A*e)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d
*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.55

method	result
default	$-\frac{A\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{(Ae-Bd)\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\,\sqrt{\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd)\left(x+\cdots\right.}{e}\cdots}}{x+\frac{d}{e}}\right.\cdots}{de\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int((B*x+A)/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+(A*e-B*d)/d/e/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e
*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(117) = 234$.

Time = 1.84 (sec) , antiderivative size = 1090, normalized size of antiderivative = 8.20

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((B*a*d - A*a*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*
e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2
+ 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*
d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2
+ 2*d*e*x + d^2)) - (A*c*d^2 - A*b*d*e + A*a*e^2)*sqrt(a)*log(-(8*a*b*x +
(b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x
^2))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), 1/2*(2*(B*a*d - A*a*e)*sqrt(-c*d^2
 + b*d*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*
x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b*d*e + a^2*e^2 + (c^2
*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e + a*b*e^2)*x)) + (A*c*d
^2 - A*b*d*e + A*a*e^2)*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt
(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/(a*c*d^3 - a*b*d^2*e
+ a^2*d*e^2), 1/2*(2*(A*c*d^2 - A*b*d*e + A*a*e^2)*sqrt(-a)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)) - (B*a*d
- A*a*e)*sqrt(c*d^2 - b*d*e + a*e^2)*log((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4
*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2
 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x) -
2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^
2)))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), ((B*a*d - A*a*e)*sqrt(-c*d^2 + b*d
*e - a*e^2)*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c*x^2 + b*x +...
```

Sympy [F]

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx}{x\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx}{x(d + ex)\sqrt{a + bx + cx^2}}\,dx = \int \frac{Bx + A}{\sqrt{cx^2 + bx + a}(ex + d)x}\,dx$$

input
```
integrate((B*x+A)/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx}{x(d + ex)\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx}{x(d + ex)\sqrt{a + bx + cx^2}}\,dx = \int \frac{A + B\,x}{x\,\left(d + e\,x\right)\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

input
```
int((A + B*x)/(x*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.53

$$\int \frac{A+Bx}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\sqrt{a\,e^2-bde+c\,d^2}\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)ae-\sqrt{a\,e^2-bd}}{}$$

input
```
int((B*x+A)/x/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*e - sqrt(a*e**2 - b*
d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2
) - 2*a*e + b*d - b*e*x + 2*c*d*x)*b*d - sqrt(a*e**2 - b*d*e + c*d**2)*log
(d + e*x)*a*e + sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*d + sqrt(a)*l
og(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*e**2 - sqrt(a)*log(2*sq
rt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*d*e + sqrt(a)*log(2*sqrt(a)*sq
rt(a + b*x + c*x**2) - 2*a - b*x)*c*d**2 - sqrt(a)*log(x)*a*e**2 + sqrt(a)
*log(x)*b*d*e - sqrt(a)*log(x)*c*d**2)/(d*(a*e**2 - b*d*e + c*d**2))
```

3.18 $\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 177

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{A\sqrt{a+bx+cx^2}}{adx} + \frac{(Abd-2aBd+2aAe)\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} - \frac{e(Bd-Ae)\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^2\sqrt{cd^2-bde+ae^2}}$$

output
```
-A*(c*x^2+b*x+a)^(1/2)/a/d/x+1/2*(2*A*a*e+A*b*d-2*B*a*d)*arctanh(1/2*(b*x+
2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d^2-e*(-A*e+B*d)*arctanh(1/2*(b*
d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2
/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\frac{2Ad\sqrt{a+x(b+cx)}}{ax}+\frac{4e(Bd-Ae)\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{-cd^2+e(bd-ae)}x}{\sqrt{a}(d+ex)-d\sqrt{a+x(b+cx)}}\right)}{cd^2+e(-bd+ae)}-\frac{(Abd-2aBd+2aAe)\log(x)}{a^{3/2}}+\frac{(Abd-2aBd+2}{}}{2d^2}$$

input
```
Integrate[(A + B*x)/(x^2*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*((2*A*d*Sqrt[a + x*(b + c*x)])/(a*x) + (4*e*(B*d - A*e)*Sqrt[-(c*d^2)
 + e*(b*d - a*e)]*ArcTan[(Sqrt[-(c*d^2) + e*(b*d - a*e)]*x)/(Sqrt[a]*(d +
e*x) - d*Sqrt[a + x*(b + c*x)])])/(c*d^2 + e*(-(b*d) + a*e)) - ((A*b*d - 2
*a*B*d + 2*a*A*e)*Log[x])/a^(3/2) + ((A*b*d - 2*a*B*d + 2*a*A*e)*Log[a*d^2
*(2*a + b*x - 2*Sqrt[a]*Sqrt[a + x*(b + c*x)])])/a^(3/2))/d^2
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.20, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Bd-Ae}{d^2x\sqrt{a+bx+cx^2}}-\frac{e(Bd-Ae)}{d^2(d+ex)\sqrt{a+bx+cx^2}}+\frac{A}{dx^2\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{Ab\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{e(Bd-Ae)\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^2\sqrt{ae^2-bde+cd^2}}-\frac{(Bd-Ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{A\sqrt{a+bx+cx^2}}{adx}$$

input
```
Int[(A + B*x)/(x^2*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x + c*x^2])/(a*d*x)) + (A*b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]
*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d) - ((B*d - A*e)*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (e*(B*d - A*e)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
b*x + c*x^2])])/(d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.42

method	result
risch	$-\frac{A\sqrt{c\,x^2+bx+a}}{adx}-\frac{-\frac{(2Aae+Abd-2Bad)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{2a(Ae-Bd)\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{a\,e^2}}{}\right.}{d\sqrt{a\,e^2-}}}{2ad}$
default	$\frac{A\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{d}+\frac{(Ae-Bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d^2\sqrt{a}}-\frac{(Ae-Bd)\ln\left(\frac{2a\,e^2-2bde+2c}{e^2}\right.}{}$

input
```
int((B*x+A)/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/a/d*(-(2*A*a*e+A*b*d-2*B*a*d)/d/a^(1/2)*l
n((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+2*a*(A*e-B*d)/d/((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(158) = 316.

Time = 8.25 (sec) , antiderivative size = 1504, normalized size of antiderivative = 8.50

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/4*(2*(B*a^2*d*e - A*a^2*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*x*log((8*a*b*
d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*
c)*e^2)*x^2 - 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2
*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*
x)/(e^2*x^2 + 2*d*e*x + d^2)) - (2*A*a^2*e^3 - (2*B*a - A*b)*c*d^3 + (2*B*
a*b - A*b^2 + 2*A*a*c)*d^2*e - (2*B*a^2 + A*a*b)*d*e^2)*sqrt(a)*x*log(-(8*
a*b*x + (b^2 + 4*a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) +
8*a^2)/x^2) + 4*(A*a*c*d^3 - A*a*b*d^2*e + A*a^2*d*e^2)*sqrt(c*x^2 + b*x +
 a))/((a^2*c*d^4 - a^2*b*d^3*e + a^3*d^2*e^2)*x), -1/4*(4*(B*a^2*d*e - A*a
^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*x*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c*d^2 - a*b
*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 - b^2*d*e +
a*b*e^2)*x)) - (2*A*a^2*e^3 - (2*B*a - A*b)*c*d^3 + (2*B*a*b - A*b^2 + 2*A
*a*c)*d^2*e - (2*B*a^2 + A*a*b)*d*e^2)*sqrt(a)*x*log(-(8*a*b*x + (b^2 + 4*
a*c)*x^2 + 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(
A*a*c*d^3 - A*a*b*d^2*e + A*a^2*d*e^2)*sqrt(c*x^2 + b*x + a))/((a^2*c*d^4
- a^2*b*d^3*e + a^3*d^2*e^2)*x), -1/2*((2*A*a^2*e^3 - (2*B*a - A*b)*c*d^3
+ (2*B*a*b - A*b^2 + 2*A*a*c)*d^2*e - (2*B*a^2 + A*a*b)*d*e^2)*sqrt(-a)*x*
arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a
^2)) + (B*a^2*d*e - A*a^2*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*x*log((8*a*b...
```

Sympy [F]

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{x^2\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**2/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**2*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)x^2}\,dx$$

input
```
integrate((B*x+A)/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{2\,(Bde-Ae^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}d^2} + \frac{(2\,Bad-Abd-2\,Aae)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ad^2} + \frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)Ab+2\,Aa\sqrt{c}}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-a\right)ad}$$

input
```
integrate((B*x+A)/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*d*e - A*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) + (2
*B*a*d - A*b*d - 2*A*a*e)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt
(-a))/(sqrt(-a)*a*d^2) + ((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*b + 2*A*a*
sqrt(c))/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)*a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+B\,x}{x^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/(x^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.02

$$\int \frac{A+Bx}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{a\,e^2-bde+c\,d^2}\log\left(2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)a^2e^2x-2\sqrt{a\,e^2-}}{}$$

input
```
int((B*x+A)/x^2/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*e**2*x - 2*sqrt(a*
e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e +
c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*b*d*e*x - 2*sqrt(a*e**2 - b*d*e
 + c*d**2)*log(d + e*x)*a**2*e**2*x + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(
d + e*x)*a*b*d*e*x - 2*sqrt(a + b*x + c*x**2)*a**2*d*e**2 + 2*sqrt(a + b*x
 + c*x**2)*a*b*d**2*e - 2*sqrt(a + b*x + c*x**2)*a*c*d**3 + 2*sqrt(a)*log(
 - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a**2*e**3*x - 3*sqrt(a)*l
og( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*b*d*e**2*x + 2*sqrt(
a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*a*c*d**2*e*x + sqr
t(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*d**2*e*x -
sqrt(a)*log( - 2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b*c*d**3*x -
2*sqrt(a)*log(x)*a**2*e**3*x + 3*sqrt(a)*log(x)*a*b*d*e**2*x - 2*sqrt(a)*l
og(x)*a*c*d**2*e*x - sqrt(a)*log(x)*b**2*d**2*e*x + sqrt(a)*log(x)*b*c*d**
3*x)/(2*a*d**2*x*(a*e**2 - b*d*e + c*d**2))
```

3.19 $\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 248

$$\begin{aligned}\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx &= -\frac{A\sqrt{a+bx+cx^2}}{2adx^2}+\frac{(3Abd-4aBd+4aAe)\sqrt{a+bx+cx^2}}{4a^2d^2x}\\ &\quad-\frac{((bd+2ae)(3Abd-4aBd+4aAe)-2aAd(2cd+3be))\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d^3}\\ &\quad+\frac{e^2(Bd-Ae)\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{d^3\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

output
```
-1/2*A*(c*x^2+b*x+a)^(1/2)/a/d/x^2+1/4*(4*A*a*e+3*A*b*d-4*B*a*d)*(c*x^2+b*
x+a)^(1/2)/a^2/d^2/x-1/8*((2*a*e+b*d)*(4*A*a*e+3*A*b*d-4*B*a*d)-2*a*A*d*(3
*b*e+2*c*d))*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d^
3+e^2*(-A*e+B*d)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2
)^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{\frac{d\sqrt{a+x(b+cx)}(3Abdx-2a(2Bdx+A(d-2ex)))}{a^2x^2} + \frac{8e^2(Bd-Ae)\sqrt{-cd^2+e(bd-ae)}\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{8Ae^2\operatorname{arctanh}\Big(\cdots}{\sqrt{\cdots}}}{4d^3}$$

input
```
Integrate[(A + B*x)/(x^3*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d*Sqrt[a + x*(b + c*x)]*(3*A*b*d*x - 2*a*(2*B*d*x + A*(d - 2*e*x))))/(a^
2*x^2) + (8*e^2*(B*d - A*e)*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]
*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*
d^2 + e*(-(b*d) + a*e)) + (8*A*e^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*
x)])/Sqrt[a]])/Sqrt[a] + (d*(4*a*B*(b*d + 2*a*e) + A*(-3*b^2*d + 4*a*c*d -
 4*a*b*e))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2
))/(4*d^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \; 2153$$
$$\int \left(\frac{e^2(Bd-Ae)}{d^3(d+ex)\sqrt{a+bx+cx^2}} - \frac{e(Bd-Ae)}{d^3x\sqrt{a+bx+cx^2}} + \frac{Bd-Ae}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{A}{dx^3\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{A(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{b(Bd-Ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3Ab\sqrt{a+bx+cx^2}}{4a^2dx}+\frac{e(Bd-Ae)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}+\frac{e^2(Bd-Ae)\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{d^3\sqrt{ae^2-bde+cd^2}}-\frac{\sqrt{a+bx+cx^2}(Bd-Ae)}{ad^2x}-\frac{A\sqrt{a+bx+cx^2}}{2adx^2}$$

input
```
Int[(A + B*x)/(x^3*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x + c*x^2])/(a*d*x^2) + (3*A*b*Sqrt[a + b*x + c*x^2])/(
4*a^2*d*x) - ((B*d - A*e)*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - (A*(3*b^2 - 4
*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d
) + (b*(B*d - A*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])
/(2*a^(3/2)*d^2) + (e*(B*d - A*e)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
b*x + c*x^2])])/(Sqrt[a]*d^3) + (e^2*(B*d - A*e)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(d^
3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-4Aaex-3xAbd+4Badx+2Aad)}{4a^2d^2x^2}+\frac{-\frac{\left(8A\,a^2e^2+4Aabde-4Ac\,d^2a+3A\,b^2d^2-8B\,a^2de-4Bb\,d^2a\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}}{}$
default	$\frac{A\left(-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{d}-\frac{(Ae-Bd)\left(-\frac{\sqrt{c\,x^2}}{}\right.}{}$

input
```
int((B*x+A)/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-4*A*a*e*x-3*A*b*d*x+4*B*a*d*x+2*A*a*d)/a^2/d^2/
x^2+1/8/d^2/a^2*(-(8*A*a^2*e^2+4*A*a*b*d*e-4*A*a*c*d^2+3*A*b^2*d^2-8*B*a^2
*d*e-4*B*a*b*d^2)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+
8*a^2*e*(A*e-B*d)/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. 2(222) = 444.

Time = 38.60 (sec) , antiderivative size = 2096, normalized size of antiderivative = 8.45

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/16*(8*(B*a^3*d*e^2 - A*a^3*e^3)*sqrt(c*d^2 - b*d*e + a*e^2)*x^2*log((8
*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 +
 4*a*c)*e^2)*x^2 + 4*sqrt(c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)*(b*
d - 2*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*
d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2)) - (8*A*a^3*e^4 - (4*A*a*c^2 + (4*B*a*b
- 3*A*b^2)*c)*d^4 + (4*B*a*b^2 - 3*A*b^3 - 8*(B*a^2 - A*a*b)*c)*d^3*e + (4
*B*a^2*b - A*a*b^2 + 4*A*a^2*c)*d^2*e^2 - 4*(2*B*a^3 + A*a^2*b)*d*e^3)*sqr
t(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x
+ 2*a)*sqrt(a) + 8*a^2)/x^2) + 4*(2*A*a^2*c*d^4 - 2*A*a^2*b*d^3*e + 2*A*a^
3*d^2*e^2 - (4*A*a^3*d*e^3 - (4*B*a^2 - 3*A*a*b)*c*d^4 + (4*B*a^2*b - 3*A*
a*b^2 + 4*A*a^2*c)*d^3*e - (4*B*a^3 + A*a^2*b)*d^2*e^2)*x)*sqrt(c*x^2 + b*
x + a))/((a^3*c*d^5 - a^3*b*d^4*e + a^4*d^3*e^2)*x^2), 1/16*(16*(B*a^3*d*e
^2 - A*a^3*e^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*x^2*arctan(-1/2*sqrt(-c*d^2 +
 b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a)*(b*d - 2*a*e + (2*c*d - b*e)*x)/(a*c
*d^2 - a*b*d*e + a^2*e^2 + (c^2*d^2 - b*c*d*e + a*c*e^2)*x^2 + (b*c*d^2 -
b^2*d*e + a*b*e^2)*x)) + (8*A*a^3*e^4 - (4*A*a*c^2 + (4*B*a*b - 3*A*b^2)*c
)*d^4 + (4*B*a*b^2 - 3*A*b^3 - 8*(B*a^2 - A*a*b)*c)*d^3*e + (4*B*a^2*b - A
*a*b^2 + 4*A*a^2*c)*d^2*e^2 - 4*(2*B*a^3 + A*a^2*b)*d*e^3)*sqrt(a)*x^2*log
(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(
a) + 8*a^2)/x^2) - 4*(2*A*a^2*c*d^4 - 2*A*a^2*b*d^3*e + 2*A*a^3*d^2*e^2...
```

Sympy [F]

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{x^3\left(d+ex\right)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**3*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)x^3}\,dx$$

input
```
integrate((B*x+A)/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(222) = 444.

Time = 0.15 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\,(Bde^2-Ae^3)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^3} - \frac{(4\,Babd^2-3\,Ab^2d^2+4\,Aacd^2+8\,Ba^2de-4\,Aabde-8\,Aa^2e^2)\arctan\left(-\frac{\sqrt{c}x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^2d^3} + \frac{4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Babd-3\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Ab^2d+4\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^3Aacd}{}$$

input
```
integrate((B*x+A)/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
2*(B*d*e^2 - A*e^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x + a))*e + sqrt(
c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^3) - 1
/4*(4*B*a*b*d^2 - 3*A*b^2*d^2 + 4*A*a*c*d^2 + 8*B*a^2*d*e - 4*A*a*b*d*e -
8*A*a^2*e^2)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-
a)*a^2*d^3) + 1/4*(4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*B*a*b*d - 3*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*b^2*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b
*x + a))^3*A*a*c*d - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))^3*A*a*b*e + 8*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2*B*a^2*sqrt(c)*d - 8*(sqrt(c)*x - sqrt
(c*x^2 + b*x + a))^2*A*a^2*sqrt(c)*e - 4*(sqrt(c)*x - sqrt(c*x^2 + b*x + a
))*B*a^2*b*d + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*A*a*b^2*d + 4*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*A*a^2*c*d + 4*(sqrt(c)*x - sqrt(c*x^2 + b*x +
 a))*A*a^2*b*e - 8*B*a^3*sqrt(c)*d + 8*A*a^2*b*sqrt(c)*d + 8*A*a^3*sqrt(c)
*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x + a))^2 - a)^2*a^2*d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+B\,x}{x^3\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/(x^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^3*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 816, normalized size of antiderivative = 3.29

$$\int \frac{A+Bx}{x^3(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
int((B*x+A)/x^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e*
*2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**3*e**3*x**2 - 8*s
qrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 -
 b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a**2*b*d*e**2*x**2 - 8*s
qrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a**3*e**3*x**2 + 8*sqrt(a*e**2 -
 b*d*e + c*d**2)*log(d + e*x)*a**2*b*d*e**2*x**2 - 4*sqrt(a + b*x + c*x**2
)*a**3*d**2*e**2 + 8*sqrt(a + b*x + c*x**2)*a**3*d*e**3*x + 4*sqrt(a + b*x
 + c*x**2)*a**2*b*d**3*e - 10*sqrt(a + b*x + c*x**2)*a**2*b*d**2*e**2*x -
4*sqrt(a + b*x + c*x**2)*a**2*c*d**4 + 8*sqrt(a + b*x + c*x**2)*a**2*c*d**
3*e*x + 2*sqrt(a + b*x + c*x**2)*a*b**2*d**3*e*x - 2*sqrt(a + b*x + c*x**2
)*a*b*c*d**4*x + 8*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a - b*
x)*a**3*e**4*x**2 - 12*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2) - 2*a
- b*x)*a**2*b*d*e**3*x**2 + 4*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x + c*x**2)
 - 2*a - b*x)*a**2*c*d**2*e**2*x**2 + 3*sqrt(a)*log(2*sqrt(a)*sqrt(a + b*x
 + c*x**2) - 2*a - b*x)*a*b**2*d**2*e**2*x**2 - 4*sqrt(a)*log(2*sqrt(a)*sq
rt(a + b*x + c*x**2) - 2*a - b*x)*a*c**2*d**4*x**2 + sqrt(a)*log(2*sqrt(a)
*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**3*d**3*e*x**2 - sqrt(a)*log(2*sqrt
(a)*sqrt(a + b*x + c*x**2) - 2*a - b*x)*b**2*c*d**4*x**2 - 8*sqrt(a)*log(x
)*a**3*e**4*x**2 + 12*sqrt(a)*log(x)*a**2*b*d*e**3*x**2 - 4*sqrt(a)*log(x)
*a**2*c*d**2*e**2*x**2 - 3*sqrt(a)*log(x)*a*b**2*d**2*e**2*x**2 + 4*sqr...
```

3.20 $\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 775

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2(24b^2Ce^2+ce(41bCd-28bBe-25aCe)+7c^2(6Cd^2-e(7Bd-5Ae)))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{105c^3e^3}$$
$$+\frac{2Cx^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{7ce}-\frac{2(6cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{35c^2e^3}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(48b^3Ce^3+8bce^2(5bCd-7bBe-13aCe)+c^3(48Cd^3-14de(4Bd-5Ae))+c^2e(40bC}{105c^4e^4\sqrt{\frac{}{2cd-}}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(24b^2Ce^3(bd-ae)+c^3(48Cd^4-14d^2e(4Bd-5Ae))+ce^2(25a^2Ce^2+b^2d(17Cd-28}{}$$

output
```
2/105*(24*b^2*C*e^2+c*e*(-28*B*b*e-25*C*a*e+41*C*b*d)+7*c^2*(6*C*d^2-e*(-5
*A*e+7*B*d)))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e^3+2/7*C*x^2*(e*x+d)^
(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-2/35*(-7*B*c*e+6*C*b*e+6*C*c*d)*(e*x+d)^(3/2
)*(c*x^2+b*x+a)^(1/2)/c^2/e^3-1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(48*b^3*C*e
^3+8*b*c*e^2*(-7*B*b*e-13*C*a*e+5*C*b*d)+c^3*(48*C*d^3-14*d*e*(-5*A*e+4*B*
d))+c^2*e*(40*b*C*d^2-7*b*e*(-10*A*e+7*B*d)-a*e*(-63*B*e+44*C*d)))*(e*x+d)
^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c
+b^2)^(1/2))*e))^(1/2))/c^4/e^4/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e
))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(24*b^2*C*e^
3*(-a*e+b*d)+c^3*(48*C*d^4-14*d^2*e*(-5*A*e+4*B*d))+c*e^2*(25*a^2*C*e^2+b^
2*d*(-28*B*e+17*C*d)-2*a*b*e*(-14*B*e+33*C*d))+c^2*e*(b*d*(16*C*d^2-7*e*(-
5*A*e+3*B*d))-a*e*(32*C*d^2-7*e*(-5*A*e+7*B*d))))*(c*(e*x+d)/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF
(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)
*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^4/(e*x+d)^(1/2)/(c*x^2+b
*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.97 (sec) , antiderivative size = 19047, normalized size of antiderivative = 24.58

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 3.97 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 2184$$

$$\frac{2\int -\frac{e^3(16cCd-7Bce+6bCe)x^3+e^2(Ce(13bd+5ae)+c(11Cd^2-7Ae^2))x^2+2Cde(cd^2+e(4bd+5ae))x+Cd^2e(bd+5ae)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{7ce^4}+\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-\frac{\int \frac{e^3(16cCd-7Bce+6bCe)x^3+e^2(Ce(13bd+5ae)+c(11Cd^2-7Ae^2))x^2+2Cde(cd^2+e(4bd+5ae))x+Cd^2e(bd+5ae)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{7ce^4}$$

$$\Big\downarrow \; 2184$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-$$

$$\frac{2\int -\frac{\left(\left(57Cd^2-7e(7Bd-5Ae)\right)c^2+e(41bCd-28bBe-25aCe)c+24b^2Ce^2\right)x^2e^5+d\left(6Cdeb^2+18aCe^2b+cd(11Cd-7Be)b+ace(23Cd-21Be)\right)e^4+\left(2c^2(11Cd-7Be\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{5ce^3}$$

$$7ce^4$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-$$

$$\frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+16cCd)}{5c}-\frac{\int \frac{\left(\left(57Cd^2-7e(7Bd-5Ae)\right)c^2+e(41bCd-28bBe-25aCe)c+24b^2Ce^2\right)x^2e^5+d\left(6Cdeb^2+18aCe^2b+cd(\right.}{}}{}$$

$$7ce^4$$

$$\Big\downarrow \; 2184$$

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-$$

$$\frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+16cCd)}{5c}-\frac{2\int -\frac{e^6\left(24Cde^2b^3+\left(24aCe^3+cd(23Cd-28Be)e\right)b^2+c\left(24cCd^3-7ce(4Bd-5Ae)d-2ae^2(19Cd+14Be)\right.}{}}{}$$

$\downarrow$ 27

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-$$

$$\frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+16cCd)}{5c}-\frac{2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ce(-25aCe-28bBe+41bCd)+c^2\left(57Cd^2-7e(7Bd-5Ae)\right)+24b^2Ce^2\right)}{3c}-\frac{e^4\int \frac{2}{}}{}$$

$\downarrow$ 1269

$$\frac{2C(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}-$$

$$\frac{2e(d+ex)^{3/2}\sqrt{a+bx+cx^2}(6bCe-7Bce+16cCd)}{5c}-\frac{2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\left(ce(-25aCe-28bBe+41bCd)+c^2\left(57Cd^2-7e(7Bd-5Ae)\right)+24b^2Ce^2\right)}{3c}-\frac{e^4\left(\left(\right.\right.}{}$$

$\downarrow$ 1172

$$\frac{2C(d+ex)^{5/2}\sqrt{cx^2+bx+a}}{7ce^3}-$$

$$\frac{2e(16cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2e^4\left(\left(57Cd^2-7e(7Bd-5Ae)\right)c^2+e(41bCd-28bBe-25aCe)c+24b^2Ce^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{e^4\left(\frac{\sqrt{2}}{}\right.}{}$$

$\downarrow$ 321

$$\frac{2C(d+ex)^{5/2}\sqrt{cx^2+bx+a}}{7ce^3}-$$

$$\frac{2e(16cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2e^4\left(\left(57Cd^2-7e(7Bd-5Ae)\right)c^2+e(41bCd-28bBe-25aCe)c+24b^2Ce^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{e^4\left(\frac{\sqrt{2}}{\cdots}\right)}{\cdots}$$

$\downarrow$ 327

$$\frac{2C(d+ex)^{5/2}\sqrt{cx^2+bx+a}}{7ce^3}-$$

$$\frac{2e(16cCd-7Bce+6bCe)(d+ex)^{3/2}\sqrt{cx^2+bx+a}}{5c}-\frac{2e^4\left(\left(57Cd^2-7e(7Bd-5Ae)\right)c^2+e(41bCd-28bBe-25aCe)c+24b^2Ce^2\right)\sqrt{d+ex}\sqrt{cx^2+bx+a}}{3c}-\frac{e^4\left(\frac{\sqrt{2}}{\cdots}\right)}{\cdots}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e^3) - ((2*e*(16*c*C*d -
7*B*c*e + 6*b*C*e)*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c) - ((2*e^4*
(24*b^2*C*e^2 + c*e*(41*b*C*d - 28*b*B*e - 25*a*C*e) + c^2*(57*C*d^2 - 7*e
*(7*B*d - 5*A*e)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (e^4*((Sqr
t[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*C*e^3 + 8*b*c*e^2*(5*b*C*d - 7*b*B*e - 13*a
*C*e) + c^3*(48*C*d^3 - 14*d*e*(4*B*d - 5*A*e)) + c^2*e*(40*b*C*d^2 - 7*b*
e*(7*B*d - 10*A*e) - a*e*(44*C*d - 63*B*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a +
b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(24*b
^2*C*e^3*(b*d - a*e) + c^3*(48*C*d^4 - 14*d^2*e*(4*B*d - 5*A*e)) + c*e^2*(
25*a^2*C*e^2 + b^2*d*(17*C*d - 28*B*e) - 2*a*b*e*(33*C*d - 14*B*e)) + c^2*
e*(b*d*(16*C*d^2 - 7*e*(3*B*d - 5*A*e)) - a*e*(32*C*d^2 - 7*e*(7*B*d - 5*A
*e))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(
a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))
)/(3*c))/(5*c*e^3))/(7*c*e^4)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 9.94 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.51

method	result	size
elliptic	Expression too large to display	1172
risch	Expression too large to display	4505
default	Expression too large to display	12988

input
```
int(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C/c/e
*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B-2/7*C/c/e*(3*b
*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(A-2/
7*C/c/e*(5/2*a*e+5/2*b*d)-2/5*(B-2/7*C/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d
))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(-2/5*(B-2/7*C/c/
e*(3*b*e+3*c*d))/c/e*d*a-2/3*(A-2/7*C/c/e*(5/2*a*e+5/2*b*d)-2/5*(B-2/7*C/c
/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(-4/7*a/c*d/e*C-2/5*(B-
2/7*C/c/e*(3*b*e+3*c*d))/c/e*(3/2*a*e+3/2*b*d)-2/3*(A-2/7*C/c/e*(5/2*a*e+5
/2*b*d)-2/5*(B-2/7*C/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*
(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2)
)/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 757, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
2/315*((48*C*c^4*d^4 + 8*(2*C*b*c^3 - 7*B*c^4)*d^3*e + (11*C*b^2*c^2 + 70*
A*c^4 - (8*C*a + 21*B*b)*c^3)*d^2*e^2 + (16*C*b^3*c + 7*(3*B*a + 5*A*b)*c^
3 - (34*C*a*b + 21*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*C*a^2
 + 147*B*a*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*sqrt(c*e)*
weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
 -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9
*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(48*C*c^4*d^3
*e + 8*(5*C*b*c^3 - 7*B*c^4)*d^2*e^2 + (40*C*b^2*c^2 + 70*A*c^4 - (44*C*a
+ 49*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 - 8*(13*C*a*b
+ 7*B*b^2)*c^2)*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + (b
^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c -
6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInverse(4/3
*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b
*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3),
 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*C*c^4*e^4*x^2 + 24*C*c^4*d^2*e^
2 + (23*C*b*c^3 - 28*B*c^4)*d*e^3 + (24*C*b^2*c^2 + 35*A*c^4 - (25*C*a + 2
8*B*b)*c^3)*e^4 - 3*(6*C*c^4*d*e^3 + (6*C*b*c^3 - 7*B*c^4)*e^4)*x)*sqrt(c*
x^2 + b*x + a)*sqrt(e*x + d))/(c^5*e^5)
```

Sympy [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2*(A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x
)
```

Maxima [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)x^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + B*x + A)*x^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2\left(C\,x^2+B\,x+A\right)}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((x^2*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2(C\,x^2+Bx+A)}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.21 $\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 573

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(7cCd-5Bce+4bCe)\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e^2}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$
$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2Ce^2+ce(7bCd-10bBe-9aCe)+c^2(8Cd^2-5e(2Bd-3Ae)))\sqrt{d+ex}\sqrt{-\frac{c(a+bx}{b^2-}}}{15c^3e^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(4bCe^2(bd-ae)+c^2(8Cd^3-5de(2Bd-3Ae))+ce(bd(3Cd-5Be)-ae(7Cd-5Be}{15c^3e^3\sqrt{d+ex}\sqrt{a+}}$$

output
```
-2/15*(-5*B*c*e+4*C*b*e+7*C*c*d)*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^2
+2/5*C*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/15*2^(1/2)*(-4*a*c+b^2)^(
1/2)*(8*b^2*C*e^2+c*e*(-10*B*b*e-9*C*a*e+7*C*b*d)+c^2*(8*C*d^2-5*e*(-3*A*e
+2*B*d)))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/
2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/
(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^3/(c*(e*x+d)/(2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2/15*2^(1/2)*(-4*a*c+b^2)^(1
/2)*(4*b*C*e^2*(-a*e+b*d)+c^2*(8*C*d^3-5*d*e*(-3*A*e+2*B*d))+c*e*(b*d*(-5*
B*e+3*C*d)-a*e*(-5*B*e+7*C*d)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e
))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a
*c+b^2)^(1/2))*e))^(1/2))/c^3/e^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.76 (sec) , antiderivative size = 12291, normalized size of antiderivative = 21.45

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 606, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 2184$$

$$\frac{2\int -\frac{e^2(7cCd-5Bce+4bCe)x^2+e\left(Ce(5bd+3ae)+c\left(2Cd^2-5Ae^2\right)\right)x+Cde(bd+3ae)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}+\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\int \frac{e^2(7cCd-5Bce+4bCe)x^2+e\left(Ce(5bd+3ae)+c\left(2Cd^2-5Ae^2\right)\right)x+Cde(bd+3ae)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}$$

$$\Big\downarrow \; 2184$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2\int -\frac{e^3\left(4Cdeb^2+4aCe^2b+cd(4Cd-5Be)b-ace(2Cd+5Be)+\left(\left(8Cd^2-5e(2Bd-3Ae)\right)c^2+e(7bCd-10bBe-9aCe)c+8b^2Ce^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3}}{5ce^3}$$

$$\Big\downarrow \; 27$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+7cCd)}{3c}-\frac{e\int \frac{4Cdeb^2+4aCe^2b+cd(4Cd-5Be)b-ace(2Cd+5Be)+\left(\left(8Cd^2-5e(2Bd-3Ae)\right)c^2+e(7bCd-10bBe-9aCe}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3c}}{5ce^3}$$

$$\Big\downarrow \; 1269$$

$$\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+7cCd)}{3c}-\frac{e\left(\frac{\left(ce(-9aCe-10bBe+7bCd)+c^2\left(8Cd^2-5e(2Bd-3Ae)\right)+8b^2Ce^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(ce(bd(3Cd-5Be}{}\right.}{3c}}{5ce^3}$$

$$\Big\downarrow \; 1172$$

$$\frac{\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+7cCd)}{3c}-\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ce(-9aCe-10bBe+7bCd)+c^2\left(8Cd^2-5e(2Bd-3Ae)\right)+8b^2Ce^2\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}}{}$$

$\downarrow$ 321

$$\frac{\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+7cCd)}{3c}-\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ce(-9aCe-10bBe+7bCd)+c^2\left(8Cd^2-5e(2Bd-3Ae)\right)+8b^2Ce^2\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}}{}$$

$\downarrow$ 327

$$\frac{\frac{2C(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}-\frac{\frac{2e\sqrt{d+ex}\sqrt{a+bx+cx^2}(4bCe-5Bce+7cCd)}{3c}-\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\left(ce(-9aCe-10bBe+7bCd)+c^2\left(8Cd^2-5e(2Bd-3Ae)\right)+8b^2Ce^2\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}}{}}{}$$

input
```
Int[(x*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e^2) - ((2*e*(7*c*C*d - 5
*B*c*e + 4*b*C*e)*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c) - (e*((Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(8*b^2*C*e^2 + c*e*(7*b*C*d - 10*b*B*e - 9*a*C*e) + c^
2*(8*C*d^2 - 5*e*(2*B*d - 3*A*e)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^
2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/
Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(4*b*C*e^2*(b*d
 - a*e) + c^2*(8*C*d^3 - 5*d*e*(2*B*d - 3*A*e)) + c*e*(b*d*(3*C*d - 5*B*e)
 - a*e*(7*C*d - 5*B*e)))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[
a + b*x + c*x^2])))/(3*c))/(5*c*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 7.94 (sec) , antiderivative size = 950, normalized size of antiderivative = 1.66

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2Cx\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}{5ce}+\frac{2\left(B-\frac{2(2be+2cd)C}{5ce}\right)\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}{3ce}+\frac{2\left(-\frac{2adC}{5ce}-\frac{2}{\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input

```
int(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output

```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*C/c/e
*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(B-2/5/c/e*(2*b*e+2
*c*d)*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(-2/5*a/c*d
/e*C-2/3*(B-2/5/c/e*(2*b*e+2*c*d)*C)/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-
4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((
x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/
2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A-2/5*C/c/e*(3/2*a*e+3
/2*b*d)-2/3*(B-2/5/c/e*(2*b*e+2*c*d)*C)/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c
+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((
x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),
((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a
*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*
(-b+(-4*a*c+b^2)^(1/2))))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.99

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx =$$
$$-\frac{2\left((8\,Cc^3d^3+(3\,Cbc^2-10\,Bc^3)d^2e+(3\,Cb^2c+15\,Ac^3-(3\,Ca+5\,Bb)c^2)de^2+(8\,Cb^3+15\,(Ba+A\right.}{}$$

input
```
integrate(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
-2/45*((8*C*c^3*d^3 + (3*C*b*c^2 - 10*B*c^3)*d^2*e + (3*C*b^2*c + 15*A*c^3
 - (3*C*a + 5*B*b)*c^2)*d*e^2 + (8*C*b^3 + 15*(B*a + A*b)*c^2 - (21*C*a*b
+ 10*B*b^2)*c)*e^3)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e +
 (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c
 - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d +
 b*e)/(c*e)) + 3*(8*C*c^3*d^2*e + (7*C*b*c^2 - 10*B*c^3)*d*e^2 + (8*C*b^2*
c + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*sqrt(c*e)*weierstrassZeta(4/3*(c
^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -
4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a
*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(3*C*c^3*e^3*x
 - 4*C*c^3*d*e^2 - (4*C*b*c^2 - 5*B*c^3)*e^3)*sqrt(c*x^2 + b*x + a)*sqrt(e
*x + d))/(c^4*e^4)
```

Sympy [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x*(C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x*(A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)x}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate((C*x^2 + B*x + A)*x/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{x\,(C\,x^2+B\,x+A)}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((x*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((x*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x(C\,x^2+Bx+A)}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.22 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 450

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4ac}\right.}\right.}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(a\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(-3*B*c*e+2*C*b*e+2*C*c*d)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(c*(e*x
+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/3*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2+C*e*(-a*e+b*d)-3*c*e*(-A*e+B*d))*(c*(e*x+d
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.40

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[
(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-
1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)
) + (I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1
 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(
-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt
[d + e*x] - (I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*
c)*e^2]) + c*(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*
B*e)))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d
/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 +
 d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +...
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \; 2184$$

$$\frac{2\int -\frac{e(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

$$\Big\downarrow \; 27$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \; 1269$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{(2bCe-3Bce+2cCd)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}$$

$$\Big\downarrow \; 1172$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\cdots}}{\cdots}}{3ce}$$

$$\Big\downarrow \; 321$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{}}{}}{3ce}$$

$\Big\downarrow$ 327

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+b}{b^2}}}{}}{3ce}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C
*e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(396) = 792$.

Time = 6.79 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.83

method	result
elliptic	$\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2C\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b}{\ldots}}{-\frac{d}{e}-\ldots}}\ldots}{\sqrt{x^3e\ldots}}\ldots\right.$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*C/c/e
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(B-2/3*C/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}Cc^2e^2+(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2)\sqrt{}\right.}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b
*c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d*e +
 (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.23 $\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 596

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}C\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b\cdots}}{2cd-(b+\cdots}\cdots\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$-\frac{4\sqrt{2}A\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\cdots}{2cd-(\cdots}\cdots\right.}{\left(b+\sqrt{b^2-4ac}\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*C*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-
4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e/(c*(e*x+d)
/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(-B*e+C*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^
2)^(1/2))*e))^(1/2))/c/e/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*A*(-4
*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1
/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a*c
+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/(b+(-4*a*c+b^2)^(1/
2))/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 30.39 (sec) , antiderivative size = 1189, normalized size of antiderivative = 1.99

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((d + e*x)^(3/2)*((4*C*d*e^2*Sqrt[(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e
 + Sqrt[(b^2 - 4*a*c)*e^2])]*(a + x*(b + c*x)))/(d + e*x)^2 - (I*Sqrt[2]*C
*d*(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*a*e^2 + 2*c*d*e*x + b*
e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*c*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + S
qrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2]
)*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-
2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[
d + e*x] + (I*Sqrt[2]*(2*B*c*d*e - b*C*d*e - 2*A*c*e^2 + C*d*Sqrt[(b^2 - 4
*a*c)*e^2])*Sqrt[(-2*a*e^2 + 2*c*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c
)*e^2]*(d + e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqr
t[(2*a*e^2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x
))/((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSi
nh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c
)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + ((2*I)*Sqrt[2]*A*c*e
^2*Sqrt[(-2*a*e^2 + 2*c*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d
 + e*x))/((2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*a*e^
2 - 2*c*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-...
```

Rubi [A] (warning: unable to verify)

Time = 2.50 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.270$, Rules used = {2154, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$A\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{B+Cx}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$A\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\frac{(Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}$$

$\downarrow$ 1172

$$A\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 321

$$A\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 327

$$A\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{1279}$$

$$\frac{A\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right), -\frac{2\sqrt{b^2-4a\cdots}}{2cd-\left(b+\sqrt{b^2\cdots}\right.}\cdots\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\cdots$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{187}$$

$$-\frac{2A\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d\cdots}}{\sqrt{a+bx+cx^2}}\cdots$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right), -\frac{2\sqrt{b^2-4a\cdots}}{2cd-\left(b+\sqrt{b^2\cdots}\right.}\cdots\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}\cdots$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{413}$$

$$-\frac{2A\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{}{2c}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 413

$$-\frac{2A\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{}{ex\sqrt{1-\frac{}{2c}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+e}{e}}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 412

$$-\frac{\sqrt{2}A\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(Cd-Be)\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4a}}{2cd-\left(b+\sqrt{b^2}\right.}\right.}{ce\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

input
```
Int[(A + B*x + C*x^2)/(x*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*C*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*S
qrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(C*d - B*e)*Sqrt[(c*(
d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*A*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*S
qrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b -
Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*EllipticPi[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*c*d), ArcSin[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]],
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/
(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*d)/e +
(2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x)
)/e])
```

Defintions of rubi rules used

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(524) = 1048$.

Time = 6.53 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.79

method	result	size
elliptic	Expression too large to display	1064
default	Expression too large to display	1394

input
```
int((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*B*(d/e-
1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/
2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF((
(x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^
2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*C*(d/e-1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*
((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))
^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))-2*A*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2
))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4
*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(
-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+
b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/d*e*EllipticPi(((x+d/e)/(d/e-1/2...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.24 $\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 650

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{A\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}$$
$$+\frac{A\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}ad\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(Ac-2aC)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b}}{2cd-(b+}\right.}{ac\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$+\frac{2\sqrt{2}\sqrt{b^2-4ac}(Abd-2aBd+aAe)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticPi}\left(\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}},\arcsin\left(\right.\right.}{a\left(b+\sqrt{b^2-4ac}\right)d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
-A*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x+1/2*A*(-4*a*c+b^2)^(1/2)*(e*x+d
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a/d/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/
2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*c-2*C*a)*(
c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/a/c/(e*
x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)+2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*a*e+A*b*d-2
*B*a*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+
a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/
2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)),(-2*(-4*a*c+b^2)^(1
/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))/d/
(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.12 (sec) , antiderivative size = 1405, normalized size of antiderivative = 2.16

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((A*Sqrt[d + e*x]*(a + b*x + c*x^2))/(a*d*x*Sqrt[a + x*(b + c*x)])) + ((d
 + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(4*A*d*Sqrt[(c*d^2 + e*(-(b*d) + a*e))
/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-1 + d/(d + e*x))^2 + (e*(b
 - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)) - (I*Sqrt[2]*A*d*(2*c*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)
/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^
2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*
d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e
*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2
- 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt[2]*(4*a*C*d^2 + 2*a*e*(-2*B*d +
A*e) + A*d*(b*e + Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2]
- (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*
x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2
] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e
*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[
2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])
/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e +
 Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - ((2*I)*Sqrt[2]*e*(A*b*d - ...
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1410 vs. $2(650) = 1300$.

Time = 4.65 (sec) , antiderivative size = 1410, normalized size of antiderivative = 2.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.432$, Rules used = {2154, 1282, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$A\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{B+Cx}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1282}$$

$$A\left(-\frac{\int \frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+\int \frac{B+Cx}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{2154}$$

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int -\frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{C}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{25}$$

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{C}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+C\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1172}$$

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{321}$$

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1269

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}\right)}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1172

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}+B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+A\left(-\frac{(bd+ae)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}\right.}{2ad}\right.$$

$\Big\downarrow$ 321

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} -\right.}{2ad}\right.$$

$$B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+$$

$$\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 327

$$A\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}}{2cd-\left(b+\sqrt{b^2-\right.}}\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\right.}{2a}\right.$$

$$B\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+$$

$$\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1279

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{B\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int\frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{cx^2+bx+a}}+$$

$$A\left(-\frac{\frac{(bd+ae)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int\frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{cx^2+bx+a}}-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(cx^2}{b^2}}}{c}\right.}{}\right.$$

$$\Big\downarrow \; 187$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2B\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int-\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+e}}{\sqrt{cx^2+bx+a}}$$

$$A\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2\right.}}}}{}\right.}{}\right.$$

$$\Big\downarrow \; 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2B\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{2}{2cd-(}}}}{\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}}$$

$$A\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2}\right.}}}{}\right)}{}\right.$$

$$\Big\downarrow \; 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2B\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\int -\frac{}{ex\sqrt{1-\frac{2}{2cd-(b}}}}{\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-}}$$

$$A\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2}\right.}}}{}\right)}{}\right.$$

$\big\downarrow$ 412

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{c\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}B\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2cd-}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}}$$

$$A\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2}\right.}}}{}\right.}{}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*C*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*Sqrt[d + e*x]*S
qrt[a + b*x + c*x^2]) - (Sqrt[2]*B*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*S
qrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c
*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e +
 Sqrt[b^2 - 4*a*c]*e)/(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqr
t[b - Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2
 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e]) + A*(-((Sqrt[d + e*x]*Sqrt[a +
 b*x + c*x^2])/(a*d*x)) - (-(c*e*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*d*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 10.00 (sec) , antiderivative size = 1122, normalized size of antiderivative = 1.73

method	result	size
risch	Expression too large to display	1122
elliptic	Expression too large to display	1130
default	Expression too large to display	4319

input
```
int((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-A*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/d/a*(2*(-A*a*e-A*b*d+2*B*a*
d)*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1
/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+
b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c
+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/d*e
*EllipticPi(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),-(-d/e+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/d*e,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-2*A*c*e*(d/e-1/2*(b+(-4*a*c+b^2)^(1/
2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-
4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3
+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/
2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1
/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(
1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2))))^(1/2)))-4*a*C*d*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/
e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1
/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{x^2\sqrt{d + ex}\sqrt{a + bx + cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**2/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**2*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^2), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{x^2\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{x^2\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.25 $\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 769

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{A\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2} - \frac{(4aBd-3A(bd+ae))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{4a^2d^2x}$$

$$+ \frac{\sqrt{b^2-4ac}(4aBd-3A(bd+ae))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{4\sqrt{2}a^2d^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{b^2-4ac}(3Abd-4aBd+aAe)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\right.}{2\sqrt{2}a^2d\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{b^2-4ac}(4ad(bBd-2aCd+aBe)-A(3b^2d^2+2abde-a(4cd^2-3ae^2)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a}{}}}{\sqrt{2}a^2\left(b+\sqrt{b^2-4ac}\right)d^2\sqrt{d+ex}\sqrt{a+}}$$

output
```
-1/2*A*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/4*(4*B*a*d-3*A*(a*e+b*d
))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/a^2/d^2/x+1/8*(-4*a*c+b^2)^(1/2)*(4*B
*a*d-3*A*(a*e+b*d))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*El
lipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2
)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/d^2/(c*(e*x
+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+1/4*(-4*a*
c+b^2)^(1/2)*(A*a*e+3*A*b*d-4*B*a*d)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/
2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*
x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+
(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2)/a^2/d/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(
1/2)+1/2*(-4*a*c+b^2)^(1/2)*(4*a*d*(B*a*e+B*b*d-2*C*a*d)-A*(3*b^2*d^2+2*a*
b*d*e-a*(-3*a*e^2+4*c*d^2)))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),2*(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))*2^(1/2
)/a^2/(b+(-4*a*c+b^2)^(1/2))/d^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 35.94 (sec) , antiderivative size = 10711, normalized size of antiderivative = 13.93

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2689 vs. $2(769) = 1538$.

Time = 10.06 (sec) , antiderivative size = 2689, normalized size of antiderivative = 3.50, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.649$, Rules used = {2154, 1282, 2154, 27, 1279, 187, 413, 413, 412, 1282, 2154, 25, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$A\int \frac{1}{x^3\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{B+Cx}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1282}$$

$$A\left(-\frac{\int \frac{cex^2+2(cd+be)x+3(bd+ae)}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)+\int \frac{B+Cx}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{2154}$$

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)+B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{C}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)+B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+C\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1279}$$

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)+\frac{C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int \frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{a+bx+cx^2}}+B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 187

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)-\frac{2C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}d\sqrt{d+\cdots}}{\sqrt{a+bx+cx^2}} B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 413

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)-\frac{2C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int -\frac{1}{ex\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1-\frac{\cdots}{2cd-\cdots}}\cdots}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}} B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 413

$$A\left(-\frac{3(ae+bd)\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int \frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)-\frac{2C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int -\frac{\cdots}{ex\sqrt{1-\frac{\cdots}{2cd-\cdots}}\cdots}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}\cdots}} B\int \frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 412

$$A\left(-\frac{3(ae+bd)\int\frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int\frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right)+$$

$$B\int\frac{1}{x^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-$$

$$\frac{\sqrt{2}C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2c}{e}}}$$

$\downarrow$ 1282

$$A\left(-\frac{\int\frac{2cd+2be+cex}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{\int\frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)}{4ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{2adx^2}\right.$$

$$B\left(-\frac{\int\frac{-cex^2+bd+ae}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)-$$

$$\frac{\sqrt{2}C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2c}{e}}}$$

$\downarrow$ 2154

$$A\left(-\frac{\int\frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int-\frac{}{\sqrt{d+}}}{2ad}\right.}{4ad}\right.$$

$$B\left(-\frac{(ae+bd)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+\int-\frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)-$$

$$\frac{\sqrt{2}C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2c}{e}}}$$

$\downarrow$ 25

$$A\left(-\frac{\int \frac{ce}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{}{\sqrt{d+e}}}{2ad}\right.}{4ad}\right.$$

$$B\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-\int \frac{cex}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)-$$

$$\frac{\sqrt{2}C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2}{}}}$$

$\big\downarrow$ 27

$$A\left(-\frac{ce\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+2(be+cd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(ae+bd)\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int}{2ad}\right.}{4ad}\right.$$

$$B\left(-\frac{(ae+bd)\int \frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int \frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{a+bx+cx^2}}{adx}\right)-$$

$$\frac{\sqrt{2}C\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2}{}}}$$

$\big\downarrow$ 1172

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{e}}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int\frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{adx}\right)+$$

$$A\left(-\frac{2(cd+be)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+3(bd+ae)\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int\frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}}{}\right.}{}\right.$$

$\downarrow$ 321

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{e}}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\int\frac{x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{2ad}-\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{adx}\right)+$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)}{4\ldots}\right.$$

$\downarrow$ 1269

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{}}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}\right)}{2ad}-\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}}{adx}\right)+$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)}{}\right.$$

$\downarrow$ 1172

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{e}}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\right.}{2ad}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)}{}\right.$$

↓ 321

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{\cdots}{2c\cdots}}}{\sqrt{cd}\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c\cdots}{e}}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\cdots\right.}{2ad\cdots}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\cdots}{\cdots}\right.$$

$\downarrow$ 327

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{\cdots}{2c\cdots}}\cdots}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c\cdots}{e}\cdots}}$$

$$B\left(-\frac{(bd+ae)\int\frac{1}{x\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\cdots}{2cd-\left(b+\sqrt{b^2-\cdots}\right)\cdots}\cdots\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}\cdots\right.}{2a\cdots}\cdots\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+2(cd+be)\cdots}{\cdots}\cdots\right.$$

$\downarrow$ 1279

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{\ldots}{2c\ldots}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c\ldots}{e}}}$$

$$B\left(-\frac{\frac{(bd+ae)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int\frac{1}{x\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}}dx}{\sqrt{cx^2+bx+a}}-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(cx^2\ldots}{b^2\ldots}}}{c\ldots}\right.}{\ldots}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}+\frac{2(cd+be)\sqrt{b\ldots}}{\ldots}}{\ldots}\right.$$

↓ 187

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{e}}}$$

$$B\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2}\right.}}}{}\right.}{}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{4(cd+be)\sqrt{b}}{}}{}\right.$$

↓ 413

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{cd}\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{}}}$$

$$B\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex}{2cd-\left(b+\sqrt{b^2\right.}}}}{}\right.}{}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{4(cd+be)\sqrt{b-}}{}}{}\right.$$

↓ 413

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{\cdots}{2c\cdots}}}{\sqrt{cd}\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c\cdots}{e\cdots}}}$$

$$B\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex\cdots}{2cd-\left(b+\sqrt{b^2\cdots}\right.}}\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{4(cd+be)\sqrt{b\cdots}}{\cdots}\cdots}{\cdots}\cdots\right.$$

↓ 412

$$-\frac{\sqrt{2}C\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{}{2c}}}{\sqrt{c}d\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2c}{}}}$$

$$B\left(-\frac{-ce\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}d\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2}\right.}}}{}\right.}{}\right.$$

$$A\left(-\frac{\frac{2\sqrt{2}\sqrt{b^2-4ac}e\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}-\frac{2\sqrt{2}(cd+be)}{}}{}\right.$$

input
```
Int[(A + B*x + C*x^2)/(x^3*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
-((Sqrt[2]*C*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4
*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/
(2*c*d), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]], (2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(Sqrt[c]*d*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c
] - (2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e
+ (2*c*(d + e*x))/e])) + B*(-((Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(a*d*x
)) - (-(c*e*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x +
 c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[(c*
(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^
2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]))) - (Sqrt[2]*(b
*d + a*e)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[b - Sqrt[b^2 - 4*a*
c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x)...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 14.41 (sec) , antiderivative size = 1205, normalized size of antiderivative = 1.57

method	result	size
risch	Expression too large to display	1205
elliptic	Expression too large to display	1259
default	Expression too large to display	8918

input
```
int((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)*(-3*A*a*e*x-3*A*b*d*x+4*B*a*d*x+2*A
*a*d)/d^2/a^2/x^2+1/8/a^2/d^2*(-2*c*e*(3*A*a*e+3*A*b*d-4*B*a*d)*(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2
)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d/e)/(d/e-1/2
*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(-3*A*a^2*e^2-2*A*a*b*d*e+4*A
*a*c*d^2-3*A*b^2*d^2+4*B*a^2*d*e+4*B*a*b*d^2-8*C*a^2*d^2)*(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1
/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/d*e*EllipticPi(((x+d/e
)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),-(-d/e+1/2*(b+(-4*a*c+b^2)^(1/
2))/c)/d*e,((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^
2)^(1/2))))^(1/2))-4*A*a*c*d*e*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/x**3/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(x**3*sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x
)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^3), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}x^3}\,dx$$

input
```
integrate((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{x^3\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/(x^3*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/(x^3*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{x^3\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{x^3\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/x^3/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.26 $\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 386

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),\frac{2\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{ce\sqrt{x}\sqrt{a+bx+cx^2}} - \frac{4\sqrt{2}\sqrt{b^2-4ac}(Bd-Ae)\sqrt{-\frac{cx}{b+\sqrt{b^2-4ac}}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e},\arcsin\left(\frac{\sqrt{1+\frac{b+2}{\sqrt{b^2-}}}}{\sqrt{2}}\right.\right.}{e\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{x}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*B*(-4*a*c+b^2)^(1/2)*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*
x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/
2))^(1/2)*2^(1/2),2^(1/2)*((-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2)))^(1/2
))/c/e/x^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*(-4*a*c+b^2)^(1/2)*(-A*e+B*d)
*(-c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)
*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+
b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e),2^(1/2)*((-4*a*c+b^2)^(1/2)/
(b+(-4*a*c+b^2)^(1/2)))^(1/2))/e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/x^(1/2)/
(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.61 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{i\sqrt{1+\frac{2a}{\left(b+\sqrt{b^2-4ac}\right)x}}\,x\sqrt{2+\frac{4a}{bx-\sqrt{b^2-4ac}\,x}}\left(Ae\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{2}\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}}{\sqrt{x}}\right),\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+(Bd-A}{\sqrt{\frac{a}{b+\sqrt{b^2-4ac}}}\,de\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*Sqrt[1 + (2*a)/((b + Sqrt[b^2 - 4*a*c])*x)]*x*Sqrt[2 + (4*a)/(b*x - Sqr
t[b^2 - 4*a*c]*x)]*(A*e*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[a/(b + Sqrt[b^2
- 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + (
B*d - A*e)*EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[(Sqrt
[2]*Sqrt[a/(b + Sqrt[b^2 - 4*a*c])])/Sqrt[x]], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])]))/(Sqrt[a/(b + Sqrt[b^2 - 4*a*c])]*d*e*Sqrt[a + x*(b
 + c*x)])
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2035, 2226, 27, 1416, 2220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{2035}$$
$$2\int \frac{A+Bx}{(d+ex)\sqrt{cx^2+bx+a}}d\sqrt{x}$$

$$\Big\downarrow \text{2226}$$

$$2\left(\frac{\sqrt{a}(Bd-Ae)\int \frac{\sqrt{c}x+\sqrt{a}}{\sqrt{a}(d+ex)\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}d-\sqrt{a}e}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{(Bd-Ae)\int \frac{\sqrt{c}x+\sqrt{a}}{(d+ex)\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}B-A\sqrt{c}\right)\int \frac{1}{\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}d-\sqrt{a}e}\right)$$

$$\Big\downarrow \text{1416}$$

$$2\left(\frac{(Bd-Ae)\int \frac{\sqrt{c}x+\sqrt{a}}{(d+ex)\sqrt{cx^2+bx+a}}d\sqrt{x}}{\sqrt{c}d-\sqrt{a}e}-\frac{\left(\sqrt{a}+\sqrt{c}x\right)\left(\sqrt{a}B-A\sqrt{c}\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\right.}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx+cx^2}\left(\sqrt{c}d-\sqrt{a}e\right)}\right.$$

$$\Big\downarrow \text{2220}$$

$$2\left(\frac{(Bd-Ae)\left(\frac{\left(\sqrt{a}+\sqrt{c}x\right)\sqrt{\frac{a+bx+cx^2}{\left(\sqrt{a}+\sqrt{c}x\right)^2}}\left(\sqrt{a}e+\sqrt{c}d\right)\text{EllipticPi}\left(-\frac{\sqrt{a}\left(\frac{\sqrt{c}d}{\sqrt{a}}-e\right)^2}{4\sqrt{c}de},2\arctan\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{a}}\right),\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{4\sqrt[4]{a}\sqrt[4]{c}de\sqrt{a+bx+cx^2}}-\frac{\left(\sqrt{c}d-\sqrt{a}e\right)\text{arc}}{2\sqrt{d}\sqrt{e}}\right.}{\sqrt{c}d-\sqrt{a}e}\right.$$

input
```
Int[(A + B*x)/(Sqrt[x]*(d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
2*(-1/2*((Sqrt[a]*B - A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x
^2)/(Sqrt[a] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/a^(1/4)]
, (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(1/4)*c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*S
qrt[a + b*x + c*x^2]) + ((B*d - A*e)*(-1/2*((Sqrt[c]*d - Sqrt[a]*e)*ArcTan
[(Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[x])/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2]) + ((Sqrt[c]*d + Sqrt[
a]*e)*(Sqrt[a] + Sqrt[c]*x)*Sqrt[(a + b*x + c*x^2)/(Sqrt[a] + Sqrt[c]*x)^2
]*EllipticPi[-1/4*(Sqrt[a]*((Sqrt[c]*d)/Sqrt[a] - e)^2)/(Sqrt[c]*d*e), 2*A
rcTan[(c^(1/4)*Sqrt[x])/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(4*a^(1/4)
*c^(1/4)*d*e*Sqrt[a + b*x + c*x^2])))/(Sqrt[c]*d - Sqrt[a]*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2035
```
Int[(Fx_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Simp[k   Subst
[Int[x^(k*(m + 1) - 1)*SubstPower[Fx, x, k], x], x, x^(1/k)], x]] /; Fracti
onQ[m] && AlgebraicFunctionQ[Fx, x]
```

rule 2220
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(1 + q^2*x^2)*(Sqrt[(a
+ b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/(4*d*e*q*Sqrt[a + b*x^2 + c*x^4]))*El
lipticPi[-(e - d*q^2)^2/(4*d*e*q^2), 2*ArcTan[q*x], 1/2 - b/(4*a*q^2)], x]]
 /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &
& EqQ[c*A^2 - a*B^2, 0] && PosQ[B/A] && PosQ[-b + c*(d/e) + a*(e/d)]
```

rule 2226
```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Simp[(A*(c*d + a*e*q)
- a*B*(e + d*q))/(c*d^2 - a*e^2)   Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Simp[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2))   Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 628, normalized size of antiderivative = 1.63

method	result
elliptic	$\frac{\sqrt{x(c\,x^2+bx+a)}\left(\frac{B\left(b+\sqrt{-4ac+b^2}\right)\sqrt{2}\sqrt{\frac{\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)c}{b+\sqrt{-4ac+b^2}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{b+\sqrt{-4ac+b^2}}{2c}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{-\frac{2cx}{b+\sqrt{-4ac+b^2}}}\ \mathrm{EllipticF}\left(\sqrt{2}\sqrt{\left(\cdots\right.}\right.}{ec\sqrt{c\,x^3+b\,x^2+ax}}\right.}{\cdots}$
default	$-\frac{2\left(A\sqrt{-4ac+b^2}\ \mathrm{EllipticPi}\left(\sqrt{\frac{b+\sqrt{-4ac+b^2}+2cx}{b+\sqrt{-4ac+b^2}}},\frac{\left(b+\sqrt{-4ac+b^2}\right)e}{e\sqrt{-4ac+b^2}+be-2cd},\frac{\sqrt{2}\sqrt{\frac{b+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)ce+A\,\mathrm{EllipticPi}\left(\sqrt{\frac{b+\sqrt{-4ac+b^2}+2c\cdots}{b+\sqrt{-4ac+b^2}}}\right.\cdots}{\cdots}$

input
```
int((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x*(c*x^2+b*x+a))^(1/2)/x^(1/2)/(c*x^2+b*x+a)^(1/2)*(B/e*(b+(-4*a*c+b^2)^(
1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c-
1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4*a*c+b^2)^(1/2)))^(1/2
)/(c*x^3+b*x^2+a*x)^(1/2)*EllipticF(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)/(b+(-4*a*c+b^2)^(1/2))*c)^(1/2),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2
*(b+(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+(A*e-B*d)
/e^2*(b+(-4*a*c+b^2)^(1/2))/c*2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-1/2*(b+
(-4*a*c+b^2)^(1/2))/c-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(-2*c*x/(b+(-4
*a*c+b^2)^(1/2)))^(1/2)/(c*x^3+b*x^2+a*x)^(1/2)/(d/e-1/2*(b+(-4*a*c+b^2)^(
1/2))/c)*EllipticPi(2^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(b+(-4*a*c+b
^2)^(1/2))*c)^(1/2),-1/2*(b+(-4*a*c+b^2)^(1/2))/c/(d/e-1/2*(b+(-4*a*c+b^2)
^(1/2))/c),1/2*(-2*(b+(-4*a*c+b^2)^(1/2))/c/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{x}\,(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/x**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(x)*(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima
")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{x}\,\left(d+e\,x\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{x}(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{x}\,\left(ex+d\right)\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((B*x+A)/x^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

3.27 $\int \frac{1+16x}{(5+x)^2(-3+2x)(1+x+x^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = -\frac{79}{273(5+x)} + \frac{451\arctan\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}} + \frac{200\log(3-2x)}{3211} + \frac{2731\log(5+x)}{24843} - \frac{481\log\left(1+x+x^2\right)}{5586}$$

output
```
-79/(1365+273*x)+451/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+200/3211*ln(
3-2*x)+2731/24843*ln(5+x)-481/5586*ln(x^2+x+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\,(1+x+x^2)}\,dx = \frac{-\frac{819546}{5+x} + 152438\sqrt{3}\arctan\left(\frac{1+2x}{\sqrt{3}}\right) + 176400\log(3-2x) + 311334\log(5+x) - 243867\log\left(1+x+x^2\right.}{2832102}$$

input
```
Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

output
```
(-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
 - 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{16x+1}{(x+5)^2(2x-3)\left(x^2+x+1\right)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{-481x-15}{2793\left(x^2+x+1\right)}+\frac{2731}{24843(x+5)}+\frac{400}{3211(2x-3)}+\frac{79}{273(x+5)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{451\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}$$

input
```
Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

output
```
-79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*
Log[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result
default	$-\frac{481 \ln\left(x^{2}+x+1\right)}{5586}+\frac{451 \arctan\left(\frac{(1+2 x)\sqrt{3}}{3}\right)\sqrt{3}}{8379}+\frac{200 \ln\left(-3+2 x\right)}{3211}-\frac{79}{273 (5+x)}+\frac{2731 \ln\left(5+x\right)}{24843}$
risch	$-\frac{79}{273 (5+x)}+\frac{200 \ln\left(-3+2 x\right)}{3211}+\frac{2731 \ln\left(5+x\right)}{24843}-\frac{481 \ln\left(203401 x^{2}+203401 x+203401\right)}{5586}+\frac{451\sqrt{3}\, \arctan\left(\frac{2\left(451 x+\frac{451}{2}\right)\sqrt{3}}{1353}\right)}{8379}$

input
```
int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
-481/5586*ln(x^2+x+1)+451/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+200/321
1*ln(-3+2*x)-79/273/(5+x)+2731/24843*ln(5+x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \frac{152438\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-243867(x+5)\log\left(x^2+x+1\right)+176400(x+5)\log(2x-}{2832102(x+5)}$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

output
```
1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(
x + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log
(x + 5) - 819546)/(x + 5)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log(x+5)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

input
```
integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

output
```
200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
 + 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{79}{273(x+5)}-\frac{481}{5586}\log\left(x^2+x+1\right)+\frac{200}{3211}\log(2x-3)+\frac{2731}{24843}\log(x+5)$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

output
```
451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586
*log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right) - \frac{79}{273\,(x+5)} - \frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right) + \frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

input
```
integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

output
```
451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/
5586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2
))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \frac{200\,\ln\left(x-\frac{3}{2}\right)}{3211} + \frac{2731\,\ln\left(x+5\right)}{24843} - \frac{79}{273\,\left(x+5\right)} - \ln\left(x+\frac{1}{2}-\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right) + \ln\left(x+\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\left(-\frac{481}{5586}+\frac{\sqrt{3}\,451\mathrm{i}}{16758}\right)$$

input
```
int((16*x + 1)/((2*x - 3)*(x + 5)^2*(x + x^2 + 1)),x)
```

output
```
(200*log(x - 3/2))/3211 + (2731*log(x + 5))/24843 - 79/(273*(x + 5)) - log
(x - (3^(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 + 481/5586) + log(x + (3^
(1/2)*1i)/2 + 1/2)*((3^(1/2)*451i)/16758 - 481/5586)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$$
$$= \frac{762190\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)x+3810950\sqrt{3}\,atan\left(\frac{2x+1}{\sqrt{3}}\right)-1219335\log\left(x^2+x+1\right)x-6096675\log\left(x^2+x-\right.}{14160510}$$

input
```
int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

output
```
(762190*sqrt(3)*atan((2*x + 1)/sqrt(3))*x + 3810950*sqrt(3)*atan((2*x + 1)
/sqrt(3)) - 1219335*log(x**2 + x + 1)*x - 6096675*log(x**2 + x + 1) + 8820
00*log(2*x - 3)*x + 4410000*log(2*x - 3) + 1556670*log(x + 5)*x + 7783350*
log(x + 5) + 819546*x)/(14160510*(x + 5))
```

3.28 $\int \frac{(d+ex)^3(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 245

$$\begin{aligned}
\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx
&= \frac{e(3eg(3Aeg-B(3ef-9dg+5eg))+C(27d^2g^2-9deg(3f+5g)+e^2(9f^2+15fg+19g^2)))\,x}{27g^3} \\
&\quad + \frac{e^2(3Beg-C(3ef-9dg+5eg))x^2}{18g^2} + \frac{Ce^3x^3}{9g} - \frac{(A-B+C)(d-e)^3\log(1+x)}{f-g} \\
&\quad + \frac{(9A-6B+4C)(3d-2e)^3\log(2+3x)}{81(3f-2g)} \\
&\quad - \frac{(ef-dg)^3\,(Cf^2-Bfg+Ag^2)\log(f+gx)}{(3f-2g)(f-g)g^4}
\end{aligned}$$

output

```
1/27*e*(3*e*g*(3*A*e*g-B*(-9*d*g+3*e*f+5*e*g))+C*(27*d^2*g^2-9*d*e*g*(3*f+
5*g)+e^2*(9*f^2+15*f*g+19*g^2)))*x/g^3+1/18*e^2*(3*B*e*g-C*(-9*d*g+3*e*f+5
*e*g))*x^2/g^2+1/9*C*e^3*x^3/g-(A-B+C)*(d-e)^3*ln(1+x)/(f-g)+(9*A-6*B+4*C)
*(3*d-2*e)^3*ln(2+3*x)/(243*f-162*g)-(-d*g+e*f)^3*(A*g^2-B*f*g+C*f^2)*ln(g
*x+f)/(3*f-2*g)/(f-g)/g^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 492 vs. $2(245) = 490$.

Time = 0.63 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{3eg(3f^2-5fg+2g^2)\,x(3eg(6Aeg+B(-6ef+18dg+eg(-10+3x)))+C(54d^2g^2+9deg(-6f+g(-$$

input
```
Integrate[((d + e*x)^3*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(3*e*g*(3*f^2 - 5*f*g + 2*g^2)*x*(3*e*g*(6*A*e*g + B*(-6*e*f + 18*d*g + e*
g*(-10 + 3*x))) + C*(54*d^2*g^2 + 9*d*e*g*(-6*f + g*(-10 + 3*x)) + e^2*(18
*f^2 + f*g*(30 - 9*x) + g^2*(38 - 15*x + 6*x^2)))) - 2*g^4*(9*A*(9*d*e^2*(
13*f - 10*g) + 9*d^3*(6*f - 5*g) - 27*d^2*e*(5*f - 4*g) + e^3*(-35*f + 26*
g)) - 3*B*(9*d*e^2*(35*f - 26*g) + 27*d^3*(5*f - 4*g) + e^3*(-97*f + 70*g)
 + d^2*e*(-351*f + 270*g)) + C*(9*d*e^2*(97*f - 70*g) + 27*d^3*(13*f - 10*
g) + e^3*(-275*f + 194*g) + d^2*e*(-945*f + 702*g)))*ArcTanh[5 + 6*x] - 16
2*(e*f - d*g)^3*(C*f^2 + g*(-(B*f) + A*g))*Log[f + g*x] + g^4*(C*(e^3*(211
*f - 130*g) - 9*d*e^2*(65*f - 38*g) + 27*d^2*e*(19*f - 10*g) - 27*d^3*(5*f
 - 2*g)) - 9*A*(-27*d^2*e*f + 9*d*e^2*(5*f - 2*g) + 9*d^3*g + e^3*(-19*f +
 10*g)) + 3*B*(27*d^3*f + 9*d*e^2*(19*f - 10*g) - 27*d^2*e*(5*f - 2*g) + e
^3*(-65*f + 38*g)))*Log[2 + 5*x + 3*x^2])/(162*g^4*(3*f^2 - 5*f*g + 2*g^2)
)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(3x^2+5x+2)(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{e\left(3eg(3Aeg-B(-9dg+3ef+5eg))+C\left(27d^2g^2-9deg(3f+5g)+e^2\left(9f^2+15fg+19g^2\right)\right)\right)}{27g^3}-\frac{(ef-dg}{g^3(3f}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{ex\left(3eg(3Aeg-B(-9dg+3ef+5eg))+C\left(27d^2g^2-9deg(3f+5g)+e^2\left(9f^2+15fg+19g^2\right)\right)\right)}{27g^3}-\frac{(ef-dg)^3\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{g^4(3f-2g)(f-g)}+\frac{(3d-2e)^3\log(3x+2)(9A-6B+4C)}{81(3f-2g)}-\frac{(d-e)^3\log(x+1)(A-B+C)}{f-g}+\frac{e^2x^2(3Beg-C(-9dg+3ef+5eg))}{18g^2}+\frac{Ce^3x^3}{9g}$$

input
```
Int[((d + e*x)^3*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(e*(3*e*g*(3*A*e*g - B*(3*e*f - 9*d*g + 5*e*g)) + C*(27*d^2*g^2 - 9*d*e*g*
(3*f + 5*g) + e^2*(9*f^2 + 15*f*g + 19*g^2)))*x)/(27*g^3) + (e^2*(3*B*e*g
- C*(3*e*f - 9*d*g + 5*e*g))*x^2)/(18*g^2) + (C*e^3*x^3)/(9*g) - ((A - B +
 C)*(d - e)^3*Log[1 + x])/(f - g) + ((9*A - 6*B + 4*C)*(3*d - 2*e)^3*Log[2
 + 3*x])/(81*(3*f - 2*g)) - ((e*f - d*g)^3*(C*f^2 - B*f*g + A*g^2)*Log[f +
 g*x])/((3*f - 2*g)*(f - g)*g^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 482 vs. $2(236) = 472$.

Time = 0.34 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.97

method	result
norman	$\frac{C\,e^{3} x^{3}}{9 g}+\frac{e \left(9 A\,e^{2} g^{2}+27 B d e\,g^{2}-9 B\,e^{2} f g-15 B\,e^{2} g^{2}+27 C\,d^{2} g^{2}-27 C d e f g-45 C d e\,g^{2}+9 C\,e^{2} f^{2}+15 C\,e^{2} f g+19 C\,e^{2} g^{2}\right) x}{27 g^{3}}$ –
default	$\frac{e \left(3 C\,e^{2} x^{3} g^{2}+\frac{9}{2} B\,e^{2} g^{2} x^{2}+\frac{27}{2} C d e\,g^{2} x^{2}-\frac{9}{2} C\,e^{2} f g\,x^{2}-\frac{15}{2} C\,e^{2} x^{2} g^{2}+9 x A\,e^{2} g^{2}+27 x B d e\,g^{2}-9 x B\,e^{2} f g-15 B\,e^{2} g^{2} x+27 x C\,d^{2}\right.}{27 g^{3}}$
risch	Expression too large to display
parallelrisch	Expression too large to display

input

```
int((e*x+d)^3*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output

```
1/9*C*e^3*x^3/g+1/27*e*(9*A*e^2*g^2+27*B*d*e*g^2-9*B*e^2*f*g-15*B*e^2*g^2+
27*C*d^2*g^2-27*C*d*e*f*g-45*C*d*e*g^2+9*C*e^2*f^2+15*C*e^2*f*g+19*C*e^2*g
^2)/g^3*x+1/18*e^2/g^2*(3*B*e*g+9*C*d*g-3*C*e*f-5*C*e*g)*x^2+1/g^4*(A*d^3*
g^5-3*A*d^2*e*f*g^4+3*A*d*e^2*f^2*g^3-A*e^3*f^3*g^2-B*d^3*f*g^4+3*B*d^2*e*
f^2*g^3-3*B*d*e^2*f^3*g^2+B*e^3*f^4*g+C*d^3*f^2*g^3-3*C*d^2*e*f^3*g^2+3*C*
d*e^2*f^4*g-C*e^3*f^5)/(f-g)/(3*f-2*g)*ln(g*x+f)-(A*d^3-3*A*d^2*e+3*A*d*e^
2-A*e^3-B*d^3+3*B*d^2*e-3*B*d*e^2+B*e^3+C*d^3-3*C*d^2*e+3*C*d*e^2-C*e^3)/(
f-g)*ln(1+x)+1/81*(243*A*d^3-486*A*d^2*e+324*A*d*e^2-72*A*e^3-162*B*d^3+32
4*B*d^2*e-216*B*d*e^2+48*B*e^3+108*C*d^3-216*C*d^2*e+144*C*d*e^2-32*C*e^3)
/(3*f-2*g)*ln(2+3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(239) = 478$.

Time = 142.75 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.72

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input

```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fric
as")
```

output
```
1/162*(18*(3*C*e^3*f^2*g^3 - 5*C*e^3*f*g^4 + 2*C*e^3*g^5)*x^3 - 9*(9*C*e^3
*f^3*g^2 - 9*(3*C*d*e^2 + B*e^3)*f^2*g^3 + (45*C*d*e^2 + (15*B - 19*C)*e^3
)*f*g^4 - 2*(9*C*d*e^2 + (3*B - 5*C)*e^3)*g^5)*x^2 + 6*(27*C*e^3*f^4*g - 2
7*(3*C*d*e^2 + B*e^3)*f^3*g^2 + 27*(3*C*d^2*e + 3*B*d*e^2 + A*e^3)*f^2*g^3
 - (135*C*d^2*e + 9*(15*B - 19*C)*d*e^2 + (45*A - 57*B + 65*C)*e^3)*f*g^4
+ 2*(27*C*d^2*e + 9*(3*B - 5*C)*d*e^2 + (9*A - 15*B + 19*C)*e^3)*g^5)*x -
162*(C*e^3*f^5 - A*d^3*g^5 - (3*C*d*e^2 + B*e^3)*f^4*g + (3*C*d^2*e + 3*B*
d*e^2 + A*e^3)*f^3*g^2 - (C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*f^2*g^3 + (B*d^3
+ 3*A*d^2*e)*f*g^4)*log(g*x + f) + 2*((27*(9*A - 6*B + 4*C)*d^3 - 54*(9*A
- 6*B + 4*C)*d^2*e + 36*(9*A - 6*B + 4*C)*d*e^2 - 8*(9*A - 6*B + 4*C)*e^3)
*f*g^4 - (27*(9*A - 6*B + 4*C)*d^3 - 54*(9*A - 6*B + 4*C)*d^2*e + 36*(9*A
- 6*B + 4*C)*d*e^2 - 8*(9*A - 6*B + 4*C)*e^3)*g^5)*log(3*x + 2) - 162*(3*(
(A - B + C)*d^3 - 3*(A - B + C)*d^2*e + 3*(A - B + C)*d*e^2 - (A - B + C)*
e^3)*f*g^4 - 2*((A - B + C)*d^3 - 3*(A - B + C)*d^2*e + 3*(A - B + C)*d*e^
2 - (A - B + C)*e^3)*g^5)*log(x + 1))/(3*f^2*g^4 - 5*f*g^5 + 2*g^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)} dx = \text{Timed out}$$

input
```
integrate((e*x+d)**3*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.68

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx =$$
$$-\frac{(Ce^3f^5 - Ad^3g^5 - (3\,Cde^2 + Be^3)f^4g + (3\,Cd^2e + 3\,Bde^2 + Ae^3)f^3g^2 - (Cd^3 + 3\,Bd^2e + 3\,Ade^2)f^2}{3\,f^2g^4 - 5\,fg^5 + 2\,g^6}$$
$$+\frac{(27\,(9\,A - 6\,B + 4\,C)d^3 - 54\,(9\,A - 6\,B + 4\,C)d^2e + 36\,(9\,A - 6\,B + 4\,C)de^2 - 8\,(9\,A - 6\,B + 4\,C)e}{81\,(3\,f - 2\,g)}$$
$$-\frac{((A - B + C)d^3 - 3\,(A - B + C)d^2e + 3\,(A - B + C)de^2 - (A - B + C)e^3)\log(x+1)}{f - g}$$
$$+\frac{6\,Ce^3g^2x^3 - 3\,(3\,Ce^3fg - (9\,Cde^2 + (3\,B - 5\,C)e^3)g^2)x^2 + 2\,(9\,Ce^3f^2 - 3\,(9\,Cde^2 + (3\,B - 5\,C)e^3)f}{54\,g^3}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxi
ma")
```

output
```
-(C*e^3*f^5 - A*d^3*g^5 - (3*C*d*e^2 + B*e^3)*f^4*g + (3*C*d^2*e + 3*B*d*e
^2 + A*e^3)*f^3*g^2 - (C*d^3 + 3*B*d^2*e + 3*A*d*e^2)*f^2*g^3 + (B*d^3 + 3
*A*d^2*e)*f*g^4)*log(g*x + f)/(3*f^2*g^4 - 5*f*g^5 + 2*g^6) + 1/81*(27*(9*
A - 6*B + 4*C)*d^3 - 54*(9*A - 6*B + 4*C)*d^2*e + 36*(9*A - 6*B + 4*C)*d*e
^2 - 8*(9*A - 6*B + 4*C)*e^3)*log(3*x + 2)/(3*f - 2*g) - ((A - B + C)*d^3
- 3*(A - B + C)*d^2*e + 3*(A - B + C)*d*e^2 - (A - B + C)*e^3)*log(x + 1)/
(f - g) + 1/54*(6*C*e^3*g^2*x^3 - 3*(3*C*e^3*f*g - (9*C*d*e^2 + (3*B - 5*C
)*e^3)*g^2)*x^2 + 2*(9*C*e^3*f^2 - 3*(9*C*d*e^2 + (3*B - 5*C)*e^3)*f*g + (
27*C*d^2*e + 9*(3*B - 5*C)*d*e^2 + (9*A - 15*B + 19*C)*e^3)*g^2)*x)/g^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(239) = 478.

Time = 0.13 (sec) , antiderivative size = 520, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^3\,(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx =$$

$$-\frac{(Ce^3f^5 - 3\,Cde^2f^4g - Be^3f^4g + 3\,Cd^2ef^3g^2 + 3\,Bde^2f^3g^2 + Ae^3f^3g^2 - Cd^3f^2g^3 - 3\,Bd^2ef^2g^3 - 3}{3\,f^2g^4 - 5\,fg^5 + 2\,g^6}$$

$$+\frac{(243\,Ad^3 - 162\,Bd^3 + 108\,Cd^3 - 486\,Ad^2e + 324\,Bd^2e - 216\,Cd^2e + 324\,Ade^2 - 216\,Bde^2 + 144\,Cde}{81\,(3\,f - 2\,g)}$$

$$-\frac{(Ad^3 - Bd^3 + Cd^3 - 3\,Ad^2e + 3\,Bd^2e - 3\,Cd^2e + 3\,Ade^2 - 3\,Bde^2 + 3\,Cde^2 - Ae^3 + Be^3 - Ce^3)\log}{f-g}$$

$$+\frac{6\,Ce^3g^2x^3 - 9\,Ce^3fgx^2 + 27\,Cde^2g^2x^2 + 9\,Be^3g^2x^2 - 15\,Ce^3g^2x^2 + 18\,Ce^3f^2x - 54\,Cde^2fgx - 18\,B}{54}$$

input
```
integrate((e*x+d)^3*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac
")
```

output
```
-(C*e^3*f^5 - 3*C*d*e^2*f^4*g - B*e^3*f^4*g + 3*C*d^2*e*f^3*g^2 + 3*B*d*e^
2*f^3*g^2 + A*e^3*f^3*g^2 - C*d^3*f^2*g^3 - 3*B*d^2*e*f^2*g^3 - 3*A*d*e^2*
f^2*g^3 + B*d^3*f*g^4 + 3*A*d^2*e*f*g^4 - A*d^3*g^5)*log(abs(g*x + f))/(3*
f^2*g^4 - 5*f*g^5 + 2*g^6) + 1/81*(243*A*d^3 - 162*B*d^3 + 108*C*d^3 - 486
*A*d^2*e + 324*B*d^2*e - 216*C*d^2*e + 324*A*d*e^2 - 216*B*d*e^2 + 144*C*d
*e^2 - 72*A*e^3 + 48*B*e^3 - 32*C*e^3)*log(abs(3*x + 2))/(3*f - 2*g) - (A*
d^3 - B*d^3 + C*d^3 - 3*A*d^2*e + 3*B*d^2*e - 3*C*d^2*e + 3*A*d*e^2 - 3*B*
d*e^2 + 3*C*d*e^2 - A*e^3 + B*e^3 - C*e^3)*log(abs(x + 1))/(f - g) + 1/54*
(6*C*e^3*g^2*x^3 - 9*C*e^3*f*g*x^2 + 27*C*d*e^2*g^2*x^2 + 9*B*e^3*g^2*x^2
- 15*C*e^3*g^2*x^2 + 18*C*e^3*f^2*x - 54*C*d*e^2*f*g*x - 18*B*e^3*f*g*x +
30*C*e^3*f*g*x + 54*C*d^2*e*g^2*x + 54*B*d*e^2*g^2*x - 90*C*d*e^2*g^2*x +
18*A*e^3*g^2*x - 30*B*e^3*g^2*x + 38*C*e^3*g^2*x)/g^3
```

Mupad [B] (verification not implemented)

Time = 27.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.47

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$$

$$= x^2\left(\frac{B\,e^3+3\,C\,d\,e^2}{6\,g}-\frac{C\,e^3\,(3\,f+5\,g)}{18\,g^2}\right)-x\left(\frac{\left(\frac{B\,e^3+3\,C\,d\,e^2}{3\,g}-\frac{C\,e^3\,(3\,f+5\,g)}{9\,g^2}\right)\,(3\,f+5\,g)}{3\,g}-\frac{3\,C\,d^2\,e+3\,B\,d\,e^2+A\,e^3}{3\,g}+\frac{C\,e^3\,(5\,f+2\,g)}{9\,g^2}\right)$$

$$-\frac{\ln(f+g\,x)\,\left(g^4\,(B\,f\,d^3+3\,A\,e\,f\,d^2)-g^3\,(C\,d^3\,f^2+3\,B\,d^2\,e\,f^2+3\,A\,d\,e^2\,f^2)+g^2\,(3\,C\,d^2\,e\,f^3+3\,B\,d\right.}{3\,f^2\,g^4-5\,f\,g^5+2\,g^6}$$

$$+\frac{C\,e^3\,x^3}{9\,g}+\frac{\ln\left(x+\frac{2}{3}\right)\,(3\,d-2\,e)^3\,(9\,A-6\,B+4\,C)}{81\,(3\,f-2\,g)}$$

$$-\frac{\ln(x+1)\,(d-e)^3\,(A-B+C)}{f-g}$$

input
```
int(((d + e*x)^3*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
x^2*((B*e^3 + 3*C*d*e^2)/(6*g) - (C*e^3*(3*f + 5*g))/(18*g^2)) - x*((((B*e
^3 + 3*C*d*e^2)/(3*g) - (C*e^3*(3*f + 5*g))/(9*g^2))*(3*f + 5*g))/(3*g) -
(A*e^3 + 3*B*d*e^2 + 3*C*d^2*e)/(3*g) + (C*e^3*(5*f + 2*g))/(9*g^2)) - (lo
g(f + g*x)*(g^4*(B*d^3*f + 3*A*d^2*e*f) - g^3*(C*d^3*f^2 + 3*A*d*e^2*f^2 +
 3*B*d^2*e*f^2) + g^2*(A*e^3*f^3 + 3*B*d*e^2*f^3 + 3*C*d^2*e*f^3) - g*(B*e
^3*f^4 + 3*C*d*e^2*f^4) - A*d^3*g^5 + C*e^3*f^5))/(2*g^6 - 5*f*g^5 + 3*f^2
*g^4) + (C*e^3*x^3)/(9*g) + (log(x + 2/3)*(3*d - 2*e)^3*(9*A - 6*B + 4*C))
/(81*(3*f - 2*g)) - (log(x + 1)*(d - e)^3*(A - B + C))/(f - g)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 1325, normalized size of antiderivative = 5.41

$$\int \frac{(d+ex)^3 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^3*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
(486*log(3*x + 2)*a*d**3*f*g**4 - 486*log(3*x + 2)*a*d**3*g**5 - 972*log(3
*x + 2)*a*d**2*e*f*g**4 + 972*log(3*x + 2)*a*d**2*e*g**5 + 648*log(3*x + 2
)*a*d*e**2*f*g**4 - 648*log(3*x + 2)*a*d*e**2*g**5 - 144*log(3*x + 2)*a*e*
*3*f*g**4 + 144*log(3*x + 2)*a*e**3*g**5 - 324*log(3*x + 2)*b*d**3*f*g**4
+ 324*log(3*x + 2)*b*d**3*g**5 + 648*log(3*x + 2)*b*d**2*e*f*g**4 - 648*lo
g(3*x + 2)*b*d**2*e*g**5 - 432*log(3*x + 2)*b*d*e**2*f*g**4 + 432*log(3*x
+ 2)*b*d*e**2*g**5 + 96*log(3*x + 2)*b*e**3*f*g**4 - 96*log(3*x + 2)*b*e**
3*g**5 + 216*log(3*x + 2)*c*d**3*f*g**4 - 216*log(3*x + 2)*c*d**3*g**5 - 4
32*log(3*x + 2)*c*d**2*e*f*g**4 + 432*log(3*x + 2)*c*d**2*e*g**5 + 288*log
(3*x + 2)*c*d*e**2*f*g**4 - 288*log(3*x + 2)*c*d*e**2*g**5 - 64*log(3*x +
2)*c*e**3*f*g**4 + 64*log(3*x + 2)*c*e**3*g**5 + 162*log(f + g*x)*a*d**3*g
**5 - 486*log(f + g*x)*a*d**2*e*f*g**4 + 486*log(f + g*x)*a*d*e**2*f**2*g*
*3 - 162*log(f + g*x)*a*e**3*f**3*g**2 - 162*log(f + g*x)*b*d**3*f*g**4 +
486*log(f + g*x)*b*d**2*e*f**2*g**3 - 486*log(f + g*x)*b*d*e**2*f**3*g**2
+ 162*log(f + g*x)*b*e**3*f**4*g + 162*log(f + g*x)*c*d**3*f**2*g**3 - 486
*log(f + g*x)*c*d**2*e*f**3*g**2 + 486*log(f + g*x)*c*d*e**2*f**4*g - 162*
log(f + g*x)*c*e**3*f**5 - 486*log(x + 1)*a*d**3*f*g**4 + 324*log(x + 1)*a
*d**3*g**5 + 1458*log(x + 1)*a*d**2*e*f*g**4 - 972*log(x + 1)*a*d**2*e*g**
5 - 1458*log(x + 1)*a*d*e**2*f*g**4 + 972*log(x + 1)*a*d*e**2*g**5 + 486*l
og(x + 1)*a*e**3*f*g**4 - 324*log(x + 1)*a*e**3*g**5 + 486*log(x + 1)*b...
```

3.29 $\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 162

$$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{e(3Beg-C(3ef-6dg+5eg))x}{9g^2}+\frac{Ce^2x^2}{6g} - \frac{(A-B+C)(d-e)^2\log(1+x)}{f-g} + \frac{(9A-6B+4C)(3d-2e)^2\log(2+3x)}{27(3f-2g)} + \frac{(ef-dg)^2(Cf^2-Bfg+Ag^2)\log(f+gx)}{(3f-2g)(f-g)g^3}$$

output
```
1/9*e*(3*B*e*g-C*(-6*d*g+3*e*f+5*e*g))*x/g^2+1/6*C*e^2*x^2/g-(A-B+C)*(d-e)
^2*ln(1+x)/(f-g)+(9*A-6*B+4*C)*(3*d-2*e)^2*ln(2+3*x)/(81*f-54*g)+(-d*g+e*f
)^2*(A*g^2-B*f*g+C*f^2)*ln(g*x+f)/(3*f-2*g)/(f-g)/g^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{e(3Beg+C(-3ef+6dg-5eg))x}{9g^2} + \frac{Ce^2x^2}{6g} - \frac{(A-B+C)(d-e)^2\log(1+x)}{f-g} + \frac{(9A-6B+4C)(3d-2e)^2\log(2+3x)}{81f-54g} + \frac{(ef-dg)^2(Cf^2+g(-Bf+Ag))\log(f+gx)}{g^3(3f^2-5fg+2g^2)}$$

input
```
Integrate[((d + e*x)^2*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(e*(3*B*e*g + C*(-3*e*f + 6*d*g - 5*e*g))*x)/(9*g^2) + (C*e^2*x^2)/(6*g) -
 ((A - B + C)*(d - e)^2*Log[1 + x])/(f - g) + ((9*A - 6*B + 4*C)*(3*d - 2*
e)^2*Log[2 + 3*x])/(81*f - 54*g) + ((e*f - d*g)^2*(C*f^2 + g*(-(B*f) + A*g
))*Log[f + g*x])/(g^3*(3*f^2 - 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(3x^2+5x+2)(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{(ef-dg)^2(Ag^2-Bfg+Cf^2)}{g^2(3f-2g)(f-g)(f+gx)} + \frac{(3d-2e)^2(9A-6B+4C)}{9(3x+2)(3f-2g)} - \frac{(d-e)^2(A-B+C)}{(x+1)(f-g)} + \frac{e(3Beg-C(-6d\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ef-dg)^2\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{g^3(3f-2g)(f-g)}+\frac{(3d-2e)^2\log(3x+2)(9A-6B+4C)}{27(3f-2g)}-\frac{(d-e)^2\log(x+1)(A-B+C)}{f-g}+\frac{ex(3Beg-C(-6dg+3ef+5eg))}{9g^2}+\frac{Ce^2x^2}{6g}$$

input
```
Int[((d + e*x)^2*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(e*(3*B*e*g - C*(3*e*f - 6*d*g + 5*e*g))*x)/(9*g^2) + (C*e^2*x^2)/(6*g) -
((A - B + C)*(d - e)^2*Log[1 + x])/(f - g) + ((9*A - 6*B + 4*C)*(3*d - 2*e
)^2*Log[2 + 3*x])/(27*(3*f - 2*g)) + ((e*f - d*g)^2*(C*f^2 - B*f*g + A*g^2
)*Log[f + g*x])/((3*f - 2*g)*(f - g)*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.77

method	result
default	$\frac{e\left(\frac{3}{2}Ceg\,x^2+3Bxeg+6Cdgx-3Cefx-5Cexg\right)}{9g^2}+\frac{\left(-A\,d^2+2Ade-A\,e^2+B\,d^2-2Bde+B\,e^2-C\,d^2+2Cde-C\,e^2\right)\ln(1+x)}{f-g}+$
norman	$\frac{C\,e^2x^2}{6g}+\frac{e(3Beg+6Cdg-3Cef-5Ceg)x}{9g^2}+\frac{\left(A\,d^2g^4-2Ade\,g^3f+A\,e^2f^2g^2-B\,d^2g^3f+2Bde\,f^2g^2-B\,e^2f^3g+C\,d^2f^2g^2-\right.}{g^3(f-g)(3f-2g)}$
risch	$\frac{2eCdx}{3g}+\frac{C\,e^2x^2}{6g}+\frac{g\ln(gx+f)A\,d^2}{3f^2-5fg+2g^2}-\frac{\ln(gx+f)B\,d^2f}{3f^2-5fg+2g^2}-\frac{\ln(-x-1)A\,d^2}{f-g}-\frac{\ln(-x-1)A\,e^2}{f-g}+\frac{\ln(-x-1)B\,d^2}{f-g}+\ln($
parallelrisch	$-\frac{45C\,e^2f\,g^3x^2-54B\,e^2f^2g^2x-72Cde\,g^4x+180Cdef\,g^3x-108A\ln(1+x)e^2g^4+162A\ln\left(x+\frac{2}{3}\right)d^2g^4+72A\ln\left(x+\frac{2}{3}\right)e^2g^4-54}{}$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/9*e/g^2*(3/2*C*e*g*x^2+3*B*x*e*g+6*C*d*g*x-3*C*e*f*x-5*C*e*x*g)+(-A*d^2+
2*A*d*e-A*e^2+B*d^2-2*B*d*e+B*e^2-C*d^2+2*C*d*e-C*e^2)/(f-g)*ln(1+x)+1/3*(
81*A*d^2-108*A*d*e+36*A*e^2-54*B*d^2+72*B*d*e-24*B*e^2+36*C*d^2-48*C*d*e+1
6*C*e^2)/(27*f-18*g)*ln(2+3*x)+1/g^3*(A*d^2*g^4-2*A*d*e*f*g^3+A*e^2*f^2*g^
2-B*d^2*f*g^3+2*B*d*e*f^2*g^2-B*e^2*f^3*g+C*d^2*f^2*g^2-2*C*d*e*f^3*g+C*e^
2*f^4)/(f-g)/(3*f-2*g)*ln(g*x+f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(156) = 312$.

Time = 44.88 (sec) , antiderivative size = 410, normalized size of antiderivative = 2.53

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$$
$$= \frac{9\,(3\,Ce^2f^2g^2 - 5\,Ce^2fg^3 + 2\,Ce^2g^4)x^2 - 6\,(9\,Ce^2f^3g - 9\,(2\,Cde + Be^2)f^2g^2 + (30\,Cde + (15\,B - 19\,C)$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fric
as")
```

output
```
1/54*(9*(3*C*e^2*f^2*g^2 - 5*C*e^2*f*g^3 + 2*C*e^2*g^4)*x^2 - 6*(9*C*e^2*f
^3*g - 9*(2*C*d*e + B*e^2)*f^2*g^2 + (30*C*d*e + (15*B - 19*C)*e^2)*f*g^3
- 2*(6*C*d*e + (3*B - 5*C)*e^2)*g^4)*x + 54*(C*e^2*f^4 + A*d^2*g^4 - (2*C*
d*e + B*e^2)*f^3*g + (C*d^2 + 2*B*d*e + A*e^2)*f^2*g^2 - (B*d^2 + 2*A*d*e)
*f*g^3)*log(g*x + f) + 2*((9*(9*A - 6*B + 4*C)*d^2 - 12*(9*A - 6*B + 4*C)*
d*e + 4*(9*A - 6*B + 4*C)*e^2)*f*g^3 - (9*(9*A - 6*B + 4*C)*d^2 - 12*(9*A
- 6*B + 4*C)*d*e + 4*(9*A - 6*B + 4*C)*e^2)*g^4)*log(3*x + 2) - 54*(3*((A
- B + C)*d^2 - 2*(A - B + C)*d*e + (A - B + C)*e^2)*f*g^3 - 2*((A - B + C)
*d^2 - 2*(A - B + C)*d*e + (A - B + C)*e^2)*g^4)*log(x + 1))/(3*f^2*g^3 -
5*f*g^4 + 2*g^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**2*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.59

$$\begin{aligned}
&\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx \\
&= \frac{(Ce^2f^4 + Ad^2g^4 - (2\,Cde + Be^2)f^3g + (Cd^2 + 2\,Bde + Ae^2)f^2g^2 - (Bd^2 + 2\,Ade)fg^3)\log(gx+f)}{3\,f^2g^3 - 5\,fg^4 + 2\,g^5} \\
&\quad + \frac{(9\,(9\,A - 6\,B + 4\,C)d^2 - 12\,(9\,A - 6\,B + 4\,C)de + 4\,(9\,A - 6\,B + 4\,C)e^2)\log(3\,x+2)}{27\,(3\,f - 2\,g)} \\
&\quad - \frac{((A - B + C)d^2 - 2\,(A - B + C)de + (A - B + C)e^2)\log(x+1)}{f - g} \\
&\quad + \frac{3\,Ce^2gx^2 - 2\,(3\,Ce^2f - (6\,Cde + (3\,B - 5\,C)e^2)g)x}{18\,g^2}
\end{aligned}$$

input
```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxi
ma")
```

output
```
(C*e^2*f^4 + A*d^2*g^4 - (2*C*d*e + B*e^2)*f^3*g + (C*d^2 + 2*B*d*e + A*e^
2)*f^2*g^2 - (B*d^2 + 2*A*d*e)*f*g^3)*log(g*x + f)/(3*f^2*g^3 - 5*f*g^4 +
2*g^5) + 1/27*(9*(9*A - 6*B + 4*C)*d^2 - 12*(9*A - 6*B + 4*C)*d*e + 4*(9*A
 - 6*B + 4*C)*e^2)*log(3*x + 2)/(3*f - 2*g) - ((A - B + C)*d^2 - 2*(A - B
+ C)*d*e + (A - B + C)*e^2)*log(x + 1)/(f - g) + 1/18*(3*C*e^2*g*x^2 - 2*(
3*C*e^2*f - (6*C*d*e + (3*B - 5*C)*e^2)*g)*x)/g^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$$
$$= \frac{(Ce^2f^4 - 2\,Cdef^3g - Be^2f^3g + Cd^2f^2g^2 + 2\,Bdef^2g^2 + Ae^2f^2g^2 - Bd^2fg^3 - 2\,Adefg^3 + Ad^2g^4)\log(}{3\,f^2g^3 - 5\,fg^4 + 2\,g^5}$$
$$+ \frac{(81\,Ad^2 - 54\,Bd^2 + 36\,Cd^2 - 108\,Ade + 72\,Bde - 48\,Cde + 36\,Ae^2 - 24\,Be^2 + 16\,Ce^2)\log(|3\,x+2|)}{27\,(3\,f-2\,g)}$$
$$- \frac{(Ad^2 - Bd^2 + Cd^2 - 2\,Ade + 2\,Bde - 2\,Cde + Ae^2 - Be^2 + Ce^2)\log(|x+1|)}{f-g}$$
$$+ \frac{3\,Ce^2gx^2 - 6\,Ce^2fx + 12\,Cdegx + 6\,Be^2gx - 10\,Ce^2gx}{18\,g^2}$$

input

```
integrate((e*x+d)^2*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac
")
```

output

```
(C*e^2*f^4 - 2*C*d*e*f^3*g - B*e^2*f^3*g + C*d^2*f^2*g^2 + 2*B*d*e*f^2*g^2
 + A*e^2*f^2*g^2 - B*d^2*f*g^3 - 2*A*d*e*f*g^3 + A*d^2*g^4)*log(abs(g*x +
f))/(3*f^2*g^3 - 5*f*g^4 + 2*g^5) + 1/27*(81*A*d^2 - 54*B*d^2 + 36*C*d^2 -
 108*A*d*e + 72*B*d*e - 48*C*d*e + 36*A*e^2 - 24*B*e^2 + 16*C*e^2)*log(abs
(3*x + 2))/(3*f - 2*g) - (A*d^2 - B*d^2 + C*d^2 - 2*A*d*e + 2*B*d*e - 2*C*
d*e + A*e^2 - B*e^2 + C*e^2)*log(abs(x + 1))/(f - g) + 1/18*(3*C*e^2*g*x^2
 - 6*C*e^2*f*x + 12*C*d*e*g*x + 6*B*e^2*g*x - 10*C*e^2*g*x)/g^2
```

Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^2 (A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = x\left(\frac{B\,e^2 + 2\,C\,d\,e}{3\,g} - \frac{C\,e^2\,(3\,f+5\,g)}{9\,g^2}\right)$$
$$+ \frac{\ln(f+g\,x)\,\left(g^2\,(C\,d^2\,f^2 + 2\,B\,d\,e\,f^2 + A\,e^2\,f^2) - g^3\,(B\,f\,d^2 + 2\,A\,e\,f\,d) - g\,(B\,e^2\,f^3 + 2\,C\,d\,e\,f^3) + A}{3\,f^2\,g^3 - 5\,f\,g^4 + 2\,g^5}$$
$$+ \frac{C\,e^2\,x^2}{6\,g} + \frac{\ln\left(x+\frac{2}{3}\right)\,(3\,d-2\,e)^2\,(9\,A - 6\,B + 4\,C)}{27\,(3\,f-2\,g)}$$
$$- \frac{\ln(x+1)\,(d-e)^2\,(A-B+C)}{f-g}$$

input
```
int(((d + e*x)^2*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
x*((B*e^2 + 2*C*d*e)/(3*g) - (C*e^2*(3*f + 5*g))/(9*g^2)) + (log(f + g*x)*
(g^2*(A*e^2*f^2 + C*d^2*f^2 + 2*B*d*e*f^2) - g^3*(B*d^2*f + 2*A*d*e*f) - g
*(B*e^2*f^3 + 2*C*d*e*f^3) + A*d^2*g^4 + C*e^2*f^4))/(2*g^5 - 5*f*g^4 + 3*
f^2*g^3) + (C*e^2*x^2)/(6*g) + (log(x + 2/3)*(3*d - 2*e)^2*(9*A - 6*B + 4*
C))/(27*(3*f - 2*g)) - (log(x + 1)*(d - e)^2*(A - B + C))/(f - g)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 815, normalized size of antiderivative = 5.03

$$\int \frac{(d+ex)^2(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
(162*log(3*x + 2)*a*d**2*f*g**3 - 162*log(3*x + 2)*a*d**2*g**4 - 216*log(3
*x + 2)*a*d*e*f*g**3 + 216*log(3*x + 2)*a*d*e*g**4 + 72*log(3*x + 2)*a*e**
2*f*g**3 - 72*log(3*x + 2)*a*e**2*g**4 - 108*log(3*x + 2)*b*d**2*f*g**3 +
108*log(3*x + 2)*b*d**2*g**4 + 144*log(3*x + 2)*b*d*e*f*g**3 - 144*log(3*x
 + 2)*b*d*e*g**4 - 48*log(3*x + 2)*b*e**2*f*g**3 + 48*log(3*x + 2)*b*e**2*
g**4 + 72*log(3*x + 2)*c*d**2*f*g**3 - 72*log(3*x + 2)*c*d**2*g**4 - 96*lo
g(3*x + 2)*c*d*e*f*g**3 + 96*log(3*x + 2)*c*d*e*g**4 + 32*log(3*x + 2)*c*e
**2*f*g**3 - 32*log(3*x + 2)*c*e**2*g**4 + 54*log(f + g*x)*a*d**2*g**4 - 1
08*log(f + g*x)*a*d*e*f*g**3 + 54*log(f + g*x)*a*e**2*f**2*g**2 - 54*log(f
 + g*x)*b*d**2*f*g**3 + 108*log(f + g*x)*b*d*e*f**2*g**2 - 54*log(f + g*x)
*b*e**2*f**3*g + 54*log(f + g*x)*c*d**2*f**2*g**2 - 108*log(f + g*x)*c*d*e
*f**3*g + 54*log(f + g*x)*c*e**2*f**4 - 162*log(x + 1)*a*d**2*f*g**3 + 108
*log(x + 1)*a*d**2*g**4 + 324*log(x + 1)*a*d*e*f*g**3 - 216*log(x + 1)*a*d
*e*g**4 - 162*log(x + 1)*a*e**2*f*g**3 + 108*log(x + 1)*a*e**2*g**4 + 162*
log(x + 1)*b*d**2*f*g**3 - 108*log(x + 1)*b*d**2*g**4 - 324*log(x + 1)*b*d
*e*f*g**3 + 216*log(x + 1)*b*d*e*g**4 + 162*log(x + 1)*b*e**2*f*g**3 - 108
*log(x + 1)*b*e**2*g**4 - 162*log(x + 1)*c*d**2*f*g**3 + 108*log(x + 1)*c*
d**2*g**4 + 324*log(x + 1)*c*d*e*f*g**3 - 216*log(x + 1)*c*d*e*g**4 - 162*
log(x + 1)*c*e**2*f*g**3 + 108*log(x + 1)*c*e**2*g**4 + 54*b*e**2*f**2*g**
2*x - 90*b*e**2*f*g**3*x + 36*b*e**2*g**4*x + 108*c*d*e*f**2*g**2*x - 1...
```

3.30 $\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 122

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{Cex}{3g} - \frac{(A-B+C)(d-e)\log(1+x)}{f-g} + \frac{(9A-6B+4C)(3d-2e)\log(2+3x)}{9(3f-2g)} - \frac{(ef-dg)(Cf^2-Bfg+Ag^2)\log(f+gx)}{(3f-2g)(f-g)g^2}$$

output
```
1/3*C*e*x/g-(A-B+C)*(d-e)*ln(1+x)/(f-g)+(9*A-6*B+4*C)*(3*d-2*e)*ln(2+3*x)/
(27*f-18*g)-(-d*g+e*f)*(A*g^2-B*f*g+C*f^2)*ln(g*x+f)/(3*f-2*g)/(f-g)/g^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{Cex}{3g} - \frac{(A-B+C)(d-e)\log(1+x)}{f-g} + \frac{(9A-6B+4C)(3d-2e)\log(2+3x)}{9(3f-2g)} + \frac{(-ef+dg)(Cf^2+g(-Bf+Ag))\log(f+gx)}{g^2(3f^2-5fg+2g^2)}$$

input
```
Integrate[((d + e*x)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(C*e*x)/(3*g) - ((A - B + C)*(d - e)*Log[1 + x])/(f - g) + ((9*A - 6*B + 4
*C)*(3*d - 2*e)*Log[2 + 3*x])/(9*(3*f - 2*g)) + ((-(e*f) + d*g)*(C*f^2 + g
*(-(B*f) + A*g))*Log[f + g*x])/(g^2*(3*f^2 - 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)\left(A+Bx+Cx^2\right)}{\left(3x^2+5x+2\right)(f+gx)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(-\frac{(ef-dg)\left(Ag^2-Bfg+Cf^2\right)}{g(3f-2g)(f-g)(f+gx)}+\frac{(3d-2e)(9A-6B+4C)}{3(3x+2)(3f-2g)}-\frac{(d-e)(A-B+C)}{(x+1)(f-g)}+\frac{Ce}{3g}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(ef-dg)\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{g^2(3f-2g)(f-g)}-\frac{(d-e)\log(x+1)(A-B+C)}{f-g}+\frac{(3d-2e)\log(3x+2)(9A-6B+4C)}{9(3f-2g)}+\frac{Cex}{3g}$$

input
```
Int[((d + e*x)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(C*e*x)/(3*g) - ((A - B + C)*(d - e)*Log[1 + x])/(f - g) + ((9*A - 6*B + 4
*C)*(3*d - 2*e)*Log[2 + 3*x])/(9*(3*f - 2*g)) - ((e*f - d*g)*(C*f^2 - B*f*
g + A*g^2)*Log[f + g*x])/((3*f - 2*g)*(f - g)*g^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.28

method	result
default	$\frac{Cex}{3g}+\frac{(-Ad+Ae+Bd-Be-Cd+Ce)\ln(1+x)}{f-g}+\frac{(27Ad-18Ae-18Bd+12Be+12Cd-8Ce)\ln(2+3x)}{27f-18g}+\frac{(Ad\,g^3-Aef\,g$
norman	$\frac{Cex}{3g}+\frac{(Ad\,g^3-Aef\,g^2-Bdf\,g^2+Be\,f^2g+Cd\,f^2g-Ce\,f^3)\ln(gx+f)}{g^2(f-g)(3f-2g)}-\frac{(Ad-Ae-Bd+Be+Cd-Ce)\ln(1+x)}{f-g}+\frac{(27Ad-$
risch	$\frac{Cex}{3g}+\frac{g\ln(gx+f)Ad}{3f^2-5fg+2g^2}-\frac{\ln(gx+f)Aef}{3f^2-5fg+2g^2}-\frac{\ln(gx+f)Bdf}{3f^2-5fg+2g^2}+\frac{\ln(gx+f)Be\,f^2}{g(3f^2-5fg+2g^2)}+\frac{\ln(gx+f)Cd\,f^2}{g(3f^2-5fg+2g^2)}-\frac{\ln(gx+f)C}{g^2(3f^2-5f}$
parallelrisch	$-\frac{-9Ce\,f^2gx-18B\ln\left(x+\frac{2}{3}\right)d\,g^3+12B\ln\left(x+\frac{2}{3}\right)e\,g^3-18C\ln(1+x)d\,g^3+18C\ln(1+x)e\,g^3+12C\ln\left(x+\frac{2}{3}\right)d\,g^3-8C\ln\left(x+\frac{2}{3}\right)e}{}$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/3*C*e*x/g+(-A*d+A*e+B*d-B*e-C*d+C*e)/(f-g)*ln(1+x)+1/3*(27*A*d-18*A*e-18
*B*d+12*B*e+12*C*d-8*C*e)/(9*f-6*g)*ln(2+3*x)+1/g^2*(A*d*g^3-A*e*f*g^2-B*d
*f*g^2+B*e*f^2*g+C*d*f^2*g-C*e*f^3)/(f-g)/(3*f-2*g)*ln(g*x+f)
```

Fricas [A] (verification not implemented)

Time = 7.06 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$$
$$= \frac{3\,(3\,Cef^2g-5\,Cefg^2+2\,Ceg^3)x-9\,(Cef^3-Adg^3-(Cd+Be)f^2g+(Bd+Ae)fg^2)\log{(gx+f)}+}{}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fricas
")
```

output
```
1/9*(3*(3*C*e*f^2*g - 5*C*e*f*g^2 + 2*C*e*g^3)*x - 9*(C*e*f^3 - A*d*g^3 -
(C*d + B*e)*f^2*g + (B*d + A*e)*f*g^2)*log(g*x + f) + ((3*(9*A - 6*B + 4*C
)*d - 2*(9*A - 6*B + 4*C)*e)*f*g^2 - (3*(9*A - 6*B + 4*C)*d - 2*(9*A - 6*B
 + 4*C)*e)*g^3)*log(3*x + 2) - 9*(3*((A - B + C)*d - (A - B + C)*e)*f*g^2
- 2*((A - B + C)*d - (A - B + C)*e)*g^3)*log(x + 1))/(3*f^2*g^2 - 5*f*g^3
+ 2*g^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{Cex}{3\,g} - \frac{(Cef^3 - Adg^3 - (Cd+Be)f^2g + (Bd+Ae)fg^2)\log(gx+f)}{3\,f^2g^2 - 5\,fg^3 + 2\,g^4} + \frac{(3\,(9\,A-6\,B+4\,C)d - 2\,(9\,A-6\,B+4\,C)e)\log(3\,x+2)}{9\,(3\,f-2\,g)} - \frac{((A-B+C)d-(A-B+C)e)\log(x+1)}{f-g}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxima
")
```

output
```
1/3*C*e*x/g - (C*e*f^3 - A*d*g^3 - (C*d + B*e)*f^2*g + (B*d + A*e)*f*g^2)*
log(g*x + f)/(3*f^2*g^2 - 5*f*g^3 + 2*g^4) + 1/9*(3*(9*A - 6*B + 4*C)*d -
2*(9*A - 6*B + 4*C)*e)*log(3*x + 2)/(3*f - 2*g) - ((A - B + C)*d - (A - B
+ C)*e)*log(x + 1)/(f - g)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.34

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{Cex}{3\,g} - \frac{(Cef^3 - Cdf^2g - Bef^2g + Bdfg^2 + Aefg^2 - Adg^3)\log(|gx+f|)}{3\,f^2g^2 - 5\,fg^3 + 2\,g^4} + \frac{(27\,Ad - 18\,Bd + 12\,Cd - 18\,Ae + 12\,Be - 8\,Ce)\log(|3\,x+2|)}{9\,(3\,f-2\,g)} - \frac{(Ad - Bd + Cd - Ae + Be - Ce)\log(|x+1|)}{f-g}$$

input
```
integrate((e*x+d)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac")
```

output
```
1/3*C*e*x/g - (C*e*f^3 - C*d*f^2*g - B*e*f^2*g + B*d*f*g^2 + A*e*f*g^2 - A
*d*g^3)*log(abs(g*x + f))/(3*f^2*g^2 - 5*f*g^3 + 2*g^4) + 1/9*(27*A*d - 18
*B*d + 12*C*d - 18*A*e + 12*B*e - 8*C*e)*log(abs(3*x + 2))/(3*f - 2*g) - (
A*d - B*d + C*d - A*e + B*e - C*e)*log(abs(x + 1))/(f - g)
```

Mupad [B] (verification not implemented)

Time = 20.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)\,(3\,d-2\,e)\,(9\,A-6\,B+4\,C)}{9\,(3\,f-2\,g)} - \frac{\ln(f+g\,x)\,\left(g^2\,(A\,e\,f+B\,d\,f)-g\,(B\,e\,f^2+C\,d\,f^2)-A\,d\,g^3+C\,e\,f^3\right)}{3\,f^2\,g^2-5\,f\,g^3+2\,g^4} + \frac{C\,e\,x}{3\,g} - \frac{\ln(x+1)\,(d-e)\,(A-B+C)}{f-g}$$

input
```
int(((d + e*x)*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
(log(x + 2/3)*(3*d - 2*e)*(9*A - 6*B + 4*C))/(9*(3*f - 2*g)) - (log(f + g*
x)*(g^2*(A*e*f + B*d*f) - g*(B*e*f^2 + C*d*f^2) - A*d*g^3 + C*e*f^3))/(2*g
^4 - 5*f*g^3 + 3*f^2*g^2) + (C*e*x)/(3*g) - (log(x + 1)*(d - e)*(A - B + C
))/(f - g)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.53

$$\int \frac{(d+ex)(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{9\log(gx+f)\,ad\,g^3-9\log(gx+f)\,ce\,f^3+18\log(x+1)\,ad\,g^3-18\log(x+1)\,ae\,g^3-18\log(x+1)\,bd\,g}{}$$

input
```
int((e*x+d)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
(27*log(3*x + 2)*a*d*f*g**2 - 27*log(3*x + 2)*a*d*g**3 - 18*log(3*x + 2)*a
*e*f*g**2 + 18*log(3*x + 2)*a*e*g**3 - 18*log(3*x + 2)*b*d*f*g**2 + 18*log
(3*x + 2)*b*d*g**3 + 12*log(3*x + 2)*b*e*f*g**2 - 12*log(3*x + 2)*b*e*g**3
 + 12*log(3*x + 2)*c*d*f*g**2 - 12*log(3*x + 2)*c*d*g**3 - 8*log(3*x + 2)*
c*e*f*g**2 + 8*log(3*x + 2)*c*e*g**3 + 9*log(f + g*x)*a*d*g**3 - 9*log(f +
 g*x)*a*e*f*g**2 - 9*log(f + g*x)*b*d*f*g**2 + 9*log(f + g*x)*b*e*f**2*g +
 9*log(f + g*x)*c*d*f**2*g - 9*log(f + g*x)*c*e*f**3 - 27*log(x + 1)*a*d*f
*g**2 + 18*log(x + 1)*a*d*g**3 + 27*log(x + 1)*a*e*f*g**2 - 18*log(x + 1)*
a*e*g**3 + 27*log(x + 1)*b*d*f*g**2 - 18*log(x + 1)*b*d*g**3 - 27*log(x +
1)*b*e*f*g**2 + 18*log(x + 1)*b*e*g**3 - 27*log(x + 1)*c*d*f*g**2 + 18*log
(x + 1)*c*d*g**3 + 27*log(x + 1)*c*e*f*g**2 - 18*log(x + 1)*c*e*g**3 + 9*c
*e*f**2*g*x - 15*c*e*f*g**2*x + 6*c*e*g**3*x)/(9*g**2*(3*f**2 - 5*f*g + 2*
g**2))
```

3.31 $\int \frac{A+Bx+Cx^2}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 91

$$\int \frac{A+Bx+Cx^2}{(f+gx)\left(2+5x+3x^2\right)}\,dx=-\frac{(A-B+C)\log(1+x)}{f-g}+\frac{(9A-6B+4C)\log(2+3x)}{3(3f-2g)}+\frac{\left(Cf^2-Bfg+Ag^2\right)\log(f+gx)}{(3f-2g)(f-g)g}$$

output
```
-(A-B+C)*ln(1+x)/(f-g)+(9*A-6*B+4*C)*ln(2+3*x)/(9*f-6*g)+(A*g^2-B*f*g+C*f^
2)*ln(g*x+f)/(3*f-2*g)/(f-g)/g
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{(f+gx)\left(2+5x+3x^2\right)}\,dx=\frac{(-A+B-C)\log(1+x)}{f-g}+\frac{(9A-6B+4C)\log(2+3x)}{3(3f-2g)}+\frac{\left(Cf^2-Bfg+Ag^2\right)\log(f+gx)}{g\left(3f^2-5fg+2g^2\right)}$$

input
```
Integrate[(A + B*x + C*x^2)/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
((-A + B - C)*Log[1 + x])/(f - g) + ((9*A - 6*B + 4*C)*Log[2 + 3*x])/(3*(3
*f - 2*g)) + ((C*f^2 - B*f*g + A*g^2)*Log[f + g*x])/(g*(3*f^2 - 5*f*g + 2*
g^2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)(f+gx)}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{Ag^2-Bfg+Cf^2}{(3f-2g)(f-g)(f+gx)}+\frac{-A+B-C}{(x+1)(f-g)}+\frac{9A-6B+4C}{(3x+2)(3f-2g)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{g(3f-2g)(f-g)}-\frac{\log(x+1)(A-B+C)}{f-g}+\frac{\log(3x+2)(9A-6B+4C)}{3(3f-2g)}$$

input
```
Int[(A + B*x + C*x^2)/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
-(((A - B + C)*Log[1 + x])/(f - g)) + ((9*A - 6*B + 4*C)*Log[2 + 3*x])/(3*
(3*f - 2*g)) + ((C*f^2 - B*f*g + A*g^2)*Log[f + g*x])/((3*f - 2*g)*(f - g)
*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

method	result
default	$\frac{(-A+B-C)\ln(1+x)}{f-g}+\frac{(9A-6B+4C)\ln(2+3x)}{9f-6g}+\frac{(A g^2-Bfg+C f^2)\ln(gx+f)}{(3f-2g)(f-g)g}$
norman	$\frac{(A g^2-Bfg+C f^2)\ln(gx+f)}{g(3f^2-5fg+2g^2)}-\frac{(A-B+C)\ln(1+x)}{f-g}+\frac{(9A-6B+4C)\ln(2+3x)}{9f-6g}$
parallelrisch	$-\frac{9A\ln(1+x)fg-6A\ln(1+x)g^2-9A\ln(x+\frac{2}{3})fg+9A\ln(x+\frac{2}{3})g^2-3A\ln(gx+f)g^2-9B\ln(1+x)fg+6B\ln(1+x)g^2+6B\ln(x}{3(3f-2}$
risch	$\frac{g\ln(gx+f)A}{3f^2-5fg+2g^2}-\frac{\ln(gx+f)Bf}{3f^2-5fg+2g^2}+\frac{\ln(gx+f)C f^2}{g(3f^2-5fg+2g^2)}-\frac{\ln(-x-1)A}{f-g}+\frac{\ln(-x-1)B}{f-g}-\frac{\ln(-x-1)C}{f-g}+\frac{3\ln(-3x-2)}{3f-2g}$

input
```
int((C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-A+B-C)/(f-g)*ln(1+x)+1/3*(9*A-6*B+4*C)/(3*f-2*g)*ln(2+3*x)+(A*g^2-B*f*g+
C*f^2)*ln(g*x+f)/(3*f-2*g)/(f-g)/g
```

Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx+Cx^2}{(f+gx)(2+5x+3x^2)}\,dx = \frac{3\,(Cf^2-Bfg+Ag^2)\log(gx+f)+((9\,A-6\,B+4\,C)fg-(9\,A-6\,B+4\,C)g^2)\log(3\,x+2)-3\,(3\,(}{3\,(3\,f^2g-5\,fg^2+2\,g^3)}$$

input
```
integrate((C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fricas")
```

output
```
1/3*(3*(C*f^2 - B*f*g + A*g^2)*log(g*x + f) + ((9*A - 6*B + 4*C)*f*g - (9*
A - 6*B + 4*C)*g^2)*log(3*x + 2) - 3*(3*(A - B + C)*f*g - 2*(A - B + C)*g^
2)*log(x + 1))/(3*f^2*g - 5*f*g^2 + 2*g^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2}{(f+gx)\left(2+5x+3x^2\right)}\,dx=\frac{(Cf^2-Bfg+Ag^2)\log(gx+f)}{3\,f^2g-5\,fg^2+2\,g^3}+\frac{(9\,A-6\,B+4\,C)\log(3\,x+2)}{3\,(3\,f-2\,g)}-\frac{(A-B+C)\log(x+1)}{f-g}$$

input
```
integrate((C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxima")
```

output
```
(C*f^2 - B*f*g + A*g^2)*log(g*x + f)/(3*f^2*g - 5*f*g^2 + 2*g^3) + 1/3*(9*
A - 6*B + 4*C)*log(3*x + 2)/(3*f - 2*g) - (A - B + C)*log(x + 1)/(f - g)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{(f+gx)(2+5x+3x^2)}\,dx = \frac{(Cf^2-Bfg+Ag^2)\log(|gx+f|)}{3\,f^2g-5\,fg^2+2\,g^3} + \frac{(9\,A-6\,B+4\,C)\log(|3\,x+2|)}{3\,(3\,f-2\,g)} - \frac{(A-B+C)\log(|x+1|)}{f-g}$$

input
```
integrate((C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac")
```

output
```
(C*f^2 - B*f*g + A*g^2)*log(abs(g*x + f))/(3*f^2*g - 5*f*g^2 + 2*g^3) + 1/
3*(9*A - 6*B + 4*C)*log(abs(3*x + 2))/(3*f - 2*g) - (A - B + C)*log(abs(x
+ 1))/(f - g)
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2}{(f+gx)(2+5x+3x^2)}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)\,(9\,A-6\,B+4\,C)}{9\,f-6\,g} - \frac{\ln(x+1)\,(A-B+C)}{f-g} + \frac{\ln(f+g\,x)\,(C\,f^2-B\,f\,g+A\,g^2)}{g\,(3\,f^2-5\,f\,g+2\,g^2)}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
(log(x + 2/3)*(9*A - 6*B + 4*C))/(9*f - 6*g) - (log(x + 1)*(A - B + C))/(f
 - g) + (log(f + g*x)*(A*g^2 + C*f^2 - B*f*g))/(g*(3*f^2 - 5*f*g + 2*g^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

$$\int \frac{A+Bx+Cx^2}{(f+gx)(2+5x+3x^2)}\,dx$$
$$= \frac{9\log(3x+2)\,afg-9\log(3x+2)\,a\,g^2-6\log(3x+2)\,bfg+6\log(3x+2)\,b\,g^2+4\log(3x+2)\,cfg-4\,\mathrm{lo}}{}$$

input
```
int((C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
(9*log(3*x + 2)*a*f*g - 9*log(3*x + 2)*a*g**2 - 6*log(3*x + 2)*b*f*g + 6*l
og(3*x + 2)*b*g**2 + 4*log(3*x + 2)*c*f*g - 4*log(3*x + 2)*c*g**2 + 3*log(
f + g*x)*a*g**2 - 3*log(f + g*x)*b*f*g + 3*log(f + g*x)*c*f**2 - 9*log(x +
 1)*a*f*g + 6*log(x + 1)*a*g**2 + 9*log(x + 1)*b*f*g - 6*log(x + 1)*b*g**2
 - 9*log(x + 1)*c*f*g + 6*log(x + 1)*c*g**2)/(3*g*(3*f**2 - 5*f*g + 2*g**2
))
```

3.32 $\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 161

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx=-\frac{(A-B+C)\log(1+x)}{(d-e)(f-g)}+\frac{(9A-6B+4C)\log(2+3x)}{(3d-2e)(3f-2g)}+\frac{\left(Cd^2-Bde+Ae^2\right)\log(d+ex)}{(3d-2e)(d-e)(ef-dg)}-\frac{\left(Cf^2-Bfg+Ag^2\right)\log(f+gx)}{(3f-2g)(f-g)(ef-dg)}$$

output
```
-(A-B+C)*ln(1+x)/(d-e)/(f-g)+(9*A-6*B+4*C)*ln(2+3*x)/(3*d-2*e)/(3*f-2*g)+(
A*e^2-B*d*e+C*d^2)*ln(e*x+d)/(3*d-2*e)/(d-e)/(-d*g+e*f)-(A*g^2-B*f*g+C*f^2
)*ln(g*x+f)/(3*f-2*g)/(f-g)/(-d*g+e*f)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{(-A+B-C)\log(1+x)}{(d-e)(f-g)} + \frac{(9A-6B+4C)\log(2+3x)}{(3d-2e)(3f-2g)} + \frac{(-Cd^2+Bde-Ae^2)\log(d+ex)}{(3d^2-5de+2e^2)\,(-ef+dg)} - \frac{(Cf^2-Bfg+Ag^2)\log(f+gx)}{(ef-dg)\,(3f^2-5fg+2g^2)}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
((-A + B - C)*Log[1 + x])/((d - e)*(f - g)) + ((9*A - 6*B + 4*C)*Log[2 + 3
*x])/((3*d - 2*e)*(3*f - 2*g)) + ((-(C*d^2) + B*d*e - A*e^2)*Log[d + e*x])
/((3*d^2 - 5*d*e + 2*e^2)*(-(e*f) + d*g)) - ((C*f^2 - B*f*g + A*g^2)*Log[f
 + g*x])/((e*f - d*g)*(3*f^2 - 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)\,(d+ex)(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(-\frac{e(Ae^2-Bde+Cd^2)}{(3d-2e)(d-e)(d+ex)(dg-ef)} - \frac{g(Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)(f+gx)(ef-dg)} + \frac{-A+B-C}{(x+1)(d-e)(f-g)} + \frac{}{(3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(d+ex)\left(Ae^2-Bde+Cd^2\right)}{(3d-2e)(d-e)(ef-dg)}-\frac{\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{(3f-2g)(f-g)(ef-dg)}-\frac{\log(x+1)(A-B+C)}{(d-e)(f-g)}+\frac{\log(3x+2)(9A-6B+4C)}{(3d-2e)(3f-2g)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
-(((A - B + C)*Log[1 + x])/((d - e)*(f - g))) + ((9*A - 6*B + 4*C)*Log[2 +
 3*x])/((3*d - 2*e)*(3*f - 2*g)) + ((C*d^2 - B*d*e + A*e^2)*Log[d + e*x])/
((3*d - 2*e)*(d - e)*(e*f - d*g)) - ((C*f^2 - B*f*g + A*g^2)*Log[f + g*x])
/((3*f - 2*g)*(f - g)*(e*f - d*g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02

method	result
default	$\frac{(-A+B-C)\ln(1+x)}{(d-e)(f-g)}-\frac{\left(A\,e^2-Bde+C\,d^2\right)\ln(ex+d)}{(dg-ef)(d-e)(3d-2e)}+\frac{(27A-18B+12C)\ln(2+3x)}{3(3d-2e)(3f-2g)}+\frac{\left(A\,g^2-Bfg+C\,f^2\right)\ln(gx+f)}{(dg-ef)(f-g)(3f-2g)}$
norman	$\frac{\left(A\,g^2-Bfg+C\,f^2\right)\ln(gx+f)}{3d\,f^2g-5df\,g^2+2d\,g^3-3e\,f^3+5e\,f^2g-2ef\,g^2}+\frac{(9A-6B+4C)\ln(2+3x)}{(3d-2e)(3f-2g)}-\frac{(A-B+C)\ln(1+x)}{(d-e)(f-g)}-\frac{\left(A\,e^2-Bde+C\,d^2\right)\ln}{(3d^2-5de+2e^2)(dg}$
risch	$\frac{\ln(-gx-f)A\,g^2}{3d\,f^2g-5df\,g^2+2d\,g^3-3e\,f^3+5e\,f^2g-2ef\,g^2}-\frac{\ln(-gx-f)Bfg}{3d\,f^2g-5df\,g^2+2d\,g^3-3e\,f^3+5e\,f^2g-2ef\,g^2}+\frac{\ln(-gx-f}{3d\,f^2g-5df\,g^2+2d\,g^3-3}$
parallelrisch	$-\frac{5B\ln(ex+d)defg-5B\ln(gx+f)defg-9A\ln\left(x+\frac{2}{3}\right)e^2f^2+3A\ln(ex+d)e^2f^2+2A\ln(ex+d)e^2g^2-3A\ln(gx+f)d^2g^2-2A\ln}{}$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-A+B-C)/(d-e)/(f-g)*ln(1+x)-(A*e^2-B*d*e+C*d^2)/(d*g-e*f)/(d-e)/(3*d-2*e)
*ln(e*x+d)+1/3*(27*A-18*B+12*C)/(3*d-2*e)/(3*f-2*g)*ln(2+3*x)+(A*g^2-B*f*g
+C*f^2)/(d*g-e*f)/(f-g)/(3*f-2*g)*ln(g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fricas
")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{(Cd^2-Bde+Ae^2)\log(ex+d)}{(3\,d^2e-5\,de^2+2\,e^3)f-(3\,d^3-5\,d^2e+2\,de^2)g} - \frac{(Cf^2-Bfg+Ag^2)\log(gx+f)}{3\,ef^3-(3\,d+5\,e)f^2g+(5\,d+2\,e)fg^2-2\,dg^3} + \frac{(9\,A-6\,B+4\,C)\log(3\,x+2)}{3\,(3\,d-2\,e)f-2\,(3\,d-2\,e)g} - \frac{(A-B+C)\log(x+1)}{(d-e)f-(d-e)g}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxima
")
```

output
```
(C*d^2 - B*d*e + A*e^2)*log(e*x + d)/((3*d^2*e - 5*d*e^2 + 2*e^3)*f - (3*d
^3 - 5*d^2*e + 2*d*e^2)*g) - (C*f^2 - B*f*g + A*g^2)*log(g*x + f)/(3*e*f^3
 - (3*d + 5*e)*f^2*g + (5*d + 2*e)*f*g^2 - 2*d*g^3) + (9*A - 6*B + 4*C)*lo
g(3*x + 2)/(3*(3*d - 2*e)*f - 2*(3*d - 2*e)*g) - (A - B + C)*log(x + 1)/((
d - e)*f - (d - e)*g)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{(Cd^2e-Bde^2+Ae^3)\log(|ex+d|)}{3\,d^2e^2f-5\,de^3f+2\,e^4f-3\,d^3eg+5\,d^2e^2g-2\,de^3g} - \frac{(Cf^2g-Bfg^2+Ag^3)\log(|gx+f|)}{3\,ef^3g-3\,df^2g^2-5\,ef^2g^2+5\,dfg^3+2\,efg^3-2\,dg^4} + \frac{(9\,A-6\,B+4\,C)\log(|3\,x+2|)}{9\,df-6\,ef-6\,dg+4\,eg} - \frac{(A-B+C)\log(|x+1|)}{df-ef-dg+eg}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac")
```

output
```
(C*d^2*e - B*d*e^2 + A*e^3)*log(abs(e*x + d))/(3*d^2*e^2*f - 5*d*e^3*f + 2
*e^4*f - 3*d^3*e*g + 5*d^2*e^2*g - 2*d*e^3*g) - (C*f^2*g - B*f*g^2 + A*g^3
)*log(abs(g*x + f))/(3*e*f^3*g - 3*d*f^2*g^2 - 5*e*f^2*g^2 + 5*d*f*g^3 + 2
*e*f*g^3 - 2*d*g^4) + (9*A - 6*B + 4*C)*log(abs(3*x + 2))/(9*d*f - 6*e*f -
 6*d*g + 4*e*g) - (A - B + C)*log(abs(x + 1))/(d*f - e*f - d*g + e*g)
```

Mupad [B] (verification not implemented)

Time = 25.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{\ln\left(x+\frac{2}{3}\right)\,\left(9\,A-6\,B+4\,C\right)}{\left(3\,d-2\,e\right)\,\left(3\,f-2\,g\right)} - \ln\left(d+e\,x\right)\,\left(\frac{9\,A-6\,B+4\,C}{\left(3\,d-2\,e\right)\,\left(3\,f-2\,g\right)} + \frac{C\,f^2-B\,f\,g+A\,g^2}{\left(d\,g-e\,f\right)\,\left(3\,f^2-5\,f\,g+2\,g^2\right)} - \frac{A-B+C}{\left(d-e\right)\,\left(f-g\right)}\right) - \frac{\ln\left(x+1\right)\,\left(A-B+C\right)}{\left(d-e\right)\,\left(f-g\right)} + \frac{\ln\left(f+g\,x\right)\,\left(C\,f^2-B\,f\,g+A\,g^2\right)}{\left(f-g\right)\,\left(d\,g-e\,f\right)\,\left(3\,f-2\,g\right)}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
(log(x + 2/3)*(9*A - 6*B + 4*C))/((3*d - 2*e)*(3*f - 2*g)) - log(d + e*x)*
((9*A - 6*B + 4*C)/((3*d - 2*e)*(3*f - 2*g)) + (A*g^2 + C*f^2 - B*f*g)/((d
*g - e*f)*(3*f^2 - 5*f*g + 2*g^2)) - (A - B + C)/((d - e)*(f - g))) - (log
(x + 1)*(A - B + C))/((d - e)*(f - g)) + (log(f + g*x)*(A*g^2 + C*f^2 - B*
f*g))/((f - g)*(d*g - e*f)*(3*f - 2*g))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 851, normalized size of antiderivative = 5.29

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
(9*log(3*x + 2)*a*d**2*f*g - 9*log(3*x + 2)*a*d**2*g**2 - 9*log(3*x + 2)*a
*d*e*f**2 + 9*log(3*x + 2)*a*d*e*g**2 + 9*log(3*x + 2)*a*e**2*f**2 - 9*log
(3*x + 2)*a*e**2*f*g - 6*log(3*x + 2)*b*d**2*f*g + 6*log(3*x + 2)*b*d**2*g
**2 + 6*log(3*x + 2)*b*d*e*f**2 - 6*log(3*x + 2)*b*d*e*g**2 - 6*log(3*x +
2)*b*e**2*f**2 + 6*log(3*x + 2)*b*e**2*f*g + 4*log(3*x + 2)*c*d**2*f*g - 4
*log(3*x + 2)*c*d**2*g**2 - 4*log(3*x + 2)*c*d*e*f**2 + 4*log(3*x + 2)*c*d
*e*g**2 + 4*log(3*x + 2)*c*e**2*f**2 - 4*log(3*x + 2)*c*e**2*f*g - 3*log(d
 + e*x)*a*e**2*f**2 + 5*log(d + e*x)*a*e**2*f*g - 2*log(d + e*x)*a*e**2*g*
*2 + 3*log(d + e*x)*b*d*e*f**2 - 5*log(d + e*x)*b*d*e*f*g + 2*log(d + e*x)
*b*d*e*g**2 - 3*log(d + e*x)*c*d**2*f**2 + 5*log(d + e*x)*c*d**2*f*g - 2*l
og(d + e*x)*c*d**2*g**2 + 3*log(f + g*x)*a*d**2*g**2 - 5*log(f + g*x)*a*d*
e*g**2 + 2*log(f + g*x)*a*e**2*g**2 - 3*log(f + g*x)*b*d**2*f*g + 5*log(f
+ g*x)*b*d*e*f*g - 2*log(f + g*x)*b*e**2*f*g + 3*log(f + g*x)*c*d**2*f**2
- 5*log(f + g*x)*c*d*e*f**2 + 2*log(f + g*x)*c*e**2*f**2 - 9*log(x + 1)*a*
d**2*f*g + 6*log(x + 1)*a*d**2*g**2 + 9*log(x + 1)*a*d*e*f**2 - 4*log(x +
1)*a*d*e*g**2 - 6*log(x + 1)*a*e**2*f**2 + 4*log(x + 1)*a*e**2*f*g + 9*log
(x + 1)*b*d**2*f*g - 6*log(x + 1)*b*d**2*g**2 - 9*log(x + 1)*b*d*e*f**2 +
4*log(x + 1)*b*d*e*g**2 + 6*log(x + 1)*b*e**2*f**2 - 4*log(x + 1)*b*e**2*f
*g - 9*log(x + 1)*c*d**2*f*g + 6*log(x + 1)*c*d**2*g**2 + 9*log(x + 1)*c*d
*e*f**2 - 4*log(x + 1)*c*d*e*g**2 - 6*log(x + 1)*c*e**2*f**2 + 4*log(x ...
```

3.33 $\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)(2+5x+3x^2)} \, dx$

Optimal result

Integrand size = 37, antiderivative size = 292

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\left(2+5x+3x^2\right)} \, dx = -\frac{Cd^2-Bde+Ae^2}{(3d-2e)(d-e)(ef-dg)(d+ex)} - \frac{(A-B+C)\log(1+x)}{(d-e)^2(f-g)} + \frac{3(9A-6B+4C)\log(2+3x)}{(3d-2e)^2(3f-2g)} - \frac{(Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))-Be(2e^3f+6d^3g-d}{(3d-2e)^2(d-e)^2(ef-dg)^2} + \frac{g(Cf^2-Bfg+Ag^2)\log(f+gx)}{(3f-2g)(f-g)(ef-dg)^2}$$

output
```
-(A*e^2-B*d*e+C*d^2)/(3*d-2*e)/(d-e)/(-d*g+e*f)/(e*x+d)-(A-B+C)*ln(1+x)/(d
-e)^2/(f-g)+3*(9*A-6*B+4*C)*ln(2+3*x)/(3*d-2*e)^2/(3*f-2*g)-(C*d*(4*e^3*f+
3*d^3*g-d*e^2*(5*f+2*g))+A*e^2*(9*d^2*g+e^2*(5*f+2*g)-2*d*e*(3*f+5*g))-B*e
*(2*e^3*f+6*d^3*g-d^2*e*(3*f+5*g)))*ln(e*x+d)/(3*d-2*e)^2/(d-e)^2/(-d*g+e*
f)^2+g*(A*g^2-B*f*g+C*f^2)*ln(g*x+f)/(3*f-2*g)/(f-g)/(-d*g+e*f)^2
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.76

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{Cd^2+e(-Bd+Ae)}{\left(3d^2-5de+2e^2\right)(-ef+dg)(d+ex)}$$
$$+\frac{\left(C\left(12de(5f-4g)+4e^2(-6f+5g)+d^2(-39f+30g)\right)+A\left(de(90f-78g)-9d^2(6f-5g)+e^2(-39\right.\right.}{\left(3d^2-5de+2e^2\right)^2\left(3f^2-\right.}$$
$$+\frac{\left(Cd\left(-4e^3f-3d^3g+de^2(5f+2g)\right)+Ae^2\left(-9d^2g-e^2(5f+2g)+2de(3f+5g)\right)+Be\left(2e^3f+6d^3g\right.\right.}{\left(3d^2-5de+2e^2\right)^2(ef-dg)^2}$$
$$+\frac{g\left(Cf^2+g(-Bf+Ag)\right)\log(f+gx)}{(ef-dg)^2\left(3f^2-5fg+2g^2\right)}$$
$$+\frac{\left(A\left(e^2(15f-19g)-6de(3f-5g)-9d^2g\right)+B\left(9d^2f-12deg+2e^2(-3f+5g)\right)+C\left(12def-4e^2g+\right.\right.}{2\left(3d^2-5de+2e^2\right)^2\left(3f^2-5fg+2g^2\right)}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^2*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(C*d^2 + e*(-(B*d) + A*e))/((3*d^2 - 5*d*e + 2*e^2)*(-(e*f) + d*g)*(d + e*
x)) + ((C*(12*d*e*(5*f - 4*g) + 4*e^2*(-6*f + 5*g) + d^2*(-39*f + 30*g)) +
 A*(d*e*(90*f - 78*g) - 9*d^2*(6*f - 5*g) + e^2*(-39*f + 35*g)) + B*(2*e^2
*(15*f - 13*g) + 9*d^2*(5*f - 4*g) + d*e*(-72*f + 60*g)))*ArcTanh[5 + 6*x]
)/((3*d^2 - 5*d*e + 2*e^2)^2*(3*f^2 - 5*f*g + 2*g^2)) + ((C*d*(-4*e^3*f -
3*d^3*g + d*e^2*(5*f + 2*g)) + A*e^2*(-9*d^2*g - e^2*(5*f + 2*g) + 2*d*e*(
3*f + 5*g)) + B*e*(2*e^3*f + 6*d^3*g - d^2*e*(3*f + 5*g)))*Log[d + e*x])/(
(3*d^2 - 5*d*e + 2*e^2)^2*(e*f - d*g)^2) + (g*(C*f^2 + g*(-(B*f) + A*g))*L
og[f + g*x])/((e*f - d*g)^2*(3*f^2 - 5*f*g + 2*g^2)) + ((A*(e^2*(15*f - 19
*g) - 6*d*e*(3*f - 5*g) - 9*d^2*g) + B*(9*d^2*f - 12*d*e*g + 2*e^2*(-3*f +
 5*g)) + C*(12*d*e*f - 4*e^2*g + d^2*(-15*f + 6*g)))*Log[2 + 5*x + 3*x^2])
/(2*(3*d^2 - 5*d*e + 2*e^2)^2*(3*f^2 - 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)(d+ex)^2(f+gx)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(-\frac{e(Ae^2-Bde+Cd^2)}{(3d-2e)(d-e)(d+ex)^2(dg-ef)}+\frac{e(-Ae^2(9d^2g-2de(3f+5g)+e^2(5f+2g))+Be(6d^3g-d^2e(3f\right.$$
$$(3d-2e)^2(d-e)^2(d+ex$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ae^2-Bde+Cd^2}{(3d-2e)(d-e)(d+ex)(ef-dg)}-$$
$$\frac{\log(d+ex)\left(Ae^2(9d^2g-2de(3f+5g)+e^2(5f+2g))-Be(6d^3g-d^2e(3f+5g)+2e^3f)+Cd(3d^3g-de^2(5f\right.}{(3d-2e)^2(d-e)^2(ef-dg)^2}$$
$$\frac{g\log(f+gx)(Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)(ef-dg)^2}-\frac{\log(x+1)(A-B+C)}{(d-e)^2(f-g)}+\frac{3\log(3x+2)(9A-6B+4C)}{(3d-2e)^2(3f-2g)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^2*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
-((C*d^2 - B*d*e + A*e^2)/((3*d - 2*e)*(d - e)*(e*f - d*g)*(d + e*x))) - (
(A - B + C)*Log[1 + x])/((d - e)^2*(f - g)) + (3*(9*A - 6*B + 4*C)*Log[2 +
 3*x])/((3*d - 2*e)^2*(3*f - 2*g)) - ((C*d*(4*e^3*f + 3*d^3*g - d*e^2*(5*f
 + 2*g)) + A*e^2*(9*d^2*g + e^2*(5*f + 2*g) - 2*d*e*(3*f + 5*g)) - B*e*(2*
e^3*f + 6*d^3*g - d^2*e*(3*f + 5*g)))*Log[d + e*x])/((3*d - 2*e)^2*(d - e)
^2*(e*f - d*g)^2) + (g*(C*f^2 - B*f*g + A*g^2)*Log[f + g*x])/((3*f - 2*g)*
(f - g)*(e*f - d*g)^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.06

method	result
default	$\frac{(-A+B-C)\ln(1+x)}{(d-e)^2(f-g)}-\frac{(9A\,d^2e^2g-6Ad\,e^3f-10Ad\,e^3g+5A\,e^4f+2A\,e^4g-6B\,d^3eg+3B\,d^2e^2f+5B\,d^2e^2g-2B\,e^4f+3C\,d^4}{(dg-ef)^2(d-e)^2(3d-2e)^2}$
norman	$\frac{A\,e^3-Bd\,e^2+C\,d^2e}{e(dg-ef)(3d^2-5de+2e^2)(ex+d)}+\frac{g(A\,g^2-Bfg+C\,f^2)\ln(gx+f)}{3d^2f^2g^2-5d^2f\,g^3+2d^2g^4-6de\,f^3g+10de\,f^2g^2-4def\,g^3+3e^2f^4-5e^2f^3g+2e^2f^2g^2}-$
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-A+B-C)/(d-e)^2/(f-g)*ln(1+x)-(9*A*d^2*e^2*g-6*A*d*e^3*f-10*A*d*e^3*g+5*A
*e^4*f+2*A*e^4*g-6*B*d^3*e*g+3*B*d^2*e^2*f+5*B*d^2*e^2*g-2*B*e^4*f+3*C*d^4
*g-5*C*d^2*e^2*f-2*C*d^2*e^2*g+4*C*d*e^3*f)/(d*g-e*f)^2/(d-e)^2/(3*d-2*e)^
2*ln(e*x+d)+(A*e^2-B*d*e+C*d^2)/(d*g-e*f)/(d-e)/(3*d-2*e)/(e*x+d)+1/3*(81*
A-54*B+36*C)/(3*d-2*e)^2/(3*f-2*g)*ln(2+3*x)+(A*g^2-B*f*g+C*f^2)*g/(d*g-e*
f)^2/(f-g)/(3*f-2*g)*ln(g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**2/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.83

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2(f + gx)\,(2 + 5x + 3x^2)}\,dx =$$
$$-\frac{(((3\,B - 5\,C)d^2e^2 - 2\,(3\,A - 2\,C)de^3 + (5\,A - 2\,B)e^4)f + (3\,Cd^4 - 6\,Bd^3e + (9\,A + 5\,B - 2\,C)}{(9\,d^4e^2 - 30\,d^3e^3 + 37\,d^2e^4 - 20\,de^5 + 4\,e^6)f^2 - 2\,(9\,d^5e - 30\,d^4e^2 + 37\,d^3e^3 - 20\,d^2e^4 + 4\,de^5)fg + (9}$$
$$+ \frac{(Cf^2g - Bfg^2 + Ag^3)\log(gx + f)}{3\,e^2f^4 + 2\,d^2g^4 - (6\,de + 5\,e^2)f^3g + (3\,d^2 + 10\,de + 2\,e^2)f^2g^2 - (5\,d^2 + 4\,de)fg^3}$$
$$+ \frac{3\,(9\,A - 6\,B + 4\,C)\log(3\,x + 2)}{3\,(9\,d^2 - 12\,de + 4\,e^2)f - 2\,(9\,d^2 - 12\,de + 4\,e^2)g}$$
$$- \frac{(A - B + C)\log(x + 1)}{(d^2 - 2\,de + e^2)f - (d^2 - 2\,de + e^2)g}$$
$$- \frac{Cd^2 - Bde + Ae^2}{(3\,d^3e - 5\,d^2e^2 + 2\,de^3)f - (3\,d^4 - 5\,d^3e + 2\,d^2e^2)g + ((3\,d^2e^2 - 5\,de^3 + 2\,e^4)f - (3\,d^3e - 5\,d^2e^2 + 2\,d}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxi
ma")
```

output
```
-((((3*B - 5*C)*d^2*e^2 - 2*(3*A - 2*C)*d*e^3 + (5*A - 2*B)*e^4)*f + (3*C*d
^4 - 6*B*d^3*e + (9*A + 5*B - 2*C)*d^2*e^2 - 10*A*d*e^3 + 2*A*e^4)*g)*log(
e*x + d)/((9*d^4*e^2 - 30*d^3*e^3 + 37*d^2*e^4 - 20*d*e^5 + 4*e^6)*f^2 - 2
*(9*d^5*e - 30*d^4*e^2 + 37*d^3*e^3 - 20*d^2*e^4 + 4*d*e^5)*f*g + (9*d^6 -
 30*d^5*e + 37*d^4*e^2 - 20*d^3*e^3 + 4*d^2*e^4)*g^2) + (C*f^2*g - B*f*g^2
 + A*g^3)*log(g*x + f)/(3*e^2*f^4 + 2*d^2*g^4 - (6*d*e + 5*e^2)*f^3*g + (3
*d^2 + 10*d*e + 2*e^2)*f^2*g^2 - (5*d^2 + 4*d*e)*f*g^3) + 3*(9*A - 6*B + 4
*C)*log(3*x + 2)/(3*(9*d^2 - 12*d*e + 4*e^2)*f - 2*(9*d^2 - 12*d*e + 4*e^2
)*g) - (A - B + C)*log(x + 1)/((d^2 - 2*d*e + e^2)*f - (d^2 - 2*d*e + e^2)
*g) - (C*d^2 - B*d*e + A*e^2)/((3*d^3*e - 5*d^2*e^2 + 2*d*e^3)*f - (3*d^4
- 5*d^3*e + 2*d^2*e^2)*g + ((3*d^2*e^2 - 5*d*e^3 + 2*e^4)*f - (3*d^3*e - 5
*d^2*e^2 + 2*d*e^3)*g)*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9098 vs. $2(290) = 580$.

Time = 0.34 (sec) , antiderivative size = 9098, normalized size of antiderivative = 31.16

$$\int \frac{A + Bx + Cx^2}{(d + ex)^2(f + gx)\,(2 + 5x + 3x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^2/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac
")
```

output
```
1/3*(3*B*d^2*e^2*f - 5*C*d^2*e^2*f - 6*A*d*e^3*f + 4*C*d*e^3*f + 5*A*e^4*f
 - 2*B*e^4*f + 3*C*d^4*g - 6*B*d^3*e*g + 9*A*d^2*e^2*g + 5*B*d^2*e^2*g - 2
*C*d^2*e^2*g - 10*A*d*e^3*g + 2*A*e^4*g)*log(abs(3*e*f/(e*x + d) - 6*d*e*f
/(e*x + d)^2 + 3*d^2*e*f/(e*x + d)^3 + 5*e^2*f/(e*x + d)^2 - 5*d*e^2*f/(e*
x + d)^3 + 2*e^3*f/(e*x + d)^3 - 9*d*g/(e*x + d) + 9*d^2*g/(e*x + d)^2 - 3
*d^3*g/(e*x + d)^3 + 5*e*g/(e*x + d) - 10*d*e*g/(e*x + d)^2 + 5*d^2*e*g/(e
*x + d)^3 + 2*e^2*g/(e*x + d)^2 - 2*d*e^2*g/(e*x + d)^3 + 3*g))/(9*d^4*e^2
*f^2 - 30*d^3*e^3*f^2 + 37*d^2*e^4*f^2 - 20*d*e^5*f^2 + 4*e^6*f^2 - 18*d^5
*e*f*g + 60*d^4*e^2*f*g - 74*d^3*e^3*f*g + 40*d^2*e^4*f*g - 8*d*e^5*f*g +
9*d^6*g^2 - 30*d^5*e*g^2 + 37*d^4*e^2*g^2 - 20*d^3*e^3*g^2 + 4*d^2*e^4*g^2
) - (C*d^2*e^3/(e*x + d) - B*d*e^4/(e*x + d) + A*e^5/(e*x + d))/(3*d^2*e^4
*f - 5*d*e^5*f + 2*e^6*f - 3*d^3*e^3*g + 5*d^2*e^4*g - 2*d*e^5*g) - 1/3*((
27*B*d^5*e^15*f^3*g/(e^2*f - d*e*g) - 45*C*d^5*e^15*f^3*g/(e^2*f - d*e*g)
- 135*A*d^4*e^16*f^3*g/(e^2*f - d*e*g) + 90*C*d^4*e^16*f^3*g/(e^2*f - d*e*
g) + 450*A*d^3*e^17*f^3*g/(e^2*f - d*e*g) - 183*B*d^3*e^17*f^3*g/(e^2*f -
d*e*g) + 5*C*d^3*e^17*f^3*g/(e^2*f - d*e*g) - 561*A*d^2*e^18*f^3*g/(e^2*f
- d*e*g) + 300*B*d^2*e^18*f^3*g/(e^2*f - d*e*g) - 126*C*d^2*e^18*f^3*g/(e^
2*f - d*e*g) + 310*A*d*e^19*f^3*g/(e^2*f - d*e*g) - 184*B*d*e^19*f^3*g/(e^
2*f - d*e*g) + 100*C*d*e^19*f^3*g/(e^2*f - d*e*g) - 64*A*e^20*f^3*g/(e^2*f
 - d*e*g) + 40*B*e^20*f^3*g/(e^2*f - d*e*g) - 24*C*e^20*f^3*g/(e^2*f - ...
```

Mupad [B] (verification not implemented)

Time = 52.74 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\,(2+5x+3x^2)}\,dx$$
$$= \frac{C\,d^2-B\,d\,e+A\,e^2}{(d+e\,x)\,\left(3\,d^3\,g-2\,e^3\,f+5\,d\,e^2\,f-3\,d^2\,e\,f+2\,d\,e^2\,g-5\,d^2\,e\,g\right)}$$
$$-\frac{\ln\left(d+e\,x\right)\,\left(3\,C\,g\,d^4-6\,B\,g\,d^3\,e+\left(9\,A\,g+3\,B\,f+5\,B\,g-5\,C\,f-2\,C\,g\right)\,d^2\,e^2 -\right.}{9\,d^6\,g^2-18\,d^5\,e\,f\,g-30\,d^5\,e\,g^2+9\,d^4\,e^2\,f^2+60\,d^4\,e^2\,f\,g+37\,d^4\,e^2\,g^2-30\,d^3\,e^3\,f^2-74\,d^3\,e^3\,f\,g-20}$$
$$-\frac{\ln\left(x+1\right)\,\left(A-B+C\right)}{d^2\,f-d^2\,g+e^2\,f-e^2\,g-2\,d\,e\,f+2\,d\,e\,g}$$
$$+\frac{3\,\ln\left(x+\frac{2}{3}\right)\,\left(9\,A-6\,B+4\,C\right)}{{\left(3\,d-2\,e\right)}^2\,\left(3\,f-2\,g\right)}+\frac{g\,\ln\left(f+g\,x\right)\,\left(C\,f^2-B\,f\,g+A\,g^2\right)}{\left(f-g\right)\,{\left(d\,g-e\,f\right)}^2\,\left(3\,f-2\,g\right)}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^2*(5*x + 3*x^2 + 2)),x)
```

output
```
(A*e^2 + C*d^2 - B*d*e)/((d + e*x)*(3*d^3*g - 2*e^3*f + 5*d*e^2*f - 3*d^2*
e*f + 2*d*e^2*g - 5*d^2*e*g)) - (log(d + e*x)*(e^4*(5*A*f + 2*A*g - 2*B*f)
 + d^2*e^2*(9*A*g + 3*B*f + 5*B*g - 5*C*f - 2*C*g) + 3*C*d^4*g - d*e^3*(6*
A*f + 10*A*g - 4*C*f) - 6*B*d^3*e*g))/(9*d^6*g^2 + 4*e^6*f^2 - 20*d*e^5*f^
2 - 30*d^5*e*g^2 + 37*d^2*e^4*f^2 - 30*d^3*e^3*f^2 + 9*d^4*e^2*f^2 + 4*d^2
*e^4*g^2 - 20*d^3*e^3*g^2 + 37*d^4*e^2*g^2 - 8*d*e^5*f*g - 18*d^5*e*f*g +
40*d^2*e^4*f*g - 74*d^3*e^3*f*g + 60*d^4*e^2*f*g) - (log(x + 1)*(A - B + C
))/(d^2*f - d^2*g + e^2*f - e^2*g - 2*d*e*f + 2*d*e*g) + (3*log(x + 2/3)*(
9*A - 6*B + 4*C))/((3*d - 2*e)^2*(3*f - 2*g)) + (g*log(f + g*x)*(A*g^2 + C
*f^2 - B*f*g))/((f - g)*(d*g - e*f)^2*(3*f - 2*g))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5126, normalized size of antiderivative = 17.55

$$\int \frac{A+Bx+Cx^2}{(d+ex)^2(f+gx)\,(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^2/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
(27*log(3*x + 2)*a*d**6*f*g**2 - 27*log(3*x + 2)*a*d**6*g**3 - 54*log(3*x
+ 2)*a*d**5*e*f**2*g + 27*log(3*x + 2)*a*d**5*e*f*g**2*x - 27*log(3*x + 2)
*a*d**5*e*g**3*x + 54*log(3*x + 2)*a*d**5*e*g**3 + 27*log(3*x + 2)*a*d**4*
e**2*f**3 - 54*log(3*x + 2)*a*d**4*e**2*f**2*g*x + 81*log(3*x + 2)*a*d**4*
e**2*f**2*g - 81*log(3*x + 2)*a*d**4*e**2*f*g**2 + 54*log(3*x + 2)*a*d**4*
e**2*g**3*x - 27*log(3*x + 2)*a*d**4*e**2*g**3 + 27*log(3*x + 2)*a*d**3*e*
*3*f**3*x - 54*log(3*x + 2)*a*d**3*e**3*f**3 + 81*log(3*x + 2)*a*d**3*e**3
*f**2*g*x - 81*log(3*x + 2)*a*d**3*e**3*f*g**2*x + 54*log(3*x + 2)*a*d**3*
e**3*f*g**2 - 27*log(3*x + 2)*a*d**3*e**3*g**3*x - 54*log(3*x + 2)*a*d**2*
e**4*f**3*x + 27*log(3*x + 2)*a*d**2*e**4*f**3 - 27*log(3*x + 2)*a*d**2*e*
*4*f**2*g + 54*log(3*x + 2)*a*d**2*e**4*f*g**2*x + 27*log(3*x + 2)*a*d*e**
5*f**3*x - 27*log(3*x + 2)*a*d*e**5*f**2*g*x - 18*log(3*x + 2)*b*d**6*f*g*
*2 + 18*log(3*x + 2)*b*d**6*g**3 + 36*log(3*x + 2)*b*d**5*e*f**2*g - 18*lo
g(3*x + 2)*b*d**5*e*f*g**2*x + 18*log(3*x + 2)*b*d**5*e*g**3*x - 36*log(3*
x + 2)*b*d**5*e*g**3 - 18*log(3*x + 2)*b*d**4*e**2*f**3 + 36*log(3*x + 2)*
b*d**4*e**2*f**2*g*x - 54*log(3*x + 2)*b*d**4*e**2*f**2*g + 54*log(3*x + 2
)*b*d**4*e**2*f*g**2 - 36*log(3*x + 2)*b*d**4*e**2*g**3*x + 18*log(3*x + 2
)*b*d**4*e**2*g**3 - 18*log(3*x + 2)*b*d**3*e**3*f**3*x + 36*log(3*x + 2)*
b*d**3*e**3*f**3 - 54*log(3*x + 2)*b*d**3*e**3*f**2*g*x + 54*log(3*x + 2)*
b*d**3*e**3*f*g**2*x - 36*log(3*x + 2)*b*d**3*e**3*f*g**2 + 18*log(3*x ...
```

3.34 $\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\left(2+5x+3x^2\right)}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 581

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\left(2+5x+3x^2\right)}\,dx = -\frac{Cd^2-Bde+Ae^2}{2(3d-2e)(d-e)(ef-dg)(d+ex)^2}$$
$$+\frac{Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))-Be(2e^3f+6d^3g-d^2}{(3d-2e)^2(d-e)^2(ef-dg)^2(d+ex)}$$
$$-\frac{(A-B+C)\log(1+x)}{(d-e)^3(f-g)}+\frac{9(9A-6B+4C)\log(2+3x)}{(3d-2e)^3(3f-2g)}$$
$$+\frac{(C(4e^6f^2+9d^6g^2-9d^4e^2g(5f+2g)-6d^2e^4f(3f+5g)+d^3e^3(15f^2+73fg+10g^2))-Be(36d^2e^3f}{}$$
$$-\frac{g^2(Cf^2-Bfg+Ag^2)\log(f+gx)}{(3f-2g)(f-g)(ef-dg)^3}$$

output
```
-1/2*(A*e^2-B*d*e+C*d^2)/(3*d-2*e)/(d-e)/(-d*g+e*f)/(e*x+d)^2+(C*d*(4*e^3*
f+3*d^3*g-d*e^2*(5*f+2*g))+A*e^2*(9*d^2*g+e^2*(5*f+2*g)-2*d*e*(3*f+5*g))-B
*e*(2*e^3*f+6*d^3*g-d^2*e*(3*f+5*g)))/(3*d-2*e)^2/(d-e)^2/(-d*g+e*f)^2/(e*
x+d)-(A-B+C)*ln(1+x)/(d-e)^3/(f-g)+9*(9*A-6*B+4*C)*ln(2+3*x)/(3*d-2*e)^3/(
3*f-2*g)+(C*(4*e^6*f^2+9*d^6*g^2-9*d^4*e^2*g*(5*f+2*g)-6*d^2*e^4*f*(3*f+5*
g)+d^3*e^3*(15*f^2+73*f*g+10*g^2))-B*e*(36*d^2*e^3*f*g+27*d^5*g^2+2*e^5*f*
(5*f+2*g)-6*d*e^4*f*(3*f+5*g)-9*d^4*e*g*(3*f+5*g)+d^3*e^2*(9*f^2+15*f*g+19
*g^2))+A*e^2*(54*d^4*g^2-24*d^3*e*g*(3*f+5*g)-15*d*e^3*(3*f^2+5*f*g+2*g^2)
+e^4*(19*f^2+10*f*g+4*g^2)+3*d^2*e^2*(9*f^2+45*f*g+31*g^2)))*ln(e*x+d)/(3*
d-2*e)^3/(d-e)^3/(-d*g+e*f)^3-g^2*(A*g^2-B*f*g+C*f^2)*ln(g*x+f)/(3*f-2*g)/
(f-g)/(-d*g+e*f)^3
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 899, normalized size of antiderivative = 1.55

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{Cd^2+e(-Bd+Ae)}{2\left(3d^2-5de+2e^2\right)(-ef+dg)(d+ex)^2}$$
$$+\frac{Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))+Be(-2e^3f-6d^3g+\cdots}{\left(3d^2-5de+2e^2\right)^2(ef-dg)^2(d+ex)}$$
$$-\frac{(B(2e^3(39f-35g)+54d^2e(6f-5g)-27d^3(5f-4g)+18de^2(-15f+13g))+C(9d^3(13f-10g)+\cdots}{\cdots}$$
$$+\frac{(C(-4e^6f^2-9d^6g^2+9d^4e^2g(5f+2g)+6d^2e^4f(3f+5g)-d^3e^3(15f^2+73fg+10g^2))+Be(36d^2e^3\cdots}{\cdots}$$
$$+\frac{g^2(Cf^2+g(-Bf+Ag))\log(f+gx)}{(-ef+dg)^3\left(3f^2-5fg+2g^2\right)}$$
$$+\frac{(B(27d^3f+2e^3(15f-19g)-54d^2eg+18de^2(-3f+5g))+C(54d^2ef-9d^3(5f-2g)-36de^2g+4\cdots}{2\left(3d^2-5de+2e^2\cdots\right.}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^3*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(C*d^2 + e*(-(B*d) + A*e))/(2*(3*d^2 - 5*d*e + 2*e^2)*(-(e*f) + d*g)*(d +
e*x)^2) + (C*d*(4*e^3*f + 3*d^3*g - d*e^2*(5*f + 2*g)) + A*e^2*(9*d^2*g +
e^2*(5*f + 2*g) - 2*d*e*(3*f + 5*g)) + B*e*(-2*e^3*f - 6*d^3*g + d^2*e*(3*
f + 5*g)))/((3*d^2 - 5*d*e + 2*e^2)^2*(e*f - d*g)^2*(d + e*x)) - ((B*(2*e^
3*(39*f - 35*g) + 54*d^2*e*(6*f - 5*g) - 27*d^3*(5*f - 4*g) + 18*d*e^2*(-1
5*f + 13*g)) + C*(9*d^3*(13*f - 10*g) + 36*d*e^2*(6*f - 5*g) - 54*d^2*e*(5
*f - 4*g) + 4*e^3*(-15*f + 13*g)) + A*(9*d*e^2*(39*f - 35*g) + 27*d^3*(6*f
 - 5*g) + e^3*(-105*f + 97*g) + d^2*e*(-405*f + 351*g)))*ArcTanh[5 + 6*x])
/((3*d^2 - 5*d*e + 2*e^2)^3*(3*f^2 - 5*f*g + 2*g^2)) + ((C*(-4*e^6*f^2 - 9
*d^6*g^2 + 9*d^4*e^2*g*(5*f + 2*g) + 6*d^2*e^4*f*(3*f + 5*g) - d^3*e^3*(15
*f^2 + 73*f*g + 10*g^2)) + B*e*(36*d^2*e^3*f*g + 27*d^5*g^2 + 2*e^5*f*(5*f
 + 2*g) - 6*d*e^4*f*(3*f + 5*g) - 9*d^4*e*g*(3*f + 5*g) + d^3*e^2*(9*f^2 +
 15*f*g + 19*g^2)) - A*e^2*(54*d^4*g^2 - 24*d^3*e*g*(3*f + 5*g) - 15*d*e^3
*(3*f^2 + 5*f*g + 2*g^2) + e^4*(19*f^2 + 10*f*g + 4*g^2) + 3*d^2*e^2*(9*f^
2 + 45*f*g + 31*g^2)))*Log[d + e*x])/((3*d^2 - 5*d*e + 2*e^2)^3*(-(e*f) +
d*g)^3) + (g^2*(C*f^2 + g*(-(B*f) + A*g))*Log[f + g*x])/((-(e*f) + d*g)^3*
(3*f^2 - 5*f*g + 2*g^2)) + ((B*(27*d^3*f + 2*e^3*(15*f - 19*g) - 54*d^2*e*
g + 18*d*e^2*(-3*f + 5*g)) + C*(54*d^2*e*f - 9*d^3*(5*f - 2*g) - 36*d*e^2*
g + 4*e^3*(-3*f + 5*g)) + A*(9*d*e^2*(15*f - 19*g) - 27*d^2*e*(3*f - 5*g)
- 27*d^3*g + e^3*(-57*f + 65*g)))*Log[2 + 5*x + 3*x^2])/(2*(3*d^2 - 5*d...
```

Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)(d+ex)^3(f+gx)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(-\frac{e(Ae^2-Bde+Cd^2)}{(3d-2e)(d-e)(d+ex)^3(dg-ef)}+\frac{e(-Ae^2(9d^2g-2de(3f+5g)+e^2(5f+2g))+Be(6d^3g-d^2e(3f}{(3d-2e)^2(d-e)^2(d+ex}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ae^2-Bde+Cd^2}{2(3d-2e)(d-e)(d+ex)^2(ef-dg)}+$$
$$\frac{Ae^2\left(9d^2g-2de(3f+5g)+e^2(5f+2g)\right)-Be\left(6d^3g-d^2e(3f+5g)+2e^3f\right)+Cd\left(3d^3g-de^2(5f+2g)+4e^3f\right.}{(3d-2e)^2(d-e)^2(d+ex)(ef-dg)^2}$$
$$\frac{\log(d+ex)\left(Ae^2\left(54d^4g^2-24d^3eg(3f+5g)+3d^2e^2\left(9f^2+45fg+31g^2\right)-15de^3\left(3f^2+5fg+2g^2\right)+e^4\left(19f^2\right.\right.\right.}{}$$
$$\frac{g^2\log(f+gx)\left(Ag^2-Bfg+Cf^2\right)}{(3f-2g)(f-g)(ef-dg)^3}-\frac{\log(x+1)(A-B+C)}{(d-e)^3(f-g)}+\frac{9\log(3x+2)(9A-6B+4C)}{(3d-2e)^3(3f-2g)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^3*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
-1/2*(C*d^2 - B*d*e + A*e^2)/((3*d - 2*e)*(d - e)*(e*f - d*g)*(d + e*x)^2)
 + (C*d*(4*e^3*f + 3*d^3*g - d*e^2*(5*f + 2*g)) + A*e^2*(9*d^2*g + e^2*(5*
f + 2*g) - 2*d*e*(3*f + 5*g)) - B*e*(2*e^3*f + 6*d^3*g - d^2*e*(3*f + 5*g)
))/((3*d - 2*e)^2*(d - e)^2*(e*f - d*g)^2*(d + e*x)) - ((A - B + C)*Log[1
+ x])/((d - e)^3*(f - g)) + (9*(9*A - 6*B + 4*C)*Log[2 + 3*x])/((3*d - 2*e
)^3*(3*f - 2*g)) + ((C*(4*e^6*f^2 + 9*d^6*g^2 - 9*d^4*e^2*g*(5*f + 2*g) -
6*d^2*e^4*f*(3*f + 5*g) + d^3*e^3*(15*f^2 + 73*f*g + 10*g^2)) - B*e*(36*d^
2*e^3*f*g + 27*d^5*g^2 + 2*e^5*f*(5*f + 2*g) - 6*d*e^4*f*(3*f + 5*g) - 9*d
^4*e*g*(3*f + 5*g) + d^3*e^2*(9*f^2 + 15*f*g + 19*g^2)) + A*e^2*(54*d^4*g^
2 - 24*d^3*e*g*(3*f + 5*g) - 15*d*e^3*(3*f^2 + 5*f*g + 2*g^2) + e^4*(19*f^
2 + 10*f*g + 4*g^2) + 3*d^2*e^2*(9*f^2 + 45*f*g + 31*g^2)))*Log[d + e*x])/
((3*d - 2*e)^3*(d - e)^3*(e*f - d*g)^3) - (g^2*(C*f^2 - B*f*g + A*g^2)*Log
[f + g*x])/((3*f - 2*g)*(f - g)*(e*f - d*g)^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.18

method	result
default	$\frac{(-A+B-C)\ln(1+x)}{(d-e)^3(f-g)}+\frac{9A\,d^2e^2g-6Ad\,e^3f-10Ad\,e^3g+5A\,e^4f+2A\,e^4g-6B\,d^3eg+3B\,d^2e^2f+5B\,d^2e^2g-2B\,e^4f+3C\,d^4g}{(dg-ef)^2(d-e)^2(3d-2e)^2(ex+d)}$
norman	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERBOSE)
```

output
```
(-A+B-C)/(d-e)^3/(f-g)*ln(1+x)+(9*A*d^2*e^2*g-6*A*d*e^3*f-10*A*d*e^3*g+5*A
*e^4*f+2*A*e^4*g-6*B*d^3*e*g+3*B*d^2*e^2*f+5*B*d^2*e^2*g-2*B*e^4*f+3*C*d^4
*g-5*C*d^2*e^2*f-2*C*d^2*e^2*g+4*C*d*e^3*f)/(d*g-e*f)^2/(d-e)^2/(3*d-2*e)^
2/(e*x+d)-(54*A*d^4*e^2*g^2-72*A*d^3*e^3*f*g-120*A*d^3*e^3*g^2+27*A*d^2*e^
4*f^2+135*A*d^2*e^4*f*g+93*A*d^2*e^4*g^2-45*A*d*e^5*f^2-75*A*d*e^5*f*g-30*
A*d*e^5*g^2+19*A*e^6*f^2+10*A*e^6*f*g+4*A*e^6*g^2-27*B*d^5*e*g^2+27*B*d^4*
e^2*f*g+45*B*d^4*e^2*g^2-9*B*d^3*e^3*f^2-15*B*d^3*e^3*f*g-19*B*d^3*e^3*g^2
-36*B*d^2*e^4*f*g+18*B*d*e^5*f^2+30*B*d*e^5*f*g-10*B*e^6*f^2-4*B*e^6*f*g+9
*C*d^6*g^2-45*C*d^4*e^2*f*g-18*C*d^4*e^2*g^2+15*C*d^3*e^3*f^2+73*C*d^3*e^3
*f*g+10*C*d^3*e^3*g^2-18*C*d^2*e^4*f^2-30*C*d^2*e^4*f*g+4*C*e^6*f^2)/(d*g-
e*f)^3/(d-e)^3/(3*d-2*e)^3*ln(e*x+d)+1/2*(A*e^2-B*d*e+C*d^2)/(d*g-e*f)/(d-
e)/(3*d-2*e)/(e*x+d)^2+1/3*(243*A-162*B+108*C)/(3*d-2*e)^3/(3*f-2*g)*ln(2+
3*x)+(A*g^2-B*f*g+C*f^2)*g^2/(d*g-e*f)^3/(f-g)/(3*f-2*g)*ln(g*x+f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\,(2+5x+3x^2)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(g*x+f)/(3*x^2+5*x+2),x, algorithm="fric
as")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\,(2+5x+3x^2)}\,dx = \text{Timed out}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**3/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. 2(579) = 1158.

Time = 0.13 (sec) , antiderivative size = 1353, normalized size of antiderivative = 2.33

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\,(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(g*x+f)/(3*x^2+5*x+2),x, algorithm="maxi
ma")
```

output

```
-((3*(3*B - 5*C)*d^3*e^3 - 9*(3*A - 2*C)*d^2*e^4 + 9*(5*A - 2*B)*d*e^5 - (
19*A - 10*B + 4*C)*e^6)*f^2 - (9*(3*B - 5*C)*d^4*e^2 - (72*A + 15*B - 73*C
)*d^3*e^3 + 3*(45*A - 12*B - 10*C)*d^2*e^4 - 15*(5*A - 2*B)*d*e^5 + 2*(5*A
 - 2*B)*e^6)*f*g - (9*C*d^6 - 27*B*d^5*e + 9*(6*A + 5*B - 2*C)*d^4*e^2 - (
120*A + 19*B - 10*C)*d^3*e^3 + 93*A*d^2*e^4 - 30*A*d*e^5 + 4*A*e^6)*g^2)*l
og(e*x + d)/((27*d^6*e^3 - 135*d^5*e^4 + 279*d^4*e^5 - 305*d^3*e^6 + 186*d
^2*e^7 - 60*d*e^8 + 8*e^9)*f^3 - 3*(27*d^7*e^2 - 135*d^6*e^3 + 279*d^5*e^4
 - 305*d^4*e^5 + 186*d^3*e^6 - 60*d^2*e^7 + 8*d*e^8)*f^2*g + 3*(27*d^8*e -
 135*d^7*e^2 + 279*d^6*e^3 - 305*d^5*e^4 + 186*d^4*e^5 - 60*d^3*e^6 + 8*d^
2*e^7)*f*g^2 - (27*d^9 - 135*d^8*e + 279*d^7*e^2 - 305*d^6*e^3 + 186*d^5*e
^4 - 60*d^4*e^5 + 8*d^3*e^6)*g^3) - (C*f^2*g^2 - B*f*g^3 + A*g^4)*log(g*x
+ f)/(3*e^3*f^5 - 2*d^3*g^5 - (9*d*e^2 + 5*e^3)*f^4*g + (9*d^2*e + 15*d*e^
2 + 2*e^3)*f^3*g^2 - 3*(d^3 + 5*d^2*e + 2*d*e^2)*f^2*g^3 + (5*d^3 + 6*d^2*
e)*f*g^4) + 9*(9*A - 6*B + 4*C)*log(3*x + 2)/(3*(27*d^3 - 54*d^2*e + 36*d*
e^2 - 8*e^3)*f - 2*(27*d^3 - 54*d^2*e + 36*d*e^2 - 8*e^3)*g) - (A - B + C)
*log(x + 1)/((d^3 - 3*d^2*e + 3*d*e^2 - e^3)*f - (d^3 - 3*d^2*e + 3*d*e^2
- e^3)*g) - 1/2*((3*C*d^4*e - (9*B - 5*C)*d^3*e^2 + (15*A + 5*B - 6*C)*d^2
*e^3 - (15*A - 2*B)*d*e^4 + 2*A*e^5)*f - (9*C*d^5 - 5*(3*B + C)*d^4*e + (2
1*A + 15*B - 2*C)*d^3*e^2 - (25*A + 2*B)*d^2*e^3 + 6*A*d*e^4)*g - 2*(((3*B
 - 5*C)*d^2*e^3 - 2*(3*A - 2*C)*d*e^4 + (5*A - 2*B)*e^5)*f + (3*C*d^4*e...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(579) = 1158$.

Time = 0.27 (sec) , antiderivative size = 1885, normalized size of antiderivative = 3.24

$$\int \frac{A + Bx + Cx^2}{(d + ex)^3 (f + gx) (2 + 5x + 3x^2)} \, dx = \text{Too large to display}$$

input

```
integrate((C*x^2+B*x+A)/(e*x+d)^3/(g*x+f)/(3*x^2+5*x+2),x, algorithm="giac
")
```

output
```
-(9*B*d^3*e^4*f^2 - 15*C*d^3*e^4*f^2 - 27*A*d^2*e^5*f^2 + 18*C*d^2*e^5*f^2
 + 45*A*d*e^6*f^2 - 18*B*d*e^6*f^2 - 19*A*e^7*f^2 + 10*B*e^7*f^2 - 4*C*e^7
*f^2 - 27*B*d^4*e^3*f*g + 45*C*d^4*e^3*f*g + 72*A*d^3*e^4*f*g + 15*B*d^3*e
^4*f*g - 73*C*d^3*e^4*f*g - 135*A*d^2*e^5*f*g + 36*B*d^2*e^5*f*g + 30*C*d^
2*e^5*f*g + 75*A*d*e^6*f*g - 30*B*d*e^6*f*g - 10*A*e^7*f*g + 4*B*e^7*f*g -
 9*C*d^6*e*g^2 + 27*B*d^5*e^2*g^2 - 54*A*d^4*e^3*g^2 - 45*B*d^4*e^3*g^2 +
18*C*d^4*e^3*g^2 + 120*A*d^3*e^4*g^2 + 19*B*d^3*e^4*g^2 - 10*C*d^3*e^4*g^2
 - 93*A*d^2*e^5*g^2 + 30*A*d*e^6*g^2 - 4*A*e^7*g^2)*log(abs(e*x + d))/(27*
d^6*e^4*f^3 - 135*d^5*e^5*f^3 + 279*d^4*e^6*f^3 - 305*d^3*e^7*f^3 + 186*d^
2*e^8*f^3 - 60*d*e^9*f^3 + 8*e^10*f^3 - 81*d^7*e^3*f^2*g + 405*d^6*e^4*f^2
*g - 837*d^5*e^5*f^2*g + 915*d^4*e^6*f^2*g - 558*d^3*e^7*f^2*g + 180*d^2*e
^8*f^2*g - 24*d*e^9*f^2*g + 81*d^8*e^2*f*g^2 - 405*d^7*e^3*f*g^2 + 837*d^6
*e^4*f*g^2 - 915*d^5*e^5*f*g^2 + 558*d^4*e^6*f*g^2 - 180*d^3*e^7*f*g^2 + 2
4*d^2*e^8*f*g^2 - 27*d^9*e*g^3 + 135*d^8*e^2*g^3 - 279*d^7*e^3*g^3 + 305*d
^6*e^4*g^3 - 186*d^5*e^5*g^3 + 60*d^4*e^6*g^3 - 8*d^3*e^7*g^3) - (C*f^2*g^
3 - B*f*g^4 + A*g^5)*log(abs(-g*x - f))/(3*e^3*f^5*g - 9*d*e^2*f^4*g^2 - 5
*e^3*f^4*g^2 + 9*d^2*e*f^3*g^3 + 15*d*e^2*f^3*g^3 + 2*e^3*f^3*g^3 - 3*d^3*
f^2*g^4 - 15*d^2*e*f^2*g^4 - 6*d*e^2*f^2*g^4 + 5*d^3*f*g^5 + 6*d^2*e*f*g^5
 - 2*d^3*g^6) - (A - B + C)*log(abs(-x - 1))/(d^3*f - 3*d^2*e*f + 3*d*e^2*
f - e^3*f - d^3*g + 3*d^2*e*g - 3*d*e^2*g + e^3*g) + 9*(9*A - 6*B + 4*C...
```

Mupad [B] (verification not implemented)

Time = 55.72 (sec) , antiderivative size = 131190, normalized size of antiderivative = 225.80

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^3*(5*x + 3*x^2 + 2)),x)
```

output
```
symsum(log((3*(108*A^3*e^8*g^6 + 2673*A^3*d^2*e^6*g^6 - 3645*A^3*d^3*e^5*g
^6 + 1701*A^3*d^4*e^4*g^6 + 513*A^3*e^8*f^2*g^4 - 72*A^2*C*e^8*g^6 - 810*A
^3*d*e^7*g^6 + 270*A^3*e^8*f*g^5 + 729*A^3*d^2*e^6*f^2*g^4 - 270*B^3*d^2*e
^6*f^2*g^4 + 162*B^3*d^3*e^5*f^2*g^4 + 72*C^3*d^2*e^6*f^2*g^4 + 180*C^3*d^
2*e^6*f^3*g^3 + 360*C^3*d^3*e^5*f^2*g^4 + 216*C^3*d^3*e^5*f^3*g^3 - 1206*C
^3*d^4*e^4*f^2*g^4 - 270*C^3*d^4*e^4*f^3*g^3 + 810*C^3*d^5*e^3*f^2*g^4 - 1
08*A^2*B*d*e^7*g^6 + 405*A*C^2*d^7*e*g^6 + 720*A^2*C*d*e^7*g^6 - 108*A^2*B
*e^8*f*g^5 - 360*A^2*C*e^8*f*g^5 - 2187*A^3*d*e^7*f*g^5 - 162*C^3*d^7*e*f*
g^5 + 108*A*B^2*d^2*e^6*g^6 - 270*A*B^2*d^3*e^5*g^6 + 675*A*B^2*d^4*e^4*g^
6 - 1215*A*B^2*d^5*e^3*g^6 + 729*A*B^2*d^6*e^2*g^6 + 540*A^2*B*d^2*e^6*g^6
 - 2349*A^2*B*d^3*e^5*g^6 + 4050*A^2*B*d^4*e^4*g^6 - 2187*A^2*B*d^5*e^3*g^
6 + 72*A*C^2*d^2*e^6*g^6 - 360*A*C^2*d^3*e^5*g^6 + 882*A*C^2*d^4*e^4*g^6 -
 810*A*C^2*d^5*e^3*g^6 - 162*A*C^2*d^6*e^2*g^6 - 2790*A^2*C*d^2*e^6*g^6 +
6030*A^2*C*d^3*e^5*g^6 - 6507*A^2*C*d^4*e^4*g^6 + 2430*A^2*C*d^5*e^3*g^6 +
 243*A^2*C*d^6*e^2*g^6 - 270*A^2*B*e^8*f^2*g^4 - 72*A*C^2*e^8*f^2*g^4 - 18
0*A*C^2*e^8*f^3*g^3 - 684*A^2*C*e^8*f^2*g^4 - 855*A^2*C*e^8*f^3*g^3 + 72*B
*C^2*e^8*f^3*g^3 - 72*B^2*C*e^8*f^2*g^4 - 180*B^2*C*e^8*f^3*g^3 - 1215*A^3
*d*e^7*f^2*g^4 + 4050*A^3*d^2*e^6*f*g^5 - 2187*A^3*d^3*e^5*f*g^5 + 108*B^3
*d*e^7*f^2*g^4 - 108*B^3*d^2*e^6*f*g^5 + 270*B^3*d^3*e^5*f*g^5 - 162*B^3*d
^4*e^4*f*g^5 - 144*C^3*d*e^7*f^3*g^3 - 180*C^3*d^4*e^4*f*g^5 + 324*C^3*...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15095, normalized size of antiderivative = 25.98

$$\int \frac{A+Bx+Cx^2}{(d+ex)^3(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^3/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
(162*log(3*x + 2)*a*d**9*f*g**3 - 162*log(3*x + 2)*a*d**9*g**4 - 486*log(3
*x + 2)*a*d**8*e*f**2*g**2 + 324*log(3*x + 2)*a*d**8*e*f*g**3*x - 324*log(
3*x + 2)*a*d**8*e*g**4*x + 486*log(3*x + 2)*a*d**8*e*g**4 + 486*log(3*x +
2)*a*d**7*e**2*f**3*g - 972*log(3*x + 2)*a*d**7*e**2*f**2*g**2*x + 972*log
(3*x + 2)*a*d**7*e**2*f**2*g**2 + 162*log(3*x + 2)*a*d**7*e**2*f*g**3*x**2
 - 972*log(3*x + 2)*a*d**7*e**2*f*g**3 - 162*log(3*x + 2)*a*d**7*e**2*g**4
*x**2 + 972*log(3*x + 2)*a*d**7*e**2*g**4*x - 486*log(3*x + 2)*a*d**7*e**2
*g**4 - 162*log(3*x + 2)*a*d**6*e**3*f**4 + 972*log(3*x + 2)*a*d**6*e**3*f
**3*g*x - 1296*log(3*x + 2)*a*d**6*e**3*f**3*g - 486*log(3*x + 2)*a*d**6*e
**3*f**2*g**2*x**2 + 1944*log(3*x + 2)*a*d**6*e**3*f**2*g**2*x - 1944*log(
3*x + 2)*a*d**6*e**3*f*g**3*x + 1296*log(3*x + 2)*a*d**6*e**3*f*g**3 + 486
*log(3*x + 2)*a*d**6*e**3*g**4*x**2 - 972*log(3*x + 2)*a*d**6*e**3*g**4*x
+ 162*log(3*x + 2)*a*d**6*e**3*g**4 - 324*log(3*x + 2)*a*d**5*e**4*f**4*x
+ 486*log(3*x + 2)*a*d**5*e**4*f**4 + 486*log(3*x + 2)*a*d**5*e**4*f**3*g*
x**2 - 2592*log(3*x + 2)*a*d**5*e**4*f**3*g*x + 972*log(3*x + 2)*a*d**5*e*
*4*f**3*g + 972*log(3*x + 2)*a*d**5*e**4*f**2*g**2*x**2 - 972*log(3*x + 2)
*a*d**5*e**4*f**2*g**2 - 972*log(3*x + 2)*a*d**5*e**4*f*g**3*x**2 + 2592*l
og(3*x + 2)*a*d**5*e**4*f*g**3*x - 486*log(3*x + 2)*a*d**5*e**4*f*g**3 - 4
86*log(3*x + 2)*a*d**5*e**4*g**4*x**2 + 324*log(3*x + 2)*a*d**5*e**4*g**4*
x - 162*log(3*x + 2)*a*d**4*e**5*f**4*x**2 + 972*log(3*x + 2)*a*d**4*e*...
```

3.35 $\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 599

$$\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2(ef-dg)^2(Cf^2-Bfg+Ag^2)\sqrt{d+ex}}{(3f-2g)(f-g)g^3}$$
$$-\frac{2(C(e^2(65f-38g)-6de(19f-10g)+9d^2(5f-2g))-3B(9d^2f+e^2(19f-10g)-6de(5f-2g))-}{27(3f^2-5fg+2g^2)}$$
$$+\frac{2(9Bdf-15Cdf+9Aef-15Bef+19Cef-9Adg+6Cdg+6Beg-10Ceg)(d+ex)^{3/2}}{27(3f-2g)(f-g)}$$
$$-\frac{2(ef-dg)(Cf^2-Bfg+Ag^2)(d+ex)^{3/2}}{3g^2(3f^2-5fg+2g^2)}$$
$$+\frac{2(3Bf-5Cf-3Ag+2Cg)(d+ex)^{5/2}}{15(3f-2g)(f-g)}+\frac{2(Cf^2-Bfg+Ag^2)(d+ex)^{5/2}}{5(3f-2g)(f-g)g}$$
$$-\frac{2(ef-dg)^{5/2}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)g^{7/2}}$$
$$-\frac{2(9A-6B+4C)(3d-2e)^{5/2}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{27\sqrt{3}(3f-2g)}$$
$$+\frac{2(A-B+C)(d-e)^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}$$

output
```
2*(-d*g+e*f)^2*(A*g^2-B*f*g+C*f^2)*(e*x+d)^(1/2)/(3*f-2*g)/(f-g)/g^3-2*(C*
(e^2*(65*f-38*g)-6*d*e*(19*f-10*g)+9*d^2*(5*f-2*g))-3*B*(9*d^2*f+e^2*(19*f
-10*g)-6*d*e*(5*f-2*g))-9*A*(6*d*e*f-e^2*(5*f-2*g)-3*d^2*g))*(e*x+d)^(1/2)
/(81*f^2-135*f*g+54*g^2)+2/27*(-9*A*d*g+9*A*e*f+9*B*d*f-15*B*e*f+6*B*e*g-1
5*C*d*f+6*C*d*g+19*C*e*f-10*C*e*g)*(e*x+d)^(3/2)/(3*f-2*g)/(f-g)-2/3*(-d*g
+e*f)*(A*g^2-B*f*g+C*f^2)*(e*x+d)^(3/2)/g^2/(3*f^2-5*f*g+2*g^2)+2/15*(-3*A
*g+3*B*f-5*C*f+2*C*g)*(e*x+d)^(5/2)/(3*f-2*g)/(f-g)+2/5*(A*g^2-B*f*g+C*f^2
)*(e*x+d)^(5/2)/(3*f-2*g)/(f-g)/g-2*(-d*g+e*f)^(5/2)*(A*g^2-B*f*g+C*f^2)*a
rctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/(3*f-2*g)/(f-g)/g^(7/2)-2/81
*(9*A-6*B+4*C)*(3*d-2*e)^(5/2)*arctanh(3^(1/2)*(e*x+d)^(1/2)/(3*d-2*e)^(1/
2))*3^(1/2)/(3*f-2*g)+2*(A-B+C)*(d-e)^(5/2)*arctanh((e*x+d)^(1/2)/(d-e)^(1
/2))/(f-g)
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.50

$$\int \frac{(d+ex)^{5/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{2\sqrt{d+ex}(15eg(3Aeg+B(-3ef+7dg+eg(-5+x)))+C(69d^2g^2}{}$$

$$+\frac{2(A-B+C)(-d+e)^{5/2}\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{f-g}$$

$$-\frac{2(9A-6B+4C)(3d-2e)^3\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{27\sqrt{-9d+6e}(3f-2g)}$$

$$-\frac{2(ef-dg)^{5/2}\left(Cf^2+g(-Bf+Ag)\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{7/2}\left(3f^2-5fg+2g^2\right)}$$

input
```
Integrate[((d + e*x)^(5/2)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)
),x]
```

output
```
(2*Sqrt[d + e*x]*(15*e*g*(3*A*e*g + B*(-3*e*f + 7*d*g + e*g*(-5 + x))) + C
*(69*d^2*g^2 + d*e*g*(-105*f + g*(-175 + 33*x)) + e^2*(45*f^2 - 15*f*g*(-5
 + x) + g^2*(95 - 25*x + 9*x^2)))))/(135*g^3) + (2*(A - B + C)*(-d + e)^(5
/2)*ArcTan[Sqrt[d + e*x]/Sqrt[-d + e]])/(f - g) - (2*(9*A - 6*B + 4*C)*(3*
d - 2*e)^3*ArcTan[(Sqrt[-9*d + 6*e]*Sqrt[d + e*x])/(3*d - 2*e)])/(27*Sqrt[
-9*d + 6*e]*(3*f - 2*g)) - (2*(e*f - d*g)^(5/2)*(C*f^2 + g*(-(B*f) + A*g))
*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(g^(7/2)*(3*f^2 - 5*f*g
+ 2*g^2))
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.92, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2} (A+Bx+Cx^2)}{(3x^2+5x+2) (f+gx)} dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{(d+ex)^{5/2} (Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)(f+gx)}+\frac{(-A+B-C)(d+ex)^{5/2}}{(x+1)(f-g)}+\frac{(9A-6B+4C)(d+ex)^{5/2}}{(3x+2)(3f-2g)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(ef-dg)^{5/2}\left(Ag^2-Bfg+Cf^2\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{7/2}(3f-2g)(f-g)}-\frac{2(3d-2e)^{5/2}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{27\sqrt{3}(3f-2g)}+\frac{2(d-e)^{5/2}(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}+\frac{2(d+ex)^{5/2}\left(Ag^2-Bfg+Cf^2\right)}{5g(3f-2g)(f-g)}-\frac{2(d+ex)^{3/2}(ef-dg)\left(Ag^2-Bfg+Cf^2\right)}{3g^2(3f-2g)(f-g)}+\frac{2\sqrt{d+ex}(ef-dg)^2\left(Ag^2-Bfg+Cf^2\right)}{g^3(3f-2g)(f-g)}+\frac{2(3d-2e)^2(9A-6B+4C)\sqrt{d+ex}}{27(3f-2g)}+\frac{2(3d-2e)(9A-6B+4C)(d+ex)^{3/2}}{27(3f-2g)}+\frac{2(9A-6B+4C)(d+ex)^{5/2}}{15(3f-2g)}-\frac{2(A-B+C)(d+ex)^{5/2}}{5(f-g)}-\frac{2(d-e)(A-B+C)(d+ex)^{3/2}}{3(f-g)}-\frac{2(d-e)^2(A-B+C)\sqrt{d+ex}}{f-g}$$

input
```
Int[((d + e*x)^(5/2)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C)*(3*d - 2*e)^2*Sqrt[d + e*x])/(27*(3*f - 2*g)) - (2*(A
 - B + C)*(d - e)^2*Sqrt[d + e*x])/(f - g) + (2*(e*f - d*g)^2*(C*f^2 - B*f
*g + A*g^2)*Sqrt[d + e*x])/((3*f - 2*g)*(f - g)*g^3) + (2*(9*A - 6*B + 4*C
)*(3*d - 2*e)*(d + e*x)^(3/2))/(27*(3*f - 2*g)) - (2*(A - B + C)*(d - e)*(
d + e*x)^(3/2))/(3*(f - g)) - (2*(e*f - d*g)*(C*f^2 - B*f*g + A*g^2)*(d +
e*x)^(3/2))/(3*(3*f - 2*g)*(f - g)*g^2) + (2*(9*A - 6*B + 4*C)*(d + e*x)^(
5/2))/(15*(3*f - 2*g)) - (2*(A - B + C)*(d + e*x)^(5/2))/(5*(f - g)) + (2*
(C*f^2 - B*f*g + A*g^2)*(d + e*x)^(5/2))/(5*(3*f - 2*g)*(f - g)*g) - (2*(e
*f - d*g)^(5/2)*(C*f^2 - B*f*g + A*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqr
t[e*f - d*g]])/((3*f - 2*g)*(f - g)*g^(7/2)) - (2*(9*A - 6*B + 4*C)*(3*d -
 2*e)^(5/2)*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d - 2*e]])/(27*Sqrt[3]*
(3*f - 2*g)) + (2*(A - B + C)*(d - e)^(5/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d -
 e]])/(f - g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{\frac{2\sqrt{-d+e}\,(dg-ef)^3\sqrt{-9d+6e}\,\left(A\,g^2-Bfg+C\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{3}+\frac{2\left(9\left(d-\frac{2e}{3}\right)^3(f-g)\sqrt{-d+e}\,\left(A-\frac{2B}{3}+\frac{4C}{9}\right)g^3\operatorname{arctan}\right.}{}}{}$
risch	$\frac{2\left(9C\,e^2x^2g^2+15B\,e^2g^2x+33Cde\,g^2x-15C\,e^2fgx-25C\,e^2g^2x+45A\,e^2g^2+105Bde\,g^2-45B\,e^2fg-75B\,e^2g^2+69C\,d^2\right.}{135g^3}$
derivativedivides	$\frac{2C(ex+d)^{\frac{5}{2}}g^2}{15}+\frac{2Be\,g^2(ex+d)^{\frac{3}{2}}}{9}+\frac{2Cd\,g^2(ex+d)^{\frac{3}{2}}}{9}-\frac{2Cefg(ex+d)^{\frac{3}{2}}}{9}-\frac{10Ce\,g^2(ex+d)^{\frac{3}{2}}}{27}+\frac{2\sqrt{ex+d}\,A\,e^2g^2}{3}+\frac{4\sqrt{ex+d}\,Bde\,g^2}{3}-$
default	$\frac{2C(ex+d)^{\frac{5}{2}}g^2}{15}+\frac{2Be\,g^2(ex+d)^{\frac{3}{2}}}{9}+\frac{2Cd\,g^2(ex+d)^{\frac{3}{2}}}{9}-\frac{2Cefg(ex+d)^{\frac{3}{2}}}{9}-\frac{10Ce\,g^2(ex+d)^{\frac{3}{2}}}{27}+\frac{2\sqrt{ex+d}\,A\,e^2g^2}{3}+\frac{4\sqrt{ex+d}\,Bde\,g^2}{3}-$

input
```
int((e*x+d)^(5/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
2/3/((d*g-e*f)*g)^(1/2)*(-(-d+e)^(1/2)*(d*g-e*f)^3*(-9*d+6*e)^(1/2)*(A*g^2
-B*f*g+C*f^2)*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+(9*(d-2/3*e)^3*
(f-g)*(-d+e)^(1/2)*(A-2/3*B+4/9*C)*g^3*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(
1/2))+(-3*g^3*(d-e)^3*(A-B+C)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))+((((1/5*x
^2-5/9*x+19/9)*C+1/3*(-5+x)*B+A)*g^2-(1/3*(-5+x)*C+B)*f*g+C*f^2)*e^2+7/3*(
((11/35*x-5/3)*C+B)*g-C*f)*d*g*e+23/15*C*d^2*g^2)*(e*x+d)^(1/2)*(f-g)*(-d+
e)^(1/2))*(-9*d+6*e)^(1/2)*(f-2/3*g))*((d*g-e*f)*g)^(1/2))/(-9*d+6*e)^(1/2
)/(-d+e)^(1/2)/(f-g)/(f-2/3*g)/g^3
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(5/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 145.49 (sec) , antiderivative size = 450, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^{5/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx=\begin{cases} \frac{2\left(\frac{Ce(d+ex)^{\frac{5}{2}}}{15g}-\frac{e(d-e)^3(A-B+C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}+\frac{e(3d-2e)^3\cdot(9A-6B+4C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+\frac{2e}{3}}}\right)}{81\sqrt{-d+\frac{2e}{3}}\cdot(3f-2g)}\right.}{} \\ d^{\frac{5}{2}}\left(\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)}-\frac{(A-B+C)\log(x+1)}{f-g}+\frac{\left(Ag^2-Bfg+Cf^2\right)\left(\begin{cases}\frac{x}{f}\\ \frac{\log(f}{g}\end{cases}\right.}{(f-g)(3f}\right. \end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(C*e*(d + e*x)**(5/2)/(15*g) - e*(d - e)**3*(A - B + C)*atan(
sqrt(d + e*x)/sqrt(-d + e))/(sqrt(-d + e)*(f - g)) + e*(3*d - 2*e)**3*(9*A
 - 6*B + 4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2*e/3))/(81*sqrt(-d + 2*e/3)*(3
*f - 2*g)) + e*(d*g - e*f)**3*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e*x)
/sqrt(-(d*g - e*f)/g))/(g**4*sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)) + (
d + e*x)**(3/2)*(3*B*e**2*g + 3*C*d*e*g - 3*C*e**2*f - 5*C*e**2*g)/(27*g**
2) + sqrt(d + e*x)*(9*A*e**3*g**2 + 18*B*d*e**2*g**2 - 9*B*e**3*f*g - 15*B
*e**3*g**2 + 9*C*d**2*e*g**2 - 18*C*d*e**2*f*g - 30*C*d*e**2*g**2 + 9*C*e*
*3*f**2 + 15*C*e**3*f*g + 19*C*e**3*g**2)/(27*g**3))/e, Ne(e, 0)), (d**(5/
2)*((9*A - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*g)) - (A - B + C)*log(x + 1
)/(f - g) + (A*g**2 - B*f*g + C*f**2)*Piecewise((x/f, Eq(g, 0)), (log(f +
g*x)/g, True))/((f - g)*(3*f - 2*g))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 665, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2\sqrt{3}(243\,Ad^3-162\,Bd^3+108\,Cd^3-486\,Ad^2e+324\,Bd^2e-216\,C}{81}$$
$$-\frac{2\,(Ce^3f^5-3\,Cde^2f^4g-Be^3f^4g+3\,Cd^2ef^3g^2+3\,Bde^2f^3g^2+Ae^3f^3g^2-Cd^3f^2g^3-3\,Bd^2ef^2g^3-3\,A}{(3\,f^2g^3-5\,fg^4+2\,g^5)\sqrt{efg-dg^2}}$$
$$-\frac{2\,(Ad^3-Bd^3+Cd^3-3\,Ad^2e+3\,Bd^2e-3\,Cd^2e+3\,Ade^2-3\,Bde^2+3\,Cde^2-Ae^3+Be^3-Ce^3)\,\mathrm{arct}}{\sqrt{-d+e}(f-g)}$$
$$+\frac{2\left(45\sqrt{ex+d}Ce^2f^2g^2-15\,(ex+d)^{\frac{3}{2}}Cefg^3-90\sqrt{ex+d}Cdefg^3-45\sqrt{ex+d}Be^2fg^3+75\sqrt{ex+d}C\right.}{}$$

input

```
integrate((e*x+d)^(5/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output

```
2/81*sqrt(3)*(243*A*d^3 - 162*B*d^3 + 108*C*d^3 - 486*A*d^2*e + 324*B*d^2*
e - 216*C*d^2*e + 324*A*d*e^2 - 216*B*d*e^2 + 144*C*d*e^2 - 72*A*e^3 + 48*
B*e^3 - 32*C*e^3)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e))/(sqrt(-3*
d + 2*e)*(3*f - 2*g)) - 2*(C*e^3*f^5 - 3*C*d*e^2*f^4*g - B*e^3*f^4*g + 3*C
*d^2*e*f^3*g^2 + 3*B*d*e^2*f^3*g^2 + A*e^3*f^3*g^2 - C*d^3*f^2*g^3 - 3*B*d
^2*e*f^2*g^3 - 3*A*d*e^2*f^2*g^3 + B*d^3*f*g^4 + 3*A*d^2*e*f*g^4 - A*d^3*g
^5)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/((3*f^2*g^3 - 5*f*g^4 + 2*
g^5)*sqrt(e*f*g - d*g^2)) - 2*(A*d^3 - B*d^3 + C*d^3 - 3*A*d^2*e + 3*B*d^2
*e - 3*C*d^2*e + 3*A*d*e^2 - 3*B*d*e^2 + 3*C*d*e^2 - A*e^3 + B*e^3 - C*e^3
)*arctan(sqrt(e*x + d)/sqrt(-d + e))/(sqrt(-d + e)*(f - g)) + 2/135*(45*sq
rt(e*x + d)*C*e^2*f^2*g^2 - 15*(e*x + d)^(3/2)*C*e*f*g^3 - 90*sqrt(e*x + d
)*C*d*e*f*g^3 - 45*sqrt(e*x + d)*B*e^2*f*g^3 + 75*sqrt(e*x + d)*C*e^2*f*g^
3 + 9*(e*x + d)^(5/2)*C*g^4 + 15*(e*x + d)^(3/2)*C*d*g^4 + 45*sqrt(e*x + d
)*C*d^2*g^4 + 15*(e*x + d)^(3/2)*B*e*g^4 - 25*(e*x + d)^(3/2)*C*e*g^4 + 90
*sqrt(e*x + d)*B*d*e*g^4 - 150*sqrt(e*x + d)*C*d*e*g^4 + 45*sqrt(e*x + d)*
A*e^2*g^4 - 75*sqrt(e*x + d)*B*e^2*g^4 + 95*sqrt(e*x + d)*C*e^2*g^4)/g^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Hanged}$$

input
```
int(((d + e*x)^(5/2)*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 3422, normalized size of antiderivative = 5.71

$$\int \frac{(d+ex)^{5/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
( - 810*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( -
 d*g + e*f)))*a*d**2*g**4 + 1620*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d +
 e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d*e*f*g**3 - 810*sqrt(g)*sqrt( -
d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*e**2*f**
2*g**2 + 810*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sq
rt( - d*g + e*f)))*b*d**2*f*g**3 - 1620*sqrt(g)*sqrt( - d*g + e*f)*atan((s
qrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*d*e*f**2*g**2 + 810*sqrt(g
)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*
b*e**2*f**3*g - 810*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqr
t(g)*sqrt( - d*g + e*f)))*c*d**2*f**2*g**2 + 1620*sqrt(g)*sqrt( - d*g + e*
f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*d*e*f**3*g - 810
*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g +
e*f)))*c*e**2*f**4 + 405*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) +
sqrt(d + e*x)*sqrt(3))*a*d**2*f*g**4 - 405*sqrt(3*d - 2*e)*sqrt(3)*log( -
sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*d**2*g**5 - 540*sqrt(3*d - 2*e)
*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*d*e*f*g**4 + 54
0*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*
a*d*e*g**5 + 180*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d +
 e*x)*sqrt(3))*a*e**2*f*g**4 - 180*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d
 - 2*e) + sqrt(d + e*x)*sqrt(3))*a*e**2*g**5 - 270*sqrt(3*d - 2*e)*sqrt...
```

3.36 $\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 414

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2(9Bdf-15Cdf+9Aef-15Bef+19Cef-9Adg+6Cdg+6Be}{9(3f-2g)(f-g)}$$

$$-\frac{2(ef-dg)(Cf^2-Bfg+Ag^2)\sqrt{d+ex}}{g^2(3f^2-5fg+2g^2)}$$

$$+\frac{2(3Bf-5Cf-3Ag+2Cg)(d+ex)^{3/2}}{9(3f-2g)(f-g)}+\frac{2(Cf^2-Bfg+Ag^2)(d+ex)^{3/2}}{3(3f-2g)(f-g)g}$$

$$+\frac{2(ef-dg)^{3/2}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)g^{5/2}}$$

$$-\frac{2(9A-6B+4C)(3d-2e)^{3/2}\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{9\sqrt{3}(3f-2g)}$$

$$+\frac{2(A-B+C)(d-e)^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}$$

output
```
2/9*(-9*A*d*g+9*A*e*f+9*B*d*f-15*B*e*f+6*B*e*g-15*C*d*f+6*C*d*g+19*C*e*f-1
0*C*e*g)*(e*x+d)^(1/2)/(3*f-2*g)/(f-g)-2*(-d*g+e*f)*(A*g^2-B*f*g+C*f^2)*(e
*x+d)^(1/2)/g^2/(3*f^2-5*f*g+2*g^2)+2/9*(-3*A*g+3*B*f-5*C*f+2*C*g)*(e*x+d)
^(3/2)/(3*f-2*g)/(f-g)+2/3*(A*g^2-B*f*g+C*f^2)*(e*x+d)^(3/2)/(3*f-2*g)/(f-
g)/g+2*(-d*g+e*f)^(3/2)*(A*g^2-B*f*g+C*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(
-d*g+e*f)^(1/2))/(3*f-2*g)/(f-g)/g^(5/2)-2/27*(9*A-6*B+4*C)*(3*d-2*e)^(3/2
)*arctanh(3^(1/2)*(e*x+d)^(1/2)/(3*d-2*e)^(1/2))*3^(1/2)/(3*f-2*g)+2*(A-B+
C)*(d-e)^(3/2)*arctanh((e*x+d)^(1/2)/(d-e)^(1/2))/(f-g)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.57

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2(4Cdg+3Beg+Ce(-3f+g(-5+x)))\sqrt{d+ex}}{9g^2} - \frac{2(A-B+C)(-d+e)^{3/2}\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{f-g} - \frac{2(9A-6B+4C)(3d-2e)^2\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{9\sqrt{-9d+6e}(3f-2g)} + \frac{2(ef-dg)^{3/2}\left(Cf^2+g(-Bf+Ag)\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{5/2}\left(3f^2-5fg+2g^2\right)}$$

input
```
Integrate[((d + e*x)^(3/2)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)
),x]
```

output
```
(2*(4*C*d*g + 3*B*e*g + C*e*(-3*f + g*(-5 + x)))*Sqrt[d + e*x])/(9*g^2) -
(2*(A - B + C)*(-d + e)^(3/2)*ArcTan[Sqrt[d + e*x]/Sqrt[-d + e]])/(f - g)
- (2*(9*A - 6*B + 4*C)*(3*d - 2*e)^2*ArcTan[(Sqrt[-9*d + 6*e]*Sqrt[d + e*x
])/(3*d - 2*e)])/(9*Sqrt[-9*d + 6*e]*(3*f - 2*g)) + (2*(e*f - d*g)^(3/2)*(
C*f^2 + g*(-(B*f) + A*g))*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])
/(g^(5/2)*(3*f^2 - 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{\left(3x^2+5x+2\right)(f+gx)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{(d+ex)^{3/2}\left(Ag^2-Bfg+Cf^2\right)}{(3f-2g)(f-g)(f+gx)}+\frac{(-A+B-C)(d+ex)^{3/2}}{(x+1)(f-g)}+\frac{(9A-6B+4C)(d+ex)^{3/2}}{(3x+2)(3f-2g)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(ef-dg)^{3/2}\left(Ag^2-Bfg+Cf^2\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{5/2}(3f-2g)(f-g)}-\frac{2(3d-2e)^{3/2}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{9\sqrt{3}(3f-2g)}+\frac{2(d-e)^{3/2}(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}+\frac{2(d+ex)^{3/2}\left(Ag^2-Bfg+Cf^2\right)}{3g(3f-2g)(f-g)}-\frac{2\sqrt{d+ex}(ef-dg)\left(Ag^2-Bfg+Cf^2\right)}{g^2(3f-2g)(f-g)}+\frac{2(3d-2e)(9A-6B+4C)\sqrt{d+ex}}{9(3f-2g)}+\frac{2(9A-6B+4C)(d+ex)^{3/2}}{9(3f-2g)}-\frac{2(A-B+C)(d+ex)^{3/2}}{3(f-g)}-\frac{2(d-e)(A-B+C)\sqrt{d+ex}}{f-g}$$

input
```
Int[((d + e*x)^(3/2)*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C)*(3*d - 2*e)*Sqrt[d + e*x])/(9*(3*f - 2*g)) - (2*(A -
B + C)*(d - e)*Sqrt[d + e*x])/(f - g) - (2*(e*f - d*g)*(C*f^2 - B*f*g + A*
g^2)*Sqrt[d + e*x])/((3*f - 2*g)*(f - g)*g^2) + (2*(9*A - 6*B + 4*C)*(d +
e*x)^(3/2))/(9*(3*f - 2*g)) - (2*(A - B + C)*(d + e*x)^(3/2))/(3*(f - g))
+ (2*(C*f^2 - B*f*g + A*g^2)*(d + e*x)^(3/2))/(3*(3*f - 2*g)*(f - g)*g) +
(2*(e*f - d*g)^(3/2)*(C*f^2 - B*f*g + A*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x]
)/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*g^(5/2)) - (2*(9*A - 6*B + 4*C)*(
3*d - 2*e)^(3/2)*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d - 2*e]])/(9*Sqrt
[3]*(3*f - 2*g)) + (2*(A - B + C)*(d - e)^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d - e]])/(f - g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{-2(dg-ef)^2\sqrt{-9d+6e}\,(A g^2-Bfg+C f^2)\sqrt{-d+e}\,\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)+2\left(9\left(d-\frac{2e}{3}\right)^2(f-g)\left(A-\frac{2B}{3}+\frac{4C}{9}\right)\sqrt{-d+e} \cdots}{\sqrt{(d \cdots}}$
risch	$\frac{2(Cexg+3Beg+4Cdg-3Cef-5Ceg)\sqrt{ex+d}}{9g^2}+\frac{-\frac{18g^2\left(A d^2-2Ade+A e^2-B d^2+2Bde-B e^2+C d^2-2Cde+C e^2\right)\arctan\left(\cdots\right)}{(f-g)\sqrt{-d+e}} \cdots}{\cdots}$
derivativedivides	$\frac{\frac{2C(ex+d)^{\frac{3}{2}}g}{9}+\frac{2\sqrt{ex+d}\,Beg}{3}+\frac{2\sqrt{ex+d}\,Cdg}{3}-\frac{2\sqrt{ex+d}\,Cef}{3}-\frac{10\sqrt{ex+d}\,Ceg}{9}}{g^2}+\frac{2\left(-A d^2+2Ade-A e^2+B d^2-2Bde+B e^2 \cdots\right)}{(f-g)\sqrt{- \cdots}}$
default	$\frac{\frac{2C(ex+d)^{\frac{3}{2}}g}{9}+\frac{2\sqrt{ex+d}\,Beg}{3}+\frac{2\sqrt{ex+d}\,Cdg}{3}-\frac{2\sqrt{ex+d}\,Cef}{3}-\frac{10\sqrt{ex+d}\,Ceg}{9}}{g^2}+\frac{2\left(-A d^2+2Ade-A e^2+B d^2-2Bde+B e^2 \cdots\right)}{(f-g)\sqrt{- \cdots}}$

input
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
2/((d*g-e*f)*g)^(1/2)*(-(d*g-e*f)^2*(-9*d+6*e)^(1/2)*(A*g^2-B*f*g+C*f^2)*(
-d+e)^(1/2)*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+(9*(d-2/3*e)^2*(f
-g)*(A-2/3*B+4/9*C)*(-d+e)^(1/2)*g^2*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(1/
2))+(-3*g^2*(d-e)^2*(A-B+C)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))+(((1/3*(-5+
x)*C+B)*g-C*f)*e+4/3*C*d*g)*(e*x+d)^(1/2)*(f-g)*(-d+e)^(1/2))*(-9*d+6*e)^(
1/2)*(f-2/3*g))*((d*g-e*f)*g)^(1/2))/(-9*d+6*e)^(1/2)/(-d+e)^(1/2)/(3*f^2*
g^2-5*f*g^3+2*g^4)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\,(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 99.78 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx = \begin{cases} \frac{2\left(\frac{Ce(d+ex)^{\frac{3}{2}}}{9g} - \frac{e(d-e)^2(A-B+C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)} + \frac{e(3d-2e)^2\cdot(9A-6B+4C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+\frac{2e}{3}}}\right)}{27\sqrt{-d+\frac{2e}{3}}\cdot(3f-2g)}\right.}{e} \\ d^{\frac{3}{2}}\left(\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)} - \frac{(A-B+C)\log(x+1)}{f-g} + \frac{\left(Ag^2-Bfg+Cf^2\right)\left(\begin{cases}\frac{x}{f} \\ \frac{\log(f}{g}\end{cases}\right.}{(f-g)(3f}\right. \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(C*e*(d + e*x)**(3/2)/(9*g) - e*(d - e)**2*(A - B + C)*atan(s
qrt(d + e*x)/sqrt(-d + e))/(sqrt(-d + e)*(f - g)) + e*(3*d - 2*e)**2*(9*A
- 6*B + 4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2*e/3))/(27*sqrt(-d + 2*e/3)*(3*
f - 2*g)) + e*(d*g - e*f)**2*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e*x)/
sqrt(-(d*g - e*f)/g))/(g**3*sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)) + sq
rt(d + e*x)*(3*B*e**2*g + 3*C*d*e*g - 3*C*e**2*f - 5*C*e**2*g)/(9*g**2))/e
, Ne(e, 0)), (d**(3/2)*((9*A - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*g)) - (
A - B + C)*log(x + 1)/(f - g) + (A*g**2 - B*f*g + C*f**2)*Piecewise((x/f,
Eq(g, 0)), (log(f + g*x)/g, True))/((f - g)*(3*f - 2*g))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^{3/2}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2\sqrt{3}(81\,Ad^2-54\,Bd^2+36\,Cd^2-108\,Ade+72\,Bde-48\,Cde+36\,\ldots}{27\sqrt{-3\,d+2\,e}(3\,f-2\,g)}$$
$$+\frac{2\left(Ce^2f^4-2\,Cdef^3g-Be^2f^3g+Cd^2f^2g^2+2\,Bdef^2g^2+Ae^2f^2g^2-Bd^2fg^3-2\,Adefg^3+Ad^2g^4\right)\mathrm{arct}\ldots}{(3\,f^2g^2-5\,fg^3+2\,g^4)\sqrt{efg-dg^2}}$$
$$-\frac{2\left(Ad^2-Bd^2+Cd^2-2\,Ade+2\,Bde-2\,Cde+Ae^2-Be^2+Ce^2\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}$$
$$-\frac{2\left(3\sqrt{ex+d}Cefg-(ex+d)^{\frac{3}{2}}Cg^2-3\sqrt{ex+d}Cdg^2-3\sqrt{ex+d}Beg^2+5\sqrt{ex+d}Ceg^2\right)}{9\,g^3}$$

input
```
integrate((e*x+d)^(3/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
2/27*sqrt(3)*(81*A*d^2 - 54*B*d^2 + 36*C*d^2 - 108*A*d*e + 72*B*d*e - 48*C
*d*e + 36*A*e^2 - 24*B*e^2 + 16*C*e^2)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-
3*d + 2*e))/(sqrt(-3*d + 2*e)*(3*f - 2*g)) + 2*(C*e^2*f^4 - 2*C*d*e*f^3*g
- B*e^2*f^3*g + C*d^2*f^2*g^2 + 2*B*d*e*f^2*g^2 + A*e^2*f^2*g^2 - B*d^2*f*
g^3 - 2*A*d*e*f*g^3 + A*d^2*g^4)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2
))/((3*f^2*g^2 - 5*f*g^3 + 2*g^4)*sqrt(e*f*g - d*g^2)) - 2*(A*d^2 - B*d^2
+ C*d^2 - 2*A*d*e + 2*B*d*e - 2*C*d*e + A*e^2 - B*e^2 + C*e^2)*arctan(sqrt
(e*x + d)/sqrt(-d + e))/(sqrt(-d + e)*(f - g)) - 2/9*(3*sqrt(e*x + d)*C*e*
f*g - (e*x + d)^(3/2)*C*g^2 - 3*sqrt(e*x + d)*C*d*g^2 - 3*sqrt(e*x + d)*B*
e*g^2 + 5*sqrt(e*x + d)*C*e*g^2)/g^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Hanged}$$

input
```
int(((d + e*x)^(3/2)*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 2036, normalized size of antiderivative = 4.92

$$\int \frac{(d+ex)^{3/2}\left(A+Bx+Cx^2\right)}{(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
( - 54*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( -
d*g + e*f)))*a*d*g**3 + 54*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*
g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*e*f*g**2 + 54*sqrt(g)*sqrt( - d*g + e*f
)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*d*f*g**2 - 54*sqr
t(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)
))*b*e*f**2*g - 54*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt
(g)*sqrt( - d*g + e*f)))*c*d*f**2*g + 54*sqrt(g)*sqrt( - d*g + e*f)*atan((
sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*e*f**3 + 27*sqrt(3*d - 2*
e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*d*f*g**3 - 27
*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a
*d*g**4 - 18*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x
)*sqrt(3))*a*e*f*g**3 + 18*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e)
+ sqrt(d + e*x)*sqrt(3))*a*e*g**4 - 18*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt
(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*b*d*f*g**3 + 18*sqrt(3*d - 2*e)*sqrt(
3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*b*d*g**4 + 12*sqrt(3*d
- 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*b*e*f*g**3
- 12*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3
))*b*e*g**4 + 12*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d +
 e*x)*sqrt(3))*c*d*f*g**3 - 12*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2
*e) + sqrt(d + e*x)*sqrt(3))*c*d*g**4 - 8*sqrt(3*d - 2*e)*sqrt(3)*log( ...
```

3.37 $\int \frac{\sqrt{d+ex}\,(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 282

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx = \frac{2(3Bf-5Cf-3Ag+2Cg)\sqrt{d+ex}}{3(3f-2g)(f-g)} + \frac{2(Cf^2-Bfg+Ag^2)\,\sqrt{d+ex}}{(3f-2g)(f-g)g} - \frac{2\sqrt{ef-dg}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)g^{3/2}} - \frac{2(9A-6B+4C)\sqrt{3d-2e}\,\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{3\sqrt{3}(3f-2g)} + \frac{2(A-B+C)\sqrt{d-e}\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}$$

output
```
2/3*(-3*A*g+3*B*f-5*C*f+2*C*g)*(e*x+d)^(1/2)/(3*f-2*g)/(f-g)+2*(A*g^2-B*f*
g+C*f^2)*(e*x+d)^(1/2)/(3*f-2*g)/(f-g)/g-2*(-d*g+e*f)^(1/2)*(A*g^2-B*f*g+C
*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/(3*f-2*g)/(f-g)/g^(3/
2)-2/9*(9*A-6*B+4*C)*(3*d-2*e)^(1/2)*arctanh(3^(1/2)*(e*x+d)^(1/2)/(3*d-2*
e)^(1/2))*3^(1/2)/(3*f-2*g)+2*(A-B+C)*(d-e)^(1/2)*arctanh((e*x+d)^(1/2)/(d
-e)^(1/2))/(f-g)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \frac{2}{9}\left(\frac{3C\sqrt{d+ex}}{g}+\frac{9(A-B+C)\sqrt{-d+e}\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{f-g} - \frac{(9A-6B+4C)(3d-2e)\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{\sqrt{-d+\frac{2e}{3}}(3f-2g)} - \frac{9\sqrt{ef-dg}(Cf^2+g(-Bf+Ag))\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(3f^2-5fg+2g^2)}\right)$$

input
```
Integrate[(Sqrt[d + e*x]*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),
x]
```

output
```
(2*((3*C*Sqrt[d + e*x])/g + (9*(A - B + C)*Sqrt[-d + e]*ArcTan[Sqrt[d + e*
x]/Sqrt[-d + e]])/(f - g) - ((9*A - 6*B + 4*C)*(3*d - 2*e)*ArcTan[(Sqrt[-9
*d + 6*e]*Sqrt[d + e*x])/(3*d - 2*e)])/(Sqrt[-d + (2*e)/3]*(3*f - 2*g)) -
(9*Sqrt[e*f - d*g]*(C*f^2 + g*(-(B*f) + A*g))*ArcTan[(Sqrt[g]*Sqrt[d + e*x
])/Sqrt[e*f - d*g]])/(g^(3/2)*(3*f^2 - 5*f*g + 2*g^2))))/9
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(3x^2+5x+2)(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{\sqrt{d+ex}(Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)(f+gx)}+\frac{(-A+B-C)\sqrt{d+ex}}{(x+1)(f-g)}+\frac{(9A-6B+4C)\sqrt{d+ex}}{(3x+2)(3f-2g)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{ef-dg}(Ag^2-Bfg+Cf^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(3f-2g)(f-g)}+\frac{2\sqrt{d-e}(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{f-g}-\frac{2\sqrt{3d-2e}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{3\sqrt{3}(3f-2g)}+\frac{2\sqrt{d+ex}(Ag^2-Bfg+Cf^2)}{g(3f-2g)(f-g)}-\frac{2(A-B+C)\sqrt{d+ex}}{f-g}+\frac{2(9A-6B+4C)\sqrt{d+ex}}{3(3f-2g)}$$

input
```
Int[(Sqrt[d + e*x]*(A + B*x + C*x^2))/((f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C)*Sqrt[d + e*x])/(3*(3*f - 2*g)) - (2*(A - B + C)*Sqrt[
d + e*x])/(f - g) + (2*(C*f^2 - B*f*g + A*g^2)*Sqrt[d + e*x])/((3*f - 2*g)
*(f - g)*g) - (2*Sqrt[e*f - d*g]*(C*f^2 - B*f*g + A*g^2)*ArcTan[(Sqrt[g]*S
qrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*g^(3/2)) - (2*(9*A -
6*B + 4*C)*Sqrt[3*d - 2*e]*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d - 2*e]
])/(3*Sqrt[3]*(3*f - 2*g)) + (2*(A - B + C)*Sqrt[d - e]*ArcTanh[Sqrt[d + e
*x]/Sqrt[d - e]])/(f - g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2\sqrt{ex+d}\,C}{3g}-\frac{2\left(Ad\,g^3-Aef\,g^2-Bdf\,g^2+Be\,f^2g+Cd\,f^2g-Ce\,f^3\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{g(3f-2g)(f-g)\sqrt{(dg-ef)g}}+\frac{2(27Ad-18Ae-18Bd}{}$
default	$\frac{2\sqrt{ex+d}\,C}{3g}-\frac{2\left(Ad\,g^3-Aef\,g^2-Bdf\,g^2+Be\,f^2g+Cd\,f^2g-Ce\,f^3\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{g(3f-2g)(f-g)\sqrt{(dg-ef)g}}+\frac{2(27Ad-18Ae-18Bd}{}$
risch	$\frac{2\sqrt{ex+d}\,C}{3g}+\frac{-\frac{6g(Ad-Ae-Bd+Be+Cd-Ce)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)\sqrt{-d+e}}+\frac{2g(27Ad-18Ae-18Bd+12Be+12Cd-8Ce)\arctan\left(\frac{3\sqrt{ex+}}{\sqrt{-9d+}}\right.}{(3f-2g)\sqrt{-9d+6e}}}{3g}$
pseudoelliptic	$-\frac{\frac{2\sqrt{-9d+6e}\,(dg-ef)\sqrt{-d+e}\,\left(A\,g^2-Bfg+C\,f^2\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{3}+2\left(9\left(d-\frac{2e}{3}\right)(f-g)\left(A-\frac{2B}{3}+\frac{4C}{9}\right)\sqrt{-d+e}\,g\arctan\left(\frac{3}{\sqrt{-}}\right.\right.}{\sqrt{(dg-ef)g}\,\sqrt{-9d+6e}\,\sqrt{-d+e}\,(}$

input
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(e*x+d)^(1/2)*C/g-2/g*(A*d*g^3-A*e*f*g^2-B*d*f*g^2+B*e*f^2*g+C*d*f^2*g
-C*e*f^3)/(3*f-2*g)/(f-g)/((d*g-e*f)*g)^(1/2)*arctanh(g*(e*x+d)^(1/2)/((d*
g-e*f)*g)^(1/2))+2*(27*A*d-18*A*e-18*B*d+12*B*e+12*C*d-8*C*e)/(9*f-6*g)/(-
9*d+6*e)^(1/2)*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(1/2))+2*(-A*d+A*e+B*d-B*
e-C*d+C*e)/(f-g)/(-d+e)^(1/2)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx=\text{Timed out}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 88.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx = \begin{cases} \frac{2\left(\frac{Ce\sqrt{d+ex}}{3g}-\frac{e(d-e)(A-B+C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}+\frac{e(3d-2e)(9A-6B+4C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+\frac{2e}{3}}}\right)}{9\sqrt{-d+\frac{2e}{3}}\cdot(3f-2g)}+\frac{e(dg-ef)\left(Ag^2-Bfg+Cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g^2\sqrt{-\frac{dg-ef}{g}}(f-g)(3f-2g)}\right)}{e} & \text{f} \\ \sqrt{d}\left(\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)}-\frac{(A-B+C)\log(x+1)}{f-g}+\frac{(Ag^2-Bfg+Cf^2)\left(\begin{cases}\frac{x}{f} & \text{for } g=0\\ \frac{\log(f+gx)}{g} & \text{otherwise}\end{cases}\right)}{(f-g)(3f-2g)}\right) & \text{o} \end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(C*x**2+B*x+A)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(C*e*sqrt(d + e*x)/(3*g) - e*(d - e)*(A - B + C)*atan(sqrt(d
+ e*x)/sqrt(-d + e))/(sqrt(-d + e)*(f - g)) + e*(3*d - 2*e)*(9*A - 6*B + 4
*C)*atan(sqrt(d + e*x)/sqrt(-d + 2*e/3))/(9*sqrt(-d + 2*e/3)*(3*f - 2*g))
+ e*(d*g - e*f)*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e*x)/sqrt(-(d*g -
e*f)/g))/(g**2*sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)))/e, Ne(e, 0)), (s
qrt(d)*((9*A - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*g)) - (A - B + C)*log(x
 + 1)/(f - g) + (A*g**2 - B*f*g + C*f**2)*Piecewise((x/f, Eq(g, 0)), (log(
f + g*x)/g, True))/((f - g)*(3*f - 2*g))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)\,(2+5x+3x^2)}\,dx$$
$$= \frac{2\sqrt{3}(27\,Ad-18\,Bd+12\,Cd-18\,Ae+12\,Be-8\,Ce)\arctan\left(\frac{\sqrt{3}\sqrt{ex+d}}{\sqrt{-3\,d+2\,e}}\right)}{9\sqrt{-3\,d+2\,e}(3\,f-2\,g)}$$
$$+\frac{2\sqrt{ex+d}C}{3\,g}$$
$$-\frac{2\,(Cef^3-Cdf^2g-Bef^2g+Bdfg^2+Aefg^2-Adg^3)\arctan\left(\frac{\sqrt{ex+d}g}{\sqrt{efg-dg^2}}\right)}{\sqrt{efg-dg^2}(3\,f^2g-5\,fg^2+2\,g^3)}$$
$$-\frac{2\,(Ad-Bd+Cd-Ae+Be-Ce)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}$$

input
```
integrate((e*x+d)^(1/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
2/9*sqrt(3)*(27*A*d - 18*B*d + 12*C*d - 18*A*e + 12*B*e - 8*C*e)*arctan(sq
rt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e))/(sqrt(-3*d + 2*e)*(3*f - 2*g)) + 2/3
*sqrt(e*x + d)*C/g - 2*(C*e*f^3 - C*d*f^2*g - B*e*f^2*g + B*d*f*g^2 + A*e*
f*g^2 - A*d*g^3)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/(sqrt(e*f*g -
 d*g^2)*(3*f^2*g - 5*f*g^2 + 2*g^3)) - 2*(A*d - B*d + C*d - A*e + B*e - C*
e)*arctan(sqrt(e*x + d)/sqrt(-d + e))/(sqrt(-d + e)*(f - g))
```

Mupad [B] (verification not implemented)

Time = 34.11 (sec) , antiderivative size = 179516, normalized size of antiderivative = 636.58

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int(((d + e*x)^(1/2)*(A + B*x + C*x^2))/((f + g*x)*(5*x + 3*x^2 + 2)),x)
```

output
```
(2*C*(d + e*x)^(1/2))/(3*g) - (atan(((((32*(d + e*x)^(1/2)*(324*A^4*e^12*g
^6 + 144*B^4*e^12*g^6 + 793*C^4*e^12*f^6 + 64*C^4*e^12*g^6 + 1332*A^2*B^2*
e^12*g^6 + 1053*A^2*C^2*e^12*f^6 + 964*A^2*C^2*e^12*g^6 + 873*B^2*C^2*e^12
*f^6 + 592*B^2*C^2*e^12*g^6 + 4050*A^4*d^2*e^10*g^6 - 4860*A^4*d^3*e^9*g^6
 + 2187*A^4*d^4*e^8*g^6 + 1332*B^4*d^2*e^10*g^6 - 1080*B^4*d^3*e^9*g^6 + 3
24*B^4*d^4*e^8*g^6 + 873*C^4*d^2*e^10*f^6 + 1053*A^4*e^12*f^2*g^4 + 592*C^
4*d^2*e^10*g^6 - 480*C^4*d^3*e^9*g^6 + 144*C^4*d^4*e^8*g^6 + 873*B^4*e^12*
f^4*g^2 - 720*A*B^3*e^12*g^6 + 1746*A*C^3*e^12*f^6 - 1080*A^3*B*e^12*g^6 +
 416*A*C^3*e^12*g^6 - 1650*B*C^3*e^12*f^6 + 936*A^3*C*e^12*g^6 - 320*B*C^3
*e^12*g^6 - 480*B^3*C*e^12*g^6 - 1620*A^4*d*e^11*g^6 - 720*B^4*d*e^11*g^6
- 1650*C^4*d*e^11*f^6 - 320*C^4*d*e^11*g^6 - 1890*A*B*C^2*e^12*f^6 - 1520*
A*B*C^2*e^12*g^6 + 1824*A*B^2*C*e^12*g^6 - 2280*A^2*B*C*e^12*g^6 + 1458*A^
4*d^2*e^10*f^2*g^4 + 873*B^4*d^2*e^10*f^2*g^4 + 3780*B^4*d^2*e^10*f^3*g^3
+ 1053*B^4*d^2*e^10*f^4*g^2 - 1890*B^4*d^3*e^9*f^2*g^4 - 2106*B^4*d^3*e^9*
f^3*g^3 + 1053*B^4*d^4*e^8*f^2*g^4 + 793*C^4*d^2*e^10*f^4*g^2 - 1650*C^4*d
^3*e^9*f^4*g^2 + 873*C^4*d^4*e^8*f^4*g^2 + 3600*A*B^3*d*e^11*g^6 - 3780*A*
C^3*d*e^11*f^6 + 5400*A^3*B*d*e^11*g^6 - 2080*A*C^3*d*e^11*g^6 + 3492*B*C^
3*d*e^11*f^6 - 4680*A^3*C*d*e^11*g^6 + 1600*B*C^3*d*e^11*g^6 + 2400*B^3*C*
d*e^11*g^6 - 1586*B*C^3*e^12*f^5*g - 1746*B^3*C*e^12*f^5*g - 2106*A^4*d*e^
11*f*g^5 - 1586*C^4*d*e^11*f^5*g - 6660*A*B^3*d^2*e^10*g^6 + 5400*A*B^3...
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 951, normalized size of antiderivative = 3.37

$$\int \frac{\sqrt{d+ex}(A+Bx+Cx^2)}{(f+gx)(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)*(C*x^2+B*x+A)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
( - 18*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( -
d*g + e*f)))*a*g**2 + 18*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)
/(sqrt(g)*sqrt( - d*g + e*f)))*b*f*g - 18*sqrt(g)*sqrt( - d*g + e*f)*atan(
(sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*f**2 + 9*sqrt(3*d - 2*e)
*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*f*g**2 - 9*sqrt
(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*g**3
 - 6*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3
))*b*f*g**2 + 6*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d +
e*x)*sqrt(3))*b*g**3 + 4*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) +
sqrt(d + e*x)*sqrt(3))*c*f*g**2 - 4*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*
d - 2*e) + sqrt(d + e*x)*sqrt(3))*c*g**3 - 9*sqrt(3*d - 2*e)*sqrt(3)*log(s
qrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*f*g**2 + 9*sqrt(3*d - 2*e)*sqrt(
3)*log(sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*g**3 + 6*sqrt(3*d - 2*e)
*sqrt(3)*log(sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*b*f*g**2 - 6*sqrt(3*
d - 2*e)*sqrt(3)*log(sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*b*g**3 - 4*s
qrt(3*d - 2*e)*sqrt(3)*log(sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*c*f*g*
*2 + 4*sqrt(3*d - 2*e)*sqrt(3)*log(sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3)
)*c*g**3 - 27*sqrt(d - e)*log( - sqrt(d - e) + sqrt(d + e*x))*a*f*g**2 + 1
8*sqrt(d - e)*log( - sqrt(d - e) + sqrt(d + e*x))*a*g**3 + 27*sqrt(d - e)*
log( - sqrt(d - e) + sqrt(d + e*x))*b*f*g**2 - 18*sqrt(d - e)*log( - sq...
```

3.38 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)(2+5x+3x^2)} \, dx$

Optimal result

Integrand size = 39, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\left(2+5x+3x^2\right)} \, dx = \frac{2(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)\sqrt{g}\sqrt{ef-dg}} - \frac{2(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{\sqrt{3}\sqrt{3d-2e}(3f-2g)} + \frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{\sqrt{d-e}(f-g)}$$

output
```
2*(A*g^2-B*f*g+C*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/(3*f-
2*g)/(f-g)/g^(1/2)/(-d*g+e*f)^(1/2)-2/3*(9*A-6*B+4*C)*arctanh(3^(1/2)*(e*x
+d)^(1/2)/(3*d-2*e)^(1/2))*3^(1/2)/(3*d-2*e)^(1/2)/(3*f-2*g)+2*(A-B+C)*arc
tanh((e*x+d)^(1/2)/(d-e)^(1/2))/(d-e)^(1/2)/(f-g)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\,(2+5x+3x^2)}\,dx = -\frac{2(A-B+C)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)} - \frac{2(9A-6B+4C)\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{\sqrt{-9d+6e}(3f-2g)} + \frac{2(Cf^2+g(-Bf+Ag))\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{g}\sqrt{ef-dg}\,(3f^2-5fg+2g^2)}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(-2*(A - B + C)*ArcTan[Sqrt[d + e*x]/Sqrt[-d + e]])/(Sqrt[-d + e]*(f - g))
 - (2*(9*A - 6*B + 4*C)*ArcTan[(Sqrt[-9*d + 6*e]*Sqrt[d + e*x])/(3*d - 2*e
)])/(Sqrt[-9*d + 6*e]*(3*f - 2*g)) + (2*(C*f^2 + g*(-(B*f) + A*g))*ArcTan[
(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/(Sqrt[g]*Sqrt[e*f - d*g]*(3*f^2
- 5*f*g + 2*g^2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)\sqrt{d+ex}(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Ag^2-Bfg+Cf^2}{(3f-2g)(f-g)\sqrt{d+ex}(f+gx)} + \frac{-A+B-C}{(x+1)(f-g)\sqrt{d+ex}} + \frac{9A-6B+4C}{(3x+2)(3f-2g)\sqrt{d+ex}}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2(Ag^2-Bfg+Cf^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{\sqrt{g}(3f-2g)(f-g)\sqrt{ef-dg}}-\frac{2(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{\sqrt{3}\sqrt{3d-2e}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{\sqrt{d-e}(f-g)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(C*f^2 - B*f*g + A*g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]]
)/((3*f - 2*g)*(f - g)*Sqrt[g]*Sqrt[e*f - d*g]) - (2*(9*A - 6*B + 4*C)*Arc
Tanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d - 2*e]])/(Sqrt[3]*Sqrt[3*d - 2*e]*(3
*f - 2*g)) + (2*(A - B + C)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e]])/(Sqrt[d -
e]*(f - g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{2\left(A\,g^{2}-B f g+C\,f^{2}\right) \operatorname{arctanh}\left(\frac{g\sqrt{e x+d}}{\sqrt{(d g-e f) g}}\right)}{(3 f-2 g)(f-g)\sqrt{(d g-e f) g}}+\frac{(18 A-12 B+8 C) \arctan\left(\frac{3\sqrt{e x+d}}{\sqrt{-9 d+6 e}}\right)}{\sqrt{-9 d+6 e}\,(3 f-2 g)}-\frac{2(A-B+C) \arctan\left(\frac{\sqrt{ex\cdots}}{\sqrt{-d\cdots}}\right)}{(f-g)\sqrt{-d+e}}$
derivativedivides	$-\frac{2\left(A\,g^{2}-B f g+C\,f^{2}\right) \operatorname{arctanh}\left(\frac{g\sqrt{e x+d}}{\sqrt{(d g-e f) g}}\right)}{(3 f-2 g)(f-g)\sqrt{(d g-e f) g}}+\frac{2(9 A-6 B+4 C) \arctan\left(\frac{3\sqrt{e x+d}}{\sqrt{-9 d+6 e}}\right)}{(3 f-2 g)\sqrt{-9 d+6 e}}+\frac{2(-A+B-C) \arctan\left(\frac{\sqrt{ex\cdots}}{\sqrt{-\cdots}}\right)}{(f-g)\sqrt{-d+e}}$
default	$-\frac{2\left(A\,g^{2}-B f g+C\,f^{2}\right) \operatorname{arctanh}\left(\frac{g\sqrt{e x+d}}{\sqrt{(d g-e f) g}}\right)}{(3 f-2 g)(f-g)\sqrt{(d g-e f) g}}+\frac{2(9 A-6 B+4 C) \arctan\left(\frac{3\sqrt{e x+d}}{\sqrt{-9 d+6 e}}\right)}{(3 f-2 g)\sqrt{-9 d+6 e}}+\frac{2(-A+B-C) \arctan\left(\frac{\sqrt{ex\cdots}}{\sqrt{-\cdots}}\right)}{(f-g)\sqrt{-d+e}}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
-2*(A*g^2-B*f*g+C*f^2)/(3*f-2*g)/(f-g)/((d*g-e*f)*g)^(1/2)*arctanh(g*(e*x+
d)^(1/2)/((d*g-e*f)*g)^(1/2))+(18*A-12*B+8*C)/(-9*d+6*e)^(1/2)*arctan(3*(e
*x+d)^(1/2)/(-9*d+6*e)^(1/2))/(3*f-2*g)-2*(A-B+C)*arctan((e*x+d)^(1/2)/(-d
+e)^(1/2))/(f-g)/(-d+e)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 114.72 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\left(2+5x+3x^2\right)}\,dx = \begin{cases} \frac{2\left(-\frac{e(A-B+C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}+\frac{e(9A-6B+4C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+\frac{2e}{3}}}\right)}{3\sqrt{-d+\frac{2e}{3}}\cdot(3f-2g)}+\frac{e\left(Ag^2-Bfg+Cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{g\sqrt{-\frac{dg-ef}{g}}(f-g)(3f-2g)}\right)}{e} & \text{for } e\neq 0 \\ \frac{\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)}-\frac{(A-B+C)\log(x+1)}{f-g}+\frac{\left(Ag^2-Bfg+Cf^2\right)\left(\begin{cases}\frac{x}{f} & \text{for } g=0\\ \frac{\log(f+gx)}{g} & \text{otherwise}\end{cases}\right)}{(f-g)(3f-2g)}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(-e*(A - B + C)*atan(sqrt(d + e*x)/sqrt(-d + e))/(sqrt(-d + e
)*(f - g)) + e*(9*A - 6*B + 4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2*e/3))/(3*s
qrt(-d + 2*e/3)*(3*f - 2*g)) + e*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e
*x)/sqrt(-(d*g - e*f)/g))/(g*sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)))/e,
 Ne(e, 0)), (((9*A - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*g)) - (A - B + C)
*log(x + 1)/(f - g) + (A*g**2 - B*f*g + C*f**2)*Piecewise((x/f, Eq(g, 0)),
 (log(f + g*x)/g, True))/((f - g)*(3*f - 2*g)))/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87

$$\int \frac{A + Bx + Cx^2}{\sqrt{d+ex}(f+gx)\,(2+5x+3x^2)}\,dx = \frac{2\sqrt{3}(9\,A - 6\,B + 4\,C)\arctan\left(\frac{\sqrt{3}\sqrt{ex+d}}{\sqrt{-3\,d+2\,e}}\right)}{3\sqrt{-3\,d+2\,e}(3\,f - 2\,g)} + \frac{2\,(Cf^2 - Bfg + Ag^2)\arctan\left(\frac{\sqrt{ex+d}g}{\sqrt{efg-dg^2}}\right)}{\sqrt{efg-dg^2}(3\,f^2 - 5\,fg + 2\,g^2)} - \frac{2\,(A - B + C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(f-g)}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
2/3*sqrt(3)*(9*A - 6*B + 4*C)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e
))/(sqrt(-3*d + 2*e)*(3*f - 2*g)) + 2*(C*f^2 - B*f*g + A*g^2)*arctan(sqrt(
e*x + d)*g/sqrt(e*f*g - d*g^2))/(sqrt(e*f*g - d*g^2)*(3*f^2 - 5*f*g + 2*g^
2)) - 2*(A - B + C)*arctan(sqrt(e*x + d)/sqrt(-d + e))/(sqrt(-d + e)*(f -
g))
```

Mupad [B] (verification not implemented)

Time = 70.66 (sec) , antiderivative size = 34205, normalized size of antiderivative = 181.94

$$\int \frac{A + Bx + Cx^2}{\sqrt{d+ex}(f+gx)\,(2+5x+3x^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(1/2)*(5*x + 3*x^2 + 2)),x)
```

output
```
(atan((((g*(d*g - e*f))^(1/2)*((d + e*x)^(1/2)*(23328*A^4*e^8*g^5 + 3456*B
^4*e^8*g^5 + 1536*C^4*e^8*g^5 + 43200*A^2*B^2*e^8*g^5 + 32448*A^2*C^2*e^8*
g^5 + 14208*B^2*C^2*e^8*g^5 + 11232*B^4*e^8*f^2*g^3 - 17280*A*B^3*e^8*g^5
- 51840*A^3*B*e^8*g^5 + 9984*A*C^3*e^8*g^5 + 44928*A^3*C*e^8*g^5 - 7680*B*
C^3*e^8*g^5 - 11520*B^3*C*e^8*g^5 + 9312*C^4*e^8*f^4*g - 36480*A*B*C^2*e^8
*g^5 + 43776*A*B^2*C*e^8*g^5 - 74880*A^2*B*C*e^8*g^5 - 22464*A*B^3*e^8*f*g
^4 - 31104*A^3*B*e^8*f*g^4 + 22464*A*C^3*e^8*f^4*g - 20160*B*C^3*e^8*f^4*g
 - 25920*A*B^3*e^8*f^2*g^3 + 51840*A^2*B^2*e^8*f*g^4 + 18624*A*C^3*e^8*f^2
*g^3 + 15552*A^2*C^2*e^8*f^4*g + 31104*A^3*C*e^8*f^2*g^3 - 18624*B*C^3*e^8
*f^3*g^2 + 11232*B^2*C^2*e^8*f^4*g - 20160*B^3*C*e^8*f^2*g^3 - 22464*B^3*C
*e^8*f^3*g^2 + 15552*A^2*B^2*e^8*f^2*g^3 + 44928*A^2*C^2*e^8*f^2*g^3 + 931
2*B^2*C^2*e^8*f^2*g^3 + 40320*B^2*C^2*e^8*f^3*g^2 - 18624*A*B*C^2*e^8*f*g^
4 - 25920*A*B*C^2*e^8*f^4*g + 40320*A*B^2*C*e^8*f*g^4 - 44928*A^2*B*C*e^8*
f*g^4 - 40320*A*B*C^2*e^8*f^2*g^3 - 44928*A*B*C^2*e^8*f^3*g^2 + 44928*A*B^
2*C*e^8*f^2*g^3 + 51840*A*B^2*C*e^8*f^3*g^2 - 51840*A^2*B*C*e^8*f^2*g^3 -
31104*A^2*B*C*e^8*f^3*g^2) + ((g*(d*g - e*f))^(1/2)*(A*g^2 + C*f^2 - B*f*g
)*(38880*A^3*e^9*f^2*g^4 - 4320*A^3*e^9*g^6 - 23328*A^3*e^9*f^3*g^3 - 4838
4*B^3*e^9*f^2*g^4 + 51840*B^3*e^9*f^3*g^3 - 15552*B^3*e^9*f^4*g^2 + 49920*
C^3*e^9*f^2*g^4 - 50880*C^3*e^9*f^3*g^3 + 14400*C^3*e^9*f^4*g^2 + 2880*A*B
^2*e^9*g^6 - 2016*A^2*B*e^9*g^6 + 1920*A*C^2*e^9*g^6 + 2880*A^2*C*e^9*g...
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 3015, normalized size of antiderivative = 16.04

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
( - 18*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( -
d*g + e*f)))*a*d**2*g**2 + 30*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*
x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d*e*g**2 - 12*sqrt(g)*sqrt( - d*g +
e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*e**2*g**2 + 18
*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g +
e*f)))*b*d**2*f*g - 30*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(
sqrt(g)*sqrt( - d*g + e*f)))*b*d*e*f*g + 12*sqrt(g)*sqrt( - d*g + e*f)*ata
n((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*e**2*f*g - 18*sqrt(g)*
sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*
d**2*f**2 + 30*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*
sqrt( - d*g + e*f)))*c*d*e*f**2 - 12*sqrt(g)*sqrt( - d*g + e*f)*atan((sqrt
(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*e**2*f**2 + 9*sqrt(3*d - 2*e)
*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*d**2*f*g**2 - 9
*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a
*d**2*g**3 - 9*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e
*x)*sqrt(3))*a*d*e*f**2*g + 9*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*
e) + sqrt(d + e*x)*sqrt(3))*a*d*e*g**3 + 9*sqrt(3*d - 2*e)*sqrt(3)*log( -
sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*e**2*f**2*g - 9*sqrt(3*d - 2*e)
*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3))*a*e**2*f*g**2 - 6
*sqrt(3*d - 2*e)*sqrt(3)*log( - sqrt(3*d - 2*e) + sqrt(d + e*x)*sqrt(3)...
```

3.39 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 242

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx =&-\frac{2(Cd^2-Bde+Ae^2)}{(3d^2-5de+2e^2)\,(ef-dg)\sqrt{d+ex}}\\&-\frac{2\sqrt{g}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{3/2}}\\&-\frac{2\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{3/2}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{3/2}(f-g)}\end{aligned}$$

output
```
(-2*A*e^2+2*B*d*e-2*C*d^2)/(3*d^2-5*d*e+2*e^2)/(-d*g+e*f)/(e*x+d)^(1/2)-2*
g^(1/2)*(A*g^2-B*f*g+C*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))
/(3*f-2*g)/(f-g)/(-d*g+e*f)^(3/2)-2*3^(1/2)*(9*A-6*B+4*C)*arctanh(3^(1/2)*
(e*x+d)^(1/2)/(3*d-2*e)^(1/2))/(3*d-2*e)^(3/2)/(3*f-2*g)+2*(A-B+C)*arctanh
((e*x+d)^(1/2)/(d-e)^(1/2))/(d-e)^(3/2)/(f-g)
```

Mathematica [A] (verified)

Time = 2.61 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = 2\left(\frac{Cd^2+e(-Bd+Ae)}{\left(3d^2-5de+2e^2\right)(-ef+dg)\sqrt{d+ex}} + \frac{(A-B+C)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{(-d+e)^{3/2}(f-g)} + \frac{\sqrt{3}(9A-6B+4C)\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{(-3d+2e)^{3/2}(3f-2g)} - \frac{\sqrt{g}\left(Cf^2+g(-Bf+Ag)\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{3/2}}\right)$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)*(2 + 5*x + 3*x^2)),
x]
```

output
```
2*((C*d^2 + e*(-(B*d) + A*e))/((3*d^2 - 5*d*e + 2*e^2)*(-(e*f) + d*g)*Sqrt
[d + e*x]) + ((A - B + C)*ArcTan[Sqrt[d + e*x]/Sqrt[-d + e]])/((-d + e)^(3
/2)*(f - g)) + (Sqrt[3]*(9*A - 6*B + 4*C)*ArcTan[(Sqrt[-9*d + 6*e]*Sqrt[d
+ e*x])/(3*d - 2*e)])/((-3*d + 2*e)^(3/2)*(3*f - 2*g)) - (Sqrt[g]*(C*f^2 +
 g*(-(B*f) + A*g))*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f
- 2*g)*(f - g)*(e*f - d*g)^(3/2)))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.29, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\left(3x^2+5x+2\right)(d+ex)^{3/2}(f+gx)}dx$$

$$\Big\downarrow \text{ 2153}$$

$$\int \left(\frac{Ag^2-Bfg+Cf^2}{(3f-2g)(f-g)(d+ex)^{3/2}(f+gx)}+\frac{-A+B-C}{(x+1)(f-g)(d+ex)^{3/2}}+\frac{9A-6B+4C}{(3x+2)(3f-2g)(d+ex)^{3/2}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\sqrt{g}(Ag^2-Bfg+Cf^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{3/2}}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{3/2}(f-g)}-\frac{2\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{3/2}(3f-2g)}-\frac{2(Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)\sqrt{d+ex}(ef-dg)}-\frac{2(A-B+C)}{(d-e)(f-g)\sqrt{d+ex}}+\frac{2(9A-6B+4C)}{(3d-2e)(3f-2g)\sqrt{d+ex}}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C))/((3*d - 2*e)*(3*f - 2*g)*Sqrt[d + e*x]) - (2*(A - B
+ C))/((d - e)*(f - g)*Sqrt[d + e*x]) - (2*(C*f^2 - B*f*g + A*g^2))/((3*f
- 2*g)*(f - g)*(e*f - d*g)*Sqrt[d + e*x]) - (2*Sqrt[g]*(C*f^2 - B*f*g + A*
g^2)*ArcTan[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)
*(e*f - d*g)^(3/2)) - (2*Sqrt[3]*(9*A - 6*B + 4*C)*ArcTanh[(Sqrt[3]*Sqrt[d
 + e*x])/Sqrt[3*d - 2*e]])/((3*d - 2*e)^(3/2)*(3*f - 2*g)) + (2*(A - B + C
)*ArcTanh[Sqrt[d + e*x]/Sqrt[d - e]])/((d - e)^(3/2)*(f - g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{2(A-B+C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(-d+e)^{\frac{3}{2}}}+\frac{(54A-36B+24C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{\sqrt{-9d+6e}\,(3f-2g)(3d-2e)}-\frac{2(A\,g^{2}-Bfg+C\,f^{2})g\ \operatorname{arctanh}\left(\frac{g\sqrt{ex+}}{\sqrt{(dg-ef}}\right.}{(f-g)(3f-2g)(dg-ef)\sqrt{(dg-ef)g}}$
derivativedivides	$\frac{2(-A+B-C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(d-e)\sqrt{-d+e}}+\frac{2(27A-18B+12C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{(3f-2g)(3d-2e)\sqrt{-9d+6e}}-\frac{2(A\,g^{2}-Bfg+C\,f^{2})g\ \operatorname{arctanh}\left(\frac{g\sqrt{e}}{\sqrt{(dg}}\right.}{(f-g)(3f-2g)(dg-ef)\sqrt{(dg-ef}}$
default	$\frac{2(-A+B-C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(d-e)\sqrt{-d+e}}+\frac{2(27A-18B+12C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{(3f-2g)(3d-2e)\sqrt{-9d+6e}}-\frac{2(A\,g^{2}-Bfg+C\,f^{2})g\ \operatorname{arctanh}\left(\frac{g\sqrt{e}}{\sqrt{(dg}}\right.}{(f-g)(3f-2g)(dg-ef)\sqrt{(dg-ef}}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
2*(A-B+C)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))/(f-g)/(-d+e)^(3/2)+(54*A-36*B
+24*C)/(-9*d+6*e)^(1/2)*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(1/2))/(3*f-2*g)
/(3*d-2*e)-2*(A*g^2-B*f*g+C*f^2)*g/(f-g)/(3*f-2*g)/(d*g-e*f)/((d*g-e*f)*g)
^(1/2)*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+2*(A*e^2-B*d*e+C*d^2)/
(d*g-e*f)/(3*d-2*e)/(d-e)/(e*x+d)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 80.93 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \begin{cases} \frac{2\left(\frac{e\left(Ae^2-Bde+Cd^2\right)}{(d-e)\sqrt{d+ex}(3d-2e)(dg-ef)}-\frac{e(A-B+C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{\sqrt{-d+e}(d-e)(f-g)}+\frac{e(9A-6B+4C)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d+\frac{2e}{3}}}\right)}{\sqrt{-d+\frac{2e}{3}}\cdot(3d-2e)(3f-2g)}+\dots\right)}{e} & \\ \frac{\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)}-\frac{(A-B+C)\log(x+1)}{f-g}+\frac{\left(Ag^2-Bfg+Cf^2\right)\left(\begin{cases}\frac{x}{f} & \text{for } g=0\\ \frac{\log(f+gx)}{g} & \text{otherwise}\end{cases}\right)}{(f-g)(3f-2g)}}{d^{\frac{3}{2}}} & \end{cases}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(e*(A*e**2 - B*d*e + C*d**2)/((d - e)*sqrt(d + e*x)*(3*d - 2*
e)*(d*g - e*f)) - e*(A - B + C)*atan(sqrt(d + e*x)/sqrt(-d + e))/(sqrt(-d
+ e)*(d - e)*(f - g)) + e*(9*A - 6*B + 4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2
*e/3))/(sqrt(-d + 2*e/3)*(3*d - 2*e)*(3*f - 2*g)) + e*(A*g**2 - B*f*g + C*
f**2)*atan(sqrt(d + e*x)/sqrt(-(d*g - e*f)/g))/(sqrt(-(d*g - e*f)/g)*(f -
g)*(3*f - 2*g)*(d*g - e*f)))/e, Ne(e, 0)), (((9*A - 6*B + 4*C)*log(3*x + 2
)/(3*(3*f - 2*g)) - (A - B + C)*log(x + 1)/(f - g) + (A*g**2 - B*f*g + C*f
**2)*Piecewise((x/f, Eq(g, 0)), (log(f + g*x)/g, True))/((f - g)*(3*f - 2*
g)))/d**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{2\sqrt{3}(9A-6B+4C)\arctan\left(\frac{\sqrt{3}\sqrt{ex+d}}{\sqrt{-3d+2e}}\right)}{(9df-6ef-6dg+4eg)\sqrt{-3d+2e}} - \frac{2\left(Cf^2g-Bfg^2+Ag^3\right)\arctan\left(\frac{\sqrt{ex+d}g}{\sqrt{efg-dg^2}}\right)}{(3ef^3-3df^2g-5ef^2g+5dfg^2+2efg^2-2dg^3)\sqrt{efg-dg^2}} - \frac{2(A-B+C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(df-ef-dg+eg)\sqrt{-d+e}} - \frac{2\left(Cd^2-Bde+Ae^2\right)}{(3d^2ef-5de^2f+2e^3f-3d^3g+5d^2eg-2de^2g)\sqrt{ex+d}}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
2*sqrt(3)*(9*A - 6*B + 4*C)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e))
/((9*d*f - 6*e*f - 6*d*g + 4*e*g)*sqrt(-3*d + 2*e)) - 2*(C*f^2*g - B*f*g^2
 + A*g^3)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/((3*e*f^3 - 3*d*f^2*
g - 5*e*f^2*g + 5*d*f*g^2 + 2*e*f*g^2 - 2*d*g^3)*sqrt(e*f*g - d*g^2)) - 2*
(A - B + C)*arctan(sqrt(e*x + d)/sqrt(-d + e))/((d*f - e*f - d*g + e*g)*sq
rt(-d + e)) - 2*(C*d^2 - B*d*e + A*e^2)/((3*d^2*e*f - 5*d*e^2*f + 2*e^3*f
- 3*d^3*g + 5*d^2*e*g - 2*d*e^2*g)*sqrt(e*x + d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(3/2)*(5*x + 3*x^2 + 2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7571, normalized size of antiderivative = 31.29

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)/(3*x^2+5*x+2),x)
```

output
```
( - 18*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sq
rt(g)*sqrt( - d*g + e*f)))*a*d**4*g**2 + 60*sqrt(g)*sqrt(d + e*x)*sqrt( -
d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**3*e*g
**2 - 74*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(
sqrt(g)*sqrt( - d*g + e*f)))*a*d**2*e**2*g**2 + 40*sqrt(g)*sqrt(d + e*x)*s
qrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d
*e**3*g**2 - 8*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x
)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*e**4*g**2 + 18*sqrt(g)*sqrt(d + e*x)*
sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*
d**4*f*g - 60*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)
*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*d**3*e*f*g + 74*sqrt(g)*sqrt(d + e*x)*
sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*
d**2*e**2*f*g - 40*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d +
 e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*b*d*e**3*f*g + 8*sqrt(g)*sqrt(d + e
*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f))
)*b*e**4*f*g - 18*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d +
e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*d**4*f**2 + 60*sqrt(g)*sqrt(d + e*
x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))
*c*d**3*e*f**2 - 74*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d
+ e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*c*d**2*e**2*f**2 + 40*sqrt(g)*s...
```

3.40 $\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 376

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = -\frac{2(Cd^2-Bde+Ae^2)}{3\left(3d^2-5de+2e^2\right)(ef-dg)(d+ex)^{3/2}}$$
$$+\frac{2(Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))-Be(2e^3f+6d^3g-d^2e}{\left(3d^2-5de+2e^2\right)^2(ef-dg)^2\sqrt{d+ex}}$$
$$+\frac{2g^{3/2}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{5/2}}$$
$$-\frac{6\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{5/2}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{5/2}(f-g)}$$

output

```
1/3*(-2*A*e^2+2*B*d*e-2*C*d^2)/(3*d^2-5*d*e+2*e^2)/(-d*g+e*f)/(e*x+d)^(3/2
)+2*(C*d*(4*e^3*f+3*d^3*g-d*e^2*(5*f+2*g))+A*e^2*(9*d^2*g+e^2*(5*f+2*g)-2*
d*e*(3*f+5*g))-B*e*(2*e^3*f+6*d^3*g-d^2*e*(3*f+5*g)))/(3*d^2-5*d*e+2*e^2)^
2/(-d*g+e*f)^2/(e*x+d)^(1/2)+2*g^(3/2)*(A*g^2-B*f*g+C*f^2)*arctan(g^(1/2)*
(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/(3*f-2*g)/(f-g)/(-d*g+e*f)^(5/2)-6*3^(1/2)
*(9*A-6*B+4*C)*arctanh(3^(1/2)*(e*x+d)^(1/2)/(3*d-2*e)^(1/2))/(3*d-2*e)^(5
/2)/(3*f-2*g)+2*(A-B+C)*arctanh((e*x+d)^(1/2)/(d-e)^(1/2))/(d-e)^(5/2)/(f-
g)
```

Mathematica [A] (verified)

Time = 7.93 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \frac{2(Cd(12d^4g-2d^2e^2(5f+2g)+12e^4fx+de^3(10f-15fx-}{}$$
$$-\frac{2(A-B+C)\arctan\left(\frac{\sqrt{d+ex}}{\sqrt{-d+e}}\right)}{(-d+e)^{5/2}(f-g)}-\frac{6\sqrt{3}(9A-6B+4C)\arctan\left(\frac{\sqrt{-9d+6e}\sqrt{d+ex}}{3d-2e}\right)}{(-3d+2e)^{5/2}(3f-2g)}$$
$$+\frac{2g^{3/2}(Cf^2+g(-Bf+Ag))\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{5/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(5/2)*(f + g*x)*(2 + 5*x + 3*x^2)),
x]
```

output
```
(2*(C*d*(12*d^4*g - 2*d^2*e^2*(5*f + 2*g) + 12*e^4*f*x + d*e^3*(10*f - 15*
f*x - 6*g*x) + d^3*e*(-3*f - 5*g + 9*g*x)) + e*(-(B*(4*d*e^3*f + 21*d^4*g
+ 6*e^4*f*x + d^2*e^2*(5*f + 2*g - 9*f*x - 15*g*x) - 2*d^3*e*(6*f + 10*g -
 9*g*x))) + A*e*(30*d^3*g + 2*d*e^2*(10*f + 4*g - 9*f*x - 15*g*x) + e^3*(-
2*f + 15*f*x + 6*g*x) + d^2*e*(-21*f - 35*g + 27*g*x)))))/(3*(3*d - 2*e)^2
*(d - e)^2*(e*f - d*g)^2*(d + e*x)^(3/2)) - (2*(A - B + C)*ArcTan[Sqrt[d +
 e*x]/Sqrt[-d + e]])/((-d + e)^(5/2)*(f - g)) - (6*Sqrt[3]*(9*A - 6*B + 4*
C)*ArcTan[(Sqrt[-9*d + 6*e]*Sqrt[d + e*x])/(3*d - 2*e)])/((-3*d + 2*e)^(5/
2)*(3*f - 2*g)) + (2*g^(3/2)*(C*f^2 + g*(-(B*f) + A*g))*ArcTan[(Sqrt[g]*Sq
rt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*(e*f - d*g)^(5/2))
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)\,(d+ex)^{5/2}(f+gx)}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Ag^2-Bfg+Cf^2}{(3f-2g)(f-g)(d+ex)^{5/2}(f+gx)}+\frac{-A+B-C}{(x+1)(f-g)(d+ex)^{5/2}}+\frac{9A-6B+4C}{(3x+2)(3f-2g)(d+ex)^{5/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2g^{3/2}\left(Ag^2-Bfg+Cf^2\right)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{5/2}}-\frac{6\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{5/2}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{5/2}(f-g)}+\frac{2g\left(Ag^2-Bfg+Cf^2\right)}{(3f-2g)(f-g)\sqrt{d+ex}(ef-dg)^2}-\frac{2\left(Ag^2-Bfg+Cf^2\right)}{3(3f-2g)(f-g)(d+ex)^{3/2}(ef-dg)}+\frac{6(9A-6B+4C)}{(3d-2e)^2(3f-2g)\sqrt{d+ex}}-\frac{2(A-B+C)}{(d-e)^2(f-g)\sqrt{d+ex}}+\frac{2(9A-6B+4C)}{3(3d-2e)(3f-2g)(d+ex)^{3/2}}-\frac{2(A-B+C)}{3(d-e)(f-g)(d+ex)^{3/2}}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(5/2)*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C))/(3*(3*d - 2*e)*(3*f - 2*g)*(d + e*x)^(3/2)) - (2*(A
- B + C))/(3*(d - e)*(f - g)*(d + e*x)^(3/2)) - (2*(C*f^2 - B*f*g + A*g^2)
)/(3*(3*f - 2*g)*(f - g)*(e*f - d*g)*(d + e*x)^(3/2)) + (6*(9*A - 6*B + 4*
C))/((3*d - 2*e)^2*(3*f - 2*g)*Sqrt[d + e*x]) - (2*(A - B + C))/((d - e)^2
*(f - g)*Sqrt[d + e*x]) + (2*g*(C*f^2 - B*f*g + A*g^2))/((3*f - 2*g)*(f -
g)*(e*f - d*g)^2*Sqrt[d + e*x]) + (2*g^(3/2)*(C*f^2 - B*f*g + A*g^2)*ArcTa
n[(Sqrt[g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*(e*f - d*
g)^(5/2)) - (6*Sqrt[3]*(9*A - 6*B + 4*C)*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/S
qrt[3*d - 2*e]])/((3*d - 2*e)^(5/2)*(3*f - 2*g)) + (2*(A - B + C)*ArcTanh[
Sqrt[d + e*x]/Sqrt[d - e]])/((d - e)^(5/2)*(f - g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{2(A-B+C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(-d+e)^{\frac{5}{2}}}-\frac{2g^{2}\left(A\,g^{2}-Bfg+C\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^{2}\sqrt{(dg-ef)g}}+\frac{\frac{2}{9}A\,e^{2}-\frac{2}{9}Bde+\frac{2}{9}C\,d^{2}}{(ex+d)^{\frac{3}{2}}(d-e)(dg-ef)(d-\frac{2e}{3}}$
derivativedivides	$\frac{2(-A+B-C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(d-e)^{2}\sqrt{-d+e}}-\frac{2g^{2}\left(A\,g^{2}-Bfg+C\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^{2}\sqrt{(dg-ef)g}}-\frac{2\left(-A\,e^{2}+Bde-C\,d^{2}\right)}{3(dg-ef)(3d-2e)(d-e)(ex+d}$
default	$\frac{2(-A+B-C)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d+e}}\right)}{(f-g)(d-e)^{2}\sqrt{-d+e}}-\frac{2g^{2}\left(A\,g^{2}-Bfg+C\,f^{2}\right)\operatorname{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^{2}\sqrt{(dg-ef)g}}-\frac{2\left(-A\,e^{2}+Bde-C\,d^{2}\right)}{3(dg-ef)(3d-2e)(d-e)(ex+d}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
-2*(A-B+C)*arctan((e*x+d)^(1/2)/(-d+e)^(1/2))/(f-g)/(-d+e)^(5/2)-2*g^2*(A*
g^2-B*f*g+C*f^2)/(f-g)/(3*f-2*g)/(d*g-e*f)^2/((d*g-e*f)*g)^(1/2)*arctanh(g
*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+2/9*(A*e^2-B*d*e+C*d^2)/(e*x+d)^(3/2)/
(d-e)/(d*g-e*f)/(d-2/3*e)+2/9*(3*C*d^4*g-6*B*d^3*e*g+9*d^2*((A+5/9*B-2/9*C
)*g+1/3*(B-5/3*C)*f)*e^2-6*d*(5/3*A*g+f*(A-2/3*C))*e^3+5*e^4*(2/5*A*g+f*(A
-2/5*B)))/(e*x+d)^(1/2)/(d-e)^2/(d*g-e*f)^2/(d-2/3*e)^2+(162*A-108*B+72*C)
/(-9*d+6*e)^(1/2)*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(1/2))/(3*f-2*g)/(3*d-
2*e)^2
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 61.07 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\begin{cases} \frac{2\left(\frac{eg\left(Ag^2-Bfg+Cf^2\right)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-\frac{dg-ef}{g}}}\right)}{\sqrt{-\frac{dg-ef}{g}}(f-g)(3f-2g)(dg-ef)^2}+\frac{e\left(Ae^2-Bde+Cd^2\right)}{3(d-e)(d+ex)^{\frac{3}{2}}\cdot(3d-2e)(dg-ef)}+\frac{e\left(9Ad^2\right.}{}\cdots\right.}{} \\ \frac{\frac{(9A-6B+4C)\log(3x+2)}{3\cdot(3f-2g)}-\frac{(A-B+C)\log(x+1)}{f-g}+\frac{\left(Ag^2-Bfg+Cf^2\right)\left(\begin{cases}\frac{x}{f} & \text{fo}\cdots \\ \frac{\log(f+gx)}{g} & \text{o}\cdots\end{cases}\right.}{(f-g)(3f-2g)}\cdots}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(5/2)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(e*g*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e*x)/sqrt(-(d*g
- e*f)/g))/(sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)*(d*g - e*f)**2) + e*(
A*e**2 - B*d*e + C*d**2)/(3*(d - e)*(d + e*x)**(3/2)*(3*d - 2*e)*(d*g - e*
f)) + e*(9*A*d**2*e**2*g - 6*A*d*e**3*f - 10*A*d*e**3*g + 5*A*e**4*f + 2*A
*e**4*g - 6*B*d**3*e*g + 3*B*d**2*e**2*f + 5*B*d**2*e**2*g - 2*B*e**4*f +
3*C*d**4*g - 5*C*d**2*e**2*f - 2*C*d**2*e**2*g + 4*C*d*e**3*f)/((d - e)**2
*sqrt(d + e*x)*(3*d - 2*e)**2*(d*g - e*f)**2) - e*(A - B + C)*atan(sqrt(d
+ e*x)/sqrt(-d + e))/(sqrt(-d + e)*(d - e)**2*(f - g)) + 3*e*(9*A - 6*B +
4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2*e/3))/(sqrt(-d + 2*e/3)*(3*d - 2*e)**2
*(3*f - 2*g)))/e, Ne(e, 0)), (((9*A - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*
g)) - (A - B + C)*log(x + 1)/(f - g) + (A*g**2 - B*f*g + C*f**2)*Piecewise
((x/f, Eq(g, 0)), (log(f + g*x)/g, True))/((f - g)*(3*f - 2*g)))/d**(5/2),
 True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(342) = 684$.

Time = 0.30 (sec) , antiderivative size = 789, normalized size of antiderivative = 2.10

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(5/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
6*sqrt(3)*(9*A - 6*B + 4*C)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e))
/((27*d^2*f - 36*d*e*f + 12*e^2*f - 18*d^2*g + 24*d*e*g - 8*e^2*g)*sqrt(-3
*d + 2*e)) + 2*(C*f^2*g^2 - B*f*g^3 + A*g^4)*arctan(sqrt(e*x + d)*g/sqrt(e
*f*g - d*g^2))/((3*e^2*f^4 - 6*d*e*f^3*g - 5*e^2*f^3*g + 3*d^2*f^2*g^2 + 1
0*d*e*f^2*g^2 + 2*e^2*f^2*g^2 - 5*d^2*f*g^3 - 4*d*e*f*g^3 + 2*d^2*g^4)*sqr
t(e*f*g - d*g^2)) - 2*(A - B + C)*arctan(sqrt(e*x + d)/sqrt(-d + e))/((d^2
*f - 2*d*e*f + e^2*f - d^2*g + 2*d*e*g - e^2*g)*sqrt(-d + e)) - 2/3*(3*C*d
^4*e*f - 9*(e*x + d)*B*d^2*e^2*f + 15*(e*x + d)*C*d^2*e^2*f - 3*B*d^3*e^2*
f - 5*C*d^3*e^2*f + 18*(e*x + d)*A*d*e^3*f - 12*(e*x + d)*C*d*e^3*f + 3*A*
d^2*e^3*f + 5*B*d^2*e^3*f + 2*C*d^2*e^3*f - 15*(e*x + d)*A*e^4*f + 6*(e*x
+ d)*B*e^4*f - 5*A*d*e^4*f - 2*B*d*e^4*f + 2*A*e^5*f - 9*(e*x + d)*C*d^4*g
 - 3*C*d^5*g + 18*(e*x + d)*B*d^3*e*g + 3*B*d^4*e*g + 5*C*d^4*e*g - 27*(e*
x + d)*A*d^2*e^2*g - 15*(e*x + d)*B*d^2*e^2*g + 6*(e*x + d)*C*d^2*e^2*g -
3*A*d^3*e^2*g - 5*B*d^3*e^2*g - 2*C*d^3*e^2*g + 30*(e*x + d)*A*d*e^3*g + 5
*A*d^2*e^3*g + 2*B*d^2*e^3*g - 6*(e*x + d)*A*e^4*g - 2*A*d*e^4*g)/((9*d^4*
e^2*f^2 - 30*d^3*e^3*f^2 + 37*d^2*e^4*f^2 - 20*d*e^5*f^2 + 4*e^6*f^2 - 18*
d^5*e*f*g + 60*d^4*e^2*f*g - 74*d^3*e^3*f*g + 40*d^2*e^4*f*g - 8*d*e^5*f*g
 + 9*d^6*g^2 - 30*d^5*e*g^2 + 37*d^4*e^2*g^2 - 20*d^3*e^3*g^2 + 4*d^2*e^4*
g^2)*(e*x + d)^(3/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(5/2)*(5*x + 3*x^2 + 2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26571, normalized size of antiderivative = 70.67

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{5/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(5/2)/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
( - 162*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(s
qrt(g)*sqrt( - d*g + e*f)))*a*d**7*g**3 - 162*sqrt(g)*sqrt(d + e*x)*sqrt(
- d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**6*e
*g**3*x + 810*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)
*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**6*e*g**3 + 810*sqrt(g)*sqrt(d + e*x
)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*
a*d**5*e**2*g**3*x - 1674*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((s
qrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**5*e**2*g**3 - 1674*sqrt
(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt(
 - d*g + e*f)))*a*d**4*e**3*g**3*x + 1830*sqrt(g)*sqrt(d + e*x)*sqrt( - d*
g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**4*e**3*
g**3 + 1830*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g
)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**3*e**4*g**3*x - 1116*sqrt(g)*sqrt(d +
 e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f
)))*a*d**3*e**4*g**3 - 1116*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan(
(sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**2*e**5*g**3*x + 360*s
qrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sq
rt( - d*g + e*f)))*a*d**2*e**5*g**3 + 360*sqrt(g)*sqrt(d + e*x)*sqrt( - d*
g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d*e**6*g**
3*x - 48*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g...
```

3.41 $\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)(2+5x+3x^2)}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 667

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = -\frac{2(Cd^2-Bde+Ae^2)}{5\left(3d^2-5de+2e^2\right)(ef-dg)(d+ex)^{5/2}}$$
$$+\frac{2(Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))-Be(2e^3f+6d^3g-d^2e}{3\left(3d^2-5de+2e^2\right)^2(ef-dg)^2(d+ex)^{3/2}}$$
$$-\frac{2(C(4e^6f^2+9d^6g^2-9d^4e^2g(5f+2g)-6d^2e^4f(3f+5g)+d^3e^3(15f^2+73fg+10g^2))-Be(36d^2e^3fg}{}$$
$$-\frac{2g^{5/2}(Cf^2-Bfg+Ag^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{7/2}}$$
$$-\frac{18\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{7/2}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{7/2}(f-g)}$$

output
```
1/5*(-2*A*e^2+2*B*d*e-2*C*d^2)/(3*d^2-5*d*e+2*e^2)/(-d*g+e*f)/(e*x+d)^(5/2
)+2/3*(C*d*(4*e^3*f+3*d^3*g-d*e^2*(5*f+2*g))+A*e^2*(9*d^2*g+e^2*(5*f+2*g)-
2*d*e*(3*f+5*g))-B*e*(2*e^3*f+6*d^3*g-d^2*e*(3*f+5*g)))/(3*d^2-5*d*e+2*e^2
)^2/(-d*g+e*f)^2/(e*x+d)^(3/2)-2*(C*(4*e^6*f^2+9*d^6*g^2-9*d^4*e^2*g*(5*f+
2*g)-6*d^2*e^4*f*(3*f+5*g)+d^3*e^3*(15*f^2+73*f*g+10*g^2))-B*e*(36*d^2*e^3
*f*g+27*d^5*g^2+2*e^5*f*(5*f+2*g)-6*d*e^4*f*(3*f+5*g)-9*d^4*e*g*(3*f+5*g)+
d^3*e^2*(9*f^2+15*f*g+19*g^2))+A*e^2*(54*d^4*g^2-24*d^3*e*g*(3*f+5*g)-15*d
*e^3*(3*f^2+5*f*g+2*g^2)+e^4*(19*f^2+10*f*g+4*g^2)+3*d^2*e^2*(9*f^2+45*f*g
+31*g^2)))/(3*d^2-5*d*e+2*e^2)^3/(-d*g+e*f)^3/(e*x+d)^(1/2)-2*g^(5/2)*(A*g
^2-B*f*g+C*f^2)*arctan(g^(1/2)*(e*x+d)^(1/2)/(-d*g+e*f)^(1/2))/(3*f-2*g)/(
f-g)/(-d*g+e*f)^(7/2)-18*3^(1/2)*(9*A-6*B+4*C)*arctanh(3^(1/2)*(e*x+d)^(1/
2)/(3*d-2*e)^(1/2))/(3*d-2*e)^(7/2)/(3*f-2*g)+2*(A-B+C)*arctanh((e*x+d)^(1
/2)/(d-e)^(1/2))/(d-e)^(7/2)/(f-g)
```

Mathematica [A] (verified)

Time = 17.53 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx = 2\left(-\frac{Cd^2-Bde+Ae^2}{5(3d-2e)(d-e)(ef-dg)(d+ex)^{5/2}}\right.$$
$$+\frac{Cd(4e^3f+3d^3g-de^2(5f+2g))+Ae^2(9d^2g+e^2(5f+2g)-2de(3f+5g))-Be(2e^3f+6d^3g-d^2e(3}{3(3d-2e)^2(d-e)^2(ef-dg)^2(d+ex)^{3/2}}$$
$$-\frac{9Bd^3e^3f^2-15Cd^3e^3f^2-27Ad^2e^4f^2+18Cd^2e^4f^2+45Ade^5f^2-18Bde^5f^2-19Ae^6f^2+10Be^6f^2-4}{}$$
$$-\frac{g^{5/2}(Cf^2-g(Bf-Ag))\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{7/2}}$$
$$\left.-\frac{9\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{7/2}(3f-2g)}+\frac{(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{7/2}(f-g)}\right)$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(7/2)*(f + g*x)*(2 + 5*x + 3*x^2)),
x]
```

output
```
2*(-1/5*(C*d^2 - B*d*e + A*e^2)/((3*d - 2*e)*(d - e)*(e*f - d*g)*(d + e*x)
^(5/2)) + (C*d*(4*e^3*f + 3*d^3*g - d*e^2*(5*f + 2*g)) + A*e^2*(9*d^2*g +
e^2*(5*f + 2*g) - 2*d*e*(3*f + 5*g)) - B*e*(2*e^3*f + 6*d^3*g - d^2*e*(3*f
 + 5*g)))/(3*(3*d - 2*e)^2*(d - e)^2*(e*f - d*g)^2*(d + e*x)^(3/2)) - (9*B
*d^3*e^3*f^2 - 15*C*d^3*e^3*f^2 - 27*A*d^2*e^4*f^2 + 18*C*d^2*e^4*f^2 + 45
*A*d*e^5*f^2 - 18*B*d*e^5*f^2 - 19*A*e^6*f^2 + 10*B*e^6*f^2 - 4*C*e^6*f^2
- 27*B*d^4*e^2*f*g + 45*C*d^4*e^2*f*g + 72*A*d^3*e^3*f*g + 15*B*d^3*e^3*f*
g - 73*C*d^3*e^3*f*g - 135*A*d^2*e^4*f*g + 36*B*d^2*e^4*f*g + 30*C*d^2*e^4
*f*g + 75*A*d*e^5*f*g - 30*B*d*e^5*f*g - 10*A*e^6*f*g + 4*B*e^6*f*g - 9*C*
d^6*g^2 + 27*B*d^5*e*g^2 - 54*A*d^4*e^2*g^2 - 45*B*d^4*e^2*g^2 + 18*C*d^4*
e^2*g^2 + 120*A*d^3*e^3*g^2 + 19*B*d^3*e^3*g^2 - 10*C*d^3*e^3*g^2 - 93*A*d
^2*e^4*g^2 + 30*A*d*e^5*g^2 - 4*A*e^6*g^2)/((3*d - 2*e)^3*(d - e)^3*(-(e*f
) + d*g)^3*Sqrt[d + e*x]) - (g^(5/2)*(C*f^2 - g*(B*f - A*g))*ArcTan[(Sqrt[
g]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*(e*f - d*g)^(7/2)
) - (9*Sqrt[3]*(9*A - 6*B + 4*C)*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d
- 2*e]])/((3*d - 2*e)^(7/2)*(3*f - 2*g)) + ((A - B + C)*ArcTanh[Sqrt[d + e
*x]/Sqrt[d - e]])/((d - e)^(7/2)*(f - g)))
```

Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 571, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(3x^2+5x+2)(d+ex)^{7/2}(f+gx)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{Ag^2-Bfg+Cf^2}{(3f-2g)(f-g)(d+ex)^{7/2}(f+gx)}+\frac{-A+B-C}{(x+1)(f-g)(d+ex)^{7/2}}+\frac{9A-6B+4C}{(3x+2)(3f-2g)(d+ex)^{7/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2g^{5/2}(Ag^2-Bfg+Cf^2)\arctan\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{(3f-2g)(f-g)(ef-dg)^{7/2}}-\frac{18\sqrt{3}(9A-6B+4C)\text{arctanh}\left(\frac{\sqrt{3}\sqrt{d+ex}}{\sqrt{3d-2e}}\right)}{(3d-2e)^{7/2}(3f-2g)}+\frac{2(A-B+C)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d-e}}\right)}{(d-e)^{7/2}(f-g)}-\frac{2g^2(Ag^2-Bfg+Cf^2)}{(3f-2g)(f-g)\sqrt{d+ex}(ef-dg)^3}+\frac{2g(Ag^2-Bfg+Cf^2)}{3(3f-2g)(f-g)(d+ex)^{3/2}(ef-dg)^2}-\frac{2(Ag^2-Bfg+Cf^2)}{5(3f-2g)(f-g)(d+ex)^{5/2}(ef-dg)}+\frac{18(9A-6B+4C)}{(3d-2e)^3(3f-2g)\sqrt{d+ex}}-\frac{2(A-B+C)}{(d-e)^3(f-g)\sqrt{d+ex}}+\frac{2(9A-6B+4C)}{(3d-2e)^2(3f-2g)(d+ex)^{3/2}}-\frac{2(A-B+C)}{3(d-e)^2(f-g)(d+ex)^{3/2}}+\frac{2(9A-6B+4C)}{5(3d-2e)(3f-2g)(d+ex)^{5/2}}-\frac{2(A-B+C)}{5(d-e)(f-g)(d+ex)^{5/2}}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(7/2)*(f + g*x)*(2 + 5*x + 3*x^2)),x]
```

output
```
(2*(9*A - 6*B + 4*C))/(5*(3*d - 2*e)*(3*f - 2*g)*(d + e*x)^(5/2)) - (2*(A
- B + C))/(5*(d - e)*(f - g)*(d + e*x)^(5/2)) - (2*(C*f^2 - B*f*g + A*g^2)
)/(5*(3*f - 2*g)*(f - g)*(e*f - d*g)*(d + e*x)^(5/2)) + (2*(9*A - 6*B + 4*
C))/((3*d - 2*e)^2*(3*f - 2*g)*(d + e*x)^(3/2)) - (2*(A - B + C))/(3*(d -
e)^2*(f - g)*(d + e*x)^(3/2)) + (2*g*(C*f^2 - B*f*g + A*g^2))/(3*(3*f - 2*
g)*(f - g)*(e*f - d*g)^2*(d + e*x)^(3/2)) + (18*(9*A - 6*B + 4*C))/((3*d -
 2*e)^3*(3*f - 2*g)*Sqrt[d + e*x]) - (2*(A - B + C))/((d - e)^3*(f - g)*Sq
rt[d + e*x]) - (2*g^2*(C*f^2 - B*f*g + A*g^2))/((3*f - 2*g)*(f - g)*(e*f -
 d*g)^3*Sqrt[d + e*x]) - (2*g^(5/2)*(C*f^2 - B*f*g + A*g^2)*ArcTan[(Sqrt[g
]*Sqrt[d + e*x])/Sqrt[e*f - d*g]])/((3*f - 2*g)*(f - g)*(e*f - d*g)^(7/2))
 - (18*Sqrt[3]*(9*A - 6*B + 4*C)*ArcTanh[(Sqrt[3]*Sqrt[d + e*x])/Sqrt[3*d
- 2*e]])/((3*d - 2*e)^(7/2)*(3*f - 2*g)) + (2*(A - B + C)*ArcTanh[Sqrt[d +
 e*x]/Sqrt[d - e]])/((d - e)^(7/2)*(f - g))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 211.90 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{(486A-324B+216C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{\sqrt{-9d+6e}\,(3f-2g)(3d-2e)^3}-\frac{2g^3(A\,g^2-Bfg+C\,f^2)\,\mathrm{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^3\sqrt{(dg-ef)g}}+\frac{\frac{2}{15}A\,e^2-\frac{2}{15}Bde+}{(ex+d)^{\frac{5}{2}}(d-e)(dg-e}$
derivativedivides	$\frac{2(243A-162B+108C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{(3f-2g)(3d-2e)^3\sqrt{-9d+6e}}-\frac{2g^3(A\,g^2-Bfg+C\,f^2)\,\mathrm{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^3\sqrt{(dg-ef)g}}-\frac{2(-A\,e^2+Bde}{5(dg-ef)(3d-2e)(c}$
default	$\frac{2(243A-162B+108C)\arctan\left(\frac{3\sqrt{ex+d}}{\sqrt{-9d+6e}}\right)}{(3f-2g)(3d-2e)^3\sqrt{-9d+6e}}-\frac{2g^3(A\,g^2-Bfg+C\,f^2)\,\mathrm{arctanh}\left(\frac{g\sqrt{ex+d}}{\sqrt{(dg-ef)g}}\right)}{(f-g)(3f-2g)(dg-ef)^3\sqrt{(dg-ef)g}}-\frac{2(-A\,e^2+Bde}{5(dg-ef)(3d-2e)(c}$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(g*x+f)/(3*x^2+5*x+2),x,method=_RETURNVERB
OSE)
```

output
```
(486*A-324*B+216*C)/(-9*d+6*e)^(1/2)*arctan(3*(e*x+d)^(1/2)/(-9*d+6*e)^(1/
2))/(3*f-2*g)/(3*d-2*e)^3-2*g^3*(A*g^2-B*f*g+C*f^2)/(f-g)/(3*f-2*g)/(d*g-e
*f)^3/((d*g-e*f)*g)^(1/2)*arctanh(g*(e*x+d)^(1/2)/((d*g-e*f)*g)^(1/2))+2/1
5*(A*e^2-B*d*e+C*d^2)/(e*x+d)^(5/2)/(d-e)/(d*g-e*f)/(d-2/3*e)+2/3*(1/3*C*d
^4*g-2/3*B*d^3*e*g+d^2*((A+5/9*B-2/9*C)*g+1/3*(B-5/3*C)*f)*e^2-2/3*d*(5/3*
A*g+f*(A-2/3*C))*e^3+5/9*e^4*(2/5*A*g+f*(A-2/5*B)))/(e*x+d)^(3/2)/(d-2/3*e
)^2/(d*g-e*f)^2/(d-e)^2+2/27*((4*A*g^2+10*f*(A-2/5*B)*g+19*(A-10/19*B+4/19
*C)*f^2)*e^6-45*d*(2/3*A*g^2+5/3*f*(A-2/5*B)*g+f^2*(A-2/5*B))*e^5+27*(31/9
*A*g^2+5*(A-4/15*B-2/9*C)*f*g+f^2*(A-2/3*C))*d^2*e^4-72*(1/3*(5*A+19/24*B-
5/12*C)*g^2+f*(A+5/24*B-73/72*C)*g+1/8*(B-5/3*C)*f^2)*d^3*e^3+54*((A+5/6*B
-1/3*C)*g+1/2*(B-5/3*C)*f)*d^4*g*e^2-27*B*d^5*e*g^2+9*C*d^6*g^2)/(d-e)^3/(
e*x+d)^(1/2)/(d*g-e*f)^3/(d-2/3*e)^3+2*(A-B+C)*arctan((e*x+d)^(1/2)/(-d+e)
^(1/2))/(f-g)/(-d+e)^(7/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 96.53 (sec) , antiderivative size = 910, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(7/2)/(g*x+f)/(3*x**2+5*x+2),x)
```

output
```
Piecewise((2*(e*g**2*(A*g**2 - B*f*g + C*f**2)*atan(sqrt(d + e*x)/sqrt(-(d
*g - e*f)/g))/(sqrt(-(d*g - e*f)/g)*(f - g)*(3*f - 2*g)*(d*g - e*f)**3) +
e*(A*e**2 - B*d*e + C*d**2)/(5*(d - e)*(d + e*x)**(5/2)*(3*d - 2*e)*(d*g -
 e*f)) + e*(9*A*d**2*e**2*g - 6*A*d*e**3*f - 10*A*d*e**3*g + 5*A*e**4*f +
2*A*e**4*g - 6*B*d**3*e*g + 3*B*d**2*e**2*f + 5*B*d**2*e**2*g - 2*B*e**4*f
 + 3*C*d**4*g - 5*C*d**2*e**2*f - 2*C*d**2*e**2*g + 4*C*d*e**3*f)/(3*(d -
e)**2*(d + e*x)**(3/2)*(3*d - 2*e)**2*(d*g - e*f)**2) + e*(54*A*d**4*e**2*
g**2 - 72*A*d**3*e**3*f*g - 120*A*d**3*e**3*g**2 + 27*A*d**2*e**4*f**2 + 1
35*A*d**2*e**4*f*g + 93*A*d**2*e**4*g**2 - 45*A*d*e**5*f**2 - 75*A*d*e**5*
f*g - 30*A*d*e**5*g**2 + 19*A*e**6*f**2 + 10*A*e**6*f*g + 4*A*e**6*g**2 -
27*B*d**5*e*g**2 + 27*B*d**4*e**2*f*g + 45*B*d**4*e**2*g**2 - 9*B*d**3*e**
3*f**2 - 15*B*d**3*e**3*f*g - 19*B*d**3*e**3*g**2 - 36*B*d**2*e**4*f*g + 1
8*B*d*e**5*f**2 + 30*B*d*e**5*f*g - 10*B*e**6*f**2 - 4*B*e**6*f*g + 9*C*d*
*6*g**2 - 45*C*d**4*e**2*f*g - 18*C*d**4*e**2*g**2 + 15*C*d**3*e**3*f**2 +
 73*C*d**3*e**3*f*g + 10*C*d**3*e**3*g**2 - 18*C*d**2*e**4*f**2 - 30*C*d**
2*e**4*f*g + 4*C*e**6*f**2)/((d - e)**3*sqrt(d + e*x)*(3*d - 2*e)**3*(d*g
- e*f)**3) - e*(A - B + C)*atan(sqrt(d + e*x)/sqrt(-d + e))/(sqrt(-d + e)*
(d - e)**3*(f - g)) + 9*e*(9*A - 6*B + 4*C)*atan(sqrt(d + e*x)/sqrt(-d + 2
*e/3))/(sqrt(-d + 2*e/3)*(3*d - 2*e)**3*(3*f - 2*g)))/e, Ne(e, 0)), (((9*A
 - 6*B + 4*C)*log(3*x + 2)/(3*(3*f - 2*g)) - (A - B + C)*log(x + 1)/(f ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{7/2}(f + gx)\,(2 + 5x + 3x^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d-4*e>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2365 vs. $2(630) = 1260$.

Time = 0.87 (sec) , antiderivative size = 2365, normalized size of antiderivative = 3.55

$$\int \frac{A + Bx + Cx^2}{(d + ex)^{7/2}(f + gx)\,(2 + 5x + 3x^2)}\,dx = \text{Too large to display}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(7/2)/(g*x+f)/(3*x^2+5*x+2),x, algorithm="
giac")
```

output
```
18*sqrt(3)*(9*A - 6*B + 4*C)*arctan(sqrt(3)*sqrt(e*x + d)/sqrt(-3*d + 2*e)
)/((81*d^3*f - 162*d^2*e*f + 108*d*e^2*f - 24*e^3*f - 54*d^3*g + 108*d^2*e
*g - 72*d*e^2*g + 16*e^3*g)*sqrt(-3*d + 2*e)) - 2*(C*f^2*g^3 - B*f*g^4 + A
*g^5)*arctan(sqrt(e*x + d)*g/sqrt(e*f*g - d*g^2))/((3*e^3*f^5 - 9*d*e^2*f^
4*g - 5*e^3*f^4*g + 9*d^2*e*f^3*g^2 + 15*d*e^2*f^3*g^2 + 2*e^3*f^3*g^2 - 3
*d^3*f^2*g^3 - 15*d^2*e*f^2*g^3 - 6*d*e^2*f^2*g^3 + 5*d^3*f*g^4 + 6*d^2*e*
f*g^4 - 2*d^3*g^5)*sqrt(e*f*g - d*g^2)) - 2*(A - B + C)*arctan(sqrt(e*x +
d)/sqrt(-d + e))/((d^3*f - 3*d^2*e*f + 3*d*e^2*f - e^3*f - d^3*g + 3*d^2*e
*g - 3*d*e^2*g + e^3*g)*sqrt(-d + e)) - 2/15*(27*C*d^6*e^2*f^2 - 135*(e*x
+ d)^2*B*d^3*e^3*f^2 + 225*(e*x + d)^2*C*d^3*e^3*f^2 - 45*(e*x + d)*B*d^4*
e^3*f^2 + 75*(e*x + d)*C*d^4*e^3*f^2 - 27*B*d^5*e^3*f^2 - 90*C*d^5*e^3*f^2
 + 405*(e*x + d)^2*A*d^2*e^4*f^2 - 270*(e*x + d)^2*C*d^2*e^4*f^2 + 90*(e*x
 + d)*A*d^3*e^4*f^2 + 75*(e*x + d)*B*d^3*e^4*f^2 - 185*(e*x + d)*C*d^3*e^4
*f^2 + 27*A*d^4*e^4*f^2 + 90*B*d^4*e^4*f^2 + 111*C*d^4*e^4*f^2 - 675*(e*x
+ d)^2*A*d*e^5*f^2 + 270*(e*x + d)^2*B*d*e^5*f^2 - 225*(e*x + d)*A*d^2*e^5
*f^2 + 150*(e*x + d)*C*d^2*e^5*f^2 - 90*A*d^3*e^5*f^2 - 111*B*d^3*e^5*f^2
- 60*C*d^3*e^5*f^2 + 285*(e*x + d)^2*A*e^6*f^2 - 150*(e*x + d)^2*B*e^6*f^2
 + 60*(e*x + d)^2*C*e^6*f^2 + 185*(e*x + d)*A*d*e^6*f^2 - 50*(e*x + d)*B*d
*e^6*f^2 - 40*(e*x + d)*C*d*e^6*f^2 + 111*A*d^2*e^6*f^2 + 60*B*d^2*e^6*f^2
 + 12*C*d^2*e^6*f^2 - 50*(e*x + d)*A*e^7*f^2 + 20*(e*x + d)*B*e^7*f^2 -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Hanged}$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(7/2)*(5*x + 3*x^2 + 2)),x)
```

output
```
\text{Hanged}
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 62380, normalized size of antiderivative = 93.52

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{7/2}(f+gx)\left(2+5x+3x^2\right)}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(7/2)/(g*x+f)/(3*x^2+5*x+2),x)
```

output

```
( - 2430*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(
sqrt(g)*sqrt( - d*g + e*f)))*a*d**10*g**4 - 4860*sqrt(g)*sqrt(d + e*x)*sqr
t( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**
9*e*g**4*x + 16200*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d +
 e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**9*e*g**4 - 2430*sqrt(g)*sqrt(d
 + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e
*f)))*a*d**8*e**2*g**4*x**2 + 32400*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*
f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**8*e**2*g**4*x
 - 46980*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(
sqrt(g)*sqrt( - d*g + e*f)))*a*d**8*e**2*g**4 + 16200*sqrt(g)*sqrt(d + e*x
)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*
a*d**7*e**3*g**4*x**2 - 93960*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*ata
n((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**7*e**3*g**4*x + 774
00*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g
)*sqrt( - d*g + e*f)))*a*d**7*e**3*g**4 - 46980*sqrt(g)*sqrt(d + e*x)*sqrt
( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**6
*e**4*g**4*x**2 + 154800*sqrt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sq
rt(d + e*x)*g)/(sqrt(g)*sqrt( - d*g + e*f)))*a*d**6*e**4*g**4*x - 79230*sq
rt(g)*sqrt(d + e*x)*sqrt( - d*g + e*f)*atan((sqrt(d + e*x)*g)/(sqrt(g)*sqr
t( - d*g + e*f)))*a*d**6*e**4*g**4 + 77400*sqrt(g)*sqrt(d + e*x)*sqrt( ...
```

3.42 $\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}(1+x-x^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = \sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{1+x}}{\sqrt{-1+x}}\right) + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{\sqrt{-2+\sqrt{5}}\sqrt{1+x}}{\sqrt{-1+x}}\right)$$

output
```
1/5*(-10+10*5^(1/2))^(1/2)*arctan((2+5^(1/2))^(1/2)*(1+x)^(1/2)/(-1+x)^(1/
2))+1/5*(10+10*5^(1/2))^(1/2)*arctanh((-2+5^(1/2))^(1/2)*(1+x)^(1/2)/(-1+x
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = -\sqrt{\frac{2}{5}\left(-1+\sqrt{5}\right)}\arctan\left(\sqrt{-2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right) + \sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{2+\sqrt{5}}\sqrt{\frac{-1+x}{1+x}}\right)$$

input
```
Integrate[x/(Sqrt[-1 + x]*Sqrt[1 + x]*(1 + x - x^2)),x]
```

output
```
-(Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[-2 + Sqrt[5]]*Sqrt[(-1 + x)/(1 +
x)]]) + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2 + Sqrt[5]]*Sqrt[(-1 + x)/
(1 + x)]]
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{x-1}\sqrt{x+1}\left(-x^2+x+1\right)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{1-\frac{1}{\sqrt{5}}}{\left(-2x-\sqrt{5}+1\right)\sqrt{x-1}\sqrt{x+1}}+\frac{1+\frac{1}{\sqrt{5}}}{\left(-2x+\sqrt{5}+1\right)\sqrt{x-1}\sqrt{x+1}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\frac{2}{5}\left(\sqrt{5}-1\right)}\arctan\left(\frac{\sqrt{2+\sqrt{5}}\sqrt{x+1}}{\sqrt{x-1}}\right)+\sqrt{\frac{2}{5}\left(1+\sqrt{5}\right)}\text{arctanh}\left(\frac{\sqrt{\sqrt{5}-2}\sqrt{x+1}}{\sqrt{x-1}}\right)$$

input
```
Int[x/(Sqrt[-1 + x]*Sqrt[1 + x]*(1 + x - x^2)),x]
```

output
```
Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[(Sqrt[2 + Sqrt[5]]*Sqrt[1 + x])/Sqrt[-1
+ x]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[(Sqrt[-2 + Sqrt[5]]*Sqrt[1 + x])
/Sqrt[-1 + x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(61) = 122$.

Time = 0.39 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.57

method	result
default	$\frac{16\sqrt{x-1}\,\sqrt{1+x}\,\left(\sqrt{2+2\sqrt{5}}\,\arctan\left(\frac{x\sqrt{5}-x+2}{\sqrt{2\sqrt{5}-2}\,\sqrt{x^{2}-1}}\right)\sqrt{5}-\operatorname{arctanh}\left(\frac{x\sqrt{5}+x-2}{\sqrt{2+2\sqrt{5}}\,\sqrt{x^{2}-1}}\right)\sqrt{5}\,\sqrt{2\sqrt{5}-2}-5\sqrt{2+2\sqrt{5}}\,\arctan\left(\frac{x\sqrt{5}-}{\sqrt{2\sqrt{5}-2}}\right.\right.}{5\sqrt{x^{2}-1}\,\left(3+\sqrt{5}\right)\left(\sqrt{5}-3\right)\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)\sqrt{2\sqrt{5}-2}\,\sqrt{2+2\sqrt{5}}}$

input
```
int(x/(x-1)^(1/2)/(1+x)^(1/2)/(-x^2+x+1),x,method=_RETURNVERBOSE)
```

output
```
16/5*(x-1)^(1/2)*(1+x)^(1/2)*((2+2*5^(1/2))^(1/2)*arctan((x*5^(1/2)-x+2)/(
2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))*5^(1/2)-arctanh((x*5^(1/2)+x-2)/(2+2*5^(
1/2))^(1/2)/(x^2-1)^(1/2))*5^(1/2)*(2*5^(1/2)-2)^(1/2)-5*(2+2*5^(1/2))^(1/
2)*arctan((x*5^(1/2)-x+2)/(2*5^(1/2)-2)^(1/2)/(x^2-1)^(1/2))-5*arctanh((x*
5^(1/2)+x-2)/(2+2*5^(1/2))^(1/2)/(x^2-1)^(1/2))*(2*5^(1/2)-2)^(1/2))/(x^2-
1)^(1/2)/(3+5^(1/2))/(5^(1/2)-3)/(5^(1/2)-1)/(5^(1/2)+1)/(2*5^(1/2)-2)^(1/
2)/(2+2*5^(1/2))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.64

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = -2\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\arctan\left(\frac{1}{2}\left(\sqrt{x+1}\sqrt{x-1}\left(\sqrt{5}+5\right)-\sqrt{5}(x+2)-5x\right)\sqrt{\frac{1}{10}\sqrt{5}-\frac{1}{10}}\right) + \sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x+2\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}+\sqrt{5}+1\right) - \sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}\log\left(2\sqrt{x+1}\sqrt{x-1}-2x-2\sqrt{5}\sqrt{\frac{1}{10}\sqrt{5}+\frac{1}{10}}+\sqrt{5}+1\right)$$

input
```
integrate(x/(x-1)^(1/2)/(1+x)^(1/2)/(-x^2+x+1),x, algorithm="fricas")
```

output
```
-2*sqrt(1/10*sqrt(5) - 1/10)*arctan(1/2*(sqrt(x + 1)*sqrt(x - 1)*(sqrt(5)
+ 5) - sqrt(5)*(x + 2) - 5*x)*sqrt(1/10*sqrt(5) - 1/10)) + sqrt(1/10*sqrt(
5) + 1/10)*log(2*sqrt(x + 1)*sqrt(x - 1) - 2*x + 2*sqrt(5)*sqrt(1/10*sqrt(
5) + 1/10) + sqrt(5) + 1) - sqrt(1/10*sqrt(5) + 1/10)*log(2*sqrt(x + 1)*sq
rt(x - 1) - 2*x - 2*sqrt(5)*sqrt(1/10*sqrt(5) + 1/10) + sqrt(5) + 1)
```

Sympy [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = -\int \frac{x}{x^2\sqrt{x-1}\sqrt{x+1}-x\sqrt{x-1}\sqrt{x+1}-\sqrt{x-1}\sqrt{x+1}}\,dx$$

input
```
integrate(x/(x-1)**(1/2)/(1+x)**(1/2)/(-x**2+x+1),x)
```

output
```
-Integral(x/(x**2*sqrt(x - 1)*sqrt(x + 1) - x*sqrt(x - 1)*sqrt(x + 1) - sq
rt(x - 1)*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = \int -\frac{x}{(x^2-x-1)\sqrt{x+1}\sqrt{x-1}}\,dx$$

input
```
integrate(x/(x-1)^(1/2)/(1+x)^(1/2)/(-x^2+x+1),x, algorithm="maxima")
```

output
```
-integrate(x/((x^2 - x - 1)*sqrt(x + 1)*sqrt(x - 1)), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.76

$$\begin{aligned}
&\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx \\
&= -\frac{1}{5}\left(3\sqrt{5\sqrt{5}+10}-5\sqrt{\sqrt{5}+2}\right)\arctan\left(\frac{\sqrt{-\frac{2}{x+1}+1}}{\sqrt{\sqrt{5}+2}}\right) \\
&\quad+\frac{1}{5}\left(3i\sqrt{5\sqrt{5}-10}+5i\sqrt{\sqrt{5}-2}\right)\arctan\left(-\frac{i\sqrt{-\frac{2}{x+1}+1}}{\sqrt{\sqrt{5}-2}}\right) \\
&\quad+\frac{3}{10}\sqrt{5\sqrt{5}+10}\arctan\left(\frac{1}{\sqrt{\sqrt{5}+2}}\right)+\frac{3}{10}i\sqrt{5\sqrt{5}-10}\arctan\left(\frac{1}{\sqrt{\sqrt{5}+2}}\right) \\
&\quad-\frac{1}{2}\sqrt{\sqrt{5}+2}\arctan\left(\frac{1}{\sqrt{\sqrt{5}+2}}\right)-\frac{1}{2}i\sqrt{\sqrt{5}-2}\arctan\left(\frac{1}{\sqrt{\sqrt{5}+2}}\right) \\
&\quad-\frac{3}{10}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{i}{\sqrt{\sqrt{5}-2}}\right)-\frac{3}{10}i\sqrt{5\sqrt{5}-10}\arctan\left(-\frac{i}{\sqrt{\sqrt{5}-2}}\right) \\
&\quad-\frac{1}{2}\sqrt{\sqrt{5}+2}\arctan\left(-\frac{i}{\sqrt{\sqrt{5}-2}}\right)-\frac{1}{2}i\sqrt{\sqrt{5}-2}\arctan\left(-\frac{i}{\sqrt{\sqrt{5}-2}}\right)
\end{aligned}$$

input
```
integrate(x/(x-1)^(1/2)/(1+x)^(1/2)/(-x^2+x+1),x, algorithm="giac")
```

output
```
-1/5*(3*sqrt(5*sqrt(5) + 10) - 5*sqrt(sqrt(5) + 2))*arctan(sqrt(-2/(x + 1)
 + 1)/sqrt(sqrt(5) + 2)) + 1/5*(3*I*sqrt(5*sqrt(5) - 10) + 5*I*sqrt(sqrt(5
) - 2))*arctan(-I*sqrt(-2/(x + 1) + 1)/sqrt(sqrt(5) - 2)) + 3/10*sqrt(5*sq
rt(5) + 10)*arctan(1/sqrt(sqrt(5) + 2)) + 3/10*I*sqrt(5*sqrt(5) - 10)*arct
an(1/sqrt(sqrt(5) + 2)) - 1/2*sqrt(sqrt(5) + 2)*arctan(1/sqrt(sqrt(5) + 2)
) - 1/2*I*sqrt(sqrt(5) - 2)*arctan(1/sqrt(sqrt(5) + 2)) - 3/10*sqrt(5*sqrt
(5) + 10)*arctan(-I/sqrt(sqrt(5) - 2)) - 3/10*I*sqrt(5*sqrt(5) - 10)*arcta
n(-I/sqrt(sqrt(5) - 2)) - 1/2*sqrt(sqrt(5) + 2)*arctan(-I/sqrt(sqrt(5) - 2
)) - 1/2*I*sqrt(sqrt(5) - 2)*arctan(-I/sqrt(sqrt(5) - 2))
```

Mupad [B] (verification not implemented)

Time = 21.56 (sec) , antiderivative size = 1482, normalized size of antiderivative = 17.03

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = \text{Too large to display}$$

input
```
int(x/((x - 1)^(1/2)*(x + 1)^(1/2)*(x - x^2 + 1)),x)
```

output
```
(10^(1/2)*atan((25769803776*(2*5^(1/2) + 2)^(1/2))/((34359738368*((x - 1)^
(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - 12884901888*5^(1/2) + (((x - 1)^(1/
2) - 1i)*4294967296i)/((x + 1)^(1/2) - 1) + (5^(1/2)*((x - 1)^(1/2) - 1i)*
21474836480i)/((x + 1)^(1/2) - 1) - (12884901888*5^(1/2)*((x - 1)^(1/2) -
1i)^2)/((x + 1)^(1/2) - 1)^2 + 34359738368) - (6442450944*(10*5^(1/2) + 10
)^(1/2))/((34359738368*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - 128
84901888*5^(1/2) + (((x - 1)^(1/2) - 1i)*4294967296i)/((x + 1)^(1/2) - 1)
+ (5^(1/2)*((x - 1)^(1/2) - 1i)*21474836480i)/((x + 1)^(1/2) - 1) - (12884
901888*5^(1/2)*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 34359738368
) - (8589934592*(- 2*5^(1/2) - 2)^(1/2)*((x - 1)^(1/2) - 1i))/(((x + 1)^(1
/2) - 1)*((34359738368*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - 128
84901888*5^(1/2) + (((x - 1)^(1/2) - 1i)*4294967296i)/((x + 1)^(1/2) - 1)
+ (5^(1/2)*((x - 1)^(1/2) - 1i)*21474836480i)/((x + 1)^(1/2) - 1) - (12884
901888*5^(1/2)*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 34359738368
)) + (8589934592*(- 10*5^(1/2) - 10)^(1/2)*((x - 1)^(1/2) - 1i))/(((x + 1)
^(1/2) - 1)*((34359738368*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 -
12884901888*5^(1/2) + (((x - 1)^(1/2) - 1i)*4294967296i)/((x + 1)^(1/2) -
1) + (5^(1/2)*((x - 1)^(1/2) - 1i)*21474836480i)/((x + 1)^(1/2) - 1) - (12
884901888*5^(1/2)*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 34359738
368)) + (25769803776*(2*5^(1/2) + 2)^(1/2)*((x - 1)^(1/2) - 1i)^2)/(((x...
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\,(1+x-x^2)}\,dx = -\left(\int \frac{x}{\sqrt{x+1}\,\sqrt{x-1}\,x^2-\sqrt{x+1}\,\sqrt{x-1}\,x-\sqrt{x+1}\,\sqrt{x-1}}dx\right)$$

input
```
int(x/(x-1)^(1/2)/(1+x)^(1/2)/(-x^2+x+1),x)
```

output
```
 - int(x/(sqrt(x + 1)*sqrt(x - 1)*x**2 - sqrt(x + 1)*sqrt(x - 1)*x - sqrt(
x + 1)*sqrt(x - 1)),x)
```

3.43 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}(a+bx+cx^2)}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 504

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \frac{2(Cf^2-g(Bf-Ag))\sqrt{d+ex}}{(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{f+gx}}$$
$$-\frac{2\left(2c(Acf-aCf-Abg+aBg)-\left(b-\sqrt{b^2-4ac}\right)(Bcf-bCf-Acg+aCg)\right)\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b-\sqrt{b^2-\cdots}\right.}}{\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\left(cf^2-bfg+ag^2\right)}$$
$$+\frac{2\left(2c(Acf-aCf-Abg+aBg)-\left(b+\sqrt{b^2-4ac}\right)(Bcf-bCf-Acg+aCg)\right)\operatorname{arctanh}\left(\frac{\sqrt{2cf-\left(b+\sqrt{b^2-\cdots}\right.}}{\sqrt{2cd-\left(b+\sqrt{b^2-\cdots}\right.}}\right.}{\sqrt{b^2-4ac}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\left(cf^2-bfg+ag^2\right)}$$

output
```
2*(C*f^2-g*(-A*g+B*f))*(e*x+d)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g*x+f
)^(1/2)-2*(2*c*(-A*b*g+A*c*f+B*a*g-C*a*f)-(b-(-4*a*c+b^2)^(1/2))*(-A*c*g+B
*c*f+C*a*g-C*b*f))*arctanh((2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)*(e*x+d)^
(1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(g*x+f)^(1/2))/(-4*a*c+b^2)^(
1/2)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*
g)^(1/2)/(a*g^2-b*f*g+c*f^2)+2*(2*c*(-A*b*g+A*c*f+B*a*g-C*a*f)-(b+(-4*a*c+
b^2)^(1/2))*(-A*c*g+B*c*f+C*a*g-C*b*f))*arctanh((2*c*f-(b+(-4*a*c+b^2)^(1/
2))*g)^(1/2)*(e*x+d)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)/(g*x+f)^
(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*f-(b
+(-4*a*c+b^2)^(1/2))*g)^(1/2)/(a*g^2-b*f*g+c*f^2)
```

Mathematica [A] (verified)

Time = 13.47 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.88

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+bx+cx^2\right)}\,dx = -\frac{2\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(Bc-bC)g\sqrt{d+ex}}{c\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)(ef-dg)\sqrt{f+gx}} - \frac{2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(Bc-bC)g\sqrt{d+ex}}{c\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(ef-dg)\sqrt{f+gx}} + \frac{4(Ac-aC)g\sqrt{d+ex}}{\sqrt{b^2-4ac}\left(2cf+\left(-b+\sqrt{b^2-4ac}\right)g\right)(-ef+dg)\sqrt{f+gx}} + \frac{4(Ac-aC)g\sqrt{d+ex}}{\sqrt{b^2-4ac}\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)(-ef+dg)\sqrt{f+gx}} + \frac{2C\sqrt{d+ex}}{(cef-cdg)\sqrt{f+gx}} - \frac{8c(Ac-aC)\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)^{3/2}} - \frac{4\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(Bc-bC)\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}\left(-2cf+\left(b-\sqrt{b^2-4ac}\right)g\right)^{3/2}} + \frac{8c(Ac-aC)\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)^{3/2}} - \frac{4\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(Bc-bC)\text{arctanh}\left(\frac{\sqrt{-2cf+\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\sqrt{f+gx}}\right)}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}\left(-2cf+\left(b+\sqrt{b^2-4ac}\right)g\right)^{3/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + b*x + c*x^
2)),x]
```

output

```
(-2*(1 - b/Sqrt[b^2 - 4*a*c])*(B*c - b*C)*g*Sqrt[d + e*x])/(c*(2*c*f + (-b
 + Sqrt[b^2 - 4*a*c])*g)*(e*f - d*g)*Sqrt[f + g*x]) - (2*(1 + b/Sqrt[b^2 -
 4*a*c])*(B*c - b*C)*g*Sqrt[d + e*x])/(c*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*
g)*(e*f - d*g)*Sqrt[f + g*x]) + (4*(A*c - a*C)*g*Sqrt[d + e*x])/(Sqrt[b^2
- 4*a*c]*(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)*(-(e*f) + d*g)*Sqrt[f + g*x]
) + (4*(A*c - a*C)*g*Sqrt[d + e*x])/(Sqrt[b^2 - 4*a*c]*(-2*c*f + (b + Sqrt
[b^2 - 4*a*c])*g)*(-(e*f) + d*g)*Sqrt[f + g*x]) + (2*C*Sqrt[d + e*x])/((c*
e*f - c*d*g)*Sqrt[f + g*x]) - (8*c*(A*c - a*C)*ArcTanh[(Sqrt[-2*c*f + (b -
 Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c
])*e]*Sqrt[f + g*x])])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*
a*c])*e]*(-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g)^(3/2)) - (4*(1 - b/Sqrt[b^2
- 4*a*c])*(B*c - b*C)*ArcTanh[(Sqrt[-2*c*f + (b - Sqrt[b^2 - 4*a*c])*g]*Sq
rt[d + e*x])/(Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(S
qrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]*(-2*c*f + (b - Sqrt[b^2 - 4*a*c])*
g)^(3/2)) + (8*c*(A*c - a*C)*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c]
)*g]*Sqrt[d + e*x])/(Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x
])])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*(-2*c*f +
 (b + Sqrt[b^2 - 4*a*c])*g)^(3/2)) - (4*(1 + b/Sqrt[b^2 - 4*a*c])*(B*c - b
*C)*ArcTanh[(Sqrt[-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt
[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])])/(Sqrt[-2*c*d + (b...
```

Rubi [A] (verified)

Time = 3.47 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+bx+cx^2\right)}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{\frac{-2acC+2Ac^2+b^2C-bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B}{\sqrt{d+ex}(f+gx)^{3/2}\left(-\sqrt{b^2-4ac}+b+2cx\right)}+\frac{-\frac{-2acC+2Ac^2+b^2C-bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B}{\sqrt{d+ex}(f+gx)^{3/2}\left(\sqrt{b^2-4ac}+b+2cx\right)}+\frac{C}{c\sqrt{d+ex}(\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4c\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(2cf-g\left(b-\sqrt{b^2-4ac}\right)\right)^{3/2}}-\frac{4c\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{f+gx}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)^{3/2}}-\frac{2g\sqrt{d+ex}\left(-\frac{-2c(Ac-aC)+b^2(-C)+bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B\right)}{\sqrt{f+gx}(ef-dg)\left(2cf-g\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{2g\sqrt{d+ex}\left(\frac{-2c(Ac-aC)+b^2(-C)+bBc}{c\sqrt{b^2-4ac}}-\frac{bC}{c}+B\right)}{\sqrt{f+gx}(ef-dg)\left(2cf-g\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{2C\sqrt{d+ex}}{c\sqrt{f+gx}(ef-dg)}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)^(3/2)*(a + b*x + c*x^2)),x]
```

output
```
(2*C*Sqrt[d + e*x])/(c*(e*f - d*g)*Sqrt[f + g*x]) - (2*(B - (b*C)/c - (b*B
*c - b^2*C - 2*c*(A*c - a*C))/(c*Sqrt[b^2 - 4*a*c]))*g*Sqrt[d + e*x])/((2*
c*f - (b - Sqrt[b^2 - 4*a*c])*g)*(e*f - d*g)*Sqrt[f + g*x]) - (2*(B - (b*C
)/c + (b*B*c - b^2*C - 2*c*(A*c - a*C))/(c*Sqrt[b^2 - 4*a*c]))*g*Sqrt[d +
e*x])/((2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)*(e*f - d*g)*Sqrt[f + g*x]) - (4
*c*(B - (b*C)/c - (b*B*c - b^2*C - 2*c*(A*c - a*C))/(c*Sqrt[b^2 - 4*a*c]))
*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[2*c*d - (b - Sqrt[b
^2 - 4*a*c])*e]*(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)^(3/2)) - (4*c*(B - (b*
C)/c + (b*B*c - b^2*C - 2*c*(A*c - a*C))/(c*Sqrt[b^2 - 4*a*c]))*ArcTanh[(S
qrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + S
qrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x])]])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164166 vs. $2(452) = 904$.

Time = 2.49 (sec) , antiderivative size = 164167, normalized size of antiderivative = 325.73

method	result	size
default	Expression too large to display	164167

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a),x,method=_RETU
RNVERBOSE)
```

output
```
result too large to display
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}\left(a+bx+cx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}(a+bx+cx^2)}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(f+gx)^{\frac{3}{2}}\,(a+bx+cx^2)}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+b*x+a),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*(f + g*x)**(3/2)*(a + b*x + c*x
**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^{3/2}(a+bx+cx^2)}\,dx=\int \frac{Cx^2+Bx+A}{(cx^2+bx+a)\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/((c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)^(3/
2)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^{3/2} (a + bx + cx^2)} \, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a),x, algor
ithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^{3/2} (a + bx + cx^2)} \, dx = \int \frac{C\,x^2 + B\,x + A}{(f + g\,x)^{3/2}\,\sqrt{d + e\,x}\,(c\,x^2 + b\,x + a)} \, dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(3/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),
x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(3/2)*(d + e*x)^(1/2)*(a + b*x + c*x^2)),
 x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.12

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^{3/2} (a + bx + cx^2)} \, dx = \frac{-2\sqrt{gx + f}\,\sqrt{ex + d}\,g - 2\sqrt{g}\,\sqrt{e}\,f - 2\sqrt{g}\,\sqrt{e}\,gx}{g\,(d\,g^2x - efgx + dfg - e\,f^2)}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^(3/2)/(c*x^2+b*x+a),x)
```

output
```
( - 2*(sqrt(f + g*x)*sqrt(d + e*x)*g + sqrt(g)*sqrt(e)*f + sqrt(g)*sqrt(e)
*g*x))/(g*(d*f*g + d*g**2*x - e*f**2 - e*f*g*x))
```

3.44 $\int \frac{(f+gx)^2(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 504

$$\int \frac{(f+gx)^2(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{(15b^2Ce^2g^2-2ceg(8aCeg+9b(2Cef-Cdg+Beg))+8c^2(3eg(2Bef-Bdg+Aeg)+C(2e^2f^2-6de}{24c^3e^3}$$
$$-\frac{g(5bCeg-c(4Cef-6Cdg+6Beg))x\sqrt{a+bx+cx^2}}{12c^2e^2}+\frac{C(f+gx)^2\sqrt{a+bx+cx^2}}{3ce}$$
$$-\frac{(5b^3Ce^3g^2-6bce^2g(2aCeg+b(2Cef-Cdg+Beg))+8c^2e(bC(ef-dg)^2+beg(2Bef-Bdg+Ae}{}$$
$$+\frac{(Cd^2-e(Bd-Ae))(ef-dg)^2\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^4\sqrt{cd^2-bde+ae^2}}$$

output
```
1/24*(15*b^2*C*e^2*g^2-2*c*e*g*(8*a*C*e*g+9*b*(B*e*g-C*d*g+2*C*e*f))+8*c^2
*(3*e*g*(A*e*g-B*d*g+2*B*e*f)+C*(3*d^2*g^2-6*d*e*f*g+2*e^2*f^2)))*(c*x^2+b
*x+a)^(1/2)/c^3/e^3-1/12*g*(5*b*C*e*g-c*(6*B*e*g-6*C*d*g+4*C*e*f))*x*(c*x^
2+b*x+a)^(1/2)/c^2/e^2+1/3*C*(g*x+f)^2*(c*x^2+b*x+a)^(1/2)/c/e-1/16*(5*b^3
*C*e^3*g^2-6*b*c*e^2*g*(2*a*C*e*g+b*(B*e*g-C*d*g+2*C*e*f))+8*c^2*e*(b*C*(-
d*g+e*f)^2+b*e*g*(A*e*g-B*d*g+2*B*e*f)+a*e*g*(B*e*g-C*d*g+2*C*e*f))+16*c^3
*(C*d*(-d*g+e*f)^2-e*(B*(-d*g+e*f)^2+A*e*g*(-d*g+2*e*f))))*arctanh(1/2*(2*
c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/e^4+(C*d^2-e*(-A*e+B*d))*(-d*g
+e*f)^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(
c*x^2+b*x+a)^(1/2))/e^4/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 5.43 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.90

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{2e\sqrt{a+x(b+cx)}\left(15b^2Ce^2g^2-2ceg(9bBeg+8aCeg+bC(18ef-9dg+5egx))+4c^2\left(3eg(2Aeg+B(4ef-2dg+egx))+C\left(6d^2g^2-3deg(4f+gx)+2e^2\right.\right.\right.}{c^3}}{}$$

input
```
Integrate[((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(15*b^2*C*e^2*g^2 - 2*c*e*g*(9*b*B*e*g + 8*a*C
*e*g + b*C*(18*e*f - 9*d*g + 5*e*g*x)) + 4*c^2*(3*e*g*(2*A*e*g + B*(4*e*f
- 2*d*g + e*g*x)) + C*(6*d^2*g^2 - 3*d*e*g*(4*f + g*x) + 2*e^2*(3*f^2 + 3*
f*g*x + g^2*x^2)))))/c^3 + (96*Sqrt[-(c*d^2) + e*(b*d - a*e)]*(C*d^2 + e*(
-(B*d) + A*e))*(e*f - d*g)^2*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b +
 c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + (3*(
5*b^3*C*e^3*g^2 - 6*b*c*e^2*g*(2*a*C*e*g + b*(2*C*e*f - C*d*g + B*e*g)) +
8*c^2*e*(b*C*(e*f - d*g)^2 + b*e*g*(2*B*e*f - B*d*g + A*e*g) + a*e*g*(2*C*
e*f - C*d*g + B*e*g)) + 16*c^3*(C*d*(e*f - d*g)^2 - e*(B*(e*f - d*g)^2 + A
*e*g*(2*e*f - d*g))))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/c^
(7/2))/(48*e^4)
```

Rubi [A] (verified)

Time = 1.80 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(A+Bx+Cx^2\right)}{(d+ex)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{(ef-dg)^2\left(Ae^2-Bde+Cd^2\right)}{e^4(d+ex)\sqrt{a+bx+cx^2}}+\frac{e\left(Aeg(2ef-dg)+B(ef-dg)^2\right)-Cd(ef-dg)^2}{e^4\sqrt{a+bx+cx^2}}+\frac{x\left(eg(Aeg-Bdg+\right.}{e^3\sqrt{a+}}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(eg(Aeg-Bdg+2Bef)+C(ef-dg)^2\right)}{2c^{3/2}e^3}+\frac{(ef-dg)^2\left(Cd^2-e(Bd-Ae)\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(Cd(ef-dg)^2-e\left(Aeg(2ef-dg)+B(ef-dg)^2\right)\right)}{\sqrt{c}e^4}+\frac{\sqrt{a+bx+cx^2}\left(eg(Aeg-Bdg+2Bef)+C(ef-dg)^2\right)}{ce^3}+\frac{g\left(3b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(Beg-Cdg+2Cef)}{8c^{5/2}e^2}-\frac{bCg^2\left(5b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{7/2}e}+\frac{Cg^2\left(-16ac+15b^2-10bcx\right)\sqrt{a+bx+cx^2}}{24c^3e}-\frac{3bg\sqrt{a+bx+cx^2}(Beg-Cdg+2Cef)}{4c^2e^2}+\frac{gx\sqrt{a+bx+cx^2}(Beg-Cdg+2Cef)}{2ce^2}+\frac{Cg^2x^2\sqrt{a+bx+cx^2}}{3ce}$$

input
```
Int[((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-3*b*g*(2*C*e*f - C*d*g + B*e*g)*Sqrt[a + b*x + c*x^2])/(4*c^2*e^2) + ((C
*(e*f - d*g)^2 + e*g*(2*B*e*f - B*d*g + A*e*g))*Sqrt[a + b*x + c*x^2])/(c*
e^3) + (g*(2*C*e*f - C*d*g + B*e*g)*x*Sqrt[a + b*x + c*x^2])/(2*c*e^2) + (
C*g^2*x^2*Sqrt[a + b*x + c*x^2])/(3*c*e) + (C*g^2*(15*b^2 - 16*a*c - 10*b*
c*x)*Sqrt[a + b*x + c*x^2])/(24*c^3*e) - (b*(5*b^2 - 12*a*c)*C*g^2*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(7/2)*e) + ((3*b^2
- 4*a*c)*g*(2*C*e*f - C*d*g + B*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
 + b*x + c*x^2])])/(8*c^(5/2)*e^2) - (b*(C*(e*f - d*g)^2 + e*g*(2*B*e*f -
B*d*g + A*e*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2
*c^(3/2)*e^3) - ((C*d*(e*f - d*g)^2 - e*(B*(e*f - d*g)^2 + A*e*g*(2*e*f -
d*g)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e^
4) + ((C*d^2 - e*(B*d - A*e))*(e*f - d*g)^2*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^4*Sqr
t[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.52

method	result
default	$\frac{(A e^2 g^2 - Bde g^2 + 2B e^2 fg + C d^2 g^2 - 2Cdefg + C e^2 f^2)\left(\frac{\sqrt{c x^2+bx+a}}{c} - \frac{b \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}} + \sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{e^3} - \frac{(A d^2 e^2 g^2 - 2Ad e^3 fg + A \cdots}{\cdots}$
risch	$\frac{(8C g^2 e^2 c^2 x^2 + 12B c^2 e^2 g^2 x - 10Cbc e^2 g^2 x - 12C c^2 de g^2 x + 24C c^2 e^2 fgx + 24A c^2 e^2 g^2 - 18Bbc e^2 g^2 - 24B c^2 de g^2 + 48B c^2 e^2 fg - 16 \cdots}{24c^3 e^3}$

input

```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output

```
1/e^3*(A*e^2*g^2-B*d*e*g^2+2*B*e^2*f*g+C*d^2*g^2-2*C*d*e*f*g+C*e^2*f^2)*(1
/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2)))-(A*d^2*e^2*g^2-2*A*d*e^3*f*g+A*e^4*f^2-B*d^3*e*g^2+2*B*d^2*e^2*f*g
-B*d*e^3*f^2+C*d^4*g^2-2*C*d^3*e*f*g+C*d^2*e^2*f^2)/e^5/((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^
2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x+d/e))+C/e*g^2*(1/3*x^2/c*(c*x^2+b*x+a)^(1/2)-5/6*b/c
*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-2/3*a/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^
(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))))+1/e^2*g*(B*e*g-C*d*g+2
*C*e*f)*(1/2*x/c*(c*x^2+b*x+a)^(1/2)-3/4*b/c*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*
b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-1/2*a/c^(3/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))-(A*d*e^2*g^2-2*A*e^3*f*g-B*d^2*e*g
^2+2*B*d*e^2*f*g-B*e^3*f^2+C*d^3*g^2-2*C*d^2*e*f*g+C*d*e^2*f^2)/e^4*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**2*(C*x**2+B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)**2*(A + B*x + C*x**2)/((d + e*x)*sqrt(a + b*x + c*x**2)
), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^2\,(C\,x^2+B\,x+A)}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x
)
```

Reduce [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 13191, normalized size of antiderivative = 26.17

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
( - 48*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a
*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*
d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)
*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*s
qrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*
d*e - 8*c**2*d**2))*b*c**3*d**2*e*g**2 + 96*sqrt(4*sqrt(c)*sqrt(a*e**2 - b
*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e
**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*a
tan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sq
rt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*
c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*b*c**3*d*e**2*f*g
 - 48*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*b*c**3*e**3*f**2 + 96*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*
e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2
 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan
((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sq...
```

3.45 $\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 296

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = -\frac{(3bCeg-4c(Cef-Cdg+Beg))\sqrt{a+bx+cx^2}}{4c^2e^2}+\frac{Cgx\sqrt{a+bx+cx^2}}{2ce}$$
$$+\frac{(3b^2Ce^2g-8c^2(Cd(ef-dg)-e(Bef-Bdg+Aeg))-4ce(aCeg+b(Cef-Cdg+Beg)))\,\mathrm{arctanh}}{8c^{5/2}e^3}$$
$$+\frac{(Cd^2-e(Bd-Ae))\,(ef-dg)\mathrm{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^3\sqrt{cd^2-bde+ae^2}}$$

output

```
-1/4*(3*b*C*e*g-4*c*(B*e*g-C*d*g+C*e*f))*(c*x^2+b*x+a)^(1/2)/c^2/e^2+1/2*C
*g*x*(c*x^2+b*x+a)^(1/2)/c/e+1/8*(3*b^2*C*e^2*g-8*c^2*(C*d*(-d*g+e*f)-e*(A
*e*g-B*d*g+B*e*f))-4*c*e*(a*C*e*g+b*(B*e*g-C*d*g+C*e*f)))*arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/e^3+(C*d^2-e*(-A*e+B*d))*(-d*g+
e*f)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^2+b*x+a)^(1/2))/e^3/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.93

$$\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{2e\sqrt{a+x(b+cx)}(4Bceg-3bCeg+2cC(2ef-2dg+egx))}{c^2} - \frac{16\sqrt{-cd^2+bde-ae^2}(Cd^2+e(-Bd+Ae))(-ef+dg)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}}{8e^3}$$

input
```
Integrate[((f + g*x)*(A + B*x + C*x^2))/((d + e*x)*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
((2*e*Sqrt[a + x*(b + c*x)]*(4*B*c*e*g - 3*b*C*e*g + 2*c*C*(2*e*f - 2*d*g
+ e*g*x)))/c^2 - (16*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(C*d^2 + e*(-(B*d) + A
*e))*(-(e*f) + d*g)*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/S
qrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + ((-3*b^2*C*e^
2*g - 8*c^2*(C*d*(-(e*f) + d*g) + e*(B*e*f - B*d*g + A*e*g)) + 4*c*e*(a*C*
e*g + b*(C*e*f - C*d*g + B*e*g)))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b
+ c*x)]])/c^(5/2))/(8*e^3)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2153}$$

$$\int \left(\frac{(ef-dg)(Ae^2-Bde+Cd^2)}{e^3(d+ex)\sqrt{a+bx+cx^2}} + \frac{e(Aeg-Bdg+Bef)-Cd(ef-dg)}{e^3\sqrt{a+bx+cx^2}} + \frac{x(Beg-Cdg+Cef)}{e^2\sqrt{a+bx+cx^2}} + \frac{Cg}{e\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(ef-dg)\left(Cd^2-e(Bd-Ae)\right)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(Cd(ef-dg)-e(Aeg-Bdg+Bef)\right)}{\sqrt{c}e^3}+\frac{Cg\left(3b^2-4ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}e}-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(Beg-Cdg+Cef)}{2c^{3/2}e^2}+\frac{\sqrt{a+bx+cx^2}(Beg-Cdg+Cef)}{ce^2}-\frac{3bCg\sqrt{a+bx+cx^2}}{4c^2e}+\frac{Cgx\sqrt{a+bx+cx^2}}{2ce}$$

input
```
Int[((f + g*x)*(A + B*x + C*x^2))/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-3*b*C*g*Sqrt[a + b*x + c*x^2])/(4*c^2*e) + ((C*e*f - C*d*g + B*e*g)*Sqrt
[a + b*x + c*x^2])/(c*e^2) + (C*g*x*Sqrt[a + b*x + c*x^2])/(2*c*e) + ((3*b
^2 - 4*a*c)*C*g*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8
*c^(5/2)*e) - (b*(C*e*f - C*d*g + B*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sq
rt[a + b*x + c*x^2])])/(2*c^(3/2)*e^2) - ((C*d*(e*f - d*g) - e*(B*e*f - B*
d*g + A*e*g))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[c]*e^3) + ((C*d^2 - e*(B*d - A*e))*(e*f - d*g)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^
3*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) && !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.32

method	result
risch	$\frac{(2gCcex+4Bgce-3bCeg-4Ccdg+4Cfce)\sqrt{c\,x^2+bx+a}}{4c^2e^2}+\frac{\frac{\left(8A\,c^2e^2g-4Bbc\,e^2g-8B\,c^2deg+8B\,c^2e^2f-4Cac\,e^2g+3b^2C\,e^2g+4Cbcdeg-\cdots\right.}{e\sqrt{c}}}{\cdots}$
default	$\frac{e(Beg-Cdg+Cef)\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)+\frac{A\,e^2g\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+\frac{B\,e^2f\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+\cdots}{\cdots}$

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
1/4*(2*C*c*e*g*x+4*B*c*e*g-3*C*b*e*g-4*C*c*d*g+4*C*c*e*f)*(c*x^2+b*x+a)^(1
/2)/c^2/e^2+1/8/c^2/e^2*((8*A*c^2*e^2*g-4*B*b*c*e^2*g-8*B*c^2*d*e*g+8*B*c^
2*e^2*f-4*C*a*c*e^2*g+3*C*b^2*e^2*g+4*C*b*c*d*e*g-4*C*b*c*e^2*f+8*C*c^2*d^
2*g-8*C*c^2*d*e*f)/e*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+8
*(A*d*e^2*g-A*e^3*f-B*d^2*e*g+B*d*e^2*f+C*d^3*g-C*d^2*e*f)*c^2/e^2/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)*(C*x**2+B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)*(A + B*x + C*x**2)/((d + e*x)*sqrt(a + b*x + c*x**2)),
x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)\,\left(C\,x^2+B\,x+A\right)}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 8335, normalized size of antiderivative = 28.16

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(8*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**
2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2
)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e +
 b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(
c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e
- 8*c**2*d**2))*b*c**2*d*e*g - 8*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*
e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt
(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 -
 b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c
*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*b*c**2*e**2*f - 16*sqrt(4*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e +
c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**
2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e
*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e*
*2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**
2))*c**3*d**2*g + 16*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*
sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c
*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a +
 b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + ...
```

3.46 $\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx = \frac{C\sqrt{a+bx+cx^2}}{ce} - \frac{(2cCd-2Bce+bCe)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}e^2} + \frac{(Cd^2-e(Bd-Ae))\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e^2\sqrt{cd^2-bde+ae^2}}$$

output
```
C*(c*x^2+b*x+a)^(1/2)/c/e-1/2*(-2*B*c*e+C*b*e+2*C*c*d)*arctanh(1/2*(2*c*x+
b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/e^2+(C*d^2-e*(-A*e+B*d))*arctanh(1
/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2
))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$=\frac{\frac{2Ce\sqrt{a+x(b+cx)}}{c}+\frac{4\sqrt{-cd^2+bde-ae^2}\left(Cd^2+e(-Bd+Ae)\right)\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}-\frac{(2cCd-2Bce+bCe)\text{arctanh}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{c^{3/2}}}{2e^2}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((2*C*e*Sqrt[a + x*(b + c*x)])/c + (4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(C*d^
2 + e*(-(B*d) + A*e))*ArcTan[(Sqrt[c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])
/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) - ((2*c*C*d -
 2*B*c*e + b*C*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/
c^(3/2))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2184, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \ 2184$$
$$\frac{\int -\frac{e(bCd-2Ace+(2cCd-2Bce+bCe)x)}{2(d+ex)\sqrt{cx^2+bx+a}}dx}{ce^2}+\frac{C\sqrt{a+bx+cx^2}}{ce}$$
$$\Big\downarrow \ 27$$
$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\int \frac{bCd-2Ace+(2cCd-2Bce+bCe)x}{(d+ex)\sqrt{cx^2+bx+a}}dx}{2ce}$$

$$\Big\downarrow \text{1269}$$

$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{(bCe-2Bce+2cCd)\int\frac{1}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{2c(Ae^2-Bde+Cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{1092}$$

$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{2(bCe-2Bce+2cCd)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{e}-\frac{2c(Ae^2-Bde+Cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bCe-2Bce+2cCd)}{\sqrt{ce}}-\frac{2c(Ae^2-Bde+Cd^2)\int\frac{1}{(d+ex)\sqrt{cx^2+bx+a}}dx}{e}}{2ce}$$

$$\Big\downarrow \text{1154}$$

$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{4c(Ae^2-Bde+Cd^2)\int\frac{1}{4\left(cd^2-bed+ae^2\right)-\frac{(bd-2ae+(2cd-be)x)^2}{cx^2+bx+a}}d\left(-\frac{bd-2ae+(2cd-be)x}{\sqrt{cx^2+bx+a}}\right)}{e}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bCe-2Bce+2cCd)}{\sqrt{ce}}}{2ce}$$

$$\Big\downarrow \text{219}$$

$$\frac{C\sqrt{a+bx+cx^2}}{ce}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(bCe-2Bce+2cCd)}{\sqrt{ce}}-\frac{2c(Ae^2-Bde+Cd^2)\text{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e\sqrt{ae^2-bde+cd^2}}}{2ce}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(C*Sqrt[a + b*x + c*x^2])/(c*e) - (((2*c*C*d - 2*B*c*e + b*C*e)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e) - (2*c*(C*d^2 -
B*d*e + A*e^2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d
*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*e^2]))/(2*c
*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.41

method	result
risch	$\frac{C\sqrt{c\,x^2+bx+a}}{ce}+\frac{\frac{(2Bce-Cbe-2Ccd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{e\sqrt{c}}-\frac{2\left(A\,e^2-Bde+C\,d^2\right)c\ln\left(\frac{\frac{2a\,e^2-2bde+2c\,d^2}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{a\,e^2 \ldots}}{\ldots}\right)}{e^2\sqrt{\frac{a\,e^2-bde+\ldots}{e^2}}}}{2ce}$
default	$\frac{\frac{Be\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+Ce\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)-\frac{Cd\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}}{e^2}-\frac{\left(A\,e^2-Bde+C\,d^2\right)\ldots}{\ldots}$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
C*(c*x^2+b*x+a)^(1/2)/c/e+1/2/c/e*((2*B*c*e-C*b*e-2*C*c*d)/e*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-2*(A*e^2-B*d*e+C*d^2)*c/e^2/((a*e^2
-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{a + bx + cx^2}}\,dx = \int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{a + bx + cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume((b/e-(2*c*d)/e^2)^2>0)', see `as
sume?` for
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A + Bx + Cx^2}{(d + ex)\sqrt{a + bx + cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 3976, normalized size of antiderivative = 22.21

$$\int \frac{A+Bx+Cx^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

output

```
( - 2*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*
e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d
**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*sqrt(a + b*x + c*x**2)*
e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d
*e - 8*c**2*d**2))*b*c*e + 4*sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*
b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2
 + 8*b*c*d*e - 8*c**2*d**2)*sqrt(a*e**2 - b*d*e + c*d**2)*atan((2*sqrt(c)*
sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b*d
*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e**
2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*c**2*d - 4*sqrt(c)*sqrt(4*sqrt(c
)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d*
*2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sqrt(c
)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2 - b
*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - 4*a*c*e
**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2))*a*c*e**2 + 4*sqrt(c)*sqrt(4*sq
rt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e +
c*d**2)*c*d - 4*a*c*e**2 - b**2*e**2 + 8*b*c*d*e - 8*c**2*d**2)*atan((2*sq
rt(c)*sqrt(a + b*x + c*x**2)*e + b*e + 2*c*e*x)/sqrt(4*sqrt(c)*sqrt(a*e**2
 - b*d*e + c*d**2)*b*e - 8*sqrt(c)*sqrt(a*e**2 - b*d*e + c*d**2)*c*d - ...
```

3.47 $\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 263

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{C\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}eg}+\frac{(Cd^2-e(Bd-Ae))\operatorname{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{e\sqrt{cd^2-bde+ae^2}(ef-dg)} - \frac{(Cf^2-g(Bf-Ag))\operatorname{arctanh}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)\sqrt{cf^2-bfg+ag^2}}$$

output

```
C*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(1/2)/e/g+(C*d^2-e*
(-A*e+B*d))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b*d*e+c*d^2)^(1/
2)/(c*x^2+b*x+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e*f)-(C*f^2-g*(-
A*g+B*f))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)
/(c*x^2+b*x+a)^(1/2))/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.09

$$\begin{aligned}&\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx\\&=\frac{2\sqrt{-cd^2+e(bd-ae)}(Cd^2+e(-Bd+Ae))\arctan\left(\frac{\sqrt{c}(d+ex)-e\sqrt{a+x(b+cx)}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\left(cd^2+e(-bd+ae)\right)(ef-dg)}\\&\quad+\frac{2\sqrt{-cf^2+bfg-ag^2}(Cf^2+g(-Bf+Ag))\arctan\left(\frac{\sqrt{c}(f+gx)-g\sqrt{a+x(b+cx)}}{\sqrt{-cf^2+g(bf-ag)}}\right)}{g(-ef+dg)\left(cf^2+g(-bf+ag)\right)}\\&\quad-\frac{C\log\left(eg\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}eg}\end{aligned}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[-(c*d^2) + e*(b*d - a*e)]*(C*d^2 + e*(-(B*d) + A*e))*ArcTan[(Sqrt[
c]*(d + e*x) - e*Sqrt[a + x*(b + c*x)])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(
e*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)) + (2*Sqrt[-(c*f^2) + b*f*g - a*g
^2]*(C*f^2 + g*(-(B*f) + A*g))*ArcTan[(Sqrt[c]*(f + g*x) - g*Sqrt[a + x*(b
 + c*x)])/Sqrt[-(c*f^2) + g*(b*f - a*g)]])/(g*(-(e*f) + d*g)*(c*f^2 + g*(-
(b*f) + a*g))) - (C*Log[e*g*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
)/(Sqrt[c]*e*g)
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 2153$$

$$\int \left(\frac{Ae^2 - Bde + Cd^2}{e(d+ex)\sqrt{a+bx+cx^2}(ef-dg)} + \frac{Ag^2 - Bfg + Cf^2}{g(f+gx)\sqrt{a+bx+cx^2}(dg-ef)} + \frac{C}{eg\sqrt{a+bx+cx^2}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(Cd^2 - e(Bd-Ae))\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef-dg)\sqrt{ae^2-bde+cd^2}} - \frac{(Cf^2 - g(Bf-Ag))\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)\sqrt{ag^2-bfg+cf^2}} + \frac{C\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}eg}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(C*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*e*g) +
 ((C*d^2 - e*(B*d - A*e))*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[
c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e*Sqrt[c*d^2 - b*d*e + a*
e^2]*(e*f - d*g)) - ((C*f^2 - g*(B*f - A*g))*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f
 - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.52

method	result
default	$\frac{C\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{eg\sqrt{c}}+\frac{(A\,e^{2}-Bde+C\,d^{2})\ln\left(\frac{\frac{2a\,e^{2}-2bde+2c\,d^{2}}{e^{2}}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}\sqrt{\left(x+\frac{d}{e}\right)^{2}c+\frac{(be-2cd}{e}}}{x+\frac{d}{e}}\right.}{e^{2}(dg-ef)\sqrt{\frac{a\,e^{2}-bde+c\,d^{2}}{e^{2}}}}$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
C/e/g*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+1/e^2*(A*e^2-B*d
*e+C*d^2)/(d*g-e*f)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*((x+d/e)^2
*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/e))-1/g^2*(A
*g^2-B*f*g+C*f^2)/(d*g-e*f)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b
*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((
x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)),
x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(f+g\,x)\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{a\,e^2-bde+c\,d^2}\log\left(-2\sqrt{c\,x^2+bx+a}\,\sqrt{a\,e^2-bde+c\,d^2}-2ae+bd-bex+2cdx\right)g-\sqrt{a\,e^2-bde}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
(sqrt(a*e**2 - b*d*e + c*d**2)*log( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*g - sqrt(a*e**2 - b*d*
e + c*d**2)*log(d + e*x)*g + sqrt(a*g**2 - b*f*g + c*f**2)*log(2*sqrt(a +
b*x + c*x**2)*sqrt(a*g**2 - b*f*g + c*f**2) - 2*a*g + b*f - b*g*x + 2*c*f*
x)*e - sqrt(a*g**2 - b*f*g + c*f**2)*log(f + g*x)*e + sqrt(c)*log( - 2*sqr
t(c)*sqrt(a + b*x + c*x**2) - b - 2*c*x)*d*g - sqrt(c)*log( - 2*sqrt(c)*sq
rt(a + b*x + c*x**2) - b - 2*c*x)*e*f)/(e*g*(d*g - e*f))
```

3.48 $\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 367

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{(Cf^2-g(Bf-Ag))\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)} + \frac{(Cd^2-e(Bd-Ae))\,\text{arctanh}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}}\right)}{\sqrt{cd^2-bde+ae^2}(ef-dg)^2} + \frac{(2a(Bd-Ae)g^3-bCf^2(ef-3dg)+2aCfg(ef-2dg)-2cf(Cdf^2-Bef^2+Ag(2ef-dg))+bg(A}{2(ef-dg)^2\left(cf^2-bfg+ag^2\right)^{3/2}}$$

output
```
(C*f^2-g*(-A*g+B*f))*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(g
*x+f)+(C*d^2-e*(-A*e+B*d))*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x)/(a*e^2-b
*d*e+c*d^2)^(1/2)/(c*x^2+b*x+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)/(-d*g+e*f
)^2+1/2*(2*a*(-A*e+B*d)*g^3-b*C*f^2*(-3*d*g+e*f)+2*a*C*f*g*(-2*d*g+e*f)-2*
c*f*(C*d*f^2-B*e*f^2+A*g*(-d*g+2*e*f))+b*g*(A*g*(-d*g+3*e*f)-B*f*(d*g+e*f)
))*arctanh(1/2*(b*f-2*a*g+(-b*g+2*c*f)*x)/(a*g^2-b*f*g+c*f^2)^(1/2)/(c*x^2
+b*x+a)^(1/2))/(-d*g+e*f)^2/(a*g^2-b*f*g+c*f^2)^(3/2)
```

Mathematica [A] (verified)

Time = 11.57 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{\frac{2(ef-dg)(Cf^2+g(-Bf+Ag))\sqrt{a+x(b+cx)}}{(cf^2+g(-bf+ag))(f+gx)} + \frac{2(Cd^2+e(-Bd+Ae))\text{arctanh}\left(\frac{-2ae+2cdx+b(d-ex)}{2\sqrt{cd^2+e(-bd+ae)}\sqrt{a+x(b+cx)}}\right)}{\sqrt{cd^2+e(-bd+ae)}} - \frac{(2cf-bg)(ef-dg)(Cf}{}}{2(ef-}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
((2*(e*f - d*g)*(C*f^2 + g*(-(B*f) + A*g))*Sqrt[a + x*(b + c*x)])/((c*f^2
+ g*(-(b*f) + a*g))*(f + g*x)) + (2*(C*d^2 + e*(-(B*d) + A*e))*ArcTanh[(-2
*a*e + 2*c*d*x + b*(d - e*x))/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c*d^2 + e*(-(b*d) + a*e)] - ((2*c*f - b*g)*(e*f - d*g)
*(C*f^2 + g*(-(B*f) + A*g))*ArcTanh[(-2*a*g + 2*c*f*x + b*(f - g*x))/(2*Sq
rt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b + c*x)])])/(g*(c*f^2 + g*(-(b*f
) + a*g))^(3/2)) + (2*((B*d - A*e)*g^2 + C*f*(e*f - 2*d*g))*ArcTanh[(-2*a*
g + 2*c*f*x + b*(f - g*x))/(2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a + x*(b
 + c*x)])])/(g*Sqrt[c*f^2 + g*(-(b*f) + a*g)]))/(2*(e*f - d*g)^2)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {2153, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 2153$$

$$\int \left(\frac{Ae^2-Bde+Cd^2}{(d+ex)\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{Ag^2-Bfg+Cf^2}{g(f+gx)^2\sqrt{a+bx+cx^2}(dg-ef)}+\frac{g^2(Bd-Ae)+Cf(ef-2dg}{g(f+gx)\sqrt{a+bx+cx^2}(ef-d}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Cd^2-e(Bd-Ae))\operatorname{arctanh}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}}-\frac{(2cf-bg)\left(Cf^2-g(Bf-Ag)\right)\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)\left(ag^2-bfg+cf^2\right)^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)\left(g^2(Bd-Ae)+Cf(ef-2dg)\right)}{g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}}+\frac{\sqrt{a+bx+cx^2}\left(Cf^2-g(Bf-Ag)\right)}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
((C*f^2 - g*(B*f - A*g))*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*
g + a*g^2)*(f + g*x)) + ((C*d^2 - e*(B*d - A*e))*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(Sq
rt[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^2) - ((2*c*f - b*g)*(C*f^2 - g*(B*f
- A*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*
g^2]*Sqrt[a + b*x + c*x^2])])/(2*g*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^(3/
2)) + (((B*d - A*e)*g^2 + C*f*(e*f - 2*d*g))*ArcTanh[(b*f - 2*a*g + (2*c*f
 - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g*(e*f
 - d*g)^2*Sqrt[c*f^2 - b*f*g + a*g^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2153
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*
x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && PolyQ[Px, x] && (IntegerQ[p] || (IntegerQ[2*p] && IntegerQ
[m] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] && IGtQ[n, 0])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.83

method	result
default	$-\frac{(A e^2-B d e+C d^2)\ln\left(\frac{\frac{2a e^2-2b d e+2c d^2}{e^2}+\frac{(b e-2c d)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-b d e+c d^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c+\frac{(b e-2c d)\left(x+\frac{d}{e}\right)}{e}+\frac{a e^2-b d e+c d^2}{e^2}}}{x+\frac{d}{e}}\right)}{(d g-e f)^2 e\sqrt{\frac{a e^2-b d e+c d^2}{e^2}}}+$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-(A*e^2-B*d*e+C*d^2)/(d*g-e*f)^2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x
+d/e))+(A*g^2-B*f*g+C*f^2)/g^3/(d*g-e*f)*(-1/(a*g^2-b*f*g+c*f^2)*g^2/(x+f/
g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*(
b*g-2*c*f)*g/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*
g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/
2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f
/g)))+(A*e*g^2-B*d*g^2+2*C*d*f*g-C*e*f^2)/(d*g-e*f)^2/g^2/((a*g^2-b*f*g+c*
f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*
g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*
g+c*f^2)/g^2)^(1/2))/(x+f/g))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)\,(f+gx)^2\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)*(f + g*x)**2*sqrt(a + b*x + c*x**2)
), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)(gx+f)^2}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2),
 x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{(f+g\,x)^2\,(d+e\,x)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1758, normalized size of antiderivative = 4.79

$$\int \frac{A+Bx+Cx^2}{(d+ex)(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int((C*x^2+B*x+A)/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

output

```
(2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2
- b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*f*g**2 + 2*sqrt(a*e**
2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d
**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*a*g**3*x - 2*sqrt(a*e**2 - b*d*e + c
*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e
+ b*d - b*e*x + 2*c*d*x)*b*f**2*g - 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*
sqrt(a + b*x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x
 + 2*c*d*x)*b*f*g**2*x + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*
x + c*x**2)*sqrt(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)
*c*f**3 + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(2*sqrt(a + b*x + c*x**2)*sqr
t(a*e**2 - b*d*e + c*d**2) - 2*a*e + b*d - b*e*x + 2*c*d*x)*c*f**2*g*x - 2
*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*a*f*g**2 - 2*sqrt(a*e**2 - b*d
*e + c*d**2)*log(d + e*x)*a*g**3*x + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d
 + e*x)*b*f**2*g + 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*b*f*g**2*x
 - 2*sqrt(a*e**2 - b*d*e + c*d**2)*log(d + e*x)*c*f**3 - 2*sqrt(a*e**2 - b
*d*e + c*d**2)*log(d + e*x)*c*f**2*g*x + 2*sqrt(a*g**2 - b*f*g + c*f**2)*l
og( - 2*sqrt(a + b*x + c*x**2)*sqrt(a*g**2 - b*f*g + c*f**2) - 2*a*g + b*f
 - b*g*x + 2*c*f*x)*a*e*f*g + 2*sqrt(a*g**2 - b*f*g + c*f**2)*log( - 2*sqr
t(a + b*x + c*x**2)*sqrt(a*g**2 - b*f*g + c*f**2) - 2*a*g + b*f - b*g*x +
2*c*f*x)*a*e*g**2*x - sqrt(a*g**2 - b*f*g + c*f**2)*log( - 2*sqrt(a + b...
```

3.49 $\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 434

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}} - \frac{4\sqrt{2}\sqrt{b^2-4ac}(Bf-Ag)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4ac}g}{2cf-\left(b+\sqrt{b^2-4ac}\right)g},\arcsin\left(\frac{\sqrt{1}}{}\right.\right.}{g\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2*2^(1/2)*B*(-4*a*c+b^2)^(1/2)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)
)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(
-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*
c+b^2)^(1/2))*e))^(1/2))/c/g/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-4*2^(1/2)*(
-4*a*c+b^2)^(1/2)*(-A*g+B*f)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-
4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)*g/(2*c*f-(b+(-4*a*c+
b^2)^(1/2))*g),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^
(1/2))/g/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.78 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx$$

$$= \frac{i(d+ex)\sqrt{1-\frac{2(cd^2+e(-bd+ae))}{\left(2cd-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{2+\frac{4(cd^2+e(-bd+ae))}{\left(-2cd+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\left((-Bdg+Aeg)\,\text{EllipticF}\left(ia\right.\right.}{}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x]*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(I*(d + e*x)*Sqrt[1 - (2*(c*d^2 + e*(-(b*d) + a*e)))/((2*c*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(c*d^2 + e*(-(b*d) + a*e)))/((
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((-(B*d*g) + A*e*g)*El
lipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + e*(B*f - A*g)*Ellipt
icPi[((-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(e*f - d*g))/(2*(c*d^2 + e*
(-(b*d) + a*e))*g), I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e + Sqr
t[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(e*Sqrt[
(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*g*(e*
f - d*g)*Sqrt[a + x*(b + c*x)])
```

Rubi [A] (warning: unable to verify)

Time = 2.38 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.39, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2154, 27, 1172, 321, 1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}dx$$

$\Big\downarrow$ 2154

$$\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{B}{g\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$\Big\downarrow$ 27

$$\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\frac{B\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{g}$$

$\Big\downarrow$ 1172

$$\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+ \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 321

$$\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+ \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 1279

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}(f+gx)}dx}{\sqrt{a+bx+cx^2}}+ \frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\Big\downarrow$ 187

$$\frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\int -\frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}}{\sqrt{a+bx+cx^2}}$$

$\downarrow$ 25

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\int \frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}(e}}{\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int \frac{}{\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$$\frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{}{\sqrt{1}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}+b}}$$

$$\frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 412

$$\frac{\sqrt{2}\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A-\frac{Bf}{g}\right)\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)\sqrt{-\sqrt{b^2-4ac}+b+2}}$$

$$\frac{2\sqrt{2}B\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x]*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*B*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*g*Sqrt[d + e*x]
*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
*(A - (B*f)/g)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a
*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[
-1/2*((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*g)/(c*(e*f - d*g)), ArcSin[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt
[c]*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*d)
/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d +
 e*x))/e])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 15.94 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2B\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x+\frac{b+\sqrt{-4ac+b^2}}{2c}}{-\frac{d}{e}+\frac{b+\sqrt{-4ac+b^2}}{2c}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x \ldots}{\frac{d}{e}-\frac{b+\ldots}{\ldots}}}\ldots\right.}{g\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}\ldots\right)}{\ldots}$
default	Expression too large to display

input
```
int((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*B/g*(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c
))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^
(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*EllipticF
(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(A*g-B*f)/g^
2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(f/g
-d/e)*EllipticPi(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),(-d/e+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(f/g-d/e),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{\sqrt{d+ex}\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(1/2)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)), x)
```

Giac [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+B\,x}{(f+g\,x)\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/((f + g*x)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((f + g*x)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{ex+d}\,(gx+f)\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.50 $\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 418

$$
\begin{aligned}
&\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \\
&-\frac{\sqrt{1+2x}(54857259443+10865116404x)\sqrt{2+7x+4x^2}}{175134960} \\
&-\frac{\sqrt{1+2x}(5684701+1459136x)(2+7x+4x^2)^{3/2}}{972972} \\
&-\frac{(175-132x)\sqrt{1+2x}(2+7x+4x^2)^{5/2}}{2574} \\
&-\frac{132358116469\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)\middle|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{70053984\sqrt{-1-2x}\sqrt{2+7x+4x^2}} \\
&-\frac{2987675052409\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{75057840\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}} \\
&+\frac{13484913472\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{1}\right.\right.}{6561\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}
\end{aligned}
$$

output
```
-1/175134960*(1+2*x)^(1/2)*(54857259443+10865116404*x)*(4*x^2+7*x+2)^(1/2)
-1/972972*(1+2*x)^(1/2)*(5684701+1459136*x)*(4*x^2+7*x+2)^(3/2)-1/2574*(17
5-132*x)*(1+2*x)^(1/2)*(4*x^2+7*x+2)^(5/2)-132358116469/70053984*(3+17^(1/
2))^(1/2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^(1/2)*(
7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4*x^2+7*
x+2)^(1/2)-2987675052409/75057840*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*Elli
pticF(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))
/(3+17^(1/2))^(1/2)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+13484913472/6561*(-1
-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/
2)*34^(1/2),-153/1784+183/1784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(
1/2))^(1/2)/(61+3*17^(1/2))/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.45 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.85

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \frac{12\sqrt{3+\sqrt{17}}(-718741962148-2648350226628x-194627585}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(2 + 7*x + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 +
2*x]),x]
```

output
```
(12*Sqrt[3 + Sqrt[17]]*(-718741962148 - 2648350226628*x - 1946275855429*x^
2 - 434091365288*x^3 - 139410462456*x^4 - 34156062720*x^5 - 932037120*x^6
+ 2543063040*x^7 + 574801920*x^8) - (1985371747035*I)*(3 + Sqrt[17])*(1 +
2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*ArcSinh[Sqrt[2/
(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4] + (3*I)*(18873993
33713 + 661790582345*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 +
 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4
 + (3*Sqrt[17])/4] - (41533533493760*I)*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*
x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/(-3
 + Sqrt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4])/(1050809760*Sqrt[3 +
Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\left(4x^2+7x+2\right)^{5/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{1231}$$

$$-\frac{5\int \frac{2(2304x+571)\left(4x^2+7x+2\right)^{3/2}}{9\sqrt{2x+1}}dx}{1144}+\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\int \frac{(2304x+571)\left(4x^2+7x+2\right)^{3/2}}{\sqrt{2x+1}}dx}{5148}+\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{1}{168}\int \frac{36(11678x+4485)\sqrt{4x^2+7x+2}}{\sqrt{2x+1}}dx\right)}{5148}+\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\int \frac{(11678x+4485)\sqrt{4x^2+7x+2}}{\sqrt{2x+1}}dx\right)}{5148}+\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{1231}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{30}\sqrt{2x+1}(70068x+39011)\sqrt{4x^2+7x+2}-\frac{1}{120}\int\frac{2(43158}{\sqrt{2x+1}}\right.\right.}{5148}$$

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{30}\sqrt{2x+1}(70068x+39011)\sqrt{4x^2+7x+2}-\frac{1}{60}\int\frac{431580}{\sqrt{2x+1}\sqrt{}}\right.\right.}{5148}$$

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{1269}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{60}\left(44597\int\frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-215790\int\frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx\right)\right.\right.}{5148}$$

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{1172}$$

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{60}\left(\frac{89194\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\int\frac{1}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}d\frac{\sqrt{8x+}}{\sqrt{2}}}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.\right.}{}$$

$$\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \text{321}$$

$$-\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{60}\left(\frac{89194\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(\right.\right.}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.\right.}{}$$

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow 327$$

$$\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{60}\left(\frac{89194\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17\right.\right.}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.\right.}{}$$

$$\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow 1292$$

$$\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{7}\sqrt{2x+1}(1792x+891)\left(4x^2+7x+2\right)^{3/2}-\frac{3}{14}\left(\frac{1}{60}\left(\frac{89194\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17\right.\right.}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.\right.}{}$$

$$\frac{(175-132x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}{2574}$$

input
```
Int[((4 + 6*x - 2*x^2)*(2 + 7*x + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.24

method	result
risch	$\frac{\left(143700480x^5+312439680x^4-1133586720x^3-6453975240x^2-18850599504x-56951379803\right)\sqrt{4x^2+7x+2}\,\sqrt{1+2x}}{175134960}+\frac{2\left(-\frac{226222\cdots}{\cdots}\right.}{\cdots}$
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(\frac{32x^5\sqrt{8x^3+18x^2+11x+2}}{39}+\frac{2296x^4\sqrt{8x^3+18x^2+11x+2}}{1287}-\frac{224918x^3\sqrt{8x^3+18x^2+11x+2}}{34749}-\frac{5975903x^2\sqrt{8x^3+18x^2+1\cdots}}{162162}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{4x^2+7x+2}\,\sqrt{1+2x}\left(-32803994766528-191279916530208x+20913725366863\sqrt{-(1+2x)\left(3+\sqrt{17}\right)}\,\sqrt{\left(-8x-7+\sqrt{17}\right)\left(-3+\sqrt{\cdots}\right.}\cdots\right.}{\cdots}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/175134960*(143700480*x^5+312439680*x^4-1133586720*x^3-6453975240*x^2-188
50599504*x-56951379803)*(4*x^2+7*x+2)^(1/2)*(1+2*x)^(1/2)+2*(-226222444190
3/95528160*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/
8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*
17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticF((-(x+1/2)/(3/8+1/8*
17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))-1323581
16469/17513496*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(
x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+
1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*((3/8-1/8*17^(1/2))*Ellip
ticE((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*1
7^(1/2)))^(1/2))+(-7/8+1/8*17^(1/2))*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)
))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))+1685614184/852
93*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17
^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)
))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticPi((-(x+1/2)/(3/8+1/8*17^(1/2
)))^(1/2),-9/52-3/52*17^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(
1/2)))*((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\,(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+7\,x+2)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*(16*x^6 + 8*x^5 - 135*x^4 - 279*x^3 - 210*x^2 - 68*x - 8)*sqrt(
4*x^2 + 7*x + 2)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\left(2+7x+4x^2\right)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+7*x+2)**(5/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\left(2+7x+4x^2\right)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\left(4\,x^2+7\,x+2\right)^{\frac{5}{2}}\left(x^2-3\,x-2\right)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((4*x^2 + 7*x + 2)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)\left(2+7x+4x^2\right)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\left(4\,x^2+7\,x+2\right)^{\frac{5}{2}}\left(x^2-3\,x-2\right)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(4*x^2 + 7*x + 2)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(-2\,x^2+6\,x+4)\,(4\,x^2+7\,x+2)^{5/2}}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(5/2))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(5/2))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)\,(4x^2+7x+2)^{\frac{5}{2}}}{(5-3x)\,\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.51 $\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 386

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{\sqrt{1+2x}(108598+22689x)\sqrt{2+7x+4x^2}}{8505} - \frac{1}{378}(43-28x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{3/2}$$
$$-\frac{2072593\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{27216\sqrt{-1-2x}\sqrt{2+7x+4x^2}}$$
$$-\frac{23422931\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{14580\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$
$$+\frac{60470464\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{729\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$

output
```
-1/8505*(1+2*x)^(1/2)*(108598+22689*x)*(4*x^2+7*x+2)^(1/2)-1/378*(43-28*x)
*(1+2*x)^(1/2)*(4*x^2+7*x+2)^(3/2)-2072593/27216*(3+17^(1/2))^(1/2)*(1+2*x
)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34
^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)-23422
931/14580*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticF(1/34*(17+17^(1/2)*
(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(1+2
*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+60470464/729*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(
1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),-153/1784+183/17
84*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(61+3*17^(1/2))/
(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.60 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.90

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \frac{78\sqrt{3+\sqrt{17}}(-22494458-82718883x-59841704x^2-110472}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(2 + 7*x + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 +
2*x]),x]
```

output
```
(78*Sqrt[3 + Sqrt[17]]*(-22494458 - 82718883*x - 59841704*x^2 - 11047288*x
^3 - 1505376*x^4 + 397440*x^5 + 161280*x^6) - (404155635*I)*(3 + Sqrt[17])
*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*ArcSinh[S
qrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4] + (3*I)*(38
4128327 + 134718545*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 +
2*x)^2]*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4
+ (3*Sqrt[17])/4] - (8465864960*I)*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/
(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/(-3 + Sq
rt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4])/(5307120*Sqrt[3 + Sqrt[17]
]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{76}{9}\int \frac{(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(4x^2+7x+2)^{3/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{ 1231}$$

$$-\frac{1}{168}\int \frac{2(524x+127)\sqrt{4x^2+7x+2}}{3\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}(4x^2+7x+2)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{252}\int \frac{(524x+127)\sqrt{4x^2+7x+2}}{\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}(4x^2+7x+2)^{3/2}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{1}{252}\left(\frac{1}{120}\int \frac{24(1985x+702)}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{4}{5}\sqrt{2x+1}(131x+42)\sqrt{4x^2+7x+2}\right)+\frac{76}{9}\int \frac{(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}(4x^2+7x+2)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{252}\left(\frac{1}{5}\int \frac{1985x+702}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{4}{5}\sqrt{2x+1}(131x+42)\sqrt{4x^2+7x+2}\right)+\frac{76}{9}\int \frac{(4x^2+7x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}(4x^2+7x+2)^{3/2}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{1}{252}\left(\frac{1}{5}\left(\frac{1985}{2}\int\frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx-\frac{581}{2}\int\frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx\right)-\frac{4}{5}\sqrt{2x+1}(131x+42)\sqrt{4x^2+7x+\right.$$
$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}$$

$\big\downarrow$ 1172

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+$$
$$\frac{1}{252}\left(\frac{1}{5}\left(\frac{1985\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int\frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{581\sqrt{-2x-1}\sqrt{-4x^2-7x-}}{\sqrt{3+\sqrt{17}}\sqrt{}}\right.\right.$$
$$\frac{1}{378}(43-28x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}$$

$\big\downarrow$ 321

$$\frac{1}{252}\left(\frac{1}{5}\left(\frac{1985\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int\frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{581\sqrt{-2x-1}\sqrt{-4x^2-7x-}}{\sqrt{3+}}\right.\right.$$
$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{378}(43-28x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}$$

$\big\downarrow$ 327

$$\frac{76}{9}\int\frac{\left(4x^2+7x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+$$
$$\frac{1}{252}\left(\frac{1}{5}\left(\frac{1985\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{581\sqrt{-2x-1}\sqrt{}}{}\right.\right.$$
$$\frac{1}{378}(43-28x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}$$

$\big\downarrow$ 1292

$$\frac{76}{9}\int \frac{\left(4x^2+7x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+$$

$$\frac{1}{252}\left(\frac{1}{5}\left(\frac{1985\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{581\sqrt{-2x-1}\sqrt{}}{}\right.\right.$$

$$\frac{1}{378}(43-28x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}$$

input
```
Int[((4 + 6*x - 2*x^2)*(2 + 7*x + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(5040x^3+1080x^2-56403x-221066)\sqrt{4x^2+7x+2}\sqrt{1+2x}}{17010}+\frac{2\left(-\frac{48762353\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{-\frac{x+\frac{7}{8}-\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{\frac{x+\frac{7}{8}+\frac{\sqrt{17}}{8}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\,\mathrm{El}}{51030\sqrt{8x^3+18x^2+11x+2}}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(\frac{8x^3\sqrt{8x^3+18x^2+11x+2}}{27}+\frac{4x^2\sqrt{8x^3+18x^2+11x+2}}{63}-\frac{2089x\sqrt{8x^3+18x^2+11x+2}}{630}-\frac{110533\sqrt{8x^3+18x^2+11x+2}}{8505}-\frac{487}{}\right.}{}$
default	$\frac{\sqrt{4x^2+7x+2}\sqrt{1+2x}\left(-3310684416-19053455616x-1103561472\sqrt{17}+2131486721\sqrt{-(1+2x)\left(3+\sqrt{17}\right)}\sqrt{\left(-8x-7+\sqrt{17}\right)\left(-3\right.}\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/17010*(5040*x^3+1080*x^2-56403*x-221066)*(4*x^2+7*x+2)^(1/2)*(1+2*x)^(1/
2)+2*(-48762353/51030*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1
/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2)
)/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticF((-(x+1/2
)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/
2))-2072593/6804*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(
-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/
8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*((3/8-1/8*17^(1/2))*Ell
ipticE((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8
*17^(1/2)))^(1/2))+(-7/8+1/8*17^(1/2))*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/
2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))+7558808/9477
*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(
1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))
^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticPi((-(x+1/2)/(3/8+1/8*17^(1/2))
)^(1/2),-9/52-3/52*17^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/
2)))*((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+7\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*(4*x^4 - 5*x^3 - 27*x^2 - 20*x - 4)*sqrt(4*x^2 + 7*x + 2)*sqrt(
2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{4\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{20x\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{27x^2\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{5x^3\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{4x^4\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+7*x+2)**(3/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-4*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)
), x) + Integral(-20*x*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(
2*x + 1)), x) + Integral(-27*x**2*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1
) - 5*sqrt(2*x + 1)), x) + Integral(-5*x**3*sqrt(4*x**2 + 7*x + 2)/(3*x*sq
rt(2*x + 1) - 5*sqrt(2*x + 1)), x) + Integral(4*x**4*sqrt(4*x**2 + 7*x + 2
)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\,(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+7\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((4*x^2 + 7*x + 2)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)\,(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+7\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(4*x^2 + 7*x + 2)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\,(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(-2\,x^2+6\,x+4)\,(4\,x^2+7\,x+2)^{3/2}}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(3/2))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(3/2))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)(2+7x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)(4x^2+7x+2)^{\frac{3}{2}}}{(5-3x)\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.52 $\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 354

$$\begin{aligned}\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx &= -\frac{1}{270}(83-36x)\sqrt{1+2x}\sqrt{2+7x+4x^2} \\ &- \frac{173\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{54\sqrt{-1-2x}\sqrt{2+7x+4x^2}} \\ &- \frac{52513\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{810\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}} \\ &+ \frac{271168\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{81\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\end{aligned}$$

output
```
-1/270*(83-36*x)*(1+2*x)^(1/2)*(4*x^2+7*x+2)^(1/2)-173/54*(3+17^(1/2))^(1/
2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^(1/2)*(7+8*x))
^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4*x^2+7*x+2)^(1
/2)-52513/810*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticF(1/34*(17+17^(1
/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/
(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+271168/81*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^
(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),-153/1784+183/1
784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(61+3*17^(1/2))
/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 23.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.24

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx$$

$$= \frac{-141414-505089x-312702x^2-624x^3+11232x^4-33735i\sqrt{3+\sqrt{17}}(1+2x)^{3/2}\sqrt{\frac{2+7x+4x^2}{(1+2x)^2}}E\left(i\text{arcs}\right.}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*Sqrt[2 + 7*x + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*
x]),x]
```

output
```
(-141414 - 505089*x - 312702*x^2 - 624*x^3 + 11232*x^4 - (33735*I)*Sqrt[3
+ Sqrt[17]]*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticE[
I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4]
+ ((3*I)*(32152 + 11245*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(
1 + 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -1
3/4 + (3*Sqrt[17])/4])/Sqrt[3 + Sqrt[17]] - ((677920*I)*Sqrt[1 + 2*x]*Sqrt
[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcS
inh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4])/Sqrt[3
+ Sqrt[17]] - ((1355840*I)*x*Sqrt[1 + 2*x]*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x
)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]
/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4])/Sqrt[3 + Sqrt[17]])/(10530*Sqrt[1
+ 2*x]*Sqrt[2 + 7*x + 4*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.92 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.69, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.415$, Rules used = {2154, 1231, 27, 1269, 1172, 321, 327, 1274, 1269, 1172, 321, 327, 1279, 186, 25, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\sqrt{4x^2+7x+2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{1231}$$

$$-\frac{1}{120}\int \frac{2(840x+191)}{9\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{540}\int \frac{840x+191}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \text{1269}$$

$$\frac{1}{540}\left(229\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-420\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx\right)+\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \text{1172}$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx+$$
$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\int \frac{1}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2 \cdots}}{\sqrt{-2x-1 \cdots}}\right.$$
$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 321

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+\cdots}}{\cdots}\right.$$
$$\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 327

$$\frac{76}{9}\int \frac{\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx+$$
$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+\cdots}}{\cdots}\right.$$
$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 1274

$$\frac{76}{9}\left(\frac{223}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{1}{9}\int \frac{12x+41}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx\right)+$$
$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+\cdots}}{\cdots}\right.$$
$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 1269

$$\frac{76}{9}\left(\frac{223}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{1}{9}\left(-35\int\frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-6\int\frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 1172

$$\frac{76}{9}\left(\frac{223}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{1}{9}\left(-\frac{70\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\int\frac{1}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 321

$$\frac{76}{9}\left(\frac{1}{9}\left(-\frac{3\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int\frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{70\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{Ell}}{\sqrt{3+\sqrt{17}}}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$\downarrow$ 327

$$\frac{76}{9}\left(\frac{223}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{1}{9}\left(-\frac{70\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{}}}{\sqrt{2}\sqrt[4]{}}\right.\right.}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \; 1279$$

$$\frac{76}{9}\left(\frac{223\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}}dx}{9\sqrt{4x^2+7x+2}}+\frac{1}{9}\left(-\frac{70\sqrt{-2x-1}\sqrt{-4x^2-7}}{}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \; 186$$

$$\frac{76}{9}\left(\frac{1}{9}\left(-\frac{70\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{3\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \; 25$$

$$\frac{76}{9}\left(\frac{446\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-\sqrt{17}+3}\sqrt{4(2x+1)+\sqrt{17}+3}}d\sqrt{2x+1}}{9\sqrt{4x^2+7x+2}}+\frac{1}{9}\left(-\frac{70\sqrt{-2}}{}\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \text{413}$$

$$\frac{76}{9}\left(\frac{446\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)+\sqrt{17}+3}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+7x+2}\sqrt{4(2x+1)-\sqrt{17}+3}}+\frac{1}{9}\left(-\right.\right.$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

$$\Big\downarrow \text{412}$$

$$\frac{1}{540}\left(\frac{458\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}-\frac{210\sqrt{3+\sqrt{17}}\sqrt{2x+}}{}\right.$$

$$\frac{76}{9}\left(\frac{223\sqrt{13-3\sqrt{17}}\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}\,\text{EllipticPi}\left(-\frac{3}{52}\left(3-\sqrt{17}\right),\arcsin\left(\frac{2\sqrt{2x+1}}{\sqrt{-3+\sqrt{17}}}\right.\right.}{234\sqrt{4x^2+7x+2}\sqrt{4(2x+1)-\sqrt{17}+3}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+7x+2}(83-36x)$$

input
```
Int[((4 + 6*x - 2*x^2)*Sqrt[2 + 7*x + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
-1/270*((83 - 36*x)*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2]) + ((-210*Sqrt[3 +
 Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[-2 - 7*x - 4*x^2]*EllipticE[ArcSin[Sqrt[7 +
Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17 - 3*Sqrt[17])/4])/(Sqrt[-1 - 2*x]
*Sqrt[2 + 7*x + 4*x^2]) + (458*Sqrt[-1 - 2*x]*Sqrt[-2 - 7*x - 4*x^2]*Ellip
ticF[ArcSin[Sqrt[7 + Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17 - 3*Sqrt[17]
)/4])/(Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2]))/540 + (76*
(((-3*Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[-2 - 7*x - 4*x^2]*EllipticE[Ar
cSin[Sqrt[7 + Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17 - 3*Sqrt[17])/4])/(
Sqrt[-1 - 2*x]*Sqrt[2 + 7*x + 4*x^2]) - (70*Sqrt[-1 - 2*x]*Sqrt[-2 - 7*x -
 4*x^2]*EllipticF[ArcSin[Sqrt[7 + Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17
 - 3*Sqrt[17])/4])/(Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2]
))/9 + (223*Sqrt[13 - 3*Sqrt[17]]*Sqrt[7 - Sqrt[17] + 8*x]*Sqrt[7 + Sqrt[1
7] + 8*x]*Sqrt[1 + (4*(1 + 2*x))/(3 - Sqrt[17])]*EllipticPi[(-3*(3 - Sqrt[
17]))/52, ArcSin[(2*Sqrt[1 + 2*x])/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17]
)/4])/(234*Sqrt[2 + 7*x + 4*x^2]*Sqrt[3 - Sqrt[17] + 4*(1 + 2*x)])))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1274
```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/e^2   Int[1/((d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Simp[1/e^2   Int[(c*
d - b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.40

method	result
risch	$\frac{(-83+36x)\sqrt{4x^2+7x+2}\,\sqrt{1+2x}}{270}+\frac{2\left(-\frac{62893\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{-\frac{x+\frac{7}{8}-\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{\frac{x+\frac{7}{8}+\frac{\sqrt{17}}{8}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\,\mathrm{EllipticF}\left(\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}},i\sqrt{\frac{\frac{3}{8}+}{-\frac{3}{8}+}}\right.}{1620\sqrt{8x^3+18x^2+11x+2}}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(\frac{2x\sqrt{8x^3+18x^2+11x+2}}{15}-\frac{83\sqrt{8x^3+18x^2+11x+2}}{270}-\frac{62893\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{-\frac{x+\frac{7}{8}-\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{\frac{x+\frac{7}{8}+\frac{\sqrt{17}}{8}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}}{810\sqrt{8x^3+18x^2+11x+2}}\right.}{}$
default	$\frac{\sqrt{4x^2+7x+2}\,\sqrt{1+2x}\left(359424\sqrt{17}\,x^4-19968\sqrt{17}\,x^3+1078272x^4+682669\sqrt{-(1+2x)\left(3+\sqrt{17}\right)}\,\sqrt{\left(-8x-7+\sqrt{17}\right)\left(-3+\sqrt{17}\right)}\,\sqrt{(\right.}{}$

input

```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
1/270*(-83+36*x)*(4*x^2+7*x+2)^(1/2)*(1+2*x)^(1/2)+2*(-62893/1620*(-3/8-1/
8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3
/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8
*x^3+18*x^2+11*x+2)^(1/2)*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*
((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))-346/27*(-3/8-1/8*17^(1/2))
*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(
1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2
+11*x+2)^(1/2)*((3/8-1/8*17^(1/2))*EllipticE((-(x+1/2)/(3/8+1/8*17^(1/2)))
^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))+(-7/8+1/8*17^(1/2
))*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3
/8+1/8*17^(1/2)))^(1/2)))+33896/1053*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/
8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7
/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*Ell
ipticPi((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),-9/52-3/52*17^(1/2),I*((3/8+1/
8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))*((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*
x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4x^2+7x+2}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(6*x^2 - 7*
x - 5), x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{2\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{3x\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{x^2\sqrt{4x^2+7x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+7*x+2)**(1/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-2*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)
), x) + Integral(-3*x*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2
*x + 1)), x) + Integral(x**2*sqrt(4*x**2 + 7*x + 2)/(3*x*sqrt(2*x + 1) - 5
*sqrt(2*x + 1)), x))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4x^2+7x+2}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate(sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)
), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4x^2+7x+2}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(-2x^2+6x+4)\sqrt{4x^2+7x+2}}{\sqrt{2x+1}\,(3x-5)}\,dx$$

input
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(1/2))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((6*x - 2*x^2 + 4)*(7*x + 4*x^2 + 2)^(1/2))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+7x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)\sqrt{4x^2+7x+2}}{(5-3x)\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+7*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.53 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 322

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = \frac{\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{6\sqrt{-1-2x}\sqrt{2+7x+4x^2}} - \frac{22\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}} + \frac{1216\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$

output
```
1/6*(3+17^(1/2))^(1/2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(
17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1
/2)/(4*x^2+7*x+2)^(1/2)-22/9*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticF
(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+1
7^(1/2))^(1/2)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+1216/9*(-1-2*x)^(1/2)*(-4
*x^2-7*x-2)^(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),-15
3/1784+183/1784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(61
+3*17^(1/2))/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.16 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.06

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx$$

$$= \frac{(3+6x)^{3/2}\left(78-\frac{39}{(1+2x)^2}+\frac{117}{1+2x}+\frac{39i\sqrt{3+\sqrt{17}}\sqrt{\frac{2+7x+4x^2}{(1+2x)^2}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{\frac{2}{-3+\sqrt{17}}}}{\sqrt{1+2x}}\right)|-\frac{13}{4}+\frac{3\sqrt{17}}{4}\right)}{\sqrt{1+2x}}-\frac{3i\left(27+13\sqrt{17}\right)\sqrt{\cdots}}{\cdots}\right)}{702\cdots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2]
),x]
```

output
```
((3 + 6*x)^(3/2)*(78 - 39/(1 + 2*x)^2 + 117/(1 + 2*x) + ((39*I)*Sqrt[3 + S
qrt[17]]*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*ArcSinh[Sqrt[2/(-
3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4])/Sqrt[1 + 2*x] - ((
3*I)*(27 + 13*Sqrt[17])*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticF[I*Ar
cSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4])/(Sq
rt[3 + Sqrt[17]]*Sqrt[1 + 2*x]) - ((608*I)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x
)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]
/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4])/(Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x])
))/(702*Sqrt[3]*Sqrt[2 + 7*x + 4*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.244$, Rules used = {2154, 1269, 1172, 321, 327, 1279, 186, 25, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx$$

$$\Big\downarrow \text{ 1269}$$

$$-\frac{11}{9}\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{1}{3}\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx$$

$$\Big\downarrow \text{ 1172}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\int \frac{1}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 1279}$$

$$\frac{76\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}}dx}{9\sqrt{4x^2+7x+2}}-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 186}$$

$$-\frac{152\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\int -\frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-\sqrt{17}+3}\sqrt{4(2x+1)+\sqrt{17}+3}}d\sqrt{2x+1}}{9\sqrt{4x^2+7x+2}}-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{152\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-\sqrt{17}+3}\sqrt{4(2x+1)+\sqrt{17}+3}}d\sqrt{2x+1}}{9\sqrt{4x^2+7x+2}}-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 413}$$

$$\frac{152\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)+\sqrt{17}+3}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+7x+2}\sqrt{4(2x+1)-\sqrt{17}+3}}-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{412}$$

$$-\frac{22\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right),\frac{1}{4}(17-3\sqrt{17})\right)}{9\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{4x^2+7x+2}}+\frac{\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{6\sqrt{-2x-1}\sqrt{4x^2+7x+2}}+\frac{38\sqrt{13-3\sqrt{17}}\sqrt{8x-\sqrt{17}+7}\sqrt{8x+\sqrt{17}+7}\sqrt{\frac{4(2x+1)}{3-\sqrt{17}}+1}\,\text{EllipticPi}\left(-\frac{3}{52}(3-\sqrt{17}),\arcsin\left(\frac{2\sqrt{2x+1}}{\sqrt{-3+\sqrt{17}}}\right),\frac{1}{4}(\right.}{117\sqrt{4x^2+7x+2}\sqrt{4(2x+1)-\sqrt{17}+3}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2]),x]
```

output
```
(Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[-2 - 7*x - 4*x^2]*EllipticE[ArcSin[
Sqrt[7 + Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17 - 3*Sqrt[17])/4])/(6*Sqr
t[-1 - 2*x]*Sqrt[2 + 7*x + 4*x^2]) - (22*Sqrt[-1 - 2*x]*Sqrt[-2 - 7*x - 4*
x^2]*EllipticF[ArcSin[Sqrt[7 + Sqrt[17] + 8*x]/(Sqrt[2]*17^(1/4))], (17 -
3*Sqrt[17])/4])/(9*Sqrt[3 + Sqrt[17]]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2])
 + (38*Sqrt[13 - 3*Sqrt[17]]*Sqrt[7 - Sqrt[17] + 8*x]*Sqrt[7 + Sqrt[17] +
8*x]*Sqrt[1 + (4*(1 + 2*x))/(3 - Sqrt[17])]*EllipticPi[(-3*(3 - Sqrt[17]))
/52, ArcSin[(2*Sqrt[1 + 2*x])/Sqrt[-3 + Sqrt[17]]], (-13 + 3*Sqrt[17])/4])
/(117*Sqrt[2 + 7*x + 4*x^2]*Sqrt[3 - Sqrt[17] + 4*(1 + 2*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 186
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.11

method	result
default	$-\frac{\sqrt{1+2x}\,\sqrt{4x^2+7x+2}\,\sqrt{-(1+2x)\left(3+\sqrt{17}\right)}\,\sqrt{\left(-8x-7+\sqrt{17}\right)\left(-3+\sqrt{17}\right)}\,\sqrt{\left(8x+7+\sqrt{17}\right)\left(3+\sqrt{17}\right)}\,\left(39\,\text{EllipticF}\left(\frac{2\sqrt{-(1+2x}}{3+}\right.\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(-\frac{16\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\,\sqrt{-\frac{x+\frac{7}{8}-\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\,\sqrt{\frac{x+\frac{7}{8}+\frac{\sqrt{17}}{8}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\,\text{EllipticF}\left(\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}},i\sqrt{\frac{\frac{3}{8}+\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\right)}{9\sqrt{8x^3+18x^2+11x+2}}+\frac{4\left(-\frac{3}{8}-\right.}{}\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/936*(1+2*x)^(1/2)*(4*x^2+7*x+2)^(1/2)/(3+17^(1/2))*(-(1+2*x)*(3+17^(1/2
)))^(1/2)/(-3+17^(1/2))*((-8*x-7+17^(1/2))*(-3+17^(1/2)))^(1/2)*((8*x+7+17
^(1/2))*(3+17^(1/2)))^(1/2)*(39*EllipticF(2/(3+17^(1/2))*(-(1+2*x)*(3+17^(
1/2)))^(1/2),I/(-3+17^(1/2))*((3+17^(1/2))*(-3+17^(1/2)))^(1/2))*17^(1/2)-
39*EllipticE(2/(3+17^(1/2))*(-(1+2*x)*(3+17^(1/2)))^(1/2),I/(-3+17^(1/2))*
((3+17^(1/2))*(-3+17^(1/2)))^(1/2))*17^(1/2)-689*EllipticF(2/(3+17^(1/2))*
(-(1+2*x)*(3+17^(1/2)))^(1/2),I/(-3+17^(1/2))*((3+17^(1/2))*(-3+17^(1/2)))
^(1/2))+117*EllipticE(2/(3+17^(1/2))*(-(1+2*x)*(3+17^(1/2)))^(1/2),I/(-3+1
7^(1/2))*((3+17^(1/2))*(-3+17^(1/2)))^(1/2))+608*EllipticPi(2/(3+17^(1/2))
*(-(1+2*x)*(3+17^(1/2)))^(1/2),-9/52-3/52*17^(1/2),I/(-3+17^(1/2))*((3+17^
(1/2))*(-3+17^(1/2)))^(1/2)))/(8*x^3+18*x^2+11*x+2)
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+7\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(24*x^4 + 1
4*x^3 - 57*x^2 - 49*x - 10), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = 2\left(\int \left(-\frac{3x}{3x\sqrt{2x+1}\sqrt{4x^2+7x+2}-5\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right)dx + \int \frac{x^2}{3x\sqrt{2x+1}\sqrt{4x^2+7x+2}-5\sqrt{2x+1}\sqrt{4x^2+7x+2}}\,dx + \int \left(-\frac{2}{3x\sqrt{2x+1}\sqrt{4x^2+7x+2}-5\sqrt{2x+1}\sqrt{4x^2+7x+2}}\right)dx\right)$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+7*x+2)**(1/2),x)
```

output
```
2*(Integral(-3*x/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 5*sqrt(2*x +
1)*sqrt(4*x**2 + 7*x + 2)), x) + Integral(x**2/(3*x*sqrt(2*x + 1)*sqrt(4*x
**2 + 7*x + 2) - 5*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x) + Integral(-2
/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 5*sqrt(2*x + 1)*sqrt(4*x**2 +
 7*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+7\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/(sqrt(4*x^2 + 7*x + 2)*(3*x - 5)*sqrt(2*x + 1)
), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+7\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/(sqrt(4*x^2 + 7*x + 2)*(3*x - 5)*sqrt(2*x + 1)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,(3\,x-5)\,\sqrt{4\,x^2+7\,x+2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(1/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(1/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,\sqrt{4x^2+7x+2}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2),x)
```

3.54 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 354

$$\begin{aligned}\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx &= -\frac{8\sqrt{1+2x}(328+319x)}{3791\sqrt{2+7x+4x^2}} \\ &+ \frac{638\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)\middle|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{3791\sqrt{-1-2x}\sqrt{2+7x+4x^2}} \\ &+ \frac{1132\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{3791\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}} \\ &+ \frac{1216\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{223\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}\end{aligned}$$

output
```
-8/3791*(1+2*x)^(1/2)*(328+319*x)/(4*x^2+7*x+2)^(1/2)+638/3791*(3+17^(1/2)
)^(1/2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^(1/2)*(7+
8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4*x^2+7*x+
2)^(1/2)+1132/3791*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticF(1/34*(17+
17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(
1/2)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+1216/223*(-1-2*x)^(1/2)*(-4*x^2-7*x
-2)^(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),-153/1784+1
83/1784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(61+3*17^(1
/2))/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.21 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.19

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \frac{2\left(-468\sqrt{3+\sqrt{17}}+7358\sqrt{3+\sqrt{17}}x+4147i\left(3+\sqrt{17}\right)(1-\right.}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(3/
2)),x]
```

output
```
(2*(-468*Sqrt[3 + Sqrt[17]] + 7358*Sqrt[3 + Sqrt[17]]*x + (4147*I)*(3 + Sq
rt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticE[I*Ar
cSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17])/4] - I*
(2499 + 4147*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]
*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sq
rt[17])/4] - (2584*I)*Sqrt[1 + 2*x]*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*El
lipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1
 + 2*x]], (-13 + 3*Sqrt[17])/4] - (5168*I)*x*Sqrt[1 + 2*x]*Sqrt[(2 + 7*x +
 4*x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2/
(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4]))/(49283*Sqrt[3 + S
qrt[17]]*Sqrt[1 + 2*x]*Sqrt[2 + 7*x + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx+\frac{1}{17}\int -\frac{2(108x+1)}{9\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx-\frac{2}{153}\int \frac{108x+1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx-\frac{2}{153}\left(54\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx-53\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx\right)+\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \text{ 1172}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx-\frac{2}{153}\left(\frac{27\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{106\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\int \frac{\cdots}{\sqrt{1\cdots}}}{\sqrt{3+\sqrt{17}}\sqrt{2x+\cdots}}\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \; 321$$

$$-\frac{2}{153}\left(\frac{27\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{106\sqrt{-2x-1}\sqrt{-4x^2-7x-2}\,\text{Elli}}{\sqrt{3+\sqrt{17}}}\right.$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \; 327$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx-$$

$$\frac{2}{153}\left(\frac{27\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{106\sqrt{-2x-1}\sqrt{-4x^2}}{}\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

$$\Big\downarrow \; 1292$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{3/2}}dx-$$

$$\frac{2}{153}\left(\frac{27\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{106\sqrt{-2x-1}\sqrt{-4x^2}}{}\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{153\sqrt{4x^2+7x+2}}$$

input

```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(3/2)),x]
```

output

```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.40

method	result
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(-\frac{2(4+8x)\left(\frac{328}{3791}+\frac{319x}{3791}\right)}{\sqrt{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)(4+8x)}}+\frac{3684\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{-\frac{x+\frac{7}{8}-\frac{\sqrt{17}}{8}}{-\frac{3}{8}+\frac{\sqrt{17}}{8}}}\sqrt{\frac{x+\frac{7}{8}+\frac{\sqrt{17}}{8}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\operatorname{EllipticF}\left(\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}},i\right.}{3791\sqrt{8x^3+18x^2+11x+2}}\right.}{}$
default	$-\frac{\sqrt{1+2x}\sqrt{4x^2+7x+2}\left(3679\sqrt{-(1+2x)\left(3+\sqrt{17}\right)}\sqrt{\left(-8x-7+\sqrt{17}\right)\left(-3+\sqrt{17}\right)}\sqrt{\left(8x+7+\sqrt{17}\right)\left(3+\sqrt{17}\right)}\operatorname{EllipticF}\left(\frac{2\sqrt{-(1-}}{3}\right.\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)*(-2*(4+8*x
)*(328/3791+319/3791*x)/((x^2+7/4*x+1/2)*(4+8*x))^(1/2)+3684/3791*(-3/8-1/
8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3
/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8
*x^3+18*x^2+11*x+2)^(1/2)*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*
((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))+5104/3791*(-3/8-1/8*17^(1/
2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*1
7^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*
x^2+11*x+2)^(1/2)*((3/8-1/8*17^(1/2))*EllipticE((-(x+1/2)/(3/8+1/8*17^(1/2
)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))+(-7/8+1/8*17^(
1/2))*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/
(-3/8+1/8*17^(1/2)))^(1/2)))+304/2899*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1
/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+
7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*El
lipticPi((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),-9/52-3/52*17^(1/2),I*((3/8+1
/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(96*x^6 + 2
24*x^5 - 82*x^4 - 567*x^3 - 497*x^2 - 168*x - 20), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = 2\Bigg(\int \Bigg(-\frac{}{12x^3\sqrt{2x+1}\sqrt{4x^2+7x+2}+x^2\sqrt{2x+1}\sqrt{4x^2+7x}}$$
$$+\int \frac{x^2}{12x^3\sqrt{2x+1}\sqrt{4x^2+7x+2}+x^2\sqrt{2x+1}\sqrt{4x^2+7x+2}-29x\sqrt{2x+1}\sqrt{4x^2+7x+2}-10\sqrt{2x+1}}$$
$$+\int \Bigg(-\frac{2}{12x^3\sqrt{2x+1}\sqrt{4x^2+7x+2}+x^2\sqrt{2x+1}\sqrt{4x^2+7x+2}-29x\sqrt{2x+1}\sqrt{4x^2+7x+2}-10\sqrt{2x}}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+7*x+2)**(3/2),x)
```

output
```
2*(Integral(-3*x/(12*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) + x**2*sqrt
(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 29*x*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x +
2) - 10*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x) + Integral(x**2/(12*x**3
*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) + x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 7
*x + 2) - 29*x*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 10*sqrt(2*x + 1)*sqr
t(4*x**2 + 7*x + 2)), x) + Integral(-2/(12*x**3*sqrt(2*x + 1)*sqrt(4*x**2
+ 7*x + 2) + x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 29*x*sqrt(2*x + 1
)*sqrt(4*x**2 + 7*x + 2) - 10*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(3/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(3/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,(3\,x-5)\,(4\,x^2+7\,x+2)^{3/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(3/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(3/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{3/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+7x+2)^{\frac{3}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(3/2),x)
```

3.55 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 386

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{5/2}}\,dx = -\frac{8\sqrt{1+2x}(328+319x)}{11373\left(2+7x+4x^2\right)^{3/2}}+\frac{4\sqrt{1+2x}(3731851+3842916x)}{43115043\sqrt{2+7x+4x^2}}$$
$$-\frac{1280972\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{14371681\sqrt{-1-2x}\sqrt{2+7x+4x^2}}$$
$$-\frac{8574352\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{43115043\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$
$$+\frac{10944\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{49729\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$

output
```
-8/11373*(1+2*x)^(1/2)*(328+319*x)/(4*x^2+7*x+2)^(3/2)+4/43115043*(1+2*x)^
(1/2)*(3731851+3842916*x)/(4*x^2+7*x+2)^(1/2)-1280972/14371681*(3+17^(1/2)
)^(1/2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^(1/2)*(7+
8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4*x^2+7*x+
2)^(1/2)-8574352/43115043*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticF(1/
34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(
1/2))^(1/2)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+10944/49729*(-1-2*x)^(1/2)*(
-4*x^2-7*x-2)^(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),-
153/1784+183/1784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(3+17^(1/2))^(1/2)/(
61+3*17^(1/2))/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.13 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.94

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = \frac{4\left(13(1+2x)\left(4976806+31390131x+41827816x^2+153716\right.\right.}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(5/
2)),x]
```

output
```
(4*(13*(1 + 2*x)*(4976806 + 31390131*x + 41827816*x^2 + 15371664*x^3) - ((
2 + 7*x + 4*x^2)*(24978954*Sqrt[3 + Sqrt[17]]*(2 + 7*x + 4*x^2) + (1248947
7*I)*(3 + Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*El
lipticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[
17])/4] - I*(10194815 + 12489477*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x +
 4*x^2)/(1 + 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 +
2*x]], -13/4 + (3*Sqrt[17])/4] + (593028*I)*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x
+ 4*x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[Sqrt[2
/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4]))/Sqrt[3 + Sqrt[17
]]))/(560495559*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(3/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{1}{51}\int \frac{2(324x+589)}{9\sqrt{2x+1}\,(4x^2+7x+2)^{3/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{459\,(4x^2+7x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{2}{459}\int \frac{324x+589}{\sqrt{2x+1}\,(4x^2+7x+2)^{3/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{459\,(4x^2+7x+2)^{3/2}}$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{2}{459}\left(\frac{1}{17}\int \frac{8(1605x+497)}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{2\sqrt{2x+1}(6420x+9247)}{17\sqrt{4x^2+7x+2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{459\,(4x^2+7x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{2}{459}\left(\frac{8}{17}\int \frac{1605x+497}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{2\sqrt{2x+1}(6420x+9247)}{17\sqrt{4x^2+7x+2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{459\,(4x^2+7x+2)^{3/2}}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}dx+$$
$$\frac{2}{459}\left(\frac{8}{17}\left(\frac{1605}{2}\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx-\frac{611}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx\right)-\frac{2\sqrt{2x+1}(6420x+9247)}{17\sqrt{4x^2+7x+2}}\right)+$$
$$\frac{8\sqrt{2x+1}(27x+47)}{459\left(4x^2+7x+2\right)^{3/2}}$$

$\big\downarrow$ 1172

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}dx+$$
$$\frac{2}{459}\left(\frac{8}{17}\left(\frac{1605\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{611\sqrt{-2x-1}\sqrt{-4x^2-7x-}}{\sqrt{3+\sqrt{17}}}\right.\right.$$
$$\frac{8\sqrt{2x+1}(27x+47)}{459\left(4x^2+7x+2\right)^{3/2}}$$

$\big\downarrow$ 321

$$\frac{2}{459}\left(\frac{8}{17}\left(\frac{1605\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{611\sqrt{-2x-1}\sqrt{-4x^2-7x-}}{\sqrt{3+}}\right.\right.$$
$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{459\left(4x^2+7x+2\right)^{3/2}}$$

$\big\downarrow$ 327

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{5/2}}dx+$$
$$\frac{2}{459}\left(\frac{8}{17}\left(\frac{1605\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{611\sqrt{-2x-1}}{}\right.\right.$$
$$\frac{8\sqrt{2x+1}(27x+47)}{459\left(4x^2+7x+2\right)^{3/2}}$$

$\big\downarrow$ 1292

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+$$
$$\frac{2}{459}\left(\frac{8}{17}\left(\frac{1605\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}(17-3\sqrt{17})\right)}{4\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{611\sqrt{-2x-1}\sqrt{}}{}\right.\right.$$
$$\frac{8\sqrt{2x+1}(27x+47)}{459\,(4x^2+7x+2)^{3/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.37

method	result
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(\frac{\left(-\frac{164}{11373}-\frac{319x}{22746}\right)\sqrt{8x^3+18x^2+11x+2}}{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)^2}-\frac{2(4+8x)\left(-\frac{3731851}{86230086}-\frac{640486x}{14371681}\right)}{\sqrt{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)(4+8x)}}-\frac{23946016\left(-\frac{3}{8}-\frac{\sqrt{17}}{8}\right)\sqrt{-\frac{x+\frac{1}{2}}{\frac{3}{8}+\frac{\sqrt{17}}{8}}}\cdots}{\cdots}\cdots\right.}{\cdots}$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)*((-164/113
73-319/22746*x)*(8*x^3+18*x^2+11*x+2)^(1/2)/(x^2+7/4*x+1/2)^2-2*(4+8*x)*(-
3731851/86230086-640486/14371681*x)/((x^2+7/4*x+1/2)*(4+8*x))^(1/2)-239460
16/43115043*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7
/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8
*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticF((-(x+1/2)/(3/8+1/8
*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))-102477
76/14371681*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7
/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8
*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*((3/8-1/8*17^(1/2))*Elliptic
E((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(
1/2)))^(1/2))+(-7/8+1/8*17^(1/2))*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^
(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))+2736/646477*(-3/8
-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/
(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)
/(8*x^3+18*x^2+11*x+2)^(1/2)*EllipticPi((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2
),-9/52-3/52*17^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(384*x^8 +
1568*x^7 + 1432*x^6 - 2394*x^5 - 6121*x^4 - 5285*x^3 - 2250*x^2 - 476*x -
40), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = 2\left(\int \left(-\frac{}{48x^5\sqrt{2x+1}\sqrt{4x^2+7x+2}+88x^4\sqrt{2x+1}\sqrt{4x^2+}}\right.\right.$$
$$+\int \frac{x^2}{48x^5\sqrt{2x+1}\sqrt{4x^2+7x+2}+88x^4\sqrt{2x+1}\sqrt{4x^2+7x+2}-85x^3\sqrt{2x+1}\sqrt{4x^2+7x+2}-241x^2\sqrt{}}$$
$$+\int \left(-\frac{2}{48x^5\sqrt{2x+1}\sqrt{4x^2+7x+2}+88x^4\sqrt{2x+1}\sqrt{4x^2+7x+2}-85x^3\sqrt{2x+1}\sqrt{4x^2+7x+2}-241x}\right.$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+7*x+2)**(5/2),x)
```

output
```
2*(Integral(-3*x/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) + 88*x**4*s
qrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 85*x**3*sqrt(2*x + 1)*sqrt(4*x**2 +
7*x + 2) - 241*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 128*x*sqrt(2*x
+ 1)*sqrt(4*x**2 + 7*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x)
 + Integral(x**2/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) + 88*x**4*s
qrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 85*x**3*sqrt(2*x + 1)*sqrt(4*x**2 +
7*x + 2) - 241*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 128*x*sqrt(2*x
+ 1)*sqrt(4*x**2 + 7*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x)
 + Integral(-2/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) + 88*x**4*sqr
t(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 85*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 7*
x + 2) - 241*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2) - 128*x*sqrt(2*x +
1)*sqrt(4*x**2 + 7*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 7*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(5/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(5/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+7\,x+2\right)^{5/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(5/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(5/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{5/2}}\,dx=\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{\frac{5}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(5/2),x)
```

3.56 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 418

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+7x+4x^2\right)^{7/2}}\,dx = -\frac{8\sqrt{1+2x}(328+319x)}{18955\left(2+7x+4x^2\right)^{5/2}} + \frac{4\sqrt{1+2x}(10115413+9883660x)}{215575215\left(2+7x+4x^2\right)^{3/2}} - \frac{8\sqrt{1+2x}(7809126509+7301723844x)}{163449128013\sqrt{2+7x+4x^2}} + \frac{4867815896\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{-2-7x-4x^2}\,E\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{54483042671\sqrt{-1-2x}\sqrt{2+7x+4x^2}} + \frac{21088452736\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{163449128013\sqrt{3+\sqrt{17}}\sqrt{1+2x}\sqrt{2+7x+4x^2}} + \frac{98496\sqrt{-1-2x}\sqrt{-2-7x-4x^2}\,\text{EllipticPi}\left(-\frac{3\left(51-61\sqrt{17}\right)}{1784},\arcsin\left(\frac{\sqrt{17+\sqrt{17}(7+8x)}}{\sqrt{34}}\right),\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{11089567\sqrt{3+\sqrt{17}}\left(61+3\sqrt{17}\right)\sqrt{1+2x}\sqrt{2+7x+4x^2}}$$

output
```
-8/18955*(1+2*x)^(1/2)*(328+319*x)/(4*x^2+7*x+2)^(5/2)+4/215575215*(1+2*x)
^(1/2)*(10115413+9883660*x)/(4*x^2+7*x+2)^(3/2)-8/163449128013*(1+2*x)^(1/
2)*(7809126509+7301723844*x)/(4*x^2+7*x+2)^(1/2)+4867815896/54483042671*(3
+17^(1/2))^(1/2)*(1+2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticE(1/34*(17+17^
(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2))^(1/2))/(-1-2*x)^(1/2)/(4
*x^2+7*x+2)^(1/2)+21088452736/163449128013*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(
1/2)*EllipticF(1/34*(17+17^(1/2)*(7+8*x))^(1/2)*34^(1/2),1/2*(17-3*17^(1/2
))^(1/2))/(3+17^(1/2))^(1/2)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)+98496/11089
567*(-1-2*x)^(1/2)*(-4*x^2-7*x-2)^(1/2)*EllipticPi(1/34*(17+17^(1/2)*(7+8*
x))^(1/2)*34^(1/2),-153/1784+183/1784*17^(1/2),1/2*(17-3*17^(1/2))^(1/2))/
(3+17^(1/2))^(1/2)/(61+3*17^(1/2))/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 24.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.89

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \frac{4\left(-13(1+2x)\,(263953467202+2162761148813x+6704742\right.}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(7/
2)),x]
```

output
```
(4*(-13*(1 + 2*x)*(263953467202 + 2162761148813*x + 6704742099018*x^2 + 89
69355523400*x^3 + 5338425594080*x^4 + 1168275815040*x^5) + (10*(2 + 7*x +
4*x^2)^2*(47461204986*Sqrt[3 + Sqrt[17]]*(2 + 7*x + 4*x^2) + (23730602493*
I)*(3 + Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7*x + 4*x^2)/(1 + 2*x)^2]*Elli
pticE[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], -13/4 + (3*Sqrt[17
])/4] - I*(36877705141 + 23730602493*Sqrt[17])*(1 + 2*x)^(3/2)*Sqrt[(2 + 7
*x + 4*x^2)/(1 + 2*x)^2]*EllipticF[I*ArcSinh[Sqrt[2/(-3 + Sqrt[17])]/Sqrt[
1 + 2*x]], -13/4 + (3*Sqrt[17])/4] - (45366642*I)*(1 + 2*x)^(3/2)*Sqrt[(2
+ 7*x + 4*x^2)/(1 + 2*x)^2]*EllipticPi[(-13*(-3 + Sqrt[17]))/6, I*ArcSinh[
Sqrt[2/(-3 + Sqrt[17])]/Sqrt[1 + 2*x]], (-13 + 3*Sqrt[17])/4]))/Sqrt[3 + S
qrt[17]]))/(10624193320845*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(5/2))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx+\frac{1}{85}\int \frac{2(84x+131)}{\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{765\,(4x^2+7x+2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx+\frac{2}{85}\int \frac{84x+131}{\sqrt{2x+1}\,(4x^2+7x+2)^{5/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{765\,(4x^2+7x+2)^{5/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx+\frac{2}{85}\left(\frac{1}{51}\int -\frac{4(2106x+2681)}{\sqrt{2x+1}\,(4x^2+7x+2)^{3/2}}dx-\frac{2\sqrt{2x+1}(1404x+1985)}{51\,(4x^2+7x+2)^{3/2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{765\,(4x^2+7x+2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+7x+2)^{7/2}}dx+\frac{2}{85}\left(-\frac{4}{51}\int \frac{2106x+2681}{\sqrt{2x+1}\,(4x^2+7x+2)^{3/2}}dx-\frac{2\sqrt{2x+1}(1404x+1985)}{51\,(4x^2+7x+2)^{3/2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{765\,(4x^2+7x+2)^{5/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+\frac{2}{85}\left(-\frac{4}{51}\left(\frac{1}{17}\int \frac{2(27960x+10627)}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{2\sqrt{2x+1}(27960x+38303)}{17\sqrt{4x^2+7x+2}}\right)-\frac{2\sqrt{2x+1}(1404x+1985)}{51\left(4x^2+7x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\int \frac{27960x+10627}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx-\frac{2\sqrt{2x+1}(27960x+38303)}{17\sqrt{4x^2+7x+2}}\right)-\frac{2\sqrt{2x+1}(1404x+1985)}{51\left(4x^2+7x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\left(13980\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+7x+2}}dx-3353\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+7x+2}}dx\right)-\frac{2\sqrt{2x+1}(27960x+38303)}{17\sqrt{4x^2+7x+2}}\right.\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$$\Big\downarrow \text{1172}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\left(\frac{6990\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{6706\sqrt{-2x-1}\sqrt{-4x^2}}{\sqrt{3}}\right.\right.\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$$\Big\downarrow \text{321}$$

$$\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\left(\frac{6990\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}\int \frac{\sqrt{1-\frac{8x+\sqrt{17}+7}{3+\sqrt{17}}}}{\sqrt{1-\frac{8x+\sqrt{17}+7}{2\sqrt{17}}}}d\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{6706\sqrt{-2x-1}\sqrt{-4x^2}}{}\right.\right.\right.$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$\downarrow$ 327

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+$$

$$\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\left(\frac{6990\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{6706\sqrt{-2}}{}\right.\right.\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

$\downarrow$ 1292

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+7x+2\right)^{7/2}}dx+$$

$$\frac{2}{85}\left(-\frac{4}{51}\left(\frac{2}{17}\left(\frac{6990\sqrt{3+\sqrt{17}}\sqrt{2x+1}\sqrt{-4x^2-7x-2}E\left(\arcsin\left(\frac{\sqrt{8x+\sqrt{17}+7}}{\sqrt{2}\sqrt[4]{17}}\right)|\frac{1}{4}\left(17-3\sqrt{17}\right)\right)}{\sqrt{-2x-1}\sqrt{4x^2+7x+2}}-\frac{6706\sqrt{-2}}{}\right.\right.\right.$$

$$\frac{8\sqrt{2x+1}(27x+47)}{765\left(4x^2+7x+2\right)^{5/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 7*x + 4*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.35

method	result
elliptic	$\frac{\sqrt{(4x^2+7x+2)(1+2x)}\left(\frac{\left(-\frac{41}{18955}-\frac{319x}{151640}\right)\sqrt{8x^3+18x^2+11x+2}}{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)^3}+\frac{\left(\frac{10115413}{862300860}+\frac{494183x}{43115043}\right)\sqrt{8x^3+18x^2+11x+2}}{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)^2}-\frac{2(4+8x)\left(\frac{7809126509}{163449128013}+\right.}{\sqrt{\left(x^2+\frac{7}{4}x+\frac{1}{2}\right)}}\right.}{}$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+7*x+2)*(1+2*x))^(1/2)/(4*x^2+7*x+2)^(1/2)/(1+2*x)^(1/2)*((-41/1895
5-319/151640*x)*(8*x^3+18*x^2+11*x+2)^(1/2)/(x^2+7/4*x+1/2)^3+(10115413/86
2300860+494183/43115043*x)*(8*x^3+18*x^2+11*x+2)^(1/2)/(x^2+7/4*x+1/2)^2-2
*(4+8*x)*(7809126509/163449128013+2433907948/54483042671*x)/((x^2+7/4*x+1/
2)*(4+8*x))^(1/2)+79502243488/163449128013*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(
3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)
*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/
2)*EllipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3
/8+1/8*17^(1/2)))^(1/2))+38942527168/54483042671*(-3/8-1/8*17^(1/2))*(-(x+
1/2)/(3/8+1/8*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))
^(1/2)*((x+7/8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+
2)^(1/2)*((3/8-1/8*17^(1/2))*EllipticE((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2)
,I*((3/8+1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2))+(-7/8+1/8*17^(1/2))*Ell
ipticF((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),I*((3/8+1/8*17^(1/2))/(-3/8+1/8
*17^(1/2)))^(1/2)))+24624/144164371*(-3/8-1/8*17^(1/2))*(-(x+1/2)/(3/8+1/8
*17^(1/2)))^(1/2)*(-(x+7/8-1/8*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)*((x+7/
8+1/8*17^(1/2))/(3/8+1/8*17^(1/2)))^(1/2)/(8*x^3+18*x^2+11*x+2)^(1/2)*Elli
pticPi((-(x+1/2)/(3/8+1/8*17^(1/2)))^(1/2),-9/52-3/52*17^(1/2),I*((3/8+1/8
*17^(1/2))/(-3/8+1/8*17^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 7*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(1536*x^10
+ 8960*x^9 + 17472*x^8 + 3584*x^7 - 38378*x^6 - 68775*x^5 - 58237*x^4 - 28
224*x^3 - 7992*x^2 - 1232*x - 80), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+7*x+2)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(7/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+7\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 7*x + 2)^(7/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+7\,x+2\right)^{7/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(7/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(7*x + 4*x^2 + 2)^(7/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+7x+4x^2)^{7/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+7x+2)^{\frac{7}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+7*x+2)^(7/2),x)
```

3.57 $\int \frac{(f+gx)^2(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 1068

$$\int \frac{(f+gx)^2\,(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2/105*(24*b^2*C*e^2*g^2-c*e*g*(25*a*C*e*g+b*(28*B*e*g-41*C*d*g+56*C*e*f))+
c^2*(7*e*g*(5*A*e*g-7*B*d*g+10*B*e*f)+C*(42*d^2*g^2-68*d*e*f*g+20*e^2*f^2)
))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^3/e^3-2/35*g*(6*b*C*e*g-c*(7*B*e*g-
6*C*d*g+4*C*e*f))*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c^2/e^3+2/7*C*(e*x+d)^
(1/2)*(g*x+f)^2*(c*x^2+b*x+a)^(1/2)/c/e-1/105*2^(1/2)*(-4*a*c+b^2)^(1/2)*(
48*b^3*C*e^3*g^2-8*b*c*e^2*g*(13*a*C*e*g+b*(7*B*e*g-5*C*d*g+14*C*e*f))+c^2
*e*(7*b*e*g*(10*A*e*g-7*B*d*g+20*B*e*f)+a*e*g*(63*B*e*g-44*C*d*g+126*C*e*f
)+b*C*(40*d^2*g^2-98*d*e*f*g+70*e^2*f^2))+c^3*(2*C*d*(24*d^2*g^2-56*d*e*f*
g+35*e^2*f^2)-7*e*(10*A*e*g*(-d*g+3*e*f)+B*(8*d^2*g^2-20*d*e*f*g+15*e^2*f^
2))))*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1
+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c
*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^4/e^4/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+
b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/105*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(24*b^2*C*e^3*(-a*e+b*d)*g^2+c*e^2*g*(25*a^2*C*e^2*g+2*a*b*e*(14*B*e*g-33
*C*d*g+28*C*e*f)-b^2*d*(28*B*e*g-17*C*d*g+56*C*e*f))+c^3*(2*C*d^2*(24*d^2*
g^2-56*d*e*f*g+35*e^2*f^2)+7*e*(5*A*e*(2*d^2*g^2-6*d*e*f*g+3*e^2*f^2)-B*d*
(8*d^2*g^2-20*d*e*f*g+15*e^2*f^2)))+c^2*e*(b*d*(7*e*g*(5*A*e*g-3*B*d*g+10*
B*e*f)+C*(16*d^2*g^2-42*d*e*f*g+35*e^2*f^2))-a*e*(7*e*g*(5*A*e*g-7*B*d*g+1
0*B*e*f)+C*(32*d^2*g^2-98*d*e*f*g+35*e^2*f^2))))*(c*(e*x+d)/(2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*Ellipti...
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 38.92 (sec) , antiderivative size = 37177, normalized size of antiderivative = 34.81

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f + g*x)^2*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*
x^2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 5.47 (sec) , antiderivative size = 1109, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {2184, 27, 2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2184}$$

$$\frac{2\int \frac{e^3g(14cCef-16cCdg+7Bceg-6bCeg)x^3-e^2\left(-7cg(2Bf+Ag)e^2+C(13bd+5ae)g^2e-cC\left(7e^2f^2-11d^2g^2\right)\right)x^2+e\left(7Bcf^2e^3+2g(7Ace^3f-Cd(\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{7ce^4}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{e^3g(14cCef-16cCdg+7Bceg-6bCeg)x^3-e^2\left(-7cg(2Bf+Ag)e^2+C(13bd+5ae)g^2e-cC\left(7e^2f^2-11d^2g^2\right)\right)x^2+e\left(7Bcf^2e^3+2g(7Ace^3f-Cd(\right.}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{7ce^4}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$$\Big\downarrow \text{ 2184}$$

$$2\int \frac{\left(\left(7eg(10Bef-7Bdg+5Aeg)+C\left(35e^2f^2-98degf+57d^2g^2\right)\right)c^2-eg(25aCeg+b(56Cef-41Cdg+28Beg))c+24b^2Ce^2g^2\right)x^2e^5+\left(35Ac^2f^2e^3+dg\left(6Cdegb^2+18\right.\right.}{}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\left(\left(7eg(10Bef-7Bdg+5Aeg)+C\left(35e^2f^2-98degf+57d^2g^2\right)\right)c^2-eg(25aCeg+b(56Cef-41Cdg+28Beg))c+24b^2Ce^2g^2\right)x^2e^5+\left(35Ac^2f^2e^3+dg\left(6Cdegb^2+18a\right.\right.}{}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$$\Big\downarrow \text{ 2184}$$

$$2\int -\frac{e^6\Big(24Cde^2g^2b^3+eg\Big(24aCe^2g-cd(56Cef-23Cdg+28Beg)\Big)b^2-c\Big(2ae^2g(28Cef+19Cdg+14Beg)-cd\Big(7eg(10Bef-4Bdg+5Aeg)+C\Big(35e^2f^2-56degf+24d^2$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$\downarrow$ 27

$$\frac{2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\Big(-ceg(25aCeg+b(28Beg-41Cdg+56Cef))+c^2\Big(7eg(5Aeg-7Bdg+10Bef)+C\Big(57d^2g^2-98defg+35e^2f^2\Big)\Big)+24b^2Ce^2g^2\Big)}{3c}-\frac{e^4\int\frac{24Cde^2g}{}}{}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$\downarrow$ 1269

$$\frac{2e^4\sqrt{d+ex}\sqrt{a+bx+cx^2}\Big(-ceg(25aCeg+b(28Beg-41Cdg+56Cef))+c^2\Big(7eg(5Aeg-7Bdg+10Bef)+C\Big(57d^2g^2-98defg+35e^2f^2\Big)\Big)+24b^2Ce^2g^2\Big)}{3c}-\frac{e^4\Bigg(\frac{\Big(c^2e\Big(aeg}{}}{}$$

$$\frac{2Cg^2(d+ex)^{5/2}\sqrt{a+bx+cx^2}}{7ce^3}$$

$\downarrow$ 1172

$$\frac{2Cg^2\sqrt{cx^2+bx+a}(d+ex)^{5/2}}{7ce^3}+$$

$$\frac{2eg(14cCef-16cCdg+7Bceg-6bCeg)\sqrt{cx^2+bx+a}(d+ex)^{3/2}}{5c}+\frac{2e^4\Big(\Big(7eg(10Bef-7Bdg+5Aeg)+C\Big(35e^2f^2-98degf+57d^2g^2\Big)\Big)c^2-eg(25aCeg+b(56C}{3c}$$

$\downarrow$ 321

$$\frac{2Cg^2\sqrt{cx^2+bx+a}(d+ex)^{5/2}}{7ce^3}+$$

$$\frac{\frac{2eg(14cCef-16cCdg+7Bceg-6bCeg)\sqrt{cx^2+bx+a}(d+ex)^{3/2}}{5c}+\frac{\frac{2e^4\left(\left(7eg(10Bef-7Bdg+5Aeg)+C\left(35e^2f^2-98degf+57d^2g^2\right)\right)c^2-eg(25aCeg+b(56C}{3c}}{}}{}$$

$$\Big\downarrow \text{327}$$

$$\frac{2Cg^2\sqrt{cx^2+bx+a}(d+ex)^{5/2}}{7ce^3}+$$

$$\frac{\frac{2eg(14cCef-16cCdg+7Bceg-6bCeg)\sqrt{cx^2+bx+a}(d+ex)^{3/2}}{5c}+\frac{\frac{2e^4\left(\left(7eg(10Bef-7Bdg+5Aeg)+C\left(35e^2f^2-98degf+57d^2g^2\right)\right)c^2-eg(25aCeg+b(56C}{3c}}{}}{}$$

input

```
Int[((f + g*x)^2*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
(2*C*g^2*(d + e*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(7*c*e^3) + ((2*e*g*(14*c*
C*e*f - 16*c*C*d*g + 7*B*c*e*g - 6*b*C*e*g)*(d + e*x)^(3/2)*Sqrt[a + b*x +
 c*x^2])/(5*c) + ((2*e^4*(24*b^2*C*e^2*g^2 - c*e*g*(25*a*C*e*g + b*(56*C*e
*f - 41*C*d*g + 28*B*e*g)) + c^2*(7*e*g*(10*B*e*f - 7*B*d*g + 5*A*e*g) + C
*(35*e^2*f^2 - 98*d*e*f*g + 57*d^2*g^2)))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x
^2])/(3*c) - (e^4*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*C*e^3*g^2 - 8*b*c*e^
2*g*(13*a*C*e*g + b*(14*C*e*f - 5*C*d*g + 7*B*e*g)) + c^2*e*(7*b*e*g*(20*B
*e*f - 7*B*d*g + 10*A*e*g) + a*e*g*(126*C*e*f - 44*C*d*g + 63*B*e*g) + b*C
*(70*e^2*f^2 - 98*d*e*f*g + 40*d^2*g^2)) + c^3*(2*C*d*(35*e^2*f^2 - 56*d*e
*f*g + 24*d^2*g^2) - 7*e*(10*A*e*g*(3*e*f - d*g) + B*(15*e^2*f^2 - 20*d*e*
f*g + 8*d^2*g^2))))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*
c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*
c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b
*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(24*b^2*C*e^3*(b*d - a*e)*g^2
+ c*e^2*g*(25*a^2*C*e^2*g + 2*a*b*e*(28*C*e*f - 33*C*d*g + 14*B*e*g) - b^2
*d*(56*C*e*f - 17*C*d*g + 28*B*e*g)) + c^3*(2*C*d^2*(35*e^2*f^2 - 56*d*e*f
*g + 24*d^2*g^2) + 7*e*(5*A*e*(3*e^2*f^2 - 6*d*e*f*g + 2*d^2*g^2) - B*d*(1
5*e^2*f^2 - 20*d*e*f*g + 8*d^2*g^2))) + c^2*e*(b*d*(7*e*g*(10*B*e*f - 3*B*
d*g + 5*A*e*g) + C*(35*e^2*f^2 - 42*d*e*f*g + 16*d^2*g^2)) - a*e*(7*e*g...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 18.72 (sec) , antiderivative size = 1316, normalized size of antiderivative = 1.23

method	result	size
elliptic	Expression too large to display	1316
risch	Expression too large to display	7433
default	Expression too large to display	25662

input
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/7*C*g^2
/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/5*(B*g^2+2*C*f*
g-2/7*C*g^2/c/e*(3*b*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+
a*d)^(1/2)+2/3*(A*g^2+2*B*f*g+C*f^2-2/7*C*g^2/c/e*(5/2*a*e+5/2*b*d)-2/5*(B
*g^2+2*C*f*g-2/7*C*g^2/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+
b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A*f^2-2/5*(B*g^2+2*C*f*g-2/7*C*g
^2/c/e*(3*b*e+3*c*d))/c/e*d*a-2/3*(A*g^2+2*B*f*g+C*f^2-2/7*C*g^2/c/e*(5/2*
a*e+5/2*b*d)-2/5*(B*g^2+2*C*f*g-2/7*C*g^2/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*
c*d))/c/e*(1/2*a*e+1/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(
d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+
a*e*x+b*d*x+a*d)^(1/2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/
c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2
)^(1/2))))^(1/2))+2*(2*A*f*g+B*f^2-4/7*C*g^2/c/e*d*a-2/5*(B*g^2+2*C*f*g-2/
7*C*g^2/c/e*(3*b*e+3*c*d))/c/e*(3/2*a*e+3/2*b*d)-2/3*(A*g^2+2*B*f*g+C*f^2-
2/7*C*g^2/c/e*(5/2*a*e+5/2*b*d)-2/5*(B*g^2+2*C*f*g-2/7*C*g^2/c/e*(3*b*e+3*
c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)
*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+
b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4...
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1121, normalized size of antiderivative = 1.05

$$\int \frac{(f+gx)^2\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\text{Too large to display}$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="fricas")
```

output
```
2/315*((35*(2*C*c^4*d^2*e^2 + (C*b*c^3 - 3*B*c^4)*d*e^3 + (2*C*b^2*c^2 + 9
*A*c^4 - 3*(C*a + B*b)*c^3)*e^4)*f^2 - 14*(8*C*c^4*d^3*e + (3*C*b*c^3 - 10
*B*c^4)*d^2*e^2 + (3*C*b^2*c^2 + 15*A*c^4 - (3*C*a + 5*B*b)*c^3)*d*e^3 + (
8*C*b^3*c + 15*(B*a + A*b)*c^3 - (21*C*a*b + 10*B*b^2)*c^2)*e^4)*f*g + (48
*C*c^4*d^4 + 8*(2*C*b*c^3 - 7*B*c^4)*d^3*e + (11*C*b^2*c^2 + 70*A*c^4 - (8
*C*a + 21*B*b)*c^3)*d^2*e^2 + (16*C*b^3*c + 7*(3*B*a + 5*A*b)*c^3 - (34*C*
a*b + 21*B*b^2)*c^2)*d*e^3 + (48*C*b^4 - 105*A*a*c^3 + (75*C*a^2 + 147*B*a
*b + 70*A*b^2)*c^2 - 8*(22*C*a*b^2 + 7*B*b^3)*c)*e^4)*g^2)*sqrt(c*e)*weier
strassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/2
7*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*
c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(35*(2*C*c^4*d*e^3
 + (2*C*b*c^3 - 3*B*c^4)*e^4)*f^2 - 14*(8*C*c^4*d^2*e^2 + (7*C*b*c^3 - 10*
B*c^4)*d*e^3 + (8*C*b^2*c^2 + 15*A*c^4 - (9*C*a + 10*B*b)*c^3)*e^4)*f*g +
(48*C*c^4*d^3*e + 8*(5*C*b*c^3 - 7*B*c^4)*d^2*e^2 + (40*C*b^2*c^2 + 70*A*c
^4 - (44*C*a + 49*B*b)*c^3)*d*e^3 + (48*C*b^3*c + 7*(9*B*a + 10*A*b)*c^3 -
 8*(13*C*a*b + 7*B*b^2)*c^2)*e^4)*g^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*
d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weier
strassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/2
7*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a...
```

Sympy [F]

$$\int \frac{(f+gx)^2\,(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)^2\,(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)**2*(C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x
)
```

output
```
Integral((f + g*x)**2*(A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x
**2)), x)
```

Maxima [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x +
d)), x)
```

Giac [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(Cx^2+Bx+A)(gx+f)^2}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)^2/(sqrt(c*x^2 + b*x + a)*sqrt(e*x +
d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+g\,x)^2\,(C\,x^2+B\,x+A)}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/
2)),x)
```

output
```
int(((f + g*x)^2*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/
2)), x)
```

Reduce [F]

$$\int \frac{(f+gx)^2 (A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(gx+f)^2 (C\,x^2+Bx+A)}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x+f)^2*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.58 $\int \frac{(f+gx)(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 39, antiderivative size = 650

$$\int \frac{(f+gx)(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = -\frac{2(4bCeg-c(5Cef-7Cdg+5Beg))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{15c^2e^2} + \frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}(8b^2Ce^2g-c^2(2Cd(5ef-4dg)-5e(3Bef-2Bdg+3Aeg))-ce(9aCeg+b(10Cef-}{15c^3e^3\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(4bCe^2(bd-ae)g+ce(ae(5Cef-7Cdg+5Beg)-bd(5Cef-3Cdg+5Beg))-c^2(}{}$$

output
```
-2/15*(4*b*C*e*g-c*(5*B*e*g-7*C*d*g+5*C*e*f))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^
(1/2)/c^2/e^2+2/5*C*g*(e*x+d)^(3/2)*(c*x^2+b*x+a)^(1/2)/c/e^2+1/15*2^(1/2)
*(-4*a*c+b^2)^(1/2)*(8*b^2*C*e^2*g-c^2*(2*C*d*(-4*d*g+5*e*f)-5*e*(3*A*e*g-
2*B*d*g+3*B*e*f))-c*e*(9*a*C*e*g+b*(10*B*e*g-7*C*d*g+10*C*e*f)))*(e*x+d)^(
1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*
a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b
^2)^(1/2))*e))^(1/2))/c^3/e^3/(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))
^(1/2)/(c*x^2+b*x+a)^(1/2)-2/15*2^(1/2)*(-4*a*c+b^2)^(1/2)*(4*b*C*e^2*(-a*
e+b*d)*g+c*e*(a*e*(5*B*e*g-7*C*d*g+5*C*e*f)-b*d*(5*B*e*g-3*C*d*g+5*C*e*f))
-c^2*(2*C*d^2*(-4*d*g+5*e*f)-5*e*(B*d*(-2*d*g+3*e*f)-3*A*e*(-d*g+e*f))))*(
c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*
c+b^2))^(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)
,(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^3/e^3
/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.50 (sec) , antiderivative size = 18202, normalized size of antiderivative = 28.00

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((f + g*x)*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 683, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {2184, 27, 2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2184}$$

$$\frac{2\int \frac{-e^2(4bCeg-c(5Cef-7Cdg+5Beg))x^2+e\left(5Bce^2f-\left(Ce(5bd+3ae)+c\left(2Cd^2-5Ae^2\right)\right)g\right)x+e\left(5Ace^2f-Cd(bd+3ae)g\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}+\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{-e^2(4bCeg-c(5Cef-7Cdg+5Beg))x^2+e\left(5Bce^2f-\left(Ce(5bd+3ae)+c\left(2Cd^2-5Ae^2\right)\right)g\right)x+e\left(5Ace^2f-Cd(bd+3ae)g\right)}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{5ce^3}+\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$\Big\downarrow \text{2184}$$

$$\frac{\frac{2\int \frac{e^3\left(4Cdegb^2+\left(4aCe^2g-cd(5Cef-4Cdg+5Beg)\right)b+ce(15Acef-a(5Cef+2Cdg+5Beg))+\left(-\left((2Cd(5ef-4dg)-5e(3Bef-2Bdg+3Aeg))c^2\right)-e(9aCeg+b(10C\cdots\right.\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3ce^2}}{5ce^3}+\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{e\int \frac{4Cdegb^2+\left(4aCe^2g-cd(5Cef-4Cdg+5Beg)\right)b+ce(15Acef-a(5Cef+2Cdg+5Beg))+\left(-\left((2Cd(5ef-4dg)-5e(3Bef-2Bdg+3Aeg))c^2\right)-e(9aCeg+b(10Cef\cdots\right.}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}}{3c}}{5ce^3}+\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{e\left(\frac{\left(-ce(9aCeg+b(10Beg-7Cdg+10Cef))-\left(c^2(2Cd(5ef-4dg)-5e(3Aeg-2Bdg+3Bef))\right)+8b^2Ce^2g\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(ce(ae(5Beg-7Cdg+5Cef)-bd(5B\cdots\right.}{\cdots}\right.}{3c}$$

$$\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$\Big\downarrow$ 1172

$$\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCeg+b(10Beg-7Cdg+10Cef))-\left(c^2(2Cd(5ef-4dg)-5e(3Aeg-2Bdg+3Bef))\right)+8b^2Ce^2g\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b\cdots}\right)}{2cd-\left(b+\sqrt{b^2\cdots}\right)}}}{\sqrt{1-\frac{b+2cx+\cdots}{2\sqrt{b^2\cdots}}}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\cdots\right.}{\cdots}$$

$$\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$\Big\downarrow$ 321

$$\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\left(-ce(9aCeg+b(10Beg-7Cdg+10Cef))-\left(c^2(2Cd(5ef-4dg)-5e(3Aeg-2Bdg+3Bef))\right)+8b^2Ce^2g\right)\int\frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b\cdots}\right)}{2cd-\left(b+\sqrt{b^2\cdots}\right)}}}{\sqrt{1-\frac{b+2cx+\cdots}{2\sqrt{b^2\cdots}}}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\cdots\right.}{\cdots}$$

$$\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

$\Big\downarrow$ 327

$$\frac{e\left(\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\left(-ce(9aCeg+b(10Beg-7Cdg+10Cef))-\left(c^2(2Cd(5ef-4\cdots\right.\right.}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}\cdots\right.}{\cdots}$$

$$\frac{2Cg(d+ex)^{3/2}\sqrt{a+bx+cx^2}}{5ce^2}$$

input
```
Int[((f + g*x)*(A + B*x + C*x^2))/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*g*(d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(5*c*e^2) + ((-2*e*(4*b*C*e*
g - c*(5*C*e*f - 7*C*d*g + 5*B*e*g))*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/
(3*c) + (e*((Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^2*C*e^2*g - c^2*(2*C*d*(5*e*f
- 4*d*g) - 5*e*(3*B*e*f - 2*B*d*g + 3*A*e*g)) - c*e*(9*a*C*e*g + b*(10*C*e
*f - 7*C*d*g + 10*B*e*g)))*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2
 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqr
t[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(4*b*C*e^2*(b*d - a*e)*
g + c*e*(a*e*(5*C*e*f - 7*C*d*g + 5*B*e*g) - b*d*(5*C*e*f - 3*C*d*g + 5*B*
e*g)) - c^2*(2*C*d^2*(5*e*f - 4*d*g) - 5*e*(B*d*(3*e*f - 2*d*g) - 3*A*e*(e
*f - d*g))))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[
-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d + e*x]*Sqrt[a + b*x + c*
x^2])))/(3*c))/(5*c*e^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [A] (verified)

Time = 12.45 (sec) , antiderivative size = 979, normalized size of antiderivative = 1.51

method	result
elliptic	$\frac{\sqrt{(ex+d)(c\,x^2+bx+a)}\left(\frac{2gCx\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}{5ce}+\frac{2\left(Bg+Cf-\frac{2(2be+2cd)gC}{5ce}\right)\sqrt{x^3ec+be\,x^2+cd\,x^2+aex+bdx+da}}{3ce}+\frac{2\left(Af\right.}{}\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/5*g*C/c
/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2/3*(B*g+C*f-2/5/c/e*
(2*b*e+2*c*d)*g*C)/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(
A*f-2/5*a/c*d/e*g*C-2/3*(B*g+C*f-2/5/c/e*(2*b*e+2*c*d)*g*C)/c/e*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(A*g+B*f-2/5*g*C/c/e*(3/2*a*e+3/2*b*d)-2/3*(B*g+C*f-2/5/c/e*(2*b*e+2*c*d)*
g*C)/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(
b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1
/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d
/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d
*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/
e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c
)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2
))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))...
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 686, normalized size of antiderivative = 1.06

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
2/45*(sqrt(c*e)*(5*(2*C*c^3*d^2*e + (C*b*c^2 - 3*B*c^3)*d*e^2 + (2*C*b^2*c
 + 9*A*c^3 - 3*(C*a + B*b)*c^2)*e^3)*f - (8*C*c^3*d^3 + (3*C*b*c^2 - 10*B*
c^3)*d^2*e + (3*C*b^2*c + 15*A*c^3 - (3*C*a + 5*B*b)*c^2)*d*e^2 + (8*C*b^3
 + 15*(B*a + A*b)*c^2 - (21*C*a*b + 10*B*b^2)*c)*e^3)*g)*weierstrassPInver
se(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3
*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*sqrt(c*e)*(5*(2*C*c^3*d*e^2 +
(2*C*b*c^2 - 3*B*c^3)*e^3)*f - (8*C*c^3*d^2*e + (7*C*b*c^2 - 10*B*c^3)*d*e
^2 + (8*C*b^2*c + 15*A*c^3 - (9*C*a + 10*B*b)*c^2)*e^3)*g)*weierstrassZeta
(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e
^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*
e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^
3 - 9*a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*C*c^
3*e^3*g*x + 5*C*c^3*e^3*f - (4*C*c^3*d*e^2 + (4*C*b*c^2 - 5*B*c^3)*e^3)*g)
*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d))/(c^4*e^4)
```

Sympy [F]

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((g*x+f)*(C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((f + g*x)*(A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2
)), x)
```

Maxima [F]

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(gx+f)}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)
), x)
```

Giac [F]

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(Cx^2+Bx+A)(gx+f)}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(g*x + f)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)\left(A+Bx+Cx^2\right)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(f+g\,x)\,\left(C\,x^2+B\,x+A\right)}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)
),x)
```

output
```
int(((f + g*x)*(A + B*x + C*x^2))/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{(f+gx)\,(A+Bx+Cx^2)}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{(gx+f)\,(C\,x^2+Bx+A)}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((g*x+f)*(C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.59 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 450

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\sqrt{2}\sqrt{b^2-4ac}(2cCd-3Bce+2bCe)\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ace}}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{3c^2e^2\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cCd^2+Ce(bd-ae)-3ce(Bd-Ae))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(a\ldots\right.}{3c^2e^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2/3*C*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/c/e-1/3*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(-3*B*c*e+2*C*b*e+2*C*c*d)*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(c*(e*x
+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)+2/3*2^(1/2
)*(-4*a*c+b^2)^(1/2)*(2*c*C*d^2+C*e*(-a*e+b*d)-3*c*e*(-A*e+B*d))*(c*(e*x+d
)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^
(1/2)*EllipticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4
*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c^2/e^2/(e*x+d)
^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.40

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*(a + b*x + c*x^2))/(3*c*e*Sqrt[a + x*(b + c*x)]) + ((d
+ e*x)^(3/2)*Sqrt[a + b*x + c*x^2]*(-4*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[
(c*d^2 + e*(-(b*d) + a*e))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c*(-
1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)
) + (I*Sqrt[2]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*(2*c*d - b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1
 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4
*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(
-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(
-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt
[d + e*x] - (I*Sqrt[2]*(-2*b^2*C*e^2 + b*e*(3*B*c*e + 2*C*Sqrt[(b^2 - 4*a*
c)*e^2]) + c*(-6*A*c*e^2 + 2*a*C*e^2 + Sqrt[(b^2 - 4*a*c)*e^2]*(2*C*d - 3*
B*e)))*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*a*e^2)/(d + e*x) - 2*c*d*(-1 + d
/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*c*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*a*e^2)/(d + e*x) + 2*c*d*(-1 +
 d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*c*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*d^2 - b*d*e + a*e^2)/(-2*c
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*c*d + b*e +...
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2184, 27, 1269, 1172, 321, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \ 2184$$

$$\frac{2\int -\frac{e(bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce^2}+\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}$$

$$\Big\downarrow \ 27$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\int \frac{bCd-3Ace+aCe+(2cCd-3Bce+2bCe)x}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{3ce}$$

$$\Big\downarrow \ 1269$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{(2bCe-3Bce+2cCd)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{e}-\frac{\left(Ce(bd-ae)-3ce(Bd-Ae)+2cCd^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{e}}{3ce}$$

$$\Big\downarrow \ 1172$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce}-\frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\cdots}}{\cdots}}{3ce}$$

$$\Big\downarrow \ 321$$

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}\sqrt{\ }}{}}{3ce}$$

$\Big\downarrow$ 327

$$\frac{2C\sqrt{d+ex}\sqrt{a+bx+cx^2}}{3ce} - \frac{\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c\left(a+bx+cx^2\right)}{b^2-4ac}}(2bCe-3Bce+2cCd)E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+b}{b^2}}}{}}{3ce}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*C*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) - ((Sqrt[2]*Sqrt[b^2 - 4
*a*c]*(2*c*C*d - 3*B*c*e + 2*b*C*e)*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)])/(c*e*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*c*C*d^2 + C
*e*(b*d - a*e) - 3*c*e*(B*d - A*e))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[
b^2 - 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-
2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*Sqrt[d +
 e*x]*Sqrt[a + b*x + c*x^2]))/(3*c*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 2184
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Simp[1/(c*e^q*(m + q + 2*p + 1))   Int[(d + e*x)^m*(a +
b*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p +
1)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c
*d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[
q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Pol
yQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGt
Q[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(396) = 792$.

Time = 6.76 (sec) , antiderivative size = 823, normalized size of antiderivative = 1.83

method	result
elliptic	$\frac{\sqrt{(ex+d)(cx^2+bx+a)}\left(\frac{2C\sqrt{x^3ec+bex^2+cdx^2+aex+bdx+da}}{3ce}+\frac{2\left(A-\frac{2C\left(\frac{ae}{2}+\frac{bd}{2}\right)}{3ce}\right)\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2c}}}\sqrt{\frac{x-\frac{-b \cdots}{\cdots}}{-\frac{d}{e}-\cdots}}\cdots}{\sqrt{x^3e \cdots}}\cdots\right)}{\cdots}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2/3*C/c/e
*(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)+2*(A-2/3*C/c/e*(1/2*a*e+1
/2*b*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-
4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/
2)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*
(B-2/3*C/c/e*(b*e+c*d))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1
/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)
/(-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x
+b*d*x+a*d)^(1/2)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)
/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2
))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^
(1/2))*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$
$$= \frac{2\left(3\sqrt{cx^2+bx+a}\sqrt{ex+d}Cc^2e^2+(2\,Cc^2d^2+(Cbc-3\,Bc^2)de+(2\,Cb^2+9\,Ac^2-3\,(Ca+Bb)c)e^2)\sqrt{\right.}}{}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="fr
icas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*C*c^2*e^2 + (2*C*c^2*d^2 + (C*b
*c - 3*B*c^2)*d*e + (2*C*b^2 + 9*A*c^2 - 3*(C*a + B*b)*c)*e^2)*sqrt(c*e)*w
eierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2),
-4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*
a*b*c)*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*C*c^2*d*e +
 (2*C*b*c - 3*B*c^2)*e^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e
 + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*(b^2
*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + (b^2 - 3*a*c)*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*(b^2*c - 6*a*c^2)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))))/(c^3*e^3)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="ma
xima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="gi
ac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{\sqrt{ex+d}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.60 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 643

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{2}\sqrt{b^2-4ac}C\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$
$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(Cef+Cdg-Beg)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$+\frac{4\sqrt{2}\sqrt{b^2-4ac}(Cf^2-Bfg+Ag^2)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\,\text{EllipticPi}\left(-\frac{2\sqrt{b^2-4acg}}{2cf-\left(b+\sqrt{b^2-4ac}\right)g},a\right.}{g^2\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

output
```
2^(1/2)*(-4*a*c+b^2)^(1/2)*C*(e*x+d)^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))
^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-
4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e/g/(c*(e*x+
d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(
-4*a*c+b^2)^(1/2)*(-B*e*g+C*d*g+C*e*f)*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticF(1/2*(1+(2*
c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(
b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/e/g^2/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)
+4*2^(1/2)*(-4*a*c+b^2)^(1/2)*(A*g^2-B*f*g+C*f^2)*(c*(e*x+d)/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticP
i(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),-2*(-4*a*c+b^2)^(1/2)
*g/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e))^(1/2))/g^2/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d)^(
1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 36.72 (sec) , antiderivative size = 19347, normalized size of antiderivative = 30.09

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 2.75 (sec) , antiderivative size = 822, normalized size of antiderivative = 1.28, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.268$, Rules used = {2154, 1269, 1172, 321, 327, 1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1269}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\frac{(Beg-C(dg+ef))\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{eg^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx+a}}dx}{eg}$$

$$\Big\downarrow \text{1172}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\sqrt{}}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}+\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$$\Big\downarrow \text{321}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+$$
$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\int \frac{\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}+$$
$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 327

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+$$
$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 1279

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}(f+gx)}dx}{\sqrt{a+bx+cx^2}}$$
$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$
$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 187

$$-\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int -\frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-}}}}{\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\Big\downarrow$ 25

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}}}}{\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\Big\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\int \frac{}{\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 413

$$\frac{2\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{\sqrt{b^2-4ac}+b+2cx}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\sqrt{1-\frac{2c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right.}}}{\sqrt{a+bx+cx^2}\sqrt{-\sqrt{b^2-4ac}+b+\frac{2c(d+ex)}{e}-\frac{2cd}{e}}\sqrt{\sqrt{b^2-4ac}-}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}(Beg-C(dg+ef))\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$\frac{\sqrt{2}C\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{a+bx+cx^2}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

$\downarrow$ 412

$$\frac{\sqrt{2}\sqrt{b^2-4ac}C\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{ceg\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2+bx+a}}+$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}(Beg-C(ef+dg))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{}{2}\right.}{ceg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(A+\frac{f(Cf-Bg)}{g^2}\right)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-\right.}}}}{\sqrt{c}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{}}}$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(Sqrt[2]*Sqrt[b^2 - 4*a*c]*C*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b
^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e)])/(c*e*g*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(B*e*g - C*(e*f + d
*g))*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((c*(a
+ b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(c*e*g^2*Sqrt[d + e*x]*Sqrt[a + b*x + c*x^2]
) + (Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(A + (f*(C*f - B*g))/
g^2)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*
x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 -
(2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[-1/2*((2*c
*d - b*e + Sqrt[b^2 - 4*a*c]*e)*g)/(c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c
]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[c]*(e*f -
 d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*
(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*c*(d + e*x))/e])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
 - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 24.26 (sec) , antiderivative size = 1114, normalized size of antiderivative = 1.73

method	result	size
elliptic	Expression too large to display	1114
default	Expression too large to display	3400

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(2*(B*g-C*
f)/g^2*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)
*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(
b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*C/
g*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4*a*c+
b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)*((-d
/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b
^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+
(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(((x+d
/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(
1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)))+2*(A*g^2-B*f*g+C*f^
2)/g^3*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{\sqrt{d+ex}\,(f+gx)\,\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*(f + g*x)*sqrt(a + b*x + c*x**2
)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)
), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}\sqrt{ex+d}(gx+f)}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)\sqrt{a + bx + cx^2}}\,dx = \int \frac{C\,x^2 + B\,x + A}{(f + g\,x)\,\sqrt{d + e\,x}\,\sqrt{c\,x^2 + b\,x + a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),
x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)\sqrt{a + bx + cx^2}}\,dx = \int \frac{C\,x^2 + Bx + A}{\sqrt{ex + d}\,(gx + f)\,\sqrt{c\,x^2 + bx + a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)
```

3.61 $\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 916

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \frac{(Cf^2-g(Bf-Ag))\sqrt{d+ex}\sqrt{a+bx+cx^2}}{(ef-dg)\left(cf^2-bfg+ag^2\right)(f+gx)}$$

$$-\frac{\sqrt{b^2-4ac}(Cf^2-g(Bf-Ag))\sqrt{d+ex}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{1+\frac{b+2cx}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}g(ef-dg)\left(cf^2-bfg+ag^2\right)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{a+bx+cx^2}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}(2Cg(bf-ag)-c(Cf^2+g(Bf-Ag)))\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\operatorname{EllipticF}\left(\text{ar}\right.}{cg^2\left(cf^2-bfg+ag^2\right)\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$$-\frac{2\sqrt{2}\sqrt{b^2-4ac}(c(Cf^3(ef-2dg)+fg(Bef^2-Ag(3ef-2dg)))-g(b(Cf^2(2ef-3dg)+g^2(Bdf-}{g^2\left(2c\right.}$$

output
```
(C*f^2-g*(-A*g+B*f))*(e*x+d)^(1/2)*(c*x^2+b*x+a)^(1/2)/(-d*g+e*f)/(a*g^2-b
*f*g+c*f^2)/(g*x+f)-1/2*(-4*a*c+b^2)^(1/2)*(C*f^2-g*(-A*g+B*f))*(e*x+d)^(1
/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*EllipticE(1/2*(1+(2*c*x+b)/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^
2)^(1/2))*e))^(1/2))*2^(1/2)/g/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(c*(e*x+d)/(
2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)/(c*x^2+b*x+a)^(1/2)-2^(1/2)*(-4*a*c
+b^2)^(1/2)*(2*C*g*(-a*g+b*f)-c*(C*f^2+g*(-A*g+B*f)))*(c*(e*x+d)/(2*c*d-(b
+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-4*a*c+b^2))^(1/2)*Ellip
ticF(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*(-4*a*c+b^2)^(
1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/c/g^2/(a*g^2-b*f*g+c*f^2)/
(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)-2*2^(1/2)*(-4*a*c+b^2)^(1/2)*(c*(C*f^3*(
-2*d*g+e*f)+f*g*(B*e*f^2-A*g*(-2*d*g+3*e*f)))-g*(b*(C*f^2*(-3*d*g+2*e*f)+g
^2*(A*d*g-2*A*e*f+B*d*f))-a*g*(C*f*(-4*d*g+3*e*f)-g*(A*e*g-2*B*d*g+B*e*f))
))*(c*(e*x+d)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2)*(-c*(c*x^2+b*x+a)/(-
4*a*c+b^2))^(1/2)*EllipticPi(1/2*(1+(2*c*x+b)/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),-2*(-4*a*c+b^2)^(1/2)*g/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g),(-2*(-4*a*c
+b^2)^(1/2)*e/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e))^(1/2))/g^2/(2*c*f-(b+(-4*a
*c+b^2)^(1/2))*g)/(-d*g+e*f)/(a*g^2-b*f*g+c*f^2)/(e*x+d)^(1/2)/(c*x^2+b*x+
a)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 39.07 (sec) , antiderivative size = 32746, normalized size of antiderivative = 35.75

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)^2*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
Result too large to show
```

Rubi [A] (warning: unable to verify)

Time = 5.63 (sec) , antiderivative size = 1534, normalized size of antiderivative = 1.67, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.390, Rules used = {2154, 1282, 2154, 27, 1172, 321, 1269, 1172, 321, 327, 1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\int \frac{1}{\sqrt{d+ex}(f+gx)^2\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1282}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\int \frac{-ceg^2x^2-2cefgx+2cf(ef-dg)-g(2bef-bdg-aeg)}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}+\frac{g^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)\left(ag^2-bfg+cf^2\right)}\right)+\int \frac{\frac{B}{g}+\frac{Cx}{g}-\frac{Cf}{g^2}}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{(cf(3ef-2dg)-g(-aeg-bdg+2bef))\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{-cef-cegx}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}\ldots}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}\ldots\right.$$
$$\frac{(2Cf-Bg)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\int \frac{C}{g^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{(cf(3ef-2dg)-g(-aeg-bdg+2bef))\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{-cef-cegx}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}\right.$$
$$\frac{(2Cf-Bg)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+\frac{C\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx}{g^2}$$

$\big\downarrow$ 1172

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{(cf(3ef-2dg)-g(-aeg-bdg+2bef))\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{-cef-cegx}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}\right.$$
$$\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\int \frac{1}{\sqrt{1-\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}}\sqrt{\frac{e\left(b+2cx+\sqrt{b^2-4ac}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}+1}}d\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}}{cg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}-$$
$$\frac{(2Cf-Bg)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}$$

$\big\downarrow$ 321

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{(cf(3ef-2dg)-g(-aeg-bdg+2bef))\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx+\int \frac{-cef-cegx}{\sqrt{d+ex}\sqrt{cx^2+bx+}}}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}\right.$$
$$\frac{(2Cf-Bg)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+$$
$$\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\big\downarrow$ 1269

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{-c(ef-dg)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx+a}}dx+(cf(3ef-2dg)-g(-aeg-bdg+2bef))\int \frac{}{\sqrt{d+ex}(f}}{2(ef-dg)\left(ag^2-bfg+cf^2\right)}\right.$$
$$\frac{(2Cf-Bg)\int \frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+$$
$$\frac{2\sqrt{2}C\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(d+ex)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{a+bx+cx^2}}$$

$\downarrow$ 1172

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{(2Cf-Bg)\int\frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{(cf(3ef-2dg)-g(2bef-bdg-aeg))\int\frac{}{\sqrt{d+ex}}}{}\right.$$

$\downarrow$ 321

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{(2Cf-Bg)\int\frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c\left(cx^2+bx+\right.}{b^2-4ac}}}{\sqrt{d+e}}}{}\right.$$

$\downarrow$ 327

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{(2Cf-Bg)\int\frac{1}{\sqrt{d+ex}(f+gx)\sqrt{cx^2+bx+a}}dx}{g^2}+$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx}{\sqrt{}}}}{}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2}}}{}\right.$$

$\downarrow$ 1279

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int\frac{1}{\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{d+ex}(f+gx)}dx}{g^2\sqrt{cx^2+bx+a}}+$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx}{\sqrt{}}}}{}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2}}}{}\right.$$

$\downarrow$ 187

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int -\frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}}}{g^2\sqrt{cx^2+bx+a}}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\sqrt{\frac{b+2cx}{\sqrt{}}}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2-}}}{}\right.$$

$$\Big\downarrow \; 25$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\int \frac{1}{\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}(e\ldots}}{g^2\sqrt{cx^2+bx+a}}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\sqrt{\frac{b+2cx}{\sqrt{}}}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2-}}}{}\right.$$

$$\Big\downarrow \; 413$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\int\frac{}{\sqrt{b+\frac{2c(d+ex)}{e}+\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{1}}}{g^2\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)(cf^2-bgf+ag^2)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx}{\sqrt{}}}}{}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2}}}{}\right.$$

$\downarrow$ 413

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{2(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\int\frac{}{\sqrt{1}}}{g^2\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2-4ac}-\frac{2cd}{e}}\sqrt{b+\frac{2c(d+ex)}{e}+}}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)(cf^2-bgf+ag^2)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx}{\sqrt{}}}}{}\right.\right.}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2}}}{}\right.$$

$\downarrow$ 412

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}C\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right),-\frac{2\sqrt{b^2-4ac}e}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{cg^2\sqrt{d+ex}\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}(2Cf-Bg)\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{b+2cx+\sqrt{b^2-4ac}}\sqrt{1-\frac{2c(d+ex)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}}{\sqrt{c}g^2(ef-dg)\sqrt{cx^2+bx+a}\sqrt{b+\frac{2c(d+ex)}{e}-\sqrt{b^2}}}$$

$$\left(A+\frac{f(Cf-Bg)}{g^2}\right)\left(\frac{\sqrt{d+ex}\sqrt{cx^2+bx+a}g^2}{(ef-dg)\left(cf^2-bgf+ag^2\right)(f+gx)}+\frac{-\frac{\sqrt{2}\sqrt{b^2-4ac}g\sqrt{d+ex}\sqrt{-\frac{c\left(cx^2+bx+a\right)}{b^2-4ac}}E\left(\arcsin\left(\frac{\sqrt{\frac{b+2cx}{\sqrt{}}}}{}\right)\right)}{\sqrt{\frac{c(d+ex)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{cx^2}}}{}\right.$$

input
```
Int[(A + B*x + C*x^2)/(Sqrt[d + e*x]*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*C*Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin
[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqr
t[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(c*g^2*Sqrt[d + e*
x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]*(2*C*f - B*g)*Sqrt[b - Sqrt[b^2 - 4*a*c] + 2*c*x]*Sqrt[b + Sqrt[b^2 - 4
*a*c] + 2*c*x]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e
)]*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
i[-1/2*((2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)*g)/(c*(e*f - d*g)), ArcSin[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]], (2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[c]*g^2*(e*f - d*g)*Sqrt[a + b*x + c*x^2]*Sqrt[b - Sqrt[b^2 - 4*a*c] - (
2*c*d)/e + (2*c*(d + e*x))/e]*Sqrt[b + Sqrt[b^2 - 4*a*c] - (2*c*d)/e + (2*
c*(d + e*x))/e]) + (A + (f*(C*f - B*g))/g^2)*((g^2*Sqrt[d + e*x]*Sqrt[a +
b*x + c*x^2])/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x)) + (-((Sqrt[2
]*Sqrt[b^2 - 4*a*c]*g*Sqrt[d + e*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*
a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e)])/(Sqrt[(c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[a + b*x
 + c*x^2])) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(e*f - d*g)*Sqrt[(c*(d + e*x...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1282
```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))   Int[((d + e*x)^
(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*
e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*
g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e,
f, g}, x] && IntegerQ[2*m] && LeQ[m, -2]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [A] (verified)

Time = 47.53 (sec) , antiderivative size = 1508, normalized size of antiderivative = 1.65

method	result	size
elliptic	Expression too large to display	1508
default	Expression too large to display	50098

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((e*x+d)*(c*x^2+b*x+a))^(1/2)/(e*x+d)^(1/2)/(c*x^2+b*x+a)^(1/2)*(-1/(a*d*g
^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*f^3)*(A*g^2-B*f*g+C*f^2)*(c
*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)/(g*x+f)+2*(C/g^2+1/2*c*e*f/g
^2*(A*g^2-B*f*g+C*f^2)/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*
e*f^3))*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^
2)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4
*a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-
4*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2
)*EllipticF(((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*
(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+c*e
*(A*g^2-B*f*g+C*f^2)/(a*d*g^3-a*e*f*g^2-b*d*f*g^2+b*e*f^2*g+c*d*f^2*g-c*e*
f^3)/g*(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2
)^(1/2))/c))^(1/2)*((x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(-d/e-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*((x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e+1/2*(b+(-4
*a*c+b^2)^(1/2))/c))^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+a*e*x+b*d*x+a*d)^(1/2)
*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*EllipticE(((x+d/e)/(d/e-1/2*(b+(-4*
a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-d/e-1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+1/2/c*(-b+(-4*a*c+b^2)^(1/2))*EllipticF(
((x+d/e)/(d/e-1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2),((-d/e+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))-(A*a*e*g^4+...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx=\text{Timed out}$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^2\sqrt{a + bx + cx^2}} \, dx = \int \frac{A + Bx + Cx^2}{\sqrt{d + ex}\,(f + gx)^2\,\sqrt{a + bx + cx^2}} \, dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(1/2)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x
)
```

output
```
Integral((A + B*x + C*x**2)/(sqrt(d + e*x)*(f + g*x)**2*sqrt(a + b*x + c*x
**2)), x)
```

Maxima [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^2\sqrt{a + bx + cx^2}} \, dx = \int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\sqrt{ex + d}(gx + f)^2} \, dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)
^2), x)
```

Giac [F]

$$\int \frac{A + Bx + Cx^2}{\sqrt{d + ex}(f + gx)^2\sqrt{a + bx + cx^2}} \, dx = \int \frac{Cx^2 + Bx + A}{\sqrt{cx^2 + bx + a}\sqrt{ex + d}(gx + f)^2} \, dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*(g*x + f)
^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+B\,x+A}{\left(f+g\,x\right)^2\,\sqrt{d+e\,x}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)
),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^2*(d + e*x)^(1/2)*(a + b*x + c*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{d+ex}(f+gx)^2\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2+Bx+A}{\sqrt{ex+d}\,\left(gx+f\right)^2\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(1/2)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

3.62 $\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 512

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{\sqrt{1+2x}(39998722199+8990086284x)\sqrt{2+5x+4x^2}}{175134960} - \frac{\sqrt{1+2x}(4620643+1253336x)(2+5x+4x^2)^{3/2}}{972972} - \frac{(205-132x)\sqrt{1+2x}(2+5x+4x^2)^{5/2}}{2574} - \frac{31634927315\sqrt{1+2x}\sqrt{2+5x+4x^2}}{5837832\sqrt{2}\left(1+\sqrt{2}(1+2x)\right)} + \frac{2830924\sqrt{\frac{193}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)}{2187} + \frac{31634927315\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)\middle|\frac{1}{8}(4-\sqrt{2})\right)}{11675664\sqrt[4]{2}\sqrt{2+5x+4x^2}} + \frac{(7455591967279-47304438629395\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+\right.\right.}{16462686240\ 2^{3/4}\sqrt{2+5x+4x^2}} - \frac{273184166\sqrt[4]{2}(347-78\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticPi}\left(\frac{1}{312}(156+347\sqrt{2}),2\arctan\left(\right.\right.}{28061397\sqrt{2+5x+4x^2}}$$

output
```
-1/175134960*(1+2*x)^(1/2)*(39998722199+8990086284*x)*(4*x^2+5*x+2)^(1/2)-
1/972972*(1+2*x)^(1/2)*(4620643+1253336*x)*(4*x^2+5*x+2)^(3/2)-1/2574*(205
-132*x)*(1+2*x)^(1/2)*(4*x^2+5*x+2)^(5/2)-31634927315/11675664*(1+2*x)^(1/
2)*(4*x^2+5*x+2)^(1/2)*2^(1/2)/(1+2^(1/2)*(1+2*x))+2830924/85293*7527^(1/2
)*arctanh(1/39*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))+31634927315/2
3351328*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*El
lipticE(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3
/4)/(4*x^2+5*x+2)^(1/2)+1/32925372480*(7455591967279-47304438629395*2^(1/2
))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*Inverse
JacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2)),1/4*(8-2*2^(1/2))^(1/2))*2^(1/4)/
(4*x^2+5*x+2)^(1/2)-273184166/28061397*2^(1/4)*(347-78*2^(1/2))*((4*x^2+5*
x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticPi(sin(2*arc
tan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),1/4*(8-2*2^(1/2))^(1/2))/(
4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.47 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.25

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \frac{\sqrt{1+2x}\left(24(2+5x+4x^2)(-41717946479-13842904824x-\right.}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(2 + 5*x + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 +
2*x]),x]
```

output
```
(Sqrt[1 + 2*x]*(24*(2 + 5*x + 4*x^2)*(-41717946479 - 13842904824*x - 48471
15960*x^2 - 1092097440*x^3 + 136080000*x^4 + 143700480*x^5) - ((1 + 2*x)*(
(1423571729175*(I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((
-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + S
qrt[7])*(1 + 2*x))]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1
 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1 + 2*x] + (5694286916700*Sqr
t[(-2*I)/(-I + Sqrt[7])]*(2 + 5*x + 4*x^2) - 3*(546266138719*I + 474523909
725*Sqrt[7])*(1 + 2*x)^(3/2)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((
-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + S
qrt[7])*(1 + 2*x))]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1
 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])] + (26925031400960*I)*(1 + 2*x)^(3/2
)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*S
qrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*Ellipti
cPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1
+ 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/(1 + 2*x)^2))/Sqrt[I/(2*I - 2*Sqrt[
7])]))/(4203239040*Sqrt[2 + 5*x + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)(4x^2+5x+2)^{5/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \; 2154$$

$$\frac{76}{9}\int \frac{(4x^2+5x+2)^{5/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(4x^2+5x+2)^{5/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \; 1231$$

$$-\frac{5\int \frac{2(344x+747)(4x^2+5x+2)^{3/2}}{9\sqrt{2x+1}}dx}{1144}+\frac{76}{9}\int \frac{(4x^2+5x+2)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}(4x^2+5x+2)^{5/2}}{2574}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\int \frac{(344x+747)\left(4x^2+5x+2\right)^{3/2}}{\sqrt{2x+1}}dx}{5148}+\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 1231$$

$$-\frac{5\left(\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}-\frac{1}{168}\int -\frac{4(14478x+17503)\sqrt{4x^2+5x+2}}{\sqrt{2x+1}}dx\right)}{5148}+\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\left(\frac{1}{42}\int \frac{(14478x+17503)\sqrt{4x^2+5x+2}}{\sqrt{2x+1}}dx+\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}\right)}{5148}+\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 1231$$

$$-\frac{5\left(\frac{1}{42}\left(\frac{1}{30}\sqrt{2x+1}(86868x+153313)\sqrt{4x^2+5x+2}-\frac{1}{120}\int -\frac{2(350060x+373071)}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx\right)+\frac{1}{63}\sqrt{2x+1}(2408x+6637\right.}{5148}$$
$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 27$$

$$-\frac{5\left(\frac{1}{42}\left(\frac{1}{60}\int \frac{350060x+373071}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+153313)\right)+\frac{1}{63}\sqrt{2x+1}(2408x+6637)\right.}{5148}$$
$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 1269$$

$$-\frac{5\left(\frac{1}{42}\left(\frac{1}{60}\left(198041\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+175030\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)+\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+1533\right.\right.}{5148}$$
$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$$\Big\downarrow \; 1172$$

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}+\frac{1}{42}\left(\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+153313)+\frac{1}{60}\left(\frac{39608}{\ }\right.\right.\right.}{5148}$$

$$\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$\downarrow$ 321

$$-\frac{5\left(\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}+\frac{1}{42}\left(\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+153313)+\frac{1}{60}\left(\frac{875}{\ }\right.\right.\right.}{5148}$$

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$\downarrow$ 327

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}+\frac{1}{42}\left(\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+153313)+\frac{1}{60}\left(\frac{39608}{\ }\right.\right.\right.}{5148}$$

$$\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

$\downarrow$ 1292

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{5/2}}{(5-3x)\sqrt{2x+1}}dx-$$

$$\frac{5\left(\frac{1}{63}\sqrt{2x+1}(2408x+6637)\left(4x^2+5x+2\right)^{3/2}+\frac{1}{42}\left(\frac{1}{30}\sqrt{2x+1}\sqrt{4x^2+5x+2}(86868x+153313)+\frac{1}{60}\left(\frac{39608}{}\right.\right.\right.}{5148}$$

$$\frac{(205-132x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}{2574}$$

input
```
Int[((4 + 6*x - 2*x^2)*(2 + 5*x + 4*x^2)^(5/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.03

method	result
risch	$\frac{(143700480x^5+136080000x^4-1092097440x^3-4847115960x^2-13842904824x-41717946479)\sqrt{4x^2+5x+2}\,\sqrt{1+2x}}{175134960}+\frac{2\left(-\frac{162020}{}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(\frac{32x^5\sqrt{8x^3+14x^2+9x+2}}{39}+\frac{1000x^4\sqrt{8x^3+14x^2+9x+2}}{1287}-\frac{216686x^3\sqrt{8x^3+14x^2+9x+2}}{34749}-\frac{13464211x^2\sqrt{8x^3+14x^2+9x}}{486486}\right.}{}$
default	$\frac{\sqrt{4x^2+5x+2}\,\sqrt{1+2x}\left(13795246080x^8+37205360640x^7+13354902356989i\sqrt{7}\,\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\,\sqrt{\frac{i\sqrt{7}-8x-5}{i\sqrt{7}-1}}\,\sqrt{\frac{i\sqrt{7}+8x+5}{1+i\sqrt{7}}}\,\mathrm{EllipticF}\left(2\sqrt{}\right.\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/175134960*(143700480*x^5+136080000*x^4-1092097440*x^3-4847115960*x^2-138
42904824*x-41717946479)*(4*x^2+5*x+2)^(1/2)*(1+2*x)^(1/2)+2*(-162020458153
39/1050809760*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((
x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/
8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticF(((x+1/2)/(-1/
8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))-3
1634927315/5837832*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/
2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2)
)/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*((1/8-1/8*I*7^(1/2
))*EllipticE(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/
8-1/8*I*7^(1/2)))^(1/2))+(-5/8+1/8*I*7^(1/2))*EllipticF(((x+1/2)/(-1/8-1/8
*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))+10927
36664/85293*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+
5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+
1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticPi(((x+1/2)/(-1/8
-1/8*I*7^(1/2)))^(1/2),-3/52-3/52*I*7^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*
I*7^(1/2)))^(1/2)))*((4*x^2+5*x+2)*(1+2*x))^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2
*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\,(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+5\,x+2)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*(16*x^6 - 8*x^5 - 111*x^4 - 183*x^3 - 138*x^2 - 52*x - 8)*sqrt(
4*x^2 + 5*x + 2)*sqrt(2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \text{Timed out}$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+5*x+2)**(5/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+5\,x+2)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((4*x^2 + 5*x + 2)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+5\,x+2)^{\frac{5}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(4*x^2 + 5*x + 2)^(5/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(4\,x^2+5\,x+2)^{5/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((5*x + 4*x^2 + 2)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((5*x + 4*x^2 + 2)^(5/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{5/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)(4x^2+5x+2)^{\frac{5}{2}}}{(5-3x)\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(5/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.63 $\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 480

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{\sqrt{1+2x}(12877+2817x)\sqrt{2+5x+4x^2}}{1215}$$
$$-\frac{1}{54}(7-4x)\sqrt{1+2x}\left(2+5x+4x^2\right)^{3/2}-\frac{81851\sqrt{1+2x}\sqrt{2+5x+4x^2}}{324\sqrt{2}\left(1+\sqrt{2}(1+2x)\right)}$$
$$+\frac{14668}{243}\sqrt{\frac{193}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)$$
$$+\frac{81851\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}\left(4-\sqrt{2}\right)\right)}{648\sqrt[4]{2}\sqrt{2+5x+4x^2}}$$
$$-\frac{\left(856882525-67550501\sqrt{2}\right)\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}(4-\right.}{6395760\sqrt[4]{2}\sqrt{2+5x+4x^2}}$$
$$-\frac{1415462\sqrt[4]{2}\left(347-78\sqrt{2}\right)\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticPi}\left(\frac{1}{312}\left(156+347\sqrt{2}\right),2\arctan\left(\sqrt[4]{\right.\right.}{3117933\sqrt{2+5x+4x^2}}$$

output
```
-1/1215*(1+2*x)^(1/2)*(12877+2817*x)*(4*x^2+5*x+2)^(1/2)-1/54*(7-4*x)*(1+2
*x)^(1/2)*(4*x^2+5*x+2)^(3/2)-81851/648*(1+2*x)^(1/2)*(4*x^2+5*x+2)^(1/2)*
2^(1/2)/(1+2^(1/2)*(1+2*x))+14668/9477*7527^(1/2)*arctanh(1/39*7527^(1/2)*
(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))+81851/1296*((4*x^2+5*x+2)/(1+2^(1/2)*(1
+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticE(sin(2*arctan(2^(1/4)*(1+2*x)
^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/(4*x^2+5*x+2)^(1/2)-1/12791520*(
856882525-67550501*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1
+2^(1/2)*(1+2*x))*InverseJacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2)),1/4*(8-2
*2^(1/2))^(1/2))*2^(3/4)/(4*x^2+5*x+2)^(1/2)-1415462/3117933*2^(1/4)*(347-
78*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x)
)*EllipticPi(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),1/4*
(8-2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.00 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.33

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\frac{\sqrt{1+2x}\sqrt{2+5x+4x^2}(-26384-6849x-360x^2+720x^3)}{4860}\right.$$

$$-\frac{(1+2x)^{3/2}\left(\frac{383062680\sqrt{-\frac{i}{-i+\sqrt{7}}}(2+5x+4x^2)}{(1+2x)^2}+\frac{47882835\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}+2\left(-i+\sqrt{7}\right)x}{\left(-i+\sqrt{7}\right)(1+2x)}}\sqrt{\frac{3i+\sqrt{7}+2\left(i+\sqrt{7}\right)x}{\left(i+\sqrt{7}\right)(1+2x)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\cdots}}{\sqrt{1+\cdots}}\right.\right.}{\sqrt{\frac{1}{2}+x}}\right.}{}$$

input
```
Integrate[((4 + 6*x - 2*x^2)*(2 + 5*x + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 +
2*x]),x]
```

output
```
2*((Sqrt[1 + 2*x]*Sqrt[2 + 5*x + 4*x^2]*(-26384 - 6849*x - 360*x^2 + 720*x
^3))/4860 - ((1 + 2*x)^(3/2)*((383062680*Sqrt[(-I)/(-I + Sqrt[7])]*(2 + 5*
x + 4*x^2))/(1 + 2*x)^2 + (47882835*(I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] + 2
*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I
+ Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(
-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x]
 - (3*(18377173*I + 15960945*Sqrt[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[
7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x
)/((I + Sqrt[7])*(1 + 2*x))]*EllipticF[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7]
)]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x] + ((9058956
80*I)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x)
)]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*Ell
ipticPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqr
t[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x]))/(6065280*Sqrt[(
-I)/(-I + Sqrt[7])]*Sqrt[2 + 5*x + 4*x^2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-2x^2+6x+4)(4x^2+5x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \; 2154$$

$$\frac{76}{9}\int \frac{(4x^2+5x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)(4x^2+5x+2)^{3/2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \; 1231$$

$$-\frac{1}{168}\int \frac{14(12x+25)\sqrt{4x^2+5x+2}}{3\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(4x^2+5x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}(4x^2+5x+2)^{3/2}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{36}\int \frac{(12x+25)\sqrt{4x^2+5x+2}}{\sqrt{2x+1}}dx+\frac{76}{9}\int \frac{(4x^2+5x+2)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}(4x^2+5x+2)^{3/2}$$

$$\Big\downarrow \; 1231$$

$$\frac{1}{36}\left(\frac{1}{120}\int -\frac{8(125x+156)}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+2}\right)+ \frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{36}\left(-\frac{1}{15}\int \frac{125x+156}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{4}{15}\sqrt{2x+1}\sqrt{4x^2+5x+2}(9x+29)\right)+ \frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \; 1269$$

$$\frac{1}{36}\left(\frac{1}{15}\left(-\frac{187}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{125}{2}\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+\right. \frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \; 1172$$

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+ \frac{1}{36}\left(-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+2}+\frac{1}{15}\left(-\frac{187i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}\right.}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{2x+1}}\right.\right. \frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \; 321$$

$$\frac{1}{36}\left(-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+2}+\frac{1}{15}\left(-\frac{125i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{4\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}-\frac{187i\sqrt{-}}{}\right.\right. \frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{327}$$

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\frac{1}{36}\left(-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+2}+\frac{1}{15}\left(-\frac{187i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}\right.\right.$$

$$\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

$$\Big\downarrow \text{1292}$$

$$\frac{76}{9}\int \frac{\left(4x^2+5x+2\right)^{3/2}}{(5-3x)\sqrt{2x+1}}dx+\frac{1}{36}\left(-\frac{4}{15}\sqrt{2x+1}(9x+29)\sqrt{4x^2+5x+2}+\frac{1}{15}\left(-\frac{187i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}\right.\right.$$

$$\frac{1}{54}(7-4x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}$$

input
```
Int[((4 + 6*x - 2*x^2)*(2 + 5*x + 4*x^2)^(3/2))/((5 - 3*x)*Sqrt[1 + 2*x]),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.08

method	result
risch	$\frac{\left(720x^3-360x^2-6849x-26384\right)\sqrt{4x^2+5x+2}\,\sqrt{1+2x}}{2430}+\frac{2\left(-\frac{2620621\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\,\sqrt{\frac{x+\frac{5}{8}-\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\,\sqrt{\frac{x+\frac{5}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}+\frac{i\sqrt{7}}{8}}}\,\mathrm{EllipticF}\left(\right.}{3645\sqrt{8x^3+14x^2+9x+2}}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(\frac{8x^3\sqrt{8x^3+14x^2+9x+2}}{27}-\frac{4x^2\sqrt{8x^3+14x^2+9x+2}}{27}-\frac{761x\sqrt{8x^3+14x^2+9x+2}}{270}-\frac{13192\sqrt{8x^3+14x^2+9x+2}}{1215}-\frac{5241242\left(\right.}{}\right.}{}$
default	$\frac{\sqrt{4x^2+5x+2}\,\sqrt{1+2x}\left(224661749i\sqrt{7}\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\sqrt{\frac{i\sqrt{7}-8x-5}{i\sqrt{7}-1}}\sqrt{\frac{i\sqrt{7}+8x+5}{1+i\sqrt{7}}}\,\mathrm{EllipticF}\left(2\sqrt{-\frac{1+2x}{1+i\sqrt{7}}},\sqrt{-\frac{1+i\sqrt{7}}{i\sqrt{7}-1}}\right)-226473920i\sqrt{7}\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output

```
1/2430*(720*x^3-360*x^2-6849*x-26384)*(4*x^2+5*x+2)^(1/2)*(1+2*x)^(1/2)+2*
(-2620621/3645*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*(
(x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1
/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticF(((x+1/2)/(-1
/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))-
81851/324*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/
8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/
8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*((1/8-1/8*I*7^(1/2))*Ellipt
icE(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7
^(1/2)))^(1/2))+(-5/8+1/8*I*7^(1/2))*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2
)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))+5661848/9477*(
-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(
1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)
))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticPi(((x+1/2)/(-1/8-1/8*I*7^(1/2
)))^(1/2),-3/52-3/52*I*7^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(
1/2)))*((4*x^2+5*x+2)*(1+2*x))^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\,(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx=\int \frac{2\,(4\,x^2+5\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input

```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output

```
integral(2*(4*x^4 - 7*x^3 - 21*x^2 - 16*x - 4)*sqrt(4*x^2 + 5*x + 2)*sqrt(
2*x + 1)/(6*x^2 - 7*x - 5), x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)\,(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int \left(-\frac{4\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{16x\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{21x^2\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \left(-\frac{7x^3\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int \frac{4x^4\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+5*x+2)**(3/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-4*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)
), x) + Integral(-16*x*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(
2*x + 1)), x) + Integral(-21*x**2*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1
) - 5*sqrt(2*x + 1)), x) + Integral(-7*x**3*sqrt(4*x**2 + 5*x + 2)/(3*x*sq
rt(2*x + 1) - 5*sqrt(2*x + 1)), x) + Integral(4*x**4*sqrt(4*x**2 + 5*x + 2
)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)), x))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\,(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+5\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((4*x^2 + 5*x + 2)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\,(4\,x^2+5\,x+2)^{\frac{3}{2}}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(4*x^2 + 5*x + 2)^(3/2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int -\frac{(4\,x^2+5\,x+2)^{3/2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((5*x + 4*x^2 + 2)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((5*x + 4*x^2 + 2)^(3/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)(2+5x+4x^2)^{3/2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{(-2x^2+6x+4)\,(4x^2+5x+2)^{\frac{3}{2}}}{(5-3x)\,\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(3/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.64 $\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 448

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = -\frac{1}{270}(89-36x)\sqrt{1+2x}\sqrt{2+5x+4x^2}$$
$$-\frac{52\sqrt{2}\sqrt{1+2x}\sqrt{2+5x+4x^2}}{9\left(1+\sqrt{2}(1+2x)\right)}+\frac{76}{27}\sqrt{\frac{193}{39}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)$$
$$+\frac{26\,2^{3/4}\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}(4-\sqrt{2})\right)}{9\sqrt{2+5x+4x^2}}$$
$$+\frac{(177433-1099960\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}(4-\sqrt{\ldots}\right.}{177660\,2^{3/4}\sqrt{2+5x+4x^2}}$$
$$-\frac{7334\sqrt[4]{2}(347-78\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\operatorname{EllipticPi}\left(\frac{1}{312}(156+347\sqrt{2}),2\arctan\left(\sqrt[4]{2}\ldots\right.\right.}{346437\sqrt{2+5x+4x^2}}$$

output
```
-1/270*(89-36*x)*(1+2*x)^(1/2)*(4*x^2+5*x+2)^(1/2)-52*2^(1/2)*(1+2*x)^(1/2
)*(4*x^2+5*x+2)^(1/2)/(9+9*2^(1/2)*(1+2*x))+76/1053*7527^(1/2)*arctanh(1/3
9*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))+26/9*((4*x^2+5*x+2)/(1+2^(
1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticE(sin(2*arctan(2^(1/4)*
(1+2*x)^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/(4*x^2+5*x+2)^(1/2)+1/355
320*(177433-1099960*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(
1+2^(1/2)*(1+2*x))*InverseJacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2)),1/4*(8-
2*2^(1/2))^(1/2))*2^(1/4)/(4*x^2+5*x+2)^(1/2)-7334/346437*2^(1/4)*(347-78*
2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*E
llipticPi(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),1/4*(8-
2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 25.99 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.41

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\frac{1}{540}\sqrt{1+2x}(-89+36x)\sqrt{2+5x+4x^2} - \frac{(1+2x)^{3/2}\left(\frac{30420\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}+2\left(-i+\sqrt{7}\right)x}{\left(-i+\sqrt{7}\right)(1+2x)}}\sqrt{\frac{3i+\sqrt{7}+2\left(i+\sqrt{7}\right)x}{\left(i+\sqrt{7}\right)(1+2x)}}E\left(i\operatorname{arcsinh}\left(\frac{\sqrt{-\frac{2i}{-i+\sqrt{7}}}}{\sqrt{1+2x}}\right)|\frac{i-\sqrt{7}}{i+\sqrt{7}}\right)}{\sqrt{1+2x}} - \frac{3\left(11801i+10140\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Integrate[((4 + 6*x - 2*x^2)*Sqrt[2 + 5*x + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*
x]),x]
```

output
```
2*((Sqrt[1 + 2*x]*(-89 + 36*x)*Sqrt[2 + 5*x + 4*x^2])/540 - ((1 + 2*x)^(3/
2)*((30420*(I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I +
 Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[
7])*(1 + 2*x))]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2
*x]], (I - Sqrt[7])/(I + Sqrt[7])]])/Sqrt[1 + 2*x] - (3*(11801*I + 10140*Sq
rt[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x
))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*El
lipticF[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7]
)/(I + Sqrt[7])]])/Sqrt[1 + 2*x] + (80*(1521*Sqrt[(-2*I)/(-I + Sqrt[7])]*(2
 + 5*x + 4*x^2) + (7334*I)*(1 + 2*x)^(3/2)*Sqrt[(-3*I + Sqrt[7] + 2*(-I +
Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[
7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticPi[(-13*(1 + I*Sqrt[7]))/6, I*Ar
cSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[
7])]))/(1 + 2*x)^2))/(42120*Sqrt[(-2*I)/(-I + Sqrt[7])]*Sqrt[2 + 5*x + 4*x
^2]))
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.99 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.44, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.439$, Rules used = {2154, 1231, 27, 1269, 1172, 321, 327, 1274, 1269, 1172, 321, 327, 1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(-2x^2+6x+4\right)\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx+\int \frac{\left(\frac{2x}{3}-\frac{8}{9}\right)\sqrt{4x^2+5x+2}}{\sqrt{2x+1}}dx$$

$$\Big\downarrow \text{1231}$$

$$-\frac{1}{120}\int \frac{2(160x+303)}{9\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \ 27$$

$$-\frac{1}{540}\int \frac{160x+303}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \ 1269$$

$$\frac{1}{540}\left(-223\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-80\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)+\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \ 1172$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx+\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x\right.}}{\sqrt{2}}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \ 321$$

$$\frac{1}{540}\left(-\frac{40i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\right.}{\sqrt{2x+1}}\right.$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx-\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \ 327$$

$$\frac{76}{9}\int \frac{\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx+$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 1274$$

$$\frac{76}{9}\left(\frac{193}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{1}{9}\int \frac{12x+35}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx\right)+$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 1269$$

$$\frac{76}{9}\left(\frac{193}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{9}\left(-29\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-6\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 1172$$

$$\frac{76}{9}\left(\frac{193}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{9}\left(-\frac{58i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\right.}}{\sqrt{2}\sqrt[4]{}}}{\sqrt{2x+1}}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\downarrow 321$$

$$\frac{76}{9}\left(\frac{193}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{9}\left(-\frac{3i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}-\frac{58i\sqrt{-\frac{}{1}}}{}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\downarrow 327$$

$$\frac{76}{9}\left(\frac{193}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{9}\left(-\frac{58i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2}{i-}\right.}{\sqrt{2x+1}}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \text{1279}$$

$$\frac{76}{9}\left(\frac{193\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}}dx}{9\sqrt{4x^2+5x+2}}+\frac{1}{9}\left(-\frac{58i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\right.}{}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \text{187}$$

$$\frac{76}{9}\left(\frac{1}{9}\left(-\frac{58i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{3i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 25$$

$$\frac{76}{9}\left(\frac{386\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\int\frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}}+\frac{1}{9}\left(-\frac{58i\sqrt{-\frac{2}{1}}}{}\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 413$$

$$\frac{76}{9}\left(\frac{386\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)+i\sqrt{7}+1}\sqrt{\frac{4(2x+1)}{1-i\sqrt{7}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}}+\frac{1}{9}\left(-\right.\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 413$$

$$\frac{76}{9}\left(\frac{386\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\sqrt{1+\frac{4(2x+1)}{1+i\sqrt{7}}}\int \frac{1}{(13-3(2x+1))\sqrt{\frac{4(2x+1)}{1-i\sqrt{7}}+1}\sqrt{\frac{4(2x+1)}{1+i\sqrt{7}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

$$\Big\downarrow \; 412$$

$$\frac{76}{9}\left(\frac{193\sqrt{-1+i\sqrt{7}}\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\sqrt{1+\frac{4(2x+1)}{1+i\sqrt{7}}}\,\text{EllipticPi}\left(-\frac{3}{52}\left(1-i\sqrt{7}\right),\text{arcsi}\right.}{117\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}\right.$$

$$\frac{1}{540}\left(-\frac{446i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}-\frac{40i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right.}}{\sqrt{2}\sqrt[4]{7}}\right.\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{1}{270}\sqrt{2x+1}\sqrt{4x^2+5x+2}(89-36x)$$

input
```
Int[((4 + 6*x - 2*x^2)*Sqrt[2 + 5*x + 4*x^2])/((5 - 3*x)*Sqrt[1 + 2*x]),x]
```

output
```
-1/270*((89 - 36*x)*Sqrt[1 + 2*x]*Sqrt[2 + 5*x + 4*x^2]) + (((-40*I)*Sqrt[
1 + 2*x]*EllipticE[ArcSin[Sqrt[(-I)*(5 + I*Sqrt[7] + 8*x)]/(Sqrt[2]*7^(1/4
))], (-2*Sqrt[7])/(I - Sqrt[7])])/Sqrt[-((1 + 2*x)/(1 + I*Sqrt[7]))] - ((4
46*I)*Sqrt[-((1 + 2*x)/(1 + I*Sqrt[7]))]*EllipticF[ArcSin[Sqrt[(-I)*(5 + I
*Sqrt[7] + 8*x)]/(Sqrt[2]*7^(1/4))], (-2*Sqrt[7])/(I - Sqrt[7])])/Sqrt[1 +
 2*x])/540 + (76*((((-3*I)*Sqrt[1 + 2*x]*EllipticE[ArcSin[Sqrt[(-I)*(5 + I
*Sqrt[7] + 8*x)]/(Sqrt[2]*7^(1/4))], (-2*Sqrt[7])/(I - Sqrt[7])])/Sqrt[-((
1 + 2*x)/(1 + I*Sqrt[7]))] - ((58*I)*Sqrt[-((1 + 2*x)/(1 + I*Sqrt[7]))]*El
lipticF[ArcSin[Sqrt[(-I)*(5 + I*Sqrt[7] + 8*x)]/(Sqrt[2]*7^(1/4))], (-2*Sq
rt[7])/(I - Sqrt[7])])/Sqrt[1 + 2*x])/9 + (193*Sqrt[-1 + I*Sqrt[7]]*Sqrt[5
 - I*Sqrt[7] + 8*x]*Sqrt[5 + I*Sqrt[7] + 8*x]*Sqrt[1 + (4*(1 + 2*x))/(1 -
I*Sqrt[7])]*Sqrt[1 + (4*(1 + 2*x))/(1 + I*Sqrt[7])]*EllipticPi[(-3*(1 - I*
Sqrt[7]))/52, ArcSin[(2*Sqrt[1 + 2*x])/Sqrt[-1 + I*Sqrt[7]]], (I + Sqrt[7]
)/(I - Sqrt[7])])/(117*Sqrt[2 + 5*x + 4*x^2]*Sqrt[1 - I*Sqrt[7] + 4*(1 + 2
*x)]*Sqrt[1 + I*Sqrt[7] + 4*(1 + 2*x)])))/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1274
```
Int[Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]/(((d_.) + (e_.)*(x_))*Sqrt[(f_.
) + (g_.)*(x_)]), x_Symbol] :> Simp[(c*d^2 - b*d*e + a*e^2)/e^2   Int[1/((d
 + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] - Simp[1/e^2   Int[(c*
d - b*e - c*e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.13

method	result
risch	$\frac{(-89+36x)\sqrt{4x^2+5x+2}\,\sqrt{1+2x}}{270}+\frac{2\left(-\frac{54109\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}-\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}+\frac{i\sqrt{7}}{8}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}},\sqrt{\frac{\frac{1}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\right.}{1620\sqrt{8x^3+14x^2+9x+2}}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(\frac{2x\sqrt{8x^3+14x^2+9x+2}}{15}-\frac{89\sqrt{8x^3+14x^2+9x+2}}{270}-\frac{54109\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}-\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}+\frac{i\sqrt{7}}{8}}}\,\mathrm{Ellipti}}{810\sqrt{8x^3+14x^2+9x+2}}\right.}{}$
default	$\frac{\sqrt{4x^2+5x+2}\,\sqrt{1+2x}\left(581737i\sqrt{7}\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\sqrt{\frac{i\sqrt{7}-8x-5}{i\sqrt{7}-1}}\sqrt{\frac{i\sqrt{7}+8x+5}{1+i\sqrt{7}}}\,\mathrm{EllipticF}\left(2\sqrt{-\frac{1+2x}{1+i\sqrt{7}}},\sqrt{-\frac{1+i\sqrt{7}}{i\sqrt{7}-1}}\right)-586720i\sqrt{7}\sqrt{-\frac{1+2}{1+i}}\right.}{}$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
1/270*(-89+36*x)*(4*x^2+5*x+2)^(1/2)*(1+2*x)^(1/2)+2*(-54109/1620*(-1/8-1/
8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(
1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2
)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2
),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))-104/9*(-1/8-1/8*I*7^(1/
2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I
*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+
14*x^2+9*x+2)^(1/2)*((1/8-1/8*I*7^(1/2))*EllipticE(((x+1/2)/(-1/8-1/8*I*7^
(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))+(-5/8+1/8*I
*7^(1/2))*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/
2))/(1/8-1/8*I*7^(1/2)))^(1/2)))+29336/1053*(-1/8-1/8*I*7^(1/2))*((x+1/2)/
(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1
/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)
^(1/2)*EllipticPi(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),-3/52-3/52*I*7^(1/2
),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))*((4*x^2+5*x+2)*(1+2*x)
)^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2*x)^(1/2)
```

Fricas [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4\,x^2+5\,x+2}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(6*x^2 - 7*
x - 5), x)
```

Sympy [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = 2\left(\int\left(-\frac{2\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int\left(-\frac{3x\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\right)dx + \int\frac{x^2\sqrt{4x^2+5x+2}}{3x\sqrt{2x+1}-5\sqrt{2x+1}}\,dx\right)$$

input
```
integrate((-2*x**2+6*x+4)*(4*x**2+5*x+2)**(1/2)/(5-3*x)/(1+2*x)**(1/2),x)
```

output
```
2*(Integral(-2*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2*x + 1)
), x) + Integral(-3*x*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1) - 5*sqrt(2
*x + 1)), x) + Integral(x**2*sqrt(4*x**2 + 5*x + 2)/(3*x*sqrt(2*x + 1) - 5
*sqrt(2*x + 1)), x))
```

Maxima [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx = \int \frac{2\sqrt{4x^2+5x+2}(x^2-3x-2)}{(3x-5)\sqrt{2x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate(sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)
), x)
```

Giac [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int \frac{2\,\sqrt{4\,x^2+5\,x+2}(x^2-3\,x-2)}{(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)/((3*x - 5)*sqrt(2*x + 1)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int -\frac{\sqrt{4\,x^2+5\,x+2}\,(-2\,x^2+6\,x+4)}{\sqrt{2\,x+1}\,(3\,x-5)}\,dx$$

input
```
int(-((5*x + 4*x^2 + 2)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)),x)
```

output
```
int(-((5*x + 4*x^2 + 2)^(1/2)*(6*x - 2*x^2 + 4))/((2*x + 1)^(1/2)*(3*x - 5
)), x)
```

Reduce [F]

$$\int \frac{(4+6x-2x^2)\sqrt{2+5x+4x^2}}{(5-3x)\sqrt{1+2x}}\,dx=\int \frac{(-2x^2+6x+4)\sqrt{4x^2+5x+2}}{(5-3x)\sqrt{2x+1}}dx$$

input
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)*(4*x^2+5*x+2)^(1/2)/(5-3*x)/(1+2*x)^(1/2),x)
```

3.65 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 414

$$
\begin{aligned}
&\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx \\
&= \frac{\sqrt{2}\sqrt{1+2x}\sqrt{2+5x+4x^2}}{3\left(1+\sqrt{2}(1+2x)\right)}+\frac{76\,\mathrm{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)}{3\sqrt{7527}} \\
&\quad - \frac{\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}\left(4-\sqrt{2}\right)\right)}{3\sqrt[4]{2}\sqrt{2+5x+4x^2}} \\
&\quad + \frac{\left(25+111\sqrt{2}\right)\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\mathrm{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}\left(4-\sqrt{2}\right)\right)}{1974\sqrt[4]{2}\sqrt{2+5x+4x^2}} \\
&\quad - \frac{38\sqrt[4]{2}\left(347-78\sqrt{2}\right)\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\mathrm{EllipticPi}\left(\frac{1}{312}\left(156+347\sqrt{2}\right),2\arctan\left(\sqrt[4]{2}\sqrt{\right.\right.}{38493\sqrt{2+5x+4x^2}}
\end{aligned}
$$

output
```
2^(1/2)*(1+2*x)^(1/2)*(4*x^2+5*x+2)^(1/2)/(3+3*2^(1/2)*(1+2*x))+76/22581*7
527^(1/2)*arctanh(1/39*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))-1/6*(
(4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticE(s
in(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/(4*x^
2+5*x+2)^(1/2)+1/3948*(25+111*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^
2)^(1/2)*(1+2^(1/2)*(1+2*x))*InverseJacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2
)),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/(4*x^2+5*x+2)^(1/2)-38/38493*2^(1/4)*(
347-78*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+
2*x))*EllipticPi(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),
1/4*(8-2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 27.09 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.43

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx$$

$$= \frac{(3+6x)^{3/2}\Bigg(\frac{312\sqrt{-\frac{i}{-i+\sqrt{7}}}(2+5x+4x^2)}{(1+2x)^2}+\frac{39\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}+2\left(-i+\sqrt{7}\right)x}{\left(-i+\sqrt{7}\right)(1+2x)}}\sqrt{\frac{3i+\sqrt{7}+2\left(i+\sqrt{7}\right)x}{\left(i+\sqrt{7}\right)(1+2x)}}E\Bigg(i\operatorname{arcsinh}\Bigg(\frac{\sqrt{-\frac{2i}{-i+\sqrt{7}}}}{\sqrt{1+2x}}\Bigg)|\frac{i-\sqrt{7}}{i+\sqrt{7}}}{\sqrt{\frac{1}{2}+x}}}{}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[2 + 5*x + 4*x^2]
),x]
```

output
```
((3 + 6*x)^(3/2)*((312*Sqrt[(-I)/(-I + Sqrt[7])]*(2 + 5*x + 4*x^2))/(1 + 2
*x)^2 + (39*(I + Sqrt[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I
+ Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt
[7])*(1 + 2*x))]*EllipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 +
2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x] - (3*(I + 13*Sqrt[7])*S
qrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt
[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticF[
I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + S
qrt[7])])/Sqrt[1/2 + x] - ((608*I)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])
*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/(
(I + Sqrt[7])*(1 + 2*x))]*EllipticPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sq
rt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sq
rt[1/2 + x]))/(1404*Sqrt[(-6*I)/(-I + Sqrt[7])]*Sqrt[4 + 10*x + 8*x^2])
```

Rubi [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.268$, Rules used = {2154, 1269, 1172, 321, 327, 1279, 187, 25, 413, 413, 412}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx$$

$$\Big\downarrow \text{1269}$$

$$-\frac{11}{9}\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{1}{3}\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx$$

$$\Big\downarrow \text{1172}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$$\Big\downarrow \text{ 321}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$$\Big\downarrow \text{ 1279}$$

$$\frac{76\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\int \frac{1}{(5-3x)\sqrt{2x+1}\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}}dx}{9\sqrt{4x^2+5x+2}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$$\Big\downarrow \; 187$$

$$-\frac{152\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\int -\frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$$\Big\downarrow \; 25$$

$$\frac{152\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\int \frac{1}{(13-3(2x+1))\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$$\Big\downarrow \; 413$$

$$\frac{152\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\int\frac{1}{(13-3(2x+1))\sqrt{4(2x+1)+i\sqrt{7}+1}\sqrt{\frac{4(2x+1)}{1-i\sqrt{7}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$\downarrow$ 413

$$\frac{152\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\sqrt{1+\frac{4(2x+1)}{1+i\sqrt{7}}}\int\frac{1}{(13-3(2x+1))\sqrt{\frac{4(2x+1)}{1-i\sqrt{7}}+1}\sqrt{\frac{4(2x+1)}{1+i\sqrt{7}}+1}}d\sqrt{2x+1}}{9\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}-\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

$\downarrow$ 412

$$\frac{76\sqrt{-1+i\sqrt{7}}\sqrt{8x-i\sqrt{7}+5}\sqrt{8x+i\sqrt{7}+5}\sqrt{1+\frac{4(2x+1)}{1-i\sqrt{7}}}\sqrt{1+\frac{4(2x+1)}{1+i\sqrt{7}}}\text{EllipticPi}\left(-\frac{3}{52}\left(1-i\sqrt{7}\right),\arcsin\left(\frac{2\sqrt{}}{\sqrt{-}}\right.\right.}{117\sqrt{4x^2+5x+2}\sqrt{4(2x+1)-i\sqrt{7}+1}\sqrt{4(2x+1)+i\sqrt{7}+1}}$$

$$\frac{22i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{9\sqrt{2x+1}}+\frac{i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)|-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{6\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*Sqrt[2 + 5*x + 4*x^2]),x]
```

output
```
((I/6)*Sqrt[1 + 2*x]*EllipticE[ArcSin[Sqrt[(-I)*(5 + I*Sqrt[7] + 8*x)]/(Sq
rt[2]*7^(1/4))], (-2*Sqrt[7])/(I - Sqrt[7])])/Sqrt[-((1 + 2*x)/(1 + I*Sqrt
[7]))] - (((22*I)/9)*Sqrt[-((1 + 2*x)/(1 + I*Sqrt[7]))]*EllipticF[ArcSin[S
qrt[(-I)*(5 + I*Sqrt[7] + 8*x)]/(Sqrt[2]*7^(1/4))], (-2*Sqrt[7])/(I - Sqrt
[7])])/Sqrt[1 + 2*x] + (76*Sqrt[-1 + I*Sqrt[7]]*Sqrt[5 - I*Sqrt[7] + 8*x]*
Sqrt[5 + I*Sqrt[7] + 8*x]*Sqrt[1 + (4*(1 + 2*x))/(1 - I*Sqrt[7])]*Sqrt[1 +
 (4*(1 + 2*x))/(1 + I*Sqrt[7])]*EllipticPi[(-3*(1 - I*Sqrt[7]))/52, ArcSin
[(2*Sqrt[1 + 2*x])/Sqrt[-1 + I*Sqrt[7]]], (I + Sqrt[7])/(I - Sqrt[7])])/(1
17*Sqrt[2 + 5*x + 4*x^2]*Sqrt[1 - I*Sqrt[7] + 4*(1 + 2*x)]*Sqrt[1 + I*Sqrt
[7] + 4*(1 + 2*x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 187
```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_] :> Simp[-2   Subst[Int[1/(Simp[b*c - a*d
- b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g - c*h)/
d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d*x]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 412
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

rule 413
```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2]   Int[1/((a +
 b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] &&  !GtQ[c, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1279
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2])   Int[1/((d + e*x
)*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.87

method	result
default	$-\frac{\sqrt{1+2x}\,\sqrt{4x^2+5x+2}\left(1+i\sqrt{7}\right)\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\,\sqrt{\frac{i\sqrt{7}-8x-5}{i\sqrt{7}-1}}\,\sqrt{\frac{i\sqrt{7}+8x+5}{1+i\sqrt{7}}}\left(39i\,\mathrm{EllipticF}\left(2\sqrt{-\frac{1+2x}{1+i\sqrt{7}}},\sqrt{-\frac{1+i\sqrt{7}}{i\sqrt{7}-1}}\right)\sqrt{7}-39i\,\mathrm{EllipticE}\right.}{}$
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(-\frac{16\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\,\sqrt{\frac{x+\frac{5}{8}-\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\,\sqrt{\frac{x+\frac{5}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}+\frac{i\sqrt{7}}{8}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}},\sqrt{\frac{\frac{1}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\right)}{9\sqrt{8x^3+14x^2+9x+2}}+\frac{4\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{}}{}\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/936*(1+2*x)^(1/2)*(4*x^2+5*x+2)^(1/2)*(1+I*7^(1/2))*(-(1+2*x)/(1+I*7^(1
/2)))^(1/2)*((I*7^(1/2)-8*x-5)/(I*7^(1/2)-1))^(1/2)*((I*7^(1/2)+8*x+5)/(1+
I*7^(1/2)))^(1/2)*(39*I*EllipticF(2*(-(1+2*x)/(1+I*7^(1/2)))^(1/2),(-(1+I*
7^(1/2))/(I*7^(1/2)-1))^(1/2))*7^(1/2)-39*I*EllipticE(2*(-(1+2*x)/(1+I*7^(
1/2)))^(1/2),(-(1+I*7^(1/2))/(I*7^(1/2)-1))^(1/2))*7^(1/2)-611*EllipticF(2
*(-(1+2*x)/(1+I*7^(1/2)))^(1/2),(-(1+I*7^(1/2))/(I*7^(1/2)-1))^(1/2))+39*E
llipticE(2*(-(1+2*x)/(1+I*7^(1/2)))^(1/2),(-(1+I*7^(1/2))/(I*7^(1/2)-1))^(
1/2))+608*EllipticPi(2*(-(1+2*x)/(1+I*7^(1/2)))^(1/2),-3/52-3/52*I*7^(1/2)
,(-(1+I*7^(1/2))/(I*7^(1/2)-1))^(1/2)))/(8*x^3+14*x^2+9*x+2)
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+5\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(24*x^4 + 2
*x^3 - 43*x^2 - 39*x - 10), x)
```

Sympy [F]

$$\begin{aligned}&\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx\\&=2\left(\int\left(-\frac{3x}{3x\sqrt{2x+1}\sqrt{4x^2+5x+2}-5\sqrt{2x+1}\sqrt{4x^2+5x+2}}\right)dx\right.\\&\quad+\int\frac{x^2}{3x\sqrt{2x+1}\sqrt{4x^2+5x+2}-5\sqrt{2x+1}\sqrt{4x^2+5x+2}}\,dx\\&\quad\left.+\int\left(-\frac{2}{3x\sqrt{2x+1}\sqrt{4x^2+5x+2}-5\sqrt{2x+1}\sqrt{4x^2+5x+2}}\right)dx\right)\end{aligned}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+5*x+2)**(1/2),x)
```

output
```
2*(Integral(-3*x/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 5*sqrt(2*x +
1)*sqrt(4*x**2 + 5*x + 2)), x) + Integral(x**2/(3*x*sqrt(2*x + 1)*sqrt(4*x
**2 + 5*x + 2) - 5*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x) + Integral(-2
/(3*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 5*sqrt(2*x + 1)*sqrt(4*x**2 +
 5*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+5\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/(sqrt(4*x^2 + 5*x + 2)*(3*x - 5)*sqrt(2*x + 1)
), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{\sqrt{4\,x^2+5\,x+2}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/(sqrt(4*x^2 + 5*x + 2)*(3*x - 5)*sqrt(2*x + 1)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\sqrt{4\,x^2+5\,x+2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(1/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(1/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\sqrt{2+5x+4x^2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,\sqrt{4x^2+5x+2}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2),x)
```

3.66 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+5x+4x^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 448

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \frac{8\sqrt{1+2x}(146+221x)}{1351\sqrt{2+5x+4x^2}}$$
$$-\frac{884\sqrt{2}\sqrt{1+2x}\sqrt{2+5x+4x^2}}{1351\left(1+\sqrt{2}(1+2x)\right)}+\frac{76}{193}\sqrt{\frac{3}{2509}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)$$
$$+\frac{442\ 2^{3/4}\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}(4-\sqrt{2})\right)}{1351\sqrt{2+5x+4x^2}}$$
$$-\frac{\sqrt[4]{2}(9+55\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}(4-\sqrt{2})\right)}{329\sqrt{2+5x+4x^2}}$$
$$-\frac{38\sqrt[4]{2}(347-78\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\text{EllipticPi}\left(\frac{1}{312}(156+347\sqrt{2}),2\arctan\left(\sqrt[4]{2}\sqrt{1+\right.\right.}{825461\sqrt{2+5x+4x^2}}$$

output
```
8/1351*(1+2*x)^(1/2)*(146+221*x)/(4*x^2+5*x+2)^(1/2)-884*2^(1/2)*(1+2*x)^(
1/2)*(4*x^2+5*x+2)^(1/2)/(1351+1351*2^(1/2)*(1+2*x))+76/484237*7527^(1/2)*
arctanh(1/39*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))+442/1351*((4*x^
2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticE(sin(2*
arctan(2^(1/4)*(1+2*x)^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/(4*x^2+5*x
+2)^(1/2)-1/329*2^(1/4)*(9+55*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^
2)^(1/2)*(1+2^(1/2)*(1+2*x))*InverseJacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2
)),1/4*(8-2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)-38/825461*2^(1/4)*(347-78*
2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*E
llipticPi(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),1/4*(8-
2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 28.31 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.39

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \frac{\sqrt{1+2x}\left(1248\sqrt{3}(146+221x)+\frac{6(1+2x)\left(-\frac{2873\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}+2(\cdots}{\left(-i+\sqrt{7}\right)\cdots}}\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(3/
2)),x]
```

output
```
(Sqrt[1 + 2*x]*(1248*Sqrt[3]*(146 + 221*x) + (6*(1 + 2*x)*((-2873*(I + Sqr
t[7])*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x)
)]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*Ell
ipticE[I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])
/(I + Sqrt[7])])/Sqrt[1/2 + x] + ((1883*I + 2873*Sqrt[7])*Sqrt[(-3*I + Sqr
t[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7]
 + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticF[I*ArcSinh[Sqrt[
(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[
1/2 + x] - 4*((5746*Sqrt[(-I)/(-I + Sqrt[7])]*(2 + 5*x + 4*x^2))/(1 + 2*x)
^2 + ((266*I)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(
1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*
x))]*EllipticPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[
7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x])))/Sqrt[I
/(3*I - 3*Sqrt[7])]))/(210756*Sqrt[3]*Sqrt[2 + 5*x + 4*x^2])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}\,dx$$

$$\Big\downarrow \; 2154$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx$$

$$\Big\downarrow \; 1235$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx+\frac{1}{7}\int -\frac{2(68x+81)}{9\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$$\Big\downarrow \; 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx-\frac{2}{63}\int \frac{68x+81}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$\Big\downarrow$ 1269

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx- \frac{2}{63}\left(47\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+34\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)+\frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$\Big\downarrow$ 1172

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx- \frac{2}{63}\left(\frac{94i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{2x+1}}+\frac{17i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}\right.}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right. \frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$\Big\downarrow$ 321

$$-\frac{2}{63}\left(\frac{17i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}+\frac{94i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2}{i-}\right.}{\sqrt{2x+1}}\right. \frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$\Big\downarrow$ 327

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx- \frac{2}{63}\left(\frac{94i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}+\frac{17i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right. \frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

$\Big\downarrow$ 1292

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx -$$

$$\frac{2}{63}\left(\frac{94i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}+\frac{17i\sqrt{2x+1}E\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right)\right.}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.$$

$$\frac{8\sqrt{2x+1}(17x+1)}{63\sqrt{4x^2+5x+2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(3/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(-\frac{2(4+8x)\left(-\frac{146}{1351}-\frac{221x}{1351}\right)}{\sqrt{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)(4+8x)}}-\frac{2084\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}-\frac{i\sqrt{7}}{8}}{\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}+\frac{i\sqrt{7}}{8}}{\frac{1}{8}+\frac{i\sqrt{7}}{8}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}},\sqrt{\right.}{1351\sqrt{8x^3+14x^2+9x+2}}$
default	$\frac{\sqrt{1+2x}\,\sqrt{4x^2+5x+2}\left(1027i\sqrt{7}\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\sqrt{\frac{i\sqrt{7}-8x-5}{i\sqrt{7}-1}}\sqrt{\frac{i\sqrt{7}+8x+5}{1+i\sqrt{7}}}\,\mathrm{EllipticF}\left(2\sqrt{-\frac{1+2x}{1+i\sqrt{7}}},\sqrt{-\frac{1+i\sqrt{7}}{i\sqrt{7}-1}}\right)-532i\sqrt{7}\sqrt{-\frac{1+2x}{1+i\sqrt{7}}}\sqrt{\right.}{}$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+5*x+2)*(1+2*x))^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2*x)^(1/2)*(-2*(4+8*x
)*(-146/1351-221/1351*x)/((x^2+5/4*x+1/2)*(4+8*x))^(1/2)-2084/1351*(-1/8-1
/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/
(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/
2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/
2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))-3536/1351*(-1/8-1/8*I*
7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-
1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8
*x^3+14*x^2+9*x+2)^(1/2)*((1/8-1/8*I*7^(1/2))*EllipticE(((x+1/2)/(-1/8-1/8
*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))+(-5/8+
1/8*I*7^(1/2))*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*
7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))+304/2509*(-1/8-1/8*I*7^(1/2))*((x+1/
2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))
^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x
+2)^(1/2)*EllipticPi(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),-3/52-3/52*I*7^(
1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(96*x^6 + 1
28*x^5 - 114*x^4 - 367*x^3 - 321*x^2 - 128*x - 20), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = 2\Bigg(\int \Bigg(-\frac{}{12x^3\sqrt{2x+1}\sqrt{4x^2+5x+2}-5x^2\sqrt{2x+1}\sqrt{4x^2+5}}$$
$$+\int \frac{x^2}{12x^3\sqrt{2x+1}\sqrt{4x^2+5x+2}-5x^2\sqrt{2x+1}\sqrt{4x^2+5x+2}-19x\sqrt{2x+1}\sqrt{4x^2+5x+2}-10\sqrt{2x+}}$$
$$+\int \Bigg(-\frac{2}{12x^3\sqrt{2x+1}\sqrt{4x^2+5x+2}-5x^2\sqrt{2x+1}\sqrt{4x^2+5x+2}-19x\sqrt{2x+1}\sqrt{4x^2+5x+2}-10\sqrt{2x}}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+5*x+2)**(3/2),x)
```

output
```
2*(Integral(-3*x/(12*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 5*x**2*sq
rt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 19*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x
+ 2) - 10*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x) + Integral(x**2/(12*x*
*3*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 5*x**2*sqrt(2*x + 1)*sqrt(4*x**2
 + 5*x + 2) - 19*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 10*sqrt(2*x + 1)
*sqrt(4*x**2 + 5*x + 2)), x) + Integral(-2/(12*x**3*sqrt(2*x + 1)*sqrt(4*x
**2 + 5*x + 2) - 5*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 19*x*sqrt(2
*x + 1)*sqrt(4*x**2 + 5*x + 2) - 10*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)),
 x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(3/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{3}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(3/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+5\,x+2\right)^{3/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(3/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(3/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{3/2}}\,dx=\int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+5x+2)^{\frac{3}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(3/2),x)
```

3.67 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\left(2+5x+4x^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 480

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \frac{8\sqrt{1+2x}(146+221x)}{4053\,(2+5x+4x^2)^{3/2}}$$
$$+\frac{4\sqrt{1+2x}(1335743+1956796x)}{5475603\sqrt{2+5x+4x^2}}$$
$$-\frac{3913592\sqrt{2}\sqrt{1+2x}\sqrt{2+5x+4x^2}}{5475603\,(1+\sqrt{2}(1+2x))}+\frac{684\sqrt{\frac{3}{2509}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)}{37249}$$
$$+\frac{1956796\,2^{3/4}\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\,(1+\sqrt{2}(1+2x))\,E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}(4-\sqrt{2})\right)}{5475603\sqrt{2+5x+4x^2}}$$
$$-\frac{2\,2^{3/4}(119243+15571\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\,(1+\sqrt{2}(1+2x))\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}(4-\right.}{1333437\sqrt{2+5x+4x^2}}$$
$$-\frac{342\sqrt[4]{2}(347-78\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\,(1+\sqrt{2}(1+2x))\,\text{EllipticPi}\left(\frac{1}{312}(156+347\sqrt{2})\,,2\arctan\left(\sqrt[4]{2}\sqrt{1\right.\right.}{159313973\sqrt{2+5x+4x^2}}$$

output
```
8/4053*(1+2*x)^(1/2)*(146+221*x)/(4*x^2+5*x+2)^(3/2)+4/5475603*(1+2*x)^(1/
2)*(1335743+1956796*x)/(4*x^2+5*x+2)^(1/2)-3913592*2^(1/2)*(1+2*x)^(1/2)*(
4*x^2+5*x+2)^(1/2)/(5475603+5475603*2^(1/2)*(1+2*x))+684/93457741*7527^(1/
2)*arctanh(1/39*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))+1956796/5475
603*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*Ellipt
icE(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/4*(8-2*2^(1/2))^(1/2))*2^(3/4)/
(4*x^2+5*x+2)^(1/2)-2/1333437*2^(3/4)*(119243+15571*2^(1/2))*((4*x^2+5*x+2
)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*InverseJacobiAM(2*arcta
n(2^(1/4)*(1+2*x)^(1/2)),1/4*(8-2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)-342/
159313973*2^(1/4)*(347-78*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(
1/2)*(1+2^(1/2)*(1+2*x))*EllipticPi(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1
/2+347/312*2^(1/2),1/4*(8-2*2^(1/2))^(1/2))/(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.91 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.33

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = 2\left(\frac{2\sqrt{1+2x}(3065978+11189449x+15126952x^2+7827184\ldots}{5475603\,(2+5x+4x^2)^{3/2}}\right.$$

$$-\frac{(1+2x)^{3/2}\left(\frac{50876696\sqrt{-\frac{i}{-i+\sqrt{7}}}(2+5x+4x^2)}{(1+2x)^2}+\frac{6359587\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{7}+2\left(-i+\sqrt{7}\right)x}{\left(-i+\sqrt{7}\right)(1+2x)}}\sqrt{\frac{3i+\sqrt{7}+2\left(i+\sqrt{7}\right)x}{\left(i+\sqrt{7}\right)(1+2x)}}E\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{2i}{-i+\ldots}}}{\sqrt{1+2x}}\ldots\right.\right.}{\sqrt{\frac{1}{2}+x}}\ldots}{\ldots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(5/
2)),x]
```

output
```
2*((2*Sqrt[1 + 2*x]*(3065978 + 11189449*x + 15126952*x^2 + 7827184*x^3))/(
5475603*(2 + 5*x + 4*x^2)^(3/2)) - ((1 + 2*x)^(3/2)*((50876696*Sqrt[(-I)/(
-I + Sqrt[7])]*(2 + 5*x + 4*x^2))/(1 + 2*x)^2 + (6359587*(I + Sqrt[7])*Sqr
t[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(
3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticE[I*
ArcSinh[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqr
t[7])])/Sqrt[1/2 + x] - ((3031931*I + 6359587*Sqrt[7])*Sqrt[(-3*I + Sqrt[7
] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] +
2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticF[I*ArcSinh[Sqrt[(-2
*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2
 + x] + ((100548*I)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt
[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(
1 + 2*x))]*EllipticPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sqrt[(-2*I)/(-I +
 Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x]))/(
142365678*Sqrt[(-I)/(-I + Sqrt[7])]*Sqrt[2 + 5*x + 4*x^2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{5/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\,(4x^2+5x+2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{5/2}}dx+\frac{1}{21}\int -\frac{2(33-68x)}{3\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{189\,(4x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{5/2}}dx-\frac{2}{63}\int \frac{33-68x}{\sqrt{2x+1}\,(4x^2+5x+2)^{3/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{189\,(4x^2+5x+2)^{3/2}}$$

$$\Big\downarrow \ 1235$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-\frac{2}{63}\left(\frac{1}{7}\int \frac{8(135x+143)}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-\frac{2}{63}\left(\frac{8}{7}\int \frac{135x+143}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1269$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-\frac{2}{63}\left(\frac{8}{7}\left(\frac{151}{2}\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+\frac{135}{2}\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 1172$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-\frac{2}{63}\left(-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}+\frac{8}{7}\left(\frac{151i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{\sqrt{2x+1}}+\frac{135i\sqrt{2x}\cdots}{\cdots}\right.\right.$$

$$\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$$\Big\downarrow \ 321$$

$$-\frac{2}{63}\left(-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}+\frac{8}{7}\left(\frac{135i\sqrt{2x+1}\int\frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}}{4\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}+\frac{151i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\right.\right.$$

$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 327

$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-$$

$$\frac{2}{63}\left(-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}+\frac{8}{7}\left(\frac{151i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}+\frac{135i\sqrt{2x\right.\right.$$

$$\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

$\downarrow$ 1292

$$\frac{76}{9}\int\frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx-$$

$$\frac{2}{63}\left(-\frac{2\sqrt{2x+1}(540x+103)}{7\sqrt{4x^2+5x+2}}+\frac{8}{7}\left(\frac{151i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-i\left(8x+i\sqrt{7}+5\right)}}{\sqrt{2}\sqrt[4]{7}}\right),-\frac{2\sqrt{7}}{i-\sqrt{7}}\right)}{\sqrt{2x+1}}+\frac{135i\sqrt{2x\right.\right.$$

$$\frac{8\sqrt{2x+1}(17x+1)}{189\left(4x^2+5x+2\right)^{3/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(5/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(\frac{\left(\frac{73}{4053}+\frac{221x}{8106}\right)\sqrt{8x^3+14x^2+9x+2}}{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)^2}-\frac{2(4+8x)\left(-\frac{1335743}{10951206}-\frac{978398x}{5475603}\right)}{\sqrt{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)(4+8x)}}-\frac{2960672\left(-\frac{1}{8}-\frac{i\sqrt{7}}{8}\right)\sqrt{\frac{x+\frac{1}{2}}{-\frac{1}{8}-\frac{i\sqrt{7}}{8}}}\sqrt{\frac{x+\frac{5}{8}-}{\frac{1}{8}-i}}}{182520}\right.}{}$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+5*x+2)*(1+2*x))^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2*x)^(1/2)*((73/4053+
221/8106*x)*(8*x^3+14*x^2+9*x+2)^(1/2)/(x^2+5/4*x+1/2)^2-2*(4+8*x)*(-13357
43/10951206-978398/5475603*x)/((x^2+5/4*x+1/2)*(4+8*x))^(1/2)-2960672/1825
201*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*
I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^
(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticF(((x+1/2)/(-1/8-1/8*I*7^
(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))-15654368/54
75603*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/
8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*
7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*((1/8-1/8*I*7^(1/2))*EllipticE(
((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/
2)))^(1/2))+(-5/8+1/8*I*7^(1/2))*EllipticF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^
(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))+2736/484237*(-1/8-
1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))
/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1
/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*EllipticPi(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(
1/2),-3/52-3/52*I*7^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))
)
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(384*x^8 +
992*x^7 + 376*x^6 - 1782*x^5 - 3347*x^4 - 2851*x^3 - 1362*x^2 - 356*x - 40
), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = 2\left(\int \left(-\frac{}{48x^5\sqrt{2x+1}\sqrt{4x^2+5x+2}+40x^4\sqrt{2x+1}\sqrt{4x^2+}}\right.\right.$$
$$+\int \frac{x^2}{48x^5\sqrt{2x+1}\sqrt{4x^2+5x+2}+40x^4\sqrt{2x+1}\sqrt{4x^2+5x+2}-77x^3\sqrt{2x+1}\sqrt{4x^2+5x+2}-145x^2\sqrt{}}$$
$$+\int \left(-\frac{2}{48x^5\sqrt{2x+1}\sqrt{4x^2+5x+2}+40x^4\sqrt{2x+1}\sqrt{4x^2+5x+2}-77x^3\sqrt{2x+1}\sqrt{4x^2+5x+2}-145x}\right.$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+5*x+2)**(5/2),x)
```

output
```
2*(Integral(-3*x/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) + 40*x**4*s
qrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 77*x**3*sqrt(2*x + 1)*sqrt(4*x**2 +
5*x + 2) - 145*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 88*x*sqrt(2*x +
 1)*sqrt(4*x**2 + 5*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x)
+ Integral(x**2/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) + 40*x**4*sq
rt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 77*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 5
*x + 2) - 145*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 88*x*sqrt(2*x +
1)*sqrt(4*x**2 + 5*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x) +
 Integral(-2/(48*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) + 40*x**4*sqrt(
2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 77*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x
+ 2) - 145*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 88*x*sqrt(2*x + 1)*
sqrt(4*x**2 + 5*x + 2) - 20*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(5/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{5}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(5/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+5\,x+2\right)^{5/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(5/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(5/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{5/2}}\,dx = \int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+5x+2)^{\frac{5}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(5/2),x)
```

3.68 $\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 41, antiderivative size = 512

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx = \frac{8\sqrt{1+2x}(146+221x)}{6755\,(2+5x+4x^2)^{5/2}}$$

$$+\frac{4\sqrt{1+2x}(3097569+4553140x)}{27378015\,(2+5x+4x^2)^{3/2}}+\frac{8\sqrt{1+2x}(1346256517+2000747516x)}{7397539653\sqrt{2+5x+4x^2}}$$

$$-\frac{8002990064\sqrt{2}\sqrt{1+2x}\sqrt{2+5x+4x^2}}{7397539653\left(1+\sqrt{2}(1+2x)\right)}+\frac{6156\sqrt{\frac{3}{2509}}\operatorname{arctanh}\left(\frac{\sqrt{\frac{193}{39}}\sqrt{1+2x}}{\sqrt{2+5x+4x^2}}\right)}{7189057}$$

$$+\frac{4001495032\ 2^{3/4}\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)E\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right)|\frac{1}{8}(4-\sqrt{2})\right)}{7397539653\sqrt{2+5x+4x^2}}$$

$$-\frac{8\sqrt[4]{2}(18773087+60905342\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\text{EllipticF}\left(2\arctan\left(\sqrt[4]{2}\sqrt{1+2x}\right),\frac{1}{8}(\right.}{1801473387\sqrt{2+5x+4x^2}}$$

$$-\frac{3078\sqrt[4]{2}(347-78\sqrt{2})\sqrt{\frac{2+5x+4x^2}{\left(1+\sqrt{2}(1+2x)\right)^2}}\left(1+\sqrt{2}(1+2x)\right)\text{EllipticPi}\left(\frac{1}{312}(156+347\sqrt{2}),2\arctan\left(\sqrt[4]{2}\sqrt{\right.\right.}}{30747596789\sqrt{2+5x+4x^2}}$$

output
```
8/6755*(1+2*x)^(1/2)*(146+221*x)/(4*x^2+5*x+2)^(5/2)+4/27378015*(1+2*x)^(1
/2)*(3097569+4553140*x)/(4*x^2+5*x+2)^(3/2)+8/7397539653*(1+2*x)^(1/2)*(13
46256517+2000747516*x)/(4*x^2+5*x+2)^(1/2)-8002990064*2^(1/2)*(1+2*x)^(1/2
)*(4*x^2+5*x+2)^(1/2)/(7397539653+7397539653*2^(1/2)*(1+2*x))+6156/1803734
4013*7527^(1/2)*arctanh(1/39*7527^(1/2)*(1+2*x)^(1/2)/(4*x^2+5*x+2)^(1/2))
+4001495032/7397539653*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1
/2)*(1+2*x))*EllipticE(sin(2*arctan(2^(1/4)*(1+2*x)^(1/2))),1/4*(8-2*2^(1/
2))^(1/2))*2^(3/4)/(4*x^2+5*x+2)^(1/2)-8/1801473387*2^(1/4)*(18773087+6090
5342*2^(1/2))*((4*x^2+5*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*
x))*InverseJacobiAM(2*arctan(2^(1/4)*(1+2*x)^(1/2)),1/4*(8-2*2^(1/2))^(1/2
))/(4*x^2+5*x+2)^(1/2)-3078/30747596789*2^(1/4)*(347-78*2^(1/2))*((4*x^2+5
*x+2)/(1+2^(1/2)*(1+2*x))^2)^(1/2)*(1+2^(1/2)*(1+2*x))*EllipticPi(sin(2*ar
ctan(2^(1/4)*(1+2*x)^(1/2))),1/2+347/312*2^(1/2),1/4*(8-2*2^(1/2))^(1/2))/
(4*x^2+5*x+2)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 26.84 (sec) , antiderivative size = 687, normalized size of antiderivative = 1.34

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx = 2\left(\frac{2\sqrt{1+2x}(63818768194+384928083441x+99961039874\ldots}{3698769826\ldots}\right.$$

$$-\frac{(1+2x)^{3/2}\left(52019435416\sqrt{-\frac{i}{-i+\sqrt{7}}}+\frac{26009717708\sqrt{-\frac{i}{-i+\sqrt{7}}}}{(1+2x)^2}+\frac{26009717708\sqrt{-\frac{i}{-i+\sqrt{7}}}}{1+2x}+\frac{6502429427\left(i+\sqrt{7}\right)\sqrt{\frac{-3i+\sqrt{\ldots}}{\left(-i\ldots\right.}}}{\ldots}\right.}{\ldots}$$

input
```
Integrate[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(7/
2)),x]
```

output
```
2*((2*Sqrt[1 + 2*x]*(63818768194 + 384928083441*x + 999610398746*x^2 + 138
3414256920*x^3 + 1015700049120*x^4 + 320119602560*x^5))/(36987698265*(2 +
5*x + 4*x^2)^(5/2)) - ((1 + 2*x)^(3/2)*(52019435416*Sqrt[(-I)/(-I + Sqrt[7
])] + (26009717708*Sqrt[(-I)/(-I + Sqrt[7])])/(1 + 2*x)^2 + (26009717708*S
qrt[(-I)/(-I + Sqrt[7])])/(1 + 2*x) + (6502429427*(I + Sqrt[7])*Sqrt[(-3*I
 + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + S
qrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticE[I*ArcSinh
[Sqrt[(-2*I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])
/Sqrt[1/2 + x] - ((2493689569*I + 6502429427*Sqrt[7])*Sqrt[(-3*I + Sqrt[7]
 + 2*(-I + Sqrt[7])*x)/((-I + Sqrt[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2
*(I + Sqrt[7])*x)/((I + Sqrt[7])*(1 + 2*x))]*EllipticF[I*ArcSinh[Sqrt[(-2*
I)/(-I + Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2
+ x] + ((3167262*I)*Sqrt[(-3*I + Sqrt[7] + 2*(-I + Sqrt[7])*x)/((-I + Sqrt
[7])*(1 + 2*x))]*Sqrt[(3*I + Sqrt[7] + 2*(I + Sqrt[7])*x)/((I + Sqrt[7])*(
1 + 2*x))]*EllipticPi[(-13*(1 + I*Sqrt[7]))/6, I*ArcSinh[Sqrt[(-2*I)/(-I +
 Sqrt[7])]/Sqrt[1 + 2*x]], (I - Sqrt[7])/(I + Sqrt[7])])/Sqrt[1/2 + x]))/(
96168015489*Sqrt[(-I)/(-I + Sqrt[7])]*Sqrt[2 + 5*x + 4*x^2]))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{-2x^2+6x+4}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx+\int \frac{\frac{2x}{3}-\frac{8}{9}}{\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx+\frac{1}{35}\int -\frac{2(117-476x)}{9\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-\frac{2}{315}\int \frac{117-476x}{\sqrt{2x+1}\left(4x^2+5x+2\right)^{5/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-\frac{2}{315}\left(\frac{1}{21}\int \frac{4(821-4986x)}{\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}}dx-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-\frac{2}{315}\left(\frac{4}{21}\int \frac{821-4986x}{\sqrt{2x+1}\left(4x^2+5x+2\right)^{3/2}}dx-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-\frac{2}{315}\left(\frac{4}{21}\left(\frac{1}{7}\int \frac{2(33200x+32349)}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}\right)-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-\frac{2}{315}\left(\frac{4}{21}\left(\frac{2}{7}\int \frac{33200x+32349}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}\right)-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}\right)+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-$$
$$\frac{2}{315}\left(\frac{4}{21}\left(\frac{2}{7}\left(15749\int \frac{1}{\sqrt{2x+1}\sqrt{4x^2+5x+2}}dx+16600\int \frac{\sqrt{2x+1}}{\sqrt{4x^2+5x+2}}dx\right)-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}\right)\right.$$
$$\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$\downarrow$ 1172

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-$$
$$\frac{2}{315}\left(-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}+\frac{4}{21}\left(-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}+\frac{2}{7}\left(\frac{31498i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\int \frac{1}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}\sqrt{1}}}{\sqrt{2x+1}}\right.\right.\right.$$
$$\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$\downarrow$ 321

$$-\frac{2}{315}\left(-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}+\frac{4}{21}\left(-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}+\frac{2}{7}\left(\frac{8300i\sqrt{2x+1}\int \frac{\sqrt{1-\frac{i\left(8x+i\sqrt{7}+5\right)}{i-\sqrt{7}}}}{\sqrt{\frac{i\left(8x+i\sqrt{7}+5\right)}{2\sqrt{7}}+1}}d\sqrt{}}{\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}}\right.\right.\right.$$
$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx+\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$\downarrow$ 327

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\left(4x^2+5x+2\right)^{7/2}}dx-$$
$$\frac{2}{315}\left(-\frac{2\sqrt{2x+1}(3324x+835)}{21\left(4x^2+5x+2\right)^{3/2}}+\frac{4}{21}\left(-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}+\frac{2}{7}\left(\frac{31498i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\operatorname{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{2x+1}}\right.\right.\right.$$
$$\frac{8\sqrt{2x+1}(17x+1)}{315\left(4x^2+5x+2\right)^{5/2}}$$

$\downarrow$ 1292

$$\frac{76}{9}\int \frac{1}{(5-3x)\sqrt{2x+1}\,(4x^2+5x+2)^{7/2}}dx-$$

$$\frac{2}{315}\left(-\frac{2\sqrt{2x+1}(3324x+835)}{21\,(4x^2+5x+2)^{3/2}}+\frac{4}{21}\left(-\frac{2\sqrt{2x+1}(33200x+9151)}{7\sqrt{4x^2+5x+2}}+\frac{2}{7}\left(\frac{31498i\sqrt{-\frac{2x+1}{1+i\sqrt{7}}}\,\text{EllipticF}\left(\arcsin\left(\right.\right.}{\sqrt{2x+1}}\right.\right.\right.$$

$$\frac{8\sqrt{2x+1}(17x+1)}{315\,(4x^2+5x+2)^{5/2}}$$

input
```
Int[(4 + 6*x - 2*x^2)/((5 - 3*x)*Sqrt[1 + 2*x]*(2 + 5*x + 4*x^2)^(7/2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 321
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 1172
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Simp[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m))   Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^
2 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{(4x^2+5x+2)(1+2x)}\left(\frac{\left(\frac{73}{27020}+\frac{221x}{54040}\right)\sqrt{8x^3+14x^2+9x+2}}{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)^3}+\frac{\left(\frac{1032523}{36504020}+\frac{227657x}{5475603}\right)\sqrt{8x^3+14x^2+9x+2}}{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)^2}-\frac{2(4+8x)\left(-\frac{1346256517}{7397539653}-\frac{200074}{73975}\right.}{\sqrt{\left(x^2+\frac{5}{4}x+\frac{1}{2}\right)(4+8x)}}\right.}{}$
default	Expression too large to display

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x,method=_RET
URNVERBOSE)
```

output
```
((4*x^2+5*x+2)*(1+2*x))^(1/2)/(4*x^2+5*x+2)^(1/2)/(1+2*x)^(1/2)*((73/27020
+221/54040*x)*(8*x^3+14*x^2+9*x+2)^(1/2)/(x^2+5/4*x+1/2)^3+(1032523/365040
20+227657/5475603*x)*(8*x^3+14*x^2+9*x+2)^(1/2)/(x^2+5/4*x+1/2)^2-2*(4+8*x
)*(-1346256517/7397539653-2000747516/7397539653*x)/((x^2+5/4*x+1/2)*(4+8*x
))^(1/2)-6158282016/2465846551*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7
^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1
/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*Ellipt
icF(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7
^(1/2)))^(1/2))-32011960256/7397539653*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8
-1/8*I*7^(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*(
(x+5/8+1/8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2
)*((1/8-1/8*I*7^(1/2))*EllipticE(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/
8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2))+(-5/8+1/8*I*7^(1/2))*Elliptic
F(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),((1/8+1/8*I*7^(1/2))/(1/8-1/8*I*7^(
1/2)))^(1/2)))+24624/93457741*(-1/8-1/8*I*7^(1/2))*((x+1/2)/(-1/8-1/8*I*7^
(1/2)))^(1/2)*((x+5/8-1/8*I*7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)*((x+5/8+1/
8*I*7^(1/2))/(1/8+1/8*I*7^(1/2)))^(1/2)/(8*x^3+14*x^2+9*x+2)^(1/2)*Ellipti
cPi(((x+1/2)/(-1/8-1/8*I*7^(1/2)))^(1/2),-3/52-3/52*I*7^(1/2),((1/8+1/8*I*
7^(1/2))/(1/8-1/8*I*7^(1/2)))^(1/2)))
```

Fricas [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x, algo
rithm="fricas")
```

output
```
integral(2*sqrt(4*x^2 + 5*x + 2)*(x^2 - 3*x - 2)*sqrt(2*x + 1)/(1536*x^10
+ 5888*x^9 + 7232*x^8 - 3264*x^7 - 21546*x^6 - 31703*x^5 - 26397*x^4 - 139
36*x^3 - 4664*x^2 - 912*x - 80), x)
```

Sympy [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx = 2\Bigg(\int\Bigg(-\frac{}{192x^7\sqrt{2x+1}\sqrt{4x^2+5x+2}+400x^6\sqrt{2x+1}\sqrt{4x^2}}$$
$$+\int \frac{}{192x^7\sqrt{2x+1}\sqrt{4x^2+5x+2}+400x^6\sqrt{2x+1}\sqrt{4x^2+5x+2}-12x^5\sqrt{2x+1}\sqrt{4x^2+5x+2}-885x^4}$$
$$+\int\Bigg(-\frac{}{192x^7\sqrt{2x+1}\sqrt{4x^2+5x+2}+400x^6\sqrt{2x+1}\sqrt{4x^2+5x+2}-12x^5\sqrt{2x+1}\sqrt{4x^2+5x+2}-88}$$

input
```
integrate((-2*x**2+6*x+4)/(5-3*x)/(1+2*x)**(1/2)/(4*x**2+5*x+2)**(7/2),x)
```

output
```
2*(Integral(-3*x/(192*x**7*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) + 400*x**6
*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 12*x**5*sqrt(2*x + 1)*sqrt(4*x**2
+ 5*x + 2) - 885*x**4*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 1231*x**3*sqr
t(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 810*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5
*x + 2) - 276*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 40*sqrt(2*x + 1)*sq
rt(4*x**2 + 5*x + 2)), x) + Integral(x**2/(192*x**7*sqrt(2*x + 1)*sqrt(4*x
**2 + 5*x + 2) + 400*x**6*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 12*x**5*s
qrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 885*x**4*sqrt(2*x + 1)*sqrt(4*x**2 +
 5*x + 2) - 1231*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 810*x**2*sqrt
(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 276*x*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x +
 2) - 40*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x) + Integral(-2/(192*x**7
*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) + 400*x**6*sqrt(2*x + 1)*sqrt(4*x**2
 + 5*x + 2) - 12*x**5*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 885*x**4*sqrt
(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 1231*x**3*sqrt(2*x + 1)*sqrt(4*x**2 + 5
*x + 2) - 810*x**2*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2) - 276*x*sqrt(2*x +
 1)*sqrt(4*x**2 + 5*x + 2) - 40*sqrt(2*x + 1)*sqrt(4*x**2 + 5*x + 2)), x))
```

Maxima [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx = \int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x, algo
rithm="maxima")
```

output
```
2*integrate((x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(7/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Giac [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx=\int \frac{2\,(x^2-3\,x-2)}{(4\,x^2+5\,x+2)^{\frac{7}{2}}(3\,x-5)\sqrt{2\,x+1}}\,dx$$

input
```
integrate((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x, algo
rithm="giac")
```

output
```
integrate(2*(x^2 - 3*x - 2)/((4*x^2 + 5*x + 2)^(7/2)*(3*x - 5)*sqrt(2*x +
1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx=\int -\frac{-2\,x^2+6\,x+4}{\sqrt{2\,x+1}\,\left(3\,x-5\right)\,\left(4\,x^2+5\,x+2\right)^{7/2}}\,dx$$

input
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(7/2))
,x)
```

output
```
int(-(6*x - 2*x^2 + 4)/((2*x + 1)^(1/2)*(3*x - 5)*(5*x + 4*x^2 + 2)^(7/2))
, x)
```

Reduce [F]

$$\int \frac{4+6x-2x^2}{(5-3x)\sqrt{1+2x}\,(2+5x+4x^2)^{7/2}}\,dx=\int \frac{-2x^2+6x+4}{(5-3x)\,\sqrt{2x+1}\,(4x^2+5x+2)^{\frac{7}{2}}}dx$$

input
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x)
```

output
```
int((-2*x^2+6*x+4)/(5-3*x)/(1+2*x)^(1/2)/(4*x^2+5*x+2)^(7/2),x)
```

3.69 $\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 1363

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*d^2*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g
+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d
))^(1/2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)/(2
*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)
^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*
e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e/(2*c*d-(b-(-4*a*c+b^2)^(1/2)
)*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*
g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4
*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+4*d*(c*d-(b+(-4*a*c
+b^2)^(1/2))*e)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(
1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticF((2*c
*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1
/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-
4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^
2)^(1/2))*g))^(1/2))/e^2/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+
(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*
(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/
2)/(c*x^2+b*x+a)^(1/2)+2*(-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*
f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^
(1/2)*EllipticPi((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)/(...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3222 vs. 2(1363) = 2726.

Time = 35.51 (sec) , antiderivative size = 3222, normalized size of antiderivative = 2.36

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x^2/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(-2*d^2*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(e*f - d
*g)*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[a + b*x + c*x^2]*((-2*d^2
*(d + e*x)^(5/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/
(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g + (e*f)/(d + e*x) -
(d*g)/(d + e*x)))/(Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*
d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2]*Sqrt[f + ((d + e*x)*(g -
 (d*g)/(d + e*x)))/e]) - (2*(c*d^2 - b*d*e + a*e^2)*(-(e*f) + d*g)*(d + e*
x)^(3/2)*Sqrt[(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d
+ e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g + (e*f)/(d + e*x) - (d*
g)/(d + e*x))]*((-2*d*(-1/2*(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(c*d
^2 - b*d*e + a*e^2) + (d + e*x)^(-1))*Sqrt[(-1/2*(2*c*d - b*e + Sqrt[b^2*e
^2 - 4*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d + e*x)^(-1))/((2*c*d - b*e -
 Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)) - (2*c*d - b*e + S
qrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)))]*Sqrt[(-(g/(-(e*f)
+ d*g)) + (d + e*x)^(-1))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c
*d^2 - b*d*e + a*e^2)) - g/(-(e*f) + d*g))]*EllipticF[ArcSin[Sqrt[(-2*c*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*d^2)/(d + e*x) - (2*b*d*e)/(d + e*x
) + (2*a*e^2)/(d + e*x))/Sqrt[(b^2 - 4*a*c)*e^2]]/Sqrt[2]], (2*Sqrt[(b^2 -
 4*a*c)*e^2]*(e*f - d*g))/(-2*c*d*e*f + b*e^2*f + e*Sqrt[(b^2 - 4*a*c)*e^2
]*f + b*d*e*g - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g)])/(Sqrt[(-1/2*...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2154}$$

$$\frac{d^2\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1281}$$

$$\frac{d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e^2}+\int \frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1280}$$

$$\frac{d^2\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int\frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right.}{e^2}$$

$$\int\frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1292}$$

$$\frac{d^2\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int\frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right.}{e^2}$$

$$\int\frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1416}$$

$$\frac{d^2\left(\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{(f+gx)^2(ae^2-\cdots}{(d+ex)^2(cf^2-g\cdots}}}{\sqrt{a+bx+cx^2}(ef-dg)^2\sqrt[4]{ae^2-ba\cdots}}\right.}{e^2}$$

$$\int\frac{\frac{x}{e}-\frac{d}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{2154}$$

$$d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2-\right.}{(d+ex)^2\left(cf^2-g\right.}}{\left(\right.}}}{\sqrt{a+bx+cx^2}(ef-dg)^2\sqrt[4]{ae^2-ba}}\right.$$
$$\overline{\qquad\qquad e^2}$$
$$\frac{2d\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 27

$$d^2\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2-\right.}{(d+ex)^2\left(cf^2-g\right.}}{\left(\right.}}}{\sqrt{a+bx+cx^2}(ef-dg)^2\sqrt[4]{ae^2-ba}}\right.$$
$$\overline{\qquad\qquad e^2}$$
$$\frac{2d\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$\downarrow$ 1276

$$\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f-}{\left(cf^2-bgf+ag^2\right)(d+ex}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}}}\right.$$
$$\overline{\qquad\qquad e^2}$$
$$\frac{2\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dxd}{e^2}+$$
$$\frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(d+}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)eg}}$$

$$\Big\downarrow \; 1280$$

$$\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f-}{\left(cf^2-bgf+ag^2\right)(d+ex}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}}}\right.$$

$$e^2$$

$$\frac{4(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}d}{e^2(ef-dg)\sqrt{cx^2+bx+a}}+$$

$$\frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(d+}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)eg}}$$

$$\Big\downarrow \; 1416$$

$$\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f-}{\left(cf^2-bgf+ag^2\right)(d+ex}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}}}\right.$$

$$e^2$$

$$\frac{2\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(}{\left(cf^2-bgf+\right.}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+\right.}}}{e^2\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd^2-bed}{(cf^2-g(b}}}$$

$$\frac{\sqrt{2}\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(d+}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)eg}}$$

input
```
Int[x^2/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3125 vs. $2(1222) = 2444$.

Time = 434.05 (sec) , antiderivative size = 3126, normalized size of antiderivative = 2.29

method	result	size
elliptic	Expression too large to display	3126
default	Expression too large to display	104836

input
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*d^2/((x+d/e)*(c*e*g
*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(-d/e^2+1/e^2*(a*
e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*d^2/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*
d*e^2*f+c*d^3*g-c*d^2*e*f)-(a*e*g+b*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*
e^2*f+c*d^3*g-c*d^2*e*f)/e*d^2)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(
-4*a*c+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f
/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*Ell
ipticF(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a
*c+b^2)^(1/2))/c)*(-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c
*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(1/e+1/e*(b*e*g-c*d*g+c*e*f)*d^2/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e*f)/(a*
d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*d^2)*(-f/g+d/e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x**2/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="maxima")
```

output
```
integrate(x^2/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm
="giac")
```

output
```
integrate(x^2/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^2/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^2/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x^2}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x^2/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.70 $\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 882

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2d\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\,E\left(\arcsin\left(\frac{\sqrt{2cd-be+\sqrt{b^2}}}{\sqrt{2cf-\left(b-\sqrt{b^2}\right.}}\right.\right.}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}(ef-dg)}$$

$$+\frac{2\left(b+\sqrt{b^2-4ac}\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)}}\right.\right.}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{\frac{(ef-}{(2cf-}}}$$

output
```
-2*d*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+
e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d)
)^(1/2)*EllipticE(((-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)^(1/2)*(g*x+f)^(1/2)/(2*
c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^
(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e
)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e
)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^
(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*
c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*(b+(-4*a*c+b^2)^(1/2
))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(
2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticF(((-4*a*c+b^2)^(1/
2)*e-b*e+2*c*d)^(1/2)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)
/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1
/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))
^(1/2))/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2
))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(
1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^
(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2445 vs. 2(882) = 1764.

Time = 35.36 (sec) , antiderivative size = 2445, normalized size of antiderivative = 2.77

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[x/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
(2*d*e*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e^2)*(e*f - d*
g)*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]) - (2*Sqrt[a + b*x + c*x^2]*(-((d*(
d + e*x)^(5/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d
 + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g + (e*f)/(d + e*x) - (d
*g)/(d + e*x)))/(Sqrt[((d + e*x)^2*(c*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)
/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2]*Sqrt[f + ((d + e*x)*(g - (
d*g)/(d + e*x)))/e])) - ((c*d^2 - b*d*e + a*e^2)*(-(e*f) + d*g)*(d + e*x)^
(3/2)*Sqrt[(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(d + e*x)^2 + (a*e^2)/(d + e
*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g + (e*f)/(d + e*x) - (d*g)/
(d + e*x))]*(-(((-1/2*(2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(c*d^2 - b
*d*e + a*e^2) + (d + e*x)^(-1))*Sqrt[(-1/2*(2*c*d - b*e + Sqrt[b^2*e^2 - 4
*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d + e*x)^(-1))/((2*c*d - b*e - Sqrt[
b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)) - (2*c*d - b*e + Sqrt[b^
2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)))]*Sqrt[(-(g/(-(e*f) + d*g)
) + (d + e*x)^(-1))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 -
 b*d*e + a*e^2)) - g/(-(e*f) + d*g))]*EllipticF[ArcSin[Sqrt[(-2*c*d + b*e
+ Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*d^2)/(d + e*x) - (2*b*d*e)/(d + e*x) + (2
*a*e^2)/(d + e*x))/Sqrt[(b^2 - 4*a*c)*e^2]]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c
)*e^2]*(e*f - d*g))/(-2*c*d*e*f + b*e^2*f + e*Sqrt[(b^2 - 4*a*c)*e^2]*f +
b*d*e*g - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)*e^2]*g)])/(Sqrt[(-1/2*(2*c*d...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\int \frac{1}{e\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{ 1280}$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}-\frac{d\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{ 1281}$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}-\frac{d\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \text{ 1280}$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}-\frac{d\left(\frac{2g(d+ex)\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \text{ 1292}$$

$$-\frac{2(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}-\frac{d\left(\frac{2g(d+ex)\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e}$$

$$\Big\downarrow \text{ 1416}$$

$$-\frac{\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b}{(cf^2-bgf+}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+\right.}}}{e\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd^2-bed}{(cf^2-g(b}}}$$

$$\frac{d\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(}{(cf^2-bgf+ag^2)(d+e}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)^2}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{(cd^2-bed+ae^2)(f+gx}{(cf^2-g(bf-ag))(d+ex}}}\right.}{e}$$

input
```
Int[x/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3096 vs. $2(788) = 1576$.

Time = 14.48 (sec) , antiderivative size = 3097, normalized size of antiderivative = 3.51

method	result	size
elliptic	Expression too large to display	3097
default	Expression too large to display	35044

input
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(-2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d
*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*d/((x+d/e)*(c*e*g*x^
3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(1/e-1/e*(a*e^2*g-b*
d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*d/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*
d^3*g-c*d^2*e*f)+(a*e*g+b*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^
3*g-c*d^2*e*f)*d)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)
/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b
^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^
(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2
/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1
/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)
^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)
^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)*(-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2))+2*(-(b*e*g-c*d*g+c*e*f)*d/(a*d*e^2*g-a*e^3*f-b*d^2*e*g
+b*d*e^2*f+c*d^3*g-c*d^2*e*f)+(2*b*e*g+2*c*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e
*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*d)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+...
```

Fricas [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)*x/(c*e^2*g*x^5
+ (c*e^2*f + (2*c*d*e + b*e^2)*g)*x^4 + a*d^2*f + ((2*c*d*e + b*e^2)*f + (
c*d^2 + 2*b*d*e + a*e^2)*g)*x^3 + ((c*d^2 + 2*b*d*e + a*e^2)*f + (b*d^2 +
2*a*d*e)*g)*x^2 + (a*d^2*g + (b*d^2 + 2*a*d*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(x/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(x/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(x/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{x}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(x/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.71 $\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 869

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \frac{2e\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg\cdots}{\left(2cf-\left(b\cdots\right.\right.}}\cdots}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac\cdots\right.\right.}\cdots}$$

$$-\frac{4c\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}e}\sqrt{f+gx}}{\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{d+ex}}\right),\frac{\left(2cd-\cdots\right.}{\left(2cd-\cdots\right.}\cdots\right.}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{\frac{(ef-dg)\left(b-\sqrt{b\cdots\right.}{\left(2cf-\left(b-\sqrt{b^2-\cdots\right.\right.}}\cdots}$$

output
```
2*e*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e
*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))
^(1/2)*EllipticE(((-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)^(1/2)*(g*x+f)^(1/2)/(2*c
*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^(
1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)
/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)
^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(
1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c
+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)-4*c*(b-(-4*a*c+b^2)^(1/
2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^
(1/2))*g)/(e*x+d))^(1/2)*EllipticF(((-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)^(1/2)*
(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*
a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-(b-(-4*
a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*
c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-
(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5981 vs. $2(869) = 1738$.

Time = 6.40 (sec) , antiderivative size = 5981, normalized size of antiderivative = 6.88

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1281}$$

$$\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}$$

$$\Big\downarrow \text{ 1280}$$

$$\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}$$

$$\Big\downarrow \text{ 1292}$$

$$\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}$$

$$\Big\downarrow \text{ 1416}$$

$$\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+$$

$$\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2\left(ae^2-bde+cd^2\right)}{(d+ex)^2\left(cf^2-g(bf-ag)\right)}-\frac{(f+gx)(2aeg-b}{(d+ex)\left(ag^2\right.}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+\right.}}}{\sqrt{a+bx+cx^2}(ef-dg)^2\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(}{(d+ex)^2(c}}}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3090 vs. $2(777) = 1554$.

Time = 14.47 (sec) , antiderivative size = 3091, normalized size of antiderivative = 3.56

method	result	size
elliptic	Expression too large to display	3091
default	Expression too large to display	18084

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*e/((x+d/e)*(c*e*g*x^3
+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*((a*e^2*g-b*d*e*g+b*e
^2*f+c*d^2*g-c*d*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2
*e*f)-(a*e*g+b*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e
*f)*e)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/
(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(
-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2
)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(-
4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x
+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c*(-b+(
-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(
1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+d
/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2))+2*(e*(b*e*g-c*d*g+c*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c
*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f
+c*d^3*g-c*d^2*e*f)*e)*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))...
```

Fricas [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^2*g*x^5 +
(c*e^2*f + (2*c*d*e + b*e^2)*g)*x^4 + a*d^2*f + ((2*c*d*e + b*e^2)*f + (c*
d^2 + 2*b*d*e + a*e^2)*g)*x^3 + ((c*d^2 + 2*b*d*e + a*e^2)*f + (b*d^2 + 2*
a*d*e)*g)*x^2 + (a*d^2*g + (b*d^2 + 2*a*d*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral(1/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algorithm="
giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{\sqrt{f+g\,x}\,{(d+e\,x)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{1}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int(1/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.72 $\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 897

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{2(Bd-Ae)\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\,E\left(\arcsin\left(\frac{\sqrt{\ldots}}{\sqrt{2\ldots}}\right.\right.}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\,(ef-\ldots}$$

$$+\frac{2\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{\frac{(ef-dg)\left(b+\sqrt{b^2-4ac}+2cx\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{2cd-\ldots}}{\sqrt{2cf-\ldots}}\right.\right.}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\sqrt{\ldots}}$$

output
```
-2*(-A*e+B*d)*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*x
)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g
)/(e*x+d))^(1/2)*EllipticE(((-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)^(1/2)*(g*x+f)^
(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*
a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)
^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-(b-(-4*a*c+b^2)
^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(
1/2))*g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-
(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*(b*B-2*A*c+
B*(-4*a*c+b^2)^(1/2))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+
b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*Elliptic
F(((-4*a*c+b^2)^(1/2)*e-b*e+2*c*d)^(1/2)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b
^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f
-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*
a*c+b^2)^(1/2))*g))^(1/2))/(2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(
b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f
)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(
1/2)/(c*x^2+b*x+a)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 3628 vs. $2(897) = 1794$.

Time = 36.09 (sec) , antiderivative size = 3628, normalized size of antiderivative = 4.04

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x]
```

output
```
(-2*e*(-(B*d) + A*e)*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*
e^2)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[a + b*x + c*
x^2]*((-2*(-(B*d) + A*e)*(d + e*x)^(5/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d*e
)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))
*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(Sqrt[((d + e*x)^2*(c*(-1 + d/(d
 + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2]*
Sqrt[f + ((d + e*x)*(g - (d*g)/(d + e*x)))/e]) + (2*(c*d^2 - b*d*e + a*e^2
)*(-(e*f) + d*g)*(d + e*x)^(3/2)*Sqrt[(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(
d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g
 + (e*f)/(d + e*x) - (d*g)/(d + e*x))]*(-((B*(-1/2*(2*c*d - b*e - Sqrt[b^2
*e^2 - 4*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d + e*x)^(-1))*Sqrt[(-1/2*(2
*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d + e*x
)^(-1))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e
^2)) - (2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2
)))]*Sqrt[(-(g/(-(e*f) + d*g)) + (d + e*x)^(-1))/((2*c*d - b*e - Sqrt[b^2*
e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)) - g/(-(e*f) + d*g))]*Ellipti
cF[ArcSin[Sqrt[(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*d^2)/(d + e*
x) - (2*b*d*e)/(d + e*x) + (2*a*e^2)/(d + e*x))/Sqrt[(b^2 - 4*a*c)*e^2]]/S
qrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f - d*g))/(-2*c*d*e*f + b*e^2*f + e
*Sqrt[(b^2 - 4*a*c)*e^2]*f + b*d*e*g - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a*c)...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{B}{e\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\frac{B\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e}$$

$$\Big\downarrow \text{1280}$$

$$\frac{\left(A-\frac{Bd}{e}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx - 2B(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}$$

$\Big\downarrow$ 1281

$$\left(A-\frac{Bd}{e}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)-\frac{2B(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}$$

$\Big\downarrow$ 1280

$$\left(A-\frac{Bd}{e}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \ldots}{\ldots}\right.$$
$$\frac{2B(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}$$

$\Big\downarrow$ 1292

$$\left(A-\frac{Bd}{e}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \ldots}{\ldots}\right.$$
$$\frac{2B(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e\sqrt{a+bx+cx^2}(ef-dg)}$$

$\Big\downarrow$ 1416

$$\left(A-\frac{Bd}{e}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)}{\left(cf^2-g(bf-ag)\right)(d+ex}}{\left(\frac{\sqrt{cd^2-}}{\sqrt{cf^2-}}\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx}}\right.$$

$$\frac{B\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b}{(cf^2-bgf-}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+\right.}}}{e\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd^2-bed}{(cf^2-g(bf}}}$$

input
```
Int[(A + B*x)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3140 vs. $2(803) = 1606$.

Time = 14.86 (sec) , antiderivative size = 3141, normalized size of antiderivative = 3.50

method	result	size
elliptic	Expression too large to display	3141
default	Expression too large to display	70227

input
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RETU
RNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(A*e-B*d)/((x+d/e)*(c
*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(B/e+1/e*(a*e
^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*(A*e-B*d)/(a*d*e^2*g-a*e^3*f-b*d^2*e
*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(a*e*g+b*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g
+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(A*e-B*d))*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)
*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*
(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/
2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(
x+d/e)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+
(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^
(1/2)*EllipticF(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x
-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2
*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(
d/e+1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*((b*e*g-c*d*g+c*e*f)*(A*e-B*d
)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e
*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)*(A*e-B*d)...
```

Fricas [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(B*x + A)*sqrt(e*x + d)*sqrt(g*x + f)/(c*e^
2*g*x^5 + (c*e^2*f + (2*c*d*e + b*e^2)*g)*x^4 + a*d^2*f + ((2*c*d*e + b*e^
2)*f + (c*d^2 + 2*b*d*e + a*e^2)*g)*x^3 + ((c*d^2 + 2*b*d*e + a*e^2)*f + (
b*d^2 + 2*a*d*e)*g)*x^2 + (a*d^2*g + (b*d^2 + 2*a*d*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((A + B*x)/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2))
, x)
```

Maxima [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),
 x)
```

Giac [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{A+B\,x}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)), x
)
```

Reduce [F]

$$\int \frac{A+Bx}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Bx+A}{(ex+d)^{\frac{3}{2}}\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.73 $\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 1384

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*(A*e^2+C*d^2)*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c
*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))
*g)/(e*x+d))^(1/2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f
)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-
4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^
2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e/(2*c*d-(b-(-4*a*c+
b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^
2)^(1/2))*g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*
c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)-4*((b+(-4
*a*c+b^2)^(1/2))*C*d*e-c*(-A*e^2+C*d^2))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d
*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x
+d))^(1/2)*EllipticF((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)/
(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b^
2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2)
)*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e^2/(2*c*d-(b-(-4*a*c+b^2)^(
1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1/
2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b^
2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*C*(-d*g+e*f)*(b-(-4*a*c+
b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a
*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticPi((2*c*d-(b-(-4*a*c+b^2)^(1/2...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 4412 vs. $2(1384) = 2768$.

Time = 36.36 (sec) , antiderivative size = 4412, normalized size of antiderivative = 3.19

$$\int \frac{A + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a + bx + cx^2}}\, dx = \text{Result too large to show}$$

input
```
Integrate[(A + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
(-2*(C*d^2 + A*e^2)*Sqrt[f + g*x]*(a + b*x + c*x^2))/((c*d^2 - b*d*e + a*e
^2)*(e*f - d*g)*Sqrt[d + e*x]*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[a + b*x + c
*x^2]*(((-(C*d^2) - A*e^2)*(d + e*x)^(5/2)*(c + (c*d^2)/(d + e*x)^2 - (b*d
*e)/(d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x
))*(g + (e*f)/(d + e*x) - (d*g)/(d + e*x)))/(Sqrt[((d + e*x)^2*(c*(-1 + d/
(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (a*e)/(d + e*x)))/(d + e*x)))/e^2
]*Sqrt[f + ((d + e*x)*(g - (d*g)/(d + e*x)))/e]) - ((c*d^2 - b*d*e + a*e^2
)*(-(e*f) + d*g)*(d + e*x)^(3/2)*Sqrt[(c + (c*d^2)/(d + e*x)^2 - (b*d*e)/(
d + e*x)^2 + (a*e^2)/(d + e*x)^2 - (2*c*d)/(d + e*x) + (b*e)/(d + e*x))*(g
 + (e*f)/(d + e*x) - (d*g)/(d + e*x))]*((-2*C*d*(-1/2*(2*c*d - b*e - Sqrt[
b^2*e^2 - 4*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d + e*x)^(-1))*Sqrt[(-1/2
*(2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])/(c*d^2 - b*d*e + a*e^2) + (d +
e*x)^(-1))/((2*c*d - b*e - Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e +
a*e^2)) - (2*c*d - b*e + Sqrt[b^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*
e^2)))]*Sqrt[(-(g/(-(e*f) + d*g)) + (d + e*x)^(-1))/((2*c*d - b*e - Sqrt[b
^2*e^2 - 4*a*c*e^2])/(2*(c*d^2 - b*d*e + a*e^2)) - g/(-(e*f) + d*g))]*Elli
pticF[ArcSin[Sqrt[(-2*c*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*d^2)/(d +
 e*x) - (2*b*d*e)/(d + e*x) + (2*a*e^2)/(d + e*x))/Sqrt[(b^2 - 4*a*c)*e^2]
]/Sqrt[2]], (2*Sqrt[(b^2 - 4*a*c)*e^2]*(e*f - d*g))/(-2*c*d*e*f + b*e^2*f
+ e*Sqrt[(b^2 - 4*a*c)*e^2]*f + b*d*e*g - 2*a*e^2*g - d*Sqrt[(b^2 - 4*a...
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 2154}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1281}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)+$$
$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 1280

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 1292

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 1416

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+}}{(d+ex)\sqrt{cf^2-g(bf-}}\right.}{\sqrt{a+bx+}}\right.$$

$$\int \frac{\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$\downarrow$ 2154

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+}}{(d+ex)\sqrt{cf^2-g(bf-}}\right.}{\sqrt{a+bx+}}\right.$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\left(A+\frac{Cd^2}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+}}{(d+ex)\sqrt{cf^2-g(bf-}}\right.}{\sqrt{a+bx+}}\right.$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\Big\downarrow \text{1276}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}\left(\right.}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\frac{2Cd\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+g}{\left(cf^2-g(bf-ag)\right)(d+e}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx}}\right.$$

$$\Big\downarrow \text{1280}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+g\cdots}{\left(cf^2-g(bf-ag)\right)(d+e\cdots}}{\left(\frac{\sqrt{cd\cdots}}{\sqrt{cf^2\cdots}}\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx\cdots}}\right.$$

$$\frac{4Cd(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{cx^2+bx+a}}$$

$\downarrow$ 1416

$$\frac{2Cd\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg\cdots}{(cf^2-b\cdots}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx\cdots}{\sqrt{cf^2-g(bf-ag)}(d+ex\cdots}\right.}}}{e^2\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd^2-be\cdots}{(cf^2-g(\cdots}}}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$\left(\frac{Cd^2}{e^2}+A\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+g\cdots}{\left(cf^2-g(bf-ag)\right)(d+e\cdots}}{\left(\frac{\sqrt{cd\cdots}}{\sqrt{cf^2\cdots}}\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx\cdots}}\right.$$

input
```
Int[(A + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3176 vs. $2(1243) = 2486$.

Time = 15.03 (sec) , antiderivative size = 3177, normalized size of antiderivative = 2.30

method	result	size
elliptic	Expression too large to display	3177
default	Expression too large to display	140616

input
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_RE
TURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2+C*d^2)/((x+d
/e)*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(-C*d/e
^2+1/e^2*(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*(A*e^2+C*d^2)/(a*d*e^2*
g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(a*e*g+b*e*f)/(a*d*e^2*g-
a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2+C*d^2))*(-f/g+d/e)
*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-
4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b
^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4
*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))
+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2),((1/2/c*(-b+
(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+d/e)/(1/2*(b+(-4*a
*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e+1
/e*(b*e*g-c*d*g+c*e*f)*(A*e^2+C*d^2)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*
f+c*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Timed out}$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x
)
```

output
```
Integral((A + C*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**
2)), x)
```

Maxima [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="maxima")
```

output
```
integrate((C*x^2 + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)
), x)
```

Giac [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, alg
orithm="giac")
```

output
```
integrate((C*x^2 + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+A}{\sqrt{f+g\,x}\,{\left(d+e\,x\right)}^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),
x)
```

output
```
int((A + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)),
 x)
```

Reduce [F]

$$\int \frac{A+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.74 $\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 1382

$$\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*d*(-B*e+C*d)*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/2)+2*c*
x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*
g)/(e*x+d))^(1/2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)
^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4
*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2
)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e/(2*c*d-(b-(-4*a*c+b
^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2
)^(1/2))*g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c
*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*(2*c*C*d
^2-(b+(-4*a*c+b^2)^(1/2))*e*(-B*e+2*C*d))*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-
d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*
x+d))^(1/2)*EllipticF((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*(g*x+f)^(1/2)
/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d-(b+(-4*a*c+b
^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*a*c+b^2)^(1/2
))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e^2/(2*c*d-(b-(-4*a*c+b^2)^
(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*a*c+b^2)^(1
/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b-(-4*a*c+b
^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*C*(-d*g+e*f)*(b-(-4*a*c
+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*
a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticPi((2*c*d-(b-(-4*a*c+b^2)^(1/...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 5152 vs. 2(1382) = 2764.

Time = 36.42 (sec) , antiderivative size = 5152, normalized size of antiderivative = 3.73

$$\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^
2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(B+Cx)}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 2154}$$

$$\frac{d(Cd-Be)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1281}$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\frac{d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)}{e^2}$$

$$\Big\downarrow \text{ 1280}$$

$$\frac{d(Cd-Be)\left(\frac{2g(d+ex)\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int \frac{\sqrt{f+\cdots}}{(d+ex)^{3/2}\sqrt{\cdots}}}{ef-\cdots}\cdots\right.}{e^2}$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{ 1292}$$

$$\frac{d(Cd-Be)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int\frac{1}{\sqrt{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{(cf^2-bgf+ag^2)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}+\frac{e\int\frac{\sqrt{f+}}{(d+ex)^{3/2}\sqrt{}}}{ef-}\right.}{e^2}$$

$$\int\frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 1416$$

$$d(Cd-Be)\left(\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\ldots}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\int\frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 2154$$

$$d(Cd-Be)\left(\frac{e\int\frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\ldots}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int\frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow 27$$

$$d(Cd-Be)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{\left(a+bx+cx^2\right)(ef-dg)^2}{(d+ex)^2\left(ag^2-bfg+cf^2\right)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\cdots}}{\sqrt{a+bx+cx^2}(ef-dg)^2\cdots}\right.$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$\downarrow$ 1276

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+$$

$$d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2\cdots}{(cf^2\cdots}\cdots}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+\cdots}{\sqrt{cf^2-g(bf-ag)}(d+\cdots}\cdots\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-\cdots\right.}{\left(cf^2-\cdots\right.}}}\right.$$

$\downarrow$ 1280

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(e\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2\cdots}{\left(cf^2\cdots\right.}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+\cdots}{\sqrt{cf^2-g(bf-ag)}(d+\cdots}\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-\cdots\right.}{\left(cf^2-\cdots\right.}}}\right.$$

$$\frac{2(2Cd-Be)(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{cx^2+bx+a}}$$

$\big\downarrow$ 1416

$$\frac{(2Cd-Be)\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\cdots}{\left(\frac{\sqrt{cd^2-bed\cdots}}{\sqrt{cf^2-g(bf\cdots}}\right.}}}{e^2\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(c\cdots}{(cf\cdots}}}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(e\cdots}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e\cdots}}$$

$$d(Cd-Be)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2\left(cx^2+bx+a\right)}{\left(cf^2-bgf+ag^2\right)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2\cdots}{\left(cf^2\cdots\right.}}{\left(\frac{\sqrt{cd^2-bed+ae^2}(f+\cdots}{\sqrt{cf^2-g(bf-ag)}(d+\cdots}\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{cx^2+bx+a}\sqrt{\frac{\left(cd^2-\cdots\right.}{\left(cf^2-\cdots\right.}}}\right.$$

input
```
Int[(B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3169 vs. 2(1241) = 2482.

Time = 15.10 (sec) , antiderivative size = 3170, normalized size of antiderivative = 2.29

method	result	size
elliptic	Expression too large to display	3170
default	Expression too large to display	140756

input
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(-2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d
*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*d*(B*e-C*d)/((x+d/
e)*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*((B*e-C*
d)/e^2-1/e^2*(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*d*(B*e-C*d)/(a*d*e^
2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)+(a*e*g+b*e*f)/(a*d*e^2*
g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*d*(B*e-C*d))*(-f/g+d/e)
*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*
a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*((1/2/c*(-b+(-
4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)/(-1/2*(b+(-4*a*c+b
^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*((1/2/c*(-b+(-4
*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*(-b+(-4*a*c+b^2)^(1/2))
+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)
))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2),((1/2/c*(-b+
(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(-f/g+d/e)/(1/2*(b+(-4*a
*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2))+2*(C/e-1
/e*(b*e*g-c*d*g+c*e*f)*d*(B*e-C*d)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+
c*d^3*g-c*d^2*e*f)+(2*b*e*g+2*c*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e...
```

Fricas [F(-1)]

Timed out.

$$\int \frac{Bx + Cx^2}{(d + ex)^{3/2}\sqrt{f + gx}\sqrt{a + bx + cx^2}}\, dx = \text{Timed out}$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, a
lgorithm="fricas")
```

output
```
Timed out
```

Sympy [F]

$$\int \frac{Bx + Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{x(B+Cx)}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2)
,x)
```

output
```
Integral(x*(B + C*x)/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2
)), x)
```

Maxima [F]

$$\int \frac{Bx + Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, a
lgorithm="maxima")
```

output
```
integrate((C*x^2 + B*x)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x +
f)), x)
```

Giac [F]

$$\int \frac{Bx + Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x, a
lgorithm="giac")
```

output
```
integrate((C*x^2 + B*x)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x +
f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{Bx + Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2 + B\,x}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
),x)
```

output
```
int((B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(1/2)
), x)
```

Reduce [F]

$$\int \frac{Bx + Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{C\,x^2 + Bx}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

3.75 $\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 1396

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Too large to display}$$

output
```
2*(A*e^2-B*d*e+C*d^2)*(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)*(b-(-4*a*c+b^2)^(1/
2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^2)^
(1/2))*g)/(e*x+d))^(1/2)*EllipticE((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)*
(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*c*d
-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(-4*
a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e/(2*c*d-(b-(-
4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b-(-4*
a*c+b^2)^(1/2))*g)^(1/2)/(-d*g+e*f)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*
x)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)-2*(
(b+(-4*a*c+b^2)^(1/2))*e*(-B*e+2*C*d)-2*c*(-A*e^2+C*d^2))*(b-(-4*a*c+b^2)^
(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/(2*c*f-(b+(-4*a*c+b^
2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticF((2*c*d-(b-(-4*a*c+b^2)^(1/2))*e)^(1/
2)*(g*x+f)^(1/2)/(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)^(1/2)/(e*x+d)^(1/2),((2*
c*d-(b+(-4*a*c+b^2)^(1/2))*e)*(2*c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(2*c*d-(b-(
-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g))^(1/2))/e^2/(2*c*d-
(b-(-4*a*c+b^2)^(1/2))*e)^(1/2)/(2*c*d-(b+(-4*a*c+b^2)^(1/2))*e)/(2*c*f-(b
-(-4*a*c+b^2)^(1/2))*g)^(1/2)/((-d*g+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/(2*
c*f-(b-(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)/(c*x^2+b*x+a)^(1/2)+2*C*(-d*g
+e*f)*(b-(-4*a*c+b^2)^(1/2)+2*c*x)*((-d*g+e*f)*(b+(-4*a*c+b^2)^(1/2)+2*c*x
)/(2*c*f-(b+(-4*a*c+b^2)^(1/2))*g)/(e*x+d))^(1/2)*EllipticPi((2*c*d-(b-...
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 6336 vs. $2(1396) = 2792$.

Time = 6.67 (sec) , antiderivative size = 6336, normalized size of antiderivative = 4.54

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + 
c*x^2]),x]
```

output
```
Result too large to show
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1281}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}-\frac{g\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{ef-dg}\right)+\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1280}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1292}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{2g(d+ex)\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\int \frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{\sqrt{a+bx+cx^2}(ef-dg)^2}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow \text{1416}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{}}{(d+ex)\sqrt{c}}\right.}{\sqrt{a}}\right.$$

$$\int \frac{\frac{B}{e}+\frac{Cx}{e}-\frac{Cd}{e^2}}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow \text{2154}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{}}{(d+ex)\sqrt{c}}\right.}{\sqrt{a}}\right.$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\int \frac{C\sqrt{d+ex}}{e^2\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx$$

$$\downarrow \text{27}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{e\int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}\sqrt{cx^2+bx+a}}dx}{ef-dg}+\frac{g(d+ex)\sqrt[4]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{}}{(d+ex)\sqrt{c}}\right.}{\sqrt{a}}\right.$$

$$\frac{(2Cd-Be)\int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+\frac{C\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}$$

$$\downarrow \text{1276}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\frac{(2Cd-Be)\int\frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{cx^2+bx+a}}dx}{e^2}+$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed-\right.}{\left(cf^2-g(bf\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c}}\right.$$

$\Big\downarrow$ 1280

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\left(cd^2-bed-\right.}{\left(cf^2-g(bf\right.}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c}}\right.$$

$$\frac{2(2Cd-Be)(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\int\frac{1}{\sqrt{\frac{\left(cd^2-bed+ae^2\right)(f+gx)^2}{\left(cf^2-g(bf-ag)\right)(d+ex)^2}-\frac{(2cdf+2aeg-b(ef+dg))(f+gx)}{\left(cf^2-bgf+ag^2\right)(d+ex)}+1}}d\frac{\sqrt{f+gx}}{\sqrt{d+ex}}}{e^2(ef-dg)\sqrt{cx^2+bx+a}}$$

$\Big\downarrow$ 1416

$$\frac{(2Cd-Be)\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{\frac{(cd^2-bed+ae^2)(f+gx)^2}{(cf^2-g(bf-ag))(d+ex)^2}-}{\left(\frac{\sqrt{cd^2-bed}}{\sqrt{cf^2-g(bf}}\right.}}}{e^2\sqrt[4]{cd^2-bed+ae^2}(ef-dg)\sqrt{cx^2+bx+a}\sqrt{\frac{(cd}{(cf}}}$$

$$\frac{\sqrt{2}C\sqrt{2cf-\left(b+\sqrt{b^2-4ac}\right)g}\sqrt{b+2cx-\sqrt{b^2-4ac}}\sqrt{\frac{(ef-dg)\left(b+2cx+\sqrt{b^2-4ac}\right)}{\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)(d+ex)}}\sqrt{\frac{(ef-dg)\left(2a+\left(b+\sqrt{b^2-4ac}\right)x\right)}{\left(bf+\sqrt{b^2-4ac}f-2ag\right)(d+ex)}}(}{e^2\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

$$\left(A+\frac{d(Cd-Be)}{e^2}\right)\left(\frac{g\sqrt[4]{cf^2-g(bf-ag)}(d+ex)\sqrt{\frac{(ef-dg)^2(cx^2+bx+a)}{(cf^2-bgf+ag^2)(d+ex)^2}}\left(\frac{\sqrt{cd^2-bed+ae^2}(f+gx)}{\sqrt{cf^2-g(bf-ag)}(d+ex)}+1\right)\sqrt{\frac{(cd^2-bed}{(cf^2-g(bf}}}{\sqrt[4]{cd^2-bed+ae^2}(ef-dg)^2\sqrt{c}}\right.$$

input
```
Int[(A + B*x + C*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]
),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1276
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(
x_) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[Sqrt
[2]*Sqrt[2*c*f - g*(b + q)]*Sqrt[b - q + 2*c*x]*(d + e*x)*Sqrt[(e*f - d*g)*
((b + q + 2*c*x)/((2*c*f - g*(b + q))*(d + e*x)))]*(Sqrt[(e*f - d*g)*((2*a
+ (b + q)*x)/((b*f + q*f - 2*a*g)*(d + e*x)))]/(g*Sqrt[2*c*d - e*(b + q)]*S
qrt[2*a*(c/(b + q)) + c*x]*Sqrt[a + b*x + c*x^2]))*EllipticPi[e*((2*c*f - g
*(b + q))/(g*(2*c*d - e*(b + q)))), ArcSin[Sqrt[2*c*d - e*(b + q)]*(Sqrt[f
+ g*x]/(Sqrt[2*c*f - g*(b + q)]*Sqrt[d + e*x]))], (b*d + q*d - 2*a*e)*((2*c
*f - g*(b + q))/((b*f + q*f - 2*a*g)*(2*c*d - e*(b + q))))], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x]
```

rule 1280
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-2*(d + e*x)*(Sqrt[(e*f - d*g)^2*
((a + b*x + c*x^2)/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2))]/((e*f - d*g)*Sqr
t[a + b*x + c*x^2]))   Subst[Int[1/Sqrt[1 - (2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*(x^2/(c*f^2 - b*f*g + a*g^2)) + (c*d^2 - b*d*e + a*e^2)*(x^4/(c*f^2 -
b*f*g + a*g^2))], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

rule 1281
```
Int[1/(((d_.) + (e_.)*(x_))^(3/2)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_
.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[-g/(e*f - d*g)   Int[1/(Sqrt[d
+ e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] + Simp[e/(e*f - d*g)
Int[Sqrt[f + g*x]/((d + e*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[
{a, b, c, d, e, f, g}, x]
```

rule 1292
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Unintegrable[(d + e*x)^m*(f + g*x)^n*
(a + b*x + c*x^2)^p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 1416
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

rule 2154
```
Int[(Px_)*((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b
_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[PolynomialQuotient[Px, d +
 e*x, x]*(d + e*x)^(m + 1)*(f + g*x)^n*(a + b*x + c*x^2)^p, x] + Simp[Polyn
omialRemainder[Px, d + e*x, x]   Int[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x
^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && PolynomialQ[Px, x
] && LtQ[m, 0] &&  !IntegerQ[n] && IntegersQ[2*m, 2*n, 2*p]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3211 vs. 2(1255) = 2510.

Time = 15.45 (sec) , antiderivative size = 3212, normalized size of antiderivative = 2.30

method	result	size
elliptic	Expression too large to display	3212
default	Expression too large to display	176190

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,method
=_RETURNVERBOSE)
```

output
```
((g*x+f)*(c*x^2+b*x+a)*(e*x+d))^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2)/(e
*x+d)^(1/2)*(2*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f)/(a*d*
e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2-B*d*e+C*d^2)
/((x+d/e)*(c*e*g*x^3+b*e*g*x^2+c*e*f*x^2+a*e*g*x+b*e*f*x+a*e*f))^(1/2)+2*(
(B*e-C*d)/e^2+1/e^2*(a*e^2*g-b*d*e*g+b*e^2*f+c*d^2*g-c*d*e*f)*(A*e^2-B*d*e
+C*d^2)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(a*e*g+b
*e*f)/(a*d*e^2*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)/e*(A*e^2-B
*d*e+C*d^2))*(-f/g+d/e)*((-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g
-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)*(x-1/2/c*(-b+(-4*a*c+b^2)^(
1/2)))^2*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+1/2*(b+(-4*a*c+b^2)^(1/2)
)/c)/(-1/2*(b+(-4*a*c+b^2)^(1/2))/c+f/g)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))
)^(1/2)*((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+f/g)*(x+d/e)/(f/g-d/e)/(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2))))^(1/2)/(-d/e-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))/(1/2/c*
(-b+(-4*a*c+b^2)^(1/2))+f/g)/(c*e*g*(x+f/g)*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(x+d/e))^(1/2)*EllipticF(((-d/e-1/2/c
*(-b+(-4*a*c+b^2)^(1/2)))*(x+f/g)/(f/g-d/e)/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
))))^(1/2),((1/2/c*(-b+(-4*a*c+b^2)^(1/2))+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*(
-f/g+d/e)/(1/2*(b+(-4*a*c+b^2)^(1/2))/c-f/g)/(d/e+1/2/c*(-b+(-4*a*c+b^2)^(
1/2))))^(1/2))+2*(C/e+1/e*(b*e*g-c*d*g+c*e*f)*(A*e^2-B*d*e+C*d^2)/(a*d*e^2
*g-a*e^3*f-b*d^2*e*g+b*d*e^2*f+c*d^3*g-c*d^2*e*f)-(2*b*e*g+2*c*e*f)/(a*...
```

Fricas [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*sqrt(c*x^2 + b*x + a)*sqrt(e*x + d)*sqrt(g*x +
f)/(c*e^2*g*x^5 + (c*e^2*f + (2*c*d*e + b*e^2)*g)*x^4 + a*d^2*f + ((2*c*d*
e + b*e^2)*f + (c*d^2 + 2*b*d*e + a*e^2)*g)*x^3 + ((c*d^2 + 2*b*d*e + a*e^
2)*f + (b*d^2 + 2*a*d*e)*g)*x^2 + (a*d^2*g + (b*d^2 + 2*a*d*e)*f)*x), x)
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{A+Bx+Cx^2}{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(e*x+d)**(3/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/
2),x)
```

output
```
Integral((A + B*x + C*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)*sqrt(a + b*x +
 c*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*
x + f)), x)
```

Giac [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx = \int \frac{Cx^2+Bx+A}{\sqrt{cx^2+bx+a}(ex+d)^{\frac{3}{2}}\sqrt{gx+f}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x,
 algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*
x + f)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+B\,x+A}{\sqrt{f+g\,x}\,(d+e\,x)^{3/2}\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)),x)
```

output
```
int((A + B*x + C*x^2)/((f + g*x)^(1/2)*(d + e*x)^(3/2)*(a + b*x + c*x^2)^(
1/2)), x)
```

Reduce [F]

$$\int \frac{A+Bx+Cx^2}{(d+ex)^{3/2}\sqrt{f+gx}\sqrt{a+bx+cx^2}}\,dx=\int \frac{C\,x^2+Bx+A}{(ex+d)^{\frac{3}{2}}\,\sqrt{gx+f}\,\sqrt{c\,x^2+bx+a}}dx$$

input
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
int((C*x^2+B*x+A)/(e*x+d)^(3/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:  #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file