Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.2-Trinomial/1.2.1-Quadratic-trinomial/113-1.2.1.9

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**32**]. This is test number [113].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	90.62 (29)	9.38 (3)
Mathematica	90.62 (29)	9.38 (3)
Maple	78.12 (25)	21.88 (7)
Fricas	78.12 (25)	21.88 (7)
Giac	71.88 (23)	28.12 (9)
Reduce	59.38 (19)	40.62 (13)
Maxima	53.12 (17)	46.88 (15)
Sympy	31.25 (10)	68.75 (22)
Mupad	12.50 (4)	87.50 (28)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

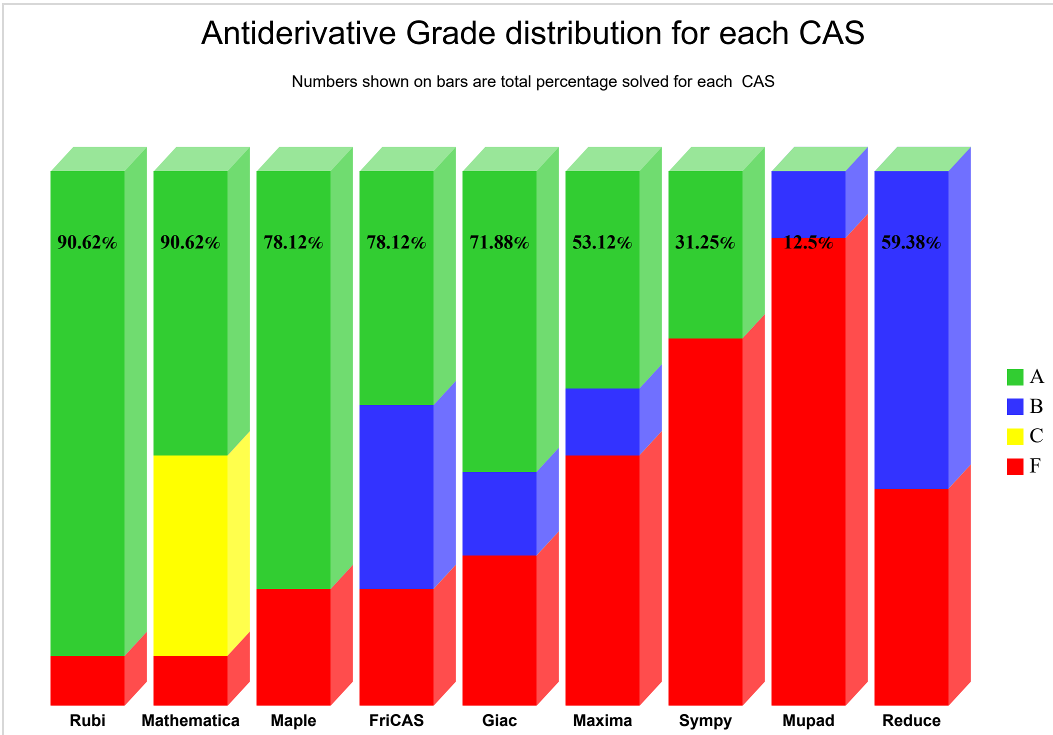
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

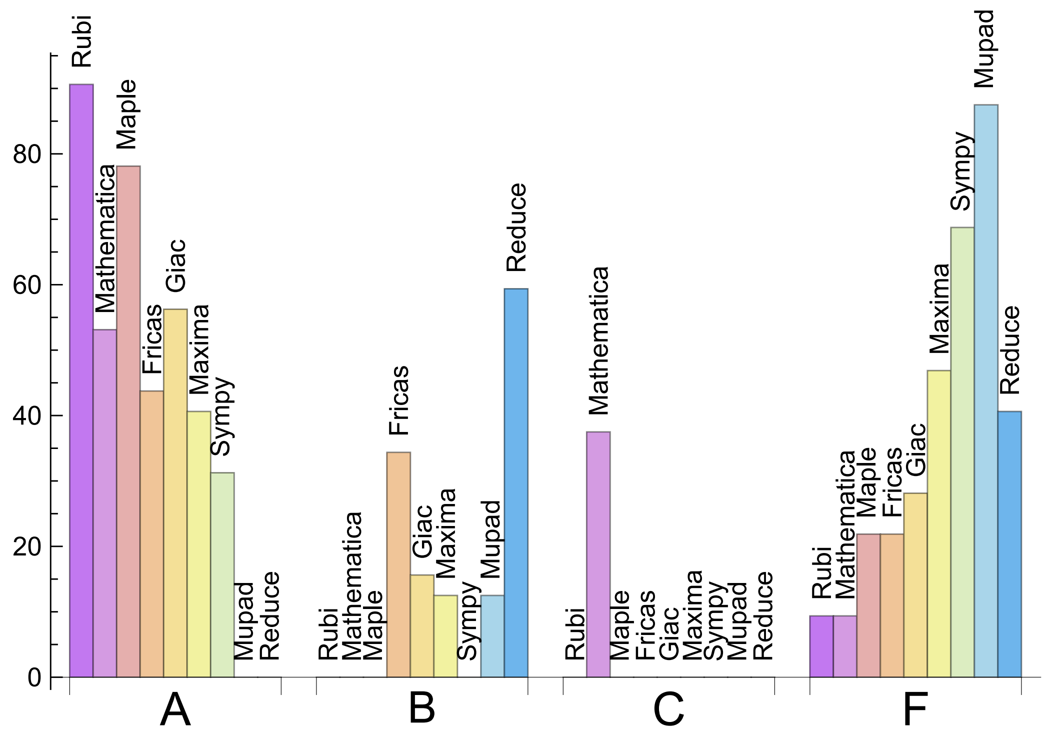
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.625	0.000	0.000	9.375
Maple	78.125	0.000	0.000	21.875
Giac	56.250	15.625	0.000	28.125
Mathematica	53.125	0.000	37.500	9.375
Fricas	43.750	34.375	0.000	21.875
Maxima	40.625	12.500	0.000	46.875
Sympy	31.250	0.000	0.000	68.750
Mupad	0.000	12.500	0.000	87.500
Reduce	0.000	59.375	0.000	40.625

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	3	100.00	0.00	0.00
Mathematica	3	100.00	0.00	0.00
Fricas	7	100.00	0.00	0.00
Maple	7	100.00	0.00	0.00
Giac	9	77.78	0.00	22.22
Reduce	13	100.00	0.00	0.00
Maxima	15	100.00	0.00	0.00
Sympy	22	95.45	4.55	0.00
Mupad	28	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Maxima	0.12
Giac	0.15
Sympy	0.41
Maple	0.73
Mathematica	0.85
Rubi	1.01
Mupad	14.53
Reduce	21.51

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	81.60	0.58	85.00	0.52
Maple	141.52	0.76	85.00	0.85
Giac	159.13	0.86	92.00	0.63
Mupad	162.25	1.33	170.00	1.18
Rubi	211.21	1.09	215.00	1.11
Fricas	213.92	1.14	112.00	1.12
Maxima	232.00	1.38	148.00	0.89
Mathematica	239.83	1.16	184.00	0.84
Reduce	753.11	3.79	185.00	1.04

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

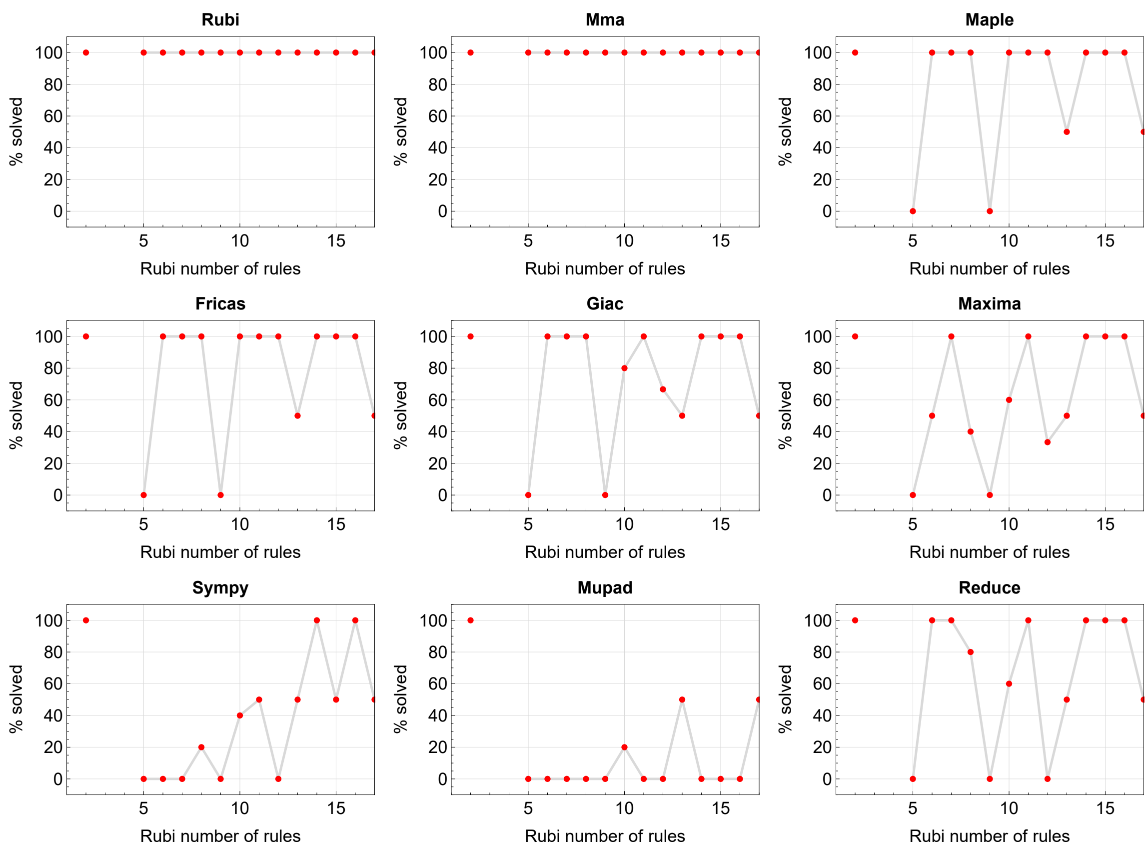


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

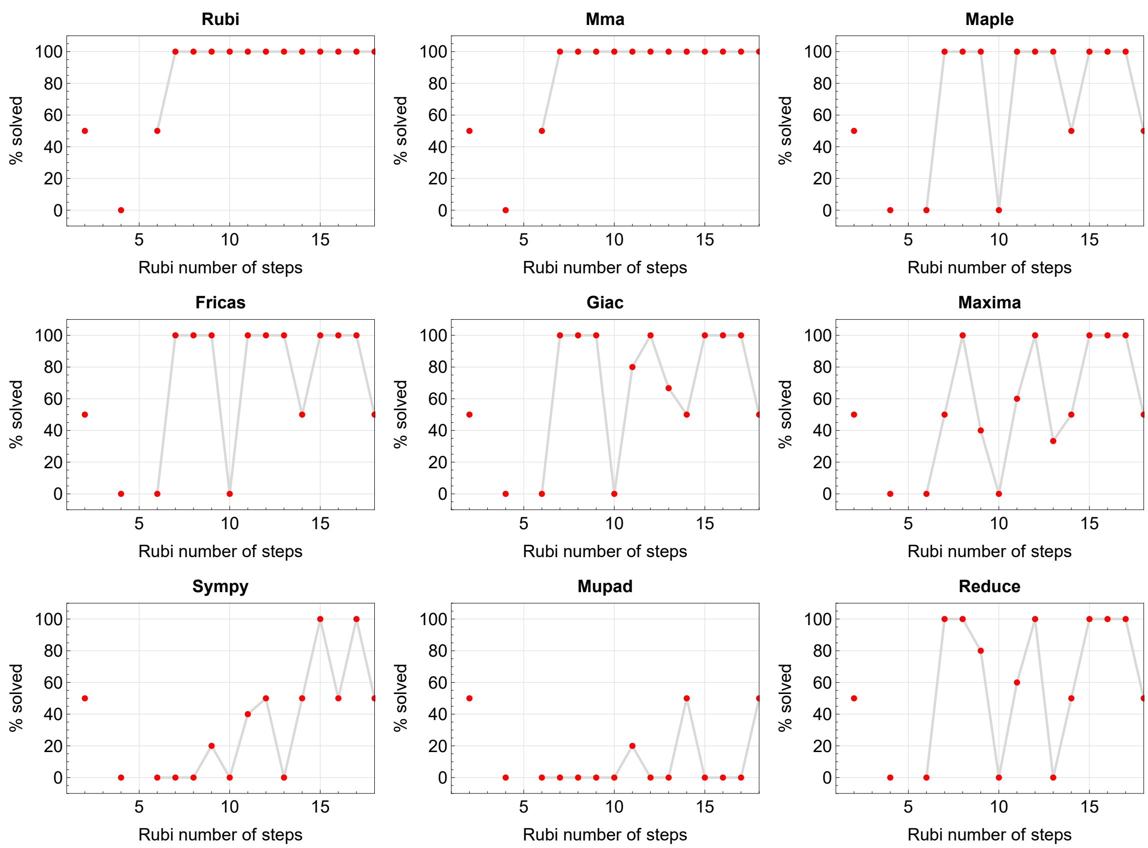


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

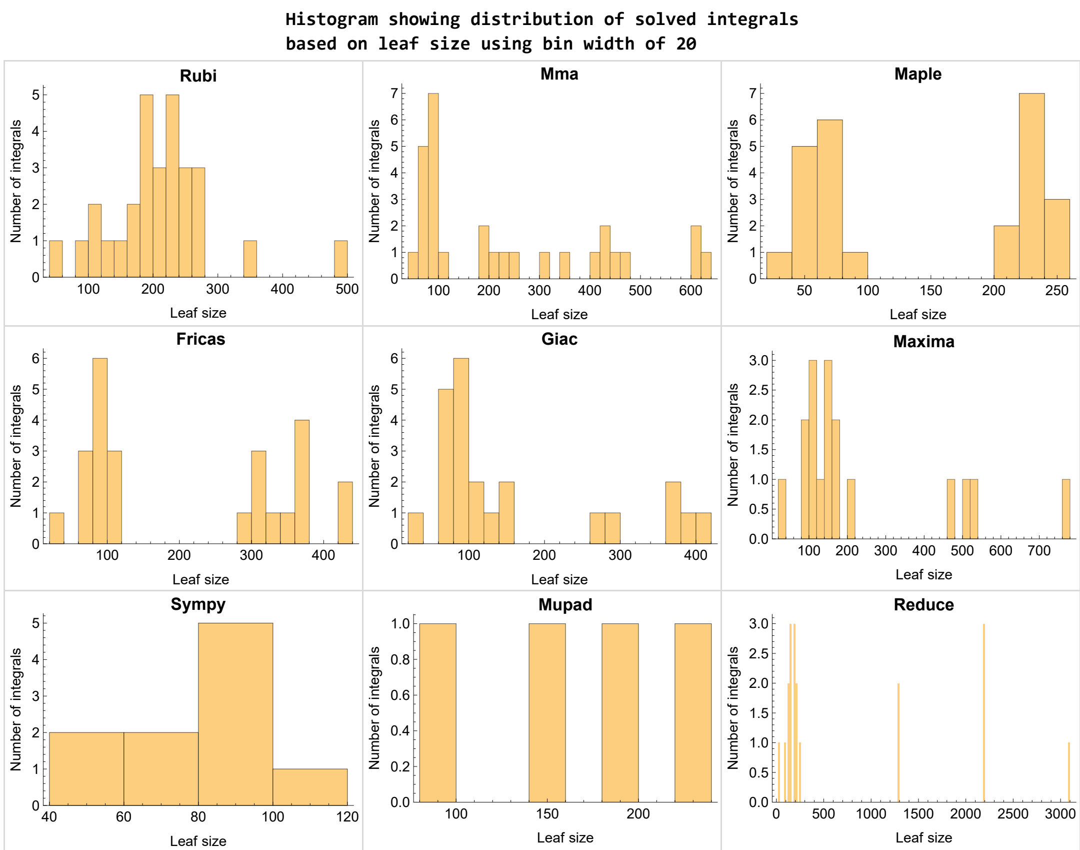


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

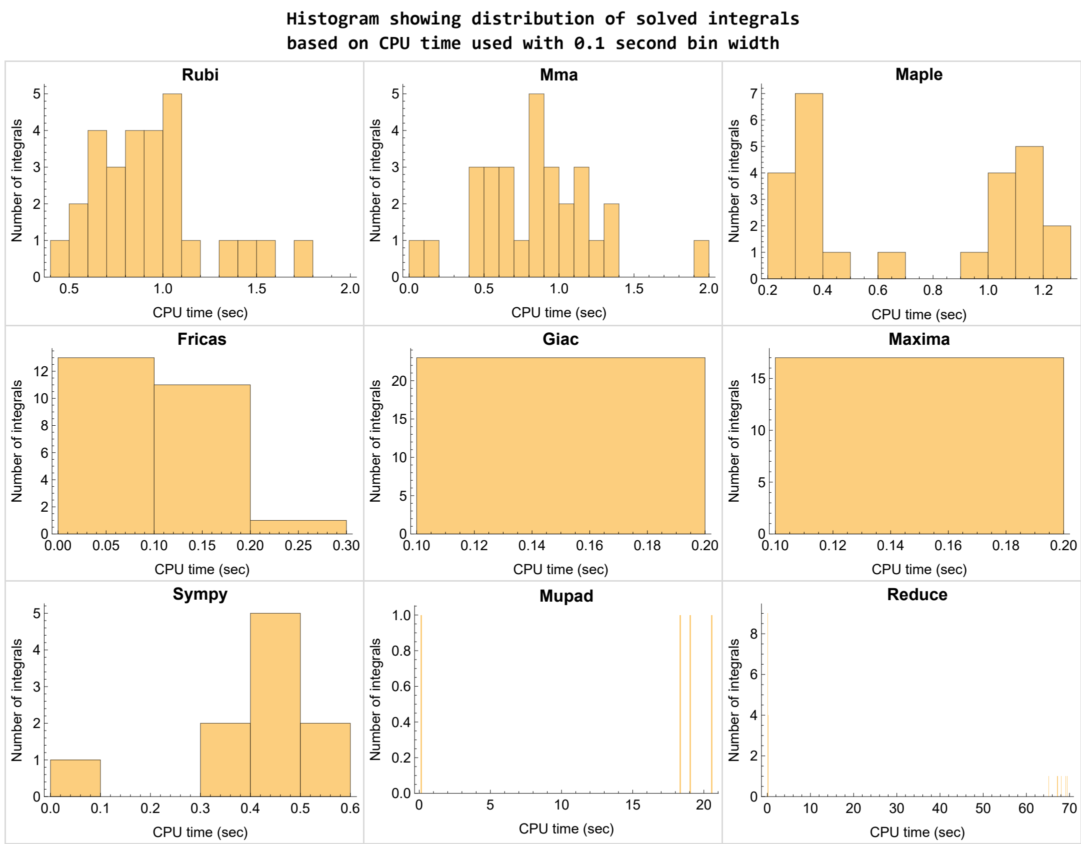


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

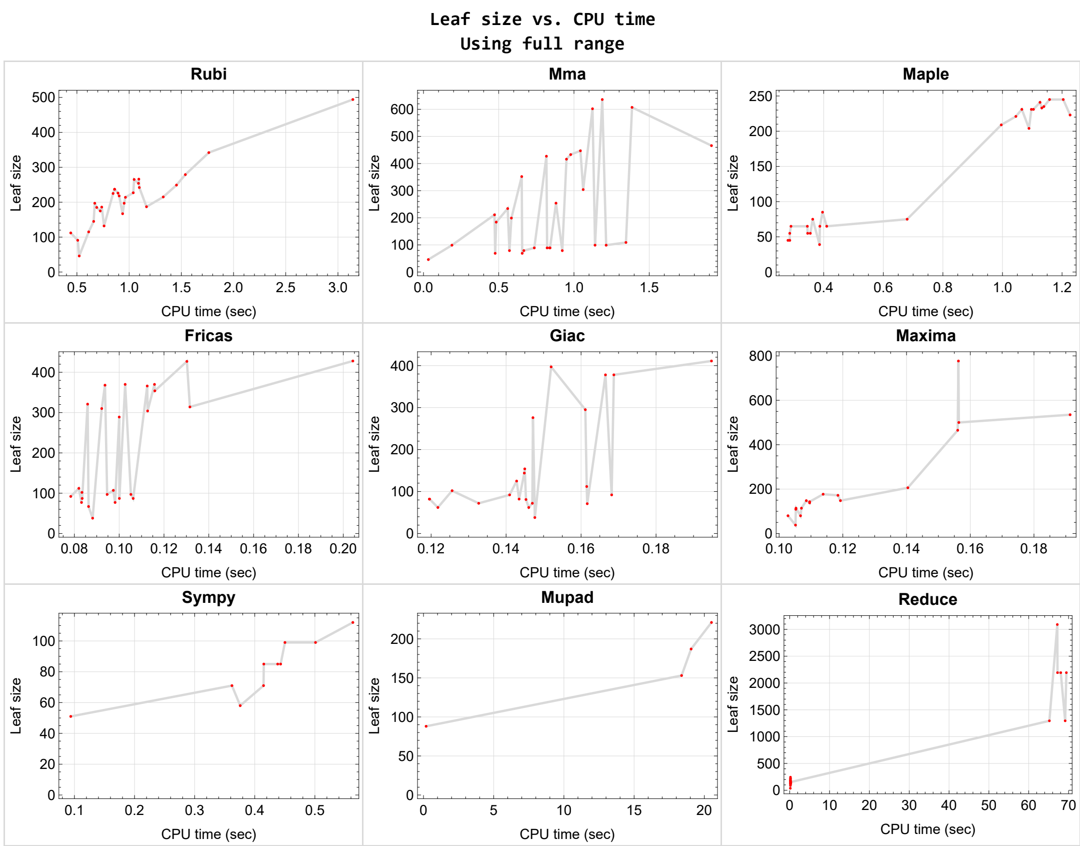


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

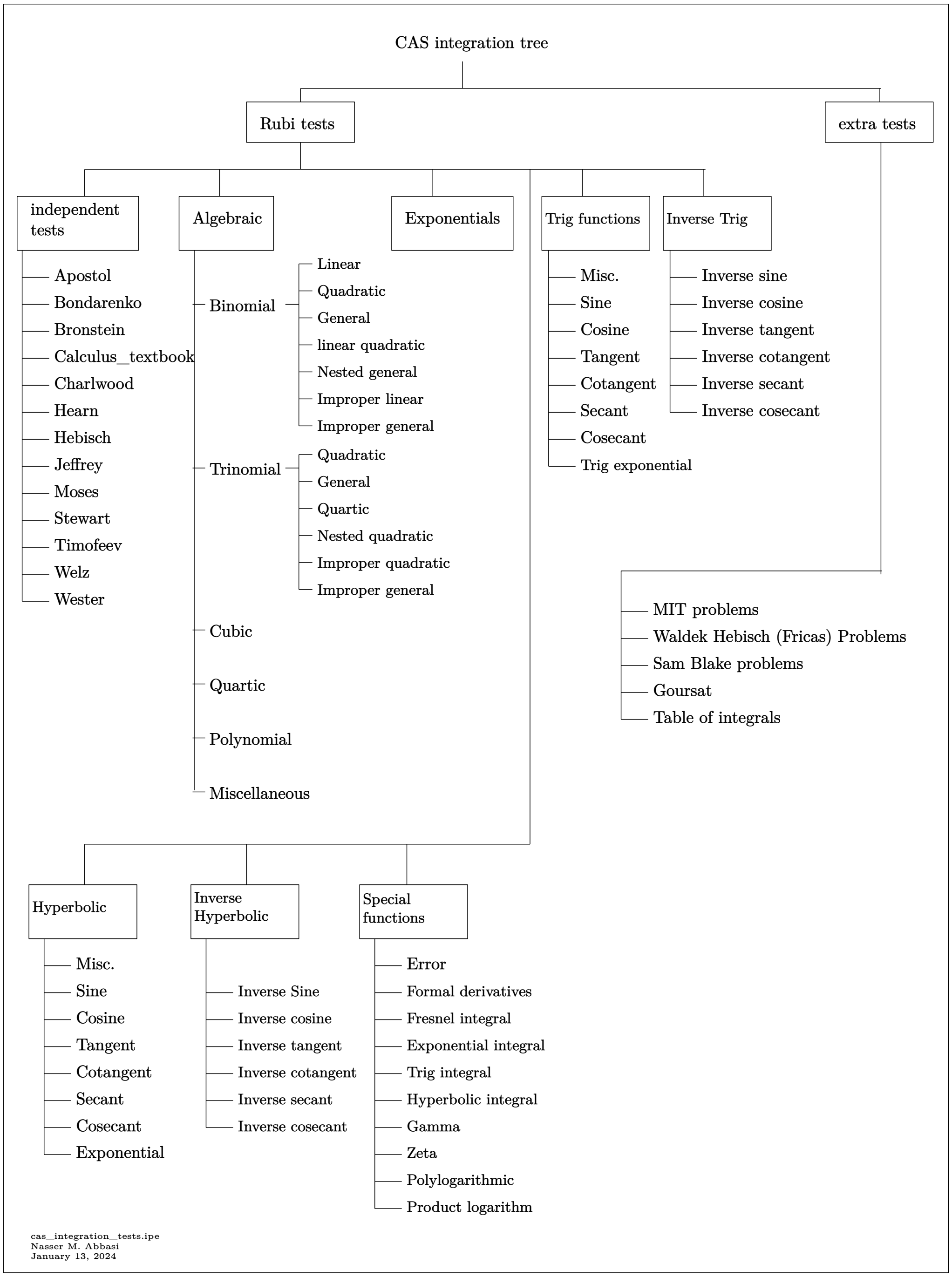
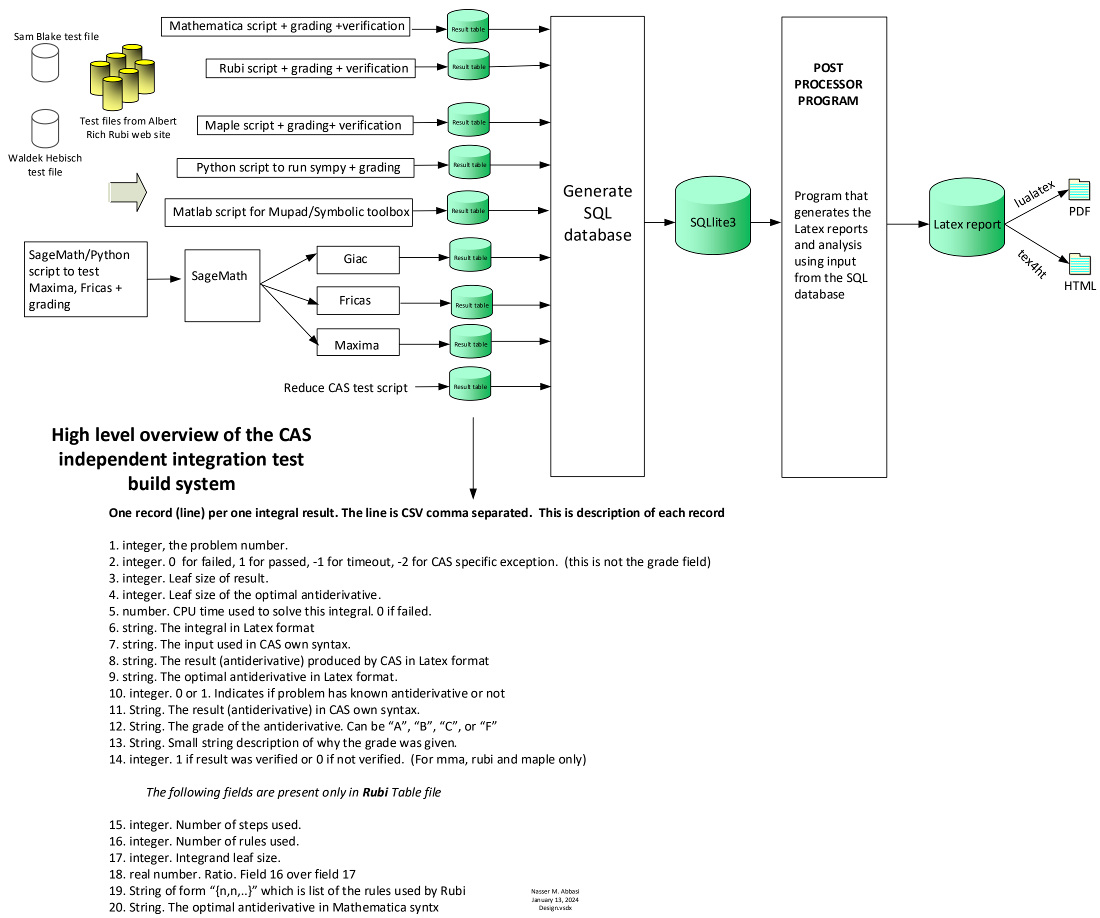


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 }

B grade { }

C grade { }

F normal fail { 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 8, 9, 10, 14, 15, 16, 20, 21, 22, 26, 27, 28, 29 }

B grade { }

C grade { 5, 6, 7, 11, 12, 13, 17, 18, 19, 23, 24, 25 }

F normal fail { 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25 }

B grade { }

C grade { }

F normal fail { 26, 27, 28, 29, 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 8, 9, 10, 11, 14, 15, 16, 20, 21, 22 }

B grade { 5, 6, 7, 12, 13, 17, 18, 19, 23, 24, 25 }

C grade { }

F normal fail { 26, 27, 28, 29, 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 8, 9, 10, 14, 15, 16, 20, 21, 22 }

B grade { 5, 11, 17, 23 }

C grade { }

F normal fail { 6, 7, 12, 13, 18, 19, 24, 25, 26, 27, 28, 29, 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade $\{ 1, 2, 3, 4, 5, 8, 9, 10, 11, 14, 15, 16, 17, 20, 21, 22, 23, 24 \}$

B grade $\{ 7, 13, 18, 19, 25 \}$

C grade $\{ \}$

F normal fail $\{ 26, 27, 28, 29, 30, 31, 32 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ 6, 12 \}$

Mupad

A grade $\{ \}$

B grade $\{ 1, 2, 3, 4 \}$

C grade $\{ \}$

F normal fail $\{ \}$

F(-1) timedout fail $\{ 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32 \}$

F(-2) exception fail $\{ \}$

Sympy

A grade $\{ 1, 2, 3, 4, 8, 9, 10, 14, 15, 16 \}$

B grade $\{ \}$

C grade $\{ \}$

F normal fail $\{ 5, 6, 7, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31 \}$

F(-1) timedout fail $\{ 32 \}$

F(-2) exception fail $\{ \}$

Reduce

A grade { }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25 }

C grade { }

F normal fail { 5, 6, 11, 12, 13, 17, 26, 27, 28, 29, 30, 31, 32 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	39	38	38	51	38	36	88
N.S.	1	1.00	1.00	0.85	0.83	0.83	1.11	0.83	0.78	1.91
time (sec)	N/A	0.522	0.032	0.386	0.105	0.088	0.094	0.148	0.154	0.187

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	249	99	75	177	97	99	92	185	221
N.S.	1	1.20	0.48	0.36	0.85	0.47	0.48	0.44	0.89	1.06
time (sec)	N/A	1.454	1.214	0.680	0.114	0.105	0.450	0.141	0.172	20.510

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	197	89	65	143	87	85	82	153	187
N.S.	1	1.19	0.54	0.39	0.86	0.52	0.51	0.49	0.92	1.13
time (sec)	N/A	0.952	0.841	0.345	0.110	0.106	0.415	0.143	0.154	19.059

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	145	79	55	109	77	71	72	121	153
N.S.	1	1.17	0.64	0.44	0.88	0.62	0.57	0.58	0.98	1.23
time (sec)	N/A	0.660	0.571	0.286	0.105	0.098	0.362	0.147	0.146	18.381

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	214	234	223	500	304	0	144	35	0
N.S.	1	1.06	1.16	1.11	2.49	1.51	0.00	0.72	0.17	0.00
time (sec)	N/A	0.965	0.560	1.227	0.156	0.113	0.000	0.145	200.042	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	226	427	235	0	354	0	0	35	0
N.S.	1	1.14	2.15	1.18	0.00	1.78	0.00	0.00	0.18	0.00
time (sec)	N/A	0.893	0.817	1.139	0.000	0.116	0.000	0.000	200.043	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	237	602	231	0	366	0	378	2193	0
N.S.	1	1.11	2.83	1.08	0.00	1.72	0.00	1.77	10.30	0.00
time (sec)	N/A	0.861	1.123	1.104	0.000	0.113	0.000	0.169	69.451	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	279	109	85	206	107	112	102	217	0
N.S.	1	1.21	0.47	0.37	0.89	0.46	0.48	0.44	0.94	0.00
time (sec)	N/A	1.539	1.345	0.396	0.140	0.097	0.563	0.126	0.212	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	227	99	75	172	97	99	92	185	0
N.S.	1	1.20	0.52	0.40	0.91	0.51	0.52	0.49	0.98	0.00
time (sec)	N/A	1.040	1.140	0.363	0.118	0.095	0.501	0.168	0.203	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	175	89	65	138	87	85	82	153	0
N.S.	1	1.19	0.61	0.44	0.94	0.59	0.58	0.56	1.04	0.00
time (sec)	N/A	0.723	0.838	0.290	0.110	0.100	0.438	0.120	0.196	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	242	254	233	535	314	0	154	35	0
N.S.	1	1.00	1.05	0.96	2.20	1.29	0.00	0.63	0.14	0.00
time (sec)	N/A	1.098	0.881	1.132	0.191	0.132	0.000	0.145	200.023	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	254	447	245	0	370	0	0	35	0
N.S.	1	1.08	1.89	1.04	0.00	1.57	0.00	0.00	0.15	0.00
time (sec)	N/A	1.089	1.043	1.158	0.000	0.116	0.000	0.000	200.027	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	266	636	245	0	427	0	411	35	0
N.S.	1	1.14	2.72	1.05	0.00	1.82	0.00	1.76	0.15	0.00
time (sec)	N/A	1.093	1.188	1.204	0.000	0.130	0.000	0.195	200.032	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	215	89	65	148	87	85	82	153	0
N.S.	1	1.16	0.48	0.35	0.80	0.47	0.46	0.44	0.83	0.00
time (sec)	N/A	1.327	0.822	0.387	0.109	0.083	0.443	0.120	0.291	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	167	79	55	114	77	71	72	121	0
N.S.	1	1.17	0.55	0.38	0.80	0.54	0.50	0.50	0.85	0.00
time (sec)	N/A	0.938	0.667	0.346	0.105	0.083	0.415	0.133	0.277	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	115	69	45	80	67	58	62	89	0
N.S.	1	1.14	0.68	0.45	0.79	0.66	0.57	0.61	0.88	0.00
time (sec)	N/A	0.613	0.476	0.280	0.107	0.086	0.376	0.122	0.189	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	186	211	204	465	289	0	125	35	0
N.S.	1	1.13	1.29	1.24	2.84	1.76	0.00	0.76	0.21	0.00
time (sec)	N/A	0.738	0.472	1.089	0.156	0.100	0.000	0.143	200.029	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	197	352	221	0	310	0	276	1295	0
N.S.	1	1.11	1.98	1.24	0.00	1.74	0.00	1.55	7.28	0.00
time (sec)	N/A	0.671	0.653	1.045	0.000	0.092	0.000	0.147	65.186	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	237	433	231	0	370	0	378	2193	0
N.S.	1	1.04	1.91	1.02	0.00	1.63	0.00	1.67	9.66	0.00
time (sec)	N/A	0.861	0.978	1.097	0.000	0.103	0.000	0.167	68.040	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	187	89	65	148	112	0	81	244	0
N.S.	1	1.13	0.54	0.39	0.89	0.67	0.00	0.49	1.47	0.00
time (sec)	N/A	1.166	0.736	0.410	0.119	0.082	0.000	0.145	0.193	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	132	79	55	114	102	0	71	212	0
N.S.	1	1.06	0.64	0.44	0.92	0.82	0.00	0.57	1.71	0.00
time (sec)	N/A	0.760	0.922	0.355	0.107	0.083	0.000	0.162	0.183	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	91	69	45	80	92	0	62	180	0
N.S.	1	1.11	0.84	0.55	0.98	1.12	0.00	0.76	2.20	0.00
time (sec)	N/A	0.508	0.656	0.286	0.103	0.078	0.000	0.146	0.181	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	185	199	209	777	321	0	112	1295	0
N.S.	1	1.11	1.20	1.26	4.68	1.93	0.00	0.67	7.80	0.00
time (sec)	N/A	0.690	0.584	0.996	0.156	0.086	0.000	0.162	69.145	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	225	416	231	0	368	0	295	2193	0
N.S.	1	1.05	1.93	1.07	0.00	1.71	0.00	1.37	10.20	0.00
time (sec)	N/A	0.846	0.950	1.065	0.000	0.094	0.000	0.161	67.225	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	265	607	241	0	428	0	397	3091	0
N.S.	1	1.06	2.43	0.96	0.00	1.71	0.00	1.59	12.36	0.00
time (sec)	N/A	1.048	1.386	1.126	0.000	0.204	0.000	0.152	67.177	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	612	494	466	0	0	0	0	0	0	0
N.S.	1	0.81	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.143	1.914	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	402	342	304	0	0	0	0	0	0	0
N.S.	1	0.85	0.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.763	1.061	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	218	184	0	0	0	0	0	1244	0
N.S.	1	0.92	0.78	0.00	0.00	0.00	0.00	0.00	5.27	0.00
time (sec)	N/A	0.904	0.483	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	112	99	0	0	0	0	0	547	0
N.S.	1	0.98	0.87	0.00	0.00	0.00	0.00	0.00	4.80	0.00
time (sec)	N/A	0.441	0.189	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	396	0	0	0	0	0	0	0	667	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.68	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	448	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	515	0	0	0	0	0	0	0	0	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.256	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [2] had the largest ratio of [.485713999999999979]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	29	0.069
2	A	18	17	1.20	35	0.486
3	A	14	13	1.19	35	0.371
4	A	11	10	1.17	33	0.303
5	A	11	10	1.06	35	0.286
6	A	11	10	1.14	35	0.286
7	A	9	8	1.11	35	0.229
8	A	17	16	1.21	35	0.457
9	A	15	14	1.20	35	0.400
10	A	11	10	1.19	33	0.303
11	A	13	12	1.00	35	0.343
12	A	13	12	1.08	35	0.343
13	A	13	12	1.14	35	0.343
14	A	16	15	1.16	35	0.429
15	A	12	11	1.17	35	0.314
16	A	9	8	1.14	33	0.242
17	A	9	8	1.13	35	0.229
18	A	7	6	1.11	35	0.171
19	A	9	8	1.04	35	0.229
20	A	16	15	1.13	35	0.429
21	A	12	11	1.06	35	0.314

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	7	1.11	33	0.212
23	A	7	6	1.11	35	0.171
24	A	9	8	1.05	35	0.229
25	A	11	10	1.06	35	0.286
26	A	18	17	0.81	35	0.486
27	A	14	13	0.85	35	0.371
28	A	10	9	0.92	33	0.273
29	A	6	5	0.98	23	0.217
30	F	0	0	N/A	0.000	N/A
31	F	0	0	N/A	0.000	N/A
32	F	0	0	N/A	0.000	N/A

CHAPTER 3

Listing of integrals

3.1 $\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

output
```
-5^(1/2)*arctan(1/5*x*5^(1/2))+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)+1/2*l
n(x^2+2*x+3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = -\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{1+x}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(3+2x+x^2\right)$$

input
```
Integrate[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3+x^2-5x+15}{(x^2+5)(x^2+2x+3)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{x+6}{x^2+2x+3}-\frac{5}{x^2+5}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\sqrt{5}\arctan\left(\frac{x}{\sqrt{5}}\right)+\frac{5\arctan\left(\frac{x+1}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
Int[(15 - 5*x + x^2 + x^3)/((5 + x^2)*(3 + 2*x + x^2)),x]
```

output
```
-(Sqrt[5]*ArcTan[x/Sqrt[5]]) + (5*ArcTan[(1 + x)/Sqrt[2]])/Sqrt[2] + Log[3
 + 2*x + x^2]/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

method	result	size
risch	$-\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)+\frac{5\arctan\left(\frac{(1+x)\sqrt{2}}{2}\right)\sqrt{2}}{2}+\frac{\ln\left(x^{2}+2x+3\right)}{2}$	39
default	$-\sqrt{5}\,\arctan\left(\frac{x\sqrt{5}}{5}\right)+\frac{\ln\left(x^{2}+2x+3\right)}{2}+\frac{5\sqrt{2}\,\arctan\left(\frac{(2x+2)\sqrt{2}}{4}\right)}{2}$	41

input
```
int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x,method=_RETURNVERBOSE)
```

output
```
-5^(1/2)*arctan(1/5*x*5^(1/2))+5/2*arctan(1/2*(1+x)*2^(1/2))*2^(1/2)+1/2*l
n(x^2+2*x+3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)\,(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)+\frac{1}{2}\log\left(x^2+2\,x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="fricas")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\log(x^2+2x+3)}{2} - \sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right) + \frac{5\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}+\frac{\sqrt{2}}{2}\right)}{2}$$

input
```
integrate((x**3+x**2-5*x+15)/(x**2+5)/(x**2+2*x+3),x)
```

output
```
log(x**2 + 2*x + 3)/2 - sqrt(5)*atan(sqrt(5)*x/5) + 5*sqrt(2)*atan(sqrt(2)
*x/2 + sqrt(2)/2)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="maxima")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x+1)\right) - \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) + \frac{1}{2}\log\left(x^2+2x+3\right)$$

input
```
integrate((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x, algorithm="giac")
```

output
```
5/2*sqrt(2)*arctan(1/2*sqrt(2)*(x + 1)) - sqrt(5)*arctan(1/5*sqrt(5)*x) +
1/2*log(x^2 + 2*x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)}{2} + \frac{\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)}{2} + \sqrt{5}\,\mathrm{atan}\left(\frac{2000\sqrt{5}}{2000\,x+1120} - \frac{224\sqrt{5}\,x}{2000\,x+1120}\right) - \frac{\sqrt{2}\,\ln\left(x+1-\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4} + \frac{\sqrt{2}\,\ln\left(x+1+\sqrt{2}\,1\mathrm{i}\right)\,5\mathrm{i}}{4}$$

input
```
int((x^2 - 5*x + x^3 + 15)/((x^2 + 5)*(2*x + x^2 + 3)),x)
```

output
```
log(x - 2^(1/2)*1i + 1)/2 + log(x + 2^(1/2)*1i + 1)/2 + 5^(1/2)*atan((2000
*5^(1/2))/(2000*x + 1120) - (224*5^(1/2)*x)/(2000*x + 1120)) - (2^(1/2)*lo
g(x - 2^(1/2)*1i + 1)*5i)/4 + (2^(1/2)*log(x + 2^(1/2)*1i + 1)*5i)/4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{15-5x+x^2+x^3}{(5+x^2)(3+2x+x^2)}\,dx = \frac{5\sqrt{2}\,atan\left(\frac{x+1}{\sqrt{2}}\right)}{2} - \sqrt{5}\,atan\left(\frac{x}{\sqrt{5}}\right) + \frac{\log(x^2+2x+3)}{2}$$

input
```
int((x^3+x^2-5*x+15)/(x^2+5)/(x^2+2*x+3),x)
```

output
```
(5*sqrt(2)*atan((x + 1)/sqrt(2)) - 2*sqrt(5)*atan(x/sqrt(5)) + log(x**2 +
2*x + 3))/2
```

3.2 $\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx$

Optimal result

Integrand size = 35, antiderivative size = 208

$$\begin{aligned}\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx &= -\frac{77159983(1+5x)\sqrt{3+2x+5x^2}}{31250000} - \frac{1968340667(3+2x+5x^2)^{3/2}}{131250000} \\ &+ \frac{1045360143x(3+2x+5x^2)^{3/2}}{43750000} + \frac{98060877x^2(3+2x+5x^2)^{3/2}}{4375000} \\ &- \frac{90960857x^3(3+2x+5x^2)^{3/2}}{1575000} - \frac{888751x^4(3+2x+5x^2)^{3/2}}{105000} \\ &+ \frac{190939x^5(3+2x+5x^2)^{3/2}}{3000} - \frac{50519x^6(3+2x+5x^2)^{3/2}}{2250} \\ &- \frac{343}{50}x^7 (3+2x+5x^2)^{3/2} - \frac{540119881\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{15625000\sqrt{5}}\end{aligned}$$

output
```
-77159983/31250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)-1968340667/131250000*(5*x^2
+2*x+3)^(3/2)+1045360143/43750000*x*(5*x^2+2*x+3)^(3/2)+98060877/4375000*x
^2*(5*x^2+2*x+3)^(3/2)-90960857/1575000*x^3*(5*x^2+2*x+3)^(3/2)-888751/105
000*x^4*(5*x^2+2*x+3)^(3/2)+190939/3000*x^5*(5*x^2+2*x+3)^(3/2)-50519/2250
*x^6*(5*x^2+2*x+3)^(3/2)-343/50*x^7*(5*x^2+2*x+3)^(3/2)-540119881/78125000
*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-93436408944+57768004650x+78839046795x^2-17642392275x^3-56757413000x^4}{196875000}$$
$$+\frac{540119881\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{15625000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-93436408944 + 57768004650*x + 78839046795*x^2 - 1
7642392275*x^3 - 56757413000*x^4 - 225922362500*x^5 + 34674656250*x^6 + 49
7593468750*x^7 - 248031875000*x^8 - 67528125000*x^9))/1968750000 + (540119
881*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625000*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.20, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^3 \left(x^2+5x+2\right) \sqrt{5x^2+2x+3}\,dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{50}\int \sqrt{5x^2+2x+3}\left(-50519x^7+110453x^6+6350x^5-43550x^4-3050x^3+5750x^2+1450x+100\right)dx- \frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{50}\left(\frac{1}{45}\int 6\sqrt{5x^2+2x+3}\left(954695x^6+199182x^5-326625x^4-22875x^3+43125x^2+10875x+750\right)dx-\frac{5051}{45}\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{50}\left(\frac{2}{15}\int \sqrt{5x^2+2x+3}\left(954695x^6+199182x^5-326625x^4-22875x^3+43125x^2+10875x+750\right)dx-\frac{50519}{45}\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{40}\int 5\sqrt{5x^2+2x+3}\left(-888751x^5-5477085x^4-183000x^3+345000x^2+87000x+6000\right)dx+\frac{19093}{8}\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\int \sqrt{5x^2+2x+3}\left(-888751x^5-5477085x^4-183000x^3+345000x^2+87000x+6000\right)dx+\frac{190939}{8}\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{1}{35}\int 2\sqrt{5x^2+2x+3}\left(-90960857x^4+2130006x^3+6037500x^2+1522500x+105000\right)dx-\frac{88875}{35}\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\int \sqrt{5x^2+2x+3}\left(-90960857x^4+2130006x^3+6037500x^2+1522500x+105000\right)dx-\frac{888751}{35}\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{1}{30}\int 9\sqrt{5x^2+2x+3}\left(98060877x^3+111085857x^2+5075000x+350000\right)dx-\frac{90960857}{30}x^3(5\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\int\sqrt{5x^2+2x+3}(98060877x^3+111085857x^2+5075000x+350000)\,dx-\frac{90960857}{30}x^3(5x\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}(1045360143x^2-230745131x+4375000)\,dx+\frac{98060877}{25}(5x^2+2x\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\int\sqrt{5x^2+2x+3}(1045360143x^2-230745131x+4375000)\,dx+\frac{98060877}{25}(5x^2+2x\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int-\left((9841703335x+3048580429)\sqrt{5x^2+2x+3}\right)dx+\frac{1045360143}{20}x(5x^2+2x\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1045360143}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int(9841703335x+3048580429)\sqrt{5x^2+2x+3}d\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\int\sqrt{5x^2+2x+3}dx-\frac{1968340667}{3}\left(5x^2+2x+3\right)^{3/2}\right)+\frac{1045360}{20}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)-\frac{19683406}{3}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2+2x+3}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{50}\left(\frac{2}{15}\left(\frac{1}{8}\left(\frac{2}{35}\left(\frac{3}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(-1080239762\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)-\frac{196834066}{3}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{50}x^7\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-343*x^7*(3 + 2*x + 5*x^2)^(3/2))/50 + ((-50519*x^6*(3 + 2*x + 5*x^2)^(3/
2))/45 + (2*((190939*x^5*(3 + 2*x + 5*x^2)^(3/2))/8 + ((-888751*x^4*(3 + 2
*x + 5*x^2)^(3/2))/35 + (2*((-90960857*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + (
3*((98060877*x^2*(3 + 2*x + 5*x^2)^(3/2))/25 + (2*((1045360143*x*(3 + 2*x
+ 5*x^2)^(3/2))/20 + ((-1968340667*(3 + 2*x + 5*x^2)^(3/2))/3 - 1080239762
*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14]
)])/(5*Sqrt[5])))/20))/25))/10))/35)/8))/15)/50
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.36

method	result
risch	$-\frac{(67528125000x^9+248031875000x^8-497593468750x^7-34674656250x^6+225922362500x^5+56757413000x^4+17642392275x^3-78}{1968750000}$
trager	$\left(-\frac{343}{10}x^9-\frac{56693}{450}x^8+\frac{2274713}{9000}x^7+\frac{369863}{21000}x^6-\frac{18073789}{157500}x^5-\frac{56757413}{1968750}x^4-\frac{235231897}{26250000}x^3+\frac{5255936453}{131250000}x^2\right.$
default	$-\frac{77159983(10x+2)\sqrt{5x^2+2x+3}}{62500000}-\frac{540119881\sqrt{5}\,\mathrm{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}-\frac{1968340667(5x^2+2x+3)^{\frac{3}{2}}}{131250000}+\frac{1045360143x(5x^2}{4375000}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/1968750000*(67528125000*x^9+248031875000*x^8-497593468750*x^7-346746562
50*x^6+225922362500*x^5+56757413000*x^4+17642392275*x^3-78839046795*x^2-57
768004650*x+93436408944)*(5*x^2+2*x+3)^(1/2)-540119881/78125000*5^(1/2)*ar
csinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.47

$$\int \left(1+4x-7x^2\right)^3\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx =$$
$$-\frac{1}{1968750000}\left(67528125000\,x^9+248031875000\,x^8-497593468750\,x^7-34674656250\,x^6+225922362\right.$$
$$+\frac{540119881}{156250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/1968750000*(67528125000*x^9 + 248031875000*x^8 - 497593468750*x^7 - 346
74656250*x^6 + 225922362500*x^5 + 56757413000*x^4 + 17642392275*x^3 - 7883
9046795*x^2 - 57768004650*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 5401198
81/156250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
- 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.48

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = \sqrt{5x^2+2x+3}\left(-\frac{343x^9}{10}-\frac{56693x^8}{450}+\frac{2274713x^7}{9000}+\frac{369863x^6}{21000}-\frac{18073789x^5}{157500}-\frac{56757413x^4}{1968750}-\frac{235231897x^3}{26250000}+\frac{5255936453x^2}{131250000}+\frac{385120031x}{13125000}-\frac{648863951}{13671875}\right)-\frac{540119881\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{78125000}$$

input

```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output

```
sqrt(5*x**2 + 2*x + 3)*(-343*x**9/10 - 56693*x**8/450 + 2274713*x**7/9000
+ 369863*x**6/21000 - 18073789*x**5/157500 - 56757413*x**4/1968750 - 23523
1897*x**3/26250000 + 5255936453*x**2/131250000 + 385120031*x/13125000 - 64
8863951/13671875) - 540119881*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/78125
000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.85

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\, dx = -\frac{343}{50}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^7-\frac{50519}{2250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^6+\frac{190939}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5-\frac{888751}{105000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4-\frac{90960857}{1575000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3+\frac{98060877}{4375000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2+\frac{1045360143}{43750000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x-\frac{1968340667}{131250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{77159983}{6250000}\sqrt{5x^2+2x+3}x-\frac{540119881}{78125000}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{77159983}{31250000}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-343/50*(5*x^2 + 2*x + 3)^(3/2)*x^7 - 50519/2250*(5*x^2 + 2*x + 3)^(3/2)*x
^6 + 190939/3000*(5*x^2 + 2*x + 3)^(3/2)*x^5 - 888751/105000*(5*x^2 + 2*x
+ 3)^(3/2)*x^4 - 90960857/1575000*(5*x^2 + 2*x + 3)^(3/2)*x^3 + 98060877/4
375000*(5*x^2 + 2*x + 3)^(3/2)*x^2 + 1045360143/43750000*(5*x^2 + 2*x + 3)
^(3/2)*x - 1968340667/131250000*(5*x^2 + 2*x + 3)^(3/2) - 77159983/6250000
*sqrt(5*x^2 + 2*x + 3)*x - 540119881/78125000*sqrt(5)*arcsinh(1/14*sqrt(14
)*(5*x + 1)) - 77159983/31250000*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.44

$$\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx =$$
$$-\frac{1}{1968750000}(5((5(10(25(5(49(140(315x+1157)x-324959)x-1109589)x+36147578)x+2270$$
$$+\frac{540119881}{78125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
-1/1968750000*(5*((5*(10*(25*(5*(49*(140*(315*x + 1157)*x - 324959)*x - 11
09589)*x + 36147578)*x + 227029652)*x + 705695691)*x - 15767809359)*x - 11
553600930)*x + 93436408944)*sqrt(5*x^2 + 2*x + 3) + 540119881/78125000*sqr
t(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 20.51 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.06

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx = \frac{98060877\,x^2\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{4375000}-\frac{90960857\,x^3\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{1575000}-\frac{888751\,x^4\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{105000}+\frac{190939\,x^5\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{3000}-\frac{50519\,x^6\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{2250}-\frac{343\,x^7\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{50}-\frac{3048580429\,\sqrt{5}\,\ln\left(\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(5\,x+1\right)}{5}\right)}{156250000}-\frac{3048580429\,\left(\frac{x}{2}+\frac{1}{10}\right)\,\sqrt{5\,x^2+2\,x+3}}{43750000}-\frac{1968340667\,\sqrt{5\,x^2+2\,x+3}\,\left(200\,x^2+20\,x+108\right)}{5250000000}+\frac{1045360143\,x\,{\left(5\,x^2+2\,x+3\right)}^{3/2}}{43750000}+\frac{1968340667\,\sqrt{5}\,\ln\left(2\,\sqrt{5\,x^2+2\,x+3}+\frac{\sqrt{5}\,\left(10\,x+2\right)}{5}\right)}{156250000}$$

input

```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3,x)
```

output

```
(98060877*x^2*(2*x + 5*x^2 + 3)^(3/2))/4375000 - (90960857*x^3*(2*x + 5*x^
2 + 3)^(3/2))/1575000 - (888751*x^4*(2*x + 5*x^2 + 3)^(3/2))/105000 + (190
939*x^5*(2*x + 5*x^2 + 3)^(3/2))/3000 - (50519*x^6*(2*x + 5*x^2 + 3)^(3/2)
)/2250 - (343*x^7*(2*x + 5*x^2 + 3)^(3/2))/50 - (3048580429*5^(1/2)*log((2
*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(5*x + 1))/5))/156250000 - (3048580429*(x
/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/43750000 - (1968340667*(2*x + 5*x^2 +
3)^(1/2)*(20*x + 200*x^2 + 108))/5250000000 + (1045360143*x*(2*x + 5*x^2 +
 3)^(3/2))/43750000 + (1968340667*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) +
(5^(1/2)*(10*x + 2))/5))/156250000
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.89

$$\int (1+4x-7x^2)^3 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx$$
$$= -\frac{343\sqrt{5x^2+2x+3}\,x^9}{10} - \frac{56693\sqrt{5x^2+2x+3}\,x^8}{450} + \frac{2274713\sqrt{5x^2+2x+3}\,x^7}{9000}$$
$$+ \frac{369863\sqrt{5x^2+2x+3}\,x^6}{21000} - \frac{18073789\sqrt{5x^2+2x+3}\,x^5}{157500}$$
$$- \frac{56757413\sqrt{5x^2+2x+3}\,x^4}{1968750} - \frac{235231897\sqrt{5x^2+2x+3}\,x^3}{26250000}$$
$$+ \frac{5255936453\sqrt{5x^2+2x+3}\,x^2}{131250000} + \frac{385120031\sqrt{5x^2+2x+3}\,x}{13125000}$$
$$- \frac{648863951\sqrt{5x^2+2x+3}}{13671875} - \frac{540119881\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{78125000}$$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 337640625000*sqrt(5*x**2 + 2*x + 3)*x**9 - 1240159375000*sqrt(5*x**2 +
 2*x + 3)*x**8 + 2487967343750*sqrt(5*x**2 + 2*x + 3)*x**7 + 173373281250*
sqrt(5*x**2 + 2*x + 3)*x**6 - 1129611812500*sqrt(5*x**2 + 2*x + 3)*x**5 -
283787065000*sqrt(5*x**2 + 2*x + 3)*x**4 - 88211961375*sqrt(5*x**2 + 2*x +
 3)*x**3 + 394195233975*sqrt(5*x**2 + 2*x + 3)*x**2 + 288840023250*sqrt(5*
x**2 + 2*x + 3)*x - 467182044720*sqrt(5*x**2 + 2*x + 3) - 68055105006*sqrt
(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)))/9843750000
```

3.3 $\int \left(1+4 x-7 x^2\right)^2 \left(2+5 x+x^2\right) \sqrt{3+2 x+5 x^2} \, dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \left(1+4 x-7 x^2\right)^2 \left(2+5 x+x^2\right) \sqrt{3+2 x+5 x^2} \, dx = -\frac{2521723(1+5 x) \sqrt{3+2 x+5 x^2}}{1250000} + \frac{198439 \left(3+2 x+5 x^2\right)^{3/2}}{750000} + \frac{1781669 x \left(3+2 x+5 x^2\right)^{3/2}}{250000} - \frac{77509 x^2 \left(3+2 x+5 x^2\right)^{3/2}}{25000} - \frac{25277 x^3 \left(3+2 x+5 x^2\right)^{3/2}}{3000} + \frac{989}{200} x^4 \left(3+2 x+5 x^2\right)^{3/2} + \frac{49}{40} x^5 \left(3+2 x+5 x^2\right)^{3/2} - \frac{17652061 \text{arcsinh}\left(\frac{1+5 x}{\sqrt{14}}\right)}{625000 \sqrt{5}}$$

output
```
-2521723/1250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)+198439/750000*(5*x^2+2*x+3)^(
3/2)+1781669/250000*x*(5*x^2+2*x+3)^(3/2)-77509/25000*x^2*(5*x^2+2*x+3)^(3
/2)-25277/3000*x^3*(5*x^2+2*x+3)^(3/2)+989/200*x^4*(5*x^2+2*x+3)^(3/2)+49/
40*x^5*(5*x^2+2*x+3)^(3/2)-17652061/3125000*arcsinh(1/14*(1+5*x)*14^(1/2))
*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-4588584+44333650x+23531995x^2+15583725x^3-65693000x^4-107112500x^5+1}{3750000}$$
$$+\frac{17652061\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-4588584 + 44333650*x + 23531995*x^2 + 15583725*x^
3 - 65693000*x^4 - 107112500*x^5 + 101906250*x^6 + 22968750*x^7))/3750000
+ (17652061*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sqrt[5]
)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.371, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^2 \left(x^2+5x+2\right) \sqrt{5x^2+2x+3}\,dx$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{40}\int \sqrt{5x^2+2x+3}\left(6923x^5-7935x^4-3760x^3+1800x^2+840x+80\right)dx+\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{40}\left(\frac{1}{35}\int 14\sqrt{5x^2+2x+3}\left(-25277x^4-15334x^3+4500x^2+2100x+200\right)dx+\frac{989}{5}\left(5x^2+2x+3\right)^{3/2}x^4\right)+\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(\frac{2}{5}\int\sqrt{5x^2+2x+3}\left(-25277x^4-15334x^3+4500x^2+2100x+200\right)dx+\frac{989}{5}\left(5x^2+2x+3\right)^{3/2}x^4\right)+\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{30}\int 3\sqrt{5x^2+2x+3}\left(-77509x^3+120831x^2+21000x+2000\right)dx-\frac{25277}{30}x^3\left(5x^2+2x+3\right)^{3/2}\right)+\frac{98}{5}\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\int\sqrt{5x^2+2x+3}\left(-77509x^3+120831x^2+21000x+2000\right)dx-\frac{25277}{30}x^3\left(5x^2+2x+3\right)^{3/2}\right)+\frac{989}{5}\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}\left(1781669x^2+495027x+25000\right)dx-\frac{77509}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{25277}{30}\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\int\sqrt{5x^2+2x+3}\left(1781669x^2+495027x+25000\right)dx-\frac{77509}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{25277}{30}x^3\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int-\left((4845007-992195x)\sqrt{5x^2+2x+3}\right)dx+\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{77509}{25}x^2\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow 25$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int (4845007-992195x)\sqrt{5x^2+2x+3}dx\right)-\frac{77509}{25}x^2\left(5x^2+\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow 1160$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\int \sqrt{5x^2+2x+3}dx\right)+\frac{1781669}{20}x\left(5x^2+2x+3\right)^{3/2}\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow 1087$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{7}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow 1090$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

$$\Big\downarrow 222$$

$$\frac{1}{40}\left(\frac{2}{5}\left(\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{198439}{3}\left(5x^2+2x+3\right)^{3/2}-5043446\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right.\right.\right.\right.\right.$$
$$\frac{49}{40}\left(5x^2+2x+3\right)^{3/2}x^5$$

input
```
Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(49*x^5*(3 + 2*x + 5*x^2)^(3/2))/40 + ((989*x^4*(3 + 2*x + 5*x^2)^(3/2))/5
 + (2*((-25277*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + ((-77509*x^2*(3 + 2*x + 5
*x^2)^(3/2))/25 + (2*((1781669*x*(3 + 2*x + 5*x^2)^(3/2))/20 + ((198439*(3
 + 2*x + 5*x^2)^(3/2))/3 - 5043446*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 +
 (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5])))/20))/25)/10))/5)/40
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$\frac{(22968750x^7+101906250x^6-107112500x^5-65693000x^4+15583725x^3+23531995x^2+44333650x-4588584)\sqrt{5x^2+2x+3}}{3750000} - \frac{1765}{}$
trager	$\left(\frac{49}{8}x^7+\frac{1087}{40}x^6-\frac{8569}{300}x^5-\frac{65693}{3750}x^4+\frac{207783}{50000}x^3+\frac{4706399}{750000}x^2+\frac{886673}{75000}x-\frac{191191}{156250}\right)\sqrt{5x^2+2x+3}+$
default	$-\frac{2521723(10x+2)\sqrt{5x^2+2x+3}}{2500000}-\frac{17652061\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}+\frac{198439(5x^2+2x+3)^{\frac{3}{2}}}{750000}+\frac{1781669x(5x^2+2x+3)^{\frac{3}{2}}}{250000}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/3750000*(22968750*x^7+101906250*x^6-107112500*x^5-65693000*x^4+15583725*
x^3+23531995*x^2+44333650*x-4588584)*(5*x^2+2*x+3)^(1/2)-17652061/3125000*
5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$$
$$=\frac{1}{3750000}\left(22968750\,x^7+101906250\,x^6-107112500\,x^5-65693000\,x^4+15583725\,x^3+23531995\,x^2+4\right.$$
$$+\frac{17652061}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
1/3750000*(22968750*x^7 + 101906250*x^6 - 107112500*x^5 - 65693000*x^4 + 1
5583725*x^3 + 23531995*x^2 + 44333650*x - 4588584)*sqrt(5*x^2 + 2*x + 3) +
 17652061/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25
*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.51

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx$$
$$=\sqrt{5x^2+2x+3}\cdot\left(\frac{49x^7}{8}+\frac{1087x^6}{40}-\frac{8569x^5}{300}-\frac{65693x^4}{3750}+\frac{207783x^3}{50000}+\frac{4706399x^2}{750000}+\frac{886673x}{75000}-\frac{191191}{156250}\right)-\frac{17652061\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**7/8 + 1087*x**6/40 - 8569*x**5/300 - 65693*x
**4/3750 + 207783*x**3/50000 + 4706399*x**2/750000 + 886673*x/75000 - 1911
91/156250) - 17652061*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.86

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{49}{40}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^5 + \frac{989}{200}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^4 - \frac{25277}{3000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3 - \frac{77509}{25000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2 + \frac{1781669}{250000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x + \frac{198439}{750000}\left(5x^2+2x+3\right)^{\frac{3}{2}} - \frac{2521723}{250000}\sqrt{5x^2+2x+3}x - \frac{17652061}{3125000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{2521723}{1250000}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
49/40*(5*x^2 + 2*x + 3)^(3/2)*x^5 + 989/200*(5*x^2 + 2*x + 3)^(3/2)*x^4 -
25277/3000*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 77509/25000*(5*x^2 + 2*x + 3)^(3/
2)*x^2 + 1781669/250000*(5*x^2 + 2*x + 3)^(3/2)*x + 198439/750000*(5*x^2 +
 2*x + 3)^(3/2) - 2521723/250000*sqrt(5*x^2 + 2*x + 3)*x - 17652061/312500
0*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2521723/1250000*sqrt(5*x^2 +
2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{1}{3750000}\left(5\left(\left(5\left(10\left(25\left(15\left(245x+1087\right)x-17138\right)x-262772\right)x+623349\right)x+4706399\right)x+8866730\right)\right. + \frac{17652061}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/3750000*(5*((5*(10*(25*(15*(245*x + 1087)*x - 17138)*x - 262772)*x + 623
349)*x + 4706399)*x + 8866730)*x - 4588584)*sqrt(5*x^2 + 2*x + 3) + 176520
61/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 19.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2}\, dx = \frac{989\, x^4\, (5\, x^2+2\, x+3)^{3/2}}{200} - \frac{25277\, x^3\, (5\, x^2+2\, x+3)^{3/2}}{3000} - \frac{77509\, x^2\, (5\, x^2+2\, x+3)^{3/2}}{25000} + \frac{49\, x^5\, (5\, x^2+2\, x+3)^{3/2}}{40} - \frac{33915049\, \sqrt{5}\, \ln\left(\sqrt{5\, x^2+2\, x+3}+\frac{\sqrt{5}\, (5\, x+1)}{5}\right)}{6250000} - \frac{4845007\, \left(\frac{x}{2}+\frac{1}{10}\right)\, \sqrt{5\, x^2+2\, x+3}}{250000} + \frac{198439\, \sqrt{5\, x^2+2\, x+3}\, (200\, x^2+20\, x+108)}{30000000} + \frac{1781669\, x\, (5\, x^2+2\, x+3)^{3/2}}{250000} - \frac{1389073\, \sqrt{5}\, \ln\left(2\, \sqrt{5\, x^2+2\, x+3}+\frac{\sqrt{5}\, (10\, x+2)}{5}\right)}{6250000}$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2,x)
```

output
```
(989*x^4*(2*x + 5*x^2 + 3)^(3/2))/200 - (25277*x^3*(2*x + 5*x^2 + 3)^(3/2)
)/3000 - (77509*x^2*(2*x + 5*x^2 + 3)^(3/2))/25000 + (49*x^5*(2*x + 5*x^2
+ 3)^(3/2))/40 - (33915049*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*
(5*x + 1))/5))/6250000 - (4845007*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/25
0000 + (198439*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/30000000 +
(1781669*x*(2*x + 5*x^2 + 3)^(3/2))/250000 - (1389073*5^(1/2)*log(2*(2*x +
 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x + 2))/5))/6250000
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int (1+4x-7x^2)^2 (2+5x+x^2) \sqrt{3+2x+5x^2}\,dx$$
$$= \frac{49\sqrt{5x^2+2x+3}\,x^7}{8} + \frac{1087\sqrt{5x^2+2x+3}\,x^6}{40} - \frac{8569\sqrt{5x^2+2x+3}\,x^5}{300} - \frac{65693\sqrt{5x^2+2x+3}\,x^4}{3750} + \frac{207783\sqrt{5x^2+2x+3}\,x^3}{50000} + \frac{4706399\sqrt{5x^2+2x+3}\,x^2}{750000} + \frac{886673\sqrt{5x^2+2x+3}\,x}{75000} - \frac{191191\sqrt{5x^2+2x+3}}{156250} - \frac{17652061\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{3125000}$$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

output
```
(114843750*sqrt(5*x**2 + 2*x + 3)*x**7 + 509531250*sqrt(5*x**2 + 2*x + 3)*
x**6 - 535562500*sqrt(5*x**2 + 2*x + 3)*x**5 - 328465000*sqrt(5*x**2 + 2*x
 + 3)*x**4 + 77918625*sqrt(5*x**2 + 2*x + 3)*x**3 + 117659975*sqrt(5*x**2
+ 2*x + 3)*x**2 + 221668250*sqrt(5*x**2 + 2*x + 3)*x - 22942920*sqrt(5*x**
2 + 2*x + 3) - 105912366*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x
 + 1)/sqrt(14)))/18750000
```

3.4 $\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 124

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = -\frac{4633(1+5x)\sqrt{3+2x+5x^2}}{12500}+\frac{7819(3+2x+5x^2)^{3/2}}{7500}+\frac{2149x(3+2x+5x^2)^{3/2}}{2500} - \frac{289}{250}x^2(3+2x+5x^2)^{3/2}-\frac{7}{30}x^3(3+2x+5x^2)^{3/2}-\frac{32431\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{6250\sqrt{5}}$$

output
```
-4633/12500*(1+5*x)*(5*x^2+2*x+3)^(1/2)+7819/7500*(5*x^2+2*x+3)^(3/2)+2149
/2500*x*(5*x^2+2*x+3)^(3/2)-289/250*x^2*(5*x^2+2*x+3)^(3/2)-7/30*x^3*(5*x^
2+2*x+3)^(3/2)-32431/31250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \frac{\sqrt{3+2x+5x^2}(103386+105400x+129895x^2+48225x^3-234250x^4-43750x^5)}{37500} + \frac{32431\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{6250\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(103386 + 105400*x + 129895*x^2 + 48225*x^3 - 23425
0*x^4 - 43750*x^5))/37500 + (32431*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5
*x^2]])/(6250*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = $\{2192, 27, 2192, 27, 2192, 25, 1160, 1087, 1090, 222\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)\left(x^2+5x+2\right)\sqrt{5x^2+2x+3}\,dx$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{30}\int 3\sqrt{5x^2+2x+3}\left(-289x^3+91x^2+130x+20\right)dx-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\int \sqrt{5x^2+2x+3}\left(-289x^3+91x^2+130x+20\right)dx-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{10}\left(\frac{1}{25}\int 2\sqrt{5x^2+2x+3}\left(2149x^2+2492x+250\right)dx-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{10}\left(\frac{2}{25}\int \sqrt{5x^2+2x+3}\left(2149x^2+2492x+250\right)dx-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 2192$$

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\int -\left((1447-39095x)\sqrt{5x^2+2x+3}\right)dx+\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 25

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}-\frac{1}{20}\int (1447-39095x)\sqrt{5x^2+2x+3}dx\right)-\frac{289}{25}x^2\left(5x^2+2x+3\right)^{3/2}\right)-$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 1160

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\int \sqrt{5x^2+2x+3}dx\right)+\frac{2149}{20}x\left(5x^2+2x+3\right)^{3/2}\right)-\frac{289}{25}x^2\left(5x^2+\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 1087

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{7}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right)+\frac{2149}{20}x\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 1090

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2+2x+3}(5\right.\right.\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

$\downarrow$ 222

$$\frac{1}{10}\left(\frac{2}{25}\left(\frac{1}{20}\left(\frac{7819}{3}\left(5x^2+2x+3\right)^{3/2}-9266\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)\right)+\frac{2149}{20}x(5x\right.\right.$$
$$\frac{7}{30}x^3\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-7*x^3*(3 + 2*x + 5*x^2)^(3/2))/30 + ((-289*x^2*(3 + 2*x + 5*x^2)^(3/2))/
25 + (2*((2149*x*(3 + 2*x + 5*x^2)^(3/2))/20 + ((7819*(3 + 2*x + 5*x^2)^(3
/2))/3 - 9266*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x
)/(2*Sqrt[14])])/(5*Sqrt[5])))/20))/25)/10
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{(43750x^5+234250x^4-48225x^3-129895x^2-105400x-103386)\sqrt{5x^2+2x+3}}{37500}-\frac{32431\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}$
trager	$\left(-\frac{7}{6}x^5-\frac{937}{150}x^4+\frac{643}{500}x^3+\frac{25979}{7500}x^2+\frac{1054}{375}x+\frac{17231}{6250}\right)\sqrt{5x^2+2x+3}-\frac{32431\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(5\,\mathrm{Root}\right.}{}$
default	$-\frac{4633(10x+2)\sqrt{5x^2+2x+3}}{25000}-\frac{32431\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}+\frac{7819(5x^2+2x+3)^{\frac{3}{2}}}{7500}+\frac{2149x(5x^2+2x+3)^{\frac{3}{2}}}{2500}-\frac{289x^2(5}{}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/37500*(43750*x^5+234250*x^4-48225*x^3-129895*x^2-105400*x-103386)*(5*x^
2+2*x+3)^(1/2)-32431/31250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int (1+4x-7x^2)\left(2+5x+x^2\right)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{37500}\left(43750\,x^5+234250\,x^4-48225\,x^3-129895\,x^2-105400\,x-103386\right)\sqrt{5\,x^2+2\,x+3}+\frac{32431}{62500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/37500*(43750*x^5 + 234250*x^4 - 48225*x^3 - 129895*x^2 - 105400*x - 103
386)*sqrt(5*x^2 + 2*x + 3) + 32431/62500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 +
2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{7x^5}{6}-\frac{937x^4}{150}+\frac{643x^3}{500}+\frac{25979x^2}{7500}+\frac{1054x}{375}+\frac{17231}{6250}\right) - \frac{32431\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{31250}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-7*x**5/6 - 937*x**4/150 + 643*x**3/500 + 25979*x*
*2/7500 + 1054*x/375 + 17231/6250) - 32431*sqrt(5)*asinh(5*sqrt(14)*(x + 1
/5)/14)/31250
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = -\frac{7}{30}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^3-\frac{289}{250}\left(5x^2+2x+3\right)^{\frac{3}{2}}x^2 + \frac{2149}{2500}\left(5x^2+2x+3\right)^{\frac{3}{2}}x+\frac{7819}{7500}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{4633}{2500}\sqrt{5x^2+2x+3}\,x - \frac{32431}{31250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{4633}{12500}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-7/30*(5*x^2 + 2*x + 3)^(3/2)*x^3 - 289/250*(5*x^2 + 2*x + 3)^(3/2)*x^2 +
2149/2500*(5*x^2 + 2*x + 3)^(3/2)*x + 7819/7500*(5*x^2 + 2*x + 3)^(3/2) -
4633/2500*sqrt(5*x^2 + 2*x + 3)*x - 32431/31250*sqrt(5)*arcsinh(1/14*sqrt(
14)*(5*x + 1)) - 4633/12500*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = -\frac{1}{37500}\left(5\left(\left(5\left(10\left(175\,x+937\right)x-1929\right)x-25979\right)x-21080\right)x-103386\right)\sqrt{5\,x^2+2\,x+3} + \frac{32431}{31250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
-1/37500*(5*((5*(10*(175*x + 937)*x - 1929)*x - 25979)*x - 21080)*x - 1033
86)*sqrt(5*x^2 + 2*x + 3) + 32431/31250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x -
sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [B] (verification not implemented)

Time = 18.38 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.23

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = \frac{7819\sqrt{5x^2+2x+3}\,(200x^2+20x+108)}{300000} - \frac{7x^3(5x^2+2x+3)^{3/2}}{30} - \frac{10129\sqrt{5}\ln\left(\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(5x+1)}{5}\right)}{62500} - \frac{1447\left(\frac{x}{2}+\frac{1}{10}\right)\sqrt{5x^2+2x+3}}{2500} - \frac{289x^2(5x^2+2x+3)^{3/2}}{250} + \frac{2149x(5x^2+2x+3)^{3/2}}{2500} - \frac{54733\sqrt{5}\ln\left(2\sqrt{5x^2+2x+3}+\frac{\sqrt{5}(10x+2)}{5}\right)}{62500}$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1),x)
```

output
```
(7819*(2*x + 5*x^2 + 3)^(1/2)*(20*x + 200*x^2 + 108))/300000 - (7*x^3*(2*x
 + 5*x^2 + 3)^(3/2))/30 - (10129*5^(1/2)*log((2*x + 5*x^2 + 3)^(1/2) + (5^
(1/2)*(5*x + 1))/5))/62500 - (1447*(x/2 + 1/10)*(2*x + 5*x^2 + 3)^(1/2))/2
500 - (289*x^2*(2*x + 5*x^2 + 3)^(3/2))/250 + (2149*x*(2*x + 5*x^2 + 3)^(3
/2))/2500 - (54733*5^(1/2)*log(2*(2*x + 5*x^2 + 3)^(1/2) + (5^(1/2)*(10*x
+ 2))/5))/62500
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int (1+4x-7x^2)(2+5x+x^2)\sqrt{3+2x+5x^2}\,dx = -\frac{7\sqrt{5x^2+2x+3}\,x^5}{6} - \frac{937\sqrt{5x^2+2x+3}\,x^4}{150} + \frac{643\sqrt{5x^2+2x+3}\,x^3}{500} + \frac{25979\sqrt{5x^2+2x+3}\,x^2}{7500} + \frac{1054\sqrt{5x^2+2x+3}\,x}{375} + \frac{17231\sqrt{5x^2+2x+3}}{6250} - \frac{32431\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{31250}$$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 218750*sqrt(5*x**2 + 2*x + 3)*x**5 - 1171250*sqrt(5*x**2 + 2*x + 3)*x*
*4 + 241125*sqrt(5*x**2 + 2*x + 3)*x**3 + 649475*sqrt(5*x**2 + 2*x + 3)*x*
*2 + 527000*sqrt(5*x**2 + 2*x + 3)*x + 516930*sqrt(5*x**2 + 2*x + 3) - 194
586*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)))/1875
00
```

3.5 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 201

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx &= -\frac{397}{490}\sqrt{3+2x+5x^2}-\frac{1}{14}x\sqrt{3+2x+5x^2}-\frac{8233\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1715\sqrt{5}}\\ &\quad-\frac{3}{343}\sqrt{\frac{1}{11}\left(497041-146555\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)\\ &\quad+\frac{3}{343}\sqrt{\frac{1}{11}\left(497041+146555\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)\end{aligned}$$

output

```
-397/490*(5*x^2+2*x+3)^(1/2)-1/14*x*(5*x^2+2*x+3)^(1/2)-8233/8575*arcsinh(
1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/3773*(5467451-1612105*11^(1/2))^(1/2)*arc
tanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*x+3)
^(1/2))+3/3773*(5467451+1612105*11^(1/2))^(1/2)*arctanh((23+11^(1/2)+(17+5
*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx$$
$$= \frac{1}{490}(-397-35x)\sqrt{3+2x+5x^2}+\frac{8233\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1715\sqrt{5}}$$
$$+\frac{6}{343}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&,\frac{3317\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+676\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

output
```
((-397 - 35*x)*Sqrt[3 + 2*x + 5*x^2])/490 + (8233*Log[-1 - 5*x + Sqrt[5]*S
qrt[3 + 2*x + 5*x^2]])/(1715*Sqrt[5]) + (6*RootSum[83 - 16*Sqrt[5]*#1 - 70
*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (3317*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
+ 5*x^2] - #1] + 676*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1
]*#1 - 1331*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[
5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/343
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}\,dx$$
$$\Big\downarrow \text{2138}$$

$$-\frac{1}{490}\int -\frac{2\left(8233x^2+6704x+1721\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{245}\int \frac{8233x^2+6704x+1721}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{245}\left(-\frac{8233}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{60(1331x+338)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{8233}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{8233\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{245}\left(\frac{60}{7}\int \frac{1331x+338}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{8233\operatorname{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}\right)-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{1}{11}\left(14641-5028\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(14641+5028\sqrt{11}\right)\int \frac{}{2\left(-7x\right.}\right.\right.$$
$$-\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{1}{22}\left(14641-5028\sqrt{11}\right)\int\frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(14641+5028\sqrt{11}\right)\int\frac{}{\left(-7x+\right.}\right.\right.$$
$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$\Big\downarrow$ 1154

$$\frac{1}{245}\left(\frac{60}{7}\left(-\frac{1}{11}\left(14641-5028\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}\right.}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$
$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

$\Big\downarrow$ 219

$$\frac{1}{245}\left(\frac{60}{7}\left(\frac{\left(14641-5028\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(14641+5028\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{}\right.}{\sqrt{2\left(125+17\right.}}\right.}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.\right.$$
$$\frac{1}{490}(35x+397)\sqrt{5x^2+2x+3}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2),x]
```

output
```
-1/490*((397 + 35*x)*Sqrt[3 + 2*x + 5*x^2]) + ((-8233*ArcSinh[(2 + 10*x)/(
2*Sqrt[14])])/(7*Sqrt[5]) + (60*(((14641 - 5028*Sqrt[11])*ArcTanh[(23 - Sq
rt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x +
5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((14641 + 5028*Sqrt[11])*ArcT
anh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqr
t[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/245
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2138

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143

```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{(35x+397)\sqrt{5x^2+2x+3}}{490}-\frac{8233\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{8575}+\frac{6\left(-5028+1331\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{1}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\right.}}\right.}{3773\sqrt{250-34}}$
default	$-\frac{(10x+2)\sqrt{5x^2+2x+3}}{140}-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{25}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\right.}}{49}\right.}{}$
trager	$\left(-\frac{x}{14}-\frac{397}{490}\right)\sqrt{5x^2+2x+3}+\frac{\operatorname{RootOf}\left(\_Z^2+5929\operatorname{RootOf}\left(5929\_Z^4-2460352950\_Z^2+11145463008750\right)^2-2460\right.}{}$

input

```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE
)
```

output

```
-1/490*(35*x+397)*(5*x^2+2*x+3)^(1/2)-8233/8575*5^(1/2)*arcsinh(5/14*14^(1
/2)*(x+1/5))+6/3773*(-5028+1331*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)
*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1
/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11
^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+6/3773*(5028+1331*11^
(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2
)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*
(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34
*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(143) = 286$.

Time = 0.11 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.51

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx =$$

$$-\frac{3}{686}\sqrt{\frac{1}{11}}\sqrt{146555\sqrt{11}+497041}\log\left(\frac{3\left(\sqrt{\frac{1}{11}}\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}\left(265\sqrt{11}-9\right.\right.}{x}\right.$$

$$+\frac{3}{686}\sqrt{\frac{1}{11}}\sqrt{146555\sqrt{11}+497041}\log\left(-\frac{3\left(\sqrt{\frac{1}{11}}\sqrt{5x^2+2x+3}\sqrt{146555\sqrt{11}+497041}\left(265\sqrt{11}-\right.\right.}{x}\right.$$

$$-\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397)$$

$$+\frac{8233}{17150}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

$$-\frac{1}{686}\sqrt{-\frac{1318995}{11}\sqrt{11}+\frac{4473369}{11}}\log\left(-\frac{\sqrt{5x^2+2x+3}\left(265\sqrt{11}+957\right)\sqrt{-\frac{1318995}{11}\sqrt{11}+\frac{4473369}{11}}+}{x}\right.$$

$$+\frac{1}{686}\sqrt{-\frac{1318995}{11}\sqrt{11}+\frac{4473369}{11}}\log\left(\frac{\sqrt{5x^2+2x+3}\left(265\sqrt{11}+957\right)\sqrt{-\frac{1318995}{11}\sqrt{11}+\frac{4473369}{11}}-1}{x}\right.$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="fri
cas")
```

output
```
-3/686*sqrt(1/11)*sqrt(146555*sqrt(11) + 497041)*log(3*(sqrt(1/11)*sqrt(5*
x^2 + 2*x + 3)*sqrt(146555*sqrt(11) + 497041)*(265*sqrt(11) - 957) + 6517*
sqrt(11)*(x + 3) + 19551*x - 32585)/x) + 3/686*sqrt(1/11)*sqrt(146555*sqrt
(11) + 497041)*log(-3*(sqrt(1/11)*sqrt(5*x^2 + 2*x + 3)*sqrt(146555*sqrt(1
1) + 497041)*(265*sqrt(11) - 957) - 6517*sqrt(11)*(x + 3) - 19551*x + 3258
5)/x) - 1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/17150*sqrt(5)*log(
sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 1/686*sqrt(
-1318995/11*sqrt(11) + 4473369/11)*log(-(sqrt(5*x^2 + 2*x + 3)*(265*sqrt(1
1) + 957)*sqrt(-1318995/11*sqrt(11) + 4473369/11) + 19551*sqrt(11)*(x + 3)
 - 58653*x + 97755)/x) + 1/686*sqrt(-1318995/11*sqrt(11) + 4473369/11)*log
((sqrt(5*x^2 + 2*x + 3)*(265*sqrt(11) + 957)*sqrt(-1318995/11*sqrt(11) + 4
473369/11) - 19551*sqrt(11)*(x + 3) + 58653*x - 97755)/x)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\int \frac{2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1),x)
```

output
```
-Integral(2*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x*s
qrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(x**2*sqrt(5*x**2 +
 2*x + 3)/(7*x**2 - 4*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(143) = 286$.

Time = 0.16 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.49

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="max
ima")
```

output
```
1/188650*sqrt(11)*(975*sqrt(11)*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/
7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqr
t(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x
- 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) - 122
5*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 16466*sqrt(11)*sqrt(5)*arcsinh(5/14*s
qrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) - 6825*sqrt(11)*sqrt(-34/49*sqrt(
11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*s
qrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
+ 2*sqrt(11) - 4)) + 4575*sqrt(2)*sqrt(17*sqrt(11) + 125)*arcsinh(5/7*sqrt
(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x
/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sq
rt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4)) + 32025*sqr
t(-34/49*sqrt(11) + 250/49)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*
x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) +
1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqr
t(2)/abs(14*x + 2*sqrt(11) - 4)) - 13895*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = -\frac{1}{490}\sqrt{5x^2+2x+3}(35x+397) + \frac{8233}{8575}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right) + 2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right) - 0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right) - 2.61475869687464\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right) + 0.276245077121866\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x, algorithm="gia
c")
```

output
```
-1/490*sqrt(5*x^2 + 2*x + 3)*(35*x + 397) + 8233/8575*sqrt(5)*log(-5*sqrt(
5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 2.61475869687464*log(-sqrt(5)*
x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.276245077121866*log(-sqr
t(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 2.61475869687464*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.276245077121866
*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = \int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{-7\,x^2+4\,x+1}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1),x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{1+4x-7x^2}\,dx = \int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}dx$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)
```

output
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1),x)
```

3.6 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 199

$$\begin{aligned}\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx &= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{154\,(1+4x-7x^2)} + \frac{1}{49}\sqrt{5}\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)\\ &\quad - \frac{\sqrt{\frac{325022311+39132731\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}\\ &\quad + \frac{\sqrt{\frac{325022311-39132731\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2156}\end{aligned}$$

output

```
3*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-1078*x^2+616*x+154)+1/49*arcsinh(1/14*(1+
5*x)*14^(1/2))*5^(1/2)-1/3011932*(454056168467+54668425207*11^(1/2))^(1/2)
*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*
x+3)^(1/2))+1/3011932*(454056168467-54668425207*11^(1/2))^(1/2)*arctanh((2
3+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.15

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$$
$$= \frac{-\frac{5145(3+61x)\sqrt{3+2x+5x^2}}{-1-4x+7x^2} - 5390\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right) - 55\text{RootSum}\left[83-16\sqrt{5}\#1-70\right.}{}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

output
```
((-5145*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(-1 - 4*x + 7*x^2) - 5390*Sqrt[5
]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]] - 55*RootSum[83 - 16*Sqrt[
5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-314239*Log[-(Sqrt[5]*x) +
Sqrt[3 + 2*x + 5*x^2] - #1] + 28462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*
x + 5*x^2] - #1]*#1 - 11221*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]
*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*Root
Sum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (599633*Sqr
t[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 391895*Log[-(Sqrt[5]
*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 21462*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#
1^3) & ])/264110
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2132, 27, 2143, 25, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^2}\,dx$$

$$\Big\downarrow \text{2132}$$

$$\frac{3(61x+3)\sqrt{5x^2+2x+3}}{154\left(-7x^2+4x+1\right)}-\frac{1}{308}\int -\frac{4\left(-55x^2+47x+237\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{77}\int \frac{-55x^2+47x+237}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{77}\left(\frac{55}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{109x+1604}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{77}\left(\frac{55}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{7}\int \frac{109x+1604}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{77}\left(\frac{1}{7}\int \frac{109x+1604}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx+\frac{11}{14}\sqrt{\frac{5}{14}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{77}\left(\frac{1}{7}\int \frac{109x+1604}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx+\frac{11}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{77}\left(\frac{1}{7}\left(\frac{1}{11}\left(1199-11446\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(1199+11446\sqrt{11}\right)\int \frac{}{2\left(-7x+\right.}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{77}\left(\frac{1}{7}\left(\frac{1}{22}\left(1199-11446\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(1199+11446\sqrt{11}\right)\int \frac{}{\left(-7x+\sqrt{1}\right.}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 1154$$

$$\frac{1}{77}\left(\frac{1}{7}\left(-\frac{1}{11}\left(1199-11446\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+\right.}{\sqrt{5x^2+2x+3}}\right.\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{77}\left(\frac{11}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)+\frac{1}{7}\left(\frac{\left(1199-11446\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(1199+11446\right.}{}\right.\right.$$
$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{154\left(-7x^2+4x+1\right)}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^2,x]
```

output
```
(3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(154*(1 + 4*x - 7*x^2)) + ((11*Sqrt[5
]*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/7 + (((1199 - 11446*Sqrt[11])*ArcTanh[
(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3
+ 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((1199 + 11446*Sqrt[1
1])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[1
1])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/7)/77
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{3(3+61x)\sqrt{5x^2+2x+3}}{154(7x^2-4x-1)}+\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{49}+\frac{\left(-11446+109\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49(\cdots}{\cdots}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2\cdots}}\cdots\right.}{11858\sqrt{250-34\sqrt{11}}}\cdots$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBO
SE)
```

output
```
-3/154*(3+61*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/49*5^(1/2)*arcsinh(5/1
4*14^(1/2)*(x+1/5))+1/11858*(-11446+109*11^(1/2))*11^(1/2)/(250-34*11^(1/2
))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1
/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7
-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+1/11858*(1144
6+109*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/4
9*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1
/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(145) = 290$.

Time = 0.12 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.78

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx$$

$$= \frac{44\sqrt{5}(7x^2-4x-1)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)-(7x^2-4x-1)\sqrt{\frac{3557521}{127}}}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="f
ricas")
```

output
```
1/4312*(44*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5
*x + 1) - 25*x^2 - 10*x - 8) - (7*x^2 - 4*x - 1)*sqrt(3557521/127*sqrt(11)
 + 325022311/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(3557521/127*sqrt(11) +
 325022311/1397)*(16943*sqrt(11) + 235367) + 18697175*sqrt(11)*(x + 3) - 5
6091525*x + 93485875)/x) + (7*x^2 - 4*x - 1)*sqrt(3557521/127*sqrt(11) + 3
25022311/1397)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(3557521/127*sqrt(11) + 3250
22311/1397)*(16943*sqrt(11) + 235367) - 18697175*sqrt(11)*(x + 3) + 560915
25*x - 93485875)/x) - (7*x^2 - 4*x - 1)*sqrt(-3557521/127*sqrt(11) + 32502
2311/1397)*log((sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) - 235367)*sqrt(-3557
521/127*sqrt(11) + 325022311/1397) + 18697175*sqrt(11)*(x + 3) + 56091525*
x - 93485875)/x) + (7*x^2 - 4*x - 1)*sqrt(-3557521/127*sqrt(11) + 32502231
1/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*(16943*sqrt(11) - 235367)*sqrt(-355752
1/127*sqrt(11) + 325022311/1397) - 18697175*sqrt(11)*(x + 3) - 56091525*x
+ 93485875)/x) - 84*sqrt(5*x^2 + 2*x + 3)*(61*x + 3))/(7*x^2 - 4*x - 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx=\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**2,x)
```

output
```
Integral((x**2 + 5*x + 2)*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1)**2, x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx=\int \frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="m
axima")
```

output
```
integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{184473632,[8]%%%}+%%%{%%{[421654016,0]:[1,0,-5]%%},[7]%%%}
+%%%{-248
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{(-7\,x^2+4\,x+1)^2}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^2}dx$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x)
```

output
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2,x)
```

3.7 $\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 213

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3} dx \\
&= \frac{3(3+61x)\sqrt{3+2x+5x^2}}{308(1+4x-7x^2)^2} - \frac{(272941-813113x)\sqrt{3+2x+5x^2}}{1721104(1+4x-7x^2)} \\
&- \frac{\sqrt{\frac{6492253020949-11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+(17-5\sqrt{11})x}{\sqrt{2(125-17\sqrt{11})}\sqrt{3+2x+5x^2}}\right)}{491744} \\
&+ \frac{\sqrt{\frac{6492253020949+11879169071\sqrt{11}}{1397}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+(17+5\sqrt{11})x}{\sqrt{2(125+17\sqrt{11})}\sqrt{3+2x+5x^2}}\right)}{491744}
\end{aligned}$$

output
```
3/308*(3+61*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-(272941-813113*x)*(5*x
^2+2*x+3)^(1/2)/(-12047728*x^2+6884416*x+1721104)-1/686966368*(90696774702
65753-16595199192187*11^(1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2))*
x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))+1/686966368*(9069677470265
753+16595199192187*11^(1/2))^(1/2)*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)
/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.12 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.83

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$= \frac{\frac{5764801\sqrt{3+2x+5x^2}(-31807+106279x+737577x^2-813113x^3)}{(1+4x-7x^2)^2} - 60545521580434\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+\right.}{}$$

input
```
Integrate[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

output
```
((5764801*Sqrt[3 + 2*x + 5*x^2]*(-31807 + 106279*x + 737577*x^2 - 813113*x
^3))/(1 + 4*x - 7*x^2)^2 - 60545521580434*RootSum[83 - 16*Sqrt[5]*#1 - 70*
#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2
] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 20661853520*R
ootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-465*Lo
g[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7
*#1^3) & ] + 22*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*
#1^4 & , (3751778663030*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]*#1 + 2597308755559*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^
2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*RootSum[83 - 16*
Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-11648778057271*Log[-(
Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 13372446682211*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 9645047011740*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ])/1417403151472
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {2132, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\sqrt{5x^2+2x+3}}{(-7x^2+4x+1)^3}dx$$

$$\Big\downarrow \text{2132}$$

$$\frac{3(61x+3)\sqrt{5x^2+2x+3}}{308(-7x^2+4x+1)^2}-\frac{1}{616}\int -\frac{4(805x^2+391x+753)}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\int \frac{805x^2+391x+753}{(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}dx+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{154}\left(-\frac{\int -\frac{56(126542x+212417)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{\sqrt{5x^2+2x+3}(272941-813113x)}{11176(-7x^2+4x+1)}\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\left(\frac{7\int \frac{126542x+212417}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{(272941-813113x)\sqrt{5x^2+2x+3}}{11176(-7x^2+4x+1)}\right)+\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308(-7x^2+4x+1)^2}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{154}\left(\frac{7\left(\frac{1}{11}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(1391962+1740003\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right.}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

$\big\downarrow$ 27

$$\frac{1}{154}\left(\frac{7\left(\frac{1}{22}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(1391962+1740003\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

$\big\downarrow$ 1154

$$\frac{1}{154}\left(\frac{7\left(-\frac{1}{11}\left(1391962-1740003\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{1}{11}\left(\right.\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

$\big\downarrow$ 219

$$\frac{1}{154}\left(\frac{7\left(\frac{\left(1391962-1740003\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(1391962+1740003\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+2}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+}}\right.}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.}{5588}\right.$$

$$\frac{3\sqrt{5x^2+2x+3}(61x+3)}{308\left(-7x^2+4x+1\right)^2}$$

input
```
Int[((2 + 5*x + x^2)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(3*(3 + 61*x)*Sqrt[3 + 2*x + 5*x^2])/(308*(1 + 4*x - 7*x^2)^2) + (-1/11176
*((272941 - 813113*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + (7*(((139
1962 - 1740003*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sq
rt[11])]) + ((1391962 + 1740003*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5
*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sq
rt[2*(125 + 17*Sqrt[11])])))/5588)/154
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2135

```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{(813113x^3-737577x^2-106279x+31807)\sqrt{5x^2+2x+3}}{245872(7x^2-4x-1)^2}+\frac{\left(-1740003+126542\sqrt{11}\right)\sqrt{11}\ \mathrm{arctanh}\left(\frac{250-34\sqrt{11}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{}}{}\right.}}\right.}{2704592\sqrt{250-34\sqrt{1}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/245872*(813113*x^3-737577*x^2-106279*x+31807)/(7*x^2-4*x-1)^2*(5*x^2+2*
x+3)^(1/2)+1/2704592*(-1740003+126542*11^(1/2))*11^(1/2)/(250-34*11^(1/2))
^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+1/2704592*(1740
003+126542*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49
+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2
))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11
^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(160) = 320.

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.72

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx =$$

$$-\frac{(49\,x^4-56\,x^3+2\,x^2+8\,x+1)\sqrt{\frac{1079924461}{127}\,\sqrt{11}+\frac{6492253020949}{1397}}\log\left(\frac{\sqrt{5\,x^2+2\,x+3}\sqrt{\frac{1079924461}{127}\,\sqrt{11}+\frac{64922530209}{1397}}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="f
ricas")
```

output
```
-1/983488*((49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(1079924461/127*sqrt(11
) + 6492253020949/1397)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(1079924461/127*sqr
t(11) + 6492253020949/1397)*(4822219*sqrt(11) - 37335441) + 407352683515*s
qrt(11)*(x + 3) + 1222058050545*x - 2036763417575)/x) - (49*x^4 - 56*x^3 +
 2*x^2 + 8*x + 1)*sqrt(1079924461/127*sqrt(11) + 6492253020949/1397)*log(-
(sqrt(5*x^2 + 2*x + 3)*sqrt(1079924461/127*sqrt(11) + 6492253020949/1397)*
(4822219*sqrt(11) - 37335441) - 407352683515*sqrt(11)*(x + 3) - 1222058050
545*x + 2036763417575)/x) + (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1079
924461/127*sqrt(11) + 6492253020949/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*(482
2219*sqrt(11) + 37335441)*sqrt(-1079924461/127*sqrt(11) + 6492253020949/13
97) + 407352683515*sqrt(11)*(x + 3) - 1222058050545*x + 2036763417575)/x)
- (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-1079924461/127*sqrt(11) + 6492
253020949/1397)*log((sqrt(5*x^2 + 2*x + 3)*(4822219*sqrt(11) + 37335441)*s
qrt(-1079924461/127*sqrt(11) + 6492253020949/1397) - 407352683515*sqrt(11)
*(x + 3) + 1222058050545*x - 2036763417575)/x) + 4*(813113*x^3 - 737577*x^
2 - 106279*x + 31807)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^2 + 8*
x + 1)
```

Sympy [F]

$$\begin{aligned}
&\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx \\
&= -\int \frac{2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\
&\quad -\int \frac{5x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx \\
&\quad -\int \frac{x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx
\end{aligned}$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(1/2)/(-7*x**2+4*x+1)**3,x)
```

output
```
-Integral(2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x
**3 - 27*x**2 - 12*x - 1), x) - Integral(5*x*sqrt(5*x**2 + 2*x + 3)/(343*x
**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral(
x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 2
7*x**2 - 12*x - 1), x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \int -\frac{\sqrt{5\,x^2+2\,x+3}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^3}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="m
axima")
```

output
```
-integrate(sqrt(5*x^2 + 2*x + 3)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(160) = 320$.

Time = 0.17 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.77

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx$$
$$= \frac{6200558\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7-835775\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-190947036\left(\sqrt{5}x-\sqrt{5\,x^2\right.}{430276\left(7\left(\sqrt{5}x-\sqrt{5\right.\right.}$$
$$+0.139051039089282\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.138209741946053\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.139051039089282\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.138209741946053\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x, algorithm="g
iac")
```

output
```
1/430276*(6200558*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 835775*sqrt(5)*(
sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 190947036*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3))^5 - 92732607*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 81
6321374*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 419437335*sqrt(5)*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 - 765111048*sqrt(5)*x - 376983161*sqrt(5) +
765111048*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4
- 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5
*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^
2 + 0.139051039089282*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.419247364
59000) - 0.138209741946053*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2529
5163054000) - 0.139051039089282*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1
.02258038113000) + 0.138209741946053*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3
) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx=\int \frac{(x^2+5\,x+2)\,\sqrt{5\,x^2+2\,x+3}}{{(-7\,x^2+4\,x+1)}^3}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(1/2))/(4*x - 7*x^2 + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 69.45 (sec) , antiderivative size = 2193, normalized size of antiderivative = 10.30

$$\int \frac{(2+5x+x^2)\sqrt{3+2x+5x^2}}{(1+4x-7x^2)^3}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^3,x)
```

output
```
( - 1356859147*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x
 + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(
17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) -
 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(
2))/(8890*x**2 + 3556*x + 5334))*x**4 + 1550696168*sqrt(17*sqrt(11) - 125)
*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)
*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(
5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*
x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x**3
- 55382006*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3
)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*s
qrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125
)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/
(8890*x**2 + 3556*x + 5334))*x**2 - 221528024*sqrt(17*sqrt(11) - 125)*sqrt
(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x -
19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**
2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3
)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x - 276910
03*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(1
7*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(1...
```

3.8 $\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx$

Optimal result

Integrand size = 35, antiderivative size = 231

$$\begin{aligned}
&\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = \\
&-\frac{479652579(1+5x)\sqrt{3+2x+5x^2}}{312500000} \\
&-\frac{22840599(1+5x)(3+2x+5x^2)^{3/2}}{62500000}-\frac{6133820867(3+2x+5x^2)^{5/2}}{1203125000} \\
&+\frac{837379699x(3+2x+5x^2)^{5/2}}{72187500}+\frac{2173004363x^2(3+2x+5x^2)^{5/2}}{173250000} \\
&-\frac{190236913x^3(3+2x+5x^2)^{5/2}}{4950000}-\frac{796559x^4(3+2x+5x^2)^{5/2}}{123750} \\
&+\frac{1031177x^5(3+2x+5x^2)^{5/2}}{20625}-\frac{61103x^6(3+2x+5x^2)^{5/2}}{3300} \\
&-\frac{343}{60}x^7(3+2x+5x^2)^{5/2}-\frac{3357568053\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250000\sqrt{5}}
\end{aligned}$$

output
```
-479652579/312500000*(1+5*x)*(5*x^2+2*x+3)^(1/2)-22840599/62500000*(1+5*x)
*(5*x^2+2*x+3)^(3/2)-6133820867/1203125000*(5*x^2+2*x+3)^(5/2)+837379699/7
2187500*x*(5*x^2+2*x+3)^(5/2)+2173004363/173250000*x^2*(5*x^2+2*x+3)^(5/2)
-190236913/4950000*x^3*(5*x^2+2*x+3)^(5/2)-796559/123750*x^4*(5*x^2+2*x+3)
^(5/2)+1031177/20625*x^5*(5*x^2+2*x+3)^(5/2)-61103/3300*x^6*(5*x^2+2*x+3)^
(5/2)-343/60*x^7*(5*x^2+2*x+3)^(5/2)-3357568053/781250000*arcsinh(1/14*(1+
5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.47

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \frac{\sqrt{3+2x+5x^2}(-10506617068392+6352777129950x+15865844408685x^2+1904168823\ldots}{} + \frac{3357568053 \log \left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-10506617068392 + 6352777129950*x + 15865844408685
*x^2 + 19041688239675*x^3 + 2573089891000*x^4 - 85130334087500*x^5 - 52106
830406250*x^6 + 72918247281250*x^7 + 30505457500000*x^8 + 148393743750000*
x^9 - 125007421875000*x^10 - 30950390625000*x^11))/216562500000 + (3357568
053*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(156250000*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.457$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^3 \left(x^2+5x+2\right) \left(5x^2+2x+3\right)^{3/2} \, dx$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{60}\int \left(5x^2+2x+3\right)^{3/2} \left(-61103x^7+131103x^6+7620x^5-52260x^4-3660x^3+6900x^2+1740x+120\right) dx - \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{60}\left(\frac{1}{55}\int 2\left(5x^2+2x+3\right)^{3/2} \left(4124708x^6+759477x^5-1437150x^4-100650x^3+189750x^2+47850x+3300\right) d\right. \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{60}\left(\frac{2}{55}\int \left(5x^2+2x+3\right)^{3/2} \left(4124708x^6+759477x^5-1437150x^4-100650x^3+189750x^2+47850x+3300\right) dx\right. \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{1}{50}\int 30\left(5x^2+2x+3\right)^{3/2} \left(-796559x^5-4457604x^4-167750x^3+316250x^2+79750x+5500\right) dx + \right.\right. \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\int \left(5x^2+2x+3\right)^{3/2} \left(-796559x^5-4457604x^4-167750x^3+316250x^2+79750x+5500\right) dx + \frac{2062}{25}\right.\right. \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\int \left(5x^2+2x+3\right)^{3/2} \left(-190236913x^4+2009958x^3+14231250x^2+3588750x+247500\right) dx - \frac{79}{}\right.\right.\right. \frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\int \left(5x^2+2x+3\right)^{3/2}\left(2173004363x^3+2281382217x^2+143550000x+9900000\right)dx-\frac{19023}{4}\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{1}{35}\int 6\left(5x^2+2x+3\right)^{3/2}\left(10048556388x^2-1335629363x+57750000\right)dx+\frac{2173004363}{35}x\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\int \left(5x^2+2x+3\right)^{3/2}\left(10048556388x^2-1335629363x+57750000\right)dx+\frac{2173004363}{35}x^2\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 2192

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{30}\int -18(6133820867x+1578509398)\left(5x^2+2x+3\right)^{3/2}dx+\frac{1674759398}{5}x\left(5x^2+2\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 27

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\int(6133820867x+1578509398)\left(5x^2+2x+3\right)^{3/}\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 1160

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\int \left(5x^2+2x+3\right)^{3/2}dx+\frac{6133820}{25}\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 1087

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\int \sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x\right.\right.\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{60}x^7\left(5x^2+2x+3\right)^{5/2}$$

$\downarrow$ 1087

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x(5x^2+2x+3)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{60}x^7(5x^2+2x+3)^{5/2}$$

$\downarrow$ 1090

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x(5x^2+2x+3)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2}}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{60}x^7(5x^2+2x+3)^{5/2}$$

$\downarrow$ 222

$$\frac{1}{60}\left(\frac{2}{55}\left(\frac{3}{5}\left(\frac{1}{45}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1674759398}{5}x(5x^2+2x+3)^{5/2}-\frac{3}{5}\left(\frac{1758726123}{5}\left(\frac{21}{10}\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5}\right.\right.\right.\right.\right.\right.\right.\right.\right.$$

$$\frac{343}{60}x^7(5x^2+2x+3)^{5/2}$$

input
```
Int[(1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-343*x^7*(3 + 2*x + 5*x^2)^(5/2))/60 + ((-61103*x^6*(3 + 2*x + 5*x^2)^(5/
2))/55 + (2*((2062354*x^5*(3 + 2*x + 5*x^2)^(5/2))/25 + (3*((-796559*x^4*(
3 + 2*x + 5*x^2)^(5/2))/45 + ((-190236913*x^3*(3 + 2*x + 5*x^2)^(5/2))/40
+ ((2173004363*x^2*(3 + 2*x + 5*x^2)^(5/2))/35 + (6*((1674759398*x*(3 + 2*
x + 5*x^2)^(5/2))/5 - (3*((6133820867*(3 + 2*x + 5*x^2)^(5/2))/25 + (17587
26123*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/20 + (21*(((1 + 5*x)*Sqrt[3 + 2
*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5])))/10))/
5))/5))/35)/40)/45))/5))/55)/60
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.37

method	result
risch	$-\frac{(30950390625000x^{11}+125007421875000x^{10}-148393743750000x^{9}-30505457500000x^{8}-72918247281250x^{7}+52106830406250x$ ... ${216}$
trager	$\left(-\frac{1715}{12}x^{11}-\frac{76195}{132}x^{10}+\frac{376873}{550}x^{9}+\frac{1743169}{12375}x^{8}+\frac{333340559}{990000}x^{7}-\frac{555806191}{2310000}x^{6}-\frac{6810426727}{17325000}x^{5}+\frac{25730898}{21656250}\right.$
default	$-\frac{22840599(10x+2)(5x^{2}+2x+3)^{\frac{3}{2}}}{125000000}-\frac{479652579(10x+2)\sqrt{5x^{2}+2x+3}}{625000000}-\frac{3357568053\sqrt{5}\,\mathrm{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250000}-\frac{61338208}{1}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/216562500000*(30950390625000*x^11+125007421875000*x^10-148393743750000*
x^9-30505457500000*x^8-72918247281250*x^7+52106830406250*x^6+8513033408750
0*x^5-2573089891000*x^4-19041688239675*x^3-15865844408685*x^2-635277712995
0*x+10506617068392)*(5*x^2+2*x+3)^(1/2)-3357568053/781250000*5^(1/2)*arcsi
nh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.46

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx =$$
$$-\frac{1}{216562500000}\left(30950390625000\,x^{11}+125007421875000\,x^{10}-148393743750000\,x^{9}-30505457500000\,x\right.$$
$$+\frac{3357568053}{1562500000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/216562500000*(30950390625000*x^11 + 125007421875000*x^10 - 148393743750
000*x^9 - 30505457500000*x^8 - 72918247281250*x^7 + 52106830406250*x^6 + 8
5130334087500*x^5 - 2573089891000*x^4 - 19041688239675*x^3 - 1586584440868
5*x^2 - 6352777129950*x + 10506617068392)*sqrt(5*x^2 + 2*x + 3) + 33575680
53/1562500000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2
 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.48

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \sqrt{5x^2+2x+3}\left(-\frac{1715x^{11}}{12} - \frac{76195x^{10}}{132} + \frac{376873x^9}{550} + \frac{1743169x^8}{12375} + \frac{333340559x^7}{990000} - \frac{555806191x^6}{2310000} - \frac{6810426727x^5}{17325000} + \frac{2573089891x^4}{216562500} + \frac{253889176529x^3}{2887500000} + \frac{352574320193x^2}{4812500000} + \frac{14117282511x}{481250000} - \frac{145925237061}{3007812500}\right) - \frac{3357568053\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}(x+\frac{1}{5})}{14}\right)}{781250000}$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-1715*x**11/12 - 76195*x**10/132 + 376873*x**9/550
 + 1743169*x**8/12375 + 333340559*x**7/990000 - 555806191*x**6/2310000 - 6
810426727*x**5/17325000 + 2573089891*x**4/216562500 + 253889176529*x**3/28
87500000 + 352574320193*x**2/4812500000 + 14117282511*x/481250000 - 145925
237061/3007812500) - 3357568053*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/781
250000
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.89

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = -\frac{343}{60} (5x^2+2x+3)^{\frac{5}{2}} x^7 - \frac{61103}{3300} (5x^2+2x+3)^{\frac{5}{2}} x^6 + \frac{1031177}{20625} (5x^2+2x+3)^{\frac{5}{2}} x^5 - \frac{796559}{123750} (5x^2+2x+3)^{\frac{5}{2}} x^4 - \frac{190236913}{4950000} (5x^2+2x+3)^{\frac{5}{2}} x^3 + \frac{2173004363}{173250000} (5x^2+2x+3)^{\frac{5}{2}} x^2 + \frac{837379699}{72187500} (5x^2+2x+3)^{\frac{5}{2}} x - \frac{6133820867}{1203125000} (5x^2+2x+3)^{\frac{5}{2}} - \frac{22840599}{12500000} (5x^2+2x+3)^{\frac{3}{2}} x - \frac{22840599}{62500000} (5x^2+2x+3)^{\frac{3}{2}} - \frac{479652579}{62500000} \sqrt{5x^2+2x+3}\,x - \frac{3357568053}{781250000} \sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{479652579}{312500000} \sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-343/60*(5*x^2 + 2*x + 3)^(5/2)*x^7 - 61103/3300*(5*x^2 + 2*x + 3)^(5/2)*x
^6 + 1031177/20625*(5*x^2 + 2*x + 3)^(5/2)*x^5 - 796559/123750*(5*x^2 + 2*
x + 3)^(5/2)*x^4 - 190236913/4950000*(5*x^2 + 2*x + 3)^(5/2)*x^3 + 2173004
363/173250000*(5*x^2 + 2*x + 3)^(5/2)*x^2 + 837379699/72187500*(5*x^2 + 2*
x + 3)^(5/2)*x - 6133820867/1203125000*(5*x^2 + 2*x + 3)^(5/2) - 22840599/
12500000*(5*x^2 + 2*x + 3)^(3/2)*x - 22840599/62500000*(5*x^2 + 2*x + 3)^(
3/2) - 479652579/62500000*sqrt(5*x^2 + 2*x + 3)*x - 3357568053/781250000*s
qrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 479652579/312500000*sqrt(5*x^2 +
 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.44

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx =$$
$$-\frac{1}{216562500000}(5((5(10(25(5(7(20(105(875(77x+311)x-323034)x-6972676)x-333340559)x+$$
$$+\frac{3357568053}{781250000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-1/216562500000*(5*((5*(10*(25*(5*(7*(20*(105*(875*(77*x + 311)*x - 323034
)*x - 6972676)*x - 333340559)*x + 1667418573)*x + 13620853454)*x - 1029235
9564)*x - 761667529587)*x - 3173168881737)*x - 1270555425990)*x + 10506617
068392)*sqrt(5*x^2 + 2*x + 3) + 3357568053/781250000*sqrt(5)*log(-sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \left(1+4x-7x^2\right)^3 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} dx = \int \left(x^2+5x+2\right) \left(5x^2+2x+3\right)^{3/2} \left(-7x^2+4x+1\right)^3 dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3,x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

$$\int (1+4x-7x^2)^3 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = -\frac{1715\sqrt{5x^2+2x+3}\,x^{11}}{12} - \frac{76195\sqrt{5x^2+2x+3}\,x^{10}}{132} + \frac{376873\sqrt{5x^2+2x+3}\,x^9}{550} + \frac{1743169\sqrt{5x^2+2x+3}\,x^8}{12375} + \frac{333340559\sqrt{5x^2+2x+3}\,x^7}{990000} - \frac{555806191\sqrt{5x^2+2x+3}\,x^6}{2310000} - \frac{6810426727\sqrt{5x^2+2x+3}\,x^5}{17325000} + \frac{2573089891\sqrt{5x^2+2x+3}\,x^4}{216562500} + \frac{253889176529\sqrt{5x^2+2x+3}\,x^3}{2887500000} + \frac{352574320193\sqrt{5x^2+2x+3}\,x^2}{4812500000} + \frac{14117282511\sqrt{5x^2+2x+3}\,x}{481250000} - \frac{145925237061\sqrt{5x^2+2x+3}}{3007812500} - \frac{3357568053\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{781250000}$$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 154751953125000*sqrt(5*x**2 + 2*x + 3)*x**11 - 625037109375000*sqrt(5*
x**2 + 2*x + 3)*x**10 + 741968718750000*sqrt(5*x**2 + 2*x + 3)*x**9 + 1525
27287500000*sqrt(5*x**2 + 2*x + 3)*x**8 + 364591236406250*sqrt(5*x**2 + 2*
x + 3)*x**7 - 260534152031250*sqrt(5*x**2 + 2*x + 3)*x**6 - 42565167043750
0*sqrt(5*x**2 + 2*x + 3)*x**5 + 12865449455000*sqrt(5*x**2 + 2*x + 3)*x**4
 + 95208441198375*sqrt(5*x**2 + 2*x + 3)*x**3 + 79329222043425*sqrt(5*x**2
 + 2*x + 3)*x**2 + 31763885649750*sqrt(5*x**2 + 2*x + 3)*x - 5253308534196
0*sqrt(5*x**2 + 2*x + 3) - 4653589321458*sqrt(5)*log((sqrt(5*x**2 + 2*x +
3)*sqrt(5) + 5*x + 1)/sqrt(14)))/1082812500000
```

3.9 $\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx$

Optimal result

Integrand size = 35, antiderivative size = 189

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = -\frac{14501781(1+5x)\sqrt{3+2x+5x^2}}{6250000} - \frac{690561(1+5x)(3+2x+5x^2)^{3/2}}{1250000} + \frac{505667(3+2x+5x^2)^{5/2}}{2187500} + \frac{86721x(3+2x+5x^2)^{5/2}}{21875} - \frac{219271x^2(3+2x+5x^2)^{5/2}}{105000} - \frac{18379x^3(3+2x+5x^2)^{5/2}}{3000} + \frac{581}{150}x^4(3+2x+5x^2)^{5/2} + \frac{49}{50}x^5(3+2x+5x^2)^{5/2} - \frac{101512467\,\mathrm{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{3125000\sqrt{5}}$$

output
```
-14501781/6250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)-690561/1250000*(1+5*x)*(5*x^
2+2*x+3)^(3/2)+505667/2187500*(5*x^2+2*x+3)^(5/2)+86721/21875*x*(5*x^2+2*x
+3)^(5/2)-219271/105000*x^2*(5*x^2+2*x+3)^(5/2)-18379/3000*x^3*(5*x^2+2*x+
3)^(5/2)+581/150*x^4*(5*x^2+2*x+3)^(5/2)+49/50*x^5*(5*x^2+2*x+3)^(5/2)-101
512467/15625000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2}\,dx = \frac{\sqrt{3+2x+5x^2}(-249003936+2291675850x+3721040355x^2+5959365525x^3-322759700}{1312} + \frac{101512467\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{3125000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-249003936 + 2291675850*x + 3721040355*x^2 + 59593
65525*x^3 - 3227597000*x^4 - 12554262500*x^5 - 4105593750*x^6 - 5561281250
*x^7 + 15281875000*x^8 + 3215625000*x^9))/131250000 + (101512467*Log[-1 -
5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(3125000*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.20, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-7x^2+4x+1\right)^2 \left(x^2+5x+2\right) \left(5x^2+2x+3\right)^{3/2}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{50}\int 5\left(5x^2+2x+3\right)^{3/2}\left(1743x^5-1947x^4-940x^3+450x^2+210x+20\right)dx + \frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\int \left(5x^2+2x+3\right)^{3/2}\left(1743x^5-1947x^4-940x^3+450x^2+210x+20\right)dx+\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{10}\left(\frac{1}{45}\int 6\left(5x^2+2x+3\right)^{3/2}\left(-18379x^4-10536x^3+3375x^2+1575x+150\right)dx+\frac{581}{15}\left(5x^2+2x+3\right)^{5/2}x^4\right)+\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}\left(\frac{2}{15}\int \left(5x^2+2x+3\right)^{3/2}\left(-18379x^4-10536x^3+3375x^2+1575x+150\right)dx+\frac{581}{15}\left(5x^2+2x+3\right)^{5/2}x^4\right)+\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\int \left(5x^2+2x+3\right)^{3/2}\left(-219271x^3+300411x^2+63000x+6000\right)dx-\frac{18379}{40}x^3\left(5x^2+2x+3\right)^{5/2}\right)\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{1}{35}\int 6\left(5x^2+2x+3\right)^{3/2}\left(2081304x^2+586771x+35000\right)dx-\frac{219271}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\int \left(5x^2+2x+3\right)^{3/2}\left(2081304x^2+586771x+35000\right)dx-\frac{219271}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \text{ 2192}$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{30}\int -6(865652-505667x)\left(5x^2+2x+3\right)^{3/2}dx+\frac{346884}{5}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{219271}{35}x\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{346884}{5}x\left(5x^2+2x+3\right)^{5/2}-\frac{1}{5}\int (865652-505667x)\left(5x^2+2x+3\right)^{3/2}dx\right)-\frac{219271}{35}x^2(5x\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{4833927}{5}\int \left(5x^2+2x+3\right)^{3/2}dx\right)+\frac{346884}{5}x\left(5x^2+2x+3\right)^{5}\right.\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\int \sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x+1)\left(5x^2+2x+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

$$\Big\downarrow \; 222$$

$$\frac{1}{10}\left(\frac{2}{15}\left(\frac{1}{40}\left(\frac{6}{35}\left(\frac{1}{5}\left(\frac{505667}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{4833927}{5}\left(\frac{21}{10}\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{49}{50}\left(5x^2+2x+3\right)^{5/2}x^5$$

input
```
Int[(1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(49*x^5*(3 + 2*x + 5*x^2)^(5/2))/50 + ((581*x^4*(3 + 2*x + 5*x^2)^(5/2))/1
5 + (2*((-18379*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 + ((-219271*x^2*(3 + 2*x +
 5*x^2)^(5/2))/35 + (6*((346884*x*(3 + 2*x + 5*x^2)^(5/2))/5 + ((505667*(3
 + 2*x + 5*x^2)^(5/2))/25 - (4833927*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/
20 + (21*(((1 + 5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*
Sqrt[14])])/(5*Sqrt[5])))/10))/5)/5))/35)/40))/15)/10
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.40

method	result
risch	$\frac{(3215625000x^9+15281875000x^8-5561281250x^7-4105593750x^6-12554262500x^5-3227597000x^4+5959365525x^3+3721040355x^2}{131250000}$
trager	$\left(\frac{49}{2}x^9+\frac{3493}{30}x^8-\frac{25423}{600}x^7-\frac{43793}{1400}x^6-\frac{1004341}{10500}x^5-\frac{3227597}{131250}x^4+\frac{79458207}{1750000}x^3+\frac{248069357}{8750000}x^2+\frac{15277839}{875000}\right.$
default	$-\frac{690561(10x+2)(5x^2+2x+3)^{\frac{3}{2}}}{2500000}-\frac{14501781(10x+2)\sqrt{5x^2+2x+3}}{12500000}-\frac{101512467\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{15625000}+\frac{505667(5x^2+}{218750}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/131250000*(3215625000*x^9+15281875000*x^8-5561281250*x^7-4105593750*x^6-
12554262500*x^5-3227597000*x^4+5959365525*x^3+3721040355*x^2+2291675850*x-
249003936)*(5*x^2+2*x+3)^(1/2)-101512467/15625000*5^(1/2)*arcsinh(5/14*14^
(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.51

$$\int \left(1+4x-7x^2\right)^2\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx=\frac{1}{131250000}\left(3215625000\,x^9+15281875000\,x^8-5561281250\,x^7-4105593750\,x^6-1255426\right.$$
$$+\frac{101512467}{31250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/131250000*(3215625000*x^9 + 15281875000*x^8 - 5561281250*x^7 - 410559375
0*x^6 - 12554262500*x^5 - 3227597000*x^4 + 5959365525*x^3 + 3721040355*x^2
 + 2291675850*x - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/31250000*sq
rt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.52

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} dx = \sqrt{5x^2+2x+3} \cdot \left(\frac{49x^9}{2} + \frac{3493x^8}{30} - \frac{25423x^7}{600} - \frac{43793x^6}{1400} - \frac{1004341x^5}{10500} - \frac{3227597x^4}{131250} + \frac{79458207x^3}{1750000} + \frac{248069357x^2}{8750000} + \frac{15277839x}{875000} - \frac{5187582}{2734375}\right) - \frac{101512467\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}(x+\frac{1}{5})}{14}\right)}{15625000}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**9/2 + 3493*x**8/30 - 25423*x**7/600 - 43793*
x**6/1400 - 1004341*x**5/10500 - 3227597*x**4/131250 + 79458207*x**3/17500
00 + 248069357*x**2/8750000 + 15277839*x/875000 - 5187582/2734375) - 10151
2467*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/15625000
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{49}{50} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} x^5 + \frac{581}{150} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} x^4 - \frac{18379}{3000} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} x^3 - \frac{219271}{105000} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} x^2 + \frac{86721}{21875} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} x + \frac{505667}{2187500} \left(5 x^2 + 2 x + 3\right)^{\frac{5}{2}} - \frac{690561}{250000} \left(5 x^2 + 2 x + 3\right)^{\frac{3}{2}} x - \frac{690561}{1250000} \left(5 x^2 + 2 x + 3\right)^{\frac{3}{2}} - \frac{14501781}{1250000} \sqrt{5 x^2 + 2 x + 3} x - \frac{101512467}{15625000} \sqrt{5} \operatorname{arsinh}\left(\frac{1}{14} \sqrt{14} (5 x + 1)\right) - \frac{14501781}{6250000} \sqrt{5 x^2 + 2 x + 3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
49/50*(5*x^2 + 2*x + 3)^(5/2)*x^5 + 581/150*(5*x^2 + 2*x + 3)^(5/2)*x^4 -
18379/3000*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 219271/105000*(5*x^2 + 2*x + 3)^(
5/2)*x^2 + 86721/21875*(5*x^2 + 2*x + 3)^(5/2)*x + 505667/2187500*(5*x^2 +
 2*x + 3)^(5/2) - 690561/250000*(5*x^2 + 2*x + 3)^(3/2)*x - 690561/1250000
*(5*x^2 + 2*x + 3)^(3/2) - 14501781/1250000*sqrt(5*x^2 + 2*x + 3)*x - 1015
12467/15625000*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 14501781/6250000
*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \left(1+4x-7x^2\right)^2 \left(2+5x+x^2\right) \left(3+2x+5x^2\right)^{3/2} \, dx = \frac{1}{131250000} (5 ((5 (10 (25 (5 (7 (140 (105 x + 499)x - 25423)x - 131379)x - 2008682)x - 12 + \frac{101512467}{15625000} \sqrt{5} \log\left(-\sqrt{5}\left(\sqrt{5}x - \sqrt{5 x^2 + 2 x + 3}\right) - 1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/131250000*(5*((5*(10*(25*(5*(7*(140*(105*x + 499)*x - 25423)*x - 131379)
*x - 2008682)*x - 12910388)*x + 238374621)*x + 744208071)*x + 458335170)*x
 - 249003936)*sqrt(5*x^2 + 2*x + 3) + 101512467/15625000*sqrt(5)*log(-sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \int (x^2+5\,x+2)\,\left(5\,x^2+2\,x+3\right)^{3/2}\,\left(-7\,x^2+4\,x+1\right)^2 \, dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2,x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98

$$\int (1+4x-7x^2)^2 (2+5x+x^2) (3+2x+5x^2)^{3/2} \, dx = \frac{49\sqrt{5x^2+2x+3}\,x^9}{2} + \frac{3493\sqrt{5x^2+2x+3}\,x^8}{30} - \frac{25423\sqrt{5x^2+2x+3}\,x^7}{600} - \frac{43793\sqrt{5x^2+2x+3}\,x^6}{1400} - \frac{1004341\sqrt{5x^2+2x+3}\,x^5}{10500} - \frac{3227597\sqrt{5x^2+2x+3}\,x^4}{131250} + \frac{79458207\sqrt{5x^2+2x+3}\,x^3}{1750000} + \frac{248069357\sqrt{5x^2+2x+3}\,x^2}{8750000} + \frac{15277839\sqrt{5x^2+2x+3}\,x}{875000} - \frac{5187582\sqrt{5x^2+2x+3}}{2734375} - \frac{101512467\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{15625000}$$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

output
```
(16078125000*sqrt(5*x**2 + 2*x + 3)*x**9 + 76409375000*sqrt(5*x**2 + 2*x +
 3)*x**8 - 27806406250*sqrt(5*x**2 + 2*x + 3)*x**7 - 20527968750*sqrt(5*x*
*2 + 2*x + 3)*x**6 - 62771312500*sqrt(5*x**2 + 2*x + 3)*x**5 - 16137985000
*sqrt(5*x**2 + 2*x + 3)*x**4 + 29796827625*sqrt(5*x**2 + 2*x + 3)*x**3 + 1
8605201775*sqrt(5*x**2 + 2*x + 3)*x**2 + 11458379250*sqrt(5*x**2 + 2*x + 3
)*x - 1245019680*sqrt(5*x**2 + 2*x + 3) - 4263523614*sqrt(5)*log((sqrt(5*x
**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)))/656250000
```

3.10 $\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 147

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = -\frac{128779(1+5x)\sqrt{3+2x+5x^2}}{250000}-\frac{18397(1+5x)(3+2x+5x^2)^{3/2}}{150000}+\frac{149509(3+2x+5x^2)^{5/2}}{262500}+\frac{2809x(3+2x+5x^2)^{5/2}}{5250}-\frac{1163x^2(3+2x+5x^2)^{5/2}}{1400}-\frac{7}{40}x^3(3+2x+5x^2)^{5/2}-\frac{901453\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{125000\sqrt{5}}$$

output

```
-128779/250000*(1+5*x)*(5*x^2+2*x+3)^(1/2)-18397/150000*(1+5*x)*(5*x^2+2*x
+3)^(3/2)+149509/262500*(5*x^2+2*x+3)^(5/2)+2809/5250*x*(5*x^2+2*x+3)^(5/2
)-1163/1400*x^2*(5*x^2+2*x+3)^(5/2)-7/40*x^3*(5*x^2+2*x+3)^(5/2)-901453/62
5000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \frac{\sqrt{3+2x+5x^2}(22275576+36695150x+86464445x^2+78608475x^3-28373000x^4-4823}{5250000} + \frac{901453\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{125000\sqrt{5}}$$

input
```
Integrate[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(22275576 + 36695150*x + 86464445*x^2 + 78608475*x^
3 - 28373000*x^4 - 48237500*x^5 - 127406250*x^6 - 22968750*x^7))/5250000 +
 (901453*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(125000*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {2192, 2192, 27, 2192, 27, 1160, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-7x^2+4x+1)(x^2+5x+2)(5x^2+2x+3)^{3/2}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\int (5x^2+2x+3)^{3/2}(-1163x^3+343x^2+520x+80)\,dx - \frac{7}{40}x^3(5x^2+2x+3)^{5/2}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{1}{35}\int 2(5x^2+2x+3)^{3/2}(11236x^2+12589x+1400)\,dx - \frac{1163}{35}x^2(5x^2+2x+3)^{5/2}\right) - \frac{7}{40}x^3(5x^2+2x+3)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(\frac{2}{35}\int \left(5x^2+2x+3\right)^{3/2}\left(11236x^2+12589x+1400\right)dx-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right)-\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{30}\int 2(149509x+4146)\left(5x^2+2x+3\right)^{3/2}dx+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/}\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\int(149509x+4146)\left(5x^2+2x+3\right)^{3/2}dx+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}x^2\left(5x^2+2x+3\right)^{5/2}\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\int\left(5x^2+2x+3\right)^{3/2}dx\right)+\frac{5618}{15}x\left(5x^2+2x+3\right)^{5/2}\right)-\frac{1163}{35}\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\int\sqrt{5x^2+2x+3}dx+\frac{1}{20}(5x+1)\left(5x^2+2x+3\right)^{3/2}\right)\right)\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 1087$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{7}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)+\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{1}{10}\sqrt{\frac{7}{10}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)+\frac{1}{10}\sqrt{5x^2\right.\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{40}\left(\frac{2}{35}\left(\frac{1}{15}\left(\frac{149509}{25}\left(5x^2+2x+3\right)^{5/2}-\frac{128779}{5}\left(\frac{21}{10}\left(\frac{7\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{1}{10}\sqrt{5x^2+2x+3}(5x+1)\right)+\frac{1}{20}\right.\right.\right.\right.$$
$$\frac{7}{40}x^3\left(5x^2+2x+3\right)^{5/2}$$

input
```
Int[(1 + 4*x - 7*x^2)*(2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-7*x^3*(3 + 2*x + 5*x^2)^(5/2))/40 + ((-1163*x^2*(3 + 2*x + 5*x^2)^(5/2))
/35 + (2*((5618*x*(3 + 2*x + 5*x^2)^(5/2))/15 + ((149509*(3 + 2*x + 5*x^2)
^(5/2))/25 - (128779*(((1 + 5*x)*(3 + 2*x + 5*x^2)^(3/2))/20 + (21*(((1 +
5*x)*Sqrt[3 + 2*x + 5*x^2])/10 + (7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*S
qrt[5])))/10))/5)/15))/35)/40
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result
risch	$-\frac{\left(22968750 x^{7}+127406250 x^{6}+48237500 x^{5}+28373000 x^{4}-78608475 x^{3}-86464445 x^{2}-36695150 x -22275576\right) \sqrt{5 x^{2}+2 x +3}}{5250000}-\frac{90}{}$
trager	$\left(-\frac{35}{8} x^{7}-\frac{1359}{56} x^{6}-\frac{3859}{420} x^{5}-\frac{28373}{5250} x^{4}+\frac{1048113}{70000} x^{3}+\frac{17292889}{1050000} x^{2}+\frac{733903}{105000} x +\frac{928149}{218750}\right) \sqrt{5 x^{2}+2 x +3}$
default	$-\frac{18397 \left(10 x +2\right) \left(5 x^{2}+2 x +3\right)^{\frac{3}{2}}}{300000}-\frac{128779 \left(10 x +2\right) \sqrt{5 x^{2}+2 x +3}}{500000}-\frac{901453 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x +\frac{1}{5}\right)}{14}\right)}{625000}+\frac{149509 \left(5 x^{2}+2 x +3\right)}{262500}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/5250000*(22968750*x^7+127406250*x^6+48237500*x^5+28373000*x^4-78608475*
x^3-86464445*x^2-36695150*x-22275576)*(5*x^2+2*x+3)^(1/2)-901453/625000*5^
(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.59

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx =$$
$$-\frac{1}{5250000}\left(22968750\,x^7+127406250\,x^6+48237500\,x^5+28373000\,x^4-78608475\,x^3-86464445\,x^2-36\right.$$
$$+\frac{901453}{1250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/5250000*(22968750*x^7 + 127406250*x^6 + 48237500*x^5 + 28373000*x^4 - 7
8608475*x^3 - 86464445*x^2 - 36695150*x - 22275576)*sqrt(5*x^2 + 2*x + 3)
+ 901453/1250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*
x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.58

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{35x^7}{8}-\frac{1359x^6}{56}-\frac{3859x^5}{420}-\frac{28373x^4}{5250}+\frac{1048113x^3}{70000}+\frac{17292889x^2}{1050000}+\frac{733903x}{105000}+\frac{928149}{218750}\right)-\frac{901453\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{625000}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)*(5*x**2+2*x+3)**(3/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-35*x**7/8 - 1359*x**6/56 - 3859*x**5/420 - 28373*
x**4/5250 + 1048113*x**3/70000 + 17292889*x**2/1050000 + 733903*x/105000 +
 928149/218750) - 901453*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/625000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{7}{40}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^3-\frac{1163}{1400}\left(5x^2+2x+3\right)^{\frac{5}{2}}x^2+\frac{2809}{5250}\left(5x^2+2x+3\right)^{\frac{5}{2}}x+\frac{149509}{262500}\left(5x^2+2x+3\right)^{\frac{5}{2}}-\frac{18397}{30000}\left(5x^2+2x+3\right)^{\frac{3}{2}}x-\frac{18397}{150000}\left(5x^2+2x+3\right)^{\frac{3}{2}}-\frac{128779}{50000}\sqrt{5x^2+2x+3}x-\frac{901453}{625000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right)-\frac{128779}{250000}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-7/40*(5*x^2 + 2*x + 3)^(5/2)*x^3 - 1163/1400*(5*x^2 + 2*x + 3)^(5/2)*x^2
+ 2809/5250*(5*x^2 + 2*x + 3)^(5/2)*x + 149509/262500*(5*x^2 + 2*x + 3)^(5
/2) - 18397/30000*(5*x^2 + 2*x + 3)^(3/2)*x - 18397/150000*(5*x^2 + 2*x +
3)^(3/2) - 128779/50000*sqrt(5*x^2 + 2*x + 3)*x - 901453/625000*sqrt(5)*ar
csinh(1/14*sqrt(14)*(5*x + 1)) - 128779/250000*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.56

$$\int \left(1+4x-7x^2\right)\left(2+5x+x^2\right)\left(3+2x+5x^2\right)^{3/2}\,dx = -\frac{1}{5250000}\left(5\left(\left(5\left(10\left(25\left(15\left(245x+1359\right)x+7718\right)x+113492\right)x-3144339\right)x-17292889\right)x-7339030\right)\right. + \frac{901453}{625000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
-1/5250000*(5*((5*(10*(25*(15*(245*x + 1359)*x + 7718)*x + 113492)*x - 314
4339)*x - 17292889)*x - 7339030)*x - 22275576)*sqrt(5*x^2 + 2*x + 3) + 901
453/625000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = \int (x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)\,dx$$

input
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1),x)
```

output
```
int((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.04

$$\int (1+4x-7x^2)(2+5x+x^2)(3+2x+5x^2)^{3/2}\,dx = -\frac{35\sqrt{5x^2+2x+3}\,x^7}{8} - \frac{1359\sqrt{5x^2+2x+3}\,x^6}{56} - \frac{3859\sqrt{5x^2+2x+3}\,x^5}{420} - \frac{28373\sqrt{5x^2+2x+3}\,x^4}{5250} + \frac{1048113\sqrt{5x^2+2x+3}\,x^3}{70000} + \frac{17292889\sqrt{5x^2+2x+3}\,x^2}{1050000} + \frac{733903\sqrt{5x^2+2x+3}\,x}{105000} + \frac{928149\sqrt{5x^2+2x+3}}{218750} - \frac{901453\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{625000}$$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)*(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 114843750*sqrt(5*x**2 + 2*x + 3)*x**7 - 637031250*sqrt(5*x**2 + 2*x +
3)*x**6 - 241187500*sqrt(5*x**2 + 2*x + 3)*x**5 - 141865000*sqrt(5*x**2 +
2*x + 3)*x**4 + 393042375*sqrt(5*x**2 + 2*x + 3)*x**3 + 432322225*sqrt(5*x
**2 + 2*x + 3)*x**2 + 183475750*sqrt(5*x**2 + 2*x + 3)*x + 111377880*sqrt(
5*x**2 + 2*x + 3) - 37861026*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) +
 5*x + 1)/sqrt(14)))/26250000
```

3.11 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 243

$$\begin{aligned}
\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx &= -\frac{477777\sqrt{3+2x+5x^2}}{60025} \\
&- \frac{10641x\sqrt{3+2x+5x^2}}{3430} - \frac{281}{196}x^2\sqrt{3+2x+5x^2} \\
&- \frac{5}{28}x^3\sqrt{3+2x+5x^2} - \frac{34425687\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{840350\sqrt{5}} \\
&- \frac{6\sqrt{\frac{2}{11}\left(8098902607-2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807} \\
&+ \frac{6\sqrt{\frac{2}{11}\left(8098902607+2434122235\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{16807}
\end{aligned}$$

output
```
-477777/60025*(5*x^2+2*x+3)^(1/2)-10641/3430*x*(5*x^2+2*x+3)^(1/2)-281/196
*x^2*(5*x^2+2*x+3)^(1/2)-5/28*x^3*(5*x^2+2*x+3)^(1/2)-34425687/4201750*arc
sinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-6/184877*(178175857354-53550689170*11^
(1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/
2)/(5*x^2+2*x+3)^(1/2))+6/184877*(178175857354+53550689170*11^(1/2))^(1/2)
*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*
x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.88 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.05

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \frac{\sqrt{3+2x+5x^2}(-1911108-744870x-344225x^2-42875x^3)}{240100}$$
$$+\frac{34425687\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{840350\sqrt{5}}$$
$$-\frac{12\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-648783\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-533850\log\left(-\right.}{-4\sqrt{5}-}\right.}{16807\sqrt{5}}$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-1911108 - 744870*x - 344225*x^2 - 42875*x^3))/240
100 + (34425687*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(840350*Sqr
t[5]) - (12*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4
 & , (-648783*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 533
850*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 251851*Sqrt[5]*Log
[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*
Sqrt[5]*#1^2 + 7*#1^3) & ])/(16807*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {2138, 27, 2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\left(5x^2+2x+3\right)^{3/2}}{-7x^2+4x+1}dx$$

$$\Big\downarrow \text{2138}$$

$$-\frac{\int -\frac{18\sqrt{5x^2+2x+3}(5603x^2+4404x+1131)}{-7x^2+4x+1}dx}{2940}-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{490}\int \frac{\sqrt{5x^2+2x+3}(5603x^2+4404x+1131)}{-7x^2+4x+1}dx-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2138}$$

$$\frac{3}{490}\left(-\frac{1}{490}\int -\frac{2(11475229x^2+7834212x+1411253)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}\sqrt{5x^2+2x+3}(196105x+571621)\right)-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{490}\left(\frac{1}{245}\int \frac{11475229x^2+7834212x+1411253}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{1}{490}(196105x+571621)\sqrt{5x^2+2x+3}\right)-\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{2143}$$

$$\frac{3}{490}\left(\frac{1}{245}\left(-\frac{11475229}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{400(251851x+53385)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(196105x+57\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{1}{490}(196105x+571\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 1090$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}\right)-\frac{1}{490}(196105\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 222$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\int \frac{251851x+53385}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{11475229\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}\right)-\frac{1}{490}(196105x+571621)\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 1365$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{1}{11}\left(2770361-877397\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(2770361+877397\right.\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{1}{22}\left(2770361-877397\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(2770361+877397\sqrt{\right.\right.\right.\right.$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \ 1154$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(-\frac{1}{11}\left(2770361-877397\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{\right.\right.}{\sqrt{5x\right.\right.\right.\right.}$$
$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{490}\left(\frac{1}{245}\left(\frac{400}{7}\left(\frac{\left(2770361-877397\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(2770361+877397\sqrt{11}\right)\text{arc}}{22\sqrt{2\left(1\right.}}\right.\right.\right.$$

$$\frac{1}{980}(35x+267)\left(5x^2+2x+3\right)^{3/2}$$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2),x]
```

output
```
-1/980*((267 + 35*x)*(3 + 2*x + 5*x^2)^(3/2)) + (3*(-1/490*((571621 + 1961
05*x)*Sqrt[3 + 2*x + 5*x^2]) + ((-11475229*ArcSinh[(2 + 10*x)/(2*Sqrt[14])
])/(7*Sqrt[5]) + (400*(((2770361 - 877397*Sqrt[11])*ArcTanh[(23 - Sqrt[11]
 + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2]
)])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((2770361 + 877397*Sqrt[11])*ArcTan
h[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[
3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/245))/490
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{(42875x^3+344225x^2+744870x+1911108)\sqrt{5x^2+2x+3}}{240100}-\frac{34425687\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{4201750}+ 12\left(-877397+251851\sqrt{11}\right.$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x,method=_RETURNVERBOSE
)
```

output
```
-1/240100*(42875*x^3+344225*x^2+744870*x+1911108)*(5*x^2+2*x+3)^(1/2)-3442
5687/4201750*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+12/184877*(-877397+251
851*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*
11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2
)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))
+250-34*11^(1/2))^(1/2))+12/184877*(877397+251851*11^(1/2))*11^(1/2)/(250+
34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2)
)*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^
2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.29

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx =$$

$$-\frac{3}{16807}\sqrt{\frac{2}{11}}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(\frac{6\left(\sqrt{\frac{2}{11}}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+809890\cdots}\right.}{\cdots}\right.$$

$$+\frac{3}{16807}\sqrt{\frac{2}{11}}\sqrt{2434122235\sqrt{11}+8098902607}\log\left(-\frac{6\left(\sqrt{\frac{2}{11}}\sqrt{5x^2+2x+3}\sqrt{2434122235\sqrt{11}+80989\cdots}\right.}{\cdots}\right.$$

$$-\frac{1}{240100}\left(42875x^3+344225x^2+744870x+1911108\right)\sqrt{5x^2+2x+3}$$

$$+\frac{34425687}{8403500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2-10x-8\right)$$

$$-\frac{1}{33614}\sqrt{-\frac{175256800920}{11}\sqrt{11}+\frac{583120987704}{11}}\log\left(-\frac{\sqrt{5x^2+2x+3}\left(24697\sqrt{11}+84590\right)\sqrt{-\frac{1752568\cdots}{11}}}{\cdots}\right.$$

$$+\frac{1}{33614}\sqrt{-\frac{175256800920}{11}\sqrt{11}+\frac{583120987704}{11}}\log\left(\frac{\sqrt{5x^2+2x+3}\left(24697\sqrt{11}+84590\right)\sqrt{-\frac{1752568009\cdots}{11}}}{\cdots}\right.$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="fri
cas")
```

output
```
-3/16807*sqrt(2/11)*sqrt(2434122235*sqrt(11) + 8098902607)*log(6*(sqrt(2/1
1)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(24697*sqr
t(11) - 84590) + 40555291*sqrt(11)*(x + 3) + 121665873*x - 202776455)/x) +
 3/16807*sqrt(2/11)*sqrt(2434122235*sqrt(11) + 8098902607)*log(-6*(sqrt(2/
11)*sqrt(5*x^2 + 2*x + 3)*sqrt(2434122235*sqrt(11) + 8098902607)*(24697*sq
rt(11) - 84590) - 40555291*sqrt(11)*(x + 3) - 121665873*x + 202776455)/x)
- 1/240100*(42875*x^3 + 344225*x^2 + 744870*x + 1911108)*sqrt(5*x^2 + 2*x
+ 3) + 34425687/8403500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1
) - 25*x^2 - 10*x - 8) - 1/33614*sqrt(-175256800920/11*sqrt(11) + 58312098
7704/11)*log(-(sqrt(5*x^2 + 2*x + 3)*(24697*sqrt(11) + 84590)*sqrt(-175256
800920/11*sqrt(11) + 583120987704/11) + 243331746*sqrt(11)*(x + 3) - 72999
5238*x + 1216658730)/x) + 1/33614*sqrt(-175256800920/11*sqrt(11) + 5831209
87704/11)*log((sqrt(5*x^2 + 2*x + 3)*(24697*sqrt(11) + 84590)*sqrt(-175256
800920/11*sqrt(11) + 583120987704/11) - 243331746*sqrt(11)*(x + 3) + 72999
5238*x - 1216658730)/x)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = -\int \frac{6\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{7x^2-4x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1),x)
```

output
```
-Integral(6*sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(19*x*
sqrt(5*x**2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(23*x**2*sqrt(5*x*
*2 + 2*x + 3)/(7*x**2 - 4*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x
+ 3)/(7*x**2 - 4*x - 1), x) - Integral(5*x**4*sqrt(5*x**2 + 2*x + 3)/(7*x*
*2 - 4*x - 1), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(177) = 354$.

Time = 0.19 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.20

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{1+4x-7x^2}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="max
ima")
```

output
```
1/92438500*sqrt(11)*(19500*sqrt(11)*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcs
inh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs
(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))
 - 300125*sqrt(11)*(5*x^2 + 2*x + 3)^(3/2)*x - 3344250*sqrt(11)*(-34/49*sq
rt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2
*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*s
qrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/
abs(14*x + 2*sqrt(11) - 4)) + 91500*sqrt(2)*(17*sqrt(11) + 125)^(3/2)*arcs
inh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(
7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs
(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))
 + 15692250*(-34/49*sqrt(11) + 250/49)^(3/2)*arcsinh(5/7*sqrt(11)*sqrt(7)*
sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2
*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) -
 23/7*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4)) - 2289525*sqrt(11)*(5*x^
2 + 2*x + 3)^(3/2) - 20591025*sqrt(11)*sqrt(5*x^2 + 2*x + 3)*x - 68851374*
sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/14*sqrt(7)*sqrt(2)) -
60020205*sqrt(11)*sqrt(5*x^2 + 2*x + 3))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.63

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx =$$
$$-\frac{1}{240100}(35(35(35x+281)x+21282)x+1911108)\sqrt{5x^2+2x+3}$$
$$+\frac{34425687}{4201750}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right)$$
$$+19.3580321168561\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+4.41924736459000\right)$$
$$-0.773682164624264\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}+1.25295163054000\right)$$
$$-19.3580321168561\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-1.02258038113000\right)$$
$$+0.773682164625454\log\left(-\sqrt{5}x+\sqrt{5x^2+2x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x, algorithm="gia
c")
```

output
```
-1/240100*(35*(35*(35*x + 281)*x + 21282)*x + 1911108)*sqrt(5*x^2 + 2*x +
3) + 34425687/4201750*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 +
2*x + 3)) + 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41
924736459000) - 0.773682164624264*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) +
 1.25295163054000) - 19.3580321168561*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x +
3) - 1.02258038113000) + 0.773682164625454*log(-sqrt(5)*x + sqrt(5*x^2 + 2
*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx = \int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{3/2}}{-7x^2+4x+1}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1),x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{1+4x-7x^2}\,dx=\int \frac{(x^2+5x+2)(5x^2+2x+3)^{\frac{3}{2}}}{-7x^2+4x+1}dx$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x)
```

output
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1),x)
```

3.12 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 236

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{465}{686}\sqrt{3+2x+5x^2} + \frac{5}{98}x\sqrt{3+2x+5x^2} + \frac{6(655+5028x)\sqrt{3+2x+5x^2}}{3773(1+4x-7x^2)} + \frac{16691\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{2401\sqrt{5}} - \frac{\sqrt{\frac{1}{22}\left(52175400311-13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411} - \frac{\sqrt{\frac{1}{22}\left(52175400311+13155376531\sqrt{11}\right)}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{26411}$$

output

```
465/686*(5*x^2+2*x+3)^(1/2)+5/98*x*(5*x^2+2*x+3)^(1/2)+6*(655+5028*x)*(5*x
^2+2*x+3)^(1/2)/(-26411*x^2+15092*x+3773)+16691/12005*arcsinh(1/14*(1+5*x)
*14^(1/2))*5^(1/2)-1/581042*(1147858806842-289418283682*11^(1/2))^(1/2)*ar
ctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*x+3
)^(1/2))-1/581042*(1147858806842+289418283682*11^(1/2))^(1/2)*arctanh((23+
11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.04 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.89

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2} dx = \frac{\frac{1715\sqrt{3+2x+5x^2}(-12975-81181x+34265x^2+2695x^3)}{-1-4x+7x^2} - 17992898\sqrt{5}\log(-1}{}$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

output
```
((1715*Sqrt[3 + 2*x + 5*x^2]*(-12975 - 81181*x + 34265*x^2 + 2695*x^3))/(-
1 - 4*x + 7*x^2) - 17992898*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]] + 44*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3
 + 7*#1^4 & , (25954129*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1] - 19416530*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 271709
9*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
 - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 6*Sqrt[5]*RootSum[83 - 16*Sqrt[5
]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (225782939*Sqrt[5]*Log[-(Sqrt
[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 137400830*Log[-(Sqrt[5]*x) + Sqrt[3
 + 2*x + 5*x^2] - #1]*#1 + 7775369*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x
 + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/
12941390
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {2132, 27, 2138, 27, 2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)(5x^2+2x+3)^{3/2}}{(-7x^2+4x+1)^2} dx$$

$$\Big\downarrow \; 2132$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}-\frac{1}{308}\int -\frac{4\left(-970x^2-181x+228\right)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{77}\int \frac{\left(-970x^2-181x+228\right)\sqrt{5x^2+2x+3}}{-7x^2+4x+1}dx+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 2138$$

$$\frac{1}{77}\left(\frac{1}{49}(3395x+5826)\sqrt{5x^2+2x+3}-\frac{1}{490}\int -\frac{10\left(-183601x^2-107622x+17505\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{77}\left(\frac{1}{49}\int \frac{-183601x^2-107622x+17505}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+5826)\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 2143$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int \frac{2(743879x+30533)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+\right. \\ \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{2}{7}\int \frac{743879x+30533}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+\right. \\ \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}-\frac{2}{7}\int \frac{743879x+30533}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3\right. \\ \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\int\frac{743879x+30533}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx\right)+\frac{1}{49}\sqrt{5x^2+2x+3}(3395x+582\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{1}{11}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{1}{22}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(-\frac{1}{11}\left(8182669-1701489\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}\right.}{5x^2+2x+3}}\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{77}\left(\frac{1}{49}\left(\frac{183601\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}-\frac{2}{7}\left(\frac{\left(8182669-1701489\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{(8182\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{154\left(-7x^2+4x+1\right)}$$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^2,x]
```

output
```
(3*(3 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(154*(1 + 4*x - 7*x^2)) + (((5826 +
 3395*x)*Sqrt[3 + 2*x + 5*x^2])/49 + ((183601*ArcSinh[(2 + 10*x)/(2*Sqrt[1
4])])/(7*Sqrt[5]) - (2*(((8182669 - 1701489*Sqrt[11])*ArcTanh[(23 - Sqrt[1
1] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^
2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((8182669 + 1701489*Sqrt[11])*Arc
Tanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sq
rt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7)/49)/77
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.04

method	result
risch	$\frac{\left(2695 x^{3}+34265 x^{2}-81181 x-12975\right) \sqrt{5 x^{2}+2 x+3}}{52822 x^{2}-30184 x-7546}+\frac{16691 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x+\frac{1}{5}\right)}{14}\right)}{12005}-\frac{\left(-1701489+743879 \sqrt{11}\right) \sqrt{11}\, \operatorname{arctan}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x,method=_RETURNVERBO
SE)
```

output
```
1/7546*(2695*x^3+34265*x^2-81181*x-12975)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2
)+16691/12005*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))-1/290521*(-1701489+74
3879*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49
*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/
2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)
)+250-34*11^(1/2))^(1/2))-1/290521*(1701489+743879*11^(1/2))*11^(1/2)/(250
+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2
))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))
^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(174) = 348$.

Time = 0.12 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.57

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \frac{183601\sqrt{5}(7x^2-4x-1)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-2\right.}{}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="f
ricas")
```

output
```
1/264110*(183601*sqrt(5)*(7*x^2 - 4*x - 1)*log(-sqrt(5)*sqrt(5*x^2 + 2*x +
 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(7*x^2 - 4*x - 1)*sqrt(1195943321/2
*sqrt(11) + 52175400311/22)*log((2*sqrt(5*x^2 + 2*x + 3)*sqrt(1195943321/2
*sqrt(11) + 52175400311/22)*(68441*sqrt(11) - 178266) + 1795191685*sqrt(11
)*(x + 3) + 5385575055*x - 8975958425)/x) + 5*(7*x^2 - 4*x - 1)*sqrt(11959
43321/2*sqrt(11) + 52175400311/22)*log(-(2*sqrt(5*x^2 + 2*x + 3)*sqrt(1195
943321/2*sqrt(11) + 52175400311/22)*(68441*sqrt(11) - 178266) - 1795191685
*sqrt(11)*(x + 3) - 5385575055*x + 8975958425)/x) - 5*(7*x^2 - 4*x - 1)*sq
rt(-1195943321/2*sqrt(11) + 52175400311/22)*log(-(2*sqrt(5*x^2 + 2*x + 3)*
(68441*sqrt(11) + 178266)*sqrt(-1195943321/2*sqrt(11) + 52175400311/22) +
1795191685*sqrt(11)*(x + 3) - 5385575055*x + 8975958425)/x) + 5*(7*x^2 - 4
*x - 1)*sqrt(-1195943321/2*sqrt(11) + 52175400311/22)*log((2*sqrt(5*x^2 +
2*x + 3)*(68441*sqrt(11) + 178266)*sqrt(-1195943321/2*sqrt(11) + 521754003
11/22) - 1795191685*sqrt(11)*(x + 3) + 5385575055*x - 8975958425)/x) + 35*
(2695*x^3 + 34265*x^2 - 81181*x - 12975)*sqrt(5*x^2 + 2*x + 3))/(7*x^2 - 4
*x - 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx=\int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{\frac{3}{2}}}{(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**2,x)
```

output
```
Integral((x**2 + 5*x + 2)*(5*x**2 + 2*x + 3)**(3/2)/(7*x**2 - 4*x - 1)**2,
 x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx=\int \frac{(5\,x^2+2\,x+3)^{\frac{3}{2}}(x^2+5\,x+2)}{(7\,x^2-4\,x-1)^2}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="m
axima")
```

output
```
integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^2, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{63274455776,[8]%%%}+%%%{%%{[144627327488,0]:[1,0,-5]%%},[7
]%%%}+%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}}{(-7\,x^2+4\,x+1)^2}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^2}\,dx = \int \frac{(x^2+5x+2)\,(5x^2+2x+3)^{\frac{3}{2}}}{(-7x^2+4x+1)^2}dx$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

output
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^2,x)
```

3.13 $\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} dx$

Optimal result

Integrand size = 35, antiderivative size = 234

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3} dx = \frac{3(655+5028x)\sqrt{3+2x+5x^2}}{3773(1+4x-7x^2)^2} - \frac{(138372-189161x)\sqrt{3+2x+5x^2}}{166012(1+4x-7x^2)} - \frac{5}{343}\sqrt{5}\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right) - \frac{\sqrt{\frac{62294197250171-2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024} + \frac{\sqrt{\frac{62294197250171+2085440742055\sqrt{11}}{2794}}\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{332024}$$

output
```
3/3773*(655+5028*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-(138372-189161*x)
*(5*x^2+2*x+3)^(1/2)/(-1162084*x^2+664048*x+166012)-5/343*arcsinh(1/14*(1+
5*x)*14^(1/2))*5^(1/2)-1/927675056*(174049987116977774-5826721433301670*11
^(1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1
/2)/(5*x^2+2*x+3)^(1/2))+1/927675056*(174049987116977774+5826721433301670*
11^(1/2))^(1/2)*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^
(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.19 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.72

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{\sqrt{3+2x+5x^2}(-7416+42767x+246464x^2-189161x^3)}{23716(-1-4x+7x^2)^2}$$
$$+\frac{5}{343}\sqrt{5}\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)$$
$$-\frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{4506829\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1320270\log\left(-\sqrt{5}\right.}{-4\sqrt{5}-35}\right.}{33614\sqrt{5}}$$
$$+\frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-16323208013227\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+1511207}{}\right.}{71748713246\sqrt{5}}$$
$$-\frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&,\frac{-4192656948824863\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+245}{}\right.}{34726377211064\sqrt{}}$$

input
```
Integrate[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-7416 + 42767*x + 246464*x^2 - 189161*x^3))/(23716
*(-1 - 4*x + 7*x^2)^2) + (5*Sqrt[5]*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x +
5*x^2]])/343 - RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#
1^4 & , (4506829*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] -
1320270*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 64435*Sqrt[5]*
Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 +
 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(33614*Sqrt[5]) + RootSum[83 - 16*Sqrt[5]*#1
 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-16323208013227*Sqrt[5]*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 151120773150070*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 21832390993791*Sqrt[5]*Log[-(Sqrt[5]*x)
 + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2
+ 7*#1^3) & ]/(71748713246*Sqrt[5]) - (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#
1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-4192656948824863*Sqrt[5]*Log[-(Sqrt[5]
*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 24518831643829090*Log[-(Sqrt[5]*x) + S
qrt[3 + 2*x + 5*x^2] - #1]*#1 + 3523608887504055*Sqrt[5]*Log[-(Sqrt[5]*x)
+ Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 +
 7*#1^3) & ])/(34726377211064*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.343, Rules used = {2132, 27, 2132, 27, 2143, 25, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+5x+2)\left(5x^2+2x+3\right)^{3/2}}{(-7x^2+4x+1)^3}\,dx$$

$$\Big\downarrow \; 2132$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}-\frac{1}{616}\int -\frac{4\left(-110x^2+163x+744\right)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{1}{154}\int \frac{\left(-110x^2+163x+744\right)\sqrt{5x^2+2x+3}}{\left(-7x^2+4x+1\right)^2}dx+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 2132$$

$$\frac{1}{154}\left(-\frac{1}{308}\int -\frac{2\left(12100x^2+89403x+128019\right)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{\sqrt{5x^2+2x+3}(9495-37088x)}{154\left(-7x^2+4x+1\right)}\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{154}\left(\frac{1}{154}\int \frac{12100x^2+89403x+128019}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{(9495-37088x)\sqrt{5x^2+2x+3}}{154\left(-7x^2+4x+1\right)}\right)+\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 2143$$

$$\frac{1}{154}\left(\frac{1}{154}\left(-\frac{12100}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int -\frac{674221x+908233}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{(9495-37088x)\sqrt{5x^2}}{154\left(-7x^2+4x\right.}\right.\quad \frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{12100}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx\right)-\frac{(9495-37088x)\sqrt{5x^2+}}{154(-7x^2+4x+1}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{605}{7}\sqrt{\frac{10}{7}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)\right)-\frac{(9495-}{154}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\int \frac{674221x+908233}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{2420}{7}\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)\right)-\frac{(9495-37088x)\sqrt{5x^2+}}{154(-7x^2+4x+1}\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{1}{11}\left(7416431-7706073\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(7416431+7706073\right.\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{1}{22}\left(7416431-7706073\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(7416431+7706073\sqrt{\right.\right.\right.\right.$$
$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(-\frac{1}{11}\left(7416431-7706073\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{1\ldots}\right.\right.}{\sqrt{5x^2\ldots}}\right.\right.\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{154}\left(\frac{1}{154}\left(\frac{1}{7}\left(\frac{\left(7416431-7706073\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(7416431+7706073\sqrt{11}\right)\text{arc}\ldots}{22\sqrt{2\left(1\ldots\right.}}\right.\right.\right.$$

$$\frac{3(61x+3)\left(5x^2+2x+3\right)^{3/2}}{308\left(-7x^2+4x+1\right)^2}$$

input
```
Int[((2 + 5*x + x^2)*(3 + 2*x + 5*x^2)^(3/2))/(1 + 4*x - 7*x^2)^3,x]
```

output
```
(3*(3 + 61*x)*(3 + 2*x + 5*x^2)^(3/2))/(308*(1 + 4*x - 7*x^2)^2) + (-1/154
*((9495 - 37088*x)*Sqrt[3 + 2*x + 5*x^2])/(1 + 4*x - 7*x^2) + ((-2420*Sqrt
[5]*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/7 + (((7416431 - 7706073*Sqrt[11])*A
rcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((7416431 + 77
06073*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])
)/7)/154)/154
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2132
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(A*b*c - 2*a*B*c + a*b*C - (c*(b*B - 2*A*c) -
C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^q/(c*(b^2
- 4*a*c)*(p + 1))), x] - Simp[1/(c*(b^2 - 4*a*c)*(p + 1))   Int[(a + b*x +
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[e*q*(A*b*c - 2*a*B*c + a*b*C)
 - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*c - b^2*(p + 2))) + (2*f*q*(A*b*c
- 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(2*p + q + 3) + C*(2*a*c*(q + 1) -
b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)*(2*p + 2*q + 3) + C*(2*a*c*(2*q +
 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
&& PolyQ[Px, x, 2] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{(189161x^3-246464x^2-42767x+7416)\sqrt{5x^2+2x+3}}{23716(7x^2-4x-1)^2}-\frac{5\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\,\left(x+\frac{1}{5}\right)}{14}\right)}{343}+\frac{\left(-7706073+674221\sqrt{11}\right)\sqrt{11}\,\operatorname{arctan}}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x,method=_RETURNVERBO
SE)
```

output
```
-1/23716*(189161*x^3-246464*x^2-42767*x+7416)/(7*x^2-4*x-1)^2*(5*x^2+2*x+3
)^(1/2)-5/343*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+1/3652264*(-7706073+6
74221*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/4
9*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1
/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2
))+250-34*11^(1/2))^(1/2))+1/3652264*(7706073+674221*11^(1/2))*11^(1/2)/(2
50+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1
/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2
))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(176) = 352$.

Time = 0.13 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.82

$$\int \frac{(2+5x+x^2)\,(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="f
ricas")
```

output
```
1/664048*(4840*sqrt(5)*(49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*log(sqrt(5)*sqr
t(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - (49*x^4 - 56*x^3 + 2*x
^2 + 8*x + 1)*sqrt(189585522005/254*sqrt(11) + 62294197250171/2794)*log((2
*sqrt(5*x^2 + 2*x + 3)*sqrt(189585522005/254*sqrt(11) + 62294197250171/279
4)*(11840590*sqrt(11) - 83479737) + 3884517682577*sqrt(11)*(x + 3) + 11653
553047731*x - 19422588412885)/x) + (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqr
t(189585522005/254*sqrt(11) + 62294197250171/2794)*log(-(2*sqrt(5*x^2 + 2*
x + 3)*sqrt(189585522005/254*sqrt(11) + 62294197250171/2794)*(11840590*sqr
t(11) - 83479737) - 3884517682577*sqrt(11)*(x + 3) - 11653553047731*x + 19
422588412885)/x) - (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-189585522005/
254*sqrt(11) + 62294197250171/2794)*log(-(2*sqrt(5*x^2 + 2*x + 3)*(1184059
0*sqrt(11) + 83479737)*sqrt(-189585522005/254*sqrt(11) + 62294197250171/27
94) + 3884517682577*sqrt(11)*(x + 3) - 11653553047731*x + 19422588412885)/
x) + (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-189585522005/254*sqrt(11) +
 62294197250171/2794)*log((2*sqrt(5*x^2 + 2*x + 3)*(11840590*sqrt(11) + 83
479737)*sqrt(-189585522005/254*sqrt(11) + 62294197250171/2794) - 388451768
2577*sqrt(11)*(x + 3) + 11653553047731*x - 19422588412885)/x) - 28*(189161
*x^3 - 246464*x^2 - 42767*x + 7416)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^
3 + 2*x^2 + 8*x + 1)
```

Sympy [F]

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^3}\,dx = -\int \frac{6\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{19x\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{23x^2\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{27x^3\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx - \int \frac{5x^4\sqrt{5x^2+2x+3}}{343x^6-588x^5+189x^4+104x^3-27x^2-12x-1}\,dx$$

input
```
integrate((x**2+5*x+2)*(5*x**2+2*x+3)**(3/2)/(-7*x**2+4*x+1)**3,x)
```

output
```
-Integral(6*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x
**3 - 27*x**2 - 12*x - 1), x) - Integral(19*x*sqrt(5*x**2 + 2*x + 3)/(343*
x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral
(23*x**2*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3
 - 27*x**2 - 12*x - 1), x) - Integral(27*x**3*sqrt(5*x**2 + 2*x + 3)/(343*
x**6 - 588*x**5 + 189*x**4 + 104*x**3 - 27*x**2 - 12*x - 1), x) - Integral
(5*x**4*sqrt(5*x**2 + 2*x + 3)/(343*x**6 - 588*x**5 + 189*x**4 + 104*x**3
- 27*x**2 - 12*x - 1), x)
```

Maxima [F]

$$\int \frac{(2+5x+x^2)\left(3+2x+5x^2\right)^{3/2}}{\left(1+4x-7x^2\right)^3}\,dx = \int -\frac{\left(5\,x^2+2\,x+3\right)^{\frac{3}{2}}\left(x^2+5\,x+2\right)}{\left(7\,x^2-4\,x-1\right)^3}\,dx$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="m
axima")
```

output
```
-integrate((5*x^2 + 2*x + 3)^(3/2)*(x^2 + 5*x + 2)/(7*x^2 - 4*x - 1)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(176) = 352$.

Time = 0.19 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.76

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \frac{5}{343}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)+\frac{264327\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7-3224225\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-87069759\left(\sqrt{5}x-\sqrt{5}\right.}{83006\left(7\left(\sqrt{5}x-\right.\right.}$$
$$+0.474028359165602\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.424017987132256\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.474028359165602\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.424017987132256\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x, algorithm="g
iac")
```

output
```
5/343*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) + 1/83
006*(264327*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 - 3224225*sqrt(5)*(sqrt(
5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 87069759*(sqrt(5)*x - sqrt(5*x^2 + 2*x +
 3))^5 - 36535763*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 41681814
9*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 204858869*sqrt(5)*(sqrt(5)*x - s
qrt(5*x^2 + 2*x + 3))^2 - 411908789*sqrt(5)*x - 187277977*sqrt(5) + 411908
789*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sq
rt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83)^2 + 0.
474028359165602*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000)
 - 0.424017987132256*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.2529516305
4000) - 0.474028359165602*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258
038113000) + 0.424017987132256*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.
09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \int \frac{(x^2+5\,x+2)\,(5\,x^2+2\,x+3)^{3/2}}{(-7\,x^2+4\,x+1)^3}\,dx$$

input
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3,x)
```

output
```
int(((5*x + x^2 + 2)*(2*x + 5*x^2 + 3)^(3/2))/(4*x - 7*x^2 + 1)^3, x)
```

Reduce [F]

$$\int \frac{(2+5x+x^2)(3+2x+5x^2)^{3/2}}{(1+4x-7x^2)^3}\,dx = \int \frac{(x^2+5x+2)(5x^2+2x+3)^{\frac{3}{2}}}{(-7x^2+4x+1)^3}dx$$

input
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

output
```
int((x^2+5*x+2)*(5*x^2+2*x+3)^(3/2)/(-7*x^2+4*x+1)^3,x)
```

3.14 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 185

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{16515809\sqrt{3+2x+5x^2}}{156250} + \frac{5793077x\sqrt{3+2x+5x^2}}{75000} + \frac{40722851x^2\sqrt{3+2x+5x^2}}{750000} - \frac{5160533x^3\sqrt{3+2x+5x^2}}{50000} - \frac{47807x^4\sqrt{3+2x+5x^2}}{3750} + \frac{26159}{300}x^5\sqrt{3+2x+5x^2} - \frac{1141}{40}x^6\sqrt{3+2x+5x^2} - \frac{343}{40}x^7\sqrt{3+2x+5x^2} - \frac{77513689\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{625000\sqrt{5}}$$

output
```
-16515809/156250*(5*x^2+2*x+3)^(1/2)+5793077/75000*x*(5*x^2+2*x+3)^(1/2)+4
0722851/750000*x^2*(5*x^2+2*x+3)^(1/2)-5160533/50000*x^3*(5*x^2+2*x+3)^(1/
2)-47807/3750*x^4*(5*x^2+2*x+3)^(1/2)+26159/300*x^5*(5*x^2+2*x+3)^(1/2)-11
41/40*x^6*(5*x^2+2*x+3)^(1/2)-343/40*x^7*(5*x^2+2*x+3)^(1/2)-77513689/3125
000*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.48

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\sqrt{3+2x+5x^2}(-396379416+289653850x+203614255x^2-387039975x^3-47807000x^4+326987500x}{3750000}$$
$$+\frac{77513689\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{625000\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-396379416 + 289653850*x + 203614255*x^2 - 3870399
75*x^3 - 47807000*x^4 + 326987500*x^5 - 106968750*x^6 - 32156250*x^7))/375
0000 + (77513689*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(625000*Sq
rt[5])
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2192, 2192, 27, 2192, 27, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^3\,(x^2+5x+2)}{\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\int \frac{-39935x^7+89803x^6+5080x^5-34840x^4-2440x^3+4600x^2+1160x+80}{\sqrt{5x^2+2x+3}}dx-\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(\frac{1}{35}\int \frac{70\left(52318x^6+12809x^5-17420x^4-1220x^3+2300x^2+580x+40\right)}{\sqrt{5x^2+2x+3}}dx-1141x^6\sqrt{5x^2+2x+3}\right)-\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(2\int \frac{52318x^6+12809x^5-17420x^4-1220x^3+2300x^2+580x+40}{\sqrt{5x^2+2x+3}}dx-1141x^6\sqrt{5x^2+2x+3}\right)-\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(2\left(\frac{1}{30}\int \frac{2\left(-95614x^5-653685x^4-18300x^3+34500x^2+8700x+600\right)}{\sqrt{5x^2+2x+3}}dx+\frac{26159}{15}\sqrt{5x^2+2x+3}x^5\right)-\right.\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\int \frac{-95614x^5-653685x^4-18300x^3+34500x^2+8700x+600}{\sqrt{5x^2+2x+3}}dx+\frac{26159}{15}\sqrt{5x^2+2x+3}x^5\right)-1141\right.\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{1}{25}\int \frac{3\left(-5160533x^4+229956x^3+287500x^2+72500x+5000\right)}{\sqrt{5x^2+2x+3}}dx-\frac{95614}{25}x^4\sqrt{5x^2+2x+3}\right)+\right.\right.\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\int\frac{-5160533x^4+229956x^3+287500x^2+72500x+5000}{\sqrt{5x^2+2x+3}}dx-\frac{95614}{25}x^4\sqrt{5x^2+2x+3}\right)+\frac{2615}{15}\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 2192

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\int\frac{40722851x^3+52194797x^2+1450000x+100000}{\sqrt{5x^2+2x+3}}dx-\frac{5160533}{20}x^3\sqrt{5x^2+2x+3}\right)-\frac{956}{25}\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 2192

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{1}{15}\int\frac{2(289653850x^2-111293553x+750000)}{\sqrt{5x^2+2x+3}}dx+\frac{40722851}{15}\sqrt{5x^2+2x+3}x^2\right)-\frac{5160}{20}\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 27

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\int\frac{289653850x^2-111293553x+750000}{\sqrt{5x^2+2x+3}}dx+\frac{40722851}{15}\sqrt{5x^2+2x+3}x^2\right)-\frac{5160533}{20}\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 2192

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int-\frac{30(66063236x+28715385)}{\sqrt{5x^2+2x+3}}dx+28965385\sqrt{5x^2+2x+3}x\right)+\frac{40722851}{15}\sqrt{5x\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 27

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\int\frac{66063236x+28715385}{\sqrt{5x^2+2x+3}}dx\right)+\frac{40722851}{15}\sqrt{5x^2+2x\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$\downarrow$ 1160

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{66063236}{5}\sqrt{5x^2+2x\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}+\frac{66063236}{5}\sqrt{\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{40}\left(2\left(\frac{1}{15}\left(\frac{3}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(28965385x\sqrt{5x^2+2x+3}-3\left(\frac{77513689\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{66063236}{5}\sqrt{5x^2+2x+\right.\right.\right.\right.\right.\right.\right.$$
$$\frac{343}{40}x^7\sqrt{5x^2+2x+3}$$

input
```
Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-343*x^7*Sqrt[3 + 2*x + 5*x^2])/40 + (-1141*x^6*Sqrt[3 + 2*x + 5*x^2] + 2
*((26159*x^5*Sqrt[3 + 2*x + 5*x^2])/15 + ((-95614*x^4*Sqrt[3 + 2*x + 5*x^2
])/25 + (3*((-5160533*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((40722851*x^2*Sqrt[
3 + 2*x + 5*x^2])/15 + (2*(28965385*x*Sqrt[3 + 2*x + 5*x^2] - 3*((66063236
*Sqrt[3 + 2*x + 5*x^2])/5 + (77513689*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5
*Sqrt[5]))))/15)/20))/25)/15))/40
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{\left(32156250 x^{7}+106968750 x^{6}-326987500 x^{5}+47807000 x^{4}+387039975 x^{3}-203614255 x^{2}-289653850 x +396379416\right) \sqrt{5 x^{2}+2 x +3}}{3750000}$
trager	$\left(-\frac{343}{40} x^{7}-\frac{1141}{40} x^{6}+\frac{26159}{300} x^{5}-\frac{47807}{3750} x^{4}-\frac{5160533}{50000} x^{3}+\frac{40722851}{750000} x^{2}+\frac{5793077}{75000} x -\frac{16515809}{156250}\right) \sqrt{5 x^{2}+2x$
default	$-\frac{77513689 \sqrt{5}\, \operatorname{arcsinh}\left(\frac{5 \sqrt{14}\, \left(x +\frac{1}{5}\right)}{14}\right)}{3125000}-\frac{16515809 \sqrt{5 x^{2}+2 x +3}}{156250}+\frac{5793077 x \sqrt{5 x^{2}+2 x +3}}{75000}+\frac{40722851 x^{2} \sqrt{5 x^{2}+2 x +3}}{750000}-51$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/3750000*(32156250*x^7+106968750*x^6-326987500*x^5+47807000*x^4+38703997
5*x^3-203614255*x^2-289653850*x+396379416)*(5*x^2+2*x+3)^(1/2)-77513689/31
25000*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{3750000}\left(32156250\,x^7+106968750\,x^6-326987500\,x^5+47807000\,x^4+387039975\,x^3-203614255\,x\right. + \frac{77513689}{6250000}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/3750000*(32156250*x^7 + 106968750*x^6 - 326987500*x^5 + 47807000*x^4 +
387039975*x^3 - 203614255*x^2 - 289653850*x + 396379416)*sqrt(5*x^2 + 2*x
+ 3) + 77513689/6250000*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1
) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.46

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{343x^7}{40}-\frac{1141x^6}{40}+\frac{26159x^5}{300}-\frac{47807x^4}{3750}-\frac{5160533x^3}{50000}+\frac{40722851x^2}{750000}+\frac{5793077x}{75000}-\frac{16515809}{156250}\right)-\frac{77513689\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{3125000}$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-343*x**7/40 - 1141*x**6/40 + 26159*x**5/300 - 478
07*x**4/3750 - 5160533*x**3/50000 + 40722851*x**2/750000 + 5793077*x/75000
 - 16515809/156250) - 77513689*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/3125
000
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343}{40}\sqrt{5x^2+2x+3}x^7 - \frac{1141}{40}\sqrt{5x^2+2x+3}x^6 + \frac{26159}{300}\sqrt{5x^2+2x+3}x^5 - \frac{47807}{3750}\sqrt{5x^2+2x+3}x^4 - \frac{5160533}{50000}\sqrt{5x^2+2x+3}x^3 + \frac{40722851}{750000}\sqrt{5x^2+2x+3}x^2 + \frac{5793077}{75000}\sqrt{5x^2+2x+3}x - \frac{77513689}{3125000}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{16515809}{156250}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-343/40*sqrt(5*x^2 + 2*x + 3)*x^7 - 1141/40*sqrt(5*x^2 + 2*x + 3)*x^6 + 26
159/300*sqrt(5*x^2 + 2*x + 3)*x^5 - 47807/3750*sqrt(5*x^2 + 2*x + 3)*x^4 -
 5160533/50000*sqrt(5*x^2 + 2*x + 3)*x^3 + 40722851/750000*sqrt(5*x^2 + 2*
x + 3)*x^2 + 5793077/75000*sqrt(5*x^2 + 2*x + 3)*x - 77513689/3125000*sqrt
(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 16515809/156250*sqrt(5*x^2 + 2*x +
3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.44

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx =$$
$$-\frac{1}{3750000}(5((5(10(175(15(49x+163)x-7474)x+191228)x+15481599)x-40722851)x-57930$$
$$+\frac{77513689}{3125000}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
-1/3750000*(5*((5*(10*(175*(15*(49*x + 163)*x - 7474)*x + 191228)*x + 1548
1599)*x - 40722851)*x - 57930770)*x + 396379416)*sqrt(5*x^2 + 2*x + 3) + 7
7513689/3125000*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) -
 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5x+2)\,(-7x^2+4x+1)^3}{\sqrt{5x^2+2x+3}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.83

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{343\sqrt{5x^2+2x+3}\,x^7}{40} - \frac{1141\sqrt{5x^2+2x+3}\,x^6}{40} + \frac{26159\sqrt{5x^2+2x+3}\,x^5}{300} - \frac{47807\sqrt{5x^2+2x+3}\,x^4}{3750} - \frac{5160533\sqrt{5x^2+2x+3}\,x^3}{50000} + \frac{40722851\sqrt{5x^2+2x+3}\,x^2}{750000} + \frac{5793077\sqrt{5x^2+2x+3}\,x}{75000} - \frac{16515809\sqrt{5x^2+2x+3}}{156250} - \frac{77513689\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{3125000}$$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 160781250*sqrt(5*x**2 + 2*x + 3)*x**7 - 534843750*sqrt(5*x**2 + 2*x +
3)*x**6 + 1634937500*sqrt(5*x**2 + 2*x + 3)*x**5 - 239035000*sqrt(5*x**2 +
 2*x + 3)*x**4 - 1935199875*sqrt(5*x**2 + 2*x + 3)*x**3 + 1018071275*sqrt(
5*x**2 + 2*x + 3)*x**2 + 1448269250*sqrt(5*x**2 + 2*x + 3)*x - 1981897080*
sqrt(5*x**2 + 2*x + 3) - 465082134*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqr
t(5) + 5*x + 1)/sqrt(14)))/18750000
```

3.15 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 143

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{22053\sqrt{3+2x+5x^2}}{31250}+\frac{36073x\sqrt{3+2x+5x^2}}{1875} - \frac{207427x^2\sqrt{3+2x+5x^2}}{37500} - \frac{33259x^3\sqrt{3+2x+5x^2}}{2500} + \frac{5131}{750}x^4\sqrt{3+2x+5x^2}+\frac{49}{30}x^5\sqrt{3+2x+5x^2} - \frac{1719097\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{31250\sqrt{5}}$$

output
```
-22053/31250*(5*x^2+2*x+3)^(1/2)+36073/1875*x*(5*x^2+2*x+3)^(1/2)-207427/3
7500*x^2*(5*x^2+2*x+3)^(1/2)-33259/2500*x^3*(5*x^2+2*x+3)^(1/2)+5131/750*x
^4*(5*x^2+2*x+3)^(1/2)+49/30*x^5*(5*x^2+2*x+3)^(1/2)-1719097/156250*arcsin
h(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.55

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{\sqrt{3+2x+5x^2}(-132318+3607300x-1037135x^2-2494425x^3+1282750x^4+306250x^5)}{187500} + \frac{1719097\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{31250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(-132318 + 3607300*x - 1037135*x^2 - 2494425*x^3 +
1282750*x^4 + 306250*x^5))/187500 + (1719097*Log[-1 - 5*x + Sqrt[5]*Sqrt[3
 + 2*x + 5*x^2]])/(31250*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.17, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2192, 2192, 27, 2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^2(x^2+5x+2)}{\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{30}\int \frac{5131x^5-6135x^4-2820x^3+1350x^2+630x+60}{\sqrt{5x^2+2x+3}}dx+\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{30}\left(\frac{1}{25}\int \frac{6\left(-33259x^4-22012x^3+5625x^2+2625x+250\right)}{\sqrt{5x^2+2x+3}}dx+\frac{5131}{25}\sqrt{5x^2+2x+3}x^4\right)+\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\left(\frac{6}{25}\int \frac{-33259x^4-22012x^3+5625x^2+2625x+250}{\sqrt{5x^2+2x+3}}dx+\frac{5131}{25}\sqrt{5x^2+2x+3}x^4\right)+\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\int \frac{-207427x^3+411831x^2+52500x+5000}{\sqrt{5x^2+2x+3}}dx-\frac{33259}{20}x^3\sqrt{5x^2+2x+3}\right)+\frac{5131}{25}\sqrt{5x^2+2x+3x}\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{1}{15}\int \frac{2\left(3607300x^2+1016031x+37500\right)}{\sqrt{5x^2+2x+3}}dx-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x^3\sqrt{5x^2+2x}\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\int \frac{3607300x^2+1016031x+37500}{\sqrt{5x^2+2x+3}}dx-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x^3\sqrt{5x^2+2x+3}\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 2192$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int -\frac{30(22053x+348230)}{\sqrt{5x^2+2x+3}}dx+360730\sqrt{5x^2+2x+3}x\right)-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\int \frac{22053x+348230}{\sqrt{5x^2+2x+3}}dx\right)-\frac{207427}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{33259}{20}x\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097}{5}\int \frac{1}{\sqrt{5x^2+2x+3}}dx+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{207}{1}\right.\right.\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 1090$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right.\right.\right.\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

$$\Big\downarrow \; 222$$

$$\frac{1}{30}\left(\frac{6}{25}\left(\frac{1}{20}\left(\frac{2}{15}\left(360730x\sqrt{5x^2+2x+3}-3\left(\frac{1719097\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}+\frac{22053}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{20742}{15}\right.\right.\right.\right.$$
$$\frac{49}{30}\sqrt{5x^2+2x+3}x^5$$

input
```
Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(49*x^5*Sqrt[3 + 2*x + 5*x^2])/30 + ((5131*x^4*Sqrt[3 + 2*x + 5*x^2])/25 +
 (6*((-33259*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((-207427*x^2*Sqrt[3 + 2*x +
5*x^2])/15 + (2*(360730*x*Sqrt[3 + 2*x + 5*x^2] - 3*((22053*Sqrt[3 + 2*x +
 5*x^2])/5 + (1719097*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5]))))/15)
/20))/25)/30
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.38

method	result
risch	$\frac{\left(306250x^5+1282750x^4-2494425x^3-1037135x^2+3607300x-132318\right)\sqrt{5x^2+2x+3}}{187500}-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$
trager	$\left(\frac{49}{30}x^5+\frac{5131}{750}x^4-\frac{33259}{2500}x^3-\frac{207427}{37500}x^2+\frac{36073}{1875}x-\frac{22053}{31250}\right)\sqrt{5x^2+2x+3}-\frac{1719097\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\right.}{}$
default	$-\frac{1719097\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}-\frac{22053\sqrt{5x^2+2x+3}}{31250}+\frac{36073x\sqrt{5x^2+2x+3}}{1875}-\frac{207427x^2\sqrt{5x^2+2x+3}}{37500}-\frac{33259x^3\sqrt{5}}{250}$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
1/187500*(306250*x^5+1282750*x^4-2494425*x^3-1037135*x^2+3607300*x-132318)
*(5*x^2+2*x+3)^(1/2)-1719097/156250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{1}{187500}\left(306250\,x^5+1282750\,x^4-2494425\,x^3-1037135\,x^2+3607300\,x-132318\right)\sqrt{5\,x^2+2\,x+3} + \frac{1719097}{312500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
1/187500*(306250*x^5 + 1282750*x^4 - 2494425*x^3 - 1037135*x^2 + 3607300*x
 - 132318)*sqrt(5*x^2 + 2*x + 3) + 1719097/312500*sqrt(5)*log(sqrt(5)*sqrt
(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\cdot\left(\frac{49x^5}{30}+\frac{5131x^4}{750}-\frac{33259x^3}{2500}-\frac{207427x^2}{37500}+\frac{36073x}{1875}-\frac{22053}{31250}\right) - \frac{1719097\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{156250}$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(49*x**5/30 + 5131*x**4/750 - 33259*x**3/2500 - 207
427*x**2/37500 + 36073*x/1875 - 22053/31250) - 1719097*sqrt(5)*asinh(5*sqr
t(14)*(x + 1/5)/14)/156250
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49}{30}\sqrt{5x^2+2x+3}x^5 + \frac{5131}{750}\sqrt{5x^2+2x+3}x^4 - \frac{33259}{2500}\sqrt{5x^2+2x+3}x^3 - \frac{207427}{37500}\sqrt{5x^2+2x+3}x^2 + \frac{36073}{1875}\sqrt{5x^2+2x+3}x - \frac{1719097}{156250}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) - \frac{22053}{31250}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
49/30*sqrt(5*x^2 + 2*x + 3)*x^5 + 5131/750*sqrt(5*x^2 + 2*x + 3)*x^4 - 332
59/2500*sqrt(5*x^2 + 2*x + 3)*x^3 - 207427/37500*sqrt(5*x^2 + 2*x + 3)*x^2
 + 36073/1875*sqrt(5*x^2 + 2*x + 3)*x - 1719097/156250*sqrt(5)*arcsinh(1/1
4*sqrt(14)*(5*x + 1)) - 22053/31250*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.50

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{1}{187500}(5((5(70(175x+733)x-99777)x-207427)x+721460)x-132318)\sqrt{5x^2+2x+3} + \frac{1719097}{156250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/187500*(5*((5*(70*(175*x + 733)*x - 99777)*x - 207427)*x + 721460)*x - 1
32318)*sqrt(5*x^2 + 2*x + 3) + 1719097/156250*sqrt(5)*log(-sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^2}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.85

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{49\sqrt{5x^2+2x+3}\,x^5}{30} + \frac{5131\sqrt{5x^2+2x+3}\,x^4}{750} - \frac{33259\sqrt{5x^2+2x+3}\,x^3}{2500} - \frac{207427\sqrt{5x^2+2x+3}\,x^2}{37500} + \frac{36073\sqrt{5x^2+2x+3}\,x}{1875} - \frac{22053\sqrt{5x^2+2x+3}}{31250} - \frac{1719097\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\,\sqrt{5}+5x+1}{\sqrt{14}}\right)}{156250}$$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
(1531250*sqrt(5*x**2 + 2*x + 3)*x**5 + 6413750*sqrt(5*x**2 + 2*x + 3)*x**4
 - 12472125*sqrt(5*x**2 + 2*x + 3)*x**3 - 5185675*sqrt(5*x**2 + 2*x + 3)*x
**2 + 18036500*sqrt(5*x**2 + 2*x + 3)*x - 661590*sqrt(5*x**2 + 2*x + 3) -
10314582*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)))
/937500
```

3.16 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 101

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{463}{125}\sqrt{3+2x+5x^2}+\frac{59}{30}x\sqrt{3+2x+5x^2} - \frac{571}{300}x^2\sqrt{3+2x+5x^2} - \frac{7}{20}x^3\sqrt{3+2x+5x^2}-\frac{1901\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{250\sqrt{5}}$$

output
```
463/125*(5*x^2+2*x+3)^(1/2)+59/30*x*(5*x^2+2*x+3)^(1/2)-571/300*x^2*(5*x^2
+2*x+3)^(1/2)-7/20*x^3*(5*x^2+2*x+3)^(1/2)-1901/1250*arcsinh(1/14*(1+5*x)*
14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \frac{\sqrt{3+2x+5x^2}(5556+2950x-2855x^2-525x^3)}{1500} + \frac{1901\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(Sqrt[3 + 2*x + 5*x^2]*(5556 + 2950*x - 2855*x^2 - 525*x^3))/1500 + (1901*
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(250*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2192, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)(x^2+5x+2)}{\sqrt{5x^2+2x+3}}dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\int \frac{-571x^3+203x^2+260x+40}{\sqrt{5x^2+2x+3}}dx-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\left(\frac{1}{15}\int \frac{2(2950x^2+3663x+300)}{\sqrt{5x^2+2x+3}}dx-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\frac{2}{15}\int \frac{2950x^2+3663x+300}{\sqrt{5x^2+2x+3}}dx-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{20}\left(\frac{2}{15}\left(\frac{1}{10}\int -\frac{30(195-926x)}{\sqrt{5x^2+2x+3}}dx+295\sqrt{5x^2+2x+3}x\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\int\frac{195-926x}{\sqrt{5x^2+2x+3}}dx\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$\big\downarrow$ 1160

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901}{5}\int\frac{1}{\sqrt{5x^2+2x+3}}dx-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$\big\downarrow$ 1090

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{10\sqrt{70}}-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

$\big\downarrow$ 222

$$\frac{1}{20}\left(\frac{2}{15}\left(295x\sqrt{5x^2+2x+3}-3\left(\frac{1901\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{5\sqrt{5}}-\frac{926}{5}\sqrt{5x^2+2x+3}\right)\right)-\frac{571}{15}x^2\sqrt{5x^2+2x+3}\right)-\frac{7}{20}x^3\sqrt{5x^2+2x+3}$$

input
```
Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/Sqrt[3 + 2*x + 5*x^2],x]
```

output
```
(-7*x^3*Sqrt[3 + 2*x + 5*x^2])/20 + ((-571*x^2*Sqrt[3 + 2*x + 5*x^2])/15 +
 (2*(295*x*Sqrt[3 + 2*x + 5*x^2] - 3*((-926*Sqrt[3 + 2*x + 5*x^2])/5 + (19
01*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/(5*Sqrt[5]))))/15)/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.45

method	result
risch	$-\frac{\left(525x^{3}+2855x^{2}-2950x-5556\right)\sqrt{5x^{2}+2x+3}}{1500}-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}$
trager	$\left(-\frac{7}{20}x^{3}-\frac{571}{300}x^{2}+\frac{59}{30}x+\frac{463}{125}\right)\sqrt{5x^{2}+2x+3}+\frac{1901\operatorname{RootOf}\left(\_Z^{2}-5\right)\ln\left(-5\operatorname{RootOf}\left(\_Z^{2}-5\right)x-\operatorname{RootOf}\left(\_\right.\right.}{1250}$
default	$-\frac{1901\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}+\frac{463\sqrt{5x^{2}+2x+3}}{125}+\frac{59x\sqrt{5x^{2}+2x+3}}{30}-\frac{571x^{2}\sqrt{5x^{2}+2x+3}}{300}-\frac{7x^{3}\sqrt{5x^{2}+2x+3}}{20}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/1500*(525*x^3+2855*x^2-2950*x-5556)*(5*x^2+2*x+3)^(1/2)-1901/1250*5^(1/
2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.66

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{1500}\left(525\,x^3+2855\,x^2-2950\,x-5556\right)\sqrt{5\,x^2+2\,x+3} + \frac{1901}{2500}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output
```
-1/1500*(525*x^3 + 2855*x^2 - 2950*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/
2500*sqrt(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x -
 8)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \sqrt{5x^2+2x+3}\left(-\frac{7x^3}{20}-\frac{571x^2}{300}+\frac{59x}{30}+\frac{463}{125}\right) - \frac{1901\sqrt{5}\operatorname{asinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{1250}$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(1/2),x)
```

output
```
sqrt(5*x**2 + 2*x + 3)*(-7*x**3/20 - 571*x**2/300 + 59*x/30 + 463/125) - 1
901*sqrt(5)*asinh(5*sqrt(14)*(x + 1/5)/14)/1250
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{7}{20}\sqrt{5x^2+2x+3}x^3 - \frac{571}{300}\sqrt{5x^2+2x+3}x^2 + \frac{59}{30}\sqrt{5x^2+2x+3}x - \frac{1901}{1250}\sqrt{5}\operatorname{arsinh}\left(\frac{1}{14}\sqrt{14}(5x+1)\right) + \frac{463}{125}\sqrt{5x^2+2x+3}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-7/20*sqrt(5*x^2 + 2*x + 3)*x^3 - 571/300*sqrt(5*x^2 + 2*x + 3)*x^2 + 59/3
0*sqrt(5*x^2 + 2*x + 3)*x - 1901/1250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x +
 1)) + 463/125*sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.61

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{1}{1500}(5((105x+571)x-590)x-5556)\sqrt{5x^2+2x+3} + \frac{1901}{1250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5x^2+2x+3}\right)-1\right)$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
-1/1500*(5*((105*x + 571)*x - 590)*x - 5556)*sqrt(5*x^2 + 2*x + 3) + 1901/
1250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{\sqrt{3+2x+5x^2}}\,dx = -\frac{7\sqrt{5x^2+2x+3}\,x^3}{20} - \frac{571\sqrt{5x^2+2x+3}\,x^2}{300} + \frac{59\sqrt{5x^2+2x+3}\,x}{30} + \frac{463\sqrt{5x^2+2x+3}}{125} - \frac{1901\sqrt{5}\log\left(\frac{\sqrt{5x^2+2x+3}\sqrt{5}+5x+1}{\sqrt{14}}\right)}{1250}$$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 2625*sqrt(5*x**2 + 2*x + 3)*x**3 - 14275*sqrt(5*x**2 + 2*x + 3)*x**2 +
 14750*sqrt(5*x**2 + 2*x + 3)*x + 27780*sqrt(5*x**2 + 2*x + 3) - 11406*sqr
t(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)))/7500
```

3.17 $\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 164

$$\begin{aligned}\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx &= -\frac{\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{7\sqrt{5}} \\ &\quad -\frac{3}{14}\sqrt{\frac{4091-1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right) \\ &\quad +\frac{3}{14}\sqrt{\frac{4091+1055\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)\end{aligned}$$

output

```
-1/35*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)-3/39116*(11430254-2947670*11^
(1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/
2)/(5*x^2+2*x+3)^(1/2))+3/39116*(11430254+2947670*11^(1/2))^(1/2)*arctanh(
(23+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2
))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \frac{\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{7\sqrt{5}} + \frac{3}{14}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3 +7\#1^4\&, \frac{29\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+10\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-13}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]]/(7*Sqrt[5]) + (3*RootSum[83
- 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (29*Log[-(Sqrt[5]*
x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 10*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 +
2*x + 5*x^2] - #1]*#1 - 13*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*
#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/14
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2143, 27, 1090, 222, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \text{2143}$$

$$-\frac{1}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx - \frac{1}{7}\int -\frac{3(13x+5)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx$$

$\big\downarrow$ 27

$$\frac{3}{7}\int \frac{13x+5}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{1}{7}\int \frac{1}{\sqrt{5x^2+2x+3}}dx$$

$\big\downarrow$ 1090

$$\frac{3}{7}\int \frac{13x+5}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{14\sqrt{70}}$$

$\big\downarrow$ 222

$$\frac{3}{7}\int \frac{13x+5}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$\big\downarrow$ 1365

$$\frac{3}{7}\left(\frac{1}{11}\left(143-61\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(143+61\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x\right.}$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$\big\downarrow$ 27

$$\frac{3}{7}\left(\frac{1}{22}\left(143-61\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(143+61\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+\right.}$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$\big\downarrow$ 1154

$$\frac{3}{7}\left(-\frac{1}{11}\left(143-61\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\right.$$
$$\frac{\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

$\big\downarrow$ 219

$$\frac{3}{7}\left(\frac{\left(143-61\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(143+61\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right.$$

$$\frac{\operatorname{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{7\sqrt{5}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
-1/7*ArcSinh[(2 + 10*x)/(2*Sqrt[14])]/Sqrt[5] + (3*(((143 - 61*Sqrt[11])*A
rcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*
Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + ((143 + 61*Sqr
t[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{35}+\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\,\operatorname{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^{2}+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+}}\right.}{154\sqrt{250-34\sqrt{11}}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-5\right)\ln\left(-5\operatorname{RootOf}\left(\_Z^{2}-5\right)x-\operatorname{RootOf}\left(\_Z^{2}-5\right)+5\sqrt{5x^{2}+2x+3}\right)}{35}+\frac{\operatorname{RootOf}\left(\_Z^{2}+382515364\operatorname{RootOf}\left(240\right.\right.}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/35*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))+3/154*(-61+13*11^(1/2))*11^(1
/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7
*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*1
1^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(
1/2))+3/154*(61+13*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2
*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34
*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(114) = 228$.

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.76

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{3}{28}\sqrt{\frac{1}{2794}}\sqrt{1055\sqrt{11}+4091}\log\left(\frac{3\left(2\sqrt{\frac{1}{2794}}\sqrt{5\,x^2+2\,x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)+\right.}{x}\right.$$

$$+\frac{3}{28}\sqrt{\frac{1}{2794}}\sqrt{1055\sqrt{11}+4091}\log\left(-\frac{3\left(2\sqrt{\frac{1}{2794}}\sqrt{5\,x^2+2\,x+3}\sqrt{1055\sqrt{11}+4091}(172\sqrt{11}-715)\right.}{x}\right.$$

$$+\frac{1}{70}\sqrt{5}\log\left(\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-25\,x^2-10\,x-8\right)$$

$$-\frac{1}{28}\sqrt{-\frac{9495}{2794}\sqrt{11}+\frac{36819}{2794}}\log\left(-\frac{2\sqrt{5\,x^2+2\,x+3}(172\sqrt{11}+715)\sqrt{-\frac{9495}{2794}\sqrt{11}+\frac{36819}{2794}}+399\sqrt{11}(}{x}\right.$$

$$+\frac{1}{28}\sqrt{-\frac{9495}{2794}\sqrt{11}+\frac{36819}{2794}}\log\left(\frac{2\sqrt{5\,x^2+2\,x+3}(172\sqrt{11}+715)\sqrt{-\frac{9495}{2794}\sqrt{11}+\frac{36819}{2794}}-399\sqrt{11}(x}{x}\right.$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="fri
cas")
```

output

```
-3/28*sqrt(1/2794)*sqrt(1055*sqrt(11) + 4091)*log(3*(2*sqrt(1/2794)*sqrt(5
*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091)*(172*sqrt(11) - 715) + 133*sqrt
(11)*(x + 3) + 399*x - 665)/x) + 3/28*sqrt(1/2794)*sqrt(1055*sqrt(11) + 40
91)*log(-3*(2*sqrt(1/2794)*sqrt(5*x^2 + 2*x + 3)*sqrt(1055*sqrt(11) + 4091
)*(172*sqrt(11) - 715) - 133*sqrt(11)*(x + 3) - 399*x + 665)/x) + 1/70*sqr
t(5)*log(sqrt(5)*sqrt(5*x^2 + 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 1/
28*sqrt(-9495/2794*sqrt(11) + 36819/2794)*log(-(2*sqrt(5*x^2 + 2*x + 3)*(1
72*sqrt(11) + 715)*sqrt(-9495/2794*sqrt(11) + 36819/2794) + 399*sqrt(11)*(
x + 3) - 1197*x + 1995)/x) + 1/28*sqrt(-9495/2794*sqrt(11) + 36819/2794)*l
og((2*sqrt(5*x^2 + 2*x + 3)*(172*sqrt(11) + 715)*sqrt(-9495/2794*sqrt(11)
+ 36819/2794) - 399*sqrt(11)*(x + 3) + 1197*x - 1995)/x)
```

Sympy [F]

$$\begin{aligned}
&\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx \\
&= -\int \frac{5x}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx \\
&\quad -\int \frac{x^2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx \\
&\quad -\int \frac{2}{7x^2\sqrt{5x^2+2x+3}-4x\sqrt{5x^2+2x+3}-\sqrt{5x^2+2x+3}}\,dx
\end{aligned}$$

input

```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(1/2),x)
```

output

```
-Integral(5*x/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3)
- sqrt(5*x**2 + 2*x + 3)), x) - Integral(x**2/(7*x**2*sqrt(5*x**2 + 2*x +
3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2
/(7*x**2*sqrt(5*x**2 + 2*x + 3) - 4*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2
 + 2*x + 3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(114) = 228$.

Time = 0.16 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.84

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="max
ima")
```

output
```
-1/10780*sqrt(11)*(28*sqrt(11)*sqrt(5)*arcsinh(5/14*sqrt(7)*sqrt(2)*x + 1/
14*sqrt(7)*sqrt(2)) - 1365*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*s
qrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*
sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) +
23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt(17*sqrt(11) + 125) +
 390*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11)
 - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1/7*sqrt(11)*s
qrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt(2)/abs(14*x
+ 2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49) - 6405*sqrt(2)*arcsinh(5
/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 17/7*sqrt(7)*sq
rt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x
 - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4))/sqrt
(17*sqrt(11) + 125) - 1830*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x
 + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) + 1
/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqrt
(2)/abs(14*x + 2*sqrt(11) - 4))/sqrt(-34/49*sqrt(11) + 250/49))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx = \frac{1}{35}\sqrt{5}\log\left(-5\sqrt{5}x-\sqrt{5}+5\sqrt{5x^2+2x+3}\right) + 0.353184817631429\log\left(-\sqrt{5}x + \sqrt{5x^2+2x+3}+4.41924736459000\right) - 0.0986339689905714\log\left(-\sqrt{5}x + \sqrt{5x^2+2x+3}+1.25295163054000\right) - 0.353184817631429\log\left(-\sqrt{5}x + \sqrt{5x^2+2x+3}-1.02258038113000\right) + 0.0986339689905714\log\left(-\sqrt{5}x + \sqrt{5x^2+2x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x, algorithm="gia
c")
```

output
```
1/35*sqrt(5)*log(-5*sqrt(5)*x - sqrt(5) + 5*sqrt(5*x^2 + 2*x + 3)) + 0.353
184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) -
0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.252951630540
00) - 0.353184817631429*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803
8113000) + 0.0986339689905714*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.0
9411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx=\int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,\left(-7\,x^2+4\,x+1\right)}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)), x)
```

Reduce [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\sqrt{3+2x+5x^2}}\,dx=\int \frac{x^2+5x+2}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x)
```

output
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2),x)
```

3.18 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 178

$$\begin{aligned}
\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx
&= -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{5588\,(1+4x-7x^2)} \\
&\quad - \frac{\sqrt{\frac{3027900955+14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176} \\
&\quad + \frac{\sqrt{\frac{3027900955-14035681\sqrt{11}}{2794}}\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{11176}
\end{aligned}$$

output
```
-3*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-39116*x^2+22352*x+5588)-1/31225744*(84
59955268270+39215692714*11^(1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2
))*x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))+1/31225744*(84599552682
70-39215692714*11^(1/2))^(1/2)*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)/(25
0+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.98

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$
$$= -\frac{3(-40+371x)\sqrt{3+2x+5x^2}}{5588(-1-4x+7x^2)} - \frac{1}{49}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{-397\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+7\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}\&\Big]$$
$$+\frac{3\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{-1510889\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+238966\sqrt{5}\log}{-4\sqrt{5}}}{547624}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(-3*(-40 + 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(-1 - 4*x + 7*x^2)) - RootS
um[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-397*Log[-(
Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 7*Sqrt[5]*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^
3) & ]/49 + (3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#
1^4 & , (-1510889*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 238966*
Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 - 60319*Log[-(Sq
rt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[
5]*#1^2 + 7*#1^3) & ])/547624
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{\left(-7x^2+4x+1\right)^2 \sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \ 2135$$

$$-\frac{\int -\frac{8(3693x+6517)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3693x+6517}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 1365$$

$$\frac{\frac{1}{11}\left(40623-53005\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{11}\left(40623+53005\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{22}\left(40623-53005\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{1}{22}\left(40623+53005\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 1154$$

$$\frac{-\frac{1}{11}\left(40623-53005\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{1}{11}\left(40623+53005\right.}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{\left(40623-53005\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{\left(40623+53005\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}}{5588}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
(-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) + (((4062
3 - 53005*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*
(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11
])]) + ((40623 + 53005*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11]
)*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125
 + 17*Sqrt[11])]))/5588
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.24

method	result
risch	$-\frac{3(-40+371x)\sqrt{5x^2+2x+3}}{5588(7x^2-4x-1)}+\frac{\left(-53005+3693\sqrt{11}\right)\sqrt{11}\,\mathrm{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{1}}{7}\right.}}\right.}{122936\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	$\frac{161\sqrt{11}\,\mathrm{arctanh}\left(\frac{250+34\sqrt{11}+\frac{49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250+34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}+\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}-\frac{\sqrt{11}}{7}\right)+250+34\sqrt{11}}}\right)}{484\sqrt{250+34\sqrt{11}}}-\frac{161\sqrt{11}\,\mathrm{arctanh}\left(\frac{}{\sqrt{250-34\sqrt{1}}}\right.}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-3/5588*(-40+371*x)/(7*x^2-4*x-1)*(5*x^2+2*x+3)^(1/2)+1/122936*(-53005+369
3*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11
^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/
(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+2
50-34*11^(1/2))^(1/2))+1/122936*(53005+3693*11^(1/2))*11^(1/2)/(250+34*11^
(1/2))^(1/2)*arctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2
/7-1/7*11^(1/2)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(
34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(129) = 258$.

Time = 0.09 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.74

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{(7\,x^2-4\,x-1)\sqrt{\frac{1275971}{254}\sqrt{11}+\frac{3027900955}{2794}}\log\left(-\frac{2\sqrt{5\,x^2+2\,x+3}\left(71796\sqrt{11}+567523\right)\sqrt{\frac{1275971}{254}\sqrt{11}+\frac{3027900955}{2794}}+1899}{x}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/22352*((7*x^2 - 4*x - 1)*sqrt(1275971/254*sqrt(11) + 3027900955/2794)*l
og(-(2*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) + 567523)*sqrt(1275971/254*sq
rt(11) + 3027900955/2794) + 189964949*sqrt(11)*(x + 3) - 569894847*x + 949
824745)/x) - (7*x^2 - 4*x - 1)*sqrt(1275971/254*sqrt(11) + 3027900955/2794
)*log((2*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) + 567523)*sqrt(1275971/254*
sqrt(11) + 3027900955/2794) - 189964949*sqrt(11)*(x + 3) + 569894847*x - 9
49824745)/x) + (7*x^2 - 4*x - 1)*sqrt(-1275971/254*sqrt(11) + 3027900955/2
794)*log((2*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-1275971/
254*sqrt(11) + 3027900955/2794) + 189964949*sqrt(11)*(x + 3) + 569894847*x
 - 949824745)/x) - (7*x^2 - 4*x - 1)*sqrt(-1275971/254*sqrt(11) + 30279009
55/2794)*log(-(2*sqrt(5*x^2 + 2*x + 3)*(71796*sqrt(11) - 567523)*sqrt(-127
5971/254*sqrt(11) + 3027900955/2794) - 189964949*sqrt(11)*(x + 3) - 569894
847*x + 949824745)/x) + 12*sqrt(5*x^2 + 2*x + 3)*(371*x - 40))/(7*x^2 - 4*
x - 1)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5x+2}{\sqrt{5x^2+2x+3}\,(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(1/2),x)
```

output
```
Integral((x**2 + 5*x + 2)/(sqrt(5*x**2 + 2*x + 3)*(7*x**2 - 4*x - 1)**2),
x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^2\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*sqrt(5*x^2 + 2*x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(129) = 258$.

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.55

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{3\left(1231\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3+1735\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2-3913\,\sqrt{5}x-3989 \cdots\right)}{2794\left(7\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2+ \cdots\right)}$$
$$+ 0.0924287071109520\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$- 0.0938608034602720\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$- 0.0924287071109520\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+ 0.0938608034602720\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
3/2794*(1231*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 1735*sqrt(5)*(sqrt(5)
*x - sqrt(5*x^2 + 2*x + 3))^2 - 3913*sqrt(5)*x - 3989*sqrt(5) + 3913*sqrt(
5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sq
rt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)
)^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.09242870711
09520*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.09386
08034602720*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0
.0924287071109520*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.0225803811300
0) + 0.0938608034602720*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.0941123
5400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,{\left(-7\,x^2+4\,x+1\right)}^2}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 65.19 (sec) , antiderivative size = 1295, normalized size of antiderivative = 7.28

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 2967517*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x +
3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*
sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 12
5)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))
/(8890*x**2 + 3556*x + 5334))*x**2 + 1695724*sqrt(17*sqrt(11) - 125)*sqrt(
22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 1
9*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2
 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)
*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x + 423931*
sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*s
qrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 1
25)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x
 - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2
+ 3556*x + 5334)) - 2417030*sqrt(17*sqrt(11) - 125)*sqrt(2)*atan((24*sqrt(
5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*
x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(1
7*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11)
- 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x**2 + 1381160*sqrt(17*sqrt(1
1) - 125)*sqrt(2)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*
sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)...
```

3.19 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 227

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = -\frac{3(40-371x)\sqrt{3+2x+5x^2}}{11176\,(1+4x-7x^2)^2} - \frac{7(409769-1189370x)\sqrt{3+2x+5x^2}}{62451488\,(1+4x-7x^2)} - \frac{7\left(39370231-2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(39370231+2538725\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{124902976\sqrt{22\left(125+17\sqrt{11}\right)}}$$

output

```
-3/11176*(40-371*x)*(5*x^2+2*x+3)^(1/2)/(-7*x^2+4*x+1)^2-7*(409769-1189370
*x)*(5*x^2+2*x+3)^(1/2)/(-437160416*x^2+249805952*x+62451488)-7/124902976*
(39370231-2538725*11^(1/2))*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-3
4*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))/(2750-374*11^(1/2))^(1/2)+7/1249029
76*(39370231+2538725*11^(1/2))*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)/(25
0+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.91

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{\frac{235298\sqrt{3+2x+5x^2}(-3538943+3071502x+53381041x^2-58279130x^3)}{(1+4x-7x^2)^2} - 1796775175713\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1\right.}{}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output
```
((235298*Sqrt[3 + 2*x + 5*x^2]*(-3538943 + 3071502*x + 53381041*x^2 - 5827
9130*x^3))/(1 + 4*x - 7*x^2)^2 - 1796775175713*RootSum[83 - 16*Sqrt[5]*#1
- 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 11176*Ro
otSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (10486671
792*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 6928653865
*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1
+ 6*Sqrt[5]*#1^2 + 7*#1^3) & ] - 3*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 +
8*Sqrt[5]*#1^3 + 7*#1^4 & , (36376673721218*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqr
t[3 + 2*x + 5*x^2] - #1]*#1 + 26508461599305*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]
)/14694710223424
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^3\sqrt{5x^2+2x+3}}\,dx$$

$$\Big\downarrow \; 2135$$

$$-\frac{\int -\frac{8\left(11130x^2+10125x+16253\right)}{\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}dx}{89408}-\frac{3\sqrt{5x^2+2x+3}(40-371x)}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11130x^2+10125x+16253}{\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}dx}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 2135$$

$$\frac{-\frac{\int -\frac{8(17771075x+34292781)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{44704}-\frac{7\sqrt{5x^2+2x+3}(409769-1189370x)}{5588\left(-7x^2+4x+1\right)}}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{17771075x+34292781}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-1189370x)\sqrt{5x^2+2x+3}}{5588\left(-7x^2+4x+1\right)}}{11176}-\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{\frac{7}{11}\left(27925975-39370231\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(27925975+39370231\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-11\ldots}{5588(\ldots}}{11176}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{7}{22}\left(27925975-39370231\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(27925975+39370231\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx}{5588}-\frac{7(409769-118\ldots}{5588(-\ldots}}{11176}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{-\frac{7}{11}\left(27925975-39370231\sqrt{11}\right)\int\frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(27925975+39370231\sqrt{11}\right)\int -}{5588}}{11176}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{7\left(27925975-39370231\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(27925975+39370231\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}}{5588}-\frac{7(40}{}}{11176}$$

$$\frac{3(40-371x)\sqrt{5x^2+2x+3}}{11176\left(-7x^2+4x+1\right)^2}$$

input

```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*Sqrt[3 + 2*x + 5*x^2]),x]
```

output

```
(-3*(40 - 371*x)*Sqrt[3 + 2*x + 5*x^2])/(11176*(1 + 4*x - 7*x^2)^2) + ((-7
*(409769 - 1189370*x)*Sqrt[3 + 2*x + 5*x^2])/(5588*(1 + 4*x - 7*x^2)) + ((
7*(27925975 - 39370231*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11]
)*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125
 - 17*Sqrt[11])]) + (7*(27925975 + 39370231*Sqrt[11])*ArcTanh[(23 + Sqrt[1
1] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^
2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/5588)/11176
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{(58279130x^3-53381041x^2-3071502x+3538943)\sqrt{5x^2+2x+3}}{62451488(7x^2-4x-1)^2}+\frac{7\left(-39370231+2538725\sqrt{11}\right)\sqrt{11}\ \operatorname{arctanh}\left(\frac{}{\sqrt{250-34\sqrt{11}}\sqrt{2}}\right.}{1373932736\sqrt{}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/62451488*(58279130*x^3-53381041*x^2-3071502*x+3538943)/(7*x^2-4*x-1)^2*
(5*x^2+2*x+3)^(1/2)+7/1373932736*(-39370231+2538725*11^(1/2))*11^(1/2)/(25
0-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/
2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2)
)^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7
/1373932736*(39370231+2538725*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*a
rctanh(49/2*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2
)))/(250+34*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(
1/2))*(x-2/7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(174) = 348.

Time = 0.10 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.63

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\sqrt{3+2x+5x^2}}\,dx =$$

$$-\frac{\left(49\,x^4-56\,x^3+2\,x^2+8\,x+1\right)\sqrt{\frac{116724881545921}{254}\sqrt{11}+\frac{826162807691484 25}{2794}}\log\left(-\frac{2\sqrt{5\,x^2+2\,x+3}\sqrt{\frac{11672488154592}{254}}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="f
ricas")
```

output
```
-1/249805952*((49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(116724881545921/254
*sqrt(11) + 82616280769148425/2794)*log(-(2*sqrt(5*x^2 + 2*x + 3)*sqrt(116
724881545921/254*sqrt(11) + 82616280769148425/2794)*(358684877*sqrt(11) +
2940638404) + 5176915513390201*sqrt(11)*(x + 3) - 15530746540170603*x + 25
884577566951005)/x) - (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(11672488154
5921/254*sqrt(11) + 82616280769148425/2794)*log((2*sqrt(5*x^2 + 2*x + 3)*s
qrt(116724881545921/254*sqrt(11) + 82616280769148425/2794)*(358684877*sqrt
(11) + 2940638404) - 5176915513390201*sqrt(11)*(x + 3) + 15530746540170603
*x - 25884577566951005)/x) + (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*sqrt(-116
724881545921/254*sqrt(11) + 82616280769148425/2794)*log((2*sqrt(5*x^2 + 2*
x + 3)*(358684877*sqrt(11) - 2940638404)*sqrt(-116724881545921/254*sqrt(11
) + 82616280769148425/2794) + 5176915513390201*sqrt(11)*(x + 3) + 15530746
540170603*x - 25884577566951005)/x) - (49*x^4 - 56*x^3 + 2*x^2 + 8*x + 1)*
sqrt(-116724881545921/254*sqrt(11) + 82616280769148425/2794)*log(-(2*sqrt(
5*x^2 + 2*x + 3)*(358684877*sqrt(11) - 2940638404)*sqrt(-116724881545921/2
54*sqrt(11) + 82616280769148425/2794) - 5176915513390201*sqrt(11)*(x + 3)
- 15530746540170603*x + 25884577566951005)/x) + 4*(58279130*x^3 - 53381041
*x^2 - 3071502*x + 3538943)*sqrt(5*x^2 + 2*x + 3))/(49*x^4 - 56*x^3 + 2*x^
2 + 8*x + 1)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = -\int \frac{5x}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x}$$
$$-\int \frac{x^2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x}$$
$$-\int \frac{2}{343x^6\sqrt{5x^2+2x+3}-588x^5\sqrt{5x^2+2x+3}+189x^4\sqrt{5x^2+2x+3}+104x^3\sqrt{5x^2+2x+3}-27x}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(1/2),x)
```

output
```
-Integral(5*x/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*
x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3)
 - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x
**2 + 2*x + 3)), x) - Integral(x**2/(343*x**6*sqrt(5*x**2 + 2*x + 3) - 588
*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2*x + 3) + 104*x**3*
sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3) - 12*x*sqrt(5*x**2
 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(343*x**6*sqrt(5*x*
*2 + 2*x + 3) - 588*x**5*sqrt(5*x**2 + 2*x + 3) + 189*x**4*sqrt(5*x**2 + 2
*x + 3) + 104*x**3*sqrt(5*x**2 + 2*x + 3) - 27*x**2*sqrt(5*x**2 + 2*x + 3)
 - 12*x*sqrt(5*x**2 + 2*x + 3) - sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3\sqrt{5\,x^2+2\,x+3}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="m
axima")
```

output
```
-integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*sqrt(5*x^2 + 2*x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(174) = 348$.

Time = 0.17 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.67

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx$$
$$= \frac{124397525\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7+26796567\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-3595807617\left(\sqrt{5}x-\right.}{31225744\left(7\left(\sqrt{\right.\right.}}$$
$$+0.0423989586659649\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.0446437606655585\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.0423989586659649\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.0446437606655585\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x, algorithm="g
iac")
```

output
```
1/31225744*(124397525*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 26796567*sqr
t(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^6 - 3595807617*(sqrt(5)*x - sqrt(
5*x^2 + 2*x + 3))^5 - 1719888775*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3
))^4 + 17096132999*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 8328401413*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 16383202915*sqrt(5)*x - 780062
3485*sqrt(5) + 16383202915*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5*x
^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*(s
qrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 +
 2*x + 3)) + 83)^2 + 0.0423989586659649*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x
+ 3) + 4.41924736459000) - 0.0446437606655585*log(-sqrt(5)*x + sqrt(5*x^2
+ 2*x + 3) + 1.25295163054000) - 0.0423989586659649*log(-sqrt(5)*x + sqrt(
5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0446437606655585*log(-sqrt(5)*x +
 sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \int \frac{x^2+5\,x+2}{\sqrt{5\,x^2+2\,x+3}\,(-7\,x^2+4\,x+1)^3}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(1/2)*(4*x - 7*x^2 + 1)^3), x)
```

Reduce [B] (verification not implemented)

Time = 68.04 (sec) , antiderivative size = 2193, normalized size of antiderivative = 9.66

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\sqrt{3+2x+5x^2}}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(1/2),x)
```

output
```
( - 108940163850*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2
*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqr
t(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11)
 - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqr
t(2))/(8890*x**2 + 3556*x + 5334))*x**4 + 124503044400*sqrt(17*sqrt(11) -
125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt
(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*s
qrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2
+ 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x
**3 - 4446537300*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2
*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqr
t(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11)
 - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqr
t(2))/(8890*x**2 + 3556*x + 5334))*x**2 - 17786149200*sqrt(17*sqrt(11) - 1
25)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(
22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sq
rt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 +
 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x
- 2223268650*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x +
 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt...
```

3.20 $\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{16(6122807-5338217x)}{546875\sqrt{3+2x+5x^2}} + \frac{15715799\sqrt{3+2x+5x^2}}{156250} - \frac{3192602x\sqrt{3+2x+5x^2}}{46875} - \frac{2583293x^2\sqrt{3+2x+5x^2}}{187500} + \frac{393659x^3\sqrt{3+2x+5x^2}}{12500} - \frac{25921x^4\sqrt{3+2x+5x^2}}{3750} - \frac{343}{150}x^5\sqrt{3+2x+5x^2} + \frac{50047657\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{156250\sqrt{5}}$$

output
```
16/546875*(6122807-5338217*x)/(5*x^2+2*x+3)^(1/2)+15715799/156250*(5*x^2+2
*x+3)^(1/2)-3192602/46875*x*(5*x^2+2*x+3)^(1/2)-2583293/187500*x^2*(5*x^2+
2*x+3)^(1/2)+393659/12500*x^3*(5*x^2+2*x+3)^(1/2)-25921/3750*x^4*(5*x^2+2*
x+3)^(1/2)-343/150*x^5*(5*x^2+2*x+3)^(1/2)+50047657/781250*arcsinh(1/14*(1
+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.54

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{3155769618-1045703388x+2135143465x^2-1795638985x^3-17\ldots}{6562500\sqrt{3+2x\ldots}}$$
$$-\frac{50047657\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{156250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(3155769618 - 1045703388*x + 2135143465*x^2 - 1795638985*x^3 - 174819575*x
^4 + 897612625*x^5 - 256821250*x^6 - 75031250*x^7)/(6562500*Sqrt[3 + 2*x +
 5*x^2]) - (50047657*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(15625
0*Sqrt[5])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^3(x^2+5x+2)}{(5x^2+2x+3)^{3/2}}dx$$

$$\Big\downarrow \; 2191$$

$$\frac{1}{28}\int \frac{28(-5359375x^6-15465625x^5+41667500x^4-5403250x^3-36448575x^2+16868255x+16918718)}{78125\sqrt{5x^2+2x+3}}dx+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{-5359375x^6-15465625x^5+41667500x^4-5403250x^3-36448575x^2+16868255x+16918718}{\sqrt{5x^2+2x+3}}dx}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{30}\int \frac{5(-81003125x^5+266083125x^4-32419500x^3-218691450x^2+101209530x+101512308)}{\sqrt{5x^2+2x+3}}dx-\frac{1071875}{6}x^5\sqrt{5x^2+2x+3}}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\int \frac{-81003125x^5+266083125x^4-32419500x^3-218691450x^2+101209530x+101512308}{\sqrt{5x^2+2x+3}}dx-\frac{1071875}{6}x^5\sqrt{5x^2+2x+3}}{78125}+\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{6}\left(\frac{1}{25}\int \frac{150(49207375x^4+1077000x^3-36448575x^2+16868255x+16918718)}{\sqrt{5x^2+2x+3}}dx-3240125x^4\sqrt{5x^2+2x+3}\right)-\frac{1071875}{6}x^5\sqrt{5x^2-}}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\int \frac{49207375x^4+1077000x^3-36448575x^2+16868255x+16918718}{\sqrt{5x^2+2x+3}}dx-3240125x^4\sqrt{5x^2+2x+3}\right)-\frac{1071875}{6}x^5\sqrt{5x^2+2x+}}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{20}\int \frac{5(-64582325x^3-234367575x^2+67473020x+67674872)}{\sqrt{5x^2+2x+3}}dx+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3240125x^4\sqrt{5x^2+2x+}\right.}{78125}$$
$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\int\frac{-64582325x^3-234367575x^2+67473020x+67674872}{\sqrt{5x^2+2x+3}}dx+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3240125x^4\sqrt{5x^2+2x+3}\right)}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{1}{15}\int\frac{10(-319260200x^2+139958925x+101512308)}{\sqrt{5x^2+2x+3}}dx-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\int\frac{-319260200x^2+139958925x+101512308}{\sqrt{5x^2+2x+3}}dx-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+2x+3}x^3\right)-3240\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 2192

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(\frac{1}{10}\int\frac{330(7143545x+5978496)}{\sqrt{5x^2+2x+3}}dx-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x}\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 27

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\int\frac{7143545x+5978496}{\sqrt{5x^2+2x+3}}dx-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5x^2+2x+3}\right)+\frac{9841475}{4}\sqrt{5x^2+}\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 1160

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(4549787\int\frac{1}{\sqrt{5x^2+2x+3}}dx+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{5}\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$\downarrow$ 1090

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(\frac{4549787\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)}{2\sqrt{70}}+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916}{3}\right.\right.\right.}{7812}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 222$$

$$\frac{\frac{1}{6}\left(6\left(\frac{1}{4}\left(\frac{2}{3}\left(33\left(\frac{4549787\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)}{\sqrt{5}}+1428709\sqrt{5x^2+2x+3}\right)-31926020x\sqrt{5x^2+2x+3}\right)-\frac{12916465}{3}x^2\sqrt{}\right.\right.\right.}{78125}$$

$$\frac{16(6122807-5338217x)}{546875\sqrt{5x^2+2x+3}}$$

input
```
Int[((1 + 4*x - 7*x^2)^3*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(16*(6122807 - 5338217*x))/(546875*Sqrt[3 + 2*x + 5*x^2]) + ((-1071875*x^5
*Sqrt[3 + 2*x + 5*x^2])/6 + (-3240125*x^4*Sqrt[3 + 2*x + 5*x^2] + 6*((9841
475*x^3*Sqrt[3 + 2*x + 5*x^2])/4 + ((-12916465*x^2*Sqrt[3 + 2*x + 5*x^2])/
3 + (2*(-31926020*x*Sqrt[3 + 2*x + 5*x^2] + 33*(1428709*Sqrt[3 + 2*x + 5*x
^2] + (4549787*ArcSinh[(2 + 10*x)/(2*Sqrt[14])])/Sqrt[5])))/3)/4))/6)/7812
5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}} + \frac{50047}{}$
trager	$-\frac{75031250x^7+256821250x^6-897612625x^5+174819575x^4+1795638985x^3-2135143465x^2+1045703388x-3155769618}{6562500\sqrt{5x^2+2x+3}} - \frac{50047}{}$
default	$\frac{\frac{176049701x}{1093750}+\frac{176049701}{5468750}}{\sqrt{5x^2+2x+3}} + \frac{175268451}{390625\sqrt{5x^2+2x+3}} - \frac{50047657x}{156250\sqrt{5x^2+2x+3}} + \frac{50047657\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{781250} + \frac{6100409}{187500\sqrt{5x}}$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/6562500*(75031250*x^7+256821250*x^6-897612625*x^5+174819575*x^4+1795638
985*x^3-2135143465*x^2+1045703388*x-3155769618)/(5*x^2+2*x+3)^(1/2)+500476
57/781250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1051000797\,\sqrt{5}(5\,x^2+2\,x+3)\log\left(-\sqrt{5}\sqrt{5\,x^2+2\,x+3}(5\,x+1)-\right.}{}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/32812500*(1051000797*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 +
 2*x + 3)*(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(75031250*x^7 + 256821250*x^6
 - 897612625*x^5 + 174819575*x^4 + 1795638985*x^3 - 2135143465*x^2 + 10457
03388*x - 3155769618)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$\int \frac{(1+4x-7x^2)^3\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\int \left(-\frac{29x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \left(-\frac{115x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \frac{61x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \frac{871x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \left(-\frac{127x^5}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \left(-\frac{2065x^6}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int \frac{1127x^7}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \frac{343x^8}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int \left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

input
```
integrate((-7*x**2+4*x+1)**3*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(-29*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-115*x**2/(5*x**2*sqrt(5*x**2
 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) -
 Integral(61*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x +
 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(871*x**4/(5*x**2*sqrt(5*x**
2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x)
- Integral(-127*x**5/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*
x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-2065*x**6/(5*x**2*sqrt(
5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3))
, x) - Integral(1127*x**7/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2
 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(343*x**8/(5*x**2*sq
rt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x +
3)), x) - Integral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2
*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.89

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{343\,x^7}{30\,\sqrt{5\,x^2+2\,x+3}} - \frac{29351\,x^6}{750\,\sqrt{5\,x^2+2\,x+3}} + \frac{1025843\,x^5}{7500\,\sqrt{5\,x^2+2\,x+3}} - \frac{998969\,x^4}{37500\,\sqrt{5\,x^2+2\,x+3}} - \frac{51303971\,x^3}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{61004099\,x^2}{187500\,\sqrt{5\,x^2+2\,x+3}} + \frac{50047657}{781250}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{87141949\,x}{546875\,\sqrt{5\,x^2+2\,x+3}} + \frac{525961603}{1093750\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-343/30*x^7/sqrt(5*x^2 + 2*x + 3) - 29351/750*x^6/sqrt(5*x^2 + 2*x + 3) +
1025843/7500*x^5/sqrt(5*x^2 + 2*x + 3) - 998969/37500*x^4/sqrt(5*x^2 + 2*x
 + 3) - 51303971/187500*x^3/sqrt(5*x^2 + 2*x + 3) + 61004099/187500*x^2/sq
rt(5*x^2 + 2*x + 3) + 50047657/781250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x +
 1)) - 87141949/546875*x/sqrt(5*x^2 + 2*x + 3) + 525961603/1093750/sqrt(5*
x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.49

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\frac{50047657}{781250}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right)$$
$$-\frac{(35\,((5\,(35\,(70\,(175\,x+599)x-146549)x+998969)x+51303971)x-61004099)x+1045703388)x-3}{6562500\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-50047657/781250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))
- 1) - 1/6562500*((35*((5*(35*(70*(175*x + 599)*x - 146549)*x + 998969)*x
+ 51303971)*x - 61004099)*x + 1045703388)*x - 3155769618)/sqrt(5*x^2 + 2*x
 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^3}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^3)/(2*x + 5*x^2 + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.47

$$\int \frac{(1+4x-7x^2)^3(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-1500625000\sqrt{5x^2+2x+3}\,x^7 - 5136425000\sqrt{5x^2+2x+3}\,x^6 + \cdots}{}$$

input
```
int((-7*x^2+4*x+1)^3*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 1500625000*sqrt(5*x**2 + 2*x + 3)*x**7 - 5136425000*sqrt(5*x**2 + 2*x
+ 3)*x**6 + 17952252500*sqrt(5*x**2 + 2*x + 3)*x**5 - 3496391500*sqrt(5*x*
*2 + 2*x + 3)*x**4 - 35912779700*sqrt(5*x**2 + 2*x + 3)*x**3 + 42702869300
*sqrt(5*x**2 + 2*x + 3)*x**2 - 20914067760*sqrt(5*x**2 + 2*x + 3)*x + 6311
5392360*sqrt(5*x**2 + 2*x + 3) + 42040031880*sqrt(5)*log((sqrt(5*x**2 + 2*
x + 3)*sqrt(5) + 5*x + 1)/sqrt(14))*x**2 + 16816012752*sqrt(5)*log((sqrt(5
*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14))*x + 25224019128*sqrt(5)*log(
(sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)) - 26833542615*sqrt(5)
*x**2 - 10733417046*sqrt(5)*x - 16100125569*sqrt(5))/(131250000*(5*x**2 +
2*x + 3))
```

3.21 $\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 124

$$\begin{aligned}\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx &= -\frac{8(12983+136602x)}{21875\sqrt{3+2x+5x^2}} \\ &\quad -\frac{5086\sqrt{3+2x+5x^2}}{3125}-\frac{8749x\sqrt{3+2x+5x^2}}{1250}+\frac{203}{100}x^2\sqrt{3+2x+5x^2} \\ &\quad +\frac{49}{100}x^3\sqrt{3+2x+5x^2}+\frac{89583\,\text{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{1250\sqrt{5}}\end{aligned}$$

output

```
1/21875*(-103864-1092816*x)/(5*x^2+2*x+3)^(1/2)-5086/3125*(5*x^2+2*x+3)^(1
/2)-8749/1250*x*(5*x^2+2*x+3)^(1/2)+203/100*x^2*(5*x^2+2*x+3)^(1/2)+49/100
*x^3*(5*x^2+2*x+3)^(1/2)+89583/6250*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-168536-1298674x-280805x^2-515655x^3+194775x^4+42875}{17500\sqrt{3+2x+5x^2}} - \frac{89583\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{1250\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-168536 - 1298674*x - 280805*x^2 - 515655*x^3 + 194775*x^4 + 42875*x^5)/(
17500*Sqrt[3 + 2*x + 5*x^2]) - (89583*Log[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x
+ 5*x^2]])/(1250*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2191, 27, 2192, 27, 2192, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)^2(x^2+5x+2)}{(5x^2+2x+3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{28}\int \frac{28(30625x^4+105875x^3-173225x^2-52985x+153254)}{3125\sqrt{5x^2+2x+3}}dx - \frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{30625x^4+105875x^3-173225x^2-52985x+153254}{\sqrt{5x^2+2x+3}}dx}{3125} - \frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{1}{20}\int \frac{5\left(380625x^3-748025x^2-211940x+613016\right)}{\sqrt{5x^2+2x+3}}dx+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\int \frac{380625x^3-748025x^2-211940x+613016}{\sqrt{5x^2+2x+3}}dx+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{1}{4}\left(\frac{1}{15}\int \frac{30\left(-437450x^2-182095x+306508\right)}{\sqrt{5x^2+2x+3}}dx+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\left(2\int \frac{-437450x^2-182095x+306508}{\sqrt{5x^2+2x+3}}dx+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{1}{4}\left(2\left(\frac{1}{10}\int \frac{10(437743-50860x)}{\sqrt{5x^2+2x+3}}dx-43745x\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\left(2\left(\int \frac{437743-50860x}{\sqrt{5x^2+2x+3}}dx-43745x\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}\sqrt{5x^2+2x+3}x^3}{3125}-\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}\left(2\left(447915\int \frac{1}{\sqrt{5x^2+2x+3}}dx-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+\frac{6125}{4}}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1090$$

$$\frac{\frac{1}{4}\left(2\left(\frac{89583}{2}\sqrt{\frac{5}{14}}\int \frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2}\right.}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

$\big\downarrow$ 222

$$\frac{\frac{1}{4}\left(2\left(89583\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)-43745\sqrt{5x^2+2x+3}x-10172\sqrt{5x^2+2x+3}\right)+25375\sqrt{5x^2+2x+3}x^2\right)+6}{3125}$$
$$\frac{8(136602x+12983)}{21875\sqrt{5x^2+2x+3}}$$

input
```
Int[((1 + 4*x - 7*x^2)^2*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-8*(12983 + 136602*x))/(21875*Sqrt[3 + 2*x + 5*x^2]) + ((6125*x^3*Sqrt[3
+ 2*x + 5*x^2])/4 + (25375*x^2*Sqrt[3 + 2*x + 5*x^2] + 2*(-10172*Sqrt[3 +
2*x + 5*x^2] - 43745*x*Sqrt[3 + 2*x + 5*x^2] + 89583*Sqrt[5]*ArcSinh[(2 +
10*x)/(2*Sqrt[14])]))/4)/3125
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

method	result
risch	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{6250}$
trager	$\frac{42875x^5+194775x^4-515655x^3-280805x^2-1298674x-168536}{17500\sqrt{5x^2+2x+3}}-\frac{89583\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-5\,\mathrm{RootOf}\left(\_Z^2-5\right)x-\mathrm{RootOf}\right.}{6250}$
default	$-\frac{5564(10x+2)}{21875\sqrt{5x^2+2x+3}}-\frac{28506}{3125\sqrt{5x^2+2x+3}}-\frac{89583x}{1250\sqrt{5x^2+2x+3}}+\frac{89583\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{6250}-\frac{8023x^2}{500\sqrt{5x^2+2x+3}}-$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
1/17500*(42875*x^5+194775*x^4-515655*x^3-280805*x^2-1298674*x-168536)/(5*x
^2+2*x+3)^(1/2)+89583/6250*5^(1/2)*arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{627081\sqrt{5}(5x^2+2x+3)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25\right.}{}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
1/87500*(627081*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x +
3)*(5*x + 1) - 25*x^2 - 10*x - 8) + 5*(42875*x^5 + 194775*x^4 - 515655*x^3
 - 280805*x^2 - 1298674*x - 168536)*sqrt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x +
3)
```

Sympy [F]

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5x+2)(7x^2-4x-1)^2}{(5x^2+2x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-7*x**2+4*x+1)**2*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
Integral((x**2 + 5*x + 2)*(7*x**2 - 4*x - 1)**2/(5*x**2 + 2*x + 3)**(3/2),
 x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.92

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{49\,x^5}{20\,\sqrt{5\,x^2+2\,x+3}} + \frac{1113\,x^4}{100\,\sqrt{5\,x^2+2\,x+3}} - \frac{14733\,x^3}{500\,\sqrt{5\,x^2+2\,x+3}} - \frac{8023\,x^2}{500\,\sqrt{5\,x^2+2\,x+3}} + \frac{89583}{6250}\,\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\,\sqrt{14}(5\,x+1)\right) - \frac{649337\,x}{8750\,\sqrt{5\,x^2+2\,x+3}} - \frac{42134}{4375\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
49/20*x^5/sqrt(5*x^2 + 2*x + 3) + 1113/100*x^4/sqrt(5*x^2 + 2*x + 3) - 147
33/500*x^3/sqrt(5*x^2 + 2*x + 3) - 8023/500*x^2/sqrt(5*x^2 + 2*x + 3) + 89
583/6250*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 649337/8750*x/sqrt(5*x
^2 + 2*x + 3) - 42134/4375/sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{(1+4x-7x^2)^2(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{89583}{6250}\,\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right) + \frac{(35\,((35\,(35\,x+159)x-14733)x-8023)x-1298674)x-168536}{17500\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
-89583/6250*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1)
+ 1/17500*((35*((35*(35*x + 159)*x - 14733)*x - 8023)*x - 1298674)*x - 168
536)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)^2}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1)^2)/(2*x + 5*x^2 + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.71

$$\int \frac{(1+4x-7x^2)^2\,(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1071875\sqrt{5x^2+2x+3}\,x^5+4869375\sqrt{5x^2+2x+3}\,x^4-12891375}{}$$

input
```
int((-7*x^2+4*x+1)^2*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

output
```
(1071875*sqrt(5*x**2 + 2*x + 3)*x**5 + 4869375*sqrt(5*x**2 + 2*x + 3)*x**4
 - 12891375*sqrt(5*x**2 + 2*x + 3)*x**3 - 7020125*sqrt(5*x**2 + 2*x + 3)*x
**2 - 32466850*sqrt(5*x**2 + 2*x + 3)*x - 4213400*sqrt(5*x**2 + 2*x + 3) +
 31354050*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14))
*x**2 + 12541620*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sq
rt(14))*x + 18812430*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1
)/sqrt(14)) - 24272510*sqrt(5)*x**2 - 9709004*sqrt(5)*x - 14563506*sqrt(5)
)/(437500*(5*x**2 + 2*x + 3))
```

3.22 $\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 82

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{2(2321+2449x)}{875\sqrt{3+2x+5x^2}} - \frac{261}{250}\sqrt{3+2x+5x^2} - \frac{7}{50}x\sqrt{3+2x+5x^2} + \frac{149\operatorname{arcsinh}\left(\frac{1+5x}{\sqrt{14}}\right)}{25\sqrt{5}}$$

output
```
1/875*(-4642-4898*x)/(5*x^2+2*x+3)^(1/2)-261/250*(5*x^2+2*x+3)^(1/2)-7/50*
x*(5*x^2+2*x+3)^(1/2)+149/125*arcsinh(1/14*(1+5*x)*14^(1/2))*5^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-2953-2837x-1925x^2-245x^3}{350\sqrt{3+2x+5x^2}} - \frac{149\log\left(-1-5x+\sqrt{5}\sqrt{3+2x+5x^2}\right)}{25\sqrt{5}}$$

input
```
Integrate[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-2953 - 2837*x - 1925*x^2 - 245*x^3)/(350*Sqrt[3 + 2*x + 5*x^2]) - (149*L
og[-1 - 5*x + Sqrt[5]*Sqrt[3 + 2*x + 5*x^2]])/(25*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2191, 27, 2192, 27, 1160, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(-7x^2+4x+1)(x^2+5x+2)}{(5x^2+2x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2191}$$

$$\frac{1}{28}\int \frac{28(-175x^2-705x+562)}{125\sqrt{5x^2+2x+3}}dx-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\int \frac{-175x^2-705x+562}{\sqrt{5x^2+2x+3}}dx-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{1}{125}\left(\frac{1}{10}\int \frac{5(1229-1305x)}{\sqrt{5x^2+2x+3}}dx-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{125}\left(\frac{1}{2}\int \frac{1229-1305x}{\sqrt{5x^2+2x+3}}dx-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{1}{125}\left(\frac{1}{2}\left(1490\int \frac{1}{\sqrt{5x^2+2x+3}}dx-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{125}\left(\frac{1}{2}\left(149\sqrt{\frac{5}{14}}\int\frac{1}{\sqrt{\frac{1}{56}(10x+2)^2+1}}d(10x+2)-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \ 222$$

$$\frac{1}{125}\left(\frac{1}{2}\left(298\sqrt{5}\text{arcsinh}\left(\frac{10x+2}{2\sqrt{14}}\right)-261\sqrt{5x^2+2x+3}\right)-\frac{35}{2}x\sqrt{5x^2+2x+3}\right)-\frac{2(2449x+2321)}{875\sqrt{5x^2+2x+3}}$$

input
```
Int[((1 + 4*x - 7*x^2)*(2 + 5*x + x^2))/(3 + 2*x + 5*x^2)^(3/2),x]
```

output
```
(-2*(2321 + 2449*x))/(875*Sqrt[3 + 2*x + 5*x^2]) + ((-35*x*Sqrt[3 + 2*x +
5*x^2])/2 + (-261*Sqrt[3 + 2*x + 5*x^2] + 298*Sqrt[5]*ArcSinh[(2 + 10*x)/(
2*Sqrt[14])])/2)/125
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}$
trager	$-\frac{245x^3+1925x^2+2837x+2953}{350\sqrt{5x^2+2x+3}}-\frac{149\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-5\operatorname{RootOf}\left(\_Z^2-5\right)x-\operatorname{RootOf}\left(\_Z^2-5\right)+5\sqrt{5x^2+2x+3}\right)}{125}$
default	$-\frac{751(10x+2)}{3500\sqrt{5x^2+2x+3}}-\frac{1001}{125\sqrt{5x^2+2x+3}}-\frac{149x}{25\sqrt{5x^2+2x+3}}+\frac{149\sqrt{5}\,\operatorname{arcsinh}\left(\frac{5\sqrt{14}\left(x+\frac{1}{5}\right)}{14}\right)}{125}-\frac{11x^2}{2\sqrt{5x^2+2x+3}}-\frac{7x^3}{10\sqrt{5x^2+}}$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-1/350*(245*x^3+1925*x^2+2837*x+2953)/(5*x^2+2*x+3)^(1/2)+149/125*5^(1/2)*
arcsinh(5/14*14^(1/2)*(x+1/5))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{1043\sqrt{5}(5x^2+2x+3)\log\left(-\sqrt{5}\sqrt{5x^2+2x+3}(5x+1)-25x^2\right.}{1750\,(5x^2}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
1/1750*(1043*sqrt(5)*(5*x^2 + 2*x + 3)*log(-sqrt(5)*sqrt(5*x^2 + 2*x + 3)*
(5*x + 1) - 25*x^2 - 10*x - 8) - 5*(245*x^3 + 1925*x^2 + 2837*x + 2953)*sq
rt(5*x^2 + 2*x + 3))/(5*x^2 + 2*x + 3)
```

Sympy [F]

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\int\left(-\frac{13x}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int\left(-\frac{7x^2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx - \int\frac{31x^3}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int\frac{7x^4}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\,dx - \int\left(-\frac{2}{5x^2\sqrt{5x^2+2x+3}+2x\sqrt{5x^2+2x+3}+3\sqrt{5x^2+2x+3}}\right)dx$$

input
```
integrate((-7*x**2+4*x+1)*(x**2+5*x+2)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(-13*x/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(-7*x**2/(5*x**2*sqrt(5*x**2 +
 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - I
ntegral(31*x**3/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3
) + 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(7*x**4/(5*x**2*sqrt(5*x**2 +
2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*sqrt(5*x**2 + 2*x + 3)), x) - In
tegral(-2/(5*x**2*sqrt(5*x**2 + 2*x + 3) + 2*x*sqrt(5*x**2 + 2*x + 3) + 3*
sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{7\,x^3}{10\,\sqrt{5\,x^2+2\,x+3}} - \frac{11\,x^2}{2\,\sqrt{5\,x^2+2\,x+3}} + \frac{149}{125}\sqrt{5}\,\mathrm{arsinh}\left(\frac{1}{14}\sqrt{14}(5\,x+1)\right) - \frac{2837\,x}{350\,\sqrt{5\,x^2+2\,x+3}} - \frac{2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-7/10*x^3/sqrt(5*x^2 + 2*x + 3) - 11/2*x^2/sqrt(5*x^2 + 2*x + 3) + 149/125
*sqrt(5)*arcsinh(1/14*sqrt(14)*(5*x + 1)) - 2837/350*x/sqrt(5*x^2 + 2*x +
3) - 2953/350/sqrt(5*x^2 + 2*x + 3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = -\frac{149}{125}\sqrt{5}\log\left(-\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)-1\right) - \frac{(35\,(7\,x+55)x+2837)x+2953}{350\,\sqrt{5\,x^2+2\,x+3}}$$

input
```
integrate((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
-149/125*sqrt(5)*log(-sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) - 1) - 1
/350*((35*(7*x + 55)*x + 2837)*x + 2953)/sqrt(5*x^2 + 2*x + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \int \frac{(x^2+5\,x+2)\,(-7\,x^2+4\,x+1)}{(5\,x^2+2\,x+3)^{3/2}}\,dx$$

input
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2),x)
```

output
```
int(((5*x + x^2 + 2)*(4*x - 7*x^2 + 1))/(2*x + 5*x^2 + 3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.20

$$\int \frac{(1+4x-7x^2)(2+5x+x^2)}{(3+2x+5x^2)^{3/2}}\,dx = \frac{-6125\sqrt{5x^2+2x+3}\,x^3-48125\sqrt{5x^2+2x+3}\,x^2-70925\sqrt{5x^2+}}{}$$

input
```
int((-7*x^2+4*x+1)*(x^2+5*x+2)/(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 6125*sqrt(5*x**2 + 2*x + 3)*x**3 - 48125*sqrt(5*x**2 + 2*x + 3)*x**2 -
 70925*sqrt(5*x**2 + 2*x + 3)*x - 73825*sqrt(5*x**2 + 2*x + 3) + 52150*sqr
t(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14))*x**2 + 20860
*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14))*x + 3129
0*sqrt(5)*log((sqrt(5*x**2 + 2*x + 3)*sqrt(5) + 5*x + 1)/sqrt(14)) - 48980
*sqrt(5)*x**2 - 19592*sqrt(5)*x - 29388*sqrt(5))/(8750*(5*x**2 + 2*x + 3))
```

3.23 $\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 166

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = -\frac{131-605x}{3556\sqrt{3+2x+5x^2}} - \frac{3\sqrt{\frac{281693-25015\sqrt{11}}{1397}}\,\text{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016} + \frac{3\sqrt{\frac{281693+25015\sqrt{11}}{1397}}\,\text{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{1016}$$

output
```
-1/3556*(131-605*x)/(5*x^2+2*x+3)^(1/2)-3/1419352*(393525121-34945955*11^(
1/2))^(1/2)*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2
)/(5*x^2+2*x+3)^(1/2))+3/1419352*(393525121+34945955*11^(1/2))^(1/2)*arcta
nh((23+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(
1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.20

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \frac{-131+605x}{3556\sqrt{3+2x+5x^2}}$$
$$+\frac{3}{254}\text{RootSum}\Big[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3$$
$$+7\#1^4\&, \frac{22\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+41\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)\#1-21\log}{-4\sqrt{5}-35\#1+6\sqrt{5}\#1^2+7\#1^3}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(-131 + 605*x)/(3556*Sqrt[3 + 2*x + 5*x^2]) + (3*RootSum[83 - 16*Sqrt[5]*#
1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (22*Log[-(Sqrt[5]*x) + Sqrt[3 +
2*x + 5*x^2] - #1] + 41*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] -
 #1]*#1 - 21*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt
[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/254
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}\,dx$$
$$\Big\downarrow \text{2135}$$
$$\frac{\int \frac{336(42x+41)}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx}{28448}-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{3}{254}\int \frac{42x+41}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{1365}$$

$$\frac{3}{254}\left(\frac{7}{11}\left(66-53\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(66+53\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{3}{254}\left(\frac{7}{22}\left(66-53\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(66+53\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{1154}$$

$$\frac{3}{254}\left(-\frac{7}{11}\left(66-53\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\cdots\right)-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{3}{254}\left(\frac{7\left(66-53\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(66+53\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right)-\frac{131-605x}{3556\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
-1/3556*(131 - 605*x)/Sqrt[3 + 2*x + 5*x^2] + (3*((7*(66 - 53*Sqrt[11])*Ar
cTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]*S
qrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + (7*(66 + 53*Sqr
t[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqr
t[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])))/254
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.26

method	result
risch	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}+\frac{21\left(-53+6\sqrt{11}\right)\sqrt{11}\,\mathrm{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)}{2}}{\sqrt{250-34\sqrt{11}}\,\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+250-34\sqrt{11}}}\right.}{5588\sqrt{250-34\sqrt{11}}}$
trager	$\frac{-131+605x}{3556\sqrt{5x^2+2x+3}}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+2529285264\,\mathrm{RootOf}\left(12905329536\_Z^4-4015815408\_Z^2+285305881\right)^2-787050242\right)}{}$
default	$-\frac{10x+2}{196\sqrt{5x^2+2x+3}}-\frac{3\left(-61+13\sqrt{11}\right)\sqrt{11}\left(\frac{1}{7\left(\frac{250}{49}-\frac{34\sqrt{11}}{49}\right)\sqrt{5\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)+\frac{250}{49}-\frac{34\sqrt{11}}{49}}}-\frac{}{7\left(\frac{250}{49}\right.}\right.}{}$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
1/3556*(-131+605*x)/(5*x^2+2*x+3)^(1/2)+21/5588*(-53+6*11^(1/2))*11^(1/2)/
(250-34*11^(1/2))^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^
(1/2))*(x-2/7+1/7*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1
/2))^2+49*(34/7-10/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2)
)+21/5588*(53+6*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(5
00/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11
^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1
/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(117) = 234$.

Time = 0.09 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.93

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\frac{21\sqrt{\frac{1}{1397}}(5x^2+2x+3)\sqrt{25015\sqrt{11}+281693}\log\left(\frac{3\left(\sqrt{\frac{1}{1397}}\sqrt{5x^2+2x+3}\sqrt{25015\sqrt{11}+281693}\left(1335\sqrt{11}-8173\right)+168\right.}{x}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="fri
cas")
```

output
```
-1/14224*(21*sqrt(1/1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*
log(3*(sqrt(1/1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1
335*sqrt(11) - 8173) + 16891*sqrt(11)*(x + 3) + 50673*x - 84455)/x) - 21*s
qrt(1/1397)*(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*log(-3*(sqrt(1
/1397)*sqrt(5*x^2 + 2*x + 3)*sqrt(25015*sqrt(11) + 281693)*(1335*sqrt(11)
- 8173) - 16891*sqrt(11)*(x + 3) - 50673*x + 84455)/x) + 7*(5*x^2 + 2*x +
3)*sqrt(-225135/1397*sqrt(11) + 2535237/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*
(1335*sqrt(11) + 8173)*sqrt(-225135/1397*sqrt(11) + 2535237/1397) + 50673*
sqrt(11)*(x + 3) - 152019*x + 253365)/x) - 7*(5*x^2 + 2*x + 3)*sqrt(-22513
5/1397*sqrt(11) + 2535237/1397)*log((sqrt(5*x^2 + 2*x + 3)*(1335*sqrt(11)
+ 8173)*sqrt(-225135/1397*sqrt(11) + 2535237/1397) - 50673*sqrt(11)*(x + 3
) + 152019*x - 253365)/x) - 4*sqrt(5*x^2 + 2*x + 3)*(605*x - 131))/(5*x^2
+ 2*x + 3)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx =$$
$$-\int \frac{5x}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}$$
$$-\int \frac{x^2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}$$
$$-\int \frac{2}{35x^4\sqrt{5x^2+2x+3}-6x^3\sqrt{5x^2+2x+3}+8x^2\sqrt{5x^2+2x+3}-14x\sqrt{5x^2+2x+3}-3\sqrt{5x^2+2x+}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(5*x/(35*x**4*sqrt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x +
 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt
(5*x**2 + 2*x + 3)), x) - Integral(x**2/(35*x**4*sqrt(5*x**2 + 2*x + 3) -
6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2 + 2*x + 3) - 14*x*sqrt(
5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(35*x**4*sq
rt(5*x**2 + 2*x + 3) - 6*x**3*sqrt(5*x**2 + 2*x + 3) + 8*x**2*sqrt(5*x**2
+ 2*x + 3) - 14*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. $2(117) = 234$.

Time = 0.16 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.68

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)\,(3+2x+5x^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="max
ima")
```

output
```
-1/4312*sqrt(11)*(20*sqrt(11)*x/sqrt(5*x^2 + 2*x + 3) - 7890*sqrt(11)*x/(1
7*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) + 7890*sqrt(
11)*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 13
377*sqrt(11)*sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*s
qrt(11) - 4) + 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqr
t(11)*sqrt(7)*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/ab
s(14*x - 2*sqrt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 4*sqrt(11)/sqrt(5*x^
2 + 2*x + 3) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) + 125*sqrt(5*x^2
 + 2*x + 3)) - 26280*x/(17*sqrt(11)*sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2
 + 2*x + 3)) + 156*sqrt(11)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*
x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) +
1/7*sqrt(11)*sqrt(7)*sqrt(2)/abs(14*x + 2*sqrt(11) - 4) - 23/7*sqrt(7)*sqr
t(2)/abs(14*x + 2*sqrt(11) - 4))/(-34/49*sqrt(11) + 250/49)^(3/2) - 62769*
sqrt(2)*arcsinh(5/7*sqrt(11)*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4)
+ 17/7*sqrt(7)*sqrt(2)*x/abs(14*x - 2*sqrt(11) - 4) + 1/7*sqrt(11)*sqrt(7)
*sqrt(2)/abs(14*x - 2*sqrt(11) - 4) + 23/7*sqrt(7)*sqrt(2)/abs(14*x - 2*sq
rt(11) - 4))/(17*sqrt(11) + 125)^(3/2) + 2244*sqrt(11)/(17*sqrt(11)*sqrt(5
*x^2 + 2*x + 3) + 125*sqrt(5*x^2 + 2*x + 3)) - 2244*sqrt(11)/(17*sqrt(11)*
sqrt(5*x^2 + 2*x + 3) - 125*sqrt(5*x^2 + 2*x + 3)) - 732*arcsinh(5/7*sqrt(
11)*sqrt(7)*sqrt(2)*x/abs(14*x + 2*sqrt(11) - 4) - 17/7*sqrt(7)*sqrt(2)...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.67

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \frac{605\,x-131}{3556\,\sqrt{5\,x^2+2\,x+3}}$$
$$+ 0.0477059376662992\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$- 0.0352174957837795\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$- 0.0477059376662992\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+ 0.0352174957837795\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x, algorithm="gia
c")
```

output
```
1/3556*(605*x - 131)/sqrt(5*x^2 + 2*x + 3) + 0.0477059376662992*log(-sqrt(
5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0352174957837795*log(
-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.047705937666299
2*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.035217495
7837795*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)), x)
```

Reduce [B] (verification not implemented)

Time = 69.14 (sec) , antiderivative size = 1295, normalized size of antiderivative = 7.80

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)(3+2x+5x^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 82545*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)
*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sq
rt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)
*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(
8890*x**2 + 3556*x + 5334))*x**2 - 33018*sqrt(17*sqrt(11) - 125)*sqrt(22)*
atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sq
rt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2
*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqr
t(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x - 49527*sqrt(
17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(1
1) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*s
qrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 19
2*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 355
6*x + 5334)) - 24915*sqrt(17*sqrt(11) - 125)*sqrt(2)*atan((24*sqrt(5*x**2
+ 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*
sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(
11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*
sqrt(2))/(8890*x**2 + 3556*x + 5334))*x**2 - 9966*sqrt(17*sqrt(11) - 125)*
sqrt(2)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x
 - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt...
```

3.24 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 215

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = -\frac{76567+22755x}{19870928\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{5588(1+4x-7x^2)\sqrt{3+2x+5x^2}} - \frac{7\left(541543-5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125-17\sqrt{11}\right)}} + \frac{7\left(541543+5144\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{2838704\sqrt{22\left(125+17\sqrt{11}\right)}}$$

output
```
-1/19870928*(76567+22755*x)/(5*x^2+2*x+3)^(1/2)-3/5588*(40-371*x)/(-7*x^2+
4*x+1)/(5*x^2+2*x+3)^(1/2)-7/2838704*(541543-5144*11^(1/2))*arctanh((23-11
^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))/(27
50-374*11^(1/2))^(1/2)+7/2838704*(541543+5144*11^(1/2))*arctanh((23+11^(1/
2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(1/2)/(5*x^2+2*x+3)^(1/2))/(2750+3
74*11^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.93

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \frac{503287-3628805x-444949x^2-159285x^3}{19870928\sqrt{3+2x+5x^2}(-1-4x+7x^2)} + \frac{\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{116685\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)+205710\log\left(-\sqrt{5}x+\right.}{-4\sqrt{5}-35\#}\right.}{258064\sqrt{5}} - \frac{3\text{RootSum}\left[83-16\sqrt{5}\#1-70\#1^2+8\sqrt{5}\#1^3+7\#1^4\&, \frac{746007\sqrt{5}\log\left(-\sqrt{5}x+\sqrt{3+2x+5x^2}-\#1\right)-1016580\log\left(-\sqrt{}\right.}{-4\sqrt{5}-3}\right.}{2838704\sqrt{5}}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(503287 - 3628805*x - 444949*x^2 - 159285*x^3)/(19870928*Sqrt[3 + 2*x + 5*
x^2]*(-1 - 4*x + 7*x^2)) + RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5
]*#1^3 + 7*#1^4 & , (116685*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^
2] - #1] + 205710*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 8351
*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5]
- 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]/(258064*Sqrt[5]) - (3*RootSum[83 -
16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (746007*Sqrt[5]*Log[
-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] - 1016580*Log[-(Sqrt[5]*x) + Sq
rt[3 + 2*x + 5*x^2] - #1]*#1 + 42623*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2
*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ]
)/(2838704*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}\,dx$$

$$\Big\downarrow \; 2135$$

$$-\frac{\int -\frac{8(11130x^2+4719x+6277)}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}dx}{44704}-\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{11130x^2+4719x+6277}{(-7x^2+4x+1)(5x^2+2x+3)^{3/2}}dx}{5588}-\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 2135$$

$$\frac{\frac{\int \frac{112(36008x+531255)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{28448}-\frac{22755x+76567}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{254}\int \frac{36008x+531255}{(-7x^2+4x+1)\sqrt{5x^2+2x+3}}dx-\frac{22755x+76567}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{1}{254}\left(\frac{7}{11}\left(56584-541543\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(56584+541543\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+}}\right.}{5588}$$

$$\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{254}\left(\frac{7}{22}\left(56584-541543\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(56584+541543\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}d\right.}{5588}$$

$$\frac{3(40-371x)}{5588\,(-7x^2+4x+1)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{1}{254}\left(-\frac{7}{11}\left(56584-541543\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(56584+\right.\right.}{5588}$$

$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{254}\left(\frac{7\left(56584-541543\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(56584+541543\sqrt{11}\right)\text{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125+17\sqrt{11}\right)}}\right)}{5588}$$

$$\frac{3(40-371x)}{5588\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^2*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(-3*(40 - 371*x))/(5588*(1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) + (-1/355
6*(76567 + 22755*x)/Sqrt[3 + 2*x + 5*x^2] + ((7*(56584 - 541543*Sqrt[11])*
ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sqrt[2*(125 - 17*Sqrt[11])]
*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sqrt[11])]) + (7*(56584 + 5
41543*Sqrt[11])*ArcTanh[(23 + Sqrt[11] + (17 + 5*Sqrt[11])*x)/(Sqrt[2*(125
 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 + 17*Sqrt[11])])
)/254)/5588
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{159285x^3+444949x^2+3628805x-503287}{19870928(7x^2-4x-1)\sqrt{5x^2+2x+3}}+\frac{7\left(-541543+5144\sqrt{11}\right)\sqrt{11}\,\text{arctanh}\left(\frac{250-34\sqrt{11}+\frac{49\left(\frac{34}{7}-\frac{10\sqrt{11}}{7}\right)}{}}{\sqrt{250-34\sqrt{11}}\sqrt{245\left(x-\frac{2}{7}+\frac{\sqrt{11}}{7}\right)^2+49\left(\frac{34}{7}-\right.}}\right.}{31225744\sqrt{250-34\sqrt{11}}}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/19870928*(159285*x^3+444949*x^2+3628805*x-503287)/(7*x^2-4*x-1)/(5*x^2+
2*x+3)^(1/2)+7/31225744*(-541543+5144*11^(1/2))*11^(1/2)/(250-34*11^(1/2))
^(1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7
*11^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-1
0/7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7/31225744*(541
543+5144*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2*(500/49+6
8/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34*11^(1/2))
^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(
1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(162) = 324.

Time = 0.09 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.71

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\frac{7\,(35\,x^4-6\,x^3+8\,x^2-14\,x-3)\sqrt{\frac{390372164915}{127}\,\sqrt{11}+\frac{35653135368317}{1397}}\log\left(-\frac{\sqrt{5\,x^2+2\,x+3}\sqrt{\frac{390372164915}{127}\,\sqrt{11}+\frac{35653}{}}}{}\right.}{}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/79483712*(7*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(390372164915/127*s
qrt(11) + 35653135368317/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*sqrt(3903721649
15/127*sqrt(11) + 35653135368317/1397)*(5609479*sqrt(11) + 77949905) + 205
0844269271*sqrt(11)*(x + 3) - 6152532807813*x + 10254221346355)/x) - 7*(35
*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqrt(390372164915/127*sqrt(11) + 35653135
368317/1397)*log((sqrt(5*x^2 + 2*x + 3)*sqrt(390372164915/127*sqrt(11) + 3
5653135368317/1397)*(5609479*sqrt(11) + 77949905) - 2050844269271*sqrt(11)
*(x + 3) + 6152532807813*x - 10254221346355)/x) + 7*(35*x^4 - 6*x^3 + 8*x^
2 - 14*x - 3)*sqrt(-390372164915/127*sqrt(11) + 35653135368317/1397)*log((
sqrt(5*x^2 + 2*x + 3)*(5609479*sqrt(11) - 77949905)*sqrt(-390372164915/127
*sqrt(11) + 35653135368317/1397) + 2050844269271*sqrt(11)*(x + 3) + 615253
2807813*x - 10254221346355)/x) - 7*(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)*sqr
t(-390372164915/127*sqrt(11) + 35653135368317/1397)*log(-(sqrt(5*x^2 + 2*x
 + 3)*(5609479*sqrt(11) - 77949905)*sqrt(-390372164915/127*sqrt(11) + 3565
3135368317/1397) - 2050844269271*sqrt(11)*(x + 3) - 6152532807813*x + 1025
4221346355)/x) + 4*(159285*x^3 + 444949*x^2 + 3628805*x - 503287)*sqrt(5*x
^2 + 2*x + 3))/(35*x^4 - 6*x^3 + 8*x^2 - 14*x - 3)
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5x+2}{(5x^2+2x+3)^{\frac{3}{2}}(7x^2-4x-1)^2}\,dx$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**2/(5*x**2+2*x+3)**(3/2),x)
```

output
```
Integral((x**2 + 5*x + 2)/((5*x**2 + 2*x + 3)**(3/2)*(7*x**2 - 4*x - 1)**2
), x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5x+2}{(7x^2-4x-1)^2(5x^2+2x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^2*(5*x^2 + 2*x + 3)^(3/2)), x
)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.37

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \frac{25230\,x+13397}{903224\,\sqrt{5\,x^2+2\,x+3}}$$
$$+\frac{3\left(42623\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3+77302\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2-275511\sqrt{5}x-2198\right.}{709676\left(7\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^4-8\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^3-70\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^2\right.}$$
$$+0.0218058276253952\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.0218058276253952\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.0332874364433911\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/903224*(25230*x + 13397)/sqrt(5*x^2 + 2*x + 3) + 3/709676*(42623*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3))^3 + 77302*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2
*x + 3))^2 - 275511*sqrt(5)*x - 219860*sqrt(5) + 275511*sqrt(5*x^2 + 2*x +
 3))/(7*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqr
t(5*x^2 + 2*x + 3))^3 - 70*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt
(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3)) + 83) + 0.0218058276253952*log(-sq
rt(5)*x + sqrt(5*x^2 + 2*x + 3) + 4.41924736459000) - 0.0332874364433911*l
og(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) + 1.25295163054000) - 0.021805827625
3952*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.033287
4364433911*log(-sqrt(5)*x + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)^2}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^2), x)
```

Reduce [B] (verification not implemented)

Time = 67.23 (sec) , antiderivative size = 2193, normalized size of antiderivative = 10.20

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^2(3+2x+5x^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 2335583145*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x
 + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(
17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) -
 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(
2))/(8890*x**2 + 3556*x + 5334))*x**4 + 400385682*sqrt(17*sqrt(11) - 125)*
sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*
x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5
*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x
 + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x**3 -
 533847576*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3
)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*s
qrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125
)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/
(8890*x**2 + 3556*x + 5334))*x**2 + 934233258*sqrt(17*sqrt(11) - 125)*sqrt
(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x -
19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**
2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3
)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x + 200192
841*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(
17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(...
```

3.25 $\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 250

$$\begin{aligned}&\int \frac{2+5x+x^2}{(1+4x-7x^2)^3(3+2x+5x^2)^{3/2}}\,dx =\\&-\frac{5(461370781+1118731375x)}{222077491328\sqrt{3+2x+5x^2}}-\frac{3(40-371x)}{11176(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}\\&-\frac{2701733-9148874x}{62451488(1+4x-7x^2)\sqrt{3+2x+5x^2}}\\&-\frac{7\left(2792860024-84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23-\sqrt{11}+\left(17-5\sqrt{11}\right)x}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125-17\sqrt{11}\right)}}\\&+\frac{7\left(2792860024+84865895\sqrt{11}\right)\operatorname{arctanh}\left(\frac{23+\sqrt{11}+\left(17+5\sqrt{11}\right)x}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{3+2x+5x^2}}\right)}{31725355904\sqrt{22\left(125+17\sqrt{11}\right)}}\end{aligned}$$

output
```
1/222077491328*(-2306853905-5593656875*x)/(5*x^2+2*x+3)^(1/2)-3/11176*(40-
371*x)/(-7*x^2+4*x+1)^2/(5*x^2+2*x+3)^(1/2)-1/62451488*(2701733-9148874*x)
/(-7*x^2+4*x+1)/(5*x^2+2*x+3)^(1/2)-7/31725355904*(2792860024-84865895*11^
(1/2))*arctanh((23-11^(1/2)+(17-5*11^(1/2))*x)/(250-34*11^(1/2))^(1/2)/(5*
x^2+2*x+3)^(1/2))/(2750-374*11^(1/2))^(1/2)+7/31725355904*(2792860024+8486
5895*11^(1/2))*arctanh((23+11^(1/2)+(17+5*11^(1/2))*x)/(250+34*11^(1/2))^(
1/2)/(5*x^2+2*x+3)^(1/2))/(2750+374*11^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 607, normalized size of antiderivative = 2.43

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}}\,dx = \frac{-\frac{1715(14298727813+7828199499x-148022158802x^2+109737266678x^3-2002089436}{(1+4x-7x^2)^2\sqrt{3+2x+5x^2}}}{}$$

input
```
Integrate[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
((-1715*(14298727813 + 7828199499*x - 148022158802*x^2 + 109737266678*x^3
- 200208943655*x^4 + 274089186875*x^5))/((1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x
+ 5*x^2]) + 2324168*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[
5]*#1^3 + 7*#1^4 & , (-4989740*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5
*x^2] - #1] + 3790865*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 +
400449*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sq
rt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ] + 22*Sqrt[5]*RootSum[83 - 16*
Sqrt[5]*#1 - 70*#1^2 + 8*Sqrt[5]*#1^3 + 7*#1^4 & , (-3200991286865*Sqrt[5]
*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1] + 18470877323690*Log[-(Sqr
t[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1 + 2296522946389*Sqrt[5]*Log[-(Sqr
t[5]*x) + Sqrt[3 + 2*x + 5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5
]*#1^2 + 7*#1^3) & ] - 9*Sqrt[5]*RootSum[83 - 16*Sqrt[5]*#1 - 70*#1^2 + 8*
Sqrt[5]*#1^3 + 7*#1^4 & , (-8189062651053*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[
3 + 2*x + 5*x^2] - #1] + 39132066594240*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1 + 5875617407695*Sqrt[5]*Log[-(Sqrt[5]*x) + Sqrt[3 + 2*x +
5*x^2] - #1]*#1^2)/(-4*Sqrt[5] - 35*#1 + 6*Sqrt[5]*#1^2 + 7*#1^3) & ])/380
862897627520
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2135, 27, 2135, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+5x+2}{(-7x^2+4x+1)^3(5x^2+2x+3)^{3/2}}dx$$

$$\Big\downarrow \text{2135}$$

$$-\frac{\int -\frac{8(22260x^2+11151x+16013)}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}dx}{89408}-\frac{3(40-371x)}{11176(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{22260x^2+11151x+16013}{(-7x^2+4x+1)^2(5x^2+2x+3)^{3/2}}dx}{11176}-\frac{3(40-371x)}{11176(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{-\frac{\int -\frac{8\left(91488740x^2+13060267x+26911493\right)}{\left(-7x^2+4x+1\right)\left(5x^2+2x+3\right)^{3/2}}dx}{44704}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{91488740x^2+13060267x+26911493}{\left(-7x^2+4x+1\right)\left(5x^2+2x+3\right)^{3/2}}dx}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176(-7x^2+4x+1)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{\frac{\frac{\int \frac{112(594061265x+2623128234)}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx}{28448}-\frac{5(1118731375x+461370781)}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{254}\int \frac{594061265x+2623128234}{\left(-7x^2+4x+1\right)\sqrt{5x^2+2x+3}}dx-\frac{5(1118731375x+461370781)}{3556\sqrt{5x^2+2x+3}}}{5588}-\frac{2701733-9148874x}{5588(-7x^2+4x+1)\sqrt{5x^2+2x+3}}}{11176}-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1365$$

$$\frac{\frac{\frac{1}{254}\left(\frac{7}{11}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{2\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{11}\left(933524845+2792860024\sqrt{11}\right)\int \frac{1}{2\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-5}{5588}}{11176}$$

$$-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{1}{254}\left(\frac{7}{22}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{\left(-7x-\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx+\frac{7}{22}\left(933524845+2792860024\sqrt{11}\right)\int \frac{1}{\left(-7x+\sqrt{11}+2\right)\sqrt{5x^2+2x+3}}dx\right)-5(1}{5588}}{11176}$$

$$-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\frac{1}{254}\left(-\frac{7}{11}\left(933524845-2792860024\sqrt{11}\right)\int \frac{1}{8\left(125-17\sqrt{11}\right)-\frac{4\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)^2}{5x^2+2x+3}}d\left(-\frac{2\left(\left(17-5\sqrt{11}\right)x-\sqrt{11}+23\right)}{\sqrt{5x^2+2x+3}}\right)-\frac{7}{11}\left(933524845+2792860\right.}{5588}}{}$$

$$-\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{1}{254}\left(\frac{7\left(933524845-2792860024\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17-5\sqrt{11}\right)x-\sqrt{11}+23}{\sqrt{2\left(125-17\sqrt{11}\right)}\sqrt{5x^2+2x+3}}\right)}{22\sqrt{2\left(125-17\sqrt{11}\right)}}+\frac{7\left(933524845+2792860024\sqrt{11}\right)\operatorname{arctanh}\left(\frac{\left(17+5\sqrt{11}\right)x+\sqrt{11}+23}{\sqrt{2\left(125+17\sqrt{11}\right)}\sqrt{5x^2+2x+\ldots}}\right.}{22\sqrt{2\left(125+17\sqrt{11}\right)}}}{5588}}{11176}$$

$$\frac{3(40-371x)}{11176\left(-7x^2+4x+1\right)^2\sqrt{5x^2+2x+3}}$$

input
```
Int[(2 + 5*x + x^2)/((1 + 4*x - 7*x^2)^3*(3 + 2*x + 5*x^2)^(3/2)),x]
```

output
```
(-3*(40 - 371*x))/(11176*(1 + 4*x - 7*x^2)^2*Sqrt[3 + 2*x + 5*x^2]) + (-1/
5588*(2701733 - 9148874*x)/((1 + 4*x - 7*x^2)*Sqrt[3 + 2*x + 5*x^2]) + ((-
5*(461370781 + 1118731375*x))/(3556*Sqrt[3 + 2*x + 5*x^2]) + ((7*(93352484
5 - 2792860024*Sqrt[11])*ArcTanh[(23 - Sqrt[11] + (17 - 5*Sqrt[11])*x)/(Sq
rt[2*(125 - 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])/(22*Sqrt[2*(125 - 17*Sq
rt[11])]) + (7*(933524845 + 2792860024*Sqrt[11])*ArcTanh[(23 + Sqrt[11] +
(17 + 5*Sqrt[11])*x)/(Sqrt[2*(125 + 17*Sqrt[11])]*Sqrt[3 + 2*x + 5*x^2])])
/(22*Sqrt[2*(125 + 17*Sqrt[11])]))/254)/5588)/11176
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{274089186875x^5-200208943655x^4+109737266678x^3-148022158802x^2+7828199499x+14298727813}{222077491328(7x^2-4x-1)^2\sqrt{5x^2+2x+3}}+\frac{7\left(-2792860024+848\right.}{}$
trager	Expression too large to display
default	Expression too large to display

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x,method=_RETURNVERBO
SE)
```

output
```
-1/222077491328*(274089186875*x^5-200208943655*x^4+109737266678*x^3-148022
158802*x^2+7828199499*x+14298727813)/(7*x^2-4*x-1)^2/(5*x^2+2*x+3)^(1/2)+7
/348978914944*(-2792860024+84865895*11^(1/2))*11^(1/2)/(250-34*11^(1/2))^(
1/2)*arctanh(49/2*(500/49-68/49*11^(1/2)+(34/7-10/7*11^(1/2))*(x-2/7+1/7*1
1^(1/2)))/(250-34*11^(1/2))^(1/2)/(245*(x-2/7+1/7*11^(1/2))^2+49*(34/7-10/
7*11^(1/2))*(x-2/7+1/7*11^(1/2))+250-34*11^(1/2))^(1/2))+7/348978914944*(2
792860024+84865895*11^(1/2))*11^(1/2)/(250+34*11^(1/2))^(1/2)*arctanh(49/2
*(500/49+68/49*11^(1/2)+(34/7+10/7*11^(1/2))*(x-2/7-1/7*11^(1/2)))/(250+34
*11^(1/2))^(1/2)/(245*(x-2/7-1/7*11^(1/2))^2+49*(34/7+10/7*11^(1/2))*(x-2/
7-1/7*11^(1/2))+250+34*11^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(193) = 386$.

Time = 0.20 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.71

$$\int \frac{2+5x+x^2}{\left(1+4x-7x^2\right)^3\left(3+2x+5x^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="f
ricas")
```

output
```
-1/888309965312*(7*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x +
 3)*sqrt(6790301337330876497/127*sqrt(11) + 896266498377233657855/1397)*lo
g(-(sqrt(5*x^2 + 2*x + 3)*sqrt(6790301337330876497/127*sqrt(11) + 89626649
8377233657855/1397)*(37271563201*sqrt(11) + 407780707037) + 54045898845271
335107*sqrt(11)*(x + 3) - 162137696535814005321*x + 270229494226356675535)
/x) - 7*(245*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)*sqrt(67
90301337330876497/127*sqrt(11) + 896266498377233657855/1397)*log((sqrt(5*x
^2 + 2*x + 3)*sqrt(6790301337330876497/127*sqrt(11) + 89626649837723365785
5/1397)*(37271563201*sqrt(11) + 407780707037) - 54045898845271335107*sqrt(
11)*(x + 3) + 162137696535814005321*x - 270229494226356675535)/x) + 7*(245
*x^6 - 182*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)*sqrt(-6790301337330
876497/127*sqrt(11) + 896266498377233657855/1397)*log((sqrt(5*x^2 + 2*x +
3)*(37271563201*sqrt(11) - 407780707037)*sqrt(-6790301337330876497/127*sqr
t(11) + 896266498377233657855/1397) + 54045898845271335107*sqrt(11)*(x + 3
) + 162137696535814005321*x - 270229494226356675535)/x) - 7*(245*x^6 - 182
*x^5 + 45*x^4 - 124*x^3 + 27*x^2 + 26*x + 3)*sqrt(-6790301337330876497/127
*sqrt(11) + 896266498377233657855/1397)*log(-(sqrt(5*x^2 + 2*x + 3)*(37271
563201*sqrt(11) - 407780707037)*sqrt(-6790301337330876497/127*sqrt(11) + 8
96266498377233657855/1397) - 54045898845271335107*sqrt(11)*(x + 3) - 16213
7696535814005321*x + 270229494226356675535)/x) + 4*(274089186875*x^5 - ...
```

Sympy [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx =$$

$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640}$$

$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640}$$

$$-\int \frac{}{1715x^8\sqrt{5x^2+2x+3} - 2254x^7\sqrt{5x^2+2x+3} + 798x^6\sqrt{5x^2+2x+3} - 866x^5\sqrt{5x^2+2x+3} + 640}$$

input
```
integrate((x**2+5*x+2)/(-7*x**2+4*x+1)**3/(5*x**2+2*x+3)**(3/2),x)
```

output
```
-Integral(5*x/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 +
2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x +
3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 1
10*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x*
*2 + 2*x + 3)), x) - Integral(x**2/(1715*x**8*sqrt(5*x**2 + 2*x + 3) - 225
4*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2*x + 3) - 866*x**5
*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3) + 198*x**3*sqrt(
5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38*x*sqrt(5*x**2 + 2
*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x) - Integral(2/(1715*x**8*sqrt(5*x**
2 + 2*x + 3) - 2254*x**7*sqrt(5*x**2 + 2*x + 3) + 798*x**6*sqrt(5*x**2 + 2
*x + 3) - 866*x**5*sqrt(5*x**2 + 2*x + 3) + 640*x**4*sqrt(5*x**2 + 2*x + 3
) + 198*x**3*sqrt(5*x**2 + 2*x + 3) - 110*x**2*sqrt(5*x**2 + 2*x + 3) - 38
*x*sqrt(5*x**2 + 2*x + 3) - 3*sqrt(5*x**2 + 2*x + 3)), x)
```

Maxima [F]

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx=\int -\frac{x^2+5\,x+2}{(7\,x^2-4\,x-1)^3(5\,x^2+2\,x+3)^{\frac{3}{2}}}\,dx$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="m
axima")
```

output
```
-integrate((x^2 + 5*x + 2)/((7*x^2 - 4*x - 1)^3*(5*x^2 + 2*x + 3)^(3/2)),
x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(193) = 386$.

Time = 0.15 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.59

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3\,(3+2x+5x^2)^{3/2}}\,dx = \frac{501205\,x+1702037}{458837792\,\sqrt{5\,x^2+2\,x+3}}$$
$$+\frac{6871871279\,\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^7+4012856750\,\sqrt{5}\left(\sqrt{5}x-\sqrt{5\,x^2+2\,x+3}\right)^6-223088535693\,(\sqrt{5}}{793133}$$
$$+0.0107382277384459\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+4.41924736459000\right)$$
$$-0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}+1.25295163054000\right)$$
$$-0.0107382277384459\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-1.02258038113000\right)$$
$$+0.0142619066316905\,\log\left(-\sqrt{5}x+\sqrt{5\,x^2+2\,x+3}-2.09411235400000\right)$$

input
```
integrate((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x, algorithm="g
iac")
```

output
```
1/458837792*(501205*x + 1702037)/sqrt(5*x^2 + 2*x + 3) + 1/7931338976*(687
1871279*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^7 + 4012856750*sqrt(5)*(sqrt(5
)*x - sqrt(5*x^2 + 2*x + 3))^6 - 223088535693*(sqrt(5)*x - sqrt(5*x^2 + 2*
x + 3))^5 - 100577598176*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^4 + 1
255097956673*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 + 566810398070*sqrt(5)*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 - 1246245909011*sqrt(5)*x - 56129965
4796*sqrt(5) + 1246245909011*sqrt(5*x^2 + 2*x + 3))/(7*(sqrt(5)*x - sqrt(5
*x^2 + 2*x + 3))^4 - 8*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^3 - 70*
(sqrt(5)*x - sqrt(5*x^2 + 2*x + 3))^2 + 16*sqrt(5)*(sqrt(5)*x - sqrt(5*x^2
 + 2*x + 3)) + 83)^2 + 0.0107382277384459*log(-sqrt(5)*x + sqrt(5*x^2 + 2*
x + 3) + 4.41924736459000) - 0.0142619066316905*log(-sqrt(5)*x + sqrt(5*x^
2 + 2*x + 3) + 1.25295163054000) - 0.0107382277384459*log(-sqrt(5)*x + sqr
t(5*x^2 + 2*x + 3) - 1.02258038113000) + 0.0142619066316905*log(-sqrt(5)*x
 + sqrt(5*x^2 + 2*x + 3) - 2.09411235400000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}}\,dx = \int \frac{x^2+5\,x+2}{(5\,x^2+2\,x+3)^{3/2}\,(-7\,x^2+4\,x+1)^3}\,dx$$

input
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3),x)
```

output
```
int((5*x + x^2 + 2)/((2*x + 5*x^2 + 3)^(3/2)*(4*x - 7*x^2 + 1)^3), x)
```

Reduce [B] (verification not implemented)

Time = 67.18 (sec) , antiderivative size = 3091, normalized size of antiderivative = 12.36

$$\int \frac{2+5x+x^2}{(1+4x-7x^2)^3 (3+2x+5x^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((x^2+5*x+2)/(-7*x^2+4*x+1)^3/(5*x^2+2*x+3)^(3/2),x)
```

output
```
( - 81643207255575*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 +
 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*s
qrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(1
1) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*s
qrt(2))/(8890*x**2 + 3556*x + 5334))*x**6 + 60649239675570*sqrt(17*sqrt(11
) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*
sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) -
85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x
**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x + 5334
))*x**5 - 14995691128575*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*
x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x
+ 3)*sqrt(17*sqrt(11) - 125)*sqrt(22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*
sqrt(11) - 125)*sqrt(2)*x - 192*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) -
125)*sqrt(2))/(8890*x**2 + 3556*x + 5334))*x**4 + 41321459998740*sqrt(17*s
qrt(11) - 125)*sqrt(22)*atan((24*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) -
 125)*sqrt(22)*x - 19*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(
22) - 85*sqrt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2)*x - 192*sq
rt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(2))/(8890*x**2 + 3556*x
+ 5334))*x**3 - 8997414677145*sqrt(17*sqrt(11) - 125)*sqrt(22)*atan((24*sq
rt(5*x**2 + 2*x + 3)*sqrt(17*sqrt(11) - 125)*sqrt(22)*x - 19*sqrt(5*x**...
```

3.26 $\int (7+3x-6x^2)^q (1+5x-2x^2)^3 (3+2x+4x^2)\, dx$

Optimal result

Integrand size = 35, antiderivative size = 612

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^3 (3+2x+4x^2)\, dx =$$

$$-\frac{(2412977088+5623437568q+5434627508q^2+2842212284q^3+871543171q^4+156578109q^5+1518}{3456(1+q)(2+q)(3+q)(4+q)(3+2q)(5+2q)(7+2q)(9+2q}$$

$$-\frac{(238010208+569038544q+528541446q^2+246189277q^3+61144323q^4+7732179q^5+391383q^6)\,x(7}{864(2+q)(3+q)(4+q)(3+2q)(5+2q)(7+2q)(9+2q)}$$

$$-\frac{(6468096+9304496q+5312930q^2+1517005q^3+217914q^4+12639q^5)\,x^2(7+3x-6x^2)^{1+q}}{48(2+q)(3+q)(4+q)(5+2q)(7+2q)(9+2q)}$$

$$+\frac{(3553404+2916686q+774803q^2+64434q^3-567q^4)\,x^3(7+3x-6x^2)^{1+q}}{108(3+q)(4+q)(5+2q)(7+2q)(9+2q)}$$

$$-\frac{(67668+41534q+7637q^2+381q^3)\,x^4(7+3x-6x^2)^{1+q}}{18(3+q)(4+q)(7+2q)(9+2q)}$$

$$+\frac{2(10618+5089q+603q^2)\,x^5(7+3x-6x^2)^{1+q}}{9(4+q)(7+2q)(9+2q)}$$

$$-\frac{4(118+27q)x^6(7+3x-6x^2)^{1+q}}{3(4+q)(9+2q)}+\frac{16x^7(7+3x-6x^2)^{1+q}}{3(9+2q)}$$

$$-\frac{2^{-3(3+q)}59^q(589098472+754659810q+286430679q^2+42093666q^3+1989765q^4)\,(1-4x)\,\text{Hypergeon}}{27(3+2q)(5+2q)(7+2q)(9+2q)}$$

output
```
-1/3456*(607959*q^7+15186393*q^6+156578109*q^5+871543171*q^4+2842212284*q^
3+5434627508*q^2+5623437568*q+2412977088)*(-6*x^2+3*x+7)^(1+q)/(1+q)/(2+q)
/(3+q)/(4+q)/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)-1/864*(391383*q^6+7732179*q^5
+61144323*q^4+246189277*q^3+528541446*q^2+569038544*q+238010208)*x*(-6*x^2
+3*x+7)^(1+q)/(2+q)/(3+q)/(4+q)/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)-1/48*(1263
9*q^5+217914*q^4+1517005*q^3+5312930*q^2+9304496*q+6468096)*x^2*(-6*x^2+3*
x+7)^(1+q)/(2+q)/(3+q)/(4+q)/(5+2*q)/(7+2*q)/(9+2*q)+1/108*(-567*q^4+64434
*q^3+774803*q^2+2916686*q+3553404)*x^3*(-6*x^2+3*x+7)^(1+q)/(3+q)/(4+q)/(5
+2*q)/(7+2*q)/(9+2*q)-1/18*(381*q^3+7637*q^2+41534*q+67668)*x^4*(-6*x^2+3*
x+7)^(1+q)/(3+q)/(4+q)/(7+2*q)/(9+2*q)+2/9*(603*q^2+5089*q+10618)*x^5*(-6*
x^2+3*x+7)^(1+q)/(4+q)/(7+2*q)/(9+2*q)-4/3*(118+27*q)*x^6*(-6*x^2+3*x+7)^(
1+q)/(4+q)/(9+2*q)+16*x^7*(-6*x^2+3*x+7)^(1+q)/(27+6*q)-1/27*59^q*(1989765
*q^4+42093666*q^3+286430679*q^2+754659810*q+589098472)*(1-4*x)*hypergeom([
1/2, -q],[3/2],3/59*(1-4*x)^2)/(2^(9+3*q))/(3+2*q)/(5+2*q)/(7+2*q)/(9+2*q)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.76

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^3 (3+2x+4x^2)\,dx = \frac{8^{-3-q}\left(2^{2+3q}(7+3x-6x^2)^{1+q}\left(9q^7(-67551-173948x-202224x^2-4032x^3-32512x^4+205824x^5-\right.\right.}{}$$

input
```
Integrate[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)^3*(3 + 2*x + 4*x^2),x]
```

output
```
(8^(-3 - q)*(2^(2 + 3*q)*(7 + 3*x - 6*x^2)^(1 + q)*(9*q^7*(-67551 - 173948
*x - 202224*x^2 - 4032*x^3 - 32512*x^4 + 205824*x^5 - 110592*x^6 + 16384*x
^7) + 192*(-12567589 - 4958546*x - 7276608*x^2 + 3553404*x^3 - 2030040*x^4
 + 3822480*x^5 - 1784160*x^6 + 241920*x^7) + 3*q^6*(-5062131 - 10831416*x
- 11976552*x^2 + 1320160*x^3 - 2637824*x^4 + 11385856*x^5 - 5928960*x^6 +
860160*x^7) + 64*q*(-87866212 - 50440547*x - 67785714*x^2 + 31847184*x^3 -
 19369368*x^4 + 38751696*x^5 - 18276192*x^6 + 2493504*x^7) + q^5*(-1565781
09 - 275506008*x - 299627784*x^2 + 67908512*x^3 - 78148416*x^4 + 260733696
*x^5 - 132129792*x^6 + 18837504*x^7) + 4*q^3*(-710553071 - 774730723*x - 8
95043826*x^2 + 350528200*x^3 - 261268656*x^4 + 629564736*x^5 - 305710848*x
^6 + 42448896*x^7) + 4*q^2*(-1358656877 - 1097579990*x - 1357154316*x^2 +
596370976*x^3 - 395119488*x^4 + 856650048*x^5 - 409334400*x^6 + 56286720*x
^7) + q^4*(-871543171 - 1229334400*x - 1358253144*x^2 + 436506848*x^3 - 38
4996096*x^4 + 1066279680*x^5 - 527961600*x^6 + 74188800*x^7)) + 59^q*(1413
8363328 + 47566759040*q + 65225773316*q^2 + 47635860004*q^3 + 20313207997*
q^4 + 5191733160*q^5 + 777009114*q^6 + 61991316*q^7 + 1989765*q^8)*(-1 + 4
*x)*Hypergeometric2F1[1/2, -q, 3/2, (3*(1 - 4*x)^2)/59]))/(27*(1 + q)*(2 +
 q)*(3 + q)*(4 + q)*(3 + 2*q)*(5 + 2*q)*(7 + 2*q)*(9 + 2*q))
```

Rubi [A] (verified)

Time = 3.14 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.81, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.486$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2+5x+1\right)^3 \left(4x^2+2x+3\right) \left(-6x^2+3x+7\right)^q \, dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}-\frac{\int -2\left(-6x^2+3x+7\right)^q \left(48(27q+118)x^7-8(342q+1637)x^6+492(2q+9)x^5-24(2q+9)x^4+1179(2q+9)x^3}{6(2q+9)}$$

$$\Big\downarrow \text{27}$$

$$\int \left(-6x^2+3x+7\right)^q \left(48(27q+118)x^7-8(342q+1637)x^6+492(2q+9)x^5-24(2q+9)x^4+1179(2q+9)x^3+$$

$$3(2q+9)$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\int -12\left(-6x^2+3x+7\right)^q\left(-4\left(603q^2+5089q+10618\right)x^6+12\left(82q^2+1075q+3128\right)x^5-24(q+4)(2q+9)x^4+1179(q+4)(2q+9)x^3+723(q+4)(2q+9)x}{12(q+4)}$$

$$3(2q+9)$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int\left(-6x^2+3x+7\right)^q\left(-4\left(603q^2+5089q+10618\right)x^6+12\left(82q^2+1075q+3128\right)x^5-24(q+4)(2q+9)x^4+1179(q+4)(2q+9)x^3+723(q+4)(2q+9)x^2+141}{q+4}$$

$$3(2q+9)$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$\frac{2\left(603q^2+5089q+10618\right)x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}-\frac{\int -2\left(-6x^2+3x+7\right)^q\left(6\left(381q^3+7637q^2+41534q+67668\right)x^5-2\left(144q^3+22833q^2+184991q+380702\right)x^4+3537(q+}{6(2q+7)}$$

$$q+4$$

$$3(2q+9)$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int\left(-6x^2+3x+7\right)^q\left(6\left(381q^3+7637q^2+41534q+67668\right)x^5-2\left(144q^3+22833q^2+184991q+380702\right)x^4+3537(q+4)(2q+7)(2q+9)x^3+2169(q+4)(2q+7)(2q+9)x^2+423}{3(2q+7)}$$

$$q+4$$

$$3(2q+9)$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\int -6\left(-6x^2+3x+7\right)^q\left(-\left(\left(-567q^4+64434q^3+774803q^2+2916686q+3553404\right)x^4\right)+2\left(14148q^4+217554q^3+1291813q^2+3499501q+3621324\right)x^3+4338(q+3)(q+4)\right.}{12(q+3)} \quad \frac{}{3(2q+7)}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(-6x^2+3x+7\right)^q\left(-\left(\left(-567q^4+64434q^3+774803q^2+2916686q+3553404\right)x^4\right)+2\left(14148q^4+217554q^3+1291813q^2+3499501q+3621324\right)x^3+4338(q+3)(q+4)(2q+\right.}{2(q+3)} \quad \frac{}{3(2q+7)}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\left(-567q^4+64434q^3+774803q^2+2916686q+3553404\right)x^3\left(-6x^2+3x+7\right)^{q+1}}{6(2q+5)}-\frac{\int -3\left(-6x^2+3x+7\right)^q\left(9\left(12639q^5+217914q^4+1517005q^3+5312930q^2+9304496q+646\right.\right.}{2}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(-6x^2+3x+7\right)^q\left(9\left(12639q^5+217914q^4+1517005q^3+5312930q^2+9304496q+6468096\right)x^3+\left(69408q^5+1218609q^4+7964682q^3+23424079q^2+28489810q+7921452\right.\right.}{2(2q+5)} \quad \frac{}{2(q}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\int -3\left(-6x^2+3x+7\right)^q\left(\left(391383q^6+7732179q^5+61144323q^4+246189277q^3+528541446q^2+569038544q+238010208\right)x^2+6\left(9024q^6+264441q^5+2935398q^4+16557\right.\right.}{12(q+2)}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(-6x^2+3x+7\right)^q \left(\left(391383q^6+7732179q^5+61144323q^4+246189277q^3+528541446q^2+569038544q+238010208\right)x^2+6\left(9024q^6+264441q^5+2935398q^4+16557955q^3 \cdots}{4(q+2)} \cdots$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\int -\left(\left(41472q^7+3610593q^6+61818309q^5+465024021q^4+1827961387q^3+3873373770q^2+4139520752q+3\left(607959q^7+15186393q^6+156578109q^5+871543171q^4+2 \cdots}{6(2q+3)} \cdots$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \left(41472q^7+3610593q^6+61818309q^5+465024021q^4+1827961387q^3+3873373770q^2+4139520752q+3\left(607959q^7+15186393q^6+156578109q^5+871543171q^4+28422 \cdots}{6(2q+3)} \cdots$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{1}{4}(q+2)(q+3)(q+4)\left(1989765q^4+42093666q^3+286430679q^2+754659810q+589098472\right)\int \left(-6x^2+3x+7\right)^q dx-\frac{\left(607959q^7+15186393q^6+156578109q^5+871543171q \cdots}{6(2q+3)} \cdots}{\cdots}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$$\Big\downarrow \; 1090$$

$$-\frac{1}{3}2^{-3q-4}59^q(q+2)(q+3)(q+4)\left(1989765q^4+42093666q^3+286430679q^2+754659810q+589098472\right)\int\left(1-\frac{1}{177}(3-12x)^2\right)^q d(3-12x)-\frac{\left(607959q^7+15186393q^6+1\cdots\right.}{6(2q+3)}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

$\downarrow$ 237

$$-\frac{1}{3}2^{-3q-4}59^q(q+2)(q+3)(q+4)\left(1989765q^4+42093666q^3+286430679q^2+754659810q+589098472\right)(3-12x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{1}{177}(3-12x)^2\right)-\frac{\left(6079\cdots\right.}{6(2q+3)}$$

$$\frac{16x^7\left(-6x^2+3x+7\right)^{q+1}}{3(2q+9)}$$

input

```
Int[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)^3*(3 + 2*x + 4*x^2),x]
```

output

```
(16*x^7*(7 + 3*x - 6*x^2)^(1 + q))/(3*(9 + 2*q)) + ((-4*(118 + 27*q)*x^6*(
7 + 3*x - 6*x^2)^(1 + q))/(4 + q) + ((2*(10618 + 5089*q + 603*q^2)*x^5*(7
+ 3*x - 6*x^2)^(1 + q))/(3*(7 + 2*q)) + (-1/2*((67668 + 41534*q + 7637*q^2
 + 381*q^3)*x^4*(7 + 3*x - 6*x^2)^(1 + q))/(3 + q) + (((3553404 + 2916686*
q + 774803*q^2 + 64434*q^3 - 567*q^4)*x^3*(7 + 3*x - 6*x^2)^(1 + q))/(6*(5
 + 2*q)) + ((-3*(6468096 + 9304496*q + 5312930*q^2 + 1517005*q^3 + 217914*
q^4 + 12639*q^5)*x^2*(7 + 3*x - 6*x^2)^(1 + q))/(4*(2 + q)) + (-1/6*((2380
10208 + 569038544*q + 528541446*q^2 + 246189277*q^3 + 61144323*q^4 + 77321
79*q^5 + 391383*q^6)*x*(7 + 3*x - 6*x^2)^(1 + q))/(3 + 2*q) + (-1/4*((2412
977088 + 5623437568*q + 5434627508*q^2 + 2842212284*q^3 + 871543171*q^4 +
156578109*q^5 + 15186393*q^6 + 607959*q^7)*(7 + 3*x - 6*x^2)^(1 + q))/(1 +
 q) - (2^(-4 - 3*q)*59^q*(2 + q)*(3 + q)*(4 + q)*(589098472 + 754659810*q
+ 286430679*q^2 + 42093666*q^3 + 1989765*q^4)*(3 - 12*x)*Hypergeometric2F1
[1/2, -q, 3/2, (3 - 12*x)^2/177])/3)/(6*(3 + 2*q)))/(4*(2 + q)))/(2*(5 + 2
*q)))/(2*(3 + q)))/(3*(7 + 2*q)))/(4 + q))/(3*(9 + 2*q))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(-6x^2+3x+7\right)^q \left(-2x^2+5x+1\right)^3 \left(4x^2+2x+3\right) dx$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x)
```

output
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x)
```

Fricas [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^3 \left(3+2x+4x^2\right) dx$$
$$= \int -\left(4\,x^2+2\,x+3\right)\left(2\,x^2-5\,x-1\right)^3\left(-6\,x^2+3\,x+7\right)^q dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
integral(-(32*x^8 - 224*x^7 + 456*x^6 - 164*x^5 + 8*x^4 - 393*x^3 - 241*x^
2 - 47*x - 3)*(-6*x^2 + 3*x + 7)^q, x)
```

Sympy [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^3 \left(3+2x+4x^2\right) dx$$
$$= -\int \left(-47x\left(-6x^2+3x+7\right)^q\right) dx - \int \left(-241x^2\left(-6x^2+3x+7\right)^q\right) dx$$
$$- \int \left(-393x^3\left(-6x^2+3x+7\right)^q\right) dx$$
$$- \int 8x^4\left(-6x^2+3x+7\right)^q dx - \int \left(-164x^5\left(-6x^2+3x+7\right)^q\right) dx$$
$$- \int 456x^6\left(-6x^2+3x+7\right)^q dx - \int \left(-224x^7\left(-6x^2+3x+7\right)^q\right) dx$$
$$- \int 32x^8\left(-6x^2+3x+7\right)^q dx - \int \left(-3\left(-6x^2+3x+7\right)^q\right) dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(-2*x**2+5*x+1)**3*(4*x**2+2*x+3),x)
```

output
```
-Integral(-47*x*(-6*x**2 + 3*x + 7)**q, x) - Integral(-241*x**2*(-6*x**2 +
 3*x + 7)**q, x) - Integral(-393*x**3*(-6*x**2 + 3*x + 7)**q, x) - Integra
l(8*x**4*(-6*x**2 + 3*x + 7)**q, x) - Integral(-164*x**5*(-6*x**2 + 3*x +
7)**q, x) - Integral(456*x**6*(-6*x**2 + 3*x + 7)**q, x) - Integral(-224*x
**7*(-6*x**2 + 3*x + 7)**q, x) - Integral(32*x**8*(-6*x**2 + 3*x + 7)**q,
x) - Integral(-3*(-6*x**2 + 3*x + 7)**q, x)
```

Maxima [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^3 (3+2x+4x^2)\,dx = \int -(4\,x^2+2\,x+3)(2\,x^2-5\,x-1)^3(-6\,x^2+3\,x+7)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
-integrate((4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)^3*(-6*x^2 + 3*x + 7)^q, x)
```

Giac [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^3 (3+2x+4x^2)\,dx = \int -(4\,x^2+2\,x+3)(2\,x^2-5\,x-1)^3(-6\,x^2+3\,x+7)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
integrate(-(4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)^3*(-6*x^2 + 3*x + 7)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^3 \left(3+2x+4x^2\right)\,dx$$
$$=\int \left(-2\,x^2+5\,x+1\right)^3\,\left(4\,x^2+2\,x+3\right)\,\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
int((5*x - 2*x^2 + 1)^3*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q,x)
```

output
```
int((5*x - 2*x^2 + 1)^3*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q, x)
```

Reduce [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^3 \left(3+2x+4x^2\right)\,dx = \text{too large to display}$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^3*(4*x^2+2*x+3),x)
```

output

```
( - 5308416*( - 6*x**2 + 3*x + 7)**q*q**8*x**9 + 38486016*( - 6*x**2 + 3*x
 + 7)**q*q**8*x**8 - 78409728*( - 6*x**2 + 3*x + 7)**q*q**8*x**7 + 2073600
*( - 6*x**2 + 3*x + 7)**q*q**8*x**6 + 73840896*( - 6*x**2 + 3*x + 7)**q*q*
*8*x**5 + 52577856*( - 6*x**2 + 3*x + 7)**q*q**8*x**4 + 22074768*( - 6*x**
2 + 3*x + 7)**q*q**8*x**3 - 82733724*( - 6*x**2 + 3*x + 7)**q*q**8*x**2 -
70726311*( - 6*x**2 + 3*x + 7)**q*q**8*x + 2322432*( - 6*x**2 + 3*x + 7)**
q*q**8 - 95551488*( - 6*x**2 + 3*x + 7)**q*q**7*x**9 + 706019328*( - 6*x**
2 + 3*x + 7)**q*q**7*x**8 - 1480660992*( - 6*x**2 + 3*x + 7)**q*q**7*x**7
+ 153709056*( - 6*x**2 + 3*x + 7)**q*q**7*x**6 + 1186518528*( - 6*x**2 + 3
*x + 7)**q*q**7*x**5 + 1058679360*( - 6*x**2 + 3*x + 7)**q*q**7*x**4 + 700
436664*( - 6*x**2 + 3*x + 7)**q*q**7*x**3 - 1588598730*( - 6*x**2 + 3*x +
7)**q*q**7*x**2 - 1490487345*( - 6*x**2 + 3*x + 7)**q*q**7*x + 217282779*(
 - 6*x**2 + 3*x + 7)**q*q**7 - 724598784*( - 6*x**2 + 3*x + 7)**q*q**6*x**
9 + 5439135744*( - 6*x**2 + 3*x + 7)**q*q**6*x**8 - 11694302208*( - 6*x**2
 + 3*x + 7)**q*q**6*x**7 + 2033434368*( - 6*x**2 + 3*x + 7)**q*q**6*x**6 +
 7674236352*( - 6*x**2 + 3*x + 7)**q*q**6*x**5 + 9242915184*( - 6*x**2 + 3
*x + 7)**q*q**6*x**4 + 7721773320*( - 6*x**2 + 3*x + 7)**q*q**6*x**3 - 126
80279082*( - 6*x**2 + 3*x + 7)**q*q**6*x**2 - 12877687701*( - 6*x**2 + 3*x
 + 7)**q*q**6*x + 3982932765*( - 6*x**2 + 3*x + 7)**q*q**6 - 3009871872*(
- 6*x**2 + 3*x + 7)**q*q**5*x**9 + 22889889792*( - 6*x**2 + 3*x + 7)**q...
```

3.27 $\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx$

Optimal result

Integrand size = 35, antiderivative size = 402

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx =$$
$$-\frac{(944892+2110084q+1816573q^2+745487q^3+145127q^4+10701q^5)(7+3x-6x^2)^{1+q}}{288(1+q)(2+q)(3+q)(3+2q)(5+2q)(7+2q)}$$
$$-\frac{(282522+413083q+221705q^2+52101q^3+4557q^4)\,x(7+3x-6x^2)^{1+q}}{72(2+q)(3+q)(3+2q)(5+2q)(7+2q)}$$
$$+\frac{(1238-157q-582q^2-127q^3)\,x^2(7+3x-6x^2)^{1+q}}{12(2+q)(3+q)(5+2q)(7+2q)}$$
$$-\frac{(1329+806q+117q^2)\,x^3(7+3x-6x^2)^{1+q}}{9(3+q)(5+2q)(7+2q)}$$
$$+\frac{2(57+17q)x^4(7+3x-6x^2)^{1+q}}{3(3+q)(7+2q)}-\frac{8x^5(7+3x-6x^2)^{1+q}}{3(7+2q)}$$
$$-\frac{2^{-7-3q}59^q(1881602+1613049q+454302q^2+39015q^3)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-\right.}{9(3+2q)(5+2q)(7+2q)}$$

output
```
-1/288*(10701*q^5+145127*q^4+745487*q^3+1816573*q^2+2110084*q+944892)*(-6*
x^2+3*x+7)^(1+q)/(1+q)/(2+q)/(3+q)/(3+2*q)/(5+2*q)/(7+2*q)-1/72*(4557*q^4+
52101*q^3+221705*q^2+413083*q+282522)*x*(-6*x^2+3*x+7)^(1+q)/(2+q)/(3+q)/(
3+2*q)/(5+2*q)/(7+2*q)+1/12*(-127*q^3-582*q^2-157*q+1238)*x^2*(-6*x^2+3*x+
7)^(1+q)/(2+q)/(3+q)/(5+2*q)/(7+2*q)-1/9*(117*q^2+806*q+1329)*x^3*(-6*x^2+
3*x+7)^(1+q)/(3+q)/(5+2*q)/(7+2*q)+2/3*(57+17*q)*x^4*(-6*x^2+3*x+7)^(1+q)/
(3+q)/(7+2*q)-8*x^5*(-6*x^2+3*x+7)^(1+q)/(21+6*q)-1/9*2^(-7-3*q)*59^q*(390
15*q^3+454302*q^2+1613049*q+1881602)*(1-4*x)*hypergeom([1/2, -q],[3/2],3/5
9*(1-4*x)^2)/(3+2*q)/(5+2*q)/(7+2*q)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.76

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx$$
$$= \frac{2^{-7-3q}\left(-2^{2+3q}(7+3x-6x^2)^{1+q}\left(3q^5(3567+6076x+2032x^2+2496x^3-4352x^4+1024x^5)+12(7874\right.\right.}{}$$

input
```
Integrate[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)^2*(3 + 2*x + 4*x^2),x]
```

output
```
(2^(-7 - 3*q)*(-(2^(2 + 3*q)*(7 + 3*x - 6*x^2)^(1 + q)*(3*q^5*(3567 + 6076
*x + 2032*x^2 + 2496*x^3 - 4352*x^4 + 1024*x^5) + 12*(78741 + 94174*x - 74
28*x^2 + 21264*x^3 - 27360*x^4 + 5760*x^5) + q^4*(145127 + 226632*x + 4317
6*x^2 + 85280*x^3 - 135168*x^4 + 30720*x^5) + 4*q*(527521 + 695605*x - 343
14*x^2 + 176904*x^3 - 235152*x^4 + 50112*x^5) + q^3*(745487 + 1095224*x +
86520*x^2 + 365856*x^3 - 538176*x^4 + 119040*x^5) + q^2*(1816573 + 2539152
*x + 1320*x^2 + 740512*x^3 - 1028352*x^4 + 222720*x^5))) + 59^q*(11289612
+ 30375916*q + 31758963*q^2 + 16791308*q^3 + 4768026*q^4 + 688392*q^5 + 39
015*q^6)*(-1 + 4*x)*Hypergeometric2F1[1/2, -q, 3/2, (3*(1 - 4*x)^2)/59]))/
(9*(1 + q)*(2 + q)*(3 + q)*(3 + 2*q)*(5 + 2*q)*(7 + 2*q))
```

Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.85, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {2192, 27, 2192, 27, 2192, 27, 2192, 27, 2192, 25, 1160, 1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-2x^2+5x+1\right)^2 \left(4x^2+2x+3\right) \left(-6x^2+3x+7\right)^q \, dx$$

$$\Big\downarrow \; 2192$$

$$-\frac{\int -2\left(-6x^2+3x+7\right)^q \left(-24(17q+57)x^5+112(3q+13)x^4+66(2q+7)x^3+261(2q+7)x^2+96(2q+7)x+\right.}{6(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \left(-6x^2+3x+7\right)^q \left(-24(17q+57)x^5+112(3q+13)x^4+66(2q+7)x^3+261(2q+7)x^2+96(2q+7)x+9(2q\right.}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{2(17q+57)x^4\left(-6x^2+3x+7\right)^{q+1}}{q+3}-\frac{\int -12\left(-6x^2+3x+7\right)^q \left(2\left(117q^2+806q+1329\right)x^4-2\left(-66q^2+47q+903\right)x^3+261(q+3)(2q+7)x^2+96(q+3)(2q\right.}{12(q+3)}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \left(-6x^2+3x+7\right)^q \left(2\left(117q^2+806q+1329\right)x^4-2\left(-66q^2+47q+903\right)x^3+261(q+3)(2q+7)x^2+96(q+3)(2q+7)x+9(q+3)(2q+7)\right)dx}{q+3}+\frac{2(17q+57)x}{}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\frac{\int -6\left(-6x^2+3x+7\right)^q\left(-3\left(-127q^3-582q^2-157q+1238\right)x^3+\left(1044q^3+10215q^2+33569q+36708\right)x^2+96(q+3)(2q+5)(2q+7)x+9(q+3)(2q+5)(2q+7)\right)dx}{6(2q+5)}-\frac{(117q^2 \ldots}{\ldots}}{q+3}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \left(-6x^2+3x+7\right)^q\left(-3\left(-127q^3-582q^2-157q+1238\right)x^3+\left(1044q^3+10215q^2+33569q+36708\right)x^2+96(q+3)(2q+5)(2q+7)x+9(q+3)(2q+5)(2q+7)\right)dx}{2q+5}-\frac{(117q^2+806 \ldots}{\ldots}}{q+3}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$\frac{\frac{\frac{\left(-127q^3-582q^2-157q+1238\right)x^2\left(-6x^2+3x+7\right)^{q+1}}{4(q+2)}-\frac{\int -3\left(-6x^2+3x+7\right)^q\left(\left(\left(4557q^4+52101q^3+221705q^2+413083q+282522\right)x^2+2\left(768q^4+9337q^3+38442q^2+6 \ldots\right.\right.\right.}{12(q+2)}}{2q+5}}{q+3}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \left(-6x^2+3x+7\right)^q\left(\left(\left(4557q^4+52101q^3+221705q^2+413083q+282522\right)x^2+2\left(768q^4+9337q^3+38442q^2+62347q+31654\right)x+36(q+2)(q+3)(2q+5)(2q+7)\right)dx}{4(q+2)}+\frac{(-127q \ldots}{\ldots}}{2q+5}}{q+3}}{3(2q+7)}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 2192$$

$$-\frac{\frac{\frac{\int -\left(\left(1728q^5+53499q^4+470547q^3+1805735q^2+3189013q+3\left(10701q^5+145127q^4+745487q^3+1816573q^2+2110084q+944892\right)x+2113734\right)\left(-6x^2+3x+7\right)^q\right)dx}{6(2q+3)}-\ldots}{4(q+2)}}{2q+5}}{\ldots}$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 25$$

$$\int \left(1728q^5+53499q^4+470547q^3+1805735q^2+3189013q+3\left(10701q^5+145127q^4+745487q^3+1816573q^2+2110084q+944892\right)x+2113734\right)\left(-6x^2+3x+7\right)^q dx - \left(4557\right.$$

$$6(2q+3)$$

$$4(q+2)$$

$$2q+5$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 1160$$

$$\frac{1}{4}(q+2)(q+3)\left(39015q^3+454302q^2+1613049q+1881602\right)\int \left(-6x^2+3x+7\right)^q dx - \frac{\left(10701q^5+145127q^4+745487q^3+1816573q^2+2110084q+944892\right)\left(-6x^2+3x+7\right)^{q+}}{4(q+1)}$$

$$6(2q+3)$$

$$4(q+2)$$

$$2q+5$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 1090$$

$$-\frac{1}{3}2^{-3q-4}59^q(q+2)(q+3)\left(39015q^3+454302q^2+1613049q+1881602\right)\int \left(1-\frac{1}{177}(3-12x)^2\right)^q d(3-12x) - \frac{\left(10701q^5+145127q^4+745487q^3+1816573q^2+2110084q+9\right.}{4(q+1)}$$

$$6(2q+3)$$

$$4(q+2)$$

$$2q+5$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

$$\Big\downarrow \; 237$$

$$-\frac{1}{3}2^{-3q-4}59^q(q+2)(q+3)\left(39015q^3+454302q^2+1613049q+1881602\right)(3-12x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{1}{177}(3-12x)^2\right) - \left(10701q^5+145127q^4+745487q^3+18\right.$$

$$6(2q+3)$$

$$4(q+2)$$

$$\frac{8x^5\left(-6x^2+3x+7\right)^{q+1}}{3(2q+7)}$$

input
```
Int[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)^2*(3 + 2*x + 4*x^2),x]
```

output
```
(-8*x^5*(7 + 3*x - 6*x^2)^(1 + q))/(3*(7 + 2*q)) + ((2*(57 + 17*q)*x^4*(7
+ 3*x - 6*x^2)^(1 + q))/(3 + q) + (-1/3*((1329 + 806*q + 117*q^2)*x^3*(7 +
 3*x - 6*x^2)^(1 + q))/(5 + 2*q) + (((1238 - 157*q - 582*q^2 - 127*q^3)*x^
2*(7 + 3*x - 6*x^2)^(1 + q))/(4*(2 + q)) + (-1/6*((282522 + 413083*q + 221
705*q^2 + 52101*q^3 + 4557*q^4)*x*(7 + 3*x - 6*x^2)^(1 + q))/(3 + 2*q) + (
-1/4*((944892 + 2110084*q + 1816573*q^2 + 745487*q^3 + 145127*q^4 + 10701*
q^5)*(7 + 3*x - 6*x^2)^(1 + q))/(1 + q) - (2^(-4 - 3*q)*59^q*(2 + q)*(3 +
q)*(1881602 + 1613049*q + 454302*q^2 + 39015*q^3)*(3 - 12*x)*Hypergeometri
c2F1[1/2, -q, 3/2, (3 - 12*x)^2/177])/3)/(6*(3 + 2*q)))/(4*(2 + q)))/(5 +
2*q))/(3 + q))/(3*(7 + 2*q))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int (-6x^2+3x+7)^q (-2x^2+5x+1)^2 (4x^2+2x+3)\,dx$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x)
```

output
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x)
```

Fricas [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx = \int (4\,x^2+2\,x+3)(2\,x^2-5\,x-1)^2(-6\,x^2+3\,x+7)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x, algorithm="fr
icas")
```

output
```
integral((16*x^6 - 72*x^5 + 56*x^4 + 22*x^3 + 87*x^2 + 32*x + 3)*(-6*x^2 +
 3*x + 7)^q, x)
```

Sympy [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^2 \left(3+2x+4x^2\right)\,dx = \int \left(-6x^2+3x+7\right)^q \left(2x^2-5x-1\right)^2 \cdot \left(4x^2+2x+3\right)\,dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(-2*x**2+5*x+1)**2*(4*x**2+2*x+3),x)
```

output
```
Integral((-6*x**2 + 3*x + 7)**q*(2*x**2 - 5*x - 1)**2*(4*x**2 + 2*x + 3),
x)
```

Maxima [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^2 \left(3+2x+4x^2\right)\,dx = \int \left(4\,x^2+2\,x+3\right)\left(2\,x^2-5\,x-1\right)^2\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x, algorithm="ma
xima")
```

output
```
integrate((4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)^2*(-6*x^2 + 3*x + 7)^q, x)
```

Giac [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right)^2 \left(3+2x+4x^2\right)\,dx = \int \left(4\,x^2+2\,x+3\right)\left(2\,x^2-5\,x-1\right)^2\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x, algorithm="gi
ac")
```

output
```
integrate((4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)^2*(-6*x^2 + 3*x + 7)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx$$
$$= \int (-2\,x^2+5\,x+1)^2\,(4\,x^2+2\,x+3)\,(-6\,x^2+3\,x+7)^q\,dx$$

input
```
int((5*x - 2*x^2 + 1)^2*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q,x)
```

output
```
int((5*x - 2*x^2 + 1)^2*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q, x)
```

Reduce [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)^2 (3+2x+4x^2)\,dx = \text{too large to display}$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)^2*(4*x^2+2*x+3),x)
```

output

```
(110592*( - 6*x**2 + 3*x + 7)**q*q**6*x**7 - 525312*( - 6*x**2 + 3*x + 7)*
*q*q**6*x**6 + 375552*( - 6*x**2 + 3*x + 7)**q*q**6*x**5 + 633024*( - 6*x*
*2 + 3*x + 7)**q*q**6*x**4 + 231984*( - 6*x**2 + 3*x + 7)**q*q**6*x**3 - 1
98900*( - 6*x**2 + 3*x + 7)**q*q**6*x**2 - 841149*( - 6*x**2 + 3*x + 7)**q
*q**6*x + 96768*( - 6*x**2 + 3*x + 7)**q*q**6 + 1161216*( - 6*x**2 + 3*x +
 7)**q*q**5*x**7 - 5681664*( - 6*x**2 + 3*x + 7)**q*q**5*x**6 + 4400640*(
- 6*x**2 + 3*x + 7)**q*q**5*x**5 + 6012864*( - 6*x**2 + 3*x + 7)**q*q**5*x
**4 + 3915816*( - 6*x**2 + 3*x + 7)**q*q**5*x**3 - 767646*( - 6*x**2 + 3*x
 + 7)**q*q**5*x**2 - 10544751*( - 6*x**2 + 3*x + 7)**q*q**5*x + 3317433*(
- 6*x**2 + 3*x + 7)**q*q**5 + 4838400*( - 6*x**2 + 3*x + 7)**q*q**4*x**7 -
 24226560*( - 6*x**2 + 3*x + 7)**q*q**4*x**6 + 19964736*( - 6*x**2 + 3*x +
 7)**q*q**4*x**5 + 21980880*( - 6*x**2 + 3*x + 7)**q*q**4*x**4 + 24404664*
( - 6*x**2 + 3*x + 7)**q*q**4*x**3 + 3155562*( - 6*x**2 + 3*x + 7)**q*q**4
*x**2 - 51179511*( - 6*x**2 + 3*x + 7)**q*q**4*x + 32394243*( - 6*x**2 + 3
*x + 7)**q*q**4 + 10160640*( - 6*x**2 + 3*x + 7)**q*q**3*x**7 - 51788160*(
 - 6*x**2 + 3*x + 7)**q*q**3*x**6 + 44743680*( - 6*x**2 + 3*x + 7)**q*q**3
*x**5 + 39475440*( - 6*x**2 + 3*x + 7)**q*q**3*x**4 + 71536584*( - 6*x**2
+ 3*x + 7)**q*q**3*x**3 + 21381282*( - 6*x**2 + 3*x + 7)**q*q**3*x**2 - 11
8677861*( - 6*x**2 + 3*x + 7)**q*q**3*x + 143127019*( - 6*x**2 + 3*x + 7)*
*q*q**3 + 11225088*( - 6*x**2 + 3*x + 7)**q*q**2*x**7 - 57984768*( - 6*...
```

3.28 $\int (7+3x-6x^2)^q (1+5x-2x^2) (3+2x+4x^2) \, dx$

Optimal result

Integrand size = 33, antiderivative size = 236

$$\begin{aligned}
&\int (7+3x-6x^2)^q (1+5x-2x^2) (3+2x+4x^2) \, dx \\
&= -\frac{(4836+6980q+3221q^2+477q^3)(7+3x-6x^2)^{1+q}}{72(1+q)(2+q)(3+2q)(5+2q)} \\
&\quad - \frac{(72+81q+23q^2)\, x(7+3x-6x^2)^{1+q}}{6(2+q)(3+2q)(5+2q)} \\
&\quad - \frac{(16+7q)x^2(7+3x-6x^2)^{1+q}}{3(2+q)(5+2q)} + \frac{4x^3(7+3x-6x^2)^{1+q}}{3(5+2q)} \\
&\quad - \frac{2^{-5-3q}59^q(4506+4063q+765q^2)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)}{3(3+2q)(5+2q)}
\end{aligned}$$

output
```
-1/72*(477*q^3+3221*q^2+6980*q+4836)*(-6*x^2+3*x+7)^(1+q)/(1+q)/(2+q)/(3+2
*q)/(5+2*q)-1/6*(23*q^2+81*q+72)*x*(-6*x^2+3*x+7)^(1+q)/(2+q)/(3+2*q)/(5+2
*q)-1/3*(16+7*q)*x^2*(-6*x^2+3*x+7)^(1+q)/(2+q)/(5+2*q)+4*x^3*(-6*x^2+3*x+
7)^(1+q)/(15+6*q)-1/3*2^(-5-3*q)*59^q*(765*q^2+4063*q+4506)*(1-4*x)*hyperg
eom([1/2, -q],[3/2],3/59*(1-4*x)^2)/(3+2*q)/(5+2*q)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.78

$$\int (7+3x-6x^2)^q (1+5x-2x^2)(3+2x+4x^2)\,dx$$
$$= \frac{2^{-5-3q}\left(2^{2+3q}(7+3x-6x^2)^{1+q}(12(-403-72x-96x^2+48x^3)+3q^3(-159-92x-112x^2+64x^3)+\cdots\right.}{}$$

input
```
Integrate[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)*(3 + 2*x + 4*x^2),x]
```

output
```
(2^(-5 - 3*q)*(2^(2 + 3*q)*(7 + 3*x - 6*x^2)^(1 + q)*(12*(-403 - 72*x - 96
*x^2 + 48*x^3) + 3*q^3*(-159 - 92*x - 112*x^2 + 64*x^3) + 4*q*(-1745 - 459
*x - 606*x^2 + 312*x^3) + q^2*(-3221 - 1248*x - 1608*x^2 + 864*x^3)) + 3*5
9^q*(9012 + 21644*q + 18225*q^2 + 6358*q^3 + 765*q^4)*(-1 + 4*x)*Hypergeom
etric2F1[1/2, -q, 3/2, (3*(1 - 4*x)^2)/59]))/(9*(1 + q)*(2 + q)*(3 + 2*q)*
(5 + 2*q))
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2192, 27, 2192, 27, 2192, 27, 1160, 1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2x^2+5x+1)(4x^2+2x+3)(-6x^2+3x+7)^q\,dx$$

$$\Big\downarrow \text{2192}$$

$$\frac{4x^3(-6x^2+3x+7)^{q+1}}{3(2q+5)} - \frac{\int -6(-6x^2+3x+7)^q(4(7q+16)x^3+4(4q+3)x^2+17(2q+5)x+3(2q+5))\,dx}{6(2q+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(-6x^2+3x+7\right)^q \left(4(7q+16)x^3+4(4q+3)x^2+17(2q+5)x+3(2q+5)\right)dx}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 2192

$$\frac{-\frac{\int -4\left(-6x^2+3x+7\right)^q\left(3\left(23q^2+81q+72\right)x^2+\left(102q^2+557q+734\right)x+9(q+2)(2q+5)\right)dx}{12(q+2)}-\frac{(7q+16)x^2\left(-6x^2+3x+7\right)^{q+1}}{3(q+2)}}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \left(-6x^2+3x+7\right)^q\left(3\left(23q^2+81q+72\right)x^2+\left(102q^2+557q+734\right)x+9(q+2)(2q+5)\right)dx}{3(q+2)}-\frac{(7q+16)x^2\left(-6x^2+3x+7\right)^{q+1}}{3(q+2)}}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 2192

$$\frac{-\frac{\int -3\left(72q^3+593q^2+1413q+\left(477q^3+3221q^2+6980q+4836\right)x+1044\right)\left(-6x^2+3x+7\right)^q dx}{6(2q+3)}-\frac{\left(23q^2+81q+72\right)x\left(-6x^2+3x+7\right)^{q+1}}{2(2q+3)}}{3(q+2)}-\frac{(7q+16)x^2\left(-6x^2+3x+7 \cdots\right.}{3(q+2)}}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \left(72q^3+593q^2+1413q+\left(477q^3+3221q^2+6980q+4836\right)x+1044\right)\left(-6x^2+3x+7\right)^q dx}{2(2q+3)}-\frac{\left(23q^2+81q+72\right)x\left(-6x^2+3x+7\right)^{q+1}}{2(2q+3)}}{3(q+2)}-\frac{(7q+16)x^2\left(-6x^2+3x+7\right)^{q+1}\cdots}{3(q+2)}}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 1160

$$\frac{\frac{\frac{\frac{1}{4}(q+2)\left(765q^2+4063q+4506\right)\int \left(-6x^2+3x+7\right)^q dx-\frac{\left(477q^3+3221q^2+6980q+4836\right)\left(-6x^2+3x+7\right)^{q+1}}{12(q+1)}}{2(2q+3)}-\frac{\left(23q^2+81q+72\right)x\left(-6x^2+3x+7\right)^{q+1}}{2(2q+3)}}{3(q+2)}-(7q+16)x^2\cdots}{2q+5}+\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 1090

$$\frac{\frac{\frac{-\frac{1}{3}2^{-3q-4}59^q(q+2)\left(765q^2+4063q+4506\right)\int\left(1-\frac{1}{177}(3-12x)^2\right)^q d(3-12x)-\frac{\left(477q^3+3221q^2+6980q+4836\right)\left(-6x^2+3x+7\right)^{q+1}}{12(q+1)}}{2(2q+3)}-\frac{\left(23q^2+81q+72\right)x\left(-6x^2+3x+\right.}{2(2q+3)}}{3(q+2)}}{2q+5}$$

$$\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

$\Big\downarrow$ 237

$$\frac{\frac{\frac{-\frac{1}{3}2^{-3q-4}59^q(q+2)\left(765q^2+4063q+4506\right)(3-12x)\text{ Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{1}{177}(3-12x)^2\right)-\frac{\left(477q^3+3221q^2+6980q+4836\right)\left(-6x^2+3x+7\right)^{q+1}}{12(q+1)}}{2(2q+3)}-\frac{\left(23q^2+8\right.}{}}{3(q+2)}}{2q+5}$$

$$\frac{4x^3\left(-6x^2+3x+7\right)^{q+1}}{3(2q+5)}$$

input
```
Int[(7 + 3*x - 6*x^2)^q*(1 + 5*x - 2*x^2)*(3 + 2*x + 4*x^2),x]
```

output
```
(4*x^3*(7 + 3*x - 6*x^2)^(1 + q))/(3*(5 + 2*q)) + (-1/3*((16 + 7*q)*x^2*(7
 + 3*x - 6*x^2)^(1 + q))/(2 + q) + (-1/2*((72 + 81*q + 23*q^2)*x*(7 + 3*x
- 6*x^2)^(1 + q))/(3 + 2*q) + (-1/12*((4836 + 6980*q + 3221*q^2 + 477*q^3)
*(7 + 3*x - 6*x^2)^(1 + q))/(1 + q) - (2^(-4 - 3*q)*59^q*(2 + q)*(4506 + 4
063*q + 765*q^2)*(3 - 12*x)*Hypergeometric2F1[1/2, -q, 3/2, (3 - 12*x)^2/1
77])/3)/(2*(3 + 2*q)))/(3*(2 + q)))/(5 + 2*q)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(-6x^2+3x+7\right)^q \left(-2x^2+5x+1\right) \left(4x^2+2x+3\right) dx$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x)
```

output
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x)
```

Fricas [F]

$$\int (7+3x-6x^2)^q \left(1+5x-2x^2\right) \left(3+2x+4x^2\right) \, dx = \int -\left(4\,x^2+2\,x+3\right)\left(2\,x^2-5\,x-1\right)\left(-6\,x^2+3\,x+7\right)^q \, dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x, algorithm="fric
as")
```

output
```
integral(-(8*x^4 - 16*x^3 - 8*x^2 - 17*x - 3)*(-6*x^2 + 3*x + 7)^q, x)
```

Sympy [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)(3+2x+4x^2)\,dx$$
$$= -\int \left(-17x(-6x^2+3x+7)^q\right) dx - \int \left(-8x^2(-6x^2+3x+7)^q\right) dx - \int \left(-16x^3(-6x^2+3x+7)^q\right) dx - \int 8x^4(-6x^2+3x+7)^q\,dx - \int \left(-3(-6x^2+3x+7)^q\right) dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(-2*x**2+5*x+1)*(4*x**2+2*x+3),x)
```

output
```
-Integral(-17*x*(-6*x**2 + 3*x + 7)**q, x) - Integral(-8*x**2*(-6*x**2 + 3
*x + 7)**q, x) - Integral(-16*x**3*(-6*x**2 + 3*x + 7)**q, x) - Integral(8
*x**4*(-6*x**2 + 3*x + 7)**q, x) - Integral(-3*(-6*x**2 + 3*x + 7)**q, x)
```

Maxima [F]

$$\int (7+3x-6x^2)^q (1+5x-2x^2)(3+2x+4x^2)\,dx = \int -(4x^2+2x+3)(2x^2-5x-1)(-6x^2+3x+7)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x, algorithm="maxi
ma")
```

output
```
-integrate((4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)*(-6*x^2 + 3*x + 7)^q, x)
```

Giac [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right) \left(3+2x+4x^2\right) \, dx = \int -\left(4\,x^2+2\,x+3\right)\left(2\,x^2-5\,x-1\right)\left(-6\,x^2+3\,x+7\right)^q \, dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x, algorithm="giac
")
```

output
```
integrate(-(4*x^2 + 2*x + 3)*(2*x^2 - 5*x - 1)*(-6*x^2 + 3*x + 7)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right) \left(3+2x+4x^2\right) \, dx = \int \left(-2\,x^2+5\,x+1\right)\,\left(4\,x^2+2\,x+3\right)\,\left(-6\,x^2+3\,x+7\right)^q \, dx$$

input
```
int((5*x - 2*x^2 + 1)*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q,x)
```

output
```
int((5*x - 2*x^2 + 1)*(2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q, x)
```

Reduce [F]

$$\int \left(7+3x-6x^2\right)^q \left(1+5x-2x^2\right) \left(3+2x+4x^2\right) \, dx = \text{Too large to display}$$

input
```
int((-6*x^2+3*x+7)^q*(-2*x^2+5*x+1)*(4*x^2+2*x+3),x)
```

output

```
( - 2304*( - 6*x**2 + 3*x + 7)**q*q**4*x**5 + 5184*( - 6*x**2 + 3*x + 7)**
q*q**4*x**4 + 3984*( - 6*x**2 + 3*x + 7)**q*q**4*x**3 - 636*( - 6*x**2 + 3
*x + 7)**q*q**4*x**2 - 4431*( - 6*x**2 + 3*x + 7)**q*q**4*x + 4032*( - 6*x
**2 + 3*x + 7)**q*q**4 - 11520*( - 6*x**2 + 3*x + 7)**q*q**3*x**5 + 27072*
( - 6*x**2 + 3*x + 7)**q*q**3*x**4 + 19416*( - 6*x**2 + 3*x + 7)**q*q**3*x
**3 + 8334*( - 6*x**2 + 3*x + 7)**q*q**3*x**2 - 21087*( - 6*x**2 + 3*x + 7
)**q*q**3*x + 40579*( - 6*x**2 + 3*x + 7)**q*q**3 - 20160*( - 6*x**2 + 3*x
 + 7)**q*q**2*x**5 + 48816*( - 6*x**2 + 3*x + 7)**q*q**2*x**4 + 33672*( -
6*x**2 + 3*x + 7)**q*q**2*x**3 + 43134*( - 6*x**2 + 3*x + 7)**q*q**2*x**2
- 31308*( - 6*x**2 + 3*x + 7)**q*q**2*x + 134883*( - 6*x**2 + 3*x + 7)**q*
q**2 - 14400*( - 6*x**2 + 3*x + 7)**q*q*x**5 + 35568*( - 6*x**2 + 3*x + 7)
**q*q*x**4 + 24000*( - 6*x**2 + 3*x + 7)**q*q*x**3 + 56124*( - 6*x**2 + 3*
x + 7)**q*q*x**2 - 9972*( - 6*x**2 + 3*x + 7)**q*q*x + 186452*( - 6*x**2 +
 3*x + 7)**q*q - 3456*( - 6*x**2 + 3*x + 7)**q*x**5 + 8640*( - 6*x**2 + 3*
x + 7)**q*x**4 + 5760*( - 6*x**2 + 3*x + 7)**q*x**3 + 18360*( - 6*x**2 + 3
*x + 7)**q*x**2 + 6480*( - 6*x**2 + 3*x + 7)**q*x + 92316*( - 6*x**2 + 3*x
 + 7)**q - 1083240*int((( - 6*x**2 + 3*x + 7)**q*x)/(48*q**3*x**2 - 24*q**
3*x - 56*q**3 + 216*q**2*x**2 - 108*q**2*x - 252*q**2 + 276*q*x**2 - 138*q
*x - 322*q + 90*x**2 - 45*x - 105),x)*q**8 - 13877508*int((( - 6*x**2 + 3*
x + 7)**q*x)/(48*q**3*x**2 - 24*q**3*x - 56*q**3 + 216*q**2*x**2 - 108*...
```

3.29 $\int (7+3x-6x^2)^q (3+2x+4x^2)\, dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int (7+3x-6x^2)^q (3+2x+4x^2)\, dx = -\frac{(5+3q)(7+3x-6x^2)^{1+q}}{6(1+q)(3+2q)} - \frac{2x(7+3x-6x^2)^{1+q}}{3(3+2q)} - \frac{2^{-3(1+q)}59^q(97+45q)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)}{3(3+2q)}$$

output
```
-1/6*(5+3*q)*(-6*x^2+3*x+7)^(1+q)/(1+q)/(3+2*q)-2*x*(-6*x^2+3*x+7)^(1+q)/(
9+6*q)-1/3*59^q*(97+45*q)*(1-4*x)*hypergeom([1/2, -q],[3/2],3/59*(1-4*x)^2
)/(2^(3+3*q))/(3+2*q)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.87

$$\int (7+3x-6x^2)^q (3+2x+4x^2)\, dx = \frac{8^{-1-q}\left(-2^{2+3q}(7+3x-6x^2)^{1+q}(5+4x+q(3+4x)) + 59^q(97+142q+45q^2)(-1+4x)\,\text{Hypergeomet}\right.}{3(1+q)(3+2q)}$$

input
```
Integrate[(7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2),x]
```

output
```
(8^(-1 - q)*(-(2^(2 + 3*q)*(7 + 3*x - 6*x^2)^(1 + q)*(5 + 4*x + q*(3 + 4*x
))) + 59^q*(97 + 142*q + 45*q^2)*(-1 + 4*x)*Hypergeometric2F1[1/2, -q, 3/2
, (3*(1 - 4*x)^2)/59]))/(3*(1 + q)*(3 + 2*q))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2192, 27, 1160, 1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+2x+3\right)\left(-6x^2+3x+7\right)^q dx$$

$$\Big\downarrow \text{2192}$$

$$-\frac{\int -2(18q+6(3q+5)x+41)\left(-6x^2+3x+7\right)^q dx}{6(2q+3)}-\frac{2x\left(-6x^2+3x+7\right)^{q+1}}{3(2q+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (18q+6(3q+5)x+41)\left(-6x^2+3x+7\right)^q dx}{3(2q+3)}-\frac{2x\left(-6x^2+3x+7\right)^{q+1}}{3(2q+3)}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{2}(45q+97)\int \left(-6x^2+3x+7\right)^q dx-\frac{(3q+5)\left(-6x^2+3x+7\right)^{q+1}}{2(q+1)}}{3(2q+3)}-\frac{2x\left(-6x^2+3x+7\right)^{q+1}}{3(2q+3)}$$

$$\Big\downarrow \text{1090}$$

$$\frac{-\frac{1}{3}2^{-3q-3}59^q(45q+97)\int \left(1-\frac{1}{177}(3-12x)^2\right)^q d(3-12x)-\frac{(3q+5)\left(-6x^2+3x+7\right)^{q+1}}{2(q+1)}}{3(2q+3)}-\frac{2x\left(-6x^2+3x+7\right)^{q+1}}{3(2q+3)}$$

$$\Big\downarrow \text{237}$$

$$\frac{-\frac{1}{3}2^{-3q-3}59^q(45q+97)(3-12x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{1}{177}(3-12x)^2\right)-\frac{(3q+5)(-6x^2+3x+7)^{q+1}}{2(q+1)}}{3(2q+3)}-\frac{2x\left(-6x^2+3x+7\right)^{q+1}}{3(2q+3)}$$

input
```
Int[(7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2),x]
```

output
```
(-2*x*(7 + 3*x - 6*x^2)^(1 + q))/(3*(3 + 2*q)) + (-1/2*((5 + 3*q)*(7 + 3*x
 - 6*x^2)^(1 + q))/(1 + q) - (2^(-3 - 3*q)*59^q*(97 + 45*q)*(3 - 12*x)*Hyp
ergeometric2F1[1/2, -q, 3/2, (3 - 12*x)^2/177])/3)/(3*(3 + 2*q))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [F]

$$\int \left(-6x^2+3x+7\right)^q \left(4x^2+2x+3\right) dx$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x)
```

output
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x)
```

Fricas [F]

$$\int \left(7+3x-6x^2\right)^q \left(3+2x+4x^2\right) \, dx = \int \left(4\,x^2+2\,x+3\right)\left(-6\,x^2+3\,x+7\right)^q \, dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x, algorithm="fricas")
```

output
```
integral((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q, x)
```

Sympy [F]

$$\int \left(7+3x-6x^2\right)^q \left(3+2x+4x^2\right) \, dx = \int \left(-6x^2+3x+7\right)^q \left(4x^2+2x+3\right) \, dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(4*x**2+2*x+3),x)
```

output
```
Integral((-6*x**2 + 3*x + 7)**q*(4*x**2 + 2*x + 3), x)
```

Maxima [F]

$$\int \left(7+3x-6x^2\right)^q \left(3+2x+4x^2\right)\,dx=\int \left(4\,x^2+2\,x+3\right)\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x, algorithm="maxima")
```

output
```
integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q, x)
```

Giac [F]

$$\int \left(7+3x-6x^2\right)^q \left(3+2x+4x^2\right)\,dx=\int \left(4\,x^2+2\,x+3\right)\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x, algorithm="giac")
```

output
```
integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(7+3x-6x^2\right)^q \left(3+2x+4x^2\right)\,dx=\int \left(4\,x^2+2\,x+3\right)\,\left(-6\,x^2+3\,x+7\right)^q\,dx$$

input
```
int((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q,x)
```

output
```
int((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q, x)
```

Reduce [F]

$$\int (7+3x-6x^2)^q (3+2x+4x^2)\,dx$$
$$= \frac{144(-6x^2+3x+7)^q q^2x^3 + 36(-6x^2+3x+7)^q q^2x^2 - 87(-6x^2+3x+7)^q q^2x + 504(-6x^2+3x+7)}{}$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3),x)
```

output
```
(144*( - 6*x**2 + 3*x + 7)**q*q**2*x**3 + 36*( - 6*x**2 + 3*x + 7)**q*q**2
*x**2 - 87*( - 6*x**2 + 3*x + 7)**q*q**2*x + 504*( - 6*x**2 + 3*x + 7)**q*
q**2 + 216*( - 6*x**2 + 3*x + 7)**q*q*x**3 + 126*( - 6*x**2 + 3*x + 7)**q*
q*x**2 + 57*( - 6*x**2 + 3*x + 7)**q*q*x + 1715*( - 6*x**2 + 3*x + 7)**q*q
 + 72*( - 6*x**2 + 3*x + 7)**q*x**3 + 54*( - 6*x**2 + 3*x + 7)**q*x**2 + 1
62*( - 6*x**2 + 3*x + 7)**q*x + 1253*( - 6*x**2 + 3*x + 7)**q - 31860*int(
(( - 6*x**2 + 3*x + 7)**q*x)/(24*q**2*x**2 - 12*q**2*x - 28*q**2 + 48*q*x*
*2 - 24*q*x - 56*q + 18*x**2 - 9*x - 21),x)*q**5 - 164256*int((( - 6*x**2
+ 3*x + 7)**q*x)/(24*q**2*x**2 - 12*q**2*x - 28*q**2 + 48*q*x**2 - 24*q*x
- 56*q + 18*x**2 - 9*x - 21),x)*q**4 - 293643*int((( - 6*x**2 + 3*x + 7)**
q*x)/(24*q**2*x**2 - 12*q**2*x - 28*q**2 + 48*q*x**2 - 24*q*x - 56*q + 18*
x**2 - 9*x - 21),x)*q**3 - 212754*int((( - 6*x**2 + 3*x + 7)**q*x)/(24*q**
2*x**2 - 12*q**2*x - 28*q**2 + 48*q*x**2 - 24*q*x - 56*q + 18*x**2 - 9*x -
 21),x)*q**2 - 51507*int((( - 6*x**2 + 3*x + 7)**q*x)/(24*q**2*x**2 - 12*q
**2*x - 28*q**2 + 48*q*x**2 - 24*q*x - 56*q + 18*x**2 - 9*x - 21),x)*q)/(1
8*(4*q**3 + 12*q**2 + 11*q + 3))
```

3.30 $\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{1+5x-2x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 396

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{1+5x-2x^2}\,dx = \frac{3^{-\frac{1}{2}+2q}(20-3\sqrt{33})\left(\frac{3-\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}(7+3x-6x^2)^q\,\text{AppellF1}\left(-2q,-q,-q,1-2q,\frac{12}{3(}\right.}{2\sqrt{11}q} - \frac{3^{-\frac{1}{2}+2q}(20+3\sqrt{33})\left(\frac{3-\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}(7+3x-6x^2)^q\,\text{AppellF1}\left(-2q,-q,-q,1-2q,\right.}{2\sqrt{11}q} + 2^{-1-3q}59^q(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)$$

output
```
1/22*3^(-1/2+2*q)*(20-3*33^(1/2))*(-6*x^2+3*x+7)^q*AppellF1(-2*q,-q,-q,1-2
*q,(12-3*33^(1/2)-177^(1/2))/(15-3*33^(1/2)-12*x),(12-3*33^(1/2)+177^(1/2)
)/(15-3*33^(1/2)-12*x))*11^(1/2)/q/(((3-177^(1/2)-12*x)/(5-33^(1/2)-4*x))^
q)/(((3+177^(1/2)-12*x)/(5-33^(1/2)-4*x))^q)-1/22*3^(-1/2+2*q)*(20+3*33^(1
/2))*(-6*x^2+3*x+7)^q*AppellF1(-2*q,-q,-q,1-2*q,(12+3*33^(1/2)-177^(1/2))/
(15+3*33^(1/2)-12*x),(12+3*33^(1/2)+177^(1/2))/(15+3*33^(1/2)-12*x))*11^(1
/2)/q/(((3-177^(1/2)-12*x)/(5+33^(1/2)-4*x))^q)/(((3+177^(1/2)-12*x)/(5+33
^(1/2)-4*x))^q)+2^(-1-3*q)*59^q*(1-4*x)*hypergeom([1/2, -q],[3/2],3/59*(1-
4*x)^2)
```

Mathematica [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{1+5x-2x^2}\,dx = \int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{1+5x-2x^2}\,dx$$

input
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2),x]
```

output
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^2+2x+3)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{(12x+5)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}-2\left(-6x^2+3x+7\right)^q\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{(12x+5)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}dx+2^{-3q-1}59^q(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)$$

input
```
Int[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(-6x^2+3x+7)^q (4x^2+2x+3)}{-2x^2+5x+1}dx$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x)
```

output
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x)
```

Fricas [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{1+5x-2x^2} dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{2\,x^2-5\,x-1} dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x, algorithm="fric
as")
```

output
```
integral(-(4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1), x)
```

Sympy [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{1+5x-2x^2}\,dx = -\int \frac{3(-6x^2+3x+7)^q}{2x^2-5x-1}\,dx - \int \frac{2x(-6x^2+3x+7)^q}{2x^2-5x-1}\,dx - \int \frac{4x^2(-6x^2+3x+7)^q}{2x^2-5x-1}\,dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(4*x**2+2*x+3)/(-2*x**2+5*x+1),x)
```

output
```
-Integral(3*(-6*x**2 + 3*x + 7)**q/(2*x**2 - 5*x - 1), x) - Integral(2*x*(
-6*x**2 + 3*x + 7)**q/(2*x**2 - 5*x - 1), x) - Integral(4*x**2*(-6*x**2 +
3*x + 7)**q/(2*x**2 - 5*x - 1), x)
```

Maxima [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{1+5x-2x^2}\,dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{2\,x^2-5\,x-1}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x, algorithm="maxi
ma")
```

output
```
-integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1), x)
```

Giac [F]

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{1+5x-2x^2}\,dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{2\,x^2-5\,x-1}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x, algorithm="giac
")
```

output
```
integrate(-(4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{1+5x-2x^2}\,dx = \int \frac{(4\,x^2+2\,x+3)\,(-6\,x^2+3\,x+7)^q}{-2\,x^2+5\,x+1}\,dx$$

input
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1),x)
```

output
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1), x)
```

Reduce [F]

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{1+5x-2x^2}\,dx$$
$$= \frac{-66(-6x^2+3x+7)^q\,qx - 19(-6x^2+3x+7)^q\,q - 3(-6x^2+3x+7)^q + 2886\left(\int \frac{(-6x^2}{24q\,x^4-72q\,x^3+12x^4-10q\,x^2}\right.}{}$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1),x)
```

output

```
( - 66*( - 6*x**2 + 3*x + 7)**q*q*x - 19*( - 6*x**2 + 3*x + 7)**q*q - 3*(
- 6*x**2 + 3*x + 7)**q + 2886*int(( - 6*x**2 + 3*x + 7)**q/(24*q*x**4 - 72
*q*x**3 - 10*q*x**2 + 76*q*x + 14*q + 12*x**4 - 36*x**3 - 5*x**2 + 38*x +
7),x)*q**3 + 3771*int(( - 6*x**2 + 3*x + 7)**q/(24*q*x**4 - 72*q*x**3 - 10
*q*x**2 + 76*q*x + 14*q + 12*x**4 - 36*x**3 - 5*x**2 + 38*x + 7),x)*q**2 +
 1164*int(( - 6*x**2 + 3*x + 7)**q/(24*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76
*q*x + 14*q + 12*x**4 - 36*x**3 - 5*x**2 + 38*x + 7),x)*q - 7800*int((( -
6*x**2 + 3*x + 7)**q*x**3)/(24*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 1
4*q + 12*x**4 - 36*x**3 - 5*x**2 + 38*x + 7),x)*q**3 - 8508*int((( - 6*x**
2 + 3*x + 7)**q*x**3)/(24*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 14*q +
 12*x**4 - 36*x**3 - 5*x**2 + 38*x + 7),x)*q**2 - 2304*int((( - 6*x**2 + 3
*x + 7)**q*x**3)/(24*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 14*q + 12*x
**4 - 36*x**3 - 5*x**2 + 38*x + 7),x)*q + 3546*int((( - 6*x**2 + 3*x + 7)*
*q*x)/(24*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 14*q + 12*x**4 - 36*x*
*3 - 5*x**2 + 38*x + 7),x)*q**3 + 8325*int((( - 6*x**2 + 3*x + 7)**q*x)/(2
4*q*x**4 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 14*q + 12*x**4 - 36*x**3 - 5*x
**2 + 38*x + 7),x)*q**2 + 3276*int((( - 6*x**2 + 3*x + 7)**q*x)/(24*q*x**4
 - 72*q*x**3 - 10*q*x**2 + 76*q*x + 14*q + 12*x**4 - 36*x**3 - 5*x**2 + 38
*x + 7),x)*q)/(33*q*(2*q + 1))
```

3.31 $\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 448

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = -\frac{(23+14x)(7+3x-6x^2)^{1+q}}{264(1+5x-2x^2)}$$

$$+\frac{3^{-\frac{3}{2}+2q}(224-1665q+327\sqrt{33}q)\left(\frac{3-\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}(7+3x-6x^2)^q\,\text{AppellF1}\left(-2q,-\right.}{352\sqrt{11}q}$$

$$-\frac{3^{-\frac{3}{2}+2q}(224-3(555+109\sqrt{33})q)\left(\frac{3-\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}(7+3x-6x^2)^q\,\text{AppellF1}\left(-2q,\right.}{352\sqrt{11}q}$$

$$-\frac{7}{11}2^{-4-3q}59^q(1+2q)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)$$

output
```
-1/264*(23+14*x)*(-6*x^2+3*x+7)^(1+q)/(-2*x^2+5*x+1)+1/3872*3^(-3/2+2*q)*(
224-1665*q+327*33^(1/2)*q)*(-6*x^2+3*x+7)^q*AppellF1(-2*q,-q,-q,1-2*q,(12-
3*33^(1/2)-177^(1/2))/(15-3*33^(1/2)-12*x),(12-3*33^(1/2)+177^(1/2))/(15-3
*33^(1/2)-12*x))*11^(1/2)/q/(((3-177^(1/2)-12*x)/(5-33^(1/2)-4*x))^q)/(((3
+177^(1/2)-12*x)/(5-33^(1/2)-4*x))^q)-1/3872*3^(-3/2+2*q)*(224-3*(555+109*
33^(1/2))*q)*(-6*x^2+3*x+7)^q*AppellF1(-2*q,-q,-q,1-2*q,(12+3*33^(1/2)-177
^(1/2))/(15+3*33^(1/2)-12*x),(12+3*33^(1/2)+177^(1/2))/(15+3*33^(1/2)-12*x
))*11^(1/2)/q/(((3-177^(1/2)-12*x)/(5+33^(1/2)-4*x))^q)/(((3+177^(1/2)-12*
x)/(5+33^(1/2)-4*x))^q)-7/11*2^(-4-3*q)*59^q*(1+2*q)*(1-4*x)*hypergeom([1/
2, -q],[3/2],3/59*(1-4*x)^2)
```

Mathematica [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx$$

input
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^2,x]
```

output
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^2, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^2+2x+3)\,(-6x^2+3x+7)^q}{(-2x^2+5x+1)^2}\,dx$$

$$\Big\downarrow \; 2135$$

$$\frac{\int \frac{88(-6x^2+3x+7)^q(-84(2q+1)x^2+6(35-39q)x+69q+154)}{-2x^2+5x+1}dx}{23232}-\frac{(14x+23)\,(-6x^2+3x+7)^{q+1}}{264\,(-2x^2+5x+1)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{264}\int \frac{\left(-6x^2+3x+7\right)^q \left(-84(2q+1)x^2+6(35-39q)x+69q+154\right)}{-2x^2+5x+1}dx-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{264\left(-2x^2+5x+1\right)}$$

$$\Big\downarrow \text{7279}$$

$$\frac{1}{264}\int \left(42(2q+1)\left(-6x^2+3x+7\right)^q+\frac{(-654xq-15q+112)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}\right)dx-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{264\left(-2x^2+5x+1\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{264}\left(\int \frac{(-654xq-15q+112)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}dx-21\,2^{-3q-1}59^q(2q+1)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-\right.\right.$$
$$\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{264\left(-2x^2+5x+1\right)}$$

input
```
Int[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(-6x^2 + 3x + 7)^q (4x^2 + 2x + 3)}{(-2x^2 + 5x + 1)^2} dx$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x)
```

output
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x)
```

Fricas [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^2}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x, algorithm="fr
icas")
```

output
```
integral((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(4*x^4 - 20*x^3 + 21*x^2 +
 10*x + 1), x)
```

Sympy [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(-6x^2+3x+7)^q (4x^2+2x+3)}{(2x^2-5x-1)^2}\,dx$$

input
```
integrate((-6*x**2+3*x+7)**q*(4*x**2+2*x+3)/(-2*x**2+5*x+1)**2,x)
```

output
```
Integral((-6*x**2 + 3*x + 7)**q*(4*x**2 + 2*x + 3)/(2*x**2 - 5*x - 1)**2,
x)
```

Maxima [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^2}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x, algorithm="ma
xima")
```

output
```
integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1)^2, x)
```

Giac [F]

$$\int \frac{(7+3x-6x^2)^q\,(3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^2}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x, algorithm="gi
ac")
```

output
```
integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+3x-6x^2)^q\,(3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \int \frac{(4\,x^2+2\,x+3)\,(-6\,x^2+3\,x+7)^q}{(-2\,x^2+5\,x+1)^2}\,dx$$

input
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1)^2,x)
```

output
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1)^2, x)
```

Reduce [F]

$$\int \frac{(7+3x-6x^2)^q\,(3+2x+4x^2)}{(1+5x-2x^2)^2}\,dx = \text{too large to display}$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^2,x)
```

output
```
(66*( - 6*x**2 + 3*x + 7)**q*q*x + 19*( - 6*x**2 + 3*x + 7)**q*q - 42*( -
6*x**2 + 3*x + 7)**q*x - 63492*int(( - 6*x**2 + 3*x + 7)**q/(528*q**2*x**6
 - 2904*q**2*x**5 + 3476*q**2*x**4 + 3014*q**2*x**3 - 3762*q**2*x**2 - 160
6*q**2*x - 154*q**2 - 600*q*x**6 + 3300*q*x**5 - 3950*q*x**4 - 3425*q*x**3
 + 4275*q*x**2 + 1825*q*x + 175*q + 168*x**6 - 924*x**5 + 1106*x**4 + 959*
x**3 - 1197*x**2 - 511*x - 49),x)*q**4*x**2 + 158730*int(( - 6*x**2 + 3*x
+ 7)**q/(528*q**2*x**6 - 2904*q**2*x**5 + 3476*q**2*x**4 + 3014*q**2*x**3
- 3762*q**2*x**2 - 1606*q**2*x - 154*q**2 - 600*q*x**6 + 3300*q*x**5 - 395
0*q*x**4 - 3425*q*x**3 + 4275*q*x**2 + 1825*q*x + 175*q + 168*x**6 - 924*x
**5 + 1106*x**4 + 959*x**3 - 1197*x**2 - 511*x - 49),x)*q**4*x + 31746*int
(( - 6*x**2 + 3*x + 7)**q/(528*q**2*x**6 - 2904*q**2*x**5 + 3476*q**2*x**4
 + 3014*q**2*x**3 - 3762*q**2*x**2 - 1606*q**2*x - 154*q**2 - 600*q*x**6 +
 3300*q*x**5 - 3950*q*x**4 - 3425*q*x**3 + 4275*q*x**2 + 1825*q*x + 175*q
+ 168*x**6 - 924*x**5 + 1106*x**4 + 959*x**3 - 1197*x**2 - 511*x - 49),x)*
q**4 + 150382*int(( - 6*x**2 + 3*x + 7)**q/(528*q**2*x**6 - 2904*q**2*x**5
 + 3476*q**2*x**4 + 3014*q**2*x**3 - 3762*q**2*x**2 - 1606*q**2*x - 154*q*
*2 - 600*q*x**6 + 3300*q*x**5 - 3950*q*x**4 - 3425*q*x**3 + 4275*q*x**2 +
1825*q*x + 175*q + 168*x**6 - 924*x**5 + 1106*x**4 + 959*x**3 - 1197*x**2
- 511*x - 49),x)*q**3*x**2 - 375955*int(( - 6*x**2 + 3*x + 7)**q/(528*q**2
*x**6 - 2904*q**2*x**5 + 3476*q**2*x**4 + 3014*q**2*x**3 - 3762*q**2*x*...
```

3.32 $\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 515

$$\int \frac{(7+3x-6x^2)^q(3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx = -\frac{(23+14x)(7+3x-6x^2)^{1+q}}{528(1+5x-2x^2)^2} - \frac{(51+61q+142(1-q)x)(7+3x-6x^2)^{1+q}}{15488(1+5x-2x^2)} + \frac{3^{-\frac{1}{2}+2q}(6144-13505q+3\sqrt{33}(741-517q)q+9825q^2)\left(\frac{3-\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5-\sqrt{33}-4x}\right)^{-q}(7+3x-6x}{61952\sqrt{11}q} - \frac{3^{-\frac{1}{2}+2q}(6144-13505q-3\sqrt{33}(741-517q)q+9825q^2)\left(\frac{3-\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}\left(\frac{3+\sqrt{177}-12x}{5+\sqrt{33}-4x}\right)^{-q}(7+3x-6x}{61952\sqrt{11}q} - \frac{213}{121}2^{-8-3q}59^q(1-q)(1+2q)(1-4x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-q,\frac{3}{2},\frac{3}{59}(1-4x)^2\right)$$

output
```
-1/528*(23+14*x)*(-6*x^2+3*x+7)^(1+q)/(-2*x^2+5*x+1)^2-(51+61*q+142*(1-q)*
x)*(-6*x^2+3*x+7)^(1+q)/(-30976*x^2+77440*x+15488)+1/681472*3^(-1/2+2*q)*(
6144-13505*q+3*33^(1/2)*(741-517*q)*q+9825*q^2)*(-6*x^2+3*x+7)^q*AppellF1(
-2*q,-q,-q,1-2*q,(12-3*33^(1/2)-177^(1/2))/(15-3*33^(1/2)-12*x),(12-3*33^(
1/2)+177^(1/2))/(15-3*33^(1/2)-12*x))*11^(1/2)/q/(((3-177^(1/2)-12*x)/(5-3
3^(1/2)-4*x))^q)/(((3+177^(1/2)-12*x)/(5-33^(1/2)-4*x))^q)-1/681472*3^(-1/
2+2*q)*(6144-13505*q-3*33^(1/2)*(741-517*q)*q+9825*q^2)*(-6*x^2+3*x+7)^q*A
ppellF1(-2*q,-q,-q,1-2*q,(12+3*33^(1/2)-177^(1/2))/(15+3*33^(1/2)-12*x),(1
2+3*33^(1/2)+177^(1/2))/(15+3*33^(1/2)-12*x))*11^(1/2)/q/(((3-177^(1/2)-12
*x)/(5+33^(1/2)-4*x))^q)/(((3+177^(1/2)-12*x)/(5+33^(1/2)-4*x))^q)-213/121
*2^(-8-3*q)*59^q*(1-q)*(1+2*q)*(1-4*x)*hypergeom([1/2, -q],[3/2],3/59*(1-4
*x)^2)
```

Mathematica [F]

$$\int \frac{(7+3x-6x^2)^q\,(3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx=\int \frac{(7+3x-6x^2)^q\,(3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx$$

input
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^3,x]
```

output
```
Integrate[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^3, x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(4x^2+2x+3)\,(-6x^2+3x+7)^q}{(-2x^2+5x+1)^3}\,dx$$

$$\Big\downarrow \ 2135$$

$$\frac{\int \frac{264(-6x^2+3x+7)^q\,(28(1-2q)x^2+6(34-13q)x+23q+47)}{(-2x^2+5x+1)^2}dx}{46464}-\frac{(14x+23)\,(-6x^2+3x+7)^{q+1}}{528\,(-2x^2+5x+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{176}\int \frac{\left(-6x^2+3x+7\right)^q\left(28(1-2q)x^2+6(34-13q)x+23q+47\right)}{\left(-2x^2+5x+1\right)^2}dx-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{528\left(-2x^2+5x+1\right)^2}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{176}\left(\frac{\int \frac{264\left(-6x^2+3x+7\right)^q\left(183q^2-769q-852(1-q)(2q+1)x^2+6\left(-193q^2-386q+355\right)x+3498\right)}{-2x^2+5x+1}dx}{23232}-\frac{(142(1-q)x+61q+51)\left(-6\right.}{88\left(-2x^2+5x+\right.}\right.$$
$$-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{528\left(-2x^2+5x+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{176}\left(\frac{1}{88}\int \frac{\left(-6x^2+3x+7\right)^q\left(183q^2-769q-852(1-q)(2q+1)x^2+6\left(-193q^2-386q+355\right)x+3498\right)}{-2x^2+5x+1}dx-\right.$$
$$-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{528\left(-2x^2+5x+1\right)^2}$$

$$\Big\downarrow \text{7279}$$

$$\frac{1}{176}\left(\frac{1}{88}\int\left(426(1-q)(2q+1)\left(-6x^2+3x+7\right)^q+\frac{\left(1035q^2-6(741-517q)xq-1195q+3072\right)\left(-6x^2+3x\right.}{-2x^2+5x+1}\right.\right.$$
$$-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{528\left(-2x^2+5x+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{176}\left(\frac{1}{88}\left(\int \frac{\left(1035q^2-6(741-517q)xq-1195q+3072\right)\left(-6x^2+3x+7\right)^q}{-2x^2+5x+1}dx-213\ 2^{-3q-1}59^q(1-q)(2q+1)\right.\right.$$
$$-\frac{(14x+23)\left(-6x^2+3x+7\right)^{q+1}}{528\left(-2x^2+5x+1\right)^2}$$

input
```
Int[((7 + 3*x - 6*x^2)^q*(3 + 2*x + 4*x^2))/(1 + 5*x - 2*x^2)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{(-6x^2+3x+7)^q (4x^2+2x+3)}{(-2x^2+5x+1)^3}dx$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x)
```

output
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x)
```

Fricas [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^3}\,dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x, algorithm="fr
icas")
```

output
```
integral(-(4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(8*x^6 - 60*x^5 + 138*x^4
 - 65*x^3 - 69*x^2 - 15*x - 1), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((-6*x**2+3*x+7)**q*(4*x**2+2*x+3)/(-2*x**2+5*x+1)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3} \, dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^3} \, dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x, algorithm="ma
xima")
```

output
```
-integrate((4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1)^3, x)
```

Giac [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3} \, dx = \int -\frac{(4\,x^2+2\,x+3)(-6\,x^2+3\,x+7)^q}{(2\,x^2-5\,x-1)^3} \, dx$$

input
```
integrate((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x, algorithm="gi
ac")
```

output
```
integrate(-(4*x^2 + 2*x + 3)*(-6*x^2 + 3*x + 7)^q/(2*x^2 - 5*x - 1)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3} \, dx = \int \frac{(4\,x^2+2\,x+3)\,(-6\,x^2+3\,x+7)^q}{(-2\,x^2+5\,x+1)^3} \, dx$$

input
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1)^3,x)
```

output
```
int(((2*x + 4*x^2 + 3)*(3*x - 6*x^2 + 7)^q)/(5*x - 2*x^2 + 1)^3, x)
```

Reduce [F]

$$\int \frac{(7+3x-6x^2)^q (3+2x+4x^2)}{(1+5x-2x^2)^3} \, dx = \text{too large to display}$$

input
```
int((-6*x^2+3*x+7)^q*(4*x^2+2*x+3)/(-2*x^2+5*x+1)^3,x)
```

output
```
( - 66*( - 6*x**2 + 3*x + 7)**q*q*x - 19*( - 6*x**2 + 3*x + 7)**q*q + 84*(
 - 6*x**2 + 3*x + 7)**q*x + 3*( - 6*x**2 + 3*x + 7)**q + 126984*int(( - 6*
x**2 + 3*x + 7)**q/(1056*q**2*x**8 - 8448*q**2*x**7 + 20944*q**2*x**6 - 84
48*q**2*x**5 - 26070*q**2*x**4 + 12584*q**2*x**3 + 11484*q**2*x**2 + 2376*
q**2*x + 154*q**2 - 2928*q*x**8 + 23424*q*x**7 - 58072*q*x**6 + 23424*q*x*
*5 + 72285*q*x**4 - 34892*q*x**3 - 31842*q*x**2 - 6588*q*x - 427*q + 2016*
x**8 - 16128*x**7 + 39984*x**6 - 16128*x**5 - 49770*x**4 + 24024*x**3 + 21
924*x**2 + 4536*x + 294),x)*q**4*x**4 - 634920*int(( - 6*x**2 + 3*x + 7)**
q/(1056*q**2*x**8 - 8448*q**2*x**7 + 20944*q**2*x**6 - 8448*q**2*x**5 - 26
070*q**2*x**4 + 12584*q**2*x**3 + 11484*q**2*x**2 + 2376*q**2*x + 154*q**2
 - 2928*q*x**8 + 23424*q*x**7 - 58072*q*x**6 + 23424*q*x**5 + 72285*q*x**4
 - 34892*q*x**3 - 31842*q*x**2 - 6588*q*x - 427*q + 2016*x**8 - 16128*x**7
 + 39984*x**6 - 16128*x**5 - 49770*x**4 + 24024*x**3 + 21924*x**2 + 4536*x
 + 294),x)*q**4*x**3 + 666666*int(( - 6*x**2 + 3*x + 7)**q/(1056*q**2*x**8
 - 8448*q**2*x**7 + 20944*q**2*x**6 - 8448*q**2*x**5 - 26070*q**2*x**4 + 1
2584*q**2*x**3 + 11484*q**2*x**2 + 2376*q**2*x + 154*q**2 - 2928*q*x**8 +
23424*q*x**7 - 58072*q*x**6 + 23424*q*x**5 + 72285*q*x**4 - 34892*q*x**3 -
 31842*q*x**2 - 6588*q*x - 427*q + 2016*x**8 - 16128*x**7 + 39984*x**6 - 1
6128*x**5 - 49770*x**4 + 24024*x**3 + 21924*x**2 + 4536*x + 294),x)*q**4*x
**2 + 317460*int(( - 6*x**2 + 3*x + 7)**q/(1056*q**2*x**8 - 8448*q**2*x...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file